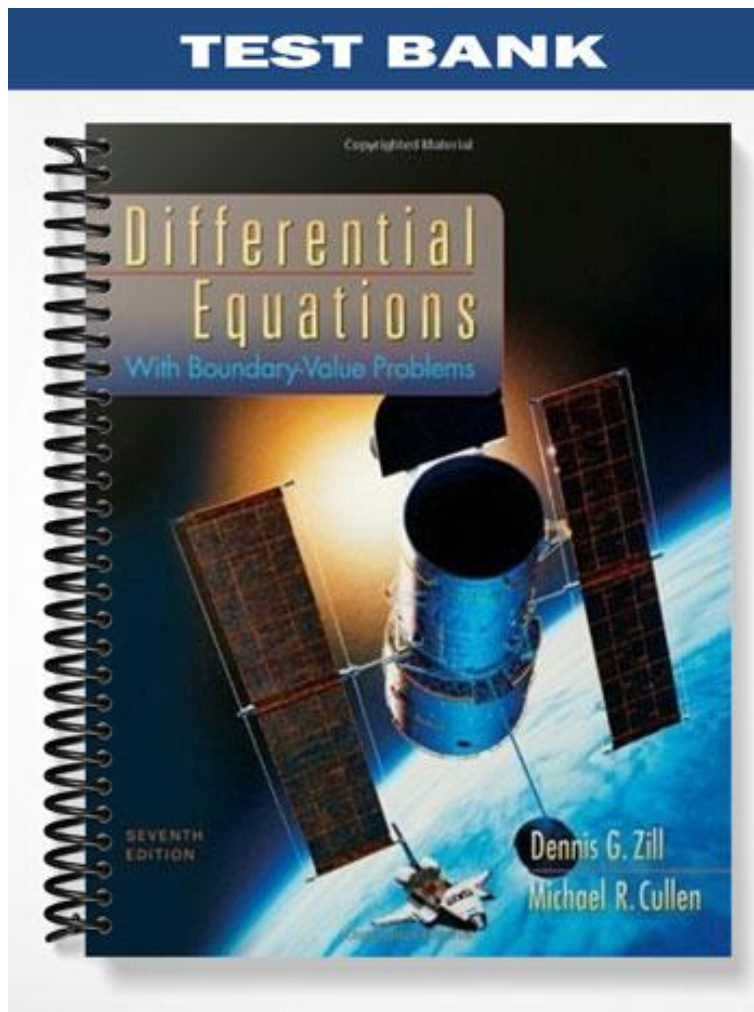


# TEST BANK



NOTE: UNDER NO CIRCUMSTANCES MAY THIS MATERIAL OR ANY PORTION THEREOF BE SOLD, LICENSED, AUCTIONED, OR OTHERWISE REDISTRIBUTED EXCEPT AS MAY BE PERMITTED BY THE LICENSE TERMS HEREIN.

NOTE: UNDER NO CIRCUMSTANCES MAY THIS MATERIAL OR ANY PORTION THEREOF BE SOLD, LICENSED, AUCTIONED, OR OTHERWISE REDISTRIBUTED EXCEPT AS MAY BE PERMITTED BY THE LICENSE TERMS HEREIN.

# Test Bank

Test Bank  
A First Course in Differential Equations 9e  
Differential Equations with Boundary-Value Problems 7e  
Zill | Cullen

## READ IMPORTANT LICENSE INFORMATION

Dear Professor or Other Supplement Recipient:

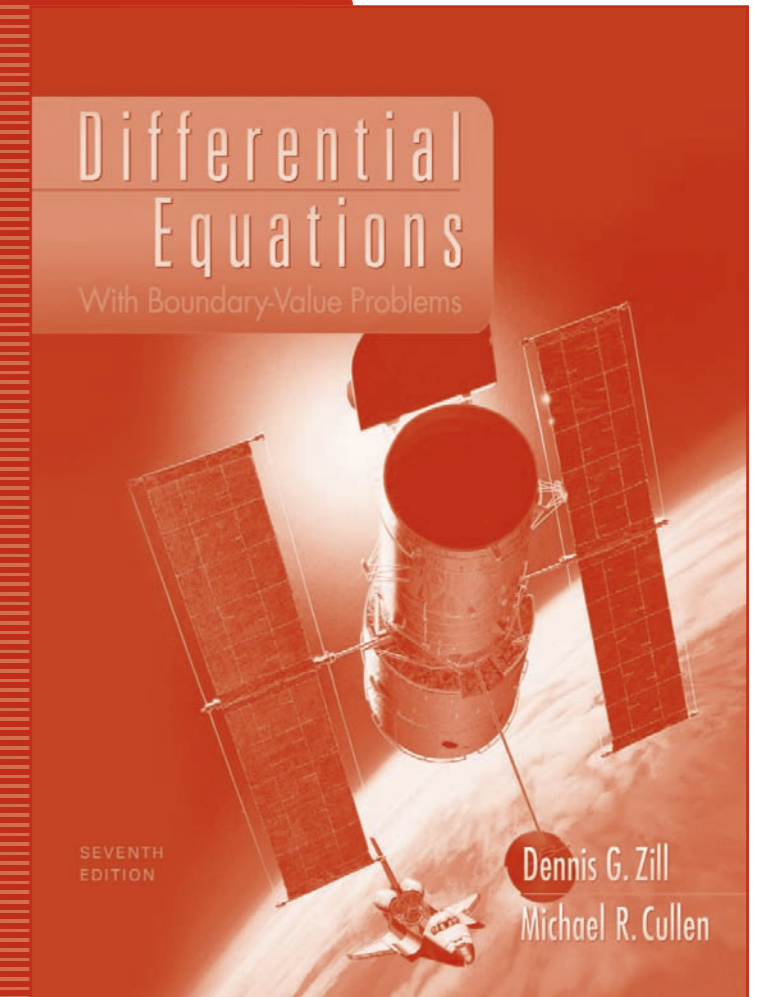
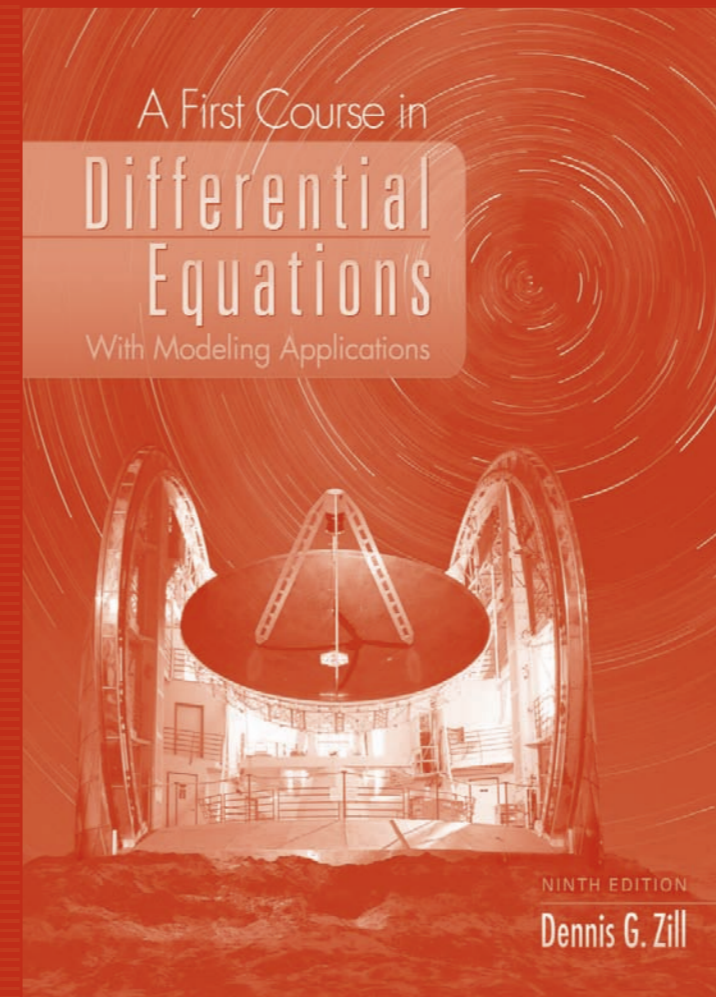
Cengage Learning has provided you with this product (the "Supplement") for your review and, to the extent that you adopt the associated textbook for use in connection with your course (the "Course"), you and your students who purchase the textbook may use the Supplement as described below. Cengage Learning has established these use limitations in response to concerns raised by authors, professors, and other users regarding the pedagogical problems stemming from unlimited distribution of Supplements.

Cengage Learning hereby grants you a nontransferable license to use the Supplement in connection with the Course, subject to the following conditions. The Supplement is for your personal, noncommercial use only and may not be reproduced, posted electronically or distributed, except that portions of the Supplement may be provided to your students IN PRINT FORM ONLY in connection with your instruction of the Course, so long as such students are advised that they may not copy or distribute any portion of the Supplement to any third party. You may not sell, license, auction, or otherwise redistribute the Supplement in any form. We ask that you take reasonable steps to protect the Supplement from unauthorized use, reproduction, or distribution. Your use of the Supplement indicates your acceptance of the conditions set forth in this Agreement. If you do not accept these conditions, you must return the Supplement unused within 30 days of receipt.

