

# CHAPTER 2 <br> TIME VALUE OF MONEY 

## (Difficulty: $\mathbf{E}=$ Easy, $\mathbf{M}=$ Medium, and $\mathbf{T}=$ Tough)

Note: Most problems assume students have a calculator with a $y^{x}$ feature (i.e., an exponential feature). Annuity problems for finding the interest rate or the number of periods are in the financial calculator section at the end of this chapter.

## True-False

Easy:

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PV versus FV Answer: b Diff: E
1. If the discount (or interest) rate is positive, the present value of an
    expected series of payments will always exceed the future value of the
    same series.
    a. True
    b. False
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PV versus FV Answer: a Diff: E
2. Disregarding risk, if money has time value, it is impossible for the
present value of a given sum to be greater than its future value.
a. True
b. False

Amortization Answer: a Diff: E
3. The payment made each period on an amortized loan is constant, and it consists of some interest and some principal. The later we are in the loan's life, the larger the principal portion of the payment.
a. True
b. False

Effective annual rate Answer: b Diff: E
4. If a bank uses quarterly compounding for savings accounts, the nominal rate will be greater than the effective annual rate.
a. True
b. False

## Retirement and compounding

Answer: a Diff: E
5. One of the potential benefits of investing early for retirement is that an investor can receive greater benefits from the compounding of interest.
a. True
b. False

## Medium:

PV of an annuity Answer: a Diff: M
6. All other factors held constant, the present value of a given annual annuity decreases as the number of discounting periods per year increases.
a. True
b. False

PV of a sum
Answer: a Diff: M
7. The present value of a future sum decreases as either the discount rate or the number of discount periods per year increases.
a. True
b. False

Compounding
Answer: b Diff: M
8. The greater the number of compounding periods within a year, the greater the future value of a lump sum invested initially, and the greater the present value of a given lump sum to be received at maturity.
a. True
b. False

Comparative compounding
Answer: a Diff: M
9. Suppose an investor can earn a steady 5 percent annually with investment $A$, while investment $B$ will yield a constant 12 percent annually. Within 11 years' time, the compounded value of investment $B$ will be more than twice the compounded value of investment A (ignore risk).
a. True
b. False

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10. When a loan is amortized, the largest portion of the periodic payment goes to reduce principal in the early years of the loan such that the accumulated interest can be spread out over the life of the loan.
a. True
b. False

Effective and nominal rates Answer: b Diff: M 11. The effective annual rate is always greater than the nominal rate as a result of compounding effects.
a. True
b. False

Periodic and nominal rates Answer: a Diff: m
12. If we calculate a periodic interest rate, say a monthly rate, in order to get the nominal annual rate, we can multiply the periodic rate by the number of periods within a year.
a. True
b. False

Lump sum and annuity
Answer: b Diff: M
13. Since we usually assume positive interest rates in time value analyses, the present value of a three-year annuity will always be less than the future value of a single lump sum, if the annuity payment equals the original lump sum investment.
a. True
b. False

## Multiple Choice: Conceptual

## Easy:

PV and discount rate
Answer: a Diff: E
14. You have determined the profitability of a planned project by finding the present value of all the cash flows from that project. Which of the following would cause the project to look more appealing in terms of the present value of those cash flows?
a. The discount rate decreases.
b. The cash flows are extended over a longer period of time, but the total amount of the cash flows remains the same.
c. The discount rate increases.
d. Answers b and c above.
e. Answers $a$ and b above.

PV versus FV
Answer: e Diff: E
15. Which of the following statements is most correct?
a. If the discount (or interest) rate is positive, the future value of an expected series of payments will always exceed the present value of the same series.
b. To increase present consumption beyond present income normally requires either the payment of interest or else an opportunity cost of interest foregone.
c. Disregarding risk, if money has time value, it is impossible for the present value of a given sum to be greater than its future value.
d. Disregarding risk, if the present value of a sum is equal to its future value, either $r=0$ or $t=0$.
e. Each of the statements above is true.

Time value concepts
Answer: e Diff: E
16. Which of the following statements is most correct?
a. A 5-year $\$ 100$ annuity due will have a higher present value than a 5year $\$ 100$ ordinary annuity.
b. A 15-year mortgage will have larger monthly payments than a 30-year mortgage of the same amount and same interest rate.
c. If an investment pays 10 percent interest compounded annually, its effective rate will also be 10 percent.
d. Statements a and c are correct.
e. All of the statements above are correct.

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17. The future value of a lump sum at the end of five years is $\$ 1,000$. The nominal interest rate is 10 percent and interest is compounded semiannually. Which of the following statements is most correct?
a. The present value of the $\$ 1,000$ is greater if interest is compounded monthly rather than semiannually.
b. The effective annual rate is greater than 10 percent.
c. The periodic interest rate is 5 percent.
d. Both statements $b$ and $c$ are correct.
e. All of the statements above are correct.

Time value concepts
Answer: d Diff: E
18. Which of the following statements is most correct?
a. The present value of an annuity due will exceed the present value of an ordinary annuity (assuming all else equal).
b. The future value of an annuity due will exceed the future value of an ordinary annuity (assuming all else equal).
c. The nominal interest rate will always be greater than or equal to the effective annual interest rate.
d. Statements $a$ and $b$ are correct.
e. All of the statements above are correct.

Effective annual rate
Answer: d Diff: E
19. Which of the following statements is most correct?
a. If annual compounding is used, the effective annual rate equals the nominal rate.
b. If annual compounding is used, the effective annual rate equals the periodic rate.
c. If a loan has a 12 percent nominal rate with semiannual compounding, its effective annual rate is equal to 11.66 percent.
d. Answers a and b are correct.
e. Answers a and c are correct. return?
a. An account which pays 10 percent nominal interest with monthly compounding.
b. An account which pays 10 percent nominal interest with daily compounding.
c. An account which pays 10 percent nominal interest with annual compounding.
d. An account which pays 9 percent nominal interest with daily compounding.
e. All of the investments above have the same effective annual return.

Effective annual rate
Answer: d Diff: E
21. You are interested in investing your money in a bank account. Which of the following banks provides you with the highest effective rate of interest?
a. Bank 1; 8 percent with monthly compounding.
b. Bank 2; 8 percent with annual compounding.
c. Bank 3; 8 percent with quarterly compounding.
d. Bank 4; 8 percent with daily (365-day) compounding.
e. Bank 5; 7.8 percent with annual compounding.

Amortization Answer: b Diff: E
22. Your family recently obtained a 30 -year ( 360 -month) $\$ 100,000$ fixed-rate mortgage. Which of the following statements is most correct? (Ignore all taxes and transactions costs.)
a. The remaining balance after three years will be $\$ 100,000$ less the total amount of interest paid during the first 36 months.
b. The proportion of the monthly payment that goes towards repayment of principal will be higher ten years from now than it will be this year.
c. The monthly payment on the mortgage will steadily decline over time.
d. All of the statements above are correct.
e. None of the statements above is correct.

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## Amortization

Answer: e Diff: E
23. Frank Lewis has a 30 -year, $\$ 100,000$ mortgage with a nominal interest rate of 10 percent and monthly compounding. Which of the following statements regarding his mortgage is most correct?
a. The monthly payments will decline over time.
b. The proportion of the monthly payment which represents interest will be lower for the last payment than for the first payment on the loan.
c. The total dollar amount of principal being paid off each month gets larger as the loan approaches maturity.
d. Statements a and c are correct.
e. Statements b and c are correct.

## Medium:

Annuities Answer: c Diff: M
24. Suppose someone offered you the choice of two equally risky annuities, each paying $\$ 10,000$ per year for five years. One is an ordinary (or deferred) annuity, the other is an annuity due. Which of the following statements is most correct?
a. The present value of the ordinary annuity must exceed the present value of the annuity due, but the future value of an ordinary annuity may be less than the future value of the annuity due.
b. The present value of the annuity due exceeds the present value of the ordinary annuity, while the future value of the annuity due is less than the future value of the ordinary annuity.
c. The present value of the annuity due exceeds the present value of the ordinary annuity, and the future value of the annuity due also exceeds the future value of the ordinary annuity.
d. If interest rates increase, the difference between the present value of the ordinary annuity and the present value of the annuity due remains the same.
e. Answers a and d are correct.

Time value concepts
Answer: e Diff: M
25. A $\$ 10,000$ loan is to be amortized over 5 years, with annual end-of-year payments. Given the following facts, which of these statements is most correct?
a. The annual payments would be larger if the interest rate were lower.
b. If the loan were amortized over 10 years rather than 5 years, and if the interest rate were the same in either case, the first payment would include more dollars of interest under the 5-year amortization plan.
c. The last payment would have a higher proportion of interest than the first payment.
d. The proportion of interest versus principal repayment would be the same for each of the 5 payments.
e. The proportion of each payment that represents interest as opposed to repayment of principal would be higher if the interest rate were higher.

Time value concepts
Answer: e Diff: M
26. Which of the following is most correct?
a. The present value of a 5-year annuity due will exceed the present value of a 5-year ordinary annuity. (Assume that both annuities pay $\$ 100$ per period and there is no chance of default.)
b. If a loan has a nominal rate of 10 percent, then the effective rate can never be less than 10 percent.
c. If there is annual compounding, then the effective, periodic, and nominal rates of interest are all the same.
d. Answers a and c are correct.
e. All of the answers above are correct.

Time value concepts Answer: c Diff: M
27. Which of the following statements is most correct?
a. An investment which compounds interest semiannually, and has a nominal rate of 10 percent, will have an effective rate less than 10 percent.
b. The present value of a three-year $\$ 100$ annuity due is less than the present value of a three-year $\$ 100$ ordinary annuity.
c. The proportion of the payment of a fully amortized loan which goes toward interest declines over time.
d. Statements a and c are correct.
e. None of the answers above is correct.

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## Tough:

Time value concepts Answer: d Diff: T
28. Which of the following statements is most correct?
a. The first payment under a 3-year, annual payment, amortized loan for $\$ 1,000$ will include a smaller percentage (or fraction) of interest if the interest rate is 5 percent than if it is 10 percent.
b. If you are lending money, then, based on effective interest rates, you should prefer to lend at a 10 percent nominal, or quoted, rate but with semiannual payments, rather than at a 10.1 percent nominal rate with annual payments. However, as a borrower you should prefer the annual payment loan.
c. The value of a perpetuity (say for $\$ 100$ per year) will approach infinity as the interest rate used to evaluate the perpetuity approaches zero.
d. Statements a, b, and c are all true.
e. Statements b and c are true.

## Multiple Choice: Problems

## Easy:

FV of a single payment
Answer: d Diff: E
29. You deposit $\$ 2,000$ in a savings account that pays 10 percent interest, compounded annually. How much will your account be worth in 15 years?
a. \$2,030.21
b. $\$ 5,000.00$
c. $\$ 8,091.12$
d. \$8,354.50
e. \$9,020.10

FV of a single payment
Answer: c Diff: E
30. You deposit $\$ 1,000$ in a savings account that pays 9 percent interest, compounded annually. How much will your account be worth in 6 years?
a. \$1,054.00
b. $\$ 1,199.00$
c. $\$ 1,677.10$
d. $\$ 1,689.48$
e. \$7,523.33

PV of a single payment
Answer: b Diff: E
31. You can earn 8 percent interest, compounded annually. How much must you deposit today to withdraw $\$ 10,000$ in 6 years?
a. \$5,402. 69
b. $\$ 6,301.70$
c. $\$ 6,756.76$
d. $\$ 8,432.10$
e. $\$ 9,259.26$

PV of a single payment
Answer: e Diff: E
32. You can earn 15 percent interest, compounded annually. How much must you deposit today to withdraw $\$ 4,000$ in 10 years?
a. \$525.11
b. $\$ 842.51$
c. $\$ 869.57$
d. $\$ 957.57$
e. \$988. 74

## Growth rate

Answer: d Diff: E
33. In 1958 the average tuition for one year at an Ivy League school was $\$ 1,800$. Thirty years later, in 1988, the average cost was $\$ 13,700$. What was the growth rate in tuition over the 30 -year period?
a. $12 \%$
b. $9 \%$
c. $6 \%$
d. $7 \%$
e. $8 \%$

Solving for the interest rate for a single payment Answer: e Diff: E 34. Suppose you invested $\$ 1,000$ in stocks 10 years ago. If your account is now worth $\$ 2,839.42$, what rate of return did your stocks earn?
a. $15 \%$
b. $14 \%$
c. $13 \%$
d. $12 \%$
e. 11\%

Time for a sum to double
Answer: d Diff: E
35. You are currently investing your money in a bank account which has a nominal annual rate of 7.23 percent, compounded annually. How many years will it take for you to double your money?
a. 8.67 years
b. 9.15 years
c. 9.50 years
d. 9.93 years
e. 10.25 years

Solving for N for a single payment
Answer: b Diff: E
36. You are currently investing your money in a bank account which has a nominal annual rate of 8 percent, compounded annually. If you invest $\$ 2,000$ today, how many years will it take for your account to grow to \$10,000?
a. 22.91 years
b. 20.91 years
c. 18.91 years
d. 16.91 years
e. 14.91 years

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FV of a sum
Answer: b Diff: E
37. You deposited $\$ 1,000$ in a savings account that pays 8 percent interest, compounded quarterly, planning to use it to finish your last year in college. Eighteen months later, you decide to go to the Rocky Mountains to become a ski instructor rather than continue in school, so you close out your account. How much money will you receive?
a. \$1,171
b. $\$ 1,126$
c. $\$ 1,082$
d. $\$ 1,163$
e. $\$ 1,008$

FV of an annuity
Answer: e Diff: E
38. What is the future value of a 5-year ordinary annuity with annual payments of $\$ 200$, evaluated at a 15 percent interest rate?
a. \$ 670.44
b. \$ 842.91
c. $\$ 1,169.56$
d. $\$ 1,522.64$
e. \$1,348.48

PV of an annuity
Answer: a Diff: E
39. What is the present value of a 5-year ordinary annuity with annual payments of $\$ 200$, evaluated at a 15 percent interest rate?
a. \$ 670.43
b. \$ 842.91
c. $\$ 1,169.56$
d. $\$ 1,348.48$
e. \$1,522.64

PV of a perpetuity
Answer: c Diff: E
40. You have the opportunity to buy a perpetuity which pays $\$ 1,000$ annually. Your required rate of return on this investment is 15 percent. You should be essentially indifferent to buying or not buying the investment if it were offered at a price of
a. $\$ 5,000.00$
b. $\$ 6,000.00$
c. $\$ 6,666.67$
d. $\$ 7,500.00$
e. $\$ 8,728.50$

PV of an uneven CF stream
Answer: c Diff: E
41. Assume that you will receive $\$ 2,000$ a year in Years 1 through 5, $\$ 3,000$ a year in Years 6 through 8, and $\$ 4,000$ in Year 9, with all cash flows to be received at the end of the year. If you require a 14 percent rate of return, what is the present value of these cash flows?
a. \$ 9,851
b. $\$ 13,250$
c. \$11,714
d. $\$ 15,129$
e. $\$ 17,353$

Required annuity payments Answer: b Diff: E
42. If a 5-year ordinary annuity has a present value of $\$ 1,000$, and if the interest rate is 10 percent, what is the amount of each annuity payment?
a. $\$ 240.42$
b. $\$ 263.80$
c. $\$ 300.20$
d. $\$ 315.38$
e. \$346.87

Quarterly compounding
Answer: a Diff: E
43. If $\$ 100$ is placed in an account that earns a nominal 4 percent, compounded quarterly, what will it be worth in 5 years?
a. $\$ 122.02$
b. $\$ 105.10$
c. \$135.41
d. $\$ 120.90$
e. \$117.48

Effect of inflation
Answer: c Diff: E
44. At an inflation rate of 9 percent, the purchasing power of $\$ 1$ would be cut in half in 8.04 years. How long to the nearest year would it take the purchasing power of $\$ 1$ to be cut in half if the inflation rate were only 4 percent?
a. 12 years
b. 15 years
c. 18 years
d. 20 years
e. 23 years

## Effective annual rate

Answer: c Diff: E
45. Gomez Electronics needs to arrange financing for its expansion program. Bank A offers to lend Gomez the required funds on a loan where interest must be paid monthly, and the quoted rate is 8 percent. Bank B will charge 9 percent, with interest due at the end of the year. What is the difference in the effective annual rates charged by the two banks?
a. $0.25 \%$
b. $0.50 \%$
c. $0.70 \%$
d. $1.00 \%$
e. $1.25 \%$

Effective annual rate
Answer: b Diff: E
46. You recently received a letter from Cut-to-the-Chase National Bank that offers you a new credit card that has no annual fee. It states that the annual percentage rate (APR) is 18 percent on outstanding balances. What is the effective annual interest rate? (Hint: Remember these companies bill you monthly.)
a. $18.81 \%$
b. $19.56 \%$
c. $19.25 \%$
d. $20.00 \%$
e. $18.00 \%$

## Effective annual return

Answer: b Diff: E
47. Which of the following investments has the highest effective return (EAR)? (Assume that all CDs are of equal risk.)
a. A bank CD which pays 10 percent interest quarterly.
b. A bank CD which pays 10 percent monthly.
c. A bank CD which pays 10.2 percent annually.
d. A bank CD which pays 10 percent semiannually.
e. A bank CD which pays 9.6 percent daily (on a 365-day basis).

