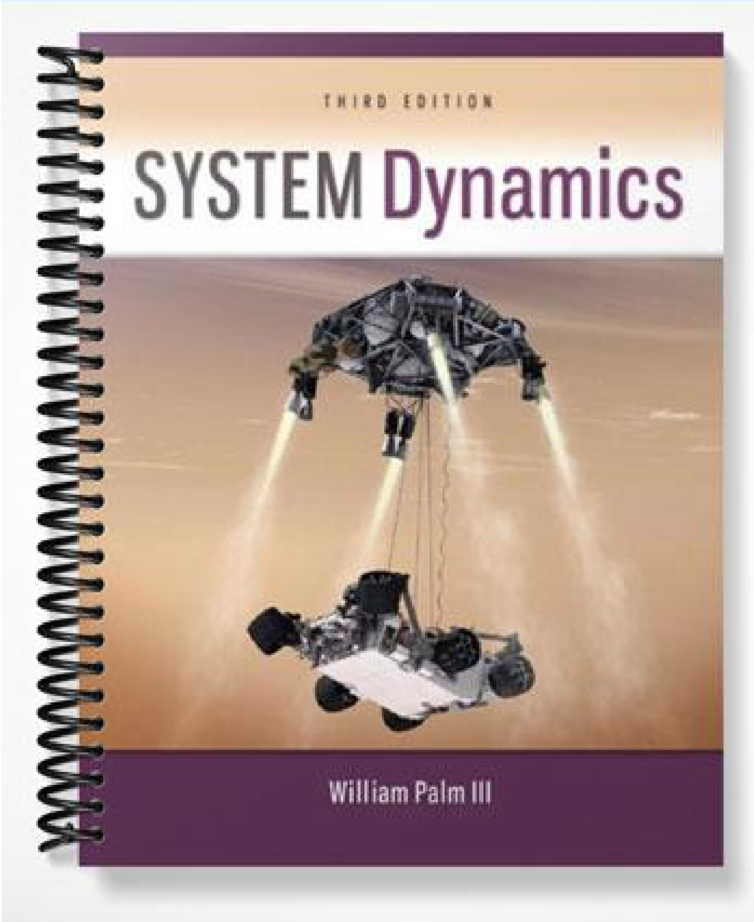
SOLUTIONS MANUAL



Solutions Manual©

to accompany

System Dynamics, Third Edition

by

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Solutions to Problems in Chapter Two

2.1 a) Nonlinear because of the $y\ddot{y}$ term. b) Nonlinear because of the $\sin y$ term. c) Nonlinear because of the $\sqrt{y}$ term. d) Variable coefficient, but Linear. e) Nonlinear because of the $\sin y$ term. f) Variable coefficient, but linear.

2.2 a)

$$4\int_2^x dx = 3\int_0^t t\,dt$$
$$x(t) = 2 + \frac{3}{8}t^2$$

b)

$$5\int_3^x dx = 2\int_0^t e^{-4t}\,dt$$
$$x(t) = 3.1 - 0.1e^{-4t}$$

c) Let $v = \dot{x}$.

$$3\int_7^v dv = 5\int_0^t t\,dt$$
$$v(t) = \frac{dx}{dt} = 7 + \frac{5}{6}t^2$$
$$\int_2^x dx = \int_0^t \left(7 + \frac{5}{6}t^2\right)\,dt$$
$$x(t) = 2 + 7t + \frac{5}{18}t^3$$

d) Let $v = \dot{x}$.

$$4\int_2^v dv = 7\int_0^t e^{-2t}\,dt$$
$$v(t) = \frac{23}{8} - \frac{7}{8}e^{-2t}$$
$$\int_4^x dx = \int_0^t \left(\frac{23}{8} - \frac{7}{8}e^{-2t}\right)\,dt$$
$$x(t) = \frac{57}{16} + \frac{23}{8}t + \frac{7}{16}e^{-2t}$$

e) $\dot{x} = C_1$, but $\ddot{x}(0) = 5$, so $C_1 = 5$. $x = 5t + C_2$, but $x(0) = 2$, so $C_2 = 2$. Thus $x = 5t + 2$.

2.3 a)

$$\int_3^x \frac{dx}{25-5x^2} = \int_0^t dt = t$$

$$\int_3^x \frac{dx}{25-5x^2} = \frac{\sqrt{5}}{25}\left[\operatorname{arctanh}\left(\frac{\sqrt{5}x}{5}\right) - \operatorname{arctanh}\left(\frac{3\sqrt{5}}{5}\right)\right] = t$$

Let

$$C = \operatorname{arctanh}\left(\frac{3\sqrt{5}}{5}\right)$$

Solve for x to obtain

$$x = \sqrt{5}\tanh(5\sqrt{5}t + C)$$

b)

$$\int_{10}^x \frac{dx}{36+4x^2} = \int_0^t dt = t$$

$$\frac{1}{12}\tan^{-1}\frac{x}{3}\bigg|_{10}^{x} = t$$

$$x(t) = 3\,\tan(12t + C) \qquad C = \tan^{-1}\frac{10}{3}$$

c)

$$\int_4^x \frac{x\,dx}{5x+25} = \int_0^t dt$$

$$\frac{x}{5} - \ln(x+5)\bigg|_4^x = \frac{x}{5} - \ln(x+5) - \frac{4}{5} + \ln 9 = t$$

$$x - 5\,\ln(x+5) = 5t + 4 - 5\,\ln 9$$

So a closed form solution does not exist.

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Problem 2.3 continued:

d)

$$\int_5^x \frac{dx}{x} = -2\int_0^t e^{-4t}\,dt$$

$$\ln x|_5^x = \frac{1}{2}\left(e^{-4t} - 1\right)$$

$$\ln \frac{x}{5} = \frac{1}{2}\left(e^{-4t} - 1\right)$$

$$x(t) = \frac{5}{\sqrt{e}} e^{\frac{1}{2}e^{-4t}}$$

2.4 From the transform definition, we have

$$\mathcal{L}[mt] = \lim_{T\to\infty}\left[\int_0^T mte^{-st}dt\right] = m\lim_{T\to\infty}\left[\int_0^T te^{-st}dt\right]$$

The method of *integration by parts* states that

$$\int_0^T u\,dv = uv|_0^T - \int_0^T v\,du$$

Choosing $u = t$ and $dv = e^{-st}dt$, we have $du = dt$, $v = -e^{-st}/s$, and

$$\mathcal{L}[mt] = m\lim_{T\to\infty}\left[\int_0^T te^{-st}dt\right] = m\lim_{T\to\infty}\left[t\frac{e^{-st}}{-s}\bigg|_0^T - \int_0^T \frac{e^{-st}}{-s}\,dt\right]$$

$$= m\lim_{T\to\infty}\left[t\frac{e^{-st}}{-s}\bigg|_0^T - \frac{e^{-st}}{(-s)^2}\bigg|_0^T\right] = m\lim_{T\to\infty}\left[\frac{Te^{-sT}}{-s} - 0 - \frac{e^{-sT}}{(-s)^2} + \frac{e^0}{(-s)^2}\right]$$

$$= \frac{m}{s^2}$$

because, if we choose the real part of s to be positive, then

$$\lim_{T\to\infty} Te^{-sT} = 0$$

2.5 From the transform definition, we have

$$\mathcal{L}[t^2] = \lim_{T\to\infty} \left[\int_0^T t^2 e^{-st}\,dt\right]$$

The method of *integration by parts* states that

$$\int_0^T u\,dv = uv|_0^T - \int_0^T v\,du$$

Choosing $u = t^2$ and $dv = e^{-st}dt$, we have $du = 2t\,dt$, $v = -e^{-st}/s$, and

$$\mathcal{L}[t^2] = \lim_{T\to\infty} \left[\int_0^T t^2 e^{-st}\,dt\right] = \lim_{T\to\infty} \left[t^2 \frac{e^{-st}}{-s}\bigg|_0^T - \int_0^T \frac{e^{-st}}{-s} 2t\,dt \right]$$

$$= \lim_{T\to\infty} \left[-T^2 \frac{e^{-st}}{s} + \frac{2}{s}\int_0^T t e^{-st}\,dt \right] = \lim_{T\to\infty} \left[-T^2 \frac{e^{-st}}{s} \right] + \frac{2}{s}\left(\frac{1}{s^2}\right)$$

$$= \frac{2}{s^3}$$

because, if we choose the real part of s to be positive, then,

$$\lim_{T\to\infty} T^2 e^{-sT} = 0$$

2.6 a)

$$X(s) = \frac{10}{s} + \frac{2}{s^3}$$

b)

$$X(s) = \frac{6}{(s+5)^2} + \frac{1}{s+3}$$

c) From Property 8,

$$X(s) = -\frac{dY(s)}{ds}$$

where $y(t) = e^{-3t}\sin 5t$. Thus

$$Y(s) = \frac{5}{(s+3)^2 + 5^2} = \frac{5}{s^2 + 6s + 34}$$

$$\frac{dY(s)}{ds} = -\frac{10s + 30}{(s^2 + 6s + 34)^2}$$

Thus

$$X(s) = \frac{10s + 30}{(s^2 + 6s + 34)^2}$$

d) $X(s) = e^{-5s}G(s)$, where $g(t) = t$. Thus $G(s) = 1/s^2$ and

$$X(s) = \frac{e^{-5s}}{s^2}$$

2.7

$$f(t) = 5u_s(t) - 7u_s(t-6) + 2u_s(t-14)$$

Thus

$$F(s) = \frac{5}{s} - 7\frac{e^{-6s}}{s} + 2\frac{e^{-14s}}{s}$$

2.8 a)

$$2\ \sin\ 3t$$

b)

$$4\ \cos\ 2t + \frac{5}{2}\sin\ 2t$$

c)

$$2\mathrm{e}^{-2\mathrm{t}}\,\sin\ 3\mathrm{t}$$

d)

$$\frac{5}{3} - \frac{5\,\mathrm{e}^{-3\,\mathrm{t}}}{3}$$

e)

$$\frac{5\,\mathrm{e}^{-3\,\mathrm{t}}}{2} - \frac{5\,\mathrm{e}^{-7\,\mathrm{t}}}{2}$$

f)

$$\frac{\mathrm{e}^{-3\,\mathrm{t}}}{2} + \frac{3\,\mathrm{e}^{-7\,\mathrm{t}}}{2}$$