All rights (including without limitation, copyrights, patents, and trade secrets) in the Supplement are and will remain the sole and exclusive property of Cengage Learning and/or its licensors. The Supplement is furnished by Cengage Learning on an "as is" basis without any warranties, express or implied. This Agreement will be governed by and construed pursuant to the laws of the State of New York, without regard to such State's conflict of law rules.

Thank you for your assistance in helping to safeguard the integrity of the content contained in this Supplement. We trust you find the Supplement a useful teaching tool.

[www.cengage.com/math](http://www.cengage.com/math)



GILBERT N. LEWIS



To learn more about Brooks/Cole, visit [www.cengage.com/brookscole](http://www.cengage.com/brookscole)

Purchase any of our products at your local college store or at our preferred online store [www.ichapters.com](http://www.ichapters.com)



# Test Bank

---

## A First Course in Differential Equations

**NINTH EDITION**

**Dennis G. Zill**

Loyola Marymount University

## Differential Equations with Boundary-Value Problems

**SEVENTH EDITION**

**Dennis G. Zill**

Loyola Marymount University

**Michael R. Cullen**

Late of Loyola Marymount University

Prepared by

**Gilbert N. Lewis**

Michigan Technological University



**BROOKS/COLE**  
CENGAGE Learning<sup>®</sup>

© 2009 Brooks/Cole, Cengage Learning

ALL RIGHTS RESERVED. No part of this work covered by the copyright herein may be reproduced, transmitted, stored, or used in any form or by any means graphic, electronic, or mechanical, including but not limited to photocopying, recording, scanning, digitizing, taping, Web distribution, information networks, or information storage and retrieval systems, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without the prior written permission of the publisher except as may be permitted by the license terms below.

For product information and technology assistance, contact us at  
**Cengage Learning Customer & Sales Support,**  
**1-800-354-9706**

For permission to use material from this text or product, submit all requests online at [www.cengage.com/permissions](http://www.cengage.com/permissions)  
Further permissions questions can be emailed to [permissionrequest@cengage.com](mailto:permissionrequest@cengage.com)

ISBN-13: 978-0-495-38606-3  
ISBN-10: 0-495-38606-5

**Brooks/Cole**  
10 Davis Drive  
Belmont, CA 94002-3098  
USA

Cengage Learning is a leading provider of customized learning solutions with office locations around the globe, including Singapore, the United Kingdom, Australia, Mexico, Brazil, and Japan. Locate your local office at [www.cengage.com/global](http://www.cengage.com/global)

Cengage Learning products are represented in Canada by Nelson Education, Ltd.

To learn more about Brooks/Cole, visit [www.cengage.com/brookscole](http://www.cengage.com/brookscole)

Purchase any of our products at your local college store or at our preferred online store [www.ichapters.com](http://www.ichapters.com)

**NOTE: UNDER NO CIRCUMSTANCES MAY THIS MATERIAL OR ANY PORTION THEREOF BE SOLD, LICENSED, AUCTIONED, OR OTHERWISE REDISTRIBUTED EXCEPT AS MAY BE PERMITTED BY THE LICENSE TERMS HEREIN.**

#### READ IMPORTANT LICENSE INFORMATION

Dear Professor or Other Supplement Recipient:

Cengage Learning has provided you with this product (the "Supplement") for your review and, to the extent that you adopt the associated textbook for use in connection with your course (the "Course"), you and your students who purchase the textbook may use the Supplement as described below. Cengage Learning has established these use limitations in response to concerns raised by authors, professors, and other users regarding the pedagogical problems stemming from unlimited distribution of Supplements.

Cengage Learning hereby grants you a nontransferable license to use the Supplement in connection with the Course, subject to the following conditions. The Supplement is for your personal, noncommercial use only and may not be reproduced, posted electronically or distributed, except that portions of the Supplement may be provided to your students IN PRINT FORM ONLY in connection with your instruction of the Course, so long as such students are advised that they may not copy or distribute

any portion of the Supplement to any third party. You may not sell, license, auction, or otherwise redistribute the Supplement in any form. We ask that you take reasonable steps to protect the Supplement from unauthorized use, reproduction, or distribution. Your use of the Supplement indicates your acceptance of the conditions set forth in this Agreement. If you do not accept these conditions, you must return the Supplement unused within 30 days of receipt.

All rights (including without limitation, copyrights, patents, and trade secrets) in the Supplement are and will remain the sole and exclusive property of Cengage Learning and/or its licensors. The Supplement is furnished by Cengage Learning on an "as is" basis without any warranties, express or implied. This Agreement will be governed by and construed pursuant to the laws of the State of New York, without regard to such State's conflict of law rules.

Thank you for your assistance in helping to safeguard the integrity of the content contained in this Supplement. We trust you find the Supplement a useful teaching tool.

## Table of Contents

1. Introduction to Differential Equations
  - Form A - Free Response
  - Form B - Free Response
  - Form C - Multiple Choice
  - Form D - Multiple Choice
  - Form E - Mixed (Free Response/Multiple Choice)
  - Form F - Mixed (Free Response/Multiple Choice)
  - Form G - Mixed (Free Response/Multiple Choice)
  - Form H - Mixed (Free Response/Multiple Choice)
2. First-Order Differential Equations
  - Form A - Free Response
  - Form B - Free Response
  - Form C - Multiple Choice
  - Form D - Multiple Choice
  - Form E - Mixed (Free Response/Multiple Choice)
  - Form F - Mixed (Free Response/Multiple Choice)
  - Form G - Mixed (Free Response/Multiple Choice)
  - Form H - Mixed (Free Response/Multiple Choice)
3. Modeling with First-Order Differential Equations
  - Form A - Free Response
  - Form B - Free Response
  - Form C - Multiple Choice
  - Form D - Multiple Choice
  - Form E - Mixed (Free Response/Multiple Choice)
  - Form F - Mixed (Free Response/Multiple Choice)
  - Form G - Mixed (Free Response/Multiple Choice)
  - Form H - Mixed (Free Response/Multiple Choice)
4. Higher-Order Differential Equations
  - Form A - Free Response
  - Form B - Free Response
  - Form C - Multiple Choice
  - Form D - Multiple Choice
  - Form E - Mixed (Free Response/Multiple Choice)
  - Form F - Mixed (Free Response/Multiple Choice)
  - Form G - Mixed (Free Response/Multiple Choice)
  - Form H - Mixed (Free Response/Multiple Choice)

5. Modeling with Higher-Order Differential Equations

Form A - Free Response

Form B - Free Response

Form C - Multiple Choice

Form D - Multiple Choice

Form E - Mixed (Free Response/Multiple Choice)

Form F - Mixed (Free Response/Multiple Choice)

Form G - Mixed (Free Response/Multiple Choice)

Form H - Mixed (Free Response/Multiple Choice)

6. Series Solutions of Linear Equations

Form A - Free Response

Form B - Free Response

Form C - Multiple Choice

Form D - Multiple Choice

Form E - Mixed (Free Response/Multiple Choice)

Form F - Mixed (Free Response/Multiple Choice)

Form G - Mixed (Free Response/Multiple Choice)

Form H - Mixed (Free Response/Multiple Choice)

7. The Laplace Transform

Form A - Free Response

Form B - Free Response

Form C - Multiple Choice

Form D - Multiple Choice

Form E - Mixed (Free Response/Multiple Choice)

Form F - Mixed (Free Response/Multiple Choice)

Form G - Mixed (Free Response/Multiple Choice)

Form H - Mixed (Free Response/Multiple Choice)

8. Systems of Linear First-Order Differential Equations

Form A - Free Response

Form B - Free Response

Form C - Multiple Choice

Form D - Multiple Choice

Form E - Mixed (Free Response/Multiple Choice)

Form F - Mixed (Free Response/Multiple Choice)

Form G - Mixed (Free Response/Multiple Choice)

Form H - Mixed (Free Response/Multiple Choice)

9. Numerical Solutions of Ordinary Differential Equations
  - Form A - Free Response
  - Form B - Free Response
  - Form C - Multiple Choice
  - Form D - Multiple Choice
  - Form E - Mixed (Free Response/Multiple Choice)
  - Form F - Mixed (Free Response/Multiple Choice)
  - Form G - Mixed (Free Response/Multiple Choice)
  - Form H - Mixed (Free Response/Multiple Choice)
10. Plane Autonomous Systems
  - Form A - Free Response
  - Form B - Free Response
  - Form C - Multiple Choice
  - Form D - Multiple Choice
  - Form E - Mixed (Free Response/Multiple Choice)
  - Form F - Mixed (Free Response/Multiple Choice)
  - Form G - Mixed (Free Response/Multiple Choice)
  - Form H - Mixed (Free Response/Multiple Choice)
11. Orthogonal Functions and Fourier Series
  - Form A - Free Response
  - Form B - Free Response
  - Form C - Multiple Choice
  - Form D - Multiple Choice
  - Form E - Mixed (Free Response/Multiple Choice)
  - Form F - Mixed (Free Response/Multiple Choice)
  - Form G - Mixed (Free Response/Multiple Choice)
  - Form H - Mixed (Free Response/Multiple Choice)
12. Boundary-Value Problems in Rectangular Coordinates
  - Form A - Free Response
  - Form B - Free Response
  - Form C - Multiple Choice
  - Form D - Multiple Choice
  - Form E - Mixed (Free Response/Multiple Choice)
  - Form F - Mixed (Free Response/Multiple Choice)
  - Form G - Mixed (Free Response/Multiple Choice)
  - Form H - Mixed (Free Response/Multiple Choice)