Effective annual return
Answer: a Diff: E
48. Which one of the following investments provides the highest effective return?
a. An investment which has a 9.9 percent nominal rate and quarterly annual compounding.
b. An investment which has a 9.7 percent nominal rate and daily (365) compounding.
c. An investment which has a 10.2 percent nominal rate and annual compounding.
d. An investment which has a 10 percent nominal rate and semiannual com-pounding.
e. An investment which has a 9.6 percent nominal rate and monthly compounding.
49. Which of the following investments would provide an investor the highest effective annual return?
a. An investment which has a 9 percent nominal rate with semiannual com-pounding.
b. An investment which has a 9 percent nominal rate with quarterly compounding.
c. An investment which has a 9.2 percent nominal rate with annual compounding.
d. An investment which has an 8.9 percent nominal rate with monthly com-pounding.
e. An investment which has an 8.9 percent nominal rate with quarterly compounding.

PV of an uneven CF stream
Answer: d Diff: E
50. You are given the following cash flows. What is the present value (t = 0) if the discount rate is 8 percent, rounded to a whole dollar?

| 0 | $8 \%$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | \| | \| | 4 |  |
| 0 | 1,000 | 2,000 | 3,000 | 4,000 |  |

a. $\$ 2,500$
b. $\$ 4,804$
c. $\$ 5,302$
d. $\$ 7,963$
e. \$10,000

Interest payment on an amortized loan Answer: b Diff: E
51. You borrow $\$ 100,000$. You make annual payments (at the end of the year)for 5 years. The interest rate is $10 \%$. How much interest do you pay in year 2?
a. $\$ 7,362$
b. $\$ 8,362$
c. \$10,000
d. \$16,380
e. $\$ 26,380$

Effective annual rate
Answer: c Diff: E
52. Suppose you pay $15 \%$ as a nominal annual rate on your credit card. If you make monthly payments with monthly compounding, what is your effective annual rate?
a. 1.25\%
b. $15.00 \%$
c. $16.08 \%$
d. $18.80 \%$
e. $19.24 \%$

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## Medium:

FV of an annuity
Answer: b Diff: M
53. Assume you are to receive a 20 -year annuity with annual payments of $\$ 50$. The first payment will be received at the end of Year 1 , and the last payment will be received at the end of Year 20. You will invest each payment in an account that pays 10 percent. What will be the value in your account at the end of Year 30 ?
a. $\$ 6,354.81$
b. \$7,427.83
c. $\$ 7,922.33$
d. \$8,591.00
e. $\$ 6,752.46$

FV of annuity due Answer: b Diff: M
54. Your uncle has agreed to deposit $\$ 3,000$ in your brokerage account at the beginning of each of the next five years ( $t=0, t=1$, $t=2$, $t=$ 3 and $t=4)$. You estimate that you can earn 9 percent a year on your investments. How much will you have in your account four years from now (at $t=4$ )? (Assume that no money is withdrawn from the account until
$\mathrm{t}=4$.
a. $\$ 13,719.39$
b. $\$ 17,954.13$
c. $\$ 19,570.00$
d. $\$ 21,430.45$
e. $\$ 22,436.12$

FV under monthly compounding
Answer: e Diff: M
55. You just put $\$ 1,000$ in a bank account which pays 6 percent nominal annual interest, compounded monthly. How much will you have in your account after 3 years?
a. \$1,006.00
b. $\$ 1,056.45$
c. \$1,180.32
d. \$1,191.00
e. \$1,196.68

FV of lump sum and annuity
Answer: e Diff: M
56. You are interested in saving money for your first house. Your plan is to make regular deposits into a brokerage account which will earn 14 percent. Your first deposit of $\$ 5,000$ will be made today. You also plan to make four additional deposits at the beginning of each of the next four years. Your plan is to increase your deposits by 10 percent a year. (That is, you plan to deposit $\$ 5,500$ at $t=1$, and $\$ 6,050$ at $t=2$, etc.) How much money will be in your account after five years?
a. $\$ 24,697.40$
b. $\$ 30,525.00$
c. $\$ 32,485.98$
d. $\$ 39,362.57$
e. $\$ 44,873.90$

PV of an uneven CF stream
Answer: a Diff: M
57. You are given the following cash flows. What is the present value ( $t=0$ ) if the discount rate is 12 percent?

a. $\$ 3,277$
b. \$4,804
c. $\$ 5,302$
d. $\$ 4,289$
e. $\$ 2,804$

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PV of uncertain cash flows
Answer: e Diff: M
58. A project with a 3-year life has the following probability distributions for possible end-of-year cash flows in each of the next three years:

| Year 1 |  | Year 2 |  | Year 3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Prob | Cash Flow | Prob | Cash Flow | Prob | Cash Flow |
| 0.30 | \$300 | 0.15 | \$100 | 0.25 | \$200 |
| 0.40 | 500 | 0.35 | 200 | 0.75 | 800 |
| 0.30 | 700 | 0.35 | 600 |  |  |
|  |  | 0.15 | 900 |  |  |

Using an interest rate of 8 percent, find the expected present value of these uncertain cash flows. (Hint: Find the expected cash flow in each year, then evaluate those cash flows.)
a. \$1,204.95
b. \$ 835.42
c. $\$ 1,519.21$
d. \$1,580.00
e. \$1,347.61

FV of an uneven CF stream
Answer: d Diff: M
59. You just graduated, and you plan to work for 10 years and then to leave for the Australian "Outback" bush country. You figure you can save $\$ 1,000$ a year for the first 5 years and $\$ 2,000$ a year for the next 5 years. These savings cash flows will start one year from now. In addition, your family has just given you a $\$ 5,000$ graduation gift. If you put the gift now, and your future savings when they start, into an account which pays 8 percent compounded annually, what will your financial "stake" be when you leave for Australia 10 years from now?
a. $\$ 21,432$
b. $\$ 28,393$
c. $\$ 16,651$
d. $\$ 31,148$
e. $\$ 20,000$

| $\frac{\text { Year }}{0}$ |  | $\frac{\text { Cash Flow }}{-\$ 300.00}$ |
| :---: | :---: | :---: |
| 1 |  | 100.00 |
| 2 |  | 125.43 |
| 3 |  | 90.12 |
| 4 |  | $?$ |

What cash flow will the project have to generate in the fourth year in order for the project to have a $15 \%$ rate of return?
a. \$ 15.55
b. \$ 58.95
c. \$100.25
d. \$103.10
e. $\$ 150.75$

Value of missing cash flow Answer: c Diff: M
61. John Keene recently invested $\$ 2,566.70$ in a project that is promising to return 12 percent per year. The cash flows are expected to be as follows:

| End of <br> Year | Cash <br> Flow |
| :---: | ---: |
| 1 | $\$ 325$ |
| 2 | 400 |
| 3 | 550 |
| 4 | $?$ |
| 5 | 750 |
| 6 | 800 |

What is the cash flow at the end of the 4 th year?
a. $\$ 1,187$
b. \$ 600
c. $\$ 1,157$
d. \$ 655
e. \$1,267

## Amortization

Answer: c Diff: M
62. If you buy a factory for $\$ 250,000$ and the terms are 20 percent down, the balance to be paid off over 30 years at a 12 percent rate of interest on the unpaid balance, what are the 30 equal annual payments?
a. $\$ 20,593$
b. \$31,036
c. $\$ 24,829$
d. $\$ 50,212$
e. \$ 6,667

## Effective annual rate

Answer: d Diff: M 63. Steaks Galore needs to arrange financing for its expansion program. One bank offers to lend the required $\$ 1,000,000$ on a loan which requires interest to be paid at the end of each quarter. The quoted rate is 10 percent, and the principal must be repaid at the end of the year. A second lender offers 9 percent, daily compounding (365-day year), with interest and principal due at the end of the year. What is the difference in the effective annual rates (EFF\%) charged by the two banks?
a. $0.31 \%$
b. $0.53 \%$
c. $0.75 \%$
d. $0.96 \%$
e. $1.25 \%$

Value of a perpetuity
Answer: c Diff: M
64. You are willing to pay $\$ 15,625$ to purchase a perpetuity which will pay you and your heirs $\$ 1,250$ each year, forever. If your required rate of return does not change, how much would you be willing to pay if this were a 20-year, annual payment, ordinary annuity instead of a perpetuity?
a. $\$ 10,342$
b. \$11,931
c. $\$ 12,273$
d. \$13,922
e. \$17,157

## Tough:

PV of an uneven CF stream Answer: C Diff: T
65. Find the present value of an income stream which has a negative flow of $\$ 100$ per year for 3 years, a positive flow of $\$ 200$ in the 4 th year, and a positive flow of $\$ 300$ per year in Years 5 through 8 . The appropriate discount rate is 4 percent for each of the first 3 years and 5 percent for each of the later years. Thus, a cash flow accruing in Year 8 should be discounted at 5 percent for some years and 4 percent in other years. All payments occur at year-end.
a. \$ 528.21
b. \$1,329.00
c. \$ 792.49
d. $\$ 1,046.41$
e. \$ 875.18

## Required annuity payments

Answer: b Diff: T
66. You are saving for the college education of your two children. One child will enter college in 5 years, while the other child will enter college in 7 years. College costs are currently $\$ 10,000$ per year and are expected to grow at a rate of 5 percent per year. All college costs are paid at the beginning of the year. You assume that each child will be in college for four years.

You currently have $\$ 50,000$ in your educational fund. Your plan is to contribute a fixed amount to the fund over each of the next 5 years. Your first contribution will come at the end of this year, and your final contribution will come at the date at which you make the first tuition payment for your oldest child. You expect to invest your contributions into various investments which are expected to earn 8 percent per year. How much should you contribute each year in order to meet the expected cost of your children's education?
a. $\$ 2,894$
b. $\$ 3,712$
c. $\$ 4,125$
d. $\$ 5,343$
e. $\$ 6,750$

## Required annuity payments

Answer: b Diff: T
67. A young couple is planning for the education of their two children. They plan to invest the same amount of money at the end of each of the next 16 years, i.e., the first contribution will be made at the end of the year and the final contribution will be made at the time the oldest child enters college.

The money will be invested in securities that are certain to earn a return of 8 percent each year. The oldest child will begin college in 16 years and the second child will begin college in 18 years. The parents anticipate college costs of $\$ 25,000$ a year (per child). These costs must be paid at the end of each year. If each child takes four years to complete their college degrees, then how much money must the couple save each year?
a. \$ 9,612.10
b. \$ 5,071.63
c. $\$ 12,507.29$
d. \$ 5,329.45
e. \$ 4,944.84

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## Required annuity payments

Answer: a Diff: T
68. Today is Rachel's 30th birthday. Five years ago, Rachel opened a brokerage account when her grandmother gave her $\$ 25,000$ for her 25 th birthday. Rachel added $\$ 2,000$ to this account on her 26 th birthday, $\$ 3,000$ on her 27 th birthday, $\$ 4,000$ on her $28 t h$ birthday, and $\$ 5,000$ on her 29th birthday. Rachel's goal is to have $\$ 400,000$ in the account by her 40th birthday.

Starting today, she plans to contribute a fixed amount to the account each year on her birthday. She will make 11 contributions, the first one will occur today, and the final contribution will occur on her 40th birthday. Complicating things somewhat is the fact that Rachel plans to withdraw $\$ 20,000$ from the account on her 35 th birthday to finance the down payment on a home. How large does each of these 11 contributions have to be for Rachel to reach her goal? Assume that the account has earned (and will continue to earn) an effective return of 12 percent a year.
a. $\$ 11,743.95$
b. \$10,037.46
c. $\$ 11,950.22$
d. \$14,783.64
e. $\$ 9,485.67$

## Amortization

Answer: b Diff: $\mathbf{T}$
69. The Florida Boosters Association has decided to build new bleachers for the football field. Total costs are estimated to be $\$ 1$ million, and financing will be through a bond issue of the same amount. The bond will have a maturity of 20 years, a coupon rate of 8 percent, and has annual payments. In addition, the Association must set up a reserve to pay off the loan by making 20 equal annual payments into an account which pays 8 percent, annual compounding. The interest-accumulated amount in the reserve will be used to retire the entire issue at its maturity 20 years hence. The Association plans to meet the payment requirements by selling season tickets at a $\$ 10$ net profit per ticket. How many tickets must be sold each year to service the debt, i.e., to meet the interest and principal repayment requirements?
a. 5,372
b. 10,186
c. 15,000
d. 20,459
e. 25,000

## Financial Calculator Section

## Multiple Choice: Problems

## Easy:

## Solving for $N$ for an annuity

Answer: b Diff: E
70. You are currently investing your money in a bank account which has a nominal annual rate of 9 percent, compounded monthly. If you invest $\$ 900$ at the end of each month, how many months will it take for your account to grow to $\$ 301,066.27$ (rounded to the nearest month)?
a. 40 months
b. 168 months
c. 175 months
d. 221 months
e. 335 months

Nominal and effective rates Answer: b Diff: E
71. An investment pays you 9 percent interest compounded semiannually. A second investment of equal risk, pays interest compounded quarterly. What nominal rate of interest would you have to receive on the second investment in order to make you indifferent between the two investments?
a. $8.71 \%$
b. $8.90 \%$
c. $9.00 \%$
d. $9.20 \%$
e. $9.31 \%$

Effective annual rate
Answer: c Diff: E
72. You want to borrow $\$ 1,000$ from a friend for one year, and you propose to pay her $\$ 1,120$ at the end of the year. She agrees to lend you the $\$ 1,000$, but she wants you to pay her $\$ 10$ of interest at the end of each of the first 11 months plus $\$ 1,010$ at the end of the 12 th month. How much higher is the effective annual rate under your friend's proposal than under your proposal?
a. $0.00 \%$
b. $0.45 \%$
c. $0.68 \%$
d. $0.89 \%$
e. $1.00 \%$

Time for a sum to double
Answer: d Diff: E
73. You are currently investing your money in a bank account which has a nominal annual rate of 7 percent, compounded monthly. How many years will it take for you to double your money?
a. 8.67 years
b. 9.15 years
c. 9.50 years
d. 9.93 years
e. 10.25 years

Monthly payments on loan
Answer: c Diff: E
74. You are considering buying a new car. The sticker price is $\$ 15,000$ and you have $\$ 2,000$ to put toward a down payment. If you can negotiate a nominal annual interest rate of 10 percent and you wish to pay for the car over a 5-year period, what are your monthly car payments?
a. $\$ 216.67$
b. $\$ 252.34$
c. $\$ 276.21$
d. $\$ 285.78$
e. \$318.71

Interest rate of an annuity Answer: b Diff: E
75. South Penn Trucking is financing a new truck with a loan of $\$ 10,000$ to be repaid in 5 annual end-of-year installments of $\$ 2,504.56$. What annual interest rate is the company paying?
a. $7 \%$
b. $8 \%$
c. $9 \%$
d. $10 \%$
e. $11 \%$

## Medium:

Effective annual rate Answer: b Diff: M
76. If it were evaluated with an interest rate of 0 percent, a 10-year regular annuity would have a present value of $\$ 3,755.50$. If the future (compounded) value of this annuity, evaluated at Year 10, is $\$ 5,440.22$, what effective annual interest rate must the analyst be using to find the future value?
a. $7 \%$
b. $8 \%$
c. $9 \%$
d. $10 \%$
e. 11\%

## Number of periods for an annuity

Answer: a Diff: M
77. Your subscription to Jogger's World Monthly is about to run out and you have the choice of renewing it by sending in the $\$ 10$ a year regular rate or of getting a lifetime subscription to the magazine by paying $\$ 100$. Your cost of capital is 7 percent. How many years would you have to live to make the lifetime subscription the better buy? Payments for the regular subscription are made at the beginning of each year. (Round up if necessary to obtain a whole number of years.)
a. 15 years
b. 10 years
c. 18 years
d. 7 years
e. 8 years

