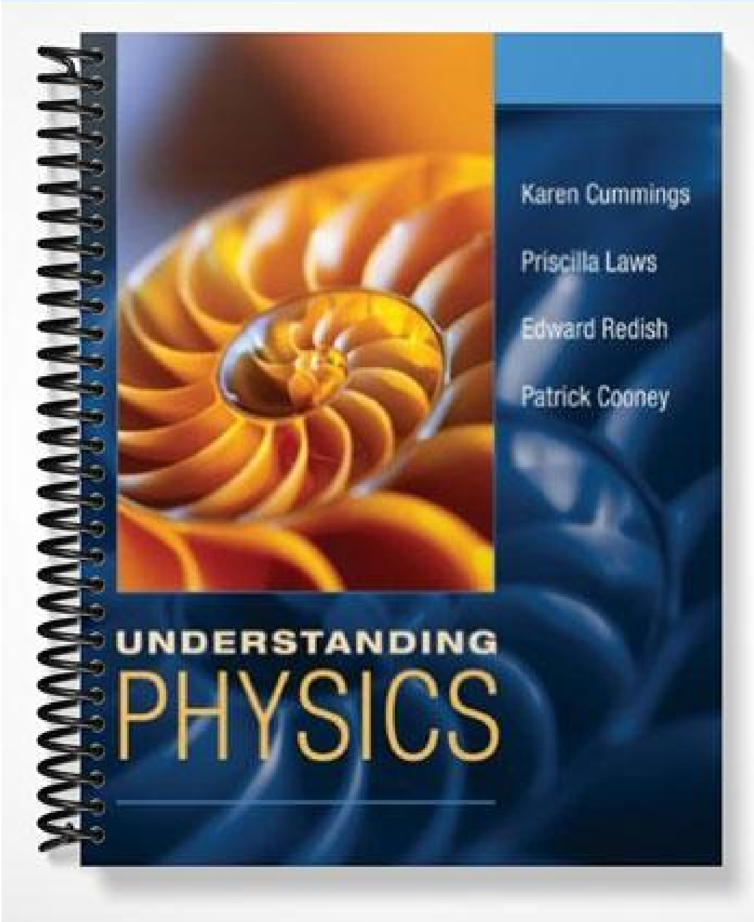
SOLUTIONS MANUAL



Chapter 2

1. Assuming the horizontal velocity of the ball is constant, the x-component of the horizontal displacement is

$$\Delta x = v_x \Delta t$$

where Δx is the horizontal distance traveled, Δt is the time, and v_x is the horizontal velocity component. Converting v_x to m/s, we have $(160 \text{ km/h})(1000 \text{ m/1 km})\,(1 \text{ h/3600 s}) = 44.4$ m/s.

Thus
$$\Delta t = \frac{\Delta x}{v_x} = \frac{18.4 \text{ m}}{44.4 \text{ m/s}} = 0.414 \text{ s.}$$

The velocity-unit conversion implemented above can also be done using the "one-step" conversion found in Appendix D (1 km/h = 0.2778 m/s).

2. Converting to SI units, we use Eq. 2-5 with d for total distance.

$$\langle s \rangle = \frac{d}{\Delta t}$$

$$(110.6 \text{ km/h})\left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}}\right) = \frac{200.0 \text{ m}}{\Delta t}$$

which yields an elapsed time of $\Delta t = 6.510$ s. As mentioned in problem 1, the velocity-unit conversion implemented above can also be done using the "one-step" conversion found in Appendix D (1 km/h = 0.2778 m/s).

3. We use Eq. 2-4 and Eq. 2-5. During a time interval Δt if the velocity remains a positive constant, speed is equivalent to the x-component of velocity, and distance is equivalent to the x-component of displacement, with $\Delta x = v_x \Delta t$.

(a) During the first part of the motion, the displacement between times t_1 and t_2 is $\Delta x_{1-2} = x_2 - x_1 = 40$ km and the time interval from t_1 to t_2 is

$$\Delta t_{1-2} = t_2 - t_1 = \frac{(40 \text{ km})}{(30 \text{ km/h})} = 1.33 \text{ h.}$$

During the second part the displacement is $\Delta x_{2-3} = x_3 - x_2 = 40$ km and the time interval is

$$\Delta t_{2-3} = t_3 - t_2 = \frac{(40 \text{ km})}{(60 \text{ km/h})} = 0.67 \text{ h.}$$

Both displacements are in the same direction, so the total displacement is $\Delta x_{1\text{-}3} = \Delta x_{1\text{-}2} + \Delta x_{2\text{-}3}$ = 40 km + 40 km = 80 km. The total elapsed time for the trip is $\Delta t_{1\text{-}3} = \Delta t_{1\text{-}2} + \Delta t_{2\text{-}3}$ = 2.00 h. Consequently, the average x-component of velocity is

$$\langle \vec{v} \rangle = \langle v_x \rangle \,\hat{\imath} = \frac{(80 \text{ km})}{(2.0 \text{ h})}\,\hat{\imath} = (40 \text{ km/h})\,\hat{\imath}.$$

(b) Since the velocity component is positive in this example, the numerical result for the average speed is the same as the x-component of the average velocity 40 km/h.

(c) In the interest of saving space, we briefly describe the graph: two contiguous line segments, the first having a slope of 30 km/hr and connecting the origin (t_1, x_1) = (0 hr, 0 km) to (t_2, x_2) = (1.33 hr, 40 km) and the second having a slope of 60 km/hr and connecting (t_2, x_2) to (t_3, x_3) = (2.00 hr, 80 km). The average velocity, from the graphical point of view, is the slope of a line drawn from the origin to (t_3, x_3).

4. If the plane (with velocity component v_x) maintains its present course, and if the terrain continues its upward slope of 4.3°, then the plane will strike the ground after traveling

$$\Delta x = \frac{h}{\tan\theta} = \frac{35 \text{ m}}{\tan 4.3°} = 465.5 \text{ m} \approx 0.465 \text{ km}.$$

This corresponds to a time of flight found from Eq. 2-4 (with $v_x = \langle v_x \rangle$ since it is constant)

$$\Delta t = \frac{\Delta x}{v_x} = \frac{0.465 \text{ km}}{1300 \text{ km/h}} = 0.000358 \text{ h} \approx 1.3 \text{ s}.$$

This, then, estimates the time available to the pilot to make his correction.

5. (a) Denoting the travel time and distance from San Antonio to Houston as T and D, respectively, the average speed is

$$\langle s_1 \rangle = \frac{D}{T} = \frac{(55 \text{ km/h})\frac{T}{2} + (90 \text{ km/h})\frac{T}{2}}{T} = 72.5 \text{ km/h}$$

which should be rounded to 73 km/h.

(b) Using the fact that time = distance/speed while the speed is constant, we find

$$\langle s_2 \rangle = \frac{D}{T} = \frac{D}{\frac{D/2}{55 \text{ km/h}} + \frac{D/2}{90 \text{ km/h}}} = 68.3 \text{ km/h}$$

which should be rounded to 68 km/h.

(c) The total distance traveled ($2D$) must not be confused with the net displacement (zero). For the two-way trip, we use Eq. 2-5

$$\langle s\rangle = \frac{2D}{\frac{D}{72.5\ \text{km/h}} + \frac{D}{68.3\ \text{km/h}}} = 70\ \text{km/h}.$$

(d) Since the net displacement vanishes, the average velocity for the trip in its entirety is zero.

(e) In asking for a *sketch*, the problem is allowing the student to arbitrarily set the distance D (the intent is *not* to make the student go to an Atlas to look it up); the student can just as easily arbitrarily set T instead of D, as will be clear in the following discussion. In the interest of saving space, we briefly describe the graph: two contiguous line segments, the first having a slope of 55 km/hr and connecting the origin $(t_1, x_1) = (0$ hr, 0 km$)$ to $(t_2, x_2) = \left(\frac{T}{2}\ \text{hr}, (55\ \text{km/hr})T/2\ \text{km}\right)$ and the second having a slope of 90 km/hr and connecting (t_2, x_2) to $(t_3, x_3) = (T, D)$ where $D = \big((55\ \text{km/hr} + 90\ \text{km/hr})T/2(\text{km})\big)$. The average velocity, from the graphical point of view, is the slope of a line drawn from the origin (0 hr, 0 km) to (T, D).

