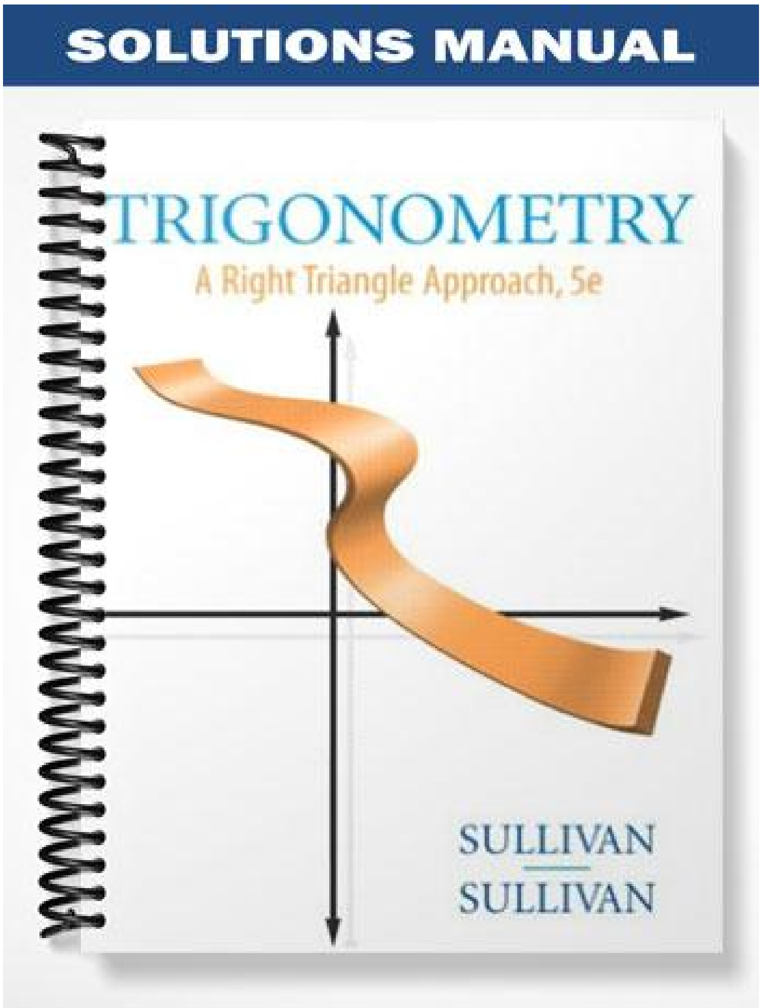

SOLUTIONS MANUAL
TRIGONOMETRY
A Right Triangle Approach, 5e
SULLIVAN
SULLIVAN

Chapter 2

Trigonometric Functions

Section 2.1

1. $C = 2\pi r$

2. $A = \pi r^2$

3. standard position

4. $r\theta$; $\frac{1}{2}r^2\theta$

5. $\frac{s}{t}$; $\frac{\theta}{t}$

6. False

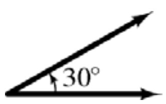
7. True

8. True

9. True

10. False

11.



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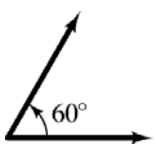

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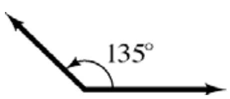

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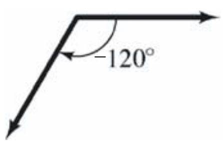

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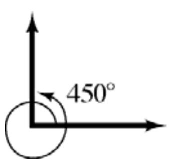

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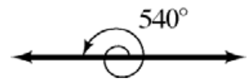

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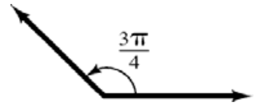

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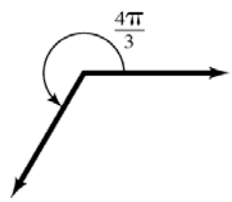

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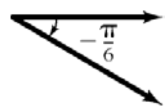

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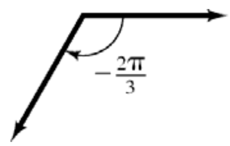

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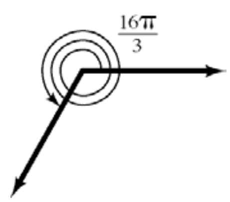

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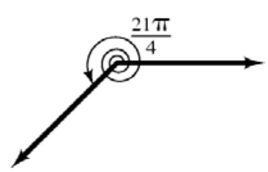


23.
$$40^\circ 10'25'' = \left(40 + 10\cdot\frac{1}{60} + 25\cdot\frac{1}{60}\cdot\frac{1}{60}\right)^\circ$$
$$\approx (40 + 0.1667 + 0.00694)^\circ$$
$$\approx 40.17^\circ$$

24.
$$61^\circ 42'21'' = \left(61 + 42\cdot\frac{1}{60} + 21\cdot\frac{1}{60}\cdot\frac{1}{60}\right)^\circ$$
$$\approx (61 + 0.7000 + 0.00583)^\circ$$
$$\approx 61.71^\circ$$

25. $1^\circ 2'3'' = \left(1 + 2\cdot\frac{1}{60} + 3\cdot\frac{1}{60}\cdot\frac{1}{60}\right)^\circ$
$\approx (1 + 0.0333 + 0.00083)^\circ$
$\approx 1.03^\circ$

26. $73^\circ 40'40'' = \left(73 + 40\cdot\frac{1}{60} + 40\cdot\frac{1}{60}\cdot\frac{1}{60}\right)^\circ$
$\approx (73 + 0.6667 + 0.0111)^\circ$
$\approx 73.68^\circ$

27. $9^\circ 9'9'' = \left(9 + 9\cdot\frac{1}{60} + 9\cdot\frac{1}{60}\cdot\frac{1}{60}\right)^\circ$
$= (9 + 0.15 + 0.0025)^\circ$
$\approx 9.15^\circ$

28. $98^\circ 22'45'' = \left(98 + 22\cdot\frac{1}{60} + 45\cdot\frac{1}{60}\cdot\frac{1}{60}\right)^\circ$
$\approx (98 + 0.3667 + 0.0125)^\circ$
$\approx 98.38^\circ$

29. $40.32^\circ = 40^\circ + 0.32^\circ$
$= 40^\circ + 0.32(60')$
$= 40^\circ + 19.2'$
$= 40^\circ + 19' + 0.2'$
$= 40^\circ + 19' + 0.2(60'')$
$= 40^\circ + 19' + 12''$
$= 40^\circ 19'12''$

30. $61.24^\circ = 61^\circ + 0.24^\circ$
$= 61^\circ + 0.24(60')$
$= 61^\circ + 14.4'$
$= 61^\circ + 14' + 0.4'$
$= 61^\circ + 14' + 0.4(60'')$
$= 61^\circ + 14' + 24''$
$= 61^\circ 14'24''$

31. $18.255^\circ = 18^\circ + 0.255^\circ$
$= 18^\circ + 0.255(60')$
$= 18^\circ + 15.3'$
$= 18^\circ + 15' + 0.3'$
$= 18^\circ + 15' + 0.3(60'')$
$= 18^\circ + 15' + 18''$
$= 18^\circ 15'18''$

32. $29.411^\circ = 29^\circ + 0.411^\circ$
$= 29^\circ + 0.411(60')$
$= 29^\circ + 24.66'$
$= 29^\circ + 24' + 0.66'$
$= 29^\circ + 0.66(60'')$
$= 29^\circ + 24' + 39.6''$
$\approx 29^\circ 24'40''$

33. $19.99^\circ = 19^\circ + 0.99^\circ$
$= 19^\circ + 0.99(60')$
$= 19^\circ + 59.4'$
$= 19^\circ + 59' + 0.4'$
$= 19^\circ + 59' + 0.4(60'')$
$= 19^\circ + 59' + 24''$
$= 19^\circ 59'24''$

34. $44.01^\circ = 44^\circ + 0.01^\circ$
$= 44^\circ + 0.01(60')$
$= 44^\circ + 0.6'$
$= 44^\circ + 0' + 0.6'$
$= 44^\circ + 0' + 0.6(60'')$
$= 44^\circ + 0' + 36''$
$= 44^\circ 0'36''$

35. $30^\circ = 30\cdot\frac{\pi}{180}$ radian $= \frac{\pi}{6}$ radian

36. $120^\circ = 120\cdot\frac{\pi}{180}$ radian $= \frac{2\pi}{3}$ radians

37. $240^\circ = 240\cdot\frac{\pi}{180}$ radian $= \frac{4\pi}{3}$ radians

38. $330^\circ = 330\cdot\frac{\pi}{180}$ radian $= \frac{11\pi}{6}$ radians

39. $-60^\circ = -60\cdot\frac{\pi}{180}$ radian $= -\frac{\pi}{3}$ radian

40. $-30^\circ = -30\cdot\frac{\pi}{180}$ radian $= -\frac{\pi}{6}$ radian

41. $180^\circ = 180\cdot\frac{\pi}{180}$ radian $= \pi$ radians

42. $270^\circ = 270\cdot\frac{\pi}{180}$ radian $= \frac{3\pi}{2}$ radians

43. $-135° = -135 \cdot \frac{\pi}{180}$ radian $= -\frac{3\pi}{4}$ radians

44. $-225° = -225 \cdot \frac{\pi}{180}$ radian $= -\frac{5\pi}{4}$ radians

45. $-90° = -90 \cdot \frac{\pi}{180}$ radian $= -\frac{\pi}{2}$ radians

46. $-180° = -180 \cdot \frac{\pi}{180}$ radian $= -\pi$ radians

47. $\frac{\pi}{3} = \frac{\pi}{3} \cdot \frac{180}{\pi}$ degrees $= 60°$

48. $\frac{5\pi}{6} = \frac{5\pi}{6} \cdot \frac{180}{\pi}$ degrees $= 150°$

49. $-\frac{5\pi}{4} = -\frac{5\pi}{4} \cdot \frac{180}{\pi}$ degrees $= -225°$

50. $-\frac{2\pi}{3} = -\frac{2\pi}{3} \cdot \frac{180}{\pi}$ degrees $= -120°$

51. $\frac{\pi}{2} = \frac{\pi}{2} \cdot \frac{180}{\pi}$ degrees $= 90°$

52. $4\pi = 4\pi \cdot \frac{180}{\pi}$ degrees $= 720°$

53. $\frac{\pi}{12} = \frac{\pi}{12} \cdot \frac{180}{\pi}$ degrees $= 15°$

54. $\frac{5\pi}{12} = \frac{5\pi}{12} \cdot \frac{180}{\pi}$ degrees $= 75°$

55. $-\frac{\pi}{2} = -\frac{\pi}{2} \cdot \frac{180}{\pi}$ degrees $= -90°$

56. $-\pi = -\pi \cdot \frac{180}{\pi}$ degrees $= -180°$

57. $-\frac{\pi}{6} = -\frac{\pi}{6} \cdot \frac{180}{\pi}$ degrees $= -30°$

58. $-\frac{3\pi}{4} = -\frac{3\pi}{4} \cdot \frac{180}{\pi}$ degrees $= -135°$

59. $17° = 17 \cdot \frac{\pi}{180}$ radian $= \frac{17\pi}{180}$ radian ≈ 0.30 radian

60. $73° = 73 \cdot \frac{\pi}{180}$ radian
$= \frac{73\pi}{180}$ radians
≈ 1.27 radians

61. $-40° = -40 \cdot \frac{\pi}{180}$ radian
$= -\frac{2\pi}{9}$ radian
≈ -0.70 radian

62. $-51° = -51 \cdot \frac{\pi}{180}$ radian
$= -\frac{17\pi}{60}$ radian
≈ -0.89 radian

63. $125° = 125 \cdot \frac{\pi}{180}$ radian
$= \frac{25\pi}{36}$ radians
≈ 2.18 radians

64. $350° = 350 \cdot \frac{\pi}{180}$ radian
$= \frac{35\pi}{18}$ radians
≈ 6.11 radians

65. 3.14 radians $= 3.14 \cdot \frac{180}{\pi}$ degrees $\approx 179.91°$

66. 0.75 radian $= 0.75 \cdot \frac{180}{\pi}$ degrees $\approx 42.97°$

67. 2 radians $= 2 \cdot \frac{180}{\pi}$ degrees $\approx 114.59°$

68. 3 radians $= 3 \cdot \frac{180}{\pi}$ degrees $\approx 171.89°$

69. 6.32 radians $= 6.32 \cdot \frac{180}{\pi}$ degrees $\approx 362.11°$

70. $\sqrt{2}$ radians $= \sqrt{2} \cdot \frac{180}{\pi}$ degrees $\approx 81.03°$

71. $r = 10$ meters; $\theta = \dfrac{1}{2}$ radian;

$s = r\theta = 10 \cdot \dfrac{1}{2} = 5$ meters

72. $r = 6$ feet; $\theta = 2$ radian; $s = r\theta = 6 \cdot 2 = 12$ feet

73. $\theta = \dfrac{1}{3}$ radian; $s = 2$ feet;

$s = r\theta$

$r = \dfrac{s}{\theta} = \dfrac{2}{(1/3)} = 6$ feet

74. $\theta = \dfrac{1}{4}$ radian; $s = 6$ cm;

$s = r\theta$

$r = \dfrac{s}{\theta} = \dfrac{6}{(1/4)} = 24$ cm

75. $r = 5$ miles; $s = 3$ miles;

$s = r\theta$

$\theta = \dfrac{s}{r} = \dfrac{3}{5} = 0.6$ radian

76. $r = 6$ meters; $s = 8$ meters;

$s = r\theta$

$\theta = \dfrac{s}{r} = \dfrac{8}{6} = \dfrac{4}{3} \approx 1.333$ radians

77. $r = 2$ inches; $\theta = 30° = 30 \cdot \dfrac{\pi}{180} = \dfrac{\pi}{6}$ radian;

$s = r\theta = 2 \cdot \dfrac{\pi}{6} = \dfrac{\pi}{3} \approx 1.047$ inches

78. $r = 3$ meters; $\theta = 120° = 120 \cdot \dfrac{\pi}{180} = \dfrac{2\pi}{3}$ radians

$s = r\theta = 3 \cdot \dfrac{2\pi}{3} = 2\pi \approx 6.283$ meters

79. $r = 10$ meters; $\theta = \dfrac{1}{2}$ radian

$A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(10)^2\left(\dfrac{1}{2}\right) = \dfrac{100}{4} = 25 \text{ m}^2$

80. $r = 6$ feet; $\theta = 2$ radians

$A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(6)^2(2) = 36 \text{ ft}^2$

81. $\theta = \dfrac{1}{3}$ radian; $A = 2 \text{ ft}^2$

$A = \dfrac{1}{2}r^2\theta$

$2 = \dfrac{1}{2}r^2\left(\dfrac{1}{3}\right)$

$2 = \dfrac{1}{6}r^2$

$12 = r^2$

$r = \sqrt{12} = 2\sqrt{3} \approx 3.464$ feet

82. $\theta = \dfrac{1}{4}$ radian; $A = 6 \text{ cm}^2$

$A = \dfrac{1}{2}r^2\theta$

$6 = \dfrac{1}{2}r^2\left(\dfrac{1}{4}\right)$

$6 = \dfrac{1}{8}r^2$

$48 = r^2$

$r = \sqrt{48} = 4\sqrt{3} \approx 6.928$ cm

83. $r = 5$ miles; $A = 3 \text{ mi}^2$

$A = \dfrac{1}{2}r^2\theta$

$3 = \dfrac{1}{2}(5)^2\theta$

$3 = \dfrac{25}{2}\theta$

$\theta = \dfrac{6}{25} = 0.24$ radian

84. $r = 6$ meters; $A = 8 \text{ m}^2$

$A = \dfrac{1}{2}r^2\theta$

$8 = \dfrac{1}{2}(6)^2\theta$

$8 = 18\theta$

$\theta = \dfrac{8}{18} = \dfrac{4}{9} \approx 0.444$ radian

85. $r = 2$ inches; $\theta = 30° = 30 \cdot \dfrac{\pi}{180} = \dfrac{\pi}{6}$ radian

$A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(2)^2\left(\dfrac{\pi}{6}\right) = \dfrac{\pi}{3} \approx 1.047 \text{ in}^2$

86. $r = 3$ meters; $\theta = 120° = 120 \cdot \frac{\pi}{180} = \frac{2\pi}{3}$ radians

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(3)^2\left(\frac{2\pi}{3}\right) = 3\pi \approx 9.425 \text{ m}^2$

87. $r = 2$ feet; $\theta = \frac{\pi}{3}$ radians

$s = r\theta = 2 \cdot \frac{\pi}{3} = \frac{2\pi}{3} \approx 2.094$ feet

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(2)^2\left(\frac{\pi}{3}\right) = = \frac{2\pi}{3} \approx 2.094 \text{ ft}^2$

88. $r = 4$ meters; $\theta = \frac{\pi}{6}$ radian

$s = r\theta = 4 \cdot \frac{\pi}{6} = \frac{2\pi}{3} \approx 2.094$ meters

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(4)^2\left(\frac{\pi}{6}\right) = \frac{4\pi}{3} \approx 4.189 \text{ m}^2$

89. $r = 12$ yards; $\theta = 70° = 70 \cdot \frac{\pi}{180} = \frac{7\pi}{18}$ radians

$s = r\theta = 12 \cdot \frac{7\pi}{18} \approx 14.661$ yards

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(12)^2\left(\frac{7\pi}{18}\right) = 28\pi \approx 87.965 \text{ yd}^2$

90. $r = 9$ cm; $\theta = 50° = 50 \cdot \frac{\pi}{180} = \frac{5\pi}{18}$ radian

$s = r\theta = 9 \cdot \frac{5\pi}{18} \approx 7.854$ cm

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(9)^2\left(\frac{5\pi}{18}\right) = \frac{45\pi}{4} \approx 35.343 \text{ cm}^2$

91. $r = 6$ inches

In 15 minutes,

$\theta = \frac{15}{60}$ rev $= \frac{1}{4} \cdot 360° = 90° = \frac{\pi}{2}$ radians

$s = r\theta = 6 \cdot \frac{\pi}{2} = 3\pi \approx 9.42$ inches

In 25 minutes,

$\theta = \frac{25}{60}$ rev $= \frac{5}{12} \cdot 360° = 150° = \frac{5\pi}{6}$ radians

$s = r\theta = 6 \cdot \frac{5\pi}{6} = 5\pi \approx 15.71$ inches

92. $r = 40$ inches; $\theta = 20° = \frac{\pi}{9}$ radian

$s = r\theta = 40 \cdot \frac{\pi}{9} = \frac{40\pi}{9} \approx 13.96$ inches

93. $r = 4$ m; $\theta = 45° = 45 \cdot \frac{\pi}{180} = \frac{\pi}{4}$ radian

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(4)^2\left(\frac{\pi}{4}\right) = 2\pi \approx 6.28 \text{ m}^2$

94. $r = 3$ cm; $\theta = 60° = 60 \cdot \frac{\pi}{180} = \frac{\pi}{3}$ radians

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(3)^2\left(\frac{\pi}{3}\right) = \frac{3\pi}{2} \approx 4.71 \text{ cm}^2$

95. $r = 30$ feet; $\theta = 135° = 135 \cdot \frac{\pi}{180} = \frac{3\pi}{4}$ radians

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(30)^2\left(\frac{3\pi}{4}\right) = \frac{675\pi}{2} \approx 1060.29 \text{ ft}^2$

96. $r = 15$ yards; $A = 100 \text{ yd}^2$

$A = \frac{1}{2}r^2\theta$

$100 = \frac{1}{2}(15)^2\theta$

$100 = 112.5\theta$

$\theta = \frac{100}{112.5} = \frac{8}{9} \approx 0.89$ radian

or $\frac{8}{9} \cdot \frac{180}{\pi} = \left(\frac{160}{\pi}\right)^{\circ} \approx 50.93°$

97. $r = 5$ cm; $t = 20$ seconds; $\theta = \frac{1}{3}$ radian

$\omega = \frac{\theta}{t} = \frac{(1/3)}{20} = \frac{1}{3} \cdot \frac{1}{20} = \frac{1}{60}$ radian/sec

$v = \frac{s}{t} = \frac{r\theta}{t} = \frac{5 \cdot (1/3)}{20} = \frac{5}{3} \cdot \frac{1}{20} = \frac{1}{12}$ cm/sec

98. $r = 2$ meters; $t = 20$ seconds; $s = 5$ meters

$\omega = \frac{\theta}{t} = \frac{(s/r)}{t} = \frac{(5/2)}{20} = \frac{5}{2} \cdot \frac{1}{20} = \frac{1}{8}$ radian/sec

$v = \frac{s}{t} = \frac{5}{20} = \frac{1}{4}$ m/sec

99. $d = 26$ inches; $r = 13$ inches; $v = 35$ mi/hr

$$v = \frac{35\text{ mi}}{\text{hr}} \cdot \frac{5280\text{ ft}}{\text{mi}} \cdot \frac{12\text{ in.}}{\text{ft}} \cdot \frac{1\text{ hr}}{60\text{ min}}$$
$$= 36{,}960\text{ in./min}$$
$$\omega = \frac{v}{r} = \frac{36{,}960\text{ in./min}}{13\text{ in.}}$$
$$\approx 2843.08\text{ radians/min}$$
$$\approx \frac{2843.08\text{ rad}}{\text{min}} \cdot \frac{1\text{ rev}}{2\pi\text{ rad}}$$
$$\approx 452.5\text{ rev/min}$$

100. $r = 15$ inches; $\omega = 3$ rev/sec $= 6\pi$ rad/sec

$v = r\omega = 15 \cdot 6\pi$ in./sec $= 90\pi \approx 282.7$ in/sec

$$v = 90\pi\frac{\text{in.}}{\text{sec}} \cdot \frac{1\text{ft}}{12\text{in.}} \cdot \frac{1\text{mi}}{5280\text{ft}} \cdot \frac{3600\text{sec}}{1\text{hr}} \approx 16.1\text{ mi/hr}$$

101. $r = 3960$ miles

$$\theta = 35^\circ 9' - 29^\circ 57'$$
$$= 5^\circ 12'$$
$$= 5.2^\circ$$
$$= 5.2 \cdot \frac{\pi}{180}$$
$$\approx 0.09076\text{ radian}$$
$$s = r\theta = 3960(0.09076) \approx 359\text{ miles}$$

102. $r = 3960$ miles

$$\theta = 38^\circ 21' - 30^\circ 20'$$
$$= 8^\circ 1'$$
$$\approx 8.017^\circ$$
$$= 8.017 \cdot \frac{\pi}{180}$$
$$\approx 0.1399\text{ radian}$$
$$s = r\theta = 3960(0.1399) \approx 554\text{ miles}$$

103. $r = 3429.5$ miles

$$\omega = 1\text{ rev/day} = 2\pi\text{ radians/day} = \frac{\pi}{12}\text{ radians/hr}$$
$$v = r\omega = 3429.5 \cdot \frac{\pi}{12} \approx 898\text{ miles/hr}$$

104. $r = 3033.5$ miles

$$\omega = 1\text{ rev/day} = 2\pi\text{ radians/day} = \frac{\pi}{12}\text{ radians/hr}$$
$$v = r\omega = 3033.5 \cdot \frac{\pi}{12} \approx 794\text{ miles/hr}$$

105. $r = 2.39 \times 10^5$ miles

$$\omega = 1\text{ rev/27.3 days}$$
$$= 2\pi\text{ radians/27.3 days}$$
$$= \frac{\pi}{12 \cdot 27.3}\text{ radians/hr}$$
$$v = r\omega = \left(2.39 \times 10^5\right) \cdot \frac{\pi}{327.6} \approx 2292\text{ miles/hr}$$

106. $r = 9.29 \times 10^7$ miles

$$\omega = 1\text{ rev/365 days}$$
$$= 2\pi\text{ radians/365 days}$$
$$= \frac{\pi}{12 \cdot 365}\text{ radians/hr}$$
$$v = r\omega = \left(9.29 \times 10^7\right) \cdot \frac{\pi}{4380} \approx 66{,}633\text{ miles/hr}$$

107. $r_1 = 2$ inches; $r_2 = 8$ inches;

$\omega_1 = 3$ rev/min $= 6\pi$ radians/min

Find ω_2:

$$v_1 = v_2$$
$$r_1\omega_1 = r_2\omega_2$$
$$2(6\pi) = 8\omega_2$$
$$\omega_2 = \frac{12\pi}{8}$$
$$= 1.5\pi\text{ radians/min}$$
$$= \frac{1.5\pi}{2\pi}\text{ rev/min}$$
$$= \frac{3}{4}\text{ rev/min}$$

108. $r = 30$ feet

$$\omega = \frac{1\text{ rev}}{70\text{ sec}} = \frac{2\pi}{70\text{ sec}} = \frac{\pi}{35} \approx 0.09\text{ radian/sec}$$
$$v = r\omega = 30\text{ feet} \cdot \frac{\pi\text{ rad}}{35\text{ sec}} = \frac{6\pi\text{ ft}}{7\text{ sec}} \approx 2.69\text{ feet/sec}$$

109. $r = 4$ feet; $\omega = 10$ rev/min $= 20\pi$ radians/min

$$v = r\omega$$
$$= 4 \cdot 20\pi$$
$$= 80\pi\ \frac{\text{ft}}{\text{min}}$$
$$= \frac{80\pi\text{ ft}}{\text{min}} \cdot \frac{1\text{ mi}}{5280\text{ ft}} \cdot \frac{60\text{ min}}{\text{hr}}$$
$$\approx 2.86\text{ mi/hr}$$

110. $d = 26$ inches; $r = 13$ inches;

$\omega = 480$ rev/min $= 960\pi$ radians/min

$$\begin{aligned} v &= r\omega \\ &= 13 \cdot 960\pi \\ &= 12480\pi \ \frac{\text{in}}{\text{min}} \\ &= \frac{12480\pi \text{ in}}{\text{min}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{60 \text{ min}}{\text{hr}} \\ &\approx 37.13 \text{ mi/hr} \end{aligned}$$

$$\begin{aligned} \omega &= \frac{v}{r} \\ &= \frac{80 \text{ mi/hr}}{13 \text{ in}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}} \\ &\approx 1034.26 \text{ rev/min} \end{aligned}$$

111. $d = 8.5$ feet; $r = 4.25$ feet; $v = 9.55$ mi/hr

$$\begin{aligned} \omega &= \frac{v}{r} = \frac{9.55 \text{ mi/hr}}{4.25 \text{ ft}} \\ &= \frac{9.55 \text{ mi}}{\text{hr}} \cdot \frac{1}{4.25 \text{ ft}} \cdot \frac{5280 \text{ ft}}{\text{mi}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ rev}}{2\pi} \\ &\approx 31.47 \text{ rev/min} \end{aligned}$$

112. Let t represent the time for the earth to rotate 90 miles.

$$\frac{t}{90} = \frac{24}{2\pi(3559)}$$

$$t = \frac{90(24)}{2\pi(3559)} \approx 0.0966 \text{ hours} \approx 5.8 \text{ minutes}$$

113. The earth makes one full rotation in 24 hours. The distance traveled in 24 hours is the circumference of the earth. At the equator the circumference is $2\pi(3960)$ miles. Therefore, the linear velocity a person must travel to keep up with the sun is:

$$v = \frac{s}{t} = \frac{2\pi(3960)}{24} \approx 1037 \text{ miles/hr}$$

114. Find s, when $r = 3960$ miles and $\theta = 1'$.

$$\theta = 1' \cdot \frac{1 \text{ degree}}{60 \text{ min}} \cdot \frac{\pi \text{ radians}}{180 \text{ degrees}} \approx 0.00029 \text{ radian}$$

$$s = r\theta = 3960(0.00029) \approx 1.15 \text{ miles}$$

Thus, 1 nautical mile is approximately 1.15 statute miles.

115. We know that the distance between Alexandria and Syene to be $s = 500$ miles. Since the measure of the Sun's rays in Alexandria is $7.2°$, the central angle formed at the center of Earth between Alexandria and Syene must also be $7.2°$. Converting to radians, we have

$7.2° = 7.2° \cdot \frac{\pi}{180°} = \frac{\pi}{25}$ radian. Therefore,

$$s = r\theta$$

$$500 = r \cdot \frac{\pi}{25}$$

$$r = \frac{25}{\pi} \cdot 500 = \frac{12{,}500}{\pi} \approx 3979 \text{ miles}$$

$$C = 2\pi r = 2\pi \cdot \frac{12{,}500}{\pi} = 25{,}000 \text{ miles.}$$

The radius of Earth is approximately 3979 miles, and the circumference is approximately 25,000 miles.

116. **a.** The length of the outfield fence is the arc length subtended by a central angle $\theta = 96°$ with $r = 200$ feet.

$$s = r \cdot \theta = 200 \cdot 96° \cdot \frac{\pi}{180°} \approx 335.10 \text{ feet}$$

The outfield fence is approximately 335.1 feet long.

b. The area of the warning track is the difference between the areas of two sectors with central angle $\theta = 96°$. One sector with $r = 200$ feet and the other with $r = 190$ feet.

$$\begin{aligned} A &= \frac{1}{2}R^2\theta - \frac{1}{2}r^2\theta = \frac{\theta}{2}\left(R^2 - r^2\right) \\ &= \frac{96°}{2} \cdot \frac{\pi}{180°}\left(200^2 - 190^2\right) \\ &= \frac{4\pi}{15}\left(3900\right) \approx 3267.26 \end{aligned}$$

The area of the warning track is about 3267.26 square feet.

117. r_1 rotates at ω_1 rev/min , so $v_1 = r_1\omega_1$.
r_2 rotates at ω_2 rev/min , so $v_2 = r_2\omega_2$.
Since the linear speed of the belt connecting the pulleys is the same, we have:

$$v_1 = v_2$$
$$r_1\omega_1 = r_2\omega_2$$
$$\frac{r_1\omega_1}{r_2\omega_1} = \frac{r_2\omega_2}{r_2\omega_1}$$
$$\frac{r_1}{r_2} = \frac{\omega_2}{\omega_1}$$

118. Answers will vary.

119. If the radius of a circle is *r* and the length of the arc subtended by the central angle is also *r*, then the measure of the angle is 1 radian. Also, $1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$.

$1° = \frac{1}{360}$ revolution

120. Note that $1° = 1° \cdot \left(\frac{\pi \text{ radians}}{180°}\right) \approx 0.017$ radian and $1 \text{ radian} \cdot \left(\frac{180°}{\pi \text{ radians}}\right) \approx 57.296°$.
Therefore, an angle whose measure is 1 radian is larger than an angle whose measure is 1 degree.

121. Linear speed measures the distance traveled per unit time, and angular speed measures the change in a central angle per unit time. In other words, linear speed describes distance traveled by a point located on the edge of a circle, and angular speed describes the turning rate of the circle itself.

122. This is a true statement. That is, since an angle measured in degrees can be converted to radian measure by using the formula $180 \text{ degrees} = \pi \text{ radians}$, the arc length formula can be rewritten as follows: $s = r\theta = \frac{\pi}{180} r\theta$.

123 – 125. Answers will vary.

Section 2.2

1. $c^2 = a^2 + b^2 = 6^2 + 10^2 = 36 + 100 = 136$
$c = \sqrt{136} = 2\sqrt{34}$

2. $f(5) = 3(5) - 7 = 15 - 7 = 8$

3. complementary

4. cosine

5. $62°$

6. 1

7. True

8. False

9. True

10. False

11. opposite = 5; adjacent = 12; hypotenuse = ?
$(\text{hypotenuse})^2 = 5^2 + 12^2 = 169$
$\text{hypotenuse} = \sqrt{169} = 13$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{5}{13}$ $\qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{13}{5}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{12}{13}$ $\qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{13}{12}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{5}{12}$ $\qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{12}{5}$

12. opposite = 3; adjacent = 4, hypotenuse = ?
$(\text{hypotenuse})^2 = 3^2 + 4^2 = 25$
$\text{hypotenuse} = \sqrt{25} = 5$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}$ $\qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{3}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}$ $\qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{4}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}$ $\qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{3}$

13. opposite = 2; adjacent = 3; hypotenuse = ?

$(\text{hypotenuse})^2 = 2^2 + 3^2 = 13$

$\text{hypotenuse} = \sqrt{13}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2}{\sqrt{13}} = \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{\sqrt{13}} = \frac{3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{2}{3}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{13}}{2}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{13}}{3}$

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{2}$

14. opposite = 3; adjacent = 3; hypotenuse = ?

$(\text{hypotenuse})^2 = 3^2 + 3^2 = 18$

$\text{hypotenuse} = \sqrt{18} = 3\sqrt{2}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{3\sqrt{2}} = \frac{3}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{3\sqrt{2}} = \frac{3}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{3} = 1$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{3\sqrt{2}}{3} = \sqrt{2}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{3\sqrt{2}}{3} = \sqrt{2}$

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{3} = 1$

15. adjacent = 2; hypotenuse = 4; opposite = ?

$(\text{opposite})^2 + 2^2 = 4^2$

$(\text{opposite})^2 = 16 - 4 = 12$

$\text{opposite} = \sqrt{12} = 2\sqrt{3}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2}{4} = \frac{1}{2}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{4}{2\sqrt{3}} = \frac{4}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{4}{2} = 2$

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{2}{2\sqrt{3}} = \frac{2}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

16. opposite = 3; hypotenuse = 4; adjacent = ?

$3^2 + (\text{adjacent})^2 = 4^2$

$(\text{adjacent})^2 = 16 - 9 = 7$

$\text{adjacent} = \sqrt{7}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{4}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{7}}{4}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{\sqrt{7}} = \frac{3}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{4}{3}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{4}{\sqrt{7}} = \frac{4}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{7}}{3}$

17. opposite = $\sqrt{2}$; adjacent = 1; hypotenuse = ?

$(\text{hypotenuse})^2 = \left(\sqrt{2}\right)^2 + 1^2 = 3$

$\text{hypotenuse} = \sqrt{3}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

18. opposite = 2; adjacent = $\sqrt{3}$; hypotenuse = ?

$(\text{hypotenuse})^2 = 2^2 + \left(\sqrt{3}\right)^2 = 7$

$\text{hypotenuse} = \sqrt{7}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2}{\sqrt{7}} = \frac{2}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{\sqrt{7}} = \frac{\sqrt{3}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{21}}{7}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{7}}{2}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{7}}{\sqrt{3}} = \frac{\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{21}}{3}$

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{3}}{2}$

19. opposite = 1; hypotenuse = $\sqrt{5}$; adjacent = ?

$1^2 + (\text{adjacent})^2 = \left(\sqrt{5}\right)^2$

$(\text{adjacent})^2 = 5 - 1 = 4$

$\text{adjacent} = \sqrt{4} = 2$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{2}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{5}}{1} = \sqrt{5}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{5}}{2}$

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{2}{1} = 2$

20. adjacent = 2; hypotenuse = $\sqrt{5}$; opposite = ?

$(\text{opposite})^2 + 2^2 = \left(\sqrt{5}\right)^2$

$(\text{opposite})^2 = 5 - 4 = 1$

$\text{opposite} = \sqrt{1} = 1$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{2}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{5}}{1} = \sqrt{5}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{5}}{2}$

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{2}{1} = 2$

21. $\sin\theta = \frac{1}{2}; \quad \cos\theta = \frac{\sqrt{3}}{2}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{1}{2}} = 1 \cdot 2 = 2$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{\sqrt{3}}{3}} = \frac{3}{\sqrt{3}} = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

22. $\sin\theta = \dfrac{\sqrt{3}}{2}; \quad \cos\theta = \dfrac{1}{2}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2}\cdot\frac{2}{1} = \sqrt{3}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{1}{2}} = 1\cdot 2 = 2$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

23. $\sin\theta = \dfrac{2}{3}; \quad \cos\theta = \dfrac{\sqrt{5}}{3}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{2}{3}}{\frac{\sqrt{5}}{3}} = \frac{2}{3}\cdot\frac{3}{\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{\sqrt{5}}{3}} = \frac{3}{\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{2\sqrt{5}}{5}} = \frac{5}{2\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{10} = \frac{\sqrt{5}}{2}$$

24. $\sin\theta = \dfrac{1}{3}; \quad \cos\theta = \dfrac{2\sqrt{2}}{3}$

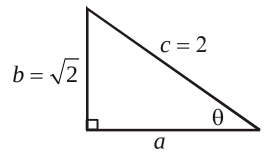
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = \frac{1}{3}\cdot\frac{3}{2\sqrt{2}} = \frac{1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{1}{3}} = 1\cdot 3 = 3$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{2\sqrt{2}}{3}} = \frac{3}{2\sqrt{2}} = \frac{3}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{\sqrt{2}}{4}} = \frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

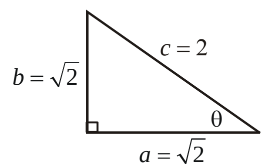
25. $\sin\theta = \dfrac{\sqrt{2}}{2}$ corresponds to the right triangle:



Using the Pythagorean Theorem:

$$a^2 + \left(\sqrt{2}\right)^2 = 2^2$$
$$a^2 = 4 - 2 = 2$$
$$a = \sqrt{2}$$

So the triangle is:



$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{2}}{2}$$

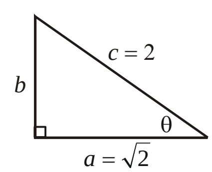
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

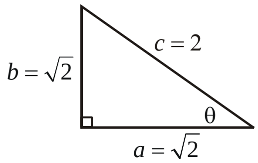
26. $\cos\theta = \dfrac{\sqrt{2}}{2}$ corresponds to the right triangle:



Using the Pythagorean Theorem:

$$b^2 + \left(\sqrt{2}\right)^2 = 2^2$$
$$b^2 = 4 - 2 = 2$$
$$b = \sqrt{2}$$

So the triangle is:



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{2}}{2}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

27. $\cos\theta = \frac{1}{3}$

Using the Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \left(\frac{1}{3}\right)^2 = 1$$

$$\sin^2\theta + \frac{1}{9} = 1$$

$$\sin^2\theta = \frac{8}{9}$$

$$\sin\theta = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

(Note: $\sin\theta$ must be positive since θ is acute.)

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = \frac{2\sqrt{2}}{3}\cdot\frac{3}{1} = 2\sqrt{2}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{2\sqrt{2}}{3}} = \frac{3}{2\sqrt{2}} = \frac{3}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{1}{3}} = 1\cdot 3 = 3$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{2\sqrt{2}} = \frac{1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$$

28. $\sin\theta = \frac{\sqrt{3}}{4}$

Using the Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{\sqrt{3}}{4}\right)^2 + \cos^2\theta = 1$$

$$\frac{3}{16} + \cos^2\theta = 1$$

$$\cos^2\theta = \frac{13}{16}$$

$$\cos\theta = \sqrt{\frac{13}{16}} = \frac{\sqrt{13}}{4}$$

(Note: $\cos\theta$ must be positive since θ is acute.)

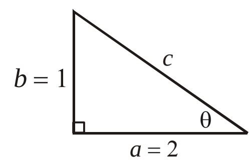
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{3}}{4}}{\frac{\sqrt{13}}{4}} = \frac{\sqrt{3}}{\sqrt{13}} = \frac{\sqrt{3}}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}} = \frac{\sqrt{39}}{13}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{\sqrt{3}}{4}} = \frac{4}{\sqrt{3}} = \frac{4}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{\sqrt{13}}{4}} = \frac{4}{\sqrt{13}} = \frac{4}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}} = \frac{4\sqrt{13}}{13}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{\sqrt{13}}{4}}{\frac{\sqrt{3}}{4}} = \frac{\sqrt{13}}{\sqrt{3}} = \frac{\sqrt{13}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{39}}{3}$$

29. $\tan\theta = \frac{1}{2}$ corresponds to the right triangle:

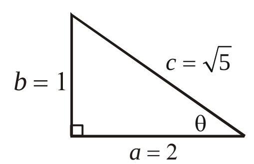


Using the Pythagorean Theorem:

$$c^2 = 1^2 + 2^2 = 5$$

$$c = \sqrt{5}$$

So, the triangle is:



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

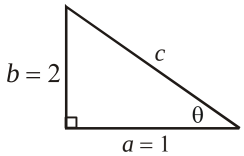
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{5}}{1} = \sqrt{5}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{5}}{2}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{2}{1} = 2$$

30. $\cot\theta = \frac{1}{2}$ corresponds to the right triangle:

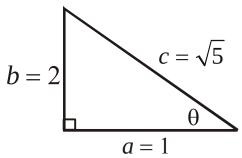


Using the Pythagorean Theorem:

$c^2 = 1^2 + 2^2 = 5$

$c = \sqrt{5}$

So the triangle is:



$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{2}{1} = 2$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{5}}{2}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{5}}{1} = \sqrt{5}$

31. $\sec\theta = 3$

Using the Pythagorean Identities:

$\tan^2\theta + 1 = \sec^2\theta$

$\tan^2\theta + 1 = 3^2$

$\tan^2\theta = 3^2 - 1 = 8$

$\tan\theta = \sqrt{8} = 2\sqrt{2}$

(Note: $\tan\theta$ must be positive since θ is acute.)

$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, so

$\sin\theta = (\tan\theta)(\cos\theta) = 2\sqrt{2} \cdot \frac{1}{3} = \frac{2\sqrt{2}}{3}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{2\sqrt{2}}{3}} = \frac{3}{2\sqrt{2}} = \frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{2\sqrt{2}} = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$

32. $\csc\theta = 5$

Using the Pythagorean Identities:

$\cot^2\theta + 1 = \csc^2\theta$

$\cot^2\theta + 1 = 5^2$

$\cot^2\theta = 5^2 - 1 = 24$

$\cot\theta = \sqrt{24} = 2\sqrt{6}$

(Note: $\cot\theta$ must be positive since θ is acute.)

$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{5}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$, so

$\cos\theta = (\cot\theta)(\sin\theta) = 2\sqrt{6} \cdot \frac{1}{5} = \frac{2\sqrt{6}}{5}$

$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{2\sqrt{6}} = \frac{1}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{12}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{2\sqrt{6}}{5}} = \frac{5}{2\sqrt{6}} = \frac{5}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{5\sqrt{6}}{12}$

33. $\tan\theta = \sqrt{2}$

Using the Pythagorean Identities:

$\sec^2\theta = \tan^2\theta + 1$

$\sec^2\theta = \left(\sqrt{2}\right)^2 + 1 = 3$

$\sec\theta = \sqrt{3}$

(Note: $\sec\theta$ must be positive since θ is acute.)

$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, so

$\sin\theta = (\tan\theta)(\cos\theta) = \sqrt{2} \cdot \frac{\sqrt{3}}{3} = \frac{\sqrt{6}}{3}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{\sqrt{6}}{3}} = \frac{3}{\sqrt{6}} = \frac{3}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{6}}{6} = \frac{\sqrt{6}}{2}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

34. $\sec\theta = \frac{5}{3}$

Using the Pythagorean Identities:

$\tan^2\theta + 1 = \sec^2\theta$

$\tan^2\theta + 1 = \left(\frac{5}{3}\right)^2$

$$\tan^2\theta = \left(\frac{5}{3}\right)^2 - 1 = \frac{25}{9} - 1 = \frac{16}{9}$$

$$\tan\theta = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

(Note: $\tan\theta$ must be positive since θ is acute.)

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\frac{5}{3}} = \frac{3}{5}$$

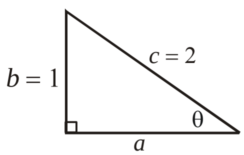
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, so

$$\sin\theta = (\tan\theta)(\cos\theta) = \frac{4}{3}\cdot\frac{3}{5} = \frac{4}{5}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

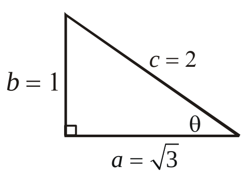
35. $\csc\theta = 2$ corresponds to the right triangle:



Using the Pythagorean Theorem:

$$a^2 + 1^2 = 2^2$$
$$a^2 + 1 = 4$$
$$a^2 = 4 - 1 = 3$$
$$a = \sqrt{3}$$

So the triangle is:



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2}$$

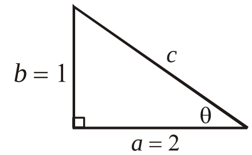
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

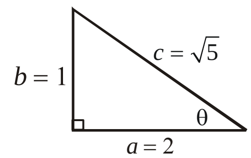
36. $\cot\theta = 2$ corresponds to the right triangle:



Using the Pythagorean Theorem:

$$c^2 = 1^2 + 2^2 = 1 + 4 = 5$$
$$c = \sqrt{5}$$

So the triangle is:



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{2}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{5}}{1} = \sqrt{5}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{5}}{2}$$

37. $\sin^2 20^\circ + \cos^2 20^\circ = 1$, using the identity $\sin^2\theta + \cos^2\theta = 1$

38. $\sec^2 28^\circ - \tan^2 28^\circ = 1$, using the identity $\tan^2\theta + 1 = \sec^2\theta$

39. $\sin 80^\circ \csc 80^\circ = \sin 80^\circ \cdot \dfrac{1}{\sin 80^\circ} = 1$, using the identity $\csc\theta = \dfrac{1}{\sin\theta}$

40. $\tan 10^\circ \cot 10^\circ = \tan 10^\circ \cdot \dfrac{1}{\tan 10^\circ} = 1$, using the identity $\cot\theta = \dfrac{1}{\tan\theta}$

41. $\tan 50^\circ - \dfrac{\sin 50^\circ}{\cos 50^\circ} = \tan 50^\circ - \tan 50^\circ = 0$, using the identity $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

42. $\cot 25^\circ - \dfrac{\cos 25^\circ}{\sin 25^\circ} = \cot 25^\circ - \cot 25^\circ = 0$, using the identity $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

43.
$$\begin{aligned}\sin 38^\circ - \cos 52^\circ &= \sin 38^\circ - \sin(90^\circ - 52^\circ)\\ &= \sin 38^\circ - \sin 38^\circ\\ &= 0\end{aligned}$$
using the identity $\cos\theta = \sin(90^\circ - \theta)$

44.
$$\begin{aligned}\tan 12^\circ - \cot 78^\circ &= \tan 12^\circ - \tan(90^\circ - 78^\circ)\\ &= \tan 12^\circ - \tan 12^\circ\\ &= 0\end{aligned}$$
using the identity $\cot\theta = \tan(90^\circ - \theta)$

45. $\dfrac{\cos 10^\circ}{\sin 80^\circ} = \dfrac{\sin(90^\circ - 10^\circ)}{\sin 80^\circ} = \dfrac{\sin 80^\circ}{\sin 80^\circ} = 1$
using the identity $\cos\theta = \sin(90^\circ - \theta)$

46. $\dfrac{\cos 40^\circ}{\sin 50^\circ} = \dfrac{\sin(90^\circ - 40^\circ)}{\sin 50^\circ} = \dfrac{\sin 50^\circ}{\sin 50^\circ} = 1$
using the identity $\cos\theta = \sin(90^\circ - \theta)$

47.
$$\begin{aligned}1 - \cos^2 20^\circ - \cos^2 70^\circ &= 1 - \cos^2 20^\circ - \sin^2(90^\circ - 70^\circ)\\ &= 1 - \cos^2 20^\circ - \sin^2(20^\circ)\\ &= 1 - \left(\cos^2 20^\circ + \sin^2(20^\circ)\right)\\ &= 1 - 1\\ &= 0\end{aligned}$$
using the identities $\cos\theta = \sin(90^\circ - \theta)$ and $\sin^2\theta + \cos^2\theta = 1$.

48.
$$\begin{aligned}1 + \tan^2 5^\circ - \csc^2 85^\circ &= \sec^2 5^\circ - \csc^2 85^\circ\\ &= \sec^2 5^\circ - \sec^2(90^\circ - 85^\circ)\\ &= \sec^2 5^\circ - \sec^2 5^\circ\\ &= 0\end{aligned}$$
using the identities $1 + \tan^2\theta = \sec^2\theta$ and $\csc\theta = \sec(90^\circ - \theta)$

49.
$$\begin{aligned}\tan 20^\circ - \frac{\cos 70^\circ}{\cos 20^\circ} &= \tan 20^\circ - \frac{\sin(90^\circ - 70^\circ)}{\cos 20^\circ}\\ &= \tan 20^\circ - \frac{\sin 20^\circ}{\cos 20^\circ}\\ &= \tan 20^\circ - \tan 20^\circ\\ &= 0\end{aligned}$$
using the identities $\cos\theta = \sin(90^\circ - \theta)$ and $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

50.
$$\begin{aligned}\cot 40^\circ - \frac{\sin 50^\circ}{\sin 40^\circ} &= \cot 40^\circ - \frac{\cos(90^\circ - 50^\circ)}{\sin 40^\circ}\\ &= \cot 40^\circ - \frac{\cos 40^\circ}{\sin 40^\circ}\\ &= \cot 40^\circ - \cot 40^\circ\\ &= 0\end{aligned}$$
using the identities $\sin\theta = \cos(90^\circ - \theta)$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$.

51.
$$\begin{aligned}\tan 35^\circ \cdot \sec 55^\circ \cdot \cos 35^\circ &= \left(\frac{\sin 35^\circ}{\cos 35^\circ}\right)\sec 55^\circ \cdot \cos 35^\circ\\ &= \sin 35^\circ \cdot \sec 55^\circ\\ &= \sin 35^\circ \cdot \csc(90^\circ - 55^\circ)\\ &= \sin 35^\circ \cdot \csc 35^\circ\\ &= \sin 35^\circ \cdot \frac{1}{\sin 35^\circ}\\ &= 1\end{aligned}$$
using the identities $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\sec\theta = \csc(90^\circ - \theta)$, and $\csc\theta = \dfrac{1}{\sin\theta}$.

52.
$$\begin{aligned}\cot 25^\circ \cdot \csc 65^\circ \cdot \sin 25^\circ &= \left(\frac{\cos 25^\circ}{\sin 25^\circ}\right) \cdot \csc 65^\circ \cdot \sin 25^\circ\\ &= \cos 25^\circ \cdot \csc 65^\circ\\ &= \cos 25^\circ \cdot \sec(90^\circ - 65^\circ)\\ &= \cos 25^\circ \cdot \sec 25^\circ\\ &= \cos 25^\circ \cdot \frac{1}{\cos 25^\circ}\\ &= 1\end{aligned}$$
using the identities $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$, $\csc\theta = \sec(90^\circ - \theta)$, and $\sec\theta = \dfrac{1}{\cos\theta}$.

53. $\cos 35^\circ \cdot \sin 55^\circ + \cos 55^\circ \cdot \sin 35^\circ$
$= \cos 35^\circ \cdot \cos(90^\circ - 55^\circ) + \sin(90^\circ - 55^\circ) \cdot \sin 35^\circ$
$= \cos 35^\circ \cdot \cos 35^\circ + \sin 35^\circ \cdot \sin 35^\circ$
$= \cos^2 35^\circ + \sin^2 35^\circ$
$= 1$
using the identities $\sin\theta = \cos(90^\circ - \theta)$, $\cos\theta = \sin(90^\circ - \theta)$, and $\sin^2\theta + \cos^2\theta = 1$.

