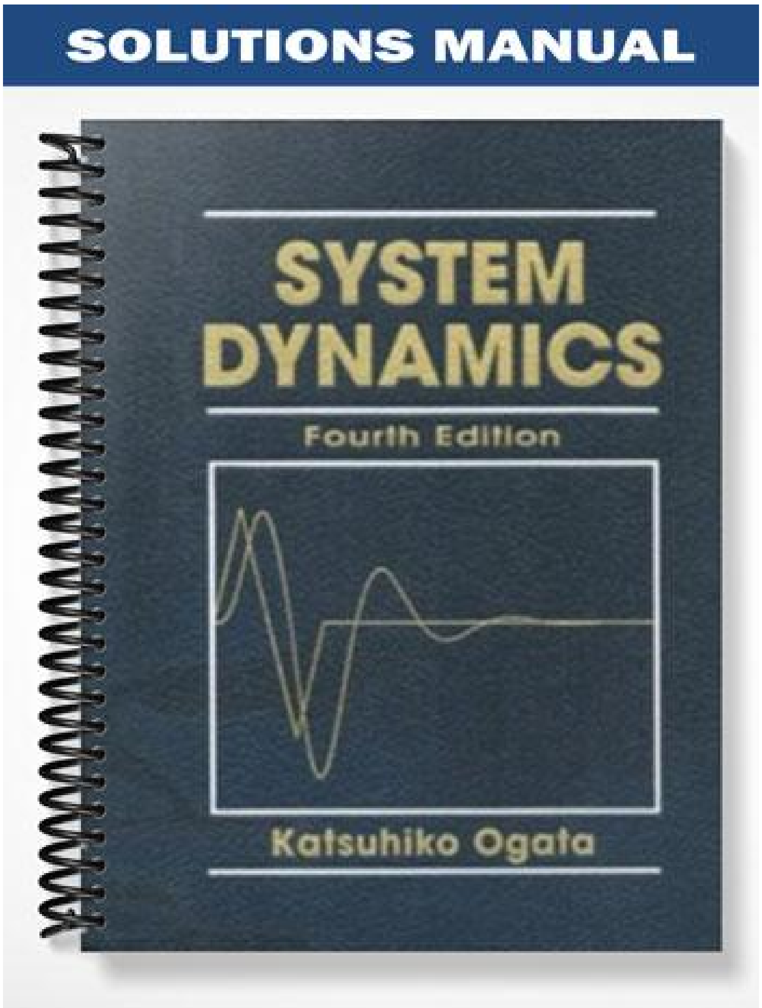
SOLUTIONS MANUAL
SYSTEM
DYNAMICS
Fourth Edition
Katsuhiko Ogata

SOLUTIONS MANUAL

SYSTEM DYNAMICS

Fourth Edition

Katsuhiko Ogata

PEARSON
Prentice Hall

Upper Saddle River, New Jersey 07458

Acquisitions Editor: *Laura Fischer*
Supplement Cover Manager: *Daniel Sandin*
Executive Managing Editor: *Vince O'Brien*
Managing Editor: *David A. George*
Production Editor: *Barbara A. Till*
Supplement Editor: *Andrea Messineo*
Manufacturing Buyer: *Ilene Kahn*

Pearson Prentice Hall
Pearson Education, Inc.
Upper Saddle River, NJ 07458

Printed in the United States of America
10 9 8 7 6 5 4 3 2 1

ISBN 0-13-142463-7

Pearson Education Ltd., *London*
Pearson Education Australia Pty. Ltd., *Sydney*
Pearson Education Singapore, Pte. Ltd.
Pearson Education North Asia Ltd., *Hong Kong*
Pearson Education Canada, Inc., *Toronto*
Pearson Educación de Mexico, S.A. de C.V.
Pearson Education—Japan, *Tokyo*
Pearson Education Malaysia, Pte. Ltd.
Pearson Education, Inc., *Upper Saddle River, New Jersey*

Contents

Preface

This Solutions Manual presents solutions to all unsolved B-problems. For some problems, solutions include more materials than are required in problem statements to aid the user of the book.

The text may be used in a few different ways depending on the course objective and the time allocated to the course.