13. Boundary-Value Problems in Other Coordinate Systems

Form A - Free Response

Form B - Free Response

Form C - Multiple Choice

Form D - Multiple Choice

Form E - Mixed (Free Response/Multiple Choice)

Form F - Mixed (Free Response/Multiple Choice)

Form G - Mixed (Free Response/Multiple Choice)

Form H - Mixed (Free Response/Multiple Choice)

14. Integral Transforms

Form A - Free Response

Form B - Free Response

Form C - Multiple Choice

Form D - Multiple Choice

Form E - Mixed (Free Response/Multiple Choice)

Form F - Mixed (Free Response/Multiple Choice)

Form G - Mixed (Free Response/Multiple Choice)

Form H - Mixed (Free Response/Multiple Choice)

15. Numerical Solutions of Partial Differential Equations

Form A - Free Response

Form B - Free Response

Form C - Multiple Choice

Form D - Multiple Choice

Form E - Mixed (Free Response/Multiple Choice)

Form F - Mixed (Free Response/Multiple Choice)

Form G - Mixed (Free Response/Multiple Choice)

Form H - Mixed (Free Response/Multiple Choice)

16. Final Exam 1

Form A - Free Response

Form B - Multiple Choice

Form C - Mixed (Free Response/Multiple Choice)

17. Final Exam 2

Form A - Free Response

Form B - Multiple Choice

Form C - Mixed (Free Response/Multiple Choice)



## Preface

This test bank accompanies Zill's *A First Course in Differential Equations*, 9<sup>th</sup> edition and Zill's *Differential Equations with Boundary-Value Problems*, 7<sup>th</sup> edition. Each chapter contains eight exams, two exams of the free response variety, two multiple choice exams, and four exams of a mixed type. Each exam has 20 questions, followed by answers for that exam. The two final exam chapters at the end contain three exams each, one each of the free response, multiple choice, and mixed varieties. Each of the final exams contain 50 questions with answers.

This test bank contains 2700 questions with answers. With such a large number, there are bound to be errors, for which I am entirely responsible. If you, the reader, find mistakes, please contact me at the e-mail address listed below. I endeavor to make this test bank as free of errors as possible, so your input is very much appreciated.

I would like to extend my appreciation to two reviewers, Dennis Lewandowski and David Olson, for their perusal of this test bank. I also thank Stacy Green and the staff at Cengage for putting this project together. I also thank Dennis Zill for giving me the opportunity to work on this project. Finally, I thank my wife, Susan, for her patience through this project.

Gilbert N. Lewis  
lewis@mtu.edu



1. State the order of the differential equation and tell whether it is linear or nonlinear.

$$y''' + 3yy'' + 2y' = 3x + 1.$$

2. State the order of the differential equation and tell whether it is linear or nonlinear.

$$y'' + 2y' - \sin x y = e^x.$$

3. Verify that the indicated function is an explicit solution of the given differential equation.

$$y'' - 3y' + 2y = 0; y = e^{2x}.$$

4. Verify that the indicated family of functions is a solution of the given differential equation.

$$y'' - 6y' + 9y = 0; y = c_1e^{3x} + c_2xe^{3x}.$$

5. Verify that the pair of functions is a solution of the system of differential equations.

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -2x + 3y$$

$$x = e^t + 3e^{2t}$$

$$y = e^t + 6e^{2t}$$

6.  $y = 1/(x^2 + c)$  is a one parameter family of solutions of the differential equation  $y' + 2xy^2 = 0$ . Find a solution satisfying the initial condition  $y(1) = 2$ .
7.  $y = c_1\cos x + c_2\sin x$  is a two parameter family of solutions of the differential equation  $y'' + y = 0$ . Find a solution that also satisfies the initial conditions  $y(\pi/4) = 2, y'(\pi/4) = 1$ .
8.  $y = c_1e^x + c_2e^{-x}$  is a two parameter family of solutions of the differential equation  $y'' - y = 0$ . Find a solution that also satisfies the initial conditions  $y(0) = 3, y'(0) = -1$ .
9. Determine by inspection at least two solutions of the initial value problem  $xy' = 2y, y(0) = 0$ .
10. Determine whether Theorem 1.1 guarantees that the differential equation  $y' = \sqrt{y^2 - 4}$  has a unique solution through the point  $(1, 3)$ .
11. Determine whether Theorem 1.1 guarantees that the differential equation  $y' = \sqrt{y^2 - 4}$  has a unique solution through the point  $(-1, 2)$ .

12. Determine a region of the  $xy$ -plane for which the given differential equation has a unique solution passing through a point  $(x_0, y_0)$  in the region.

$$y' = \sqrt{xy}.$$

13. Determine a region of the  $xy$ -plane for which the given differential equation has a unique solution passing through a point  $(x_0, y_0)$  in the region.

$$x(y - 3)y' = 1.$$

14. The population of a certain country grows at a rate proportional to its population. If immigration into the country at a rate  $r > 0$  is allowed, write down the differential equation for the population as a function of time.
15. A chicken is taken out of the freezer (at  $32^\circ\text{F}$ ) and placed on the kitchen table (at  $70^\circ\text{F}$ ). Write down the differential equation and initial condition for the temperature of the chicken at time  $t$ . Assume that Newton's Law of warming applies.
16. A large tank initially contains 100 gallons of pure water. A brine solution with a concentration of 2 pounds of salt per gallon is poured into the tank at the rate of 3 gallons per minute. The well-stirred mixture is then pumped out at the same rate. Write down the differential equation and initial condition for the amount of salt in the tank at time  $t$ .
17. Write down the differential equation for the charge on the capacitor in an  $R$ - $C$  circuit to which a 12 volt battery is attached.
18. Write down the differential equation for the charge on the capacitor in an  $L$ - $R$  circuit to which a voltage of  $100\sin t$  is applied.
19. A rock is thrown upward from the top of a 200 foot tall building with an upward speed of 20 feet per second. Write down the initial value problem for the position of the rock as a function of time.
20. A body falls toward earth under the force of gravity. Assume that there is a damping force due to air resistance that is proportional to the velocity. Write down the differential equation for the velocity as a function of time.

**ANSWER KEY****Zill Differential Equations 9e Chapter 1 Form A**

---

1. Third order, nonlinear
2. Second order, linear
3.  $y = e^{2x}$ ,  $y' = 2e^{2x}$ ,  $y'' = 4e^{2x}$ ,  
 $y'' - 3y' + 2y = 4e^{2x} - 3(2e^{2x}) + 2e^{2x} = (4 - 6 + 2)e^{2x} = 0$
4.  $y = c_1e^{3x} + c_2xe^{3x}$ ,  
 $y' = 3c_1e^{3x} + 3c_2xe^{3x} + c_2e^{3x}$ ,  
 $y'' = 9c_1e^{3x} + 9c_2xe^{3x} + 6c_2e^{3x}$ ,  
 $y'' - 6y' + 9y = 9c_1e^{3x} + 9c_2xe^{3x} + 6c_2e^{3x} - 6(3c_1e^{3x} + 3c_2xe^{3x} + c_2e^{3x}) + 9(c_1e^{3x} + c_2xe^{3x}) =$   
 $(9 - 18 + 9)c_1e^{3x} + (9 - 18 + 9)c_2xe^{3x} + (6 - 6)c_2e^{3x} = 0$
5.  $\frac{dx}{dt} = e^t + 6e^{2t} = y$ ,  
 $-2x + 3y = -2e^t - 6e^{2t} + 3e^t + 18e^{2t} = e^t + 12e^{2t} = \frac{dy}{dt}$
6.  $y = \frac{1}{x^2 - 1/2}$
7.  $y = \sqrt{2}\cos x/2 + 3\sqrt{2}\sin x/2$
8.  $y = e^x + 2e^{-x}$
9.  $y = 0, y = x^2$
10. Yes
11. No, because  $\sqrt{y^2 - 4}$  is neither continuous nor differentiable there.
12.  $\{(x, y) : x > 0, y > 0\}$  or  $\{(x, y) : x < 0, y < 0\}$
13.  $\{(x, y) : x > 0, y > 3\}$  or three other regions
14.  $\frac{dP}{dt} = kP + r$ , where  $P(t)$  is the population at time  $t$  and  $k$  is the constant of proportionality.
15.  $\frac{dT}{dt} = k(T - 70)$ ,  $T(0) = 32$ , where  $T(t)$  is the temperature of the chicken at time  $t$ , and  $k$  is the constant of proportionality.
16.  $\frac{dA}{dt} = 6 - \frac{3A}{100}$ ,  $A(0) = 0$ , where  $A(t)$  is the amount of salt in the tank at time  $t$ .
17.  $R\frac{dq}{dt} + \frac{1}{C}q = 12$ , where  $q$  is the charge on the capacitor, and  $R$  and  $C$  are the resistance and capacitance, respectively.
18.  $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} = 100\sin t$ , where  $q$  is the charge on the capacitor, and  $L$  and  $R$  are the inductance and resistance, respectively.
19.  $\frac{d^2s}{dt^2} = -32$ ,  $s(0) = 200$ ,  $\frac{ds}{dt}(0) = 20$ , where  $s(t)$  is the position of the rock at time  $t$  relative to the ground.
20.  $m\frac{dv}{dt} = mg - kv$ , where  $v$  is velocity at time  $t$ ,  $m$  is mass,  $g$  is the gravitational acceleration, and  $k$  is the constant of proportionality. The positive direction is downward.

