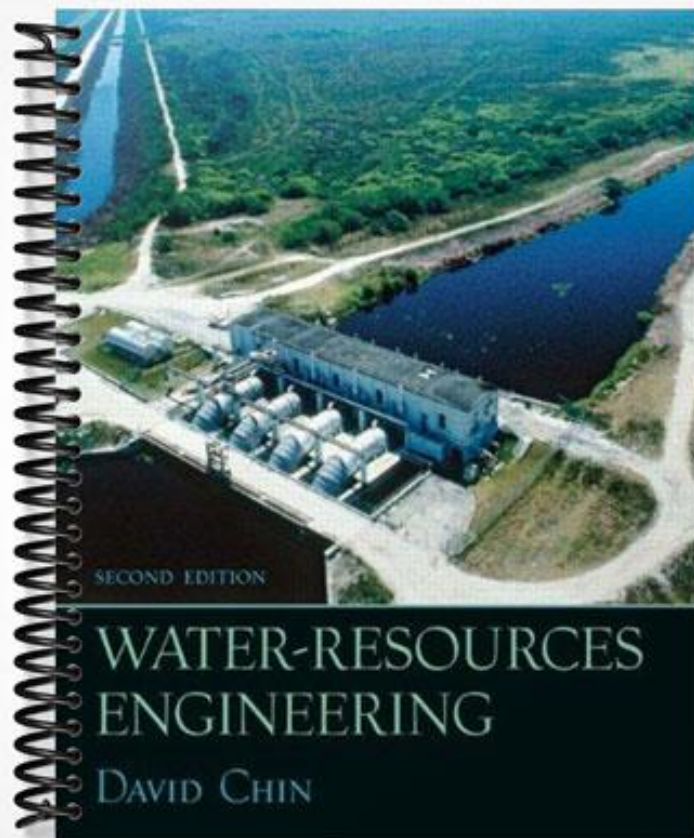


SOLUTIONS MANUAL



SECOND EDITION

WATER-RESOURCES ENGINEERING

DAVID CHIN

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by David A. Chin.
ISBN 0-13-148192-4.

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Chapter 1

Introduction

- 1.1. The mean annual rainfall in Boston is approximately 1050 mm , and the mean annual evapotranspiration is in the range of $380 - 630 \text{ mm}$ (USGS). On the basis of rainfall, this indicates a subhumid climate. The mean annual rainfall in Santa Fe is approximately 360 mm and the mean annual evapotranspiration is $< 380 \text{ mm}$. On the basis of rainfall, this indicates an arid climate.

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Chapter 2

Flow in Closed Conduits

2.1. $D_1 = 0.1$ m, $D_2 = 0.15$ m, $V_1 = 2$ m/s, and

$$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.1)^2 = 0.007854 \text{ m}^2$$

$$A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.15)^2 = 0.01767 \text{ m}^2$$

Volumetric flow rate, Q , is given by

$$Q = A_1V_1 = (0.007854)(2) = \boxed{0.0157 \text{ m}^3/\text{s}}$$

According to continuity,

$$A_1V_1 = A_2V_2 = Q$$

Therefore

$$V_2 = \frac{Q}{A_2} = \frac{0.0157}{0.01767} = \boxed{0.889 \text{ m/s}}$$

At 20°C, the density of water, ρ , is 998 kg/m³, and the mass flow rate, \dot{m} , is given by

$$\dot{m} = \rho Q = (998)(0.0157) = \boxed{15.7 \text{ kg/s}}$$

2.2. From the given data: $D_1 = 200$ mm, $D_2 = 100$ mm, $V_1 = 1$ m/s, and

$$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.2)^2 = 0.0314 \text{ m}^2$$

$$A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.1)^2 = 0.00785 \text{ m}^2$$

The flowrate, Q_1 , in the 200-mm pipe is given by

$$Q_1 = A_1V_1 = (0.0314)(1) = 0.0314 \text{ m}^3/\text{s}$$

and hence the flowrate, Q_2 , in the 100-mm pipe is

$$Q_2 = \frac{Q_1}{2} = \frac{0.0314}{2} = \boxed{0.0157 \text{ m}^3/\text{s}}$$

The average velocity, V_2 , in the 100-mm pipe is

$$V_2 = \frac{Q_2}{A_2} = \frac{0.0157}{0.00785} = \boxed{2 \text{ m/s}}$$

2.3. The velocity distribution in the pipe is

$$v(r) = V_o \left[1 - \left(\frac{r}{R} \right)^2 \right] \quad (1)$$

and the average velocity, \bar{V} , is defined as

$$\bar{V} = \frac{1}{A} \int_A V \, dA \quad (2)$$

where

$$A = \pi R^2 \quad \text{and} \quad dA = 2\pi r \, dr \quad (3)$$

Combining Equations 1 to 3 yields

$$\begin{aligned} \bar{V} &= \frac{1}{\pi R^2} \int_0^R V_o \left[1 - \left(\frac{r}{R} \right)^2 \right] 2\pi r \, dr \\ &= \frac{2V_o}{R^2} \left[\int_0^R r \, dr - \int_0^R \frac{r^3}{R^2} \, dr \right] \\ &= \frac{2V_o}{R^2} \left[\frac{R^2}{2} - \frac{R^4}{4R^2} \right] \\ &= \frac{2V_o}{R^2} \frac{R^2}{4} \\ &= \boxed{\frac{V_o}{2}} \end{aligned}$$

