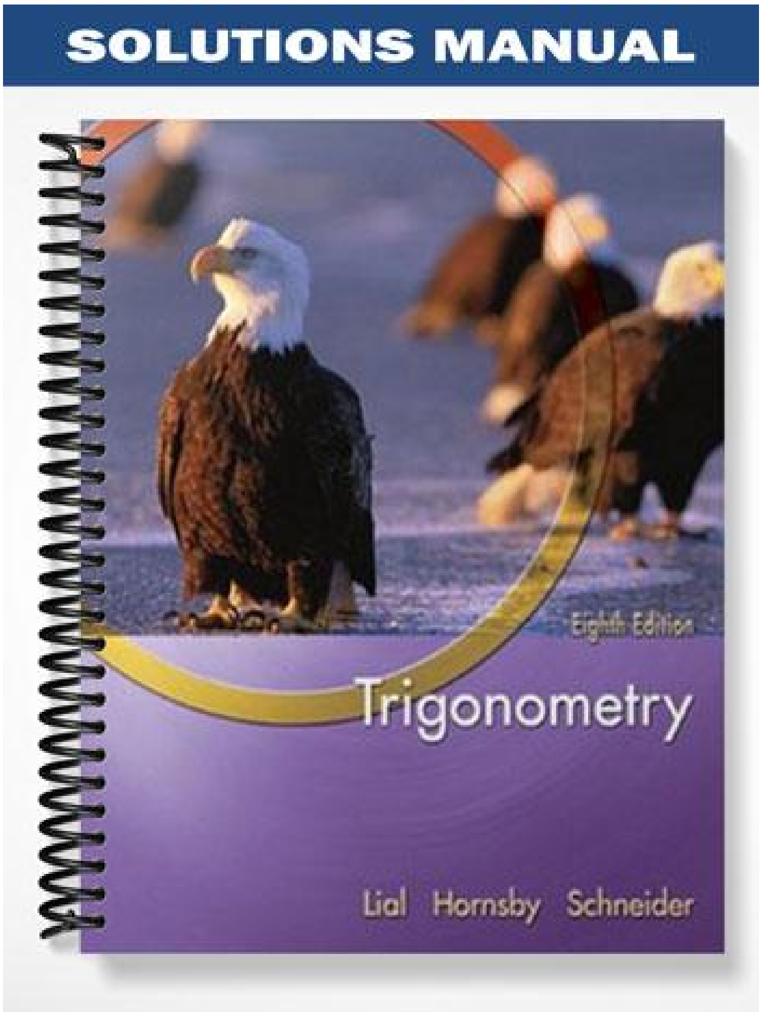

SOLUTIONS MANUAL
Eighth Edition
Trigonometry
Lial Hornsby Schneider

Chapter 2

ACUTE ANGLES AND RIGHT TRIANGLES

Section 2.1: Trigonometric Functions of Acute Angles

1. $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{21}{29}$

$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{20}{29}$

$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{21}{20}$

2. $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{45}{53}$

$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{28}{53}$

$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{45}{28}$

3. $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{n}{p}$

$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{m}{p}$

$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{n}{m}$

4. $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{k}{z}$

$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{y}{z}$

$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{k}{y}$

For Exercises 5–10, refer to the Function Values of Special Angles chart on page 50 of your text.

5. C; $\sin 30° = \frac{1}{2}$

6. H; $\cos 45° = \frac{\sqrt{2}}{2}$

7. B; $\tan 45° = 1$

8. G; $\sec 60° = \frac{1}{\cos 60°} = \frac{1}{\frac{1}{2}} = 2$

9. E; $\csc 60° = \frac{1}{\sin 60°} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

10. A; $\cot 30° = \frac{\cos 30°}{\sin 30°} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$

11. $a = 5, b = 12$

$c^2 = a^2 + b^2 \Rightarrow c^2 = 5^2 + 12^2 \Rightarrow c^2 = 25 + 144 \Rightarrow c^2 = 169 \Rightarrow c = 13$

$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{b}{c} = \frac{12}{13}$

$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{c} = \frac{5}{13}$

$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{b}{a} = \frac{12}{5}$

$\cot B = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{a}{b} = \frac{5}{12}$

$\sec B = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{a} = \frac{13}{5}$

$\csc B = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{b} = \frac{13}{12}$

12. $a = 3, b = 5$

$c^2 = a^2 + b^2 \Rightarrow c^2 = 3^2 + 5^2 \Rightarrow c^2 = 9 + 25 \Rightarrow c^2 = 34 \Rightarrow c = \sqrt{34}$

$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{b}{c} = \frac{5}{\sqrt{34}} = \frac{5}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}} = \frac{5\sqrt{34}}{34}$

$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{c} = \frac{3}{\sqrt{34}} = \frac{3}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}} = \frac{3\sqrt{34}}{34}$

$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{b}{a} = \frac{5}{3}$

$\cot B = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{a}{b} = \frac{3}{5}$

$\sec B = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{a} = \frac{\sqrt{34}}{3}$

$\csc B = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{b} = \frac{\sqrt{34}}{5}$

13. $a = 6, c = 7$

$c^2 = a^2 + b^2 \Rightarrow 7^2 = 6^2 + b^2 \Rightarrow 49 = 36 + b^2 \Rightarrow b^2 = 13 \Rightarrow b = \sqrt{13}$

$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{b}{c} = \frac{\sqrt{13}}{7}$

$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{c} = \frac{6}{7}$

$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{b}{a} = \frac{\sqrt{13}}{6}$

$\cot B = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{a}{b} = \frac{6}{\sqrt{13}} = \frac{6}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{6\sqrt{13}}{13}$

$\sec B = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{a} = \frac{7}{6}$

$\csc B = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{b} = \frac{7}{\sqrt{13}} = \frac{7}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{7\sqrt{13}}{13}$

14. $b = 7, c = 12$

$c^2 = a^2 + b^2 \Rightarrow 12^2 = a^2 + 7^2 \Rightarrow 144 = a^2 + 49 \Rightarrow a^2 = 95 \Rightarrow a = \sqrt{95}$

$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{b}{c} = \frac{7}{12}$

$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{c} = \frac{\sqrt{95}}{12}$

$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{b}{a} = \frac{7}{\sqrt{95}} = \frac{7}{\sqrt{95}} \cdot \frac{\sqrt{95}}{\sqrt{95}} = \frac{7\sqrt{95}}{95}$

$\cot B = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{a}{b} = \frac{\sqrt{95}}{7}$

$\sec B = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{a} = \frac{12}{\sqrt{95}} = \frac{12}{\sqrt{95}} \cdot \frac{\sqrt{95}}{\sqrt{95}} = \frac{12\sqrt{95}}{95}$

$\csc B = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{b} = \frac{12}{7}$

15. $\sin\theta = \cos(90° - \theta)$; $\cos\theta = \sin(90° - \theta)$;
$\tan\theta = \cot(90° - \theta)$; $\cot\theta = \tan(90° - \theta)$;
$\sec\theta = \csc(90° - \theta)$; $\csc\theta = \sec(90° - \theta)$

16. $\cot 73° = \tan(90° - 73°) = \tan 17°$

17. $\sec 39° = \csc(90° - 39°) = \csc 51°$

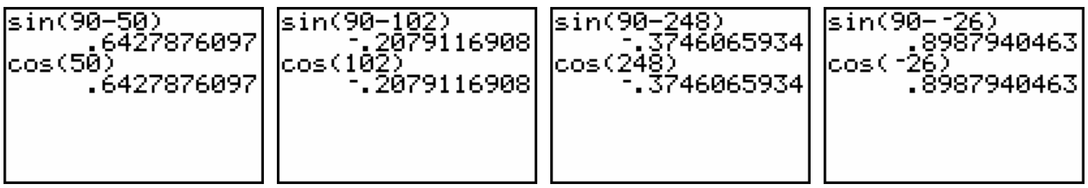
18. $\cos(\alpha+20°)=\sin[90°-(\alpha+20°)]$
$=\sin(90°-\alpha-20°)$
$=\sin(70°-\alpha)$

19. $\cot(\theta-10°)=\tan[90°-(\theta-10°)]$
$=\tan(90°-\theta+10°)$
$=\tan(100°-\theta)$

20. $\tan 25.4° = \cot(90°-25.4°)=\cot 64.6°$

21. $\sin 38.7° = \cos(90°-38.7°)=\cos 51.3°$

22. Using $A = 50°,\ 102°,\ 248°,$ and $-26°,$ we see that $\sin(90°-A)$ and $\cos A$ yield the same values.



23. $\tan\alpha = \cot(\alpha+10°)$

Since tangent and cotangent are cofunctions, this equation is true if the sum of the angles is 90°.

$$\alpha+(\alpha+10°)=90° \Rightarrow 2\alpha+10°=90° \Rightarrow 2\alpha=80° \Rightarrow \alpha=40°$$

24. $\cos\theta=\sin 2\theta$

Since sine and cosine are cofunctions, this equation is true if the sum of the angles is 90°.

$$\theta+2\theta=90° \Rightarrow 3\theta=90° \Rightarrow \theta=30°$$

25. $\sin(2\theta+10°)=\cos(3\theta-20°)$

Since sine and cosine are cofunctions, this equation is true if the sum of the angles is 90°.

$$(2\theta+10°)+(3\theta-20°)=90° \Rightarrow 5\theta-10°=90° \Rightarrow 5\theta=100° \Rightarrow \theta=20°$$

26. $\sec(\beta+10°)=\csc(2\beta+20°)$

Since secant and cosecant are cofunctions, this equation is true if the sum of the angles is 90°.

$$(\beta+10°)+(2\beta+20°)=90° \Rightarrow 3\beta+30°=90° \Rightarrow 3\beta=60° \Rightarrow \beta=20°$$

27. $\tan(3B+4°)=\cot(5B-10°)$

Since tangent and cotangent are cofunctions, this equation is true if the sum of the angles is 90°.

$$(3B+4°)+(5B-10°)=90° \Rightarrow 8B-6°=90° \Rightarrow 8B=96° \Rightarrow B=12°$$

28. $\cot(5\theta+2°)=\tan(2\theta+4°)$

Since tangent and cotangent are cofunctions, this equation is true if the sum of the angles is 90°.

$$(5\theta+2°)+(2\theta+4°)=90° \Rightarrow 7\theta+6°=90° \Rightarrow 7\theta=84° \Rightarrow \theta=12°$$

29. $\sin 50° > \sin 40°$

In the interval from 0° to 90°, as the angle increases, so does the sine of the angle, so $\sin 50° > \sin 40°$ is true.

30. $\tan 28° \le \tan 40°$
$\tan\theta$ increases as θ increases from 0° to 90°. Since 40°>28°, tan 40° > tan 28°. Therefore, the given statement is true.

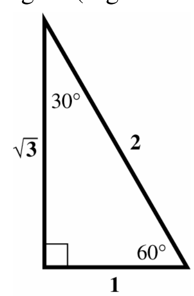
31. $\sin 46° < \cos 46°$
$\sin\theta$ increases as θ increases from 0° to 90°. Since $46° > 44°$, $\sin 46° > \sin 44°$ and $\sin 44° = \cos 46°$, we have $\sin 46° > \cos 46°$. Thus, the statement is false.

32. $\cos 28° < \sin 28°$
$\cos\theta$ decreases as θ increases from 0° to 90°. Since $28° < 62°$, $\cos 28° > \cos 62°$ and $\cos 62° = \sin 28°$, we have $\cos 28° > \sin 28°$. Thus, the statement is false.

33. $\tan 41° < \cot 41°$
$\tan\theta$ increases as θ increases from 0° to 90°. Since $49° > 41°$, $\tan 49° > \tan 41°$. Since $\tan 49° = \cot 41°$, we have $\cot 41° > \tan 41°$. Therefore, the statement is true.

34. $\cot 30° < \tan 40°$
$\tan(90° - 30°) = \tan 60° = \cot 30°$
In the interval from 0° to 90°, the tangent increases so, $\tan 60° > \tan 40°$. Therefore, $\cot 30° > \tan 40°$, and the statement is false.

For Exercises 35 – 40, refer to the following figure (Figure 6 from page 49 of your text).



35. $\tan 30° = \dfrac{\text{side opposite}}{\text{side adjacent}} = \dfrac{1}{\sqrt{3}} = \dfrac{1}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

36. $\cot 30° = \dfrac{\text{side adjacent}}{\text{side opposite}} = \dfrac{\sqrt{3}}{1} = \sqrt{3}$

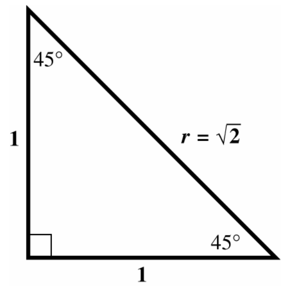
37. $\sin 30° = \dfrac{\text{side opposite}}{\text{hypotenuse}} = \dfrac{1}{2}$

38. $\cos 30° = \dfrac{\text{side adjacent}}{\text{hypotenuse}} = \dfrac{\sqrt{3}}{2}$

39. $\sec 30° = \dfrac{\text{hypotenuse}}{\text{side adjacent}} = \dfrac{2}{\sqrt{3}} = \dfrac{2}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

40. $\csc 30° = \dfrac{\text{hypotenuse}}{\text{side opposite}} = \dfrac{2}{1} = 2$

For Exercises 41 – 44, refer to the following figure (Figure 7 from page 50 of your text).



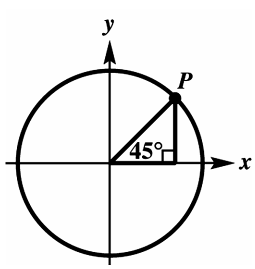
41. $\csc 45° = \dfrac{\text{hypotenuse}}{\text{side opposite}} = \dfrac{\sqrt{2}}{1} = \sqrt{2}$

42. $\sec 45° = \dfrac{\text{hypotenuse}}{\text{side adjacent}} = \dfrac{\sqrt{2}}{1} = \sqrt{2}$

43. $\cos 45° = \dfrac{\text{side adjacent}}{\text{hypotenuse}} = \dfrac{1}{\sqrt{2}}$
$= \dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

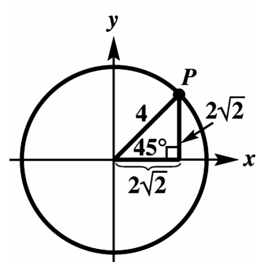
44. $\cot 45° = \dfrac{\text{side adjacent}}{\text{side opposite}} = \dfrac{\sqrt{2}}{\sqrt{2}} = 1$

45.



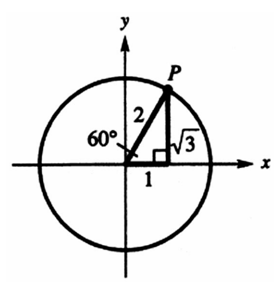
46. $\sin 45° = \dfrac{y}{4} \Rightarrow y = 4\sin 45° = 4 \cdot \dfrac{\sqrt{2}}{2} = 2\sqrt{2}$

$\cos 45° = \dfrac{x}{4} \Rightarrow x = 4\cos 45° = 4 \cdot \dfrac{\sqrt{2}}{2} = 2\sqrt{2}$



47. The legs of the right triangle provide the coordinates of *P*. *P* is $\left(2\sqrt{2}, 2\sqrt{2}\right)$.

48.



$\sin 60° = \dfrac{y}{2} \Rightarrow y = 2 \sin 60° = 2 \cdot \dfrac{\sqrt{3}}{2} = \sqrt{3}$ and $\cos 60° = \dfrac{x}{2} \Rightarrow x = 2\cos 60° = 2 \cdot \dfrac{1}{2} = 1$

The legs of the right triangle provide the coordinates of *P*. *P* is $\left(1, \sqrt{3}\right)$.

49. Y_1 is $\sin x$ and Y_2 is $\tan x$.

$\sin 0° = 0$	$\tan 0° = 0$
$\sin 30° = .5$	$\tan 30° \approx .57735$
$\sin 45° \approx .70711$	$\tan 45° = 1$
$\sin 60° \approx .86603$	$\tan 60° = 1.7321$
$\sin 90° = 1$	$\tan 90°$: undefined

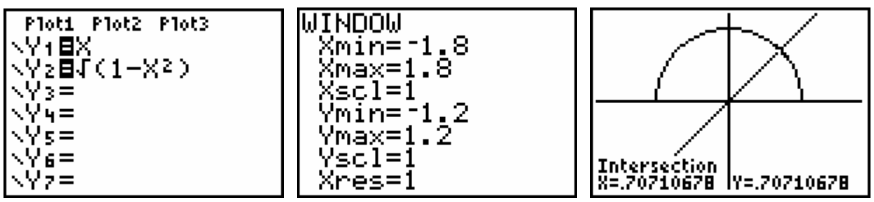
50. Y_1 is $\cos x$ and Y_2 is $\csc x$.

$\cos 0° = 1$	$\csc 0°$: undefined
$\cos 30° \approx .86603$	$\csc 30° = 2$
$\cos 45° \approx .70711$	$\csc 45° \approx 1.4142$
$\cos 60° = .5$	$\csc 60° \approx 1.1547$
$\cos 90° = 0$	$\csc 90° = 1$

51. Since $\sin 60° = \frac{\sqrt{3}}{2}$ and 60° is between 0° and 90°, $A = 60°$.

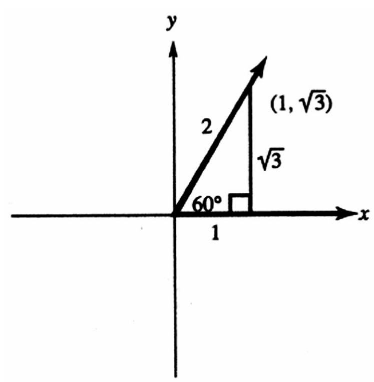
52. .7071067812 is a rational approximation for the exact value $\frac{\sqrt{2}}{2}$ (an irrational value).

53. The point of intersection is (.70710678, .70710678). This corresponds to the point $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.



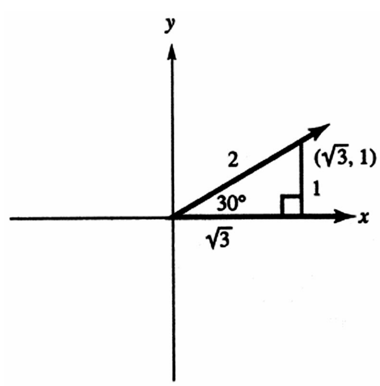
These coordinates are the sine and cosine of 45°.

54.



The line passes through $(0,0)$ and $(1,\sqrt{3})$. The slope is change in y over the change in x. Thus, $m = \frac{\sqrt{3}}{1} = \sqrt{3}$ and the equation of the line is $y = \sqrt{3}x$.

55.

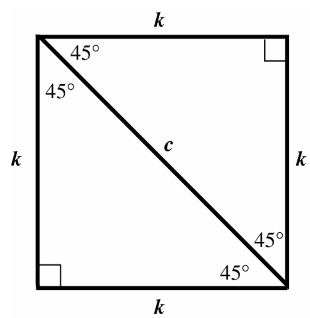


The line passes through (0, 0) and $(\sqrt{3}, 1)$. The slope is change in y over the change in x. Thus, $m = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ and the equation of the line is $y = \frac{\sqrt{3}}{3}x$.

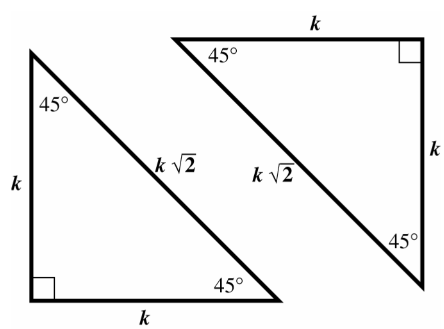
56. One point on the line $y = \frac{\sqrt{3}}{3}x$, is the origin $(0,0)$. Let (x, y) be any other point on this line. Then, by the definition of slope, $m = \frac{y-0}{x-0} = \frac{y}{x} = \frac{\sqrt{3}}{3}$, but also, by the definition of tangent, $\tan\theta = \frac{\sqrt{3}}{3}$. Because $\tan 30° = \frac{\sqrt{3}}{3}$, the line $y = \frac{\sqrt{3}}{3}x$ makes a 30° angle with the positive x-axis. (See Exercise 55).

57. One point on the line $y = \sqrt{3}x$ is the origin $(0,0)$. Let (x, y) be any other point on this line. Then, by the definition of slope, $m = \frac{y-0}{x-0} = \frac{y}{x} = \sqrt{3}$, but also, by the definition of tangent, $\tan\theta = \sqrt{3}$. Because $\tan 60° = \sqrt{3}$, the line $y = \sqrt{3}x$ makes a 60° angle with the positive x-axis (See Exercise 54).

58. **(a)** The diagonal forms two isosceles right triangles. Each angle formed by a side of the square and the diagonal measures 45°.

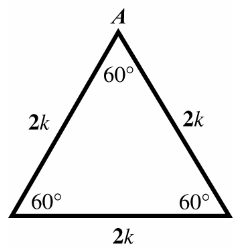


(b) By the Pythagorean theorem, $k^2 + k^2 = c^2 \Rightarrow 2k^2 = c^2 \Rightarrow c = \sqrt{2k^2} \Rightarrow c = k\sqrt{2}$. The length of the diagonal is $k\sqrt{2}$.



(c) In a $45° - 45°$ right triangle, the hypotenuse has a length that is $\underline{\sqrt{2}}$ times as long as either leg.

59. **(a)** Each of angles of the equilateral triangle has measure of $\frac{1}{3}(180°) = 60°$.

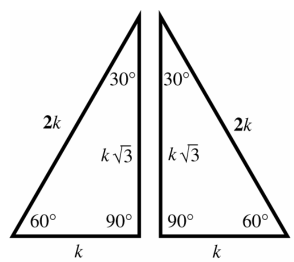


(b) The perpendicular bisects the opposite side so the length of each side opposite each 30° angle is *k*.

(c) Let *x* equal the length of the perpendicular and apply the Pythagorean theorem.

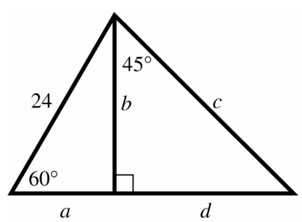
$$x^2 + k^2 = (2k)^2 \Rightarrow x^2 + k^2 = 4k^2 \Rightarrow x^2 = 3k^2 \Rightarrow x = k\sqrt{3}$$

The length of the perpendicular is $k\sqrt{3}$.



(d) In a 30° - 60° right triangle, the hypotenuse is always <u>2</u> times as long as the shorter leg, and the longer leg has a length that is $\underline{\sqrt{3}}$ times as long as that of the shorter leg. Also, the shorter leg is opposite the <u>30°</u> angle, and the longer leg is opposite the <u>60°</u> angle.

60. Apply the relationships between the lengths of the sides of a $30° - 60°$ right triangle first to the triangle on the left to find the values of *a* and *b*. In the $30° - 60°$ right triangle, the side opposite the $30°$ angle is $\frac{1}{2}$ the length of the hypotenuse. The longer leg is $\sqrt{3}$ times the shorter leg.