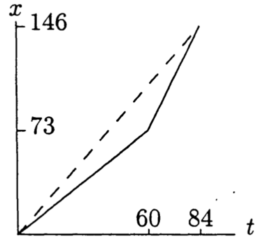
6. (a) Using the fact that time = distance/velocity while the velocity is constant, we find

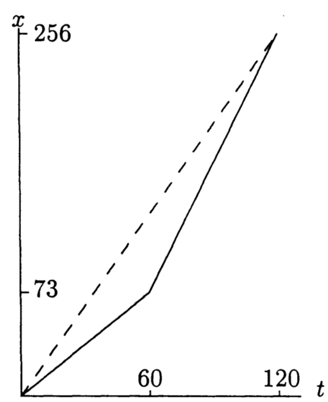
$$\langle \vec{v}\rangle = \langle v_x\rangle\,\hat{i} = \frac{73.2\ \text{m} + 73.2\ \text{m}}{73.2\ \text{m}/1.22\ \text{m/s} + 73.2\ \text{m}/3.05\ \text{m}}\,\hat{i} = (1.74\ \text{m/s})\,\hat{i}.$$

(b) Using the fact that displacement = velocity × time when the velocity $\vec{v}$ is constant, we find

$$\langle \vec{v}\rangle = \langle v_x\rangle\,\hat{i} = \frac{(1.22\ \text{m/s})(60\ \text{s}) + (3.05\ \text{m/s})(60\ \text{s})}{120\ \text{s}}\,\hat{i} = (2.14\ \text{m/s})\,\hat{i}.$$

(c) The graphs are shown below (with meters and seconds understood). The first consists of two (solid) line segments, the first having a slope of 1.22 m/s and the second having a slope of 3.05 m/s. The slope of the dashed line represents the average velocity (in both graphs). The second graph also consists of two (solid) line segments, having the same slopes as before — the main difference (compared to the first graph) being that the stage involving higher-speed motion lasts much longer.





7. We use $x = (3\text{ m/s})t - (4\text{ m/s}^2)t^2 + (1\text{ m/s}^3)t^3$. We will quote our answers to one or two significant figures, and not try to follow the significant figure rules rigorously.

(a) Substituting in $t = 1$ s yields $x = 0$ m. With $t = 2$ s we get $x = -2$ m. Similarly, $t = 3$ s yields $x = 0$ m and $t = 4$ s yields $x = 12$ m. For later reference, we also note that the position at $t = 0$ s is $x = 0$ m.

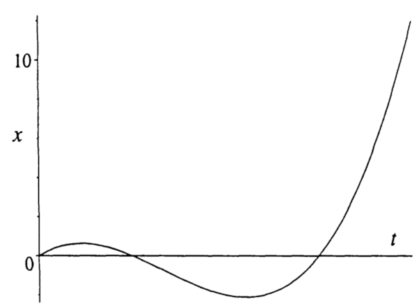
(b) The position at $t = 0$ s is subtracted from the position at $t = 4$ s to find the displacement $\Delta x = 12$ m.

(c) The position at $t = 2$ s is subtracted from the position at $t = 4$ s to give the displacement $\Delta x = 14$ m. Eq. 2-4, then, leads to a velocity of

$$\langle \vec{v} \rangle = \langle v_x \rangle\, \hat{i} = \frac{\Delta x}{\Delta t}\, \hat{i} = \frac{14\ m}{2\ s}\, \hat{i} = (7\text{ m/s})\, \hat{i}.$$

(d) The horizontal axis is $0\text{ s} \le t \le 4$ s with SI units understood.

Not shown is a straight line drawn from the point at $(t, x) = (2\text{ s}, -2\text{ m})$ to the highest point shown at $(t, x) = (4\text{ s}, 12\text{ m})$ which would represent the answer for part (c).



8. Recognizing that the gap between the trains is closing at a constant rate of 60 km/h, the total time which elapses before they crash is $\Delta t = (60\text{ km})/(60\text{ km/h}) = 1.0$ h. During this time, the bird travels a distance of $|\Delta x| = |v_x \Delta t| = (60\text{ km/h})(1.0\text{ h}) = 60$ km.

9. Converting to seconds, the elapsed running times are $\Delta t_A = 147.95$ s and $\Delta t_B = 148.15$ s, respectively. If the runners A and B were equally fast, then

$$\langle s_A \rangle = \langle s_B \rangle \ \Rightarrow\ \frac{L_A}{\Delta t_A} = \frac{L_B}{\Delta t_B}.$$

We will assume that the shorter track (L_A) is actually 1.00000 km (1000.00 m) in length. We obtain

$$L_B = \left(\frac{148.15\ \text{s}}{147.95\ \text{s}}\right) L_A = \left(\frac{148.15\ \text{s}}{147.95\ \text{s}}\right) 1000.00\ \text{m} = 1001.35\ \text{m}$$

Thus, runner A is faster than runner B as long as L_A is not shorter than L_B by more than about 1.35 m. If L_A is shorter than L_B by more than that then runner B is actually the faster.

10. Assuming a conventional horizontal x axis, the velocity component (both magnitude and sign) is determined by the slope of the x versus t curve, in accordance with Eq. 2-6.

(a) The armadillo is to the left of the coordinate origin on the axis between $t = 2.0$ s and $t = 4.0$ s.

(b) The velocity component is negative between $t = 0.0$ s and $t = 3.0$ s.

(c) The velocity component is positive between $t = 3.0$ s and $t = 7.0$ s.

(d) The velocity component is zero at the graph minimum (at $t = 3.0$ s).

11. We use Eq. 2-6.

(a) The velocity of the particle is

$$\vec{v} = v_x\,\hat{i} = \frac{dx}{dt}\,\hat{i} = \frac{d}{dt}\,[4m - (12\ \text{m/s})\ t + (3\ \text{m/s}^2)\ t^2]\,\hat{i}\ = (-12\ \text{m/s} + (6\ \text{m/s}^2)\ t)\,\hat{i}$$

Thus, at $t = 1$ s, the velocity is $\vec{v} = v_x\,\hat{i} = (-12\ \text{m/s} + (6\ \text{m/s}^2)\ 1\,s)\,\hat{i} = (-6\ \text{m/s})\,\hat{i}\, v_x = (-12\ \text{m/s} + (6\ \text{m/s}^2)(1\ \text{s})) = -6\ \text{m/s}$.

(b) Since $v_x < 0$ m/s, it is moving in the negative x direction at $t = 1$ s.

(c) At $t = 1$ s, the *speed* is $|v_x| = 6$ m/s.

(d) For $0\ \text{s} < t < 2\ \text{s}$, $|v_x|$ decreases until it vanishes. For $2\ \text{s} < t < 3\ \text{s}$, $|v_x|$ increases from zero to the value it had in part (c). Then, $|v_x|$ is larger than that value for $t > 3$ s.

(e) Yes, since v_x smoothly changes from negative values (consider the $t = 1$ result) to positive (note that as $t \to +\infty$, we have $v_x \to +\infty$). One can check that $v_x = 0$ m/s when $t = 2$ s.

(f) No. In fact, from $v_x = (-12\ \text{m/s} + (6\ \text{m/s}^2)\ \text{t})$, we know that $v_x > 0$ m/s for $t > 2$ s.

12. We use Eq. 2-4 for average velocity and Eq. 2-6 for instantaneous velocity.

(a) We substitute into the given equation for x for $t = 2.00$ s and $t = 3.00$ s and obtain $x_2 = 21.75$ m and $x_3 = 50.25$ m, respectively. The average velocity during the time interval $2.00 \text{ s} \le t \le 3.00$ s is

$$\langle \vec{v} \rangle = \langle v_x \rangle \,\hat{i} = \frac{\Delta x}{\Delta t}\,\hat{i} = \frac{50.25\text{ m} - 21.75\text{ m}}{3.00\text{ s} - 2.00\text{ s}}\,\hat{i}$$

which yields $\langle \vec{v} \rangle = (28.5\text{ m/s})\,\hat{i}$.