FV of a sum Answer: d Diff: M
78. Suppose you put $\$ 100$ into a savings account today, the account pays a nominal annual interest rate of 6 percent, but compounded semiannually, and you withdraw $\$ 100$ after 6 months. What would your ending balance be 20 years after the initial $\$ 100$ deposit was made?
a. $\$ 226.20$
b. \$115.35
c. \$ 62.91
d. $\$ 9.50$
e. \$ 3.00

FV of annuity due
Answer: d Diff: M
79. You are contributing money to an investment account so that you can purchase a house in five years. You plan to contribute six payments of $\$ 3,000$ a year--the first payment will be made today ( $t=0$ ), and the final payment will be made five years from now ( $t=5$ ). If you earn 11 percent in your investment account, how much money will you have in the account five years from now (at $t=5$ )?
a. $\$ 19,412$
b. $\$ 20,856$
c. $\$ 21,683$
d. $\$ 23,739$
e. $\$ 26,350$
80. Terry Austin is 30 years old and is saving for her retirement. She is planning on making 36 contributions to her retirement account at the beginning of each of the next 36 years. The first contribution will be made today $(t=0)$ and the final contribution will be made 35 years from today ( $t=35$ ). The retirement account will earn a return of 10 percent a year. If each contribution she makes is $\$ 3,000$, how much will be in the retirement account 35 years from now ( $t=35$ )?
a. $\$ 894,380$
b. $\$ 813,073$
c. $\$ 897,380$
d. $\$ 987,118$
e. $\$ 978,688$

Effective annual rate Answer: a Diff: M
81. You have just borrowed $\$ 20,000$ to buy a new car. The loan agreement calls for 60 monthly payments of $\$ 444.89$ each to begin one month from today. If the interest is compounded monthly, then what is the effective annual rate on this loan?
a. $12.68 \%$
b. $14.12 \%$
c. $12.00 \%$
d. $13.25 \%$
e. $15.08 \%$

Effective annual rate
Answer: e Diff: M
82. You have just taken out a 10 -year, $\$ 12,000$ loan to purchase a new car. This loan is to be repaid in 120 equal end-of-month installments. If each of the monthly installments is $\$ 150$, what is the effective annual interest rate on this car loan?
a. $6.5431 \%$
b. $7.8942 \%$
c. $8.6892 \%$
d. $8.8869 \%$
e. $9.0438 \%$

## Required annuity payments

Answer: c Diff: M
83. A baseball player is offered a 5-year contract which pays him the following amounts:

$$
\text { Year 1: } \$ 1.2 \text { million }
$$

Year 2: 1.6 million
Year 3: 2.0 million
Year 4: 2.4 million
Year 5: 2.8 million

Under the terms of the agreement all payments are made at the end of each year.

Instead of accepting the contract, the baseball player asks his agent to negotiate a contract which has a present value of $\$ 1$ million more than that which has been offered. Moreover, the player wants to receive his payments in the form of a 5-year annuity due. All cash flows are discounted at 10 percent. If the team were to agree to the player's terms, what would be the player's annual salary (in millions of dollars)?
a. $\$ 1.500$
b. \$1. 659
c. $\$ 1.989$
d. $\$ 2.343$
e. $\$ 2.500$

NPV and non-annual discounting
Answer: b Diff: M
84. Your lease calls for payments of $\$ 500$ at the end of each month for the next 12 months. Now your landlord offers you a new 1-year lease which calls for zero rent for 3 months, then rental payments of $\$ 700$ at the end of each month for the next 9 months. You keep your money in a bank time deposit that pays a nominal annual rate of 5 percent. By what amount would your net worth change if you accept the new lease? (Hint: Your return per month is $5 \% / 12=0.4166667 \%$.)
a. $-\$ 509.81$
b. $-\$ 253.62$
c. $+\$ 125.30$
d. $+\$ 253.62$
e. $+\$ 509.81$

FV under monthly compounding
Answer: d Diff: M
85. Steven just deposited $\$ 10,000$ in a bank account which has a 12 percent nominal interest rate, and the interest is compounded monthly. Steven also plans to contribute another $\$ 10,000$ to the account one year (12 months) from now and another $\$ 20,000$ to the account two years from now. How much will be in the account three years (36 months) from now?
a. $\$ 57,231$
b. $\$ 48,993$
c. $\$ 50,971$
d. $\$ 49,542$
e. $\$ 49,130$

FV under daily compounding Answer: a Diff: M
86. You have $\$ 2,000$ invested in a bank account that pays a 4 percent nominal annual interest with daily compounding. How much money will you have in the account at the end of July (i.e., in 132 days)? (Assume there are 365 days in each year.)
a. \$2,029.14
b. \$2,028.93
c. $\$ 2,040.00$
d. $\$ 2,023.44$
e. $\$ 2,023.99$

FV under non-annual compounding
Answer: d Diff: M
87. Josh and John (2 brothers) are each trying to save enough money to buy their own cars. Josh is planning to save $\$ 100$ from every paycheck (he is paid every 2 weeks.) John plans to put aside $\$ 150$ each month but has already saved $\$ 1,500$. Interest rates are currently quoted at 10 percent. Josh's bank compounds interest every two weeks while John's bank compounds interest monthly. At the end of 2 years they will each spend all their savings on a car (each brother buys a car). What is the price of the most expensive car purchased?
a. \$5,744.29
b. $\$ 5,807.48$
c. $\$ 5,703.02$
d. $\$ 5,797.63$
e. None of the answers above is correct.

Answer: c Diff: M
88. An investment pays $\$ 100$ every six months (semiannually) over the next 2.5 years. Interest, however, is compounded quarterly, at a nominal rate of 8 percent. What is the future value of the investment after 2.5 years?
a. $\$ 520.61$
b. \$541. 63
c. $\$ 542.07$
d. $\$ 543.98$
e. \$547.49

PV under monthly compounding
Answer: b Diff: M
89. You have just bought a security which pays $\$ 500$ every six months. The security lasts for ten years. Another security of equal risk also has a maturity of ten years, and pays 10 percent compounded monthly (that is, the nominal rate is 10 percent). What should be the price of the security that you just purchased?
a. \$6,108.46
b. $\$ 6,175.82$
c. $\$ 6,231.11$
d. $\$ 6,566.21$
e. $\$ 7,314.86$

PV under non-annual compounding Answer: c Diff: M
90. You have been offered an investment that pays $\$ 500$ at the end of every 6 months for the next 3 years. The nominal interest rate is 12 percent; however, interest is compounded quarterly. What is the present value of the investment?
a. \$2,458. 66
b. \$2,444.67
c. $\$ 2,451.73$
d. $\$ 2,463.33$
e. \$2,437.56

Value of missing payments Answer: d Diff: M
91. You recently purchased a 20 -year investment which pays you $\$ 100$ at $t=1, \$ 500$ at $t=2, \$ 750$ at $t=3$, and some fixed cash flow, $X$, at the end of each of the remaining 17 years. The investment cost you $\$ 5,544.87$. Alternative investments of equal risk have a required return of 9 percent. What is the annual cash flow received at the end of each of the final 17 years, that is, what is X?
a. $\$ 600$
b. \$625
c. \$650
d. $\$ 675$
e. \$700

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92. A ten-year security generates cash flows of $\$ 2,000$ a year at the end of each of the next three years ( $t=1,2,3$ ). After three years, the security pays some constant cash flow at the end of each of the next six years. ( $t=4,5,6,7,8,9)$. Ten years from now ( $t=10$ ) the security will mature and pay $\$ 10,000$. The security sells for $\$ 24,307.85$, and has a yield to maturity of 7.3 percent. What annual cash flow does the security pay for years 4 through 9?
a. \$2,995
b. $\$ 3,568$
c. $\$ 3,700$
d. $\$ 3,970$
e. $\$ 4,296$

Value of missing payments
Answer: d Diff: M
93. An investment costs $\$ 3,000$ today and provides cash flows at the end of each year for 20 years. The investment's expected return is 10 percent. The projected cash flows for years 1, 2, and 3 are $\$ 100, \$ 200$, and $\$ 300$ respectively. What is the annual cash flow received for each of the years 4 through 20 ( 17 years)? (Assume the same payment for each of these years.)
a. $\$ 285.41$
b. $\$ 313.96$
c. $\$ 379.89$
d. $\$ 417.87$
e. $\$ 459.66$

## Amortization

Answer: d Diff: M
94. You have just bought a house and have a $\$ 125,000$, 25-year mortgage with a fixed interest rate of 8.5 percent with monthly payments. Over the next five years, what percentage of your mortgage payments will go toward the repayment of principal?
a. $8.50 \%$
b. $10.67 \%$
c. $12.88 \%$
d. $14.93 \%$
e. $17.55 \%$

## Amortization

Answer: a Diff: M
95. You have just taken out an installment loan for $\$ 100,000$. Assume that the loan will be repaid in 12 equal monthly installments of $\$ 9,456$ and that the first payment will be due one month from today. How much of your third monthly payment will go toward the repayment of principal?
a. \$7,757.16
b. $\$ 6,359.12$
c. $\$ 7,212.50$
d. $\$ 7,925.88$
e. $\$ 8,333.33$

## Amortization

Answer: c Diff: M
96. A homeowner just obtained a $\$ 90,000$ mortgage. The mortgage is for 30 years (360 months) and has a fixed nominal annual rate of 9 percent, with monthly payments. What percentage of the total payments made the first two years will go toward repayment of interest?
a. $89.30 \%$
b. $91.70 \%$
c. $92.59 \%$
d. $93.65 \%$
e. $94.76 \%$

Amortization
Answer: b Diff: M
97. Robert recently borrowed $\$ 20,000$ to purchase a new car. The car loan is fully amortized over 4 years. In other words, the loan has a fixed monthly payment, and the balance on the loan will be zero after the final monthly payment is made. The loan has a nominal interest rate of 12 percent with monthly compounding. Looking ahead, Robert thinks there is a chance that he will want to pay off the loan early, after 3 years ( 36 months). What will the remaining balance be on the loan after he makes the $36^{\text {th }}$ payment?
a. \$7,915.56
b. $\$ 5,927.79$
c. \$4,746.44
d. \$4,003.85
e. \$5,541.01

Amortization
Answer: c Diff: M 98. Jerry and Faith Hudson recently obtained a 30-year (360-month), $\$ 250,000$ mortgage with a 9 percent nominal interest rate. What will be the remaining balance on the mortgage after five years (60 months)?
a. \$239,024
b. $\$ 249,307$
c. $\$ 239,700$
d. \$237,056
e. $\$ 212,386$

Remaining balance on mortgage
Answer: c Diff: M 99. You have a $\$ 175,000$, 30 -year mortgage with a 9 percent nominal rate. You make payments every month. What will be the remaining balance on your mortgage after 5 years?
a. $\$ 90,514.62$
b. $\$ 153,680.43$
c. $\$ 167,790.15$
d. \$173,804.41
e. \$174,514.83

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Answer: d Diff: M
100. You just bought a house and have a $\$ 150,000$ mortgage. The mortgage is for 30 years and has a nominal rate of 8 percent (compounded monthly). After 36 payments (3 years) what will be the remaining balance on your mortgage?
a. $\$ 110,376.71$
b. $\$ 124,565.82$
c. $\$ 144,953.86$
d. $\$ 145,920.12$
e. $\$ 148,746.95$

Remaining balance on mortgage
Answer: d Diff: M 101. Your family purchased a house three years ago. When you bought the house you financed it with a $\$ 160,000$ mortgage with an 8.5 percent nominal interest rate (compounded monthly). The mortgage was for 15 years (180 months). What is the remaining balance on your mortgage today?
a. $\$ 95,649$
b. \$103,300
c. \$125,745
d. $\$ 141,937$
e. $\$ 159,998$

## Tough:

Required annuity payments Answer: C Diff: T
102. Your father, who is 60, plans to retire in 2 years, and he expects to live independently for 3 years. He wants a retirement income which has, in the first year, the same purchasing power as $\$ 40,000$ has today. However, his retirement income will be of a fixed amount, so his real income will decline over time. His retirement income will start the day he retires, 2 years from today, and he will receive a total of 3 retirement payments.

Inflation is expected to be constant at 5 percent. Your father has $\$ 100,000$ in savings now, and he can earn 8 percent on savings now and in the future. How much must he save each year, starting today, to meet his retirement goals?
a. $\$ 1,863$
b. $\$ 2,034$
c. $\$ 2,716$
d. $\$ 5,350$
e. $\$ 6,102$

## Required annuity payments

Answer: d Diff: T
103. Your father, who is 60 , plans to retire in 2 years, and he expects to live independently for 3 years. Suppose your father wants to have a real income of $\$ 40,000$ in today's dollars in each year after he retires. His retirement income will start the day he retires, 2 years from today, and he will receive a total of 3 retirement payments.

Inflation is expected to be constant at 5 percent. Your father has $\$ 100,000$ in savings now, and he can earn 8 percent on savings now and in the future. How much must he save each year, starting today, to meet his retirement goals?
a. $\$ 1,863$
b. \$2,034
c. \$2,716
d. $\$ 5,350$
e. $\$ 6,102$

Required annuity payments
Answer: a Diff: $T$
104. Your client just turned 75 years old and plans on retiring in 10 years on her 85th birthday. She is saving money today for her retirement and is establishing a retirement account with your office. She would like to withdraw money from her retirement account on her birthday each year until she dies. She would ideally like to withdraw $\$ 50,000$ on her 85 th birthday, and increase her withdrawals 10 percent a year through her 89th birthday (i.e., she would like to withdraw $\$ 73,205$ on her $89 t h$ birthday). She plans to die on her 90 th birthday, at which time she would like to leave $\$ 200,000$ to her descendants. Your client currently has $\$ 100,000$. You estimate that the money in the retirement account will earn 8 percent a year over the next 15 years.

Your client plans to contribute an equal amount of money each year until her retirement. Her first contribution will come in 1 year; her 10th and final contribution will come in 10 years (on her 85th birthday). How much should she contribute each year to meet her objectives?
a. $\$ 12,401.59$
b. $\$ 12,998.63$
c. $\$ 13,243.18$
d. \$13,759.44
e. \$14,021.53

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105. You are considering an investment in a 40 -year security. The security will pay $\$ 25$ a year at the end of each of the first three years. The security will then pay $\$ 30$ a year at the end of each of the next 20 years. The nominal interest rate is assumed to be 8 percent, and the current price (present value) of the security is $\$ 360.39$. Given this information, what is the equal annual payment to be received from Year 24 through Year 40 (i.e., for 17 years)?
a. \$35
b. \$38
c. $\$ 40$
d. $\$ 45$
e. \$50

## Required annuity payments Answer: a Diff: T

106. John and Jessica are saving for their child's education. Their daughter is currently eight years old and will be entering college 10 years from now ( $t=10$ ). College costs are currently $\$ 15,000$ a year and are expected to increase at a rate of 5 percent a year. They expect their daughter to graduate in four years, and that all annual payments will be due at the beginning of each year ( $t=10,11,12$, and 13).

Right now, John and Jessica have $\$ 5,000$ in their college savings account. Starting today, they plan to contribute $\$ 3,000$ a year at the beginning of each of the next five years ( $t=0,1,2,3$, and 4). Then their plan is to make six equal annual contributions at the end of each of the following six years ( $t=5,6,7,8,9$, and 10). Their investment account is expected to have an annual return of 12 percent. How large of an annual payment do they have to make in the subsequent six
years (t $=5,6,7,8,9$, and 10$)$ in order to meet their child's anticipated college costs?
a. \$4,411
b. $\$ 7,643$
c. $\$ 2,925$
d. $\$ 8,015$
e. $\$ 6,798$

Required annuity payments
Answer: c Diff: T
107. John is saving for his retirement. Today is his 40 th birthday. John first started saving when he was 25 years old. On his 25 th birthday, John made the first contribution to his retirement account; he deposited $\$ 2,000$ into an account which paid 9 percent interest, compounded monthly. Each year on his birthday, John contributes another $\$ 2,000$ to the account. The 15 th (and last) contribution was made last year on his 39th birthday.

John wants to close the account today and move the money to a stock fund which is expected to earn an effective return of 12 percent a year. John's plan is to continue making contributions to this new account each year on his birthday. His next contribution will come today (age 40) and his final planned contribution will be on his 65th birthday. If John wants to accumulate $\$ 3,000,000$ in his account by age 65, how much must he contribute each year until age 65 (26 contributions in all) to achieve his goal?
a. $\$ 11,892$
b. \$13,214
c. $\$ 12,471$
d. $\$ 10,388$
e. $\$ 15,572$

## Required annuity payments

Answer: a Diff: $T$
108. Joe and Jane are interested in saving money to put their two children, John and Susy through college. John is currently 12 years old and will enter college in six years. Susy is 10 years old and will enter college in 8 years. Both children plan to finish college in four years.

