

CHAPTER 2 GRAPHS AND FUNCTIONS

Section 2.1 Rectangular Coordinates and Graphs

For the points P and Q, find (a) the distance d(P, Q) and (b) the coordinates of the midpoint of the segment PQ.

1. P(3, -3), Q(-5, 5)2. P(6, -3), Q(-3, 2)3. P(4, -6), Q(-1, 6)4. P(4, -1), Q(6, 3)5. P(-7, 5), Q(-2, -7)6. P(r, s), Q(2r, -2s)7. $P(\sqrt{5}, \sqrt{3}), Q(-\sqrt{5}, 2\sqrt{3})$ 8. $P(3\sqrt{7}, 2\sqrt{6}), Q(-\sqrt{7}, 5\sqrt{6})$

Determine whether the three points are the vertices of a right triangle.

9. (2, 5), (2, 9), (4, -1)10. (-8, 0), (-2, -5), (3, 1)11. (10, 4), (4, 2), (5, -1)12. (2, 4), (3, 7), (6, 8)

Determine whether the three points are collinear.

13.	(1, 4), (2, 5), and (4, 6)	14.	(1, 4), (2, 2), and (3, 0)
15.	(-1, 8), (-3, 1), and (4, -2)	16.	(1, 3), (-2, 2), and (3, 5)

For each equation, (a) list three ordered pairs that are solutions, and (b) graph the equation.

 17. y = 3x - 6 18. y = -x + 5

 19. $y = -x^2$ 20. xy = 9

 21. $y = \sqrt{x - 4}$ 22. y = |x| + 3

Find the coordinates of the other endpoint of each segment, given its midpoint and one endpoint.

23. Endpoint (3, -2), midpoint (7, 5) **24.** Endpoint $\left(4, -\frac{3}{2}\right)$, midpoint $\left(0, \frac{1}{2}\right)$

Solve each problem.

- **25.** Find all points (x, y) with x = y that are 3 units from (-1, 2).
- **26.** Find all points (x, y) satisfying x + y = 0 that are 4 units from (3, 1).
- 27. Find the coordinates of all points whose distance from (-2, 2) is 5 and whose distance from (5, 1) is 5.
- **28.** Find all values of y such that the distance between (-2, y) and (3, 7) is 13.

304 Chapter 2 Test Items

- **29.** Find all values of x such that the distance between (x, -4) and (-3, 2) is 7.
- **30.** Find the coordinates of the points that divide the line segment joining (-1, -7) and (2, 2) into three equal parts.

Section 2.2 Circles

In exercises 23-26, (a) find the center-radius form of the equation of each circle, and (b) graph it.

31.	Center (0, 0), radius 1	31.	Center $(-3, 1)$, radius 3
33.	Center (0, 1), radius 2	34.	Center $(-5, 1)$, radius 4

Solve each problem.

- **35.** Find the center-radius form of an equation of a circle with center (2, 2) and passes through the point (3, -5).
- **36.** Find the center-radius form of an equation of a circle with center (-7, -2) and tangent to the y-axis.
- **37.** Find the center-radius form of an equation of a circle with center (-1, 3) and passes through the point (2, 7).
- **38.** Find the center-radius form of an equation of a circle with endpoints of a diameter at (1, -3) and (7, -5).

Decide whether or not each equation has a circle as its graph. If it does, give the center and the radius.

39.	$x^2 + 8x + y^2 - 12y - 48 = 0$	40.	$x^2 + 2x - y^2 + 16y + 40 = 0$
41.	$x^2 - 3x + y^2 + 5y + 6 = 0$	42.	$x^2 - 7x + y^2 + 13y + 50 = 0$
43.	$x^2 + y^2 - 6x + 12y + 20 = 0$	44.	$4x^2 + 4y^2 + 40x + 8y + 79 = 0$
45.	$x^2 + 2x + y^2 + 5 = 0$	46.	$x^2 - 6x + y^2 + 2y = 10$

Solve each problem.

- 47. Suppose that a circle is tangent to both axes, is in the second quadrant, and has radius $\sqrt{7}$. Find the center-radius form of its equation.
- **48.** Find the center-radius form of the equation of the circle of smallest radius that contains the points (6, 7) and (8, 1) on its boundary.

Section 2.3 Functions

Decide whether each relation defines a function.

49.	$\{(-1, 1), (-2, 2), (0, 0)\}$	50.	$\{(3, 0), (2, 4), (1, 6), (3, -1)\}$
51.	$\{(-2, -1), (-1, -1), (0, 0), (1, 1)\}$	52.	$\{(3, 5), (2, 5), (1, 5)\}$
53.	$\{(1, 3), (2, -1), (-1, 4), (0, 4)\}$	54.	$\{(2, -4), (1, -2), (2, 2), (1, 3)\}$



Decide whether each relation defines y as a function of x. Give the domain and range for the functions.