2.9 a)

$$5\,\cos(3\,t)$$

b)

$$e^{3\,t} - e^{-3\,t}$$

c)

$$5 - 15\,t\,e^{-3\,t} - 5\,e^{-3\,t}$$

d)

$$\frac{2}{13} - \frac{2\,e^{-2\,t}\,\left(\cos\,3t + \frac{2\,\sin\,3\,t}{3}\right)}{13}$$

e)

$$5 - 5\,\cos\,2t$$

f)

$$5\,t\,\sin\,2t$$

2.10 a)

$$x(0+) = \lim_{s\to\infty} s\frac{5}{3s+7} = \frac{5}{3}$$

$$x(\infty) = \lim_{s\to 0} s\frac{5}{3s+7} = 0$$

b)

$$x(0+) = \lim_{s\to\infty} s\frac{10}{3s^2+7s+4} = 0$$

$$x(\infty) = \lim_{s\to 0} s\frac{10}{3s^2+7s+4} = 0$$

2.11 a)

$$X(s) = \frac{3}{2}\left(\frac{1}{s} - \frac{1}{s+4}\right)$$
$$x(t) = \frac{3}{2}\left(1 - e^{-4t}\right)$$

b)

$$X(s) = \frac{5}{3}\frac{1}{s} + \frac{31}{3}\frac{1}{s+3}$$
$$x(t) = \frac{5}{3} + \frac{31}{3}e^{-3t}$$

c)

$$X(s) = -\frac{1}{3}\frac{1}{s+2} + \frac{13}{3}\frac{1}{s+5}$$
$$x(t) = -\frac{1}{3}e^{-2t} + \frac{13}{3}e^{-5t}$$

d)

$$X(s) = \frac{5/2}{s^2(s+4)} = \frac{5}{8}\frac{1}{s^2} - \frac{5}{32}\frac{1}{s} + \frac{5}{32}\frac{1}{s+4}$$
$$x(t) = \frac{5}{8}t - \frac{5}{32} + \frac{5}{32}e^{-4t}$$

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Problem 2.11 continued:

e)

$$X(s) = \frac{2}{5}\frac{1}{s^2} + \frac{13}{25}\frac{1}{s} - \frac{13}{25}\frac{1}{s+5}$$

$$x(t) = \frac{2}{5}t + \frac{13}{25} - \frac{13}{25}e^{-5t}$$

f)

$$X(s) = -\frac{31}{4}\frac{1}{(s+3)^2} + \frac{79}{16}\frac{1}{s+3} - \frac{79}{16}\frac{1}{s+7}$$

$$x(t) = -\frac{31}{4}te^{-3t} + \frac{79}{16}e^{-3t} - \frac{79}{16}e^{-7t}$$

2.12 a)

$$X(s) = \frac{7s+2}{(s+3)^2+5^2} = C_1 \frac{5}{(s+3)^2+5^2} + C_2 \frac{s+3}{(s+3)^2+5^2}$$

or

$$X(s) = -\frac{19}{5}\frac{5}{(s+3)^2+5^2} + 7\frac{s+3}{(s+3)^2+5^2}$$

$$x(t) = -\frac{19}{5}e^{-3t}\sin 5t + 7e^{-3t}\cos 5t$$

b)

$$X(s) = \frac{4s+3}{s[(s+3)^2+5^2]} = \frac{C_1}{s} + C_2 \frac{5}{(s+3)^2+5^2} + C_3 \frac{s+3}{(s+3)^2+5^2}$$

or

$$X(s) = \frac{3}{34}\frac{1}{s} + \frac{127}{170}\frac{5}{(s+3)^2+5^2} - \frac{3}{34}\frac{s+3}{(s+3)^2+5^2}$$

$$x(t) = \frac{3}{34} + \frac{127}{170}e^{-3t}\sin 5t - \frac{3}{34}e^{-3t}\cos 5t$$

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Problem 2.12 continued:

c)

$$\begin{aligned} X(s) &= \frac{4s+9}{[(s+3)^2+5^2][(s+2)^2+4^2]} \\ &= C_1\frac{5}{(s+3)^2+5^2} + C_2\frac{s+3}{(s+3)^2+5^2} + C_3\frac{4}{(s+2)^2+4^2} + C_4\frac{s+2}{(s+2)^2+4^2} \end{aligned}$$

or

$$\begin{aligned} X(s) &= -\frac{44}{205}\frac{5}{(s+3)^2+5^2} - \frac{19}{82}\frac{s+3}{(s+3)^2+5^2} \\ &+ \frac{69}{328}\frac{4}{(s+2)^2+4^2} + \frac{19}{82}\frac{s+2}{(s+2)^2+4^2} \end{aligned}$$

$$x(t) = -\frac{44}{205}e^{-3t}\sin 5t - \frac{19}{82}e^{-3t}\cos 5t + \frac{69}{328}e^{-2t}\sin 4t + \frac{19}{82}e^{-2t}\cos 4t$$

d)

$$X(s) = 2.625\frac{1}{s+2} - 18.75\frac{1}{s+4} + 21.125\frac{1}{s+6}$$
$$x(t) = 2.625e^{-2t} - 18.75e^{-4t} + 21.125e^{-6t}$$

2.13 a) $\dot{x} = 7t/5$

$$\int_3^x dx = \frac{7}{5}\int_0^t t\,dt$$

$$x(t) = \frac{7}{10}t^2 + 3$$

b) $\dot{x} = 3e^{-5t}/4$

$$\int_4^x dx = \frac{3}{4}\int_0^t e^{-5t}\,dt$$

$$x(t) = \frac{3}{20}\left(1 - e^{-5t}\right) + 4$$

c) $\ddot{x} = 4t/7$

$$\dot{x}(t) - \dot{x}(0) = \frac{4}{7}\int_0^t t\,dt$$

$$\dot{x}(t) = \frac{4}{14}t^2 + 5$$

$$\int_3^x dx = \int_0^t \left(\frac{4}{14}t^2 + 5\right) dt$$

$$x(t) = \frac{4}{42}t^3 + 5t + 3$$

d) $\ddot{x} = 8e^{-4t}/3$

$$\dot{x}(t) - \dot{x}(0) = \frac{8}{3}\int_0^t e^{-4t}\,dt$$

$$\dot{x}(t) = \frac{17}{3} - \frac{8}{12}e^{-4t}$$

$$\int_3^x dx = \int_0^t \left(\frac{17}{3} - \frac{8}{12}e^{-4t}\right) dt$$

$$x(t) = \frac{17}{3}t + \frac{1}{6}e^{-4t} + \frac{17}{6}$$

2.14 a) The root is $-7/5$ and the form is $x(t) = Ce^{-7t/5}$. With $x(0) = 4$, $C = 4$ and $x(t) = 4e^{-7t/5}$

b) The root is $-7/5$ and the form is $x(t) = C_1 e^{-7t/5} + C_2$. At steady state, $x = 15/7 = C_2$. With $x(0) = 0$, $C_1 = -15/7$. Thus

$$x(t) = \frac{15}{7}\left(1 - e^{-7t/5}\right)$$

c) The root is $-7/5$ and the form is $x(t) = C_1 e^{-7t/5} + C_2$. At steady state, $x = 15/7 = C_2$. With $x(0) = 4$, $C_1 = 13/7$. Thus

$$x(t) = \frac{13}{7}\left(1 + e^{-7t/5}\right)$$

d)

$$sX(s) - x(0) + 7X(s) = \frac{4}{s^2}$$

$$X(s) = \frac{5s^2 + 4}{s^2(s+7)} = \frac{4}{7s^2} - \frac{4}{49} + \frac{249}{49}e^{-7t}$$

$$x(t) = \frac{4}{7}t - \frac{4}{49} + \frac{249}{49}e^{-7t}$$

2.15 a) The roots are -7 and -3. The form is

$$x(t) = C_1 e^{-7t} + C_2 e^{-3t}$$

Evaluating C_1 and C_2 for the initial conditions gives

$$x(t) = -\frac{9}{4} e^{-7t} + \frac{25}{4} e^{-3t}$$

b) The roots are -7 and -7. The form is

$$x(t) = C_1 e^{-7t} + C_2 t e^{-7t}$$

Evaluating C_1 and C_2 for the initial conditions gives

$$x(t) = e^{-7t} + 10 t e^{-7t}$$

c) The roots are $-7 \pm 3j$. The form is

$$x(t) = C_1 e^{-7t} \sin 3t + C_2 e^{-7t} \cos 3t$$

Evaluating C_1 and C_2 for the initial conditions gives

$$x(t) = \frac{20}{3} e^{-7t} \sin 3t + 4 e^{-7t} \cos 3t$$

2.16 a)

$$x = 6\,\mathrm{e}^{-2\,\mathrm{t}} - 3\,\mathrm{e}^{-5\,\mathrm{t}} + 2$$

b)

$$x = \frac{18\,\mathrm{e}^{-2\,\mathrm{t}}}{5} + \frac{76\,t\,\mathrm{e}^{-2\,\mathrm{t}}}{5} + \frac{7}{5}$$

c)

$$x = 3\,\sin\,4t - 4\,\cos\,4t + 9$$

d)

$$x = 3\,\cos\,5t\,\mathrm{e}^{-3\,\mathrm{t}} + \frac{16\,\sin\,5\mathrm{t}\,\mathrm{e}^{-3\mathrm{t}}}{5} + 2$$