College costs are currently $\$ 15,000$ a year (per child), and are expected to increase at 5 percent a year for the foreseeable future. All college costs are paid at the beginning of the school year. Up until now, Joe and Jane have saved nothing but they expect to receive $\$ 25,000$ from a favorite uncle in three years.

To provide for the additional funds that are needed, they expect to make 12 equal payments at the beginning of each of the next twelve years--the first payment will be made today and the final payment will be made on Susy's 21st birthday (which is also the day that the last payment must be made to the college). If all funds are invested in a stock fund which is expected to earn 12 percent, how large should each of the annual contributions be?
a. $\$ 7,475.60$
b. $\$ 7,798.76$
c. \$ 8,372.67
d. $\$ 9,675.98$
e. $\$ 14,731.90$

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## Required annuity payments

Answer: b Diff: T
109. John and Barbara Roberts are starting to save for their daughter's college education.

- Assume that today's date is September 1.
- College costs are currently $\$ 10,000$ a year and are expected to increase at a rate equal to 6 percent per year for the foreseeable future. All college payments are due at the beginning of the year. (So for example, college will cost $\$ 10,000$ if you start now, and $\$ 10,600$ if you start next September 1).
- Their daughter will enter college 15 years from now. She will be enrolled for four years. Therefore the Roberts will need to make four tuition payments. The first payment will be made on September 1 of the year she enters college (Year 15). The final payment will be made on September 1 of her last year in college (Year 18). Notice that because of rising tuition costs, the tuition payments will increase each year.
- The Roberts would also like to give their daughter a lump-sum payment of $\$ 50,000$ on the September 1 after she graduates (i.e., at Year 19) in order to help with a down payment on a home, or to assist with graduate school tuition.
- The Roberts currently have $\$ 10,000$ in their college account. They anticipate making 15 equal contributions to the account at the end of each of the next 15 years. (The first contribution would be made on September 1 one year from now (i.e., at Year 1) and the final contribution will be made on September 1 when she enters college (i.e., Year 15).
- All current and future investments are assumed to earn an 8 percent return. (Ignore taxes.)

How much should the Roberts contribute each year in order to reach their goal?
a. \$3,156.69
b. \$3,618.95
c. \$4,554.83
d. \$5,955.54
e. $\$ 6,279.54$

Answer: b Diff: T
110. Jim and Nancy are interested in saving money for their son's education. Today is their son's 8th birthday. Their son will enter college ten years from now on his 18th birthday, and will attend for four years. All college costs are due at the beginning of the year, so Jim and Nancy will have to make payments on their son's 18th, 19th, 20 th and 21st birthdays ( $t=10,11,12,13$ ). They estimate that the college their son wants to attend will cost $\$ 35,000$ the first year ( $t=10$ ) and that the costs will increase 7 percent each year (the final college payment will be made 13 years from now).

Currently, Jim and Nancy have $\$ 20,000$ in an investment account. They also plan to contribute a fixed amount at the end of each of the next ten years ( $t=1,2,3, \ldots 10$ ). Their invested money will be in an account which pays 9 percent interest compounded annually. How much money do Jim and Nancy need to contribute to the account in each of the next ten years?
a. $\$ 5,638$
b. $\$ 5,848$
c. $\$ 6,052$
d. $\$ 6,854$
e. $\$ 7,285$

## Required annuity payments

Answer: a Diff: T
111. Joe and June Green are planning for their children's college education. Joe would like his kids to attend his alma mater where tuition is currently $\$ 25,000$ per year. Tuition costs are expected to increase by 5 percent each year. Their children, David and Daniel, just turned 2 and 3 years old today, September 1. David will begin college in 15 years, and Daniel will begin in 16 years. Each will graduate after four years in college. College tuition must be paid at the beginning of each school year.

Grandma Green invested $\$ 10,000$ in a mutual fund the day each child was born. This was to begin the boys' college fund (a combined fund for both children). The investment has earned and is expected to continue to earn $12 \%$ per year. Joe and June will now begin adding to this fund on every birthday, beginning next year, to ensure that there is enough money to send the kids to college.

How much money must Joe and June put into the college fund each of the next 15 years if their goal is to have all of the money in the investment account by the time Daniel (the oldest son) begins college?
a. \$5,928.67
b. \$7,248.60
c. $\$ 4,822.66$
d. \$7,114.88
e. $\$ 5,538.86$

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Answer: a Diff: T
112. To save money for a new house, you want to begin contributing money to a brokerage account. Your plan is to make ten contributions to the brokerage account. Each contribution will be for $\$ 1,500$. The contributions will come at the beginning of each of the next 10 years, i.e., the first contribution will be made at $t=0$ and the final contribution will be made at $t=9$. Assume that the brokerage account pays a 9 percent return with quarterly compounding. How much money do you expect to have in the brokerage account nine years from now (t = 9) ?
a. $\$ 23,127.49$
b. $\$ 25,140.65$
c. $\$ 25,280.27$
d. $\$ 21,627.49$
e. $\$ 19,785.76$

FV of investment account
Answer: b Diff: $\mathbf{T}$
113. Kelly and Brian Johnson are a recently married couple whose parents have counseled them to start saving immediately in order to have enough money down the road to pay for their retirement and their children's college expenses. Today $(t=0)$ is their 25 th birthday (the couple shares the same birthday).

The couple plans to have two children (Dick and Jane). Dick is expected to enter college 20 years from now ( $t=20$ ); Jane is expected to enter college 22 years from now ( $t=22$ ). So in years $t=22$ and $t$ $=23$ there will be two children in college. Each child will take 4 years to complete college, and college costs are paid at the beginning of each year of college.

College costs per child will be as follows:

| Year | Cost per child | Children in college |
| :---: | :---: | :---: |
|  | $\$ 58,045$ | Dick |
| 21 | $\$ 62,108$ | Dick |
| 22 | $\$ 66,456$ | Dick and Jane |
| 23 | $\$ 71,108$ | Dick and Jane |
| 24 | $\$ 76,086$ | Jane |
| 25 | $\$ 81,411$ | Jane |

Kelly and Brian plan to retire forty years from now at age 65 (at $t=40)$. They plan to contribute $\$ 12,000$ per year at the end of each year for the next 40 years into an investment account that earns 10 percent per year. This account will be used to pay for the college costs, and also to provide a nest egg for Kelly and Brian's retirement at age 65. How big will Kelly and Brian's nest egg (the balance of the investment account) be when they retire at age 65 ( $t=40$ )?
a. $\$ 1,854,642$
b. $\$ 2,393,273$
c. $\$ 2,658,531$
d. $\$ 3,564,751$
e. $\$ 4,758,333$

## Effective annual rate

Answer: c Diff: T
114. You have some money on deposit in a bank account which pays a nominal (or quoted) rate of 8.0944 percent, but with interest compounded daily (using a 365-day year). Your friend owns a security which calls for the payment of $\$ 10,000$ after 27 months. The security is just as safe as your bank deposit, and your friend offers to sell it to you for $\$ 8,000$. If you buy the security, by how much will the effective annual rate of return on your investment change?
a. $1.87 \%$
b. $1.53 \%$
c. $2.00 \%$
d. $0.96 \%$
e. $0.44 \%$

Non-annual compounding
Answer: a Diff: T
115. A financial planner has offered you three possible options for receiving cash flows. You must choose the option that has the highest present value.
(1) $\$ 1,000$ now and another $\$ 1,000$ at the beginning of each of the 11 subsequent months during the remainder of the year, to be deposited in an account paying a 12 percent nominal annual rate, but compounded monthly (to be left on deposit for the year).
(2) $\$ 12,750$ at the end of the year (assume a 12 percent nominal interest rate with semiannual compounding).
(3) A payment scheme of 8 quarterly payments made over the next two years. The first payment of $\$ 800$ is to be made at the end of the current quarter. Payments will increase by 20 percent each quarter. The money is to be deposited in an account paying a 12 percent nominal annual rate, but compounded quarterly (to be left on deposit for the entire 2-year period).

Which one would you choose?
a. Choice 1.
b. Choice 2.
c. Choice 3.
d. Any one, since they all have the same present value.
e. Choice 1 , if the payments were made at the end of each month.

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## PMT and quarterly compounding

Answer: b Diff: T
116. Your employer has agreed to make 80 quarterly payments of $\$ 400$ each into a trust account to fund your early retirement. The first payment will be made 3 months from now. At the end of 20 years ( 80 payments), you will be paid 10 equal annual payments, with the first payment to be made at the beginning of Year 21 (or the end of Year 20). The funds will be invested at a nominal rate of 8 percent, quarterly compounding, during both the accumulation and the distribution periods. How large will each of your 10 receipts be? (Hint: You must find the EAR and use it in one of your calculations.)
a. \$ 7,561
b. \$10,789
c. $\$ 11,678$
d. $\$ 12,342$
e. \$13,119

PV of an uneven CF stream
Answer: d Diff: T
117. Hillary is trying to determine the cost of health care to college students, and parents' ability to cover those costs. She assumes that the cost of one year of health care for a college student is $\$ 1,000$ today, that the average student is 18 when he or she enters college, that inflation in health care cost is rising at the rate of 10 percent per year, and that parents can save $\$ 100$ per year to help cover their children's costs. All payments occur at the end of the relevant period, and the $\$ 100 / y e a r ~ s a v i n g s ~ w i l l ~ s t o p ~ t h e ~ d a y ~ t h e ~ c h i l d ~ e n t e r s ~$ college (hence 18 payments will be made). Savings can be invested at a nominal rate of 6 percent, annual compounding. Hillary wants a health care plan which covers the fully inflated cost of health care for a student for 4 years, during Years 19 through 22 (with payments made at the end of years 19 through 22). How much would the government have to set aside now (when a child is born), to supplement the average parent's share of a child's college health care cost? The lump sum the government sets aside will also be invested at 6 percent, annual compounding.
a. \$1,082.76
b. \$3,997.81
c. $\$ 5,674.23$
d. $\$ 7,472.08$
e. $\$ 8,554.84$

## Extensions

## Multiple Choice: Problems

## Easy:

PV continuous compounding
Answer: b Diff: E
2E-118. In six years' time, you are scheduled to receive money from a trust established for you by your grandparents. When the trust matures there will be $\$ 100,000$ in the account. If the account earns 9 percent compounded continuously, how much is in the account today?
a. \$ 23,456
b. $\$ 58,275$
c. \$171,600
d. \$ 59,627
e. $\$ 61,385$

## Medium:

FV continuous compounding
Answer: a Diff: M
2E-119. Assume one bank offers you a nominal annual interest rate of 6 percent compounded daily while another bank offers you continuous compounding at a 5.9 percent nominal annual rate. You decide to deposit $\$ 1,000$ with each bank. Exactly two years later you withdraw your funds from both banks. What is the difference in your withdrawal amounts between the two banks?
a. \$ 2.25
b. \$ 0.09
c. \$ 1.12
d. $\$ 1.58$
e. $\$ 12.58$

Continuous compounded interest rate Answer: a Diff: M 2E-120. In order to purchase your first home you need a downpayment of $\$ 19,000$ four years from today. You currently have $\$ 14,014$ to invest. In order to achieve your goal, what nominal interest rate, compounded continuously, must you earn on this investment?
a. $7.61 \%$
b. $7.26 \%$
c. $6.54 \%$
d. $30.56 \%$
e. $19.78 \%$

Payment and continuous compounding
Answer: d Diff: M
2E-121. You place $\$ 1,000$ in an account that pays 7 percent interest compounded continuously. You plan to hold the account exactly three years. Simultaneously, in another account you deposit money that earns 8 percent compounded semiannually. If the accounts are to have the same amount at the end of the three years, how much of an initial deposit do you need to make now in the account that pays 8 percent interest compounded semiannually?
a. \$1,006.42
b. \$ 986.73
c. \$ 994.50
d. \$ 975.01
e. \$ 962.68

Continuous compounding
Answer: a Diff: M
2E-122. You have the choice of placing your savings in an account paying 12.5 percent compounded annually, an account paying 12.0 percent compounded semiannually, or an account paying 11.5 percent compounded continuously. To maximize your return you would choose:
a. 12.5\% compounded annually.
b. $12.0 \%$ compounded semiannually.
c. $11.5 \%$ compounded continuously.
d. You would be indifferent since the effective rate for all three is the same.
e. You would be indifferent between choices a and c since their effective rates are the same.

## Continuous compounding

Answer: b Diff: M
2E-123. If you have $\$ 5,438$ in an account that has been paying an annual rate of 10 percent, compounded continuously, since you deposited some funds 10 years ago, how much was the original deposit?
a. $\$ 1,000$
b. \$2,000
c. \$3,000
d. \$4,000
e. $\$ 5,000$

Continuous compounding
Answer: d Diff: M
2E-124. For a 10-year deposit, what annual rate payable semiannually will produce the same effective rate as 4 percent compounded continuously?
a. $2.02 \%$
b. $2.06 \%$
c. $3.95 \%$
d. $4.04 \%$
e. $4.12 \%$

## Continuous compounding

Answer: b Diff: M
2E-125. How much should you be willing to pay for an account today that will have a value of $\$ 1,000$ in 10 years under continuous compounding if the nominal rate is 10 percent?
a. \$354
b. \$368
c. $\$ 385$
d. \$376
e. \$370

Continuous compounding Answer: b Diff: M
2E-126. If you receive $\$ 15,000$ today and can invest it at a 5 percent annual rate compounded continuously, what will be your ending value after 20 years?
a. $\$ 35,821$
b. $\$ 40,774$
c. $\$ 75,000$
d. $\$ 81,342$
e. $\$ 86,750$

1. PV versus FV
2. $\quad \mathrm{PV}$ versus FV
3. Amortization
4. Effective annual rate
5. Retirement and compounding
6. $P V$ of an annuity
7. PV of a sum
8. Compounding
9. Comparative compounding
10. Amortization
11. 
12. Periodic and nominal rates
13. Lump sum and annuity
14. PV and discount rate
15. PV versus FV
16. Time value concepts
17. Time value concepts

Statements b and c are correct; therefore, statement d is the correct choice. The present value is smaller if interest is compounded monthly rather than semiannually.
18. Time value concepts

Statements a and b are correct; therefore, statement $d$ is the correct choice. The nominal interest rate will be less than the effective rate when the number of periods per year is greater than one.

$E A R_{a}=8.30 \% ; E A R_{b}=8 \% ; E A R_{c}=8.24 \% ; E A R_{d}=8.328 \% ; E A R_{e}=7.8 \%$.

Answer: b Diff: E

Statement b is true; the others are false. The remaining balance after three years will be $\$ 100,000$ less the total amount of repaid principal during the first 36 months. On a fixed-rate mortgage the monthly payment remains the same.
23. Amortization

Answer: e Diff: E

Statements b and c are correct; therefore, statement $e$ is the correct choice. Monthly payments will remain the same over the life of the loan.

Answer: C Diff: M

By definition, an annuity due is received at the beginning of the year while an ordinary annuity is received at the end of the year. Because the payments are received earlier, both the present and future values of the annuity due are greater than those of the ordinary annuity.