1. State the order of the differential equation and tell whether it is linear or nonlinear.  
 $6y'' + 3y' - 8y = \cos x.$

2. State the order of the differential equation and tell whether it is linear or nonlinear.  
 $y'''' + 2y'y'' - e^x y = \tan x.$

3. Verify that the indicated function is an explicit solution of the given differential equation.

$$y'' - 5y' + 4y = 0; y = e^{4x}.$$

4. Verify that the indicated family of functions is a solution of the given differential equation.

$$y'' - 6y' + 5y = 0; y = c_1 e^{5x} + c_2 e^x.$$

5. Verify that the pair of functions is a solution of the system of differential equations.

$$\begin{aligned}\frac{dx}{dt} &= 2x + y \\ \frac{dy}{dt} &= 2x + 3y \\ x &= -2e^t + 2e^{4t} \\ y &= 2e^t + 4e^{4t}\end{aligned}$$

6.  $y = 1/(1 + ce^{-x})$  is a one parameter family of solutions of the differential equation  $y' = y - y^2$ . Find a solution satisfying the initial condition  $y(2) = 3$ .
7.  $y = c_1 \cos x + c_2 \sin x$  is a two parameter family of solutions of the differential equation  $y'' + y = 0$ . Find a solution that also satisfies the initial conditions  $y(\pi/2) = -1, y'(\pi/2) = 4$ .
8.  $y = c_1 e^x + c_2 e^{-x}$  is a two parameter family of solutions of the differential equation  $y'' - y = 0$ . Find a solution that also satisfies the initial conditions  $y(1) = -2, y'(1) = -3$ .
9. Determine by inspection at least two solutions of the initial value problem  $xy' = 3y, y(0) = 0$ .
10. Determine whether Theorem 1.1 guarantees that the differential equation  $y' = \sqrt{y-3}$  has a unique solution through the point  $(1, 3)$ .
11. Determine whether Theorem 1.1 guarantees that the differential equation  $y' = \sqrt{y-3}$  has a unique solution through the point  $(-1, 4)$ .
12. Determine a region of the  $xy$ -plane for which the given differential equation has a unique solution passing through a point  $(x_0, y_0)$  in the region.

$$y' = \sqrt{x(y-2)}.$$

13. Determine a region of the  $xy$ -plane for which the given differential equation has a unique solution passing through a point  $(x_0, y_0)$  in the region.

$$(x - 1)^2(y + 2)y' = 1.$$

14. The population of a certain country grows at a rate proportional to its population. If emigration out of the country at a rate  $r > 0$  is allowed, write down the differential equation for the population as a function of time.
15. A baked chicken is taken out of the oven (at  $350^\circ\text{F}$ ) and placed in the refrigerator (at  $40^\circ\text{F}$ ). Write down the differential equation and initial condition for the temperature of the chicken at time  $t$ . Assume that Newton's Law of cooling applies.
16. A large tank initially contains 50 gallons of a brine solution with a concentration of 2 pounds of salt per gallon of water. Pure water is pumped into the tank at the rate of 4 gallons per minute. The well-stirred mixture is then pumped out at the same rate. Write down the differential equation and initial condition for the amount of salt in the tank at time  $t$ .
17. Write down the differential equation for the charge on the capacitor in an  $R$ - $C$  circuit to which a voltage of  $10\cos t$  is applied.
18. Write down the differential equation for the charge on the capacitor in an  $L$ - $R$  circuit to which a 12 volt battery is attached.
19. A rock is thrown from the top of a 500 foot tall building with a downward speed of 30 feet per second. Write down the initial value problem for the position of the rock as a function of time.
20. A body falls toward earth under the force of gravity. Assume that there is a damping force due to air resistance that is proportional to the square of the velocity. Write down the differential equation for the velocity as a function of time.

**ANSWER KEY****Zill Differential Equations 9e Chapter 1 Form B**

---

1. Second order, linear
2. Fourth order, nonlinear
3.  $y = e^{4x}$ ,  $y' = 4e^{4x}$ ,  $y'' = 16e^{4x}$ ,  
 $y'' - 5y' + 4y = 16e^{4x} - 5(4e^{4x}) + 4e^{4x} = (16 - 20 + 4)e^{4x} = 0$
4.  $y = c_1e^{5x} + c_2e^x$ ,  
 $y' = 5c_1e^{5x} + c_2e^x$ ,  
 $y'' = 25c_1e^{5x} + c_2e^x$ ,  
 $y'' - 6y' + 5y = 25c_1e^{5x} + c_2e^x - 6(5c_1e^{5x} + c_2e^x) + 5(c_1e^{5x} + c_2e^x) = (25 - 30 + 5)c_1e^{5x} + (1 - 6 + 5)c_2e^x = 0$
5.  $2x + y = -4e^t + 4e^{4t} + 2e^t + 4e^{4t} = -2e^t + 8e^{4t} = \frac{dx}{dt}$ ,  
 $2x + 3y = -4e^t + 4e^{4t} + 6e^t + 12e^{4t} = 2e^t + 16e^{4t} = \frac{dy}{dt}$
6.  $y = \frac{1}{1 - 2e^{2-x}/3}$
7.  $y = -4\cos x - \sin x$
8.  $y = -5e^{x-1}/2 + e^{1-x}/2$
9.  $y = 0, y = x^3$
10. No, because  $\sqrt{y-3}$  is neither continuous nor differentiable there.
11. Yes
12.  $\{(x, y) : x > 0, y > 2\}$  or  $\{(x, y) : x < 0, y < 2\}$
13.  $\{(x, y) : x > 1, y > -2\}$  or three other regions
14.  $\frac{dP}{dt} = kP - r$ , where  $P(t)$  is the population at time  $t$  and  $k$  is the constant of proportionality.
15.  $\frac{dT}{dt} = k(T - 40)$ ,  $T(0) = 350$ , where  $T(t)$  is the temperature of the chicken at time  $t$ , and  $k$  is the constant of proportionality.
16.  $\frac{dA}{dt} = -\frac{2A}{25}$ ,  $A(0) = 100$ , where  $A(t)$  is the amount of salt in the tank at time  $t$ .
17.  $R\frac{dq}{dt} + \frac{1}{C}q = 10\cos t$ , where  $q$  is the charge on the capacitor, and  $R$  and  $C$  are the resistance and capacitance, respectively.
18.  $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} = 12$ , where  $q$  is the charge on the capacitor, and  $L$  and  $R$  are the inductance and resistance, respectively.
19.  $\frac{d^2s}{dt^2} = -32$ ,  $s(0) = 500$ ,  $\frac{ds}{dt}(0) = -30$ , where  $s(t)$  is the position of the rock at time  $t$  relative to the ground.
20.  $m\frac{dv}{dt} = mg - kv^2$ , where  $v$  is velocity at time  $t$ ,  $m$  is mass,  $g$  is the gravitational acceleration, and  $k$  is the constant of proportionality. The positive direction is downward.



1. The differential equation  $y'' + 2y' + 3y = 0$  is

Select the correct answer.

- (a) first order linear
- (b) second order linear
- (c) third order linear
- (d) first order nonlinear
- (e) second order nonlinear

2. The differential equation  $y'' + 2yy' + 3y = 0$  is

Select the correct answer.

- (a) first order linear
- (b) second order linear
- (c) third order linear
- (d) first order nonlinear
- (e) second order nonlinear

3. The differential equation  $y' + 3y = \sin x$  is

Select the correct answer.

- (a) first order linear
- (b) second order linear
- (c) third order linear
- (d) first order nonlinear
- (e) second order nonlinear

4. The differential equation  $y'' + 2y' + 3y = \sin y$  is

Select the correct answer.

- (a) first order linear
- (b) second order linear
- (c) third order linear
- (d) first order nonlinear
- (e) second order nonlinear

5. The differential equation  $y''' + 2y'' + 3xy' - 4e^x y = \sin x$  is

Select the correct answer.

- (a) first order linear
- (b) second order linear
- (c) third order linear
- (d) first order nonlinear
- (e) second order nonlinear

6. The values of  $m$  for which  $y = e^{mx}$  is a solution of  $y'' - 5y' + 6y = 0$  are

Select the correct answer.

- (a) 2 and 4
- (b) -2 and -3
- (c) 3 and 4
- (d) 2 and 3
- (e) 1 and 5

7. The values of  $m$  for which  $y = x^m$  is a solution of  $x^2 y'' - 5xy' + 8y = 0$  are

Select the correct answer.