54. $\sec 35^\circ \cdot \csc 55^\circ - \tan 35^\circ \cdot \cot 55^\circ$
$= \sec 35^\circ \cdot \sec(90^\circ - 55^\circ) - \tan 35^\circ \cdot \tan(90^\circ - 55^\circ)$
$= \sec 35^\circ \cdot \sec 35^\circ - \tan 35^\circ \cdot \tan 35^\circ$
$= \sec^2 35^\circ - \tan^2 35^\circ$
$= (1 + \tan^2 35^\circ) - \tan^2 35^\circ$
$= 1$
using the identities $\csc\theta = \sec(90^\circ - \theta)$, $\cot\theta = \tan(90^\circ - \theta)$, and $1 + \tan^2\theta = \sec^2\theta$

55. Given: $\sin 30^\circ = \frac{1}{2}$

a. $\cos 60^\circ = \sin(90^\circ - 60^\circ) = \sin 30^\circ = \frac{1}{2}$

b. $\cos^2 30^\circ = 1 - \sin^2 30^\circ = 1 - \left(\frac{1}{2}\right)^2 = \frac{3}{4}$

c. $\csc\frac{\pi}{6} = \csc 30^\circ = \frac{1}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2$

d. $\sec\frac{\pi}{3} = \csc\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = \csc\frac{\pi}{6} = \csc 30^\circ = 2$

56. Given: $\sin 60^\circ = \frac{\sqrt{3}}{2}$

a. $\cos 30^\circ = \sin(90^\circ - 30^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$

b. $\cos^2 60^\circ = 1 - \sin^2 60^\circ = 1 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4}$

c. $\sec\frac{\pi}{6} = \frac{1}{\cos\frac{\pi}{6}} = \frac{1}{\cos 30^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

d. $\csc\frac{\pi}{3} = \sec\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = \sec\left(\frac{\pi}{6}\right) = \sec 30^\circ = \frac{2\sqrt{3}}{3}$

57. Given: $\tan\theta = 4$

a. $\sec^2\theta = 1 + \tan^2\theta = 1 + 4^2 = 1 + 16 = 17$

b. $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{4}$

c. $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta = 4$

d. $\csc^2\theta = 1 + \cot^2\theta$
$= 1 + \frac{1}{\tan^2\theta} = 1 + \frac{1}{4^2} = 1 + \frac{1}{16} = \frac{17}{16}$

58. Given: $\sec\theta = 3$

a. $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{3}$

b. $\tan^2\theta = \sec^2\theta - 1 = 3^2 - 1 = 9 - 1 = 8$

c. $\csc(90^\circ - \theta) = \sec\theta = 3$

d. $\sin^2\theta = 1 - \cos^2\theta$
$= 1 - \frac{1}{\sec^2\theta} = 1 - \frac{1}{3^2} = 1 - \frac{1}{9} = \frac{8}{9}$

59. Given: $\csc\theta = 4$

a. $\sin\theta = \frac{1}{\csc\theta} = \frac{1}{4}$

b. $\cot^2\theta = \csc^2\theta - 1 = 4^2 - 1 = 16 - 1 = 15$

c. $\sec(90^\circ - \theta) = \csc\theta = 4$

d. $\sec^2\theta = 1 + \tan^2\theta = 1 + \frac{1}{\cot^2\theta} = 1 + \frac{1}{15} = \frac{16}{15}$

60. Given: $\cot\theta = 2$

a. $\tan\theta = \frac{1}{\cot\theta} = \frac{1}{2}$

b. $\csc^2\theta = \cot^2\theta + 1 = 2^2 + 1 = 4 + 1 = 5$

c. $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta = 2$

d. $\sec^2\theta = 1 + \tan^2\theta$
$= 1 + \frac{1}{\cot^2\theta} = 1 + \frac{1}{2^2} = 1 + \frac{1}{4} = \frac{5}{4}$

61. Given: $\sin 38^\circ \approx 0.62$

a. $\cos 38^\circ \approx ?$

$$\sin^2 38^\circ + \cos^2 38^\circ = 1$$
$$\cos^2 38^\circ = 1 - \sin^2 38^\circ$$
$$\cos 38^\circ = \sqrt{1-\sin^2 38^\circ} \approx \sqrt{1-(0.62)^2} \approx 0.78$$

b. $\tan 38^\circ = \dfrac{\sin 38^\circ}{\cos 38^\circ} \approx \dfrac{0.62}{0.78} \approx 0.79$

c. $\cot 38^\circ = \dfrac{\cos 38^\circ}{\sin 38^\circ} \approx \dfrac{0.79}{0.62} \approx 1.27$

d. $\sec 38^\circ = \dfrac{1}{\cos 38^\circ} \approx \dfrac{1}{0.78} \approx 1.28$

e. $\csc 38^\circ = \dfrac{1}{\sin 38^\circ} \approx \dfrac{1}{0.62} \approx 1.61$

f. $\sin 52^\circ = \cos\left(90^\circ - 52^\circ\right) = \cos 38^\circ \approx 0.78$

g. $\cos 52^\circ = \sin\left(90^\circ - 52^\circ\right) = \sin 38^\circ \approx 0.62$

h. $\tan 52^\circ = \cot\left(90^\circ - 52^\circ\right) = \cot 38^\circ \approx 1.27$

62. Given: $\cos 21^\circ \approx 0.93$

a. $\sin 21^\circ \approx ?$

$$\sin^2 21^\circ + \cos^2 21^\circ = 1$$
$$\sin^2 21^\circ = 1 - \cos^2 21^\circ$$
$$\sin 21^\circ = \sqrt{1-\cos^2 21^\circ} \approx \sqrt{1-(0.93)^2} \approx 0.37$$

b. $\tan 21^\circ = \dfrac{\sin 21^\circ}{\cos 21^\circ} \approx \dfrac{0.37}{0.93} \approx 0.40$

c. $\cot 21^\circ = \dfrac{\cos 21^\circ}{\sin 21^\circ} \approx \dfrac{0.93}{0.37} \approx 2.51$

d. $\sec 21^\circ = \dfrac{1}{\cos 21^\circ} \approx \dfrac{1}{0.93} \approx 1.08$

e. $\csc 21^\circ = \dfrac{1}{\sin 21^\circ} \approx \dfrac{1}{0.37} \approx 2.70$

f. $\sin 69^\circ = \cos\left(90^\circ - 69^\circ\right) = \cos 21^\circ \approx 0.93$

g. $\cos 69^\circ = \sin\left(90^\circ - 69^\circ\right) = \sin 21^\circ \approx 0.37$

h. $\tan 69^\circ = \cot\left(90^\circ - 69^\circ\right) = \cot 21^\circ \approx 2.51$

63. Given: $\sin\theta = 0.3$

$$\sin\theta + \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta + \sin\theta = 0.3 + 0.3 = 0.6$$

64. Given: $\tan\theta = 4$

$$\tan\theta + \tan\left(\frac{\pi}{2} - \theta\right) = \tan\theta + \cot\theta = \tan\theta + \frac{1}{\tan\theta} = 4 + \frac{1}{4} = \frac{17}{4}$$

65. The equation $\sin\theta = \cos\left(2\theta + 30^\circ\right)$ will be true when

$$\theta = 90^\circ - \left(2\theta + 30^\circ\right)$$
$$\theta = 60^\circ - 2\theta$$
$$3\theta = 60^\circ$$
$$\theta = 20^\circ$$

66. The equation $\tan\theta = \cot\left(\theta + 45^\circ\right)$ will be true when

$$\theta = 90^\circ - \left(\theta + 45^\circ\right)$$
$$\theta = 45^\circ - \theta$$
$$2\theta = 45^\circ$$
$$\theta = 22.5^\circ$$

67. a. $T = \dfrac{1500}{300} + \dfrac{500}{100} = 5 + 5 = 10$ minutes

b. $T = \dfrac{500}{100} + \dfrac{1500}{100} = 5 + 15 = 20$ minutes

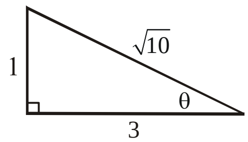
c. $\tan\theta = \dfrac{500}{x}$, so $x = \dfrac{500}{\tan\theta}$.

$\sin\theta = \dfrac{500}{\text{distance in sand}}$, so

distance in sand $= \dfrac{500}{\sin\theta}$.

$$T(\theta) = \frac{1500 - x}{300} + \frac{\text{distance in sand}}{100} = \frac{1500 - \frac{500}{\tan\theta}}{300} + \frac{\frac{500}{\sin\theta}}{100} = 5 - \frac{5}{3\tan\theta} + \frac{5}{\sin\theta} = 5\left(1 - \frac{1}{3\tan\theta} + \frac{1}{\sin\theta}\right)$$

d. $\tan\theta = \dfrac{500}{1500} = \dfrac{1}{3}$, so we can consider the triangle:

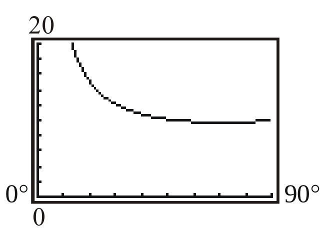


$$\begin{aligned} T &= 5 - \frac{5}{3\tan\theta} + \frac{5}{\sin\theta} \\ &= 5 - \frac{5}{3\left(\frac{1}{3}\right)} + \frac{5}{\frac{1}{\sqrt{10}}} \\ &= 5 - 5 + 5\sqrt{10} \\ &\approx 15.8 \text{ minutes} \end{aligned}$$

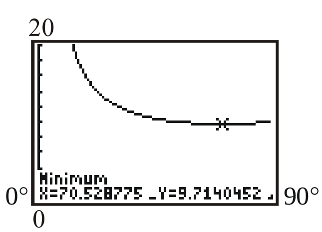
e. 1000 feet along the paved path leaves an additional 500 feet in the direction of the path, so the angle of the path across the sand is 45˚.

$$\begin{aligned} T &= 5 - \frac{5}{3\tan 45^\circ} + \frac{5}{\sin 45^\circ} \\ &= 5 - \frac{5}{3\cdot 1} + \frac{5}{\frac{\sqrt{2}}{2}} \\ &= 5 - \frac{5}{3} + \frac{10}{\sqrt{2}} \\ &\approx 10.4 \text{ minutes} \end{aligned}$$

f. Let $Y_1 = 5 - \dfrac{5}{3\tan x} + \dfrac{5}{\sin x}$ with the calculator in DEGREE mode.



Use the MINIMUM feature:



The time is least when the angle is approximately 70.5˚. The value of x for this angle is $x = \dfrac{500}{\tan 70.53^\circ} \approx 177$ feet. The least time is approximately 9.7 minutes.

g. Answers will vary.

68. a. Consider the length of the line segment in two sections, x, the portion across the hall that is 3 feet wide and y, the portion across that hall that is 4 feet wide. Then, $\cos\theta = \dfrac{3}{x}$, so $x = \dfrac{3}{\cos\theta}$ and $\sin\theta = \dfrac{4}{y}$, so $y = \dfrac{4}{\sin\theta}$.

Thus, $L(\theta) = x + y = \dfrac{3}{\cos\theta} + \dfrac{4}{\sin\theta}$.

b. Answers will vary.

69. a.

$$\begin{aligned} Z^2 &= X^2 + R^2 \\ Z &= \sqrt{X^2 + R^2} = \sqrt{400^2 + 600^2} \\ &= \sqrt{520{,}000} = 200\sqrt{13} \approx 721.1 \text{ ohms} \end{aligned}$$

The impedance is about 721.1 ohms.

b.

$$\sin\phi = \frac{X}{Z} = \frac{400}{200\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

$$\cos\phi = \frac{R}{Z} = \frac{600}{200\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\tan\phi = \frac{X}{R} = \frac{400}{600} = \frac{2}{3}$$

$$\csc\phi = \frac{Z}{X} = \frac{200\sqrt{13}}{400} = \frac{\sqrt{13}}{2}$$

$$\sec\phi = \frac{Z}{R} = \frac{200\sqrt{13}}{600} = \frac{\sqrt{13}}{3}$$

$$\cot\phi = \frac{R}{X} = \frac{600}{400} = \frac{3}{2}$$

70. a.

$$\begin{aligned} \tan\phi &= \frac{X}{R} \\ \frac{5}{12} &= \frac{X}{588} \\ \frac{5\cdot 588}{12} &= X \quad \text{or} \quad X = 245 \text{ ohms} \end{aligned}$$

The inductive reactance is 245 ohms.

$$Z = \sqrt{X^2 + R^2} = \sqrt{245^2 + 588^2} = 637$$

The impedance is 637 ohms.

b. $\sin\phi = \frac{X}{Z} = \frac{245}{637} = \frac{5}{13}$

$\cos\phi = \frac{R}{Z} = \frac{588}{637} = \frac{12}{13}$

$\csc\phi = \frac{Z}{X} = \frac{637}{245} = \frac{13}{5}$

$\sec\phi = \frac{Z}{R} = \frac{637}{588} = \frac{13}{12}$

$\cot\phi = \frac{R}{X} = \frac{588}{245} = \frac{12}{5}$

71. a. Since $|OA| = |OC| = 1$, ΔOAC is isosceles. Thus, $\angle OAC = \angle OCA$.. Now

$$\angle OAC + \angle OCA + \angle AOC = 180^\circ$$
$$\angle OAC + \angle OCA + (180^\circ - \theta) = 180^\circ$$
$$\angle OAC + \angle OCA = \theta$$
$$2(\angle OAC) = \theta$$
$$\angle OAC = \frac{\theta}{2}$$

b. $\sin\theta = \frac{|CD|}{|OC|} = \frac{|CD|}{1} = |CD|$

$\cos\theta = \frac{|OD|}{|OC|} = \frac{|OD|}{1} = |OD|$

c. $\tan\frac{\theta}{2} = \frac{|CD|}{|AD|} = \frac{|CD|}{|AO| + |OD|} = \frac{|CD|}{1 + |OD|} = \frac{\sin\theta}{1 + \cos\theta}$

72. Let h be the height of the triangle and b be the base of the triangle.

$\sin\theta = \frac{h}{a}$, so $h = a\sin\theta$

$\cos\theta = \frac{\frac{1}{2}b}{a}$, so $b = 2a\cos\theta$

$A = \frac{1}{2}bh = \frac{1}{2}(2a\cos\theta)(a\sin\theta) = a^2\sin\theta\cos\theta$

73. $h = x \cdot \frac{h}{x} = x\tan\theta$

$$h = (1-x)\cdot\frac{h}{1-x} = (1-x)\tan(n\theta)$$
$$x\tan\theta = (1-x)\tan(n\theta)$$
$$x\tan\theta = \tan(n\theta) - x\tan(n\theta)$$
$$x\tan\theta + x\tan(n\theta) = \tan(n\theta)$$
$$x\left(\tan\theta + \tan(n\theta)\right) = \tan(n\theta)$$
$$x = \frac{\tan(n\theta)}{\tan\theta + \tan(n\theta)}$$

74. Let x be the distance from O to the first circle. From the diagram, we have $\sin\theta = \frac{a}{x+a}$ and $\sin\theta = \frac{b}{x+2a+b}$.

Therefore, $\frac{a}{x+a} = \frac{b}{x+2a+b}$

$$xb + ab = xa + 2a^2 + ab$$
$$xb - xa = 2a^2$$
$$x(b-a) = 2a^2$$
$$x = \frac{2a^2}{b-a}$$

Therefore, $\sin\theta = \frac{a}{x+a}$

$$= \frac{a}{\frac{2a^2}{b-a} + a} = \frac{a}{\frac{2a^2 + ab - a^2}{b-a}}$$
$$= \frac{a}{\frac{a^2+ab}{b-a}} = \frac{a(b-a)}{a^2+ab} = \frac{a(b-a)}{a(b+a)}$$
$$= \frac{b-a}{b+a}$$

Thus, $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{b-a}{b+a}\right)^2}$

$$= \sqrt{1-\frac{b^2-2ab+a^2}{b^2+2ab+a^2}}$$
$$= \sqrt{\frac{b^2+2ab+a^2-b^2+2ab-a^2}{b^2+2ab+a^2}}$$
$$= \sqrt{\frac{4ab}{(a+b)^2}} = \frac{2\sqrt{ab}}{a+b} = \frac{\sqrt{ab}}{\frac{a+b}{2}}$$

75. a. Area $\Delta OAC = \frac{1}{2}|OC|\cdot|AC|$

$$= \frac{1}{2}\cdot\frac{|OC|}{1}\cdot\frac{|AC|}{1}$$
$$= \frac{1}{2}\cos\alpha\sin\alpha$$
$$= \frac{1}{2}\sin\alpha\cos\alpha$$

b. Area $\Delta OCB = \frac{1}{2}|OC|\cdot|BC|$

$$= \frac{1}{2}\cdot|OB|^2\cdot\frac{|OC|}{|OB|}\cdot\frac{|BC|}{|OB|}$$
$$= \frac{1}{2}|OB|^2\cos\beta\sin\beta$$
$$= \frac{1}{2}|OB|^2\sin\beta\cos\beta$$

c. Area $\Delta OAB = \frac{1}{2}|BD|\cdot|OA|$

$$= \frac{1}{2}|BD|\cdot 1$$
$$= \frac{1}{2}\cdot|OB|\cdot\frac{|BD|}{|OB|}$$
$$= \frac{1}{2}|OB|\sin(\alpha+\beta)$$

d. $\dfrac{\cos\alpha}{\cos\beta} = \dfrac{\frac{|OC|}{|OA|}}{\frac{|OC|}{|OB|}} = \dfrac{|OC|}{1}\cdot\dfrac{|OB|}{|OC|} = |OB|$

e. Area ΔOAB = Area ΔOAC + Area ΔOCB

$$\frac{1}{2}|OB|\sin(\alpha+\beta)$$
$$= \frac{1}{2}\sin\alpha\cos\alpha + \frac{1}{2}|OB|^2\sin\beta\cos\beta$$
$$\frac{\cos\alpha}{\cos\beta}\sin(\alpha+\beta)$$
$$= \sin\alpha\cos\alpha + \frac{\cos^2\alpha}{\cos^2\beta}\sin\beta\cos\beta$$
$$\sin(\alpha+\beta) = \frac{\cos\beta}{\cos\alpha}\sin\alpha\cos\alpha + \frac{\cos\alpha}{\cos\beta}\sin\beta\cos\beta$$
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

76. a. Area of $\Delta OBC = \frac{1}{2}\cdot 1\cdot\sin\theta = \frac{1}{2}\sin\theta$

b. Area of $\Delta OBD = \frac{1}{2}\cdot 1\cdot\tan\theta$

$$= \frac{1}{2}\tan\theta$$
$$= \frac{\sin\theta}{2\cos\theta}$$

c. Area $\Delta OBC <$ Area $\widehat{OBC} <$ Area ΔOBD

$$\frac{1}{2}\sin\theta < \frac{1}{2}\theta < \frac{\sin\theta}{2\cos\theta}$$
$$\frac{\sin\theta}{\sin\theta} < \frac{\theta}{\sin\theta} < \frac{\sin\theta}{\sin\theta\cos\theta}$$
$$1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$$

77. $\sin\alpha = \dfrac{\sin\alpha}{\cos\alpha}\cdot\cos\alpha$

$$= \tan\alpha\cos\alpha$$
$$= \cos\beta\cos\alpha$$
$$= \cos\beta\tan\beta$$
$$= \cos\beta\cdot\frac{\sin\beta}{\cos\beta}$$
$$= \sin\beta$$

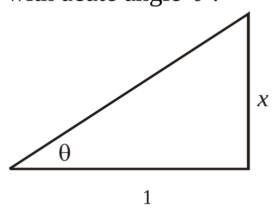
$$\sin^2\alpha + \cos^2\alpha = 1$$
$$\sin^2\alpha + \tan^2\beta = 1$$
$$\sin^2\alpha + \frac{\sin^2\beta}{\cos^2\beta} = 1$$
$$\sin^2\alpha + \frac{\sin^2\alpha}{1-\sin^2\alpha} = 1$$
$$\left(1-\sin^2\alpha\right)\left(\sin^2\alpha + \frac{\sin^2\alpha}{1-\sin^2\alpha}\right) = (1)\left(1-\sin^2\alpha\right)$$
$$\sin^2\alpha - \sin^4\alpha + \sin^2\alpha = 1-\sin^2\alpha$$
$$\sin^4\alpha - 3\sin^2\alpha + 1 = 0$$

Using the quadratic formula:

$$\sin^2\alpha = \frac{3\pm\sqrt{5}}{2}$$
$$\sin\alpha = \sqrt{\frac{3\pm\sqrt{5}}{2}}$$

But $\sqrt{\dfrac{3+\sqrt{5}}{2}} > 1$. So, $\sin\alpha = \sqrt{\dfrac{3-\sqrt{5}}{2}}$.

78. Rewrite as $\tan\theta = \frac{x}{1}$. Consider a right triangle with acute angle θ.



The hypotenuse is given by $c = \sqrt{1+x^2}$.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{\sqrt{1+x^2}} = \frac{x\sqrt{1+x^2}}{1+x^2}$$

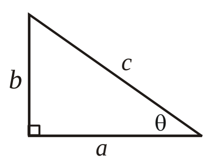
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{\sqrt{1+x^2}} = \frac{\sqrt{1+x^2}}{1+x^2}$$

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{1}{x}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{\sqrt{1+x^2}}{1} = \sqrt{1+x^2}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{\sqrt{1+x^2}}{x}$$

79. Consider the right triangle:



If θ is an acute angle in this triangle, then:

$a > 0,\ b > 0$ and $c > 0$. So $\cos\theta = \frac{a}{c} > 0$.

Also, since $a^2 + b^2 = c^2$, we know that:

$0 < a^2 < c^2$

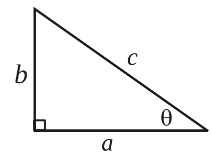
$0 < a < c$

Thus, $0 < \frac{a}{c} < 1$.

So we now know that $0 < \cos\theta < 1$ which implies that: $\frac{1}{\cos\theta} > \frac{1}{1}$

$\sec\theta > 1$

80. Consider the right triangle:



If θ is an acute angle in this triangle, then $a > 0,\ b > 0$ and $c > 0$. So $\sin\theta = \frac{b}{c} > 0$.

Also, since $a^2 + b^2 = c^2$, we know that:

$0 < b^2 < c^2$

$0 < b < c$

Thus, $0 < \frac{b}{c} < 1$

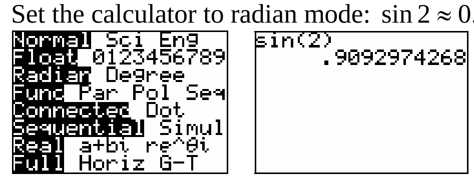
Therefore, $0 < \sin\theta < 1$.

81 – 82. Answers will vary.

Section 2.3

1. $\tan\frac{\pi}{4} + \sin 30° = 1 + \frac{1}{2} = \frac{3}{2}$

2. Set the calculator to radian mode: $\sin 2 \approx 0.91$.



3. True

4. False

5. $\sin 45° = \frac{\sqrt{2}}{2}$ $\quad$ $\csc 45° = \sqrt{2}$

$\cos 45° = \frac{\sqrt{2}}{2}$ $\quad$ $\sec 45° = \sqrt{2}$

$\tan 45° = 1$ $\quad$ $\cot 45° = 1$

6. $\sin 30° = \frac{1}{2}$ $\qquad \sin 60° = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$ $\qquad \cos 60° = \frac{1}{2}$

$\tan 30° = \frac{\sqrt{3}}{3}$ $\qquad \tan 60° = \sqrt{3}$

$\csc 30° = 2$ $\qquad \csc 60° = \frac{2\sqrt{3}}{3}$

$\sec 30° = \frac{2\sqrt{3}}{3}$ $\qquad \sec 60° = 2$

$\cot 30° = \sqrt{3}$ $\qquad \cot 60° = \frac{\sqrt{3}}{3}$

7. $f\left(60^\circ\right) = \sin 60^\circ = \frac{\sqrt{3}}{2}$

8. $g\left(60^\circ\right) = \cos 60^\circ = \frac{1}{2}$

9. $f\left(\frac{60^\circ}{2}\right) = f\left(30^\circ\right) = \sin 30^\circ = \frac{1}{2}$

10. $g\left(\frac{60^\circ}{2}\right) = g\left(30^\circ\right) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

11. $\left[f\left(60^\circ\right)\right]^2 = \left(\sin 60^\circ\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

12. $\left[g\left(60^\circ\right)\right]^2 = \left(\cos 60^\circ\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

13. $2f\left(60^\circ\right) = 2\sin 60^\circ = 2\cdot\frac{\sqrt{3}}{2} = \sqrt{3}$

14. $2g\left(60^\circ\right) = 2\cos 60^\circ = 2\cdot\frac{1}{2} = 1$

15. $\frac{f\left(60^\circ\right)}{2} = \frac{\sin 60^\circ}{2} = \frac{\frac{\sqrt{3}}{2}}{2} = \frac{\sqrt{3}}{2}\cdot\frac{1}{2} = \frac{\sqrt{3}}{4}$

16. $\frac{g\left(60^\circ\right)}{2} = \frac{\cos 60^\circ}{2} = \frac{\frac{1}{2}}{2} = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

17. $4\cos 45^\circ - 2\sin 45^\circ = 4\cdot\frac{\sqrt{2}}{2} - 2\cdot\frac{\sqrt{2}}{2}$

$= 2\sqrt{2} - \sqrt{2}$

$= \sqrt{2}$

18. $2\sin 45^\circ + 4\cos 30^\circ = 2\cdot\frac{\sqrt{2}}{2} + \frac{4\sqrt{3}}{2} = \sqrt{2} + 2\sqrt{3}$

19. $6\tan 45^\circ - 8\cos 60^\circ = 6\cdot 1 - 8\cdot\frac{1}{2} = 6 - 4 = 2$

20. $\sin 30^\circ \cdot \tan 60^\circ = \frac{1}{2}\cdot\sqrt{3} = \frac{\sqrt{3}}{2}$

21. $\sec\frac{\pi}{4} + 2\csc\frac{\pi}{3} = \sqrt{2} + 2\cdot\frac{2\sqrt{3}}{3} = \sqrt{2} + \frac{4\sqrt{3}}{3}$

22. $\tan\frac{\pi}{4} + \cot\frac{\pi}{4} = 1 + 1 = 2$

23. $\sec^2\frac{\pi}{6} - 4 = \left(\frac{2\sqrt{3}}{3}\right)^2 - 4 = \frac{12}{9} - 4 = \frac{4}{3} - 4 = -\frac{8}{3}$

24. $4 + \tan^2\frac{\pi}{3} = 4 + \left(\sqrt{3}\right)^2 = 4 + 3 = 7$

25. $\sin^2 30^\circ + \cos^2 60^\circ = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

26. $\sec^2 60^\circ - \tan^2 45^\circ = \left(2\right)^2 - \left(1\right)^2 = 4 - 1 = 3$

27. $1 - \cos^2 30^\circ - \cos^2 60^\circ = 1 - \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$= 1 - \frac{3}{4} - \frac{1}{4}$

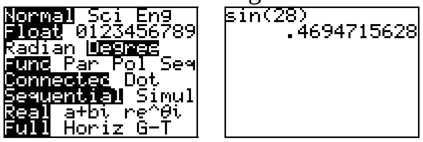
$= 0$

28. $1 + \tan^2 30^\circ - \csc^2 45^\circ = 1 + \left(\frac{\sqrt{3}}{3}\right)^2 - \left(\sqrt{2}\right)^2$

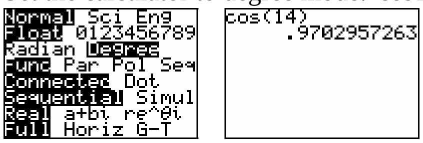
$= 1 + \frac{3}{9} - 2$

$= -\frac{2}{3}$

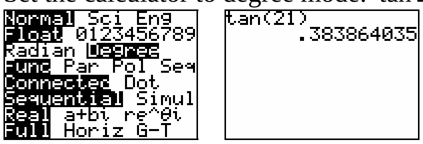
29. Set the calculator to degree mode: $\sin 28^\circ \approx 0.47$.



30. Set the calculator to degree mode: $\cos 14^\circ \approx 0.97$.

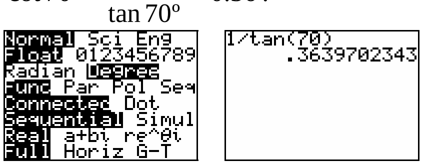


31. Set the calculator to degree mode: $\tan 21^\circ \approx 0.38$.



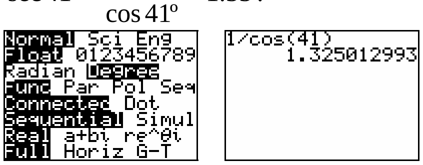
32. Set the calculator to degree mode:

$$\cot 70^\circ = \frac{1}{\tan 70^\circ} \approx 0.36 .$$



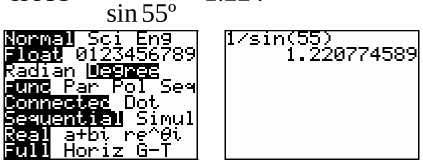
33. Set the calculator to degree mode:

$$\sec 41^\circ = \frac{1}{\cos 41^\circ} \approx 1.33 .$$

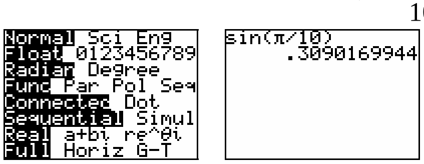


34. Set the calculator to degree mode:

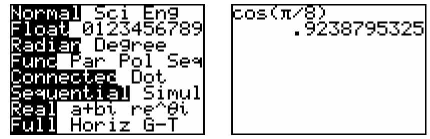
$$\csc 55^\circ = \frac{1}{\sin 55^\circ} \approx 1.22 .$$



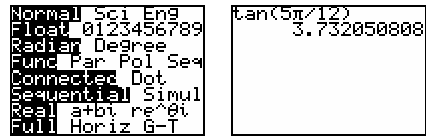
35. Set the calculator to radian mode: $\sin\frac{\pi}{10} \approx 0.31$.



36. Set the calculator to radian mode: $\cos\frac{\pi}{8} \approx 0.92$.

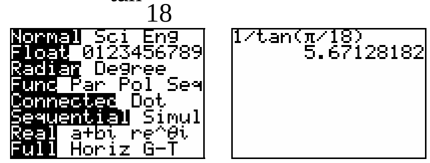


37. Set the calculator to radian mode: $\tan\frac{5\pi}{12} \approx 3.73$.



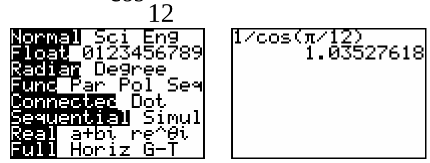
38. Set the calculator to radian mode:

$$\cot\frac{\pi}{18} = \frac{1}{\tan\frac{\pi}{18}} \approx 5.67.$$



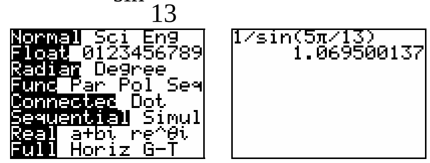
39. Set the calculator to radian mode:

$$\sec\frac{\pi}{12} = \frac{1}{\cos\frac{\pi}{12}} \approx 1.04.$$

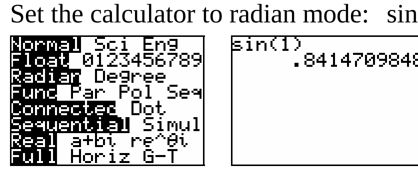


40. Set the calculator to radian mode:

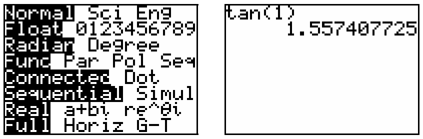
$$\csc\frac{5\pi}{13} = \frac{1}{\sin\frac{5\pi}{13}} \approx 1.07.$$



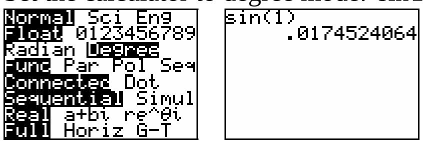
41. Set the calculator to radian mode: $\sin 1 \approx 0.84$.



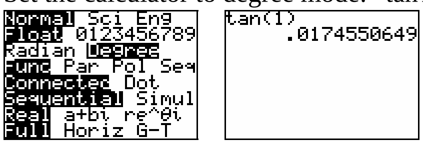
42. Set the calculator to radian mode: $\tan 1 \approx 1.56$.



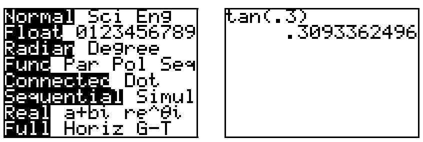
43. Set the calculator to degree mode: $\sin 1^{\circ} \approx 0.02$.



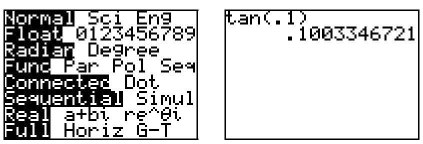
44. Set the calculator to degree mode: $\tan 1^{\circ} \approx 0.02$.



45. Set the calculator to radian mode: $\tan 0.3 \approx 0.31$.



46. Set the calculator to radian mode: $\tan 0.1 \approx 0.10$.



47.
$$\begin{aligned}(f+g)(30^\circ) &= f(30^\circ)+g(30^\circ)\\ &= \sin 30^\circ + \cos 30^\circ\\ &= \frac{1}{2}+\frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}\end{aligned}$$

48.
$$\begin{aligned}(f-g)(60^\circ) &= f(60^\circ)-g(60^\circ)\\ &= \sin 60^\circ - \cos 60^\circ\\ &= \frac{\sqrt{3}}{2}-\frac{1}{2} = \frac{\sqrt{3}-1}{2}\end{aligned}$$

49.
$$\begin{aligned}(f\cdot g)\left(\frac{\pi}{4}\right) &= f\left(\frac{\pi}{4}\right)\cdot g\left(\frac{\pi}{4}\right)\\ &= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)\\ &= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{4}}{4} = \frac{2}{4} = \frac{1}{2}\end{aligned}$$

50.
$$\begin{aligned}(f\cdot g)\left(\frac{\pi}{3}\right) &= f\left(\frac{\pi}{3}\right)\cdot g\left(\frac{\pi}{3}\right)\\ &= \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right)\\ &= \frac{\sqrt{3}}{2}\cdot\frac{1}{2} = \frac{\sqrt{3}}{4}\end{aligned}$$

51.
$$\begin{aligned}(f\circ h)\left(\frac{\pi}{6}\right) &= f\left(h\left(\frac{\pi}{6}\right)\right)\\ &= \sin\left(2\left(\frac{\pi}{6}\right)\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}\end{aligned}$$

52.
$$\begin{aligned}(g\circ p)(60^\circ) &= g\left(p(60^\circ)\right)\\ &= \cos\left(\frac{60^\circ}{2}\right) = \cos 30^\circ = \frac{\sqrt{3}}{2}\end{aligned}$$

53.
$$\begin{aligned}(p\circ g)(45^\circ) &= p\left(g(45^\circ)\right)\\ &= \frac{\cos 45^\circ}{2}\\ &= \frac{1}{2}\cos 45^\circ = \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}\end{aligned}$$

54.
$$\begin{aligned}(h\circ f)\left(\frac{\pi}{6}\right) &= h\left(f\left(\frac{\pi}{6}\right)\right)\\ &= 2\left(\sin\left(\frac{\pi}{6}\right)\right) = 2\cdot\frac{1}{2} = 1\end{aligned}$$

55. **a.** $f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

The point $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$ is on the graph of *f*.

b. The point $\left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$ is on the graph of f^{-1}.

c.
$$\begin{aligned}f\left(\frac{\pi}{4}+\frac{\pi}{4}\right)-3 &= f\left(\frac{\pi}{2}\right)-3\\ &= \sin\left(\frac{\pi}{2}\right)-3\\ &= 1-3\\ &= -2\end{aligned}$$

The point $\left(\frac{\pi}{4}, -2\right)$ is on the graph of $y = f\left(x+\frac{\pi}{4}\right)-3$.

56. **a.** $g\left(\frac{\pi}{6}\right)=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$

The point $\left(\frac{\pi}{6},\frac{\sqrt{3}}{2}\right)$ is on the graph of g.

b. The point $\left(\frac{\sqrt{3}}{2},\frac{\pi}{6}\right)$ is on the graph of g^{-1}.

c.
$$\begin{aligned}2g\left(\frac{\pi}{6}-\frac{\pi}{6}\right)&=2g(0)\\&=2\cos(0)\\&=2\cdot 1\\&=2\end{aligned}$$

Thus, the point $\left(\frac{\pi}{6},2\right)$ is on the graph of $y=2g\left(x-\frac{\pi}{6}\right)$.

57. Use the formula $R=\frac{2v_0^{\,2}\sin\theta\cos\theta}{g}$ with

$g=32.2\text{ft/sec}^2$; $\theta=45^\circ$; $v_0=100$ ft/sec:

$$R=\frac{2(100)^2\sin 45^\circ\cdot\cos 45^\circ}{32.2}\approx 310.56\text{ feet}$$

Use the formula $H=\frac{v_0^{\,2}\sin^2\theta}{2g}$ with

$g=32.2\text{ft/sec}^2$; $\theta=45^\circ$; $v_0=100$ ft/sec:

$$H=\frac{100^2\sin^2 45^\circ}{2(32.2)}\approx 77.64\text{ feet}$$

58. Use the formula $R=\frac{2v_0^{\,2}\sin\theta\cos\theta}{g}$ with

$g=9.8\text{ m/sec}^2$; $\theta=30^\circ$; $v_0=150$ m/sec:

$$R=\frac{2(150)^2\sin 30^\circ\cdot\cos 30^\circ}{g}\approx 1988.32\text{ m}$$

Use the formula $H=\frac{v_0^{\,2}\sin^2\theta}{2g}$ with

$g=9.8\text{ m/sec}^2$; $\theta=30^\circ$; $v_0=150$ m/sec:

$$H=\frac{150^2\sin^2 30^\circ}{2(9.8)}=\frac{22,500(0.5)^2}{19.6}\approx 286.99\text{ m}$$

59. Use the formula $R=\frac{2v_0^{\,2}\sin\theta\cos\theta}{g}$ with

$g=9.8\text{ m/sec}^2$; $\theta=25^\circ$; $v_0=500$ m/sec:

$$R=\frac{2(500)^2\sin 25^\circ\cdot\cos 25^\circ}{g}\approx 19,541.95\text{ m}$$

Use the formula $H=\frac{v_0^{\,2}\sin^2\theta}{2g}$ with

$g=9.8\text{ m/sec}^2$; $\theta=25^\circ$; $v_0=500$ m/sec:

$$H=\frac{500^2\sin^2 25^\circ}{2(9.8)}\approx 2278.14\text{ m}$$

60. Use the formula $R=\frac{2v_0^{\,2}\sin\theta\cos\theta}{g}$ with

$g=32.2\text{ft/sec}^2$; $\theta=50^\circ$; $v_0=200$ ft/sec:

$$R=\frac{2(200)^2\sin 50^\circ\cdot\cos 50^\circ}{g}\approx 1223.36\text{ ft}$$

Use the formula $H=\frac{v_0^{\,2}\sin^2\theta}{2g}$ with

$g=32.2\text{ft/sec}^2$; $\theta=50^\circ$; $v_0=200$ ft/sec:

$$H=\frac{200^2\sin^2 50^\circ}{2(32.2)}\approx 364.49\text{ ft}$$

61. Use the formula $t=\pm\sqrt{\frac{2a}{g\sin\theta\cos\theta}}$ with

$g=32\text{ ft/sec}^2$ and $a=10$ feet:

a. $t=\pm\sqrt{\frac{2(10)}{32\sin 30^\circ\cdot\cos 30^\circ}}\approx 1.20$ seconds

b. $t=\pm\sqrt{\frac{2(10)}{32\sin 45^\circ\cdot\cos 45^\circ}}\approx 1.12$ seconds

c. $t=\pm\sqrt{\frac{2(10)}{32\sin 60^\circ\cdot\cos 60^\circ}}\approx 1.20$ seconds

62. Use the formula

$x=\cos\theta+\sqrt{16+0.5(2\cos^2\theta-1)}$.

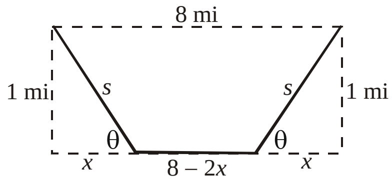
When $\theta=30^\circ$:

$x=\cos 30^\circ+\sqrt{16+0.5\left(2\cos^2 30^\circ-1\right)}\approx 4.897$ in

When $\theta=45^\circ$:

$x=\cos 45^\circ+\sqrt{16+0.5\left(2\cos^2 45^\circ-1\right)}\approx 4.707$ in

63. a. We label the diagram as follows:



Note that $\tan\theta = \frac{1}{x}$ and $\sin\theta = \frac{1}{s}$, so

$x = \frac{1}{\tan\theta}$ and $s = \frac{1}{\sin\theta}$. Also, note that

$\text{distance} = \text{rate}\cdot\text{time}$, so $\text{time} = \frac{\text{distance}}{\text{rate}}$.

Then,

$$\text{time on sand} = \frac{\text{distance on sand}}{\text{rate on sand}} = \frac{2s}{3} = \frac{2\left(\frac{1}{\sin\theta}\right)}{3} = \frac{2}{3\sin\theta}$$

and

$$\text{time on road} = \frac{\text{distance on road}}{\text{rate on road}} = \frac{8-2x}{8} = 1-\frac{x}{4} = 1-\frac{\frac{1}{\tan\theta}}{4} = 1-\frac{1}{4\tan\theta}$$

So, $\text{total time} = \text{time on sand} + \text{time on road}$

$$T(\theta) = \frac{2}{3\sin\theta} + \left(1-\frac{1}{4\tan\theta}\right) = 1+\frac{2}{3\sin\theta}-\frac{1}{4\tan\theta}$$

b. $$T(30^\circ) = 1+\frac{2}{3\sin 30^\circ}-\frac{1}{4\tan 30^\circ} = 1+\frac{2}{3\cdot\frac{1}{2}}-\frac{1}{4\cdot\frac{1}{\sqrt{3}}} = 1+\frac{4}{3}-\frac{\sqrt{3}}{4} \approx 1.9 \text{ hr}$$

Sally is on the paved road for

$1-\frac{1}{4\tan 30^\circ} \approx 0.57$ hr .

c. $$T(45^\circ) = 1+\frac{2}{3\sin 45^\circ}-\frac{1}{4\tan 45^\circ} = 1+\frac{2}{3\cdot\frac{1}{\sqrt{2}}}-\frac{1}{4\cdot 1} = 1+\frac{2\sqrt{2}}{3}-\frac{1}{4} \approx 1.69 \text{ hr}$$

Sally is on the paved road for

$1-\frac{1}{4\tan 45^\circ} = 0.75$ hr .

d. $$T(60^\circ) = 1+\frac{2}{3\sin 60^\circ}-\frac{1}{4\tan 60^\circ} = 1+\frac{2}{3\cdot\frac{\sqrt{3}}{2}}-\frac{1}{4\cdot\sqrt{3}} = 1+\frac{4}{3\sqrt{3}}-\frac{1}{4\sqrt{3}} \approx 1.63 \text{ hr}$$

Sally is on the paved road for

$1-\frac{1}{4\tan 60^\circ} \approx 0.86$ hr .

e. $T(90^\circ) = 1+\frac{2}{3\sin 90^\circ}-\frac{1}{4\tan 90^\circ}$.

But $\tan 90^\circ$ is undefined, so we can't use the function formula for this path. However, the distance would be 2 miles in the sand and 8 miles on the road. The total time would be: $\frac{2}{3}+1=\frac{5}{3}\approx 1.67$ hours. The path would be to leave the first house walking 1 mile in the sand straight to the road. Then turn and walk 8 miles on the road. Finally, turn and walk 1 mile in the sand to the second house.

f. $\tan\theta = \frac{1}{4}$, so $x = \frac{1}{\tan\theta} = \frac{1}{1/4} = 4$. Thus, the Pythagorean Theorem yields:

$s^2 = x^2 + 1^2$

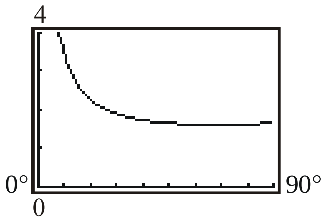
$s = \sqrt{x^2+1} = \sqrt{4^2+1} = \sqrt{17}$

Total time = time on sand + time on road

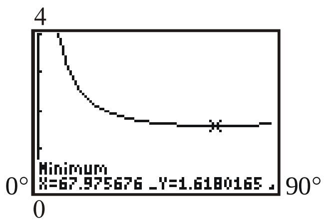
$$T = \frac{2s}{3}+\frac{8-2x}{8} = \frac{2\sqrt{17}}{3}+\frac{8-2\cdot 4}{8} = \frac{2\sqrt{17}}{3}+\frac{8-8}{8} = \frac{2\sqrt{17}}{3}+0 = \frac{2\sqrt{17}}{3} \approx 2.75 \text{ hrs}$$

The path would be to leave the first house and walk in the sand directly to the bridge. Then cross the bridge (approximately 0 miles on the road), and then walk in the sand directly to the second house.

g. Let $Y_1 = 1 + \dfrac{2}{3\sin x} - \dfrac{1}{4\tan x}$



Use the MINIMUM feature:

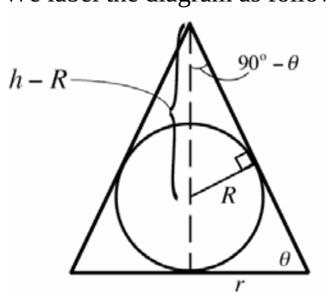


The time is least when $\theta \approx 67.98^\circ$. The least time is approximately 1.62 hour.

Sally's time on the paved road is

$1 - \dfrac{1}{4\tan\theta} \approx 1 - \dfrac{1}{4\tan 67.98^\circ} \approx 0.90$ hour.

64. a. We label the diagram as follows:



Note that $\tan\theta = \dfrac{h}{r}$, so $r = \dfrac{h}{\tan\theta} = h\cot\theta$.

Consider the smaller triangle in the figure.

From this, $\sin\left(90^\circ - \theta\right) = \dfrac{R}{h-R}$. Since

$\sin\left(90^\circ - \theta\right) = \cos\theta$, we have that:

$$\cos\theta = \frac{R}{h-R}$$

$$h - R = \frac{R}{\cos\theta}$$

$$h = \frac{R}{\cos\theta} + R = \frac{R + R\cos\theta}{\cos\theta}$$

Then $r = h\cot\theta$

$$= \left(\frac{R + R\cos\theta}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right)$$

$$= \frac{R + R\cos\theta}{\sin\theta}$$

Thus, $V = \dfrac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi\left(\frac{R + R\cos\theta}{\sin\theta}\right)^2\left(\frac{R + R\cos\theta}{\cos\theta}\right)$$

$$= \frac{\pi\left(R + R\cos\theta\right)^3}{3\sin^2\theta\cos\theta}$$

b. When $\theta = 30^\circ$:

$$V\left(30^\circ\right) = \frac{\pi\left(2 + 2\cos 30^\circ\right)^3}{3\sin^2 30^\circ \cdot \cos 30^\circ} \approx 251.4 \text{ cm}^3$$

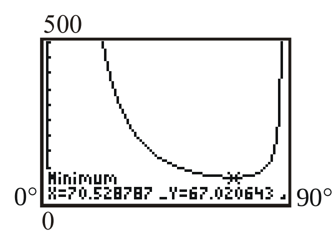
When $\theta = 45^\circ$:

$$V\left(45^\circ\right) = \frac{\pi\left(2 + 2\cos 45^\circ\right)^3}{3\sin^2 45^\circ \cdot \cos 45^\circ} \approx 117.9 \text{ cm}^3$$

When $\theta = 60^\circ$:

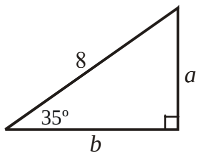
$$V\left(60^\circ\right) = \frac{\pi\left(2 + 2\cos 60^\circ\right)^3}{3\sin^2 60^\circ \cdot \cos 60^\circ} \approx 75.4 \text{ cm}^3$$

c. Let $Y_1 = \dfrac{\pi\left(2 + 2\cos x\right)^3}{3(\sin x)^2 \cos x}$.



Using a slant angle of approximately 70.5° will yield the minimum volume 67.0 cm^3.

65. $c = 8, \ \theta = 35^\circ$



$\sin(35^\circ) = \dfrac{a}{8}$

$a = 8\sin(35^\circ)$

$\approx 8(0.5736)$

≈ 4.59 in.

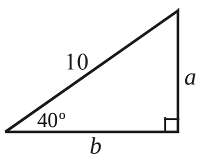
$\cos(35^\circ) = \dfrac{b}{8}$

$b = 8\cos(35^\circ)$

$\approx 8(0.8192)$

≈ 6.55 in.

66. $c = 10, \ \theta = 40^\circ$



$\sin(40^\circ) = \dfrac{a}{10}$

$a = 10\sin(40^\circ)$

$\approx 10(0.6428)$

≈ 6.43 cm.

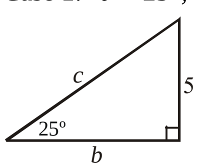
$\cos(40^\circ) = \dfrac{b}{10}$

$b = 10\cos(40^\circ)$

$\approx 10(0.7660)$

≈ 7.66 cm.

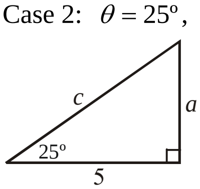
67. **a.** Case 1: $\theta = 25^\circ, \ a = 5$



$\sin(25^\circ) = \dfrac{5}{c}$

$c = \dfrac{5}{\sin(25^\circ)} \approx \dfrac{5}{0.4226} \approx 11.83$ in.

Case 2: $\theta = 25^\circ, \ b = 5$

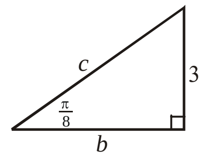


$\cos(25^\circ) = \dfrac{5}{c}$

$c = \dfrac{5}{\cos(25^\circ)} \approx \dfrac{5}{0.9063} \approx 5.52$ in.

b. There are two possible cases because the given side could be adjacent or opposite the given angle.

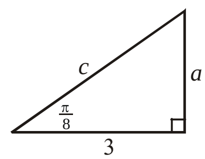
68. **a.** Case 1: $\theta = \dfrac{\pi}{8}, \ a = 3$



$\sin\left(\dfrac{\pi}{8}\right) = \dfrac{3}{c}$

$c = \dfrac{3}{\sin\left(\dfrac{\pi}{8}\right)} \approx \dfrac{3}{0.3827} \approx 7.84$ m.

Case 2: $\theta = \dfrac{\pi}{8}, \ b = 3$



$\cos\left(\dfrac{\pi}{8}\right) = \dfrac{3}{c}$

$c = \dfrac{3}{\cos\left(\dfrac{\pi}{8}\right)} \approx \dfrac{3}{0.9239} \approx 3.25$ m.

b. There are two possible cases because the given side could be adjacent or opposite the given angle.

69. $\tan(35^\circ) = \dfrac{|AC|}{100}$

$|AC| = 100\tan(35^\circ) \approx 100(0.7002) \approx 70.02$ feet

70. $\tan(40^\circ) = \dfrac{|AC|}{100}$

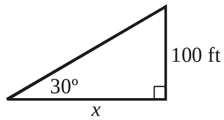
$|AC| = 100\tan(40^\circ) \approx 100(0.8391) \approx 83.91$ feet

71. Let x = the height of the Eiffel Tower.

$\tan(85.361^\circ) = \dfrac{x}{80}$

$x = 80\tan(85.361^\circ) \approx 80(12.3239) \approx 985.91$ feet

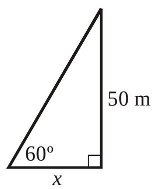
72. Let x = the distance to the shore.



$$\tan(30°) = \frac{100}{x}$$

$$x = \frac{100}{\tan(30°)} = \frac{100}{\frac{\sqrt{3}}{3}} = \frac{300}{\sqrt{3}} = 100\sqrt{3} \approx 173.21 \text{ feet}$$

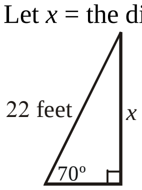
73. Let x = the distance to the base of the plateau.



$$\tan(60°) = \frac{50}{x}$$

$$x = \frac{50}{\tan(60°)} = \frac{50}{\sqrt{3}} = \frac{50\sqrt{3}}{3} \approx 28.87 \text{ meters}$$

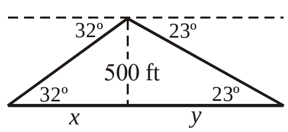
74. Let x = the distance up the building



$$\sin(70°) = \frac{x}{22}$$

$$x = 22\sin(70°) \approx 22(0.9397) \approx 20.67 \text{ feet}$$

75. We construct the figure below:



$$\tan(32°) = \frac{500}{x} \qquad \tan(23°) = \frac{500}{y}$$

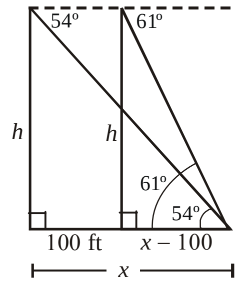
$$x = \frac{500}{\tan(32°)} \qquad y = \frac{500}{\tan(23°)}$$

$$\text{Distance} = x + y$$

$$= \frac{500}{\tan(32°)} + \frac{500}{\tan(23°)}$$

$$\approx 1978.09 \text{ feet}$$

76. Let h = the height of the balloon.



$$\tan(54°) = \frac{h}{x}$$

$$x = \frac{h}{\tan(54°)}$$

$$\tan(61°) = \frac{h}{x-100}$$

$$h = (x-100)\tan(61°)$$

$$h = \left(\frac{h}{\tan(54°)} - 100\right)\tan(61°)$$

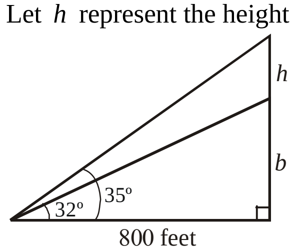
$$h - \frac{\tan(61°)}{\tan(54°)}h = -100\tan(61°)$$

$$h\left(1 - \frac{\tan(61°)}{\tan(54°)}\right) = -100\tan(61°)$$

$$h = \frac{-100\tan(61°)}{\left(1 - \frac{\tan(61°)}{\tan(54°)}\right)} \approx 580.61$$

Thus, the height of the balloon is approximately 580.61 feet.

77. Let h represent the height of Lincoln's face.



$$\tan(32°) = \frac{b}{800}$$

$$b = 800\tan(32°) \approx 499.90$$

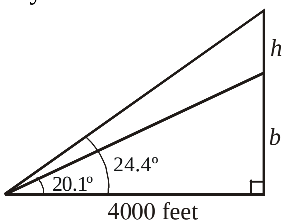
$$\tan(35°) = \frac{b+h}{800}$$

$$b + h = 800\tan(35°) \approx 560.17$$

Thus, the height of Lincoln's face is:

$$h = (b+h) - b = 560.17 - 499.90 \approx 60.27 \text{ feet}$$

78. Let h represent the height of tower above the Sky Pod.



$$\tan\left(20.1^\circ\right)=\frac{b}{4000}$$

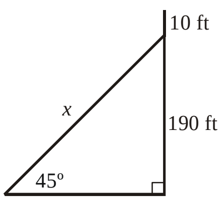
$$b=4000\tan\left(20.1^\circ\right)\approx 1463.79$$

$$\tan\left(24.4^\circ\right)=\frac{b+h}{4000}$$

$$b+h=4000\tan\left(24.4^\circ\right)\approx 1814.48$$

Thus, the height of tower above the Sky Pod is:
$h=(b+h)-b=1814.48-1463.79\approx 350.69$ feet

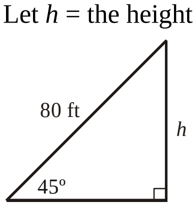
79. Let x = the length of the guy wire.



$$\sin\left(45^\circ\right)=\frac{190}{x}$$

$$x=\frac{190}{\sin\left(45^\circ\right)}=\frac{190}{\frac{\sqrt{2}}{2}}=\frac{380}{\sqrt{2}}=190\sqrt{2}\approx 268.70 \text{ ft}$$

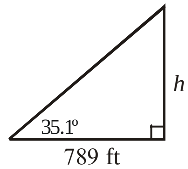
80. Let h = the height of the tower.



$$\sin\left(45^\circ\right)=\frac{h}{80}$$

$$h=80\sin\left(45^\circ\right)=80\cdot\frac{\sqrt{2}}{2}=40\sqrt{2}\approx 56.57 \text{ ft}$$

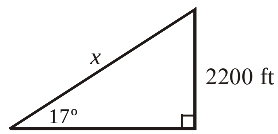
81. Let h = the height of the monument.



$$\tan\left(35.1^\circ\right)=\frac{h}{789}$$

$$h=789\tan\left(35.1^\circ\right)\approx 789(0.7028)\approx 554.52 \text{ ft}$$

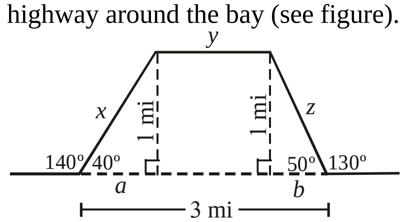
82. The elevation change is $11200-9000=2200$ ft . Let x = the length of the trail.



$$\sin 17^\circ=\frac{2200}{x}$$

$$x=\frac{2200}{\sin\left(17^\circ\right)}\approx\frac{2200}{0.2924}\approx 7524.67 \text{ ft.}$$

83. Let x, y, and z = the three segments of the highway around the bay (see figure).



The length of the highway $=x+y+z$

$$\sin\left(40^\circ\right)=\frac{1}{x}$$

$$x=\frac{1}{\sin\left(40^\circ\right)}\approx 1.5557 \text{ mi}$$

$$\sin\left(50^\circ\right)=\frac{1}{z}$$

$$z=\frac{1}{\sin\left(50^\circ\right)}\approx 1.3054 \text{ mi}$$

$$\tan\left(40^\circ\right)=\frac{1}{a}$$

$$a=\frac{1}{\tan\left(40^\circ\right)}\approx 1.1918 \text{ mi}$$

$$\tan\left(50^\circ\right)=\frac{1}{b}$$

$$b=\frac{1}{\tan\left(50^\circ\right)}\approx 0.8391 \text{ mi}$$

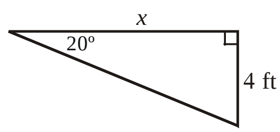
$a + y + b = 3$

$y = 3 - a - b$

$\approx 3 - 1.1918 - 0.8391 = 0.9691$ mi

The length of the highway is about:
$1.5557 + 0.9691 + 1.3054 \approx 3.83$ miles .