Sample course coverages are listed below.

If this book is used as a text for a quarter-length course (with approximately 30 lecture hours and 18 recitation hours), Chapters 1 through 7 may be covered.

If the book is used as a text for a semester-length course (with approximately 40 lecture hours and 26 recitation hours), then the first nine chapters may be covered or, alternatively, the first seven chapters plus Chapters 10 and 11 may be covered.

If the course devotes 50 to 60 hours to lectures, then the entire book may be covered in a semester.

The instructor will always have an option to omit certain subjects depending on the course objective.

Katsuhiko Ogata

Solutions to B Problems

CHAPTER 2

B-2-1.

$$f(t) = 0 \qquad t < 0$$
$$= t\, e^{-2t} \qquad t \geq 0$$

Note that

$$\mathcal{L}[t] = \frac{1}{s^2}$$

Referring to Equation (2-2), we obtain

$$F(s) = \mathcal{L}[f(t)] = \mathcal{L}[t\, e^{-2t}] = \frac{1}{(s+2)^2}$$

B-2-2.

(a)

$$f_1(t) = 0 \qquad t < 0$$
$$= 3 \sin(5t + 45^\circ) \qquad t \geq 0$$

Note that

$$3 \sin(5t + 45^\circ) = 3 \sin 5t \cos 45^\circ + 3 \cos 5t \sin 45^\circ$$
$$= \frac{3}{\sqrt{2}} \sin 5t + \frac{3}{\sqrt{2}} \cos 5t$$

So we have

$$F_1(s) = \mathcal{L}[f_1(t)] = \frac{3}{\sqrt{2}} \frac{5}{s^2 + 5^2} + \frac{3}{\sqrt{2}} \frac{s}{s^2 + 5^2}$$
$$= \frac{3}{\sqrt{2}} \frac{s + 5}{s^2 + 25}$$

(b)

$$f_2(t) = 0 \qquad t < 0$$
$$= 0.03(1 - \cos 2t) \qquad t \geq 0$$

$$F_2(s) = \mathcal{L}[f_2(t)] = 0.03 \frac{1}{s} - 0.03 \frac{s}{s^2 + 2^2} = \frac{0.12}{s(s^2 + 4)}$$

B-2-3.

$$f(t) = 0 \qquad t < 0$$
$$= t^2 e^{-at} \qquad t \geq 0$$

Note that

$$\mathcal{L}[t^2] = \frac{2}{s^3}$$

Referring to Equation (2-2), we obtain

$$F(s) = \mathcal{L}[f(t)] = \mathcal{L}[t^2 e^{-at}] = \frac{2}{(s + a)^3}$$

B-2-4.

$$f(t) = 0 \qquad t < 0$$
$$= \cos 2\omega t \cos 3\omega t \qquad t \geq 0$$

Noting that

$$\cos 2\omega t \cos 3\omega t = \tfrac{1}{2}(\cos 5\omega t + \cos \omega t)$$

we have

$$F(s) = \mathcal{L}[f(t)] = \mathcal{L}[\tfrac{1}{2}(\cos 5\omega t + \cos \omega t)]$$
$$= \frac{1}{2}\left(\frac{s}{s^2 + 25\omega^2} + \frac{s}{s^2 + \omega^2}\right) = \frac{(s^2 + 13\omega^2)s}{(s^2 + 25\omega^2)(s^2 + \omega^2)}$$

B-2-5. The function $f(t)$ can be written as

$$f(t) = (t - a)\, 1(t - a)$$

The Laplace transform of $f(t)$ is

$$F(s) = \mathcal{L}[f(t)] = \mathcal{L}[(t - a)\, 1(t - a)] = \frac{e^{-as}}{s^2}$$

B-2-6.

$$f(t) = c\, 1(t - a) - c\, 1(t - b)$$

The Laplace transform of $f(t)$ is

$$F(s) = c\,\frac{e^{-as}}{s} - c\,\frac{e^{-bs}}{s} = \frac{c}{s}\,(e^{-as} - e^{-bs})$$

B-2-7. The function f(t) can be written as

$$f(t) = \frac{10}{a^2} - \frac{12.5}{a^2} 1(t - \frac{a}{5}) + \frac{2.5}{a^2} 1(t - a)$$