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25. Time value concepts

Answer: e Diff: M
If the interest rate were higher, the payments would all be higher, and all of the increase would be attributable to interest. So, the proportion of each payment that represents interest would be higher. Note that statement $b$ is false because interest during Year 1 would be the interest rate times the beginning balance, which is $\$ 10,000$. With the same interest rate and the same beginning balance, the Year 1 interest charge will be the same, regardless of whether the loan is amortized over 5 or 10 years.
26. Time value concepts

Answer: e Diff: M
27. Time value concepts

Answer: c Diff: M

Statement $c$ is correct; the other statements are false. The effective rate of the investment in statement $a$ is $10.25 \%$. The present value of the annuity due is greater than the present value of the ordinary annuity.
28. Time value concepts

Answer: d Diff: T
29. FV of a single payment

Answer: d Diff: E

Time Line:


Numerical solution:
$F V=\$ 2,000\left(1.10^{15}\right)=\$ 1,000(4.1772)=\$ 8,354.50$.
Financial calculator solution:
Inputs: $\mathrm{N}=15 ; \mathrm{I}=10 ; \mathrm{PV}=-2,000 ; \mathrm{PMT}=0$.
Output: $\mathrm{FV}=\$ 8,354.50$.
30. FV of a single payment

Answer: c Diff: E

Time Line:


Numerical solution:
$F V=\$ 1,000\left(1.09^{6}\right)=\$ 1,000(1.6771)=\$ 1,677.10$.
Financial calculator solution:
Inputs: $\mathrm{N}=6$; $\mathrm{I}=9 ; \mathrm{PV}=-1,000 ; \mathrm{PMT}=0$.
Output: $\mathrm{FV}=\$ 1,677.10$.
31. PV of a single payment

Answer: b Diff: E


Numerical solution:
$\mathrm{PV}=\$ 10,000 /\left(1.08^{6}\right)=\$ 10,000 /(1.5869)=\$ 6,301.70$.
Financial calculator solution:
Inputs: $N=6 ; ~ I=8 ; ~ P M T=0 ; ~ F V=-10,000$.
Output: $\mathrm{FV}=\$ 6,301.70$.
32. PV of a single payment

Answer: e Diff: E

Time Line:


Numerical solution:
$\mathrm{PV}=\$ 4,000 /\left(1.15^{10}\right)=\$ 4,000 /(4.0456)=\$ 988.74$.
Financial calculator solution:
Inputs: $\mathrm{N}=10 ; \mathrm{I}=15 ; \mathrm{PMT}=0 ; \mathrm{FV}=-4,000$. Output: $\mathrm{FV}=\$ 988.74$.
33. Growth rate

Answer: d Diff: E

Time Line:


Numerical solution:
$\$ 13,700=\$ 1,800(1+i)^{30}$
$(1+i)^{30}=13,700 / 1,800=7.6111$
$(1+i)=7.6111^{(1 / 30)}=1.070$
$i \approx 7 \%$.

Financial calculator solution:
Inputs: $\mathrm{N}=30 ; \mathrm{PV}=-1,800 ; \mathrm{PMT}=0 ; \mathrm{FV}=13,700$. Output: $\mathrm{I}=7.0 \%$.
34. Solving for the interest rate for a single payment

Answer: e Diff: E

Time Line:


Numerical solution:
$\$ 2,839.42=\$ 1,000(1+i)^{10}$
$(1+i)^{10}=2,839.42 / 1,000=2.8394$
$(1+i)=2.8394^{(1 / 10)}=1.11$
$i \approx 11 \%$.

Financial calculator solution:
Inputs: $\mathrm{N}=10 ; \mathrm{PV}=-1,000 ; \mathrm{PMT}=0 ; \mathrm{FV}=2,839.42$. Output: $\mathrm{I}=$ 11.0\%.
35. Time for a sum to double

Answer: d Diff: E

Time Line:


```
Numerical solution:
2= 1(1+0.0723)n
(1+0.0723)n=2
n ln[1.0723]=ln[2]
n = ln[2]/ ln[1.0723]
n = 0.6931 / 0.0698 = 9.93.
Financial calculator solution:
    PV = -1
    FV = 2
PMT = 0
    I = 7.23
    N = ? = 119.17 months = 9.93 years.
```

36. Solving for $N$ for a single payment

Answer: b Diff: E

Time Line:


Numerical solution:
$10,0000=2,000(1+0.08)^{n}$
$(1+0.08)^{n}=10,000 / 2,000=5$
$\mathrm{n} \ln [1.08]=\ln [5]$
$\mathrm{n}=\ln [5] / \ln [1.08]$
$\mathrm{n}=1.6094 / 0.0770=20.91$.
Financial calculator solution:

$$
P V=-2,000
$$

$\mathrm{FV}=10,000$
$\mathrm{PMT}=0$
$I=8$
$\mathrm{N}=$ ? = 20.91 years.
37. FV of a sum

Answer: b Diff: E


Numerical solution:
$\mathrm{FV}=\$ 1,000\left(1.02^{6}\right)=\$ 1,000(1.1262)=\$ 1,126.20 \approx \$ 1,126$.
Financial calculator solution:
Inputs: $\mathrm{N}=6$; $\mathrm{I}=2 ; \mathrm{PV}=-1,000 ; \mathrm{PMT}=0$.
Output: $\mathrm{FV}=\$ 1,126.16 \approx \$ 1,126$.
38. FV of an annuity Answer: e Diff: E

Time Line:


Numerical solution:
$\left.\mathrm{FV}=\$ 200\left(\left(1.15^{5}-1\right) / .15\right)\right)=\$ 200 \times 6.7424=\$ 1,348.48$.

Financial calculator solution:
Inputs: $N=5 ; ~ I=15 ; ~ P V=0 ; ~ P M T=-200$. Output: $F V=\$ 1,348.48$.
39. PV of an annuity

Answer: a Diff: E

Time Line:


Numerical solution:
$P V=\$ 200\left(\left(1-\left(1 / 1.15^{5}\right)\right) / .15\right)=\$ 200 \times 3.3522=\$ 670.44$.
Financial calculator solution:
Inputs: $\mathrm{N}=5 ; \mathrm{I}=15 ; \mathrm{PMT}=200 ; \mathrm{FV}=0$. Output: $\mathrm{PV}=-\$ 670.43$.
40.

PV of a perpetuity
Answer: c Diff: E
$V=P M T / i=\$ 1,000 / 0.15=\$ 6,666.67$.
41.


Numerical solution:
$P V=\$ 2,000\left(\left(1-\left(1 / 1.14^{5}\right)\right) / .14\right)+\$ 3,000\left(\left[\left(1-\left(1 / 1.14^{3}\right)\right) / .14\right]\left[1 / 1.14^{5}\right]\right)+$
$\$ 4,000(1.149)$
$=\$ 2,000(3.4331)+\$ 3,000(2.3216)(0.5194)+\$ 4,000(0.3075)$
$=\$ 6,866.20+\$ 3,617.52+\$ 1,230.00=\$ 11,713.72 \approx \$ 11,714$.
Financial calculator solution:
Using cash flows
Inputs: $\mathrm{CF}_{0}=0 ; \mathrm{CF}_{1}=2,000 ; \mathrm{N}_{\mathrm{j}}=5 ; \mathrm{CF}_{2}=3,000 ; \mathrm{N}_{j}=3 ; \mathrm{CF}_{3}=4,000$;
$I=14$.
Output: $\quad \mathrm{NPV}=\$ 11,713.54 \approx \$ 11,714$.
42. Required annuity payments

Answer: b Diff: E
Time Line:


Numerical solution:
$\$ 1,000=\operatorname{PMT}\left(\left(1-\left(1 / 1.1^{5}\right)\right) / .1\right)$
PMT $=\$ 1,000 / 3.7908=\$ 263.80$.

Financial calculator solution:
Inputs: $\mathrm{N}=5 ; \mathrm{I}=10 ; \mathrm{PV}=-1,000 ; \mathrm{FV}=0$. Output: $\mathrm{PMT}=\$ 263.80$.
43. Quarterly compounding

Answer: a Diff: E


Numerical solution:
$\$ 100\left(1.01^{20}\right)=\$ 100(1.2202)=\$ 122.02$.
Financial calculator solution:
Inputs: $\mathrm{N}=20 ; \mathrm{I}=1 ; \mathrm{PV}=-100 ; \mathrm{PMT}=0$. Output: $\mathrm{FV}=\$ 122.02$.
44. Effect of inflation

Answer: c Diff: E

Time Line:


Numerical solution:
$0.5=\$ 1\left(1 / 1.04^{n}\right)$
(1.04) $n=1 / 0.5=2$
n LN (1.04) = LN (2)
$\mathrm{n}=0.6931 / 0.0392$
$n=17.68 \approx 18$ years.

Financial calculator solution:
Inputs: $I=4 ; ~ P V=-1 ; ~ P M T=0 ; ~ F V=0.50$.
Output: $\mathrm{N}=-17.67 \approx 18$ years.
45. Effective annual rate

Answer: c Diff: E

Bank A: 8\%, monthly.

$$
\begin{aligned}
\mathrm{EAR}_{\mathrm{A}} & =\left(1+\frac{\mathrm{r}_{\mathrm{Nom}}}{\mathrm{~m}}\right)^{\mathrm{m}}-1 \\
& =\left(1+\frac{0.08}{12}\right)^{12}-1=8.30 \%
\end{aligned}
$$

Bank B: 9\%, interest due at end of year $E A R_{B}=9 \%$.
$9.00 \%-8.30 \%=0.70 \%$.

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46. Effective annual rate

Answer: b Diff: E

Use the formula for calculating effective rates from nominal rates as follows: $\operatorname{EAR}=(1+0.18 / 12)^{12}-1=0.1956$ or $19.56 \%$.
47. Effective annual return Answer: b Diff: E

Convert each of the alternatives to an effective annual rate (EAR) for comparison.
a. $E A R=10.38 \%$.
b. $E A R=10.47 \%$.
c. $E A R=10.20 \%$.
d. $E A R=10.25 \%$.
e. $E A R=10.07 \%$. Therefore, the highest effective return is choice b.
48. Effective annual return

Answer: a Diff: E

Convert each of the alternatives to an effective annual rate (EAR) for comparison.
a. $E A R=10.2736 \%$.
b. $E A R=10.1846 \%$.
c. $E A R=10.2000 \%$.
d. $E A R=10.2500 \%$.
e. $\mathrm{EAR}=10.0339 \%$.

Therefore, the highest effective return is choice a.
49. Effective annual return

Answer: b Diff: E
Convert each of the alternatives to an effective annual rate (EAR) for comparison.
a. $\mathrm{NOM} \%=9$; $\mathrm{P} / \mathrm{YR}=2$; $\mathrm{EFF} \%=\mathrm{EAR}=9.20 \%$.
b. $\operatorname{EAR}=9.31 \%$.
c. $\operatorname{EAR}=9.20 \%$.
d. $E A R=9.27 \%$.
e. $\operatorname{EAR}=9.20 \%$.

Thus, b provides the investor with the highest EAR.
50. PV of an uneven CF stream

Answer: d Diff: E
Time Line:


Numerical solution:
$P V=\$ 1,000\left(1 / 1.08^{1}\right)=\$ 925.926$
$+\$ 2,000\left(1 / 1.08^{2}\right)=\$ 1,714.678$
$+\$ 3,000\left(1 / 1.08^{3}\right)=\$ 2,381.497$
$+\$ 4,000\left(1 / 1.08^{4}\right)=\$ 2,940.119$
$P V=\underline{\$ 7,962.22}$
Financial calculator solution:
Inputs: $\mathrm{CF}_{0}=0 ; \mathrm{CF}_{1}=1,000 ; \mathrm{CF}_{2}=2,000 ; \mathrm{CF}_{3}=3,000$;
$C F_{4}=4,000 ; ~ I=8$.
Output: $\mathrm{NPV}=\$ 7,962.22$.
51. Interest payment on an amortized loan

Answer: b Diff: E

```
Solve for the payment as follows:
N = 5
I = 10
PV = -100,000
FV = 0
Solve for PMT = $26,379.748.
Interest in Year 1 is $10,000 = 0.10($100,000).
Principle in Year 1 is $16,379.748 = $26,379.748 - $10,000.
Beginning balance in Year 1 is $83,620.252 = $100,000 - $16,379.748.
Interest in Year 2 is $8,362 = 0.10($83,620.252).
```

52. Effective annual rate

Answer: c Diff: E
$\operatorname{EAR}=(1+(0.15 / 12))^{12}-1.0=1.1608-1.0=0.1608=16.08 \%$.
Financial calculator solution:
Inputs: $\mathrm{P} / \mathrm{YR}=12$; $\mathrm{NOM} \%=15.0$. Output: $\mathrm{EFF} \%=\mathrm{EAR}=16.08 \%$.
53. FV of an annuity

Answer: b Diff: M

Time Line:


Numerical solution:
$F V_{\text {Year } 20}=\$ 50\left[\left(1.1^{20}-1\right) / .1\right]=\$ 50(57.275)=\$ 2,863.75$.
$\mathrm{FV}_{\text {Year } 30}=\$ 2,863.75\left[1.1^{10}\right]=\$ 2,863.75(2.5937)=\$ 7,427.71$.
Financial calculator solution:
Calculate FV at Year 20, take that lump sum forward 10 years to Year 30 at 10\%.
Inputs: $\mathrm{N}=20 ; \mathrm{I}=10 ; \mathrm{PV}=0 ; \mathrm{PMT}=-50$. Outputyear 20 : $\mathrm{FV}=\$ 2,863.75$. At Year 30
Inputs: $\mathrm{N}=10 ; \mathrm{I}=10 ; \mathrm{PV}=-2,863.75 ; \mathrm{PMT}=0$.
Outputyear $30: ~ F V=\$ 7,427.83$.
54. FV of annuity due

Answer: b Diff: M

```
One of the several ways of doing this is to treat this as a 4-year
annuity due plus a payment in Year 4.
Numerical solution:
$3,000[(1.094-1)/.09] (FVIFA9%,4)(1.09) + $3,000
$3,000(4.5731)(1.09) + $3,000 = $17,954.04.
Financial calculator solution:
BEGIN Mode
N = 4
I = 9
PV = 0
PMT = -3,000
FV = $14,954.13.
Plus the $3,000 at the end of Year 4 = $14,954.13 + $3,000=
$17,954.13.
```

55. FV under monthly compounding

Answer: e Diff: M

Numerical solution:
$\$ 1,000(1.005)^{36}=\$ 1,196.68$.
Financial calculator solution:
$\mathrm{N}=3 \times 12=36$
$I=6 / 12=0.5$
$\mathrm{PV}=-1,000$
$\mathrm{PMT}=0$
Solve for $F V=\$ 1,196.68$.
56. FV of lump sum and annuity

Answer: e Diff: M

```
Numerical solution:
PMT0 = $5,000; PMT = $5,000 < 1.10 = $5,500; PMT = $5,000 }\times(1.10)2
$6,050; PMT = $5,000 < (1.10) 3 = $6,655; PMT4 = $5,000 }\times(1.10)4
$7,320.50.
FV = $5,000(1.14) 5 + $5,500(1.14)4 + $6,050(1.14) 3 + $6,655(1.14)}\mp@subsup{}{}{2}
$7,320.50(1.14) = $44,873.90.
Financial calculator solution:
First, calculate the payment amounts:
PMT = $5,000, PMT = $5,500, PMT = $6,050, PMT = $ = 6,655, PMT = =
$7,320.50. Then, find the future value of each payment at t = 5: For
PMT0, N = 5, I = 14, PV = -5,000, PMT = 0; thus, FV = $9,627.0729.
Similarly, for PMT , FV = $9,289.2809, for PMT 2, FV = $8,963.3412, for
PMT3, FV = $8,648.8380, and for PMT4, FV = $8,345.3700. Finally,
summing the future values of the respective payments will give the
balance in the account at }t=5\mathrm{ or $44,873.90.
```


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57. PV of an uneven CF stream

Answer: a Diff: M

Time Line:


$$
\mathrm{PV}=\text { ? }
$$

Numerical solution:
$\mathrm{PV}=\quad+\$ 1\left(1 / 1.12^{1}\right)=\quad \$ 1(0.8929)=\$ 0.89$
$+\$ 2,000\left(1 / 1.12^{2}\right)=\$ 2,000(0.7972)=\$ 1,594.40$
$+\$ 2,000\left(1 / 1.12^{3}\right)=\$ 2,000(0.7118)=\$ 1,423.60$
$+\$ 2,000\left(1 / 1.12^{4}\right)=\$ 2,000(0.6355)=\$ 1,271.00$
$-\$ 2,000\left(1 / 1.12^{5}\right)=-\$ 2,000(0.5066)=-\$ 1,013.20$
$P V=\underline{\$ 3,276.69}$
Financial calculator solution:
Inputs: $C F_{0}=0 ; C F_{1}=1 ; C F_{2}=2,000 ; N_{j}=3$ times; $C F_{3}=0$;
$\mathrm{CF}_{4}=-2,000$; $\mathrm{I}=12$.
Output: NPV $=\$ 3,276.615 \approx \$ 3,277$.
Note: Numerical solution differs from calculator solution due to interest factor rounding.
58. PV of uncertain cash flows

Answer: e Diff: M

Time Line:


Calculate expected cash flows
$\mathrm{E}\left(\mathrm{CF}_{1}\right)=(0.30)(\$ 300)+(0.40)(\$ 500)+(0.30)(\$ 700)=\$ 500$.
$\mathrm{E}\left(\mathrm{CF}_{2}\right)=(0.15)(\$ 100)+(0.35)(\$ 200)+(0.35)(\$ 600)+(0.15)(\$ 900)=\$ 430$.
$\mathrm{E}\left(\mathrm{CF}_{3}\right)=(0.25)(\$ 200)+(0.75)(\$ 800)=\$ 650$.
Numerical solution:
$P V=\$ 500\left(1 / 1.08^{1}\right)+\$ 430\left(1 / 1.08^{2}\right)+\$ 650\left(1 / 1.08^{2}\right)$
$=\$ 500(0.9259)+\$ 430(0.8573)+\$ 650(0.7938)$
$=\$ 462.95+\$ 368.64+\$ 515.97=\$ 1,347.56$.
Financial calculator solution:
Using cash flows,
Inputs: $\mathrm{CF}_{0}=0 ; \mathrm{CF}_{1}=500 ; \mathrm{CF}_{2}=430 ; \mathrm{CF}_{3}=650 ; \mathrm{I}=8$.
Output: NPV = \$1,347.61.

