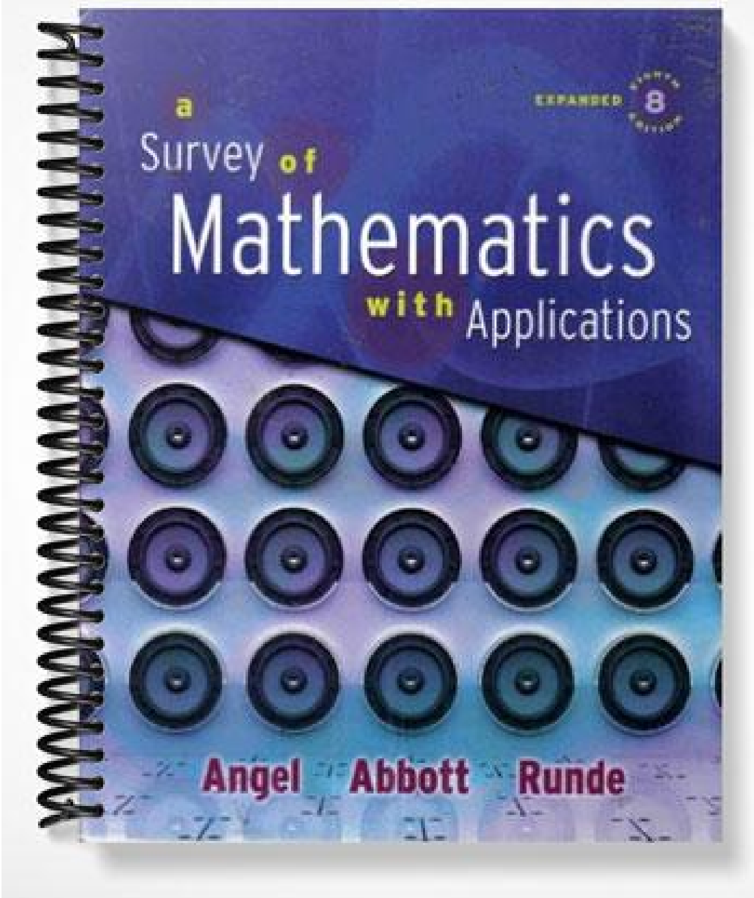
SOLUTIONS MANUAL



CHAPTER TWO

SETS

<u>Exercise Set 2.1</u>

1. A **set** is a collection of objects.
2. An **ellipsis** is three dots in a set indicating the elements continue in the same manner.
3. Description: the set of counting numbers less than 7

   Roster form: $\{1, 2, 3, 4, 5, 6\}$

   Set-builder notation: $\{x \mid x \in N \text{ and } x < 7\}$
4. An **infinite** set is a set that is not finite.
5. A set is **finite** if it either contains no elements or the number of elements in the set is a natural number.
6. Set A is **equal** to set B, symbolized by $A = B$, if and only if they contain exactly the same elements.
7. Two sets are **equivalent** if they contain the same number of elements.
8. The **cardinal number** of a set A, symbolized by $n(A)$, is the number of elements in set A.
9. A set that contains no elements is called the **empty set** or **null set**.
10. $\{\ \}, \varnothing$
11. A **universal set**, symbolized by U, is a set that contains all the elements for any specific discussion.
12. Set A and set B can be placed in **one-to-one correspondence** if every element of set A can be matched with exactly one element of set B and every element of set B can be matched with exactly one element of set A.
13. Not well defined, "best" is interpreted differently by different people.
14. Not well defined, "easiest" is interpreted differently by different people.
15. Well defined, the contents can be clearly determined.
16. Well defined, the contents can be clearly determined.
17. Well defined, the contents can be clearly determined.
18. Not well defined, "most interesting" is interpreted differently by different people.
19. Infinite, the number of elements in the set is not a natural number.
20. Finite, the number of elements in the set is a natural number.
21. Infinite, the number of elements in the set is not a natural number.
22. Infinite, the number of elements in the set is not a natural number.
23. Infinite, the number of elements in the set is not a natural number.
24. Finite, the number of elements in the set is a natural number.
25. { Maine, Maryland, Massachusetts, Michigan, Minnesota, Misssissippi, Missouri, Montana }
26. {Atlantic, Pacific, Arctic, Indian}
27. $\{11, 12, 13, 14, \ldots, 177\}$
28. $C = \{4\}$
29. $B = \{2, 4, 6, 8, \ldots\}$
30. $\{\ \}$ or $\varnothing$
31. $\{\ \}$ or $\varnothing$
32. { Alaska, Hawaii }
33. $E = \{14, 15, 16, 17, \ldots, 84\}$
34. { Alaska, Hawaii }

35. $\{$Switzerland, Denmark, Sweden, United Kingdom, Germany, New Zealand$\}$
36. $\{$Australia, Czech Republic, China$\}$
37. $\{$Switzerland, Denmark, Sweden, United Kingdom, Germany$\}$
38. $\{$Turkey, Peru, Canada, Chile, Japan, Australia, Czech Republic$\}$

39. $\{2004, 2005\}$
40. $\{1996, 1997, 1998, 1999, 2000\}$
41. $\{1998, 1999, 2000\}$
42. $\{\ \}$ or $\varnothing$
43. $B = \{x | x \in N \text{ and } 4 < x < 13\}$ or $B = \{x | x \in N \text{ and } 5 \leq x \leq 12\}$
44. $A = \{x | x \in N \text{ and } x < 10\}$ or $A = \{x | x \in N \text{ and } x \leq 9\}$
45. $C = \{x | x \in N \text{ and } x \text{ is a multiple of } 3\}$
46. $D = \{x | x \in N \text{ and } x \text{ is a multiple of } 5\}$
47. $E = \{x | x \in N \text{ and } x \text{ is odd}\}$
48. $A = \{x | x \text{ is Independence Day}\}$
49. $C = \{x | x \text{ is February}\}$
50. $F = \{x | x \in N \text{ and } 14 < x < 101\}$ or $F = \{x | x \in N \text{ and } 15 \leq x \leq 100\}$
51. Set A is the set of natural numbers less than or equal to 7.
52. Set D is the set of natural numbers that are multiples of 3.
53. Set V is the set of vowels in the English alphabet.
54. Set S is the set of the seven dwarfs in *Snow White and the Seven Dwarfs*.
55. Set T is the set of species of trees.
56. Set E is the set of natural numbers greater than or equal to 4 and less than 11.
57. Set S is the set of seasons.
58. Set B is the set of members of the Beatles.

59. $\{$Johnson & Johnson, Google, Home Depot$\}$
60. $\{$United Airlines, McDonald's$\}$
61. $\{$United Airlines$\}$
62. $\{$Johnson & Johnson, Google, Home Depot, Target, Coca-Cola, Toyota, Microsoft, Southwest Airlines$\}$
63. $\{1996, 1997, 1998, 1999\}$
64. $\{1996, 1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004\}$
65. $\{1998, 1999, 2000, 2001, 2002, 2003, 2004\}$
66. $\{\ \}$ or $\varnothing$
67. False; $\{e\}$ is a set, and not an element of the set.
68. True; b is an element of the set.
69. False; h is not an element of the set.
70. True; Mickey Mouse is an element of the set.
71. False; 3 is an element of the set.
72. False; the capital of Hawaii is Honolulu, not Maui.
73. True; *Titanic* is an element of the set.
74. False; 2 is an even natural number.
75. $n(A) = 4$
76. $n(B) = 6$
77. $n(C) = 0$
78. $n(D) = 5$
79. Both; A and B contain exactly the same elements.
80. Equivalent; both sets contain the same number of elements, 3.
81. Neither; the sets have a different number of elements.
82. Neither; not all cats are Siamese.

83. Equivalent; both sets contain the same number of elements, 3.
84. Equivalent; both sets contain the same number of elements, 50.
85. a) Set A is the set of natural numbers greater than 2. Set B is the set of all numbers greater than 2.
    b) Set A contains only natural numbers. Set B contains other types of numbers, including fractions and decimal numbers.
    c) $A = \{3, 4, 5, 6, \ldots\}$
    d) No

86. a) Set A is the set of natural numbers greater than 2 and less than or equal to 5. Set B is the set of numbers greater than 2 and less than or equal to 5.
    b) Set A contains only natural numbers. Set B contains other types of numbers, including fractions and decimal numbers.
    c) $A = \{3, 4, 5\}$
    d) No

87. Cardinal; 12 tells how many.
88. Ordinal; 25 tells the relative position of the chart.
89. Ordinal; sixteenth tells Lincoln's relative position.
90. Cardinal; 35 tells how many dollars she spent.

91. Answers will vary.
92. Answers will vary. Examples: the set of people in the class who were born on the moon, the set of automobiles that get 400 miles on a gallon of gas, the set of fish that can talk
93. Answers will vary.
94. Answers will vary. Here are some examples.
    a) The set of men. The set of actors. The set of people over 12 years old. The set of people with two legs. The set of people who have been in a movie.
    b) The set of all the people in the world.

Exercise Set 2.2

1. Set A is a **subset** of set B, symbolized by $A \subseteq B$, if and only if all the elements of set A are also elements of set B.
2. Set A is a **proper subset** of set B, symbolized by $A \subset B$, if and only if all the elements of set A are also elements of set B and set $A \neq$ set B.
3. If $A \subseteq B$, then every element of set A is also an element of set B. If $A \subset B$, then every element of set A is also an element of set B and set $A \neq$ set B.
4. 2^n, where n is the number of elements in the set.
5. $2^n - 1$, where n is the number of elements in the set.
6. No, if two sets are equal one cannot be a proper subset of the other.

7. False; Spanish is an element of the set, not a subset.
8. False; the empty set is a subset of $\{$salt, pepper, basil, garlic powder$\}$.