84. Let x = the distance from George at which the camera must be set in order to see his head and feet.

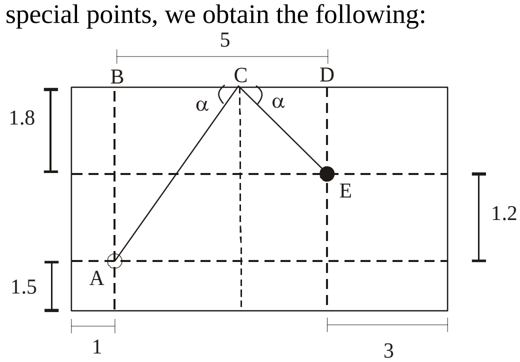


$\tan(20°) = \frac{4}{x}$

$x = \frac{4}{\tan(20°)} \approx 10.99$ feet

If the camera is set at a distance of 10 feet from George, his feet will not be seen by the lens. The camera would need to be moved back about 1 additional foot (11 feet total).

85. Adding some lines to the diagram and labeling special points, we obtain the following:



If we let x = length of side BC, we see that, in ΔABC, $\tan\alpha = \frac{3}{x}$. Also, in ΔEDC, $\tan\alpha = \frac{1.8}{5-x}$. Therefore, we have

$\frac{3}{x} = \frac{1.8}{5-x}$

$15 - 3x = 1.8x$

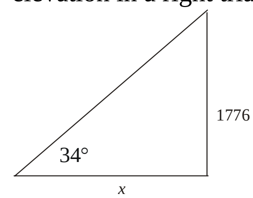
$15 = 4.8x$

$x = \frac{15}{4.8} = 3.125$ ft

$1 + 3.125 = 4.125$ ft

The player should hit the top cushion at a point that is 4.125 feet from upper left corner.

86. a. The distance between the buildings is the length of the side adjacent to the angle of elevation in a right triangle.



Since $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ and we know the angle measure, we can use the tangent to find the distance. Let x = the distance between the buildings. This gives us

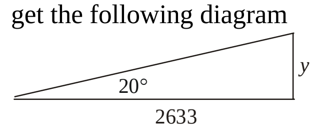
$\tan 34° = \frac{1776}{x}$

$x = \frac{1776}{\tan 34°}$

$x \approx 2633$

The office building is about 2633 feet from the base of the tower.

b. Let y = the difference in height between Freedom Tower and the office building. Together with the result from part (a), we get the following diagram



$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$\tan 20° = \frac{y}{2633}$

$y \approx 958$

The Freedom Tower is about 958 feet taller than the office building. Therefore, the office building is $1776 - 958 = 818$ feet tall.

87.

θ	$\sin\theta$	$\frac{\sin\theta}{\theta}$
0.5	0.4794	0.9589
0.4	0.3894	0.9735
0.2	0.1987	0.9933
0.1	0.0998	0.9983
0.01	0.0100	1.0000
0.001	0.0010	1.0000
0.0001	0.0001	1.0000
0.00001	0.00001	1.0000

$\frac{\sin\theta}{\theta}$ approaches 1 as θ approaches 0.

88.

θ	$\cos\theta - 1$	$\dfrac{\cos\theta - 1}{\theta}$
0.5	–0.1224	–0.2448
0.4	–0.0789	–0.1973
0.2	–0.0199	–0.0997
0.1	–0.0050	–0.0050
0.01	–0.00005	–0.0050
0.001	0.0000	–0.0005
0.0001	0.0000	–0.00005
0.00001	0.0000	–0.000005

$\dfrac{\cos\theta - 1}{\theta}$ approaches 0 as θ approaches 0.

89. We rearrange the order of the terms in this product as follows:

$$\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \cdot \ldots \cdot \tan 89^\circ$$
$$= \left(\tan 1^\circ \cdot \tan 89^\circ\right) \cdot \left(\tan 2^\circ \cdot \tan 88^\circ\right) \cdot \ldots \cdot \left(\tan 44^\circ \cdot \tan 46^\circ\right) \cdot \left(\tan 45^\circ\right)$$

Now each set of parentheses contains a pair of complementary angles. For example, using cofunction properties, we have:

$$\left(\tan 1^\circ \cdot \tan 89^\circ\right) = \left(\tan 1^\circ \cdot \tan\left(90^\circ - 1^\circ\right)\right) = \left(\tan 1^\circ \cdot \cot 1^\circ\right) = \left(\tan 1^\circ \cdot \frac{1}{\tan 1^\circ}\right) = 1$$

$$\left(\tan 2^\circ \cdot \tan 88^\circ\right) = \left(\tan 2^\circ \cdot \tan\left(90^\circ - 2^\circ\right)\right) = \left(\tan 2^\circ \cdot \cot 2^\circ\right) = \left(\tan 2^\circ \cdot \frac{1}{\tan 2^\circ}\right) = 1$$

and so on.
This result holds for each pair in our product.
Since we know that $\tan 45^\circ = 1$, our product can be rewritten as: $1 \cdot 1 \cdot 1 \cdot \ldots \cdot 1 = 1$.
Therefore, $\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \cdot \ldots \cdot \tan 89^\circ = 1$.

90. We can rearrange the order of the terms in this product as follows:

$$\cot 1^\circ \cdot \cot 2^\circ \cdot \cot 3^\circ \cdot \ldots \cdot \cot 89^\circ$$
$$= \left(\cot 1^\circ \cdot \cot 89^\circ\right) \cdot \left(\cot 2^\circ \cdot \cot 88^\circ\right) \cdot \ldots \cdot \left(\cot 44^\circ \cdot \cot 46^\circ\right) \cdot \left(\cot 45^\circ\right)$$

Now each set of parentheses contains a pair of complementary angles. For example, using cofunction properties, we have:

$$\left(\cot 1^\circ \cdot \cot 89^\circ\right) = \left(\cot 1^\circ \cdot \cot\left(90^\circ - 1^\circ\right)\right) = \left(\cot 1^\circ \cdot \tan 1^\circ\right) = 1$$

$$\left(\cot 2^\circ \cdot \cot 88^\circ\right) = \left(\cot 2^\circ \cdot \cot\left(90^\circ - 2^\circ\right)\right) = \left(\cot 2^\circ \cdot \tan 2^\circ\right) = 1$$

and so on.
This result holds for each pair in our product.
Since we know that $\cot 45^\circ = 1$, our product can be rewritten as: $1 \cdot 1 \cdot 1 \cdot \ldots \cdot 1 = 1$. Therefore, $\cot 1^\circ \cdot \cot 2^\circ \cdot \cot 3^\circ \cdot \ldots \cdot \cot 89^\circ = 1$.

91. We can rearrange the order of the terms in this product as follows:

$$\cos 1^\circ \cdot \cos 2^\circ \cdot \ldots \cdot \cos 45^\circ \cdot \csc 46^\circ \cdot \ldots \cdot \csc 89^\circ$$
$$= \left(\cos 1^\circ \cdot \csc 89^\circ\right) \cdot \left(\cos 2^\circ \cdot \csc 88^\circ\right) \cdot \ldots \cdot \left(\cos 44^\circ \cdot \csc 46^\circ\right) \cdot \left(\cos 45^\circ\right)$$

Now each set of parentheses contains a pair of complementary angles. For example, using cofunction properties, we have:

$$\left(\cos 1^\circ \cdot \csc 89^\circ\right) = \left(\cos 1^\circ \cdot \csc\left(90^\circ - 1^\circ\right)\right) = \left(\cos 1^\circ \cdot \sec 1^\circ\right) = \left(\cos 1^\circ \cdot \frac{1}{\cos 1^\circ}\right) = 1$$

$$\left(\cos 2^\circ \cdot \csc 88^\circ\right) = \left(\cos 2^\circ \cdot \csc\left(90^\circ - 2^\circ\right)\right) = \left(\cos 2^\circ \cdot \sec 2^\circ\right) = \left(\cos 2^\circ \cdot \frac{1}{\cos 2^\circ}\right) = 1$$

and so on.
This result holds for each pair in our product.
Since we know that $\cos 45^\circ = \frac{\sqrt{2}}{2}$, our product can be rewritten as $1 \cdot 1 \cdot 1 \cdots 1 \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$. Thus, $\cos 1^\circ \cdot \cos 2^\circ \cdot \ldots \cdot \cos 45^\circ \cdot \csc 46^\circ \cdot \ldots \cdot \csc 89^\circ = \frac{\sqrt{2}}{2}$.

92. We can rearrange the order of the terms in this product as follows:

$$\sin 1^\circ \cdot \sin 2^\circ \cdot \ldots \cdot \sin 45^\circ \cdot \sec 46^\circ \cdot \ldots \cdot \sec 89^\circ$$
$$= \left(\sin 1^\circ \cdot \sec 89^\circ\right) \cdot \left(\sin 2^\circ \cdot \sec 88^\circ\right) \cdot \ldots \cdot \left(\sin 44^\circ \cdot \sec 46^\circ\right) \cdot \left(\sin 45^\circ\right)$$

Now each set of parentheses contains a pair of complementary angles. For example, using cofunction properties, we have:

$$\left(\sin 1^\circ \cdot \sec 89^\circ\right) = \left(\sin 1^\circ \cdot \sec\left(90^\circ - 1^\circ\right)\right) = \left(\sin 1^\circ \cdot \csc 1^\circ\right) = \left(\sin 1^\circ \cdot \frac{1}{\sin 1^\circ}\right) = 1$$

$$\left(\sin 2^\circ \cdot \sec 88^\circ\right) = \left(\sin 2^\circ \cdot \sec\left(90^\circ - 2^\circ\right)\right) = \left(\sin 2^\circ \cdot \csc 2^\circ\right) = \left(\sin 2^\circ \cdot \frac{1}{\sin 2^\circ}\right) = 1$$

and so on.
This result holds for each pair in our product. And since we know that $\sin 45^\circ = \frac{\sqrt{2}}{2}$, our product can be rewritten as $1 \cdot 1 \cdot 1 \cdot \ldots \cdot 1 \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$. Thus,

$$\sin 1^\circ \cdot \sin 2^\circ \cdot \ldots \cdot \sin 45^\circ \cdot \sec 46^\circ \cdot \ldots \cdot \sec 89^\circ = \frac{\sqrt{2}}{2}.$$

93 – 95. Answers will vary.

Section 2.4

1. tangent, cotangent

2. coterminal

3. $240^\circ - 180^\circ = 60^\circ$

4. False

5. True

6. True

7. $600^\circ - 360^\circ = 240^\circ$; $240^\circ - 180^\circ = 60^\circ$

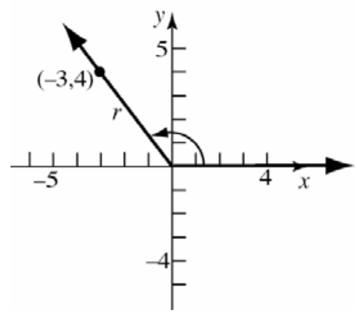
8. quadrant I and quadrant IV

9. $\frac{\pi}{2}$ and $\frac{3\pi}{2}$

10. $\frac{13\pi}{3} - \frac{12\pi}{3} = \frac{\pi}{3}$

11. $(-3, 4)$: $a = -3,\ b = 4$

$r = \sqrt{a^2 + b^2} = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$



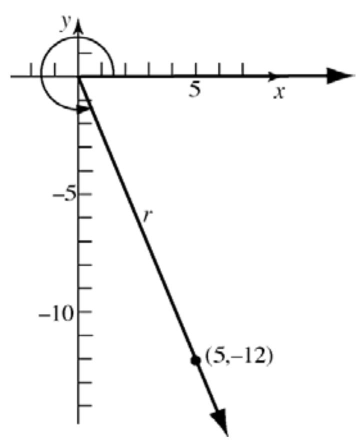
$\sin\theta = \frac{b}{r} = \frac{4}{5}$ $\quad \cos\theta = \frac{a}{r} = \frac{-3}{5} = -\frac{3}{5}$

$\tan\theta = \frac{b}{a} = \frac{4}{-3} = -\frac{4}{3}$ $\quad \cot\theta = \frac{a}{b} = \frac{-3}{4} = -\frac{3}{4}$

$\sec\theta = \frac{r}{a} = \frac{5}{-3} = -\frac{5}{3}$ $\quad \csc\theta = \frac{r}{b} = \frac{5}{4}$

12. $(5, -12)$: $a = 5,\ b = -12$

$r = \sqrt{a^2 + b^2} = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$



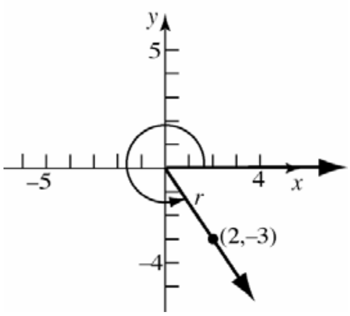
$\sin\theta = \frac{b}{r} = \frac{-12}{13} = -\frac{12}{3}$ $\quad \cos\theta = \frac{a}{r} = \frac{5}{13}$

$\tan\theta = \frac{b}{a} = \frac{-12}{5} = -\frac{12}{5}$ $\quad \cot\theta = \frac{a}{b} = \frac{5}{-12} = -\frac{5}{12}$

$\sec\theta = \frac{r}{a} = \frac{13}{5}$ $\quad \csc\theta = \frac{r}{b} = \frac{13}{-12} = -\frac{13}{12}$

13. $(2,-3)$: $a = 2,\ b = -3$

$r = \sqrt{a^2 + b^2} = \sqrt{2^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$



$\sin\theta = \frac{b}{r} = \frac{-3}{\sqrt{13}}\frac{\sqrt{13}}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$

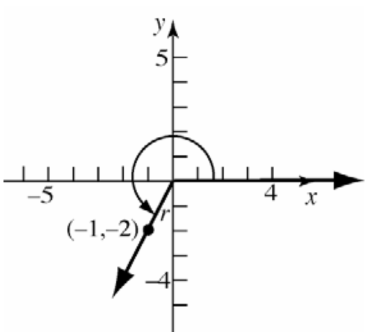
$\cos\theta = \frac{a}{r} = \frac{2}{\sqrt{13}}\frac{\sqrt{13}}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$

$\tan\theta = \frac{b}{a} = \frac{-3}{2} = -\frac{3}{2}$ $\quad$ $\cot\theta = \frac{a}{b} = \frac{2}{-3} = -\frac{2}{3}$

$\sec\theta = \frac{r}{a} = \frac{\sqrt{13}}{2}$ $\quad$ $\csc\theta = \frac{r}{b} = \frac{\sqrt{13}}{-3} = -\frac{\sqrt{13}}{3}$

14. $(-1,-2)$: $a = -1,\ b = -2$

$r = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$



$\sin\theta = \frac{b}{r} = \frac{-2}{\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

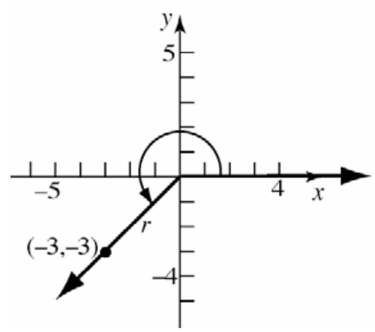
$\cos\theta = \frac{a}{r} = \frac{-1}{\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$

$\tan\theta = \frac{b}{a} = \frac{-2}{-1} = 2$ $\quad$ $\cot\theta = \frac{a}{b} = \frac{-1}{-2} = \frac{1}{2}$

$\sec\theta = \frac{r}{a} = \frac{\sqrt{5}}{-1} = -\sqrt{5}$ $\quad$ $\csc\theta = \frac{r}{b} = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2}$

15. $(-3,-3)$: $a = -3,\ b = -3$

$r = \sqrt{a^2 + b^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$



$\sin\theta = \frac{b}{r} = \frac{-3}{3\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

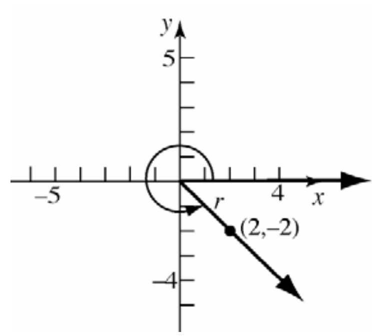
$\cos\theta = \frac{a}{r} = \frac{-3}{3\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\tan\theta = \frac{b}{a} = \frac{-3}{-3} = 1$ $\quad$ $\cot\theta = \frac{a}{b} = \frac{-3}{-3} = 1$

$\sec\theta = \frac{r}{a} = \frac{3\sqrt{2}}{-3} = -\sqrt{2}$ $\quad$ $\csc\theta = \frac{r}{b} = \frac{3\sqrt{2}}{-3} = -\sqrt{2}$

16. $(2,-2)$: $a = 2,\ b = -2$

$r = \sqrt{a^2 + b^2} = \sqrt{2^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$



$\sin\theta = \frac{b}{r} = \frac{-2}{2\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

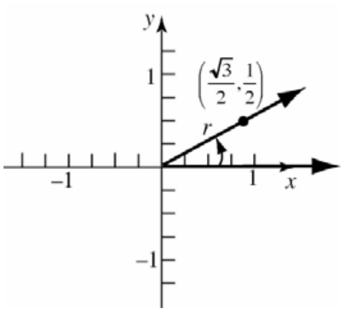
$\cos\theta = \frac{a}{r} = \frac{2}{2\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan\theta = \frac{b}{a} = \frac{-2}{2} = -1$ $\quad$ $\cot\theta = \frac{a}{b} = \frac{2}{-2} = -1$

$\sec\theta = \frac{r}{a} = \frac{2\sqrt{2}}{2} = \sqrt{2}$ $\quad$ $\csc\theta = \frac{r}{b} = \frac{2\sqrt{2}}{-2} = -\sqrt{2}$

17. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$: $a = \frac{\sqrt{3}}{2},\ b = \frac{1}{2}$

$$r = \sqrt{a^2 + b^2} = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4} + \frac{1}{4}} = \sqrt{1} = 1$$



$$\sin\theta = \frac{b}{r} = \frac{\frac{1}{2}}{1} = \frac{1}{2} \qquad \csc\theta = \frac{r}{b} = \frac{1}{\frac{1}{2}} = 2$$

$$\cos\theta = \frac{a}{r} = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$$

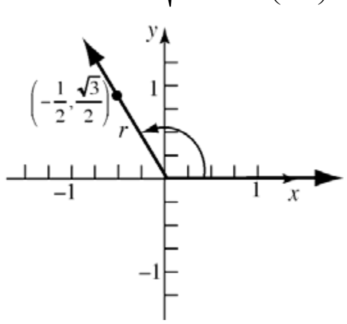
$$\tan\theta = \frac{b}{a} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sec\theta = \frac{r}{a} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cot\theta = \frac{a}{b} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

18. $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$: $a = -\frac{1}{2},\ b = \frac{\sqrt{3}}{2}$

$$r = \sqrt{a^2 + b^2} = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$$



$$\sin\theta = \frac{b}{r} = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$$

$$\cos\theta = \frac{a}{r} = \frac{-\frac{1}{2}}{1} = -\frac{1}{2}$$

$$\tan\theta = \frac{b}{a} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$$

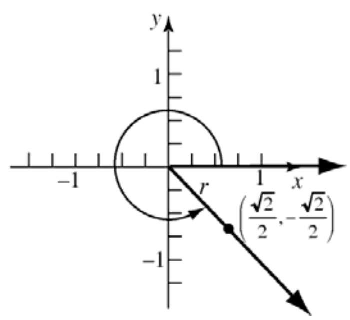
$$\csc\theta = \frac{r}{b} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\sec\theta = \frac{r}{a} = \frac{1}{-\frac{1}{2}} = -2$$

$$\cot\theta = \frac{a}{b} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

19. $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$: $a = \frac{\sqrt{2}}{2},\ b = -\frac{\sqrt{2}}{2}$

$$r = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(-\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{2}{4} + \frac{2}{4}} = \sqrt{1} = 1$$



$$\sin\theta = \frac{b}{r} = \frac{-\frac{\sqrt{2}}{2}}{1} = -\frac{\sqrt{2}}{2} \qquad \cos\theta = \frac{a}{r} = \frac{\frac{\sqrt{2}}{2}}{1} = \frac{\sqrt{2}}{2}$$

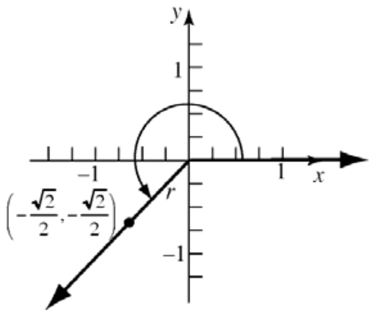
$$\tan\theta = \frac{b}{a} = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1 \qquad \cot\theta = \frac{a}{b} = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1$$

$$\sec\theta = \frac{r}{a} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

$$\csc\theta = \frac{r}{b} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\sqrt{2}$$

20. $\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$: $a=-\frac{\sqrt{2}}{2},\ b=-\frac{\sqrt{2}}{2}$

$$r=\sqrt{\left(-\frac{\sqrt{2}}{2}\right)^2+\left(-\frac{\sqrt{2}}{2}\right)^2}=\sqrt{\frac{2}{4}+\frac{2}{4}}=\sqrt{1}=1$$



$$\sin\theta=\frac{b}{r}=\frac{-\frac{\sqrt{2}}{2}}{1}=-\frac{\sqrt{2}}{2}$$

$$\cos\theta=\frac{a}{r}=\frac{-\frac{\sqrt{2}}{2}}{1}=-\frac{\sqrt{2}}{2}$$

$$\tan\theta=\frac{b}{a}=\frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}=1$$

$$\csc\theta=\frac{r}{b}=\frac{1}{-\frac{\sqrt{2}}{2}}=-\frac{2}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}=-\sqrt{2}$$

$$\sec\theta=\frac{r}{a}=\frac{1}{-\frac{\sqrt{2}}{2}}=-\frac{2}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}=-\sqrt{2}$$

$$\cot\theta=\frac{a}{b}=\frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}=1$$

21. $\sin 405^\circ=\sin(360^\circ+45^\circ)=\sin 45^\circ=\frac{\sqrt{2}}{2}$

22. $\cos 420^\circ=\cos(360^\circ+60^\circ)=\cos 60^\circ=\frac{1}{2}$

23. $\tan 405^\circ=\tan(180^\circ+180^\circ+45^\circ)=\tan 45^\circ=1$

24. $\sin 390^\circ=\sin(360^\circ+30^\circ)=\sin 30^\circ=\frac{1}{2}$

25. $\csc 450^\circ=\csc(360^\circ+90^\circ)=\csc 90^\circ=1$

26. $\sec 540^\circ=\sec(360^\circ+180^\circ)=\sec 180^\circ=-1$

27. $\cot 390^\circ=\cot(180^\circ+180^\circ+30^\circ)=\cot 30^\circ=\sqrt{3}$

28. $\sec 420^\circ=\sec(360^\circ+60^\circ)=\sec 60^\circ=2$

29.
$$\begin{aligned}\cos\frac{33\pi}{4}&=\cos\left(\frac{\pi}{4}+\frac{32\pi}{4}\right)\\&=\cos\left(\frac{\pi}{4}+8\pi\right)\\&=\cos\left(\frac{\pi}{4}+4\cdot 2\pi\right)\\&=\cos\frac{\pi}{4}\\&=\frac{\sqrt{2}}{2}\end{aligned}$$

30.
$$\begin{aligned}\sin\frac{9\pi}{4}&=\sin\left(\frac{\pi}{4}+\frac{8\pi}{4}\right)\\&=\sin\left(\frac{\pi}{4}+2\pi\right)\\&=\sin\frac{\pi}{4}\\&=\frac{\sqrt{2}}{2}\end{aligned}$$

31. $\tan 21\pi=\tan(0+21\pi)=\tan 0=0$

32.
$$\begin{aligned}\csc\frac{9\pi}{2}&=\csc\left(\frac{\pi}{2}+\frac{8\pi}{2}\right)\\&=\csc\left(\frac{\pi}{2}+4\pi\right)\\&=\csc\left(\frac{\pi}{2}+2\cdot 2\pi\right)\\&=\csc\frac{\pi}{2}\\&=1\end{aligned}$$

33. Since $\sin\theta>0$ for points in quadrants I and II, and $\cos\theta<0$ for points in quadrants II and III, the angle θ lies in quadrant II.

34. Since $\sin\theta<0$ for points in quadrants III and IV, and $\cos\theta>0$ for points in quadrants I and IV, the angle θ lies in quadrant IV.

35. Since $\sin\theta < 0$ for points in quadrants III and IV, and $\tan\theta < 0$ for points in quadrants II and IV, the angle θ lies in quadrant IV.

36. Since $\cos\theta > 0$ for points in quadrants I and IV, and $\tan\theta > 0$ for points in quadrants I and III, the angle θ lies in quadrant I.

37. Since $\cos\theta > 0$ for points in quadrants I and IV, and $\cot\theta < 0$ for points in quadrants II and IV, the angle θ lies in quadrant IV.

38. Since $\sin\theta < 0$ for points in quadrants III and IV, and $\cot\theta > 0$ for points in quadrants I and III, the angle θ lies in quadrant III.

39. Since $\sec\theta < 0$ for points in quadrants II and III, and $\tan\theta > 0$ for points in quadrants I and III, the angle θ lies in quadrant III.

40. Since $\csc\theta > 0$ for points in quadrants I and II, and $\cot\theta < 0$ for points in quadrants II and IV, the angle θ lies in quadrant II.

41. $\theta = -30^\circ$ is in quadrant IV, so the reference angle is $\alpha = 30^\circ$.

42. $\theta = -60^\circ$ is in quadrant IV, so the reference angle is $\alpha = 60^\circ$.

43. $\theta = 120^\circ$ is in quadrant II, so the reference angle is $\alpha = 180^\circ - 120^\circ = 60^\circ$.

44. $\theta = 210^\circ$ is in quadrant III, so the reference angle is $\alpha = 210^\circ - 180^\circ = 30^\circ$.

45. $\theta = 300^\circ$ is in quadrant IV, so the reference angle is $\alpha = 360^\circ - 300^\circ = 60^\circ$.

46. $\theta = 330^\circ$ is in quadrant IV, so the reference angle is $\alpha = 360^\circ - 330^\circ = 30^\circ$.

47. $\theta = \dfrac{5\pi}{4}$ is in quadrant III, so the reference angle is $\alpha = \dfrac{5\pi}{4} - \pi = \dfrac{\pi}{4}$.

48. $\theta = \dfrac{5\pi}{6}$ is in quadrant II, so the reference angle is $\alpha = \pi - \dfrac{5\pi}{6} = \dfrac{\pi}{6}$.

49. $\theta = \dfrac{8\pi}{3}$ is in quadrant II. Note that $\dfrac{8\pi}{3} - 2\pi = \dfrac{2\pi}{3}$, so the reference angle is $\alpha = \pi - \dfrac{2\pi}{3} = \dfrac{\pi}{3}$.

50. $\theta = \dfrac{7\pi}{4}$ is in quadrant IV, so the reference angle is $\alpha = 2\pi - \dfrac{7\pi}{4} = \dfrac{\pi}{4}$.

51. $\theta = -135^\circ$ is in quadrant III. Note that $-135^\circ + 360^\circ = 225^\circ$, so the reference angle is $\alpha = 225^\circ - 180^\circ = 45^\circ$.

52. $\theta = -240^\circ$ is in quadrant II. Note that $-240^\circ + 360^\circ = 120^\circ$, so the reference angle is $\alpha = 180^\circ - 120^\circ = 60^\circ$.

53. $\theta = -\dfrac{2\pi}{3}$ is in quadrant III. Note that $-\dfrac{2\pi}{3} + 2\pi = \dfrac{4\pi}{3}$, so the reference angle is $\alpha = \dfrac{4\pi}{3} - \pi = \dfrac{\pi}{3}$.

54. $\theta = -\dfrac{7\pi}{6}$ is in quadrant II. Note that $-\dfrac{7\pi}{6} + 2\pi = \dfrac{5\pi}{6}$, so the reference angle is $\alpha = \pi - \dfrac{5\pi}{6} = \dfrac{\pi}{6}$.

55. $\theta = 440^\circ$ is in quadrant I. Note that $440^\circ - 360^\circ = 80^\circ$, so the reference angle is $\alpha = 80^\circ$.

56. $\theta = 490^\circ$ is in quadrant II. Note that $490^\circ - 360^\circ = 130^\circ$, so the reference angle is $\alpha = 180^\circ - 130^\circ = 50^\circ$.

57. $\theta = \dfrac{15\pi}{4}$ is in quadrant IV. Note that $\dfrac{15\pi}{4} - 2\pi = \dfrac{7\pi}{4}$, so the reference angle is $\alpha = 2\pi - \dfrac{7\pi}{4} = \dfrac{\pi}{4}$.

58. $\theta = \dfrac{19\pi}{6}$ is in quadrant III. Note that $\dfrac{19\pi}{6} - 2\pi = \dfrac{7\pi}{6}$, so the reference angle is $\alpha = \dfrac{7\pi}{6} - \pi = \dfrac{\pi}{6}$.

59. $\sin 150^{\circ} = \sin 30^{\circ} = \dfrac{1}{2}$, since $\theta = 150^{\circ}$ has reference angle $\alpha = 30^{\circ}$ in quadrant II.

60. $\cos 210^{\circ} = -\cos 30^{\circ} = -\dfrac{\sqrt{3}}{2}$, since $\theta = 210^{\circ}$ has reference angle $\alpha = 30^{\circ}$ in quadrant III.

61. $\sin 510^{\circ} = \sin 30^{\circ} = \dfrac{1}{2}$, since $\theta = 510^{\circ}$ has reference angel $\alpha = 30^{\circ}$ in quadrant II.

62. $\cos 600^{\circ} = -\cos 60^{\circ} = -\dfrac{1}{2}$, since $\theta = 600^{\circ}$ has reference angel $\alpha = 60^{\circ}$ in quadrant III.

63. $\cos\left(-45^{\circ}\right) = \sin 45^{\circ} = \dfrac{\sqrt{2}}{2}$, since $\theta = -45^{\circ}$ has reference angel $\alpha = 45^{\circ}$ in quadrant IV.

64. $\sin\left(-240^{\circ}\right) = \sin 60^{\circ} = \dfrac{\sqrt{3}}{2}$, since $\theta = -240^{\circ}$ has reference angel $\alpha = 60^{\circ}$ in quadrant II.

65. $\sec 240^{\circ} = -\sec 60^{\circ} = -2$, since $\theta = 240^{\circ}$ has reference angle $\alpha = 60^{\circ}$ in quadrant III.

66. $\csc 300^{\circ} = -\csc 60^{\circ} = -\dfrac{2\sqrt{3}}{3}$, since $\theta = 300^{\circ}$ has reference angle $\alpha = 60^{\circ}$ in quadrant IV.

67. $\cot 330^{\circ} = -\cot 30^{\circ} = -\sqrt{3}$, since $\theta = 330^{\circ}$ has reference angle $\alpha = 30^{\circ}$ in quadrant IV.

68. $\tan 225^{\circ} = \tan 45^{\circ} = 1$, since $\theta = 225^{\circ}$ has reference angle $\alpha = 45^{\circ}$ in quadrant III.

69. $\sin\dfrac{3\pi}{4} = \sin\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$, since $\theta = \dfrac{3\pi}{4}$ has reference angle $\alpha = \dfrac{\pi}{4}$ in quadrant II.

70. $\cos\dfrac{2\pi}{3} = -\cos\dfrac{\pi}{3} = -\dfrac{1}{2}$, since $\theta = \dfrac{2\pi}{3}$ has reference angle $\alpha = \dfrac{\pi}{3}$ in quadrant II.

71. $\cos\dfrac{13\pi}{4} = -\cos\dfrac{\pi}{4} = -\dfrac{\sqrt{2}}{2}$, since $\theta = \dfrac{13\pi}{4}$ has reference angle $\alpha = \dfrac{\pi}{4}$ in quadrant III.

72. $\tan\dfrac{8\pi}{3} = -\tan\dfrac{\pi}{3} = -\sqrt{3}$, since $\theta = \dfrac{8\pi}{3}$ has reference angle $\alpha = \dfrac{\pi}{3}$ in quadrant II.

73. $\sin\left(-\dfrac{2\pi}{3}\right) = -\sin\dfrac{\pi}{3} = -\dfrac{\sqrt{3}}{2}$, since $\theta = -\dfrac{2\pi}{3}$ has reference angle $\alpha = \dfrac{\pi}{3}$ in quadrant III.

74. $\cot\left(-\dfrac{\pi}{6}\right) = -\cot\dfrac{\pi}{6} = -\sqrt{3}$, since $\theta = -\dfrac{\pi}{6}$ has reference angle $\alpha = \dfrac{\pi}{6}$ in quadrant IV.

75. $\tan\dfrac{14\pi}{3} = -\tan\dfrac{\pi}{3} = -\sqrt{3}$, since $\theta = \dfrac{14\pi}{3}$ has reference angle $\alpha = \dfrac{\pi}{3}$ in quadrant II.

76. $\sec\dfrac{11\pi}{4} = -\sec\dfrac{\pi}{4} = -\sqrt{2}$, since $\theta = \dfrac{11\pi}{4}$ has reference angle $\alpha = \dfrac{\pi}{4}$ in quadrant II.

77. $\sin(8\pi) = \sin(0 + 8\pi) = \sin(0) = 0$

78. $\cos(-2\pi) = \cos(0 - 2\pi) = \cos(0) = 1$

79. $\tan(7\pi) = \tan(\pi + 6\pi) = \tan(\pi) = 0$

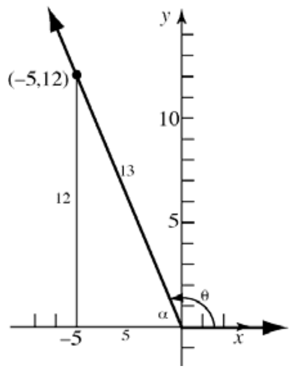
80. $\cot(5\pi) = \cot(\pi + 4\pi) = \cot(\pi)$, which is undefined

81. $\sec(-3\pi) = \sec(\pi - 4\pi) = \sec(\pi) = -1$

82. $\csc\left(-\dfrac{5\pi}{2}\right) = \csc\left(\dfrac{3\pi}{2} - 4\pi\right) = -1$

83. $\sin\theta = \frac{12}{13}$, θ in quadrant II

Since θ is in quadrant II, $\sin\theta > 0$ and $\csc\theta > 0$, while $\cos\theta < 0$, $\sec\theta < 0$, $\tan\theta < 0$, and $\cot\theta < 0$. If α is the reference angle for θ, then $\sin\alpha = \frac{12}{13}$. Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \frac{5}{13}$ $\quad\tan\alpha = \frac{12}{5}$ $\quad\sec\alpha = \frac{13}{5}$

$\csc\alpha = \frac{13}{12}$ $\quad\cot\alpha = \frac{5}{12}$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\cos\theta = -\frac{5}{13}$ $\quad\tan\theta = -\frac{12}{5}$ $\quad\sec\theta = -\frac{13}{5}$

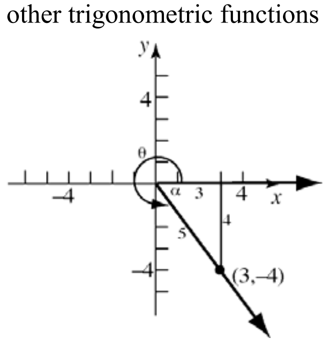
$\csc\theta = \frac{13}{12}$ $\quad\cot\theta = -\frac{5}{12}$

84. $\cos\theta = \frac{3}{5}$, θ in quadrant IV

Since θ is in quadrant IV, $\cos\theta > 0$ and $\sec\theta > 0$, while $\sin\theta < 0$, $\csc\theta < 0$, $\tan\theta < 0$, and $\cot\theta < 0$.

If α is the reference angle for θ, then $\cos\alpha = \frac{3}{5}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{4}{5}$ $\quad\tan\alpha = \frac{4}{3}$ $\quad\sec\alpha = \frac{5}{3}$

$\csc\alpha = \frac{5}{4}$ $\quad\cot\alpha = \frac{3}{4}$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\sin\theta = -\frac{4}{5}$ $\quad\tan\theta = -\frac{4}{3}$ $\quad\sec\theta = \frac{5}{3}$

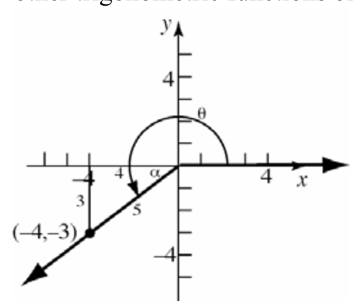
$\csc\theta = -\frac{5}{4}$ $\quad\cot\theta = -\frac{3}{4}$

85. $\cos\theta = -\frac{4}{5}$, θ in quadrant III

Since θ is in quadrant III, $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$ and $\csc\theta < 0$, while $\tan\theta > 0$ and $\cot\theta > 0$.

If α is the reference angle for θ, then $\cos\alpha = \frac{4}{5}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{3}{5}$ $\quad\tan\alpha = \frac{3}{4}$ $\quad\sec\alpha = \frac{5}{4}$

$\csc\alpha = \frac{5}{3}$ $\quad\cot\alpha = \frac{4}{3}$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\sin\theta = -\frac{3}{5}$ $\quad\tan\theta = \frac{3}{4}$ $\quad\sec\theta = -\frac{5}{4}$

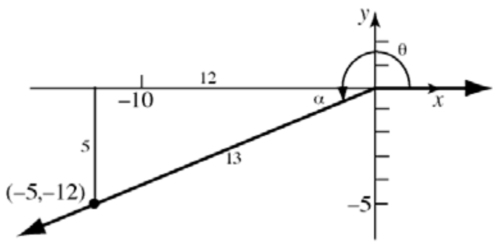
$\csc\theta = -\frac{5}{3}$ $\quad\cot\theta = \frac{4}{3}$

86. $\sin\theta = -\frac{5}{13}$, θ in quadrant III

Since θ is in quadrant III, $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$ and $\csc\theta < 0$, while $\tan\theta > 0$ and $\cot\theta > 0$.

If α is the reference angle for θ, then $\sin\alpha = \frac{5}{13}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \frac{12}{13}$ $\quad \tan\alpha = \frac{5}{12}$ $\quad \sec\alpha = \frac{13}{12}$

$\csc\alpha = \frac{13}{5}$ $\quad \cot\alpha = \frac{12}{5}$

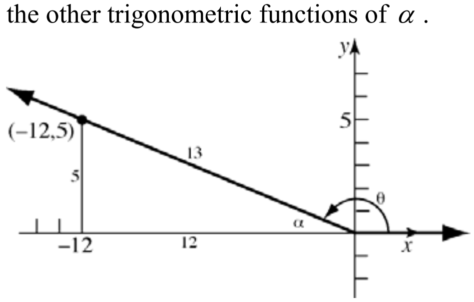
Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\cos\theta = -\frac{12}{13}$ $\quad \tan\theta = \frac{5}{12}$ $\quad \csc\theta = -\frac{13}{5}$

$\sec\theta = -\frac{13}{12}$ $\quad \cot\theta = \frac{12}{5}$

87. $\sin\theta = \frac{5}{13}$, $90^{\circ} < \theta < 180^{\circ}$, so θ in quadrant II

Since θ is in quadrant II, $\cos\theta < 0$, $\sec\theta < 0$, $\tan\theta < 0$, and $\cot\theta < 0$, while $\sin\theta > 0$ and $\csc\theta > 0$. If α is the reference angle for θ, then $\sin\alpha = \frac{5}{13}$. Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \frac{12}{13}$ $\quad \tan\alpha = \frac{5}{12}$ $\quad \csc\alpha = \frac{13}{5}$

$\sec\alpha = \frac{13}{12}$ $\quad \cot\alpha = \frac{12}{5}$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\cos\theta = -\frac{12}{13}$ $\quad \tan\theta = -\frac{5}{12}$ $\quad \csc\theta = \frac{13}{5}$

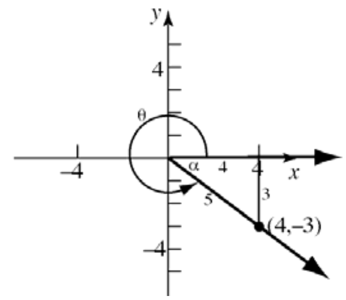
$\sec\theta = -\frac{13}{12}$ $\quad \cot\theta = -\frac{12}{5}$

88. $\cos\theta = \frac{4}{5}$, $270^{\circ} < \theta < 360^{\circ}$ (quadrant IV)

Since θ is in quadrant IV, $\sin\theta < 0$, $\csc\theta < 0$, $\tan\theta < 0$, $\cot\theta < 0$, $\cos\theta > 0$, and $\sec\theta > 0$.

If α is the reference angle for θ, then $\cos\alpha = \frac{4}{5}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{3}{5}$ $\quad \tan\alpha = \frac{3}{4}$ $\quad \sec\alpha = \frac{5}{4}$

$\csc\alpha = \frac{5}{3}$ $\quad \cot\alpha = \frac{4}{3}$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\sin\theta = -\frac{3}{5}$ $\quad \tan\theta = -\frac{3}{4}$ $\quad \sec\theta = \frac{5}{4}$

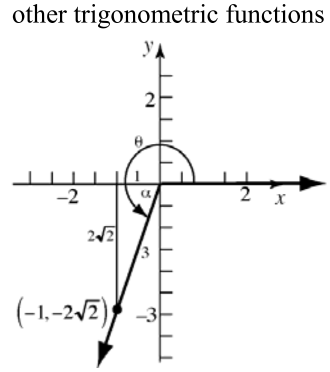
$\csc\theta = -\frac{5}{3}$ $\quad \cot\theta = -\frac{4}{3}$

89. $\cos\theta = -\frac{1}{3}$, $180^{\circ} < \theta < 270^{\circ}$ (quadrant III)

Since θ is in quadrant III, $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$, $\csc\theta < 0$, $\tan\theta > 0$, and $\cot\theta > 0$.

If α is the reference angle for θ, then $\cos\alpha = \frac{1}{3}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{2\sqrt{2}}{3}$ $\qquad \csc\alpha = \frac{3}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4}$

$\tan\alpha = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$ $\qquad \cot\alpha = \frac{1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$

$\sec\alpha = \frac{3}{1} = 3$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\sin\theta = -\frac{2\sqrt{2}}{3}$ $\qquad \csc\theta = -\frac{3\sqrt{2}}{4}$

$\tan\theta = 2\sqrt{2}$ $\qquad \cot\theta = \frac{\sqrt{2}}{4}$

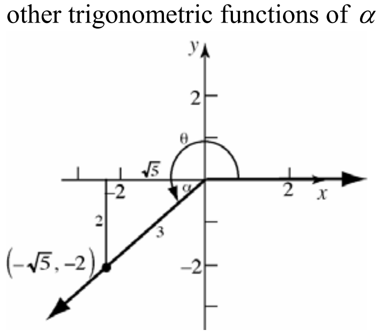
$\sec\theta = -3$

90. $\sin\theta = -\frac{2}{3}, 180^\circ < \theta < 270^\circ$ (quadrant III)

Since θ is in quadrant III, $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$, $\csc\theta < 0$, $\tan\theta > 0$, and $\cot\theta > 0$.

If α is the reference angle for θ, then $\sin\alpha = \frac{2}{3}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \frac{\sqrt{5}}{3}$ $\qquad \sec\alpha = \frac{3}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

$\tan\alpha = \frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\qquad \cot\alpha = \frac{\sqrt{5}}{2}$

$\csc\alpha = \frac{3}{2}$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\cos\theta = -\frac{\sqrt{5}}{3}$ $\qquad \sec\theta = -\frac{3\sqrt{5}}{5}$

$\tan\theta = \frac{2\sqrt{5}}{5}$ $\qquad \cot\theta = \frac{\sqrt{5}}{2}$

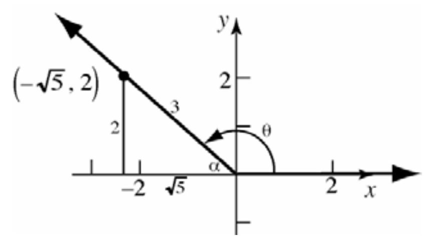
$\csc\theta = -\frac{3}{2}$

91. $\sin\theta = \frac{2}{3}$, $\tan\theta < 0$ (quadrant II)

Since θ is in quadrant II, $\cos\theta < 0$, $\sec\theta < 0$, $\tan\theta < 0$ and $\cot\theta < 0$, while $\sin\theta > 0$ and $\csc\theta > 0$.

If α is the reference angle for θ, then $\sin\alpha = \frac{2}{3}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \frac{\sqrt{5}}{3}$ $\qquad \sec\alpha = \frac{3}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

$\tan\alpha = \frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\qquad \cot\alpha = \frac{\sqrt{5}}{2}$

$\csc\alpha = \frac{3}{2}$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\cos\theta = -\frac{\sqrt{5}}{3}$ $\qquad \sec\theta = -\frac{3\sqrt{5}}{5}$

$\tan\theta = -\frac{2\sqrt{5}}{5}$ $\qquad \cot\theta = -\frac{\sqrt{5}}{2}$

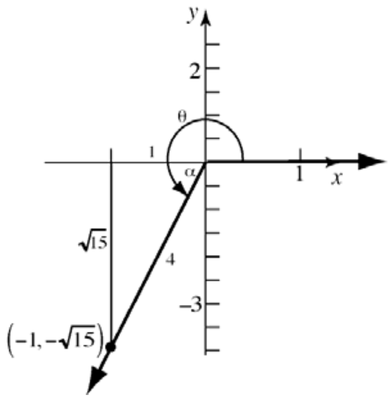
$\csc\theta = \frac{3}{2}$

92. $\cos\theta = -\frac{1}{4}$, $\tan\theta > 0$ (quadrant III)

Since θ is in quadrant III, $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$ and $\csc\theta < 0$, while $\tan\theta > 0$ and $\cot\theta > 0$.

If α is the reference angle for θ, then $\cos\alpha = \frac{1}{4}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{\sqrt{15}}{4}$ $\qquad$ $\csc\alpha = \frac{4}{\sqrt{15}}\cdot\frac{\sqrt{15}}{\sqrt{15}} = \frac{4\sqrt{15}}{15}$

$\tan\alpha = \frac{\sqrt{15}}{1} = \sqrt{15}$ $\qquad$ $\cot\alpha = \frac{1}{\sqrt{15}}\cdot\frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{15}}{15}$

$\sec\alpha = \frac{4}{1} = 4$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\sin\theta = -\frac{\sqrt{15}}{4}$ $\qquad$ $\csc\theta = -\frac{4\sqrt{15}}{15}$

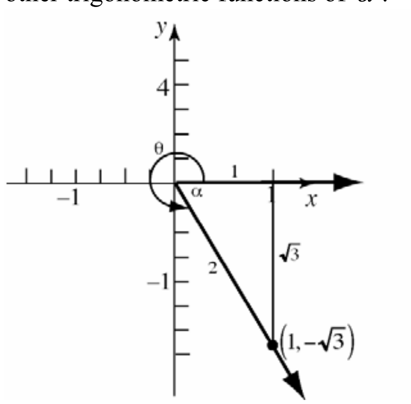
$\tan\theta = \sqrt{15}$ $\qquad$ $\cot\theta = \frac{\sqrt{15}}{15}$

$\sec\theta = -4$

93. $\sec\theta = 2, \quad \sin\theta < 0$ (quadrant IV)

Since θ is in quadrant IV, $\sin\theta < 0$, $\csc\theta < 0$, $\tan\theta < 0$ and $\cot\theta < 0$, while $\cos\theta > 0$ and $\sec\theta > 0$.

If α is the reference angle for θ, then $\sec\alpha = 2$. Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{\sqrt{3}}{2}$ $\qquad$ $\cos\alpha = \frac{1}{2}$ $\qquad$ $\tan\alpha = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\csc\alpha = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ $\qquad$ $\cot\alpha = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

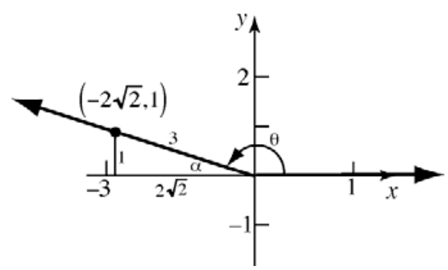
$\sin\theta = -\frac{\sqrt{3}}{2}$ $\qquad$ $\cos\theta = \frac{1}{2}$ $\qquad$ $\tan\theta = -\sqrt{3}$

$\csc\theta = -\frac{2\sqrt{3}}{3}$ $\qquad$ $\cot\theta = -\frac{\sqrt{3}}{3}$

94. $\csc\theta = 3$, $\cot\theta < 0$, (quadrant II)

Since θ is in quadrant II, $\cos\theta < 0$, $\sec\theta < 0$, $\tan\theta < 0$ and $\cot\theta < 0$, while $\sin\theta > 0$ and $\csc\theta > 0$.

If α is the reference angle for θ, then $\csc\alpha = 3$. Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{1}{3}$ $\qquad$ $\cos\alpha = \frac{2\sqrt{2}}{3}$

$\tan\alpha = \frac{1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$ $\qquad$ $\cot\alpha = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$

$\sec\alpha = \frac{3}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4}$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\sin\theta = \frac{1}{3}$ $\qquad$ $\cos\theta = -\frac{2\sqrt{2}}{3}$

$\tan\theta = -\frac{\sqrt{2}}{4}$ $\qquad$ $\cot\theta = -2\sqrt{2}$

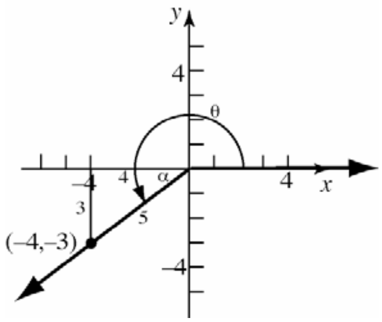
$\sec\theta = -\frac{3\sqrt{2}}{4}$

95. $\tan\theta = \frac{3}{4}$, $\sin\theta < 0$ (quadrant III)

Since θ is in quadrant III, $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$, $\csc\theta < 0$, $\tan\theta > 0$, and $\cot\theta > 0$.

If α is the reference angle for θ, then $\tan\alpha = \frac{3}{4}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{3}{5}$ $\quad\cos\alpha = \frac{4}{5}$ $\quad\cot\alpha = \frac{4}{3}$

$\csc\alpha = \frac{5}{3}$ $\quad\sec\alpha = \frac{5}{4}$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\sin\theta = -\frac{3}{5}$ $\quad\cos\theta = -\frac{4}{5}$ $\quad\cot\theta = \frac{4}{3}$

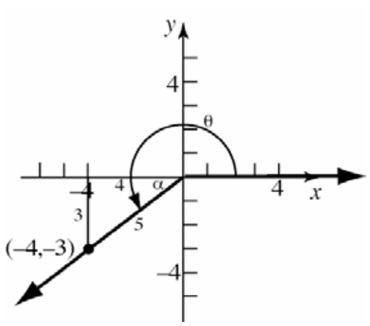
$\csc\theta = -\frac{5}{3}$ $\quad\sec\theta = -\frac{5}{4}$

96. $\cot\theta = \frac{4}{3}$, $\cos\theta < 0$ (quadrant III)

Since θ is in quadrant III, $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$, $\csc\theta < 0$, $\tan\theta > 0$, and $\cot\theta > 0$.

If α is the reference angle for θ, then $\cot\alpha = \frac{4}{3}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{3}{5}$ $\quad\cos\alpha = \frac{4}{5}$ $\quad\tan\alpha = \frac{3}{4}$

$\sec\alpha = \frac{5}{4}$ $\quad\csc\alpha = \frac{5}{3}$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\sin\theta = -\frac{3}{5}$ $\quad\cos\theta = -\frac{4}{5}$ $\quad\tan\theta = \frac{3}{4}$

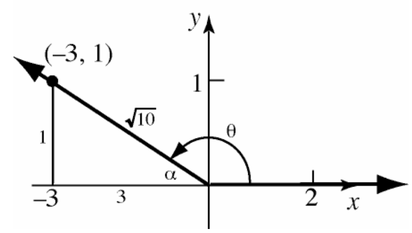
$\csc\theta = -\frac{5}{3}$ $\quad\sec\theta = -\frac{5}{4}$

97. $\tan\theta = -\frac{1}{3}$, $\sin\theta > 0$ (quadrant II)

Since θ is in quadrant II, $\cos\theta < 0$, $\sec\theta < 0$, $\tan\theta < 0$ and $\cot\theta < 0$, while $\sin\theta > 0$ and $\csc\theta > 0$.