Time Line:


Numerical solution:
$F V=\$ 1,000\left(\right.$ FVIFA $\left._{8 \%}, 10\right)+\$ 1,000\left(\right.$ FVIFA $\left._{8 \%, 5}\right)+\$ 5,000\left(\right.$ FVIF $\left._{8 \%}, 10\right)$
$=\$ 1,000\left(\left(1.08^{10}-1\right) / .08\right)+\$ 1,000\left(\left(1.08^{5}-1\right) / .08\right)+\$ 5,000\left(1.08^{10}\right)$
$=\$ 1,000(14.487)+\$ 1,000(5.866)+\$ 5,000(2.1589)$
$=\$ 14,487+\$ 5,866+\$ 10,794.50=\$ 31,147.50 \approx \$ 31,148$.
Financial calculator solution:
Solution using NFV: (Note: Some calculators do not have net future value function. Cash flows can be grouped and carried forward or PV can be used; see alternative solution below.)
Inputs: $\mathrm{CF}_{0}=5,000 ; \mathrm{CF}_{1}=1,000 ; \mathrm{N}_{\mathrm{j}}=5 ; \mathrm{CF}_{2}=2,000 ; \mathrm{N}_{\mathrm{j}}=5$; $\mathrm{I}=8$. Output: NFV = \$31,147.79 $\approx$ \$31,148.

Alternative solution: Calculate $P V$ of the cash flows, then bring them forward to $F V$ using the interest rate.
Inputs: $\mathrm{CF}_{0}=5,000 ; \mathrm{CF}_{1}=1,000 ; \mathrm{N}_{\mathrm{j}}=5 ; \mathrm{CF}_{2}=2,000 ; \mathrm{N}_{\mathrm{j}}=5 ; \mathrm{I}=8$.
Output: PV = \$14,427.45.
Inputs: $N=10 ; ~ I=8 ; ~ P V=-14,427.45$.
Output: $\mathrm{FV}=\$ 31,147.79 \approx \$ 31,148$.
60. Value of missing cash flow

Answer: d Diff: M

```
Numerical solution:
PV = -$300 + ($100)(0.8696) + ($125.43)(0.7561) + ($90.12)(0.6575)
PV = -$58.95.
Now, solve for CF4:
$58.95(1.15)4 = $103.10.
Financial calculator solution:
Enter the first 4 cash flows, enter I = 15, and solve for NPV =
$58.945. The future value of $58.945 will be the required cash flow.
PV = -58.945; N = 4; I/YR = 15; PMT = 0; solve for FV = $103.10.
```


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61. Value of missing cash flow

Answer: c Diff: M

```
Find the present value of each of the cash flows:
PV of CF1 = $325/1.12 = $290.18. PV of CF C = $400/(1.12)2 = $318.88.
PV of CF3 = $550/(1.12) 3 = $391.48. PV of CF5 = $750/(1.12) 5 = $425.57.
PV of CF6 = $800/(1.12) }\mp@subsup{}{}{6}=$405.30. Summing these values you obtai
$1,831.41. The present value of CF4 must then be $2,566.70 - $1,831.41
=$735.29. The value of CF4 is ($735.29)(1.12)4 = $1,157.
```


## 62. Amortization

Answer: c Diff: M
Time Line:


Numerical solution:
Initial balance $=0.8(\$ 250,000)=\$ 200,000$.
$\$ 200,000=$ PMT $\left(\right.$ PVIFA $\left._{12 \%}, 30\right)$
$\$ 200,000=\operatorname{PMT}\left[\left(1-\left(1 / 1.12^{30}\right) / 0.12\right]\right.$
$\$ 200,000=\operatorname{PMT}(8.0552)$
PMT $=\$ 200,000 / 8.0552=\$ 24,828.68 \approx \$ 24,829$.
Financial calculator solution:
Inputs: $\mathrm{N}=30 ; \mathrm{I}=12 ; \mathrm{PV}=-200,000 ; \mathrm{FV}=0$.
Output: $\mathrm{PMT}=\$ 24,828.73 \approx \$ 24,829$.
63. Effective annual rate

Answer: d Diff: M
$E A R_{\text {Qtr }}=\left(1+\frac{0.10}{4}\right)^{4}-1=10.38 \%$.
$E A R_{D l_{y}}=\left(1+\frac{0.09}{365}\right)^{365}-1=9.42 \%$.
Difference $=10.38 \%-9.42 \%=0.96 \%$.
Alternatively, with a financial calculator, for the quarterly loan enter $P / Y R=4, N O M \%=10$, and press EFF\% to get $E A R=10.38 \%$.

For the daily loan, enter $P / Y R=365, \mathrm{NOM} \%=9$, and press EFF\% to get $\operatorname{EAR}=9.42 \%$.
64. Value of a perpetuity

Answer: c Diff: M

Time Line:


```
Solve for required return, \(r\). We know \(V_{p}=\frac{\text { PMT }}{r}\), thus,
\(r=\frac{P M T}{V_{p}}=\frac{\$ 1,250}{\$ 15,625}=8 \%\).
Tabular solution:
Calculate the value of the annuity using \(r=8 \%\)
\(\mathrm{V}_{\text {Annuity }}=\$ 1,250\left[\left(1-\left(1 / 1.08^{20}\right) / 0.08\right]=\$ 1,250(9.8181)=\$ 12,272.63 \approx\right.\)
\$12,273.
Financial calculator solution:
Inputs: \(\mathrm{N}=20 ; \mathrm{I}=8 ; \mathrm{PMT}=-1,250\). Output: \(\mathrm{PV}=\$ 12,272.68 \approx \$ 12,273\).
```

65. PV of an uneven $C F$ stream

Answer: c Diff: T


Financial calculator solution:
Inputs: $C F_{0}=0 ; C F_{1}=-100 ; N_{j}=3 ; I=4$. Output: NPV = -277.51.
Calculate the $P V$ of CFs $4-8$ as of time $=3$ at $i=5 \%$
Inputs: $\mathrm{CF} 0=0 ; \mathrm{CF}_{1}=200 ; \mathrm{CF}_{2}=300 ; \mathrm{N}_{\mathrm{j}}=4 ; \mathrm{I}=5$.
Output: $\mathrm{NPV}_{3}=\$ 1,203.60$.
Calculate $P V$ of the $F V$ of the positive CFs at Time $=3$
Inputs: $\mathrm{N}=3 ; \mathrm{I}=4 ; \mathrm{PMT}=0 ; \mathrm{FV}=-1,203.60$. Output: $\mathrm{PV}=\$ 1,070$.
Total PV $=\$ 1,070-\$ 277.51=\$ 792.49$.
Note: Numerical solution differs from calculator solution due to interest factor rounding.

```
College Cost Today = $10,000, Inflation = 5%.
CFO = $10,000 < (1.05)5 = $12,762.82 < 1 = $12,762.82.
CF1}=$10,000\times(1.05)6=$13,400.96 < 1 = $13,400.96.
CF}2=$10,000\times(1.05)7=$14,071.00 < 2 = $28,142.00.
CF3}=$10,000\times(1.05\mp@subsup{)}{}{8}=$14,774.55\times2=$29,549.10
CF4}=$10,000\times(1.05)9=$15,513.28\times1=$15,513.28
CF5}=$10,000\times(1.05)10=$16,288.95 < 1 = $16,288.95
Numerical solution:
Find PV of college costs in Year 5:
PV = $12,762.82 + $13,400.96(0.9259) + $28,142(0.8573) +
            $29,549.10(0.7938) + $15,513.28(0.7350) + $16,288.95(0.6806)
    =$95,241.50.
Find FV of educational fund in 5 years:
$50,000(1.08)5 = $73,466.40.
Now, find net amount needed in Year 5:
$95,241.50 - $73,466.40 = $21,775.10.
Finally, find PMT needed to accumulate $21,775.10 in Year 5:
FVA5}= PMT(FVIFA 8%,5
$21,775.10 = PMT[(1.085-1)/0.08]
$21,775.10 = PMT (5.8666)
PMT = $3,711.71.
Note: Numerical solution differs from calculator solution due to
interest factor rounding.
Financial calculator solution:
Enter cash flows in CF register:
I = 8; solve for NPV = $95,244.08.
Calculate annuity:
    N = 5
    I = 8
    PV = -50,000
    FV = 95,244.08
PMT = ? = $3,712.15.
```

67. Required annuity payments

Answer: b Diff: T

```
Numerical solution:
Calculate the present value of college costs at t = 16:
PV = $25,000(0.9259) + $25,000(0.8573) + $50,000(0.7938) +
        $50,000(0.7350) + $25,000(0.6806) + $25,000(0.6302)
PV = $153,790.
Calculate the annual required deposit:
    FVA }16=\operatorname{PMT (FVIFA 
$153,790 = PMT[(1.0816-1)/0.08] (30.324)
$153,790 = PMT (30.324)
            PMT = $5,071.56.
Note: Numerical solution differs from calculator solution due to
interest factor rounding.
Financial calculator solution:
Step 1 Calculate the present value of college costs at t = 16:
    Remember, costs are incurred at end of year.
    t = 16: CF0 = 0
    t = 17: CF 
    t = 18: CF2 = 25,000
    t = 19: CFF3 = 50,000
    t = 20: CF C = 50,000
    t = 21: CF5 = 25,000
    t = 22: CF6 = 25,000
    I = 8; Solve for NPV = $153,793.54.
Step 2 Calculate the annual required deposit:
    N = 16
    I = 8
    PV = 0
    FV = -153,793.54
    Solve for PMT = $5,071.63.
```

68. Required annuity payments

```
Time Line: (In thousands)
```



```
Numcerical solution:
Long way
Annual interest: 0.08($1,000,000) = $80,000.
Annual reserve: FV = $1,000,000 = PMT (FVIFA 8%,20) =
                                    PMT[(1.0820-1)/0.08]=PMT (45.762).
                            PMT = $1,000,000/45.762 = $21,852.19 \approx $21,852 per year.
Total annual payment: $80,000 + $21,852 = $101,852.
Total number of tickets = $101,852/$10 \approx 10,186.*
*Rounded up to next whole ticket.
Short way
    PV = $1,000,000 = PMT(PVIFA 
PMT = $1,000,000/9.8181 = $101,852.70.
Note: Short way works only if bond yield = investment rate.
Financial calculator solution:
Long way Inputs: N = 20; I = 8; FV = 1,000,000.
    Output: PMT = -$21,852.21.
Add coupon interest and reserve payment together
Annual PMT Total = $80,000 + $21,852.21 = $101,852.21.
Total number of tickets = $101,852.21/$10.00 = 10,185.22 \approx 10,186.*
*Rounded up to next whole ticket.
Short way Inputs: N = 20; I = 8; PV = 1,000,000; FV = 0.
    Output: PMT = -$101,852.21.
    Total number of tickets = $101,852.21/$10.00 \approx 10,186.*
*Rounded up to next whole ticket.
```

70. Solving for $N$ for a single payment

Answer: b Diff: E

Financial calculator solution:

```
    PV = 0
    FV = 301,066.27
PMT = -900
    I = 9/12 = 0.75
    N = ? = 168 months.
```

71. Nominal and effective rates

Answer: b Diff: E

```
1st investment Enter the following:
    NOM% = 9
    P/YR = 2
    Solve for EFF% = 9.2025%.
2nd investment Enter the following:
    EFF% = 9.2025
    P/YR = 4
    Solve for NOM% = 8.90%.
```

72. Effective annual rate

Answer: c Diff: E

```
Your proposal:
EAR ( = $120/$1,000
EAR R = 12%.
Your friend's proposal:
Interest is being paid each month ($10/$1,000 = 1% per month), so it
compounds, and the EAR is higher than r rom = 12%:
EAR2}=(1+\frac{0.12}{12}\mp@subsup{)}{}{12}-1=12.68%
Difference = 12.68% - 12.00% = 0.68%.
You could also visualize your friend's proposal in a time line format:
```



```
Insert those cash flows in the cash flow register of a calculator and solve for IRR. The answer is \(1 \%\), but this is a monthly rate. The nominal rate is \(12(1 \%)=12 \%\) which converts to an EAR of \(12.68 \%\) as follows:
Input into a financial calculator the following:
P/YR = 12, NOM% = 12, and solve for EFF% = 12.68%.
```

73. Time for a sum to double

Answer: d Diff: E

```
    PV = -1
    FV = 2
PMT = 0
    I = 7/12
    N = ? = 119.17 months = 9.93 years.
```

74. Monthly payments on loan

Answer: c Diff: E
First, find the monthly interest rate $=0.10 / 12=0.8333 \% / \mathrm{month}$. Now, enter in your calculator $\mathrm{N}=60, \mathrm{I} / \mathrm{YR}=0.8333, \mathrm{PV}=-13,000, \mathrm{FV}=0$, and solve for PMT = \$276.21.

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75. Interest rate of an annuity

Answer: b Diff: E
Time Line:
0 i $=$ ?
76. Effective annual rate

Answer: b Diff: M

Time Line:


Financial calculator solution:
Calculate the PMT of the annuity
Inputs: $\mathrm{N}=10 ; \mathrm{I}=0 ; \mathrm{PV}=-3,755.50 ; \mathrm{FV}=0$. Output: $\mathrm{PMT}=\$ 375.55$.
Calculate the effective annual interest rate
Inputs: $\mathrm{N}=10 ; \mathrm{PV}=0 ; \mathrm{PMT}=-375.55 ; \mathrm{FV}=5,440.22$.
Output: $I=7.999 \approx 8.0 \%$.
77. Number of periods for an annuity Answer: a Diff: M

Time Line:

78. FV of a sum

Answer: d Diff: M


Tabular/Numerical solution:
Solve for amount on deposit at the end of 6 months.
Step $1 \quad \mathrm{FV}=\$ 100\left(\mathrm{FVIF}_{3 \circ, 1}\right)-\$ 100=\$ 3.00$. $\mathrm{FV}=\$ 100(1+0.06 / 2)-\$ 100=\$ 3.00$.
Step 2 Compound the $\$ 3.00$ for 39 periods at 3\%
$\mathrm{FV}=\$ 3.00\left(\mathrm{FVIF}_{3 \%}, 39\right)=\$ 9.50$.
Since table does not show 39 periods, use numerical/calculator exponent method. $\mathrm{FV}=\$ 3.00(1.03)^{39}=\$ 9.50$.

Financial calculator solution: (Step 2 only)
Inputs: $N=39 ; ~ I=3 ; ~ P V=-3.00 ; ~ P M T=0$. Output: $F V=\$ 9.50$.
79. FV of annuity due

Answer: d Diff: M

```
There are a few ways to do this. One way is shown below.
To get the value at t = 5 of the first 5 payments:
BEGIN mode
N = 5
I = 11
PV = 0
PMT = -3,000
FV = $20,738.58
Now add on to this the last payment that occurs at t = 5.
$20,738.58 + $3,000 = $23,738.58 \approx $23,739.
```

80. FV of an annuity

Answer: c Diff: M
To calculate the solution to this problem, change your calculator to BEGIN mode. Then enter $N=35, I=10, P V=0$, and PMT $=3000$. Solve for $F V=\$ 894,380$. Add the last payment of $\$ 3,000$, and the value at $t=35$ is $\$ 897,380$.