(b) The instantaneous velocity is $\vec{v} = \frac{dx}{dt}\,\hat{i} = (4.5\text{ m/s}^3)(t)^2\,\hat{i}$, which yields $\vec{v} = v_x\,\hat{i} = (18.0\text{ m/s})\,\hat{i}$ at time $t = 2.00$ s.

(c) At $t = 3.00$ s, the instantaneous velocity is $\vec{v} = v_x\,\hat{i} = (4.5\text{ m/s}^3)(3.00\text{ s})^2\,\hat{i} = (40.5\text{ m/s})\,\hat{i}$.

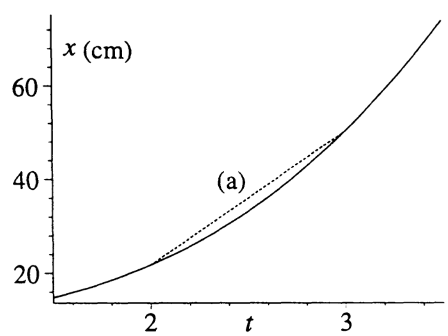
(d) At $t = 2.50$ s, the instantaneous velocity is $\vec{v} = v_x\,\hat{i} = (4.5\text{ m/s}^3)(2.50\text{ s})^2\,\hat{i} = (28.1\text{ m/s})\,\hat{i}$.

(e) Let t_m stand for the moment when the particle is midway between $x_2 = 21.75$ m and $x_3 = 50.25$ m (that is, when the particle is at $x_m = (x_2 + x_3)/2 = 36$ m). Therefore,

$$36\text{ m} = 9.75\text{ m} + (1.5\text{ m/s}^3)\,t_m^3 \quad \Rightarrow \quad t_m = 2.596\text{ s}$$

Thus, the instantaneous velocity at this time is $\vec{v} = v_x\,\hat{i} = (4.5\text{ m/s}^3)(2.596\text{ s})^2\,\hat{i} = (30.3\text{ m/s})\,\hat{i}$.

(f) The answer to part (a) is given by the slope of the straight line between $t = 2$ and $t = 3$ in this x-vs-t plot. The answers to parts (b), (c), (d) and (e) correspond to the slopes of tangent lines (not shown but easily imagined) to the curve at the appropriate points.



13. This problem involves four regions. The regions from $t = 0$ s to $t = 2$ s and from $t = 10$ s to $t = 12$ s are regions of uniformly changing velocity. The regions from $t = 2$ s to $t = 10$ s and from $t = 12$ s to $t = 16$ s are regions of constant velocity. For the regions of changing velocity, we use the alternate "primary" equation listed below Table 2-1
14.

$$\langle v_x \rangle = \frac{\Delta x}{\Delta t} = \frac{v_{1\,x} + v_{2\,x}}{2} \quad \Rightarrow \quad \Delta x = \left(\frac{v_{1\,x} + v_{2\,x}}{2}\right)\Delta t$$

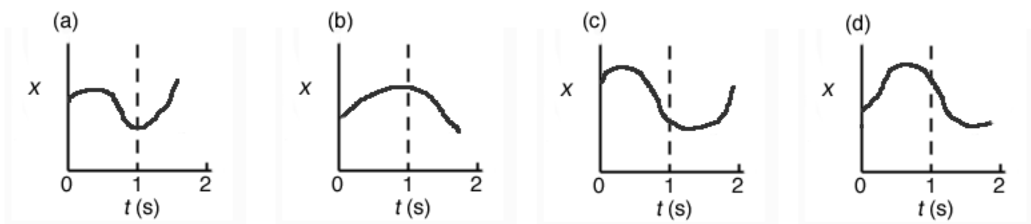
From t = 0 s to t = 2 s, Δt = 2 s – 0 s = 2 s and $v_{1\,x}$ = 0 m/s and $v_{2\,x}$ = 8 m/s so $\Delta x = (0 \text{ m/s} + 8 \text{ m/s}/2)2 \text{ s} = 8 \text{ m}$. From t = 10 s to t = 12 s, Δt = 12 s – 10 s = 2 s and $v_{1\,x}$ = 8 m/s and $v_{2\,x}$ = 4 m/s so $\Delta x = (8 \text{ m/s} + 4 \text{ m/s}/2)2 \text{ s} = 12 \text{ m}$

Since v_x is constant during the time interval between t = 2 s and t = 10 s (Δt = 10 s – 2 s = 8 s), we can use $\langle v_x \rangle = v_x = \dfrac{\Delta x}{\Delta t} \;\Rightarrow\; \Delta x = v_x \Delta t$ = (8 m/s)(8 s) = 64 m.

In the remaining region, Δt = 16 s – 12 s = 4 s and v_x = 4 m/s so Δx = (4 m/s)(4 s) = 16 m.

In this way, we obtain a total Δx = 100 m.

14. From Eq. 2-6 and Eq. 2-9, we note that the sign of the x-component of velocity is the sign of the slope in an x-vs-t plot, and the sign of the x-component of acceleration corresponds to whether such a curve is concave up (for a_x positive) or concave down (for a_x negative).



(e) Any increase in the magnitude of v_x either by becoming less negative or more positive represents increasing $|v_x|$ (speed). This will occur any time that v_x is positive and the acceleration is positive or any time both v_x is negative and the acceleration is negative. Thus, point (a) with zero velocity and positive acceleration, point (b) with zero velocity and negative acceleration and point (d) with negative velocity and negative acceleration involve increasing speed. Point (c) involves negative velocity and positive acceleration (it's magnitude is becoming less negative) so its speed is decreasing.

15. We use Eq. 2-6 and Eq. 2-9.

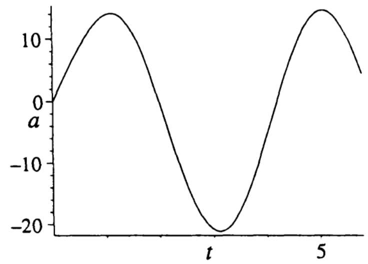
(a) This is $v_x{}^2$ — that is, the square of the x-component of velocity.

(b) This is the acceleration a_x.

(c) The SI units for these quantities are $(\text{m/s})^2 = \text{m}^2/\text{s}^2$ and m/s^2, respectively.

16. Eq. 2-9 indicates that acceleration is the slope of the v_x-vs-t graph.

Based on this, we show here a sketch of the acceleration (in m/s^2) as a function of time. The values along the acceleration axis should not be taken too seriously.



17. We represent its initial direction of motion as the $+x$ direction, so that $v_{1\,x} = +18$ m/s and $v_{2\,x} = -30$ m/s (when $t = 2.4$ s). Using Eq. 2-7 (or Eq. 2-12, suitably interpreted) we find

$$\langle a_x \rangle = \frac{(-30 \text{ m/s}) - (+18 \text{ m/s})}{2.4 \text{ s}} = -20 \text{ m/s}^2$$

which indicates that the average acceleration has magnitude 20 m/s^2 and is in the opposite direction to the particle's initial velocity.

18. We use Eq. 2-4 (average velocity) and Eq. 2-7 (average acceleration). Regarding our coordinate choices, the initial position of the man is taken as the origin and his direction of motion during 5 min $\le t \le$ 10 min is taken to be the positive x direction. We also use the fact that $\Delta x = v_x \Delta t$ when ever the velocity is constant during a given time interval Δt.