9. True; the empty set is a subset of every set.
10. False; red is an element of the set, not a proper subset.

11. True; 5 is not an element of $\{2,4,6\}$.
12. False; social worker is not in the second set.

13. False; the set $\{\varnothing\}$ contains the element $\varnothing$.

14. True; $\{$ motorboat, kayak $\}$ is a subset of $\{$ kayak, fishing boat, sailboat, motorboat $\}$.

15. True; $\{\ \}$ and $\varnothing$ each represent the empty set.

16. False; 0 is a number and $\{\ \}$ is a set.

17. False; the set $\{0\}$ contains the element 0.

18. True; $\{3,8,11\}$ is a subset of $\{3,8,11\}$.

19. False; $\{$swimming$\}$ is a set, not an element.

20. True; $\{3,5,9\}=\{3,9,5\}$.

21. True; the empty set is a subset of every set, including itself.

22. True; the elements of the set are themselves sets.

23. False; no set is a proper subset of itself.

24. True; $\{b,a,t\}$ is a subset of $\{t,a,b\}$.

25. $B\subseteq A, B\subset A$

26. $A=B, A\subseteq B, B\subseteq A$

27. $A\subseteq B,\ A\subset B$

28. None

29. $B\subseteq A, B\subset A$

30. $B\subseteq A, B\subset A$

31. $A=B, A\subseteq B, B\subseteq A$

32. $B\subseteq A, B\subset A$

33. $\{\ \}$ is the only subset.

34. $\{\ \},\{\bigcirc\}$

35. $\{\ \},\{$pen$\},\{$pencil$\},\{$pen, pencil$\}$

36. $\{\ \},\{$ steak $\},\{$ pork $\},\{$ chicken$\},\{$ steak, pork $\}$, $\{$ steak,chicken $\},\{$ pork, chicken $\}$, $\{$ steak, pork, chicken $\}$

37. a) $\{\ \},\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},$ $\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},$ $\{a,c,d\},\{b,c,d\},\{a,b,c,d\}$
    b) All the sets in part (a) are proper subsets of A except $\{a,b,c,d\}$.

38. a) $2^9=2\times2\times2\times2\times2\times2\times2\times2\times2=512$ subsets
    b) $2^9-1=512-1=511$ proper subsets

39. False; A could be equal to B.

40. True; every proper subset is a subset.

41. True; every set is a subset of itself.

42. False; no set is a proper subset of itself.

43. True; $\varnothing$ is a proper subset of every set except itself.

44. True; $\varnothing$ is a subset of every set.

45. True; every set is a subset of the universal set.

46. False; a set cannot be a proper subset of itself.

47. True; $\varnothing$ is a proper subset of every set except itself and $U\neq\varnothing$.

48. False; the only subset of $\varnothing$ is itself and $U\neq\varnothing$.

49. True; $\varnothing$ is a subset of every set.

50. False; U is not a subset of $\varnothing$.

51. The number of different variations of the house is equal to the number of subsets of $\{$deck, jacuzzi, security system, hardwood flooring$\}$, which is $2^4=2\times2\times2\times2=16$.

52. The number of options is equal to the number of subsets of $\{$ cucumber, onion, tomato, carrot, green pepper, olive, mushroom $\}$, which is $2^7=2\times2\times2\times2\times2\times2\times2=128$.

53. The number of different variations is equal to the number of subsets of $\{$call waiting, call forwarding, caller identification, three way calling, voice mail, fax line$\}$, which is $2^6=2\times2\times2\times2\times2\times2=64$.

54. The number of variations is equal to the number of subsets of $\{$ketchup, mustard, relish, hot sauce, onions, lettuce, tomato$\}$, which is $2^7=2\times2\times2\times2\times2\times2\times2=128$.

55. $E=F$ since they are both subsets of each other.

56. If there is a one-to-one correspondence between boys and girls, then the sets are equivalent.

57. a) Yes.
b) No, c is an element of set D.
c) Yes, each element of $\{a,b\}$ is an element of set D.

58. a) Each person has 2 choices, namely yes or no. $2\times2\times2\times2=16$
b) YYYY, YYYN, YYNY, YNYY, NYYY, YYNN, YNYN, YNNY, NYNY, NNYY, NYYN, YNNN, NYNN, NNYN, NNNY, NNNN
c) 5 out of 16

59. A one element set has one proper subset, namely the empty set. A one element set has two subsets, namely itself and the empty set. One is one-half of two. Thus, the set must have one element.

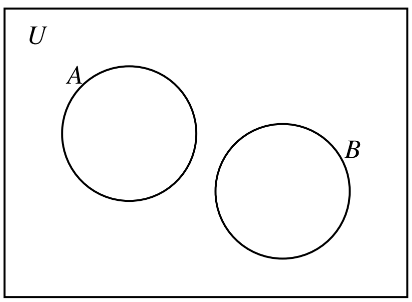
60. Yes

61. Yes

62. No

<u>Section 2.3</u>

1.



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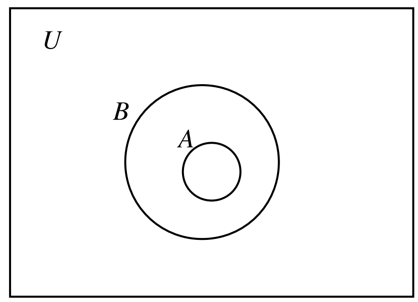

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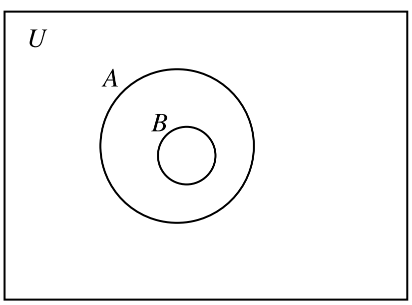

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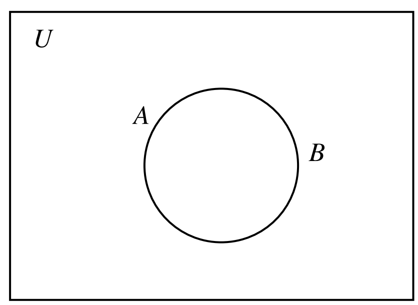

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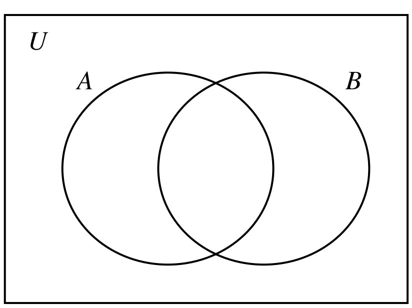
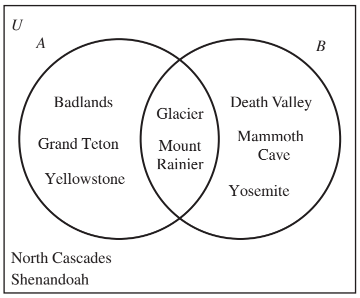


6. Combine the elements from set A and set B into one set. List any element that is contained in both sets only once.
7. Determine the elements that are in the universal set that are not in set A.
8. I, II, III
9. Select the elements common to both set A and set B.

10. II
11. a) *Or* is generally interpreted to mean *union*.
    b) *And* is generally interpreted to mean *intersection*.
12. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
13. The difference of two sets A and B is the set of elements that belong to set A but not to set B.
14. a) The Cartesian product of set A and set B is the set of all ordered pairs of the form (a, b), where $a \in A$ and $b \in B$.
    b) $m \times n$
15.



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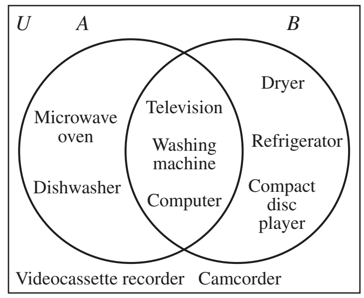
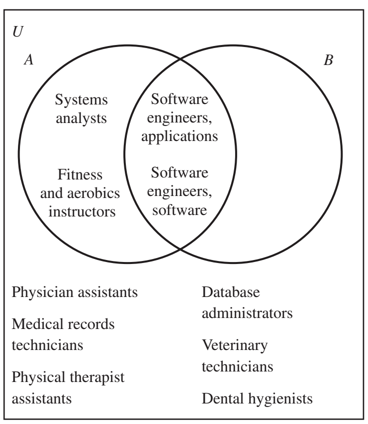


17.



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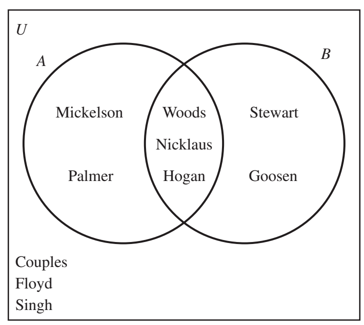


19. The set of animals in U.S. zoos that are not in the San Diego Zoo.
20. The set of U.S. colleges and universities that are not in the state of Mississippi
21. The set of insurance companies in the U.S. that do not offer life insurance
22. The set of insurance companies in the U.S. that do not offer car insurance
23. The set of insurance companies in the U.S. that offer life insurance or car insurance
24. The set of insurance companies in the U.S. that offer life insurance and car insurance
25. The set of insurance companies in the U.S. that offer life insurance and do not offer car insurance
26. The set of insurance companies in the U.S. that offer life insurance or do not offer car insurance
27. The set of furniture stores in the U.S. that sell mattresses and outdoor furniture
28. The set of furniture stores in the U.S. that sell mattresses or leather furniture
29. The set of furniture stores in the U.S. that do not sell outdoor furniture and sell leather furniture
30. The set of furniture stores in the U.S. that sell mattresses, outdoor furniture, and leather furniture
31. The set of furniture stores in the U.S. that sell mattresses or outdoor furniture or leather furniture
32. The set of furniture stores in the U.S. that do not sell mattresses or do not sell leather furniture
33. $A = \{w, b, c, t, a, h\}$
34. $B = \{a, h, f, r, d, g\}$
35. $A \cap B = \{w, b, c, t, a, h\} \cap \{a, h, f, r, d, g\} = \{a, h\}$

36. $U=\{w,b,c,t,a,h,f,r,d,g,p,m,z\}$

37. $A\cup B=\{w,b,c,t,a,h\}\cup\{a,h,f,r,d,g\}=\{w,b,c,t,a,h,f,r,d,g\}$

38. $(A\cup B)'$: From #37, $A\cup B=\{w,b,c,t,a,h,f,r,d,g\}$. $(A\cup B)'=\{w,b,c,t,a,h,f,r,d,g\}'=\{p,m,z\}$

39. $A'\cap B'=\{w,c,b,t,a,h\}\{a,h,f,r,d,g\}\cap\{w,c,b,t,p,m,z\}=\{p,m,z\}$

40. $(A\cap B)'$: From #35, $A\cap B=\{a,h\}$. $(A\cap B)'=\{a,h\}'=\{w,c,b,t,f,r,d,g,p,m,z\}$