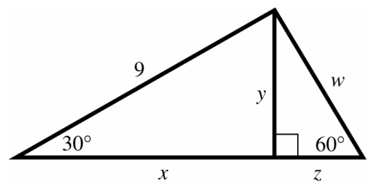


$$a = \frac{1}{2}(24) = 12 \text{ and } b = a\sqrt{3} = 12\sqrt{3}$$

Apply the relationships between the lengths of the sides of a $45° - 45°$ right triangle next to the triangle on the right to find the values of *d* and *c*. In the $45° - 45°$ right triangle, the sides opposite the $45°$ angles measure the same. The hypotenuse is $\sqrt{2}$ times the measure of a leg.

$$d = b = 12\sqrt{3} \text{ and } c = d\sqrt{2} = (12\sqrt{3})(\sqrt{2}) = 12\sqrt{6}$$

61. Apply the relationships between the lengths of the sides of a $30° - 60°$ right triangle first to the triangle on the left to find the values of *y* and *x*, and then to the triangle on the right to find the values of *z* and *w*. In the $30° - 60°$ right triangle, the side opposite the $30°$ angle is $\frac{1}{2}$ the length of the hypotenuse. The longer leg is $\sqrt{3}$ times the shorter leg.

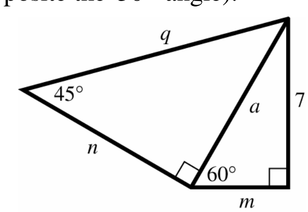


Thus, we have the following.

$$y = \frac{1}{2}(9) = \frac{9}{2} \text{ and } x = y\sqrt{3} = \frac{9\sqrt{3}}{2}$$

$$y = z\sqrt{3}, \text{ so } z = \frac{y}{\sqrt{3}} = \frac{\frac{9}{2}}{\sqrt{3}} = \frac{9\sqrt{3}}{6} = \frac{3\sqrt{3}}{2}, \text{ and } w = 2z, \text{ so } w = 2\left(\frac{3\sqrt{3}}{2}\right) = 3\sqrt{3}$$

62. Apply the relationships between the lengths of the sides of a $30° - 60°$ right triangle first to the triangle on the right to find the values of *m* and *a*. In the $30° - 60°$ right triangle, the side opposite the $60°$ angle is $\sqrt{3}$ times as long as the side opposite to the $30°$ angle. The length of the hypotenuse is 2 times as long as the shorter leg (opposite the $30°$ angle).



Thus, we have the following.

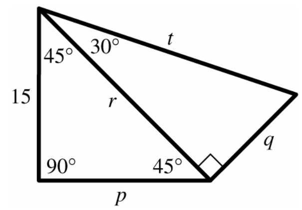
$$7 = m\sqrt{3} \Rightarrow m = \frac{7}{\sqrt{3}} = \frac{7}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{7\sqrt{3}}{3} \text{ and } a = 2m \Rightarrow a = 2\left(\frac{7\sqrt{3}}{3}\right) = \frac{14\sqrt{3}}{3}$$

Apply the relationships between the lengths of the sides of a $45° - 45°$ right triangle next to the triangle on the left to find the values of *n* and *q*. In the $45° - 45°$ right triangle, the sides opposite the $45°$ angles measure the same. The hypotenuse is $\sqrt{2}$ times the measure of a leg.

Thus, we have the following.

$$n = a = \frac{14\sqrt{3}}{3} \text{ and } q = n\sqrt{2} = \left(\frac{14\sqrt{3}}{3}\right)\sqrt{2} = \frac{14\sqrt{6}}{3}$$

63. Apply the relationships between the lengths of the sides of a $45°-45°$ right triangle to the triangle on the left to find the values of *p* and *r*. In the $45°-45°$ right triangle, the sides opposite the $45°$ angles measure the same. The hypotenuse is $\sqrt{2}$ times the measure of a leg.



Thus, we have the following.

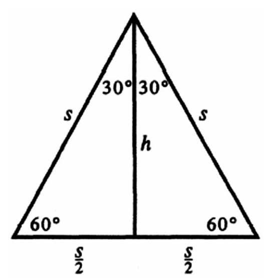
$$p = 15 \text{ and } r = p\sqrt{2} = 15\sqrt{2}$$

Apply the relationships between the lengths of the sides of a $30°-60°$ right triangle next to the triangle on the right to find the values of *q* and *t*. In the $30°-60°$ right triangle, the side opposite the $60°$ angle is $\sqrt{3}$ times as long as the side opposite to the $30°$ angle. The length of the hypotenuse is 2 times as long as the shorter leg (opposite the $30°$ angle).

Thus, we have the following.

$$r = q\sqrt{3} \Rightarrow q = \frac{r}{\sqrt{3}} = \frac{15\sqrt{2}}{\sqrt{3}} = \frac{15\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 5\sqrt{6} \text{ and } t = 2q = 2\left(5\sqrt{6}\right) = 10\sqrt{6}$$

64. Let *h* be the height of the equilateral triangle. *h* bisects the base, *s*, and forms two 30°–60° right triangles.



The formula for the area of a triangle is $A = \frac{1}{2}bh.$ In this triangle, $b = s$. The height *h* of the triangle is the side opposite the 60° angle in either 30°–60° right triangle. The side opposite the 30° angle is $\frac{s}{2}$.

The height is $\sqrt{3} \cdot \frac{s}{2}. = \frac{s\sqrt{3}}{2}$ So the area of the entire triangle is $A = \frac{1}{2}s\left(\frac{s\sqrt{3}}{2}\right) = \frac{s^2\sqrt{3}}{4}.$

65. Since $A = \frac{1}{2}bh,$ we have $A = \frac{1}{2} \cdot s \cdot s = \frac{1}{2}s^2$ or $A = \frac{s^2}{2}.$

66. Yes, the third angle can be found by subtracting the given acute angle from 90˚, and the remaining two sides can be found using a trigonometric function involving the known angle and side.

67. Answers will vary.

Section 2.2: Trigonometric Functions of Non-Acute Angles

1. C; $180° - 98° = 82°$
(98° is in quadrant II)

2. F; $212° - 180° = 32°$
(212° is in quadrant III)

3. A; $-135° + 360° = 225°$ and
$225° - 180° = 45°$
(225° is in quadrant III)

4. B; $-60° + 360° = 300°$ and
$360° - 300° = 60°$
(300° is in quadrant IV)

5. D; $750° - 2 \cdot 360° = 30°$
(30° is in quadrant I)

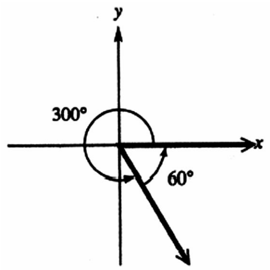
6. B; $480° - 360° = 120°$ and
$180° - 120° = 60°$
(120° is in quadrant II)

7. 2 is a good choice for r because in a $30° - 60°$ right triangle, the hypotenuse is twice the length of the shorter side (the side opposite to the $30°$ angle). By choosing 2, one avoids introducing a fraction (or decimal) when determining the length of the shorter side. Choosing any even positive integer for r would have this result; however, 2 is the most convenient value.

8. – 9. Answers will vary.

	θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
10.	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
11.	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
12.	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
13.	120°	$\frac{\sqrt{3}}{2}$	$\cos 120° = -\cos 60° = -\frac{1}{2}$	$-\sqrt{3}$	$\cot 120° = -\cot 60° = -\frac{\sqrt{3}}{3}$	$\sec 120° = -\sec 60° = -2$	$\frac{2\sqrt{3}}{3}$
14.	135°	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	$\tan 135° = -\tan 45° = -1$	$\cot 135° = -\cot 45° = -1$	$-\sqrt{2}$	$\sqrt{2}$
15.	150°	$\sin 150° = \sin 30° = \frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$\cot 150° = -\cot 30° = -\sqrt{3}$	$\sec 150° = -\sec 30° = -\frac{2\sqrt{3}}{3}$	2
16.	210°	$-\frac{1}{2}$	$\cos 210° = -\cos 30° = -\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\sec 210° = -\sec 30° = -\frac{2\sqrt{3}}{3}$	-2
17.	240°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\tan 240° = \tan 60° = \sqrt{3}$	$\cot 240° = \cot 60° = \frac{\sqrt{3}}{3}$	-2	$-\frac{2\sqrt{3}}{3}$

18. To find the reference angle for $300°$, sketch this angle in standard position.



The reference angle is $360° - 300° = 60°$. Since $300°$ lies in quadrant IV, the sine, tangent, cotangent, and cosecant are negative.

$\sin 300° = -\sin 60° = -\frac{\sqrt{3}}{2}$

$\cos 300° = \cos 60° = \frac{1}{2}$

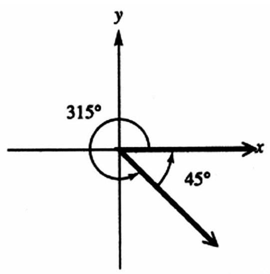
$\tan 300° = -\tan 60° = -\sqrt{3}$

$\cot 300° = -\cot 60° = -\frac{\sqrt{3}}{3}$

$\sec 300° = \sec 60° = 2$

$\csc 300° = -\csc 60° = -\frac{2\sqrt{3}}{3}$

19. To find the reference angle for $315°$, sketch this angle in standard position.



The reference angle is $360° - 315° = 45°$. Since $315°$ lies in quadrant IV, the sine, tangent, cotangent, and cosecant are negative.

$\sin 315° = -\sin 45° = -\frac{\sqrt{2}}{2}$

$\cos 315° = \cos 45° = \frac{\sqrt{2}}{2}$

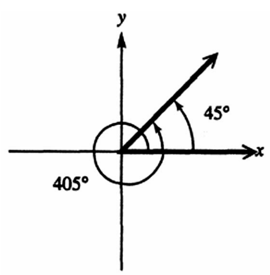
$\tan 315° = -\tan 45° = -1$

$\cot 315° = -\cot 45° = -1$

$\sec 315° = \sec 45° = \sqrt{2}$

$\csc 315° = -\csc 45° = -\sqrt{2}$

20. To find the reference angle for $405°$, sketch this angle in standard position.



The reference angle for $405°$ is $405° - 360° = 45°$. Because $405°$ lies in quadrant I, the values of all of its trigonometric functions will be positive, so these values will be identical to the trigonometric function values for $45°$. See the Function Values of Special Angles table that follows Example 5 in Section 2.1 on page 50.)

$\sin 405° = \sin 45° = \frac{\sqrt{2}}{2}$

$\cos 405° = \cos 45° = \frac{\sqrt{2}}{2}$

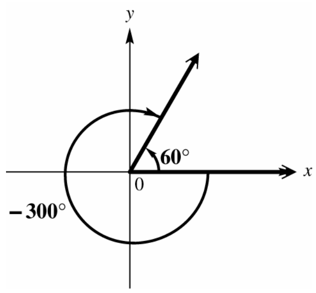
$\tan 405° = \tan 45° = 1$

$\cot 405° = \cot 45° = 1$

$\sec 405° = \sec 45° = \sqrt{2}$

$\csc 405° = \csc 45° = \sqrt{2}$

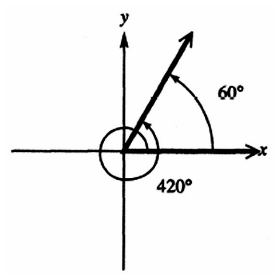
21. To find the reference angle for $-300°$, sketch this angle in standard position.



The reference angle for $-300°$ is $-300°+360°=60°$. Because $-300°$ lies in quadrant I, the values of all of its trigonometric functions will be positive, so these values will be identical to the trigonometric function values for 60°. See the Function Values of Special Angles table that follows Example 5 in Section 2.1 on page 50.)

$$\sin(-300°)=\sin 60°=\frac{\sqrt{3}}{2}$$
$$\cos(-300°)=\cos 60°=\frac{1}{2}$$
$$\tan(-300°)=\tan 60°=\sqrt{3}$$
$$\cot(-300°)=\cot 60°=\frac{\sqrt{3}}{3}$$
$$\sec(-300°)=\sec 60°=2$$
$$\csc(-300°)=\csc 60°=\frac{2\sqrt{3}}{3}$$

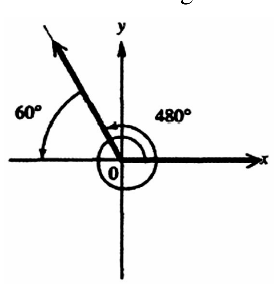
22. To find the reference angle for $420°$, sketch this angle in standard position.



The reference angle for $420°$ is $420°-360°=60°$. Because $420°$ lies in quadrant I, the values of all of its trigonometric functions will be positive, so these values will be identical to the trigonometric function values for 60°. See the Function Values of Special Angles table that follows Example 5 in Section 2.1 on page 50.)

$$\sin 420°=\sin 60°=\frac{\sqrt{3}}{2}$$
$$\cos 420°=\cos 60°=\frac{1}{2}$$
$$\tan 420°=\tan 60°=\sqrt{3}$$
$$\cot 420°=\cot 60°=\frac{\sqrt{3}}{3}$$
$$\sec 420°=\sec 60°=2$$
$$\csc 420°=\csc 60°=\frac{2\sqrt{3}}{3}$$

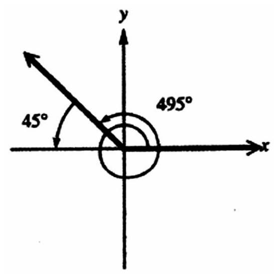
23. To find the reference angle for $480°$, sketch this angle in standard position.



$480°$ is coterminal with $480°-360°=120°$. The reference angle is $180°-120°=60°$. Since $480°$ lies in quadrant II, the cosine, tangent, cotangent, and secant are negative.

$$\sin 480°=\sin 60°=\frac{\sqrt{3}}{2}$$
$$\cos 480°=\cos 60°=-\frac{1}{2}$$
$$\tan 480°=\tan 60°=-\sqrt{3}$$
$$\cot 480°=\cot 60°=-\frac{\sqrt{3}}{3}$$
$$\sec 480°=\sec 60°=-2$$
$$\csc 480°=\csc 60°=\frac{2\sqrt{3}}{3}$$

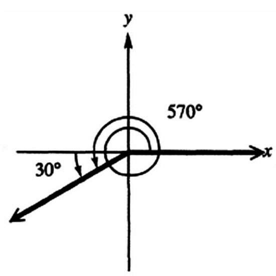
24. To find the reference angle for $495°$, sketch this angle in standard position.



$495°$ is coterminal with $495° - 360° = 135°$. The reference angle is $180° - 135° = 45°$. Since $495°$ lies in quadrant II, the cosine, tangent, cotangent, and secant are negative.

$$\sin 495° = \sin 45° = \frac{\sqrt{2}}{2}$$
$$\cos 495° = -\cos 45° = -\frac{\sqrt{2}}{2}$$
$$\tan 495° = -\tan 45° = -1$$
$$\cot 495° = -\cot 45° = -1$$
$$\sec 495° = -\sec 45° = -\sqrt{2}$$
$$\csc 495° = \csc 45° = \sqrt{2}$$

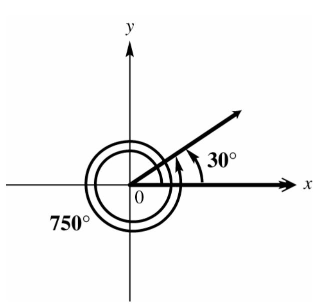
25. To find the reference angle for $570°$, sketch this angle in standard position.



$570°$ is coterminal with $570° - 360° = 210°$. The reference angle is $210° - 180° = 30°$. Since $570°$ lies in quadrant III, the sine, cosine, secant, and cosecant are negative.

$$\sin 570° = -\sin 30° = -\frac{1}{2}$$
$$\cos 570° = -\cos 30° = -\frac{\sqrt{3}}{2}$$
$$\tan 570° = \tan 30° = \frac{\sqrt{3}}{3}$$
$$\cot 570° = \cot 30° = \sqrt{3}$$
$$\sec 570° = -\sec 30° = -\frac{2\sqrt{3}}{3}$$
$$\csc 570° = -\csc 30° = -2$$

26. To find the reference angle for $750°$, sketch this angle in standard position.



$750°$ is coterminal with $30°$ because $750° - 2 \cdot 360° = 750° - 720° = 30°$. Since $750°$ lies in quadrant I, the values of all of its trigonometric functions will be positive, so these values will be identical to the trigonometric function values for $30°$.

$$\sin 750° = \sin 30° = \frac{1}{2}$$
$$\cos 750° = \cos 30° = \frac{\sqrt{3}}{2}$$
$$\tan 750° = \tan 30° = \frac{\sqrt{3}}{3}$$
$$\cot 750° = \cot 30° = \sqrt{3}$$
$$\sec 750° = \sec 30° = \frac{2\sqrt{3}}{3}$$
$$\csc 750° = \csc 30° = 2$$

27. $1305°$ is coterminal with $1305° - 3 \cdot 360° = 1305° - 1080° = 225°$. This angle lies in quadrant III and the reference angle is $225° - 180° = 45°$. Since $1305°$ lies in quadrant III, the sine, cosine, secant, and cosecant are negative.

$\sin 1305° = -\sin 45° = -\frac{\sqrt{2}}{2}$

$\cos 1305° = -\cos 45° = -\frac{\sqrt{2}}{2}$

$\tan 1305° = \tan 45° = 1$

$\cot 1305° = \cot 45° = 1$

$\sec 1305° = -\sec 45° = -\sqrt{2}$

$\csc 1305° = -\csc 45° = -\sqrt{2}$

28. $1500°$ is coterminal with $1500° - 4 \cdot 360° = 1500° - 1440° = 60°$. Because $420°$ lies in quadrant I, the values of all of its trigonometric functions will be positive, so these values will be identical to the trigonometric function values for $60°$.

$\sin 1500° = \sin 60° = \frac{\sqrt{3}}{2}$

$\cos 1500° = \cos 60° = \frac{1}{2}$

$\tan 1500° = \tan 60° = \sqrt{3}$

$\cot 1500° = \cot 60° = \frac{\sqrt{3}}{3}$

$\sec 1500° = \sec 60° = 2$

$\csc 1500° = \csc 60° = \frac{2\sqrt{3}}{3}$

29. $2670°$ is coterminal with $2670° - 7 \cdot 360° = 2670° - 2520° = 150°$. The reference angle is $180° - 150° = 30°$. Since $2670°$ lies in quadrant II, the cosine, tangent, cotangent, and secant are negative.

$\sin 2670° = \sin 30° = \frac{1}{2}$

$\cos 2670° = -\cos 30° = -\frac{\sqrt{3}}{2}$

$\tan 2670° = -\tan 30° = -\frac{\sqrt{3}}{3}$

$\cot 2670° = -\cot 30° = -\sqrt{3}$

$\sec 2670° = -\sec 30° = -\frac{2\sqrt{3}}{3}$

$\csc 2670° = \csc 30° = 2$

30. $-390°$ is coterminal with $-390° + 2 \cdot 360° = -390° + 720° = 330°$. The reference angle is $360° - 330° = 30°$. Since $-390°$ lies in quadrant IV, the sine, tangent, cotangent, and cosecant are negative.

$\sin(-390°) = -\sin 30° = -\frac{1}{2}$

$\cos(-390°) = \cos 30° = \frac{\sqrt{3}}{2}$

$\tan(-390°) = -\tan 30° = -\frac{\sqrt{3}}{3}$

$\cot(-390°) = -\cot 30° = -\sqrt{3}$

$\sec(-390°) = \sec 30° = \frac{2\sqrt{3}}{3}$

$\csc(-390°) = -\csc 30° = -2$

31. $-510°$ is coterminal with $-510° + 2 \cdot 360° = -510° + 720° = 210°$. The reference angle is $210° - 180° = 30°$. Since $-510°$ lies in quadrant III, the sine, cosine, and secant and cosecant are negative.

$\sin(-510°) = -\sin 30° = -\frac{1}{2}$

$\cos(-510°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

$\tan(-510°) = \tan 30° = \frac{\sqrt{3}}{3}$

$\cot(-510°) = \cot 30° = \sqrt{3}$

$\sec(-510°) = -\sec 30° = -\frac{2\sqrt{3}}{3}$

$\csc(-510°) = -\csc 30° = -2$

32. $-1020°$ is coterminal with $-1020° + 3 \cdot 360° = -1020° + 1080° = 60°$. Because $-1020°$ lies in quadrant I, the values of all of its trigonometric functions will be positive, so these values will be identical to the trigonometric function values for $60°$.

$\sin(-1020°) = \sin 60° = \frac{\sqrt{3}}{2}$

$\cos(-1020°) = \cos 60° = \frac{1}{2}$

$\tan(-1020°) = \tan 60° = \sqrt{3}$

$\cot(-1020°) = \cot 60° = \frac{\sqrt{3}}{3}$

$\sec(-1020°) = \sec 60° = 2$

$\csc(-1020°) = \csc 60° = \frac{2\sqrt{3}}{3}$

33. $-1290°$ is coterminal with $-1290° + 4 \cdot 360° = -1290° + 1440° = 150°$. This angle lies in quadrant II and the reference angle is $180° - 150° = 30°$. Since $-1290°$ lies in quadrant II, the cosine, tangent, cotangent, and secant are negative.

$\sin(-1290°) = \sin 30° = \frac{1}{2}$

$\cos(-1290°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

$\tan(-1290°) = -\tan 30° = -\frac{\sqrt{3}}{3}$

$\cot(-1290°) = -\cot 30° = -\sqrt{3}$

$\sec(-1290°) = -\sec 30° = -\frac{2\sqrt{3}}{3}$

$\csc(-1290°) = \csc 30° = 2$

34. Since $1305°$ is coterminal with an angle of $1305° - 3 \cdot 360° = 1305° - 1080° = 225°$, it lies in quadrant III. Its reference angle is $225° - 180° = 45°$. Since the sine is negative in quadrant III, we have $\sin 1305° = -\sin 45° = -\frac{\sqrt{2}}{2}$.

35. Since $-510°$ is coterminal with an angle of $-510° + 2 \cdot 360° = -510° + 720° = 210°$, it lies in quadrant III. Its reference angle is $210° - 180° = 30°$. Since the cosine is negative in quadrant III, we have $\cos(-510°) = -\cos 30° = -\frac{\sqrt{3}}{2}$.

36. Since $-1020°$ is coterminal with an angle of $-1020° + 3 \cdot 360° = -1020° + 1080° = 60°$, it lies in quadrant I. Because $-1020°$ lies in quadrant I, the values of all of its trigonometric functions will be positive, so $\tan(-1020°) = \tan 60° = \sqrt{3}$.

37. Since $1500°$ is coterminal with an angle of $1500° - 4 \cdot 360° = 1500° - 1440° = 60°$, it lies in quadrant I. Because $1500°$ lies in quadrant I, the values of all of its trigonometric functions will be positive, so $\sin 1500° = \sin 60° = \frac{\sqrt{3}}{2}$.