- (a) 2 and 4
- (b) -2 and -4
- (c) 3 and 5
- (d) 2 and 3
- (e) 1 and 5

8. The values of  $c$  for which  $y = c$  is a constant solution of  $y' = y^2 + 3y - 4$  are

Select the correct answer.

- (a) 1 and 4
- (b) -1 and -3
- (c) 1 and -4
- (d) -1 and 3
- (e) 1 and 3

9. The values of  $m$  for which  $y = e^{mx}$  is a solution of  $y'' - 4y' - 5y = 0$  are

Select the correct answer.

- (a) 1 and 4
- (b) -1 and 4
- (c) 2 and 3
- (d) -2 and -3
- (e) -1 and 5

10. The population of a town increases at a rate proportional to its population. its initial population is 1000. The correct initial value problem for the population,  $P(t)$ , as a function of time,  $t$ , is

Select the correct answer.

- (a)  $\frac{dP}{dt} = kP, P(0) = 1000$
- (b)  $\frac{dP}{dt} = kP^2, P(0) = 100$
- (c)  $\frac{dP}{dt} = kP, P(0) = 100$
- (d)  $\frac{dP}{dt} = kP(1 - P), P(0) = 100$
- (e)  $\frac{dP}{dt} = kP^2, P(0) = 1000$

11. The solution of the initial value problem  $y' = 3y, y(0) = 2$  is  $y = ce^{3x}$ , where  $c =$

Select the correct answer.

- (a) 2
- (b) -2
- (c) 3
- (d) -3
- (e) 1

12. The solution of the initial value problem  $y' = 2y + x, y(1) = 1/4$  is  $y = -x/2 - 1/4 + ce^{2x}$ , where  $c =$

Select the correct answer.

- (a) 2
- (b)  $e^{-2}$
- (c)  $e^{-1}$
- (d)  $e^{-2}/2$
- (e) 1

13. The initial value problem  $y' = \sqrt{y^2 - 9}$ ,  $y(x_0) = y_0$  has a unique solution guaranteed by Theorem 1.1 if

Select the correct answer.

- (a)  $y_0 = 3$
- (b)  $y_0 = -3$
- (c)  $y_0 = 5$
- (d)  $y_0 = 0$
- (e)  $y_0 = 1$

14. The temperature of a cup of coffee obeys Newton's law of cooling. The initial temperature of the coffee is  $150^\circ\text{F}$  and one minute later, it is  $135^\circ\text{F}$ . The ambient temperature of the room is  $70^\circ\text{F}$ . If  $T(t)$  represents the temperature of the coffee at time  $t$ , the correct differential equation for the temperature with side conditions is

Select the correct answer.

- (a)  $\frac{dT}{dt} = k(T - 135)$
- (b)  $\frac{dT}{dt} = k(T - 150)$
- (c)  $\frac{dT}{dt} = k(T - 70)$
- (d)  $\frac{dT}{dt} = T(T - 150)$
- (e)  $\frac{dT}{dt} = T(T - 70)$

15. In the previous problem, after a long period of time, the temperature of the coffee approaches

Select the correct answer.

- (a)  $120^\circ\text{F}$
- (b)  $100^\circ\text{F}$
- (c)  $70^\circ\text{F}$
- (d)  $65^\circ\text{F}$
- (e)  $0^\circ\text{F}$

16. In the *LRC* circuit problem in the text,  $C$  stands for

Select the correct answer.

- (a) capacitance
- (b) resistance
- (c) current
- (d) inductance
- (e) charge on the capacitor

17. A large mixing tank initially contains 100 gallons of water in which 30 pounds of salt have been dissolved. Another brine solution is pumped into the tank at the rate of 4 gallons per minute, and the resulting mixture is pumped out at the same rate. The concentration of the incoming brine solution is 2 pounds of salt per gallon. If  $A(t)$  represents the amount of salt in the tank at time  $t$ , the correct differential equation for  $A$  is

Select the correct answer.

- (a)  $\frac{dA}{dt} = 8 - .02A$
- (b)  $\frac{dA}{dt} = 8 - .04A$
- (c)  $\frac{dA}{dt} = 4 - .04A$
- (d)  $\frac{dA}{dt} = 2 - .04A$
- (e)  $\frac{dA}{dt} = 4 - .08A$

18. In the previous problem, over a long period of time, the total amount of salt in the tank will approach

Select the correct answer.

- (a) 30 pounds
- (b) 50 pounds
- (c) 100 pounds
- (d) 200 pounds
- (e) 300 pounds

19. In the  $LRC$  circuit problem in the text, the units of inductance,  $L$ , are

Select the correct answer.

- (a) ohms
- (b) farads
- (c) amperes
- (d) henrys
- (e) coulombs

20. In the falling body problem, the units of acceleration might be

Select the correct answer.

- (a) meters per second
- (b) feet per second
- (c) meters per second per second
- (d) kilograms per meter
- (e) kilograms per meter per second

**ANSWER KEY**

*Zill Differential Equations 9e Chapter 1 Form C*

---

1. b
2. e
3. a
4. e
5. c
6. d
7. a
8. c
9. e
10. a
11. a
12. b
13. c
14. c
15. c
16. a
17. b
18. d
19. d
20. c

1. The differential equation  $y''' + 2y'' + 3y' + 7y = 0$  is

Select the correct answer.

- (a) first order linear
- (b) second order linear
- (c) third order linear
- (d) first order nonlinear
- (e) second order nonlinear

2. The differential equation  $y'' + 2yy' + 3y = 0$  is

Select the correct answer.

- (a) first order linear
- (b) second order linear
- (c) third order linear
- (d) first order nonlinear
- (e) second order nonlinear

3. The differential equation  $y' + 3y = \sin x$  is

Select the correct answer.

- (a) first order linear
- (b) second order linear
- (c) third order linear
- (d) first order nonlinear
- (e) second order nonlinear

4. The differential equation  $y'' + 2y' + 3y = \sin y$  is

Select the correct answer.

- (a) first order linear
- (b) second order linear
- (c) third order linear
- (d) first order nonlinear
- (e) second order nonlinear

5. The differential equation  $y''' + 2y'' + 3xy' - 4e^x y = \sin x$  is  
Select the correct answer.
- (a) first order linear
  - (b) second order linear
  - (c) third order linear
  - (d) first order nonlinear
  - (e) second order nonlinear
6. The values of  $m$  for which  $y = e^{mx}$  is a solution of  $y'' - 9y' + 20y = 0$  are  
Select the correct answer.
- (a) 4 and -5
  - (b) -4 and -5
  - (c) 3 and 6
  - (d) 4 and 5
  - (e) 3 and 5
7. The values of  $m$  for which  $y = x^m$  is a solution of  $x^2 y'' - 7xy' + 12y = 0$  are  
Select the correct answer.
- (a) -3 and 4
  - (b) -2 and -6
  - (c) 3 and 4
  - (d) 2 and 6
  - (e) 3 and -4
8. The values of  $c$  for which  $y = c$  is a constant solution of  $y' = y^2 + 5y - 6$  are  
Select the correct answer.
- (a) 1 and 6
  - (b) -1 and 6
  - (c) 1 and -6
  - (d) -2 and 3
  - (e) 2 and 3



9. The values of  $m$  for which  $y = e^{mx}$  is a solution of  $y'' - 6y' - 7y = 0$  are

Select the correct answer.

- (a) 1 and 7
- (b) -1 and 6
- (c) 1 and 6
- (d) 1 and -6
- (e) -1 and 7

10. The population of a town increases at a rate proportional to its population. its initial population is 5000. The correct initial value problem for the population,  $P(t)$ , as a function of time,  $t$ , is

Select the correct answer.

- (a)  $\frac{dP}{dt} = kP, P(0) = 5000$
- (b)  $\frac{dP}{dt} = kP^2, P(0) = 500$
- (c)  $\frac{dP}{dt} = kP, P(0) = 500$
- (d)  $\frac{dP}{dt} = kP(1 - P), P(0) = 5000$
- (e)  $\frac{dP}{dt} = kP^2, P(0) = 5000$

11. The solution of the initial value problem  $y' = 5y, y(1) = 3$  is  $y = ce^{5x}$ , where  $c =$

Select the correct answer.

- (a)  $3e^{-5}$
- (b) 3
- (c)  $3e^5$
- (d)  $-3e^5$
- (e) -3

12. The solution of the initial value problem  $y' = 2y + x, y(-1) = 1/2$  is  $y = -x/2 - 1/4 + ce^{2x}$ , where  $c =$

Select the correct answer.

- (a) 2
- (b)  $e^2/4$
- (c)  $e^2$
- (d)  $e^2/2$
- (e) 1

13. The initial value problem  $y' = \sqrt{y^2 - 16}$ ,  $y(x_0) = y_0$  has a unique solution guaranteed by Theorem 1.1 if

Select the correct answer.