41. $A=\{\text{L},\Delta,@,*,\$\}$

42. $B=\{*,\$,\text{R},\square,\alpha\}$

43. $U=\{\text{L},\Delta,@,*,\$,\text{R},\square,\alpha,\infty,\text{Z},\Sigma\}$

44. $A\cap B=\{\text{L},\Delta,@,*,\$\}\cap\{*,\$,\text{R},\square,\alpha\}=\{*,\$\}$

45. $A'\cup B=\{\text{R},\square,\alpha,\infty,\text{Z},\Sigma\}\cup\{*,\$,\text{R},\square,\alpha\}=\{\text{R},\square,\alpha,\infty,\text{Z},\Sigma,*,\$\}$

46. $A\cup B'=\{\text{L},\Delta,@,*,\$\}\cup\{*,\$,\text{R},\square,\alpha\}'=\{\text{L},\Delta,@,*,\$\}\cup\{\text{L},\Delta,@,\infty,\Sigma,\text{Z}\}=\{\text{L},\Delta,@,*,\$,\infty,\Sigma,\text{Z}\}$

47. $A'\cap B=\{\text{L},\Delta,@,*,\$\}'\cap\{*,\$,\text{R},\square,\alpha\}=\{\text{R},\square,\alpha,\infty,\text{Z},\Sigma\}\cap\{*,\$,\text{R},\square,\alpha\}=\{\text{R},\square,\alpha\}$

48. $(A\cup B)'$: From the diagram, $(A\cup B)'=\{\infty,\text{Z},\Sigma\}$

49. $A\cup B=\{1,2,4,5,8\}\cup\{2,3,4,6\}=\{1,2,3,4,5,6,8\}$

50. $A\cap B=\{1,2,4,5,8\}\cap\{2,3,4,6\}=\{2,4\}$

51. $B'=\{2,3,4,6\}'=\{1,5,7,8\}$

52. $A\cup B'=\{1,2,4,5,8\}\cup\{2,3,4,6\}'=\{1,2,4,5,8\}\cup\{1,5,7,8\}=\{1,2,4,5,7,8\}$

53. $(A\cup B)'$ From #49, $A\cup B=\{1,2,3,4,5,6,8\}$. $(A\cup B)'=\{1,2,3,4,5,6,8\}'=\{7\}$

54. $A'\cap B'=\{1,2,4,5,8\}'\cap\{2,3,4,6\}'=\{3,6,7\}\cap\{1,5,7,8\}=\{7\}$

55. $(A\cup B)'\cap B$: From #53, $(A\cup B)'=\{7\}$. $(A\cup B)'\cap B=\{7\}\cap\{2,3,4,6\}=\{\ \}$

56. $(A\cup B)\cap(A\cup B)'=\{\ \}$ (The intersection of a set and its complement is always empty.)

57. $(B\cup A)'\cap(B'\cup A')$: From #53, $(A\cup B)'=(B\cup A)'=\{7\}$.

$(B\cup A)'\cap(B'\cup A')=\{7\}\cap\left(\{2,3,4,6\}'\cup\{1,2,4,5,8\}'\right)=\{7\}\cap(\{1,5,7,8\}\cup\{3,6,7\})$

$=\{7\}\cap\{1,3,5,6,7,8\}=\{7\}$

58. $A'\cup(A\cap B)$: From #50, $A\cap B=\{2,4\}$. $A'\cup(A\cap B)=\{1,2,4,5,8\}'\cup\{2,4\}=\{3,6,7\}\cup\{2,4\}=\{2,3,4,6,7\}$

59. $B'=\{b,c,d,f,g\}'=\{a,e,h,i,j,k\}$

60. $B\cup C=\{b,c,d,f,g\}\cup\{a,b,f,i,j\}=\{a,b,c,d,f,g,i,j\}$

61. $A\cap C=\{a,c,d,f,g,i\}\cap\{a,b,f,i,j\}=\{a,f,i\}$

62. $A'\cup B'$: $A'=\{b,e,h,j,k\}$, $B'=\{a,e,h,i,j,k\}$. $A'\cup B'=\{b,e,h,j,k\}\cup\{a,e,h,i,j,k\}=\{a,b,e,h,i,j,k\}$

63. $(A\cap C)'$: From #61, $A\cap C=\{a,f,i\}$. $(A\cap C)'=\{a,f,i\}'=\{b,c,d,e,g,h,j,k\}$

64. $(A\cap B)\cup C=(\{a,c,d,f,g,i\}\cap\{b,c,d,f,g\})\cup\{a,b,f,i,j\}=\{c,d,f,g\}\cup\{a,b,f,i,j\}$

$=\{a,b,c,d,f,g,i,j\}$

65. $A\cup(C\cap B)'=\{a,c,d,f,g,i\}\cup(\{a,b,f,i,j\}\cap\{b,c,d,f,g\})'=\{a,c,d,f,g,i\}\cup\{b,f\}'$
$=\{a,c,d,f,g,i\}\cup\{a,c,d,e,g,h,i,j,k\}=\{a,c,d,e,f,g,h,i,j,k\}$

66. $A\cup(C'\cup B')=\{a,c,d,f,g,i\}\cup\left(\{a,b,f,i,j\}'\cup\{b,c,d,f,g\}'\right)$
$=\{a,c,d,f,g,i\}\cup(\{c,d,e,g,h,k\}\cup\{a,e,h,i,j,k\})=\{a,c,d,f,g,i\}\cup\{a,c,d,e,g,h,i,j,k\}$
$=\{a,c,d,e,f,g,h,i,j,k\}$

67. $(A'\cup C)\cup(A\cap B)=\left(\{a,c,d,f,g,i\}'\cup\{a,b,f,i,j\}\right)\cup(\{a,c,d,f,g,i\}\cap\{b,c,d,f,g\})$
$=(\{b,e,h,j,k\}\cup\{a,b,f,i,j\})\cup\{c,d,f,g\}=\{a,b,e,f,h,i,j,k\}\cup\{c,d,f,g\}$
$=\{a,b,c,d,e,f,g,h,i,j,k\}$, or U

68. $(C\cap B)\cap(A'\cap B)$: From #65, $C\cap B=\{b,f\}$.
$(C\cap B)\cap(A'\cap B)=\{b,f\}\cap\left(\{a,c,d,f,g,i\}'\cap\{b,c,d,f,g\}\right)=\{b,f\}\cap(\{b,e,h,j,k\}\cap\{b,c,d,f,g\})$
$=\{b,f\}\cap\{b\}=\{b\}$

For exercises 69-76: $U=\{1,2,3,4,5,6,7,8,9,10\}$, $A=\{1,2,4,6,9\}$, $B=\{1,3,4,5,8\}$, $C=\{4,5,9\}$

69. $A-B=\{1,2,4,6,9\}-\{1,3,4,5,8\}=\{2,6,9\}$

70. $A-C=\{1,2,4,6,9\}-\{4,5,9\}=\{1,2,6\}$

71. $A-B'$: This leaves only $A\cap B$, which is $\{1,4\}$

72. $A'-C=\{3,5,7,8,10\}-\{4,5,9\}=\{3,7,8,10\}$

73. $(A-B)'=\{2,6,9\}'=\{1,3,4,5,7,8,10\}$

74. $(A-B)'-C=\{1,3,4,5,7,8,10\}-\{4,5,9\}=\{1,3,7,8,10\}$

75. $C-A'=\{4,5,9\}-\{3,5,7,8,10\}=\{4,9\}$

76. $(C-A)'-B=\{5\}'-\{1,3,4,5,8\}=\{2,6,7,9,10\}$

For exercises 77-82: $A=\{a,b,c\}$ and $B=\{1,2\}$

77. $\{(a,1),(a,2),(b,1),(b,2),(c,1),(c,2)\}$

78. $\{(1,a),(1,b),(1,c),(2,a),(2,b),(2,c)\}$

79. No; the ordered pairs are not the same.

80. 6

81. 6

82. Yes

83. $A\cap B=\{1,3,5,7,9\}\cap\{2,4,6,8\}=\{\ \}$

84. $A\cup B=\{1,3,5,7,9\}\cup\{2,4,6,8\}=\{1,2,3,4,5,6,7,8,9\}$, or U

85. $A'\cup B=\{1,3,5,7,9\}'\cup\{2,4,6,8\}=\{2,4,6,8\}\cup\{2,4,6,8\}=\{2,4,6,8\}$, or B

86. $(B\cup C)'=(\{2,4,6,8\}\cup\{1,2,3,4,5\})'=\{1,2,3,4,5,6,8\}'=\{7,9\}$

87. $A\cap C'=\{1,3,5,7,9\}\cap\{1,2,3,4,5\}'=\{1,3,5,7,9\}\cap\{6,7,8,9\}=\{7,9\}$

88. $A\cap B'=\{1,3,5,7,9\}\cap\{2,4,6,8\}'=\{1,3,5,7,9\}\cap\{1,3,5,7,9\}=\{1,3,5,7,9\}$, or A

89. $(B \cap C)' = (\{2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5\})' = \{2, 4\}' = \{1, 3, 5, 6, 7, 8, 9\}$

90. $(A \cup C) \cap B = (\{1, 3, 5, 7, 9\} \cup \{1, 2, 3, 4, 5\}) \cap \{2, 4, 6, 8\} = \{1, 2, 3, 4, 5, 7, 9\} \cap \{2, 4, 6, 8\} = \{2, 4\}$

91. $(C' \cup A) \cap B = \left(\{1, 2, 3, 4, 5\}' \cup \{1, 3, 5, 7, 9\}\right) \cap \{2, 4, 6, 8\} = (\{6, 7, 8, 9\} \cup \{1, 3, 5, 7, 9\}) \cap \{2, 4, 6, 8\}$

$= \{1, 3, 5, 6, 7, 8, 9\} \cap \{2, 4, 6, 8\} = \{6, 8\}$

92. $(C \cap B) \cup A$: From #89, $C \cap B = \{2, 4\}$. $(C \cap B) \cup A = \{2, 4\} \cup \{1, 3, 5, 7, 9\} = \{1, 2, 3, 4, 5, 7, 9\}$

93. $(A \cap B)' \cup C$: From #83, $A \cap B = \{\ \}$.

