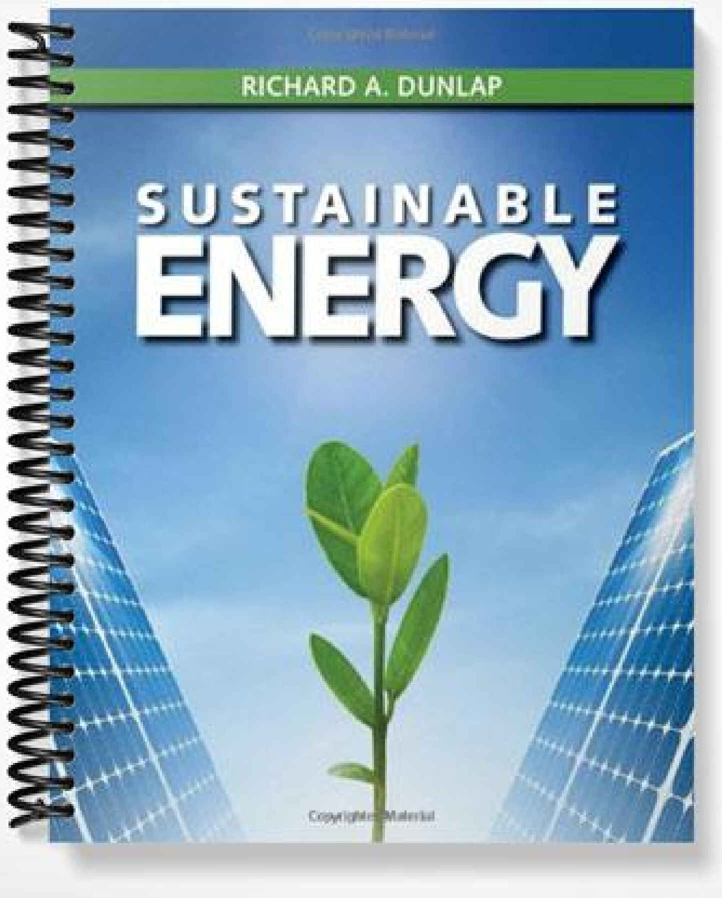
SOLUTIONS MANUAL



Chapter 2
Past, Present and Future World Energy Use

Problem 2.1 Locate information on the total primary energy consumption per capita and per dollar of GDP for five states from different geographical regions in the United States Discuss any relationships between energy use and factors such as climate, population density, types of industry, and other variables that are apparent.

Solution Per capita energy statistics are available for all states at:
http://www.statemaster.com/graph/ene_tot_ene_con_percap-total-electricity-consumption-per-capita
and per $GDP energy statistics are available for all states at:
http://www.statemaster.com/graph/ene_tot_ene_con_pergdp-energy-total-consumption-per-gdp

Choosing the following state the information from the web site is tabulated. Note that values in the table are in BBtu per year. These are converted to GJ as 1055 BBtu = 1GJ. Note that the average over all states is ~350 GJ per capita per year.

State	E(GJ)/Capita	E(GJ)/$GDP
Alaska	1171	0.0229
Alabama	450	0.0148
Maine	392	0.0120
Massachusetts	255	0.00514
Florida	245	0.00734

We expect that the energy per capita will be inversely proportional to the population density and inversely proportional to the average temperature. The population density and per capita GDP are readily available on the internet (e.g. Wikipedia) as given (for Jan 2010) in the table below.

State	population/km^2	$GDP per capita
Alaska	0.46	65,143
Alabama	30.4	36,333
Maine	16.6	40,923
Massachusetts	324	58,108
Florida	135	40,106

Per capita energy consumption in Alaska is by far the highest. This is clearly expected on the basis of a very low population density and a very cold climate. Alabama has a warm

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climate but a high per capita energy consumption. This can in fact be due to the moderate population density. This is a bit anomalous in comparison with Maine, which has a lower population density and cooler climate. Massachusetts by comparison with Maine has a slightly warmer climate but a much higher population density and a correspondingly smaller energy use. Florida has a much milder climate than Massachusetts, which compensates for its somewhat lower population density. Economic factors can be accounted for by inspecting the energy use per GDP. Note that

$$E/\text{GDP} = \left(E/\text{capita}\right) \times \left(\text{GDP/capita}\right)^{-1}$$

Alaska has a high GDP/capita but not enough to compensate for other factors. Alabama has a low GDP/capita which partly accounts for its large energy use. This may be reflected by the presence of rather energy intensive industries. Maine has a higher GDP/capita which may partially explain its lower energy use than Alabama. The order of Massachusetts and Florida are reversed when considering *E*/GDP rather than *E*/capita. This is a result of its much larger GDP per capita and is reflection of the presence of more high technology industries and businesses.

Problem 2.2 A quantity has a doubling time of 110 years. Estimate the annual percent increase in the quantity.

Solution From equation (2.9) the annual rate of increase *R* is given as

$$R = \frac{100 \ln 2}{t_D}$$

where t_D is the doubling time. If t_D is 110 years then

$$R = \frac{(100) \times (0.693)}{110 \text{ y}} = 0.63\% \text{ per year}$$

This is much less than 10% so the approximation given in equation (2.9) is valid.