If α is the reference angle for θ, then $\tan\alpha = \frac{1}{3}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{1}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}$ $\quad\csc\alpha = \frac{\sqrt{10}}{1} = \sqrt{10}$

$\cos\alpha = \frac{3}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$ $\quad\sec\alpha = \frac{\sqrt{10}}{3}$

$\cot\alpha = \frac{3}{1} = 3$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\sin\theta = \frac{\sqrt{10}}{10}$ $\quad\csc\theta = \sqrt{10}$

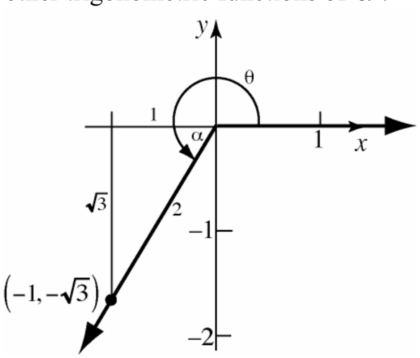
$\cos\theta = -\frac{3\sqrt{10}}{10}$ $\quad\sec\theta = -\frac{\sqrt{10}}{3}$

$\cot\theta = -3$

98. $\sec\theta = -2,\ \tan\theta > 0$ (quadrant III)

Since θ is in quadrant III, $\cos\theta < 0,\ \sec\theta < 0,$ $\sin\theta < 0$ and $\csc\theta < 0,$ while $\tan\theta > 0$ and $\cot\theta > 0.$

If α is the reference angle for θ, then $\sec\alpha = 2$. Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{\sqrt{3}}{2}$ $\cos\alpha = \frac{1}{2}$ $\tan\alpha = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\csc\alpha = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ $\cot\alpha = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

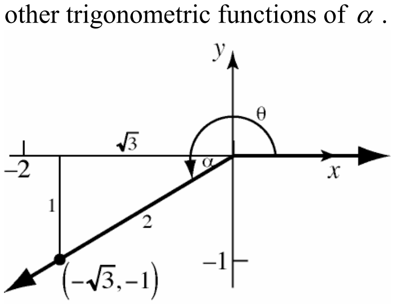
$\sin\theta = -\frac{\sqrt{3}}{2}$ $\cos\theta = -\frac{1}{2}$ $\tan\theta = \sqrt{3}$

$\csc\theta = -\frac{2\sqrt{3}}{3}$ $\cot\theta = \frac{\sqrt{3}}{3}$

99. $\csc\theta = -2,\quad \tan\theta > 0 \Rightarrow \quad \theta$ in quadrant III

Since θ is in quadrant III, $\cos\theta < 0,\ \sec\theta < 0,$ $\sin\theta < 0$ and $\csc\theta < 0,$ while $\tan\theta > 0$ and $\cot\theta > 0.$

If α is the reference angle for θ, then $\csc\alpha = 2$. Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{1}{2}$ $\cos\alpha = \frac{\sqrt{3}}{2}$

$\tan\alpha = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\sec\alpha = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cot\alpha = \frac{\sqrt{3}}{1} = \sqrt{3}$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\sin\theta = -\frac{1}{2}$ $\cos\theta = -\frac{\sqrt{3}}{2}$

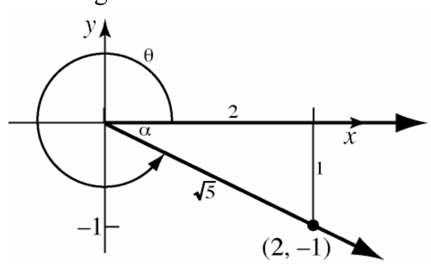
$\tan\theta = \frac{\sqrt{3}}{3}$ $\sec\theta = -\frac{2\sqrt{3}}{3}$

$\cot\theta = \sqrt{3}$

100. $\cot\theta = -2,\ \sec\theta > 0$ (quadrant IV)

Since θ is in quadrant IV, $\cos\theta > 0$ and $\sec\theta > 0,$ while $\sin\theta < 0,\ \csc\theta < 0,\ \tan\theta < 0$ and $\cot\theta < 0.$

If α is the reference angle for θ, then $\cot\alpha = 2$. Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\csc\alpha = \frac{\sqrt{5}}{1} = \sqrt{5}$

$\cos\alpha = \frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\sec\alpha = \frac{\sqrt{5}}{2}$

$\tan\alpha = \frac{1}{2}$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\sin\theta = -\frac{\sqrt{5}}{5}$ $\csc\theta = -\sqrt{5}$

$\cos\theta = \frac{2\sqrt{5}}{5}$ $\sec\theta = \frac{\sqrt{5}}{2}$

$\tan\theta = -\frac{1}{2}$

101. $\sin 40^\circ + \sin 130^\circ + \sin 220^\circ + \sin 310^\circ$
$= \sin 40^\circ + \sin(40^\circ + 90^\circ) + \sin(40^\circ + 180^\circ) + \sin(40^\circ + 270^\circ)$
$= \sin 40^\circ + \sin 40^\circ - \sin 40^\circ - \sin 40^\circ$
$= 0$

102. $\tan 40^\circ + \tan 140^\circ = \tan 40^\circ + \tan(180^\circ - 40^\circ)$
$= \sqrt{3} - \sqrt{3}$
$= 0$

103. Note: $\theta = 315^\circ$ has reference angle $\alpha = 45^\circ$ in quadrant IV.

a. $f(315^\circ) = \sin 315^\circ = -\sin 45^\circ = -\frac{\sqrt{2}}{2}$

The point $\left(315^\circ, -\frac{\sqrt{2}}{2}\right)$ is on the graph of *f*.

b. $G(315^\circ) = \sec 315^\circ = \sec 45^\circ = \sqrt{2}$

The point $\left(315^\circ, \sqrt{2}\right)$ is on the graph of *G*.

c. $h(315^\circ) = \tan 315^\circ = -\tan 45^\circ = -1$

The point $(315^\circ, -1)$ is on the graph of *h*.

104. Note: $\theta = 120^\circ$ has reference angle $\alpha = 60^\circ$ in quadrant II.

a. $g(120^\circ) = \cos 120^\circ = -\cos 60^\circ = -\frac{1}{2}$

The point $\left(120^\circ, -\frac{1}{2}\right)$ is on the graph of *g*.

b. $F(120^\circ) = \csc 120^\circ = \csc 60^\circ = \frac{2\sqrt{3}}{3}$

The point $\left(120^\circ, \frac{2\sqrt{3}}{3}\right)$ is on the graph of *F*.

c. $H(120^\circ) = \cot 120^\circ = -\cot 60^\circ = -\frac{\sqrt{3}}{3}$

The point $\left(120^\circ, -\frac{\sqrt{3}}{3}\right)$ is on the graph of *H*.

105. Note: $\theta = \frac{7\pi}{6}$ has reference angle $\alpha = \frac{\pi}{6}$ in quadrant III.

a. $g\left(\frac{7\pi}{6}\right) = \cos\frac{7\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

The point $\left(\frac{7\pi}{6}, -\frac{\sqrt{3}}{2}\right)$ is on the graph of *g*.

b. $F\left(\frac{7\pi}{6}\right) = \csc\frac{7\pi}{6} = -\csc\frac{\pi}{6} = -2$

The point $\left(\frac{7\pi}{6}, -2\right)$ is on the graph of *F*.

c. Note: $\theta = -315^\circ$ has reference angle $\alpha = 45^\circ$ in quadrant I.
$H(-315^\circ) = \cot(-315^\circ) = \cot 45^\circ = 1$
The point $(-315^\circ, 1)$ is on the graph of *H*.

106. Note: $\theta = \frac{7\pi}{4}$ has reference angle $\alpha = \frac{\pi}{4}$ in quadrant IV.

a. $f\left(\frac{7\pi}{4}\right) = \sin\frac{7\pi}{4} = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

The point $\left(\frac{7\pi}{4}, -\frac{\sqrt{2}}{2}\right)$ is on the graph of *f*.

b. $G\left(\frac{7\pi}{4}\right) = \sec\frac{7\pi}{4} = \sec\frac{\pi}{4} = \sqrt{2}$

The point $\left(\frac{7\pi}{4}, \sqrt{2}\right)$ is on the graph of *G*.

c. Note: $\theta = -225^\circ$ has reference angle $\alpha = 45^\circ$ in quadrant II.
$F(-225^\circ) = \csc(-225^\circ) = \csc 45^\circ = \sqrt{2}$
The point $\left(-225^\circ, \sqrt{2}\right)$ is on the graph of *F*.

107. Since $f(\theta) = \sin\theta = 0.2$ is positive, θ must lie either in quadrant I or II. Therefore, $\theta + \pi$ must lie either in quadrant III or IV. Thus,
$f(\theta + \pi) = \sin(\theta + \pi) = -0.2$

108. Since $g(\theta) = \cos\theta = 0.4$ is positive, θ must lie either in quadrant I or IV. Therefore, $\theta + \pi$ must lie either in quadrant II or III. Thus,
$g(\theta + \pi) = \cos(\theta + \pi) = -0.4$.

109. Since $F(\theta) = \tan\theta = 3$ is positive, θ must lie either in quadrant I or III. Therefore, $\theta + \pi$ must also lie either in quadrant I or III. Thus,
$F(\theta + \pi) = \tan(\theta + \pi) = 3$.

110. Since $G(\theta) = \cot\theta = -2$ is negative, θ must lie either in quadrant II or IV. Therefore, $\theta + \pi$ must also lie either in quadrant II or IV. Thus,
$G(\theta + \pi) = \cot(\theta + \pi) = -2$.

111. Given $\sin\theta = \frac{1}{5}$, then $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{1}{5}} = 5$

Since $\csc\theta > 0$, θ must lie in quadrant I or II. This means that $\csc(\theta + \pi)$ must lie in quadrant III or IV with the same reference angle as θ. Since cosecant is negative in quadrants III and IV, we have $\csc(\theta + \pi) = -5$.

112. Given $\cos\theta = \frac{2}{3}$, then $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$

Since $\sec\theta > 0$, θ must lie in quadrant I or IV. This means that $\csc(\theta + \pi)$ must lie in quadrant II or III with the same reference angle as θ. Since secant is negative in quadrants II and III, we have $\sec(\theta + \pi) = -\frac{3}{2}$.

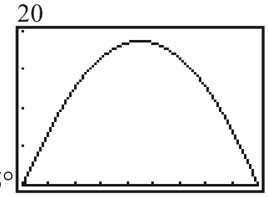
113.
$$\begin{aligned}
&\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + ... + \sin 357^\circ \\
&\qquad + \sin 358^\circ + \sin 359^\circ \\
&= \sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \cdots + \sin(360^\circ - 3^\circ) \\
&\qquad + \sin(360^\circ - 2^\circ) + \sin(360^\circ - 1^\circ) \\
&= \sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \cdots + \sin(-3^\circ) \\
&\qquad + \sin(-2^\circ) + \sin(-1^\circ) \\
&= \sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \cdots - \sin 3^\circ - \sin 2^\circ - \sin 1^\circ \\
&= \sin(180^\circ) \\
&= 0
\end{aligned}$$

114.
$$\begin{aligned}
&\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \cdots + \cos 357^\circ \\
&\qquad + \cos 358^\circ + \cos 359^\circ \\
&= \cos 1^\circ + \cos 2^\circ + \cos 3^\circ + ... + \cos(360^\circ - 3^\circ) \\
&\qquad + \cos(360^\circ - 2^\circ) + \cos(360^\circ - 1^\circ) \\
&= \cos 1^\circ + \cos 2^\circ + \cos 3^\circ + ... + \cos(-3^\circ) \\
&\qquad + \cos(-2^\circ) + \cos(-1^\circ) \\
&= \cos 1^\circ + \cos 2^\circ + \cos 3^\circ + ... + \cos 3^\circ \\
&\qquad + \cos 2^\circ + \cos 1^\circ \\
&= 2\cos 1^\circ + 2\cos 2^\circ + 2\cos 3^\circ + ... + 2\cos 178^\circ \\
&\qquad + 2\cos 179^\circ + \cos 180^\circ \\
&= 2\cos 1^\circ + 2\cos 2^\circ + 2\cos 3^\circ + ... + 2\cos(180^\circ - 2^\circ) \\
&\qquad + 2\cos(180^\circ - 1^\circ) + \cos(180^\circ) \\
&= 2\cos 1^\circ + 2\cos 2^\circ + 2\cos 3^\circ + ... - 2\cos 2^\circ \\
&\qquad - 2\cos 1^\circ + \cos 180^\circ \\
&= \cos 180^\circ \\
&= -1
\end{aligned}$$

115. a.
$$\begin{aligned}
R &= \frac{32^2\sqrt{2}}{32}\left[\sin(2(60^\circ)) - \cos(2(60^\circ)) - 1\right] \\
&\approx 32\sqrt{2}(0.866 - (-0.5) - 1) \\
&\approx 16.6 \text{ ft}
\end{aligned}$$

b. Let $Y_1 = \frac{32^2\sqrt{2}}{32}\left[\sin(2x) - \cos(2x) - 1\right]$

20

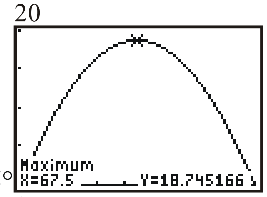


45° 90°

0

c. Using the MAXIMUM feature, we find:

20



45° 90°

0

R is largest when $\theta = 67.5^\circ$.

116 – 118. Answers will vary.

Section 2.5

1. $x^2 + y^2 = 1$

2. $\{x \mid x \neq 4\}$

3. even

4. 2π, π

5. All real number except odd multiples of $\frac{\pi}{2}$

6. All real numbers between -1 and 1, inclusive.

7. -0.2, 0.2

8. True

9. $P=\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right);\ a=\frac{\sqrt{3}}{2},\ b=-\frac{1}{2}$

$\sin t=-\frac{1}{2}$ $\qquad$ $\cos t=\frac{\sqrt{3}}{2}$

$$\tan t=\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}=\left(-\frac{1}{2}\right)\left(\frac{2}{\sqrt{3}}\right)=-\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=-\frac{\sqrt{3}}{3}$$

$$\csc t=\frac{1}{-\frac{1}{2}}=1\left(-\frac{2}{1}\right)=-2$$

$$\sec t=\frac{1}{\frac{\sqrt{3}}{2}}=1\left(\frac{2}{\sqrt{3}}\right)=\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$$

$$\cot t=\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}=\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{2}{1}\right)=-\sqrt{3}$$

10. $P=\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right);\ a=-\frac{\sqrt{3}}{2},\ b=-\frac{1}{2}$

$\sin t=-\frac{1}{2}$ $\qquad$ $\cos t=-\frac{\sqrt{3}}{2}$

$$\tan t=\frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}=\left(-\frac{1}{2}\right)\left(-\frac{2}{\sqrt{3}}\right)=\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

$$\csc t=\frac{1}{-\frac{1}{2}}=1\left(-\frac{2}{1}\right)=-2$$

$$\sec t=\frac{1}{-\frac{\sqrt{3}}{2}}=1\left(-\frac{2}{\sqrt{3}}\right)=-\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=-\frac{2\sqrt{3}}{3}$$

$$\cot t=\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}=\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{2}{1}\right)=\sqrt{3}$$

11. $P=\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right);\ a=-\frac{\sqrt{2}}{2},\ b=-\frac{\sqrt{2}}{2}$

$\sin t=-\frac{\sqrt{2}}{2}$ $\qquad$ $\cos t=-\frac{\sqrt{2}}{2}$

$\tan t=\frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}=1$ $\qquad$ $\cot t=\frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}=1$

$$\csc t=\frac{1}{-\frac{\sqrt{2}}{2}}=1\left(-\frac{2}{\sqrt{2}}\right)=-\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=-\sqrt{2}$$

$$\sec t=\frac{1}{-\frac{\sqrt{2}}{2}}=1\left(-\frac{2}{\sqrt{2}}\right)=-\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=-\sqrt{2}$$

12. $P=\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right);\ a=\frac{\sqrt{2}}{2},\ b=-\frac{\sqrt{2}}{2}$

$\sin t=-\frac{\sqrt{2}}{2}$ $\qquad$ $\cos t=\frac{\sqrt{2}}{2}$

$\tan t=\frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}=-1$ $\qquad$ $\cot t=\frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}=-1$

$$\csc t=\frac{1}{-\frac{\sqrt{2}}{2}}=1\left(-\frac{2}{\sqrt{2}}\right)=-\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=-\sqrt{2}$$

$$\sec t=\frac{1}{\frac{\sqrt{2}}{2}}=1\left(\frac{2}{\sqrt{2}}\right)=\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\sqrt{2}$$

13. $P=\left(\frac{\sqrt{5}}{3},\frac{2}{3}\right);\ a=\frac{\sqrt{5}}{3},\ b=\frac{2}{3}$

$\sin t=\frac{2}{3}$ $\qquad$ $\cos t=\frac{\sqrt{5}}{3}$

$$\tan t=\frac{\frac{2}{3}}{\frac{\sqrt{5}}{3}}=\left(\frac{2}{3}\right)\left(\frac{3}{\sqrt{5}}\right)=\frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$$

$$\csc t=\frac{1}{\frac{2}{3}}=1\left(\frac{3}{2}\right)=\frac{3}{2}$$

$$\sec t=\frac{1}{\frac{\sqrt{5}}{3}}=1\left(\frac{3}{\sqrt{5}}\right)=\frac{3}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{3\sqrt{5}}{5}$$

$$\cot t=\frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}}=\left(\frac{\sqrt{5}}{3}\right)\left(\frac{3}{2}\right)=\frac{\sqrt{5}}{2}$$

14. $P = \left(-\frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}\right);\ a = -\frac{\sqrt{5}}{5},\ b = \frac{2\sqrt{5}}{5}$

$\sin t = \frac{2\sqrt{5}}{5} \qquad \cos t = -\frac{\sqrt{5}}{5}$

$$\tan t = \frac{\frac{2\sqrt{5}}{5}}{-\frac{\sqrt{5}}{5}} = \left(\frac{2\sqrt{5}}{5}\right)\left(-\frac{5}{\sqrt{5}}\right) = -2$$

$$\csc t = \frac{1}{\frac{2\sqrt{5}}{5}} = 1\left(\frac{5}{2\sqrt{5}}\right)\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{2}$$

$$\sec t = \frac{1}{-\frac{\sqrt{5}}{5}} = 1\left(-\frac{5}{\sqrt{5}}\right)\frac{\sqrt{5}}{\sqrt{5}} = -\sqrt{5}$$

$$\cot t = \frac{-\frac{\sqrt{5}}{5}}{\frac{2\sqrt{5}}{5}} = \left(-\frac{\sqrt{5}}{5}\right)\left(\frac{5}{2\sqrt{5}}\right) = -\frac{1}{2}$$

15. For the point $(3, -4)$, $x = 3,\ y = -4,$

$r = \sqrt{x^2 + y^2} = \sqrt{9+16} = \sqrt{25} = 5$

$\sin\theta = -\frac{4}{5} \qquad \cos\theta = \frac{3}{5} \qquad \tan\theta = -\frac{4}{3}$

$\csc\theta = -\frac{5}{4} \qquad \sec\theta = \frac{5}{3} \qquad \cot\theta = -\frac{3}{4}$

16. For the point $(4, -3)$, $x = 4,\ y = -3,$

$r = \sqrt{x^2 + y^2} = \sqrt{16+9} = \sqrt{25} = 5$

$\sin\theta = -\frac{3}{5} \qquad \cos\theta = \frac{4}{5} \qquad \tan\theta = -\frac{3}{4}$

$\csc\theta = -\frac{5}{3} \qquad \sec\theta = \frac{5}{4} \qquad \cot\theta = -\frac{4}{3}$

17. For the point $(-2, 3)$, $x = -2,\ y = 3,$

$r = \sqrt{x^2 + y^2} = \sqrt{4+9} = \sqrt{13}$

$\sin\theta = \frac{3}{\sqrt{13}}\frac{\sqrt{13}}{\sqrt{13}} = \frac{3\sqrt{13}}{13} \qquad \csc\theta = \frac{\sqrt{13}}{3}$

$\cos\theta = -\frac{2}{\sqrt{13}}\frac{\sqrt{13}}{\sqrt{13}} = -\frac{2\sqrt{13}}{13} \qquad \sec\theta = -\frac{\sqrt{13}}{2}$

$\tan\theta = -\frac{3}{2} \qquad \cot\theta = -\frac{2}{3}$

18. For the point $(2, -4)$, $x = 2,\ y = -4,$

$r = \sqrt{x^2 + y^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$\sin\theta = \frac{-4}{2\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{5} \qquad \csc\theta = \frac{2\sqrt{5}}{-4} = -\frac{\sqrt{5}}{2}$

$\cos\theta = \frac{2}{2\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5} \qquad \sec\theta = \frac{5}{\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}} = \sqrt{5}$

$\tan\theta = \frac{-4}{2} = -2 \qquad \cot\theta = \frac{2}{-4} = -\frac{1}{2}$

19. For the point $(-1, -1)$, $x = -1,\ y = -1,$

$r = \sqrt{x^2 + y^2} = \sqrt{1+1} = \sqrt{2}$

$\sin\theta = \frac{-1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \qquad \csc\theta = \frac{\sqrt{2}}{-1} = -\sqrt{2}$

$\cos\theta = \frac{-1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \qquad \sec\theta = \frac{\sqrt{2}}{-1} = -\sqrt{2}$

$\tan\theta = \frac{-1}{-1} = 1 \qquad \cot\theta = \frac{-1}{-1} = 1$

20. For the point $(-3, 1)$, $x = -3,\ y = 1,$

$r = \sqrt{x^2 + y^2} = \sqrt{9+1} = \sqrt{10}$

$\sin\theta = \frac{1}{\sqrt{10}}\frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10} \qquad \csc\theta = \frac{\sqrt{10}}{1} = \sqrt{10}$

$\cos\theta = \frac{-3}{\sqrt{10}}\frac{\sqrt{10}}{\sqrt{10}} = -\frac{3\sqrt{10}}{10} \qquad \sec\theta = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3}$

$\tan\theta = \frac{1}{-3} = -\frac{1}{3} \qquad \cot\theta = \frac{-3}{1} = -3$

21. $\sin 405^\circ = \sin(360^\circ + 45^\circ) = \sin 45^\circ = \frac{\sqrt{2}}{2}$

22. $\cos 420^\circ = \cos(360^\circ + 60^\circ) = \cos 60^\circ = \frac{1}{2}$

23. $\tan 405^\circ = \tan(180^\circ + 180^\circ + 45^\circ) = \tan 45^\circ = 1$

24. $\sin 390^\circ = \sin(360^\circ + 30^\circ) = \sin 30^\circ = \frac{1}{2}$

25. $\csc 450^\circ = \csc(360^\circ + 90^\circ) = \csc 90^\circ = 1$

26. $\sec 540^\circ = \sec(360^\circ + 180^\circ) = \sec 180^\circ = -1$

27. $\cot 390^\circ = \cot(180^\circ + 180^\circ + 30^\circ) = \cot 30^\circ = \sqrt{3}$

28. $\sec 420^\circ = \sec(360^\circ + 60^\circ) = \sec 60^\circ = 2$

29. $$\begin{aligned}\cos\frac{33\pi}{4} &= \cos\left(\frac{\pi}{4}+8\pi\right)\\ &= \cos\left(\frac{\pi}{4}+4\cdot 2\pi\right)\\ &= \cos\frac{\pi}{4}\\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

30. $\sin\frac{9\pi}{4} = \sin\left(\frac{\pi}{4}+2\pi\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

31. $\tan(21\pi) = \tan(0+21\pi) = \tan(0) = 0$

32. $$\begin{aligned}\csc\frac{9\pi}{2} &= \csc\left(\frac{\pi}{2}+4\pi\right)\\ &= \csc\left(\frac{\pi}{2}+2\cdot 2\pi\right)\\ &= \csc\frac{\pi}{2}\\ &= 1\end{aligned}$$

33. $$\begin{aligned}\sec\frac{17\pi}{4} &= \sec\left(\frac{\pi}{4}+4\pi\right)\\ &= \sec\left(\frac{\pi}{4}+2\cdot 2\pi\right)\\ &= \sec\frac{\pi}{4}\\ &= \sqrt{2}\end{aligned}$$

34. $$\begin{aligned}\cot\frac{17\pi}{4} &= \cot\left(\frac{\pi}{4}+4\pi\right)\\ &= \cot\left(\frac{\pi}{4}+2\cdot 2\pi\right)\\ &= \cot\frac{\pi}{4}\\ &= 1\end{aligned}$$

35. $\tan\frac{19\pi}{6} = \tan\left(\frac{\pi}{6}+3\pi\right) = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

36. $$\begin{aligned}\sec\frac{25\pi}{6} &= \sec\left(\frac{\pi}{6}+4\pi\right)\\ &= \sec\left(\frac{\pi}{6}+2\cdot 2\pi\right)\\ &= \sec\frac{\pi}{6}\\ &= \frac{2\sqrt{3}}{3}\end{aligned}$$

37. $\sin(-60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$

38. $\cos(-30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

39. $\tan(-30^\circ) = -\tan 30^\circ = -\frac{\sqrt{3}}{3}$

40. $\sin(-135^\circ) = -\sin 135^\circ = -\frac{\sqrt{2}}{2}$

41. $\sec(-60^\circ) = \sec 60^\circ = 2$

42. $\csc(-30^\circ) = -\csc 30^\circ = -2$

43. $\sin(-90^\circ) = -\sin 90^\circ = -1$

44. $\cos(-270^\circ) = \cos 270^\circ = 0$

45. $\tan\left(-\frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -1$

46. $\sin(-\pi) = -\sin\pi = 0$

47. $\cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

48. $\sin\left(-\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

49. $\tan(-\pi) = -\tan\pi = 0$

50. $\sin\left(-\frac{3\pi}{2}\right) = -\sin\frac{3\pi}{2} = -(-1) = 1$

51. $\csc\left(-\frac{\pi}{4}\right) = -\csc\frac{\pi}{4} = -\sqrt{2}$

52. $\sec(-\pi) = \sec\pi = -1$

53. $\sec\left(-\frac{\pi}{6}\right) = \sec\frac{\pi}{6} = \frac{2\sqrt{3}}{3}$

54. $\csc\left(-\frac{\pi}{3}\right) = -\csc\frac{\pi}{3} = -\frac{2\sqrt{3}}{3}$

55. $\sin(-\pi) + \cos(5\pi) = -\sin(\pi) + \cos(\pi + 4\pi)$

$= 0 + \cos\pi$

$= -1$

56. $\tan\left(-\frac{5\pi}{6}\right) - \cot\frac{7\pi}{2} = -\tan\frac{5\pi}{6} - \cot\left(\frac{\pi}{2} + 3\pi\right)$

$= -\tan\frac{5\pi}{6} - \cot\frac{\pi}{2}$

$= -\left(-\frac{\sqrt{3}}{3}\right) - 0$

$= \frac{\sqrt{3}}{3}$

57. $\sec(-\pi) + \csc\left(-\frac{\pi}{2}\right) = \sec\pi - \csc\frac{\pi}{2}$

$= -1 - 1$

$= -2$

58. $\tan(-6\pi) + \cos\frac{9\pi}{4} = -\tan(0 + 6\pi) + \cos\left(\frac{\pi}{4} + 2\pi\right)$

$= -\tan 0 + \cos\frac{\pi}{4}$

$= 0 + \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{2}$

59. $\sin\left(-\frac{9\pi}{4}\right) - \tan\left(-\frac{9\pi}{4}\right)$

$= -\sin\frac{9\pi}{4} + \tan\frac{9\pi}{4}$

$= -\sin\left(\frac{\pi}{4} + 2\pi\right) + \tan\left(\frac{\pi}{4} + 2\pi\right)$

$= -\sin\frac{\pi}{4} + \tan\frac{\pi}{4}$

$= -\frac{\sqrt{2}}{2} + 1$, or $\frac{2 - \sqrt{2}}{2}$

60. $\cos\left(-\frac{17\pi}{4}\right) - \sin\left(-\frac{3\pi}{2}\right)$

$= \cos\frac{17\pi}{4} + \sin\frac{3\pi}{2}$

$= \cos\left(\frac{\pi}{4} + 2\cdot 2\pi\right) + \sin\frac{3\pi}{2}$

$= \cos\frac{\pi}{4} + \sin\frac{3\pi}{2}$

$= \frac{\sqrt{2}}{2} + (-1)$

$= \frac{\sqrt{2}}{2} - 1$, or $\frac{\sqrt{2} - 2}{2}$

61. The domain of the sine function is the set of all real numbers. That is, $(-\infty, \infty)$.

62. The domain of the cosine function is the set of all real numbers. That is, $(-\infty, \infty)$.

63. $f(\theta) = \tan\theta$ is not defined for numbers that are odd multiples of $\frac{\pi}{2}$.

64. $f(\theta) = \cot\theta$ is not defined for numbers that are multiples of π.

65. $f(\theta) = \sec\theta$ is not defined for numbers that are odd multiples of $\frac{\pi}{2}$.

66. $f(\theta) = \csc\theta$ is not defined for numbers that are multiples of π.

67. The range of the sine function is the set of all real numbers between –1 and 1, inclusive. That is, the interval $[-1, 1]$.

68. The range of the cosine function is the set of all real numbers between –1 and 1, inclusive. That is, the interval $[-1, 1]$.

69. The range of the tangent function is the set of all real numbers. That is, $(-\infty, \infty)$.

70. The range of the cotangent function is the set of all real numbers. That is, $(-\infty, \infty)$.

71. The range of the secant function is the set of all real number greater than or equal to 1 and all real numbers less than or equal to –1. That is, the interval $\left(-\infty,-1\right]\cup\left[1,\infty\right)$.

72. The range of the cosecant function is the set of all real number greater than or equal to 1 and all real numbers less than or equal to –1. That is, the interval $\left(-\infty,-1\right]\cup\left[1,\infty\right)$.

73. The sine function is odd because $\sin(-\theta)=-\sin\theta$. Its graph is symmetric with respect to the origin.

74. The cosine function is even because $\cos(-\theta)=\cos\theta$. Its graph is symmetric with respect to the y-axis.

75. The tangent function is odd because $\tan(-\theta)=-\tan\theta$. Its graph is symmetric with respect to the origin.

76. The cotangent function is odd because $\cot(-\theta)=-\cot\theta$. Its graph is symmetric with respect to the origin.

77. The secant function is even because $\sec(-\theta)=\sec\theta$. Its graph is symmetric with respect to the y-axis.

78. The cosecant function is odd because $\csc(-\theta)=-\csc\theta$. Its graph is symmetric with respect to the origin.

79. If $\sin\theta=0.3$, then

$$\sin\theta+\sin\left(\theta+2\pi\right)+\sin\left(\theta+4\pi\right)$$
$$=0.3+0.3+0.3=0.9$$

80. If $\cos\theta=0.2$, then

$$\cos\theta+\cos\left(\theta+2\pi\right)+\cos\left(\theta+4\pi\right)$$
$$=-0.2+0.2+0.2=0.6$$

81. If $\tan\theta=3$, then

$$\tan\theta+\tan\left(\theta+\pi\right)+\tan\left(\theta+2\pi\right)$$
$$=3+3+3$$
$$=9$$

82. If $\cot\theta=-2$, then

$$\cot\theta+\cot\left(\theta-\pi\right)+\cot\left(\theta-2\pi\right)$$
$$=-2+\left(-2\right)+\left(-2\right)$$
$$=-6$$

83. **a.** $f(-a)=-f(a)=-\frac{1}{3}$

b.
$$f(a)+f(a+2\pi)+f(a+4\pi)$$
$$=f(a)+f(a)+f(a)$$
$$=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$$
$$=1$$

84. **a.** $f(-a)=f(a)=\frac{1}{4}$

b.
$$f(a)+f(a+2\pi)+f(a-2\pi)$$
$$=f(a)+f(a)+f(a)$$
$$=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$$
$$=\frac{3}{4}$$

85. **a.** $f(-a)=-f(a)=-2$

b.
$$f(a)+f(a+\pi)+f(a+2\pi)$$
$$=f(a)+f(a)+f(a)$$
$$=2+2+2$$
$$=6$$

86. **a.** $f(-a)=-f(a)=-(-3)=3$

b.
$$f(a)+f(a+\pi)+f(a+4\pi)$$
$$=f(a)+f(a)+f(a)$$
$$=-3+(-3)+(-3)$$
$$=-9$$

87. **a.** $f(-a)=f(a)=-4$

b.
$$f(a)+f(a+2\pi)+f(a+4\pi)$$
$$=f(a)+f(a)+f(a)$$
$$=-4+(-4)+(-4)$$
$$=-12$$

88. **a.** $f(-a)=-f(a)=-2$

b.
$$f(a)+f(a+2\pi)+f(a+4\pi)$$
$$=f(a)+f(a)+f(a)$$
$$=2+2+2$$
$$=6$$

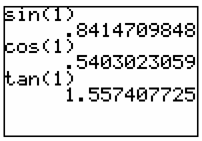
89. a. When $t = 1$, the coordinate on the unit circle is approximately $(0.5,\ 0.8)$. Thus,

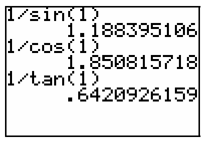
$\sin 1 \approx 0.8$ $\quad\csc 1 \approx \dfrac{1}{0.8} \approx 1.3$

$\cos 1 \approx 0.5$ $\quad\sec 1 \approx \dfrac{1}{0.5} = 2.0$

$\tan 1 \approx \dfrac{0.8}{0.5} = 1.6$ $\quad\cot 1 \approx \dfrac{0.5}{0.8} \approx 0.6$

Using a calculator on RADIAN mode:
$\sin 1 \approx 0.8$ $\quad\csc 1 \approx 1.2$
$\cos 1 \approx 0.5$ $\quad\sec 1 \approx 1.9$
$\tan 1 \approx 1.6$ $\quad\cot 1 \approx 0.6$





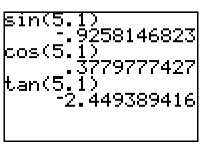
b. When $t = 5.1$, the coordinate on the unit circle is approximately $(0.4, -0.9)$. Thus,

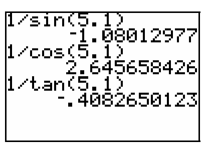
$\sin 5.1 \approx -0.9$ $\quad\csc 5.1 \approx \dfrac{1}{-0.9} \approx -1.1$

$\cos 5.1 \approx 0.4$ $\quad\sec 5.1 \approx \dfrac{1}{0.4} = 2.5$

$\tan 5.1 \approx \dfrac{-0.9}{0.4} \approx -2.3$ $\quad\cot 5.1 \approx \dfrac{0.4}{-0.9} \approx -0.4$

Using a calculator on RADIAN mode:
$\sin 5.1 \approx -0.9$ $\quad\csc 5.1 \approx -1.1$
$\cos 5.1 \approx 0.4$ $\quad\sec 5.1 \approx 2.6$
$\tan 5.1 \approx -2.4$ $\quad\cot 5.1 \approx -0.4$





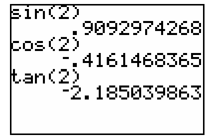
90. a. When $t = 2$, the coordinate on the unit circle is approximately $(-0.4,\ 0.9)$. Thus,

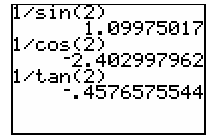
$\sin 2 \approx 0.9$ $\quad\csc 2 \approx \dfrac{1}{0.9} \approx 1.1$

$\cos 2 \approx -0.4$ $\quad\sec 2 \approx \dfrac{1}{-0.4} = -2.5$

$\tan 2 \approx \dfrac{0.9}{-0.4} = -2.3$ $\quad\cot 2 \approx \dfrac{-0.4}{0.9} \approx -0.4$

Using a calculator on RADIAN mode:
$\sin 2 \approx 0.9$ $\quad\csc 2 \approx 1.1$
$\cos 2 \approx -0.4$ $\quad\sec 2 \approx -2.4$
$\tan 2 \approx -2.2$ $\quad\cot 2 \approx -0.5$





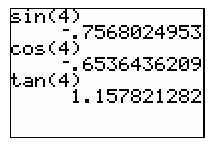
b. When $t = 4$, the coordinate on the unit circle is approximately $(-0.6, -0.8)$. Thus,

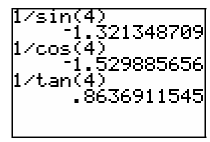
$\sin 4 \approx -0.8$ $\quad\csc 4 \approx \dfrac{1}{-0.8} \approx -1.3$

$\cos 4 \approx -0.7$ $\quad\sec 4 \approx \dfrac{1}{-0.7} \approx -1.4$

$\tan 4 \approx \dfrac{-0.8}{-0.7} \approx 1.1$ $\quad\cot 4 \approx \dfrac{-0.7}{-0.8} \approx 0.9$

Set the calculator on RADIAN mode:
$\sin 4 \approx -0.8$ $\quad\csc 4 \approx -1.3$
$\cos 4 \approx -0.7$ $\quad\sec 4 \approx -1.5$
$\tan 4 \approx 1.2$ $\quad\cot 4 \approx 0.9$





91. Let $P = (x, y)$ be the point on the unit circle that corresponds to an angle *t*. Consider the equation $\tan t = \dfrac{y}{x} = a$. Then $y = ax$. Now $x^2 + y^2 = 1$, so $x^2 + a^2x^2 = 1$. Thus, $x = \pm\dfrac{1}{\sqrt{1+a^2}}$ and $y = \pm\dfrac{a}{\sqrt{1+a^2}}$; that is, for any real number *a*, there is a point $P = (x, y)$ on the unit circle for which $\tan t = a$. In other words, $-\infty < \tan t < \infty$, and the range of the tangent function is the set of all real numbers.

92. Let $P = (x, y)$ be the point on the unit circle that corresponds to an angle *t*. Consider the equation $\cot t = \dfrac{x}{y} = a$. Then $x = ay$. Now $x^2 + y^2 = 1$, so $a^2y^2 + y^2 = 1$. Thus, $y = \pm\dfrac{1}{\sqrt{1+a^2}}$ and $x = \pm\dfrac{a}{\sqrt{1+a^2}}$; that is, for any real number *a*, there is a point $P = (x, y)$ on the unit circle for which $\cot t = a$. In other words, $-\infty < \cot t < \infty$, and the range of the cotangent function is the set of all real numbers.

93. Suppose there is a number p, $0 < p < 2\pi$, for which $\sin(\theta + p) = \sin\theta$ for all θ. If $\theta = 0$, then $\sin(0 + p) = \sin p = \sin 0 = 0$; so that $p = \pi$. If $\theta = \frac{\pi}{2}$ then $\sin\left(\frac{\pi}{2} + p\right) = \sin\left(\frac{\pi}{2}\right)$. But $p = \pi$. Thus, $\sin\left(\frac{3\pi}{2}\right) = -1 = \sin\left(\frac{\pi}{2}\right) = 1$, or $-1 = 1$. This is impossible. Therefore, the smallest positive number p for which $\sin(\theta + p) = \sin\theta$ for all θ is $p = 2\pi$.

94. Suppose there is a number p, $0 < p < 2\pi$, for which $\cos(\theta + p) = \cos\theta$ for all θ. If $\theta = \frac{\pi}{2}$, then $\cos\left(\frac{\pi}{2} + p\right) = \cos\left(\frac{\pi}{2}\right) = 0$; so that $p = \pi$. If θ=0, then $\cos(0 + p) = \cos(0)$. But $p = \pi$. Thus, $\cos(\pi) = -1 = \cos(0) = 1$, or $-1 = 1$. This is impossible. Therefore, the smallest positive number p for which $\cos(\theta + p) = \cos\theta$ for all θ is $p = 2\pi$.

95. $f(\theta) = \sec\theta = \frac{1}{\cos\theta}$: since $\cos\theta$ has period 2π, so does $f(\theta) = \sec\theta.$

96. $f(\theta) = \csc\theta = \frac{1}{\sin\theta}$: since $\sin\theta$ has period 2π, so does $f(\theta) = \csc\theta.$

97. If $P = (a, b)$ is the point on the unit circle corresponding to θ, then $Q = (-a, -b)$ is the point on the unit circle corresponding to $\theta + \pi$. Thus, $\tan(\theta + \pi) = \frac{-b}{-a} = \frac{b}{a} = \tan\theta$. If there exists a number p, $0 < p < \pi$, for which $\tan(\theta + p) = \tan\theta$ for all θ, then if $\theta = 0$, $\tan(p) = \tan(0) = 0.$ But this means that p is a multiple of π. Since no multiple of π exists in the interval $(0, \pi)$, this is impossible. Therefore, the fundamental period of $f(\theta) = \tan\theta$ is π.

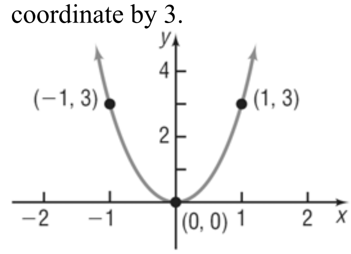
98. $f(\theta) = \cot\theta = \frac{1}{\tan\theta}$: Since $\tan\theta$ has period π, so does $f(\theta) = \cot\theta$.

99. The slope of M is $\frac{\sin\theta - 0}{\cos\theta - 0} = \frac{\sin\theta}{\cos\theta} = \tan\theta$.
Since L is parallel to M, the slope of $L = \tan\theta$.

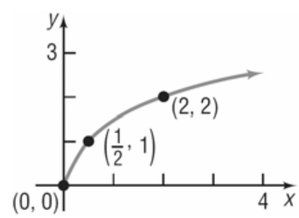
100 – 103. Answers will vary.

Section 2.6

1. $y = 3x^2$
Using the graph of $y = x^2$, vertically stretch the graph by a factor of 3. That is, multiply each y-coordinate by 3.



2. $y = \sqrt{2x}$
Using the graph of $y = \sqrt{x}$, horizontally compress the graph by a factor of $\frac{1}{2}$. That is, multiply each x-coordinate by $\frac{1}{2}$.



3. 1; $\frac{\pi}{2} + 2\pi k$, k is any integer

4. 3; π

5. 3; $\frac{2\pi}{6} = \frac{\pi}{3}$

6. True

7. False

8. True

9. The graph of $y = \sin x$ crosses the *y*-axis at the point (0, 0), so the *y*-intercept is 0.

10. The graph of $y = \cos x$ crosses the *y*-axis at the point (0, 1), so the *y*-intercept is 1.

11. The graph of $y = \sin x$ is increasing for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

12. The graph of $y = \cos x$ is decreasing for $0 < x < \pi$.

13. The largest value of $y = \sin x$ is 1.

14. The smallest value of $y = \cos x$ is -1.

15. $\sin x = 0$ when $x = 0, \pi, 2\pi$.

16. $\cos x = 0$ when $x = \frac{\pi}{2}, \frac{3\pi}{2}$.

17. $\sin x = 1$ when $x = -\frac{3\pi}{2}, \frac{\pi}{2}$;
$\sin x = -1$ when $x = -\frac{\pi}{2}, \frac{3\pi}{2}$.

18. $\cos x = 1$ when $x = -2\pi, 0, 2\pi$;
$\cos x = -1$ when $x = -\pi, \pi$.

19. $y = 2\sin x$
This is in the form $y = A\sin(\omega x)$ where $A = 2$ and $\omega = 1$. Thus, the amplitude is $|A| = |2| = 2$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 2\pi$.

20. $y = 3\cos x$
This is in the form $y = A\cos(\omega x)$ where $A = 3$ and $\omega = 1$. Thus, the amplitude is $|A| = |3| = 3$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 2\pi$.

21. $y = -4\cos(2x)$
This is in the form $y = A\cos(\omega x)$ where $A = -4$ and $\omega = 2$. Thus, the amplitude is $|A| = |-4| = 4$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$.

22. $y = -\sin\left(\frac{1}{2}x\right)$
This is in the form $y = A\sin(\omega x)$ where $A = -1$ and $\omega = \frac{1}{2}$. Thus, the amplitude is $|A| = |-1| = 1$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{1}{2}} = 4\pi$.

23. $y = 6\sin(\pi x)$
This is in the form $y = A\sin(\omega x)$ where $A = 6$ and $\omega = \pi$. Thus, the amplitude is $|A| = |6| = 6$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$.

24. $y = -3\cos(3x)$
This is in the form $y = A\cos(\omega x)$ where $A = -3$ and $\omega = 3$. Thus, the amplitude is $|A| = |-3| = 3$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{3}$.

25. $y = -\frac{1}{2}\cos\left(\frac{3}{2}x\right)$
This is in the form $y = A\cos(\omega x)$ where $A = -\frac{1}{2}$ and $\omega = \frac{3}{2}$. Thus, the amplitude is $|A| = \left|-\frac{1}{2}\right| = \frac{1}{2}$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{3}{2}} = \frac{4\pi}{3}$.

26. $y = \frac{4}{3}\sin\left(\frac{2}{3}x\right)$
This is in the form $y = A\sin(\omega x)$ where $A = \frac{4}{3}$ and $\omega = \frac{2}{3}$. Thus, the amplitude is $|A| = \left|\frac{4}{3}\right| = \frac{4}{3}$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{2}{3}} = 3\pi$.

27. $y = \frac{5}{3}\sin\left(-\frac{2\pi}{3}x\right) = -\frac{5}{3}\sin\left(\frac{2\pi}{3}x\right)$

This is in the form $y = A\sin(\omega x)$ where $A = -\frac{5}{3}$ and $\omega = \frac{2\pi}{3}$. Thus, the amplitude is $|A| = \left|-\frac{5}{3}\right| = \frac{5}{3}$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{2\pi}{3}} = 3$.

28. $y = \frac{9}{5}\cos\left(-\frac{3\pi}{2}x\right) = \frac{9}{5}\cos\left(\frac{3\pi}{2}x\right)$

This is in the form $y = A\cos(\omega x)$ where $A = \frac{9}{5}$ and $\omega = \frac{3\pi}{2}$. Thus, the amplitude is $|A| = \left|\frac{9}{5}\right| = \frac{9}{5}$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{3\pi}{2}} = \frac{4}{3}$.

29. F

30. E

31. A

32. I

33. H

34. B

35. C

36. G

37. J

38. D

39. A

40. C

41. B

42. D

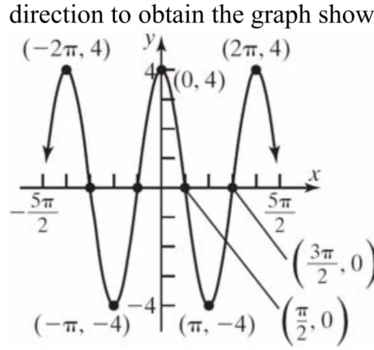
43. Comparing $y = 4\cos x$ to $y = A\cos(\omega x)$, we find $A = 4$ and $\omega = 1$. Therefore, the amplitude is $|4| = 4$ and the period is $\frac{2\pi}{1} = 2\pi$. Because the amplitude is 4, the graph of $y = 4\cos x$ will lie between -4 and 4 on the *y*-axis. Because the period is 2π, one cycle will begin at $x = 0$ and end at $x = 2\pi$. We divide the interval $[0, 2\pi]$ into four subintervals, each of length $\frac{2\pi}{4} = \frac{\pi}{2}$ by finding the following values:

$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, and 2π

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = 4\cos x$, we multiply the *y*-coordinates of the five key points for $y = \cos x$ by $A = 4$. The five key points are

$(0,4)$, $\left(\frac{\pi}{2},0\right)$, $(\pi,-4)$, $\left(\frac{3\pi}{2},0\right)$, $(2\pi,4)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -4 \le y \le 4\}$ or $[-4, 4]$.

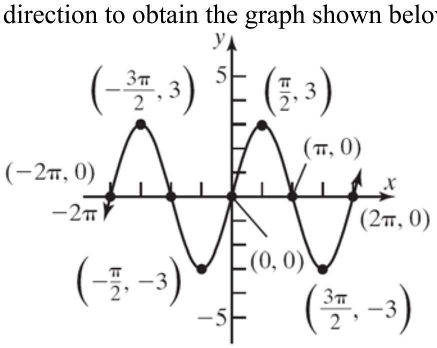
44. Comparing $y = 3\sin x$ to $y = A\sin(\omega x)$, we find $A = 3$ and $\omega = 1$. Therefore, the amplitude is $|3| = 3$ and the period is $\frac{2\pi}{1} = 2\pi$. Because the amplitude is 3, the graph of $y = 3\sin x$ will lie between -3 and 3 on the *y*-axis. Because the period is 2π, one cycle will begin at $x = 0$ and end at $x = 2\pi$. We divide the interval $[0, 2\pi]$ into four subintervals, each of length $\frac{2\pi}{4} = \frac{\pi}{2}$ by finding the following values:

0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2π

These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for $y = 3\sin x$, we multiply the y-coordinates of the five key points for $y = \sin x$ by $A = 3$. The five key points are

$(0,0)$, $\left(\frac{\pi}{2},3\right)$, $(\pi,0)$, $\left(\frac{3\pi}{2},-3\right)$, $(2\pi,0)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



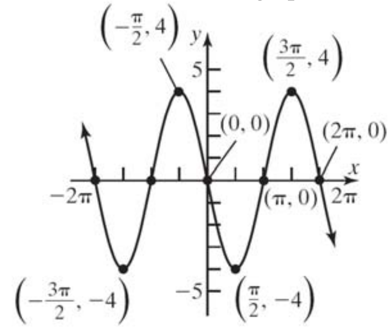
The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -3 \le y \le 3\}$ or $[-3, 3]$.

45. Comparing $y = -4\sin x$ to $y = A\sin(\omega x)$, we find $A = -4$ and $\omega = 1$. Therefore, the amplitude is $|-4| = 4$ and the period is $\frac{2\pi}{1} = 2\pi$. Because the amplitude is 4, the graph of $y = -4\sin x$ will lie between -4 and 4 on the y-axis. Because the period is 2π, one cycle will begin at $x = 0$ and end at $x = 2\pi$. We divide the interval $[0, 2\pi]$ into four subintervals, each of length $\frac{2\pi}{4} = \frac{\pi}{2}$ by finding the following values: 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, 2π

These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for $y = -4\sin x$, we multiply the y-coordinates of the five key points for $y = \sin x$ by $A = -4$. The five key points are

$(0,0)$, $\left(\frac{\pi}{2},-4\right)$, $(\pi,0)$, $\left(\frac{3\pi}{2},4\right)$, $(2\pi,0)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -4 \le y \le 4\}$ or $[-4, 4]$.

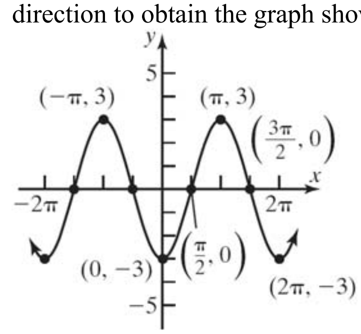
46. Comparing $y = -3\cos x$ to $y = A\cos(\omega x)$, we find $A = -3$ and $\omega = 1$. Therefore, the amplitude is $|-3| = 3$ and the period is $\frac{2\pi}{1} = 2\pi$. Because the amplitude is 3, the graph of $y = -3\cos x$ will lie between -3 and 3 on the y-axis. Because the period is 2π, one cycle will begin at $x = 0$ and end at $x = 2\pi$. We divide the interval $[0, 2\pi]$ into four subintervals, each of length $\frac{2\pi}{4} = \frac{\pi}{2}$ by finding the following values:

0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2π

These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for $y = -3\cos x$, we multiply the y-coordinates of the five key points for $y = \cos x$ by $A = -3$. The five key points are

$(0,-3)$, $\left(\frac{\pi}{2},0\right)$, $(\pi,3)$, $\left(\frac{3\pi}{2},0\right)$, $(2\pi,-3)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -3 \le y \le 3\}$ or $[-3, 3]$.

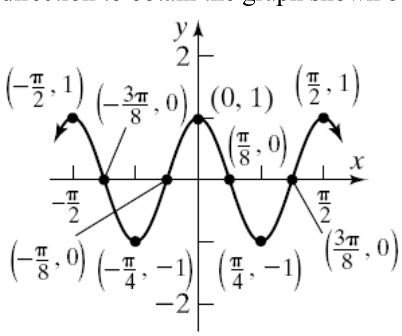
47. Comparing $y = \cos(4x)$ to $y = A\cos(\omega x)$, we find $A = 1$ and $\omega = 4$. Therefore, the amplitude is $|1| = 1$ and the period is $\frac{2\pi}{4} = \frac{\pi}{2}$. Because the amplitude is 1, the graph of $y = \cos(4x)$ will lie between -1 and 1 on the *y*-axis. Because the period is $\frac{\pi}{2}$, one cycle will begin at $x = 0$ and end at $x = \frac{\pi}{2}$. We divide the interval $\left[0, \frac{\pi}{2}\right]$ into four subintervals, each of length $\frac{\pi/2}{4} = \frac{\pi}{8}$ by finding the following values:

0, $\frac{\pi}{8}$, $\frac{\pi}{4}$, $\frac{3\pi}{8}$, and $\frac{\pi}{2}$

These values of *x* determine the *x*-coordinates of the five key points on the graph. The five key points are

$(0,1)$, $\left(\frac{\pi}{8},0\right)$, $\left(\frac{\pi}{4},-1\right)$, $\left(\frac{3\pi}{8},0\right)$, $\left(\frac{\pi}{2},1\right)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -1 \le y \le 1\}$ or $[-1, 1]$.