- **61.** $y = -x^2$ **62.** $x^2 = y^2$ **63.** y = 7x + 5**64.** x y > 0
- **65.** $y = \sqrt{x-4}$ **66.** xy = 8
- **67.** $y = \frac{3}{x+5}$ **68.** $y \le 2x+1$

Let $f(x) = x^2 - 5x + 7$ and g(x) = 4x - 9. Find the following.

- **69.** *f*(0) **70.** *f*(-3)
- **71.** *f*(4) **72.** *g*(-2)

306 Chapter 2 Test Items

73.
$$f(k)$$
 74. $g(-x)$

75. f(a-2) **76.** g(4t+5)

For each function, find (a) f(-2) and (b) f(0).

77. $f = \{(-2, 7), (0, 3), (4, 7)\}$





81.
$$f(x) = \frac{4}{x^2 + 1}$$
 82. $f(x) = x^2 - 2x$

An equation that defines y as a function of x is given. (a) Solve for y in terms of x and replace y with the function notation f(x). (b) Find f(-4).

 83. 3x + 2y = 12 84. 5x - 8y = 3

 85. -3x + 7y = 2 86. $y + 3x^2 = -4$

Determine the intervals of the domain for which each function is (a) increasing, (b) decreasing, (c) constant.



Section 2.4 Linear Functions

Graph each line. Identify any constant functions. Give the domain and range.

89.	x + y = 6	90.	x - y = 1
91.	3x + 2y = 6	92.	5x - 4y = 20
93.	5x - 3y = 15	94.	4x + y = -6
95.	y = 2	96.	y + 6 = 0

Graph each vertical line. Give the domain and range of the relation.

97.
$$x - 4 = 0$$
 98. $x - 6 = 0$

Find the slope of the line satisfying the given conditions.

99.	through $(-2, 8)$ and $(5, -1)$	100.	through $(2, -11)$ and $(-4, -11)$
101.	through (0, 4) and (-5, -8)	102.	through (7, 2) and (7, -11)
103.	through (-4, 2) and (2, -3)	104.	through (-7, 20) and (-4, 7)
105.	horizontal, through (-8, -3)	106.	vertical, through $(-8, -3)$
107.	vertical, through (0, 0)	108.	horizontal, through (6, 15)

Graph the line passing through the given point and having the indicated slope. Plot two points on the line.

109.	through $(-1, 4), m = \frac{2}{3}$	110.	through (2, -3), $m = -\frac{1}{2}$
111.	through $(2, -4), m = 2$	112.	through $(2, -1), m = -4$
113.	through $(5, -1), m = 0$	114.	through $(3, -2)$, undefined slope

Section 2.5 Equations of Lines; Curve Fitting

Write an equation for the line described. Give answers in standard form for Exercises 115 - 122 and in slope-intercept form (if possible) for Exercises 123 - 128.

115. through (-1, 4), m = 2**116.** through (-2, 5), m = -4**117.** through $(-3, 2), m = -\frac{1}{2}$ **118.** through $(0, -8), m = -\frac{4}{7}$ **119.** through (3, -4), m = 0**120.** through (5, -3), undefined slope**121.** through (9, -2) and (10, -5)**122.** through (-3, -2) and (-5, -1)**123.** through (-2, -4) and (-2, -7)**124.** through (0, 2) and (0, -6)

125.	horizontal, through (-4, 5)	126.	vertical, through $(-3, -2)$
127.	x-intercept 2, y-intercept –1	128.	<i>x</i> -intercept – 4, <i>y</i> -intercept – 6
Give i	the slope and y-intercept of each line.		
129.	2x + y = 4	130.	3x - 2y = 5
131.	2x + 7y = 14	132.	y + 5 = 0
Write	an equation (a) in standard form and (b) in slope-int	ercept	form for the line described.
133.	through $(2, -3)$; parallel to $x + 2y = 6$	134.	through (-2, 3); parallel to $4x - 3y = 8$

135. through (0, 4); parallel to 5x + y = 6 **136.** through (-1, 0); perpendicular to 2x - y = 3

 137. through (5, -3); perpendicular to 3x - 2y = 6 **138.** through (-1, 7); perpendicular to x = 4

In each problem, assume that the data can be approximated fairly closely by a straight line. Find the equation of the

line.