38. $\sin 30° + \sin 60° \stackrel{?}{=} \sin(30° + 60°)$

Evaluate each side to determine whether this statement is true or false.

$$\sin 30° + \sin 60° = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2} \text{ and } \sin(30° + 60°) = \sin 90° = 1$$

Since $\frac{1+\sqrt{3}}{2} \neq 1$, the given statement is false.

39. $\sin(30° + 60°) \stackrel{?}{=} \sin 30° \cdot \cos 60° + \sin 60° \cdot \cos 30°$

Evaluate each side to determine whether this equation is true or false.

$$\sin(30° + 60°) = \sin 90° = 1 \quad \text{and} \quad \sin 30° \cdot \cos 60° + \sin 60° \cdot \cos 30° = \frac{1}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$$

Since, 1 = 1, the statement is true.

40. $\cos 60° \stackrel{?}{=} 2\cos^2 30° - 1$

Evaluate each side to determine whether this statement is true or false.

$$\cos 60° = \frac{1}{2} \text{ and } 2\cos^2 30° - 1 = 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1 = 2\left(\frac{3}{4}\right) - 1 = \frac{3}{2} - 1 = \frac{1}{2}$$

Since $\frac{1}{2} = \frac{1}{2}$, the statement is true.

41. $\cos 60° \stackrel{?}{=} 2\cos 30°$

Evaluate each side to determine whether this statement is true or false.

$$\cos 60° = \frac{1}{2} \text{ and } 2\cos 30° = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

Since $\frac{1}{2} \neq \sqrt{3}$, the statement is false.

42. $\sin 120° \stackrel{?}{=} \sin 150° - \sin 30°$

Evaluate each side to determine whether this statement is true or false.

$$\sin 120° = \frac{\sqrt{3}}{2} \text{ and } \sin 150° - \sin 30° = \frac{1}{2} - \frac{1}{2} = 0$$

Since $\frac{\sqrt{3}}{2} \neq 0$, the statement is false.

43. $\sin 120° \stackrel{?}{=} \sin 180° \cdot \cos 60° - \sin 60° \cdot \cos 180°$

Evaluate each side to determine whether this statement is true or false.

$$\sin 120° = \frac{\sqrt{3}}{2} \text{ and } \sin 180° \cdot \cos 60° - \sin 60° \cdot \cos 180° = 0\left(\frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)(-1) = 0 - \left(-\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$$

Since $\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$, the statement is true.

44. 225° is in quadrant III, so the reference angle is $225° - 180° = 45°$.

$$\cos 45° = \frac{x}{r} \Rightarrow x = r\cos 45° = 10 \cdot \frac{\sqrt{2}}{2} = 5\sqrt{2} \text{ and } \sin 45° = \frac{y}{r} \Rightarrow y = r\sin 45° = 10 \cdot \frac{\sqrt{2}}{2} = 5\sqrt{2}$$

Since 225° is in quadrant III, both the x and y coordinate will be negative. The coordinates of P are: $\left(-5\sqrt{2}, -5\sqrt{2}\right)$.

45. 150° is in quadrant III, so the reference angle is $180° - 150° = 30°$.

$$\cos 30° = \frac{x}{r} \Rightarrow x = r\cos 30° = 6 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3} \text{ and } \sin 30° = \frac{y}{r} \Rightarrow y = r\sin 30° = 6 \cdot \frac{1}{2} = 3$$

Since 150° is in quadrant II, x will be negative and y will be positive. The coordinated of P are: $\left(-3\sqrt{3}, 3\right)$.

46. For every angle θ, $\sin^2\theta+\cos^2\theta=1$. Since $(-.8)^2+(.6)^2=.64+.36=1$, there is an angle θ for which $\cos\theta=.6$ and $\sin\theta=-.8$. Since $\cos\theta>0$ and $\sin\theta<0$, it is an angle that lies in quadrant IV.

47. For every angle θ, $\sin^2\theta+\cos^2\theta=1$. Since $\left(\frac{3}{4}\right)^2+\left(\frac{2}{3}\right)^2=\frac{9}{16}+\frac{4}{9}=\frac{145}{144}\neq 1$, there is no angle θ for which $\cos\theta=\frac{2}{3}$ and $\sin\theta=\frac{3}{4}$.

48. If θ is in the interval $(90°,180°)$, then $90°<\theta<180°\Rightarrow 45°<\frac{\theta}{2}<90°$. Thus, $\frac{\theta}{2}$ is a quadrant I angle and $\sin\frac{\theta}{2}$ is positive.

49. If θ is in the interval $(90°,180°)$, then $90°<\theta<180°\Rightarrow 45°<\frac{\theta}{2}<90°$. Thus, $\frac{\theta}{2}$ is a quadrant I angle and $\cos\frac{\theta}{2}$ is positive.

50. If θ is in the interval $(90°,180°)$, then $90°<\theta<180°\Rightarrow 270°<\theta+180°<360°$. Thus, $\theta+180°$ is a quadrant IV angle and $\cot(\theta+180°)$ is negative.

51. If θ is in the interval $(90°,180°)$, then $90°<\theta<180°\Rightarrow 270°<\theta+180°<360°$. Thus, $\theta+180°$ is a quadrant IV angle and $\sec(\theta+180°)$ is positive.

52. If θ is in the interval $(90°,180°)$, then $90°<\theta<180°\Rightarrow -90°>-\theta>-180°\Rightarrow -180°<-\theta<-90°$. Since $-180°$ is coterminal with $-180°+360°=180°$ and $-90°$ is coterminal with $-90°+360°=270°$, $-\theta$ is a quadrant III angle and $\cos(-\theta)$ is negative.

53. If θ is in the interval $(90°,180°)$, then $90°<\theta<180°\Rightarrow -90°>-\theta>-180°\Rightarrow -180°<-\theta<-90°$. Since $-180°$ is coterminal with $-180°+360°=180°$ and $-90°$ is coterminal with $-90°+360°=270°$, $-\theta$ is a quadrant III angle and $\sin(-\theta)$ is negative.

54. θ and $\theta+n\cdot 360°$ are coterminal angles, so the sine of each of these will result in the same value.

55. θ and $\theta+n\cdot 360°$ are coterminal angles, so the cosine of each of these will result in the same value.

56. The reference angle for $115°$ is $180°-115°=65°$. Since $115°$ is in quadrant II the cosine is negative. Cos θ decreases on the interval $(90°,180°)$ from 0 to –1. Therefore, $\cos 115°$ is closest to $-.4$.

57. The reference angle for $115°$ is $180°-115°=65°$. Since $115°$ is in quadrant II the sine is positive. Sin θ decreases on the interval $(90°,180°)$ from 1 to 0. Therefore, $\sin 115°$ is closest to .9.

58. When $\theta=45°, \sin\theta=\cos\theta=\frac{\sqrt{2}}{2}$. Sine and cosine are opposites in quadrants II and IV. Thus, $180°-\theta=180°-45°=135°$ in quadrant II and $360°-\theta=360°-45°=315°$ in quadrant IV.

59. When $\theta=45°, \sin\theta=\cos\theta=\frac{\sqrt{2}}{2}$. Sine and cosine are both positive in quadrant I and both negative in quadrant III. Since $\theta+180°=45°+180°=225°$, $45°$ is the quadrant I angle and $225°$ is the quadrant III angle.

60. $L = \dfrac{(\theta_2 - \theta_1) S^2}{200(h + S \tan \alpha)}$

(a) Substitute $h = 1.9$ ft, $\alpha = .9°$, $\theta_1 = -.3°$, $\theta_2 = 4°$, and $S = 336$ ft.

$$L = \frac{[4 - (-3)]336^2}{200(1.9 + 336 \tan .9°)} \approx 550\text{ft}$$

(b) Substitute $h = 1.9$ ft, $\alpha = 1.5°$, $\theta_1 = -.3°$, $\theta_2 = 4°$, and $S = 336$ ft.

$$L = \frac{[4 - (-3)]336^2}{200(1.9 + 336 \tan 1.5°)} \approx 369\text{ft}$$

(c) Answers will vary.

61. $\sin \theta = \dfrac{1}{2}$

Since $\sin \theta$ is positive, θ must lie in quadrants I or II. Since one angle, namely $30°$, lies in quadrant I, that angle is also the reference angle, θ'. The angle in quadrant II will be $180° - \theta' = 180° - 30° = 150°$.

62. $\cos \theta = \dfrac{\sqrt{3}}{2}$

Since $\cos \theta$ is positive, θ must lie in quadrants I or IV. Since one angle, namely $30°$, lies in quadrant I, that angle is also the reference angle, θ'. The angle in quadrant IV will be $360° - \theta' = 360° - 30° = 330°$.

63. $\tan \theta = -\sqrt{3}$

Since $\tan \theta$ is negative, θ must lie in quadrants II or IV. Since the absolute value of $\tan \theta$ is $\sqrt{3}$, the reference angle, θ' must be $60°$. The quadrant II angle θ equals $180° - \theta' = 180° - 60° = 120°$, and the quadrant IV angle θ equals $360° - \theta' = 360° - 60° = 300°$.

64. $\sec \theta = -\sqrt{2}$

Since $\sec \theta$ is negative, θ must lie in quadrants II or III. Since the absolute value of $\sec \theta$ is $\sqrt{2}$, the reference angle, θ' must be $45°$. The quadrant II angle θ equals $180° - \theta' = 180° - 45° = 135°$, and the quadrant III angle θ equals $180° + \theta' = 180° + 45° = 225°$.

65. $\cos \theta = \dfrac{\sqrt{2}}{2}$

Since $\cos \theta$ is positive, θ must lie in quadrants I or IV. Since one angle, namely $45°$, lies in quadrant I, that angle is also the reference angle, θ'. The angle in quadrant IV will be $360° - \theta' = 360° - 45° = 315°$.

66. $\cot \theta = -\dfrac{\sqrt{3}}{3}$

Since $\cot \theta$ is negative, θ must lie in quadrants II or IV. Since the absolute value of $\cot \theta$ is $\dfrac{\sqrt{3}}{3}$, the reference angle, θ' must be $60°$. The quadrant II angle θ equals $180° - \theta' = 180° - 60° = 120°$, and the quadrant IV angle θ equals $360° - \theta' = 360° - 60° = 300°$.

Section 2.3: Finding Trigonometric Function Values Using a Calculator

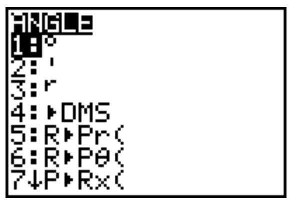
1. The CAUTION at the beginning of this section verifying that a calculator is in degree mode by finding sin 90°. If the calculator is in degree mode, the display should be 1.

2. When a scientific or graphing calculator is used to find a trigonometric function value, in most cases the result is an approximate value.

3. To find values of the cotangent, secant, and cosecant functions with a calculator, it is necessary to find the reciprocal of the reciprocal function value.

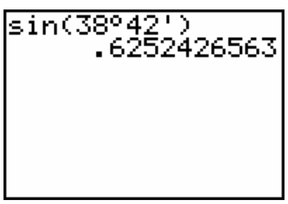
4. The reciprocal is used before the inverse function key when finding the angle, but after the function key when finding the trigonometric function value.

For Exercises 5 – 21, be sure your calculator is in degree mode. If your calculator accepts angles in degrees, minutes, and seconds, it is not necessary to change angles to decimal degrees. Keystroke sequences may vary on the type and/or model of calculator being used. Screens shown will be from a TI-83 Plus calculator. To obtain the degree (°) and (′) symbols, go to the ANGLE menu (2^{nd} APPS).



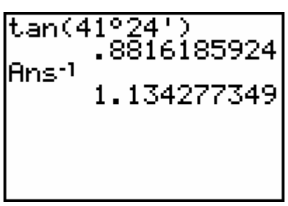
For Exercises 5 – 15, the calculation for decimal degrees is indicated for calculators that do not accept degree, minutes, and seconds.

5. $\sin 38° 42' \approx .6252427$



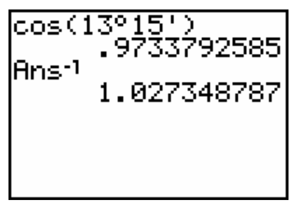
$38° 42' = \left(38 + \frac{42}{60}\right)° = 38.7°$

6. $\cot 41° 24' \approx 1.1342773$



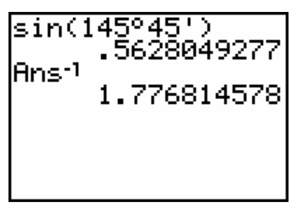
$42° 24' = \left(41 + \frac{24}{60}\right)° = 41.4°$

7. $\sec 13° 15' \approx 1.0273488$



$13°15' = \left(13 + \frac{15}{60}\right)° = 13.25°$

8. $\csc 145° 45' \approx 1.7768146$



$145°45' = \left(145 + \frac{45}{60}\right)° = 145.75°$

9. $\cot 183^\circ 48' \approx 15.055723$

```
tan(183°48')
    .0664199267
Ans⁻¹
    15.05572272
```

$183^\circ 48' = \left(183 + \frac{48}{60}\right)^\circ = 183.8^\circ$

10. $\cos 421^\circ 30' \approx .4771588$

```
cos(421°30')
    .4771587603
```

$421^\circ 30' = \left(421 + \frac{30}{60}\right)^\circ = 421.5^\circ$

11. $\sec 312^\circ 12' \approx 1.4887142$

```
cos(312°12')
    .6717205893
Ans⁻¹
     1.48871423
```

$312^\circ 12' = \left(312 + \frac{12}{60}\right)^\circ = 312.2^\circ$

12. $\tan\left(-80^\circ 6'\right) \approx -5.7297416$

```
tan(-80°6')
   -5.729741647
```

$-80^\circ 6' = -\left(80 + \frac{6}{60}\right)^\circ = -80.1^\circ$

13. $\sin\left(-317^\circ 36'\right) \approx .6743024$

```
sin(-317°36')
    .6743023876
```

$-317^\circ 36' = -\left(317 + \frac{36}{60}\right)^\circ = -317.6^\circ$

14. $\cot\left(-512^\circ 20'\right) \approx 1.9074147$

```
tan(-512°20')
    .5242698282
Ans⁻¹
    1.907414744
```

$-512^\circ 20' = -\left(512 + \frac{20}{60}\right)^\circ \approx -512.3333333^\circ$

15. $\cos\left(-15'\right) \approx .9999905$

```
cos(-0°15')
    .9999904807
```

$-15' = -\frac{15}{60}' = -.25^\circ$

16. $\dfrac{1}{\sec 14.8^\circ} = \cos 14.8^\circ \approx .9668234$

```
cos(14.8°)
    .9668233886
```

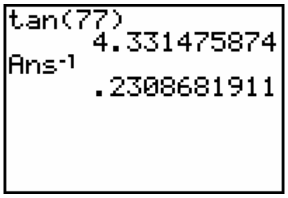
17. $\dfrac{1}{\cot 23.4^\circ} = \tan 23.4^\circ \approx .4327386$

```
tan(23.4)
    .4327386422
```

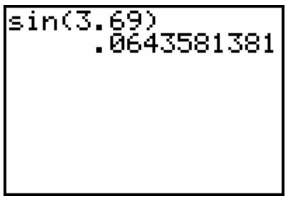
18. $\dfrac{\sin 33^\circ}{\cos 33^\circ} = \tan 33^\circ \approx .6494076$

```
tan(33)
    .6494075932
```

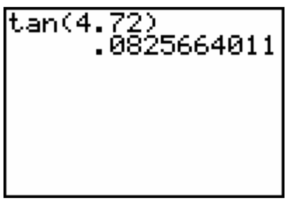
19. $\dfrac{\cos 77°}{\sin 77°} = \cot 77° \approx .2308682$



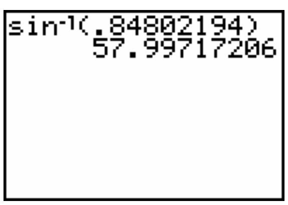
20. $\cos(90° - 3.69°) = \sin(3.69°) \approx .0643581$



21. $\cot(90° - 4.72°) = \tan 4.72° \approx .0825664$

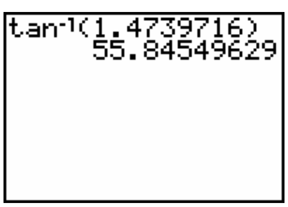


22. $\sin\theta = .84802194$



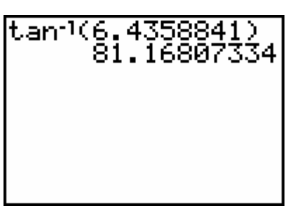
$\theta \approx 57.997172°$

23. $\tan\ \theta = 1.4739716$



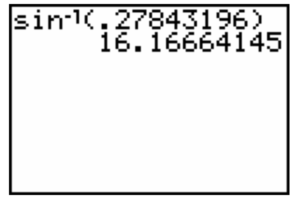
$\theta \approx 55.845496°$

24. $\tan\theta = 6.4358841$



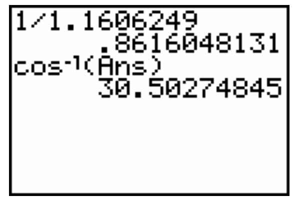
$\theta \approx 81.168073°$

25. $\sin\theta = .27843196$



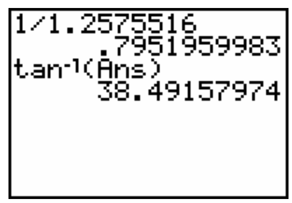
$\theta \approx 16.166641°$

26. $\sec\ \theta = 1.1606249$



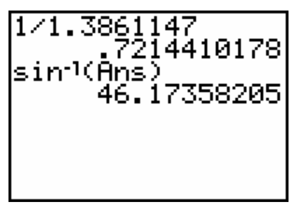
$\theta \approx 30.502748°$

27. $\cot\ \theta = 1.2575516$



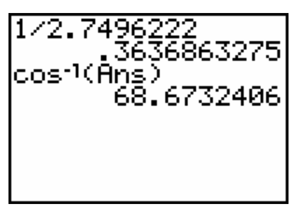
$\theta \approx 38.491580°$

28. $\csc\ \theta = 1.3861147$



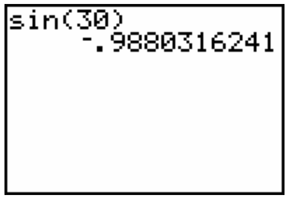
$\theta \approx 46.173582°$

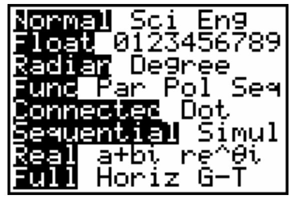
29. $\sec\ \theta = 2.7496222$



$\theta \approx 68.673241°$

30. A common mistake is to have the calculator in radian mode, when it should be in degree mode (and vice verse).

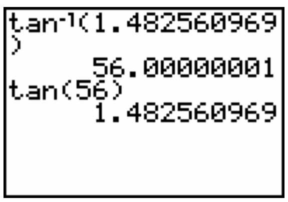




31. If the calculator allowed an angle θ where $0° \leq \theta < 360°$, then one would need to find an angle within this interval that is coterminal with $2000°$ by subtracting a multiple of $360°$, i.e. $2000° - 5 \cdot 360° = 2000° - 1800° = 200°$. If the calculator had a higher restriction on evaluating angles (such as $0 \leq \theta < 90°$) then one would need to use reference angles.

32. $\tan A = 1.482560969$

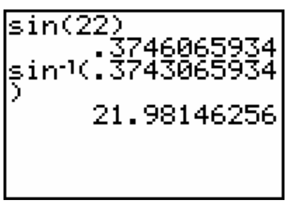
Find the arctan of 1.482560969.



$A \approx 56°$

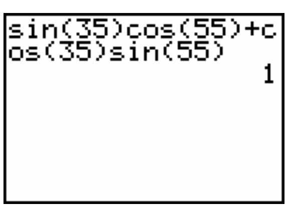
33. $\sin^{-1} A = 22$

Find the sine of $22°$.

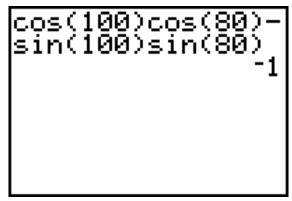


$A \approx .3746065934°$

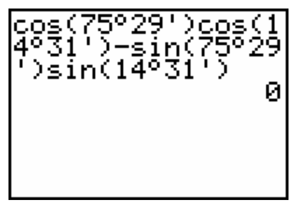
34. $\sin 35° \cos 55° + \cos 35° \sin 55° = 1$



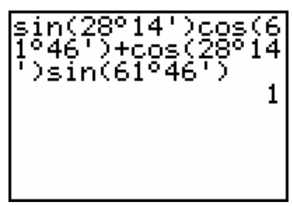
35. $\cos 100° \cos 80° - \sin 100° \sin 80° = -1$



36. $\cos 75°29' \cos 14°31' - \sin 75°29' \sin 14°31' = 0$



37. $\sin 28°14' \cos 61°46' + \cos 28°14' \sin 61°46' = 1$



38. For Auto A, calculate $70 \cdot \cos 10° \approx 68.94$. Auto A's reading is approximately 68.94 mph.

For Auto B, calculate $70 \cdot \cos 20° \approx 65.78$. Auto B's reading is approximately 65.78 mph.

39. The figure for this exercise indicates a right triangle. Because we are not considering the time involved in detecting the speed of the car, we will consider the speeds as sides of the right triangle. Given angle θ, $\cos\theta = \frac{r}{a}$. Thus, the speed that the radar detects is $r = a\cos\theta$.

40. $\cos 40° \stackrel{?}{=} 2\cos 20°$

Using a calculator gives $\cos 40° \approx .76604444$ and $2\cos 20° \approx 1.87938524$. Thus, the statement is false.

41. $\sin 10° + \sin 10° \stackrel{?}{=} \sin 20°$

Using a calculator gives $\sin 10° + \sin 10° \approx .34729636$ and $\sin 20° \approx .34202014$. Thus, the statement is false.

42. $\cos 70° \stackrel{?}{=} 2\cos^2 35° - 1$

Using a calculator gives $\cos 70° \approx .34202014$ and $2\cos^2 35° - 1 \approx .34202014$. Thus, the statement is true.

43. $\sin 50° \stackrel{?}{=} 2\sin 25° \cos 25°$

Using a calculator gives $\sin 50° \approx .76604444$ and $2\sin 25° \cos 25° \approx .76604444$. Thus, the statement is true.

44. $2\cos 38°22' \stackrel{?}{=} \cos 76°44'$

Using a calculator gives $2\cos 38°22' \approx 1.56810939$ and $\cos 76°44' \approx .22948353$. Thus, the statement is false.

45. $\cos 40° \stackrel{?}{=} 1 - 2\sin^2 80°$

Using a calculator gives $\cos 40° \approx .76604444$ and $1 - 2\sin^2 80° \approx -.93969262$. Thus, the statement is false.