- (a)  $y_0 = 4$
- (b)  $y_0 = -4$
- (c)  $y_0 = 0$
- (d)  $y_0 = 8$
- (e)  $y_0 = 1$

14. The temperature of a cup of coffee obeys Newton's law of cooling. The initial temperature of the coffee is  $140^\circ\text{F}$  and one minute later, it is  $125^\circ\text{F}$ . The ambient temperature of the room is  $65^\circ\text{F}$ . If  $T(t)$  represents the temperature of the coffee at time  $t$ , the correct differential equation for the temperature is

Select the correct answer.

- (a)  $\frac{dT}{dt} = k(T - 125)$
- (b)  $\frac{dT}{dt} = k(T - 140)$
- (c)  $\frac{dT}{dt} = k(T - 65)$
- (d)  $\frac{dT}{dt} = T(T - 140)$
- (e)  $\frac{dT}{dt} = T(T - 65)$

15. In the previous problem, after a long period of time, the temperature of the coffee approaches

Select the correct answer.

- (a)  $125^\circ\text{F}$
- (b)  $100^\circ\text{F}$
- (c)  $65^\circ\text{F}$
- (d)  $50^\circ\text{F}$
- (e)  $0^\circ\text{F}$

16. In the  $LRC$  circuit problem in the text,  $R$  stands for

Select the correct answer.

- (a) capacitance
- (b) resistance
- (c) current
- (d) inductance
- (e) charge on the capacitor

17. A large mixing tank initially contains 1000 gallons of water in which 40 pounds of salt have been dissolved. Another brine solution is pumped into the tank at the rate of 5 gallons per minute, and the resulting mixture is pumped out at the same rate. The concentration of the incoming brine solution is 3 pound of salt per gallon. If  $A(t)$  represents the amount of salt in the tank at time  $t$ , the correct differential equation for  $A$  is

Select the correct answer.

- (a)  $\frac{dA}{dt} = 3 - .005A$
- (b)  $\frac{dA}{dt} = 5 - .05A$
- (c)  $\frac{dA}{dt} = 15 - .005A$
- (d)  $\frac{dA}{dt} = 3 - .05A$
- (e)  $\frac{dA}{dt} = 15 + .05A$

18. In the previous problem, over a long period of time, the total amount of salt in the tank will approach

Select the correct answer.

- (a) 300 pounds
- (b) 500 pounds
- (c) 1000 pounds
- (d) 3000 pounds
- (e) 5000 pounds

19. In the  $LRC$  circuit problem in the text, the units for  $C$  are

Select the correct answer.

- (a) ohms
- (b) farads
- (c) amperes
- (d) henrys
- (e) coulombs

20. In the falling body problem, the units of acceleration might be

Select the correct answer.

- (a) centimeters per second
- (b) feet per second
- (c) feet per second per second
- (d) kilograms per centimeter
- (e) kilograms per centimeter per second

**ANSWER KEY**

***Zill Differential Equations 9e Chapter 1 Form D***

---

1. c
2. e
3. a
4. e
5. c
6. d
7. d
8. c
9. e
10. a
11. a
12. b
13. d
14. c
15. c
16. b
17. c
18. d
19. b
20. c

1. The differential equation  $y' + 3y = e^x$  is  
Select the correct answer.
  - (a) first order linear
  - (b) second order linear
  - (c) third order linear
  - (d) second order nonlinear
  - (e) third order nonlinear
2. The differential equation  $y''' + 2\sin x y'' + 3e^x y' - 5\ln x y = 0$  is  
Select the correct answer.
  - (a) first order linear
  - (b) second order linear
  - (c) third order linear
  - (d) first order nonlinear
  - (e) second order nonlinear
3. Verify that  $y = e^{2x}$  is a solution of  $y' = 2y$ .
4. Verify that  $y = x^2 - 2 + \cos x - \sin x$  is a solution of  $y'' + y = x^2$ .
5. Verify that  $y = \tan(2x)$  is a solution of  $y' = 2y^2 + 2$ .
6. The values of  $m$  for which  $y = e^{mx}$  is a solution of  $y'' - 3y' + 2y = 0$  are  
Select the correct answer.
  - (a) 2 and -3
  - (b) -2 and -3
  - (c) 1 and 2
  - (d) 2 and 3
  - (e) -1 and -2
7. By inspection, find a one parameter family of solutions of the differential equation  $xy' = y$ . Verify that each member of this family also satisfies the initial condition  $y(0) = 0$ .
8.  $y = \frac{1}{x^2+c}$  is a one parameter family of solutions of  $y' + 2xy^2 = 0$ . Find a solution of this differential equation that also satisfies the initial condition  $y(2) = 3$ .
9. Consider the differential equation  $y' = 1/((y^2 - 4)(x - 1))$ . Determine a region in the  $xy$ -plane for which it has a unique solution whose graph passes through the point  $(x_0, y_0)$  in the region.

10. The values of  $m$  for which  $y = x^m$  is a solution of  $x^2y'' + 2xy' - 2y = 0$  are  
Select the correct answer.
- (a) 1 and 2
  - (b) -1 and -3
  - (c) 1 and -2
  - (d) -1 and 2
  - (e) 1 and 3
11. The solution of the initial value problem  $y' = 2y + x, y(1) = 1/2$  is  $y = -x/2 - 1/4 + ce^{2x}$ , where  $c =$   
Select the correct answer.
- (a) 2
  - (b)  $5e^{-2}/4$
  - (c)  $3e^{-2}/4$
  - (d)  $e^{-2}/2$
  - (e) 1
12. The initial value problem  $y' = \sqrt{y^2 - 16}, y(x_0) = y_0$  has a unique solution guaranteed by Theorem 1.1 if  
Select the correct answer.
- (a)  $y_0 = 4$
  - (b)  $y_0 = -4$
  - (c)  $y_0 = 0$
  - (d)  $y_0 = 6$
  - (e)  $y_0 = 1$
13. Verify that  $4x^2 - 3y^2 = c$  is a one parameter family of solutions of the differential equation  $ydy/dx = 4x/3$ .
14. Determine whether Theorem 1.1 guarantees a unique solution of the initial value problem  $y' = \sqrt{1 - y^2}, y(1) = 1$ .
15. A new technology is introduced into a community of 10000 individuals. If the rate at which the technology spreads through the community is jointly proportional to the number of people who have adopted the technology and the number of individuals who have not adopted it, write down the differential equation for the number of people,  $x(t)$ , who have adopted the technology by time  $t$ .

16. A large mixing tank initially contains 100 gallons of water in which 20 pounds of salt have been dissolved. Another brine solution is pumped into the tank at the rate of 6 gallons per minute, and the resulting mixture is pumped out at the same rate. The concentration of the incoming brine solution is 3 pounds of salt per gallon. If  $A(t)$  represents the amount of salt in the tank at time  $t$ , the correct differential equation for  $A$  is

Select the correct answer.

(a)  $\frac{dA}{dt} = 6 - .03A$

(b)  $\frac{dA}{dt} = 18 - .06A$

(c)  $\frac{dA}{dt} = 3 - .06A$

(d)  $\frac{dA}{dt} = 6 + .03A$

(e)  $\frac{dA}{dt} = 18 + .06A$

17. In the previous problem, over a long period of time, the total amount of salt in the tank will approach

Select the correct answer.

(a) 30 pounds

(b) 50 pounds

(c) 100 pounds

(d) 200 pounds

(e) 300 pounds

18. In the  $LRC$  circuit problem in the text,  $C$  stands for

Select the correct answer.

(a) capacitance

(b) resistance

(c) current

(d) inductance

(e) charge on the capacitor

19. In the  $LRC$  circuit problem in the text, the units for  $R$  are

Select the correct answer.

(a) farads

(b) ohms

(c) amperes

(d) henrys

(e) coulombs

20. In the falling body problem, the units of acceleration might be  
Select the correct answer.

- (a) meters per second
- (b) feet per second
- (c) meters per second per second
- (d) kilograms per meter
- (e) kilograms per meter per second



**ANSWER KEY****Zill Differential Equations 9e Chapter 1 Form E**

---

1. a
2. c
3.  $y = e^{2x}$   
 $y' = 2e^{2x} = 2y$
4.  $y = x^2 - 2 + \cos x - \sin x$   
 $y' = 2x - \sin x - \cos x$   
 $y'' = 2 - \cos x + \sin x$   
 $y'' + y = x^2$
5.  $y = \tan(2x)$   
 $y' = 2\sec^2(2x) = 2(1 + \tan^2(2x)) = 2(1 + y^2) = 2y^2 + 2$
6. c
7.  $y = cx, y(0) = 0$
8.  $y = \frac{1}{x^2 - 11/3}$
9.  $\{(x, y) : x > 1, y > 2\}$  or three other possible regions
10. c
11. b
12. d
13.  $4x^2 - 3y^2 = c$   
 $8x - 6yy' = 0$   
 $yy' = 8x/6 = 4x/3$
14. No, the function  $\sqrt{1 - y^2}$  is not continuous or differentiable at the point  $(1, 1)$ .
15.  $\frac{dx}{dt} = kx(10000 - x)$
16. b
17. e
18. a
19. b
20. c