- **139.** When a certain industrial pollutant is introduced into a river, the reproduction of catfish declines. In a given period of time, 3 tons of pollutant results in a fish population of 37,000. Also, 12 tons of the pollutant results in a fish population of 28,000. Write an equation expressing y, the number of fish, in terms of x, the amount of the pollutant.
- 140. If the Republicans win 45% of the two party vote, they win 32.5% of the seats in the House of Representatives. If they win 60% of the vote, they get 70% of the seats. Write an equation expressing y, the percent of the seats, in terms of x, the percent of the vote.
- **141.** The radius bone goes from the wrist to the elbow. For females a radius bone 24 cm long corresponds to a height of 167 cm, while a radius bone 26 cm long corresponds to a height of 174 cm.
 - **a.** Write a linear equation showing how the height h of a female corresponds to the length r of her radius bone.
 - **b.** Estimate the height of a female having a radius bone 23 cm long; 27 cm long.
 - c. Estimate the length of a radius bone for a height of 170 cm.
- 142. The owner of an appliance store found that in 1980, year 0, the sales were \$850,000. In 1985, year 5, sales had increased to \$1,262,500.
 - **a.** Write an equation expressing the sales *y* in terms of the year *x*.
 - **b.** What were the sales in 1983?
 - **c.** Estimate the sales in 1988.
- **143.** The quantity *x* demanded for a certain commodity is 200 units when the unit price is set at \$90. The quantity demanded is 1200 units when the unit price is \$40. If *y* represents the price, find an equation for the price of an item in terms of the number of items demanded.
- **144.** The total sales of a company in millions of dollars is approximately linear with respect to time in years. Sales in 1985 were \$2.4 million, whereas sales in 1990 amounted to \$7.4 million.
 - **a.** Find an equation giving y, the amount of sales in millions of dollars, in year x, where x = 0 corresponds to 1985.
 - **b.** Estimate the sales in 1988.

Section 2.6 Graphs of Basic Functions

Graph each piecewise-defined function.

$$145. \quad y = \begin{cases} 5 \text{ if } x > 4 \\ 0 \text{ if } x = 4 \\ 3 \text{ if } x < 4 \end{cases}$$

$$146. \quad y = \begin{cases} |x| & \text{ if } x < 5 \\ x + 2 \text{ if } x \ge 5 \end{cases}$$

$$147. \quad y = \begin{cases} -3x + 4 \text{ if } x \le 1 \\ 2x - 1 & \text{ if } x > 1 \end{cases}$$

$$148. \quad y = \begin{cases} -|x| & \text{ if } x < 2 \\ x - 4 \text{ if } x \ge 2 \end{cases}$$

$$149. \quad y = \begin{cases} -\frac{x^2}{2} + 2 \text{ if } x \le 2 \\ \frac{x}{2} & \text{ if } x > 2 \end{cases}$$

$$150. \quad y = \begin{cases} 2x & \text{ if } x \le -2 \\ x + 1 & \text{ if } -2 < x \le 3 \\ x - 1 & \text{ if } x > 3 \end{cases}$$

Graph each function. Give the domain and range.

151.
$$f(x) = \llbracket x + 2 \rrbracket$$
152. $f(x) = \llbracket x \rrbracket + 2$
153. $f(x) = \llbracket \frac{1}{3}x \rrbracket$
154. $f(x) = \llbracket \frac{1}{3}x \rrbracket - 3$
155. $f(x) = 2\llbracket x \rrbracket$
156. $f(x) = -\llbracket 3x \rrbracket$

Solve each problem.

- **157.** To rent a bicycle built for two, it costs \$10 for the first hour and \$7 for each hour or part of an hour after the first hour. Graph the ordered pairs (number of hours, cost).
- **158.** A lawn care company charges a flat fee of \$20 for weekly yard service plus \$5 per hour or portion of an hour that it takes to service the yard. Graph the ordered pairs (number of hours, cost).
- **159.** A word processing company charges \$35 for each scientific paper processed plus \$6 per page for each page or portion of a page more than 5 pages. Graph the ordered pairs (number of pages, cost).
- 160. A parking garage charges \$2.50 for the first hour or portion of an hour and \$1.35 for each additional hour or portion of an hour. Let f(t) be a function that gives the parking charge (in dollars) for *t* hours. Find the following function values.

a.
$$f(1.5)$$
 b. $f(3)$ **c.** $f(4.75)$ **d.** $f(6.25)$

Section 2.7 Graphing Techniques

The graph of $y = x^2$ *is shown below.*



Explain how the graph of each equation could be obtained from the graph of $y = x^2$.

161. $y = x^2 - 4$ 162. $y = (x+5)^2$ 163. $y = (x-3)^2 - 2$ 164. $y = -(x+1)^2 + 3$

Without graphing, determine whether each equation has a graph that is symmetric with respect to the x-axis, the y-axis, the origin, or none of these.

