CALCULUS

ANTON   BIVENS   DAVIS

10TH EDITION

1. Find the average rate of change of y with respect to x for $y = f(x) = \dfrac{3}{x^4}$ over the interval [1, 7].

   A) 0.375   B) −0.500   C) −0.500   D) −17.993   E) 2.999

   Ans: B

   Difficulty: Easy

   Section: 2.1

2. Find the average rate of change of y with respect to x over the interval [1, 5]. $y = f(x) = 3x^3$

   A) 62   B) 95   C) 93   D) 74   E) 372

   Ans: C

   Difficulty: Easy

   Section: 2.1

3. Find the instantaneous rate of change of $y = 4x^2$ with respect to x at $x_0 = 7$.

   A) 8   B) 56   C) 14   D) 28   E) 22

   Ans: B

   Difficulty: Easy

   Section: 2.1

4. Find the instantaneous rate of change of $y = \dfrac{9}{x}$ with respect to x at $x_0 = 5$.

   A) −225   B) −8.9600   C) 0.3600   D) −0.3600   E) −0.0617

   Ans: D

   Difficulty: Medium

   Section: 2.1

5. Find the instantaneous rate of change of $y = -4x^7$ with respect to x at a general point x_0.

   A) $-28x_0^7$ B) $-4x_0$ C) $-4x_0^7$ D) $-4x_0^6$ E) $-28x_0^6$

   Ans: E

   Difficulty: Easy

   Section: 2.1

6. Find the instantaneous rate of change of $y = \dfrac{2}{x^3}$ with respect to x at a general point x_0.

   A) $\dfrac{-6}{x_0^3}$ B) $\dfrac{2}{x_0^4}$ C) $\dfrac{-6}{x_0^4}$ D) $\dfrac{6}{x_0^4}$ E) $\dfrac{6}{x_0^3}$

   Ans: C

   Difficulty: Medium

   Section: 2.1

7. Find the slope of the tangent line to the graph of $f(x) = 7x^4$ - 9 at a general point x_0.
   A) $28x_0^3$ - 9   B) $7x_0^3$ C) $28x_0^3$ D) $7x_0^3$ - 1   E) $7x_0^3$ - 9
   Ans: C
   Difficulty: Easy
   Section: 2.1

8. Answer true or false. The slope of the tangent line to the graph of $f(x) = -2x^2$ - 1 at $x_0 = 3$ is –13.
   Ans: False
   Difficulty: Easy
   Section: 2.1

9. Answer true or false. Use a graphing utility to graph $y = 3t^2$ on [0, 4]. If this graph represents a position versus time curve for a particle, the instantaneous velocity of the particle is increasing over the graphed domain.
   Ans: True
   Difficulty: Easy
   Section: 2.1

10. Use a graphing utility to graph $y = t^2$ - 7t + 10 on [0, 10]. If this graph represents a position versus time curve for a particle, the instantaneous velocity of the particle is zero at what time? Assume time is in seconds.
    A) 6s   B) 3s   C) 3.5s   D) 1.5s   E) 7s
    Ans: C
    Difficulty: Medium
    Section: 2.1

11. A rock is dropped from a height of 2,704 feet and falls toward earth in a straight line. In t seconds the rock drops a distance of $16t^2$ feet. What is the instantaneous velocity downward when it hits the ground?
    A)   116,985,856 feet/s   D)   32 feet/s
    B)   416 feet/s   E)   26 feet/s
    C)   208 feet/s
    Ans: B
    Difficulty: Easy
    Section: 2.1

12. Answer true or false. The magnitude of the instantaneous velocity is always less than the magnitude of the average velocity.
    Ans: False
    Difficulty: Easy
    Section: 2.1

13. Answer true or false. If a rock is thrown straight upward to a height of 26 feet from the ground, when it returns to earth its average velocity will be its initial velocity.
    Ans: False
    Difficulty: Easy
    Section: 2.1

14. Answer true or false. If an object is thrown straight upward with an instantaneous velocity of 35 m/s, its instantaneous velocity at the point where it stops rising is 0.
    Ans: True
    Difficulty: Easy
    Section: 2.1

15. An object moves in a straight line so that after t s its distance in mm from its original position is given by $s = 7t^3 + 4t$. Its instantaneous velocity at $t = 4$s is
    A) 336 mm   B) 1,348 mm   C) 5,380 mm   D) 340 mm   E) 116 mm
    Ans: D
    Difficulty: Medium
    Section: 2.1

16. Find the instantaneous rate of change of y with respect to x at $x_0 = 4$. $y = 6x^2 - 2$
    A) 48   B) 46   C) 24   D) 50   E) 96
    Ans: A
    Difficulty: Easy
    Section: 2.1