2.17 a) The roots are -3 and -7. The form is

$$x(t) = C_1 e^{-3t} + C_2 e^{-7t} + C_3$$

At steady state, $x = 5/63$ so $C_3 = 5/63$. Evaluating C_1 and C_2 for the initial conditions gives

$$x(t) = -\frac{5}{36}e^{-3t} + \frac{5}{84}e^{-7t} + \frac{5}{63}$$

b) The roots are -7 and -7. The form is

$$x(t) = C_1 e^{-7t} + C_2 t e^{-7t} + C_3$$

At steady state, $x = 98/49 = 2$ so $C_3 = 2$. Evaluating C_1 and C_2 for the initial conditions gives

$$x(t) = -2e^{-7t} - 14te^{-7t} + 2$$

c) The roots are $-7 \pm 3j$. The form is

$$x(t) = C_1 e^{-7t} \sin 3t + C_2 e^{-7t} \cos 3t + C_3$$

At steady state, $x = 174/58 = 3$ so $C_3 = 3$. Evaluating C_1 and C_2 for the initial conditions gives

$$x(t) = -7e^{-7t} \sin 3t - 3e^{-7t} \cos 3t + 3$$

2.18 a)

$$X(s) = \frac{60}{s^2 + 8s + 12}$$

$$x = 15\,\mathrm{e}^{-2\mathrm{t}} - 15\mathrm{e}^{-6\mathrm{t}}$$

b)

$$X(s) = \frac{288}{s^2 + 12s + 144}$$

$$x = 16\sqrt{3}\mathrm{e}^{-6\mathrm{t}}\ \sin\ 6\sqrt{3}\mathrm{t}$$

c)

$$X(s) = \frac{147}{s^2 + 49}$$
$$x = 21\ \sin\ 7t$$

d)

$$X(s) = \frac{170}{s^2 + 14s + 85}$$

$$x = \frac{85\,\mathrm{e}^{-7\mathrm{t}}\,\sin\ 6\mathrm{t}}{3}$$

2.19 a)

$$\frac{6}{s(s+5)} = \frac{6}{5s} - \frac{6}{5}\frac{1}{s+5}$$

$$x(t) = \frac{6}{5}\left(1 - e^{-5t}\right)$$

b)

$$\frac{4}{s+3)(s+8)} = \frac{4}{5}\frac{1}{s+3} - \frac{4}{5}\frac{1}{s+8}$$

$$x(t) = \frac{4}{5}\left(e^{-3t} - e^{-8t}\right)$$

c)

$$\frac{8s+5}{2s^2+20s+48} = \frac{1}{2}\frac{8s+5}{(s+4)(s+6)} = -\frac{27}{4}\frac{1}{s+4} + \frac{43}{4}\frac{1}{s+6}$$

$$x(t) = -\frac{27}{4}e^{-4t} + \frac{43}{4}e^{-6t}$$

d) The roots are $s = -4 \pm 10j$.

$$\begin{aligned}\frac{4s+13}{s^2+8s+116} + \frac{4s+13}{(s+4)^2+10^2} &= C_1\frac{10}{(s+4)^2+10^2} + C_2\frac{s+4}{(s+4)^2+10^2} \\ &= -\frac{3}{10}\frac{10}{(s+4)^2+10^2} + 4\frac{s+4}{(s+4)^2+10^2}\end{aligned}$$

$$x(t) = -\frac{3}{10}e^{-4t}\sin 10t + 4e^{-4t}\cos 10t$$

2.20 a)

$$\frac{3s+2}{s^2(s+10)} = \frac{1}{5}\frac{1}{s^2} + \frac{7}{25}\frac{1}{s} - \frac{7}{25}\frac{1}{s+10}$$

$$x(t) = \frac{1}{5}t + \frac{7}{25}\left(1 - e^{-10t}\right)$$

b)

$$\frac{5}{(s+4)^2(s+1)} = -\frac{15}{9}\frac{1}{(s+4)^2} - \frac{5}{9}\frac{1}{s+4} + \frac{5}{9}\frac{1}{s+1}$$

$$x(t) = -\frac{15}{9}te^{-4t} - \frac{5}{9}e^{-4t} + \frac{5}{9}e^{-t}$$

c)

$$\frac{s^2+3s+5}{s^3(s+2)} = \frac{5}{2}\frac{1}{s^3} + \frac{1}{4}\frac{1}{s^2} + \frac{3}{8}\frac{1}{s} - \frac{3}{8}\frac{1}{s+2}$$

$$x(t) = \frac{5}{4}t^2 + \frac{1}{4}t + \frac{3}{8} - \frac{3}{8}e^{-2t}$$

d)

$$\frac{s^3+s+6}{s^4(s+2)} = 3\frac{1}{s^4} - \frac{1}{s^3} + \frac{1}{2}\frac{1}{s^2} + \frac{1}{4}\frac{1}{s} - \frac{1}{4}\frac{1}{s+2}$$

$$x(t) = \frac{1}{2}t^3 - \frac{1}{2}t^2 + \frac{1}{2}t + \frac{1}{4} - \frac{1}{4}e^{-2t}$$

2.21 a)

$$5[sX(s) - 2] + 3X(s) = \frac{10}{s} + \frac{2}{s^3}$$

$$X(s) = \frac{10s^3 + 10s^2 + 2}{5s^3(s+3)} = \frac{2s^3 + 2s^2 + 2/5}{s^3(s+3/5)} = \frac{2}{3}\frac{1}{s^3} - \frac{10}{9}\frac{1}{s^2} + \frac{140}{9}\frac{1}{s} - \frac{86}{27}\frac{1}{s+3/5}$$

$$x(t) = \frac{1}{3}t^2 - \frac{10}{9}t + \frac{140}{27} - \frac{86}{27}e^{-3t/5}$$

b)

$$4[sX(s) - 5] + 7X(s) = \frac{6}{(s+5)^2} + \frac{1}{s+3}$$

$$\begin{aligned} X(s) &= \frac{1}{4}\frac{20s^3 + 261s^2 + 1116s + 1543}{(s+5)^2(s+7/4)(s+3)} \\ &= \frac{1}{4}\left[-\frac{24}{13}\frac{1}{(s+5)^2} - \frac{96}{169}\frac{1}{s+5} + \frac{18056}{845}\frac{1}{s+7/4} - \frac{4}{5}\frac{1}{s+3}\right] \end{aligned}$$

$$x(t) = -\frac{6}{13}te^{-5t} - \frac{24}{169}e^{-5t} + \frac{4514}{845}e^{-7t/4} - \frac{1}{5}e^{-3t}$$

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Problem 2.21 continued:

c) This simple-looking problem actually requires quite a lot of algebra to find the solution, and thus it serves as a good motivating example of the convenience of using MATLAB. The algebraic complexity is due to a pair of repeated complex roots.

First obtain the transform of the forcing function. Let $f(t) = te^{-3t} \sin 5t$. From Property 8,

$$F(s) = -\frac{dY(s)}{ds}$$

where $y(t) = e^{-3t} \sin 5t$. Thus

$$Y(s) = \frac{5}{(s+3)^2 + 5^2} = \frac{5}{s^2 + 6s + 34}$$

$$\frac{dY(s)}{ds} = -\frac{10s + 30}{(s^2 + 6s + 34)^2}$$

Thus

$$F(s) = \frac{10s + 30}{(s^2 + 6s + 34)^2} \qquad (1)$$

(continued on the next page)

Problem 2.21 continued:

Using the same technique, we find that the transform of $te^{-3t}\cos 5t$ is

$$\frac{2s^2+12s+18}{(s^2+6s+34)^2} - \frac{1}{s^2+6s+34} \qquad (2)$$

This fact will be useful in finding the forced response.

From the differential equation,

$$4[s^2X(s) - 10s + 2] + 3X(s) = F(s) = \frac{10s+30}{(s^2+6s+34)^2}$$

Solve for $X(s)$.

$$X(s) = \frac{40s-8}{4s^2+3} + \frac{10s+30}{[(s+3)^2+25]^2(4s^2+3)}$$

The free response is given by the first fraction, and is

$$x_{\text{free}}(t) = -\frac{4}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t + 10\cos\frac{\sqrt{3}}{2}t = -2.3094\sin 0.866t + 10\cos 0.866t \qquad (3)$$

The forced response is given by the second fraction, which can be expressed as

$$\frac{2.5s+7.5}{[(s+3)^2+25]^2(s^2+3/4)} \qquad (4)$$

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Problem 2.21 continued:

The roots of this are $s = \pm j\sqrt{3}/2$ and the repeated pair $s = -3 \pm 5j$. Thus, referring to (1), (2), and (3), we see that the form of the forced response will be

$$\begin{aligned} x_{\text{forced}}(t) &= C_1 t e^{-3t} \sin 5t + C_2 t e^{-3t} \cos 5t \\ &+ C_3 e^{-3t} \sin 5t + C_4 e^{-3t} \cos 5t \\ &+ C_5 \sin \frac{\sqrt{3}}{2} t + C_6 \cos \frac{\sqrt{3}}{2} t \qquad (5) \end{aligned}$$

The forced response can be obtained several ways. 1) You can substitute the form (5) into the differential equation and use the initial conditions to obtain equations for the C_i coefficients. 2) You can use (1) and (2) to create a partial fraction expansion of (4) in terms of the complex factors. 3) You can perform an expansion in terms of the six roots, of the form

$$\begin{aligned} \frac{A_1}{(s+3+5j)^2} &+ \frac{A_2}{s+3+5j} + \frac{A_3}{(s+3-5j)^2} + \frac{A_4}{s+3-5j} \\ &+ \frac{\sqrt{3}A_5/2}{s^2+3/4} + \frac{A_6 s}{s^2+3/4} \end{aligned}$$

4) You can use the MATLAB `residue` function.