$(A \cap B)' \cup C = \{\ \}' \cup \{1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \cup \{1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, or U

94. $(A' \cup C) \cap B = \left(\{1, 3, 5, 7, 9\}' \cup \{1, 2, 3, 4, 5\}\right) \cap \{2, 4, 6, 8\} = (\{2, 4, 6, 8\} \cup \{1, 2, 3, 4, 5\}) \cap \{2, 4, 6, 8\}$

$= \{1, 2, 3, 4, 5, 6, 8\} \cap \{2, 4, 6, 8\} = \{2, 4, 6, 8\}$, or B

95. $(A' \cup B') \cap C = \left(\{1, 3, 5, 7, 9\}' \cup \{2, 4, 6, 8\}'\right) \cap \{1, 2, 3, 4, 5\}$

$= (\{2, 4, 6, 8\} \cup \{1, 3, 5, 7, 9\}) \cap \{1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \cap \{1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5\}$, or C

96. $(A' \cap C) \cup (A \cap B)$: From #83, $A \cap B = \{\ \}$.

$(A' \cap C) \cup (A \cap B) = \left(\{1, 3, 5, 7, 9\}' \cap \{1, 2, 3, 4, 5\}\right) \cup \{\ \} = (\{2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5\}) \cup \{\ \}$

$= \{2, 4\} \cup \{\ \} = \{2, 4\}$

97. A set and its complement will always be disjoint since the complement of a set is all of the elements in the universal set that are not in the set. Therefore, a set and its complement will have no elements in common.
For example, if $U = \{1, 2, 3\}, A = \{1, 2\}$, and $A' = \{3\}$, then $A \cap A' = \{\ \}$.

98. $n(A \cap B) = 0$ when A and B are disjoint sets. For example, if $U = \{1, 2, 3, 4, 5, 6\}, A = \{1, 3\}$, $B = \{2, 4\}$, then $A \cap B = \{\ \}$. $n(A \cap B) = 0$

99. Let $A = \{$customers who owned dogs$\}$ and $B = \{$customers who owned cats$\}$.

$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 27 + 38 - 16 = 49$

100. Let $A = \{$students who sang in the chorus$\}$ and $B = \{$students who played in the stage band$\}$.

$$
\begin{aligned}
n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\
46 &= n(A) + 30 - 4 \\
46 &= n(A) + 26 \\
20 &= n(A)
\end{aligned}
$$

101. a) $A \cup B = \{a, b, c, d\} \cup \{b, d, e, f, g, h\} = \{a, b, c, d, e, f, g, h\}$, $n(A \cup B) = 8$,

$A \cap B = \{a, b, c, d\} \cap \{b, d, e, f, g, h\} = \{b, d\}, n(A \cap B) = 2$.

$n(A) + n(B) - n(A \cap B) = 4 + 6 - 2 = 8$

Therefore, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

b) Answers will vary.

c) Elements in the intersection of A and B are counted twice in $n(A) + n(B)$.

102. $A \cap B'$ defines Region I. $A \cap B$ defines Region II. $A' \cap B$ defines Region III.

$A' \cap B'$ or $(A \cup B)'$ defines Region IV.

103. $A \cup B = \{1, 2, 3, 4, \ldots\} \cup \{4, 8, 12, 16, \ldots\} = \{1, 2, 3, 4, \ldots\}$, or A

104. $A \cap B = \{1, 2, 3, 4, \ldots\} \cap \{4, 8, 12, 16, \ldots\} = \{4, 8, 12, 16, \ldots\}$, or B

105. $B \cap C = \{4, 8, 12, 16, \ldots\} \cap \{2, 4, 6, 8, \ldots\} = \{4, 8, 12, 16, \ldots\}$, or B

106. $B \cup C = \{4, 8, 12, 16, \ldots\} \cup \{2, 4, 6, 8, \ldots\} = \{2, 4, 6, 8, \ldots\}$, or C

107. $A \cap C = \{1, 2, 3, 4, \ldots\} \cap \{2, 4, 6, 8, \ldots\} = \{2, 4, 6, 8, \ldots\}$, or C

108. $A' \cap C = \{1, 2, 3, 4, \ldots\}' \cap \{2, 4, 6, 8, \ldots\} = \{0\} \cap \{2, 4, 6, 8, \ldots\} = \{\ \}$

109. $B' \cap C = \{4, 8, 12, 16, \ldots\}' \cap \{2, 4, 6, 8, \ldots\} = \{0, 1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15, \ldots\} \cap \{2, 4, 6, 8, \ldots\}$
$= \{2, 6, 10, 14, 18, \ldots\}$

110. $(B \cup C)' \cup C$: From #106, $B \cup C = C$. $(B \cup C)' \cup C = C' \cup C = \{2, 4, 6, 8, \ldots\}' \cup \{2, 4, 6, 8, \ldots\}$
$= \{0, 1, 2, 3, 4, \ldots\}$, or U

111. $(A \cap C) \cap B'$: From #107, $A \cap C = C$. $(A \cap C) \cap B' = C \cap B'$.
From #109, $B' \cap C = C \cap B' = \{2, 6, 10, 14, 18, \ldots\}$

112. $U' \cap (A \cup B)$: From #103, $A \cup B = A$. $U' \cap (A \cup B) = U' \cap A = \{\ \} \cap \{1, 2, 3, 4, \ldots\} = \{\ \}$

113. $A \cup A' = U$

114. $A \cap A' = \varnothing$

115. $A \cup \varnothing = A$

116. $A \cap \varnothing = \varnothing$

117. $A' \cup U = U$

118. $A \cap U = A$

119. $A \cup U = U$

120. $A \cup U' = A \cup \{\ \} = A$

121. If $A \cap B = B$, then $B \subseteq A$.

122. If $A \cup B = B$, then $A \subseteq B$.

123. If $A \cap B = \varnothing$, then A and B are disjoint sets.

124. If $A \cup B = A$, then $B \subseteq A$.

125. If $A \cap B = A$, then $A \subseteq B$.

126. If $A \cup B = \varnothing$, then $A = \varnothing$ and $B = \varnothing$.
Therefore, they are equal sets.

Exercise Set 2.4

1. 8
2. Region V, the intersection of all three sets
3. Regions II, IV, VI
4. $B \cap C$ is represented by regions V and VI. If $B \cap C$ contains 12 elements and region V contains 4 elements, then region VI contains $12 - 4 = 8$ elements.
5. $A \cap B$ is represented by regions II and V. If $A \cap B$ contains 9 elements and region V contains 4 elements, then region II contains $9 - 4 = 5$ elements.
6. $(A \cup B)' = A' \cap B'; (A \cap B)' = A' \cup B'$

7. a) Yes

$$A \cup B = \{1,4,5\} \cup \{1,4,5\} = \{1,4,5\}$$

$$A \cap B = \{1,4,5\} \cap \{1,4,5\} = \{1,4,5\}$$

b) No

c) No

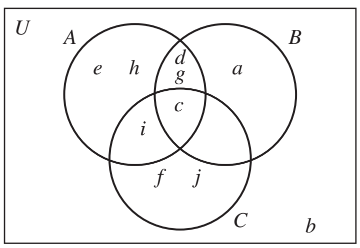
c)

$A \cup B$		$A \cap B$	
Set	Regions	Set	Regions
A	I, II	A	I, II
B	II, III	B	II, III
$A \cup B$	I, II, III	$A \cap B$	II

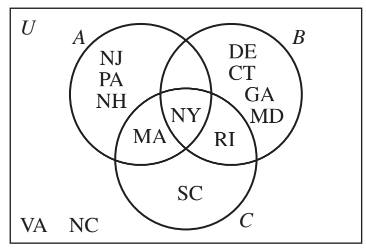
Since the two statements are not represented by the same regions, $A \cup B \neq A \cap B$ for all sets A and B.

8. Deductive reasoning.

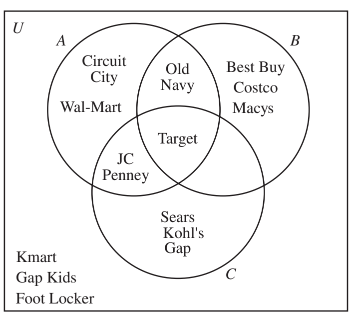
9.



10.



11.



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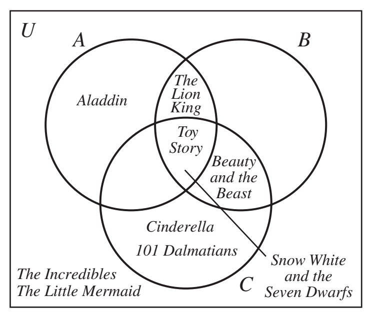
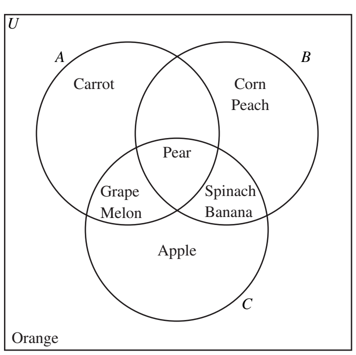
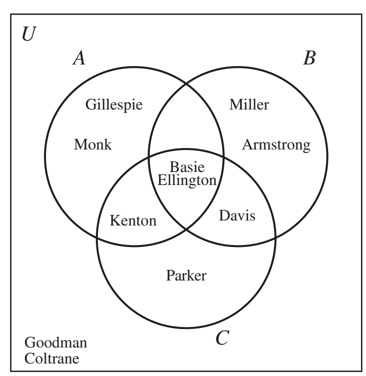


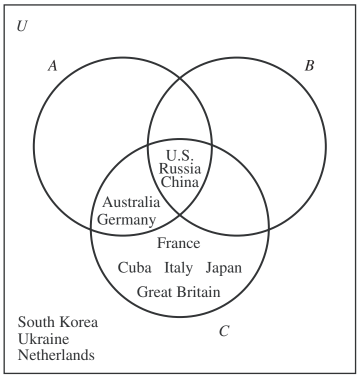
13.



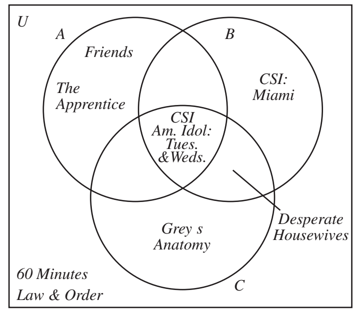
14.



15.



16.