Problem 2.3 The population of a particular country has a doubling time of 45 years. When will the population be three times its present value?

Solution From equation (2.7) the constant a can be determined from the doubling time as

$$t_D = \frac{1n2}{a}$$

so

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$$a = \frac{1n2}{t_D}$$

For t_D = 45 years then

$$a = \frac{0.693}{45} = 0.0154\text{y}^{-1}$$

From equation (2.4) the quantity of any time is given in terms of the initial value as

$$N(t) = N_0 \exp(at)$$

so solving for t we get

$$t = \frac{1}{a}\ln\left(\frac{N(t)}{N_0}\right)$$

for $N(t) = 3N_0$ then we get

$$t = \left(\frac{1}{0.0154\text{y}^{-1}}\right)\ln(3) = 71.3\text{years}$$

Problem 2.4 Assume that the historical growth rate of the human population was constant at 1.6% per year. For a population of 7 billion in 2012, determine the time in the past when the human population was 2.

Solution As the annual percentage growth rate is small then we can use the approximation of equation (2.4) to get the doubling time from R so

$$t_D = \left(\frac{100\ln 2}{R}\right) = \frac{(100)\times(0.693)}{1.6} = 43.31\text{years}$$

from equation (2.7) the constant a can be found to be

$$a = \left(\frac{\ln 2}{t_D}\right) = \frac{0.693}{43.31\text{y}} = 0.016\text{y}^{-1}$$

from equation (2.4) we start with an initial population of N_0 = 2 at t = 0 then $N(t) = 6.7 \times 10^9$ then from

$$N(t) = N_0 \exp(at)$$

so

$$t = \frac{1}{a}\ln\frac{N(t)}{N_0} = \frac{1}{0.016}\ln\frac{7\times 10^9}{2} = 1374\text{y}$$

in the past or at year 2012 – 1374 = 638 (obviously growth rate was not constant).

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Problem 2.5 What is the current average human population density (i.e., people per square kilometer) on earth?

Solution The radius of the Earth is 6378 km (assumed spherical). The total area (including oceans) is $A = 4\pi r^2 = (4)\times(3.14)\times(6378\text{km})^2 = 5.1\times10^8\,\text{km}^2$. The total current population is 6.7 × 10^9, so the population density is

$$\frac{6.7\times10^9}{5.1\times10^8\,\text{km}^2} = 13.1\text{people/km}^2$$

If only land area is included, the land area on Earth is from various values given on the web range from 1.483 × 10^8 km^2 to 1.533 × 10^8 km^2. Using 1.5 × 10^8 km^2 we find

$$\frac{6.7\times10^9}{1.5\times10^8\,\text{km}^2} = 44.7\text{people/km}^2$$

Problem 2.6 The total world population in 2012 was about 7 billion and Figure 2.11 shows that at that time the actual world population growth rate was about 1% per year. The figure also shows an anticipated roughly linear decrease in growth rate that extrapolates to zero growth in about the year 2080. Assuming an average growth rate of 0.5% between 2012 and 2080, what would the world population be in 2080? How does this compare with estimates discussed in the text for limits to human population?

Solution If R = 0.5% per year then the doubling time is found from equation (2.9) to be

$$t_D = \frac{10\ln 2}{R} = \frac{(100)\times(0.693)}{0.5} = 138.6\text{y}$$

using equation (2.7) to get the constant a

$$a = \frac{\ln 2}{t_D} = \frac{0.693}{138.6\text{y}} = 0.005\text{y}^{-1}$$

then equation (2.4) gives

$$N(t) = N_0 \exp(at)$$

so from $N_0 = 7\times10^9$ people and $t = 2080 - 2012 = 68\text{years}$ we find

$$N(t) = (7\times10^9)\exp((0.005\text{y}^{-1})\times(68\text{y})) = 9.8\times10^9\,\text{people}$$

This is consistent with comments in the text which suggest that the limit to human population can not be much more than 10 billion.

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Problem 2.7 The population of a state is 25,600 in the year 1800 and 218,900 in the year 1900. Calculate the expected population in the year 2000 if (a) the growth is linear and (b) the growth is exponential.

Solution If population growth is linear then for 100 years between 1800 and 1900 it grows by $(218.9 - 25.6)\times 10^3 = 193.3\times 10^3$, so the population would grow by another 193.3×10^3 during the 100 years from 1900 to 2000 for a total of

$$(218.9 + 193.3)\times 10^3 = 412.2\times 10^3 \text{ people}$$

If the population growth is exponential then from equation (2.4) for $N_0 = 6.7\times 10^9$ in 1800 then for t = 100 years, $N(t)$ is 218.9×10^3. From this a can be found to be

$$a = \frac{1}{t}\ln\frac{N(t)}{N_0} = \left(\frac{1}{100\text{y}}\right)\times\ln\left(\frac{218.9\times 10^3}{25.6\times 10^3}\right) = 0.0215\text{y}^{-1}$$

Then using $N_0 = 218.9\times 10^3$ in year 1900 the population at 100y (i.e. in year 2000) is

$$N(t) = (218.9\times 10^3)\times\exp((0.0215\text{y}^{-1})\times(100\text{y})) = 1.87\times 10^6$$

about 4.5 times the value for linear growth.