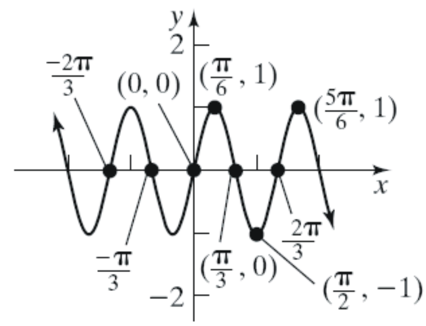
48. Comparing $y = \sin(3x)$ to $y = A\sin(\omega x)$, we find $A = 1$ and $\omega = 3$. Therefore, the amplitude is $|1| = 1$ and the period is $\frac{2\pi}{3}$. Because the amplitude is 1, the graph of $y = \sin(3x)$ will lie between -1 and 1 on the *y*-axis. Because the period is $\frac{2\pi}{3}$, one cycle will begin at $x = 0$ and end at $x = \frac{2\pi}{3}$. We divide the interval $\left[0, \frac{2\pi}{3}\right]$ into four subintervals, each of length $\frac{2\pi/3}{4} = \frac{\pi}{6}$ by finding the following values:

0, $\frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$, and $\frac{2\pi}{3}$

These values of *x* determine the *x*-coordinates of the five key points on the graph. The five key points are

$(0,0)$, $\left(\frac{\pi}{6},1\right)$, $\left(\frac{\pi}{3},0\right)$, $\left(\frac{\pi}{2},-1\right)$, $\left(\frac{2\pi}{3},0\right)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -1 \le y \le 1\}$ or $[-1, 1]$.

49. Since sine is an odd function, we can plot the equivalent form $y = -\sin(2x)$.

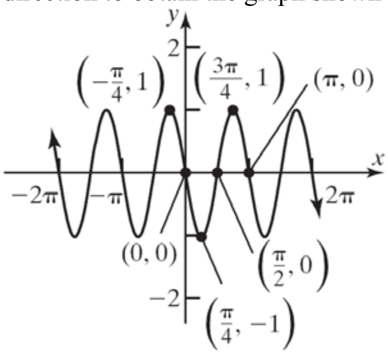
Comparing $y = -\sin(2x)$ to $y = A\sin(\omega x)$, we find $A = -1$ and $\omega = 2$. Therefore, the amplitude is $|-1| = 1$ and the period is $\frac{2\pi}{2} = \pi$. Because the amplitude is 1, the graph of $y = -\sin(2x)$ will lie between -1 and 1 on the *y*-axis. Because the period is π, one cycle will begin at $x = 0$ and end at $x = \pi$. We divide the interval $[0, \pi]$ into four subintervals, each of length $\frac{\pi}{4}$ by finding the following values:

0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, and π

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = -\sin(2x)$, we multiply the *y*-coordinates of the five key points for $y = \sin x$ by $A = -1$. The five key points are

$(0,0)$, $\left(\frac{\pi}{4},-1\right)$, $\left(\frac{\pi}{2},0\right)$, $\left(\frac{3\pi}{4},1\right)$, $(\pi,0)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -1 \le y \le 1\}$ or $[-1, 1]$.

50. Since cosine is an even function, we can plot the equivalent form $y = \cos(2x)$.

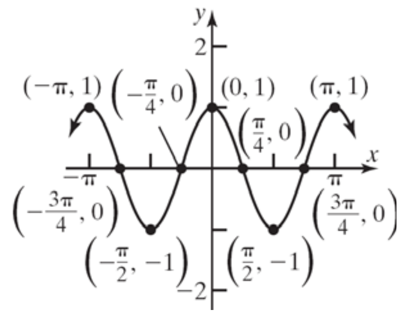
Comparing $y = \cos(2x)$ to $y = A\cos(\omega x)$, we find $A = 1$ and $\omega = 2$. Therefore, the amplitude is $|1| = 1$ and the period is $\frac{2\pi}{2} = \pi$. Because the amplitude is 1, the graph of $y = \cos(2x)$ will lie between -1 and 1 on the y-axis. Because the period is π, one cycle will begin at $x = 0$ and end at $x = \pi$. We divide the interval $[0, \pi]$ into four subintervals, each of length $\frac{\pi}{4}$ by finding the following values:

0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, and π

These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for $y = \cos(2x)$, we multiply the y-coordinates of the five key points for $y = \cos x$ by $A = 1$.The five key points are

$(0,1)$, $\left(\frac{\pi}{4},0\right)$, $\left(\frac{\pi}{2},-1\right)$, $\left(\frac{3\pi}{4},0\right)$, $(\pi,1)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -1 \le y \le 1\}$ or $[-1, 1]$.

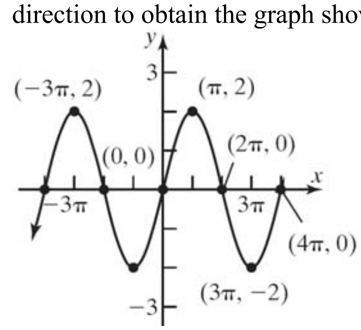
51. Comparing $y = 2\sin\left(\frac{1}{2}x\right)$ to $y = A\sin(\omega x)$, we find $A = 2$ and $\omega = \frac{1}{2}$. Therefore, the amplitude is $|2| = 2$ and the period is $\frac{2\pi}{1/2} = 4\pi$. Because the amplitude is 2, the graph of $y = 2\sin\left(\frac{1}{2}x\right)$ will lie between -2 and 2 on the y-axis. Because the period is 4π, one cycle will begin at $x = 0$ and end at $x = 4\pi$. We divide the interval $[0, 4\pi]$ into four subintervals, each of length $\frac{4\pi}{4} = \pi$ by finding the following values:

0, π, 2π, 3π, and 4π

These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for $y = 2\sin\left(\frac{1}{2}x\right)$, we multiply the y-coordinates of the five key points for $y = \sin x$ by $A = 2$. The five key points are

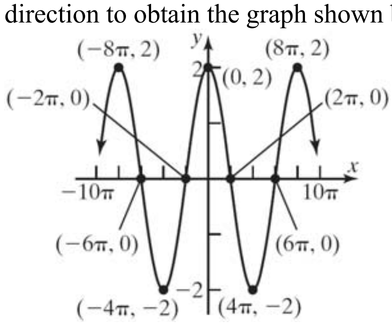
$(0,0)$, $(\pi,2)$, $(2\pi,0)$, $(3\pi,-2)$, $(4\pi,0)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



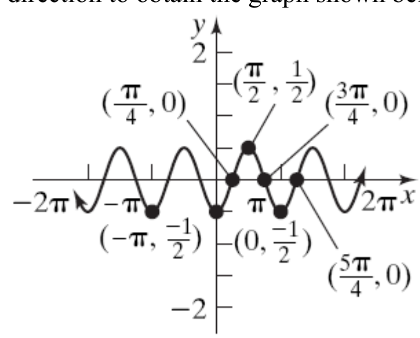
The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -2 \le y \le 2\}$ or $[-2, 2]$.

52. Comparing $y = 2\cos\left(\frac{1}{4}x\right)$ to $y = A\cos(\omega x)$, we find $A = 2$ and $\omega = \frac{1}{4}$. Therefore, the amplitude is $|2| = 2$ and the period is $\frac{2\pi}{1/4} = 8\pi$. Because the amplitude is 2, the graph of $y = 2\cos\left(\frac{1}{4}x\right)$ will lie between -2 and 2 on the *y*-axis. Because the period is 8π, one cycle will begin at $x = 0$ and end at $x = 8\pi$. We divide the interval $[0, 8\pi]$ into four subintervals, each of length $\frac{8\pi}{4} = 2\pi$ by finding the following values:
0, 2π, 4π, 6π, and 8π
These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = 2\cos\left(\frac{1}{4}x\right)$, we multiply the *y*-coordinates of the five key points for $y = \cos x$ by $A = 2$. The five key points are
$(0, 2)$, $(2\pi, 0)$, $(4\pi, -2)$, $(6\pi, 0)$, $(8\pi, 2)$
We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -2 \le y \le 2\}$ or $[-2, 2]$.

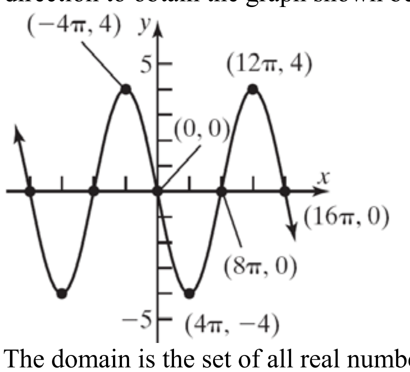
53. Comparing $y = -\frac{1}{2}\cos(2x)$ to $y = A\cos(\omega x)$, we find $A = -\frac{1}{2}$ and $\omega = 2$. Therefore, the amplitude is $\left|-\frac{1}{2}\right| = \frac{1}{2}$ and the period is $\frac{2\pi}{2} = \pi$. Because the amplitude is $\frac{1}{2}$, the graph of $y = -\frac{1}{2}\cos(2x)$ will lie between $-\frac{1}{2}$ and $\frac{1}{2}$ on the *y*-axis. Because the period is π, one cycle will begin at $x = 0$ and end at $x = \pi$. We divide the interval $[0, \pi]$ into four subintervals, each of length $\frac{\pi}{4}$ by finding the following values:
0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, and π
These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = -\frac{1}{2}\cos(2x)$, we multiply the *y*-coordinates of the five key points for $y = \cos x$ by $A = -\frac{1}{2}$. The five key points are
$\left(0, -\frac{1}{2}\right)$, $\left(\frac{\pi}{4}, 0\right)$, $\left(\frac{\pi}{2}, \frac{1}{2}\right)$, $\left(\frac{3\pi}{4}, 0\right)$, $\left(\pi, -\frac{1}{2}\right)$
We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\left\{y \mid -\frac{1}{2} \le y \le \frac{1}{2}\right\}$ or $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

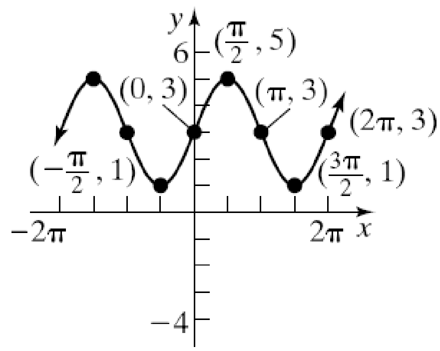
54. Comparing $y = -4\sin\left(\frac{1}{8}x\right)$ to $y = A\sin(\omega x)$, we find $A = -4$ and $\omega = \frac{1}{8}$. Therefore, the amplitude is $|-4| = 4$ and the period is $\frac{2\pi}{1/8} = 16\pi$. Because the amplitude is 4, the graph of $y = -4\sin\left(\frac{1}{8}x\right)$ will lie between -4

and 4 on the *y*-axis. Because the period is 16π, one cycle will begin at $x = 0$ and end at $x = 16\pi$. We divide the interval $[0, 16\pi]$ into four subintervals, each of length $\frac{16\pi}{4} = 4\pi$ by finding the following values:
$0,\ 4\pi,\ 8\pi,\ 12\pi$, and 16π
These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = -4\sin\left(\frac{1}{8}x\right)$, we multiply the *y*-coordinates of the five key points for $y = \sin x$ by $A = -4$. The five key points are
$(0,0)$, $(4\pi,-4)$, $(8\pi,0)$, $(12\pi,4)$, $(16\pi,0)$
We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -4 \le y \le 4\}$ or $[-4, 4]$.

55. We begin by considering $y = 2\sin x$. Comparing $y = 2\sin x$ to $y = A\sin(\omega x)$, we find $A = 2$ and $\omega = 1$. Therefore, the amplitude is $|2| = 2$ and the period is $\frac{2\pi}{1} = 2\pi$. Because the amplitude is 2, the graph of $y = 2\sin x$ will lie between -2 and 2 on the *y*-axis. Because the period is 2π, one cycle will begin at $x = 0$ and end at $x = 2\pi$. We divide the interval $[0, 2\pi]$ into four subintervals, each of length $\frac{2\pi}{4} = \frac{\pi}{2}$ by finding the following values:
$0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2}$, and 2π
These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = 2\sin x + 3$, we multiply the *y*-coordinates of the five key points for $y = \sin x$ by $A = 2$ and then add 3 units. Thus, the graph of $y = 2\sin x + 3$ will lie between 1 and 5 on the *y*-axis. The five key points are
$(0,3)$, $\left(\frac{\pi}{2},5\right)$, $(\pi,3)$, $\left(\frac{3\pi}{2},1\right)$, $(2\pi,3)$
We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.

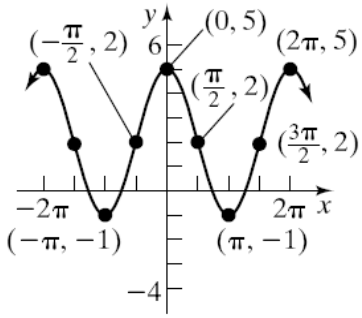


The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid 1 \le y \le 5\}$ or $[1, 5]$.

56. We begin by considering $y = 3\cos x$. Comparing $y = 3\cos x$ to $y = A\cos(\omega x)$, we find $A = 3$ and $\omega = 1$. Therefore, the amplitude is $|3| = 3$ and the period is $\frac{2\pi}{1} = 2\pi$. Because the amplitude is 3, the graph of $y = 3\cos x$ will lie between -3 and 3 on the *y*-axis. Because the period is 2π, one cycle will begin at $x = 0$ and end at $x = 2\pi$. We divide the interval $[0, 2\pi]$ into four subintervals, each of length $\frac{2\pi}{4} = \frac{\pi}{2}$ by finding the following values:
$0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2}$, and 2π
These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = 3\cos x + 2$, we multiply the *y*-coordinates of the five key points for $y = \cos x$ by $A = 3$ and then add 2 units. Thus, the graph of $y = 3\cos x + 2$ will lie between -1 and 5 on the *y*-axis. The five key points are

$(0,5)$, $\left(\frac{\pi}{2},2\right)$, $(\pi,-1)$, $\left(\frac{3\pi}{2},2\right)$, $(2\pi,5)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -1 \le y \le 5\}$ or $[-1, 5]$.

57. We begin by considering $y = 5\cos(\pi x)$.

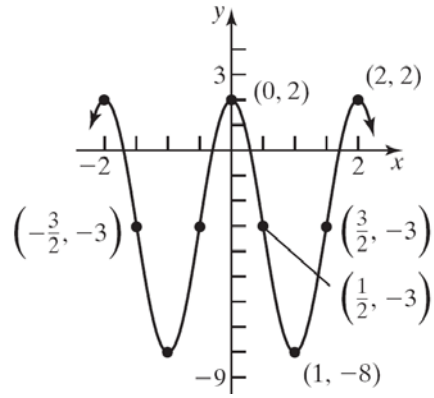
Comparing $y = 5\cos(\pi x)$ to $y = A\cos(\omega x)$, we find $A = 5$ and $\omega = \pi$. Therefore, the amplitude is $|5| = 5$ and the period is $\frac{2\pi}{\pi} = 2$. Because the amplitude is 5, the graph of $y = 5\cos(\pi x)$ will lie between -5 and 5 on the *y*-axis. Because the period is 2, one cycle will begin at $x = 0$ and end at $x = 2$. We divide the interval $[0,2]$ into four subintervals, each of length $\frac{2}{4} = \frac{1}{2}$ by finding the following values:

0, $\frac{1}{2}$, 1, $\frac{3}{2}$, and 2

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = 5\cos(\pi x) - 3$, we multiply the *y*-coordinates of the five key points for $y = \cos x$ by $A = 5$ and then subtract 3 units. Thus, the graph of $y = 5\cos(\pi x) - 3$ will lie between -8 and 2 on the *y*-axis. The five key points are

$(0,2)$, $\left(\frac{1}{2},-3\right)$, $(1,-8)$, $\left(\frac{3}{2},-3\right)$, $(2,2)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -8 \le y \le 2\}$ or $[-8, 2]$.

58. We begin by considering $y = 4\sin\left(\frac{\pi}{2}x\right)$.

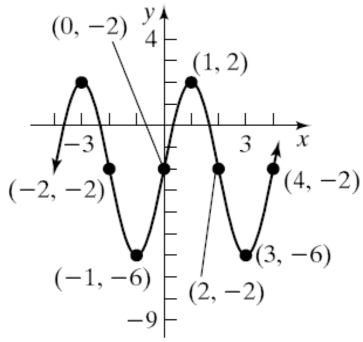
Comparing $y = 4\sin\left(\frac{\pi}{2}x\right)$ to $y = A\sin(\omega x)$, we find $A = 4$ and $\omega = \frac{\pi}{2}$. Therefore, the amplitude is $|4| = 4$ and the period is $\frac{2\pi}{\pi/2} = 4$. Because the amplitude is 4, the graph of $y = 4\sin\left(\frac{\pi}{2}x\right)$ will lie between -4 and 4 on the *y*-axis. Because the period is 4, one cycle will begin at $x = 0$ and end at $x = 4$. We divide the interval $[0,4]$ into four subintervals, each of length $\frac{4}{4} = 1$ by finding the following values:

0, 1, 2, 3, and 4

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = 4\sin\left(\frac{\pi}{2}x\right) - 2$, we multiply the *y*-coordinates of the five key points for $y = \sin x$ by $A = 4$ and then subtract 2 units. Thus, the graph of $y = 4\sin\left(\frac{\pi}{2}x\right) - 2$ will lie between -6 and 2 on the *y*-axis. The five key points are

$(0,-2)$, $(1,2)$, $(2,-2)$, $(3,-6)$, $(4,-2)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -6 \le y \le 2\}$ or $[-6, 2]$.

59. We begin by considering $y = -6\sin\left(\frac{\pi}{3}x\right)$.

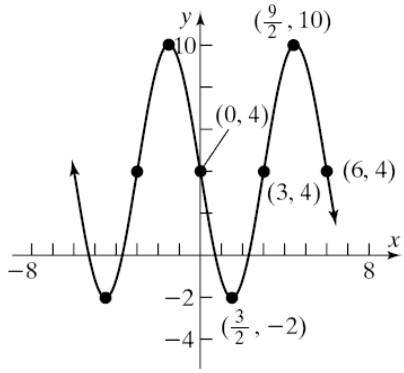
Comparing $y = -6\sin\left(\frac{\pi}{3}x\right)$ to $y = A\sin(\omega x)$, we find $A = -6$ and $\omega = \frac{\pi}{3}$. Therefore, the amplitude is $|-6| = 6$ and the period is $\frac{2\pi}{\pi/3} = 6$. Because the amplitude is 6, the graph of $y = 6\sin\left(\frac{\pi}{3}x\right)$ will lie between -6 and 6 on the *y*-axis. Because the period is 6, one cycle will begin at $x = 0$ and end at $x = 6$. We divide the interval $[0,6]$ into four subintervals, each of length $\frac{6}{4} = \frac{3}{2}$ by finding the following values:

0, $\frac{3}{2}$, 3, $\frac{9}{2}$, and 6

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = -6\sin\left(\frac{\pi}{3}x\right) + 4$, we multiply the *y*-coordinates of the five key points for $y = \sin x$ by $A = -6$ and then add 4 units. Thus, the graph of $y = -6\sin\left(\frac{\pi}{3}x\right) + 4$ will lie between -2 and 10 on the *y*-axis. The five key points are

$(0,4)$, $\left(\frac{3}{2},-2\right)$, $(3,4)$, $\left(\frac{9}{2},10\right)$, $(6,4)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -2 \le y \le 10\}$ or $[-2, 10]$.

60. We begin by considering $y = -3\cos\left(\frac{\pi}{4}x\right)$.

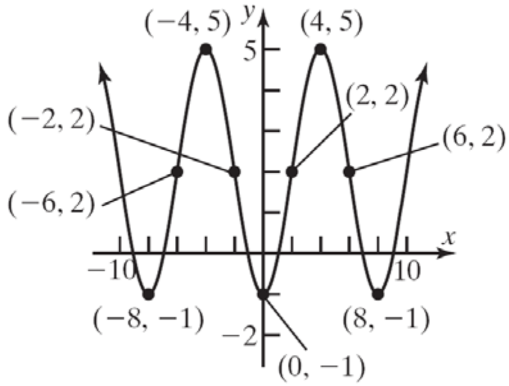
Comparing $y = -3\cos\left(\frac{\pi}{4}x\right)$ to $y = A\cos(\omega x)$, we find $A = -3$ and $\omega = \frac{\pi}{4}$. Therefore, the amplitude is $|-3| = 3$ and the period is $\frac{2\pi}{\pi/4} = 8$. Because the amplitude is 3, the graph of $y = -3\cos\left(\frac{\pi}{4}x\right)$ will lie between -3 and 3 on the *y*-axis. Because the period is 8, one cycle will begin at $x = 0$ and end at $x = 8$. We divide the interval $[0,8]$ into four subintervals, each of length $\frac{8}{4} = 2$ by finding the following values:

0, 2, 4, 6, and 8

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = -3\cos\left(\frac{\pi}{4}x\right) + 2$, we multiply the *y*-coordinates of the five key points for $y = \cos x$ by $A = -3$ and then add 2 units. Thus, the graph of $y = -3\cos\left(\frac{\pi}{4}x\right) + 2$ will lie between -1 and 5 on the *y*-axis. The five key points are

$(0,-1)$, $(2,2)$, $(4,5)$, $(6,2)$, $(8,-1)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -1 \le y \le 5\}$ or $[-1, 5]$.

61. $y = 5 - 3\sin(2x) = -3\sin(2x) + 5$

We begin by considering $y = -3\sin(2x)$.

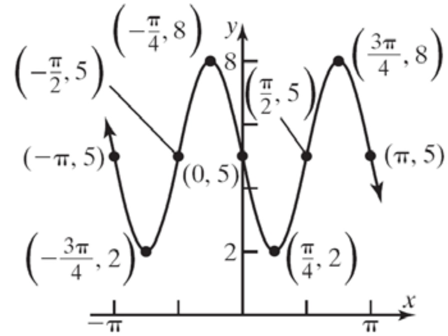
Comparing $y = -3\sin(2x)$ to $y = A\sin(\omega x)$, we find $A = -3$ and $\omega = 2$. Therefore, the amplitude is $|-3| = 3$ and the period is $\frac{2\pi}{2} = \pi$. Because the amplitude is 3, the graph of $y = -3\sin(2x)$ will lie between -3 and 3 on the *y*-axis. Because the period is π, one cycle will begin at $x = 0$ and end at $x = \pi$. We divide the interval $[0, \pi]$ into four subintervals, each of length $\frac{\pi}{4}$ by finding the following values:

$0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$, and π

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = -3\sin(2x) + 5$, we multiply the *y*-coordinates of the five key points for $y = \sin x$ by $A = -3$ and then add 5 units. Thus, the graph of $y = -3\sin(2x) + 5$ will lie between 2 and 8 on the *y*-axis. The five key points are

$(0,5)$, $\left(\frac{\pi}{4},2\right)$, $\left(\frac{\pi}{2},5\right)$, $\left(\frac{3\pi}{4},8\right)$, $(\pi,5)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid 2 \le y \le 8\}$ or $[2, 8]$.

62. $y = 2 - 4\cos(3x) = -4\cos(3x) + 2$

We begin by considering $y = -4\cos(3x)$.

Comparing $y = -4\cos(3x)$ to $y = A\cos(\omega x)$, we find $A = -4$ and $\omega = 3$. Therefore, the amplitude is $|-4| = 4$ and the period is $\frac{2\pi}{3}$.

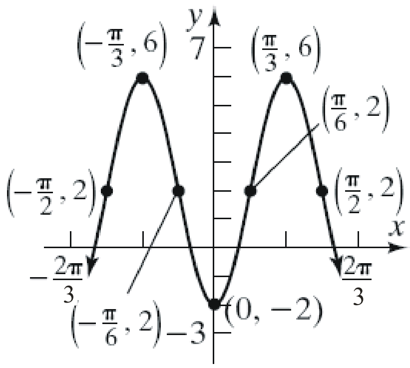
Because the amplitude is 4, the graph of $y = -4\cos(3x)$ will lie between -4 and 4 on the *y*-axis. Because the period is $\frac{2\pi}{3}$, one cycle will begin at $x = 0$ and end at $x = \frac{2\pi}{3}$. We divide the interval $\left[0, \frac{2\pi}{3}\right]$ into four subintervals, each of length $\frac{2\pi/3}{4} = \frac{\pi}{6}$ by finding the following values:

$0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$, and $\frac{2\pi}{3}$

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = -4\cos(3x) + 2$, we multiply the *y*-coordinates of the five key points for $y = \cos x$ by $A = -4$ and then adding 2 units. Thus, the graph of $y = -4\cos(3x) + 2$ will lie between -2 and 6 on the *y*-axis. The five key points are

$(0,-2)$, $\left(\frac{\pi}{6},2\right)$, $\left(\frac{\pi}{3},6\right)$, $\left(\frac{\pi}{2},2\right)$, $\left(\frac{2\pi}{3},-2\right)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -2 \le y \le 6\}$ or $[-2, 6]$.

63. Since sine is an odd function, we can plot the equivalent form $y = -\frac{5}{3}\sin\left(\frac{2\pi}{3}x\right)$.

Comparing $y = -\frac{5}{3}\sin\left(\frac{2\pi}{3}x\right)$ to $y = A\sin(\omega x)$, we find $A = -\frac{5}{3}$ and $\omega = \frac{2\pi}{3}$.

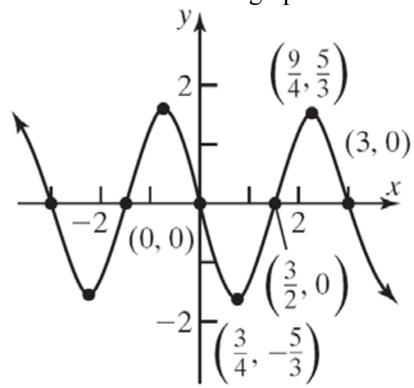
Therefore, the amplitude is $\left|-\frac{5}{3}\right| = \frac{5}{3}$ and the period is $\frac{2\pi}{2\pi/3} = 3$. Because the amplitude is $\frac{5}{3}$, the graph of $y = -\frac{5}{3}\sin\left(\frac{2\pi}{3}x\right)$ will lie between $-\frac{5}{3}$ and $\frac{5}{3}$ on the *y*-axis. Because the period is 3, one cycle will begin at $x = 0$ and end at $x = 3$. We divide the interval $[0, 3]$ into four subintervals, each of length $\frac{3}{4}$ by finding the following values:

0, $\frac{3}{4}$, $\frac{3}{2}$, $\frac{9}{4}$, and 3

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = -\frac{5}{3}\sin\left(\frac{2\pi}{3}x\right)$, we multiply the *y*-coordinates of the five key points for $y = \sin x$ by $A = -\frac{5}{3}$. The five key points are

$(0,0)$, $\left(\frac{3}{4}, -\frac{5}{3}\right)$, $\left(\frac{3}{2}, 0\right)$, $\left(\frac{9}{4}, \frac{5}{3}\right)$, $(3,0)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\left\{y \mid -\frac{5}{3} \le y \le \frac{5}{3}\right\}$ or $\left[-\frac{5}{3}, \frac{5}{3}\right]$.

64. Since cosine is an even function, we consider the equivalent form $y = \frac{9}{5}\cos\left(\frac{3\pi}{2}x\right)$. Comparing $y = \frac{9}{5}\cos\left(\frac{3\pi}{2}x\right)$ to $y = A\cos(\omega x)$, we find $A = \frac{9}{5}$ and $\omega = \frac{3\pi}{2}$. Therefore, the amplitude is $\left|\frac{9}{5}\right| = \frac{9}{5}$ and the period is $\frac{2\pi}{3\pi/2} = \frac{4}{3}$. Because the amplitude is $\frac{9}{5}$, the graph of $y = \frac{9}{5}\cos\left(\frac{3\pi}{2}x\right)$ will lie between $-\frac{9}{5}$ and $\frac{9}{5}$ on the *y*-axis. Because the period is $\frac{4}{3}$, one cycle will begin at $x = 0$ and end at $x = \frac{4}{3}$. We divide the interval $\left[0, \frac{4}{3}\right]$ into four subintervals, each of length $\frac{4/3}{4} = \frac{1}{3}$ by finding the following values:

0, $\frac{1}{3}$, $\frac{2}{3}$, 1, and $\frac{4}{3}$

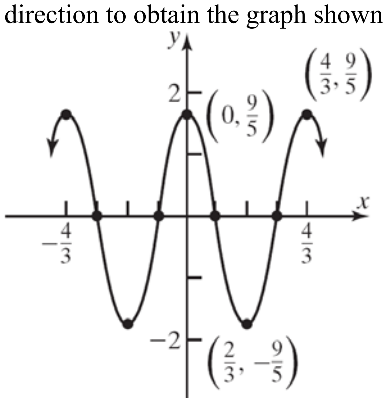
These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-

coordinates of the five key points for $y=\frac{9}{5}\cos\left(\frac{3\pi}{2}x\right)$, we multiply the y-coordinates of the five key points for $y=\cos x$ by $A=\frac{9}{5}$.

Thus, the graph of $y=\frac{9}{5}\cos\left(-\frac{3\pi}{2}x\right)$ will lie between $-\frac{9}{5}$ and $\frac{9}{5}$ on the y-axis. The five key points are

$\left(0,\frac{9}{5}\right)$, $\left(\frac{1}{3},0\right)$, $\left(\frac{2}{3},-\frac{9}{5}\right)$, $(1,0)$, $\left(\frac{4}{3},\frac{9}{5}\right)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



The domain is the set of all real number or $(-\infty,\infty)$. The range is $\left\{y\,|\,-\frac{9}{5}\le y\le\frac{9}{5}\right\}$ or $\left[-\frac{9}{5},\frac{9}{5}\right]$.

65. We begin by considering $y=-\frac{3}{2}\cos\left(\frac{\pi}{4}x\right)$.

Comparing $y=-\frac{3}{2}\cos\left(\frac{\pi}{4}x\right)$ to $y=A\cos(\omega x)$, we find $A=-\frac{3}{2}$ and $\omega=\frac{\pi}{4}$.

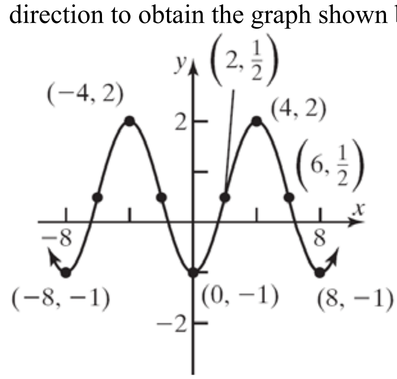
Therefore, the amplitude is $\left|-\frac{3}{2}\right|=\frac{3}{2}$ and the period is $\frac{2\pi}{\pi/4}=8$. Because the amplitude is $\frac{3}{2}$, the graph of $y=-\frac{3}{2}\cos\left(\frac{\pi}{4}x\right)$ will lie between $-\frac{3}{2}$ and $\frac{3}{2}$ on the y-axis. Because the period is 8, one cycle will begin at $x=0$ and end at $x=8$. We divide the interval $[0,8]$ into four subintervals, each of length $\frac{8}{4}=2$ by finding the following values:

0, 2, 4, 6, and 8

These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for $y=-\frac{3}{2}\cos\left(\frac{\pi}{4}x\right)+\frac{1}{2}$, we multiply the y-coordinates of the five key points for $y=\cos x$ by $A=-\frac{3}{2}$ and then add $\frac{1}{2}$ unit. Thus, the graph of $y=-\frac{3}{2}\cos\left(\frac{\pi}{4}x\right)+\frac{1}{2}$ will lie between -1 and 2 on the y-axis. The five key points are

$(0,-1)$, $\left(2,\frac{1}{2}\right)$, $(4,2)$, $\left(6,\frac{1}{2}\right)$, $(8,-1)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.

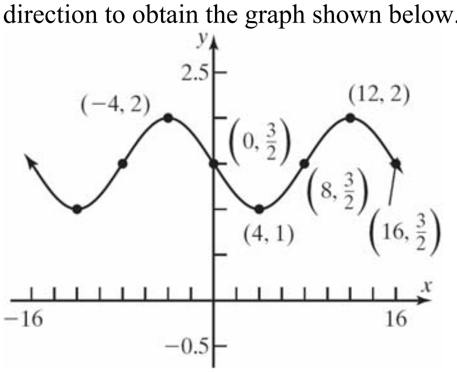


The domain is the set of all real number or $(-\infty,\infty)$. The range is $\{y\,|\,-1\le y\le 2\}$ or $[-1,2]$.

66. We begin by considering $y=-\frac{1}{2}\sin\left(\frac{\pi}{8}x\right)$.

Comparing $y=-\frac{1}{2}\sin\left(\frac{\pi}{8}x\right)$ to $y=A\sin(\omega x)$, we find $A=-\frac{1}{2}$ and $\omega=\frac{\pi}{8}$. Therefore, the amplitude is $\left|-\frac{1}{2}\right|=\frac{1}{2}$ and the period is $\frac{2\pi}{\pi/8}=16$. Because the amplitude is $\frac{1}{2}$, the

graph of $y=-\frac{1}{2}\sin\left(\frac{\pi}{8}x\right)$ will lie between $-\frac{1}{2}$ and $\frac{1}{2}$ on the *y*-axis. Because the period is 16, one cycle will begin at $x=0$ and end at $x=16$. We divide the interval $[0,16]$ into four subintervals, each of length $\frac{16}{4}=4$ by finding the following values:
0, 4, 8, 12, and 16
These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y=-\frac{1}{2}\sin\left(\frac{\pi}{8}x\right)+\frac{3}{2}$, we multiply the *y*-coordinates of the five key points for $y=\sin x$ by $A=-\frac{1}{2}$ and then add $\frac{3}{2}$ units. Thus, the graph of $y=-\frac{1}{2}\sin\left(\frac{\pi}{8}x\right)+\frac{3}{2}$ will lie between 1 and 2 on the *y*-axis. The five key points are
$\left(0,\frac{3}{2}\right)$, $(4,1)$, $\left(8,\frac{3}{2}\right)$, $(12,2)$, $\left(16,\frac{3}{2}\right)$
We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



The domain is the set of all real number or $(-\infty,\infty)$. The range is $\{y \mid 1\le y\le 2\}$ or $[1, 2]$.

67. $|A|=3; T=\pi; \omega=\frac{2\pi}{T}=\frac{2\pi}{\pi}=2$

$y=\pm 3\sin(2x)$

68. $|A|=2; T=4\pi; \omega=\frac{2\pi}{T}=\frac{2\pi}{4\pi}=\frac{1}{2}$

$y=\pm 2\sin\left(\frac{1}{2}x\right)$

69. $|A|=3; T=2; \omega=\frac{2\pi}{T}=\frac{2\pi}{2}=\pi$

$y=\pm 3\sin(\pi x)$

70. $|A|=4;\ \ T=1; \omega=\frac{2\pi}{T}=\frac{2\pi}{1}=2\pi$

$y=\pm 4\sin(2\pi x)$

71. The graph is a cosine graph with amplitude 5 and period 8.

Find ω: $8=\frac{2\pi}{\omega}$

$8\omega=2\pi$

$\omega=\frac{2\pi}{8}=\frac{\pi}{4}$

The equation is: $y=5\cos\left(\frac{\pi}{4}x\right)$.

72. The graph is a sine graph with amplitude 4 and period 8π.

Find ω: $8\pi=\frac{2\pi}{\omega}$

$8\pi\omega=2\pi$

$\omega=\frac{2\pi}{8\pi}=\frac{1}{4}$

The equation is: $y=4\sin\left(\frac{1}{4}x\right)$.

73. The graph is a reflected cosine graph with amplitude 3 and period 4π.

Find ω: $4\pi=\frac{2\pi}{\omega}$

$4\pi\omega=2\pi$

$\omega=\frac{2\pi}{4\pi}=\frac{1}{2}$

The equation is: $y=-3\cos\left(\frac{1}{2}x\right)$.

74. The graph is a reflected sine graph with amplitude 2 and period 4.

Find ω: $4=\frac{2\pi}{\omega}$

$4\omega=2\pi$

$\omega=\frac{2\pi}{4}=\frac{\pi}{2}$

The equation is: $y=-2\sin\left(\frac{\pi}{2}x\right)$.

75. The graph is a sine graph with amplitude $\frac{3}{4}$ and period 1.

Find ω: $1 = \frac{2\pi}{\omega}$

$\omega = 2\pi$

The equation is: $y = \frac{3}{4}\sin(2\pi x)$.

76. The graph is a reflected cosine graph with amplitude $\frac{5}{2}$ and period 2.

Find ω: $2 = \frac{2\pi}{\omega}$

$2\omega = 2\pi$

$\omega = \frac{2\pi}{2} = \pi$

The equation is: $y = -\frac{5}{2}\cos(\pi x)$.

77. The graph is a reflected sine graph with amplitude 1 and period $\frac{4\pi}{3}$.

Find ω: $\frac{4\pi}{3} = \frac{2\pi}{\omega}$

$4\pi\omega = 6\pi$

$\omega = \frac{6\pi}{4\pi} = \frac{3}{2}$

The equation is: $y = -\sin\left(\frac{3}{2}x\right)$.

78. The graph is a reflected cosine graph with amplitude π and period 2π.

Find ω: $2\pi = \frac{2\pi}{\omega}$

$2\pi\omega = 2\pi$

$\omega = \frac{2\pi}{2\pi} = 1$

The equation is: $y = -\pi\cos x$.

79. The graph is a reflected cosine graph, shifted up 1 unit, with amplitude 1 and period $\frac{3}{2}$.

Find ω: $\frac{3}{2} = \frac{2\pi}{\omega}$

$3\omega = 4\pi$

$\omega = \frac{4\pi}{3}$

The equation is: $y = -\cos\left(\frac{4\pi}{3}x\right) + 1$.

80. The graph is a reflected sine graph, shifted down 1 unit, with amplitude $\frac{1}{2}$ and period $\frac{4\pi}{3}$.

Find ω: $\frac{4\pi}{3} = \frac{2\pi}{\omega}$

$4\pi\omega = 6\pi$

$\omega = \frac{6\pi}{4\pi} = \frac{3}{2}$

The equation is: $y = -\frac{1}{2}\sin\left(\frac{3}{2}x\right) - 1$.

81. The graph is a sine graph with amplitude 3 and period 4.

Find ω: $4 = \frac{2\pi}{\omega}$

$4\omega = 2\pi$

$\omega = \frac{2\pi}{4} = \frac{\pi}{2}$

The equation is: $y = 3\sin\left(\frac{\pi}{2}x\right)$.

82. The graph is a reflected cosine graph with amplitude 2 and period 2.

Find ω: $2 = \frac{2\pi}{\omega}$

$2\omega = 2\pi$

$\omega = \frac{2\pi}{2} = \pi$

The equation is: $y = -2\cos(\pi x)$.

83. The graph is a reflected cosine graph with amplitude 4 and period $\frac{2\pi}{3}$.

Find ω: $\frac{2\pi}{3} = \frac{2\pi}{\omega}$

$$2\pi\omega = 6\pi$$

$$\omega = \frac{6\pi}{2\pi} = 3$$

The equation is: $y = -4\cos(3x)$.

84. The graph is a sine graph with amplitude 4 and period π.

Find ω: $\pi = \frac{2\pi}{\omega}$

$$\pi\omega = 2\pi$$

$$\omega = \frac{2\pi}{\pi} = 2$$

The equation is: $y = 4\sin(2x)$.

85. $$\frac{f(\pi/2) - f(0)}{\pi/2 - 0} = \frac{\sin(\pi/2) - \sin(0)}{\pi/2} = \frac{1-0}{\pi/2} = \frac{2}{\pi}$$

The average rate of change is $\frac{2}{\pi}$.

86. $$\frac{f(\pi/2) - f(0)}{\pi/2 - 0} = \frac{\cos(\pi/2) - \cos(0)}{\pi/2} = \frac{0-1}{\pi/2} = -\frac{2}{\pi}$$

The average rate of change is $-\frac{2}{\pi}$.

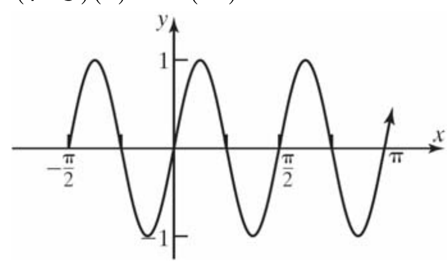
87. $$\frac{f(\pi/2) - f(0)}{\pi/2 - 0} = \frac{\sin\left(\frac{1}{2}\cdot\frac{\pi}{2}\right) - \sin\left(\frac{1}{2}\cdot 0\right)}{\pi/2} = \frac{\sin(\pi/4) - \sin(0)}{\pi/2} = \frac{\frac{\sqrt{2}}{2}}{\pi/2} = \frac{\sqrt{2}}{2}\cdot\frac{2}{\pi} = \frac{\sqrt{2}}{\pi}$$

The average rate of change is $\frac{\sqrt{2}}{\pi}$.

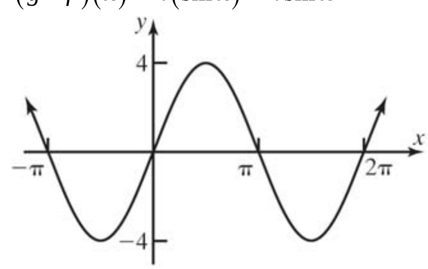
88. $$\frac{f(\pi/2) - f(0)}{\pi/2 - 0} = \frac{\cos\left(2\cdot\frac{\pi}{2}\right) - \cos(2\cdot 0)}{\pi/2} = \frac{\cos(\pi) - \cos(0)}{\pi/2} = \frac{-1-1}{\pi/2} = -2\cdot\frac{2}{\pi} = -\frac{4}{\pi}$$

The average rate of change is $-\frac{4}{\pi}$.

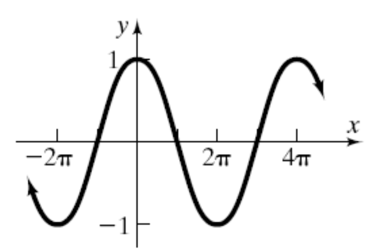
89. $(f \circ g)(x) = \sin(4x)$



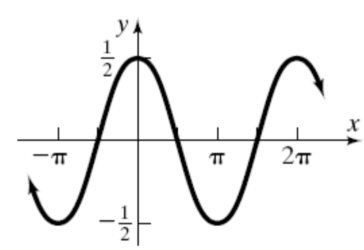
$(g \circ f)(x) = 4(\sin x) = 4\sin x$



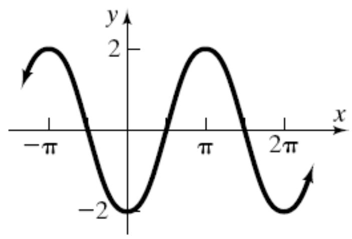
90. $(f \circ g)(x) = \cos\left(\frac{1}{2}x\right)$



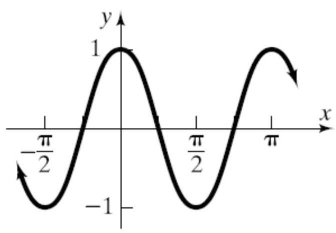
$(g \circ f)(x) = \frac{1}{2}(\cos x) = \frac{1}{2}\cos x$



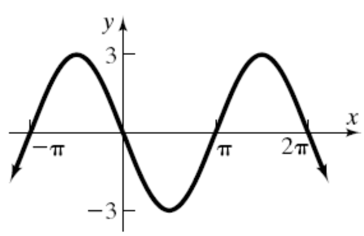
91. $(f \circ g)(x) = -2(\cos x) = -2\cos x$



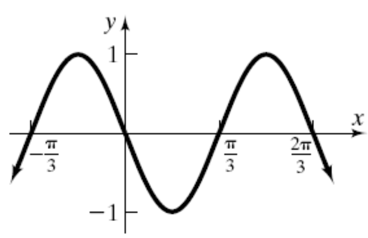
$(g \circ f)(x) = \cos(-2x)$



92. $(f \circ g)(x) = -3(\sin x) = -3\sin x$



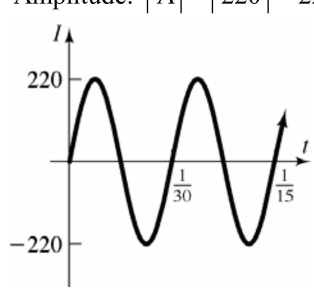
$(g \circ f)(x) = \sin(-3x)$



93. $I(t) = 220\sin(60\pi t),\ t \ge 0$

Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{60\pi} = \dfrac{1}{30}$ second

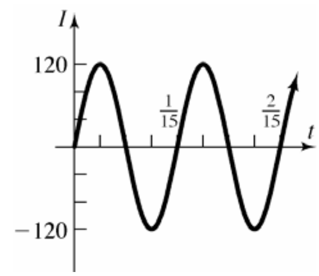
Amplitude: $|A| = |220| = 220$ amperes



94. $I(t) = 120\sin(30\pi t),\ t \ge 0$

Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{30\pi} = \dfrac{1}{15}$ second

Amplitude: $|A| = |120| = 120$ amperes

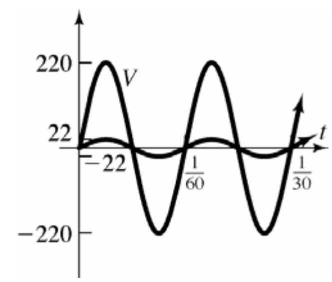


95. $V(t) = 220\sin(120\pi t)$

a. Amplitude: $|A| = |220| = 220$ volts

Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{120\pi} = \dfrac{1}{60}$ second

b, e.



c. $V = IR$

$220\sin(120\pi t) = 10I$

$22\sin(120\pi t) = I$

$I(t) = 22\sin(120\pi t)$

d. Amplitude: $|A| = |22| = 22$ amperes

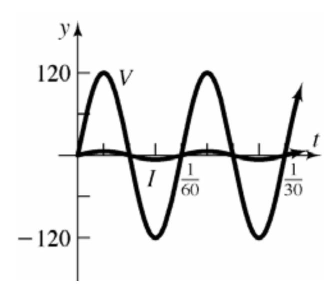
Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{120\pi} = \dfrac{1}{60}$ second

96. $V(t) = 120\sin(120\pi t)$

a. Amplitude: $|A| = |120| = 120$ volts

Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{120\pi} = \dfrac{1}{60}$ second

b, e.



c. $V = IR$

$120\sin(120\pi t) = 20I$

$6\sin(120\pi t) = I$

$I(t) = 6\sin(120\pi t)$

d. Amplitude: $|A| = |6| = 6$ amperes

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{120\pi} = \frac{1}{60}$ second

97. a. $P(t) = \frac{[V(t)]^2}{R}$

$= \frac{[V_0 \sin(2\pi ft)]^2}{R}$

$= \frac{V_0^2 \sin^2(2\pi ft)}{R}$

$= \frac{V_0^2}{R}\sin^2(2\pi ft)$

b. The graph is the reflected cosine graph translated up a distance equivalent to the amplitude. The period is $\frac{1}{2f}$, so $\omega = 4\pi f$.

The amplitude is $\frac{1}{2}\cdot\frac{V_0^2}{R} = \frac{V_0^2}{2R}$.

The equation is:

$P(t) = -\frac{V_0^2}{2R}\cos(4\pi ft) + \frac{V_0^2}{2R}$

$= \frac{V_0^2}{2R}[1 - \cos(4\pi ft)]$

c. Comparing the formulas:

$\sin^2(2\pi ft) = \frac{1}{2}(1 - \cos(4\pi ft))$

98. a. Since the tunnel is in the shape of one-half a sine cycle, the width of the tunnel at its base is one-half the period. Thus,

$T = \frac{2\pi}{\omega} = 2(28) = 56$ or $\omega = \frac{\pi}{28}$.

The tunnel has a maximum height of 15 feet so we have $A = 15$. Using the form $y = A\sin(\omega x)$, the equation for the sine curve that fits the opening is

$y = 15\sin\left(\frac{\pi x}{28}\right)$.

b. Since the shoulders are 7 feet wide and the road is 14 feet wide, the edges of the road correspond to $x = 7$ and $x = 21$.

$15\sin\left(\frac{7\pi}{28}\right) = 15\sin\left(\frac{\pi}{4}\right) = \frac{15\sqrt{2}}{2} \approx 10.6$

$15\sin\left(\frac{21\pi}{28}\right) = 15\sin\left(\frac{3\pi}{4}\right) = \frac{15\sqrt{2}}{2} \approx 10.6$

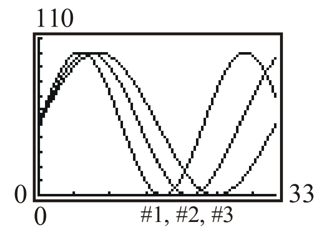
The tunnel is approximately 10.6 feet high at the edge of the road.

99. a. Physical potential: $\omega = \frac{2\pi}{23}$;

Emotional potential: $\omega = \frac{2\pi}{28} = \frac{\pi}{14}$;

Intellectual potential: $\omega = \frac{2\pi}{33}$

b.



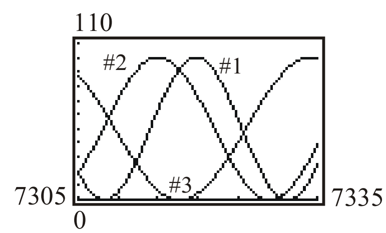
#1: $P(t) = 50\sin\left(\frac{2\pi}{23}t\right) + 50$

#2: $P(t) = 50\sin\left(\frac{\pi}{14}t\right) + 50$

#3: $P(t) = 50\sin\left(\frac{2\pi}{33}t\right) + 50$

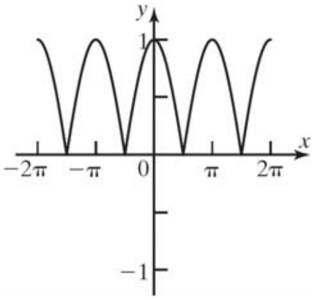
c. No.

d.

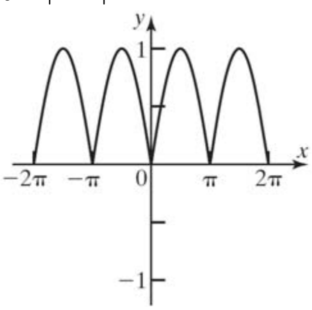


Physical potential peaks at 15 days after the 20th birthday, with minimums at the 3rd and 26th days. Emotional potential is 50% at the 17th day, with a maximum at the 10th day and a minimum at the 24th day. Intellectual potential starts fairly high, drops to a minimum at the 13th day, and rises to a maximum at the 29th day.

100. $y = |\cos x|, \quad -2\pi \le x \le 2\pi$



101. $y = |\sin x|, \quad -2\pi \le x \le 2\pi$



102 – 105. Answers will vary.

Section 2.7

1. $x = 4$

2. True

3. origin; x = odd multiples of $\frac{\pi}{2}$

4. y-axis; x = odd multiples of $\frac{\pi}{2}$

5. $y = \cos x$

6. True

7. The y-intercept of $y = \tan x$ is 0.

8. $y = \cot x$ has no y-intercept.

9. The y-intercept of $y = \sec x$ is 1.

10. $y = \csc x$ has no y-intercept.

11. $\sec x = 1$ when $x = -2\pi, 0, 2\pi$;
$\sec x = -1$ when $x = -\pi, \pi$

12. $\csc x = 1$ when $x = -\frac{3\pi}{2}, \frac{\pi}{2}$;
$\csc x = -1$ when $x = -\frac{\pi}{2}, \frac{3\pi}{2}$

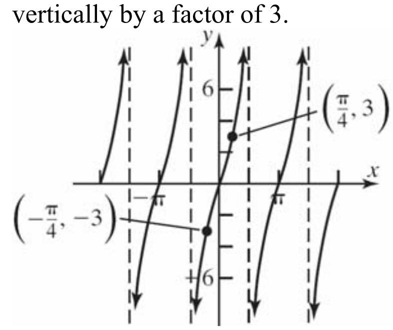
13. $y = \sec x$ has vertical asymptotes when $x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$.

14. $y = \csc x$ has vertical asymptotes when $x = -2\pi, -\pi, 0, \pi, 2\pi$.

15. $y = \tan x$ has vertical asymptotes when $x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$.

16. $y = \cot x$ has vertical asymptotes when $x = -2\pi, -\pi, 0, \pi, 2\pi$.

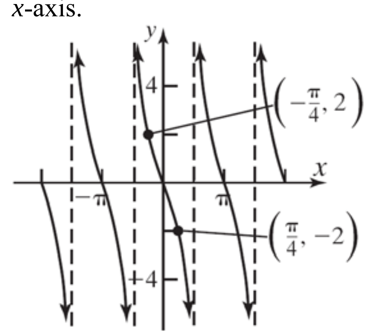
17. $y = 3\tan x$; The graph of $y = \tan x$ is stretched vertically by a factor of 3.



The domain is $\left\{x \,\middle|\, x \ne \frac{k\pi}{2}, k \text{ is an odd integer}\right\}$.

The range is the set of all real number or $(-\infty, \infty)$.

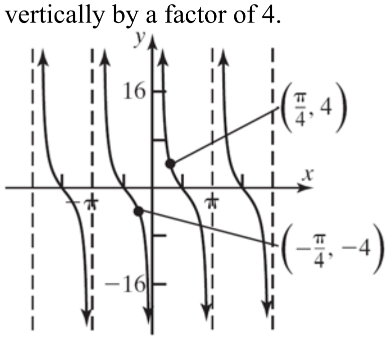
18. $y = -2\tan x$; The graph of $y = \tan x$ is stretched vertically by a factor of 2 and reflected about the x-axis.



The domain is $\left\{x \,\middle|\, x \ne \frac{k\pi}{2}, k \text{ is an odd integer}\right\}$.

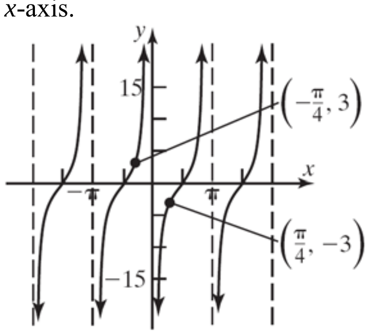
The range is the set of all real number or $(-\infty, \infty)$.

19. $y = 4\cot x$; The graph of $y = \cot x$ is stretched vertically by a factor of 4.