46. $\frac{1}{2}\sin 40° \stackrel{?}{=} \sin \frac{1}{2}(40°)$

Using a calculator gives $\frac{1}{2}\sin 40° \approx .32139380$ and $\sin \frac{1}{2}(40°) \approx .34202014$ Thus, the statement is false.

47. $\sin 39°48' + \cos 39°48' \stackrel{?}{=} 1$

Using a calculator gives $\sin 39°48' + \cos 39°48' \approx 1.40839322 \neq 1$. Thus, the statement is false.

48. $F = W \sin \theta$

$F = 2400 \sin(-2.4°) \approx -100.5$ lb

F is negative because the car is traveling downhill.

49. $F = W \sin \theta$

$F = 2100 \sin 1.8° \approx 65.96$ lb

50. $F = W \sin \theta$

$$-145 = W \sin(-3°) \Rightarrow \frac{-145}{\sin(-3°)} = W \Rightarrow W \approx 2771 \text{ lb}$$

51. $F = W\sin\theta$

$-130 = 2600\sin\theta \Rightarrow \frac{-130}{2600} = \sin\theta \Rightarrow -.05 = \sin\theta \Rightarrow \theta = \sin^{-1}(-.05) \approx -2.87°$

52. $F = W\sin\theta$

$F = 2200\sin 2° \approx 76.77889275$ lb

$F = 2000\sin 2.2° \approx 76.77561818$ lb

The 2200-lb car on a 2° uphill grade has the greater grade resistance.

53. $F = W\sin\theta$

$150 = 3000\sin\theta \Rightarrow \frac{150}{3000} = \sin\theta \Rightarrow .05 = \sin\theta \Rightarrow \theta = \sin^{-1} .05 \approx 2.87°$

54.

θ	$\sin\theta$	$\tan\theta$	$\frac{\pi\theta}{180}$
0°	.0000	.0000	.0000
.5°	.0087	.0087	.0087
1°	.0175	.0175	.0175
1.5°	.0262	.0262	.0262
2°	.0349	.0349	.0349
2.5°	.0436	.0437	.0436
3°	.0523	.0524	.0524
3.5°	.0610	.0612	.0611
4°	.0698	.0699	.0698

(a) From the table we see that if θ is small, $\sin\theta \approx \tan\theta \approx \frac{\pi\theta}{180}$.

(b) $F = W\sin\theta \approx W\tan\theta \approx \frac{W\pi\theta}{180}$

(c) $\tan\theta = \frac{4}{100} = .04$

$F \approx W\tan\theta = 2000(.04) = 80\text{lb}$

(d) Use $F \approx \frac{W\pi\theta}{180}$ from part (b)

Let $\theta = 3.75$ and $W = 1800.$

$F \approx \frac{1800\pi(3.75)}{180} \approx 117.81$ lb

55. $R = \frac{V^2}{g(f + \tan\theta)}$

(a) Since 45 mph = 66 ft/sec, $V = 66$, $\theta = 3°$, $g = 32.2$, and $f = .14$, we have the following.

$R = \frac{V^2}{g(f + \tan\theta)} = \frac{66^2}{32.2(.14 + \tan 3°)} \approx 703$ ft

Continued on next page

55. (continued)

(b) Since there are 5280 ft in one mile and 3600 sec in one min, we have the following.

$$70 \text{ mph} = 70 \text{ mph} \cdot 1 \text{ hr}/\ 3600 \text{ sec} \cdot 5280 \text{ ft}/1 \text{ mi} = 102\frac{2}{3} \text{ ft per sec} \approx 102.67 \text{ ft per sec}$$

Since $V = 102.67$, $\theta = 3°$, $g = 32.2$, and $f = .14$, we have the following.

$$R = \frac{V^2}{g(f + \tan\theta)} \approx \frac{102.67^2}{32.2(.14 + \tan 3°)} \approx 1701 \text{ ft}$$

(c) Intuitively, increasing θ would make it easier to negotiate the curve at a higher speed much like is done at a race track. Mathematically, a larger value of θ (acute) will lead to a larger value for $\tan\theta$. If $\tan\theta$ increases, then the ratio determining R will *decrease*. Thus, the radius can be smaller and the curve sharper if θ is increased.

$$R = \frac{V^2}{g(f + \tan\theta)} = \frac{66^2}{32.2(.14 + \tan 4°)} \approx 644 \text{ ft and } R = \frac{V^2}{g(f + \tan\theta)} \approx \frac{102.67^2}{32.2(.14 + \tan 4°)} \approx 1559 \text{ ft}$$

As predicted, both values are less.

56. From Exercise 55, $R = \dfrac{V^2}{g(f + \tan\theta)}$. Solving for V we have the following.

$$R = \frac{V^2}{g(f + \tan\theta)} \Rightarrow V^2 = Rg(f + \tan\theta) \Rightarrow V = \sqrt{Rg(f + \tan\theta)}$$

Since $R = 1150$, $\theta = 2.1°$, $g = 32.2$, and $f = .14$, we have the following.

$$V = \sqrt{Rg(f + \tan\theta)} = \sqrt{1150(32.2)(.14 + \tan 2.1°)} \approx 80.9 \text{ ft/sec}$$

Since $80.9 \text{ ft/sec} \cdot 3600 \text{ sec/hr} \cdot 1 \text{ mi}/5280 \text{ ft} \approx 55 \text{ mph}$, it should have a 55 mph speed limit.

57. (a) $\theta_1 = 46°$, $\theta_2 = 31°$, $c_1 = 3\times10^8$ m per sec

$$\frac{c_1}{c_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow c_2 = \frac{c_1 \sin\theta_2}{\sin\theta_1} \Rightarrow c_2 = \frac{(3\times10^8)(\sin 31°)}{\sin 46°} \approx 2\times10^8$$

Since c_1 is only given to one significant digit, c_2 can only be given to one significant digit. The speed of light in the second medium is about 2×10^8 m per sec.

(b) $\theta_1 = 39°$, $\theta_2 = 28°$, $c_1 = 3\times10^8$ m per sec

$$\frac{c_1}{c_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow c_2 = \frac{c_1 \sin\theta_2}{\sin\theta_1} \Rightarrow c_2 = \frac{(3\times10^8)(\sin 28°)}{\sin 39°} \approx 2\times10^8$$

Since c_1 is only given to one significant digit, c_2 can only be given to one significant digit. The speed of light in the second medium is about 2×10^8 m per sec.

58. (a) $\theta_1 = 40°, c_2 = 1.5\times10^8$ m per sec, and $c_1 = 3\times10^8$ m per sec

$$\frac{c_1}{c_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow \sin\theta_2 = \frac{c_2 \sin\theta_1}{c_1} \Rightarrow \sin\theta_2 = \frac{(1.5\times10^8)(\sin 40°)}{3\times10^8} \Rightarrow \theta_2 = \sin^{-1}\left[\frac{(1.5\times10^8)(\sin 40°)}{3\times10^8}\right] \approx 19°$$

(b) $\theta_1 = 62°$, $c_2 = 2.6\times10^8$ m per sec and $c_1 = 3\times10^8$ m per sec

$$\frac{c_1}{c_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow \sin\theta_2 = \frac{c_2 \sin\theta_1}{c_1} \Rightarrow \sin\theta_2 = \frac{(2.6\times10^8)(\sin 62°)}{3\times10^8} \Rightarrow \theta_2 = \sin^{-1}\left[\frac{(2.6\times10^8)(\sin 62°)}{3\times10^8}\right] \approx 50°$$

59. $\theta_1 = 90°,\ c_1 = 3\times10^8$ m per sec, and $c_2 = 2.254\times10^8$

$$\frac{c_1}{c_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow \sin\theta_2 = \frac{c_2\sin\theta_1}{c_1}$$

$$\sin\theta_2 = \frac{\left(2.254\times10^8\right)\left(\sin 90°\right)}{3\times10^8} = \frac{2.254\times10^8\,(1)}{3\times10^8} = \frac{2.254}{3} \Rightarrow \theta_2 = \sin^{-1}\left(\frac{2.254}{3}\right) \approx 48.7°$$

60. $\theta_1 = 90° - 29.6° = 60.4°,\ c_1 = 3\times10^8$ m per sec, and $c_2 = 2.254\times10^8\ c_1 = 3\times10^8$ m per sec

$$\frac{c_1}{c_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow \sin\theta_2 = \frac{c_2\sin\theta_1}{c_1}$$

$$\sin\theta_2 = \frac{\left(2.254\times10^8\right)\left(\sin 60.4°\right)}{3\times10^8} = \frac{2.254}{3}\left(\sin 60.4°\right) \Rightarrow \theta_2 = \sin^{-1}\left(\frac{2.254}{3}\left(\sin 60.4°\right)\right) \approx 40.8°$$

Light from the object is refracted at an angle of 40.8° from the vertical. Light from the horizon is refracted at an angle of 48.7° from the vertical. Therefore, the fish thinks the object lies at an angle of 48.7° – 40.8° = 7.9° above the horizon.

61. (a) Let $V_1 = 55$ mph $= 55$ mph $\cdot$ 1 hr/ 3600 sec · 5280 ft/1 mi $= 80\frac{2}{3}$ ft per sec ≈ 80.67 ft per sec, and V_2=30 mph = 30 mph $\cdot$ 1 hr/ 3600 sec · 5280 ft/1 mi = 44 ft per sec. Also, let $\theta = 3.5°$, $K_1 = .4$, and $K_2 = .02$.

$$D = \frac{1.05\left(V_1^2 - V_2^2\right)}{64.4\left(K_1 + K_2 + \sin\theta\right)} = \frac{1.05\left(80.67^2 - 44^2\right)}{64.4\left(.4 + .02 + \sin 3.5°\right)} \approx 155 \text{ ft}$$

(b) Let $V_1 \approx 80.67$ ft per sec, $V_2 = 44$ ft per sec, $\theta = -2°$, $K_1 = .4$, and $K_2 = .02$.

$$D = \frac{1.05\left(V_1^2 - V_2^2\right)}{64.4\left(K_1 + K_2 + \sin\theta\right)} = \frac{1.05\left(80.67^2 - 44^2\right)}{64.4\left[.4 + .02 + \sin\left(-2°\right)\right]} \approx 194 \text{ ft}$$

62. Using the values for K_1 and K_2 from Exercise 61, determine V_2 when D = 200, $\theta = -3.5°$, V_1=90 mph = 30 mph $\cdot$ 1 hr/ 3600 sec · 5280 ft/1 mi = 132 ft per sec.

$$D = \frac{1.05\left(V_1^2 - V_2^2\right)}{64.4\left(K_1 + K_2 + \sin\theta\right)} \Rightarrow 200 = \frac{1.05\left(132^2 - V_2^2\right)}{64.4\left[.4 + .02 + \sin\left(-3.5°\right)\right]}$$

$$200 = \frac{1.05\left(132^2\right) - 1.05V_2^2}{23.12} \Rightarrow 200\left(23.12\right) = 18{,}295.2 - 1.05V_2^2 \Rightarrow 4624 = 18{,}295.2 - 1.05V_2^2$$

$$-13{,}671.2 = -1.05V_2^2 \Rightarrow V_2^2 = \frac{-13{,}671.2}{-1.05} \Rightarrow V_2^2 = 13020.19048 \Rightarrow V_2 \approx 114.106$$

$V_2 \approx 114$ ft/sec · 3600 sec/hr · 1 mi/5280 ft ≈ 78 mph

Section 2.4: Solving Right Triangles

Connections (page 72)
Steps 1 and 2 compare to his second step. The first part of Step 3 (solving the equation) compares to his third step, and the last part of Step 3 (checking) compares to his fourth step.

Exercises

1. 20,385.5 to 20,386.5

2. 28,999.5 to 29,000.5

3. 8958.5 to 8959.5

4. Answers will vary.
No; the number of points scored will be a whole number.

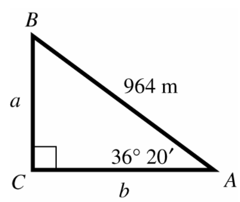
5. Answer will vary.
It would be cumbersome to write 2 as 2.00 or 2.000, for example, if the measurements had 3 or 4 significant digits (depending on the problem). In the formula, it is understood that 2 is an exact value. Since the radius measurement, 54.98 cm, has four significant digits, an appropriate answer would be 345.4 cm.

6. 23.0 ft indicates 3 significant digits and 23.00 ft indicates four significant digits.

7. If h is the actual height of a building and the height is measured as 58.6 ft, then $\left|h-58.6\right| \le \underline{.05}$.

8. If w is the actual weight of a car and the weight is measured as 15.00×10^{2} lb, then $\left|w-1500\right| \le \underline{.5}$.

9. $A=36°20',\ c=964$ m

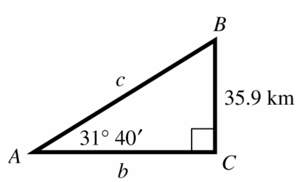


$A+B=90° \Rightarrow B=90°-A \Rightarrow B=90°-36°20'=89°60'-36°20'=53°40'$

$\sin A=\dfrac{a}{c} \Rightarrow \sin 36°20'=\dfrac{a}{964} \Rightarrow a=964\sin 36°20' \approx 571\text{ m}$ (rounded to three significant digits)

$\cos A=\dfrac{b}{c} \Rightarrow \cos 36°20'=\dfrac{b}{964} \Rightarrow b=964\cos 36°20' \approx 777\text{ m}$ (rounded to three significant digits)

10. $A=31°40',\ a=35.9$ km

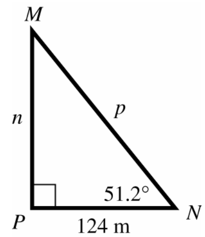


$A+B=90° \Rightarrow B=90°-A \Rightarrow B=90°-31°40'=89°60'-31°40'=58°20'$

$\sin A=\dfrac{a}{c} \Rightarrow \sin 31°40'=\dfrac{35.9}{c} \Rightarrow c=\dfrac{35.9}{\sin 31°40'} \approx 68.4$ km (rounded to three significant digits)

$\tan A=\dfrac{a}{b} \Rightarrow \tan 31°40'=\dfrac{35.9}{b} \Rightarrow b=\dfrac{35.9}{\tan 31°40'} \approx 58.2$ km (rounded to three significant digits)

11. $N = 51.2°$, $m = 124$ m

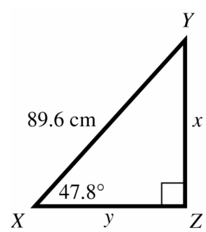


$$M + N = 90° \Rightarrow M = 90° - N \Rightarrow M = 90° - 51.2° = 38.8°$$

$$\tan N = \frac{n}{m} \Rightarrow \tan 51.2° = \frac{n}{124} \Rightarrow n = 124 \tan 51.2° \approx 154 \text{ m}$$ (rounded to three significant digits)

$$\cos N = \frac{m}{p} \Rightarrow \cos 51.2° = \frac{124}{p} \Rightarrow p = \frac{124}{\cos 51.2°} \approx 198 \text{ m}$$ (rounded to three significant digits)

12. $X = 47.8°$, $z = 89.6$ cm

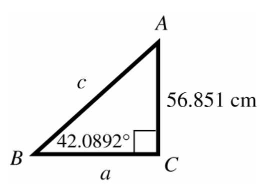


$$Y + X = 90° \Rightarrow Y = 90° - X \Rightarrow Y = 90° - 47.8° = 42.2°$$

$$\sin X = \frac{x}{z} \Rightarrow \sin 47.8° = \frac{x}{89.6} \Rightarrow x = 89.6 \sin 47.8° \approx 66.4 \text{ cm}$$ (rounded to three significant digits)

$$\cos X = \frac{y}{z} \Rightarrow \cos 47.8° = \frac{y}{89.6} \Rightarrow y = 89.6 \cos 47.8° \approx 60.2 \text{ cm}$$ (rounded to three significant digits)

13. $B = 42.0892°$, $b = 56.851$ cm

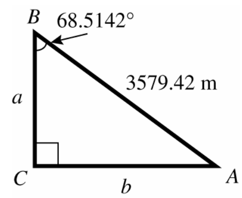


$$A + B = 90° \Rightarrow A = 90° - B \Rightarrow A = 90° - 42.0892° = 47.9108°$$

$$\sin B = \frac{b}{c} \Rightarrow \sin 42.0892° = \frac{56.851}{c} \Rightarrow c = \frac{56.851}{\sin 42.0892°} \approx 84.816 \text{ cm}$$ (rounded to five significant digits)

$$\tan B = \frac{b}{a} \Rightarrow \tan 42.0892° = \frac{56.851}{a} \Rightarrow a = \frac{56.851}{\tan 42.0892°} \approx 62.942 \text{ cm}$$ (rounded to five significant digits)

14. $B = 68.5142°,\ c = 3579.42$ m



$A + B = 90° \Rightarrow A = 90° - B \Rightarrow A = 90° - 68.5142° = 21.4858°$

$\sin B = \frac{b}{c} \Rightarrow \sin 68.5142° = \frac{b}{3579.42} \Rightarrow b = 3579.42 \sin 68.5142° \approx 3330.68$ m (rounded to six significant digits)

$\cos B = \frac{a}{c} \Rightarrow \cos 68.5142° = \frac{a}{3579.42} \Rightarrow a = 3579.42 \cos 68.5142° \approx 1311.04$ m (rounded to six significant digits)

15. No; You need to have at least one side to solve the triangle.

16. If we are given an acute angle and a side in a right triangle, the unknown part of the triangle requiring the least work to find is the other acute angle. It may be found by subtracting the given acute angle from 90°.

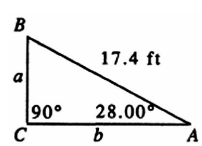
17. Answers will vary.

If you know one acute angle, the other acute angle may be found by subtracting the given acute angle from 90°. If you know one of the sides, then choose two of the trigonometric ratios involving sine, cosine or tangent that involve the known side in order to find the two unknown sides.

18. Answers will vary.

If you know the lengths of two sides, you can set up a trigonometric ratio to solve for one of the acute angles. The other acute angle may be found by subtracting the calculated acute angle from 90°. With either of the two acute angles that have been determined, you can set up a trigonometric ratio along with one of the known sides to solve for the missing side.

19. $A = 28.00°$, $c = 17.4$ ft

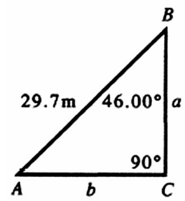


$A + B = 90° \Rightarrow B = 90° - A \Rightarrow B = 90° - 28.00° = 62.00°$

$\sin A = \frac{a}{c} \Rightarrow \sin 28.00° = \frac{a}{17.4} \Rightarrow a = 17.4 \sin 28.00° \approx 8.17$ ft (rounded to three significant digits)

$\cos A = \frac{b}{c} \Rightarrow \cos 28.00° = \frac{b}{17.4} \Rightarrow b = 17.4 \cos 28.00° \approx 15.4$ ft (rounded to three significant digits)

20. $B = 46.00°$, $c = 29.7$ m

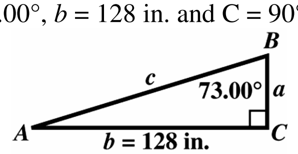


$$A + B = 90° \Rightarrow A = 90° - B \Rightarrow A = 90° - 46.00° = 44.00°$$

$$\cos B = \frac{a}{c} \Rightarrow \cos 46.00° = \frac{a}{29.7} \Rightarrow a = 29.7 \cos 46.00° \approx 20.6 \text{ m} \text{ (rounded to three significant digits)}$$

$$\sin B = \frac{b}{c} \Rightarrow \sin 46.00° = \frac{b}{29.7} \Rightarrow b = 29.7 \sin 46.00° \approx 21.4 \text{ m} \text{ (rounded to three significant digits)}$$

21. Solve the right triangle with $B = 73.00°$, $b = 128$ in. and C = 90°

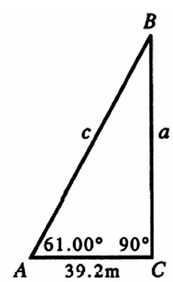


$$A + B = 90° \Rightarrow A = 90° - B \Rightarrow A = 90° - 73.00° = 17.00°$$

$$\tan B° = \frac{b}{a} \Rightarrow \tan 73.00° = \frac{128}{a} \Rightarrow a = \frac{128}{\tan 73.00°} \Rightarrow a = 39.1 \text{ in} \text{(rounded to three significant digits)}$$

$$\sin B° = \frac{b}{c} \Rightarrow \sin 73.00° = \frac{128}{c} \Rightarrow c = \frac{128}{\sin 73.00°} \Rightarrow c = 134 \text{ in} \text{(rounded to three significant digits)}$$

22. $A = 61.00°$, $b = 39.2$ cm

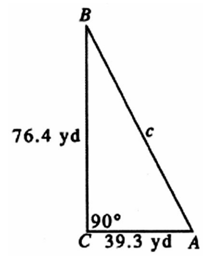


$$A + B = 90° \Rightarrow B = 90° - A \Rightarrow B = 90° - 61.00° = 29.00°$$

$$\tan A = \frac{a}{b} \Rightarrow \tan 61.00° = \frac{a}{39.2} \Rightarrow a = 39.2 \tan 61.00 \approx 70.7 \text{ cm} \text{(rounded to three significant digits)}$$

$$\cos A = \frac{b}{c} \Rightarrow \cos 61.00° = \frac{39.2}{c} \Rightarrow c = \frac{39.2}{\cos 61.00°} \approx 80.9 \text{ cm} \text{(rounded to three significant digits)}$$

23. $a = 76.4$ yd, $b = 39.3$ yd



$$c^2 = a^2 + b^2 \Rightarrow c = \sqrt{a^2 + b^2} = \sqrt{(76.4)^2 + (39.3)^2} = \sqrt{5836.96 + 1544.49} = \sqrt{7381.45} \approx 85.9 \text{ yd}$$

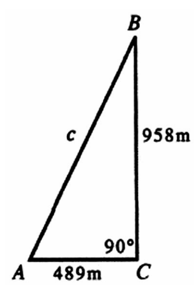
(rounded to three significant digits)

We will determine the measurements of both *A* and *B* by using the sides of the right triangle. In practice, once you find one of the measurements, subtract it from $90°$ to find the other.

$$\tan A = \frac{a}{b} \Rightarrow \tan A = \frac{76.4}{39.3} \approx 1.944020356 \Rightarrow A \approx \tan^{-1}(1.944020356) \approx 62.8° \approx 62°50'$$

$$\tan B = \frac{b}{a} \Rightarrow \tan B = \frac{39.3}{76.4} \approx .5143979058 \Rightarrow B \approx \tan^{-1}(.5143979058) \approx 27.2° \approx 27°10'$$

24. $a = 958$ m, $b = 489$ m



$$c^2 = a^2 + b^2 \Rightarrow c = \sqrt{a^2 + b^2} = \sqrt{958^2 + 489^2} = \sqrt{917{,}764 + 239{,}121} = \sqrt{1{,}156{,}885} \approx 1075.565887$$

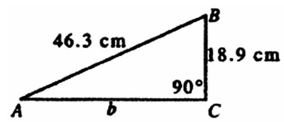
If we round to three significant digits, then $c \approx 1080$ m.