17. Find the instantaneous rate of change of y with respect to x at $x_0 = 81$. $y = \sqrt{x} - 2$
    A) $\dfrac{1}{18}$ B) $\dfrac{1}{9}$ C) $\dfrac{11}{9}$ D) $\dfrac{18}{17}$ E) $\dfrac{1}{81}$
    Ans: A
    Difficulty: Hard
    Section: 2.1

18. Let $f(x) = \dfrac{1}{x^2}$.   Find the average rate of change of y with respect to x over the interval [5, 6].
    Ans: $-\dfrac{11}{900}$
    Difficulty: Easy
    Section: 2.1

19. Let $f(x) = \dfrac{1}{x^2}$.   Find the instantaneous rate of change of y with respect to x at the point $x = 2$.

Ans: $-\dfrac{1}{4}$

Difficulty: Easy

Section: 2.1

20. Let $y = x^2 + 2$.   Find the average rate of change of y with respect to x over the interval $[-5, -1]$.

Ans: -6

Difficulty: Easy

Section: 2.1

21. Let $y = x^2 + 6$.   Find the instantaneous rate of change of y with respect to x at the point $x = -5$.

Ans: -10

Difficulty: Easy

Section: 2.1

22. Let $y = \dfrac{1}{x-1}$.   Find the average rate of change of y with respect to x over the interval $[2,4]$.

Ans: $-\dfrac{1}{3}$

Difficulty: Medium

Section: 2.1

23. Let $y = \dfrac{1}{x-3}$.   Find the instantaneous rate of change of y with respect to x at the point $x = 5$.

Ans: $-\dfrac{1}{4}$

Difficulty: Medium

Section: 2.1

24. Let $y = \dfrac{2}{x+2}$.   Find the average rate of change of y with respect to x over the given interval $[3,6]$.

Ans: $-\dfrac{1}{20}$

Difficulty: Medium

Section: 2.1

25. Let $y = \dfrac{1}{x+4}$. Find the instantaneous rate of change of y with respect to x at the point x = 1.

Ans: $-\dfrac{1}{25}$

Difficulty: Medium
Section: 2.1

26. Let $f(x) = \dfrac{1}{5-x}$. Find the slope of the tangent to the graph of f at a general point x_0 using limits and find the slope of the tangent line at $x_0 = 4$

Ans: $\displaystyle\lim_{x_1 \to x_0} \dfrac{1}{(5-x_1)(5-x_0)} = \dfrac{1}{(5-x_0)^2}$

The slope of the tangent line at $x_0 = 4$ is $\dfrac{1}{1}$.

Difficulty: Medium
Section: 2.1

27. Let $f(x) = \dfrac{1}{x-4}$. Find the slope of the tangent to the graph of f at a general point x_0 using limits and find the slope of the tangent at $x_0 = 5$.

Ans: $\displaystyle\lim_{x_1 \to x_0} \dfrac{-1}{(x_1-4)(x_0-4)} = \dfrac{-1}{(x_0-4)^2}$

The slope of the tangent line at $x_0 = 5$ is $-\dfrac{1}{1}$.

Difficulty: Medium
Section: 2.1

28. Let $f(x) = \dfrac{4}{x^4}$. Find the slope of the tangent to the graph of f at a general point x_0 using limits and find the slope of the tangent at $x_0 = -5$.

Ans: $\displaystyle\lim_{x_1 \to -5} \dfrac{\dfrac{4}{x_1^4} - \dfrac{4}{-5^4}}{x_1 + 5} = \lim_{x_1 \to -5} \dfrac{4\left(-5^4 - x_1^4\right)}{625 x_1^4 (x_1 + 5)} = -\dfrac{16}{x_0^5}$

The slope of the tangent line at $x_0 = -5$ is $\dfrac{16}{3,125}$.

Difficulty: Medium
Section: 2.1

29. Let $f(x) = 4x^3$. Find the slope of the tangent to the graph of f at a general point x_0 using limits and find the slope of the tangent at $x_0 = 2$.

    Ans: $\lim\limits_{x_1 \to x_0} 4\left(x_1^2 + x_0^2\right) = 12x_0^2$

    Slope of tangent at $x_0 = 2$ is 48
    Difficulty: Easy
    Section: 2.1

30. A rock is dropped from a height of 144 feet and falls toward the earth in a straight line. In t seconds, the rock drops a distance of $s = 16t^2$ feet. What is the average velocity of the rock while it is falling? Use limits to find the instantaneous velocity of the rock when it hits the ground.
    Ans: Average velocity: 48 feet per second

    Instantaneous velocity at ground = 96 feet per second
    Difficulty: Medium
    Section: 2.1

31. A particle moves in a straight line from its initial position so that after t seconds, its distance is given by $s = t^2 + t$ feet from its initial position. Find the average velocity of the particle over the interval [3,6] seconds. Use limits to find the instantaneous velocity of the particle at $t = 1$ second.
    Ans: Average velocity = 10 feet per second

    The instantaneous velocity at $t = 1$ second is 3 feet per second.
    Difficulty: Medium
    Section: 2.1

32. A particle moves in a straight line from its initial position so that after t seconds, its distance is given by $s = \dfrac{t}{t+1}$ feet from its initial position. Find the average velocity of the particle over the interval [4,8] seconds. Use limits to find the instantaneous velocity of the particle at $t = 4$ seconds.