The solution for the forced response is

$$\begin{aligned} x_{\text{forced}}(t) &= -0.0034 t e^{-3t} \sin 5t + 0.0066 t e^{-3t} \cos 5t \\ &- 0.0026 e^{-3t} \sin 5t + 2.308 \times 10^{-4} e^{-3t} \cos 5t \\ &+ 0.00796 \sin 0.866t - 2.308 \times 10^{-4} \cos 0.866t \end{aligned}$$

The initial condition $\dot{x}(0) = 0$ is not exactly satisfied by this expression because of the limited number of digits used to display it.

2.22 The denominator roots are $s = -3$ and $s = -5$, which are distinct. Factor the denominator so that the highest coefficients of s in each factor are unity:

$$X(s) = \frac{7s+4}{2s^2+16s+30} = \frac{1}{2}\left[\frac{7s+4}{(s+3)(s+5)}\right]$$

The partial-fraction expansion has the form

$$X(s) = \frac{1}{2}\left[\frac{7s+4}{(s+3)(s+5)}\right] = \frac{C_1}{s+3} + \frac{C_2}{s+5}$$

Using the coefficient formula, we obtain

$$C_1 = \lim_{s \to -3}\left[(s+3)\frac{7s+4}{2(s+3)(s+5)}\right] = \lim_{s \to -3}\left[\frac{7s+4}{2(s+5)}\right] = -\frac{17}{4}$$

$$C_2 = \lim_{s \to -5}\left[(s+5)\frac{7s+4}{2(s+3)(s+5)}\right] = \lim_{s \to -5}\left[\frac{7s+4}{2(s+3)}\right] = \frac{31}{4}$$

(continued on the next page)

Problem 2.22 continued:

Using the LCD method we have

$$\frac{1}{2}\frac{7s+4}{(s+3)(s+5)} = \frac{C_1}{s+3} + \frac{C_2}{s+5} = \frac{C_1(s+5)+C_2(s+3)}{(s+3)(s+5)}$$

$$= \frac{(C_1+C_2)s+5C_1+3C_2}{(s+3)(s+5)}$$

Comparing numerators, we see that $C_1 + C_2 = 7/2$ and $5C_1 + 3C_2 = 4/2 = 2$, which give $C_1 = -17/4$ and $C_2 = 31/4$.

The inverse transform is

$$x(t) = C_1 e^{-3t} + C_2 e^{-5t} = -\frac{17}{4}e^{-3t} + \frac{31}{4}e^{-5t}$$

In this example the LCD method requires more algebra, including the solution of two equations for the two unknowns C_1 and C_2.

2.23 a) The roots are -3 and -5. The form of the free response is

$$x(t) = A_1 e^{-3t} + A_2 e^{-5t}$$

Evaluating this with the given initial conditions gives

$$x(t) = 27e^{-3t} - 17e^{-5t}$$

The steady-state solution is $x_{ss} = 30/15 = 2$. Thus the form of the forced response is

$$x(t) = 2 + B_1 e^{-3t} + B_2 e^{-5t}$$

Evaluating this with zero initial conditions gives

$$x(t) = 2 - 5e^{-3t} + 3e^{-5t}$$

The total response is the sum of the free and the forced response. It is

$$x(t) = 2 + 22e^{-3t} - 14e^{-5t}$$

The transient response consists of the two exponential terms.

(continued on the next page)

Problem 2.23 continued:

b) The roots are -5 and -5. The form of the free response is

$$x(t) = A_1 e^{-5t} + A_2 t e^{-5t}$$

Evaluating this with the given initial conditions gives

$$x(t) = e^{-5t} + 9te^{-5t}$$

The steady-state solution is $x_{ss} = 75/25 = 3$. Thus the form of the forced response is

$$x(t) = 3 + B_1 e^{-5t} + B_2 t e^{-5t}$$

Evaluating this with zero initial conditions gives

$$x(t) = 3 - 3e^{-5t} - 15te^{-5t}$$

The total response is the sum of the free and the forced response. It is

$$x(t) = 3 - 2e^{-5t} - 6te^{-5t}$$

The transient response consists of the two exponential terms.

(continued on the next page)

Problem 2.23 continued:

c) The roots are $\pm 5j$. The form of the free response is

$$x(t) = A_1 \sin 5t + A_2 \cos 5t$$

Evaluating this with the given initial conditions gives

$$x(t) = \frac{4}{5} \sin 5t + 10 \cos 5t$$

The form of the forced response is

$$x(t) = B_1 + B_2 \sin 5t + B_3 \cos 5t$$

Thus the entire forced response is the steady-state forced response. There is no transient forced response. Evaluating this function with zero initial conditions shows that $B_2 = 0$ and $B_3 = -B_1$. Thus

$$x(t) = B_1 - B_1 \cos 5t$$

Substituting this into the differential equation shows that $B_1 = 4$ and the forced response is

$$x(t) = 4 - 4 \cos 5t$$

The total response is the sum of the free and the forced response. It is

$$x(t) = 4 + 6 \cos 5t + \frac{4}{5} \sin 5t$$

The entire response is the steady-state response. There is no transient response.

(continued on the next page)

Problem 2.23 continued:

d) The roots are $-4 \pm 7j$. The form of the free response is

$$x(t) = A_1 e^{-4t} \sin 7t + A_2 e^{-4t} \cos 7t$$

Evaluating this with the given initial conditions gives

$$x(t) = \frac{44}{7} e^{-4t} \sin 7t + 10 e^{-4t} \cos 7t$$

The form of the forced response is

$$x(t) = B_1 + B_2 e^{-4t} \sin 7t + B_3 e^{-4t} \cos 7t$$

The steady-state solution is $x_{ss} = 130/65 = 2$. Thus $B_1 = 2$. Evaluating this function with zero initial conditions shows that $B_2 = -8/7$ and $B_3 = -2$. Thus the forced response is

$$x(t) = 2 - \frac{8}{7} e^{-4t} \sin 7t - 2 e^{-4t} \cos 7t$$

The total response is the sum of the free and the forced response. It is

$$x(t) = 2 + \frac{36}{7} e^{-4t} \sin 7t + 8 e^{-4t} \cos 7t$$

The transient response consists of the two exponential terms.

2.24 a) The root is $s = 5/3$, which is positive. So the model is unstable.

b) The roots are $s = 5$ and -2, one of which is positive. So the model is unstable.

c) The roots are $s = 3 \pm 5j$, whose real part is positive. So the model is unstable.

d) The root is $s = 0$, so the model is neutrally stable.

e) The roots are $s = \pm 2j$, whose real part is zero. So the model is neutrally stable.

f) The roots are $s = 0$ and -5, one of which is zero and the other is negative. So the model is neutrally stable.

2.25 a) The system is stable if both of its roots are real and negative or if the roots are complex with negative real parts. Assuming that $m \neq 0$, we can divide the characteristic equation by m to obtain

$$s^2 + \frac{c}{m}s + \frac{k}{m} = s^2 + as + b = 0$$

where $a = c/m$ and $b = k/m$. The roots are given by the quadratic formula:

$$s = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

(continued on the next page)

Problem 2.25 continued:

Thus the condition that m, c, and k have the same sign is equivalent to $a > 0$ and $b > 0$. There are three cases to be considered:

1. Complex roots ($a^2 - 4b < 0$). In this case the real part of both roots is $-a/2$ and is negative if $a > 0$.

2. Repeated, real roots ($a^2 - 4b = 0$). In this case both roots are $-a/2$ and are negative if $a > 0$.

3. Distinct, real roots ($a^2 - 4b > 0$). Let the two roots be denoted r_1 and r_2. We can factor the characteristic equation as $s^2 + as + b = (s - r_1)(s - r_2) = 0$. Expanding this gives
$$(s - r_1)(s - r_2) = s^2 - (r_1 + r_2)s + r_1 r_2 = 0$$
Comparing the two forms shows that
$$r_1 r_2 = b \qquad (1) \qquad \text{and} \qquad r_1 + r_1 = -a \qquad (2)$$
If $b > 0$, condition (1) shows that both roots have the same sign. If $a < 0$, condition (2) shows that the roots must be negative. Therefore, if the roots are distinct and real, the roots will be negative if $a > 0$ and $b > 0$.

b) Neutral stability occurs if either 1) both roots are imaginary or 2) one root is zero while the other root is negative. Imaginary roots occur when $a = 0$ (the roots are $s = \pm\sqrt{b}$) In this case the free response is a constant-amplitude oscillation. Case 2 occurs when $b = 0$ and $a > 0$ (the roots are $s = 0$ and $s = -a$). In this case the free response decays to a non-zero constant.

2.26 a) $\tau = 5$

b) $\tau = 4$

c) $\tau = 3$

d) The roots is $s = 3/8$, so the model is unstable, so no time constant is defined.

2.27 a) The root is $s = -4/13$, so the model is stable, and $x_{ss} = 16/4 = 4$. Since $\tau = 13/4$, it takes about $4\tau = 13$ to reach steady state.

b) The root is $s = -4/13$, so the model is stable, and $x_{ss} = 16/4 = 4$. Since $\tau = 13/4$, it takes about $4\tau = 13$ to reach steady state.

c) The root is $s = 7/15$, so the model is unstable, and no steady state exists.