We will determine the measurements of both *A* and *B* by using the sides of the right triangle. In practice, once you find one of the measurements, subtract it from $90°$ to find the other.

$$\tan A = \frac{a}{b} \Rightarrow \tan A = \frac{958}{489} \approx 1.959100204 \Rightarrow A \approx \tan^{-1}(1.959100204) \approx 63.0° \approx 63°00'$$

$$\tan B = \frac{b}{a} \Rightarrow \tan B = \frac{489}{958} \approx .5104384134 \Rightarrow B \approx \tan^{-1}(.5104384134) \approx 27.0° \approx 27°00'$$

25. $a = 18.9$ cm, $c = 46.3$ cm



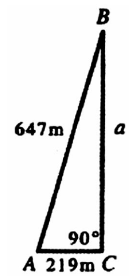
$c^2 = a^2 + b^2 \Rightarrow c^2 - a^2 = b^2 \Rightarrow b = \sqrt{c^2 - a^2} \Rightarrow b = \sqrt{(46.3)^2 - (18.9)^2}$

$b = \sqrt{2143.69 - 357.21} = \sqrt{1786.48} \approx 42.3$ cm (rounded to three significant digits)

$\sin A = \frac{a}{c} \Rightarrow \sin A = \frac{18.9}{46.3} \approx .4082073434 \Rightarrow A = \sin^{-1}(.4082073434) \approx 24.1° \approx 24°10'$

$\cos B = \frac{a}{c} \Rightarrow \cos B = \frac{18.9}{46.3} \approx .4082073434 \Rightarrow B = \cos^{-1}(.4082073434) \approx 65.9° \approx 65°50'$

26. $b = 219$ m, $c = 647$ m



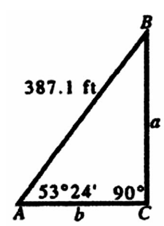
$c^2 = a^2 + b^2 \Rightarrow c^2 - b^2 = a^2 \Rightarrow a = \sqrt{c^2 - b^2} \Rightarrow a = \sqrt{(647)^2 - (219)^2}$

$b = \sqrt{418{,}609 - 47{,}961} = \sqrt{370{,}648} \approx 609$ m (rounded to three significant digits)

$\cos A = \frac{b}{c} \Rightarrow \cos A = \frac{219}{647} \approx .3384853168 \Rightarrow A = \cos^{-1}(.3384853168) \approx 70.2° \approx 70°10'$

$\sin B = \frac{b}{c} \Rightarrow \sin B = \frac{219}{647} \approx .3384853168 \Rightarrow B = \sin^{-1}(.3384853168) \approx 19.8° \approx 19°50'$

27. $A = 53°24', \; c = 387.1$ ft

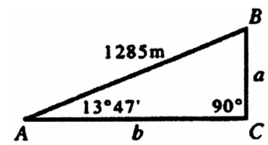


$A + B = 90° \Rightarrow B = 90° - A \Rightarrow B = 90° - 53°24' = 89°60' - 53°24° = 36°36'$

$\sin A = \frac{a}{c} \Rightarrow \sin 53°24' = \frac{a}{387.1} \Rightarrow a = 387.1 \sin 53°24' \approx 310.8$ ft (rounded to four significant digits)

$\cos A = \frac{b}{c} \Rightarrow \cos 53°24' = \frac{b}{387.1} \Rightarrow b = 387.1 \cos 53°24' \approx 230.8$ ft (rounded to four significant digits)

28. $A = 13°47', \ c = 1285$ m

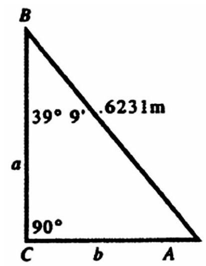


$A + B = 90° \Rightarrow B = 90° - A \Rightarrow B = 90° - 13°47' = 89°60' - 13°47' = 76°13'$

$\sin A = \frac{a}{c} \Rightarrow \sin 13°47' = \frac{a}{1285} \Rightarrow a = 1285 \sin 13°47' \Rightarrow a \approx 306.2$ m (rounded to four significant digits)

$\cos A = \frac{b}{c} \Rightarrow \cos 13°47' = \frac{b}{1285} \Rightarrow b = 1285 \cos 13°47' \approx 1248$ m (rounded to four significant digits)

29. $B = 39°9', \ c = .6231$ m

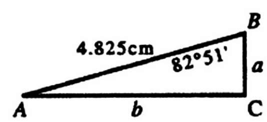


$A + B = 90° \Rightarrow A = 90° - B \Rightarrow A = 90° - 39°9' = 89°60' - 39°9' = 50°51'$

$\cos B = \frac{a}{c} \Rightarrow \cos 39°9' = \frac{a}{.6231} \Rightarrow a = .6231 \cos 39°9' \approx .4832$ m (rounded to four significant digits)

$\sin B = \frac{b}{c} \Rightarrow \sin 39°9' = \frac{b}{.6231} \Rightarrow b = .6231 \sin 39°9' \approx .3934$ m (rounded to four significant digits)

30. $B = 82°51', \ c = 4.825$ cm



$A + B = 90° \Rightarrow A = 90° - B \Rightarrow B = 90° - 82°51' = 89°60' - 82°51' = 7°9'$

$\sin B = \frac{b}{c} \Rightarrow \sin 82°51' = \frac{b}{4.825} \Rightarrow b = 4.825 \sin 82°51' \approx 4.787$ cm (rounded to three significant digits)

$\cos B = \frac{a}{c} \Rightarrow \cos 82°51' = \frac{a}{4.825} \Rightarrow a = 4.825 \cos 82°51'' \approx .6006$ cm (rounded to three significant digits)

31. The angle of elevation from *X* to *Y* is 90° whenever *Y* is directly above *X*.

32. The angle of elevation from *X* to *Y* is the acute angle formed by ray *XY* and a horizontal ray with endpoint at *X*. Therefore, the angle of elevation cannot be more than 90°.

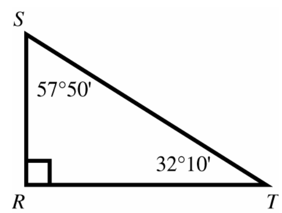
33. Answers will vary.
The angle of elevation and the angle of depression are measured between the line of sight and a horizontal line. So, in the diagram, lines *AD* and *CB* are both horizontal. Hence, they are parallel. The line formed by *AB* is a transversal and angles *DAB* and *ABC* are alternate interior angle and thus have the same measure.

34. The angle of depression is measured between the line of sight and a horizontal line. This angle is measured between the line of sight and a vertical line.

35. $\sin 43°50' = \dfrac{d}{13.5} \Rightarrow d = 13.5\sin 43°50' \approx 9.3496000$

The ladder goes up the wall 9.35 m. (rounded to three significant digits)

36. $T = 32°\ 10'$ and $S = 57°50'$

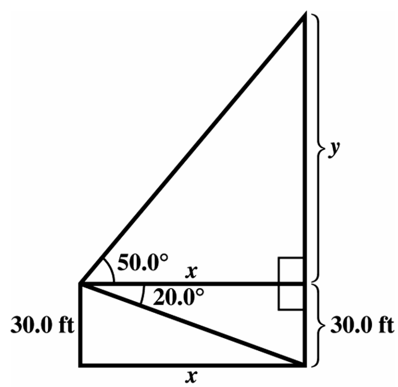


Since $S + T = 32°\ 10' + 57°50' = 89°60' = 90°$, triangle RST is a right triangle. Thus, we have

$\tan 32°10' = \dfrac{RS}{53.1} \Rightarrow RS = 53.1\tan 32°10' \approx 33.395727.$

The distance across the lake is 33.4 m. (rounded to three significant digits)

37. Let x represent the horizontal distance between the two buildings and y represent the height of the portion of the building across the street that is higher than the window.



We have the following.

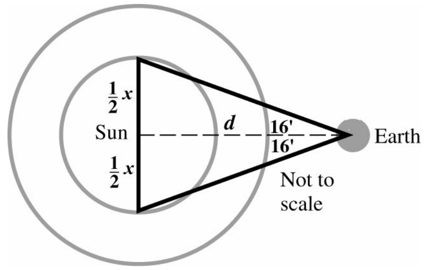
$$\tan 20.0° = \frac{30.0}{x} \Rightarrow x = \frac{30.3}{\tan 20.0°} \approx 82.4$$

$$\tan 50.0° = \frac{y}{x} \Rightarrow y = x\tan 50.0° = \left(\frac{30.0}{\tan 20.0°}\right)\tan 50.0°$$

$$\text{height} = y + 30.0 = \left(\frac{30.0}{\tan 20.0°}\right)\tan 50.0° + 30.0 \approx 128.2295$$

The height of the building across the street is about 128 ft. (rounded to three significant digits)

38. Let x = the diameter of the sun.



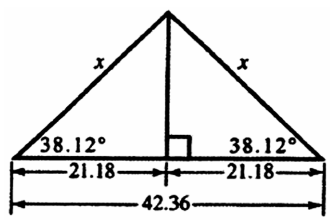
Since the included angle is $32'$, $\frac{1}{2}(32') = 16'$. We will use this angle, d, and half of the diameter to set up the following equation.

$$\frac{\frac{1}{2}x}{92{,}919{,}800} = \tan 16' \Rightarrow x = 2(92{,}919{,}800)(\tan 16') \approx 864{,}943.0189$$

The diameter of the sun is about 865,000 mi. (rounded to three significant digits)

39. The altitude of an isosceles triangle bisects the base as well as the angle opposite the base. The two right triangles formed have interior angles, which have the same measure angle. The lengths of the corresponding sides also have the same measure. Since the altitude bisects the base, each leg (base) of the right triangles is $\frac{42.36}{2} = 21.18$ in.

Let x = the length of each of the two equal sides of the isosceles triangle.

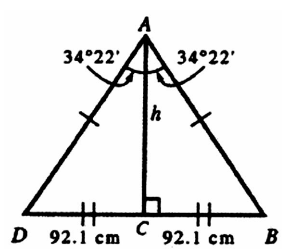


$$\cos 38.12° = \frac{21.18}{x} \Rightarrow x\cos 38.12° = 21.18 \Rightarrow x = \frac{21.18}{\cos 38.12°} \approx 26.921918$$

The length of each of the two equal sides of the triangle is 26.92 in. (rounded to four significant digits)

40. The altitude of an isosceles triangle bisects the base as well as the angle opposite the base. The two right triangles formed have interior angles, which have the same measure angle. The lengths of the corresponding sides also have the same measure. Since the altitude bisects the base, each leg (base) of the right triangles are $\frac{184.2}{2} = 92.10$ cm. Each angle opposite to the base of the right triangles measures $\frac{1}{2}(68°44') = 34°22'$.

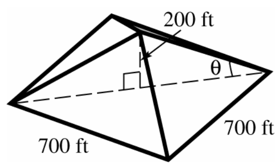
Let h = the altitude.

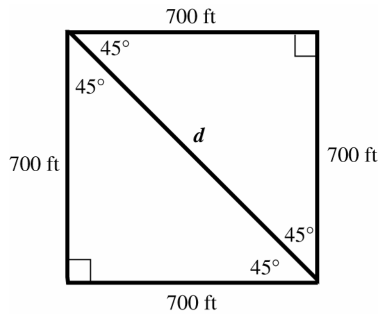


In triangle ABC, $\tan 34°22' = \frac{92.10}{h} \Rightarrow h\tan 34°22' = 92.10 \Rightarrow h = \frac{92.10}{\tan 34°22'} \approx 134.67667.$

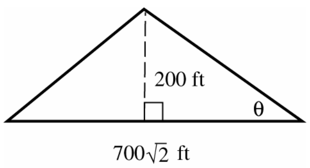
The altitude of the triangle is 134.7 cm. (rounded to four significant digits)

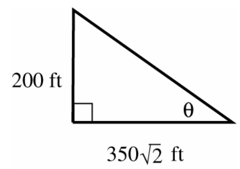
41. In order to find the angle of elevation, θ, we need to first find the length of the diagonal of the square base. The diagonal forms two isosceles right triangles. Each angle formed by a side of the square and the diagonal measures 45˚.





By the Pythagorean theorem, $700^2 + 700^2 = d^2 \Rightarrow 2 \cdot 700^2 = d^2 \Rightarrow d = \sqrt{2 \cdot 700^2} \Rightarrow d = 700\sqrt{2}$. Thus, length of the diagonal is $700\sqrt{2}$ ft. To to find the angle, θ, we consider the following isosceles triangle.





The height of the pyramid bisects the base of this triangle and forms two right triangles. We can use one of these triangles to find the angle of elevation, θ.

$$\tan\theta = \frac{200}{350\sqrt{2}} \approx .4040610178 \Rightarrow \theta \approx \tan^{-1}\left(.4040610178\right) \approx 22.0017$$

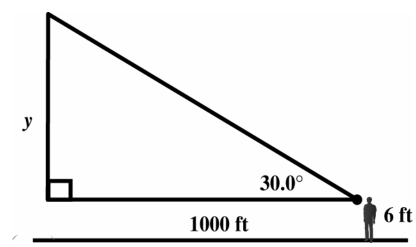
Rounding this figure to two significant digits, we have $\theta \approx 22°$.

42. Let y = the height of the spotlight (this measurement starts 6 feet above ground)

We have the following.

$$\tan 30.0° = \frac{y}{1000}$$
$$y = 1000 \cdot \tan 30.0° \approx 577.3502$$

Rounding this figure to three significant digits, we have $y \approx 577$.



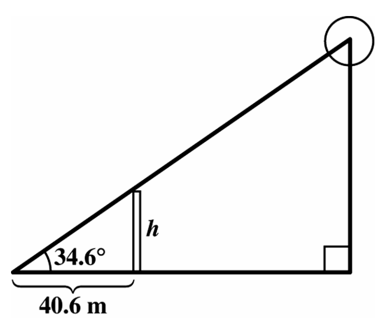
However, the observer's eye-height is 6 feet from the ground, so the cloud ceiling is $577 + 6 = 583$ ft.

43. Let h represent the height of the tower.

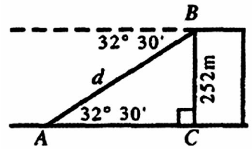
In triangle ABC we have the following.

$$\tan 34.6° = \frac{h}{40.6}$$
$$h = 40.6 \tan 34.6° \approx 28.0081$$

The height of the tower is 28.0 m. (rounded to three significant digits)



44. Let d = the distance from the top B of the building to the point on the ground A.

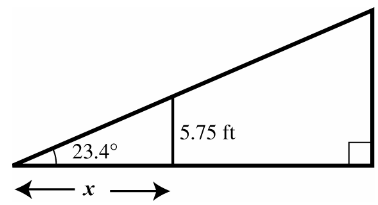


In triangle ABC, $\sin 32°30' = \dfrac{252}{d} \Rightarrow d = \dfrac{252}{\sin 32°30'} \approx 469.0121.$

The distance from the top of the building to the point on the ground is 469 m. (rounded to three significant digits)

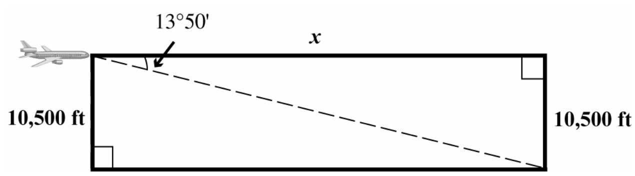
45. Let x = the length of the shadow cast by Diane Carr.

$$\tan 23.4° = \frac{5.75}{x}$$
$$x \tan 23.4° = 5.75$$
$$x = \frac{5.75}{\tan 23.4°} \approx 13.2875$$



The length of the shadow cast by Diane Carr is 13.3 ft. (rounded to three significant digits)

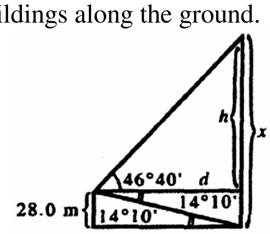
46. Let x = the horizontal distance that the plan must fly to be directly over the tree.



$$\tan 13°50' = \frac{10,500}{x} \Rightarrow x \tan 13°50' = 10,500 \Rightarrow x = \frac{10,500}{\tan 13°50'} \approx 42,641.2351$$

The horizontal distance that the plan must fly to be directly over the tree is 42,600 ft. (rounded to three significant digits)

47. Let x = the height of the taller building;
h = the difference in height between the shorter and taller buildings;
d = the distance between the buildings along the ground.



$$\frac{28.0}{d} = \tan 14°10' \Rightarrow 28.0 = d \tan 14°10' \Rightarrow d = \frac{28.0}{\tan 14°10'} \approx 110.9262493 \text{ m}$$

(We hold on to these digits for the intermediate steps.)

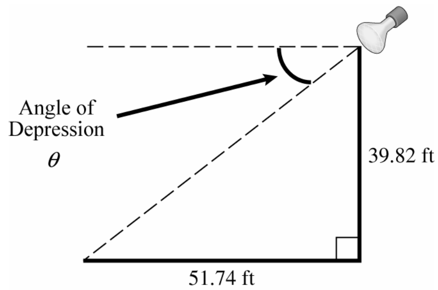
To find h, we solve the following.

$$\frac{h}{d} = \tan 46°40' \Rightarrow h = d \tan 46°40' \approx (110.9262493)\tan 46°40' \approx 117.5749$$

Thus, the value of h rounded to three significant digits is 118 m.

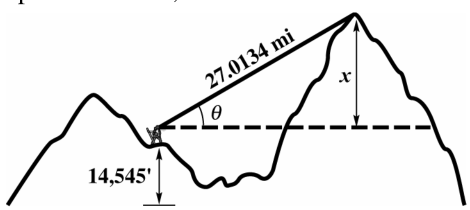
Since $x = h + 28.0 = 118 + 28.0 \approx 146\text{ m}$, the height of the taller building is 146 m.

48. Let $\theta =$ the angle of depression.



$$\tan\theta = \frac{39.82}{51.74} \approx .7696173174 \Rightarrow \theta = \tan^{-1}(.7696173174) \Rightarrow \theta \approx 37.58° \approx 37°35'$$

49. (a) Let x = the height of the peak above 14,545 ft.



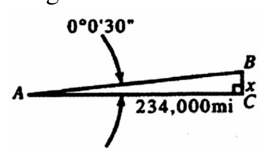
Since the diagonal of the right triangle formed is in miles, we must first convert this measurement to feet. Since there are 5280 ft in one mile, we have the length of the diagonal is $27.0134(5280) = 142,630.752$. To find the value of x, we solve the following.

$$\sin 5.82° = \frac{x}{142,630.752} \Rightarrow x = 142,630.752 \sin 5.82° \approx 14,463.2674$$

Thus, the value of x rounded to five significant digits is 14,463 ft. Thus, the total height is about 14,545 + 14,463 = 29,008 ft.

(b) The curvature of the earth would make the peak appear shorter than it actually is. Initially the surveyors did not think Mt. Everest was the tallest peak in the Himalayas. It did not look like the tallest peak because it was farther away than the other large peaks.

50. Let x = the distance from the assigned target.



In triangle ABC, we have the following.

$$\tan 0°0'30'' = \frac{x}{234,000} \Rightarrow x = 234,000 \tan 0°0'30'' \approx 34.0339$$

The distance from the assigned target is 34.0 mi. (rounded to three significant digits)

Section 2.5: Further Applications of Right Triangles

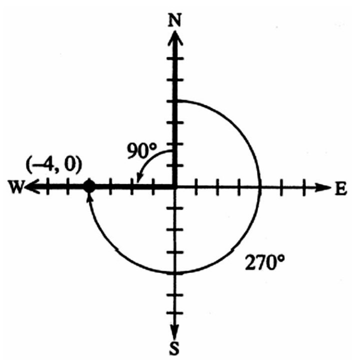
1. It should be shown as an angle measured clockwise from due north.

2. It should be shown measured from north (or south) in the east (or west) direction.

3. A sketch is important to show the relationships among the given data and the unknowns.

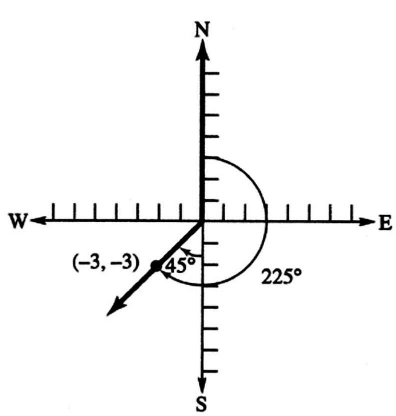
4. The angle of elevation (or depression) from X to Y is measured from the horizontal line through X to the ray XY.

5. $(-4,0)$



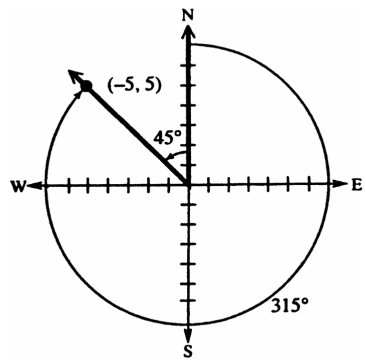
The bearing of the airplane measured in a clockwise direction from due north is 270°. The bearing can also be expressed as N 90° W, or S 90° W.

6. $(-3,-3)$



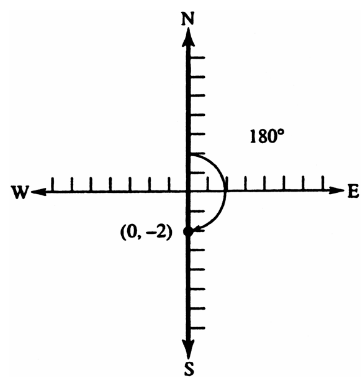
The bearing of the airplane measured in a clockwise direction from due north is 225°. The bearing can also be expressed as S 45° W.

7. $(-5,5)$



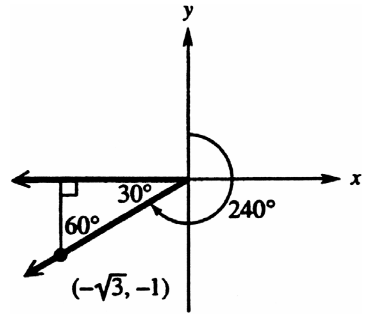
The bearing of the airplane measured in a clockwise direction from due north is 315°. The bearing can also be expressed as N 45° W.

8. $(0,-2)$



The bearing of the airplane measured in a clockwise direction from due north is 180°. The bearing can also be expressed as S 0° E or S 0° W.