(a) Here, the entire interval considered is $\Delta t_{2\text{-}8}$ = 8 min – 2 min = 6 min which is equivalent to 360 s, whereas the sub-interval in which he is *moving* is only $\Delta t_{5-8} = 8 \text{ min} - 5 \text{ min} = 3 \text{ min} = 180 \text{ s}$. His position at t = 2 min is x = 0 m and his position at t = 8 min is $x = v_x \Delta t_{5-8} = (2.20 \text{ m/s})(180 \text{ s}) = 396 \text{ m}$. Therefore,

$$\langle v_x \rangle = \frac{396 \text{ m} - 0 \text{ m}}{360 \text{ s}} = 1.10 \text{ m/s}.$$

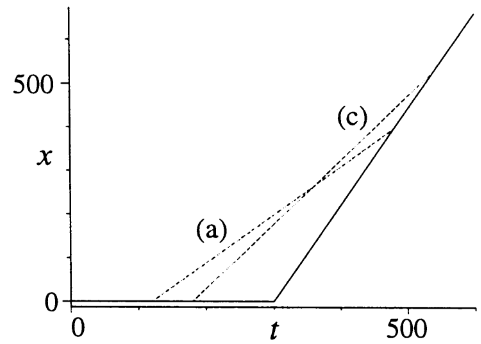
(b) The man is at rest at t = 2 min and has velocity v = +2.20 m/s at t = 8 min. Thus,

$$\langle a_x \rangle = \frac{2.20 \text{ m/s} - 0 \text{ m/s}}{360 \text{ s}} = 0.00611 \text{ m/s}^2 .$$

(c) Now, the entire interval considered is $\Delta t_{3\text{-}9}$ = 9 min – 3 min = 6 min (360 s again), whereas the sub-interval in which he is moving is $\Delta t_{5-9} = 9 \text{ min} - 5 \text{ min} = 4 \text{ min} = 240 \text{ s}$). His position at t = 3 min is x = 0 m and his position at t = 9 min is $x = v_x \Delta t_{5-9} = (2.20 \text{ m/s})(240 \text{ s}) = 528 \text{ m}$. Therefore,

$$\langle v_x \rangle = \frac{528 \text{ m} - 0 \text{ m}}{360 \text{ s}} = 1.47 \text{ m/s}.$$

(d) The horizontal line near the bottom of this x-vs-t graph represents the man standing at $x = 0$ m for $0 \text{ s} \leq t < 300$ s and the linearly rising line for $300 \text{ s} \leq t \leq 600$ s represents his constant-velocity motion. The dotted lines represent the answers to part (a) and (c) in the sense that their slopes yield those results.



The graph of v_x-vs-t is not shown here, but would consist of two horizontal "steps" (one at $v_x = 0$ m/s for $0 \text{ s} \leq t < 300$ s and the next at $v_x = 2.20$ m/s for $300 \text{ s} \leq t \leq 600$ s). The indications of the average accelerations found in parts (b) and (d) would be dotted lines connecting the "steps" at the appropriate t values (the slopes of the dotted lines representing the values of $\langle a_x \rangle$).

19. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t. The notations $v_x(t)$ and $a_x(t)$ which represent vector components have similar meanings.

(a) Since the unit of ct^2 is that of length, the unit of c must be that of length/time^2, or m/s^2 in the SI system. Since bt^3 has a unit of length, b must have a unit of length/time^3, or m/s^3.

(b) When the particle reaches its maximum (or minimum) coordinate its velocity is zero. Since the velocity is given by $v_x = dx/dt = 2ct - 3bt^2$, $v_x = 0$ m/s occurs for $t = 0$ s and for

$$t = \frac{2c}{3b} = \frac{2(3.0 \text{ m/s}^2)}{3(2.0 \text{ m/s}^3)} = 1.0 \text{ s} .$$

For $t = 0$ s, $x = x_0 = 0$ m and for $t = 1.0$ s, $x_1 = 1.0 \text{ m} > x_0$. Since we seek the maximum, we reject the first root ($t = 0$ s) and accept the second ($t = 1.0$ s).

(c) In the first 4.0 s the particle moves from the origin where $x = 0.0$ m to $x = 1.0$ m, turns around, and goes back to

$$x = (3.0 \text{ m/s}^2)(4.0 \text{ s})^2 - (2.0 \text{ m/s}^3)(4.0 \text{ s})^3 = -80 \text{ m} .$$

The total path length it travels is 1.0 m + 1.0 m + 80 m = 82 m.

(d) Its displacement is given by $\Delta x = x_4 - x_0$, where $x_0 = 0$ m and $x_4 = -80$ m. Thus, $\Delta x = -80$ m.

(e) The velocity is given by $v_x = 2ct - 3bt^2 = 2(3.0 \text{ m/s}^2)t - 3(2.0 \text{ m/s}^3)t^2$. Thus

$$\begin{aligned}
v_{1x} &= (6.0 \text{ m/s}^2)(1.0 \text{ s}) - (6.0 \text{ m/s}^3)(1.0 \text{ s})^2 = 0 \text{ m/s} \\
v_{2x} &= (6.0 \text{ m/s}^2)(2.0 \text{ s}) - (6.0 \text{ m/s}^3)(2.0 \text{ s})^2 = -12 \text{ m/s} \\
v_{3x} &= (6.0 \text{ m/s}^2)(3.0 \text{ s}) - (6.0 \text{ m/s}^3)(3.0 \text{ s})^2 = -36.0 \text{ m/s} \\
v_{4x} &= (6.0 \text{ m/s}^2)(4.0 \text{ s}) - (6.0 \text{ m/s}^3)(4.0 \text{ s})^2 = -72 \text{ m/s} \ .
\end{aligned}$$

(f) The acceleration is given by $a_x = dv_x/dt = 2c - 6b = 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)t$. Thus

$$\begin{aligned}
a_{1x} &= 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)(1.0 \text{ s}) = -6.0 \text{ m/s}^2 \\
a_{2x} &= 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)(2.0 \text{ s}) = -18 \text{ m/s}^2 \\
a_{3x} &= 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)(3.0 \text{ s}) = -30 \text{ m/s}^2 \\
a_{4x} &= 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)(4.0 \text{ s}) = -42 \text{ m/s}^2 \ .
\end{aligned}$$

20. For the automobile $\Delta v_x = 55$ km/hr – 25 km/hr = 30 km/h, which we convert to SI units:

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{(30 \text{ km/h})\left(1000 \text{ m/km} / 3600 \text{ s/h}\right)}{(0.50 \text{ min})(60 \text{ s/min})} = 0.28 \text{ m/s}^2.$$

The change of velocity for the bicycle, for the same time, is identical to that of the car, so its acceleration is also 0.28 m/s^2.

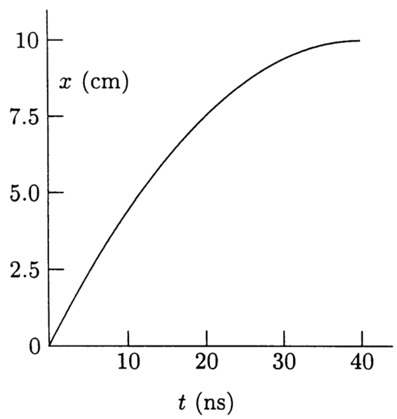
21. From the equation on the bottom of page 43 or from manipulation of the equations in Table 2-1, $v_{2x}^2 - v_{1x}^2 = 2a_x\Delta x$.

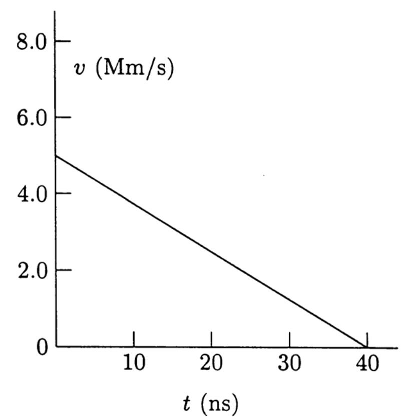
(a) Setting $v_{2x} = 0$ m/s in $v_{2x}^2 = v_{1x}^2 + 2a_x\Delta x$, we find

$$\Delta x = -\frac{1}{2}\frac{v_{1x}^2}{a_x} = -\frac{1}{2}\left(\frac{(5.00 \times 10^6 \text{ m/s})^2}{-1.25 \times 10^{14} \text{ m/s}^2}\right) = 0.100 \text{ m} .$$

Since the muon is slowing, the initial velocity and the acceleration must have opposite signs.

(b) Below are the time-plots of the position x and velocity v of the muon from the moment it enters the field at $t_1 = 0$ s to the time it stops. Using the equations in Table 2-1 where we set $t_2 - t_1 = t$ to make the plots.





22. The time required is found from Eq. 2-12 (or, suitably interpreted, Eq. 2-7). First, we convert the velocity change to SI units:

$$\Delta v_x = (100 \text{ km/h})\left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}}\right) = 27.8 \text{ m/s} .$$

Thus, $\Delta t = \Delta v_x / a_x = (27.8 \text{ m/s})/(50 \text{ m/s}^2) = 0.56$ s.