17. Mayo Clinic, V
18. UCLA Medical Center, IV
19. Methodist Hospital, I
20. Barnes-Jewish, II
21. Brigham and Women's Hospital, III
22. Yale–New Haven Hospital, VIII
23. Department of Energy, II
24. National Aeronautics and Space Administration, V
25. Department of Justice, VIII
26. Small Business Administration, VII
27. Department of Agriculture, III
28. Office of Personnel Management, VI

29. VI
30. VIII
31. III
32. IV
33. III
34. I
35. V
36. III
37. II
38. VIII
39. VII
40. VI
41. I
42. VII
43. VIII
44. V
45. VI
46. III

47. $A = \{1, 3, 4, 5, 7, 9\}$
48. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$
49. $B = \{2, 3, 4, 5, 6, 8, 12, 14\}$
50. $C = \{4, 5, 6, 7, 8, 10\}$
51. $A \cap B = \{3, 4, 5\}$
52. $A \cap C = \{4, 5, 7\}$
53. $(B \cap C)' = \{1, 2, 3, 7, 9, 10, 11, 12, 13, 14\}$
54. $A \cap B \cap C = \{4, 5\}$
55. $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 14\}$
56. $B \cup C = \{2, 3, 4, 5, 6, 7, 8, 10, 12, 14\}$
57. $(A \cup C)' = \{2, 11, 12, 13, 14\}$
58. $A \cap (B \cup C) = \{3, 4, 5, 7\}$
59. $A' = \{2, 6, 8, 10, 11, 12, 13, 14\}$
60. $(A \cup B \cup C)' = \{11, 13\}$

61. $(A \cup B)'$ $A' \cap B'$

Set	Regions	Set	Regions
A	I, II	A	I, II
B	II, III	A'	III, IV
$A \cup B$	I, II, III	B	II, III
$(A \cup B)'$	IV	B'	I, IV
		$A' \cap B'$	IV

Both statements are represented by the same region, IV, of the Venn diagram. Therefore, $(A \cup B)' = A' \cap B'$ for all sets A and B.

62. $(A \cap B)'$ $A' \cup B$

Set	Regions	Set	Regions
A	I, II	A	I, II
B	II, III	A'	III, IV
$A \cap B$	II	B	II, III
$(A \cap B)'$	I, III, IV	$A' \cup B$	II, III, IV

Since the two statements are not represented by the same regions, it is not true that $(A \cap B)' = A' \cup B$ for all sets A and B.

63. $A' \cup B'$ $A \cap B$

Set	Regions	Set	Regions
A	I, II	A	I, II
A'	III, IV	B	II, III
B	II, III	$A \cap B$	II
B'	I, IV		
$A' \cup B'$	I, III, IV		

Since the two statements are not represented by the same regions, it is not true that $A' \cup B' = A \cap B$ for all sets A and B.

64. $(A \cup B)'$ $(A \cap B)'$

Set	Regions	Set	Regions
A	I, II	A	I, II
B	II, III	B	II, III
$A \cup B$	I, II, III	$A \cap B$	II
$(A \cup B)'$	IV	$(A \cap B)'$	I, III, IV

Since the two statements are not represented by the same regions, it is not true that $(A \cup B)' = (A \cap B)'$ for all sets A and B.

65. $A' \cap B'$ $(A \cap B)'$

Set	Regions	Set	Regions
A	I, II	A	I, II
A'	III, IV	B	II, III
B	II, III	$A \cap B$	II
B'	I, IV	$(A \cap B)'$	1, III, IV
$A' \cap B'$	IV		

Since the two statements are not represented by the same regions, it is not true that $A' \cap B' = (A \cap B)'$ for all sets A and B.

66. $A' \cap B'$ $A \cup B'$

Set	Regions	Set	Regions
A	I, II	A	I, II
A'	III, IV	B	II, III
B	II, III	B'	I, IV
B'	I, IV	$A \cup B'$	I, II, IV
$A' \cap B'$	IV		

Since the two statements are not represented by the same regions, it is not true that $A' \cap B' = A \cup B'$ for all sets A and B.

67. $(A' \cap B)'$ $A \cup B'$

Set	Regions
A	I, II
A'	III, IV
B	II, III
$A' \cap B$	III
$(A' \cap B)'$	I, II, IV

Set	Regions
A	I, II
B	II, III
B'	I, IV
$A \cup B'$	I, II, IV

Both statements are represented by the same regions, I, II, IV, of the Venn diagram. Therefore, $(A' \cap B)' = A \cup B'$ for all sets A and B.

68. $A' \cap B'$ $(A' \cap B')'$

Set	Regions
A	I, II
A'	III, IV
B	II, III
B'	I, IV
$A' \cap B'$	IV

Set	Regions
A	I, II
A'	III, IV
B	II, III
B'	I, IV
$A' \cap B'$	IV
$(A' \cap B')'$	I, II, III

Since the two statements are not represented by the same regions, it is not true that $A' \cap B' = (A' \cap B')'$ for all sets A and B.

69. $A \cap (B \cup C)$ $(A \cap B) \cup C$

Set	Regions
B	II, III, V, VI
C	IV , V, VI, VII
$B \cup C$	II, III, IV, V, VI, VII
A	I, II, IV, V
$A \cap (B \cup C)$	II, IV, V

Set	Regions
A	I, II, IV, V
B	II, III, V, VI
$A \cap B$	II, V
C	IV, V, VI, VII
$(A \cap B) \cup C$	II, IV, V, VI, VII

Since the two statements are not represented by the same regions, it is not true that $A \cap (B \cup C) = (A \cap B) \cup C$ for all sets $A, B,$ and C.

70. $A \cup (B \cap C)$ $(B \cap C) \cup A$

Set	Regions
B	II, III, V, VI
C	IV , V, VI, VII
$B \cap C$	V, VI
A	I, II, IV, V
$A \cup (B \cap C)$	I, II, IV, V, VI

Set	Regions
B	II, III, V, VI
C	IV, V, VI, VII
$B \cap C$	V, VI
A	I, II, IV, V
$(B \cap C) \cup A$	I, II, IV, V, VI

Both statements are represented by the same regions, I, II, IV, V, VI, of the Venn diagram. Therefore, $A \cup (B \cap C) = (B \cap C) \cup A$ for all sets $A, B,$ and C.

71. $A \cap (B \cup C)$ $(B \cup C) \cap A$

Set	Regions
B	II, III, V, VI
C	IV , V, VI, VII
$B \cup C$	II, III, IV, V, VI, VII
A	I, II, IV, V
$A \cap (B \cup C)$	II, IV, V

Set	Regions
B	II, III, V, VI
C	IV, V, VI, VII
$B \cup C$	II, III, IV, V, VI, VII
A	I, II, IV, V
$(B \cup C) \cap A$	II, IV, V

Both statements are represented by the same regions, II, IV, V, of the Venn diagram. Therefore, $A \cap (B \cup C) = (B \cup C) \cap A$ for all sets $A, B,$ and C.

72. $A\cup(B\cap C)'$ $A'\cap(B\cup C)$

Set	Regions	Set	Regions
B	II, III, V, VI	B	II, III, V, VI
C	IV , V, VI, VII	C	IV, V, VI, VII
$B\cap C$	V, VI	$B\cup C$	II, III, IV, V, VI, VII
$(B\cap C)'$	I, II, III, IV, VII, VIII	A	I, II, IV, V
A	I, II, IV, V	A'	III, VI, VII, VIII
$A\cup(B\cap C)'$	I, II, III, IV, V, VII, VIII	$A'\cap(B\cup C)$	III, VI, VII

Since the two statements are not represented by the same regions, it is not true that $A\cup(B\cap C)'=A'\cap(B\cup C)$ for all sets $A, B,$ and C.

73. $A\cap(B\cup C)$ $(A\cap B)\cup(A\cap C)$

Set	Regions	Set	Regions
B	II, III, V, VI	A	I, II, IV, V
C	IV, V, VI, VII	B	II, III, V, VI
$B\cup C$	II, III, IV, V, VI, VII	$A\cap B$	II, V
A	I, II, IV, V	C	IV, V, VI, VII
$A\cap(B\cup C)$	II, IV, V	$A\cap C$	IV, V
		$(A\cap B)\cup(A\cap C)$	II, IV, V

Both statements are represented by the same regions, II, IV, V, of the Venn diagram.
Therefore, $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$ for all sets $A, B,$ and C.

74. $A\cup(B\cap C)$ $(A\cup B)\cap(A\cup C)$

Set	Regions	Set	Regions
B	II, III, V, VI	A	I, II, IV, V
C	IV, V, VI, VII	B	II, III, V, VI
$B\cap C$	V, VI	$A\cup B$	I, II, III, IV, V, VI
A	I, II, IV, V	C	IV, V, VI, VII
$A\cup(B\cap C)$	I, II, IV, V, VI	$A\cup C$	I, II, IV, V, VI, VII
		$(A\cup B)\cap(A\cup C)$	I, II, IV, V, VI

Both statements are represented by the same regions, I, II, IV, V, VI, of the Venn diagram.
Therefore, $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ for all sets $A, B,$ and C.

75. $A\cap(B\cup C)'$ $A\cap(B'\cap C')$

Set	Regions	Set	Regions
B	II, III, V, VI	B	II, III, V, VI
C	IV, V, VI, VII	B'	I, IV, VII, VIII
$B\cup C$	II, III, IV, V, VI, VII	C	IV, V, VI, VII
$(B\cup C)'$	I, VIII	C'	I, II, III, VIII
A	I, II, IV, V	$B'\cap C'$	I, VIII
$A\cap(B\cup C)'$	I	A	I, II, IV, V
		$A\cap(B'\cap C')$	I

Both statements are represented by the same region, I, of the Venn diagram.

Therefore, $A\cap(B\cup C)' = A\cap(B'\cap C')$ for all sets $A, B,$ and C.

76. $(A\cup B)\cap(B\cup C)$ $B\cup(A\cap C)$

Set	Regions	Set	Regions
A	I, II, IV, V	A	I, II, IV, V
B	II, III, V, VI	C	IV, V, VI, VII
$A\cup B$	I, II, III, IV, V, VI	$A\cap C$	IV, V
C	IV, V, VI, VII	B	II, III, V, VI
$B\cup C$	II, III, IV, V, VI, VII	$B\cup(A\cap C)$	II, III, IV, V, VI
$(A\cup B)\cap(B\cup C)$	II, III, IV, V, VI		

Both statements are represented by the same regions, II, III, IV, V, VI, of the Venn diagram.
Therefore, $(A\cup B)\cap(B\cup C) = B\cup(A\cap C)$ for all sets $A, B,$ and C.