9. All points whose bearing from the origin is $240°$ lie in quadrant III.



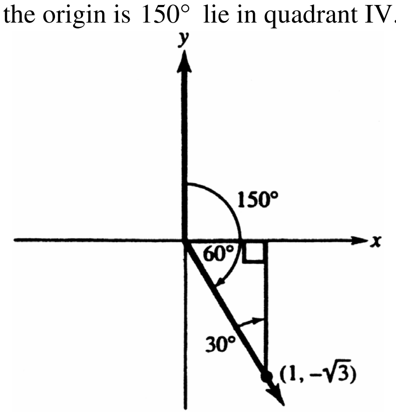
The reference angle, θ', is 30°. For any point, (x, y), on the ray $\frac{x}{r} = -\cos\theta'$ and $\frac{y}{r} = -\sin\theta'$, where r is the distance the point is from the origin. If we let $r = 2$, then we have the following.

$$\frac{x}{r} = -\cos\theta' \Rightarrow x = -r\cos\theta' = -2\cos 30° = -2\cdot\tfrac{\sqrt{3}}{2} = -\sqrt{3}$$

$$\frac{y}{r} = -\sin\theta' \Rightarrow y = -r\sin\theta' = -2\sin 30° = -2\cdot\tfrac{1}{2} = -1$$

Thus, a point on the ray is $\left(-\sqrt{3}, -1\right)$. Since the ray contains the origin, the equation is of the form $y = mx$. Substituting the point $\left(-\sqrt{3}, -1\right)$, we have $-1 = m\left(-\sqrt{3}\right) \Rightarrow m = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$. Thus, the equation of the ray is $y = \frac{\sqrt{3}}{3}x,\ x \le 0$ since the ray lies in quadrant III.

10. All points whose bearing from the origin is $150°$ lie in quadrant IV.



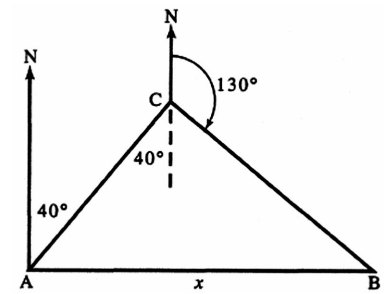
The reference angle, θ', is 60°. For any point, (x, y), on the ray $\frac{x}{r} = \cos\theta'$ and $\frac{y}{r} = -\sin\theta'$, where r is the distance the point is from the origin. If we let $r = 2$, then we have the following.

$$\frac{x}{r} = \cos\theta' \Rightarrow x = r\cos\theta' = 2\cos 60° = 2\cdot\tfrac{1}{2} = 1$$

$$\frac{y}{r} = -\sin\theta' \Rightarrow y = -r\sin\theta' = -2\sin 60° = -2\cdot\tfrac{\sqrt{3}}{2} = -\sqrt{3}$$

Thus, a point on the ray is $\left(1, -\sqrt{3}\right)$. Since the ray contains the origin, the equation is of the form $y = mx$. Substituting the point $\left(1, -\sqrt{3}\right)$, we have $-\sqrt{3} = m(1) \Rightarrow m = -\sqrt{3}$. Thus, the equation of the ray is $y = -\sqrt{3}x,\ x \ge 0$ since the ray lies in quadrant IV.

11. Let x = the distance the plane is from its starting point.
In the figure, the measure of angle *ACB* is $40° + (180° - 130°) = 40° + 50° = 90°$. Therefore, triangle *ACB* is a right triangle.



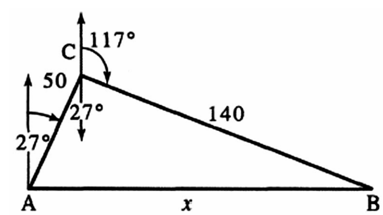
Since $d = rt$, the distance traveled in 1.5 hr is (1.5 hr)(110 mph) = 165 mi. The distance traveled in 1.3 hr is (1.3 hr)(110 mph) = 143 mi.

Using the Pythagorean theorem, we have the following.

$$x^2 = 165^2 + 143^2 \Rightarrow x^2 = 27,225 + 20,449 \Rightarrow x^2 = 47,674 \Rightarrow x \approx 218.3438$$

The plane is 220 mi from its starting point. (rounded to two significant digits)

12. Let x = the distance from the starting point.
In the figure, the measure of angle *ACB* is $27° + (180° - 117°) = 27° + 63° = 90°$. Therefore, triangle *ACB* is a right triangle.

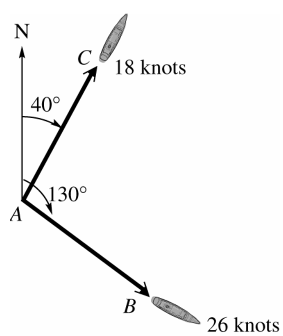


Applying the Pythagorean theorem, we have the following.

$$x^2 = 50^2 + 140^2 \Rightarrow x^2 = 2500 + 19,600 \Rightarrow x^2 = 22,100 \Rightarrow x = \sqrt{22,100} \approx 148.6607$$

The distance of the end of the trip from the starting point is 150 km. (rounded to two significant digits)

13. Let x = distance the ships are apart.
In the figure, the measure of angle *CAB* is $130° - 40° = 90°$. Therefore, triangle *CAB* is a right triangle.



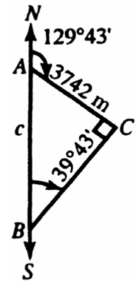
Since $d = rt$, the distance traveled by the first ship in 1.5 hr is (1.5 hr)(18 knots) = 27 nautical mi and the second ship is (1.5hr)(26 knots) = 39 nautical mi.

Applying the Pythagorean theorem, we have the following.

$$x^2 = 27^2 + 39^2 \Rightarrow x^2 = 729 + 1521 \Rightarrow x^2 = 2250 \Rightarrow x = \sqrt{2250} \approx 47.4342$$

The ships are 47 nautical mi apart. (rounded to 2 significant digits)

14. Let C = the location of the ship;
c = the distance between the lighthouses.

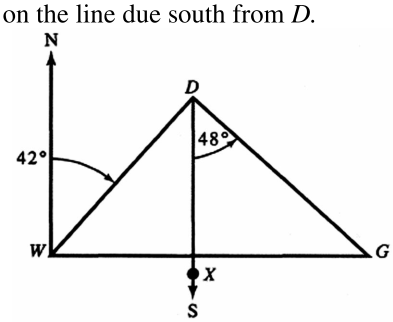


The measure of angle BAC is $180° - 129°43' = 179°60' - 129°43' = 50°17'$.
Since $50°17' + 39°43' = 89°60' = 90°$, we have a right triangle and can get set up and solve the following equation.

$$\sin 39°43 = \frac{3742}{c} \Rightarrow c \sin 39°43 = 3742 \Rightarrow c = \frac{3742}{\sin 39°43} \approx 5856.1020$$

The distance between the lighthouses is 5856 m. (rounded to four significant digits)

15. Draw triangle WDG with W representing Winston-Salem, D representing Danville, and G representing Goldsboro. Name any point X on the line due south from D.



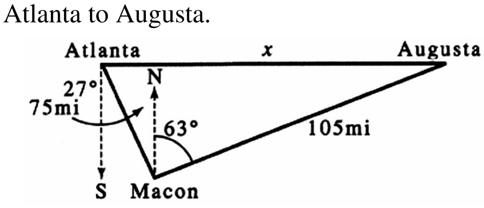
Since the bearing from W to D is 42° (equivalent to N 42° E), angle WDX measures 42°. Since angle XDG measures 48°, the measure of angle D is 42° + 48° = 90°. Thus, triangle WDG is a right triangle. Using $d = rt$ and the Pythagorean theorem, we have the following.

$$WG = \sqrt{(WD)^2 + (DG)^2} = \sqrt{[60(1)]^2 + [60(1.8)]^2}$$

$$WG = \sqrt{60^2 + 108^2} = \sqrt{3600 + 11,664} = \sqrt{15,264} \approx 123.5476$$

The distance from Winston-Salem to Goldsboro is 120 mi. (rounded to two significant digits)

16. Let x = the distance from Atlanta to Augusta.

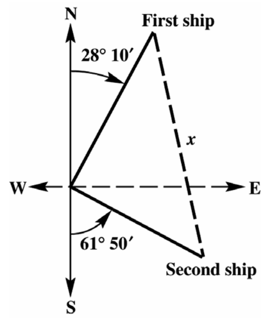


The line from Atlanta to Macon makes an angle of $27° + 63° = 90°$, with the line from Macon to Augusta. Since $d = rt$, the distance from Atlanta to Macon is $60\left(1\frac{1}{4}\right) = 75$ mi. The distance from Macon to Augusta is $60\left(1\frac{3}{4}\right) = 105$ mi.
Use the Pythagorean theorem to find x, we have the following.

$$x^2 = 75^2 + 105^2 \Rightarrow x^2 = 5635 + 11,025 \Rightarrow x^2 = 16,650 \approx 129.0349$$

The distance from Atlanta to Augusta is 130 mi. (rounded to two significant digits)

17. Let x = distance between the two ships.



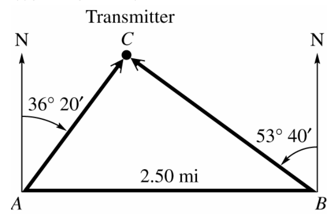
The angle between the bearings of the ships is $180° - (28°10' + 61°50') = 90°$. The triangle formed is a right triangle. The distance traveled at 24.0 mph is (4 hr) (24.0 mph) = 96 mi. The distance traveled at 28.0 mph is (4 hr)(28.0 mph) = 112 mi.

Applying the Pythagorean theorem we have the following.

$$x^2 = 96^2 + 112^2 \Rightarrow x^2 = 9216 + 12{,}544 \Rightarrow x^2 = 21{,}760 \Rightarrow x = \sqrt{21{,}760} \approx 147.5127$$

The ships are 148 mi apart. (rounded to three significant digits)

18. Let C = the location of the transmitter;
a = the distance of the transmitter from B.



The measure of angle CBA is $90° - 53°40' = 89°60' - 53°40' = 36°20'$.
The measure of angle CAB $90° - 36°20' = 89°60' - 36°20' = 53°40'$.
Since $A + B = 90°$, so $C = 90°$. Thus, we have the following.

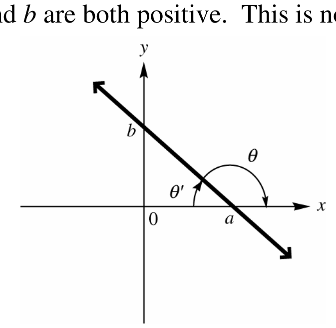
$$\sin A = \frac{a}{2.50} \Rightarrow \sin 53°40' = \frac{a}{2.50} \Rightarrow a = 2.50 \sin 53°40' \approx 2.0140$$

The distance of the transmitter from B is 2.01 mi. (rounded to 3 significant digits)

19. Solve the equation $ax = b + cx$ for x in terms of a, b, c.

$$ax = b + cx \Rightarrow ax - cx = b \Rightarrow x(a-c) = b \Rightarrow x = \frac{b}{a-c}$$

20. Suppose we have a line that has x-intercept a and y-intercept b. Assume for the following diagram that a and b are both positive. This is not a necessary condition, but it makes the visualization easier.



Now $\tan\theta = -\tan(180° - \theta)$. This is because the angle represented by $180° - \theta$ terminates in quadrant II if $0° < \theta < 90°$. If $90° < \theta < 180°$, then the angle represented by $180° - \theta$ terminates in quadrant I. Thus, $\tan\theta$ and $\tan(180° - \theta)$ are opposite in sign.

Clearly, the slope of the line is $m = -\frac{b}{a}$. and $\tan\theta = -\tan(180° - \theta) = -\tan\theta' = -\frac{b}{a}$.
Thus, $m = -\frac{b}{a} = -\tan\theta$.

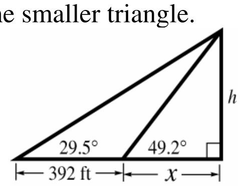
The point-slope form of the equation of a line is $y - y_1 = m(x - x_1)$. Substituting $-\tan\theta$ for m into $y - y_1 = m(x - x_1)$, we have $y - y_1 = (\tan\theta)(x - x_1)$. If the line passes through $(a, 0)$, then therefore have $y - 0 = (\tan\theta)(x - a)$ or $y = (\tan\theta)(x - a)$.

21. Using the equation $y = (\tan\theta)(x-a)$ where $(a,0)$ is a point on the line and θ is the angle the line makes with the x-axis, $y = (\tan 35°)(x-25)$.

22. Using the equation $y = (\tan\theta)(x-a)$ where $(a,0)$ is a point on the line and θ is the angle the line makes with the x-axis, $y = (\tan 15°)(x-5)$.

For Exercises 23 and 24, we will provide both the algebraic and graphing calculator solutions.

23. Algebraic Solution:
Let x = the side adjacent to 49.2° in the smaller triangle.



In the larger right triangle, we have $\tan 29.5° = \frac{h}{392+x} \Rightarrow h = (392+x)\tan 29.5°$.

In the smaller right triangle, we have $\tan 49.2° = \frac{h}{x} \Rightarrow h = x\tan 49.2°$.

Substitute the first expression for h in this equation, and solve for x.

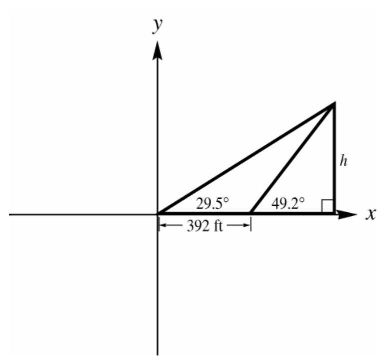
$$(392+x)\tan 29.5° = x\tan 49.2°$$
$$392\tan 29.5° + x\tan 29.5° = x\tan 49.2°$$
$$392\tan 29.5° = x\tan 49.2° - x\tan 29.5°$$
$$392\tan 29.5 = x(\tan 49.2° - \tan 29.5°)$$
$$\frac{392\tan 29.5°}{\tan 49.2° - \tan 29.5°} = x$$

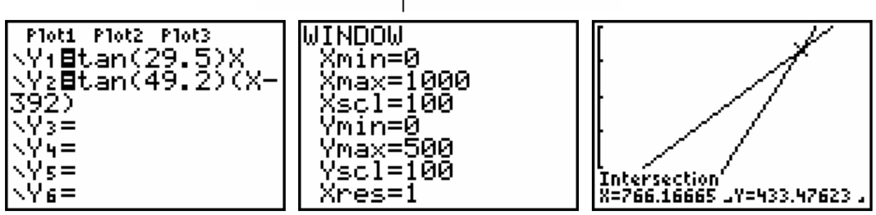
Then substitute this value for x in the equation for the smaller triangle to obtain the following.

$$h = x\tan 49.2° = \frac{392\tan 29.5°}{\tan 49.2° - \tan 29.5°}\tan 49.2° \approx 433.4762$$

Graphing Calculator Solution:

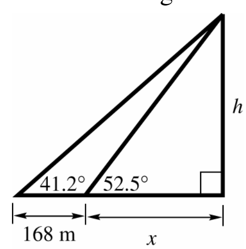
The first line considered is $y = (\tan 29.5°)x$ and the second is $y = (\tan 49.2°)(x-392)$.





The height of the triangle is 433 ft. (rounded to three significant digits)

24. Algebraic Solution:
Let x = the side adjacent to 52.5° in the smaller triangle.



In the larger triangle, we have $\tan 41.2° = \frac{h}{168 + x} \Rightarrow h = (168 + x)\tan 41.2°.$

In the smaller triangle, we have $\tan 52.5° = \frac{h}{x} \Rightarrow h = x \tan 52.5°.$

Substitute for h in this equation and solve for x.

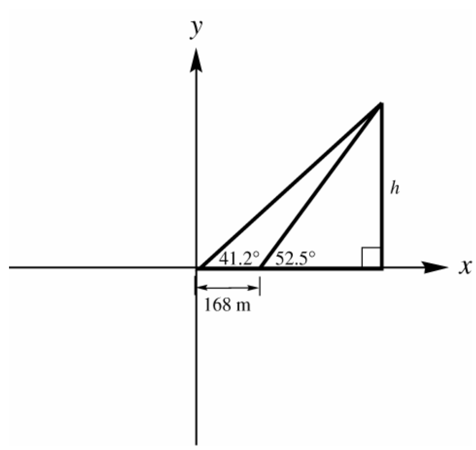
$$\begin{aligned}(168 + x)\tan 41.2° &= x \tan 52.5° \\ 168 \tan 41.2° + x \tan 41.2° &= x \tan 52.5° \\ 168 \tan 41.2° &= x \tan 52.5° - x \tan 41.2° \\ 168 \tan 41.2° &= x(\tan 52.5° - \tan 41.2°) \\ \frac{168 \tan 41.2°}{\tan 52.5° - \tan 41.2°} &= x\end{aligned}$$

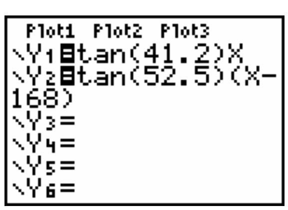
Substituting for x in the equation for the smaller triangle, we have the following.

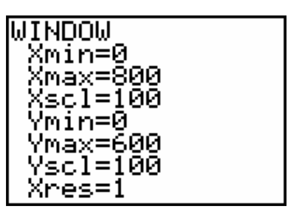
$$h = x \tan 52.5° \Rightarrow h = \frac{168 \tan 41.2° \tan 52.5°}{\tan 52.5° - \tan 41.2°} \approx 448.0432$$

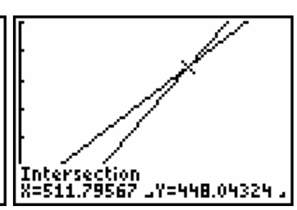
Graphing Calculator Solution:

The first line considered is $y = (\tan 41.2°)x$ and the second is $y = (\tan 52.5°)(x - 168)$.



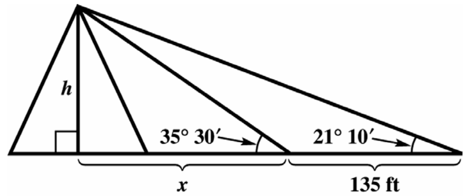






The height of the triangle is approximately 448 m. (rounded to three significant digits)

25. Let x = the distance from the closer point on the ground to the base of height h of the pyramid.



In the larger right triangle, we have $\tan 21°10' = \frac{h}{135+x} \Rightarrow h = (135+x)\tan 21°10'$.

In the smaller right triangle, we have $\tan 35°30' = \frac{h}{x} \Rightarrow h = x\tan 35°30'$.

Substitute for h in this equation, and solve for x to obtain the following.

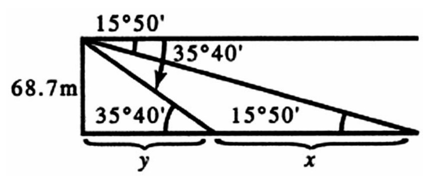
$$(135+x)\tan 21°10' = x\tan 35°30'$$
$$135\tan 21°10' + x\tan 21°10' = x\tan 35°30'$$
$$135\tan 21°10' = x\tan 35°30' - x\tan 21°10'$$
$$135\tan 21°10' = x(\tan 35°30' - \tan 21°10')$$
$$\frac{135\tan 21°10'}{\tan 35°30' - \tan 21°10'} = x$$

Substitute for x in the equation for the smaller triangle.

$$h = \frac{135\tan 21°10'}{\tan 35°30' - \tan 21°10'}\tan 35°30' \approx 114.3427$$

The height of the pyramid is 114 ft. (rounded to three significant digits)

26. Let x = the distance traveled by the whale as it approaches the tower;
y = the distance from the tower to the whale as it turns.



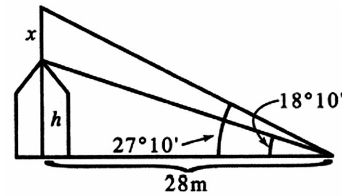
$$\frac{68.7}{y} = \tan 35°40' \Rightarrow 68.7\, y\tan 35°40' \Rightarrow y = \frac{68.7}{\tan 35°40'}$$

and

$$\frac{68.7}{x+y} = \tan 15°50' \Rightarrow 68.7 = (x+y)\tan 15°50' \Rightarrow x+y = \frac{68.7}{\tan 15°50'}$$
$$x = \frac{68.7}{\tan 15°50'} - y \Rightarrow x = \frac{68.7}{\tan 15°50'} - \frac{68.7}{\tan 35°40'} \approx 146.5190$$

The whale traveled 147 m as it approached the lighthouse. (rounded to three significant digits)

27. Let x = the height of the antenna;
h = the height of the house.



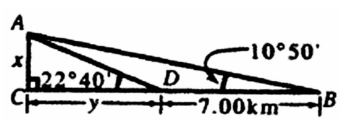
In the smaller right triangle, we have $\tan 18°10' = \frac{h}{28} \Rightarrow h = 28\tan 18°10'$.

In the larger right triangle, we have the following.

$$\tan 27°10' = \frac{x+h}{28} \Rightarrow x+h = 28\tan 27°10' \Rightarrow x = 28\tan 27°10' - h$$
$$x = 28\tan 27°10' - 28\tan 18°10' \approx 5.1816$$

The height of the antenna is 5.18 m. (rounded to three significant digits)

28. Let x = the height of Mt. Whitney above the level of the road;
y = the distance shown in the figure below.



In triangle ADC, $\tan 22°40' = \frac{x}{y} \Rightarrow y\tan 22°40' = x \Rightarrow y = \frac{x}{\tan 22°40'}$. (1)

In triangle ABC, we have the following.

$$\tan 10°50' = \frac{x}{y+7.00} \Rightarrow (y+7.00)\tan 10°50' = x$$
$$y\tan 10°50' + 7.00\tan 10°50' = x \Rightarrow y = \frac{x - 7.00\tan 10°50'}{\tan 10°50'} \quad (2)$$

Setting equations 1 and 2 equal to each other, we have the following.

$$\frac{x}{\tan 22°40'} = \frac{x - 7.00\tan 10°50'}{\tan 10°50'}$$
$$x\tan 10°50' = x\tan 22°40' - 7.00(\tan 10°50')(\tan 22°40')$$
$$7.00(\tan 10°50')(\tan 22°40') = x\tan 22°40' - x\tan 10°50'$$
$$7.00(\tan 10°50')(\tan 22°40') = x(\tan 22°40' - \tan 10°50')$$
$$\frac{7.00(\tan 10°50')(\tan 22°40')}{\tan 22°40' - \tan 10°50'} = x$$
$$x \approx 2.4725.$$

The height of the top of Mt. Whitney above road level is 2.47 km. (rounded to three significant digits)

29. **(a)** From the figure in the text, $d = \frac{b}{2}\cot\frac{\alpha}{2} + \frac{b}{2}\cot\frac{\beta}{2} \Rightarrow d = \frac{b}{2}\left(\cot\frac{\alpha}{2} + \cot\frac{\beta}{2}\right)$.