23. We use $v_{2\,x} = v_{1\,x} + a_x t$, with $t = 0$ s as the instant when the velocity equals +9.6 m/s.

(a) Since we wish to calculate the velocity for a time *before* $t = 0$ s, we set $t = -2.5$ s. Thus, Eq. 2-12 gives

$$v_{2\,x} = (9.6 \text{ m/s}) + \left(3.2 \text{ m/s}^2\right)(-2.5 \text{ s}) = 1.6 \text{ m/s}.$$

(b) Now, $t = +2.5$ s and we find

$$v_{2\,x} = (9.6 \text{ m/s}) + \left(3.2 \text{ m/s}^2\right)(2.5 \text{ s}) = 18 \text{ m/s}.$$

24. The bullet starts at rest ($v_{1\,x} = 0$ m/s) and after traveling the length of the barrel (Δx = 1.20 m) emerges with the given velocity ($v_{2\,x} = 640$ m/s), where the direction of motion is the positive direction. Turning to the constant acceleration equations 2-14 and 2-15, we use

$$\Delta x = \frac{v_{1\,x} + v_{2\,x}}{2}\,\Delta t \;\Rightarrow\; \Delta t = \frac{2(1.20 \text{ m})}{0 \text{ m/s} + 640 \text{ m/s}}$$

Thus, we find $\Delta t = 0.00375$ s (about 3.8 ms).

25. The constant acceleration stated in the problem permits the use of the equations in Table 2-1.

(a) We solve $v_{2x} = v_{1x} + a_x\Delta t$ for the time interval:

$$\Delta t = \frac{v_{2x} - v_{1x}}{a_x} = \frac{\frac{1}{10}(3.0\times10^8 \text{ m/s})}{9.8 \text{ m/s}^2} = 3.1\times10^6 \text{ s}$$

which is equivalent to 1.2 months.

(b) We evaluate $x_2 - x_1 = v_{1x}t + \frac{1}{2}a_x\Delta t^2$, with $x_1 = 0$ m and $v_{1x} = 0$ m/s. The result is

$$x_2 = \frac{1}{2}\left(9.8 \text{ m/s}^2\right)(3.1\times10^6 \text{s})^2 = 4.7\times10^{13} \text{ m} .$$

26. From the equation on the bottom of page 43 or from manipulation of the equations in Table 2-1, $v_{2x}^2 - v_{1x}^2 = 2a_x\Delta x$ is used to solve for a_x. Converting 360 km/hr to 100 m/s and 1.80 km to 1800 m, the minimum acceleration is

$$a_{x\min} = \frac{v_{2x}^2 - v_{1x}^2}{2\Delta x_{\max}} = \frac{(100 \text{ m/s})^2}{2(1800 \text{ m})} = 2.78 \text{ m/s}^2$$

27. The assumed constant acceleration permits the use of the equations in Table 2-1 or the equation on the bottom of page 43. We solve $v_{2x}^2 = v_{1x}^2 + 2a_x(x_2 - x_1)$ with $x_1 = 0$ m and $x_2 = 0.010$ m. Thus,

$$a_x = \frac{v_{2x}^2 - v_{1x}^2}{2\Delta x} = \frac{(5.7\times10^5 \text{ m/s})^2 - (1.5\times10^5 \text{ m/s})^2}{2(0.010 \text{ m})} = 1.6\times10^{15} \text{ m/s}^2.$$

28. The acceleration is found from Eq. 2-12 (or, suitably interpreted, Eq. 2-7).

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{\left(1020 \text{ km/h}\right)\left(1000 \text{ m/km}/3600 \text{ s/h}\right)}{1.4 \text{ s}} = 202.4 \text{ m/s}^2 .$$

In terms of the gravitational acceleration g, this is expressed as a multiple of 9.8 m/s^2 as follows:

$$a_x = \frac{202.4 \text{ m/s}^2}{9.8 \text{ m/s}^2}\, g = 21\, g .$$

29. We choose the positive direction to be that of the initial velocity of the car (implying that $a_x < 0$ m/s^2 since it is slowing down). We assume the acceleration is constant and use Table 2-1.

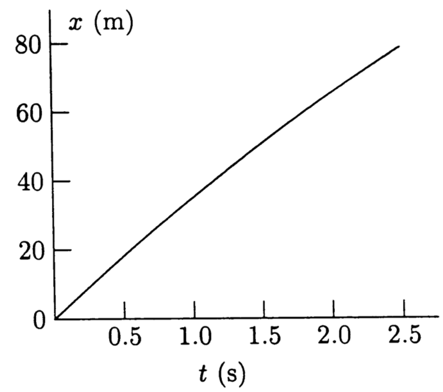
(a) Substituting $v_{1x} = 137$ km/h = 38.1 m/s, $v_{2x} = 90$ km/h = 25 m/s, and $a_x = -5.2$ m/s^2 into $v_{2x} = v_{1x} + a_x\Delta t$, we obtain

$$\Delta t = \frac{25 \text{ m/s} - 38 \text{ m/s}}{-5.2 \text{ m/s}^2} = 2.5 \text{ s}.$$

(b) We take the car to be at $x = 0$ m when the brakes are applied (at time $t = 0$ s). Thus, the coordinate of the car as a function of time is given by

$$x = (38 \text{ m/s})t + \tfrac{1}{2}(-5.2 \text{ m/s}^2)t^2$$

in SI units. This function is plotted from $t = 0$ s to $t = 2.5$ s on the graph below. We have not shown the v_x-vs-t graph here; it is a descending straight line from $v_{1\,x}$ to $v_{2\,x}$.



30. From the figure, we see that $x_1 = -2.0$ m when $t_1 = 0$ s. From Table 2-1, we can apply $x_2 - x_1 = v_{1\,x}\Delta t + \frac{1}{2}a_x\Delta t^2$ with $\Delta t_{1\text{-}2} = t_2 - t_1 = 1.0$ s, and then again for $\Delta t_{1\text{-}3} = t_3 - t_1 = 2.0$ s. This yields two equations for the two unknowns, $v_{1\,x}$ and a_x.

$$\begin{aligned} 0.0\text{ m} - (-2.0\text{ m}) &= v_{1\,x}(1.0\text{ s}) + \tfrac{1}{2}a_x(1.0\text{ s})^2 \\ 6.0\text{ m} - (-2.0\text{ m}) &= v_{1\,x}(2.0\text{ s}) + \tfrac{1}{2}a_x(2.0\text{ s})^2 \;. \end{aligned}$$

Solving these simultaneous equations yields the results $v_{1\,x} = 0.0$ m/s and $a_x = 4.0$ m/s^2. The fact that the answer is positive tells us that the acceleration vector points in the $+x$ direction.

31. The problem statement (see part (a)) indicates that a_x = constant, which allows us to use Table 2-1.

(a) We solve $\Delta x = v_{1\,x}\Delta t + \frac{1}{2}a_x\Delta t^2$ (Eq. 2-15) for the acceleration: $a_x = 2(\Delta x - v_{1\,x}\Delta t)/\Delta t^2$, Substituting $\Delta x = 24.0$ m, $v_{1\,x} = 56.0$ km/h = 15.55 m/s and $\Delta t = 2.00$ s, we find

$$a_x = \frac{2\big(24.0\text{ m} - (15.55\text{ m/s})(2.00\text{ s})\big)}{(2.00\text{ s})^2} = -3.56\text{ m/s}^2 \;.$$

The negative sign indicates that the acceleration is opposite to the direction of motion of the car. The car is slowing down.

(b) We evaluate $v_{2x} = v_{1x} + a_x\Delta t$ as follows:

$$v_{2x} = 15.55 \text{ m/s} - \left(3.56 \text{ m/s}^2\right)\left(2.00 \text{ s}\right) = 8.43 \text{ m/s}$$

which is equivalent to 30.3 km/h.