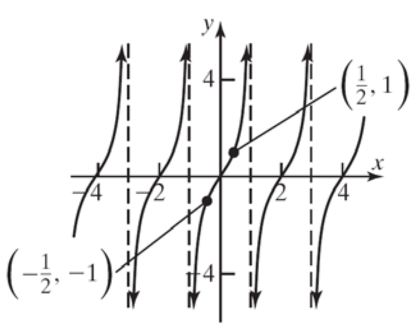
The domain is $\{x \mid x \neq k\pi,\ k \text{ is an integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

20. $y = -3\cot x$; The graph of $y = \cot x$ is stretched vertically by a factor of 3 and reflected about the x-axis.



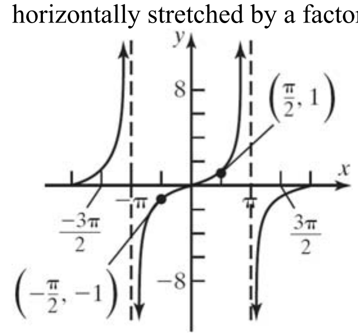
The domain is $\{x \mid x \neq k\pi,\ k \text{ is an integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

21. $y = \tan\left(\frac{\pi}{2}x\right)$; The graph of $y = \tan x$ is horizontally compressed by a factor of $\frac{2}{\pi}$.



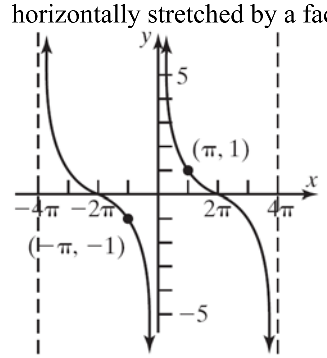
The domain is $\{x \mid x \text{ does not equal an odd integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

22. $y = \tan\left(\frac{1}{2}x\right)$; The graph of $y = \tan x$ is horizontally stretched by a factor of 2.



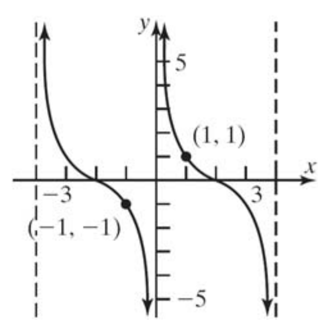
The domain is $\{x \mid x \neq k\pi,\ k \text{ is an integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

23. $y = \cot\left(\frac{1}{4}x\right)$; The graph of $y = \cot x$ is horizontally stretched by a factor of 4.



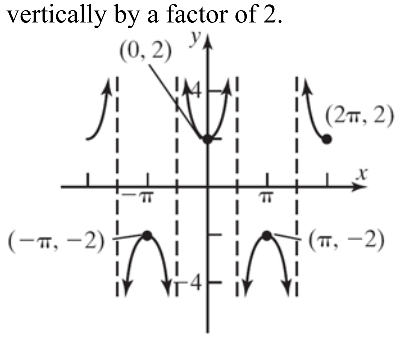
The domain is $\{x \mid x \neq 4k\pi,\ k \text{ is an integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

24. $y = \cot\left(\frac{\pi}{4}x\right)$; The graph of $y = \cot x$ is horizontally stretched by a factor of $\frac{4}{\pi}$.



The domain is $\{x \mid x \neq 4k,\ k \text{ is an integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

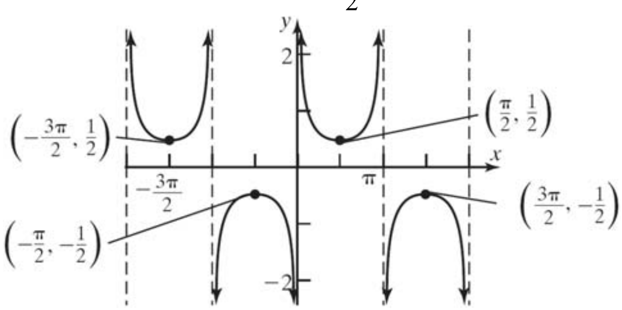
25. $y = 2\sec x$; The graph of $y = \sec x$ is stretched vertically by a factor of 2.



The domain is $\left\{x \,\middle|\, x \neq \frac{k\pi}{2}, k \text{ is an odd integer}\right\}$.

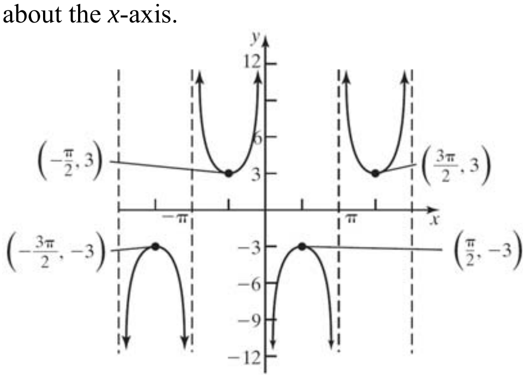
The range is $\{y \,|\, y \leq -2 \text{ or } y \geq 2\}$.

26. $y = \frac{1}{2}\csc x$; The graph of $y = \csc x$ is vertically compressed by a factor of $\frac{1}{2}$.



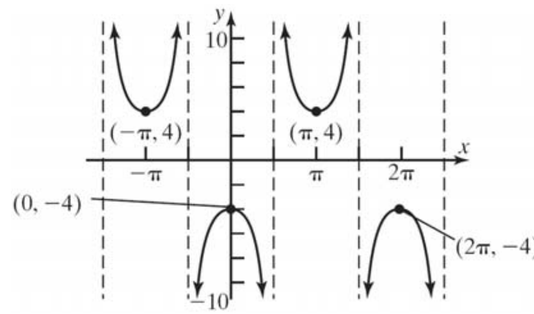
The domain is $\{x \,|\, x \neq k\pi, k \text{ is an integer}\}$. The range is $\left\{y \,\middle|\, y \leq -\frac{1}{2} \text{ or } y \geq \frac{1}{2}\right\}$.

27. $y = -3\csc x$; The graph of $y = \csc x$ is vertically stretched by a factor of 3 and reflected about the x-axis.



The domain is $\{x \,|\, x \neq k\pi, k \text{ is an integer}\}$. The range is $\{y \,|\, y \leq -3 \text{ or } y \geq 3\}$.

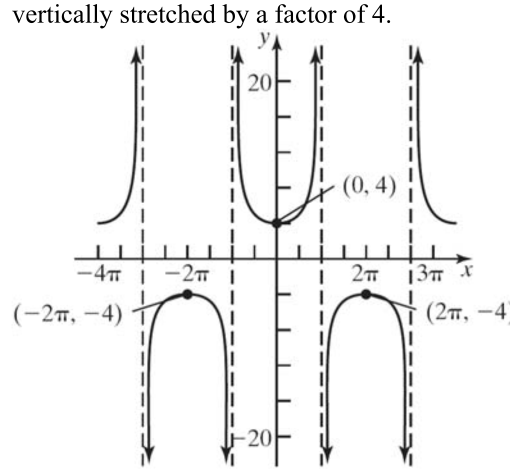
28. $y = -4\sec x$; The graph of $y = \sec x$ is vertically stretched by a factor of 4 and reflected about the x-axis.



The domain is $\left\{x \,\middle|\, x \neq \frac{k\pi}{2}, k \text{ is an odd integer}\right\}$.

The range is $\{y \,|\, y \leq -4 \text{ or } y \geq 4\}$.

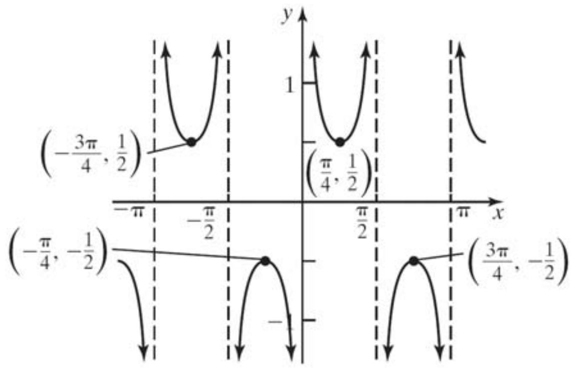
29. $y = 4\sec\left(\frac{1}{2}x\right)$; The graph of $y = \sec x$ is horizontally stretched by a factor of 2 and vertically stretched by a factor of 4.



The domain is $\{x \,|\, x \neq k\pi, k \text{ is an odd integer}\}$.

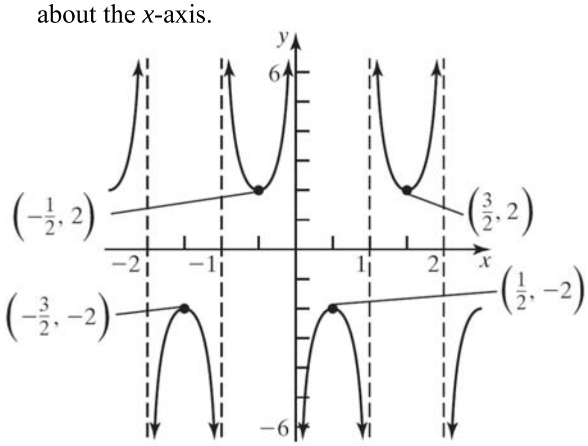
The range is $\{y \,|\, y \leq -4 \text{ or } y \geq 4\}$.

30. $y=\frac{1}{2}\csc(2x)$; The graph of $y=\csc x$ is horizontally compressed by a factor of $\frac{1}{2}$ and vertically compressed by a factor of $\frac{1}{2}$.



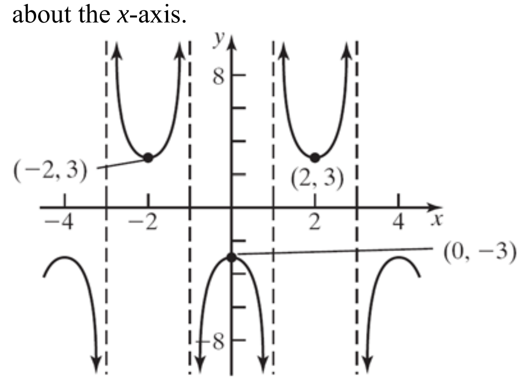
The domain is $\left\{x \middle| x \neq \frac{k\pi}{2}, k \text{ is an integer}\right\}$. The range is $\left\{y \middle| y \leq -\frac{1}{2} \text{ or } y \geq \frac{1}{2}\right\}$.

31. $y=-2\csc(\pi x)$; The graph of $y=\csc x$ is horizontally compressed by a factor of $\frac{1}{\pi}$, vertically stretched by a factor of 2, and reflected about the *x*-axis.



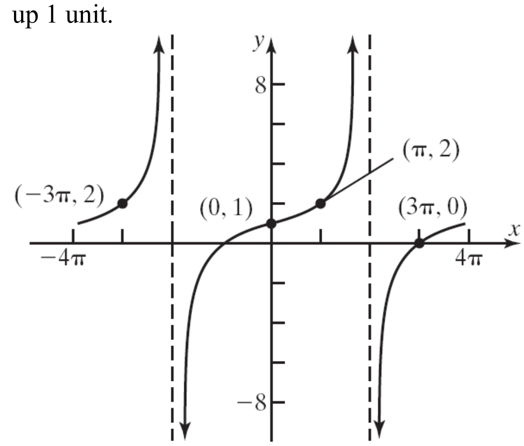
The domain is $\{x|x \text{ does not equal an integer}\}$.
The range is $\{y|y \leq -2 \text{ or } y \geq 2\}$.

32. $y=-3\sec\left(\frac{\pi}{2}x\right)$; The graph of $y=\sec x$ is horizontally compressed by a factor of $\frac{2}{\pi}$, vertically stretched by a factor of 3, and reflected about the *x*-axis.



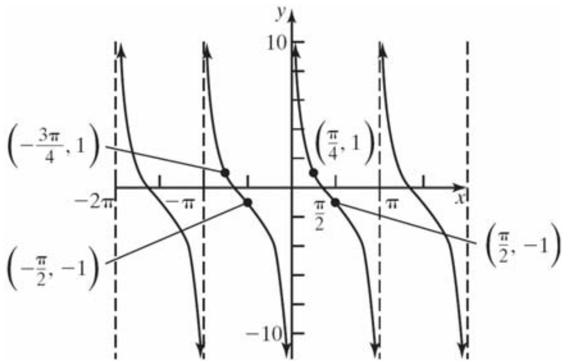
The domain is $\{x|x \text{ does not equal an odd integer}\}$.
The range is $\{y|y \leq -3 \text{ or } y \geq 3\}$.

33. $y=\tan\left(\frac{1}{4}x\right)+1$; The graph of $y=\tan x$ is horizontally stretched by a factor of 4 and shifted up 1 unit.



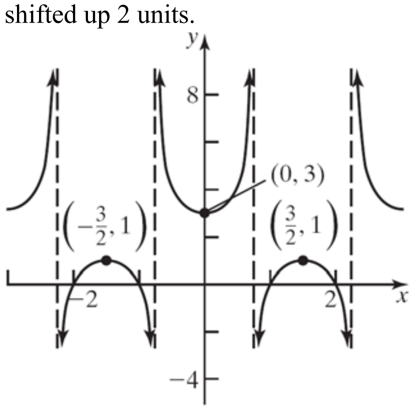
The domain is $\{x|x \neq 2k\pi, k \text{ is an odd integer}\}$.
The range is the set of all real number or $(-\infty, \infty)$.

34. $y = 2\cot x - 1$; The graph of $y = \cot x$ is vertically stretched by a factor of 2 and shifted down 1 unit.



The domain is $\{x \mid x \neq k\pi,\ k \text{ is an integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

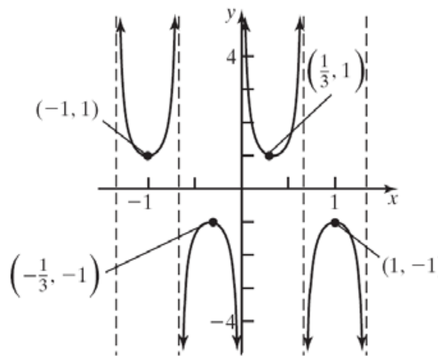
35. $y = \sec\left(\dfrac{2\pi}{3}x\right) + 2$; The graph of $y = \sec x$ is horizontally compressed by a factor of $\dfrac{3}{2\pi}$ and shifted up 2 units.



The domain is $\left\{x \,\middle|\, x \neq \dfrac{3}{4}k,\ k \text{ is an odd integer}\right\}$.

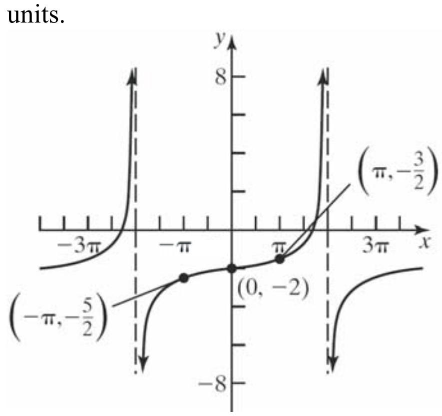
The range is $\{y \mid y \leq 1 \text{ or } y \geq 3\}$.

36. $y = \csc\left(\dfrac{3\pi}{2}x\right)$; The graph of $y = \csc x$ is horizontally compressed by a factor of $\dfrac{2}{3\pi}$.



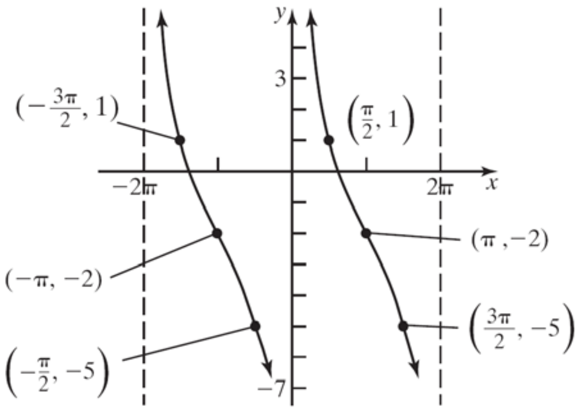
The domain is $\left\{x \,\middle|\, x \neq \dfrac{2}{3}k,\ k \text{ is an integer}\right\}$. The range is $\{y \mid y \leq -1 \text{ or } y \geq 1\}$.

37. $y = \dfrac{1}{2}\tan\left(\dfrac{1}{4}x\right) - 2$; The graph of $y = \tan x$ is horizontally stretched by a factor of 4, vertically compressed by a factor of $\dfrac{1}{2}$, and shifted down 2 units.



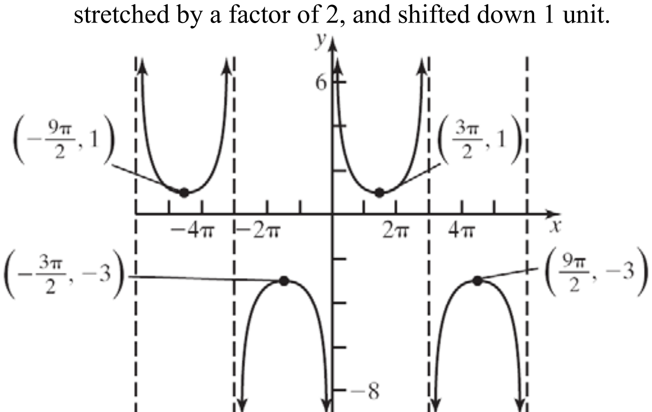
The domain is $\{x \mid x \neq 2\pi k,\ k \text{ is an odd integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

38. $y = 3\cot\left(\frac{1}{2}x\right) - 2$; The graph of $y = \cot x$ is horizontally stretched by a factor of 2, vertically stretched by a factor of 3, and shifted down 2 units.



The domain is $\{x \mid x \neq 2\pi k,\ k \text{ is an integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

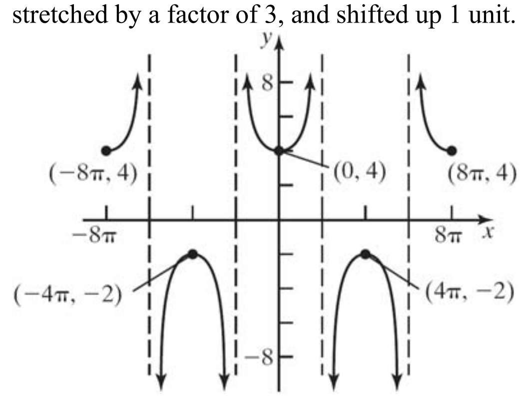
39. $y = 2\csc\left(\frac{1}{3}x\right) - 1$; The graph of $y = \csc x$ is horizontally stretched by a factor of 3, vertically stretched by a factor of 2, and shifted down 1 unit.



The domain is $\{x \mid x \neq 3\pi k,\ k \text{ is an integer}\}$.

The range is $\{y \mid y \leq -3 \text{ or } y \geq 1\}$.

40. $y = 3\sec\left(\frac{1}{4}x\right) + 1$; The graph of $y = \sec x$ is horizontally stretched by a factor of 4, vertically stretched by a factor of 3, and shifted up 1 unit.



The domain is $\{x \mid x \neq 2\pi k,\ k \text{ is an odd integer}\}$.

The range is $\{y \mid y \leq -2 \text{ or } y \geq 4\}$.

41. $$\frac{f\left(\frac{\pi}{6}\right) - f(0)}{\frac{\pi}{6} - 0} = \frac{\tan(\pi/6) - \tan(0)}{\pi/6} = \frac{\frac{\sqrt{3}}{3} - 0}{\pi/6}$$

$$= \frac{\sqrt{3}}{3} \cdot \frac{6}{\pi} = \frac{2\sqrt{3}}{\pi}$$

The average rate of change is $\frac{2\sqrt{3}}{\pi}$.

42. $$\frac{f\left(\frac{\pi}{6}\right) - f(0)}{\frac{\pi}{6} - 0} = \frac{\sec(\pi/6) - \sec(0)}{\pi/6} = \frac{\frac{2\sqrt{3}}{3} - 1}{\pi/6}$$

$$= \frac{2\sqrt{3} - 3}{3} \cdot \frac{6}{\pi} = \frac{2\sqrt{3}\left(2 - \sqrt{3}\right)}{\pi}$$

The average rate of change is $\frac{2\sqrt{3}\left(2 - \sqrt{3}\right)}{\pi}$.

43. $$\frac{f\left(\frac{\pi}{6}\right) - f(0)}{\frac{\pi}{6} - 0} = \frac{\tan(2 \cdot \pi/6) - \tan(2 \cdot 0)}{\pi/6}$$

$$= \frac{\sqrt{3} - 0}{\pi/6} = \frac{6\sqrt{3}}{\pi}$$

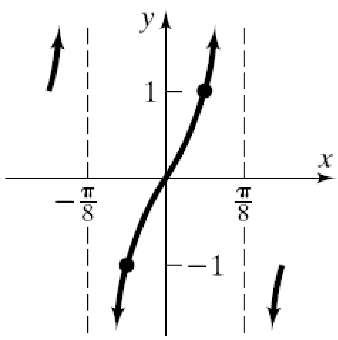
The average rate of change is $\frac{6\sqrt{3}}{\pi}$.

44. $\dfrac{f\left(\dfrac{\pi}{6}\right)-f(0)}{\dfrac{\pi}{6}-0}=\dfrac{\sec(2\cdot\pi/6)-\sec(2\cdot 0)}{\pi/6}$

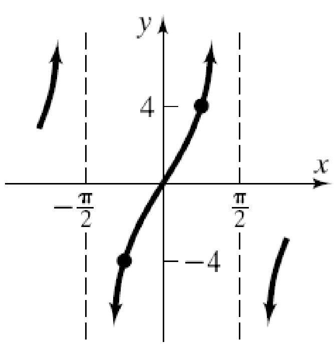
$=\dfrac{2-1}{\pi/6}=\dfrac{6}{\pi}$

The average rate of change is $\dfrac{6}{\pi}$.

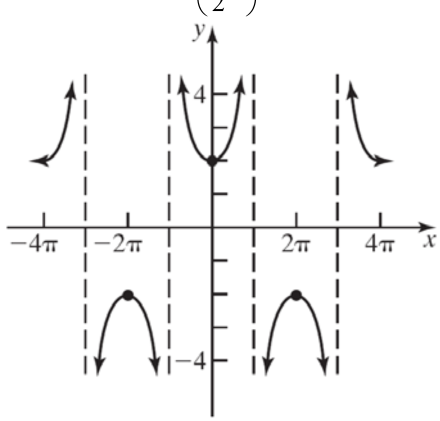
45. $(f\circ g)(x)=\tan(4x)$



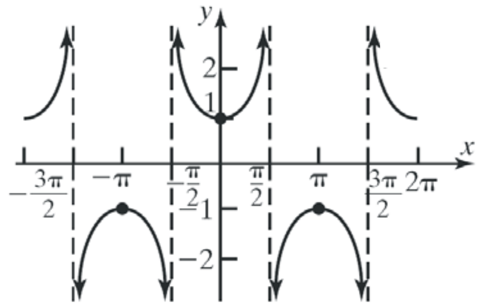
$(g\circ f)(x)=4(\tan x)=4\tan x$



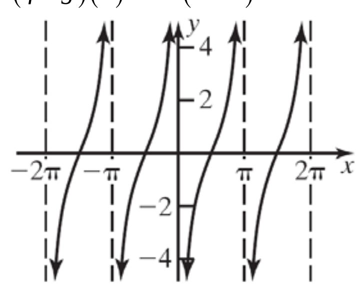
46. $(f\circ g)(x)=2\sec\left(\dfrac{1}{2}x\right)$



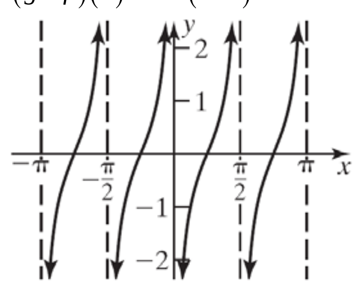
$(g\circ f)(x)=\dfrac{1}{2}(2\sec x)=\sec x$



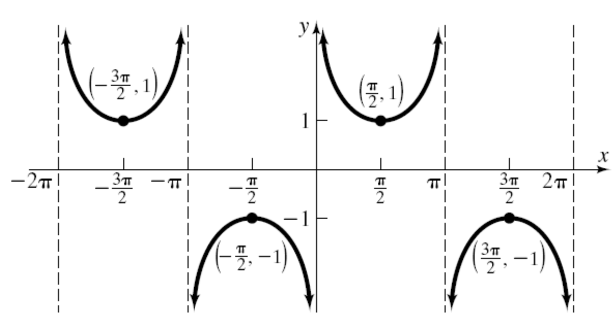
47. $(f\circ g)(x)=-2(\cot x)=-2\cot x$



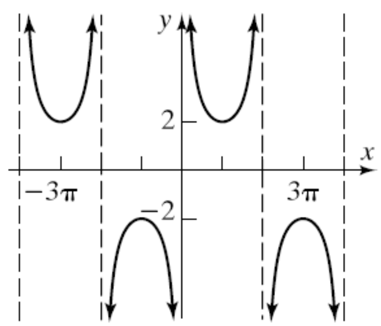
$(g\circ f)(x)=\cot(-2x)$



48. $(f\circ g)(x)=\dfrac{1}{2}(2\csc x)=\csc x$



$(g \circ f)(x) = 2\csc\left(\frac{1}{2}x\right)$



49. a. Consider the length of the line segment in two sections, *x*, the portion across the hall that is 3 feet wide and *y*, the portion across that hall that is 4 feet wide. Then,

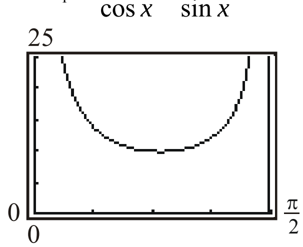
$\cos\theta = \frac{3}{x}$ and $\sin\theta = \frac{4}{y}$

$x = \frac{3}{\cos\theta}$ $y = \frac{4}{\sin\theta}$

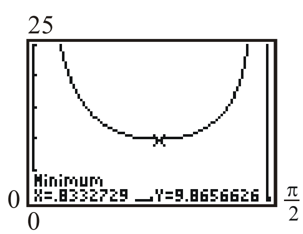
Thus,

$L = x + y = \frac{3}{\cos\theta} + \frac{4}{\sin\theta} = 3\sec\theta + 4\csc\theta$.

b. Let $Y_1 = \frac{3}{\cos x} + \frac{4}{\sin x}$.



c. Use MINIMUM to find the least value:

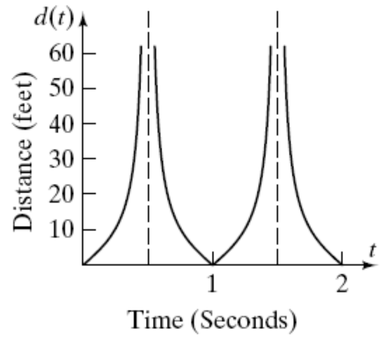


L is least when $\theta \approx 0.83$.

d. $L \approx \frac{3}{\cos(0.83)} + \frac{4}{\sin(0.83)} \approx 9.86$ feet.

Note that rounding up will result in a ladder that won't fit around the corner. Answers will vary.

50. a. $d(t) = |10\tan(\pi t)|$



b. $d(t) = |10\tan(\pi t)|$ is undefined at $t = \frac{1}{2}$ and $t = \frac{3}{2}$, or in general at $\left\{ t = \frac{k}{2} \,\middle|\, k \text{ is an odd integer} \right\}$. At these instances, the length of the beam of light approaches infinity. It is at these instances in the rotation of the beacon when the beam of light being cast on the wall changes from one side of the beacon to the other.

c.

t	$d(t) = 10\tan(\pi t)$
0	0
0.1	3.2492
0.2	7.2654
0.3	13.764
0.4	30.777

d.

$\frac{d(0.1) - d(0)}{0.1 - 0} = \frac{3.2492 - 0}{0.1 - 0} \approx 32.492$

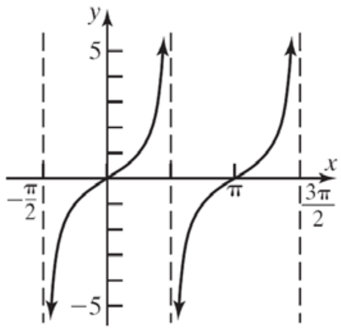
$\frac{d(0.2) - d(0.1)}{0.2 - 0.1} = \frac{7.2654 - 3.2492}{0.2 - 0.1} \approx 40.162$

$\frac{d(0.3) - d(0.2)}{0.3 - 0.2} = \frac{13.764 - 7.2654}{0.3 - 0.2} \approx 64.986$

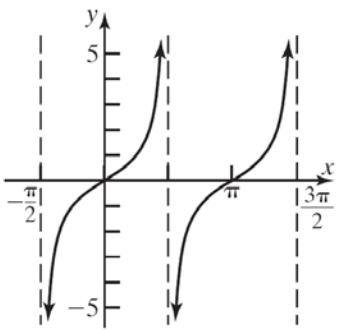
$\frac{d(0.4) - d(0.3)}{0.4 - 0.3} = \frac{30.777 - 13.764}{0.4 - 0.3} \approx 170.13$

e. The first differences represent the average rate of change of the beam of light against the wall, measured in feet per second. For example, between $t = 0$ seconds and $t = 0.1$ seconds, the average rate of change of the beam of light against the wall is 32.492 feet per second.

51.



$y = \tan x$



$y = -\cot\left(x + \frac{\pi}{2}\right)$

Yes, the two functions are equivalent.

Section 2.8

1. phase shift

2. False

3. $y = 4\sin(2x - \pi)$

   Amplitude: $|A| = |4| = 4$

   Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{2} = \pi$

   Phase Shift: $\dfrac{\phi}{\omega} = \dfrac{\pi}{2}$

   Interval defining one cycle:

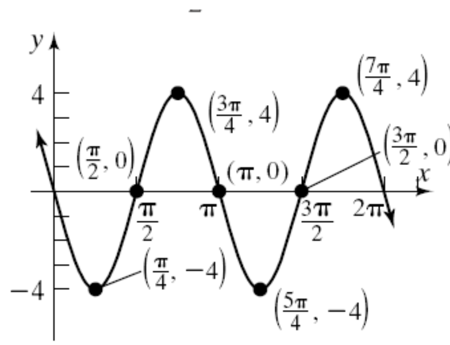
   $\left[\dfrac{\phi}{\omega}, \dfrac{\phi}{\omega} + T\right] = \left[\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right]$

   Subinterval width:

   $\dfrac{T}{4} = \dfrac{\pi}{4}$

   Key points:

   $\left(\dfrac{\pi}{2}, 0\right)$, $\left(\dfrac{3\pi}{4}, 4\right)$, $(\pi, 0)$, $\left(\dfrac{5\pi}{4}, -4\right)$, $\left(\dfrac{3\pi}{2}, 0\right)$



4. $y = 3\sin(3x - \pi)$

   Amplitude: $|A| = |3| = 3$

   Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{3}$

   Phase Shift: $\dfrac{\phi}{\omega} = \dfrac{\pi}{3}$

   Interval defining one cycle:

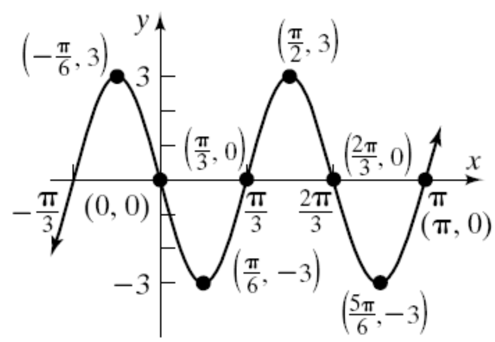
   $\left[\dfrac{\phi}{\omega}, \dfrac{\phi}{\omega} + T\right] = \left[\dfrac{\pi}{3}, \pi\right]$

   Subinterval width:

   $\dfrac{T}{4} = \dfrac{2\pi/3}{4} = \dfrac{\pi}{6}$

   Key points:

   $\left(\dfrac{\pi}{3}, 0\right)$, $\left(\dfrac{\pi}{2}, 3\right)$, $\left(\dfrac{2\pi}{3}, 0\right)$, $\left(\dfrac{5\pi}{6}, -3\right)$, $(\pi, 0)$



5. $y = 2\cos\left(3x + \dfrac{\pi}{2}\right)$

   Amplitude: $|A| = |2| = 2$

   Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{3}$

   Phase Shift: $\dfrac{\phi}{\omega} = \dfrac{\left(-\dfrac{\pi}{2}\right)}{3} = -\dfrac{\pi}{6}$

Interval defining one cycle:

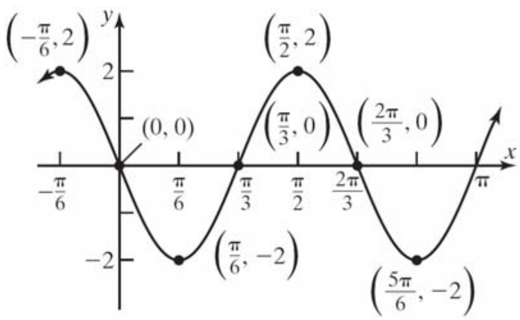
$$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega}+T\right]=\left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$$

Subinterval width:

$$\frac{T}{4}=\frac{2\pi/3}{4}=\frac{\pi}{6}$$

Key points:

$$\left(-\frac{\pi}{6},2\right),\ (0,0),\ \left(\frac{\pi}{6},-2\right),\ \left(\frac{\pi}{3},0\right),\ \left(\frac{\pi}{2},2\right)$$



6. $y=3\cos(2x+\pi)$

Amplitude: $|A|=|3|=3$

Period: $T=\frac{2\pi}{\omega}=\frac{2\pi}{2}=\pi$

Phase Shift: $\frac{\phi}{\omega}=\frac{-\pi}{2}=-\frac{\pi}{2}$

Interval defining one cycle:

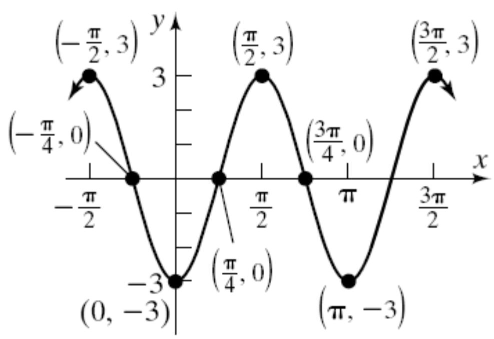
$$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega}+T\right]=\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Subinterval width:

$$\frac{T}{4}=\frac{\pi}{4}$$

Key points:

$$\left(-\frac{\pi}{2},3\right),\ \left(-\frac{\pi}{4},0\right),\ (0,-3),\ \left(\frac{\pi}{4},0\right),\ \left(\frac{\pi}{2},3\right)$$



7. $y=-3\sin\left(2x+\frac{\pi}{2}\right)$

Amplitude: $|A|=|-3|=3$

Period: $T=\frac{2\pi}{\omega}=\frac{2\pi}{2}=\pi$

Phase Shift: $\frac{\phi}{\omega}=\frac{-\frac{\pi}{2}}{2}=-\frac{\pi}{4}$

Interval defining one cycle:

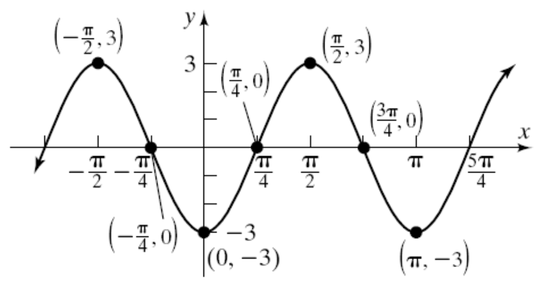
$$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega}+T\right]=\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$$

Subinterval width:

$$\frac{T}{4}=\frac{\pi}{4}$$

Key points:

$$\left(-\frac{\pi}{4},0\right),\ (0,-3),\ \left(\frac{\pi}{4},0\right),\ \left(\frac{\pi}{2},3\right),\ \left(\frac{3\pi}{4},0\right)$$



8. $y=-2\cos\left(2x-\frac{\pi}{2}\right)$

Amplitude: $|A|=|-2|=2$

Period: $T=\frac{2\pi}{\omega}=\frac{2\pi}{2}=\pi$

Phase Shift: $\frac{\phi}{\omega}=\frac{\frac{\pi}{2}}{2}=\frac{\pi}{4}$

Interval defining one cycle:

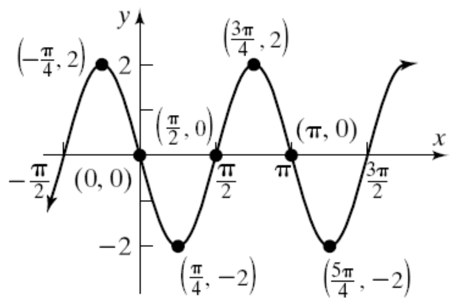
$$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega}+T\right]=\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$$

Subinterval width:

$$\frac{T}{4}=\frac{\pi}{4}$$

Key points:

$$\left(\frac{\pi}{4},-2\right),\ \left(\frac{\pi}{2},0\right),\ \left(\frac{3\pi}{4},2\right),\ (\pi,0),\ \left(\frac{5\pi}{4},-2\right)$$



9. $y = 4\sin(\pi x + 2) - 5$

Amplitude: $|A| = |4| = 4$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$

Phase Shift: $\frac{\phi}{\omega} = \frac{-2}{\pi} = -\frac{2}{\pi}$

Interval defining one cycle:

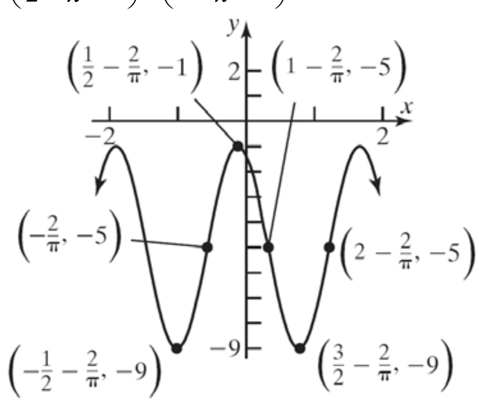
$$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + T\right] = \left[-\frac{2}{\pi}, 2 - \frac{2}{\pi}\right]$$

Subinterval width:

$$\frac{T}{4} = \frac{2}{4} = \frac{1}{2}$$

Key points:

$$\left(-\frac{2}{\pi}, -5\right), \left(\frac{1}{2} - \frac{2}{\pi}, -1\right), \left(1 - \frac{2}{\pi}, -5\right),$$
$$\left(\frac{3}{2} - \frac{2}{\pi}, -9\right), \left(2 - \frac{2}{\pi}, -5\right)$$



10. $y = 2\cos(2\pi x + 4) + 4$

Amplitude: $|A| = |2| = 2$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = 1$

Phase Shift: $\frac{\phi}{\omega} = \frac{-4}{2\pi} = -\frac{2}{\pi}$

Interval defining one cycle:

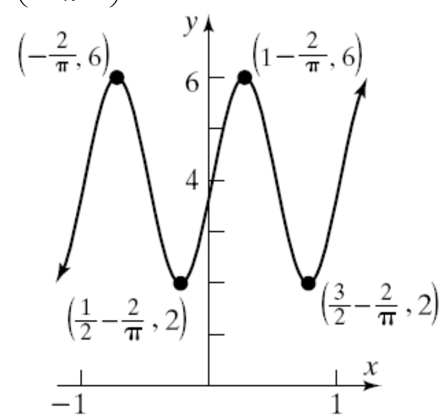
$$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + T\right] = \left[-\frac{2}{\pi}, 1 - \frac{2}{\pi}\right]$$

Subinterval width:

$$\frac{T}{4} = \frac{1}{4}$$

Key points:

$$\left(-\frac{2}{\pi}, 6\right), \left(\frac{1}{4} - \frac{2}{\pi}, 4\right), \left(\frac{1}{2} - \frac{2}{\pi}, 2\right), \left(\frac{3}{4} - \frac{2}{\pi}, 4\right),$$
$$\left(1 - \frac{2}{\pi}, 6\right)$$



11. $y = 3\cos(\pi x - 2) + 5$

Amplitude: $|A| = |3| = 3$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$

Phase Shift: $\frac{\phi}{\omega} = \frac{2}{\pi}$

Interval defining one cycle:

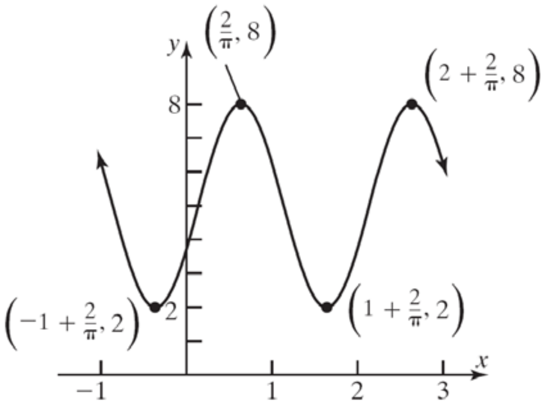
$$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + T\right] = \left[\frac{2}{\pi}, 2 + \frac{2}{\pi}\right]$$

Subinterval width:

$$\frac{T}{4} = \frac{2}{4} = \frac{1}{2}$$

Key points:

$$\left(\frac{2}{\pi}, 8\right), \left(\frac{1}{2} + \frac{2}{\pi}, 5\right), \left(1 + \frac{2}{\pi}, 2\right), \left(\frac{3}{2} + \frac{2}{\pi}, 5\right),$$
$$\left(2 + \frac{2}{\pi}, 8\right)$$



12. $y = 2\cos(2\pi x - 4) - 1$

Amplitude: $|A| = |2| = 2$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = 1$

Phase Shift: $\frac{\phi}{\omega} = \frac{4}{2\pi} = \frac{2}{\pi}$

Interval defining one cycle:

$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + T\right] = \left[\frac{2}{\pi}, 1 + \frac{2}{\pi}\right]$

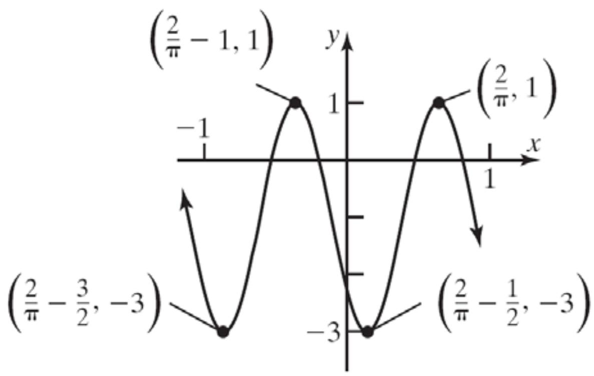
Subinterval width:

$\frac{T}{4} = \frac{1}{4}$

Key points:

$\left(\frac{2}{\pi}, 1\right)$, $\left(\frac{1}{4} + \frac{2}{\pi}, -1\right)$, $\left(\frac{1}{2} + \frac{2}{\pi}, -3\right)$,

$\left(\frac{3}{4} + \frac{2}{\pi}, -1\right)$, $\left(1 + \frac{2}{\pi}, 1\right)$



13. $y = -3\sin\left(-2x + \frac{\pi}{2}\right) = -3\sin\left(-\left(2x - \frac{\pi}{2}\right)\right)$

$= 3\sin\left(2x - \frac{\pi}{2}\right)$

Amplitude: $|A| = |3| = 3$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

Phase Shift: $\frac{\phi}{\omega} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$

Interval defining one cycle:

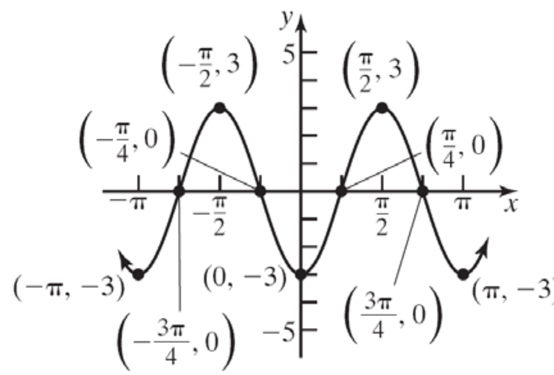
$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + T\right] = \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

Subinterval width:

$\frac{T}{4} = \frac{\pi}{4}$

Key points:

$\left(\frac{\pi}{4}, 0\right)$, $\left(\frac{\pi}{2}, 3\right)$, $\left(\frac{3\pi}{4}, 0\right)$, $(\pi, -3)$, $\left(\frac{5\pi}{4}, 0\right)$



14. $y = -3\cos\left(-2x + \frac{\pi}{2}\right) = -3\cos\left(-\left(2x - \frac{\pi}{2}\right)\right)$

$= -3\cos\left(2x - \frac{\pi}{2}\right)$

Amplitude: $|A| = |-3| = 3$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

Phase Shift: $\frac{\phi}{\omega} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$

Interval defining one cycle:

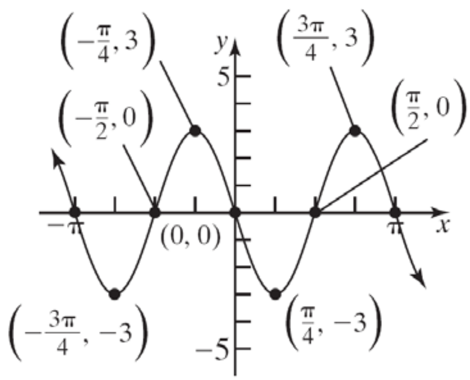
$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + T\right] = \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

Subinterval width:

$\frac{T}{4} = \frac{\pi}{4}$

Key points:

$\left(\frac{\pi}{4}, -3\right), \left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{4}, 3\right), (\pi, 0), \left(\frac{5\pi}{4}, -3\right)$



15. $|A| = 2; \quad T = \pi; \quad \frac{\phi}{\omega} = \frac{1}{2}$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2 \qquad \frac{\phi}{\omega} = \frac{\phi}{2} = \frac{1}{2}$

$\phi = 1$

Assuming A is positive, we have that

$y = A\sin(\omega x - \phi) = 2\sin(2x - 1)$

$= 2\sin\left[2\left(x - \frac{1}{2}\right)\right]$

16. $|A| = 3; \quad T = \frac{\pi}{2}; \quad \frac{\phi}{\omega} = 2$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{\frac{\pi}{2}} = 4 \qquad \frac{\phi}{\omega} = \frac{\phi}{4} = 2$

$\phi = 8$

Assuming A is positive, we have that

$y = A\sin(\omega x - \phi) = 3\sin(4x - 8)$

$= 3\sin\left[4(x - 2)\right]$

17. $|A| = 3; \quad T = 3\pi; \quad \frac{\phi}{\omega} = -\frac{1}{3}$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{3\pi} = \frac{2}{3} \qquad \frac{\phi}{\omega} = \frac{\phi}{\frac{2}{3}} = -\frac{1}{3}$

$\phi = -\frac{1}{3} \cdot \frac{2}{3} = -\frac{2}{9}$

Assuming A is positive, we have that

$y = A\sin(\omega x - \phi) = 3\sin\left(\frac{2}{3}x + \frac{2}{9}\right)$

$= 3\sin\left[\frac{2}{3}\left(x + \frac{1}{3}\right)\right]$

18. $|A| = 2; \quad T = \pi; \quad \frac{\phi}{\omega} = -2$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2 \qquad \frac{\phi}{\omega} = \frac{\phi}{2} = -2$

$\phi = -4$

Assuming A is positive, we have that

$y = A\sin(\omega x - \phi) = 2\sin(2x + 4)$

$= 2\sin\left[2(x + 2)\right]$

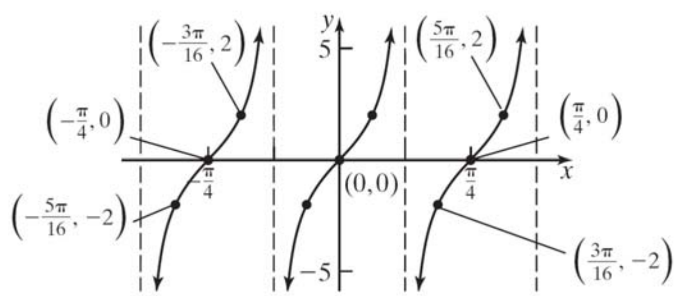
19. $y = 2\tan(4x - \pi)$

Begin with the graph of $y = \tan x$ and apply the following transformations:

1) Shift right π units $\left[y = \tan(x - \pi)\right]$

2) Horizontally compress by a factor of $\frac{1}{4}$ $\left[y = \tan(4x - \pi)\right]$

3) Vertically stretch by a factor of 2 $\left[y = 2\tan(4x - \pi)\right]$



20. $y = \frac{1}{2}\cot(2x - \pi)$

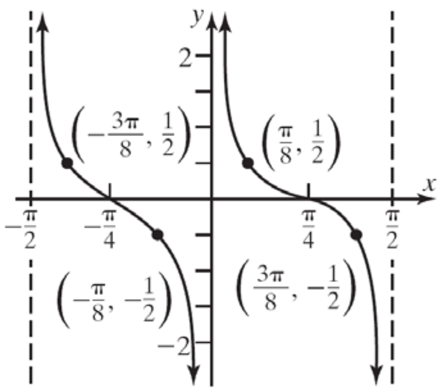
Begin with the graph of $y = \cot x$ and apply the following transformations:

1) Shift right π units $\left[y = \cot(x - \pi)\right]$

2) Horizontally compress by a factor of $\frac{1}{2}$ $\left[y = \cot(2x - \pi)\right]$

3) Vertically compress by a factor of $\frac{1}{2}$

$\left[y = \frac{1}{2}\cot\left(2x - \pi\right)\right]$



21. $y = 3\csc\left(2x - \frac{\pi}{4}\right)$

Begin with the graph of $y = \csc x$ and apply the following transformations:

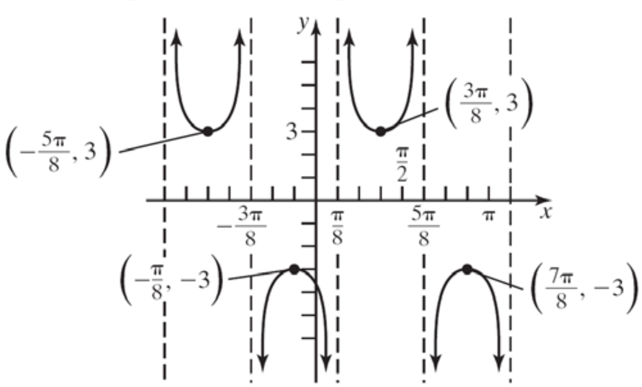
1) Shift right $\frac{\pi}{4}$ units $\left[y = \csc\left(x - \frac{\pi}{4}\right)\right]$

2) Horizontally compress by a factor of $\frac{1}{2}$

$\left[y = \csc\left(2x - \frac{\pi}{4}\right)\right]$

3) Vertically stretch by a factor of 3

$\left[y = 3\csc\left(2x - \frac{\pi}{4}\right)\right]$



22. $y = \frac{1}{2}\sec\left(3x - \pi\right)$

Begin with the graph of $y = \sec x$ and apply the following transformations:

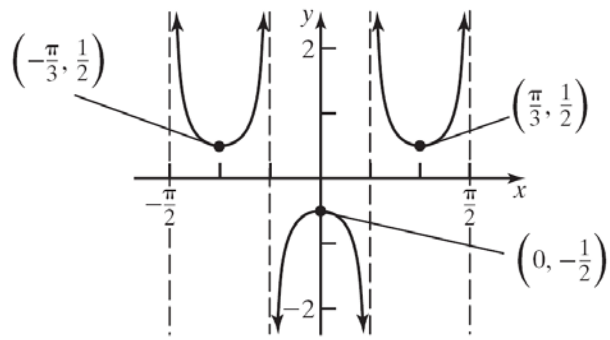
1) Shift right π units $\left[y = \sec\left(x - \pi\right)\right]$

2) Horizontally compress by a factor of $\frac{1}{3}$

$\left[y = \sec\left(3x - \pi\right)\right]$

3) Vertically compress by a factor of $\frac{1}{2}$

$\left[y = \frac{1}{2}\sec\left(3x - \pi\right)\right]$



23. $y = -\cot\left(2x + \frac{\pi}{2}\right)$

Begin with the graph of $y = \cot x$ and apply the following transformations:

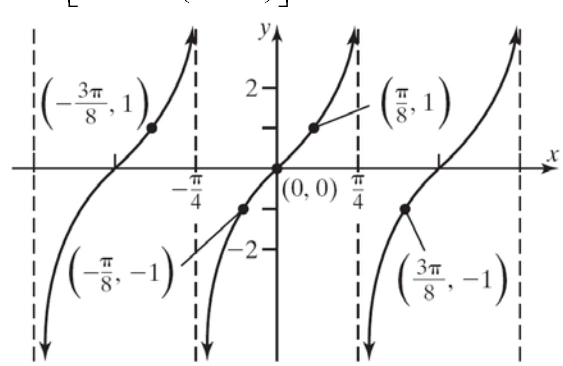
1) Shift left $\frac{\pi}{2}$ units $\left[y = \cot\left(x + \frac{\pi}{2}\right)\right]$

2) Horizontally compress by a factor of $\frac{1}{2}$

$\left[y = \cot\left(2x + \frac{\pi}{2}\right)\right]$

3) Reflect about the x-axis

$\left[y = -\cot\left(2x + \frac{\pi}{2}\right)\right]$



24. $y=-\tan\left(3x+\frac{\pi}{2}\right)$

Begin with the graph of $y=\tan x$ and apply the following transformations:

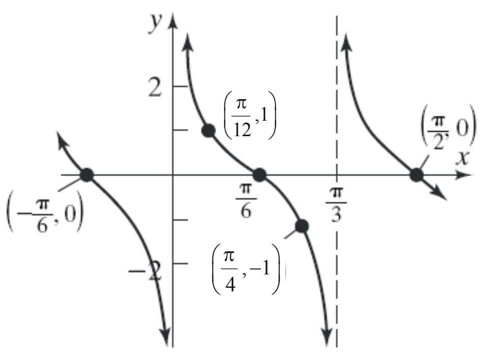
1) Shift left $\frac{\pi}{2}$ units $\left[y=\tan\left(x+\frac{\pi}{2}\right)\right]$

2) Horizontally compress by a factor of $\frac{1}{3}$

$\left[y=\tan\left(3x+\frac{\pi}{2}\right)\right]$

3) Reflect about the x-axis

$\left[y=-\tan\left(x+\frac{\pi}{2}\right)\right]$



25. $y=-\sec(2\pi x+\pi)$

Begin with the graph of $y=\sec x$ and apply the following transformations:

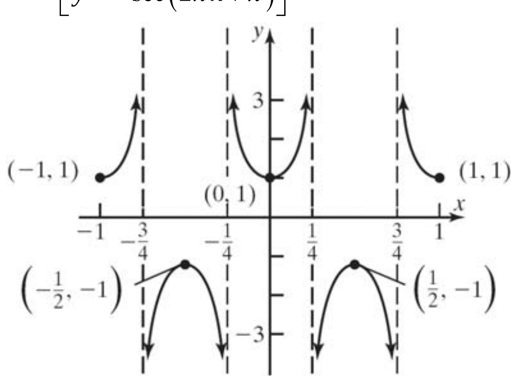
1) Shift left π units $[y=\sec(x+\pi)]$

2) Horizontally compress by a factor of $\frac{1}{2\pi}$

$[y=\sec(2\pi x+\pi)]$

3) Reflect about the x-axis

$[y=-\sec(2\pi x+\pi)]$



26. $y=-\csc\left(-\frac{1}{2}\pi x+\frac{\pi}{4}\right)$

Begin with the graph of $y=\csc x$ and apply the following transformations:

1) Shift left $\frac{\pi}{4}$ units $\left[y=\csc\left(x+\frac{\pi}{4}\right)\right]$

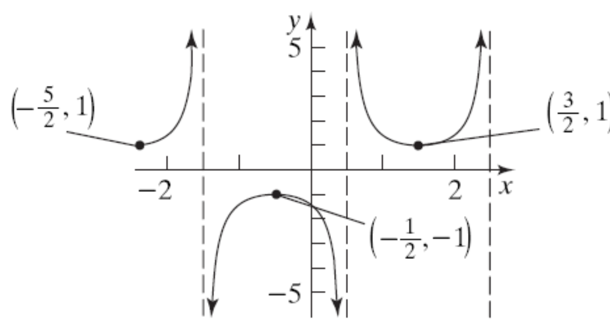
2) Reflect about the y-axis $\left[y=\csc\left(-x+\frac{\pi}{4}\right)\right]$

3) Horizontally compress by a factor of $\frac{2}{\pi}$

$\left[y=\csc\left(-\frac{1}{2}\pi x+\frac{\pi}{4}\right)\right]$

3) Reflect about the x-axis

$\left[y=-\csc\left(-\frac{1}{2}\pi x+\frac{\pi}{4}\right)\right]$

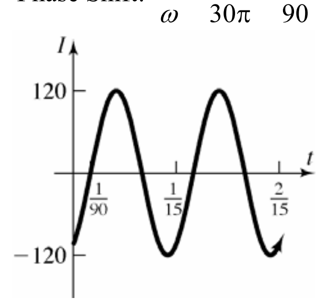


27. $I(t)=120\sin\left(30\pi t-\frac{\pi}{3}\right),\quad t\geq 0$

Period: $T=\frac{2\pi}{\omega}=\frac{2\pi}{30\pi}=\frac{1}{15}$ second

Amplitude: $|A|=|120|=120$ amperes

Phase Shift: $\frac{\phi}{\omega}=\frac{\frac{\pi}{3}}{30\pi}=\frac{1}{90}$ second

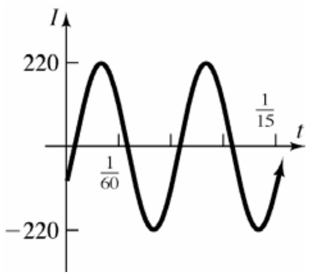


28. $I(t) = 220\sin\left(60\pi t - \frac{\pi}{6}\right), \quad t \ge 0$

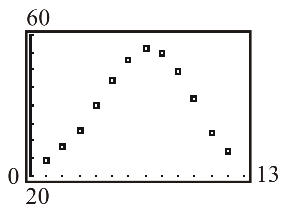
Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{60\pi} = \frac{1}{30}$ second

Amplitude: $|A| = |220| = 220$ amperes

Phase Shift: $\frac{\phi}{\omega} = \frac{\frac{\pi}{6}}{60\pi} = \frac{1}{360}$ second



29. a.



b. Amplitude: $A = \frac{56.0 - 24.2}{2} = \frac{31.8}{2} = 15.9$

Vertical Shift: $\frac{56.0 + 24.2}{2} = \frac{80.2}{2} = 40.1$

$\omega = \frac{2\pi}{12} = \frac{\pi}{6}$

Phase shift (use $y = 24.2$, $x = 1$):

$$24.2 = 15.9\sin\left(\frac{\pi}{6}\cdot 1 - \phi\right) + 40.1$$

$$-15.9 = 15.9\sin\left(\frac{\pi}{6} - \phi\right)$$

$$-1 = \sin\left(\frac{\pi}{6} - \phi\right)$$

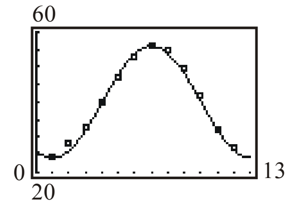
$$-\frac{\pi}{2} = \frac{\pi}{6} - \phi$$

$$\phi = \frac{2\pi}{3}$$

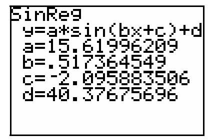
Thus, $y = 15.9\sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 40.1$ or

$y = 15.9\sin\left[\frac{\pi}{6}(x-4)\right] + 40.1$.