The flowrate, Q , is therefore given by

$$Q = A\bar{V} = \boxed{\frac{\pi R^2 V_o}{2}}$$

2.4.

$$\begin{aligned} \beta &= \frac{1}{A\bar{V}^2} \int_A v^2 \, dA \\ &= \frac{4}{\pi R^2 V_o^2} \int_0^R V_o^2 \left[1 - \frac{2r^2}{R^2} + \frac{r^4}{R^4} \right] 2\pi r \, dr \\ &= \frac{8}{R^2} \left[\int_0^R r \, dr - \int_0^R \frac{2r^3}{R^2} \, dr + \int_0^R \frac{r^5}{R^4} \, dr \right] \\ &= \frac{8}{R^2} \left[\frac{R^2}{2} - \frac{R^4}{2R^2} + \frac{R^6}{6R^4} \right] \\ &= \boxed{\frac{4}{3}} \end{aligned}$$

2.5. $D = 0.2$ m, $Q = 0.06$ m³/s, $L = 100$ m, $p_1 = 500$ kPa, $p_2 = 400$ kPa, $\gamma = 9.79$ kN/m³.

$$\begin{aligned}
 R &= \frac{D}{4} = \frac{0.2}{4} = 0.05 \text{ m} \\
 \Delta h &= \frac{p_1}{\gamma} - \frac{p_2}{\gamma} \\
 &= \frac{500 - 400}{9.79} = 10.2 \text{ m} \\
 \tau_o &= \frac{\gamma R \Delta h}{L} = \frac{(9.79 \times 10^3)(0.05)(10.2)}{100} = \boxed{49.9 \text{ N/m}^2} \\
 A &= \frac{\pi D^2}{4} = \frac{\pi(0.2)^2}{4} = 0.0314 \text{ m}^2 \\
 V &= \frac{Q}{A} = \frac{0.06}{0.0314} = 1.91 \text{ m/s} \\
 f &= \frac{8\tau_o}{\rho V^2} = \frac{8(49.9)}{(998)(1.91)^2} = \boxed{0.11}
 \end{aligned}$$

2.6. $T = 20^\circ\text{C}$, $V = 2$ m/s, $D = 0.25$ m, horizontal pipe, ductile iron. For ductile iron pipe, $k_s = 0.26$ mm, and

$$\begin{aligned}
 \frac{k_s}{D} &= \frac{0.26}{250} = 0.00104 \\
 \text{Re} &= \frac{\rho V D}{\mu} = \frac{(998.2)(2)(0.25)}{(1.002 \times 10^{-3})} = 4.981 \times 10^5
 \end{aligned}$$

From the Moody diagram:

$$\boxed{f = 0.0202 \text{ (pipe is smooth)}}$$

Using the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right)$$

Substituting for k_s/D and Re gives

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{0.00104}{3.7} + \frac{2.51}{4.981 \times 10^5 \sqrt{f}} \right)$$

By trial and error leads to

$$\boxed{f = 0.0204}$$

Using the Jain equation,

$$\begin{aligned}
 \frac{1}{\sqrt{f}} &= -2 \log \left[\frac{k_s/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}} \right] \\
 &= -2 \log \left[\frac{0.00104}{3.7} + \frac{5.74}{(4.981 \times 10^5)^{0.9}} \right]
 \end{aligned}$$

which leads to

$$\boxed{f = 0.0205}$$

The head loss, h_f , over 100 m of pipeline is given by

$$h_f = f \frac{L V^2}{D 2g} = 0.0204 \frac{100}{0.25} \frac{(2)^2}{2(9.81)} = 1.66 \text{ m}$$

Therefore the pressure drop, Δp , is given by

$$\Delta p = \gamma h_f = (9.79)(1.66) = \boxed{16.3 \text{ kPa}}$$

If the pipe is 1 m lower at the downstream end, f would not change, but the pressure drop, Δp , would then be given by

$$\Delta p = \gamma(h_f - 1.0) = 9.79(1.66 - 1) = \boxed{6.46 \text{ kPa}}$$

2.7. The Colebrook equation is given by

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{k/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right)$$

Inverting and squaring this equation gives

$$f = \frac{0.25}{\{\log[(k_s/D)/3.7 + 2.51/(\text{Re}\sqrt{f})]\}^2}$$

This equation is “slightly more convenient” than the Colebrook formula since it is quasi-explicit in f , whereas the Colebrook formula gives $1/\sqrt{f}$.