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81. Effective annual rate

Answer: a Diff: M

Time Line:

$$
\mathrm{EAR}=\text { ? }
$$



Tabular solution:
$\$ 20,000=\$ 444.89\left(\right.$ PVIFA $\left._{i}, 60\right)$
PVIFA $_{i, 60}=44.9549$
$i=1 \%$.
$\operatorname{EAR}=(1.01)^{12}-1.0=1.12681-1.0=0.1268=12.68 \%$.
Financial calculator solution:
Calculate periodic rate and nominal rate
Inputs: $\mathrm{N}=60 ; \mathrm{PV}=-20,000 ; \mathrm{PMT}=444.89 ; \mathrm{FV}=0$.
Output: $I=1.0 . \quad \mathrm{NOM} \%=1.0 \% \times 12=12.00 \%$.
Use interest rate conversion feature Inputs: $P / Y R=12 ; N O M \%=12.0$. Output: $E F F \%=E A R=12.68 \%$.
82. Effective annual rate Answer: e Diff: M

```
Given: Loan Value = $12,000; Loan Term = 10 years (120 months);
Monthly Payment = $150.
N = 120
PV = -12,000
PMT = 150
FV = 0
Solve for I/YR = 0.7241 }\times12=8.6892%. However, this is a nominal
rate. To find the effective rate, enter the following:
NOM% = 8.6892
P/YR = 12
Solve for EFF% = 9.0438%.
```

83. Required annuity payments

Answer: c Diff: M

```
Enter CFs:
CFO}=
CF
CF2 = 1.6
CF3}=2.
CF4}=2.
CF5 = 2.8
I = 10%; NPV = $7.2937 million.
$1 + $7.2937 = $8.2937 million.
Now, calculate the annual payments. BEGIN mode
N = 5; I/YR = 10; PV = -8.2937; FV = 0; PMT = ? = $1.989 million.
```

84. NPV and non-annual discounting

Answer: b Diff: M


$$
\begin{aligned}
C F_{0} & =0 \\
C F_{1-3} & =0
\end{aligned}
$$

$\mathrm{CF}_{4-12}=-700$
$I=0.4167$
Solve for NPV $=-\$ 6,094.23$.
Therefore, the PV of payments under the proposed lease would be greater than the PV of payments under the old lease by $\$ 6,094.23$ - $\$ 5,840.61=$ $\$ 253.62$. Thus, your net worth would decrease by $\$ 253.62$.
85. FV under monthly compounding

Answer: d Diff: M
Step 1 Calculate the $F V$ at $t=3$ of the first deposit. Enter $\mathrm{N}=36, \mathrm{I} / \mathrm{YR}=12 / 12=1, \mathrm{PV}=-10000$, and $\mathrm{PMT}=0$. Solve for $F V=\$ 14,308$.

Step 2 Calculate the $F V$ at $t=3$ of the second deposit. Enter $N=24, \mathrm{I} / \mathrm{YR}=12 / 12=1, \mathrm{PV}=-10000$, and $\mathrm{PMT}=0$. Solve for $F V=\$ 12,697$.

Step 3 Calculate the $F V$ at $t=3$ of the third deposit. Enter $N=12, I / Y R=12 / 12=1, \mathrm{PV}=-20000$, and $\mathrm{PMT}=0$. Solve for $\mathrm{FV}=\$ 22,537$.

Step 4 The sum of the future values gives you the answer, \$49,542.
86. FV under daily compounding

Answer: a Diff: M
The answer is a. Solve for $F V$ as $N=132, I=4 / 365=0.0110$, $\mathrm{PV}=-2,000, \mathrm{PMT}=0$, and solve for $\mathrm{FV}=?=\$ 2,029.14$.

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87. FV under non-annual compounding

Answer: d Diff: M

First, find the FV of Josh's savings as: $N=2 \times 26=52$, $=10 / 26=$ $0.3846, \mathrm{PV}=0, \mathrm{PMT}=-100$, and $\mathrm{FV}=$ ? $=\$ 5,744.29$. John's savings will have two components, a lump sum contribution of $\$ 1,500$ and his monthly contributions. The FV of his regular savings is: $N=2 \times 12=$ 24, $\mathrm{I}=10 / 12=0.8333, \mathrm{PV}=0, \mathrm{PMT}=-150$, and $\mathrm{FV}=$ ? $=\$ 3,967.04$. The FV of his previous savings is: $N=24, I=0.8333, \mathrm{PV}=-1,500$, $\mathrm{PMT}=0$, and $\mathrm{FV}=$ ? $=\$ 1,830.59$. Summing the components of John's savings yields $\$ 5,797.63$ which is greater than Josh's total savings. Thus, the most expensive car purchased costs $\$ 5,797.63$.
88. FV under quarterly compounding

Answer: c Diff: M

```
The effective rate is given by:
NOM% = 8
P/YR = 4
Solve for EFF% = 8.2432%.
The nominal rate on a semiannual basis is given by:
EFF% = 8.2432
P/YR = 2
Solve for NOM% = 8.08%.
The future value is given by:
N = 2.5 x 2 = 5
I = 8.08/2 = 4.04
PV = 0
PMT = -100
Solve for FV = $542.07.
```

89. PV under monthly compounding

Answer: b Diff: M
Start by calculating the effective rate on the second security:
$\mathrm{P} / \mathrm{YR}=12$
NOM\% = 10
Solve for $\mathrm{EFF} \%=10.4713 \%$.
Then, convert this effective rate to a semiannual rate:
$\mathrm{EFF} \%=10.4713$
$\mathrm{P} / \mathrm{YR}=2$
NOM\% = $10.2107 \%$.
Now, calculate the value of the first security as follows:
$\mathrm{N}=10 \times 2=20, \mathrm{I}=10.2107 / 2=5.1054, \mathrm{PMT}=500, \mathrm{FV}=0$, thus, $\mathrm{PV}=$ $-\$ 6,175.82$.
90. PV under non-annual compounding

Answer: c Diff: M

First, find the effective annual rate for a nominal rate of $12 \%$ with quarterly compounding: $P / Y R=4$, $N O M \%=12$, and $E F F \%=$ ? $=12.55 \%$. In order to discount the cash flows properly, it is necessary to find the nominal rate with semiannual compounding that corresponds to the effective rate calculated above. Convert the effective rate to a semiannual nominal rate as $P / Y R=2$, EFF\% $=12.55$, and $N O M \%=$ ? = 12.18\%. Finally, find the $P V$ as $N=2 \times 3=6, I=12.18 / 2=6.09$, $\mathrm{PMT}=$ 500, $\mathrm{FV}=0$, and $\mathrm{PV}=$ ? $=-\$ 2,451.73$.
91. Value of missing payments

Answer: d Diff: M
Find the $F V$ of the price and the first three cash flows at $t=3$.
To do this first find the present value of them.
$\mathrm{CF}_{0}=-5,544.87$
$\mathrm{CF}_{1}=100$
$\mathrm{CF}_{2}=500$
$\mathrm{CF}_{3}=750$
$I=9 ;$ solve for $N P V=-\$ 4,453.15$.
$\mathrm{N}=3$
$I=9$
$P V=-4,453.15$
$\mathrm{PMT}=0$
$\mathrm{FV}=\$ 5,766.96$.

Now solve for X .
$\mathrm{N}=17$
$I=9$
$P V=-5,766.96$
$\mathrm{FV}=0$
Solve for PMT = \$675.
92. Value of missing payments

Answer: c Diff: M

```
There are several different ways of doing this. One way is:
Find the future value of the first three years of the investment at
Year 3.
N = 3
I = 7.3
PV = -24,307.85
PMT = 2,000
FV = $23,580.68.
Find the value of the final $10,000 at Year 3.
N = 7
I = 7.3
PMT = 0
FV = 10,000
PV = -$6,106.63.
Add the two Year 3 values (remember to keep the signs right).
$23,580.68 + -$6,106.63 = $17,474.05.
Now solve for the PMTs over years 4 through 9 (6 years) that have a PV
of $17,474.05.
N = 6
I = 7.3
PV = -17,474.05
FV = 0
PMT = $3,700.00.
```

93. Value of missing payments Answer: d Diff: M

The project's cost should be the PV of the future cash flows. Use the cash flow key to find the $P V$ of the first 3 years of cash flows.
$C F_{0}=0, C F_{1}=100, C F_{2}=200, C F_{3}=300, I / Y R=10, N P V=\$ 481.59$.
The PV of the cash flows for Years 4-20 must be: $\$ 3,000-\$ 481.59=\$ 2,518.41$.

Take this amount forward in time 3 years:
$\mathrm{N}=3, \mathrm{I} / \mathrm{YR}=10, \mathrm{PV}=-2,518.41, \mathrm{PMT}=0$, solve for $\mathrm{FV}=\$ 3,352.00$.
This amount is also the present value of the 17-year annuity.
$\mathrm{N}=17, \mathrm{I} / \mathrm{YR}=10, \mathrm{PV}=-3,352, \mathrm{FV}=0$, solve for $\mathrm{PMT}=\$ 417.87$.
94. Amortization

Answer: d Diff: M

```
    N = 25 * 12
    I = 8.5/12
    PV = -125,000
    FV = 0
PMT = $1,006.53
Do amortization:
Enter: 1 INPUT 60 | AMORT
Interest = $51,375.85
Principal = $9,015.95
Balance = $115,984.05
Total Payments = 5 < 12 x $1,006.53
    = $60,391.80.
    % Repayment 
```

95. Amortization

Answer: a Diff: M
Given: Loan Value $=\$ 100,000$; Repayment Period $=12$ months;
Monthly Payment $=\$ 9,456$.
$\mathrm{N}=12$
$P V=-100,000$
PMT $=9,456$
$\mathrm{FV}=0$
Solve for $I / Y R=2.00 \% \times 12=24.00 \%$.
To find the amount of principal paid in the third month (or period), use the calculator's amortization feature. Enter: 3 INPUT 3 AMORT
(to activate the calculator's amortization feature).
Interest $=\$ 1,698.84$
Principal = \$7,757.16
Balance $=$ \$77,181.86
96. Amortization

Answer: c Diff: M

```
We will use the amortization feature of the HP-10B.
Enter the loan details:
N = 30 < 12 = 360
I = 9/12 = 0.75
PV = -90,000
FV = 0
PMT = $724.16.
Total payments in the first 2 years are $724.16 x 24 = $17,379.85.
Now get the interest:
I INPUT 24 AMORT
Interest = $16,092.44
Percentage of first two years that is interest is:
$16,092.44/$17,379.85 = 0.9259 = 92.59%.
```


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Find the payment of the mortgage first. $N=48 ; ~ I / Y R=12 / 12=1$; $\mathrm{PV}=$ 20000; $\mathrm{FV}=0$; solve for PMT. PMT $=\$ 526.6767$. Now use the amortization feature of your calculator. Enter 1 INPUT 36, $\square$ AMORT. Then, press $=3$ times for the remaining balance amount, which is $\$ 5,927.79$.
98. Amortization

Answer: c Diff: M
First, find the payment: Enter $N=360$, $I / Y R=9 / 12=0.75, \mathrm{PV}=$ $-250,000$, and $F V=0$. Solve for $P M T=\$ 2,011.56$. On your calculator, use the amortization feature by entering: 1 INPUT 60, $\square$ AMORT, and press the $=3$ times. The remaining balance is $\$ 239,700$.
99. Remaining balance on mortgage

Answer: c Diff: M

```
Solve for the monthly payment as follows:
N = 30 x 12 = 360
I = 9/12 = 0.75
PV = -175,000
FV = 0
Solve for PMT = $1,408.09/month.
Use the calculator's amortization feature to find the remaining
principal balance:
5 years = 5 < 12 = 60 payments.
1 INPUT 60 AMORT
Interest = $77,275.55
Principal = $ 7,209.85
Balance = $167,790.15
```

100. Remaining balance on mortgage

Answer: d Diff: M

```
Solve for the monthly payment as follows:
N = 30 x 12=360
I = 8/12 = 0.667
PV = -150,000
FV = 0
Solve for PMT = $1,100.65/month.
Use the calculator's amortization feature to find the remaining
principal balance:
3 }\times12=36 payment
1 INPUT 36 AMORT
Interest = $ 35,543.52
Principal = $ 4,079.88
Balance = $145,920.12
```

```
Solve for the monthly payment as follows:
N = 12 x 15 = 180
I = 8.5/12 = 0.7083
PV = -160,000
FV = 0
PMT = $1,575.58.
Use the calculator's amortization feature to find the remaining
principal balance:
1 INPUT 36 ■ AMORT
Interest = $ 38,658.34
Principal = $ 18,062.54
Balance = $141,937.46
```

102. Required annuity payments

Answer: C Diff: $T$


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Infl. $=5 \%$
104. Required annuity payments

Answer: a Diff: T


Value of cash outflows:
Age $85 \quad C F 0=(\$ 50,000)$
$C F_{1}=(55,000)=(-50,000) 1.1$
$C F_{2}=(60,500)=(-50,000)(1.1)^{2}$
$\mathrm{CF}_{3}=(66,550)=(-50,000)(1.1)^{3}$
$C F_{4}=(73,205)=(-50,000)(1.1)^{4}$
$\mathrm{CF}_{5}=(200,000)$
Solve for NPV at $8 \%=(\$ 395,548.96)$.
Value of $\$ 100,000$ at age 85: \$100,000(1.08) ${ }^{10}=\$ 215,892.50$.
Shortfall at age $85=\$ 215,892.50-\$ 395,548.96=(\$ 179,656.46)$.
Calculate annual payments to equal this shortfall:
$\mathrm{N}=10 ; \mathrm{I} / \mathrm{YR}=8 ; \quad \mathrm{PV}=0 ; \quad \mathrm{FV}=179,656.46$.
Solve for $\operatorname{PMT}=\$ 12,401.59$.
105. Required annuity payments Answer: c Diff: T


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106. Required annuity payments

Answer: a Diff: $T$


Step 1 Determine college costs:
College costs will be $\$ 15,000(1.05)^{10}=\$ 24,433$ at $t=10$, $\$ 15,000(1.05)^{11}=\$ 25,655$ at $t=11, \$ 15,000(1.05)^{12}=\$ 26,938$ at $t=12$, and $\$ 15,000(1.05)^{13}=\$ 28,285$ at $t=13$.

Step 2 Determine PV of college costs at $t=10$ :
Enter the cash flows into the cash flow register as follows: $C F_{0}=24433, C F_{1}=25655, C F_{2}=26938, C F_{3}=28285, I=12$, and solve for NPV = \$88,947.

Step 3 Determine the value of their savings at $t=4$ as follows: $\mathrm{N}=4, \mathrm{I}=12$, $\mathrm{PV}=8000$, and $\mathrm{PMT}=3000$. Solve for $\mathrm{FV}=$ \$26,926.