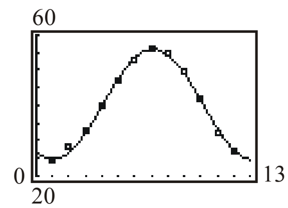
c.



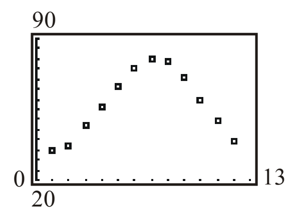
d. $y = 15.62\sin(0.517x - 2.096) + 40.377$



e.



30. a.



b. Amplitude: $A = \frac{80.0 - 34.6}{2} = \frac{45.4}{2} = 22.7$

Vertical Shift: $\frac{80.0 + 34.6}{2} = \frac{114.6}{2} = 57.3$

$\omega = \frac{2\pi}{12} = \frac{\pi}{6}$

Phase shift (use $y = 34.6$, $x = 1$):

$$34.6 = 22.7\sin\left(\frac{\pi}{6}\cdot 1 - \phi\right) + 57.3$$

$$-22.7 = 22.7\sin\left(\frac{\pi}{6} - \phi\right)$$

$$-1 = \sin\left(\frac{\pi}{6} - \phi\right)$$

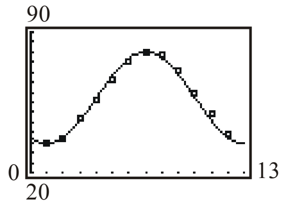
$$-\frac{\pi}{2} = \frac{\pi}{6} - \phi$$

$$\phi = \frac{2\pi}{3}$$

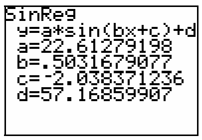
Thus, $y = 22.7\sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 57.3$ or

$y = 22.7\sin\left[\frac{\pi}{6}(x-4)\right] + 57.3$.

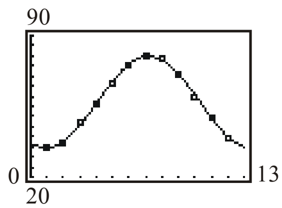
c.



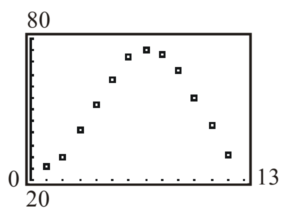
d. $y = 22.61\sin\left(0.503x - 2.038\right) + 57.17$



e.



31. a.



b. Amplitude: $A = \dfrac{75.4 - 25.5}{2} = \dfrac{49.9}{2} = 24.95$

Vertical Shift: $\dfrac{75.4+25.5}{2} = \dfrac{100.9}{2} = 50.45$

$\omega = \dfrac{2\pi}{12} = \dfrac{\pi}{6}$

Phase shift (use $y = 25.5$, $x = 1$):

$$25.5 = 24.95\sin\left(\frac{\pi}{6}\cdot 1 - \phi\right) + 50.45$$

$$-24.95 = 24.95\sin\left(\frac{\pi}{6} - \phi\right)$$

$$-1 = \sin\left(\frac{\pi}{6} - \phi\right)$$

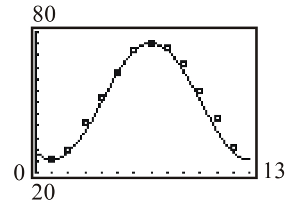
$$-\frac{\pi}{2} = \frac{\pi}{6} - \phi$$

$$\phi = \frac{2\pi}{3}$$

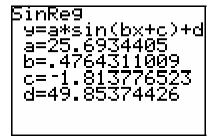
Thus, $y = 24.95\sin\left(\dfrac{\pi}{6}x - \dfrac{2\pi}{3}\right) + 50.45$ or

$y = 24.95\sin\left[\dfrac{\pi}{6}\left(x - 4\right)\right] + 50.45$.

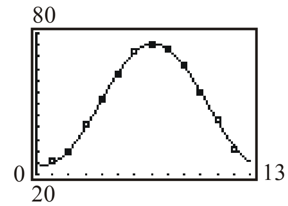
c.



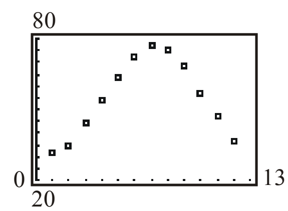
d. $y = 25.693\sin(0.476x - 1.814) + 49.854$



e.



32. a.



b. Amplitude: $A = \dfrac{77.0 - 31.8}{2} = \dfrac{45.2}{2} = 22.6$

Vertical Shift: $\dfrac{77.0+31.8}{2} = \dfrac{108.8}{2} = 54.4$

$\omega = \dfrac{2\pi}{12} = \dfrac{\pi}{6}$

Phase shift (use $y = 31.8$, $x = 1$):

$$31.8 = 22.6\sin\left(\frac{\pi}{6}\cdot 1 - \phi\right) + 54.4$$

$$-22.6 = 22.6\sin\left(\frac{\pi}{6} - \phi\right)$$

$$-1 = \sin\left(\frac{\pi}{6} - \phi\right)$$

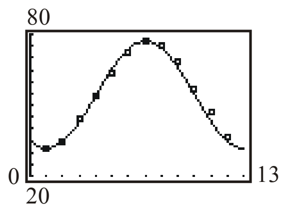
$$-\frac{\pi}{2} = \frac{\pi}{6} - \phi$$

$$\phi = \frac{2\pi}{3}$$

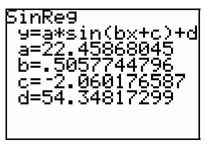
Thus, $y = 22.6\sin\left(\dfrac{\pi}{6}x - \dfrac{2\pi}{3}\right) + 54.4$ or

$y = 22.6\sin\left[\dfrac{\pi}{6}\left(x - 4\right)\right] + 54.4$.

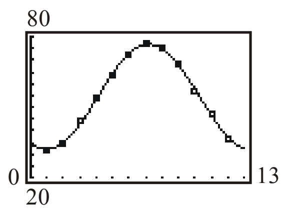
c.



d. $y = 22.46\sin(0.506x - 2.060) + 54.35$



e.



33. a. 3.6333 + 12.5 = 16.1333 hours which is at 4:08 PM.

b. Amplitude: $A = \dfrac{8.2 - (-0.6)}{2} = \dfrac{8.8}{2} = 4.4$

Vertical Shift: $\dfrac{8.2 + (-0.6)}{2} = \dfrac{7.6}{2} = 3.8$

$\omega = \dfrac{2\pi}{12.5} = \dfrac{\pi}{6.25} = \dfrac{4\pi}{25}$

Phase shift (use $y = 8.2$, $x = 3.6333$):

$8.2 = 4.4\sin\left(\dfrac{4\pi}{25}\cdot 3.6333 - \phi\right) + 3.8$

$4.4 = 4.4\sin\left(\dfrac{4\pi}{25}\cdot 3.6333 - \phi\right)$

$1 = \sin\left(\dfrac{14.5332\pi}{25} - \phi\right)$

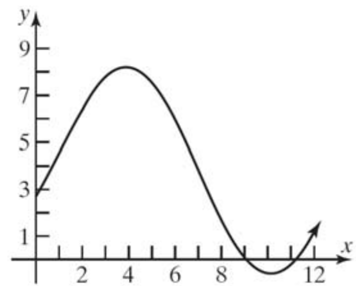
$\dfrac{\pi}{2} = \dfrac{40.5332\pi}{25} - \phi$

$\phi \approx 0.2555$

Thus, $y = 4.4\sin\left(\dfrac{4\pi}{25}x - 0.2555\right) + 3.8$ or

$y = 4.4\sin\left[\dfrac{4\pi}{25}(x - 0.5083)\right] + 3.8$.

c.



d. $y = 4.4\sin\left(\dfrac{4\pi}{25}(16.1333) - 0.2555\right) + 3.8$

≈ 8.2 feet

34. a. 8.1833 + 12.5 = 20.6833 hours which is at 8:41 PM.

b. Amplitude: $A = \dfrac{13.2 - 2.2}{2} = \dfrac{11}{2} = 5.5$

Vertical Shift: $\dfrac{13.2 + 2.2}{2} = \dfrac{15.4}{2} = 7.7$

$\omega = \dfrac{2\pi}{12.5} = \dfrac{\pi}{6.25} = \dfrac{4\pi}{25}$

Phase shift (use $y = 13.2$, $x = 8.1833$):

$13.2 = 5.5\sin\left(\dfrac{4\pi}{25}\cdot 8.1833 - \phi\right) + 7.7$

$5.5 = 5.5\sin\left(\dfrac{4\pi}{25}\cdot 8.1833 - \phi\right)$

$1 = \sin\left(\dfrac{32.7332\pi}{25} - \phi\right)$

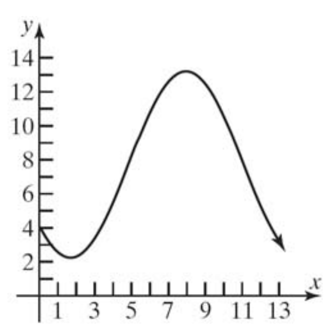
$\dfrac{\pi}{2} = \dfrac{32.7332\pi}{25} - \phi$

$\phi \approx 2.5426$

Thus, $y = 5.5\sin\left(\dfrac{4\pi}{25}x - 2.5426\right) + 7.7$ or

$y = 5.5\sin\left[\dfrac{4\pi}{25}(x - 5.0583)\right] + 7.7$.

c.



d. $y = 5.5\sin\left(\frac{4\pi}{25}(20.6833) - 2.5426\right) + 7.7$

≈ 13.2 feet

35. a. Amplitude: $A = \frac{13.75 - 10.56}{2} = 1.6$

Vertical Shift: $\frac{13.75 + 10.55}{2} = 12.15$

$\omega = \frac{2\pi}{365}$

Phase shift (use $y = 13.75$, $x = 172$):

$$13.75 = 1.6\sin\left(\frac{2\pi}{365}\cdot 172 - \phi\right) + 12.15$$

$$1.6 = 1.6\sin\left(\frac{2\pi}{365}\cdot 172 - \phi\right)$$

$$1 = \sin\left(\frac{344\pi}{365} - \phi\right)$$

$$\frac{\pi}{2} = \frac{344\pi}{365} - \phi$$

$$\phi \approx 1.3900$$

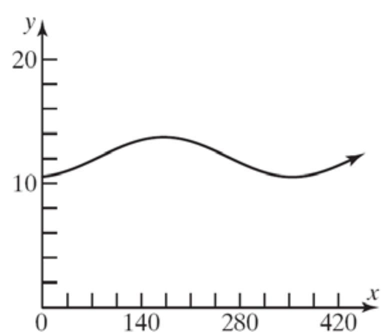
Thus, $y = 1.6\sin\left(\frac{2\pi}{365}x - 1.39\right) + 12.15$ or

$y = 1.6\sin\left[\frac{2\pi}{365}(x - 80.75)\right] + 12.15$.

b. $y = 1.6\sin\left[\frac{2\pi}{365}(91 - 80.75)\right] + 12.15$

≈ 12.43 hours

c.



d. The actual hours of sunlight on April 1, 2007 were 12.43 hours. This is the same as the predicted amount.

36. a. Amplitude: $A = \frac{15.30 - 9.10}{2} = 3.1$

Vertical Shift: $\frac{15.30 + 9.10}{2} = 12.2$

$\omega = \frac{2\pi}{365}$

Phase shift (use $y = 15.30$, $x = 172$):

$$15.30 = 3.1\sin\left(\frac{2\pi}{365}\cdot 172 - \phi\right) + 12.2$$

$$3.1 = 3.1\sin\left(\frac{2\pi}{365}\cdot 172 - \phi\right)$$

$$1 = \sin\left(\frac{344\pi}{365} - \phi\right)$$

$$\frac{\pi}{2} = \frac{344\pi}{365} - \phi$$

$$\phi \approx 1.39$$

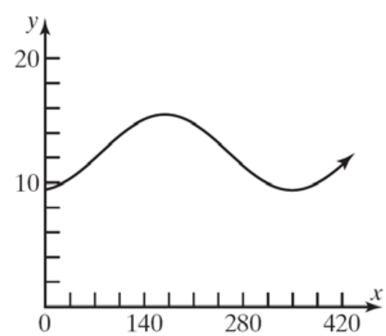
Thus, $y = 3.1\sin\left(\frac{2\pi}{365}x - 1.39\right) + 12.2$ or

$y = 3.1\sin\left[\frac{2\pi}{365}(x - 80.75)\right] + 12.2$.

b. $y = 3.1\sin\left(\frac{2\pi}{365}(91) - 1.39\right) + 12.2$

≈ 12.74 hours

c.



d. The actual hours of sunlight on April 1, 2007 were 12.72 hours. This is very close to the predicted amount of 12.74 hours.

37. a. Amplitude: $A = \frac{19.42 - 5.48}{2} = 6.97$

Vertical Shift: $\frac{19.42 + 5.48}{2} = 12.45$

$\omega = \frac{2\pi}{365}$

Phase shift (use $y = 19.42$, $x = 172$):

$$19.42 = 6.97 \sin\left(\frac{2\pi}{365} \cdot 172 - \phi\right) + 12.45$$

$$6.975 = 6.975 \sin\left(\frac{2\pi}{365} \cdot 172 - \phi\right)$$

$$1 = \sin\left(\frac{344\pi}{365} - \phi\right)$$

$$\frac{\pi}{2} = \frac{344\pi}{365} - \phi$$

$$\phi \approx 1.39$$

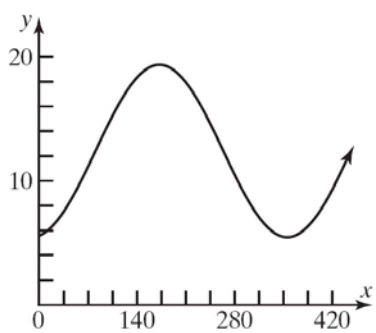
Thus, $y = 6.97 \sin\left(\frac{2\pi}{365} x - 1.39\right) + 12.45$ or

$y = 6.97 \sin\left[\frac{2\pi}{365}(x - 80.75)\right] + 12.45$.

b. $y = 6.97 \sin\left(\frac{2\pi}{365}(91) - 1.39\right) + 12.45$

≈ 13.67 hours

c.



d. The actual hours of sunlight on April 1, 2007 was 13.38 hours. This is close to the predicted amount of 13.67 hours.

38. a. Amplitude: $A = \frac{13.43 - 10.85}{2} = 1.29$

Vertical Shift: $\frac{13.43 + 10.85}{2} = 12.14$

$\omega = \frac{2\pi}{365}$

Phase shift (use $y = 13.43$, $x = 172$):

$$13.43 = 1.29 \sin\left(\frac{2\pi}{365} \cdot 172 - \phi\right) + 12.14$$

$$1.29 = 1.29 \sin\left(\frac{2\pi}{365} \cdot 172 - \phi\right)$$

$$1 = \sin\left(\frac{344\pi}{365} - \phi\right)$$

$$\frac{\pi}{2} = \frac{344\pi}{365} - \phi$$

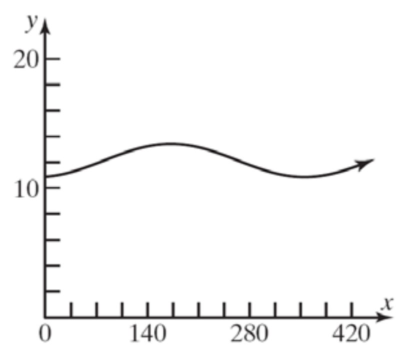
$$\phi \approx 1.39$$

Thus, $y = 1.29 \sin\left(\frac{2\pi}{365} x - 1.39\right) + 12.14$.

b. $y = 1.29 \sin\left(\frac{2\pi}{365}(91) - 1.39\right) + 12.14$

≈ 12.37 hours

c.



d. The actual hours of sunlight on April 1, 2007 were 12.37 hours. This is the same as the predicted amount.

39 – 40. Answers will vary.

Chapter 2 Review Exercises

1. $135° = 135 \cdot \frac{\pi}{180}$ radian $= \frac{3\pi}{4}$ radians

2. $210° = 210 \cdot \frac{\pi}{180}$ radian $= \frac{7\pi}{6}$ radians

3. $18° = 18 \cdot \frac{\pi}{180}$ radian $= \frac{\pi}{10}$ radian

4. $15° = 15 \cdot \frac{\pi}{180}$ radian $= \frac{\pi}{12}$ radian

5. $\frac{3\pi}{4} = \frac{3\pi}{4} \cdot \frac{180}{\pi}$ degrees $= 135°$

6. $\frac{2\pi}{3} = \frac{2\pi}{3} \cdot \frac{180}{\pi}$ degrees $= 120°$

7. $-\frac{5\pi}{2} = -\frac{5\pi}{2} \cdot \frac{180}{\pi}$ degrees $= -450°$

8. $-\frac{3\pi}{2} = -\frac{3\pi}{2} \cdot \frac{180}{\pi}$ degrees $= -270°$

9. $\tan\frac{\pi}{4} - \sin\frac{\pi}{6} = 1 - \frac{1}{2} = \frac{1}{2}$

10. $\cos\frac{\pi}{3} + \sin\frac{\pi}{2} = \frac{1}{2} + 1 = \frac{3}{2}$

11. $3\sin 45° - 4\tan\frac{\pi}{6} = 3 \cdot \frac{\sqrt{2}}{2} - 4 \cdot \frac{\sqrt{3}}{3} = \frac{3\sqrt{2}}{2} - \frac{4\sqrt{3}}{3}$

12. $4\cos 60° + 3\tan\frac{\pi}{3} = 4 \cdot \frac{1}{2} + 3 \cdot \sqrt{3} = 2 + 3\sqrt{3}$

13. $$6\cos\frac{3\pi}{4} + 2\tan\left(-\frac{\pi}{3}\right) = 6\left(-\frac{\sqrt{2}}{2}\right) + 2\left(-\sqrt{3}\right) = -3\sqrt{2} - 2\sqrt{3}$$

14. $3\sin\frac{2\pi}{3} - 4\cos\frac{5\pi}{2} = 3\left(\frac{\sqrt{3}}{2}\right) - 4(0) = \frac{3\sqrt{3}}{2}$

15. $\sec\left(-\frac{\pi}{3}\right) - \cot\left(-\frac{5\pi}{4}\right) = \sec\frac{\pi}{3} + \cot\frac{5\pi}{4} = 2 + 1 = 3$

16. $4\csc\frac{3\pi}{4} - \cot\left(-\frac{\pi}{4}\right) = 4\csc\frac{3\pi}{4} + \cot\frac{\pi}{4} = 4\sqrt{2} + 1$

17. $\tan\pi + \sin\pi = 0 + 0 = 0$

18. $\cos\frac{\pi}{2} - \csc\left(-\frac{\pi}{2}\right) = \cos\frac{\pi}{2} + \csc\frac{\pi}{2} = 0 + 1 = 1$

19. $\cos 540° - \tan(-405°) = -1 - (-1) = -1 + 1 = 0$

20. $\sin 270° + \cos(-180°) = -1 + (-1) = -2$

21. $\sin^2 20° + \frac{1}{\sec^2 20°} = \sin^2 20° + \cos^2 20° = 1$

22. $$\frac{1}{\cos^2 40°} - \frac{1}{\cot^2 40°} = \sec^2 40° - \tan^2 40° = \left(1 + \tan^2 40°\right) - \tan^2 40° = 1$$

23. $\sec 50° \cdot \cos 50° = \frac{1}{\cos 50°} \cdot \cos 50° = 1$

24. $\tan 10° \cdot \cot 10° = \tan 10° \cdot \frac{1}{\tan 10°} = 1$

25. $\frac{\sin 50°}{\cos 40°} = \frac{\sin 50°}{\sin(90° - 40°)} = \frac{\sin 50°}{\sin 50°} = 1$

26. $\frac{\tan 20°}{\cot 70°} = \frac{\tan 20°}{\tan(90° - 70°)} = \frac{\tan 20°}{\tan 20°} = 1$

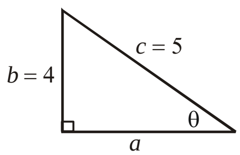
27. $\frac{\sin(-40°)}{\cos 50°} = \frac{-\sin 40°}{\sin(90° - 50°)} = \frac{-\sin 40°}{\sin 40°} = -1$

28. $\tan(-20°)\cot 20° = -\tan 20° \cdot \frac{1}{\tan 20°} = -1$

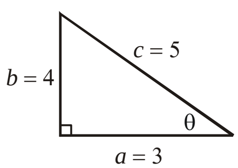
29. $$\sin 400° \cdot \sec(-50°) = \sin(40° + 360°) \cdot \sec 50° = \sin 40° \cdot \frac{1}{\cos 50°} = \sin 40° \cdot \frac{1}{\sin(90° - 50°)} = \sin 40° \cdot \frac{1}{\sin 40°} = 1$$

30. $\cot 200^\circ \cdot \cot(-70^\circ) = \cot(20^\circ + 180^\circ) \cdot (-\cot 70^\circ)$
$$= -\cot 20^\circ \cdot \cot 70^\circ$$
$$= -\cot 20^\circ \cdot \tan(90^\circ - 70^\circ)$$
$$= \frac{-1}{\tan 20^\circ} \cdot \tan(20^\circ)$$
$$= -1$$

31. θ is acute, so θ lies in quadrant I and $\sin\theta = \frac{4}{5}$ corresponds to the right triangle:



Using the Pythagorean Theorem:
$$a^2 + 4^2 = 5^2$$
$$a^2 = 25 - 16 = 9$$
$$a = \sqrt{9} = 3$$
So the triangle is:

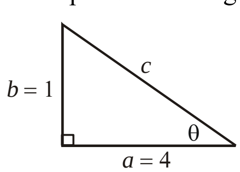


$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5}$ $\qquad$ $\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3}$

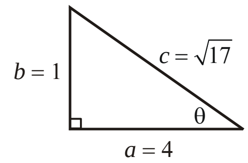
$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$ $\qquad$ $\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4}$

32. θ is acute, so θ lies in quadrant I and $\tan\theta = \frac{1}{4}$ corresponds to the right triangle:



Using the Pythagorean Theorem:
$$c^2 = 1^2 + 4^2 = 1 + 16 = 17$$
$$c = \sqrt{17}$$
So the triangle is:



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}} = \frac{\sqrt{17}}{17}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}} = \frac{4\sqrt{17}}{17}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{17}}{1} = \sqrt{17}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{17}}{4}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{1} = 4$$

33. $\tan\theta = \frac{12}{5}$ and $\sin\theta < 0$, so θ lies in quadrant III.

Using the Pythagorean Identities:
$$\sec^2\theta = \tan^2\theta + 1$$
$$\sec^2\theta = \left(\frac{12}{5}\right)^2 + 1 = \frac{144}{25} + 1 = \frac{169}{25}$$
$$\sec\theta = \pm\sqrt{\frac{169}{25}} = \pm\frac{13}{5}$$
Note that $\sec\theta$ must be negative since θ lies in quadrant III. Thus, $\sec\theta = -\frac{13}{5}$.

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{-\frac{13}{5}} = -\frac{5}{13}$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \text{ so}$$
$$\sin\theta = (\tan\theta)(\cos\theta) = \frac{12}{5}\left(-\frac{5}{13}\right) = -\frac{12}{13}$$
$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-\frac{12}{13}} = -\frac{13}{12}$$
$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{12}{5}} = \frac{5}{12}$$

34. $\cot\theta = \frac{12}{5}$ and $\cos\theta < 0$, so θ lies in quadrant III.

Using the Pythagorean Identities:

$\csc^2\theta = 1 + \cot^2\theta$

$\csc^2\theta = 1 + \left(\frac{12}{5}\right)^2 = 1 + \frac{144}{25} = \frac{169}{25}$

$\csc\theta = \pm\sqrt{\frac{169}{25}} = \pm\frac{13}{5}$

Note that $\csc\theta$ must be negative because θ lies in quadrant III. Thus, $\csc\theta = -\frac{13}{5}$.

$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{-\frac{13}{5}} = -\frac{5}{13}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$, so

$\cos\theta = (\cot\theta)(\sin\theta) = \frac{12}{5}\left(-\frac{5}{13}\right) = -\frac{12}{13}$

$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{\frac{12}{5}} = \frac{5}{12}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{12}{13}} = -\frac{13}{12}$

35. $\sec\theta = -\frac{5}{4}$ and $\tan\theta < 0$, so θ lies in quadrant II.

Using the Pythagorean Identities:

$\tan^2\theta = \sec^2\theta - 1$

$\tan^2\theta = \left(-\frac{5}{4}\right)^2 - 1 = \frac{25}{16} - 1 = \frac{9}{16}$

$\tan\theta = \pm\sqrt{\frac{9}{16}} = \pm\frac{3}{4}$

Note that $\tan\theta < 0$, so $\tan\theta = -\frac{3}{4}$.

$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{-\frac{5}{4}} = -\frac{4}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, so

$\sin\theta = (\tan\theta)(\cos\theta) = -\frac{3}{4}\left(-\frac{4}{5}\right) = \frac{3}{5}$.

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\frac{3}{4}} = -\frac{4}{3}$

36. $\csc\theta = -\frac{5}{3}$ and $\cot\theta < 0$, so θ lies in quadrant IV.

Using the Pythagorean Identities:

$\cot^2\theta = \csc^2\theta - 1$

$\cot^2\theta = \left(-\frac{5}{3}\right)^2 - 1 = \frac{25}{9} - 1 = \frac{16}{9}$

$\cot\theta = \pm\sqrt{\frac{16}{9}} = \pm\frac{4}{3}$

Note that $\cot\theta < 0$, so $\cot\theta = -\frac{4}{3}$.

$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{-\frac{5}{3}} = -\frac{3}{5}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$, so

$\cos\theta = (\cot\theta)(\sin\theta) = -\frac{4}{3}\left(-\frac{3}{5}\right) = \frac{4}{5}$

$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{-\frac{4}{3}} = -\frac{3}{4}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$

37. $\sin\theta = \frac{12}{13}$ and θ lies in quadrant II.

Using the Pythagorean Identities:

$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta = 1 - \left(\frac{12}{13}\right)^2 = 1 - \frac{144}{169} = \frac{25}{169}$

$\cos\theta = \pm\sqrt{\frac{25}{169}} = \pm\frac{5}{13}$

Note that $\cos\theta$ must be negative because θ lies in quadrant II. Thus, $\cos\theta = -\frac{5}{13}$.

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{12}{13}}{-\frac{5}{13}} = \frac{12}{13}\left(-\frac{13}{5}\right) = -\frac{12}{5}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{12}{13}} = \frac{13}{12}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{5}{13}} = -\frac{13}{5}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\frac{12}{5}} = -\frac{5}{12}$

38. $\cos\theta = -\frac{3}{5}$ and θ lies in quadrant III.

Using the Pythagorean Identities:

$\sin^2\theta = 1-\cos^2\theta$

$\sin^2\theta = 1-\left(-\frac{3}{5}\right)^2 = 1-\frac{9}{25} = \frac{16}{25}$

$\sin\theta = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$

Note that $\sin\theta$ must be negative because θ lies in quadrant III. Thus, $\sin\theta = -\frac{4}{5}$.

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{4}{5}}{-\frac{3}{5}} = -\frac{4}{5}\left(-\frac{5}{3}\right) = \frac{4}{3}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-\frac{4}{5}} = -\frac{5}{4}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{3}{5}} = -\frac{5}{3}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$

39. $\sin\theta = -\frac{5}{13}$ and $\frac{3\pi}{2} < \theta < 2\pi$ (quadrant IV)

Using the Pythagorean Identities:

$\cos^2\theta = 1-\sin^2\theta$

$\cos^2\theta = 1-\left(-\frac{5}{13}\right)^2 = 1-\frac{25}{169} = \frac{144}{169}$

$\cos\theta = \pm\sqrt{\frac{144}{169}} = \pm\frac{12}{13}$

Note that $\cos\theta$ must be positive because θ lies in quadrant IV. Thus, $\cos\theta = \frac{12}{13}$.

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{5}{13}}{\frac{12}{13}} = -\frac{5}{13}\left(\frac{13}{12}\right) = -\frac{5}{12}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-\frac{5}{13}} = -\frac{13}{5}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{12}{13}} = \frac{13}{12}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\frac{5}{12}} = -\frac{12}{5}$

40. $\cos\theta = \frac{12}{13}$ and $\frac{3\pi}{2} < \theta < 2\pi$ (quadrant IV)

Using the Pythagorean Identities:

$\sin^2\theta = 1-\cos^2\theta$

$\sin^2\theta = 1-\left(\frac{12}{13}\right)^2 = 1-\frac{144}{169} = \frac{25}{169}$

$\sin\theta = \pm\sqrt{\frac{25}{169}} = \pm\frac{5}{13}$

Note that $\sin\theta$ must be negative because θ lies in quadrant IV. Thus, $\sin\theta = -\frac{5}{13}$.

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{5}{13}}{\frac{12}{13}} = -\frac{5}{13}\left(\frac{13}{12}\right) = -\frac{5}{12}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-\frac{5}{13}} = -\frac{13}{5}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{12}{13}} = \frac{13}{12}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\frac{5}{12}} = -\frac{12}{5}$

41. $\tan\theta = \frac{1}{3}$ and $180^\circ < \theta < 270^\circ$ (quadrant III)

Using the Pythagorean Identities:

$\sec^2\theta = \tan^2\theta + 1$

$\sec^2\theta = \left(\frac{1}{3}\right)^2 + 1 = \frac{1}{9} + 1 = \frac{10}{9}$

$\sec\theta = \pm\sqrt{\frac{10}{9}} = \pm\frac{\sqrt{10}}{3}$

Note that $\sec\theta$ must be negative since θ lies in quadrant III. Thus, $\sec\theta = -\frac{\sqrt{10}}{3}$.

$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{-\frac{\sqrt{10}}{3}} = -\frac{3}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, so

$\sin\theta = (\tan\theta)(\cos\theta) = \frac{1}{3}\left(-\frac{3\sqrt{10}}{10}\right) = -\frac{\sqrt{10}}{10}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-\frac{\sqrt{10}}{10}} = -\frac{10}{\sqrt{10}} = -\sqrt{10}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{1}{3}} = 3$

42. $\tan\theta = -\frac{2}{3}$ and $90^\circ < \theta < 180^\circ$ (quadrant II)

Using the Pythagorean Identities:

$\sec^2\theta = \tan^2\theta + 1$

$\sec^2\theta = \left(-\frac{2}{3}\right)^2 + 1 = \frac{4}{9} + 1 = \frac{13}{9}$

$\sec\theta = \pm\sqrt{\frac{13}{9}} = \pm\frac{\sqrt{13}}{3}$

Note that $\sec\theta$ must be negative since θ lies in quadrant II. Thus, $\sec\theta = -\frac{\sqrt{13}}{3}$.

$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{-\frac{\sqrt{13}}{3}} = -\frac{3}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, so

$\sin\theta = (\tan\theta)(\cos\theta) = -\frac{2}{3}\left(-\frac{3\sqrt{13}}{13}\right) = \frac{2\sqrt{13}}{13}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{2\sqrt{13}}{13}} = \frac{13}{2\sqrt{13}} = \frac{\sqrt{13}}{2}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\frac{2}{3}} = -\frac{3}{2}$

43. $\sec\theta = 3$ and $\frac{3\pi}{2} < \theta < 2\pi$ (quadrant IV)

Using the Pythagorean Identities:

$\tan^2\theta = \sec^2\theta - 1$

$\tan^2\theta = 3^2 - 1 = 9 - 1 = 8$

$\tan\theta = \pm\sqrt{8} = \pm 2\sqrt{2}$

Note that $\tan\theta$ must be negative since θ lies in quadrant IV. Thus,. $\tan\theta = -2\sqrt{2}$.

$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, so

$\sin\theta = (\tan\theta)(\cos\theta) = -2\sqrt{2}\left(\frac{1}{3}\right) = -\frac{2\sqrt{2}}{3}$.

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-\frac{2\sqrt{2}}{3}} = -\frac{3}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{2} = -\frac{3\sqrt{2}}{4}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{4}$

44. $\csc\theta = -4$ and $\pi < \theta < \frac{3\pi}{2}$ (quadrant III)

Using the Pythagorean Identities:

$\cot^2\theta = \csc^2\theta - 1$

$\cot^2\theta = (-4)^2 - 1 = 16 - 1 = 15$

$\cot\theta = \pm\sqrt{15}$

Note that $\cot\theta$ must be positive since θ lies in quadrant III. Thus, $\cot\theta = \sqrt{15}$.

$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{-4} = -\frac{1}{4}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$, so

$\cos\theta = (\cot\theta)(\sin\theta) = \sqrt{15}\left(-\frac{1}{4}\right) = -\frac{\sqrt{15}}{4}$

$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{\sqrt{15}}\cdot\frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{15}}{15}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{\sqrt{15}}{4}} = -\frac{4}{\sqrt{15}}\cdot\frac{\sqrt{15}}{\sqrt{15}} = -\frac{4\sqrt{15}}{15}$

45. $\cot\theta = -2$ and $\frac{\pi}{2} < \theta < \pi$ (quadrant II)

Using the Pythagorean Identities:

$\csc^2\theta = 1 + \cot^2\theta$

$\csc^2\theta = 1 + (-2)^2 = 1 + 4 = 5$

$\csc\theta = \pm\sqrt{5}$

Note that $\csc\theta$ must be positive because θ lies in quadrant II. Thus, $\csc\theta = \sqrt{5}$.

$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$, so

$\cos\theta = (\cot\theta)(\sin\theta) = -2\left(\frac{\sqrt{5}}{5}\right) = -\frac{2\sqrt{5}}{5}$.

$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{-2} = -\frac{1}{2}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{2\sqrt{5}}{5}} = -\frac{5}{2\sqrt{5}} = -\frac{\sqrt{5}}{2}$

46. $\tan\theta = -2$ and $\frac{3\pi}{2} < \theta < 2\pi$ (quadrant IV)

Using the Pythagorean Identities:

$\sec^2\theta = \tan^2\theta + 1$

$\sec^2\theta = (-2)^2 + 1 = 4 + 1 = 5$

$\sec\theta = \pm\sqrt{5}$

Note that $\sec\theta$ must be positive since θ lies in quadrant IV. Thus, $\sec\theta = \sqrt{5}$.

$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, so

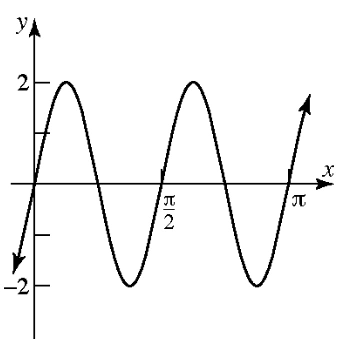
$\sin\theta = (\tan\theta)(\cos\theta) = -2\left(\frac{\sqrt{5}}{5}\right) = -\frac{2\sqrt{5}}{5}$.

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-\frac{2\sqrt{5}}{5}} = -\frac{5}{2\sqrt{5}} = -\frac{\sqrt{5}}{2}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-2} = -\frac{1}{2}$

47. $y = 2\sin(4x)$

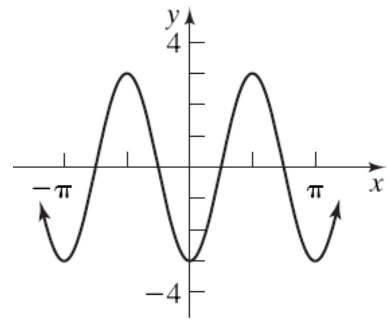
The graph of $y = \sin x$ is stretched vertically by a factor of 2 and compressed horizontally by a factor of $\frac{1}{4}$.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -2 \le y \le 2\}$ or $[-2, 2]$.

48. $y = -3\cos(2x)$

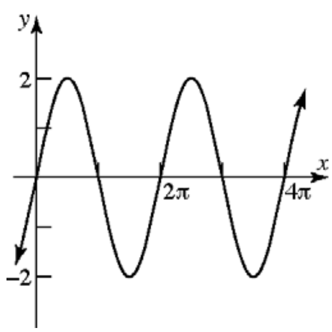
The graph of $y = \cos x$ is stretched vertically by a factor of 3, reflected across the x-axis, and compressed horizontally by a factor of $\frac{1}{2}$.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -3 \le y \le 3\}$ or $[-3, 3]$.

49. $y = -2\cos\left(x + \frac{\pi}{2}\right)$

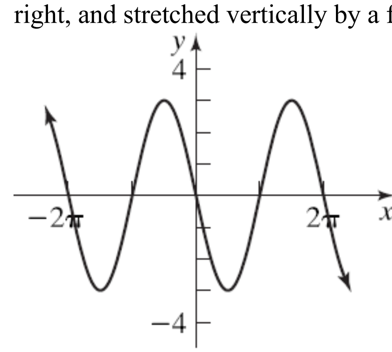
The graph of $y = \cos x$ is shifted $\frac{\pi}{2}$ units to the left, stretched vertically by a factor of 2, and reflected across the x-axis.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -2 \le y \le 2\}$ or $[-2, 2]$.

50. $y = 3\sin(x - \pi)$

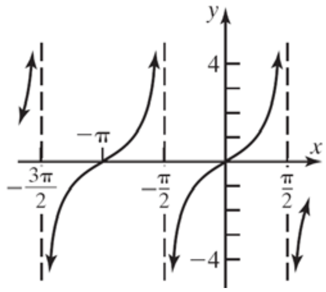
The graph of $y = \sin x$ is shifted π units to the right, and stretched vertically by a factor of 3.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -3 \le y \le 3\}$ or $[-3, 3]$.

51. $y = \tan(x + \pi)$

The graph of $y = \tan x$ is shifted π units to the left.

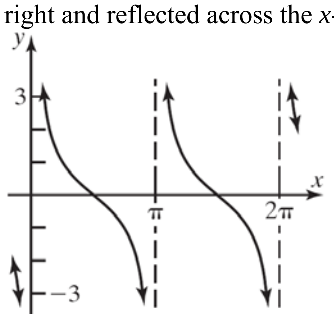


The domain is $\left\{x \middle| x \neq \frac{k\pi}{2}, k \text{ is an odd integer}\right\}$.

The range is the set of all real number or $(-\infty, \infty)$.

52. $y = -\tan\left(x - \frac{\pi}{2}\right)$

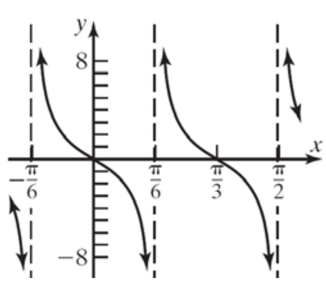
The graph of $y = \tan x$ is shifted $\frac{\pi}{2}$ units to the right and reflected across the *x*-axis.



The domain is $\{x | x \neq k\pi, k \text{ is an integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

53. $y = -2\tan(3x)$

The graph of $y = \tan x$ is stretched vertically by a factor of 2, reflected across the *x*-axis, and compressed horizontally by a factor of $\frac{1}{3}$.

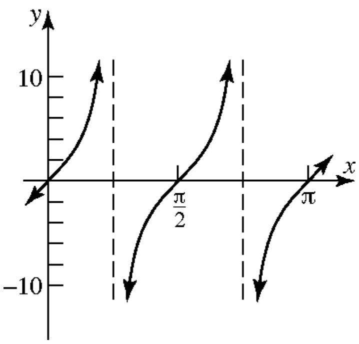


The domain is $\left\{x \middle| x \neq \frac{k\pi}{6}, k \text{ is an odd integer}\right\}$.

The range is the set of all real number or $(-\infty, \infty)$.

54. $y = 4\tan(2x)$

The graph of $y = \tan x$ is stretched vertically by a factor of 4 and compressed horizontally by a factor of $\frac{1}{2}$.

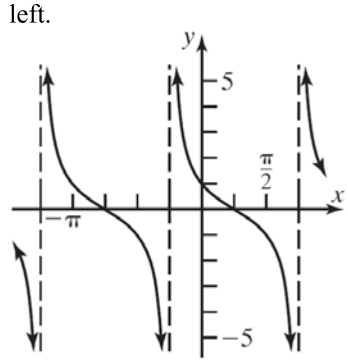


The domain is $\left\{x \middle| x \neq \frac{k\pi}{4}, k \text{ is an odd integer}\right\}$.

The range is the set of all real number or $(-\infty, \infty)$.

55. $y = \cot\left(x + \frac{\pi}{4}\right)$

The graph of $y = \cot x$ is shifted $\frac{\pi}{4}$ units to the left.

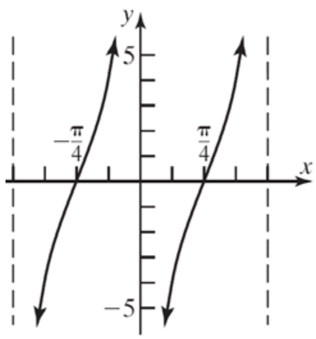


The domain is $\left\{x \middle| x \neq \frac{(4k-1)\pi}{4}, k \text{ is an integer}\right\}$.

The range is the set of all real number or $(-\infty, \infty)$.

56. $y = -4\cot(2x)$

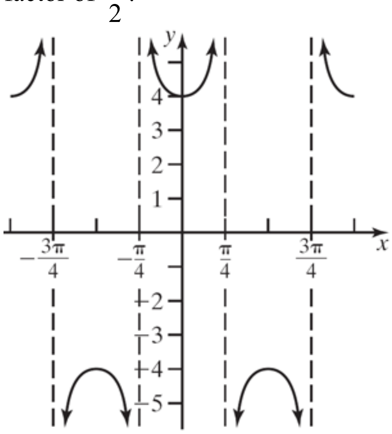
The graph of $y = \cot x$ is stretched vertically by a factor of 4, reflected across the *x*-axis and compressed horizontally by a factor of $\frac{1}{2}$.



The domain is $\left\{x \middle| x \neq \frac{k\pi}{2}, k \text{ is an integer}\right\}$. The range is the set of all real number or $(-\infty, \infty)$.

57. $y = 4\sec(2x)$

The graph of $y = \sec x$ is stretched vertically by a factor of 4 and compressed horizontally by a factor of $\frac{1}{2}$.

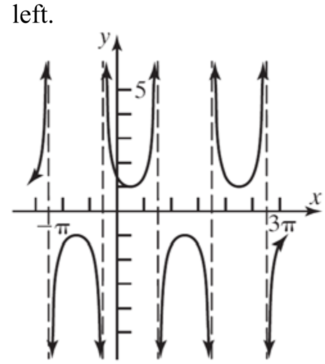


The domain is $\left\{x \middle| x \neq \frac{k\pi}{4}, k \text{ is an odd integer}\right\}$.

The range is $\{y | y \leq -4 \text{ or } y \geq 4\}$.

58. $y = \csc\left(x + \frac{\pi}{4}\right)$

The graph of $y = \csc x$ is shifted $\frac{\pi}{4}$ units to the left.

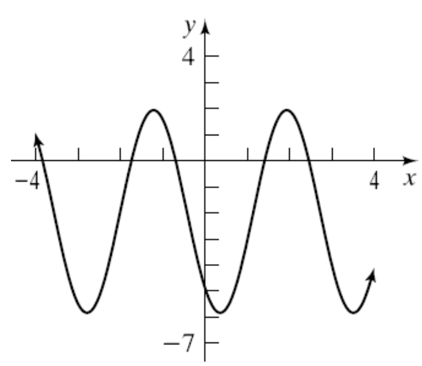


The domain is $\left\{x \middle| x \neq \frac{(4k+3)\pi}{4}, k \text{ is an integer}\right\}$.

The range is $\{y | y \leq -1 \text{ or } y \geq 1\}$.

59. $y = 4\sin(2x + 4) - 2$

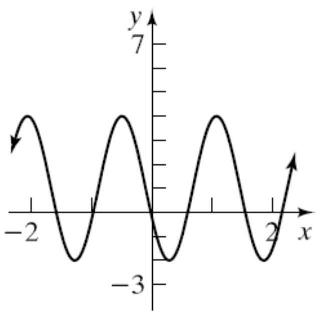
The graph of $y = \sin x$ is shifted left 4 units, compressed horizontally by a factor of $\frac{1}{2}$, stretched vertically by a factor of 4, and shifted down 2 units.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -6 \leq y \leq 2\}$ or $[-6, 2]$.

60. $y = 3\cos(4x+2)+1$

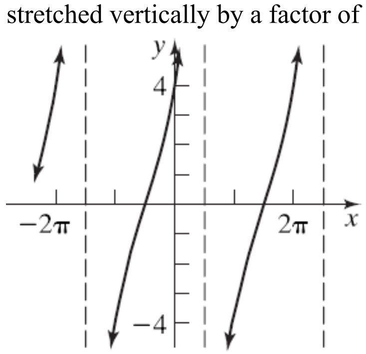
The graph of $y = \cos x$ is shifted left 2 units, compressed horizontally by a factor of $\frac{1}{4}$, stretched vertically by a factor of 3, and shifted up 1 unit.



The domain is the set of all real number or $(-\infty, \infty)$. The range is $\{y \mid -2 \le y \le 4\}$ or $[-2, 4]$.

61. $y = 4\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)$

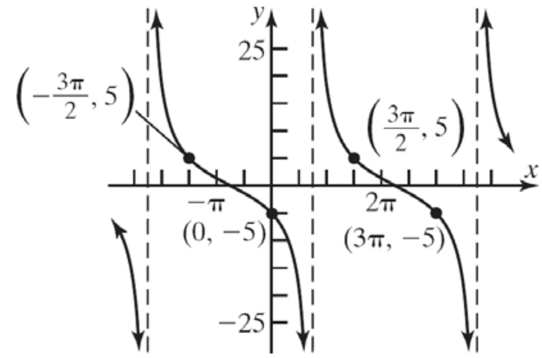
The graph of $y = \tan x$ is stretched horizontally by a factor of 2, shifted left $\frac{\pi}{4}$ units, and stretched vertically by a factor of 4.



The domain is $\left\{x \,\middle|\, x \ne \frac{(2k-1)\pi}{2}, k \text{ is an odd integer}\right\}$. The range is the set of all real number or $(-\infty, \infty)$.

62. $y = 5\cot\left(\frac{x}{3}-\frac{\pi}{4}\right)$

The graph of $y = \cot x$ is shifted right $\frac{\pi}{4}$ units, stretched horizontally by a factor of 3, and stretched vertically by a factor of 5.



The domain is $\left\{x \,\middle|\, x \ne \frac{(12k-9)\pi}{4}, k \text{ is an integer}\right\}$.

The range is the set of all real number or $(-\infty, \infty)$.