2.8. The Colebrook equation is preferable since it provides greater accuracy than interpolating from the Moody diagram.

2.9. $D = 0.5 \text{ m}$, $p_1 = 600 \text{ kPa}$, $Q = 0.50 \text{ m}^3/\text{s}$, $z_1 = 120 \text{ m}$, $z_2 = 100 \text{ m}$, $\gamma = 9.79 \text{ kN/m}^3$, $L = 1000 \text{ m}$, k_s (ductile iron) = 0.26 mm ,

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.5)^2 = 0.1963 \text{ m}^2$$

$$V = \frac{Q}{A} = \frac{0.50}{0.1963} = 2.55 \text{ m/s}$$

Using the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right)$$

where $k_s/D = 0.26/500 = 0.00052$, and at 20°C

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(998)(2.55)(0.5)}{1.00 \times 10^{-3}} = 1.27 \times 10^6$$

Substituting k_s/D and Re into the Colebrook equation gives

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{0.00052}{3.7} + \frac{2.51}{1.27 \times 10^6 \sqrt{f}} \right)$$

which leads to

$$f = 0.0172$$

Applying the energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f$$

Since $V_1 = V_2$, and h_f is given by the Darcy-Weisbach equation, then the energy equation can be written as

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + f \frac{L}{D} \frac{V^2}{2g}$$

Substituting known values leads to

$$\frac{600}{9.79} + 120 = \frac{p_2}{9.79} + 100 + 0.0172 \frac{1000}{0.5} \frac{(2.55)^2}{2(9.81)}$$

which gives

$$p_2 = 684 \text{ kPa}$$

If p is the (static) pressure at the top of a 30 m high building, then

$$\begin{aligned} p &= p_2 - 30\gamma \\ &= 684 - 30(9.79) \\ &= 390 \text{ kPa} \end{aligned}$$

This (static) water pressure is adequate for service.

2.10. The head loss, h_f , in the pipe is estimated by

$$h_f = \left(\frac{p_{\text{main}}}{\gamma} + z_{\text{main}} \right) - \left(\frac{p_{\text{outlet}}}{\gamma} + z_{\text{outlet}} \right)$$

where $p_{\text{main}} = 400 \text{ kPa}$, $z_{\text{main}} = 0 \text{ m}$, $p_{\text{outlet}} = 0 \text{ kPa}$, and $z_{\text{outlet}} = 2.0 \text{ m}$. Therefore,

$$h_f = \left(\frac{400}{9.79} + 0 \right) - (0 + 2.0) = 38.9 \text{ m}$$

Also, since $D = 25 \text{ mm}$, $L = 20 \text{ m}$, $k_s = 0.15 \text{ mm}$ (from Table 2.1), $\nu = 1.00 \times 10^{-6} \text{ m}^2/\text{s}$ (at 20°C), the Swamee-Jain equation (Equation 2.43) yields,

$$\begin{aligned} Q &= -0.965 D^2 \sqrt{\frac{g D h_f}{L}} \ln \left(\frac{k_s/D}{3.7} + \frac{1.774 \nu}{D \sqrt{g D h_f / L}} \right) \\ &= -0.965 (0.025)^2 \sqrt{\frac{(9.81)(0.025)(38.9)}{20}} \ln \left[\frac{0.15/25}{3.7} + \frac{1.774(1.00 \times 10^{-6})}{(0.025) \sqrt{(9.81)(0.025)(38.9)/20}} \right] \\ &= 0.00265 \text{ m}^3/\text{s} = 2.65 \text{ L/s} \end{aligned}$$

The faucet can therefore be expected to deliver 2.65 L/s when fully open.

2.11. Step 1: Assume $f = 0.020$

Step 2: Since $Q = 0.3 \text{ m}^3/\text{s}$, $L = 40 \text{ m}$, and $h_f = 45 \text{ m}$, then

$$D = \sqrt[5]{\left[\frac{8LQ^2}{h_f g \pi^2}\right] f} = \sqrt[5]{\left[\frac{8(40)(0.3)^2}{(45)(9.81)\pi^2}\right] (0.020)} = 0.168 \text{ m}$$

Step 3: Since $\nu = 1.00 \times 10^{-6} \text{ m}^2/\text{s}$ (at 20°C), then

$$\text{Re} = \left[\frac{4Q}{\pi\nu}\right] \frac{1}{D} = \left[\frac{4(0.3)}{\pi(1.00 \times 10^{-6})}\right] \frac{1}{0.168} = 2.27 \times 10^6$$

Step 4: Since $k_s = 0.15 \text{ mm}$ (from Table 2.1), then

$$\frac{k_s}{D} = \frac{1.5 \times 10^{-4}}{0.168} = 0.000893$$

Step 5: Using the Colebrook equation (Equation 2.35) gives

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right) = -2 \log \left(\frac{0.000893}{3.7} + \frac{2.51}{2.27 \times 10^6 \sqrt{f}} \right)$$

which leads to

$$f = 0.0192$$

Step 6: $f = 0.0192$ differs from the assumed $f (= 0.020)$, so repeat the procedure with $f = 0.0192$.

Step 2: For $f = 0.0192$, $D = 0.166 \text{ m}$

Step 3: For $D = 0.166$, $\text{Re} = 2.30 \times 10^6$

Step 4: For $D = 0.166$, $k_s/D = 0.000904$

Step 5: $f = 0.0193$

Step 6: $f = 0.0193$ differs from the assumed $f (= 0.0192)$, so repeat the procedure with $f = 0.0193$.

Step 2: For $f = 0.0193$, $D = 0.166 \text{ m}$

Step 3: For $D = 0.166$, $\text{Re} = 2.30 \times 10^6$

Step 4: For $D = 0.166$, $k_s/D = 0.000904$

Step 5: $f = 0.0193$

Step 6: This is the same value of f as originally assumed. Therefore, f can be taken as 0.0193 , and $D = \boxed{166 \text{ mm}}$.