    Ans: Average velocity = $\dfrac{1}{45}$ feet per second.

    The instantaneous velocity at $t = 4$ seconds is $\dfrac{1}{25}$ feet per second.
    Difficulty: Medium
    Section: 2.1

33. Let $f(x) = ax^2 + b$, where a and b are constant. Use the method of Section 3.1 to show that the slope of the tangent to the graph of f at $x = x_0$ is $2ax_0$.

Ans: $m_{\tan} = \lim_{x_1 \to x_0} \dfrac{\left(ax_1^2 + b\right) - \left(ax_0^2 + b\right)}{x_1 - x_0} = \lim_{x_1 \to x_0} \dfrac{a\left(x_1^2 - x_0^2\right)}{x_1 - x_0} = \lim_{x_1 \to x_0} a\left(x_1 + x_0\right) = 2ax_0$

Difficulty: Hard
Section: 2.1

34. Let $f(x) = ax^3 + b$, where a and b are constants. Use the method of Section 3.1 to show that the slope of the tangent to the graph of f at $x = x_0$ is $3ax_0^2$.
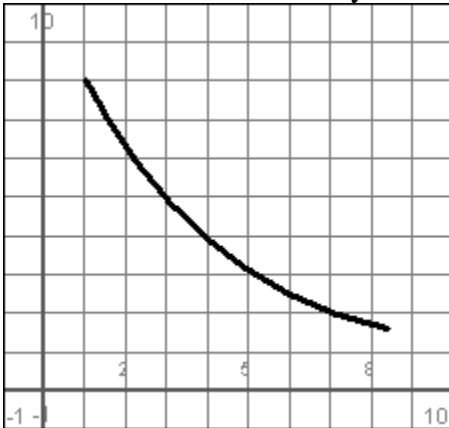
Ans:

$$m_{\tan} = \lim_{x_1 \to x_0} \frac{\left(ax_1^3 + b\right) - \left(ax_0^3 + b\right)}{x_1 - x_0} = \lim_{x_1 \to x_0} \frac{ax_1^3 - ax_0^3}{x_1 - x_0} = \lim_{x_1 \to x_0} a\left(x_1^2 + x_1 x_0 + x_0^2\right) = 3ax_0^2$$

Difficulty: Medium
Section: 2.1

35. The graph shows the position versus time curve for a particle moving on a straight line. Is the instantaneous velocity increasing or decreasing with time?
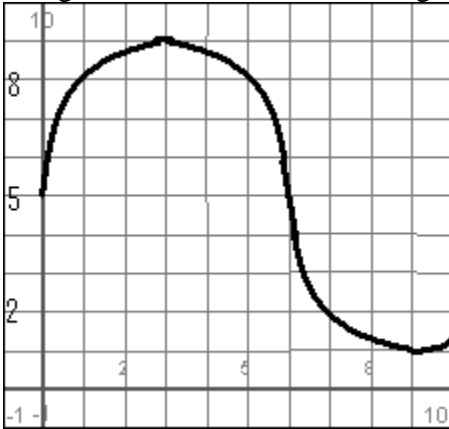

Ans: decreasing
Difficulty: Easy
Section: 2.1

36. The figure shows the position versus time curve for a certain particle moving along a straight line. Estimate, from the graph, the average velocity over the interval 3 to 9.

Ans: -4/3
Difficulty: Easy
Section: 2.1

37. Given $f(x) = x^3 - 1$, find the slope of the graph of f at the x-value $x_0 = 4$.
Ans: 48
Difficulty: Medium
Section: 2.1

38. Given $f(x) = 13\sqrt{x}$, find the slope of the graph of f at $x_0 = 1$.
Ans: $\dfrac{13}{2}$
Difficulty: Medium
Section: 2.1

39. Find the instantaneous rate of change of $f(x) = \dfrac{2}{x^3}$ at $x_0 = 5$.

Ans: $-\dfrac{6}{625}$
Difficulty: Medium
Section: 2.1

40. Find the instantaneous rate of change of $f(x) = 5x^2 - 12$ at $x_0 = 5$.
Ans: 50
Difficulty: Medium
Section: 2.1

41. Find the instantaneous rate of change of $f(x) = 5x^2 - 6x + 9$ at $x_0 = 3$.
    Ans: 24
    Difficulty: Medium
    Section: 2.1