2.28 1)

$$X(s) = \frac{s+1}{4s+1}\frac{5}{s} = \frac{1}{4}\frac{s+1}{s+1/4}\frac{5}{s} = \frac{C_1}{s} + \frac{C_2}{s+1/4}$$

$C_1 = 5$, $C_2 = -15/4$, so

$$x(t) = 5 - \frac{15}{4}e^{-t/4}$$

2)

$$X(s) = \frac{1}{4s+1}\frac{5}{s} = \frac{1}{4}\frac{1}{s+1/4}\frac{5}{s} = \frac{C_1}{s} + \frac{C_2}{s+1/4}$$

$C_1 = 5$, $C_2 = -5$, so

$$x(t) = 5 - 5e^{-t/4}$$

2.29

$$3[sX(s) - 4] + X(s) = 6$$

$$X(s) = \frac{6}{s + 1/3}$$

$$x(t) = 6e^{-t/3}$$

2.30 a)

$$\zeta = \frac{4}{2\sqrt{40}} = \frac{\sqrt{10}}{10} \qquad \omega_n = \sqrt{\frac{40}{1}} = 2\sqrt{10}$$

$$s = -2 \pm 6j$$

so $\tau = 1/2$ and $\omega_d = 6$.

b)

$$s = 1 \pm 4.7958j$$

So the model is oscillatory but unstable, and thus ζ and τ are not defined.

$$\omega_n = \sqrt{\frac{24}{1}} = 2\sqrt{6} \qquad \omega_d = 4.7958$$

c)

$$\zeta = \frac{20}{2\sqrt{100}} = 1$$

$$s = -10, -10$$

so $\tau = 1/10$. Since the roots are real, the response is not oscillatory, and ω_n and ω_d have no meaning.

d) The root is $s = -10$, so $\tau = 1/10$. Since the model is first order, ζ, ω_n and ω_d have no meaning.

2.31 a) The roots are

$$s = \frac{-10d \pm \sqrt{100d^2 - 4(29)d^2}}{2} = (-5 \pm 2j)\,d$$

So if $d > 0$, the real part is negative, and the system is stable.

b)

$$\zeta = \frac{10d}{2\sqrt{29d^2}} = \frac{10}{2\sqrt{29}} < 1$$

So the free response is always oscillatory.

2.32 a)

$$\frac{X(s)}{F(s)} = \frac{15}{5s+7}$$

The root is $s = -7/5$.

b)

$$\frac{X(s)}{F(s)} = \frac{5}{3s^2+30s+63}$$

The roots are $s = -7$ and $s = -3$.

c)

$$\frac{X(s)}{F(s)} = \frac{4}{s^2+10s+21}$$

The roots are $s = -7$ and $s = -3$.

d)

$$\frac{X(s)}{F(s)} = \frac{7}{s^2+14s+49}$$

The roots are $s = -7$ and $s = -7$.

e)

$$\frac{X(s)}{F(s)} = \frac{6s+4}{s^2+14s+58}$$

The roots are $s = -7 \pm 3j$.

f)

$$\frac{X(s)}{F(s)} = \frac{4s+15}{5s+7}$$

The root is $s = -7/5$.

2.33 Transform each equation using zero initial conditions.

$$3sX(s) = Y(s)$$

$$sY(s) = F(s) - 3Y(s) - 15X(s)$$

Solve for $X(s)/F(s)$ and $Y(s)/F(s)$.

$$\frac{X(s)}{F(s)} = \frac{1}{3s^2 + 9s + 15}$$

$$\frac{Y(s)}{F(s)} = \frac{3s}{3s^2 + 9s + 15}$$

2.34 Transform each equation using zero initial conditions.

$$sX(s) = -2X(s) + 5Y(s)$$

$$sY(s) = F(s) - 6Y(s) - 4X(s)$$

Solve for $X(s)/F(s)$ and $Y(s)/F(s)$.

$$\frac{X(s)}{F(s)} = \frac{5}{s^2 + 8s + 32}$$

$$\frac{Y(s)}{F(s)} = \frac{s + 2}{s^2 + 8s + 32}$$

2.35 a) Transform both equations to obtain $4sX(s) = Y(s)$ and $s(Y(s) = F(s) - 3Y(s) - 12X(s)$. Eliminate $X(s)$ to obtain

$$\frac{Y(s)}{F(s)} = \frac{s}{s^2 + 3s + 3}$$

Use $Y(s) = 4sX(s)$ to eliminate $Y(s)$.

$$\frac{Y(s)}{F(s)} = \frac{1}{4}\frac{1}{s^2 + 3s + 3}$$

b) The roots are

$$s = \frac{-3 \pm \sqrt{3}}{2}$$

Thus

$$\tau = \frac{2}{3} \qquad \zeta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\omega_n = \sqrt{3} \qquad \omega_d = \frac{\sqrt{3}}{2}$$

c) The response oscillates with a frequency of $\omega_d = \sqrt{3}/2$ and essentially disappears for $t > 4\tau = 8/3$.

d) With $F(s) = 1/s$,

$$X(s) = \frac{1}{4}\frac{1}{s(s^2 + 3s + 3)} = \frac{1}{4}\frac{1}{s[(s + \frac{3}{2})^2 + \frac{3}{4}]}$$

or

$$X(s) = \frac{C_1(s + \frac{3}{2}) + C_2\frac{\sqrt{3}}{2}}{(s + \frac{3}{2})^2 + \frac{3}{4}} + \frac{C_3}{s}$$

where $C_1 = -C_3 = -1/12$ and $C_2 = -\sqrt{3}/12$. Thus

$$x(t) = e^{-3t/2}\left(-\frac{1}{12}\cos\frac{\sqrt{3}}{2}t - \frac{\sqrt{3}}{12}\sin\frac{\sqrt{3}}{2}t\right) + \frac{1}{12}$$

2.36 a) Transform both equations to obtain

$$4sX(s) = -4X(s) + 2Y(s) + F(s)$$

$$sY(s) = -9Y(s) - 5X(s) + G(s)$$

These can be solved using Cramer's rule to obtan

$$\frac{X(s)}{F(s)} = \frac{s+9}{4s^2 + 40s + 46}$$

$$\frac{X(s)}{G(s)} = \frac{2}{4s^2 + 40s + 46}$$

b) The roots are $s = -1.3258$ and $s = -8.6742$. The time constants are $\tau = 0.7543$ and $\tau = 0.1153$. The response does not oscillate.

c) The free response is governed by the dominant time constant, which is $\tau = 0.7543$. The response is essentially zero for $t > 4\tau = 3.0172$.

2.37 a)

$$7[sX(s) - 3] + 5X(s) = 4$$

$$X(s) = \frac{25}{7s+5} = \frac{25/7}{s+5/7}$$

$$x(t) = \frac{25}{7}e^{-5t/7}$$

Note that this gives $x(0+) = 25/7$. From the initial value theorem

$$x(0+) = \lim_{s\to\infty} s\frac{25/7}{s+5/7} = \frac{25}{7}$$

which is not the same as $x(0-)$.

b)

$$(3s^2 + 30s + 63)X(s) = 5$$

$$X(s) = \frac{5}{3s^2+30s+63} = \frac{5/3}{s^2+10s+21} = \frac{5}{12}\frac{1}{s+3} - \frac{5}{12}\frac{1}{s+7}$$

$$x(t) = \frac{5}{12}\left(e^{-3t} - e^{-7t}\right)$$

From the initial value theorem

$$x(0+) = \lim_{s\to\infty} s\frac{5/3}{s^2+10s+21} = 0$$

which is the same as $x(0-)$. Also

$$\dot{x}(0+) = \lim_{s\to\infty} s^2\frac{5/3}{s^2+10s+21} = \frac{5}{3}$$

which is not the same as $\dot{x}(0-)$.

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Problem 2.37 continued:

c)

$$s^2X(s) - 2s - 3 + 14[sX(s) - 2] + 49X(s) = 3$$

$$X(s) = \frac{2s+34}{s^2+14s+49} = 20\frac{1}{(s+7)^2} + 2\frac{1}{s+7}$$

$$x(t) = 20te^{-7t} + 2e^{-7t}$$

From the initial value theorem

$$x(0+) = \lim_{s\to\infty} s\frac{2s+35}{s^2+14s+49} = 2$$

which is the same as $x(0-)$. However, the initial value theorem is invalid for computing $\dot{x}(0+)$ and gives an undefined result because the orders of the numerator and denominator of $sX(s)$ are equal.

d)

$$s^2X(s) - 4s - 7 + 14[sX(s) - 4] + 58X(s) = 4$$

$$X(s) = \frac{4s+67}{s^2+14s+58} = \frac{4s+67}{(s+7)^2+3^2} = 13\frac{3}{(s+7)^2+3^2} + 4\frac{s+7}{(s+7)^2+3^2}$$

$$x(t) = 13e^{-7t}\sin 3t + 4e^{-7t}\cos 3t$$

From the initial value theorem

$$x(0+) = \lim_{s\to\infty} s\frac{4s+67}{s^2+14s+58} = 4$$

which is the same as $x(0-)$. However, the initial value theorem is invalid for computing $\dot{x}(0+)$ and gives an undefined result because the order of the numerator of $sX(s)$ is greater than the denominator.

2.38 a)

$$7[sX(s) - 3] + 5X(s) = 4s\frac{1}{s} = 4$$

$$X(s) = \frac{25}{7s+5} = \frac{25/7}{s+5/7}$$

$$x(t) = \frac{25}{7}e^{-5t/7}$$

From the initial value theorem

$$x(0+) = \lim_{s\to\infty} s\frac{25/7}{s+5/7} = \frac{25}{7}$$

which is not the same as $x(0-)$.

b)

$$7[sX(s) - 3] + 5X(s) = 4s\frac{1}{s} + \frac{6}{s}$$

$$X(s) = \frac{25s+6}{s(7s+5)} = \frac{1}{7}\frac{25s+6}{s(s+5/7)} = \frac{6}{5}\frac{1}{s} + \frac{83}{35}\frac{1}{s+5/7}$$

$$x(t) = \frac{6}{5} + \frac{83}{35}e^{-5t/7}$$

which gives $x(0+) = 25/7$, which is not the same as $x(0-)$. However, the initial value theorem is invalid for computing $x(0+)$ and gives an undefined result because the orders of the numerator and denominator of $X(s)$ are equal.