2.12. Since $k_s = 0.15$ mm, $L = 40$ m, $Q = 0.3$ m³/s, $h_f = 45$ m, $\nu = 1.00 \times 10^{-6}$ m²/s, the Swamee-Jain equation gives

$$\begin{aligned} D &= 0.66 \left[k_s^{1.25} \left(\frac{LQ^2}{gh_f} \right)^{4.75} + \nu Q^{9.4} \left(\frac{L}{gh_f} \right)^{5.2} \right]^{0.04} \\ &= 0.66 \left\{ (0.00015)^{1.25} \left[\frac{(40)(0.3)^2}{(9.81)(45)} \right]^{4.75} + (1.00 \times 10^{-6})(0.3)^{9.4} \left[\frac{40}{(9.81)(45)} \right]^{5.2} \right\}^{0.04} \\ &= 0.171 \text{ m} = \boxed{171 \text{ mm}} \end{aligned}$$

The calculated pipe diameter (171 mm) is about 3% higher than calculated by the Colebrook equation (166 mm).

2.13. The kinetic energy correction factor, α , is defined by

$$\int_A \rho \frac{v^3}{2} dA = \alpha \rho \frac{V^3}{2} A$$

or

$$\alpha = \frac{\int_A v^3 dA}{V^3 A} \quad (1)$$

Using the velocity distribution in Problem 2.3 gives

$$\begin{aligned} \int_A v^3 dA &= \int_0^R V_o^3 \left[1 - \left(\frac{r}{R} \right)^2 \right]^3 2\pi r dr \\ &= 2\pi V_o^3 \int_0^R \left[1 - 3 \left(\frac{r}{R} \right)^2 + 3 \left(\frac{r}{R} \right)^4 - \left(\frac{r}{R} \right)^6 \right] r dr \\ &= 2\pi V_o^3 \int_0^R \left[r - \frac{3r^3}{R^2} + \frac{3r^5}{R^4} - \frac{r^7}{R^6} \right] dr \\ &= 2\pi V_o^3 \left[\frac{r^2}{2} - \frac{3r^4}{4R^2} + \frac{r^6}{2R^4} - \frac{r^8}{8R^6} \right]_0^R \\ &= 2\pi R^2 V_o^3 \left[\frac{1}{2} - \frac{3}{4} + \frac{1}{2} - \frac{1}{8} \right] \\ &= \frac{\pi R^2 V_o^3}{4} \end{aligned} \quad (2)$$

The average velocity, V , was calculated in Problem 2.3 as

$$V = \frac{V_o}{2}$$

hence

$$V^3 A = \left(\frac{V_o}{2} \right)^3 \pi R^2 = \frac{\pi R^2 V_o^3}{8} \quad (3)$$

Combining Equations 1 to 3 gives

$$\alpha = \frac{\pi R^2 V_o^3 / 4}{\pi R^2 V_o^3 / 8} = \boxed{2}$$

2.14. The kinetic energy correction factor, α , is defined by

$$\alpha = \frac{\int_A v^3 dA}{V^3 A} \quad (1)$$

Using the given velocity distribution gives

$$\begin{aligned} \int_A v^3 dA &= \int_0^R V_o^3 \left(1 - \frac{r}{R}\right)^{3/7} 2\pi r dr \\ &= 2\pi V_o^3 \int_0^R \left(1 - \frac{r}{R}\right)^{3/7} r dr \end{aligned} \quad (2)$$

To facilitate integration, let

$$x = 1 - \frac{r}{R} \quad (3)$$

which gives

$$r = R(1 - x) \quad (4)$$

$$dr = -R dx \quad (5)$$

Combining Equations 2 to 5 gives

$$\begin{aligned} \int_A v^3 dA &= 2\pi V_o^3 \int_0^1 x^{3/7} R(1 - x)(-R) dx \\ &= 2\pi R^2 V_o^3 \int_0^1 x^{3/7} (1 - x) dx = 2\pi R^2 V_o^3 \int_0^1 (x^{3/7} - x^{10/7}) dx \\ &= 2\pi R^2 V_o^3 \left[\frac{7}{10} x^{10/7} - \frac{7}{17} x^{17/7} \right]_0^1 \\ &= 0.576\pi R^2 V_o^3 \end{aligned} \quad (6)$$

The average velocity, V , is given by (using the same substitution as above)

$$\begin{aligned} V &= \frac{1}{A} \int_A v dA \\ &= \frac{1}{\pi R^2} \int_0^R V_o \left(1 - \frac{r}{R}\right)^{1/7} 2\pi r dr = \frac{2V_o}{R^2} \int_0^1 x^{1/7} R(1 - x)(-R) dx \\ &= 2V_o \int_0^1 (x^{1/7} - x^{8/7}) dx = 2V_o \left[\frac{7}{8} x^{8/7} - \frac{7}{15} x^{15/7} \right]_0^1 \\ &= 0.817V_o \end{aligned} \quad (7)$$