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Problem 2.8 The population of a country as a function of time is shown in the following table. Is the growth exponential?

year	population (millions)
1700	0.501
1720	0.677
1740	0.891
1760	1.202
1780	1.622
1800	2.163
1820	2.884
1840	3.890
1860	5.176
1880	6.761
1900	8.702
1920	10.23
1940	11.74
1960	13.18
1980	14.45
2000	15.49

Solution For exponential growth

$$N(t) = N_0 \exp(a(t - t_0)) \text{ so } \ln\left(\frac{N(t)}{N_0}\right) = a(t - t_0)$$

and the ln of the related population should be linear in time. Calculating $N(t)/N_0$ from the values above gives the tabulated values. They are plotted as a function of t - t_D as shown

year	population (millions)	year - 1700	ln[*N*(t)/*N*(1700)]
1700	0.501	0	0
1720	0.677	20	0.301065
1740	0.891	40	0.575738
1760	1.202	60	0.875136
1780	1.622	80	1.174809
1800	2.163	100	1.462645
1820	2.884	120	1.750327
1840	3.89	140	2.049558
1860	5.176	160	2.335182
1880	6.761	180	2.60232
1900	8.702	200	2.854702
1920	10.23	220	3.016474
1940	11.74	240	3.154151
1960	13.18	260	3.26985
1980	14.45	280	3.361844
2000	15.49	300	3.431344

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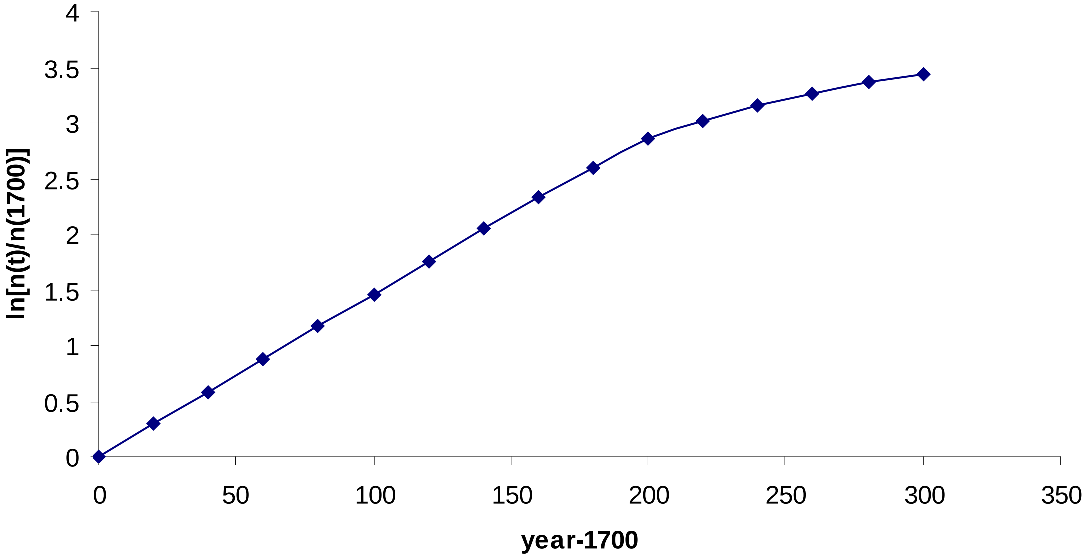
The graph shows that the ln is linear and hence the population is exponential until ~ 1900 when the increase is less than exponential.



Problem 2.9 Consider a solar photovoltaic system with a total rated output of 10 MW_e and a capacity factor of 29%. If the total installation cost is $35,000,000, calculate the decrease in the cost of electricity per kilowatt-hour if the payback period is 25 years instead of 15 years. Assume a constant interest rate of 5.8%.

Solution From Example 2.3 the contribution to the cost of electricity per kWh due to the capital cost is

$$\frac{1}{Rf\left(8760\text{h/y}\right)}\times\frac{i(1+i)^T}{\left((1+i)^T-1\right)}$$

Using I = 35,000,000, i = 0.058, R = 10^4 kW, f = 0.29, then for a payback period of 15 years the cost per kWh is

$$\frac{3.5\times10^7}{10^4\times0.29\times8760}\times\frac{0.058\times(1.058)^{15}}{\left((1.058)^{15}-1\right)}=1.378\times0.102=\$0.140\text{/kWh}$$

For a payback period of 25 years the cost is

$$1.378\times\frac{0.058\times(1.058)^{25}}{\left((1.058)^{25}-1\right)}=1.378\times0.0767=\$0.106\,/\,\text{kWh}$$

or a decrease of (0.140 – 0.106) = $0.034 per kWh.

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