(continued on the next page)

Problem 2.38 continued:

c)

$$3[s^2X(s) - 2s - 3] + 30[sX(s) - 2] + 63X(s) = 4s\frac{1}{s} = 4$$

$$X(s) = \frac{1}{3}\frac{6s+73}{(s+3)(s+7)} = \frac{55}{12}\frac{1}{s+3} - \frac{31}{12}\frac{1}{s+7}$$

$$x(t) = \frac{55}{12}e^{-3t} - \frac{31}{12}e^{-7t}$$

This gives $x(0) = 2$, which is the same as $x(0-)$, and $\dot{x}(0) = 13/2$, which is not the same as $\dot{x}(0-)$.

From the initial value theorem

$$x(0+) = \lim_{s\to\infty} s\frac{1}{3}\frac{6s+73}{(s+3)(s+7)} = 2$$

which is the same as $x(0-)$. However, the initial value theorem is invalid for computing $\dot{x}(0+)$ and gives an undefined result because the order of the numerator of $sX(s)$ is greater than the denominator.

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Problem 2.38 continued:

d)

$$3[s^2X(s) - 4s - 7] + 30[sX(s) - 4] + 63X(s) = 4s\frac{1}{s} + \frac{6}{s}$$

$$X(s) = \frac{1}{3}\frac{12s^2 + 145s + 6}{s(s^2 + 10s + 21)} = 0.0952\frac{1}{s} + 8.9167\frac{1}{s+3} - 5.0119\frac{1}{s+7}$$

$$x(t) = 0.0952 + 8.9167e^{-3t} - 5.0119e^{-7t}$$

This gives $x(0) = 4$, which is the same as $x(0-)$, and $\dot{x}(0) = 8.3332$, which is not the same as $\dot{x}(0-)$.

The initial value theorem gives $x(0+) = 4$ but is invalid for computing $\dot{x}(0+)$ because the orders of the numerator and denominator of $sX(s)$ are equal.

2.39 Transform each equation.

$$3[sX(s) - 5] = Y(s)$$

$$sY(s) - 10 = \frac{4}{s} - 3Y(s) - 15X(s)$$

Solve for $X(s)$ and $Y(s)$.

$$X(s) = \frac{15s^2 + 55s + 4}{3s^3 + 9s^2 + 15s} = \frac{1}{3}\frac{15s^2 + 55s + 4}{s(s^2 + 3s + 5)}$$

$$Y(s) = \frac{30s - 213}{3s^2 + 9s + 15} = \frac{1}{3}\frac{30s - 213}{s^2 + 3s + 5}$$

The denominator roots are $s = -1.5 \pm 1.658j$. Thus

$$X(s) = \frac{C_1}{s} + \frac{1}{3}\left[C_1\frac{1.658}{(s + 1.5)^2 + 2.75} + C_2\frac{s + 1.5}{(s + 1.5)^2 + 2.75}\right]$$

and

$$x(t) = \frac{1}{4} + \frac{1}{165}e^{-3t/2}\left[781 \cos\left(\frac{\sqrt{11}}{2}t\right) + 313\sqrt{11}\sin\left(\frac{\sqrt{11}}{2}t\right)\right]$$

Also,

$$Y(s) = C_1\frac{1.658}{(s + 1.5)^2 + 2.75} + C_2\frac{s + 1.5}{(s + 1.5)^2 + 2.75}$$

and

$$y(t) = \frac{2}{11}e^{-3t/2}\left[55 \cos\left(\frac{\sqrt{11}}{2}t\right) - 86\sqrt{11}\sin\left(\frac{\sqrt{11}}{2}t\right)\right]$$

2.40 Transform each equation.

$$sX(s) - 5 = -2X(s) + 5Y(s)$$

$$sY(s) - 2 = -6Y(s) - 4X(s) + \frac{10}{s}$$

Solve for $X(s)$ and $Y(s)$.

$$X(s) = \frac{5s^2 + 40s + 50}{s^3 + 8s^2 + 32s}$$

$$Y(s) = \frac{2s^2 - 6s + 20}{s^3 + 8s^2 + 32s}$$

The denominator roots are $s = 0$ and $s = -4 \pm 4j$. Thus

$$\begin{aligned} X(s) &= \frac{C_1}{s} + C_2 \frac{4}{(s+4)^2 + 4^2} + C_3 \frac{s+4}{(s+4)^2 + 4^2} \\ &= \frac{25}{16s} + \frac{55}{16} \frac{4}{(s+4)^2 + 4^2} + \frac{55}{16} \frac{s+4}{(s+4)^2 + 4^2} \end{aligned}$$

$$x(t) = \frac{25}{16} + \frac{55}{16} e^{-4t} \sin 4t + \frac{55}{16} e^{-4t} \cos 4t$$

Also,

$$\begin{aligned} Y(s) &= \frac{C_1}{s} + C_2 \frac{4}{(s+4)^2 + 4^2} + C_3 \frac{s+4}{(s+4)^2 + 4^2} \\ &= \frac{5}{8s} - \frac{33}{8} \frac{4}{(s+4)^2 + 4^2} + \frac{11}{8} \frac{s+4}{(s+4)^2 + 4^2} \end{aligned}$$

$$y(t) = \frac{5}{8} - \frac{33}{8} e^{-4t} \sin 4t + \frac{11}{8} e^{-4t} \cos 4t$$

2.41 Transforming both sides of the equation we obtain

$$s^2Y(s) - sy(0) - \dot{y}(0) + Y(s) = \frac{1}{s+1}$$

which gives

$$Y(s) = \frac{(s+1)\,[sy(0) + \dot{y}(0)] + 1}{(s+1)(s^2+1)} = \frac{s^2y(0) + [y(0) + \dot{y}(0)] + \dot{y}(0) + 1}{(s+1)(s^2+1)}$$

This can be expanded as follows.

$$Y(s) = C_1\frac{1}{s+1} + C_2\frac{1}{s^2+1} + C_3\frac{s}{s^2+1}$$

We find the coefficients following the usual procedure and obtain $C_1 = 1/2$, $C_2 = \dot{y}(0) + 1/2$, and $C_3 = y(0) - 1/2$. Thus the solution is

$$y(t) = \frac{1}{2}e^{-t} + \left[\dot{y}(0) + \frac{1}{2}\right]\sin t + \left[y(0) - \frac{1}{2}\right]\cos t$$

(continued on the next page)

Problem 2.41 continued:

Because the initial values can be arbitrary, the general form of the solution is

$$y(t) = \frac{1}{2}e^{-t} + A_1 \sin t + A_2 \cos t \qquad (1)$$

This form can be used to obtain a solution for cases where $y(t)$ or $\dot{y}(t)$ are specified at points other than $t = 0$. For example, suppose we are given that $y(0) = 5/2$ and $y(\pi/2) = 3$. Then evaluation of equation (1) at $t = 0$ and at $t = \pi/2$ gives

$$y(0) = \frac{1}{2} + A_2 = \frac{5}{2} \qquad y\left(\frac{\pi}{2}\right) = \frac{1}{2}e^{-\pi/2} + A_1 = 3$$

The solution of these two equations is $A_1 = 3 - e^{-\pi/2}/2 = 2.896$ and $A_2 = 2$, and the solution of the differential equation is

$$y(t) = \frac{1}{2}e^{-t} + 2.896 \sin t + 2 \cos t$$

2.42 (a) For nonzero initial conditions, the transform gives

$$s^2X(s) - sx(0) + \dot{x}(0) + 4X(s) = \frac{3}{s^2}$$

or

$$X(s) = \frac{s^3x(0) + s^2\dot{x}(0) + 3}{s^2(s^2+4)} = \frac{C_1}{s^2} + \frac{C_2}{s} + C_3\frac{2}{s^2+4} + C_4\frac{s}{s^2+4}$$

The solution form is thus

$$x(t) = C_1t + C_2 + C_3 \sin 2t + C_4 \cos 2t$$

which can be used even if the boundary conditions are not specified at $t = 0$.

(b) The form from part (a) satisfies the differential equation if $C_1 = 3/4$ and $C_2 = 0$. From $x(0) = 10$, we obtain $C_4 = 10$. From $x(5) = 30$, we obtain $C_3 = -63.675$. Thus

$$x(t) = \frac{3}{4}t - 63.675 \sin 2t + 10 \cos 2t$$

2.43 The denominator roots are $s = -3 \pm 5j$ and $s = \pm 6j$. Thus we can express $X(s)$ as follows.

$$X(s) = \frac{30}{\left[(s+3)^2 + 5^2\right](s^2 + 6^2)}$$

which can be expressed as the sum of terms that are proportional to entries 8 through 11 in Table 2.2.1.

$$X(s) = C_1 \frac{5}{(s+3)^2 + 5^2} + C_2 \frac{s+3}{(s+3)^2 + 5^2} + C_3 \frac{6}{s^2 + 6^2} + C_4 \frac{s}{s^2 + 6^2} \tag{1}$$

We can obtain the coefficients by noting that $X(s)$ can be written as

$$X(s) = \frac{5C_1(s^2 + 6^2) + C_2(s+3)(s^2 + 6^2) + 6C_3\left[(s+3)^2 + 5^2\right] + C_4 s\left[(s+3)^2 + 5^2\right]}{\left[(s+3)^2 + 5^2\right](s^2 + 6^2)} \tag{2}$$

Comparing the numerators of equations (1) and (2), and collecting powers of s, we see that

$$(C_2 + C_4)s^3 + (5C_1 + 3C_2 + 6C_3 + 6C_4)s^2 + (36C_2 + 36C_3 + 34C_4)s$$

$$+180C_1 + 108C_2 + 204C_3 = 30$$

or

$$C_2 + C_4 = 0 \qquad 5C_1 + 3C_2 + 6C_3 + 6C_4 = 0$$

$$36C_2 + 36C_3 + 34C_4 = 0 \qquad 180C_1 + 108C_2 + 204C_3 = 30$$

These are four equations in four unknowns. Note that the first equation gives $C_4 = -C_2$. Thus we can easily eliminate C_4 from the equations and obtain a set of three equations in three unknowns. The solution is $C_1 = 6/65$, $C_2 = 9/65$, and $C_3 = -1/130$, and $C_4 = -9/65$.