Using this result,

$$V^3 A = (0.817V_o)^3 \pi R^2 = 0.545\pi R^2 V_o^3 \quad (8)$$

Combining Equations 1, 6, and 8 gives

$$\alpha = \frac{0.576\pi R^2 V_o^3}{0.545\pi R^2 V_o^3} = \boxed{1.06}$$

The momentum correction factor, β , is defined by

$$\beta = \frac{\int_A v^2 dA}{AV^2} \quad (9)$$

In this case,

$$AV^2 = \pi R^2 (0.817V_o)^2 = 0.667\pi R^2 V_o^2 \quad (10)$$

and

$$\begin{aligned} \int_A v^2 dA &= \int_0^R V_o^2 \left(1 - \frac{r}{R}\right)^{2/7} 2\pi r dr \\ &= 2\pi V_o^2 \int_1^0 x^{2/7} R(1-x)(-R) dx = 2\pi R^2 V_o^2 \int_0^1 (x^{2/7} - x^{9/7}) dx \\ &= 2\pi R^2 V_o^2 \left[\frac{7}{9} x^{9/7} - \frac{7}{16} x^{16/7} \right]_0^1 = 0.681\pi R^2 V_o^2 \end{aligned} \quad (11)$$

Combining Equations 9 to 11 gives

$$\beta = \frac{0.681\pi R^2 V_o^2}{0.667\pi R^2 V_o^2} = \boxed{1.02}$$

2.15. The kinetic energy correction factor, α , is defined by

$$\alpha = \frac{\int_A v^3 dA}{V^3 A} \quad (1)$$

Using the velocity distribution given by Equation 2.75 gives

$$\begin{aligned} \int_A v^3 dA &= \int_0^R V_o^3 \left(1 - \frac{r}{R}\right)^{3/n} 2\pi r dr \\ &= 2\pi V_o^3 \int_0^R \left(1 - \frac{r}{R}\right)^{3/n} r dr \end{aligned} \quad (2)$$

Let

$$x = 1 - \frac{r}{R} \quad (3)$$

which gives

$$r = R(1-x) \quad (4)$$

$$dr = -R dx \quad (5)$$

Combining Equations 2 to 5 gives

$$\begin{aligned} \int_A v^3 dA &= 2\pi V_o^3 \int_0^1 x^{3/n} R(1-x)(-R) dx \\ &= 2\pi R^2 V_o^3 \int_0^1 x^{3/n} (1-x) dx = 2\pi R^2 V_o^3 \int_0^1 (x^{3/n} - x^{3+n/n}) dx \\ &= 2\pi R^2 V_o^3 \left[\frac{n}{3+n} x^{3+n/n} - \frac{n}{3+2n} x^{3+2n/n} \right]_0^1 \\ &= \frac{2n^2}{(3+n)(3+2n)} \pi R^2 V_o^3 \end{aligned} \quad (6)$$

The average velocity, V , is given by

$$\begin{aligned}
 V &= \frac{1}{A} \int_A v \, dA \\
 &= \frac{1}{\pi R^2} \int_0^R V_o \left(1 - \frac{r}{R}\right)^{1/n} 2\pi r \, dr = \frac{2V_o}{R^2} \int_1^0 x^{1/n} R(1-x)(-R)dx \\
 &= 2V_o \int_0^1 (x^{1/n} - x^{1+n/n})dx = 2V_o \left[\frac{n}{1+n} x^{1+n/n} - \frac{n}{1+2n} x^{1+2n/n} \right]_0^1 \\
 &= \left[\frac{2n^2}{(1+n)(1+2n)} \right] V_o
 \end{aligned} \tag{7}$$

Using this result,

$$V^3 A = \left[\frac{2n^2}{(1+n)(1+2n)} \right]^3 V_o^3 \pi R^2 = \frac{8n^6}{(1+n)^3(1+2n)^3} \pi R^2 V_o^3 \tag{8}$$

Combining Equations 1, 6, and 8 gives

$$\begin{aligned}
 \alpha &= \frac{\frac{2n^2}{(3+n)(3+2n)} \pi R^2 V_o^3}{\frac{8n^6}{(1+n)^3(1+2n)^3} \pi R^2 V_o^3} \\
 &= \frac{(1+n)^3(1+2n)^3}{4n^4(3+n)(3+2n)}
 \end{aligned}$$

Putting $n = 7$ gives $\alpha = 1.06$, the same result obtained in Problem 2.14.

2.16. $p_1 = 30$ kPa, $p_2 = 500$ kPa, therefore head, h_p , added by pump is given by

$$h_p = \frac{p_2 - p_1}{\gamma} = \frac{500 - 30}{9.79} = \boxed{48.0 \text{ m}}$$

Power, P , added by pump is given by

$$P = \gamma Q h_p = (9.79)(Q)(48.0) = \boxed{470 \text{ kW per m}^3/\text{s}}$$

2.17. $Q = 0.06$ m³/s, $D = 0.2$ m, $k_s = 0.9$ mm (riveted steel), $k_s/D = 0.9/200 = 0.00450$, for 90° bend $K = 0.3$, for the entrance $K = 1.0$, at 20°C $\rho = 998$ kg/m³, and $\mu = 1.00 \times 10^{-3}$ Pa·s, therefore

$$\begin{aligned}
 A &= \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.2)^2 = 0.0314 \text{ m}^2 \\
 V &= \frac{Q}{A} = \frac{0.06}{0.0314} = 1.91 \text{ m/s} \\
 \text{Re} &= \frac{\rho V D}{\mu} = \frac{(998)(1.91)(0.2)}{1.00 \times 10^{-3}} = 3.81 \times 10^5
 \end{aligned}$$