77. $(A\cup B)'\cap C$ $(A'\cup C)\cap(B'\cup C)$

Set	Regions	Set	Regions
A	I, II, IV, V	A	I, II, IV, V
B	II, III, V, VI	A'	III, VI, VII, VIII
$A\cup B$	I, II, III, IV, V, VI	C	IV, V, VI, VII
$(A\cup B)'$	VII, VIII	$A'\cup C$	III, IV, V, VI, VII, VIII
C	IV, V, VI, VII	B	II, III, V, VI
$(A\cup B)'\cap C$	VII	B'	I, IV, VII, VIII
		$B'\cup C$	I, IV, V, VI, VII, VIII
		$(A'\cup C)\cap(B'\cup C)$	IV, V, VI, VII, VIII

Since the two statements are not represented by the same regions, it is not true that $(A\cup B)'\cap C = (A'\cup C)\cap(B'\cup C)$ for all sets $A, B,$ and C.

78. $(C \cap B)' \cup (A \cap B)'$ $A \cap (B \cap C)$

Set	Regions	Set	Regions
C	IV, V, VI, VII	B	II, III, V, VI
B	II, III, V, VI	C	IV, V, VI, VII
$C \cap B$	V, VI	$B \cap C$	V, VI
$(C \cap B)'$	I, II, III, IV, VII, VIII	A	I, II, IV, V
A	I, II, IV, V	$A \cap (B \cap C)$	V
$A \cap B$	II, V		
$(A \cap B)'$	I, III, IV, VI, VII, VIII		
$(C \cap B)' \cup (A \cap B)'$	I, II, III, IV, VI, VII, VIII		

Since the two statements are not represented by the same regions, it is not true that $(C \cap B)' \cup (A \cap B)' = A \cap (B \cap C)$ for all sets $A, B,$ and C.

79. $(A \cup B)'$

80. $A \cap B'$

81. $(A \cup B) \cap C'$

82. $(A \cap B) \cup (B \cap C)$

83. a) $(A \cup B) \cap C = (\{1,2,3,4\} \cup \{3,6,7\}) \cap \{6,7,9\} = \{1,2,3,4,6,7\} \cap \{6,7,9\} = \{6,7\}$

$(A \cap C) \cup (B \cap C) = (\{1,2,3,4\} \cap \{6,7,9\}) \cup (\{3,6,7\} \cap \{6,7,9\}) = \varnothing \cup \{6,7\} = \{6,7\}$

Therefore, for the specific sets, $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$.

b) Answers will vary.

c) $(A \cup B) \cap C$ $(A \cap C) \cup (B \cap C)$

Set	Regions	Set	Regions
A	I, II, IV, V	A	I, II, IV, V
B	II, III, V, VI	C	IV, V, VI, VII
$A \cup B$	I, II, III, IV, V, VI	$A \cap C$	IV, V
C	IV, V, VI, VII	B	II, III, V, VI
$(A \cup B) \cap C$	IV, V, VI	$B \cap C$	V, VI
		$(A \cap C) \cup (B \cap C)$	IV, V, VI

Both statements are represented by the same regions, IV, V, VI, of the Venn diagram.
Therefore, $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ for all sets $A, B,$ and C.

84. a) $(A \cup C)' \cap B = (\{\text{a, c, d, e, f}\} \cup \{\text{a, b, c, d, e}\})' \cap \{\text{c, d}\} = \{\text{a, b, c, d, e, f}\}' \cap \{\text{c, d}\}$

$= \{\text{g, h, i}\} \cap \{\text{c, d}\} = \varnothing$

$(A \cap C)' \cap B = (\{\text{a, c, d, e, f}\} \cap \{\text{a, b, c, d, e}\})' \cap \{\text{c, d}\} = \{\text{a, c, d, e}\}' \cap \{\text{c, d}\} = \{\text{b, f, g, h, i}\} \cap \{\text{c, d}\} = \varnothing$

Therefore, for the specific sets, $(A \cup C)' \cap B = (A \cap C)' \cap B.$

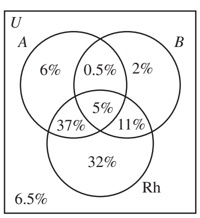
b) Answers will vary.

c) $(A \cup C)' \cap B$ $(A \cap C)' \cap B$

Set	Regions	Set	Regions
A	I, II, IV, V	A	I, II, IV, V
C	IV, V, VI, VII	C	IV, V, VI, VII
$A \cup C$	I, II, IV, V, VI, VII	$A \cap C$	IV, V
$(A \cup C)'$	III, VIII	$(A \cap C)'$	I, II, III, VI, VII, VIII
B	II, III, V, VI	B	II, III, V, VI
$(A \cup C)' \cap B$	III	$(A \cap C)' \cap B$	II, III, VI

Since the two statements are not represented by the same regions, $(A \cup C)' \cap B \neq (A \cap C)' \cap B$ for all sets $A, B,$ and C.

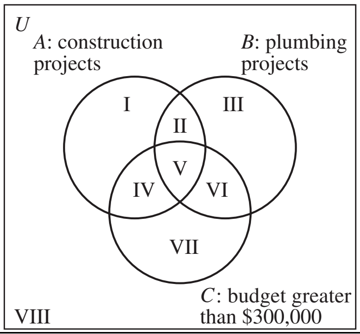
85.



86.

Region	Set	Region	Set
I	$A \cap B' \cap C'$	V	$A \cap B \cap C$
II	$A \cap B \cap C'$	VI	$A' \cap B \cap C$
III	$A' \cap B \cap C'$	VII	$A' \cap B' \cap C$
IV	$A \cap B' \cap C$	VIII	$A' \cap B' \cap C'$

87. a) A : Office Building Construction Projects, B : Plumbing Projects, C : Budget Greater Than \$300,000



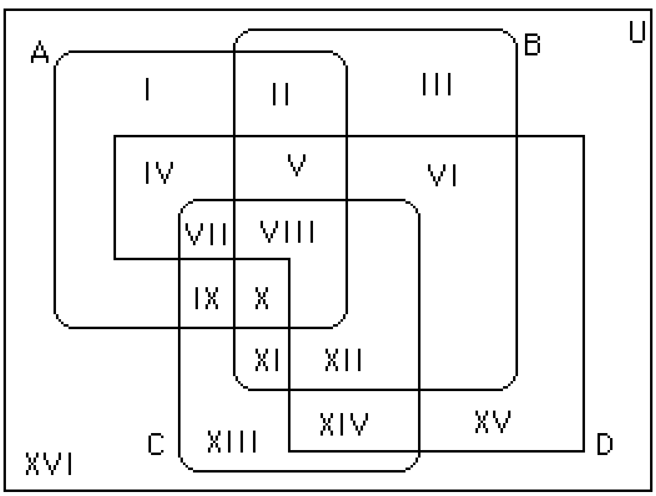
b) Region V; $A \cap B \cap C$

c) Region VI; $A' \cap B \cap C$

d) Region I; $A \cap B' \cap C'$

88. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - 2n(A \cap B \cap C) - n(A \cap B \cap C') - n(A \cap B' \cap C) - n(A' \cap B \cap C)$

89. a)

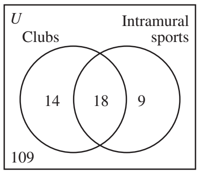


b)

Region	Set	Region	Set
I	$A \cap B' \cap C' \cap D'$	IX	$A \cap B' \cap C \cap D'$
II	$A \cap B \cap C' \cap D'$	X	$A \cap B \cap C \cap D'$
III	$A' \cap B \cap C' \cap D'$	XI	$A' \cap B \cap C \cap D'$
IV	$A \cap B' \cap C' \cap D$	XII	$A' \cap B \cap C \cap D$
V	$A \cap B \cap C' \cap D$	XIII	$A' \cap B' \cap C \cap D'$
VI	$A' \cap B \cap C' \cap D$	XIV	$A' \cap B' \cap C \cap D$
VII	$A \cap B' \cap C \cap D$	XV	$A' \cap B' \cap C' \cap D$
VIII	$A \cap B \cap C \cap D$	XVI	$A' \cap B' \cap C' \cap D'$

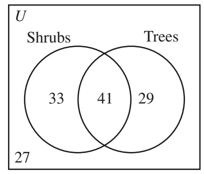
Exercise Set 2.5

1.



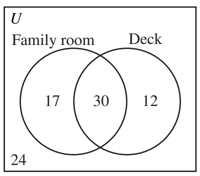
a) 14
b) 9
c) $150-(14+18+9)$, or 109

2.



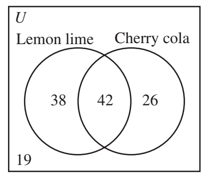
a) 33
b) 29
c) 27

3.



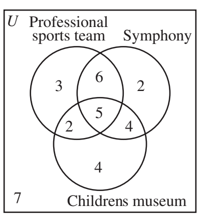
a) 17 b) 12
c) 59, the sum of the numbers in Regions I, II, III

4.



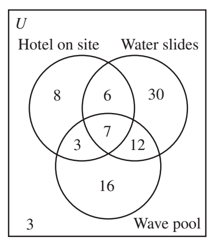
a) 38 b) 26
c) 106, the sum of the numbers in Regions I, II, III
d) $125-106$, or 19.

5.



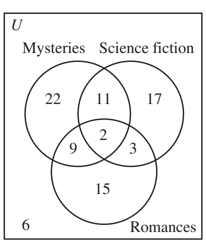
a) 3 b) 6
c) $3+2+6+5+2+4$, or 22
d) $3+6+2$, or 11
e) $2+6+4$, or 12

6.



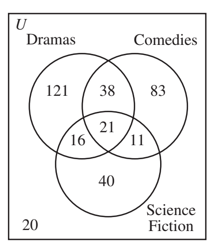
a) 30
b) $8+30+16$, or 54
c) $85-3$, or 82
d) $3+6+12$, or 21 e) 3

7.



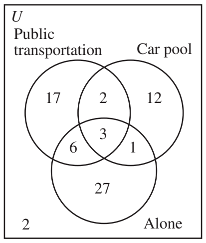
a) 22 b) 11
c) $85-15-6$, or 64
d) $22+11+17$, or 50
e) $9+11+3$, or 23

8.



a) 20 b) 121
c) $121+83+40$, or 244
d) $16+38+11$, or 65
e) $350-20-40$, or 290

9.