(continued on the next page)

Problem 2.43 continued:

The inverse transform is

$$x(t) = C_1 e^{-3t} \sin 5t + C_2 e^{-3t} \cos 5t + C_3 \sin 6t + C_2 \cos 6t$$

$$= \frac{6}{65} e^{-3t} \sin 5t + \frac{9}{65} e^{-3t} \cos 5t - \frac{1}{130} \sin 6t - \frac{9}{65} \cos 6t$$

2.44 Transform the equation.

$$(s^2 + 12s + 40)X(s) = 3\frac{5}{s^2+25}$$

The characteristic roots are $s = -6 \pm 2j$. Thus

$$\begin{aligned} X(s) &= \frac{15}{(s^2+25)(s^2+12s+40)} \\ &= C_1\frac{5}{s^2+25} + C_2\frac{s}{s^2+25} + C_3\frac{2}{(s+6)^2+4} + C_4\frac{s+6}{(s+6)^2+4} \end{aligned}$$

or

$$X(s) = \frac{1}{85}\frac{5}{s^2+25} - \frac{4}{85}\frac{s}{s^2+25} + \frac{19}{170}\frac{2}{(s+6)^2+4} + \frac{4}{85}\frac{s+6}{(s+6)^2+4}$$

Thus

$$x(t) = \frac{1}{85}\sin 5t - \frac{4}{85}\cos 5t + \frac{19}{170}e^{-6t}\sin 2t + \frac{4}{85}e^{-6t}\cos 2t$$

2.45 From the text example, the form $A \sin(\omega t + \phi)$ has the transform

$$A \frac{s \sin \phi + \omega \cos \phi}{s^2 + \omega^2}$$

For this problem, $\omega = 5$. Comparing numerators gives

$$A\,(s \sin \phi + 5 \cos \phi) = 4s + 9$$

Thus

$$A \sin \phi = 4 \qquad 5A \cos \phi = 9$$

With $A > 0$, ϕ is seen to be in the first quadrant.

$$\phi = \tan^{-1} \frac{\sin \phi}{\cos \phi} = \tan^{-1} \frac{4/A}{9/5A} = \tan^{-1} \frac{20}{9} = 1.148 \text{ rad}$$

Because $\sin^2 \phi + \cos^2 \phi = 1$,

$$\left(\frac{4}{A}\right)^2 + \left(\frac{9}{5A}\right)^2 = 1$$

which gives $A = 4.386$. Thus

$$x(t) = 4.386 \sin(5t + 1.148)$$

2.46 Taking the transform of both sides of the equation and noting that both initial conditions are zero, we obtain

$$s^2X(s) + 6sX(s) + 34X(s) = 5\frac{6}{s^2 + 6^2}$$

Solve for $X(s)$.

$$X(s) = \frac{30}{(s^2 + 6s + 34)(s^2 + 6^2)}$$

The inverse transform is

$$x(t) = \frac{6}{65}e^{-3t}\sin 5t + \frac{9}{65}e^{-3t}\cos 5t - \frac{1}{130}\sin 6t - \frac{9}{65}\cos 6t$$

2.47 Transform the equation.

$$(s^2 + 12s + 40)X(s) = \frac{10}{s}$$

or, since the characteristic roots are $s = -6 \pm 2j$,

$$X(s) = \frac{10}{s[(s+6)^2 + 2^2]} \qquad (1)$$

From the text example, the form $Ae^{-at}\sin(\omega t + \phi)$ has the transform

$$A\frac{s\ \sin\ \phi + a\ \sin\ \phi + \omega\ \cos\ \phi}{(s+a)^2 + \omega^2}$$

For this problem, $a = 6$ and $\omega = 2$. Thus

$$X(s) = \frac{10}{s[(s+6)^2 + 2^2]} = \frac{C_1}{s} + C_2\frac{s\ \sin\ \phi + 6\ \sin\ \phi + 2\ \cos\ \phi}{(s+6)^2 + 2^2}$$

or

$$X(s) = \frac{C_1(s^2 + 12s + 40) + C_2 s^2\ \sin\ \phi\ + 6C_2 s\ \sin\ \phi + 2C_2 s\ \cos\ \phi}{s[(s+6)^2 + 2^2]} \qquad (2)$$

(continued on the next page)

Problem 2.47 continued:

Collecting terms and comparing the numerators of equations (1) and (2), we have

$$(C_1 + C_2 \sin\phi)s^2 + (12C_1 + 6C_2 \sin\phi + 2C_2 \cos\phi)s + 40C_1 = 10$$

Thus comparing terms, we see that $C_1 = 1/4$ and

$$\frac{1}{4} + C_2 \sin\phi = 0$$

$$3 + 6C_2 \sin\phi + 2C_2 \cos\phi = 0$$

So

$$C_2 \sin\phi = -\frac{1}{4} \qquad C_2 \cos\phi = -\frac{3}{4}$$

Thus ϕ is in the third quadrant and

$$\phi = \tan^{-1}\frac{-1/4}{-3/4} = 0.322 + \pi = 3.463 \text{ rad}$$

Because $\sin^2\phi + \cos^2\phi = 1$,

$$\left(\frac{1}{4C_2}\right)^2 + \left(\frac{3}{4C_2}\right)^2 = 1$$

which gives $C_2 = 0.791$. Thus

$$x(t) = \frac{1}{4} + 0.791e^{-6t} \sin(2t + 3.463)$$

2.48 Transform the equation.

$$X(s) = \frac{F(s)}{s^2 + 8s + 1}$$

Thus

$$F(s) - X(s) = F(s) - \frac{F(s)}{s^2 + 8s + 1} = \frac{s^2 + 8s}{s^2 + 8s + 1} F(s)$$

Because $F(s) = 6/s^2$,

$$F(s) - X(s) = \frac{s^2 + 8s}{s^2 + 8s + 1} \frac{6}{s^2} = \frac{s + 8}{s^2 + 8s + 1} \frac{6}{s}$$

From the final value theorem,

$$f_{ss} - x_{ss} = \lim_{s \to 0} s[F(s) - X(s)] = \lim_{s \to 0} s \frac{s + 8}{s^2 + 8s + 1} \frac{6}{s} = 8$$

2.49 The roots are $s = -2$ and -4. Thus

$$X(s) = \frac{1 - e^{-3s}}{(s+2)(s+4)}$$

Let

$$F(s) = \frac{1}{(s+2)(s+4)} = \frac{1}{2}\left(\frac{1}{s+2} - \frac{1}{s+4}\right)$$

so

$$f(t) = \frac{1}{2}\left(e^{-2t} - e^{-4t}\right)$$

From Property 6 of the Laplace transform,

$$x(t) = \frac{1}{2}\left(e^{-2t} - e^{-4t}\right) - \frac{1}{2}\left[e^{-2(t-3)} - e^{-4(t-3)}\right] u_s(t-3)$$

2.50

$$f(t) = \frac{C}{D} t u_s(t) - \frac{2C}{D}(t - D)u_s(t - D) + \frac{C}{D}(t - 2D)u_s(t - 2D)$$

From Property 6 of the Laplace transform,

$$F(s) = \frac{C}{Ds^2} - \frac{2C}{Ds^2}e^{-Ds} + \frac{C}{Ds^2}e^{-2Ds} = \frac{C}{Ds^2}\left(1 - 2e^{-Ds} + e^{-2Ds}\right)$$

2.51

$$f(t) = \frac{C}{D} t u_s(t) - \frac{C}{D}(t-D) u_s(t-D) - C u_s(t-D)$$

From Property 6 of the Laplace transform,

$$F(s) = \frac{C}{Ds^2} - \frac{C}{Ds^2} e^{-Ds} - \frac{C}{s} e^{-Ds}$$

2.52

$$f(t) = Mu_s(t) - 2Mu_s(t-T) + Mu_s(t-2T)$$

From Property 6,

$$F(s) = \frac{M}{s} - \frac{2M}{s}e^{-Ts} + \frac{M}{s}e^{-2Ts}$$

2.53

$$P(t) = 3u_s(t) - 3u_s(t-5)$$

From Property 6,

$$P(s) = \frac{3}{s} - \frac{3}{s}e^{-5s}$$

$$X(s) = \frac{P(s)}{4s+1} = \frac{3\left(1-e^{-5s}\right)}{s(4s+1)} = \frac{3}{4}\frac{1-e^{-5s}}{s(s+1/4)}$$

Let

$$F(s) = \frac{3}{4}\frac{1}{s(s+1/4)} = 3\left(\frac{1}{s} - \frac{1}{s+1/4}\right)$$

Then

$$f(t) = 3\left(1-e^{-t/4}\right)$$

Since

$$X(s) = F(s)\left(1-e^{-5s}\right)$$

we have

$$x(t) = f(t) - f(t-5)u_s(t-5) = 3\left(1-e^{-t/4}\right) - 3\left[1-e^{-(t-5)/4}\right]u_s(t-5)$$

2.54 Let

$$f(t) = t + \frac{t^3}{3} + \frac{2t^5}{15}$$

Then

$$F(s) = \frac{1}{s^2} + \frac{2}{s^4} + \frac{16}{s^6} = \frac{s^4 + 2s^2 + 16}{s^6}$$

From the differential equation,

$$\begin{aligned} X(s) &= \frac{F(s)}{s+1} = \frac{s^4 + 2s^2 + 16}{s^6(s+1)} \\ &= \frac{16}{s^6} - \frac{16}{s^5} + \frac{18}{s^4} - \frac{18}{s^3} + \frac{19}{s^2} - \frac{19}{s} + \frac{19}{s+1} \end{aligned}$$

Thus

$$x(t) = \frac{2}{15}t^5 - \frac{2}{3}t^4 + 3t^3 - 9t^2 + 19t - 19 + 19e^{-t}$$

On a plot of this and the solution obtained from the lower-order approximation, the two solutions are practically indistinguishable.