Step 4 Determine the value of the annual contributions from $t=5$ through $t=10$ :
$\mathrm{N}=6, \mathrm{I}=12, \mathrm{PV}=-26926$, and $\mathrm{FV}=88947$. Solve for $\mathrm{PMT}=$ \$4,411.

```
Step 1 Convert the 9 percent monthly rate to an annual rate.
    Enter NOM% = 9, P/YR = 12, and solve for EFF% = 9.3807%.
Step 2 Compute the amount accumulated by age 40. Remember to change
    P/YR from 12 to 1. Set calculator to BEGIN mode. Then, enter
    N = 15, I = 9.3807, PV = 0, and PMT = 2000. Solve for FV =
    $66,184.35.
Step 3 John needs $3 million in 25 years. Find the PV of this amount
    today. Remember to change your calculator back from BEGIN to
    END mode. Enter N = 25, I = 12, FV = 3000000, and PMT = 0.
    Solve for PV = $176,469.92.
Step 4 Find the shortfall today, the difference between the present
    value of what he needs in 25 years and the present value of
    what he's accumulated today. $176,469.92 - $66,184.35=
    $110,285.57.
Step 5 Find the annuity needed to cover this shortfall. Since the contributions begin today this is an annuity due, so the calculator must be set up in BEGIN mode. (Remember to change your calculator back from BEGIN to END mode after working this problem.) Set calculator to BEGIN mode. Then, enter PV = -110285.57, \(\mathrm{N}=26, \mathrm{I}=12\), and \(\mathrm{FV}=0\). Solve for \(\mathrm{PMT}=\) \(\$ 12,471.31 \approx \$ 12,471\).
```

```
Step 1 Calculate the cost of tuition in each year:
    College Cost Today = $15,000, Inflation = 5%.
    $15,000(1.05)}\mp@subsup{}{}{6}=$20,101.43(1)=$20,101.43
    $15,000(1.05)7 = $21,106.51(1) = $21,106.51.
    $15,000(1.05)8 = $22,161.83(2) = $44,323.66.
    $15,000(1.05)9}=$23,269.92(2) = $46,539.85
    $15,000(1.05)10 = $24,433.42(1) = $24,433.42.
    $15,000(1.05)11=$25,655.09(1) = $25,655.09.
Step 2 Find the present value of college costs at t = 0:
    CF0 = 0
    CF
    CF6 = 20,101.43
    CF7 = 21,106.51
    CF
    CF9 = 46,539.85
    CF
    CF
    I = 12; solve for NPV = $69,657.98.
Step 3 Find the PV of the $25,000 gift received in Year 3:
    N = 3
    I = 12
    PMT = 0
    FV = 25,000
    Solve for PV = -$17,794.51.
Step 4 Calculate the PV of the net amount needed to fund college
    costs:
    $69,657.98 - $17,794.51 = $51,863.47.
Step 5 Calculate the annual contributions:
    BEGIN Mode
    N = 12
    I = 12
    PV = -51,863.47
    FV = 0
    Solve for PMT = $7,475.60.
```

```
First, what will be the present value of the college costs plus the
$50,000 nest egg as of the current date?
The first tuition payment, CFo, will equal $10,000 }\times(1.06)\mp@subsup{}{}{15}
$23,965.58. Each tuition payment will increase by 6%, hence CF = =
$25,403.52, CF C = $26,927.73, CF % = $28,543.39, and CF C = $50,000 (the
nest egg). The present value as of the time she enters college, at 8%,
is $129,983.70.
Now, what payments are needed every year until then?
N = 15
I = 8
PV = 10,000
FV = -129,983.70
Solve for PMT = $3,618.95.
```

110. Required annuity payments

Answer: b Diff: T

Find the present value of the cost of college at $t=10$. Use the cash flow register and remember that college costs increase each year by the rate of inflation. $\mathrm{t}=10: \quad \mathrm{CF}=\$ 35,000$.
$\mathrm{t}=11: \quad \mathrm{CF}_{1}=\$ 35,000 \times 1.07=\$ 37,450.00$.
$t=12: \quad C F_{2}=\$ 35,000 \times(1.07)^{2}=\$ 40,071.50$.
$t=13: \quad \mathrm{CF}_{3}=\$ 35,000 \times(1.07)^{3}=\$ 42,876.51$.
I = 9; solve for NPV = \$136,193.71.
Now figure out the amount of payments they should make:
$\mathrm{N}=10$
I = 9
$P V=20,000$
$\mathrm{FV}=-136,193.71$
$\mathrm{PMT}=\$ 5,847.88 \approx \$ 5,848$.
111. Required annuity payments

Answer: a Diff: T

```
Step 1 Calculate the cost of tuition in each year:
            t = 15: $25,000(1.05)15 = $51,973.20.
            t = 16: $25,000(1.05)16 = $54,571.86 < 2 = $109,143.73.
            t = 17: $25,000(1.05)17 = $57,300.46 < 2 = $114,600.92.
            t = 18: $25,000(1.05)18 = $60,165.48 < 2 = $120,330.96.
            t = 19: $25,000(1.05)19 = $63,173.75.
Step 2 Find the present value of these costs at t = 15:
            CF0 = 51,973.20
            CF1 = 109,143.73
            CF2 = 114,600.92
            CF3 = 120,330.96
            CF4 = 63,173.75
            I = 12; solve for NPV = $366,579.37.
Step 3 Calculate the FV of Grandma's deposits at t = 15:
            Older son: $10,000(1.12)18 = $ 76,899.66 (Deposit was made 3
                years ago.)
            Younger son: $10,000(1.12)17 = $ 68,660.41 (Deposit was made 2
                                    years ago.)
                    Total = $145,560.07
Step 4 Calculate net total amount needed at t = 15
    $366,579.37 - $145,560.07 = $221,019.30.
Step 5 Calculate the annual required deposits:
    N = 15
    I = 12
    PV = 0
    FV = 221,019.30
    Solve for PMT = -$5,928.67.
```

112. FV of annuity due

Answer: a Diff: T

First, convert the 9 percent return with quarterly compounding to an effective rate of $9.308332 \%$. With a financial calculator, $N O M \%=9$, $P / Y R=4, E F F \%=9.308332 \%$. (Don't forget to change $P / Y R=4$ back to $P / Y R=1$.$) \quad Then calculate the F V$ of all but the final payment. BEGIN $\operatorname{MODE}(1 \mathrm{P} / \mathrm{YR}) \mathrm{N}=9, \mathrm{I} / \mathrm{YR}=9.308332, \mathrm{PV}=0, \mathrm{PMT}=1500$, and solve for $\mathrm{FV}=\$ 21,627.49$. You must then add the $\$ 1,500$ at $t=9$ to find the answer, \$23,127.49.
113. FV of investment account

Answer: b Diff: T

```
We need to figure out how much money we would have saved if we didn't
pay for the college costs.
N = 40
I = 10
PV = 0
PMT = -12,000
Solve for FV = $5,311,110.67.
Now figure out how much we would use for college costs. First get the
college costs at one point in time, t = 20 using the cash flow
register.
CF0 = 58,045
CF1 = 62,108
CF2 = 66,456 x 2 = 132,912 (two kids in school)
CF3}=71,108\times2=142,21
CF4 = 76,086
CF5 = 81,411
I = 10; NPV = $433,718.02.
This is the value of the college costs at year t = 20. What we want is
to know how much this is at t = 40
N = 20
I = 10
PV = -433,718.02
PMT = 0
Solve for FV = $2,917,837.96.
The amount in the nest egg at t = 40 is the amount saved less the
amount spent on college;
$5,311,110.67 - $2,917,837.96 = $2,393,272.71 \approx $2,393,273.
```

114. Effective annual rate

Answer: c Diff: T

```
Time Line:
```



```
Numerical solution:
Step 1 Find the effective annual rate (EAR) of interest on the bank
        deposit
        EAR Daily = (1 + 0.080944/365)365-1=8.43%.
Step 2 Find the EAR of the investment
            $8,000 = $10,000/(1 + i) 2.25
            (1+i)}\mp@subsup{)}{}{2.25}=1.2
                1+i=1.25(1/2.25)
                        1+i=1.10426
                        i = 0.10426 \approx 10.43%
Step 3 Difference = 10.43%-8.43% = 2.0%
Financial calculator solution:
```



```
Inputs: P/YR = 365; NOM% = 8.0944; Output: EFF% = EAR = 8.43%.
Calculate EAR of the equal risk investment
Inputs: N = 2.25; PV = -8,000; PMT = 0; FV = 10,000.
Output: I = 10.4259 \approx 10.43%.
Difference: 10.43% - 8.43% = 2.0%.
```

```
To compare these alternatives, find the present value of each strategy
and select the option with the highest present value.
Option 1 can be valued as an annuity due. On your financial calculator
enter:
BEGIN mode (to indicate payments will be received at the start of the
period)
N = 12
I = 12/12 = 1
PMT = -1,000
FV = 0
Solve for PV = $11,367.63.
Option 2 can be valued as a lump sum payment to be received in the
future. On your financial calculator enter:
END mode (to indicate the lump sum will be received at the end of the
year)
N = 2
I = 12/2 = 6
PMT = 0
FV = 12,750
Solve for PV = $11,347.45.
Option 3 can be valued as a series of uneven cash flows. The cash
flows at the end of each period are calculated as follows:
CFO = $ 0.00.
CF
CF2 = $ 800.00 x (1.20) = $ 960.00.
CF3}=$ 960.00 x (1.20) = $1,152.00.
CF
CF5}=$1,382.40 x (1.20) = $1,658.88
CF6 = $1,658.88 x (1.20) = $1,990.66.
CF7 = $1,990.66 x (1.20) = $2,388.79.
CF8 = $2,388.79 x (1.20) = $2,866.54.
To find the present value of this cash flow stream using your financial
calculator enter:
END mode (to indicate the cash flows will occur at the end of each
period)
0 CFj; 800 CFj; 960 CFj; 1,152 CFj; 1,382.40 CFj; 1,658.88 CFj;
1,990.66 CFj; 2,388.79 CFj; 2,866.54 CFj (to enter the cash
flows);I/YR = 12/4 = 3; solve for NPV = $11,267.37.
Choose the alternative with the highest present value, and hence select
Choice 1 (Answer a).
```

116. PMT and quarterly compounding

Answer: b Diff: T
0 i $=2 \% 1$
+400

Now, determine the value of each of the receipts, remembering that this is an annuity due. With a financial calculator input the following: $\mathrm{N}=10 ; \mathrm{I}=8.2432 ; \mathrm{PV}=-77,508.78$; and $\mathrm{FV}=0$.
Solve for $\operatorname{PMT}=\$ 10,788.78 \approx \$ 10,789$.
117. $P V$ of an uneven $C F$ stream

Answer: d Diff: T

```
Parent's savings: Health Care Costs, Years 19-22:
N = 18 -$1,000(1.1) 19 = -$6,115.91
I = 6 -$1,000(1.1) 20 = -$6,727.50
PMT = 100 -$1,000(1.1) 21 = -$7,400.25
FV = 0 -$1,000(1.1) 22 = -$8,140.27
Solve for PV = $1,082.76.
```



```
-$8,554.84 PV of Health care costs
1,082.76 PV of parents' savings
\underline{-$7,472.08}}\mathrm{ Lump sum government must set aside
CF0 = 0
CF1-18}=
CF19 = -6,115.91
CF20}=-6,727.5
CF21 = -7,400.25
CF22 = - 8,140.27
I = 6
Solve for NPV = -8,554.84 = PV of Health care costs.
Consequently, the government must set aside $8,554.84 - $1,082.76 =
$7,472.08.
Alternatively,
CFo = 0
CF 1-18 = 100
CF19 = -6,115.91
CF20}=-6,727.5
CF21 = -7,400.25
CF22 = -8,140.27
I = 6
Solve for NPV = -$7,472.08 = Lump sum government must set aside now.
```

$P V=F V_{n} / e^{\text {in }}=\$ 100,000 / e^{0.09(6)}=\$ 100,000 / 1.7160=\$ 58,275$.
2E-119. FV continuous compounding
Answer: a Diff: M

Daily compounding:
$\mathrm{FV}_{2}=\mathrm{PV}(1+0.06 / 365)^{365(2)}=\$ 1,000(1.12749)=\$ 1,127.49$
Continuous compounding:
$\begin{aligned} & V_{2}=P V \mathrm{e}^{\text {in }}=\$ 1,000\left(\mathrm{e}^{0.059(2)}\right)=\$ 1,000(1.12524)= \\ & \\ & \text { Difference between accounts }\end{aligned}$
2E-120. Continuous compounded interest rate
Answer: a Diff: M

Calculate the growth factor using $P V$ and $F V$ which are given:
$F V_{n}=P V e^{i n} ; \$ 19,000=\$ 14,014 e^{i 4}$
$\mathrm{e}^{\mathrm{i} 4}=1.35579$.

Financial calculator solution:
Take the natural logarithm of both sides:
i(4) $\ln \mathrm{e}=\ln 1.35579$.
The natural $\log$ of $e=1.0$.
Inputs: 1.35579. Press LN key. Output: LN = 0.3044.
$i(4)=0.3044$
$i=0.0761=7.61 \%$.

2E-121. Payment and continuous compounding Answer: d Diff: M


Account with continuous compounding
$-1,000$
$\mathrm{FV}_{\mathrm{C}}=$ ? $=1,233.70$
Account with
semiannual
compounding

Financial calculator solution:
Step 1 Calculate the $F V$ of the $\$ 1,000$ deposit at 7\% with continuous compounding:
Using ex key:
Inputs: $X=0.21$; press $e^{x}$ key. Output: $e^{x}=1.2337$.
$F V_{n}=\$ 1,000 e^{0.07(3)}=\$ 1,000(1.2337)=\$ 1,233.70$.
Step 2 Calculate the $P V$ or initial deposit:
Inputs: $\mathrm{N}=6 ; \mathrm{I}=4 ; \mathrm{PMT}=0 ; \mathrm{FV}=1,233.70$.
Output: $P V=\$ 975.01$.

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Determine the effective annual rates.
a) $12.5 \%$ annually $=12.5 \%$.

Financial calculator solution:
Use interest rate conversion feature or $e^{x}$ exponential key on some calculators. Calculate EAR with continuous compounding.
b) Inputs: $\mathrm{P} / \mathrm{YR}=2$; $\mathrm{NOM} \%=12$. Output: $\mathrm{EFF} \%=\mathrm{EAR}=12.36 \%$.
b) $12.0 \%$ semiannually $=\left(1+\frac{0.12}{2}\right)^{2}-1.0=0.1236=12.36 \%$.
C) Inputs: Continuous compounding CONT; $\mathrm{NOM} \%=11.5$. Output: EFF\% = EAR $=12.187 \% \approx 12.19 \%$.
c) $11.5 \%$ continuously $=e^{0.115}-1.0=0.1219=12.19 \%$.

2E-123. Continuous compounding
Answer: b Diff: M
Time line:


Numerical solution:
(Constant $e=2.7183$ rounded.)

$$
\begin{aligned}
\$ 5,438 & =\mathrm{PVe}^{0.10(10)}=\mathrm{PVe}^{1.0} \\
\mathrm{PV} & =\$ 5,438 / \mathrm{e} \\
& =\$ 5,438 / 2.7183=\$ 2,000.52 \approx \$ 2,000
\end{aligned}
$$

Financial calculator solution:
Use interest rate conversion feature or $e^{x}$ exponential key on some calculators. Calculate EAR with continuous compounding.
Inputs: Continuous compounding CONT; NOM\% = 10 .
Output: EFF\% = EAR = 10.52\%.
Using ex key:
Inputs: $X=0.10 ;$ press $e^{x}$ key. Output: $e^{x}=1.1052$.
$\operatorname{EAR}=1.1052-1.0=0.1052=10.52 \%$.
Calculate $P V$ of $F V$ discounted continuously
Inputs: $\mathrm{N}=10 ; \mathrm{I}=10.52 ; \mathrm{PMT}=0 ; \mathrm{FV}=5,438$.
Output: PV = $-\$ 2,000$.

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2E-124. Continuous compounding
Answer: d Diff: M

```
Numerical solution:
\(e^{(0.04)(10)}=\left(1+\frac{i}{2}\right)^{20}\)
        \(e^{0.4}=\left(1+\frac{i}{2}\right)^{20}\)
        \(e^{0.02}=1+\frac{i}{2}\)
        \(1.0202=1+\frac{i}{2}\)
        \(\frac{i}{2}=0.0202\)
        \(i=0.0404=4.04 \%\).
Financial calculator solution:
Use interest rate conversion feature or \(e^{x}\) exponential key on some
calculators. Calculate EAR with continuous compounding.
Inputs: Continuous compounding: CONT; NOM\% = 4.
Output: EFF\% = EAR = 4.08\%.
Calculate the nominal annual rate from the effective rate using
continuous compounding:
Inputs: Semiannual compounding: \(\mathrm{P} / \mathrm{YR}=2\); \(\mathrm{EFF} \%=\mathrm{EAR}=4.08\).
Output: \(\mathrm{NOM} \%=4.04 \%\).
```

2E-125. Continuous compounding Answer: b Diff: M
Time Line:


Numerical solution:
$\$ 1,000=\mathrm{PVe}^{0.10(10)}=\mathrm{PVe}^{1.0}$
$P V=\$ 1,000 / e=\$ 1,000 / 2.7183=\$ 367.88 \approx \$ 368$.
Financial calculator solution:
Use interest rate conversion feature or $e^{x}$ exponential key on some
calculators. Calculate EAR with continuous compounding.
Inputs: Continuous compounding: CONT; NOM\% = 10
Output: EFF\% = EAR = 10.52\%.
Using ex key:
Inputs: $X=0.10$; press $e^{x}$ key. Output: $e^{x}=1.1052$.
EAR $=1.1052-1.0=0.1052=10.52 \%$.
Calculate $P V$ of $F V$ discounting at the $E A R$ :
Inputs: $\mathrm{N}=10 ; \mathrm{I}=10.52 ; \mathrm{PMT}=0 ; \mathrm{FV}=1,000$.
Output: $\mathrm{PV}=-\$ 367.78 \approx \$ 368$.

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2E-126. Continuous compounding Answer: b Diff: M

```
Time Line:
```



```
Financial calculator solution:
(Note: We carry the EAR to 5 decimal places for greater precision in
order to come closer to the correct exponential solution.)
Calculate the EAR of 5% compounded continuously
Inputs: Continuous compounding CONT; NOM% = 5.
Output: EFF% = EAR = 5.127%.
Using ex key:
Inputs: X = 0.05; press ex key. Output: }\mp@subsup{e}{}{x}=1.05127
EAR = 1.05127-1.0 = 0.05127 = 5.127%.
Calculate FV compounded continuously at EAR = 5.127%
Inputs: N = 20; I = 5.127; PV = -15,000; PMT = 0.
Output: FV = $40,773.38\approx$40,774.
```