1. Is the differential equation  $y'' + 2\sin y y' + 3e^x y = 0$  linear or nonlinear? If linear, is it homogeneous? What is its order?
2. The differential equation  $y'' + y^2 = \sin x$  is  
Select the correct answer.
  - (a) first order linear
  - (b) second order linear
  - (c) third order linear
  - (d) first order nonlinear
  - (e) second order nonlinear
3. Verify that  $y = e^{4x} + 4$  is a solution of  $y'' - 4y' = 0$ .
4. Verify that  $y = x + 3\cos x + 5\sin x$  is a solution of  $y'' + y = x$ .
5. Verify that  $y = \ln x$  is a solution of  $xy'' + y' = 0$ .
6. The values of  $m$  for which  $y = e^{mx}$  is a solution of  $y'' - 3y' - 4y = 0$  are  
Select the correct answer.
  - (a) 1 and -3
  - (b) -1 and -3
  - (c) 1 and -4
  - (d) 1 and 3
  - (e) -1 and 4
7. By inspection, find a one parameter family of solutions of the differential equation  $xy' = 4y$ . Verify that each member of this family also satisfies the initial condition  $y(0) = 0$ .
8. The values of  $m$  for which  $y = x^m$  is a solution of  $x^2y'' + 5xy' - 5y = 0$  are  
Select the correct answer.
  - (a) 2 and 3
  - (b) -1 and 5
  - (c) 2 and -3
  - (d) -2 and 3
  - (e) 1 and -5
9.  $y = \frac{1}{1+ce^{-x}}$  is a one parameter family of solutions of  $y' = y - y^2$ . Find a solution of this differential equation that also satisfies the initial condition  $y(1) = 5$ .
10. Consider the differential equation  $y' = \sqrt{y-2}/(x^2-9)$ . Determine a region in the  $xy$ -plane for which it has a unique solution whose graph passes through the point  $(x_0, y_0)$  in the region.

11. The solution of the initial value problem  $y' = 3y + x^2, y(0) = 1$  is  $y = -x^2/3 - 2x/9 - 2/27 + ce^{3x}$ , where  $c =$
- Select the correct answer.
- (a)  $1/9$
  - (b)  $8/9$
  - (c)  $25/27$
  - (d)  $29/27$
  - (e)  $1$
12. The initial value problem  $y' = \sqrt{y^2 - 25}, y(x_0) = y_0$  has a unique solution guaranteed by Theorem 1.1 if
- Select the correct answer.
- (a)  $y_0 = 5$
  - (b)  $y_0 = -5$
  - (c)  $y_0 = 0$
  - (d)  $y_0 = 8$
  - (e)  $y_0 = 4$
13. Verify that  $6x^2 - y^2 = c$  is a one parameter family of solutions of the differential equation  $ydy/dx = 6x$ .
14. Determine whether Theorem 1.1 guarantees a unique solution of the initial value problem  $y' = \sqrt{12 - y^2}, y(1) = 3$ .
15. A disease is introduced into a community of 5000 individuals. If the rate at which the disease spreads through the community is jointly proportional to the number of people who have the disease and the number of individuals who have not caught it, write down the differential equation for the number of people,  $x(t)$  who have caught the disease by time  $t$ .
16. Write down the differential equation for the charge on the capacitor in an  $LRC$  circuit where the inductance is .1 henry, the resistance is 50 ohms, and the capacitance is .001 farad.

17. A cup of coffee initially is at temperature  $90^{\circ}\text{C}$ . It is placed in a room where the temperature is  $20^{\circ}\text{C}$ . If the temperature obeys Newton's Law of cooling, the correct differential equation for the temperature,  $T$ , as a function of time,  $t$ , is

Select the correct answer.

(a)  $\frac{dT}{dt} = k(T - 90)$

(b)  $\frac{dT}{dt} = k(t - 20)$

(c)  $\frac{dT}{dt} = T(T - 90)$

(d)  $\frac{dT}{dt} = k(T - 20)$

(e)  $\frac{dT}{dt} = T(T - 20)$

18. In the previous problem, over a long period of time, the temperature of the coffee will approach

Select the correct answer.

(a)  $90^{\circ}\text{C}$

(b)  $100^{\circ}\text{C}$

(c)  $20^{\circ}\text{C}$

(d)  $55^{\circ}\text{C}$

(e)  $0^{\circ}\text{C}$

19. Assume that an object of mass  $m$  falls toward earth under the influence of gravity, and that it is also acted upon by the force of air resistance that is proportional to the square of the velocity. Take the positive direction downward. The correct differential equation for the velocity of the object is

Select the correct answer.

(a)  $\frac{dv}{dt} = mg - kv$

(b)  $\frac{dv}{dt} = mg - kv^2$

(c)  $m\frac{dv}{dt} = mg - kv$

(d)  $m\frac{dv}{dt} = mg - kv^2$

(e)  $m\frac{dv}{dt} = mg + kv^2$

20. Write down the initial value problem for the position of a falling body above ground level, if it starts from a point 200 feet above ground level with an upward velocity of 40 feet per second.

**ANSWER KEY****Zill Differential Equations 9e Chapter 1 Form F**

---

1. second order nonlinear
2. e
3.  $y = e^{4x} + 4$        $y' = 4e^{4x}$        $y'' = 16e^{4x}$   
 $y'' - 4y' = 16e^{4x} - 4(4e^{4x}) = 0$
4.  $y = x + 3\cos x + 5\sin x$   
 $y' = 1 - 3\sin x + 5\cos x$   
 $y'' = -3\cos x - 5\sin x$   
 $y'' + y = x$
5.  $y = \ln x$        $y' = 1/x$        $y'' = -1/x^2$   
 $xy'' + y' = -1/x + 1/x = 0$
6. e
7.  $y = cx^4, y(0) = 0$
8. e
9.  $y = 1/(1 - 4e^{1-x}/5)$
10.  $\{(x, y) : x > 3, y > 2\}$  or  $\{(x, y) : x < 3, y > 2\}$
11. d
12. d
13.  $6x^2 - y^2 = c$        $12x - 2yy' = 0$        $yy' = 6x$
14. Yes
15.  $\frac{dx}{dt} = kx(5000 - x)$
16.  $.1\frac{d^2q}{dt^2} + 50\frac{dq}{dt} + 1000q = 0$
17. d
18. c
19. d
20.  $\frac{d^2s}{dt^2} = -32, s(0) = 200, \frac{ds}{dt}(0) = 40$ , where  $s(t)$  is the position of the body relative to the ground at time  $t$ .

1. Is the differential equation  $y''' + 2\sin x y'' + 3e^x y' - 5\ln x y = 0$  linear or nonlinear? If linear, is it homogeneous? What is its order?
2. The differential equation  $y' + y^2 = e^x$  is  
Select the correct answer.
  - (a) first order linear
  - (b) second order linear
  - (c) third order linear
  - (d) first order nonlinear
  - (e) second order nonlinear
3. Verify that  $y = e^{3x} + 4$  is a solution of  $y'' - 3y' = 0$ .
4. Verify that  $y = x^3 - 6x + 3\cos x + 5\sin x$  is a solution of  $y'' + y = x^3$ .
5. Verify that  $y = \sec x$  is a solution of  $y' = y\sqrt{y^2 - 1}$ .
6. The values of  $m$  for which  $y = e^{mx}$  is a solution of  $y'' - 5y' + 6y = 0$  are  
Select the correct answer.
  - (a) 2 and -3
  - (b) -2 and -3
  - (c) 1 and 2
  - (d) 2 and 3
  - (e) -1 and -2
7. By inspection, find a one parameter family of solutions of the differential equation  $xy' = 3y$ . Verify that each member of this family also satisfies the initial condition  $y(0) = 0$ .
8. The values of  $m$  for which  $y = x^m$  is a solution of  $x^2y'' + 4xy' - 4y = 0$  are  
Select the correct answer.
  - (a) 1 and 2
  - (b) -1 and 4
  - (c) 1 and -3
  - (d) -1 and 3
  - (e) 1 and -4
9.  $y = \frac{1}{x^2+c}$  is a one parameter family of solutions of  $y' + 2xy^2 = 0$ . Find a solution of this differential equation that also satisfies the initial condition  $y(3) = 5$ .
10. Consider the differential equation  $y' = \sqrt{y^2 - 9}/(x - 3)$ . Determine a region in the  $xy$ -plane for which it has a unique solution whose graph passes through the point  $(x_0, y_0)$  in the region.

11. The solution of the initial value problem  $y' = 2y + x, y(-1) = 1/2$  is  $y = -x/2 - 1/4 + ce^{2x}$ , where  $c =$

Select the correct answer.

- (a) 2
- (b)  $e^2/4$
- (c)  $e^2$
- (d)  $e^2/2$
- (e) 1

12. The initial value problem  $y' = \sqrt{y^2 - 4}, y(x_0) = y_0$  has a unique solution guaranteed by Theorem 1.1 if

Select the correct answer.

- (a)  $y_0 = 2$
- (b)  $y_0 = -2$
- (c)  $y_0 = 0$
- (d)  $y_0 = 6$
- (e)  $y_0 = 1$

13. Verify that  $6x^3 + y^2 = c$  is a one parameter family of solutions of the differential equation  $ydy/dx = -9x^2$ .

14. Determine whether Theorem 1.1 guarantees a unique solution of the initial value problem  $y = \sqrt{4 - y^2}, y(1) = 2$ .