So the Laplace transform of f(t) becomes

$$F(s) = \mathcal{L}[f(t)] = \frac{10}{a^2}\frac{1}{s} - \frac{12.5}{a^2}\frac{1}{s} e^{-(a/5)s} + \frac{2.5}{a^2}\frac{1}{s} e^{-as}$$

$$= \frac{1}{a^2 s}(10 - 12.5\, e^{-(a/5)s} + 2.5\, e^{-as})$$

As a approaches zero, the limiting value of F(s) becomes as follows:

$$\lim_{a\to 0} F(s) = \lim_{a\to 0} \frac{10 - 12.5\, e^{-(a/5)s} + 2.5\, e^{-as}}{a^2 s}$$

$$= \lim_{a\to 0} \frac{\frac{d}{da}(10 - 12.5\, e^{-(a/5)s} + 2.5\, e^{-as})}{\frac{d}{da} a^2 s}$$

$$= \lim_{a\to 0} \frac{2.5\, s\, e^{-(a/5)s} - 2.5\, s\, e^{-as}}{2as}$$

$$= \lim_{a\to 0} \frac{\frac{d}{da}(2.5\, e^{-(a/5)s} - 2.5\, e^{-as})}{\frac{d}{da} 2a}$$

$$= \lim_{a\to 0} \frac{-0.5\, s\, e^{-(a/5)s} + 2.5\, s\, e^{-as}}{2}$$

$$= \frac{-0.5\, s + 2.5\, s}{2} = \frac{2s}{2} = s$$

B-2-8. The function f(t) can be written as

$$f(t) = \frac{24}{a^3} t - \frac{24}{a^2} 1(t - \frac{a}{2}) - \frac{24}{a^3}(t - a)\, 1(t - a)$$

So the Laplace transform of f(t) becomes

$$F(s) = \frac{24}{a^3}\frac{1}{s^2} - \frac{24}{a^2}\frac{1}{s} e^{-\frac{1}{2}as} - \frac{24}{a^3}\frac{e^{-as}}{s^2}$$

$$= \frac{24}{a^3}\left(\frac{1}{s^2} - \frac{a}{s} e^{-\frac{1}{2}as} - \frac{e^{-as}}{s^2}\right)$$

The limiting value of F(s) as a approaches zero is

$$\lim_{a\to 0} F(s) = \lim_{a\to 0} \frac{24(1 - as\, e^{-\frac{1}{2}as} - e^{-as})}{a^3 s^2}$$

$$= \lim_{a\to 0} \frac{\frac{d}{da} 24(1 - as\, e^{-\frac{1}{2}as} - e^{-as})}{\frac{d}{da} a^3 s^2}$$

$$= \lim_{a\to 0} \frac{24(-s\, e^{-\frac{1}{2}as} + \frac{as^2}{2} e^{-\frac{1}{2}as} + s\, e^{-as})}{3a^2 s^2}$$

$$= \lim_{a\to 0} \frac{\frac{d}{da} 8(- e^{-\frac{1}{2}as} + \frac{as}{2} e^{-\frac{1}{2}as} + e^{-as})}{\frac{d}{da} a^2 s}$$

$$= \lim_{a\to 0} \frac{8\left[\frac{s}{2} e^{-\frac{1}{2}as} + \frac{s}{2} e^{-\frac{1}{2}as} + \frac{as}{2}\left(\frac{-s}{2}\right) e^{-\frac{1}{2}as} - se^{-as}\right]}{2as}$$

$$= \lim_{a\to 0} \frac{\frac{d}{da}(4\, e^{-\frac{1}{2}as} - as\, e^{-\frac{1}{2}as} - 4\, e^{-as})}{\frac{d}{da} a}$$

$$= \lim_{a\to 0} \frac{-2s\, e^{-\frac{1}{2}as} - s\, e^{-\frac{1}{2}as} + as \frac{s}{2} e^{-\frac{1}{2}as} + 4s\, e^{-as}}{1}$$

$$= -2s - s + 4s = s$$

B-2-9.