32. We take the moment of applying brakes to be $t_1 = 0$ s. The deceleration is constant so that the Table 2-1 equations can be used. Our primed variables (such as $v'_{1x} = 72 \text{ km/h} = 20 \text{ m/s}$) refer to one train (moving in the $+x$ direction and located at the origin when $t_1 = 0$ s) and unprimed variables refer to the other (moving in the $-x$ direction and located at $x_1 = +950$ m when $t_1 = 0$ s). We note that the acceleration vector of the unprimed train points in the *positive* direction, even though the train is slowing down; its initial velocity is $v_{1x} = -144$ km/h = -40 m/s. Since the primed train has the lower initial speed, it should stop sooner than the other train would (were it not for the collision). Using the equation on the bottom of page 43 or from manipulation of the equations in Table 2-1 and knowing it should stop (meaning $v'_{2x} = 0$ m/s) at

$$\Delta x' = \frac{\left(v'_{2x}\right)^2 - \left(v'_{1x}\right)^2}{2a'_x} = \frac{(0 \text{ m/s})^2 - (20 \text{ m/s})^2}{2(-1.0 \text{ m/s}^2)} = 200 \text{ m} .$$

The speed of the other train, when it reaches that location, is

$$v_{2x} = \sqrt{v_{1x}^2 + 2a_x\Delta x} = \sqrt{\left(-40 \text{ m/s}\right)^2 + 2\left(1.0 \text{ m/s}^2\right)\left(200 \text{ m} - 950 \text{ m}\right)} = \sqrt{100 \text{ m}^2/\text{s}^2} = 10 \text{ m/s}$$

using the previous equation again. Specifically, its velocity at that moment would be -10 m/s since it is still traveling in the $-x$ direction when it crashes. If the computation of v had failed (meaning that a negative number would have been inside the square root) then we would have looked at the possibility that there was no collision and examined how far apart they finally were. A concern that can be brought up is whether the primed train collides before it comes to rest; this can be studied by computing the time it stops (Eq. 2-12 yields $\Delta t = 20$ s) and seeing where the unprimed train is at that moment (Eq. 2-17 yields $\Delta x = 350$ m, still a good distance away from contact). So $\Delta x^{total} = \Delta x + \Delta x' = 350 \text{ m} + 200 \text{ m} = 550 \text{ m} < 950 \text{ m}$.

33. The acceleration is constant and we may use the equations in Table 2-1.

(a) Since the car traveled $\Delta x = 60$ m in $\Delta t = 6.00$s, its $\langle v_x \rangle$ is 10.0 m/s and we apply Eq. 2-15:

$$\langle v_x \rangle = \frac{v_{1\,x} + v_{2\,x}}{2} \quad \Rightarrow \quad \frac{v_{1\,x} + 15\text{ m/s}}{2} = 10.0\text{ m/s}$$

We solve for the initial velocity: $v_{1\,x} = 5.00$ m/s.

(b) Substituting $v_{2\,x} = 15$ m/s, $v_{1\,x} = 5$ m/s and $\Delta t = 6.00$ s into $a_x = (v_{2\,x} - v_{1\,x})/\Delta t$ (Eq. 2-12), we find $a_x = 1.67$ m/s^2.

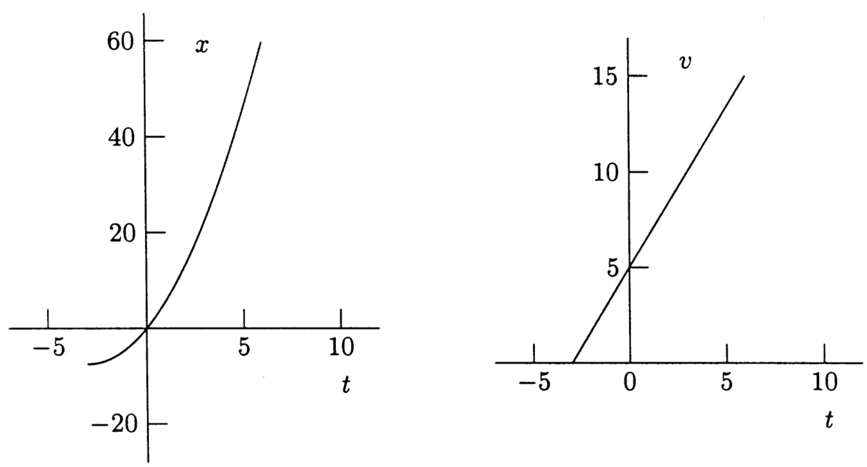
(c) Substituting $v_{2\,x} = 0$ m/s in $v_{2\,x}^2 = v_{1\,x}^2 + 2a_x\Delta x$ and solving for Δx, we obtain

$$\Delta x = -\frac{v_{1\,x}^2}{2a_x} = -\frac{(5.00\text{ m/s})^2}{2(1.67\text{ m/s}^2)} = -7.50\text{ m}\,.$$

(d) The graphs require computing the time when $v_{2\,x} = 0$ m/s, in which case, we use $v_{2\,x} = v_{1\,x} + a_x\Delta t' = 0$ m/s. Thus,

$$\Delta t' = \frac{-v_{1\,x}}{a_x} = \frac{-5.00\text{ m/s}}{1.67\text{ m/s}^2} = -3.0\text{ s}$$

indicates the moment the car was at rest. Assuming that $t_1 = 0\,s$ when $x_1 = 0$ m, then the graphs can be drawn as shown.



34. We denote the required time as t, assuming the light turns green when the clock reads zero. At this time, the distances traveled by the two vehicles must be the same.

(a) Denoting the acceleration of the automobile as $a_{\text{car}\,x}$ and the (constant) speed of the truck as $v_{\text{tr}\,x}$ then after a time interval Δt we have $\Delta x_{\text{car}} = \Delta x_{\text{tr}}$

$$\Delta x_{\text{car}} = \tfrac{1}{2} a_{\text{car}\,x}(\Delta t)^2 = v_{\text{tr}\,x}\Delta t = \Delta x_{\text{tr}}$$

which leads to

$$\Delta t = 0 \text{ s} \quad \text{and} \quad \Delta t = \frac{2v_{\text{tr } x}}{a_{\text{car } x}} = \frac{2(9.5 \text{ m/s})}{2.2 \text{ m/s}^2} = 8.6 \text{ s} .$$

We are interested in Δt = 8.6 s.

$$\Delta x_{\text{tr}} = v_{\text{tr } x}\Delta t = (9.5 \text{ m/s})(8.6 \text{ s}) = 82 \text{ m} .$$

(b) The speed of the car at that moment is

$$v_{\text{car x}} = a_{\text{car } x}\Delta t = (2.2 \text{ m/s}^2)(8.6 \text{ s}) = 19 \text{ m/s} .$$

35. We have two situations: Situation A with an initial x-component of velocity v_A and travel distance Δx_A and situation B with v_B and Δx_B. In general for each situation, we denote Δt_r as the reaction time interval and Δt_{br} as the braking time interval. The motion during Δt_r is of the constant-velocity type and the distance traveled is $v_A\Delta t_r$ or $v_B\Delta t_r$ for the two cases. The motion during Δt_{br} is constant acceleration. Then the distance the car moves in the two situations is

$$\Delta x_A = v_A\Delta t_r + v_A\Delta t_{br} + \tfrac{1}{2}a_x(\Delta t_{br})^2$$
$$\Delta x_B = v_B\Delta t_r + v_B\Delta t_{br} + \tfrac{1}{2}a_x(\Delta t_{br})^2$$

where a_x is the acceleration (which we expect to be negative-valued since we are taking the velocity in the positive direction and we know the car is slowing down). In the previous two equations, we have 3 unknowns—Δt_r, Δt_{br}, and a_x. So we need a third equation to solve for three unknowns. *After* the brakes are applied, in case A so the car is stopped, the velocity of the car is given by 0.0 m/s = $v_A + a_x\Delta t_{br}$ so that $\Delta t_{br} = -v_A/a_x$. We use this result to eliminate Δt_{br} from the first equation and obtain

$$\Delta x_A = v_A\Delta t_r - v_A^2/a_x + \frac{1}{2}\frac{v_A^2}{a_x} = v_A\Delta t_r - \frac{1}{2}\frac{v_A^2}{a_x}.$$