15. A disease is introduced into a community of 1000 individuals. If the rate at which the disease spreads through the community is jointly proportional to the number of people who have the disease and the number of individuals who have not caught it, write down the differential equation for the number of people,  $x(t)$  who have caught the disease by time  $t$ .

16. A large mixing tank initially contains 500 gallons of water in which 40 pounds of salt have been dissolved. Another brine solution is pumped into the tank at the rate of 4 gallons per minute, and the resulting mixture is pumped out at the same rate. The concentration of the incoming brine solution is 2 pounds of salt per gallon. If  $A(t)$  represents the amount of salt in the tank at time  $t$ , the correct differential equation for  $A$  is

Select the correct answer.

- (a)  $\frac{dA}{dt} = 4 - .008A$
- (b)  $\frac{dA}{dt} = 8 - .008A$
- (c)  $\frac{dA}{dt} = 4 - .08A$
- (d)  $\frac{dA}{dt} = 4 + .08A$
- (e)  $\frac{dA}{dt} = 8 + .08A$

17. In the previous problem, over a long period of time, the total amount of salt in the tank will approach

Select the correct answer.

- (a) 100 pounds
- (b) 300 pounds
- (c) 500 pounds
- (d) 800 pounds
- (e) 1000 pounds

18. In the *LRC* circuit problem in the text, *L* stands for

Select the correct answer.

- (a) capacitance
- (b) resistance
- (c) current
- (d) inductance
- (e) charge on the capacitor

19. In the *LRC* circuit problem in the text, *R* stands for

Select the correct answer.

- (a) capacitance
- (b) resistance
- (c) current
- (d) inductance
- (e) charge on the capacitor

20. In the falling body problem, the units of acceleration might be

Select the correct answer.

- (a) meters per second
- (b) feet per second
- (c) meters per second per second
- (d) kilograms per meter
- (e) kilograms per meter per second



**ANSWER KEY****Zill Differential Equations 9e Chapter 1 Form G**

---

1. third order, linear, homogeneous
2. d
3.  $y = e^{3x} + 4$        $y' = 3e^{3x}$        $y'' = 9e^{3x}$   
 $y'' - 3y' = 9e^{3x} - 3(3e^{3x}) = 0$
4.  $y = x^3 - 6x + 3\cos x + 5\sin x$   
 $y' = 3x^2 - 6 - 3\sin x + 5\cos x$   
 $y'' = 6x - 3\cos x - 5\sin x$   
 $y'' + y = x^3$
5.  $y = \sec x$        $y' = \sec x \tan x = \sec x \sqrt{\sec^2 x - 1} = y\sqrt{y^2 - 1}$
6. d
7.  $y = cx^3, y(0) = 0$
8. e
9.  $y = 1/(x^2 - 44/5)$
10.  $\{(x, y) : x > 3, y > 3\}$  or 3 other regions
11. b
12. d
13.  $6x^3 + y^2 = c$        $18x^2 + 2yy' = 0$        $yy' = -9x^2$
14. No, since  $\sqrt{4 - y^2}$  is neither continuous nor differentiable there.
15.  $\frac{dx}{dt} = kx(1000 - x)$
16. b
17. e
18. d
19. b
20. c

1. Is the differential equation  $y''' + 2\sin x y'' + 3e^y y' - 5\ln x y = 0$  linear or nonlinear? If linear, is it homogeneous? What is its order?
2. The differential equation  $y'' + y' = e^x$  is  
Select the correct answer.
  - (a) first order linear
  - (b) second order linear
  - (c) third order linear
  - (d) first order nonlinear
  - (e) second order nonlinear
3. Verify that  $y = e^{-5x} + 4$  is a solution of  $y'' + 5y' = 0$ .
4. Verify that  $y = x^4 - 12x^2 + 24 - \cos x + \sin x$  is a solution of  $y'' + y = x^4$ .
5. Verify that  $y = e^{x^2}$  is a solution of  $y' = 2xy$ .
6. The values of  $m$  for which  $y = e^{mx}$  is a solution of  $y'' - 4y' + 3y = 0$  are  
Select the correct answer.
  - (a) 1 and -3
  - (b) -1 and -3
  - (c) 1 and 2
  - (d) 1 and 3
  - (e) -1 and -2
7. By inspection, find a one parameter family of solutions of the differential equation  $xy' = y$ . Verify that each member of this family also satisfies the initial condition  $y(0) = 0$ .
8. The values of  $m$  for which  $y = x^m$  is a solution of  $x^2y'' + 5xy' - 5y = 0$  are  
Select the correct answer.
  - (a) 1 and 4
  - (b) -1 and 5
  - (c) 1 and -4
  - (d) -1 and 4
  - (e) 1 and -5
9.  $y = \frac{1}{x^2+c}$  is a one parameter family of solutions of  $y' + 2xy^2 = 0$ . Find a solution of this differential equation that also satisfies the initial condition  $y(-1) = 2$ .
10. Determine a region in the  $xy$ -plane for which the differential equation  $y' = \sqrt{y^2 - 9}/(x - 1)$  has a unique solution whose graph passes through the point  $(x_0, y_0)$  in the region.

11. The solution of the initial value problem  $y' = 2y + x, y(-1) = -1/2$  is  $y = -x/2 - 1/4 + ce^{2x}$ , where  $c =$

Select the correct answer.

- (a) 2
- (b)  $-3e^2/4$
- (c)  $5e^2/4$
- (d)  $-e^2/2$
- (e) 1

12. The initial value problem  $y' = \sqrt{y^2 - 1}, y(x_0) = y_0$  has a unique solution guaranteed by Theorem 1.1 if

Select the correct answer.

- (a)  $y_0 = 1$
- (b)  $y_0 = -1$
- (c)  $y_0 = 0$
- (d)  $y_0 = 2$
- (e)  $y_0 = 1/2$

13. Verify that  $4x^4 + y^2 = c$  is a one parameter family of solutions of the differential equation  $ydy/dx = -8x^3$ .

14. Determine whether Theorem 1.1 guarantees a unique solution of the initial value problem  $y = \sqrt{9 - y^2}, y(1) = -3$ .

15. A new technology is introduced into a community of 5000 individuals. If the rate at which the technology spreads through the community is jointly proportional to the number of people who use the technology and the number of individuals who do not use it, write down the differential equation for the number of people,  $x(t)$  who use the technology by time  $t$ .

16. A large mixing tank initially contains 50 gallons of water in which 3 pounds of salt have been dissolved. Another brine solution is pumped into the tank at the rate of 3 gallons per minute, and the resulting mixture is pumped out at the same rate. The concentration of the incoming brine solution is 3 pounds of salt per gallon. If  $A(t)$  represents the amount of salt in the tank at time  $t$ , the correct differential equation for  $A$  is

Select the correct answer.

- (a)  $\frac{dA}{dt} = 9 - .06A$
- (b)  $\frac{dA}{dt} = 3 - .6A$
- (c)  $\frac{dA}{dt} = 3 - .06A$
- (d)  $\frac{dA}{dt} = 9 - .6A$
- (e)  $\frac{dA}{dt} = 9 + .6A$

17. In the previous problem, over a long period of time, the total amount of salt in the tank will approach

Select the correct answer.

- (a) 10 pounds
- (b) 30 pounds
- (c) 50 pounds
- (d) 150 pounds
- (e) 200 pounds

18. In the *LRC* circuit problem in the text, *L* stands for

Select the correct answer.

- (a) capacitance
- (b) resistance
- (c) current
- (d) inductance
- (e) charge on the capacitor

19. In the *LRC* circuit problem in the text, *R* stands for

Select the correct answer.

- (a) capacitance
- (b) resistance
- (c) current
- (d) inductance
- (e) charge on the capacitor

20. In the falling body problem, the units of acceleration might be

Select the correct answer.

- (a) meters per second
- (b) feet per second
- (c) meters per second per second
- (d) kilograms per meter
- (e) kilograms per meter per second

**ANSWER KEY****Zill Differential Equations 9e Chapter 1 Form H**

---

1. third order, nonlinear
2. b
3.  $y = e^{-5x} + 4$        $y' = -5e^{-5x}$        $y'' = 25e^{-5x}$   
 $y'' + 5y' = 25e^{-5x} + 5(-5e^{-5x}) = 0$
4.  $y = x^4 - 12x^2 + 24 - \cos x + \sin x$   
 $y' = 4x^3 - 24x + \sin x + \cos x$   
 $y'' = 12x^2 - 24 + \cos x - \sin x$   
 $y'' + y = x^4$
5.  $y = e^{x^2}$        $y' = 2xe^{x^2} = 2xy$
6. d
7.  $y = cx, y(0) = 0$
8. e
9.  $y = 1/(x^2 - 1/2)$
10.  $\{(x, y) : x > 1, y > 3\}$  or 3 other regions
11. b
12. d
13.  $4x^4 + y^2 = c$        $16x^3 + 2yy' = 0$        $yy' = -8x^3$
14. No, since  $\sqrt{9 - y^2}$  is neither continuous nor differentiable there.
15.  $\frac{dx}{dt} = kx(5000 - x)$
16. a
17. d
18. d
19. b
20. c