$$f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

$$= \lim_{s\to 0} \frac{s\, 5(s + 2)}{s(s + 1)} = \frac{5 \times 2}{1} = 10$$

B-2-10.

$$f(0+) = \lim_{t\to 0+} f(t) = \lim_{s\to\infty} \frac{s\, 2(s + 2)}{s(s + 1)(s + 3)} = 0$$

B-2-11. Define

$$y = \dot{x}$$

Then

$$y(0+) = \dot{x}(0+)$$

The initial value of y can be obtained by use of the initial value theorem as follows:

$$y(0+) = \lim_{s \to \infty} sY(s)$$

Since

$$Y(s) = \mathcal{L}_+[y(t)] = \mathcal{L}_+[\dot{x}(t)] = sX(s) - x(0+)$$

we obtain

$$y(0+) = \lim_{s \to \infty} sY(s) = \lim_{s \to \infty} s[sX(s) - x(0+)]$$

$$= \lim_{s \to \infty} [s^2X(s) - sx(0+)]$$

B-2-12. Note that

$$\mathcal{L}\left[\frac{d}{dt} f(t)\right] = sF(s) - f(0)$$

$$\mathcal{L}\left[\frac{d^2}{dt^2} f(t)\right] = s^2F(s) - sf(0) - \dot{f}(0)$$

Define

$$g(t) = \frac{d^2}{dt^2} f(t)$$

Then

$$\mathcal{L}\left[\frac{d^3}{dt^3} f(t)\right] = \mathcal{L}\left[\frac{d}{dt} g(t)\right] = sG(s) - g(0)$$

$$= s[s^2F(s) - sf(0) - \dot{f}(0)] - \ddot{f}(0)$$

$$= s^3F(s) - s^2f(0) - s\dot{f}(0) - \ddot{f}(0)$$

B-2-13.

(a)

$$F_1(s) = \frac{s + 5}{(s + 1)(s + 3)} = \frac{a_1}{s + 1} + \frac{a_2}{s + 3}$$

where

$$a_1 = \left.\frac{s + 5}{s + 3}\right|_{s = -1} = \frac{4}{2} = 2$$

$$a_2 = \left.\frac{s + 5}{s + 1}\right|_{s = -3} = \frac{2}{-2} = -1$$

$F_1(s)$ can thus be written as

$$F_1(s) = \frac{2}{s+1} - \frac{1}{s+3}$$

and the inverse Laplace transform of $F_1(s)$ is

$$f_1(t) = 2\,e^{-t} - e^{-3t}$$

(b)

$$F_2(s) = \frac{3(s+4)}{s(s+1)(s+2)} = \frac{a_1}{s} + \frac{a_2}{s+1} + \frac{a_3}{s+2}$$

where

$$a_1 = \left.\frac{3(s+4)}{(s+1)(s+2)}\right|_{s=0} = \frac{3 \times 4}{2} = 6$$

$$a_2 = \left.\frac{3(s+4)}{s(s+2)}\right|_{s=-1} = \frac{3 \times 3}{(-1) \times 1} = -9$$

$$a_3 = \left.\frac{3(s+4)}{s(s+1)}\right|_{s=-2} = \frac{3 \times 2}{(-2)(-1)} = 3$$

$F_2(s)$ can thus be written as

$$F_2(s) = \frac{6}{s} - \frac{9}{s+1} + \frac{3}{s+2}$$

and the inverse Laplace transform of $F_2(s)$ is

$$f_2(t) = 6 - 9\,e^{-t} + 3\,e^{-2t}$$

B-2-14.

(a)

$$F_1(s) = \frac{6s+3}{s^2} = \frac{6}{s} + \frac{3}{s^2}$$

The inverse Laplace transform of $F_1(s)$ is

$$f_1(t) = 6 + 3t$$

(b)

$$F_2(s) = \frac{5s + 2}{(s + 1)(s + 2)^2} = \frac{a}{s + 1} + \frac{b_2}{(s + 2)^2} + \frac{b_1}{s + 2}$$

where

$$a = \left.\frac{5s + 2}{(s + 2)^2}\right|_{s = -1} = \frac{-5 + 2}{1^2} = -3$$

$$b_2 = \left.\frac{5s + 2}{s + 1}\right|_{s = -2} = \frac{-10 + 2}{-2 + 1} = 8$$