 165. $4y^2 + 5x^2 = 20$ 166. $3y^3 - 4 = 5x$

 167. |x| = -y 168. $2x^3 = 5y^4$

169.
$$y = -2x^3$$
 170. $y = \frac{1}{x^2 - 4}$

171.
$$xy = 9$$
 172. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

Decide whether each function is even, odd, or neither.

173. $f(x) = x^4 - 3x^2$ **174.** $f(x) = 2x^3 + 7x$

175.
$$f(x) = x^4 + x^2 + x$$
 176. $f(x) = x |x|$

Graph each relation.

177. |y| = |x| **178.** $y = -(x-1)^2 + 4$

179. y = |x-4| **180.** y = |x+2|-3

181. $y = -(x+2)^3$ **182.** $y = \frac{1}{2}x^2 - 4$

- 183. Find the function g whose graph can be obtained by translating the graph of f(x) = 3x 1 up 3 units and to the right 2 units.
- 184. Find the function g whose graph can be obtained by translating the graph $y = (x-1)^3 + 2$ down 4 units and to the left 3 units.

Section 2.8 Function Operations and Composition

- Let $f(x) = 2x^2 + 4x$ and let g(x) = 3x 5. Find each of the following.
- 185. (f+g)(2) 186. (f-g)(-3)

 187. (fg)(1) 188. $\left(\frac{f}{g}\right)(-1)$
- **189.** (g-f)(t) **190.** (f+g)(3a)

For the pair of functions defined, form the indicated function and give its domain.

- **191.** f(x) = 3x + 1, g(x) = 5x 4; (f + g)(x)
- **192.** f(x) = 6x 3, g(x) = 2x + 5; (f g)(x)
- **193.** $f(x) = x^2, g(x) = x + 6; (f + g)(x)$
- **194.** f(x) = 6 2x, g(x) = 1 4x; (f g)(x)
- **195.** $f(x) = \sqrt{x+1}, g(x) = 3x; (fg)(x)$
- **196.** $f(x) = x 3, g(x) = 2x; \left(\frac{f}{g}\right)(x)$
- **197.** $f(x) = x^2 1, g(x) = x + 1; \left(\frac{f}{g}\right)(x)$
- **198.** $f(x) = 1 \sqrt{x}, g(x) = 1 + \sqrt{x}; (f + g)(x)$
- **199.** $f(x) = \sqrt{x+4}, g(x) = -x; \left(\frac{f}{g}\right)(x)$
- **200.** $f(x) = \sqrt{x} 2, g(x) = \sqrt{x} + 2; (fg)(x)$

312 Chapter 2 Test Items

Use the graphs to evaluate each expression.



For each of the functions defined as follows, find (a) f(x+h), (b) f(x+h) - f(x), and (c) $\frac{f(x+h) - f(x)}{h}$.

207. f(x) = 2x + 7 **208.** f(x) = 3 - 9x **209.** $f(x) = x^2 - 5x$ **210.** $f(x) = 9 - 4x^2$

Let $f(x) = 3x^2 - 2x + 1$ and g(x) = -2x + 5. Find each of the following.

211. $(f \circ g)(3)$ **212.** $(f \circ g)(-1)$
213. $(g \circ f)(-2)$ **214.** $(g \circ f)(5)$

The tables below give some selected ordered pairs for functions f and g.

x	-1	0	5	9
f(x)	4	3	-2	0

x	2	3	4	5
g(x)	3	5	7	9

Find each of the following.

215.	$(f \circ g)(3)$	216.	$(f \circ g)(5)$	217.	$(g\circ f)(-1)$
218.	$(g \circ f)(0)$	219.	$(f \circ f)(9)$	220.	$(g \circ g)(3)$

Find $(f \circ g)(x)$ and $(g \circ f)(x)$ for each pair of functions. Give the domains in Exercises 227 – 228.

221.
$$f(x) = 2x - 3; g(x) = -x + 8$$

222. $f(x) = 3x + 5; g(x) = -5x$

223.
$$f(x) = 2x^2 + 5$$
; $g(x) = -6x + 1$
224. $f(x) = 3x - 2$; $g(x) = -x^2 + 2x + 4$

225.
$$f(x) = 2x^2; g(x) = \frac{4}{x}$$
 226. $f(x) = \frac{1}{x}; g(x) = -\frac{2}{x}$

227. $f(x) = \sqrt{x+3}; g(x) = 4x^2 + 6$

228.
$$f(x) = 3x^2 - 2x; g(x) = 3\sqrt{x-1}$$