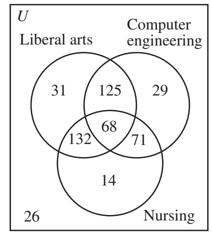


a) 17 b) 27

c) 2

d) $17+2+12$, or 31 e) 2

10.

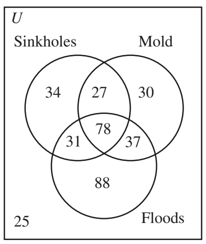


a) 496, the sum of the numbers in all the regions

b) 132 c) 29

d) $132+125+71$, , or 328 e) $496-26$, or 470

11.

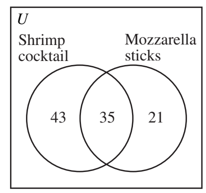


a) $30+37$, or 67

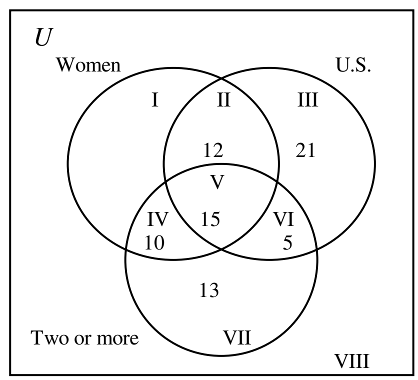
b) $350-25-88$, or 237

c) 37 d) 25

12. No. The sum of the numbers in the Venn diagram is 99. Dennis claims he surveyed 100 people.



13. The Venn diagram shows the number of cars driven by women is 37, the sum of the numbers in Regions II, IV, V. This exceeds the 35 women the agent claims to have surveyed.



14. First fill in 15, 20 and 35 on the Venn diagram. Referring to the labels in the Venn diagram and the given information, we see that

$a+c=140$

$b+c=125$

$a+b+c=185-15=170$

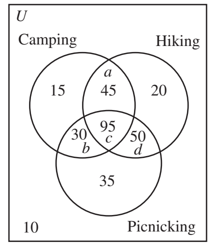
Adding the first two equations and subtracting the third from this sum gives $c=125+140-170=95$. Then $a=45$ and $b=30$. Then $d=210-45-95-20=50$. We now have labeled all the regions except the region outside the three circles, so the number of parks with at least one of the features is $15+45+20+30+95+50+35$, or 290. Thus the number with none of the features is $300-290$, or 10.

a) 290

b) 95

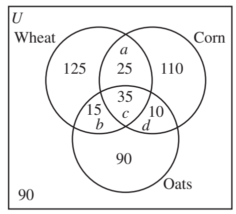
c) 10

d) $30+45+50$, or 125.

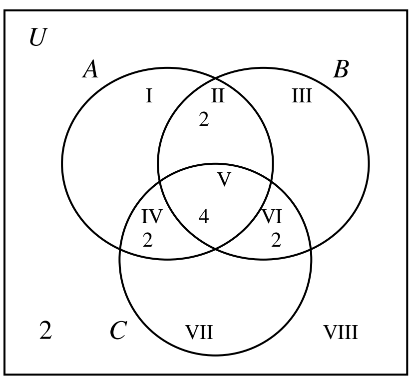


15. First fill in 15, 20 and 35 on the Venn diagram. Referring to the labels in the Venn diagram and the given information, we see that
$a+c=60$
$b+c=50$
$a+b+c=200-125=75$
Adding the first two equations and subtracting the third from this sum gives $c=60+50-75=35$. Then $a=25$ and $b=15$. Then $d=180-110-25-35=10$. We now have labeled all the regions except the region outside the three circles, so the number of farmers growing at least one of the crops is $125+25+110+15+35+10+90$, or 410. Thus the number growing none of the crops is $500-410$, or 90.

a) 410
b) 35
c) 90
d) $15+25+10$, or 50



16. From the given information we can generate the Venn diagram. First fill in 4 for Region V. Then since the intersections in pairs all have 6 elements, we can fill in 2 for each of Regions II, IV, and VI. This already accounts for the 10 elements $A \cup B \cup C$, so the remaining 2 elements in U must be in Region VIII.



a) 10, the sum of the numbers in Regions I, II, III, IV, V, VI
b) 10, the sum of the numbers in Regions III, IV, V, VI, VIII
c) 6, the sum of the numbers in Regions I, III, IV, VI, VIII

Exercise Set 2.6

1. An **infinite set** is a set that can be placed in a one-to-one correspondence with a proper subset of itself.
2. a) A set is **countable** if it is finite or if it can be placed in a one-to-one correspondence with the set of counting numbers.
b) Any set that can be placed in a one-to-one correspondence with the set of counting numbers has cardinality $\aleph_0$.

3. $\{5, 6, 7, 8, 9, \ldots, n+4, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{8, 9, 10, 11, 12, \ldots, n+7, \ldots\}$

4. $\{20, 21, 22, 23, 24, \ldots, n+19, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{23, 24, 25, 26, 27, \ldots, n+22, \ldots\}$

5. $\{3, 5, 7, 9, 11, \ldots, 2n + 1, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{5, 7, 9, 11, 13, \ldots, 2n + 3, \ldots\}$

6. $\{20, 22, 24, 26, 28, \ldots, 2n + 18, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{22, 24, 26, 28, 30, \ldots, 2n + 20, \ldots\}$

7. $\{3, 7, 11, 15, 19, \ldots, 4n - 1, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{7, 11, 15, 19, 23, \ldots, 4n + 3, \ldots\}$

8. $\{4, 8, 12, 16, 20, \ldots, 4n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{8, 12, 16, 20, 24, \ldots, 4n + 4, \ldots\}$

9. $\{6, 11, 16, 21, 26, \ldots, 5n+1, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{11, 16, 21, 26, 31, \ldots, 5n+6, \ldots\}$

10. $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots, \frac{1}{n}, \ldots\right\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots, \frac{1}{n+1}, \ldots\right\}$

11. $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots, \frac{1}{2n}, \ldots\right\}$
$\downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\left\{\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots, \frac{1}{2n+2}, \ldots\right\}$

12. $\left\{\frac{6}{13}, \frac{7}{13}, \frac{8}{13}, \frac{9}{13}, \frac{10}{13}, \ldots, \frac{n+5}{13}, \ldots\right\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\left\{\frac{7}{13}, \frac{8}{13}, \frac{9}{13}, \frac{10}{13}, \frac{11}{13}, \ldots, \frac{n+6}{13}, \ldots\right\}$

13. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{3, 6, 9, 12, 15, \ldots, 3n, \ldots\}$

14. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{50, 51, 52, 53, 54, \ldots, n + 49, \ldots\}$

15. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{4, 6, 8, 10, 12, \ldots, 2n + 2, \ldots\}$

16. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{0, 2, 4, 6, 8, \ldots, 2n - 2, \ldots\}$

17. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{2, 5, 8, 11, 14, \ldots, 3n - 1, \ldots\}$

18. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{6, 11, 16, 21, 26, \ldots, 5n + 1, \ldots\}$

19. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{5, 9, 13, 17, 21, \ldots, 4n + 1, \ldots\}$

20. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots, \frac{1}{2n}, \ldots\right\}$

21. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\left\{\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \ldots, \frac{1}{n+2}, \ldots\right\}$

22. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots, \frac{n}{n+1}, \ldots\right\}$

23. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{1, 4, 9, 16, 25, \ldots, n^2, \ldots\}$

24. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{2, 4, 8, 16, 32, \ldots, 2^n, \ldots\}$

25. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{3, 9, 27, 81, 243, \ldots, 3^n, \ldots\}$

26. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\left\{\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \frac{1}{48}, \ldots, \frac{1}{3 \times 2^{n-1}}, \ldots\right\}$

27. =

28. =

29. =

30. =

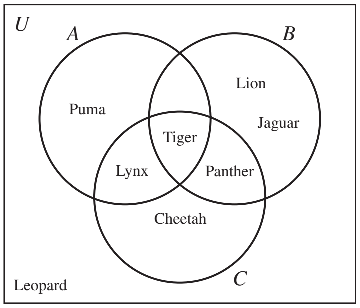
31. =

32. a) Answers will vary.
b) No

Review Exercises

1. True
2. False; the word *best* makes the statement not well defined.
3. True
4. False; no set is a proper subset of itself.
5. False; the elements 6, 12, 18, 24, … are members of both sets.
6. True
7. False; the two sets do not contain exactly the same elements.
8. True
9. True
10. True
11. True
12. True
13. True
14. True
15. $A = \{7, 9, 11, 13, 15\}$
16. $B = \{$ Colorado, Nebraska, Missouri, Oklahoma $\}$
17. $C = \{1, 2, 3, 4, \ldots, 161\}$
18. $D = \{9, 10, 11, 12, \ldots, 96\}$
19. $A = \{x | x \in N \text{ and } 52 < x < 100\}$
20. $B = \{x | x \in N \text{ and } x > 42\}$
21. $C = \{x | x \in N \text{ and } x < 5\}$
22. $D = \{x | x \in N \text{ and } 27 \le x \le 51\}$
23. A is the set of capital letters in the English alphabet from E through M, inclusive.
24. B is the set of U.S. coins with a value of less than one dollar.
25. C is the set of the last three lowercase letters in the English alphabet.
26. D is the set of numbers greater than or equal to 3 and less than 9.
27. $A \cap B = \{1, 3, 5, 7\} \cap \{5, 7, 9, 10\} = \{5, 7\}$
28. $A \cup B' = \{1, 3, 5, 7\} \cup \{5, 7, 9, 10\}' = \{1, 3, 5, 7\} \cup \{1, 2, 3, 4, 6, 8\} = \{1, 2, 3, 4, 5, 6, 7, 8\}$
29. $A' \cap B = \{1, 3, 5, 7\}' \cap \{5, 7, 9, 10\} = \{2, 4, 6, 8, 9, 10\} \cap \{5, 7, 9, 10\} = \{9, 10\}$
30. $(A \cup B)' \cup C = (\{1, 3, 5, 7\} \cup \{5, 7, 9, 10\})' \cup \{1, 7, 10\} = \{1, 3, 5, 7, 9, 10\}' \cup \{1, 7, 10\}$
    $= \{2, 4, 6, 8\} \cup \{1, 7, 10\} = \{1, 2, 4, 6, 7, 8, 10\}$
31. $A - B = \{1, 3, 5, 7\} - \{5, 7, 9, 10\} = \{1, 3\}$
32. $A - C' = \{1, 3, 5, 7\} - \{1, 7, 10\}' = \{1, 3, 5, 7\} - \{2, 3, 4, 5, 6, 8, 9\} = \{1, 7\}$
33. $\{(1,1), (1,7), (1,10), (3,1), (3,7), (3,10), (5,1), (5,7), (5,10), (7,1), (7,7), (7,10)\}$
34. $\{(5,1), (5,3), (5,5), (5,7), (7,1), (7,3), (7,5), (7,7), (9,1), (9,3), (9,5), (9,7), (10,1), (10,3), (10,5), (10,7)\}$
35. $2^4 = 2 \times 2 \times 2 \times 2 = 16$
36. $2^4 - 1 = (2 \times 2 \times 2 \times 2) - 1 = 16 - 1 = 15$

37.