$$b_1 = \left.\frac{d}{ds}\left(\frac{5s + 2}{s + 1}\right)\right|_{s = -2} = \left.\frac{5(s + 1) - (5s + 2)}{(s + 1)^2}\right|_{s = -2}$$

$$= \frac{5(-1) - (-10 + 2)}{1^2} = 3$$

$F_2(s)$ can thus be written as

$$F_2(s) = \frac{-3}{s + 1} + \frac{8}{(s + 2)^2} + \frac{3}{s + 2}$$

and the inverse Laplace transform of $F_2(s)$ is

$$f_2(t) = -3\, e^{-t} + 8t\, e^{-2t} + 3\, e^{-2t}$$

B-2-15.

$$F(s) = \frac{2s^2 + 4s + 5}{s(s + 1)} = 2 + \frac{2}{s + 1} + \frac{5}{s(s + 1)}$$

$$= 2 + \frac{2}{s + 1} + \frac{5}{s} - \frac{5}{s + 1} = 2 - \frac{3}{s + 1} + \frac{5}{s}$$

The inverse Laplace transform of $F(s)$ is

$$f(t) = 2\, \delta(t) - 3\, e^{-t} + 5$$

B-2-16.

$$F(s) = \frac{s^2 + 2s + 4}{s^2} = 1 + \frac{2}{s} + \frac{4}{s^2}$$

The inverse Laplace transform of F(s) is

$$f(t) = \delta(t) + 2 + 4t$$

B-2-17.

$$F(s) = \frac{s}{s^2 + 2s + 10} = \frac{s + 1 - 1}{(s + 1)^2 + 3^2}$$

$$= \frac{s + 1}{(s + 1)^2 + 3^2} - \frac{3}{(s + 1)^2 + 3^2} \frac{1}{3}$$

Hence

$$f(t) = e^{-t} \cos 3t - \frac{1}{3} e^{-t} \sin 3t$$

B-2-18.

$$F(s) = \frac{s^2 + 2s + 5}{s^2(s + 1)} = \frac{a}{s^2} + \frac{b}{s} + \frac{c}{s + 1}$$

where

$$a = \left. \frac{s^2 + 2s + 5}{s + 1} \right|_{s = 0} = 5$$

$$b = \left. \frac{(2s + 2)(s + 1) - (s^2 + 2s + 5)}{(s + 1)^2} \right|_{s = 0} = \frac{2 - 5}{1} = -3$$

$$c = \left. \frac{s^2 + 2s + 5}{s^2} \right|_{s = -1} = \frac{1 - 2 + 5}{1} = 4$$

Hence

$$F(s) = \frac{5}{s^2} + \frac{-3}{s} + \frac{4}{s + 1}$$

The inverse Laplace transform of F(s) is

$$f(t) = 5t - 3 + 4 e^{-t}$$

B-2-19.

$$F(s) = \frac{2s + 10}{(s + 1)^2(s + 4)} = \frac{a}{(s + 1)^2} + \frac{b}{s + 1} + \frac{c}{s + 4}$$

where

$$a = \left. \frac{2s + 10}{s + 4} \right|_{s = -1} = \frac{-2 + 10}{3} = \frac{8}{3}$$

$$b = \left. \frac{2(s + 4) - (2s + 10)}{(s + 4)^2} \right|_{s = -1} = \frac{6 - 8}{3^2} = \frac{-2}{9}$$

$$c = \left. \frac{2s + 10}{(s + 1)^2} \right|_{s = -4} = \frac{-8 + 10}{9} = \frac{2}{9}$$

Hence

$$F(s) = \frac{8}{3} \frac{1}{(s + 1)^2} - \frac{2}{9} \frac{1}{s + 1} + \frac{2}{9} \frac{1}{s + 4}$$

The inverse Laplace transform of F(s) is

$$f(t) = \frac{8}{3} te^{-t} - \frac{2}{9} e^{-t} + \frac{2}{9} e^{-4t}$$

B-2-20.