2.55 From the derivative property of the Laplace transform, we know that

$$\mathcal{L}[\dot{x}(t)] = \int_0^\infty \dot{x}(t)e^{-st}\,dt = sX(s) - x(0)$$

Therefore

$$\lim_{s\to\infty}[sX(s)] = \lim_{s\to\infty}\left[x(0) + \int_0^\infty \dot{x}(t)e^{-st}\,dt\right]$$

$$= \lim_{s\to\infty} x(0) + \lim_{s\to\infty}\left\{\lim_{\epsilon\to 0+}\left[\int_0^\epsilon \dot{x}(t)e^{-st}\,dt\right]\right\} + \lim_{\epsilon\to 0+}\left\{\int_0^\epsilon \lim_{s\to\infty}\left[\dot{x}(t)e^{-st}\,dt\right]\right\}$$

The limits on ϵ and s can be interchanged because s is independent of t. Within the interval $[0, 0+]$, $e^{-st} = 1$, and so

$$\lim_{s\to\infty}[sX(s)] = x(0) + \lim_{s\to\infty}\left\{\lim_{\epsilon\to 0+}\left[\int_0^\epsilon \dot{x}(t)\,dt\right]\right\} + \lim_{\epsilon\to 0+}\left\{\int_0^\epsilon \lim_{s\to\infty}\left[\dot{x}(t)e^{-st}\,dt\right]\right\}$$

$$= x(0) + x(t)|_{t=0}^{t=0+} + 0 = x(0+)$$

This proves the theorem.

2.56 From the derivative property of the Laplace transform, we know that

$$\mathcal{L}[\dot{x}(t)] = \int_0^\infty \dot{x}(t)e^{-st}\,dt = sX(s) - x(0)$$

Therefore,

$$\begin{aligned}\lim_{s\to 0}\,[sX(s)] &= \lim_{s\to 0} x(0) + \lim_{s\to 0}\left[\int_0^\infty \dot{x}(t)e^{-st}\,dt\right]\\ &= x(0) + \int_0^\infty \lim_{s\to 0}\left[\dot{x}(t)e^{-st}\,dt\right] = x(0) + \int_0^\infty \dot{x}(t)\,dt\end{aligned}$$

because s is independent of t and $\lim_{s\to 0} e^{-st} = 1$. Thus

$$\begin{aligned}\lim_{s\to 0}\,[sX(s)] = x(0) + \lim_{T\to\infty}\left[\int_0^T \dot{x}(t)\,dt\right] &= x(0) + \lim_{T\to\infty}\left[x(t)|_{t=0}^{t=T}\right]\\ = x(0) + \lim_{T\to\infty} x(T) - x(0) &= \lim_{T\to\infty} x(T) = \lim_{t\to\infty} x(t)\end{aligned}$$

This proves the theorem.

2.57 Let

$$g(t) = \int_0^t x(t)\,dt$$

Then

$$\mathcal{L}\left[\int_0^t x(t)\,dt\right] = \mathcal{L}[g(t)] = \int_0^t g(t)e^{-st}\,dt$$

To use integration by parts we define $u = g$ and $dv = e^{-st}dt$, which give $du = dg = x(t)\,dt$ and $v = -e^{-st}/s$. Thus

$$\int_0^t g(t)e^{-st}\,dt = \left.\frac{g(t)e^{-st}}{-s}\right|_{t=0}^{t=\infty} - \int_0^\infty \frac{e^{-st}}{-s}x(t)\,dt$$

$$= 0 + \frac{g(0)}{s} + \frac{1}{s}\int_0^\infty x(t)e^{-st}\,dt = \frac{g(0)}{s} + \frac{X(s)}{s}$$

$$= \frac{1}{s}\left.\int x(t)\,dt\right|_{t=0} + \frac{X(s)}{s}$$

This proves the property.

If there is an impulse in $x(t)$ at $t = 0$, then $g(0)$ equals the strength of the impulse. If there is no impulse at $t = 0$, then $g(0) = 0$.

2.58 a)

```
[r,p,k] = residue([8,5],[2,20,48])
```

The result is `r = [10.7500, -6.7500]`, `p = [-6.0000, -4.0000]`, and `k = [ ]`. The solution is

$$x(t) = 10.75e^{-6t} - 6.75e^{-4t}$$

b)

```
[r,p,k] = residue([4,13],[1,8,116])
```

The result is `r = [2.0000 - 0.1500i, 2.0000 + 0.1500i]`, `p = [-4.0000 + 10.0000i, -4.0000 - 10.0000i]`, and `k = [ ]`. The solution is

$$x(t) = (2 - 0.15j)e^{(-4+10j)t} + (2 + 0.15j)e^{(-4-10j)t}$$

The solution is

$$x(t) = 2e^{-4t}\left(2 \cos 10t + 0.15 \sin 10t\right)$$

c)

```
[r,p,k] = residue([3,2],[1,10,0,0])
```

The result is `r = [ -0.2800, 0.2800, 0.2000]`, `p = [-10, 0, 0]`, and `k = [ ]`. The solution is

$$x(t) = -0.28e^{-10t} + 0.28 + 0.2t$$

(continued on the next page)

Problem 2.58 continued:

d)

```
[r,p,k] = residue([1,0,1,6],[1,2,0,0,0,0])
```

The result is `r = [-0.2500, 0.2500, 0.5000, -1.0000, 3.0000]`, `p =[ -2, 0, 0, 0, 0]`, and `k = [ ]`. The solution is

$$x(t) = -0.25e^{-2t} + 0.25 + 0.5t - \frac{1}{2}t^2 + \frac{1}{2}t^3$$

e)

```
[r,p,k] = residue([4,3],[1,6,34,0])
```

The result is `r = [-0.0441 - 0.3735i, -0.0441 + 0.3735i, 0.0882]`, `p = [-3.0000 + 5.0000i, -3.0000 - 5.0000i, 0]`, and `k = [ ]`.The solution is

$$x(t) = (-0.0441 - 0.3735j)e^{(-3+5j)t} + (-0.0441 + 0.3735j)e^{(-3-5j)t} + 0.0882$$

The solution is

$$x(t) = 2e^{-3t}\left(-0.0441 \cos\, 5t + 0.3735 \,\sin\, 5t\right) + 0.0882$$

(continued on the next page)

Problem 2.58 continued:

f)

```
[r,p,k] = residue([5,3,7],[1,12,44,48])
```

The result is `r = [21.1250 -18.7500 2.6250]`, `p = [ -6, -4, -2]`, and `k = [ ]`. The solution is

$$x(t) = 21.125e^{-6t} - 18.75e^{-4t} + 2.625e^{-2}$$

2.59 a)

```
[r,p,k] = residue(5,conv([1,8,16],[1,1]))
```

The result is `r = [-0.5556, -1.6667, 0.5556]`, `p = [-4.0000, -4.0000, -1.0000]`, `k = [ ]`. The solution is

$$x(t) = -0.5556e^{-4t} - 1.6667te^{-4t} + 0.5556e^{-t}$$

b)

```
[r,p,k] = residue([4,9],conv([1,6,34],[1,4,20]))
```

The result is `r = [-0.1159 + 0.1073i, -0.1159 - 0.1073i, 0.1159 - 0.1052i, 0.1159 + 0.1052i]`, `p = -3.0000 + 5.0000i, -3.0000 - 5.0000i, -2.0000 + 4.0000i, -2.0000 - 4.0000i]`, and `k = [ ]`. The solution is

$$\begin{aligned} x(t) &= (-0.1159 + 0.1073j)e^{(-3+5j)t} + (-0.1159 - 0.1073j)e^{(-3-5j)t} \\ &+ (0.1159 - 0.1052j)e^{(-2+4j)t} + (0.1159 + 0.1052j)e^{(-2-4j)t} \end{aligned}$$

The solution is

$$x(t) = 2e^{-3t}(-0.1159 \cos 5t - 0.1073 \sin 5t) + 2e^{-2t}(0.1159 \cos 4t + 0.1052 \sin 4t)$$

2.60 a)

```
sys = tf(1,[3,21,30]);
step(sys)
```

b)

```
sys = tf(1,[5,20, 65]);
step(sys)
```

c)

```
sys = tf([3,2],[4,32,60]);
step(sys)
```

2.61 a)

```
sys = tf(1,[3,21,30]);
impulse(sys)
```

b)

```
sys = tf(1,[5,20, 65]);
impulse(sys)
```

2.62

```
sys = tf(5,[3,21,30]);
impulse(sys)
```

2.63

```
sys = tf(5,[3,21,30]);
step(sys)
```

2.64 a)

```
sys = tf(1,[3,21,30]);
t = [0:0.001:1.5];
f = 5*t;
[x,t] = lsim(sys,f,t);
plot(t,x)
```

b)

```
sys = tf(1,[5,20,65]);
t = [0:0.001:1.5];
f = 5*t;
[x,t] = lsim(sys,f,t);
plot(t,x)
```

c)

```
sys = tf([3,2],[4,32,60]);
t = [0:0.001:1.5];
f = 5*t;
[x,t] = lsim(sys,f,t);
plot(t,x)
```

2.65 a)

```
sys = tf(1,[3,21,30]);
t = [0:0.001:6];
f = 6*cos(3*t);
[x,t] = lsim(sys,f,t);
plot(t,x)
```

b)

```
sys = tf(1,[5,20,65]);
t = [0:0.001:6];
f = 6*cos(3*t);
[x,t] = lsim(sys,f,t);
plot(t,x)
```

c)

```
sys = tf([3,2],[4,32,60]);
t = [0:0.001:6];
f = 6*cos(3*t);
[x,t] = lsim(sys,f,t);
plot(t,x)
```