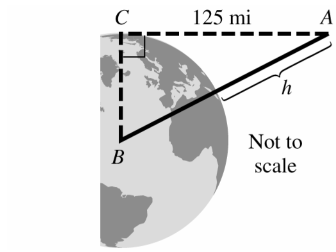
(b) Using the result of part a, let $\alpha = 37'48"$, $\beta = 42'3"$, and $b = 2.000$.

$$d = \frac{b}{2}\left(\cot\frac{\alpha}{2} + \cot\frac{\beta}{2}\right) \Rightarrow d = \frac{2}{2}\left(\cot\frac{37'48"}{2} + \cot\frac{42'3"}{2}\right)$$

$$d \approx \cot .315^\circ + \cot .3504166667^\circ = \frac{1}{\tan .315^\circ} + \frac{1}{\tan .3504166667^\circ} \approx 345.3951$$

The distance between the two points P and Q is 345.3951 cm. (rounded)

30. Let $h =$ the minimum height above the surface of the earth so a pilot at A can see an object on the horizon at C.



Using the Pythagorean theorem, we have the following.

$$\left(4.00\times10^3 + h\right)^2 = \left(4.00\times10^3\right)^2 + 125^2$$

$$\left(4000 + h\right)^2 = 4000^2 + 125^2$$

$$\left(4000 + h\right)^2 = 16{,}000{,}000 + 15{,}625$$

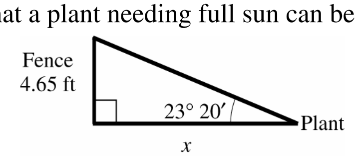
$$\left(4000 + h\right)^2 = 16{,}015{,}625$$

$$4000 + h = \sqrt{16{,}015{,}625}$$

$$h = \sqrt{16{,}015{,}625} - 4000 \approx 4001.9526 - 4000 = 1.9526$$

The minimum height above the surface of the earth would be 1.95 mi. (rounded to 3 significant digits)

31. Let x = the minimum distance that a plant needing full sun can be placed from the fence.



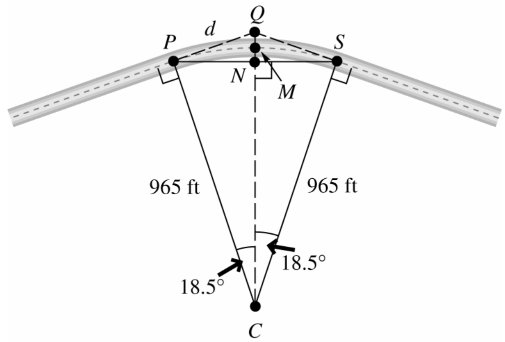
$$\tan 23^\circ 20' = \frac{4.65}{x} \Rightarrow x \tan 23^\circ 20' = 4.65 \Rightarrow x = \frac{4.65}{\tan 23^\circ 20'} \approx 10.7799$$

The minimum distance is 10.8 ft. (rounded to three significant digits)

32. $\tan A = \frac{1.0837}{1.4923} \approx .7261944649 \Rightarrow A \approx \tan^{-1}\left(.7261944649\right) \approx 35.987^\circ \approx 35^\circ 59.2' \approx 35^\circ 59' 10''$

$\tan B = \frac{1.4923}{1.0837} \approx 1.377041617 \Rightarrow B \approx \tan^{-1}\left(1.377041617\right) \approx 54.013^\circ \approx 54^\circ 00.8' \approx 54^\circ 00' 50''$

33. **(a)** If $\theta = 37°$, then $\frac{\theta}{2} = \frac{37°}{2} = 18.5°.$

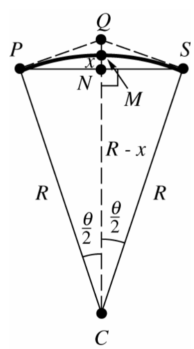


To find the distance between *P* and *Q*, *d*, we first note that angle *QPC* is a right angle. Hence, triangle *QPC* is a right triangle and we can solve the following.

$$\tan 18.5° = \frac{d}{965} \Rightarrow d = 965 \tan 18.5° \approx 322.8845$$

The distance between *P* and *Q*, is 323 ft. (rounded to three significant digits)

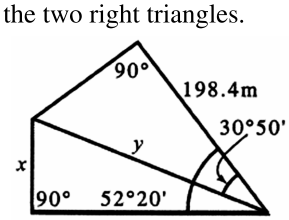
(b) Since we are dealing with a circle, the distance between *M* and *C* is *R*. If we let *x* be the distance from *N* to *M*, then the distance from C to N will be $R - x$.



Since triangle *CNP* is a right triangle, we can set up the following equation.

$$\cos\frac{\theta}{2} = \frac{R-x}{R} \Rightarrow R\cos\frac{\theta}{2} = R - x \Rightarrow x = R - R\cos\frac{\theta}{2} \Rightarrow x = R\left(1 - \cos\frac{\theta}{2}\right)$$

34. Let *y* = the common hypotenuse of the two right triangles.



$$\cos 30°50' = \frac{198.4}{y} \Rightarrow y = \frac{198.4}{\cos 30°50'} \approx 231.0571948$$

To find *x*, first find the angle opposite *x* in the right triangle by find the following.

$$52°20' - 30°50' = 51°80' - 30°50' = 21°30'$$

$$\sin 21°30' = \frac{x}{y} \Rightarrow \sin 21°30' \approx \frac{x}{231.0571948} \Rightarrow x \approx 231.0571948 \sin 21°30' \approx 84.6827$$

The length *x* is approximate 84.7 m. (rounded)

35. (a) $$\theta \approx \frac{57.3S}{R} = \frac{57.3(336)}{600} = 32.088°$$

$$d = R\left(1-\cos\frac{\theta}{2}\right) = 600(1-\cos 16.044°) \approx 23.3702 \text{ ft}$$

The distance is 23.4 ft. (rounded to three significant digits)

(b) $$\theta \approx \frac{57.3S}{R} = \frac{57.3(485)}{600} = 46.3175°$$

$$d = R\left(1-\cos\frac{\theta}{2}\right) = 600(1-\cos 23.15875°) \approx 48.3488$$

The distance is 48.3 ft. (rounded to three significant digits)

(c) The faster the speed, the more land needs to be cleared on the inside of the curve.

Chapter 2: Review Exercises

1. $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{60}{61}$ $\qquad$ $\cot A = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{11}{60}$

$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{11}{61}$ $\qquad$ $\sec A = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{61}{11}$

$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{60}{11}$ $\qquad$ $\csc A = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{61}{60}$

2. $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{40}{58} = \frac{20}{29}$ $\qquad$ $\cot A = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{42}{40} = \frac{21}{20}$

$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{42}{58} = \frac{21}{29}$ $\qquad$ $\sec A = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{58}{42} = \frac{29}{21}$

$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{40}{42} = \frac{20}{21}$ $\qquad$ $\csc A = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{58}{40} = \frac{29}{20}$

3. $\sin 4\beta = \cos 5\beta$

Since sine and cosine are cofunctions, we have the following.

$$4\beta + 5\beta = 90° \Rightarrow 9\beta = 90° \Rightarrow \beta = 10°$$

4. $\sec(2\theta + 10°) = \csc(4\theta + 20°)$

Since secant and cosecant are cofunctions, we have the following.

$$(2\theta + 10°) + (4\theta + 20°) = 90° \Rightarrow 6\theta + 30° = 90° \Rightarrow 6\theta = 60° \Rightarrow \theta = 10°$$

5. $\tan(5x + 11°) = \cot(6x + 2°)$

Since tangent and cotangent are cofunctions, we have the following.

$$(5x + 11°) + (6x + 2°) = 90° \Rightarrow 11x + 13° = 90° \Rightarrow 11x = 77° \Rightarrow x = 7°$$

6. $\cos\left(\frac{3\theta}{5}+11°\right)=\sin\left(\frac{7\theta}{10}+40°\right)$

Since sine and cosine are cofunctions, we have the following.

$$\left(\frac{3\theta}{5}+11°\right)+\left(\frac{7\theta}{10}+40°\right)=90° \Rightarrow \frac{6\theta}{10}+\frac{7\theta}{10}+51°=90° \Rightarrow \frac{13}{10}\theta+51°=90°$$
$$\frac{13}{10}\theta=39° \Rightarrow \theta=\frac{10}{13}(39°)=30°$$

7. $\sin 46° < \sin 58°$

$\text{Sin}\,\theta$ increases as θ increases from $0°$ to $90°$. Since $58° > 46°$, we have $\sin 58°$ is greater than $\sin 46°$. Thus, the statement is true.

8. $\cos 47° < \cos 58°$

$\text{Cos}\,\theta$ decreases as θ increases from $0°$ to $90°$. Since $47° < 58°$, we have $\cos 47°$ is greater than $\cos 58°$. Thus, the statement is false.

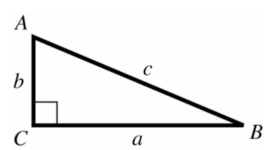
9. $\sec 48° \geq \cos 42°$

Since $48°$ and $42°$ are in quadrant I, $\sec 48°$ and $\cos 42°$ are both positive. Since $0 < \sin 42° < 1$, $\frac{1}{\sin 42°} = \csc 42° > 1$. Moreover, $0 < \cos 42° < 1$. Thus, $\sec 48° \geq \cos 42°$ and the statement is true.

10. $\sin 22° \geq \csc 68°$

Since $22°$ and $68°$ are in quadrant I, $\sin 22°$ and $\csc 68°$ are both positive. Since $0 < \sin 68° < 1$, $\frac{1}{\sin 68°} = \csc 68° > 1$. Moreover, $0 < \sin 22° < 1$. Thus, $\sin 22° < \csc 68°$ and the statement is false.

11. The measures of angles *A* and *B* sum to be $90°$, and are complementary angles. Since sine and cosine are cofunctions, we have $\sin B = \cos(90° - B) = \cos A$.



12. 120°

This angle lies in quadrant II, so the reference angle is 180° – 120° = 60°. Since 120° is in quadrant II, the cosine, tangent, cotangent and secant are negative.

$\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$

$\cos 120° = -\cos 60° = -\frac{1}{2}$

$\tan 120° = -\tan 60° = -\sqrt{3}$

$\cot 120° = -\cot 60° = -\frac{\sqrt{3}}{3}$

$\sec 120° = -\sec 60° = -2$

$\csc 120° = \csc 60° = \frac{2\sqrt{3}}{3}$

13. 300°

This angle lies in quadrant IV, so the reference angle is 360° – 300° = 60°. Since 300° is in quadrant IV, the sine, tangent, cotangent and cosecant are negative.

$\sin 300° = -\sin 60° = -\frac{\sqrt{3}}{2}$

$\cos 300° = \cos 60° = \frac{1}{2}$

$\tan 300° = -\tan 60° = -\sqrt{3}$

$\cot 300° = -\cot 60° = -\frac{\sqrt{3}}{3}$

$\sec 300° = \sec 60° = 2$

$\csc 300° = -\csc 60° = -\frac{2\sqrt{3}}{3}$

14. –225°

–225° is coterminal with –225° + 360° = 135°. This angle lies in quadrant II. The reference angle is $180° - 135° = 45°$. Since –225° is in quadrant II, the cosine, tangent, cotangent, and secant are negative.

$\sin(-225°) = \sin 45° = \frac{\sqrt{2}}{2}$

$\cos(-225°) = -\cos 45° = -\frac{\sqrt{2}}{2}$

$\tan(-225°) = -\tan 45° = -1$

$\cot(-225°) = -\cot 45° = -1$

$\sec(-225°) = -\sec 45° = -\sqrt{2}$

$\csc(-225°) = \csc 45° = \sqrt{2}$

15. –390° is coterminal with $-390° + 2 \cdot 360° = -390° + 720° = 330°$. This angle lies in quadrant IV. The reference angle is $360° - 330° = 30°$. Since –390° is in quadrant IV, the sine, tangent, cotangent, and cosecant are negative.

$\sin(-390°) = -\sin 30° = -\frac{1}{2}$

$\cos(-390°) = \cos 30° = \frac{\sqrt{3}}{2}$

$\tan(-390°) = -\tan 30° = -\frac{\sqrt{3}}{3}$

$\cot(-390°) = -\cot 30° = -\sqrt{3}$

$\sec(-390°) = \sec 30° = \frac{2\sqrt{3}}{3}$

$\csc(-390°) = -\csc 30° = -2$

16. $\sin\theta = -\frac{1}{2}$

Since $\sin\theta$ is negative, θ must lie in quadrants III or IV. Since the absolute value of $\sin\theta$ is $\frac{1}{2}$, the reference angle, θ', must be 30°. The angle in quadrant III will be $180° + \theta' = 180° + 30° = 210°$. The angle in quadrant IV will be $360° - \theta' = 360° - 30° = 330°$.

17. $\cos\theta = -\frac{1}{2}$

Since $\cos\theta$ is negative, θ must lie in quadrants II or III. Since the absolute value of $\cos\theta$ is $\frac{1}{2}$, the reference angle, θ', must be 60°. The quadrant II angle θ equals $180° - \theta' = 180° - 60° = 120°$, and the quadrant III angle θ equals $180° + \theta' = 180° + 60° = 240°$.

18. $\cot\theta = -1$

Since $\cot\theta$ is negative, θ must lie in quadrants II or IV. Since the absolute value of $\cot\theta$ is 1, the reference angle, θ', must be 45°. The quadrant II angle θ equals $180° - \theta' = 180° - 45° = 135°$, and the quadrant IV angle θ equals $360° - \theta' = 360° - 45° = 315°$.

19. $\sec\theta = -\frac{2\sqrt{3}}{3}$

Since $\sec\theta$ is negative, θ must lie in quadrants II or III. Since the absolute value of $\sec\theta$ is $\frac{2\sqrt{3}}{3}$, the reference angle, θ', must be $30°$. The quadrant II angle θ equals $180° - \theta' = 180° - 30° = 150°$, and the quadrant III angle θ equals $180° + \theta' = 180° + 30° = 210°$.

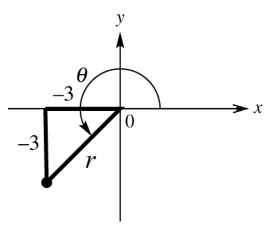
20. $\cos 60° + 2\sin^2 30° = \frac{1}{2} + 2\left(\frac{1}{2}\right)^2 = \frac{1}{2} + 2\left(\frac{1}{4}\right) = \frac{1}{2} + \frac{1}{2} = 1$

21. $\tan^2 120° - 2\cot 240° = \left(-\sqrt{3}\right)^2 - 2\left(\frac{\sqrt{3}}{3}\right) = 3 - \frac{2\sqrt{3}}{3}$

22. $\sec^2 300° - 2\cos^2 150° + \tan 45° = 2^2 - 2\left(-\frac{\sqrt{3}}{2}\right)^2 + 1 = 4 - 2\left(\frac{3}{4}\right) + 1 = 4 - \frac{3}{2} + 1 = \frac{7}{2}$

23. (a) $(-3, -3)$

Given the point (x, y), we need to determine the distance from the origin, r.

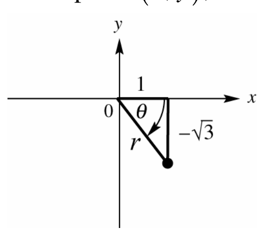


$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{(-3)^2 + (-3)^2}$$
$$r = \sqrt{9 + 9}$$
$$r = \sqrt{18}$$
$$r = 3\sqrt{2}$$

$$\sin\theta = \frac{y}{r} = \frac{-3}{3\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}; \cos\theta = \frac{x}{r} = \frac{-3}{3\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$
$$\tan\theta = \frac{y}{x} = \frac{-3}{-3} = 1$$

(b) $\left(1, -\sqrt{3}\right)$

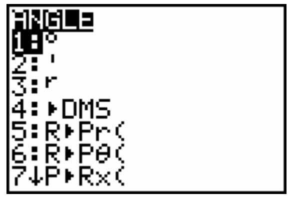
Given the point (x, y), we need to determine the distance from the origin, r.



$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{1^2 + \left(-\sqrt{3}\right)^2}$$
$$r = \sqrt{1 + 3}$$
$$r = \sqrt{4}$$
$$r = 2$$

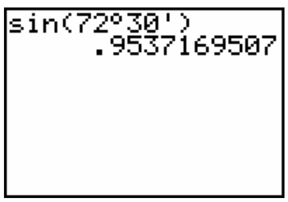
$$\sin\theta = \frac{y}{r} = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}; \cos\theta = \frac{x}{r} = \frac{1}{2}; \tan\theta = \frac{y}{x} = \frac{-\sqrt{3}}{1} = -\sqrt{3}$$

For the remainder of the exercises in this section, be sure your calculator is in degree mode. If your calculator accepts angles in degrees, minutes, and seconds, it is not necessary to change angles to decimal degrees. Keystroke sequences may vary on the type and/or model of calculator being used. Screens shown will be from a TI-83 Plus calculator. To obtain the degree $(^\circ)$ and $(')$ symbols, go to the ANGLE menu (2^{nd} APPS).



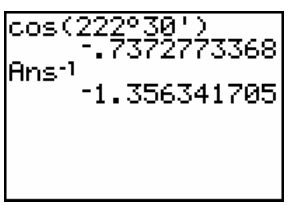
For Exercises 24, 25, and 27, the calculation for decimal degrees is indicated for calculators that do not accept degree, minutes, and seconds.

24. $\sin 72^\circ 30' \approx .95371695$



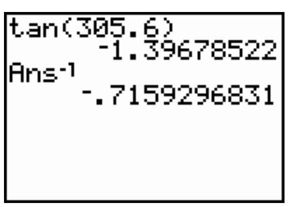
$72^\circ 30' = \left(72 + \frac{30}{60}\right)^\circ = 72.5^\circ$

25. $\sec 222^\circ 30' \approx -1.3563417$

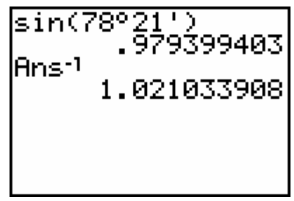


$222^\circ 30' = \left(222 + \frac{30}{60}\right)^\circ = 222.5^\circ$

26. $\cot 305.6^\circ \approx -.71592968$

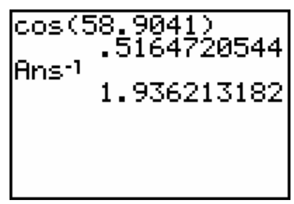


27. $\csc 78^\circ 21' \approx 1.0210339$

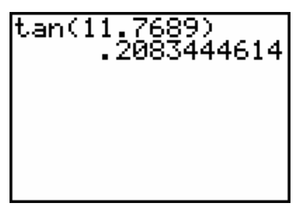


$78^\circ 21' = \left(78 + \frac{21}{60}\right)^\circ = 78.35^\circ$

28. $\sec 58.9041^\circ \approx 1.9362132$



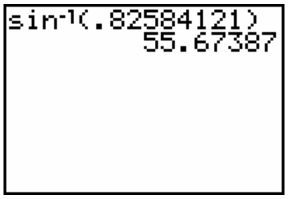
29. $\tan 11.7689^\circ \approx .20834446$



30. If $\theta = 135^\circ$, $\theta' = 45^\circ$.
If $\theta = 45^\circ$, $\theta' = 45^\circ$.
If $\theta = 300^\circ$, $\theta' = 60^\circ$.
If $\theta = 140^\circ$, $\theta' = 40^\circ$.

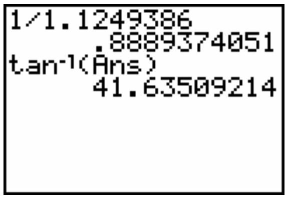
Of these reference angles, 40° is the only one which is not a special angle, so D, tan 140°, is the only one which cannot be determined exactly using the methods of this chapter.

31. $\sin\theta = .8254121$



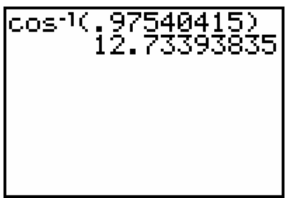
$\theta \approx 55.673870°$

32. $\cot\theta = 1.1249386$



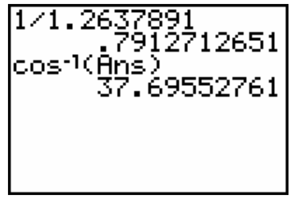
$\theta \approx 41.635092°$

33. $\cos\theta = .97540415$



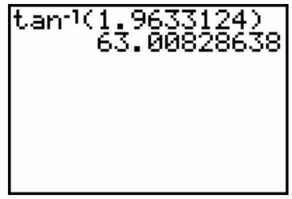
$\theta \approx 12.733938°$

34. $\sec\theta = 1.2637891$



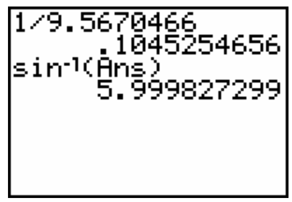
$\theta \approx 37.695528$

35. $\tan\theta = 1.9633124$



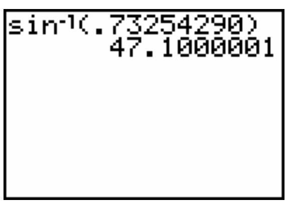
$\theta \approx 63.008286°$

36. $\csc\theta = 9.5670466$



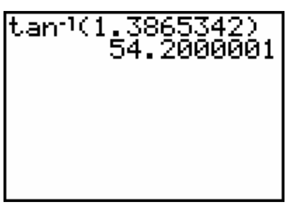
$\theta \approx 5.9998273°$

37. $\sin\theta = .73254290$



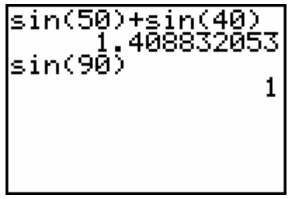
Since sin θ is positive, there will be one angle in quadrant I and one angle in quadrant II. If θ' is the reference angle, then the two angles are θ' and $180° - \theta'$. Thus, the quadrant I angle is approximately equal to $47.1°$, and the quadrant II angle is $180° - 47.1° = 132.9°$.

38. $\tan\theta = 1.3865342$



Since tan θ is positive, there will be one angle in quadrant I and one angle in quadrant III. If θ' is the reference angle, then the two angles are θ' and $180° + \theta'$. Thus, the quadrant I angle is approximately equal to $54.2°$, and the quadrant III angle is $180° + 54.2° = 234.2°$.

39. $\sin 50° + \sin 40° \overset{?}{=} \sin 90°$



Since $\sin 50° + \sin 40° \approx 1.408832053$ and $\sin 90° = 1,$ the statement is false.

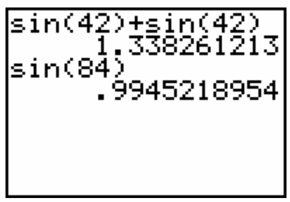
40. $\cos 210° \overset{?}{=} \cos 180° \cdot \cos 30° - \sin 180° \cdot \sin 30°$

Since $\cos 210° = -\cos 30° = -\frac{\sqrt{3}}{2}$ and $\cos 180° \cdot \cos 30° - \sin 180° \cdot \sin 30° = (-1)\left(\frac{\sqrt{3}}{2}\right) - (0)\left(\frac{1}{2}\right) = -\frac{\sqrt{3}}{2},$ the statement is true.