38. $A \cup B = \{\text{a, c, d, f, g, i, k, l}\}$

39. $A \cap B' = \{\text{i, k}\}$

40. $A \cup B \cup C = \{\text{a, b, c, d, f, g, h, i, k, l}\}$

41. $A \cap B \cap C = \{\text{f}\}$

42. $(A \cup B) \cap C = \{\text{a, f, i}\}$

43. $(A \cap B) \cup C = \{\text{a, b, d, f, h, i, l}\}$

44. $(A' \cup B')'$

Set	Regions
A	I, II
A'	III, IV
B	II, III
B'	I, IV
$A' \cup B'$	I, III, IV
$(A' \cup B')'$	II

$A \cap B$

Set	Regions
A	I, II
B	II, III
$A \cap B$	II

Both statements are represented by the same region, II, of the Venn diagram. Therefore, $(A' \cup B')' = A \cap B$ for all sets A and B.

45. $(A \cup B') \cup (A \cup C')$

Set	Regions
A	I, II, IV, V
B	II, III, V, VI
B'	I, IV, VII, VIII
$A \cup B'$	I, II, IV, V, VII, VIII
C	IV, V, VI, VII
C'	I, II, III, VIII
$A \cup C'$	I, II, III, IV, V, VIII
$(A \cup B') \cup (A \cup C')$	I, II, III, IV, V, VII, VIII

$A \cup (B \cap C)'$

Set	Regions
B	II, III, V, VI
C	IV, V, VI, VII
$B \cap C$	V, VI
$(B \cap C)'$	I, II, III, IV, VII, VIII
A	I, II, IV, V
$A \cup (B \cap C)'$	I, II, III, IV, V, VII, VIII

Both statements are represented by the same regions, I, II, III, IV, V, VII, VIII, of the Venn diagram.

Therefore, $(A \cup B') \cup (A \cup C') = A \cup (B \cap C)'$ for all sets $A, B,$ and C.

46. II

47. III

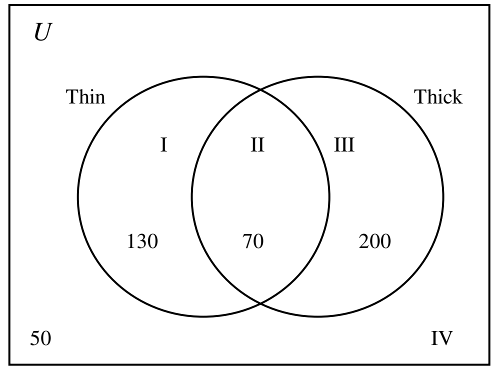
48. I

49. IV

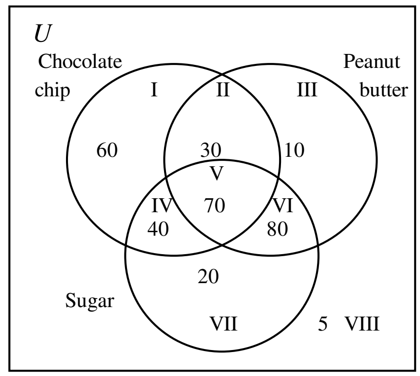
50. IV

51. II

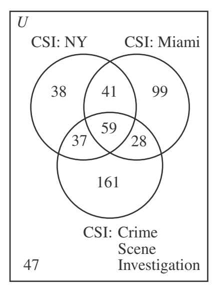
52. The company paid $450 since the sum of the numbers in Regions I through IV is 450.



53. a) 315, the sum of the numbers in Regions I through VIII
b) 10, Region III
c) 30, Region II
d) 110, the sum of the numbers in Regions III, VI, VII



54. a) 38, Region I
b) 298, the sum of the numbers in Regions I, III, VII
c) 28, Region VI
d) 236, the sum of the numbers in Regions I, IV, VII
e) 106, the sum of the numbers in Regions II, IV, VI



55. $\{2, 4, 6, 8, 10, \ldots, 2n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{4, 6, 8, 10, 12, \ldots, 2n + 2, \ldots\}$

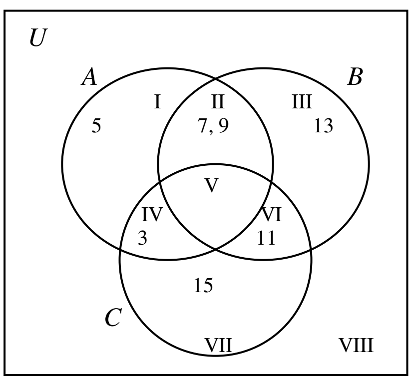
56. $\{3, 5, 7, 9, 11, \ldots, 2n + 1, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{5, 7, 9, 11, 13, \ldots, 2n + 3, \ldots\}$

57. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{5, 8, 11, 14, 17, \ldots, 3n + 2, \ldots\}$

58. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
$\{4, 9, 14, 19, 24, \ldots, 5n - 1, \ldots\}$

Chapter Test

1. True
2. False; the sets do not contain exactly the same elements.
3. True
4. False; the second set has no subset that contains the element 7.
5. False; the empty set is a subset of every set.
6. False; the set has $2^4 = 2\times 2\times 2\times 2 = 16$ subsets.
7. True
8. False; for any set A, $A\cup A' = U$, not $\{\ \}$.
9. True
10. $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$
11. Set A is the set of natural numbers less than 9.
12. $A\cap B = \{3, 5, 7, 9\}\cap\{7, 9, 11, 13\} = \{7, 9\}$
13. $A\cup C' = \{3, 5, 7, 9\}\cup\{3, 11, 15\}' = \{3, 5, 7, 9\}\cup\{5, 7, 9, 13\} = \{3, 5, 7, 9, 13\}$
14. $A\cap(B\cap C)' = \{3, 5, 7, 9\}\cap(\{7, 9, 11, 13\}\cap\{3, 11, 15\})' = \{3, 5, 7, 9\}\cap\{11\}'$
$= \{3, 5, 7, 9\}\cap\{3, 5, 7, 9, 13, 15\} = \{3, 5, 7, 9\}$, or A.
15. $n(A\cap B') = n\left(\{3, 5, 7, 9\}\cap\{7, 9, 11, 13\}'\right) = n(\{3, 5, 7, 9\}\cap\{3, 5, 15\}) = n(\{3, 5\}) = 2$
16. $A - B = \{3, 5, 7, 9\} - \{7, 9, 11, 13\} = \{3, 5\}$
17. $A\times C = \{(3, 3), (3, 11), (3, 15), (5, 3), (5, 11), (5, 15), (7, 3), (7, 11), (7, 15), (9, 3), (9, 11), (9, 15)\}$
18.

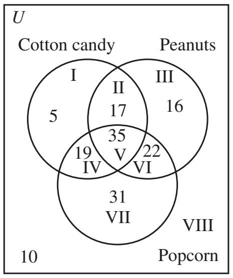


19. $A\cap(B\cup C')$ $\qquad$ $(A\cap B)\cup(A\cap C')$

Set	Regions	Set	Regions
B	II, III, V, VI	A	I, II, IV, V
C	IV, V, VI, VII	B	II, III, V, VI
C'	I, II, III, VIII	$A\cap B$	II, V
$B\cup C'$	I, II, III, V, VI, VIII	C	IV, V, VI, VII
A	I, II, IV, V	C'	I, II, III, VIII
$A\cap(B\cup C')$	I, II, V	$A\cap C'$	I, II
		$(A\cap B)\cup(A\cap C')$	I, II, V

Both statements are represented by the same regions, I, II, V, of the Venn diagram.
Therefore, $A\cap(B\cup C') = (A\cap B)\cup(A\cap C')$ for all sets $A, B,$ and C.

20.



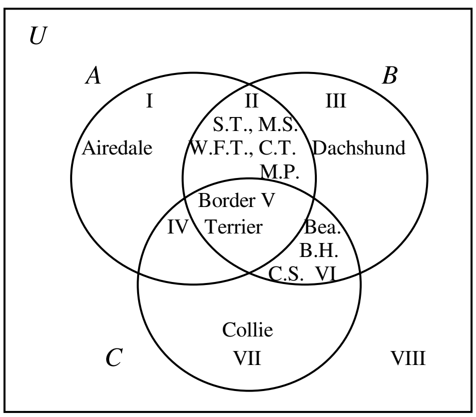
a) 52, the sum of the numbers in Regions I, III, VII
b) 10, Region VIII
c) 93, the sum of the numbers in Regions II, IV, V, VI
d) 17, Region II
e) 38, the sum of the numbers in Regions I, II, III
f) 31, Region VII

21. {7, 8, 9, 10, 11, …, $n + 6$, …}
↓ ↓ ↓ ↓ ↓ ↓
{8, 9, 10, 11, 12, …, $n + 7$, …}

22. {1, 2, 3, 4, 5, …, n, …}
↓ ↓ ↓ ↓ ↓ ↓
{1, 3, 5, 7, 9, …, $2n - 1$, …}

<u>Group Projects</u>

1. a) A : Does not shed, B : Less than 16 in. tall, C : Good with kids



b) Border terrier, Region V

2. a) Animal b) Chordate c) Mammalia d) Carnivore
e) Felidae f) Felis g) Catus

3.

	First	Second	Third	Fourth	Fifth
a) Color	yellow	blue	red	ivory	green
b) Nationality	Norwegian	Afghan.	Senegalese	Spanish	Japanese
c) Food	apple	cheese	banana	peach	fish
d) Drink	vodka	tea	milk	whiskey	ale
e) Pet	fox	horse	snail	dog	zebra
f) Ale					