We can write a similar equation for situation B to get:

$$\Delta x_B = v_B\Delta t_r - \frac{1}{2}\frac{v_B^2}{a_x}$$

Solving these equations simultaneously for Δt_r and a_x we get

$$\Delta t_r = \frac{v_B^2\Delta x_A - v_A^2\Delta x_B}{v_A v_B(v_B - v_A)} \quad \text{and} \quad a_x = -\frac{1}{2}\frac{v_B v_A^2 - v_A v_B^2}{v_B\Delta x_A - v_A\Delta x_B}$$

Substituting Δx_A = 56.7 m, v_A = 80.5 km/h = 22.4 m/s, Δx_B = 24.4 m and v_B = 48.3 km/h = 13.4 m/s, we find

(a) $$\Delta t_r = \frac{(13.4\text{ m/s})^2(56.7\text{ m}) - (22.4\text{ m/s})^2(24.4\text{ m})}{(22.4\text{ m/s})(13.4\text{ m/s})(13.4\text{ m/s} - 22.4\text{ m/s})} = 0.74\text{ s}$$

and

(b) $$a_x = -\frac{1}{2}\frac{(13.4\text{ m/s})(22.4\text{ m/s})^2 - (22.4\text{ m/s})(13.4\text{ m/s})^2}{(13.4\text{ m/s})(56.7\text{ m/s}) - (22.4\text{ m/s})(24.4\text{ m/s})} = -6.2\text{ m/s}^2.$$

The *magnitude* of the acceleration is therefore 6.2 m/s^2.

36. Constant acceleration is indicated, so we use the equations in Table 2-1. We start with Eq. 2-17 and Eq. 2-4 and denote the passenger train's initial velocity as $v_{p\,x}(t_1)$ and the locomotive's constant velocity as $v_{L\,x}$ (which is also the final velocity of the train, if the rear-end collision is barely avoided). For the displacement of the passenger train and the displacement of the locomotive where D is the initial gap between trains we have respectively:

$$\Delta x_p = v_{p\ x}(t_1)\Delta t + \tfrac{1}{2}a_{p\ x}(\Delta t)^2 \quad \text{and} \quad \Delta x_L = v_{L\ x}\Delta t + D.$$

We note that the distance the passenger train can move before colliding with the locomotive consists of the original gap between the trains (D) as well as the forward distance the locomotive travels during the time Δt.

We now use Eq. 2-12 to solve for the time it takes the passenger train to catch the locomotive remembering that to avoid a collision $v_{p\,x}(t_2) = v_{L\,x}$.

$$a_{\mathrm{p}\,x} = \frac{v_{px}(t_2) - [v_{px}(t_1)]}{\Delta t} = \frac{v_{Lx} - [v_{px}(t_1)]}{\Delta t} \quad \Rightarrow \quad \Delta t = \frac{v_{Lx} - [v_{px}(t_1)]}{a_{px}}.$$

Determining where the train and locomotive will be at the same location and substituting for Δt and solving for $a_{p\,x}$ yields:

$$\Delta x_p = \Delta x_L + D = v_{px}(t_1)\Delta t + \frac{1}{2}a_{p\ x}\Delta t^2 = v_{Lx}\Delta t + D \quad \Rightarrow$$
$$v_{px}(t_1)\left(\frac{v_{Lx} - [v_{px}(t_1)]}{a_{p\ x}}\right) + \frac{1}{2}a_{p\ x}\left(\frac{v_{Lx} - [v_{px}(t_1)]}{a_{p\ x}}\right)^2 = v_{Lx}\left(\frac{v_{Lx} - [v_{px}(t_1)]}{a_{p\ x}}\right) + D$$

which leads to

$$D = \left(\frac{v_{px}(t_1)v_{Lx} - [v_{px}(t_1)]^2}{a_{p\ x}}\right) + \left(\frac{(v_{Lx})^2 - 2v_{Lx}[v_{px}(t_1)] + [v_{px}(t_1)]^2}{2a_{p\ x}}\right) - \left(\frac{(v_{Lx})^2 - v_{Lx}[v_{px}(t_1)]}{a_{p\ x}}\right)$$

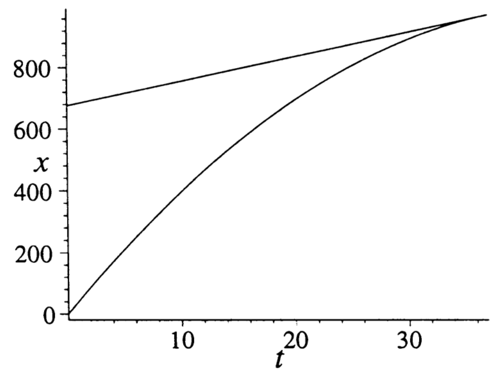
Hence, $$D = \frac{-[v_{px}(t_1)]^2 + 2(v_{Lx})[v_{px}(t_1)] - (v_{Lx})^2}{2a_{p\ x}} \quad \Rightarrow \quad a_{p\ x} = \frac{-[v_{px}(t_1)]^2 - (v_{Lx})^2}{2D}$$

Converting 161 km/hr to 44.7 m/s and 29.0 km/hr to 8.06 m/s gives

$$a_{p\,x} = -\frac{1}{2(676\text{ m})}\left(8.06\text{ m/s} - 44.7\text{ m/s}\right)^2 = -0.993\text{ m/s}^2$$

so that the *magnitude* of $a_{p\,x}$ is 0.993 m/s^2. A graph is shown below for the case where a collision is just avoided (x along the vertical axis is in meters and t along the horizontal axis is in seconds). The top (straight) line shows the motion of the locomotive and the bottom curve shows the motion of the passenger train.

The other case (where the collision is not quite avoided) would be similar except that the slope of the bottom curve would be greater than that of the top line at the point where they meet.



37. Assume that the elevator starts rising at time t_1, reaches its maximum speed $v_{2\,x}$ at time t_2, then moves at constant velocity until time t_3 and comes to rest again at t_4. If the periods of speeding up ($\Delta t_{1\text{-}2} = t_2 - t_1$) and slowing down ($\Delta t_{2\text{-}3} = t_3 - t_2$) are periods of constant a_x then the Table 2-1 equations can be used. Taking the direction of motion to be $+x$ then $a_{1\,x} = +1.22$ m/s^2 , $a_{2\,x} = 0.00$ m/s^2 and $a_{3\,x} = -1.22$ m/s^2. We use SI units so the x-component of velocity at $t = t_1$ is given by v_x = (305 m/min)/(60s/min) = 5.08 m/s.

(a) We denote Δx_1 as the distance moved during $\Delta t_{1\text{-}2}$, and use the equation on the bottom of page 43 that is derived from the equations in Table 2-1,

$$v_{2\,x}^2 = v_{1\,x}^2 + 2a_{1\,x}\Delta x_1 \;\Rightarrow\; \Delta x_1 = \frac{(5.08\text{ m/s})^2}{2(1.22\text{ m/s}^2)}$$

which yields $\Delta x_1 = 10.59\text{ m} \approx 10.6\text{ m}$.

(b) Using Eq. 2-12, we have

$$\Delta t_{1-2} = \frac{v_{2\,x} - v_{1\,x}}{a_{1\,x}} = \frac{5.08\text{ m/s}}{1.22\text{ m/s}^2} = 4.17\text{ s}.$$

Since the acceleration magnitudes are the same for speeding up and slowing down, by symmetry the slowing time $\Delta t_{3\text{-}4}$ turns out to be the same as the speeding up time $\Delta t_{1\text{-}2}$. Thus, $\Delta t_{1\text{-}2} + \Delta t_{3\text{-}4} = 8.33$ s. The distances traveled during $\Delta t_{1\text{-}2}$ and $\Delta t_{3\text{-}4}$ are the same so that they total 2(10.59 m) = 21.18 m. This implies that for a distance of 190 m – 21.18 m = 168.82 m, the elevator is traveling at constant velocity. This time interval of constant velocity motion is

$$\Delta t_{2-3} = \frac{168.82\text{ m}}{5.08\text{ m/s}} = 33.21\text{ s}.$$

Therefore, the total time for the 190 m elevator run is 8.33 s + 33.21 s ≈ 41.5 s.