Substituting k_s/D and Re into the Colebrook equation gives

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{0.00450}{3.7} + \frac{2.51}{3.81 \times 10^5 \sqrt{f}} \right)$$

which leads to

$$f = 0.0297$$

Minor head loss, h_m , is given by

$$h_m = \sum K \frac{V^2}{2g} = (1.0 + 0.3) \frac{(1.91)^2}{2(9.81)} = 0.242 \text{ m}$$

If friction losses, h_f , account for 90% of the total losses, then

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = 9h_m$$

which means that

$$0.0297 \frac{L}{0.2} \frac{(1.91)^2}{2(9.81)} = 9(0.242)$$

Solving for L gives

$$\boxed{L = 78.9 \text{ m}}$$

For pipe lengths shorter than the length calculated in this problem, the word “minor” should not be used.

- 2.18.** From the given data: $z_1 = -1.5 \text{ m}$, $z_2 = 40 \text{ m}$, $p_1 = 450 \text{ kPa}$, $\sum k = 10.0$, $Q = 20 \text{ L/s} = 0.02 \text{ m}^3/\text{s}$, $D = 150 \text{ mm}$ (PVC), $L = 60 \text{ m}$, $T = 20^\circ\text{C}$, and $p_2 = 150 \text{ kPa}$. The combined energy and Darcy-Weisbach equations give

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left[\frac{fL}{D} + \sum k \right] \frac{V^2}{2g} \quad (1)$$

where

$$V_1 = V_2 = V = \frac{Q}{A} = \frac{0.02}{\frac{\pi(0.15)^2}{4}} = 1.13 \text{ m/s} \quad (2)$$

At 20°C , $\nu = 1.00 \times 10^{-6} \text{ m}^2/\text{s}$, and

$$\text{Re} = \frac{VD}{\nu} = \frac{(1.13)(0.15)}{1.00 \times 10^{-6}} = 169500$$

Since PVC pipe is smooth ($k_s = 0$), the friction factor, f , is given by

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{2.51}{\text{Re} \sqrt{f}} \right) = -2 \log \left(\frac{2.51}{169500 \sqrt{f}} \right)$$

which yields

$$f = 0.0162 \quad (3)$$

Taking $\gamma = 9.79 \text{ kN/m}^3$ and combining Equations 1 to 3 yields

$$\frac{450}{9.79} + \frac{1.13^2}{2(9.81)} + (-1.5) + h_p = \frac{150}{9.79} + \frac{1.13^2}{2(9.81)} + 40 + \left[\frac{(0.0162)(60)}{0.15} + 10 \right] \frac{1.13^2}{2(9.81)}$$

which gives

$$h_p = 11.9 \text{ m}$$

Since $h_p > 0$, a booster pump is required. The power, P , to be supplied by the pump is given by

$$P = \gamma Q h_p = (9.79)(0.02)(11.9) = \boxed{2.3 \text{ kW}}$$

2.19. Diameter of pipe, $D = 0.75 \text{ m}$, area, A given by

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.75)^2 = 0.442 \text{ m}^2$$

and velocity, V , in pipe

$$V = \frac{Q}{A} = \frac{1}{0.442} = 2.26 \text{ m/s}$$

Energy equation between reservoir and A:

$$7 + h_p - h_f = \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A \quad (1)$$

where $p_A = 350 \text{ kPa}$, $\gamma = 9.79 \text{ kN/m}^3$, $V_A = 2.26 \text{ m/s}$, $z_A = 10 \text{ m}$, and

$$h_f = \frac{fL}{D} \frac{V^2}{2g}$$

where f depends on Re and k_s/D . At 20°C , $\nu = 1.00 \times 10^{-6} \text{ m}^2/\text{s}$ and

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} = \frac{(2.26)(0.75)}{1.00 \times 10^{-6}} = 1.70 \times 10^6 \\ \frac{k_s}{D} &= \frac{0.26}{750} = 3.47 \times 10^{-4} \end{aligned}$$

Using the Jain equation,

$$\frac{1}{\sqrt{f}} = -2 \log \left[\frac{k_s/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}} \right] = -2 \log \left[\frac{3.47 \times 10^{-4}}{3.7} + \frac{5.74}{(1.70 \times 10^6)^{0.9}} \right] = 7.93$$

which leads to

$$f = 0.0159$$

The head loss, h_f , between the reservoir and A is therefore given by

$$h_f = \frac{fL}{D} \frac{V^2}{2g} = \frac{(0.0159)(1000)}{0.75} \frac{(2.26)^2}{2(9.81)} = 5.52 \text{ m}$$

Substituting into Equation 1 yields

$$7 + h_p - 5.52 = \frac{350}{9.81} + \frac{2.26^2}{2(9.81)} + 10$$

which leads to

$$\boxed{h_p = 44.5 \text{ m}}$$

(b) Power, P , supplied by the pump is given by

$$P = \gamma Q h_p = (9.79)(1)(44.5) = \boxed{436 \text{ kW}}$$

(c) Energy equation between A and B is given by

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A - h_f = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

and since $V_A = V_B$,

$$\begin{aligned} p_B &= p_A + \gamma(z_A - z_B - h_f) = 350 + 9.79(10 - 4 - 5.52) \\ &= \boxed{355 \text{ kPa}} \end{aligned}$$