41. $\sin 240° \overset{?}{=} 2 \sin 120° \cos 120°$

Since $\sin 240° = -\sin 60° = -\frac{\sqrt{3}}{2}$ and $2 \sin 120° \cos 120° = 2(\sin 60°)(-\cos 60°) = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = -\frac{\sqrt{3}}{2},$ the statement is true.

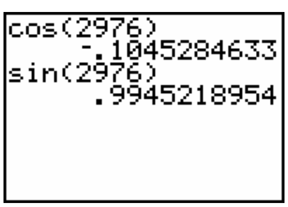
42. $\sin 42° + \sin 42° \overset{?}{=} \sin 84°$



Using a calculator, we have $\sin 42° + \sin 42° = 1.338261213$ and $\sin 84° = .9945218954.$ Thus, the statement is false.

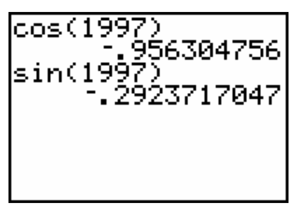
43. No, $\cot 25° = \dfrac{1}{\tan 25°} \neq \tan^{-1} 25°.$

44. $\theta = 2976°$



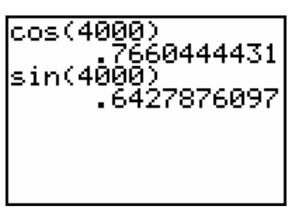
Since cosine is negative and sine is positive, the angle θ is in quadrant II.

45. $\theta = 1997°$



Since sine and cosine are both negative, the angle θ is in quadrant III.

46. $\theta = 4000°$



Since sine and cosine are both positive, the angle θ is in quadrant I.

47. $A = 58° 30', \; c = 748$

$A + B = 90° \Rightarrow B = 90° - A \Rightarrow B = 90° - 58°30' = 89°60' - 58°30' = 31°30'$

$\sin A = \frac{a}{c} \Rightarrow \sin 58°30' = \frac{a}{748} \Rightarrow a = 748 \sin 58°30' \approx 638$ (rounded to three significant digits)

$\cos A = \frac{b}{c} \Rightarrow \cos 58°30' = \frac{b}{748} \Rightarrow b = 748 \cos 58°30' \approx 391$ (rounded to three significant digits)

48. $a = 129.70, \; b = 368.10$

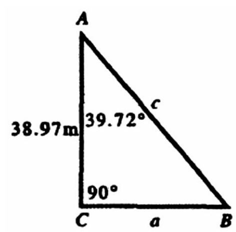
$\tan A = \frac{129.70}{368.10} \approx .3523499049 \Rightarrow A \approx \tan^{-1}(.3523499049) \approx 19.41° \approx 19°25'$

$\tan B = \frac{368.10}{129.70} \approx 2.838087895 \Rightarrow B \approx \tan^{-1}(2.838087895) \approx 70.59° \approx 70°35'$

Note: $A + B = 90°$

$c = \sqrt{a^2 + b^2} = \sqrt{129.70^2 + 368.10^2} = \sqrt{16,822.09 + 135,497.61} = \sqrt{152,319.7} \approx 390.28$ (rounded to five significant digits)

49. $A = 39.72°, \; b = 38.97$ m

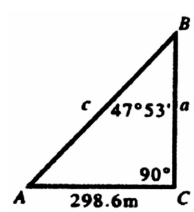


$A + B = 90° \Rightarrow B = 90° - A \Rightarrow B = 90° - 39.72° = 50.28°$

$\tan A = \frac{a}{b} \Rightarrow \tan 39.72° = \frac{a}{38.97} \Rightarrow a = 38.97 \tan 39.72° \approx 32.38$ m (rounded to four significant digits)

$\cos A = \frac{b}{c} \Rightarrow \cos 39.72° = \frac{38.97}{c} \Rightarrow c \cos 39.72° = 38.97 \Rightarrow c = \frac{38.97}{\cos 39.72°} \approx 50.66$ m (rounded to five significant digits)

50. $B = 47°53', \; b = 298.6$ m

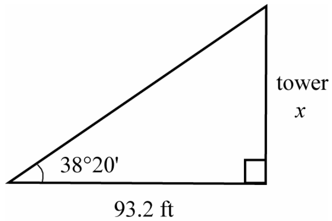


$A + B = 90° \Rightarrow A = 90° - B \Rightarrow A = 90° - 47°53' = 89°60' - 47°53' = 42°7'$

$\tan B = \frac{b}{a} \Rightarrow \tan 47°53' = \frac{298.6}{a} \Rightarrow a \tan 47°53' = 298.6 \Rightarrow a = \frac{298.6}{\tan 47°53'} \approx 270.0$ m (rounded to four significant digits)

$\sin B = \frac{b}{c} \Rightarrow \sin 47°53' = \frac{298.6}{c} \Rightarrow c \sin 47°53' = 298.6 \Rightarrow c = \frac{298.6}{\sin 47°53'} \approx 402.5$ m (rounded to four significant digits)

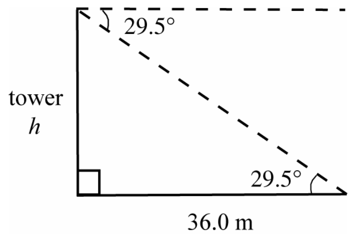
51. Let x = height of the tower.



$$\tan 38°20' = \frac{x}{93.2}$$
$$x = 93.2 \tan 38°20'$$
$$x \approx 73.6930$$

The height of the tower is 73.7 ft. (rounded to three significant digits)

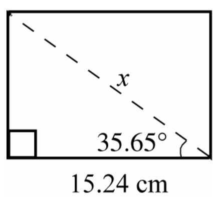
52. Let h = height of the tower.



$$\tan 29.5° = \frac{h}{36.0}$$
$$h = 36.0 \tan 29.5°$$
$$h \approx 20.3678$$

The height of the tower is 20.4 m. (rounded to three significant digits)

53. Let x = length of the diagonal.

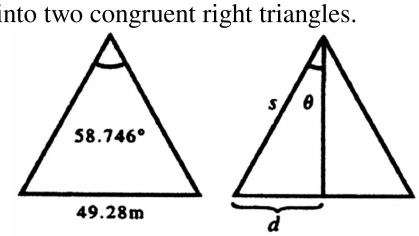


$$\cos 35.65° = \frac{15.24}{x}$$
$$x \cos 35.65° = 15.24$$
$$x = \frac{15.24}{\cos 35.65°}$$
$$x \approx 18.7548$$

The length of the diagonal of the rectangle is 18.75 cm. (rounded to three significant digits)

54. Let x = the length of the equal sides of an isosceles triangle.

Divide the isosceles triangle into two congruent right triangles.



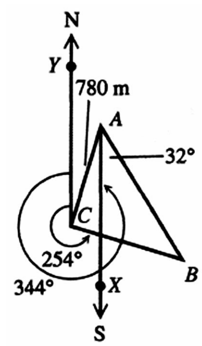
$$d = \frac{1}{2}(49.28) = 24.64 \text{ and } \theta = \frac{1}{2}(58.746°) = 29.373°$$

and

$$\sin\theta = \frac{d}{s} \Rightarrow \sin 29.373° = \frac{24.64}{s} \Rightarrow \sin 29.373° = \frac{24.64}{s} \Rightarrow s \sin 29.373° = 24.64$$
$$s = \frac{24.64}{\sin 29.373°} \Rightarrow s \approx 50.2352$$

Each side is 50.24 m long. (rounded to 4 significant digits)

55. Draw triangle ABC and extend the north-south lines to a point X south of A and S to a point Y, north of C.

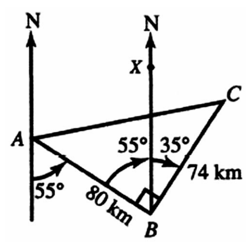


Angle ACB = 344° − 254° = 90°, so ABC is a right triangle.
Angle BAX = 32° since it is an alternate interior angle to 32°.
Angle YCA = 360° − 344° = 16°
Angle XAC = 16° since it is an alternate interior angle to angle YCA.
Angle BAC = 32° + 16° = 48°.

In triangle ABC, $\cos A = \dfrac{AC}{AB} \Rightarrow \cos 48° = \dfrac{780}{AB} \Rightarrow AB\cos 48° = 780 \Rightarrow AB = \dfrac{780}{\cos 48°} \approx 1165.6917.$

The distance from A to B is 1200 m. (rounded to two significant digits)

56. Draw triangle ABC and extend north-south lines from points A and B. Angle ABX is 55° (alternate interior angles of parallel lines cut by a transversal have the same measure) so Angle ABC is 55° + 35° = 90°.

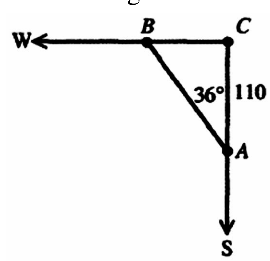


Since angle ABC is a right angle, use the Pythagorean theorem to find the distance from A to C.

$$(AC)^2 = 80^2 + 74^2 \Rightarrow (AC)^2 = 6400 + 5476 \Rightarrow (AC)^2 = 11{,}876 \Rightarrow AC = \sqrt{11{,}876} \approx 108.9771$$

It is 110 km from A to C. (rounded to two significant digits)

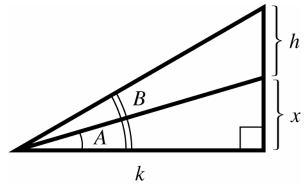
57. Suppose A is the car heading south at 55 mph, B is the car heading west, and point C is the intersection from which they start. After two hours by $d = rt$, $AC = 55(2) = 110$. Since angle ACB is a right angle, triangle ACB is a right triangle. Since the bearing of A from B is 324°, angle CAB = 360° − 324° = 36°.



$$\cos CAB = \frac{AC}{AB} \Rightarrow \cos 36° = \frac{110}{AB} \Rightarrow AB\cos 36° = 110 \Rightarrow AB = \frac{110}{\cos 36°} \approx 135.9675$$

There are 140 mi apart. (rounded to two significant digits)

58. Let x = the leg opposite angle A.



$$\tan A = \frac{x}{k} \Rightarrow x = k\tan A \text{ and } \tan B = \frac{h+x}{k} \Rightarrow x = k\tan B - h$$

Therefore, we have the following.

$$k\tan A = k\tan B - h \Rightarrow h = k\tan B - k\tan A \Rightarrow h = k\left(\tan B - \tan A\right)$$

59. – 60. Answers will vary.

61. $h = R\left(\frac{1}{\cos\left(\frac{180T}{P}\right)} - 1\right)$

(a) Let R = 3955 mi, T = 25 min, P = 140 min.

$$h = R\left(\frac{1}{\cos\left(\frac{180T}{P}\right)} - 1\right) \Rightarrow h = 3955\left(\frac{1}{\cos\left(\frac{180\cdot 25}{140}\right)} - 1\right) \approx 715.9424$$

The height of the satellite is approximately 716 mi.

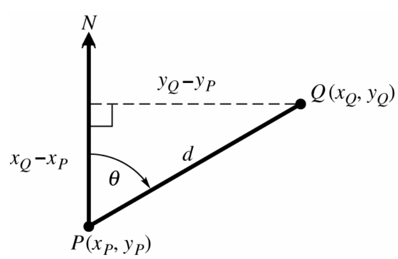
(b) Let R = 3955 mi, T = 30 min, P = 140 min.

$$h = R\left(\frac{1}{\cos\left(\frac{180T}{P}\right)} - 1\right) \Rightarrow h = 3955\left(\frac{1}{\cos\left(\frac{180\cdot 30}{140}\right)} - 1\right) \approx 1103.6349$$

The height of the satellite is approximately 1104 mi.

62. (a) From the figure we see that, $\sin\theta = \frac{x_Q - x_P}{d} \Rightarrow x_Q = x_P + d\sin\theta$. Similarly, we have

$$\cos\theta = \frac{y_Q - y_P}{d} \Rightarrow y_Q = y_P + d\cos\theta.$$



(b) Let $(x_P, y_P) = (123.62, 337.95)$, $\theta = 17°\ 19'\ 22''$, and $d = 193.86$.

$x_Q = x_P + d\sin\theta \Rightarrow x_Q = 123.62 + 193.86\sin 17°19'22'' \approx 181.3427$

$y_Q = y_P + d\cos\theta \Rightarrow= 337.95 + 193.86\cos 17°19'22'' \approx 523.0170$

The coordinates of Q are (181.34, 523.02). (rounded to five significant digits)

Chapter 2: Test

1. $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{12}{13}$

$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{5}{13}$

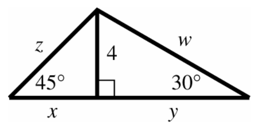
$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{12}{5}$

$\cot A = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{5}{12}$

$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{13}{5}$

$\csc A = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{13}{12}$

2. Apply the relationships between the lengths of the sides of a $30°-60°$ right triangle first to the triangle on the right to find the values of y and w. In the $30°-60°$ right triangle, the side opposite the $60°$ angle is $\sqrt{3}$ times as long as the side opposite to the $30°$ angle. The length of the hypotenuse is 2 times as long as the shorter leg (opposite the $30°$ angle).



Thus, we have the following.

$$y = 4\sqrt{3} \text{ and } w = 2(4) = 8$$

Apply the relationships between the lengths of the sides of a $45°-45°$ right triangle next to the triangle on the left to find the values of x and z. In the $45°-45°$ right triangle, the sides opposite the $45°$ angles measure the same. The hypotenuse is $\sqrt{2}$ times the measure of a leg.

Thus, we have the following.

$$x = 4 \text{ and } z = 4\sqrt{2}$$

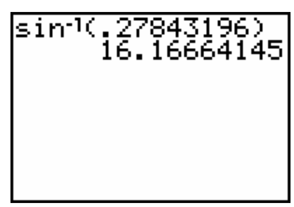
3. $\sin(B+15°) = \cos(2B+30°)$

Since sine and cosine are cofunctions, the equation is true when the following holds.

$$(B+15°)+(2B+30°) = 90°$$
$$3B+45° = 90°$$
$$3B = 45°$$
$$B = 15°$$

This is one solution; others are possible.

4. $\sin\theta = .27843196$



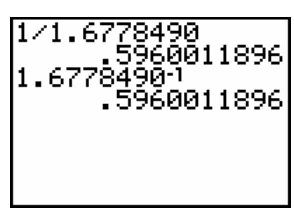
$\theta \approx 16.16664145°$

5. $\cos\theta = -\frac{\sqrt{2}}{2}$

Since $\cos\theta$ is negative, θ must lie in quadrants II or III. Since the absolute value of $\cos\theta$ is $\frac{\sqrt{2}}{2}$, the reference angle, θ', must be $45°$. The quadrant II angle θ equals $180° - \theta' = 180° - 45° = 135°$, and the quadrant III angle θ equals $180° + \theta' = 180° + 45° = 225°$.

6. $\tan\theta = 1.6778490$

Since $\cot\theta = \frac{1}{\tan\theta} = (\tan\theta)^{-1}$, we can use division or the inverse key (multiplicative inverse).



$\cot\theta \approx .5960011896$

7. **(a)** $\sin 24° < \sin 48°$

Sine increases from 0 to 1 in the interval $0° \le \theta \le 90°$. Therefore, $\sin 24°$ is less than $\sin 48°$ and the statement is true.

(b) $\cos 24° < \cos 48°$

Cosine decreases from 1 to 0 in the interval $0° \le \theta \le 90°$ Therefore, $\cos 24°$ is not less than $\cos 48°$ and the statement is false.

(c) $\tan 24° < \tan 48°$

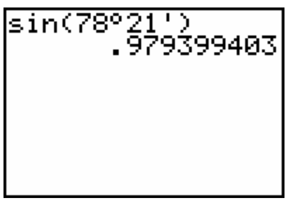
Tangent increases in the interval $0° \le \theta \le 90°$. Therefore, $\tan 24°$ is less than $\tan 48°$ and the statement is true.

8. $\cot(-750°)$

$-750°$ is coterminal with $-750° + 3 \cdot 360° = -750° + 1080° = 330°$, which is in quadrant IV. The cotangent is negative in quadrant IV and the reference angle is $360° - 330° = 30°$.

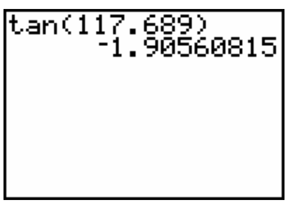
$$\cot(-750°) = -\cot 30° = -\sqrt{3}$$

9. **(a)** $\sin 78° 21' \approx .97939940$

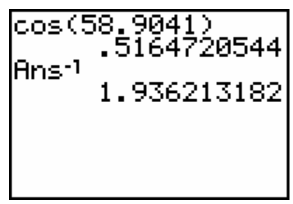


$78° 21' = \left(78 + \frac{21}{60}\right)° = 78.35°$

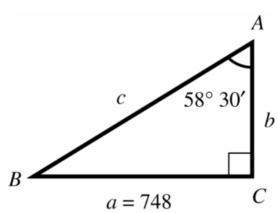
(b) $\tan 117.689° \approx -1.9056082$



(c) $\sec 58.9041° \approx 1.9362132$



10. $A = 58°30'$, $a = 748$

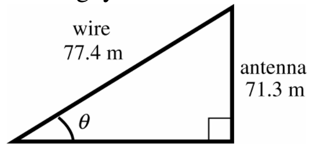


$A + B = 90° \Rightarrow B = 90° - A \Rightarrow B = 90° - 58°30' = 89°60' - 58°30' = 31°30'$

$\tan A = \frac{a}{b} \Rightarrow \tan 58°30' = \frac{748}{b} \Rightarrow b \tan 58°30' = 748 \Rightarrow b = \frac{748}{\tan 58°30'} \approx 458$ (rounded to three significant digits)

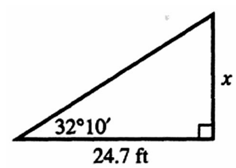
$\sin A = \frac{a}{c} \Rightarrow \sin 58°30' = \frac{748}{c} \Rightarrow c \sin 58°30' = 748 \Rightarrow c = \frac{748}{\sin 58°30'} \approx 877$ (rounded to three significant digits)

11. Let $\theta =$ the measure of the angle that the guy wire makes with the ground.



$$\sin\theta = \frac{71.3}{77.4} \approx .9211886305 \Rightarrow \theta \approx \sin^{-1}(.9211886305) \approx 61.1° \approx 61°10'$$

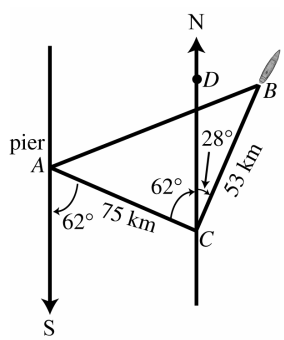
12. Let x = the height of the flagpole.



$$\tan 32°10' = \frac{x}{24.7} \Rightarrow x = 24.7 \tan 32°10' \approx 15.5344$$

The flagpole is approximately 15.5 ft high. (rounded to three significant digits)

13. Draw triangle ACB and extend north-south lines from points A and C. Angle ACD is 62° (alternate interior angles of parallel lines cut by a transversal have the same measure), so Angle ACB is 62° + 28° = 90°.

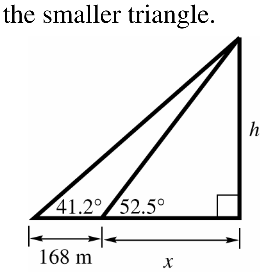


Since angle ACB is a right angle, use the Pythagorean theorem to find the distance from A to B.

$$(AB)^2 = 75^2 + 53^2 \Rightarrow (AB)^2 = 5625 + 2809 \Rightarrow (AB)^2 = 8434 \Rightarrow AB = \sqrt{8434} \approx 91.8368$$

It is 92 km from the pier to the boat. (rounded to two significant digits)

14. Let x = the side adjacent to 52.5° in the smaller triangle.



In the larger triangle, we have $\tan 41.2° = \dfrac{h}{168 + x} \Rightarrow h = (168 + x)\tan 41.2°$.

In the smaller triangle, we have $\tan 52.5° = \dfrac{h}{x} \Rightarrow h = x \tan 52.5°$.

Continued on next page

14. (continued)

Substitute for h in this equation and solve for x.

$$(168+x)\tan 41.2° = x\tan 52.5°$$
$$168\tan 41.2° + x\tan 41.2° = x\tan 52.5°$$
$$168\tan 41.2° = x\tan 52.5° - x\tan 41.2°$$
$$168\tan 41.2° = x(\tan 52.5° - \tan 41.2°)$$
$$\frac{168\tan 41.2°}{\tan 52.5° - \tan 41.2°} = x$$

Substituting for x in the equation for the smaller triangle, we have the following.

$$h = x\tan 52.5° \Rightarrow h = \frac{168\tan 41.2°\tan 52.5°}{\tan 52.5° - \tan 41.2°} \approx 448.0432$$

The height of the triangle is approximately 448 m. (rounded to three significant digits)

Chapter 2: Quantitative Reasoning

$$D = \frac{v^2\sin\theta\cos\theta + v\cos\theta\sqrt{(v\sin\theta)^2 + 64h}}{32}$$

All answers are rounded to four significant digits.

1. Since $v = 44$ ft per sec and $h = 7$ ft, we have $D = \dfrac{44^2\sin\theta\cos\theta + 44\cos\theta\sqrt{(44\sin\theta)^2 + 64\cdot 7}}{32}$.

If $\theta = 40°$, $D = \dfrac{1936\sin 40\cos 40 + 44\cos 40\sqrt{(44\sin 40)^2 + 448}}{32} \approx 67.00$ ft.

If $\theta = 42°$, $D = \dfrac{1936\sin 42\cos 42 + 44\cos 42\sqrt{(44\sin 42)^2 + 448}}{32} \approx 67.14$ ft.

If $\theta = 45°$, $D = \dfrac{1936\sin 45\cos 45 + 44\cos 45\sqrt{(44\sin 45)^2 + 448}}{32} \approx 66.84$ ft.

As θ increases, D increases and then decreases.

2. Since $h = 7$ ft and $\theta = 42°$, we have $D = \dfrac{v^2\sin 42\cos 42 + v\cos 42\sqrt{(v\sin 42)^2 + 64h}}{32}$.

If $v = 43$, $D = \dfrac{43^2\sin 42\cos 42 + 43\cos 42\sqrt{(43\sin 42)^2 + 448}}{32} \approx 64.40$ ft.

If $v = 44$, $D = \dfrac{44^2\sin 42\cos 42 + 44\cos 42\sqrt{(44\sin 42)^2 + 448}}{32} \approx 67.14$ ft.

If $v = 45$, $D = \dfrac{45^2\sin 42\cos 42 + 45\cos 42\sqrt{(45\sin 42)^2 + 448}}{32} \approx 69.93$ ft.

As v increases, D increases.

3. The velocity affects the distance more. The shot-putter should concentrate on achieving as large a value of v as possible.