38. The problem consists of two constant-acceleration parts: part 1 with $v_{1x} = 0$ m/s, $v_{2x} = 6.0$ m/s, $x_2 = 1.8$ m, and $x_1 = 0$ m (if we take its original position to be the coordinate origin); and, part 2 with $v_{2x} = 6.0$ m/s, $v_{3x} = 0$ m/s, and $a_{2x} = -2.5\text{ m/s}^2$ (negative because we are taking the positive direction to be the direction of motion).

(a) We can use Eq. 2-15 combined with Eq. 2-4 to find the time for the first part

$$x_2 - x_1 = \frac{1}{2}(v_{1x} + v_{2x})\,\Delta t_{1-2} \;\Rightarrow\; 1.8\text{ m} - 0\text{ m} = \frac{1}{2}\,(0\text{ m/s} + 6.0\text{ m/s})\Delta t_{1-2}$$

so that $\Delta t_{1-2} = 0.60$ s. And Eq. 2-12 is used to obtain the time for the second part

$$v_{3x} = v_{2x} + a_{2x}t \;\Rightarrow\; 0\text{ m/s} = 6.0\text{ m/s} + (-2.5\text{ m/s}^2)\Delta t_{2-3}$$

from which $\Delta t_{2-3} = 2.4$ s is computed. Thus, the elapsed time is $\Delta t_{1-2} + \Delta t_{2-3} = 3.0$ s.

(b) We already know the distance for part 1. We could find the distance for part 2 from several of the equations, but let's use the equation on the bottom of page 43.

$$v_{3x}^2 = v_{2x}^2 + 2a_{2x}\,\Delta x_{2-3} \;\Rightarrow\; (0\text{ m/s})^2 = (6.0\text{ m/s})^2 + 2(-2.5\text{ m/s}^2)\,\Delta x_{2-3}$$

which leads to $\Delta x_{2-3} = 7.2$ m. Therefore, the total distance traveled by the shuffleboard disk is 1.8 m + 7.2 m = 9.0 m.

39. We separate the motion into two parts, and take the direction of motion to be positive. In part 1, the vehicle accelerates from rest to its highest speed between time t_1 and t_2; we are given $v_{1x} = 0$ m/s; $v_{2x} = 20$ m/s and $a_{1x} = 2.0\text{ m/s}^2$. In part 2, the vehicle decelerates from its highest speed to a halt between time t_2 and t_3; we are given $v_{2x} = 20$ m/s; $v_{3x} = 0$ m/s and $a_{2x} = -1.0\text{ m/s}^2$ (negative because the acceleration vector points opposite to the direction of motion).

(a) From Table 2-1, we find $\Delta t_{1-2} = \Delta t_2 - \Delta t_1$ (the duration of part 1) from $v_{2x} = v_{1x} + a_{1x}\Delta t_{1-2}$. In this way, 20 m/s = 0 m/s + (2.0 m/s$^{2)}\Delta t_{1-2}$ yields Δt_{1-2} = 10 s. We obtain the duration Δt_{2-3} of part 2 from the same equation. Thus, 0 m/s = 20 m/s + (–1.0 m/s^2)Δt_{2-3} leads to Δt_{2-3} = 20 s. Finally, the total elapsed time is 20 s + 10 s = 30 s.

(b) For part 1, we use the equation $v_{2x}^2 = v_{1x}^2 + 2\, a_{1x}(x_2 - x_1)$ from the bottom of page 43 and we find

$$\Delta x_{1-2} = \frac{v_{2x}^2 - v_{1x}^2}{2a_{1x}} = \frac{(20\ \text{m/s})^2 - (0\ \text{m/s})^2}{2(2.0\ \text{m/s}^2)} = 100\ \text{m}.$$

This position is then the *initial* position for part 2, so that when the same equation is used in part 2 we obtain

$$\Delta x_{2-3} = \frac{v_{3x}^2 - v_{2x}^2}{2a_{2x}} = \frac{(0\ \text{m/s})^2 - (20\ \text{m/s})^2}{2\cdot(-1.0\ \text{m/s}^2)} = 200\ \text{m}.$$

Thus, the total displacement is $\Delta x_{tot} = \Delta x_{1-2} + \Delta x_{2-3}$ = 100m + 200 m = 300 m. That this is also the total distance traveled should be evident (the vehicle did not "backtrack" or reverse its direction of motion).

40. At t_r = 0 s, x_{r1} = 0 m, v_{rx} = 20 km/hr = 5.56 m/s, and x_{g1} = 220 m.

Case A: When passing at x = 44.5 m, the red car has traveled a distance of $\Delta x_r = x_2 - x_1$ = (44.5 m – 0 m). Since its velocity component is v_{rx} = 5.56 m/s and $v_{rx} = (\Delta x_r)/(\Delta t_r)$ then $\Delta t_r = (\Delta x_r)/(v_{rx})$ = (44.5 m)/(5.56 m/s) = 8.00 s so the elapsed travel time for both cars is ($\Delta t_r = \Delta t_g$ = 8.00 s).

Case B: When passing at x = 76.6 m, the red car has traveled a distance of $\Delta x_r = x_2 - x_1$ = (76.6 m – 0 m) at a velocity with x-component is v_{rx} = 40 km/hr = 11.12 m/s so the elaspsed travel time is $\Delta t_r = \Delta t_g = (\Delta x_r)/(v_{rx})$ = (77.6 m)/(11.12 m/s) = 6.89 s.

This gives us two simultaneous equations of the form $\Delta x_g = v_{gx}(t_1)\Delta t_g + \frac{1}{2}a_{gx}(\Delta t_g)^2$ describing the acceleration and initial velocity components of the green car.

For Case A: $(220\ \text{m} - 44.5\ \text{m}) = (v_{gx}(t_1))(8.00\ \text{s}) + \frac{1}{2}(a_{gx})(8.00\ \text{s})^2$

For Case B: $(220\ \text{m} - 76.6\ \text{m}) = (v_{gx}(t_1))(6.89\ \text{s}) + \frac{1}{2}(a_{gx})(6.89\ \text{s})^2$

Solving the equations simultaneously gives (a_{gx} = 1.99 m/s^2 and $v_{gx}(t_1)$ = 13.9 m/s (approximately 50 km/hr).

41. (a) Taking derivatives of $x = (12\ \text{m}^2/\text{s}^2)t^2 - (2\ \text{m/s}^3)t^3$ we obtain the functions for the x-components of velocity and acceleration.:

$$v_x = (24\ \text{m}^2/\text{s}^2)t - (6\ \text{m/s}^3)t^2 \quad \text{and } a_x = 24\ \text{m}^2/\text{s}^2 - (12\ \text{m/s}^3)t$$

Substituting in the value $t = 3.0$ s yields $x = 54$ m, $v_x = 18$ m/s, and $a_x = -12\ \text{m/s}^2$.

(b) At the maximum x, we must have $v_x = 0$ m/s; eliminating the $t = 0$ s root, the velocity equation reveals $t = (24\ \text{m}^2/\text{s}^2)/(6\ \text{m/s}^3) = 4$ s for the time of maximum x. Substituting $t =$ 4 s into the equation for x leads to $x = 64$ m for the largest x value reached by the particle.

(c) A maximum v_x requires $a_x = 0\ \text{m/s}^2$, which occurs when $t = (24\ \text{m}^2/\text{s}^2)/(12\ \text{m/s}^3) = 2.0$ s. This, inserted into the velocity equation, gives $(v_x)^{\max} = 24$ m/s.

(d) In part (b), the particle was (momentarily) motionless at $t = 4$ s. The acceleration at that time is readily found to be $24\ \text{m}^2/\text{s}^2 - (12\ \text{m/s}^3)(4\ \text{s}) = -24\ \text{m/s}^2$.

(e) The *average velocity* is defined by Eq. 2-4, so we see that the values of x at $t_1 = 0$ s and $t_2 = 3.0$ s are needed; these are, respectively, $x_1 = 0$ m and $x_2 = 54$ m (found in part (a)). Thus,

$$\langle \vec{v} \rangle = \langle v_x \rangle\,\hat{\text{i}} = \left(\frac{54\ \text{m} - 0\ \text{m}}{3.0\ \text{s} - 0\ \text{s}} \right) \hat{\text{i}} = (18\ \text{m/s})\,\hat{\text{i}}.$$