$$F(s) = \frac{1}{s^2(s^2 + \omega^2)} = \left(\frac{1}{s^2} - \frac{1}{s^2 + \omega^2} \right) \frac{1}{\omega^2}$$

The inverse Laplace transform of F(s) is

$$f(t) = \frac{1}{\omega^2} (t - \frac{1}{\omega} \sin \omega t)$$

B-2-21.

$$F(s) = \frac{c}{s^2} (1 - e^{-as}) - \frac{b}{s} e^{-as} \qquad a > 0$$

The inverse Laplace transform of F(s) is

$$f(t) = ct - c(t - a)1(t - a) - b\,1(t - a)$$

B-2-22. $\ddot{x} + 4x = 0, \quad x(0) = 5, \quad \dot{x}(0) = 0$

The Laplace transform of the given differential equation is

$$[s^2X(s) - sx(0) - \dot{x}(0)] + 4X(s) = 0$$

Substitution of the initial conditions into this last equation gives

$$(s^2 + 4)X(s) = 5s$$

Solving for X(s), we obtain

$$X(s) = \frac{5s}{s^2 + 4}$$

The inverse Laplace transform of X(s) is

$$x(t) = 5 \cos 2t$$

This is the solution of the given differential equation.

B-2-23. $\ddot{x} + \omega_n^2 x = t, \quad x(0) = 0, \quad \dot{x}(0) = 0$

The Laplace transform of this differential equation is

$$s^2X(s) + \omega_n^2 X(s) = \frac{1}{s^2}$$

Solving this equation for X(s), we obtain

$$X(s) = \frac{1}{s^2(s^2 + \omega_n^2)} = \left(\frac{1}{s^2} - \frac{1}{s^2 + \omega_n^2}\right)\frac{1}{\omega_n^2}$$

The inverse Laplace transform of X(s) is

$$x(t) = \frac{1}{\omega_n^2}\left(t - \frac{1}{\omega_n} \sin \omega_n t\right)$$

This is the solution of the given differential equation.

B-2-24. $2\ddot{x} + 2\dot{x} + x = 1, \quad x(0) = 0, \quad \dot{x}(0) = 2$

The Laplace transform of this differential equation is

$$2[s^2X(s) - sx(0) - \dot{x}(0)] + 2[sX(s) - x(0)] + X(s) = \frac{1}{s}$$

Substitution of the initial conditions into this equation gives

$$2[s^2X(s) - 2] + 2[sX(s)] + X(s) = \frac{1}{s}$$

or

$$(2s^2 + 2s + 1)X(s) = 4 + \frac{1}{s}$$

Solving this last equation for X(s), we get

$$X(s) = \frac{4s + 1}{s(2s^2 + 2s + 1)}$$

$$= \frac{4}{2s^2 + 2s + 1} + \frac{1}{s(2s^2 + 2s + 1)}$$

$$= \frac{2}{(s + 0.5)^2 + 0.25} + \frac{0.5}{s[(s + 0.5)^2 + 0.25]}$$

$$= \frac{4 \times 0.5}{(s + 0.5)^2 + 0.5^2} + \frac{1}{s} - \frac{(s + 0.5) + 0.5}{(s + 0.5)^2 + 0.5^2}$$

The inverse Laplace transform of X(s) gives

$$x(t) = 4\, e^{-0.5t} \sin 0.5t + 1 - e^{-0.5t} \cos 0.5t - e^{-0.5t} \sin 0.5t$$

$$= 1 + 3\, e^{-0.5t} \sin 0.5t - e^{-0.5t} \cos 0.5t$$

B-2-25. $\ddot{x} + x = \sin 3t, \qquad x(0) = 0, \qquad \dot{x}(0) = 0$

The Laplace transform of this differential equation is

$$s^2X(s) + X(s) = \frac{3}{s^2 + 3^2}$$

Solving this equation for X(s), we get

$$X(s) = \frac{3}{(s^2 + 1)(s^2 + 9)} = \frac{3}{8}\frac{1}{s^2 + 1} - \frac{1}{8}\frac{3}{s^2 + 9}$$

The inverse Laplace transform of X(s) gives

$$x(t) = \frac{3}{8} \sin t - \frac{1}{8} \sin 3t$$