2.20. From the given data: $L = 3 \text{ km} = 3000 \text{ m}$, $Q_{\text{ave}} = 0.0175 \text{ m}^3/\text{s}$, and $Q_{\text{peak}} = 0.578 \text{ m}^3/\text{s}$. If the velocity, V_{peak} , during peak flow conditions is 2.5 m/s , then

$$2.5 = \frac{Q_{\text{peak}}}{\pi D^2/4} = \frac{0.578}{\pi D^2/4}$$

which gives

$$D = \sqrt{\frac{0.578}{\pi(2.5)/4}} = 0.543 \text{ m}$$

Rounding to the nearest 25 mm gives

$$\boxed{D = 550 \text{ mm}}$$

with a cross-sectional area, A , given by

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.550)^2 = 0.238 \text{ m}^2$$

During average demand conditions, the head, h_{ave} , at the suburban development is given by

$$h_{\text{ave}} = \frac{p_{\text{ave}}}{\gamma} + \frac{V_{\text{ave}}^2}{2g} + z_o \quad (1)$$

where $p_{\text{ave}} = 340 \text{ kPa}$, $\gamma = 9.79 \text{ kN/m}^3$, $V_{\text{ave}} = Q_{\text{ave}}/A = 0.0175/0.238 = 0.0735 \text{ m/s}$, and $z_o = 8.80 \text{ m}$. Substituting into Equation 1 gives

$$h_{\text{ave}} = \frac{340}{9.79} + \frac{0.0735^2}{2(9.81)} + 8.80 = 43.5 \text{ m}$$

For ductile-iron pipe, $k_s = 0.26 \text{ mm}$, $k_s/D = 0.26/550 = 4.73 \times 10^{-4}$, at 20°C $\nu = 1.00 \times 10^{-6} \text{ m}^2/\text{s}$, and therefore

$$\text{Re} = \frac{V_{\text{ave}} D}{\nu} = \frac{(0.0735)(0.550)}{1.00 \times 10^{-6}} = 4.04 \times 10^4$$

and the Jain equation gives

$$\frac{1}{\sqrt{f_{\text{ave}}}} = -2 \log \left[\frac{k_s/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}} \right] = -2 \log \left[\frac{4.73 \times 10^{-4}}{3.7} + \frac{5.74}{(4.04 \times 10^4)^{0.9}} \right]$$

and yields

$$f_{\text{ave}} = 0.0234$$

The head loss between the water treatment plant and the suburban development is therefore given by

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = (0.0234) \frac{3000}{0.550} \frac{0.0735^2}{2(9.81)} = 0.035 \text{ m}$$

Since the head at the water treatment plant is 10.00 m, the pump head, h_p , that must be added is

$$h_p = (43.5 + 0.035) - 10.00 = 33.5 \text{ m}$$

and the power requirement, P , is given by

$$P = \gamma Q h_p = (9.79)(0.0175)(33.5) = \boxed{5.74 \text{ kW}}$$

During peak demand conditions, the head, h_{peak} , at the suburban development is given by

$$h_{\text{peak}} = \frac{p_{\text{peak}}}{\gamma} + \frac{V_{\text{peak}}^2}{2g} + z_o \quad (2)$$

where $p_{\text{peak}} = 140 \text{ kPa}$, $\gamma = 9.79 \text{ kN/m}^3$, $V_{\text{peak}} = Q_{\text{peak}}/A = 0.578/0.238 = 2.43 \text{ m/s}$, and $z_o = 8.80 \text{ m}$. Substituting into Equation 2 gives

$$h_{\text{peak}} = \frac{140}{9.79} + \frac{2.43^2}{2(9.81)} + 8.80 = 23.4 \text{ m}$$

For pipe, $k_s/D = 4.73 \times 10^{-4}$, and

$$\text{Re} = \frac{V_{\text{peak}} D}{\nu} = \frac{(2.43)(0.550)}{1.00 \times 10^{-6}} = 1.34 \times 10^6$$

and the Jain equation gives

$$\frac{1}{\sqrt{f_{\text{peak}}}} = -2 \log \left[\frac{k_s/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}} \right] = -2 \log \left[\frac{4.73 \times 10^{-4}}{3.7} + \frac{5.74}{(1.34 \times 10^6)^{0.9}} \right]$$

and yields

$$f_{\text{peak}} = 0.0170$$

The head loss between the water treatment plant and the suburban development is therefore given by

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = (0.0170) \frac{3000}{0.550} \frac{2.43^2}{2(9.81)} = 27.9 \text{ m}$$

Since the head at the water treatment plant is 10.00 m, the pump head, h_p , that must be added is

$$h_p = (23.4 + 27.9) - 10.00 = 41.3 \text{ m}$$

and the power requirement, P , is given by

$$P = \gamma Q h_p = (9.79)(0.578)(41.3) = \boxed{234 \text{ kW}}$$

2.21. The head loss is calculated using Equation 2.80. The hydraulic radius, R , is given by

$$R = \frac{A}{P} = \frac{(2)(1)}{2(2+1)} = 0.333 \text{ m}$$

and the mean velocity, V , is given by

$$V = \frac{Q}{A} = \frac{5}{(2)(1)} = 2.5 \text{ m/s}$$

At 20°C, $\rho = 998.2 \text{ kg/m}^3$, $\mu = 1.002 \times 10^{-3} \text{ N}\cdot\text{s/m}^2$, and therefore the Reynolds number, Re , is given by

$$Re = \frac{\rho V(4R)}{\nu} = \frac{(998.2)(2.5)(4 \times 0.333)}{1.002 \times 10^{-3}} = 3.32 \times 10^6$$