63. $y = 4\cos x$

Amplitude $= |4| = 4$; Period $= 2\pi$

64. $y = \sin(2x)$

Amplitude $= |1| = 1$; Period $= \frac{2\pi}{2} = \pi$

65. $y = -8\sin\left(\frac{\pi}{2}x\right)$

Amplitude $= |-8| = 8$; Period $= \frac{2\pi}{\frac{\pi}{2}} = 4$

66. $y = -2\cos(3\pi x)$

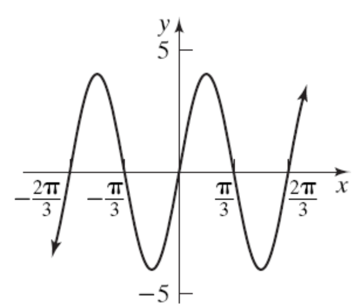
Amplitude $= |-2| = 2$; Period $= \frac{2\pi}{3\pi} = \frac{2}{3}$

67. $y = 4\sin(3x)$

Amplitude: $|A| = |4| = 4$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{3}$

Phase Shift: $\frac{\phi}{\omega} = \frac{0}{3} = 0$

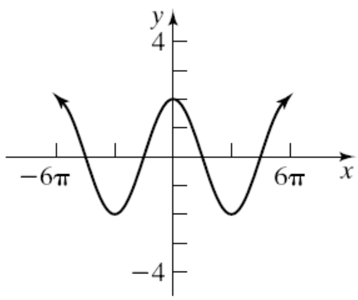


68. $y = 2\cos\left(\frac{1}{3}x\right)$

Amplitude: $|A| = |2| = 2$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{1}{3}} = 6\pi$

Phase Shift: $\frac{\phi}{\omega} = \frac{0}{\frac{1}{3}} = 0$

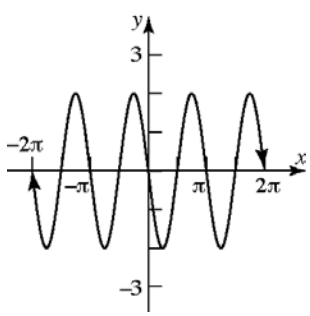


69. $y = 2\sin(2x - \pi)$

Amplitude: $|A| = |2| = 2$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

Phase Shift: $\frac{\phi}{\omega} = \frac{\pi}{2} = \frac{\pi}{2}$

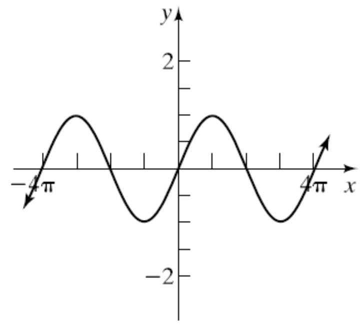


70. $y = -\cos\left(\frac{1}{2}x + \frac{\pi}{2}\right)$

Amplitude: $|A| = |-1| = 1$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{1}{2}} = 4\pi$

Phase Shift: $\frac{\phi}{\omega} = \frac{-\frac{\pi}{2}}{\frac{1}{2}} = -\pi$

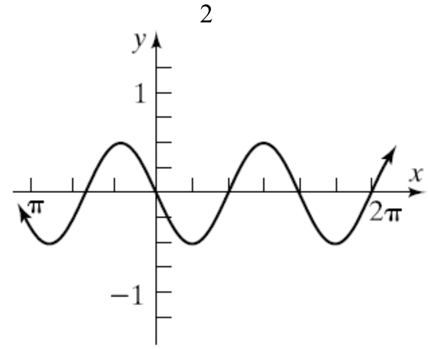


71. $y = \frac{1}{2}\sin\left(\frac{3}{2}x - \pi\right)$

Amplitude: $|A| = \left|\frac{1}{2}\right| = \frac{1}{2}$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{3}{2}} = \frac{4\pi}{3}$

Phase Shift: $\frac{\phi}{\omega} = \frac{\pi}{\frac{3}{2}} = \frac{2\pi}{3}$

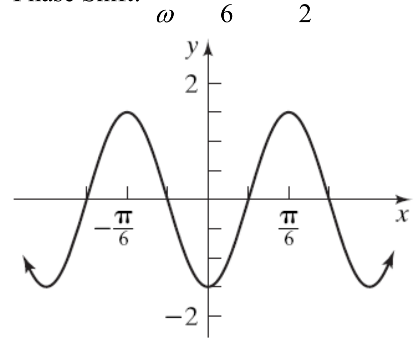


72. $y = \frac{3}{2}\cos(6x + 3\pi)$

Amplitude: $|A| = \left|\frac{3}{2}\right| = \frac{3}{2}$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{6} = \frac{\pi}{3}$

Phase Shift: $\frac{\phi}{\omega} = \frac{-3\pi}{6} = -\frac{\pi}{2}$

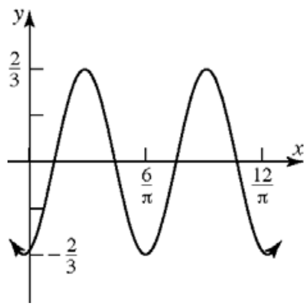


73. $y=-\frac{2}{3}\cos(\pi x-6)$

Amplitude: $|A|=\left|-\frac{2}{3}\right|=\frac{2}{3}$

Period: $T=\frac{2\pi}{\omega}=\frac{2\pi}{\pi}=2$

Phase Shift: $\frac{\phi}{\omega}=\frac{6}{\pi}$

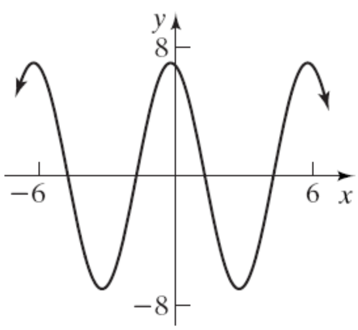


74. $y=-7\sin\left(\frac{\pi}{3}x+\frac{4}{3}\right)$

Amplitude: $|A|=|-7|=7$

Period: $T=\frac{2\pi}{\omega}=\frac{2\pi}{\frac{\pi}{3}}=6$

Phase Shift: $\frac{\phi}{\omega}=\frac{\frac{4}{3}}{\frac{\pi}{3}}=\frac{4}{\pi}$



75. The graph is a cosine graph with amplitude 5 and period 8π.

Find ω: $8\pi=\frac{2\pi}{\omega}$

$8\pi\omega=2\pi$

$\omega=\frac{2\pi}{8\pi}=\frac{1}{4}$

The equation is: $y=5\cos\left(\frac{x}{4}\right)$.

76. The graph is a sine graph with amplitude 4 and period 8π.

Find ω: $8\pi=\frac{2\pi}{\omega}$

$8\pi\omega=2\pi$

$\omega=\frac{2\pi}{8\pi}=\frac{1}{4}$

The equation is: $y=4\sin\left(\frac{x}{4}\right)$.

77. The graph is a reflected cosine graph with amplitude 6 and period 8.

Find ω: $8=\frac{2\pi}{\omega}$

$8\omega=2\pi$

$\omega=\frac{2\pi}{8}=\frac{\pi}{4}$

The equation is: $y=-6\cos\left(\frac{\pi}{4}x\right)$.

78. The graph is a reflected sine graph with amplitude 7 and period 8.

Find ω: $8=\frac{2\pi}{\omega}$

$8\omega=2\pi$

$\omega=\frac{2\pi}{8}=\frac{\pi}{4}$

The equation is: $y=-7\sin\left(\frac{\pi}{4}x\right)$.

79. hypotenuse = 13; adjacent = 12
Find the opposite side:

$12^2+(\text{opposite})^2=13^2$

$(\text{opposite})^2=169-144=25$

$\text{opposite}=\sqrt{25}=5$

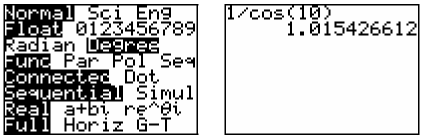
$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{5}{13}$ $\quad\csc\theta=\frac{\text{hyp}}{\text{opp}}=\frac{13}{5}$

$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{12}{13}$ $\quad\sec\theta=\frac{\text{hyp}}{\text{adj}}=\frac{13}{12}$

$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{5}{12}$ $\quad\cot\theta=\frac{\text{adj}}{\text{opp}}=\frac{12}{5}$

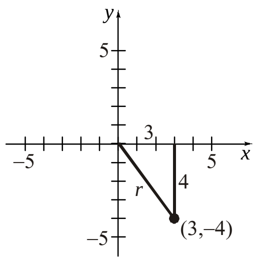
80. Set the calculator to degree mode:

$\sec 10^{\circ} \approx 1.02$.



81. $(3,-4)$; $a=3,\ b=-4$;

$r=\sqrt{a^2+b^2}=\sqrt{3^2+(-4)^2}=\sqrt{9+16}=\sqrt{25}=5$



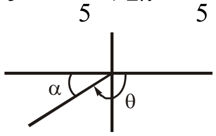
$\sin\theta=\dfrac{b}{r}=-\dfrac{4}{5}$ $\qquad \csc\theta=\dfrac{r}{b}=-\dfrac{5}{4}$

$\cos\theta=\dfrac{a}{r}=\dfrac{3}{5}$ $\qquad \sec\theta=\dfrac{r}{a}=\dfrac{5}{3}$

$\tan\theta=\dfrac{b}{a}=-\dfrac{4}{3}$ $\qquad \cot\theta=\dfrac{a}{b}=-\dfrac{3}{4}$

82. $\cos\theta>0,\ \tan\theta<0$; θ lies in quadrant IV.

83. $\theta=-\dfrac{4\pi}{5}+2\pi=\dfrac{6\pi}{5}$, so θ lies in quadrant III.



Reference angle: $\alpha=\dfrac{6\pi}{5}-\pi=\dfrac{\pi}{5}$

84. $P=\left(-\dfrac{3}{5},\dfrac{4}{5}\right)$

$\sin t=\dfrac{4}{5}$ $\qquad \cos t=-\dfrac{3}{5}$ $\qquad \tan t=\dfrac{\frac{4}{5}}{-\frac{3}{5}}=-\dfrac{4}{3}$

85. The domain of $y=\sec x$ is $\left\{x \,\middle|\, x\neq\dfrac{k\pi}{2},\ k \text{ is an odd integer}\right\}$.

The range of $y=\sec x$ is $\{y \mid y\le -1 \text{ or } y\ge 1\}$.

86. a. $32^{\circ}20'35''=32+\dfrac{20}{60}+\dfrac{35}{3600}\approx 32.34^{\circ}$

b. 63.18°

$0.18^{\circ}=(0.18)(60')=10.8'$

$0.8'=(0.8)(60'')=48''$

Thus, $63.18^{\circ}=63^{\circ}10'48''$

87. $r=2$ feet, $\theta=30^{\circ}$ or $\theta=\dfrac{\pi}{6}$

$s=r\theta=2\cdot\dfrac{\pi}{6}=\dfrac{\pi}{3}\approx 1.047$ feet

$A=\dfrac{1}{2}\cdot r^2\theta=\dfrac{1}{2}\cdot(2)^2\cdot\dfrac{\pi}{6}=\dfrac{\pi}{3}\approx 1.047$ square feet

88. $r=8$ inches, $\theta=180^{\circ}$ or $\theta=\pi$

$s=r\theta=8\cdot\pi=8\pi\approx 25.13$ inches in 30 minutes

$r=8$ inches, $\theta=120^{\circ}$ or $\theta=\dfrac{2\pi}{3}$

$s=r\theta=8\cdot\dfrac{2\pi}{3}=\dfrac{16\pi}{3}\approx 16.76$ inches in 20 minutes

89. $v=180$ mi/hr; $\quad d=\dfrac{1}{2}$ mile

$r=\dfrac{1}{4}=0.25$ mile

$$\begin{aligned}\omega&=\frac{v}{r}=\frac{180\text{ mi/hr}}{0.25\text{ mi}}\\ &=720\text{ rad/hr}\\ &=\frac{720\text{ rad}}{\text{hr}}\cdot\frac{1\text{ rev}}{2\pi\text{ rad}}\\ &=\frac{360\text{ rev}}{\pi\text{ hr}}\\ &\approx 114.59\text{ rev/hr}\end{aligned}$$

90. $r=25$ feet;

$\omega=\dfrac{1\text{ rev}}{30\text{ sec}}=\dfrac{1\text{ rev}}{30\text{ sec}}\cdot\dfrac{2\pi\text{ radians}}{1\text{ rev}}=\dfrac{\pi}{15}$ rad/sec

$v=r\omega=25\cdot\dfrac{\pi}{15}=\dfrac{5\pi}{3}$ ft/sec ≈ 5.24 ft/sec.

The linear speed is approximately 5.24 feet per second; the angular speed is $\dfrac{1}{30}$ revolution per second, or $\dfrac{\pi}{15}$ radian per second.

91. Since there are two lights on opposite sides and the light is seen every 5 seconds, the beacon makes 1 revolution every 10 seconds:

$$\omega = \frac{1 \text{ rev}}{10 \text{ sec}} \cdot \frac{2\pi \text{ radians}}{1 \text{ rev}} = \frac{\pi}{5} \text{ radians/second}$$

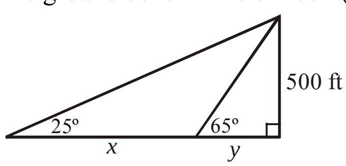
92. $r = 16$ inches; $v = 90$ mi/hr

$$\omega = \frac{v}{r}$$
$$= \frac{90 \text{ mi/hr}}{16 \text{ in}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}}$$
$$\approx 945.38 \text{ rev/min}$$

Yes, the setting will be different for a wheel of radius 14 inches:
$r = 14$ inches; $v = 90$ mi/hr

$$\omega = \frac{v}{r}$$
$$= \frac{90 \text{ mi/hr}}{14 \text{ in}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}}$$
$$\approx 1080.43 \text{ rev/min}$$

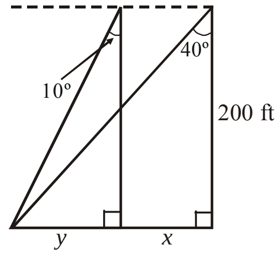
93. Let x = the length of the lake, and let y = the distance from the edge of the lake to the point on the ground beneath the balloon (see figure).



$$\tan(65^\circ) = \frac{500}{x}$$
$$x = \frac{500}{\tan(65^\circ)}$$
$$\tan(25^\circ) = \frac{500}{x+y}$$
$$x + y = \frac{500}{\tan(25^\circ)}$$
$$x = \frac{500}{\tan(25^\circ)} - y$$
$$= \frac{500}{\tan(25^\circ)} - \frac{500}{\tan(65^\circ)}$$
$$\approx 1072.25 - 233.15 = 839.10$$

Thus, the length of the lake is approximately 839.10 feet.

94. Let x = the distance traveled by the glider between the two sightings, and let y = the distance from the stationary object to a point on the ground beneath the glider at the time of the second sighting (see figure).



$$\tan(10^\circ) = \frac{y}{200}$$
$$y = 200\tan(10^\circ)$$
$$\tan(40^\circ) = \frac{x+y}{200}$$
$$x + y = 200\tan(40^\circ)$$
$$x = 200\tan(40^\circ) - y$$
$$= 200\tan(40^\circ) - 200\tan(10^\circ)$$
$$\approx 167.82 - 35.27 = 132.55$$

The glider traveled 132.55 feet in 1 second, so the speed of the glider is 132.55 feet per second.

95. Let x = the distance across the river.

$$\tan(25^\circ) = \frac{x}{50}$$
$$x = 50\tan(25^\circ) \approx 23.32$$

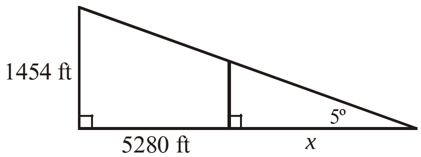
Thus, the distance across the river is 23.32 feet.

96. Let h = the height of the building.

$$\tan(25^\circ) = \frac{h}{80}$$
$$h = 80\tan(25^\circ) \approx 37.30$$

Thus, the height of the building is 37.30 feet.

97. Let x = the distance the boat is from shore (see figure). Note that 1 mile = 5280 feet.



$$\tan(5°) = \frac{1454}{x + 5280}$$

$$x + 5280 = \frac{1454}{\tan(5°)}$$

$$x = \frac{1454}{\tan(5°)} - 5280$$

$$\approx 16{,}619.30 - 5280 = 11{,}339.30$$

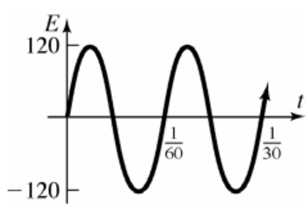
Thus, the boat is approximately 11,339.30 feet, or $\frac{11{,}339.30}{5280} \approx 2.15$ miles, from shore.

98. $E(t) = 120\sin(120\pi t), \quad t \ge 0$

a. The maximum value of E is the amplitude, which is 120 volts.

b. Period $= \frac{2\pi}{120\pi} = \frac{1}{60}$ second

c.



99. $I(t) = 220\sin\left(30\pi t + \frac{\pi}{6}\right), \quad t \ge 0$

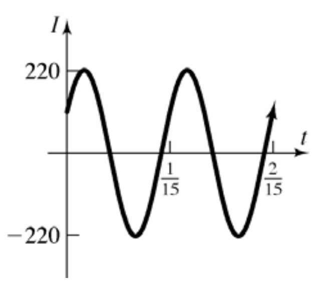
a. Period $= \frac{2\pi}{30\pi} = \frac{1}{15}$ second

b. The amplitude is 220 amperes.

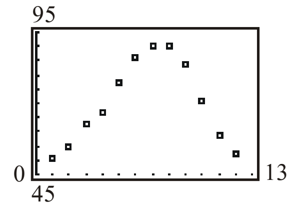
c. The phase shift is:

$$\frac{\phi}{\omega} = \frac{-\frac{\pi}{6}}{30\pi} = -\frac{\pi}{6} \cdot \frac{1}{30\pi} = -\frac{1}{180} \text{ second}$$

d.



100. a.



b. Amplitude: $A = \frac{90 - 51}{2} = \frac{39}{2} = 19.5$

Vertical Shift: $\frac{90+51}{2} = \frac{141}{2} = 70.5$

$\omega = \frac{2\pi}{12} = \frac{\pi}{6}$

Phase shift (use $y = 51$, $x = 1$):

$$51 = 19.5\sin\left(\frac{\pi}{6} \cdot 1 - \phi\right) + 70.5$$

$$-19.5 = 19.5\sin\left(\frac{\pi}{6} - \phi\right)$$

$$-1 = \sin\left(\frac{\pi}{6} - \phi\right)$$

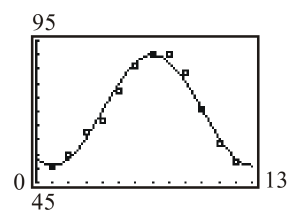
$$-\frac{\pi}{2} = \frac{\pi}{6} - \phi$$

$$\phi = \frac{2\pi}{3}$$

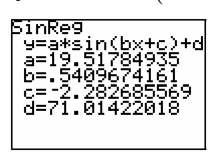
Thus, $y = 19.5\sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 70.5$, or

$y = 19.5\sin\left[\frac{\pi}{6}(x - 4)\right] + 70.5$.

c.

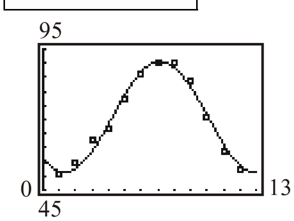


d. $y = 19.52\sin(0.54x - 2.28) + 71.01$



SinReg
y=a*sin(bx+c)+d
a=19.51784935
b=.5409674161
c=-2.282685569
d=71.01422018

e.



101. a. Amplitude: $A = \frac{14.63 - 9.70}{2} = 2.465$

Vertical Shift: $\frac{14.63 + 9.70}{2} = 12.165$

$\omega = \frac{2\pi}{365}$

Phase shift (use $y = 14.63$, $x = 172$):

$$14.63 = 2.465\sin\left(\frac{2\pi}{365}\cdot 172 - \phi\right) + 12.165$$

$$2.465 = 2.465\sin\left(\frac{2\pi}{365}\cdot 172 - \phi\right)$$

$$1 = \sin\left(\frac{344\pi}{365} - \phi\right)$$

$$\frac{\pi}{2} = \frac{344\pi}{365} - \phi$$

$$\phi \approx 1.39$$

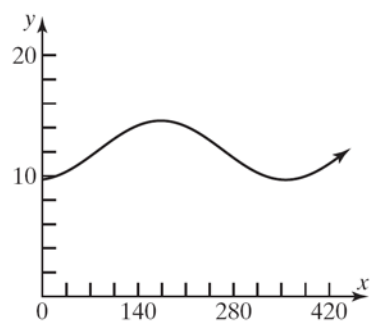
Thus, $y = 2.465\sin\left(\frac{2\pi}{365}x - 1.39\right) + 12.165$,

or $y = 2.465\sin\left[\frac{2\pi}{365}(x - 80.75)\right] + 12.165$.

b. $y = 2.465\sin\left(\frac{2\pi}{365}(91) - 1.39\right) + 12.165$

≈ 12.60 hours

c.



d. The actual hours of sunlight on April 1, 2007 were 12.6 hours. This is the same as the predicted amount.

Chapter 2 Test

1. $260° = 260 \cdot 1$ degree

$= 260 \cdot \frac{\pi}{180}$ radian

$= \frac{260\pi}{180}$ radian $= \frac{13\pi}{9}$ radian

2. $-400° = -400 \cdot 1$ degree

$= -400 \cdot \frac{\pi}{180}$ radian

$= -\frac{400\pi}{180}$ radian $= -\frac{20\pi}{9}$ radian

3. $13° = 13 \cdot 1$ degree $= 13 \cdot \frac{\pi}{180}$ radian $= \frac{13\pi}{180}$ radian

4. $-\frac{\pi}{8}$ radian $= -\frac{\pi}{8} \cdot 1$ radian

$= -\frac{\pi}{8} \cdot \frac{180}{\pi}$ degrees $= -22.5°$

5. $\frac{9\pi}{2}$ radian $= \frac{9\pi}{2} \cdot 1$ radian

$= \frac{9\pi}{2} \cdot \frac{180}{\pi}$ degrees $= 810°$

6. $\frac{3\pi}{4}$ radian $= \frac{3\pi}{4} \cdot 1$ radian

$= \frac{3\pi}{4} \cdot \frac{180}{\pi}$ degrees $= 135°$

7. $\sin\frac{\pi}{6} = \frac{1}{2}$

8. $\cos\left(-\frac{5\pi}{4}\right) - \cos\left(\frac{3\pi}{4}\right) = \cos\left(-\frac{5\pi}{4} + 2\pi\right) - \cos\left(\frac{3\pi}{4}\right)$

$= \cos\left(\frac{3\pi}{4}\right) - \cos\left(\frac{3\pi}{4}\right) = 0$

9. $\cos(-120°) = \cos(120°) = -\frac{1}{2}$

10. $\tan 330° = \tan(150° + 180°) = \tan(150°) = -\frac{\sqrt{3}}{3}$

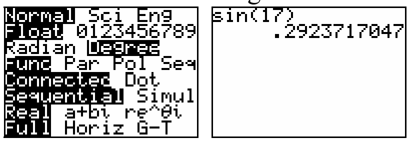
11. $\sin\frac{\pi}{2} - \tan\frac{19\pi}{4} = \sin\frac{\pi}{2} - \tan\left(\frac{3\pi}{4} + 4\pi\right)$

$= \sin\frac{\pi}{2} - \tan\left(\frac{3\pi}{4}\right) = 1 - (-1) = 2$

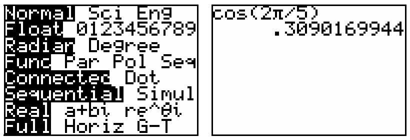
12. $2\sin^2 60° - 3\cos 45° = 2\left(\frac{\sqrt{3}}{2}\right)^2 - 3\left(\frac{\sqrt{2}}{2}\right)$

$= 2\left(\frac{3}{4}\right) - \frac{3\sqrt{2}}{2} = \frac{3}{2} - \frac{3\sqrt{2}}{2} = \frac{3\left(1 - \sqrt{2}\right)}{2}$

13. Set the calculator to degree mode: $\sin 17° \approx 0.292$

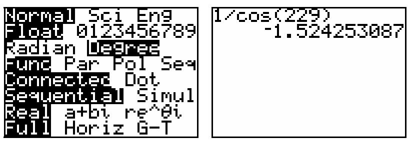


14. Set the calculator to radian mode: $\cos\frac{2\pi}{5} \approx 0.309$



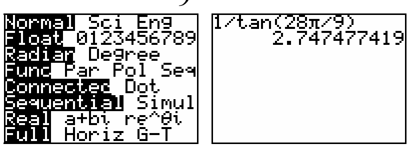
15. Set the calculator to degree mode:

$$\sec 229° = \frac{1}{\cos 229°} \approx -1.524$$



16. Set the calculator to radian mode:

$$\cot\frac{28\pi}{9} = \frac{1}{\tan\frac{28\pi}{9}} \approx 2.747$$



17. To remember the sign of each trig function, we primarily need to remember that $\sin\theta$ is positive in quadrants I and II, while $\cos\theta$ is positive in quadrants I and IV. The sign of the other four trig functions can be determined directly from sine and cosine by knowing $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\csc\theta = \frac{1}{\sin\theta}$, and $\cot\theta = \frac{\cos\theta}{\sin\theta}$.

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\sec\theta$	$\csc\theta$	$\cot\theta$
θ in QI	+	+	+	+	+	+
θ in QII	+	–	–	–	+	–
θ in QIII	–	–	+	–	–	+
θ in QIV	–	+	–	+	–	–

18. Because $f(x) = \sin x$ is an odd function and since $f(a) = \sin a = \frac{3}{5}$, then

$$f(-a) = \sin(-a) = -\sin a = -\frac{3}{5}.$$

19. $\sin\theta = \frac{5}{7}$ and θ in quadrant II.

Using the Pythagorean Identities:

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{5}{7}\right)^2 = 1 - \frac{25}{49} = \frac{24}{49}$$

$$\cos\theta = \pm\sqrt{\frac{24}{49}} = \pm\frac{2\sqrt{6}}{7}$$

Note that $\cos\theta$ must be negative because θ lies in quadrant II. Thus, $\cos\theta = -\frac{2\sqrt{6}}{7}$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{5}{7}}{-\frac{2\sqrt{6}}{7}} = \frac{5}{7}\left(-\frac{7}{2\sqrt{6}}\right)\cdot\frac{\sqrt{6}}{\sqrt{6}} = -\frac{5\sqrt{6}}{12}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{5}{7}} = \frac{7}{5}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{2\sqrt{6}}{7}} = -\frac{7}{2\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}} = -\frac{7\sqrt{6}}{12}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\frac{5\sqrt{6}}{12}} = -\frac{12}{5\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}} = -\frac{2\sqrt{6}}{5}$$

20. $\cos\theta = \frac{2}{3}$ and $\frac{3\pi}{2} < \theta < 2\pi$ (in quadrant IV).

Using the Pythagorean Identities:

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{2}{3}\right)^2 = 1 - \frac{4}{9} = \frac{5}{9}$$

$$\sin\theta = \pm\sqrt{\frac{5}{9}} = \pm\frac{\sqrt{5}}{3}$$

Note that $\sin\theta$ must be negative because θ lies in quadrant IV. Thus, $\sin\theta = -\frac{\sqrt{5}}{3}$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{\sqrt{5}}{3}}{\frac{2}{3}} = -\frac{\sqrt{5}}{3}\cdot\frac{3}{2} = -\frac{\sqrt{5}}{2}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-\frac{\sqrt{5}}{3}} = -\frac{3}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\frac{\sqrt{5}}{2}} = -\frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

21. $\tan\theta = -\frac{12}{5}$ and $\frac{\pi}{2} < \theta < \pi$ (in quadrant II)

Using the Pythagorean Identities:

$\sec^2\theta = \tan^2\theta + 1 = \left(-\frac{12}{5}\right)^2 + 1 = \frac{144}{25} + 1 = \frac{169}{25}$

$\sec\theta = \pm\sqrt{\frac{169}{25}} = \pm\frac{13}{5}$

Note that $\sec\theta$ must be negative since θ lies in quadrant II. Thus, $\sec\theta = -\frac{13}{5}$.

$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{-\frac{13}{5}} = -\frac{5}{13}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, so

$\sin\theta = (\tan\theta)(\cos\theta) = -\frac{12}{5}\left(-\frac{5}{13}\right) = \frac{12}{13}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{12}{13}} = \frac{13}{12}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\frac{12}{5}} = -\frac{5}{12}$

22. The point $(2,7)$ lies in quadrant I with $x = 2$ and $y = 7$. Since $x^2 + y^2 = r^2$, we have $r = \sqrt{2^2 + 7^2} = \sqrt{53}$. So,

$\sin\theta = \frac{y}{r} = \frac{7}{\sqrt{53}} = \frac{7}{\sqrt{53}} \cdot \frac{\sqrt{53}}{\sqrt{53}} = \frac{7\sqrt{53}}{53}$.

23. The point $(-5,11)$ lies in quadrant II with $x = -5$ and $y = 11$. Since $x^2 + y^2 = r^2$, we have $r = \sqrt{(-5)^2 + 11^2} = \sqrt{146}$. So,

$\cos\theta = \frac{x}{r} = \frac{-5}{\sqrt{146}} = \frac{-5}{\sqrt{146}} \cdot \frac{\sqrt{146}}{\sqrt{146}} = -\frac{5\sqrt{146}}{146}$.

24. The point $(6,-3)$ lies in quadrant IV with $x = 6$ and $y = -3$. Since $x^2 + y^2 = r^2$, we have $r = \sqrt{6^2 + (-3)^2} = \sqrt{45} = 3\sqrt{5}$. So,

$\tan\theta = \frac{y}{x} = \frac{-3}{6} = -\frac{1}{2}$

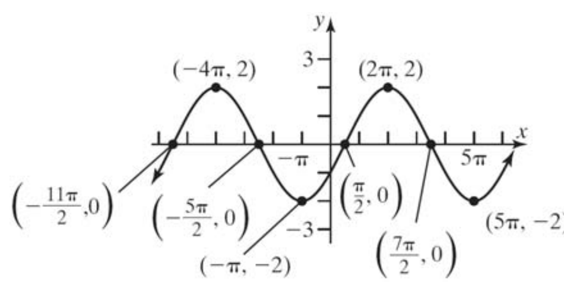
25. Comparing $y = 2\sin\left(\frac{x}{3} - \frac{\pi}{6}\right)$ to $y = A\sin(\omega x - \phi)$ we see that $A = 2$, $\omega = \frac{1}{3}$, and $\phi = \frac{\pi}{6}$. The graph is a sine curve with amplitude $|A| = 2$, period $T = \frac{2\pi}{\omega} = \frac{2\pi}{1/3} = 6\pi$, and phase shift $= \frac{\phi}{\omega} = \frac{\frac{\pi}{6}}{1/3} = \frac{\pi}{2}$. The graph of $y = 2\sin\left(\frac{x}{3} - \frac{\pi}{6}\right)$ will lie between -2 and 2 on the y-axis. One period will begin at $x = \frac{\phi}{\omega} = \frac{\pi}{2}$ and end at $x = \frac{2\pi}{\omega} + \frac{\phi}{\omega} = 6\pi + \frac{\pi}{2} = \frac{13\pi}{2}$. We divide the interval $\left[\frac{\pi}{2}, \frac{13\pi}{2}\right]$ into four subintervals, each of length $\frac{6\pi}{4} = \frac{3\pi}{2}$.

$\left[\frac{\pi}{2}, 2\pi\right], \left[2\pi, \frac{7\pi}{2}\right], \left[\frac{7\pi}{2}, 5\pi\right], \left[5\pi, \frac{13\pi}{2}\right]$

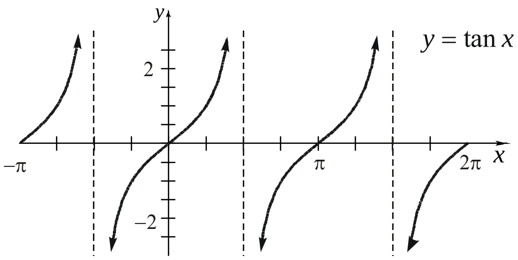
The five key points on the graph are

$\left(\frac{\pi}{2}, 0\right), (2\pi, 2), \left(\frac{7\pi}{2}, 0\right), (5\pi, -2), \left(\frac{13\pi}{2}, 0\right)$

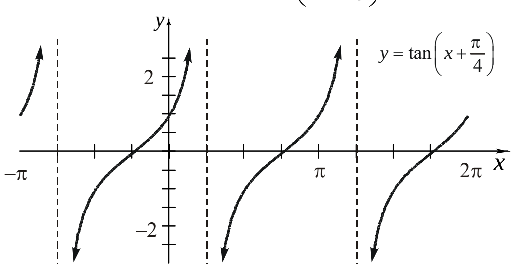
We plot these five points and fill in the graph of the sine function. The graph can then be extended in both directions.



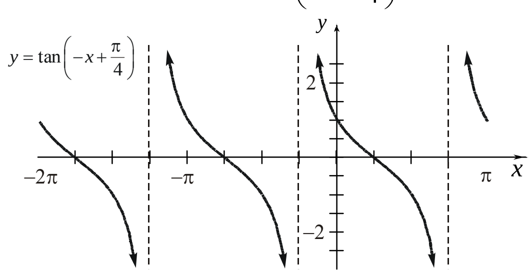
26. To graph $y = \tan\left(-x + \frac{\pi}{4}\right) + 2$ we will start with the graph of $y = \tan x$ and use transformations to obtain the desired graph.



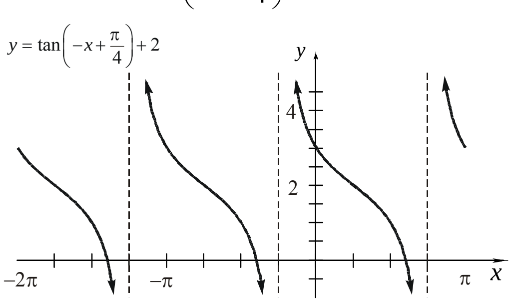
Next we shift the graph $\frac{\pi}{4}$ units to the left to obtain the graph of $y = \tan\left(x + \frac{\pi}{4}\right)$.



Now we reflect the graph about the *y*-axis to obtain the graph of $y = \tan\left(-x + \frac{\pi}{4}\right)$.

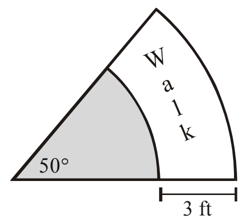


Lastly, we shift the graph up 2 units to obtain the graph of $y = \tan\left(-x + \frac{\pi}{4}\right) + 2$.



27. For a sinusoidal graph of the form $y = A\sin(\omega x - \phi)$, the amplitude is given by $|A|$, the period is given by $\frac{2\pi}{\omega}$, and the phase shift is given by $\frac{\phi}{\omega}$. Therefore, we have $A = -3$, $\omega = 3$, and $\phi = 3\left(-\frac{\pi}{4}\right) = -\frac{3\pi}{4}$. The equation for the graph is $y = -3\sin\left(3x + \frac{3\pi}{4}\right)$.

28. The area of the walk is the difference between the area of the larger sector and the area of the smaller shaded sector.



The area of the walk is given by

$$A = \frac{1}{2}R^2\theta - \frac{1}{2}r^2\theta,$$
$$= \frac{\theta}{2}\left(R^2 - r^2\right)$$

where *R* is the radius of the larger sector and *r* is the radius of the smaller sector. The larger radius is 3 feet longer than the smaller radius because the walk is to be 3 feet wide. Therefore, $R = r + 3$, and

$$A = \frac{\theta}{2}\left((r+3)^2 - r^2\right)$$
$$= \frac{\theta}{2}\left(r^2 + 6r + 9 - r^2\right)$$
$$= \frac{\theta}{2}(6r + 9)$$

The shaded sector has an arc length of 25 feet and a central angle of $50° = \frac{5\pi}{18}$ radians. The radius of this sector is $r = \frac{s}{\theta} = \frac{25}{\frac{5\pi}{18}} = \frac{90}{\pi}$ feet.

Thus, the area of the walk is given by

$$A = \frac{\frac{5\pi}{18}}{2}\left(6\left(\frac{90}{\pi}\right) + 9\right)$$
$$= \frac{5\pi}{36}\left(\frac{540}{\pi} + 9\right)$$
$$= 75 + \frac{5\pi}{4} \text{ ft}^2 \approx 78.93 \text{ ft}^2$$

29. To throw the hammer 83.19 meters, we need

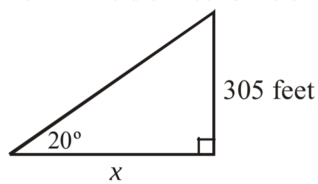
$$s = \frac{v_0^{\,2}}{g}$$
$$83.19 \text{ m} = \frac{v_0^{\,2}}{9.8 \text{ m/s}^2}$$
$$v_0^{\,2} = 815.262 \text{ m}^2/s^2$$
$$v_0 = 28.553 \text{ m/s}$$

Linear speed and angular speed are related according to the formula $v = r \cdot \omega$. The radius is $r = 190 \text{ cm} = 1.9 \text{ m}$. Thus, we have

$$28.553 = r \cdot \omega$$
$$28.553 = (1.9)\omega$$
$$\omega = 15.028 \text{ radians per second}$$
$$\omega = 15.028 \frac{\text{radians}}{\text{sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{1 \text{ revolution}}{2\pi \text{ radians}}$$
$$\approx 143.5 \text{ revolutions per minute (rpm)}$$

To throw the hammer 83.19 meters, Adrian must have been swinging it at a rate of 143.5 rpm upon release.

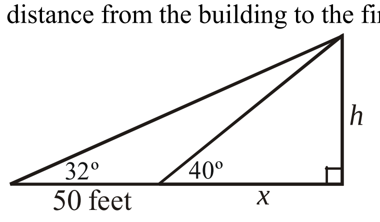
30. Let *x* = the distance to the base of the statue.



$$\tan(20^\circ) = \frac{305}{x}$$
$$x = \frac{305}{\tan(20^\circ)} \approx \frac{305}{0.3640} \approx 837.98 \text{ feet}$$

The ship is about 838 feet from the base of the Statue of Liberty.

31. Let *h* = the height of the building and let *x* = the distance from the building to the first sighting.



$$\tan(40^\circ) = \frac{h}{x}$$
$$x = \frac{h}{\tan(40^\circ)}$$
$$\tan(32^\circ) = \frac{h}{x+50}$$
$$h = (x+50)\tan(32^\circ)$$
$$h = \left(\frac{h}{\tan(40^\circ)} + 50\right)\tan(32^\circ)$$
$$h = \left(\frac{h}{0.8391} + 50\right)0.6249$$
$$h = 0.7447h + 31.245$$
$$0.2553h = 31.245$$
$$h \approx 122.39 \text{ feet}$$

The building is roughly 122.4 feet tall.

Chapter 2 Cumulative Review

1.
$$2x^2 + x - 1 = 0$$
$$(2x-1)(x+1) = 0$$
$$x = \frac{1}{2} \text{ or } x = -1$$
$$\left\{-1, \frac{1}{2}\right\}$$

2. Slope $= -3$, containing $(-2,5)$

Using $y - y_1 = m(x - x_1)$
$$y - 5 = -3(x - (-2))$$
$$y - 5 = -3(x+2)$$
$$y - 5 = -3x - 6$$
$$y = -3x - 1$$

3. radius = 4, center (0,–2)

Using $(x-h)^2 + (y-k)^2 = r^2$
$$(x-0)^2 + (y-(-2))^2 = 4^2$$
$$x^2 + (y+2)^2 = 16$$

4. $2x - 3y = 12$

This equation yields a line.
$$2x - 3y = 12$$
$$-3y = -2x + 12$$
$$y = \frac{2}{3}x - 4$$

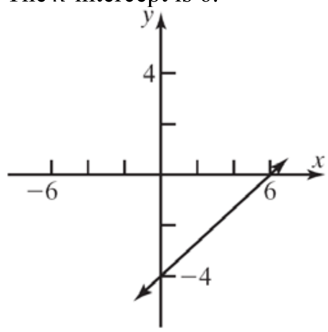
The slope is $m = \frac{2}{3}$ and the *y*-intercept is -4.

Let $y = 0$: $2x - 3(0) = 12$

$$2x = 12$$

$$x = 6$$

The x-intercept is 6.



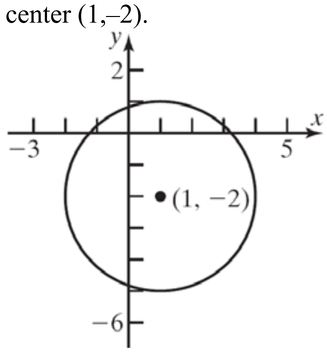
5. $x^2 + y^2 - 2x + 4y - 4 = 0$

$x^2 - 2x + 1 + y^2 + 4y + 4 = 4 + 1 + 4$

$(x-1)^2 + (y+2)^2 = 9$

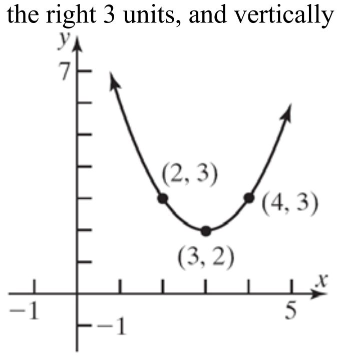
$(x-1)^2 + (y+2)^2 = 3^2$

This equation yields a circle with radius 3 and center (1,–2).

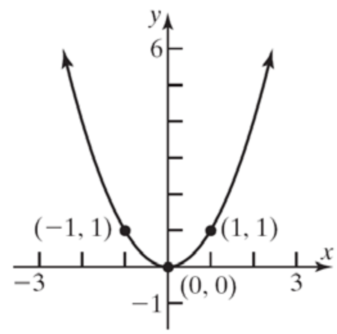


6. $y = (x-3)^2 + 2$

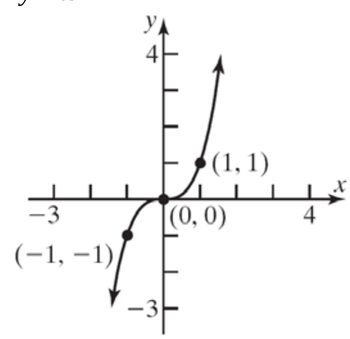
Using the graph of $y = x^2$, horizontally shift to the right 3 units, and vertically shift up 2 units.



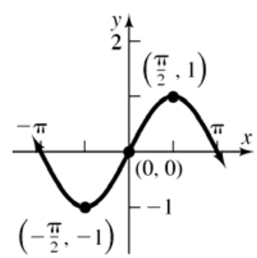
7. a. $y = x^2$



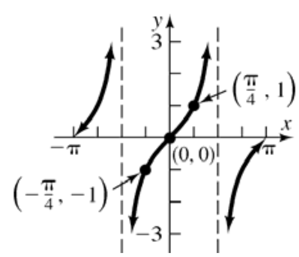
b. $y = x^3$



c. $y = \sin x$



d. $y = \tan x$



8. $f(x) = 3x - 2$

$y = 3x - 2$

$x = 3y - 2$ Inverse

$x + 2 = 3y$

$\frac{x+2}{3} = y$

$f^{-1}(x) = \frac{x+2}{3} = \frac{1}{3}(x+2)$

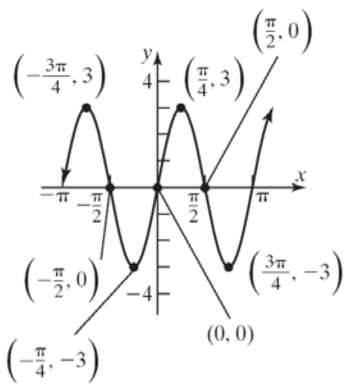
9. $(\sin 14^\circ)^2 + (\cos 14^\circ)^2 - 3 = 1 - 3 = -2$

10. $y = 3\sin(2x)$

Amplitude: $|A| = |3| = 3$

Period: $T = \frac{2\pi}{2} = \pi$

Phase Shift: $\frac{\phi}{\omega} = \frac{0}{2} = 0$



11. $\tan\frac{\pi}{4} - 3\cos\frac{\pi}{6} + \csc\frac{\pi}{6} = 1 - 3\left(\frac{\sqrt{3}}{2}\right) + 2$

$= 3 - \frac{3\sqrt{3}}{2}$

$= \frac{6 - 3\sqrt{3}}{2}$

12. The graph is a cosine graph with amplitude 3 and period 12.

Find ω: $12 = \frac{2\pi}{\omega}$

$12\omega = 2\pi$

$\omega = \frac{2\pi}{12} = \frac{\pi}{6}$

The equation is: $y = 3\cos\left(\frac{\pi}{6}x\right)$.

13. Given points $(-2, 3)$ and $(1, -6)$, we compute the slope as follows:

$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-3}{1-(-2)} = \frac{-9}{3} = -3$

Using $y - y_1 = m(x - x_1)$:

$y - 3 = -3(x - (-2))$

$y - 3 = -3(x + 2)$

$y = -3x - 6 + 3$

$y = -3x - 3$

The linear function is $f(x) = -3x - 3$.

Slope: $m = -3$;

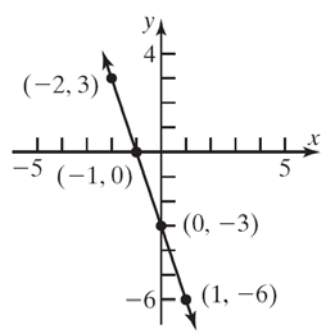
y-intercept: $f(0) = -3(0) - 3 = -3$

x-intercept: $0 = -3x - 3$

$3x = -3$

$x = -1$

Intercepts: $(-1, 0)$, $(0, -3)$



Chapter 2 Projects

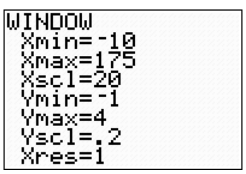
Project I

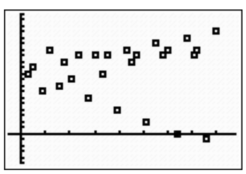
1. November 15: High tide: 11:18 am and 11:15 pm
November 19: low tide: 7:17 am and 8:38 pm

2. The low tide was below sea level. It is measured against calm water at sea level.

3.

Nov	Low Tide			Low Tide			High Tide			High Tide		
	Time	Ht (ft)	t	Time	Ht (ft)	t	Time	Ht (ft)	t	Time	Ht (ft)	t
14 0-24	6:26a	2.0	6.43	4:38p	1.4	16.63	9:29a	2.2	9.48	11:14p	2.8	23.23
15 24-48	6:22a	1.6	30.37	5:34p	1.8	41.57	11:18a	2.4	35.3	11:15p	2.6	47.25
16 48-72	6:28a	1.2	54.47	6:25p	2.0	66.42	12:37p	2.6	60.62	11:16p	2.6	71.27
17 72-96	6:40a	0.8	78.67	7:12p	2.4	91.2	1:38p	2.8	85.63	11:16p	2.6	95.27
18 96-120	6:56a	0.4	102.93	7:57p	2.6	115.95	2:27p	3.0	110.45	11:14p	2.8	119.23
19 120-144	7:17a	0.0	127.28	8:38p	2.6	140.63	3:10p	3.2	135.17	11:05p	2.8	143.08
20 144-168	7:43a	-0.2	151.72				3:52p	3.4	159.87			





4. The data seems to take on a sinusoidal shape (oscillates). The period is approximately 12 hours. The amplitude varies each day:
Nov 14: 0.1, 0.7
Nov 15: 0.4, 0.4
Nov 16: 0.7, 0.3
Nov 17: 1.0, 0.1
Nov 18: 1.3, 0.1
Nov 19: 1.6, 0.1
Nov 20: 1.8

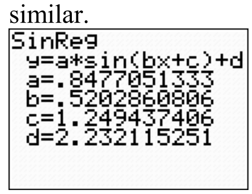
5. Average of the amplitudes: 0.66
Period : 12
Average of vertical shifts: 2.15 (approximately)
There is no phase shift. However, keeping in mind the vertical shift, the amplitude
$y = A\sin\left(Bx\right) + D$

$A = 0.66 \qquad 12 = \dfrac{2\pi}{B} \qquad D = 2.15$

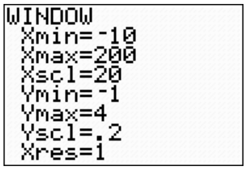
$B = \dfrac{\pi}{6} \approx 0.52$

Thus, $y = 0.66\sin\left(0.52x\right) + 2.15$
(Answers may vary)

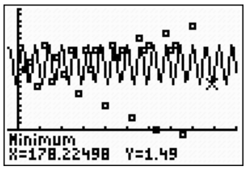
6. $y = 0.848\sin\left(0.52x + 1.25\right) + 2.23$
The two functions are not the same, but they are similar.

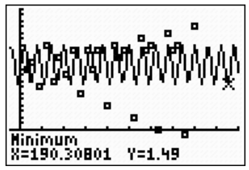


7. Find the high and low tides on November 21 which are the min and max that lie between $t = 168$ and $t = 192$. Looking at the graph of the equation for part (5) and using MAX/MIN for values between $t = 168$ and $t = 192$:

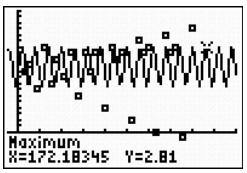


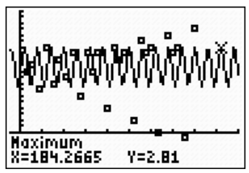
Low tides of 1.49 feet when t = 178.2 and t = 190.3.





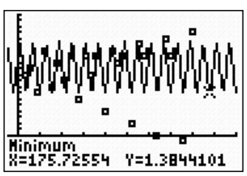
High tides of 2.81 feet occur when t = 172.2 and t = 184.3.

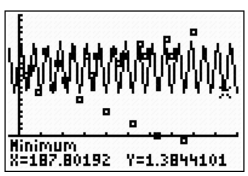




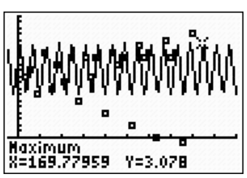
Looking at the graph for the equation in part (6) and using MAX/MIN for values between t = 168 and t = 192:

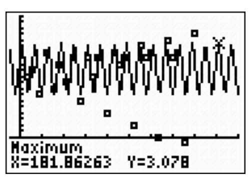
A low tide of 1.38 feet occurs when t = 175.7 and $t = 187.8$.





A high tide of 3.08 feet occurs when t = 169.8 and $t = 181.9$.





8. The low and high tides vary because of the moon phase. The moon has a gravitational pull on the water on Earth.

Project II

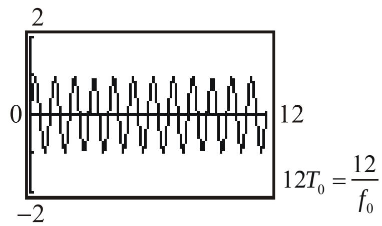
1. $s(t) = 1\sin\left(2\pi f_0 t\right)$

2. $T_0 = \dfrac{2\pi}{2\pi f_0} = \dfrac{1}{f_0}$

3.

t	0	$\frac{1}{4f_0}$	$\frac{1}{2f_0}$	$\frac{3}{4f_0}$	$\frac{1}{f_0}$
$s(t)$	0	1	0	–1	0

4. Let $f_0 = 1 = 1$. Let $0 \le x \le 12$, with $\Delta x = 0.5$. Label the graph as $0 \le x \le 12T_0$, and each tick mark is at $\Delta x = \dfrac{1}{2f_0}$.

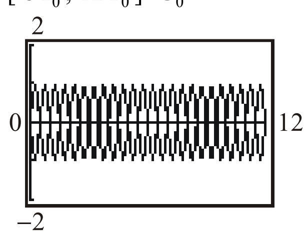


5. $t = \dfrac{1}{4f_0},\ t = \dfrac{5}{4f_0},\ t = \dfrac{9}{4f_0}, \ldots,\ t = \dfrac{45}{4f_0}$

6. M = 0 1 0 $\rightarrow$ P = 0 π 0

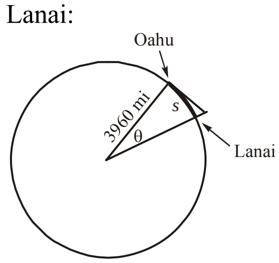
7. $S_0(t) = 1\sin(2\pi f_0 t + 0)$, $S_1(t) = 1\sin(2\pi f_0 t + \pi)$

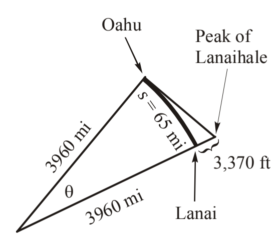
8. $[0, 4T_0]$ S_0
$[4T_0, 8T_0]$ S_1
$[8T_0, 12T_0]$ S_0



Project III

1. Lanai:





2. $s = r\theta$

$$\theta = \frac{s}{r} = \frac{65}{3960} = 0.0164$$

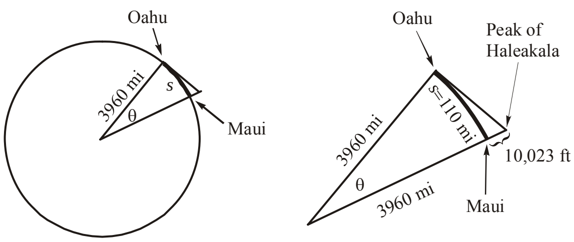
3. $$\frac{3960}{3960+h} = \cos(0.164)$$

$$3960 = 0.9999(3960+h)$$

$$h = 0.396 \text{ miles}$$

$$0.396 \times 5280 = 2090 \text{ feet}$$

4. Maui:



$$\theta = \frac{s}{r} = \frac{110}{3960} = 0.0278$$

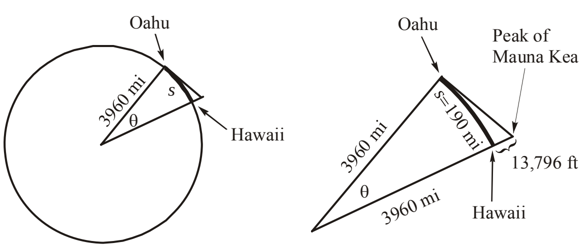
$$\frac{3960}{3960+h} = \cos(0.278)$$

$$3960 = 0.9996(3960+h)$$

$$h = 1.584 \text{ miles}$$

$$h = 1.584 \times 5280 = 8364 \text{ feet}$$

Hawaii:



$$\theta = \frac{s}{r} = \frac{190}{3960} = 0.0480$$

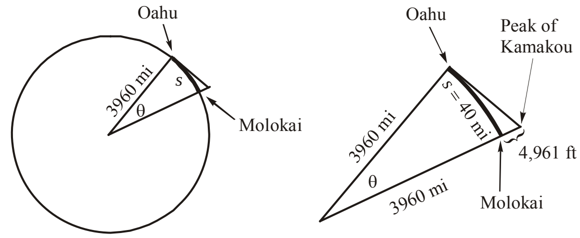
$$\frac{3960}{3960+h} = \cos(0.480)$$

$$3960 = 0.9988(3960+h)$$

$$h = 4.752 \text{ miles}$$

$$h = 4.752 \times 5280 = 25,091 \text{ feet}$$

Molokai:



$$\theta = \frac{s}{r} = \frac{40}{3960} = 0.0101$$

$$\frac{3960}{3960+h} = \cos(0.0101)$$

$$3960 = 0.9999(3960+h)$$

$$h = 0.346 \text{ miles}$$

$$h = 0.346x5280 = 2090 \text{ feet}$$

5. Kamakou, Haleakala, and Lanaihale are all visible from Oahu.

Project IV

Answers will vary.