A median equivalent sand roughness for concrete can be taken as $k_s = 1.6 \text{ mm}$ (Table 2.1), and therefore the relative roughness, $k_s/4R$, is given by

$$\frac{k_s}{4R} = \frac{1.6 \times 10^{-3}}{4(0.333)} = 0.00120$$

Substituting Re and $k_s/4R$ into the Jain equation (Equation 2.38) for the friction factor yields

$$\frac{1}{\sqrt{f}} = -2 \log \left[\frac{k_s/4R}{3.7} + \frac{5.74}{Re^{0.9}} \right] = -2 \log \left[\frac{0.00120}{3.7} + \frac{5.74}{(3.32 \times 10^6)^{0.9}} \right] = 6.96$$

which yields

$$f = 0.0206$$

The frictional head loss in the culvert, h_f , is therefore given by the Darcy-Weisbach equation as

$$h_f = \frac{fL}{4R} \frac{V^2}{2g} = \frac{(0.0206)(100)}{(4 \times 0.333)} \frac{2.5^2}{2(9.81)} = \boxed{0.493 \text{ m}}$$

2.22. The frictional head loss is calculated using Equation 2.80. The hydraulic radius, R , is given by

$$R = \frac{A}{P} = \frac{(2)(2)}{2(2+2)} = 0.500 \text{ m}$$

and the mean velocity, V , is given by

$$V = \frac{Q}{A} = \frac{10}{(2)(2)} = 2.5 \text{ m/s}$$

At 20°C, $\rho = 998 \text{ kg/m}^3$, $\mu = 1.00 \times 10^{-3} \text{ N}\cdot\text{s/m}^2$, and therefore the Reynolds number, Re , is given by

$$Re = \frac{\rho V(4R)}{\mu} = \frac{(998)(2.5)(4 \times 0.500)}{1.00 \times 10^{-3}} = 4.99 \times 10^6$$

A median equivalent sand roughness for concrete can be taken as $k_s = 1.6 \text{ mm}$ (Table 2.1), and therefore the relative roughness, $k_s/4R$, is given by

$$\frac{k_s}{4R} = \frac{1.6 \times 10^{-3}}{4(0.500)} = 0.0008$$

Substituting Re and $k_s/4R$ into the Jain equation (Equation 2.39) for the friction factor yields

$$\frac{1}{\sqrt{f}} = -2 \log \left[\frac{k_s/4R}{3.7} + \frac{5.74}{Re^{0.9}} \right] = -2 \log \left[\frac{0.0008}{3.7} + \frac{5.74}{(4.99 \times 10^6)^{0.9}} \right] = 7.31$$

which yields

$$f = 0.0187$$

The frictional head loss in the culvert, h_f , is therefore given by the Darcy-Weisbach equation as

$$h_f = \frac{fL}{4R} \frac{V^2}{2g} = \frac{(0.0187)(500)}{(4 \times 0.500)} \frac{2.5^2}{2(9.81)} = 1.49 \text{ m}$$

Applying the energy equation between the upstream and downstream sections (Sections 1 and 2 respectively),

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f$$

which gives

$$\frac{p_1}{9.79} + \frac{2.5^2}{2(9.81)} + (0.002)(500) = \frac{p_2}{9.79} + \frac{2.5^2}{2(9.81)} + 0 + 1.49$$

Re-arranging this equation gives

$$\boxed{p_1 - p_2 = 4.80 \text{ kPa}}$$

2.23. The Hazen-Williams formula is given by

$$V = 0.849 C_H R^{0.63} S_f^{0.54} \quad (1)$$

where

$$S_f = \frac{h_f}{L} \quad (2)$$

Combining Equations 1 and 2, and taking $R = D/4$ gives

$$V = 0.849 C_H \left(\frac{D}{4} \right)^{0.63} \left(\frac{h_f}{L} \right)^{0.54}$$

which simplifies to

$$\boxed{h_f = 6.82 \frac{L}{D^{1.17}} \left(\frac{V}{C_H} \right)^{1.85}}$$

2.24. Comparing the Hazen-Williams and Darcy-Weisbach equations for head loss gives

$$h_f = 6.82 \frac{L}{D^{1.17}} \left(\frac{V}{C_H} \right)^{1.85} = f \frac{L}{D} \frac{V^2}{2g}$$

which leads to

$$\boxed{f = \frac{134}{C_H^{1.85} D^{0.17} V^{0.15}}}$$

For laminar flow, Equation 2.36 gives $f \sim 1/Re \sim 1/V$, and for fully-turbulent flow Equation 2.35 gives $f \sim 1/V^0$. Since the Hazen-Williams formula requires that $f \sim 1/V^{0.15}$, this indicates that the flow must be in the transition regime.