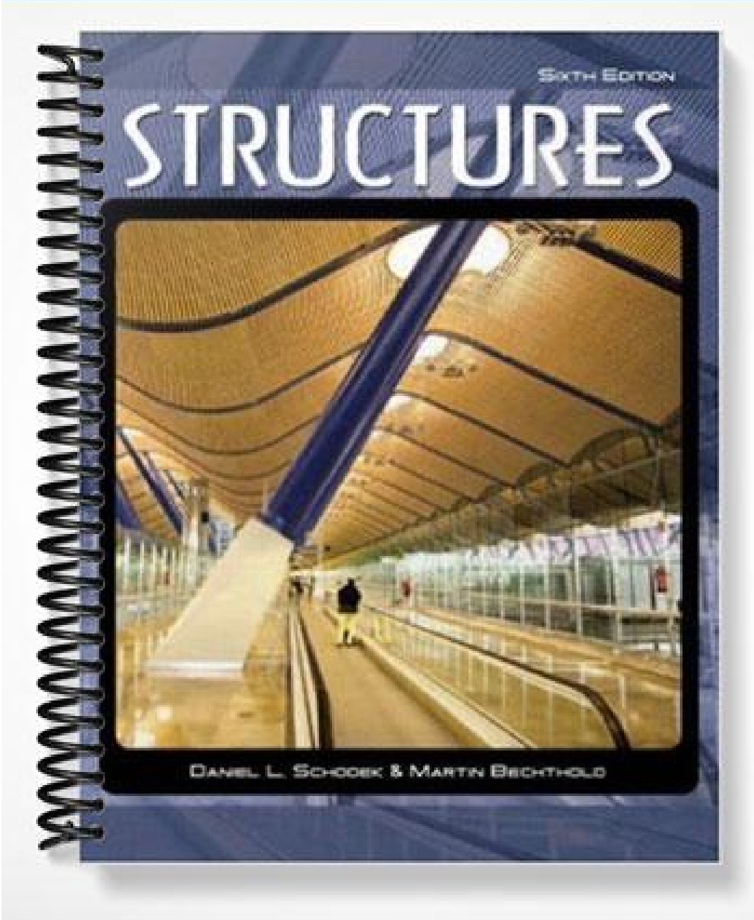
SOLUTIONS MANUAL



Structures Instructor's Manual and Answer Guide

D.L. Schodek, M. Bechthold

NOTES TO THE INSTRUCTOR

This Instructor's Manual is designed to accompany *Structures* and to facilitate the use of the textbook in a classroom circumstance. The book is designed to support several different types of teaching approaches, ranging from a basic and purely qualitative approach through to fairly rigorous treatments (including computer-based techniques).

Comprehensive, Semi-quantitative Coverage

The book is fundamentally targeted to support a comprehensive coverage of structural analysis and design issues in which traditional subject distinctions (such as statics, strength of materials, indeterminate structures, etc.), have been integrated into a single continuous treatment.

The initial chapter in Part I provides an overview introduction to fundamental issues, Part II covers the analysis and design of structural elements, and Part III places emphasis on broader design issues within the building context. A semi-quantitative approach is used that balances extensive discussions of structural behavior with simple but powerful numerical methods. A typical chapter in Part II introduces semi-quantitative methods of analysis, and culminates in a review of design issues.

It is suggested that the instructor lead off with one or two overview lectures briefly covering the topics in Chapter 1 which are extensively illustrated with structural systems in real buildings. The extent to which 'basic equilibrium' is next covered depends largely on the specific background of students. Note that it is strongly suggested that for novice students several topics in this chapter (for example shear and moment diagrams, mechanical properties of materials) be introduced at a time considered most sensible by the instructor, and not necessarily when they appear in the book. The various topics in Chapter 2 were collected in one place as a concise overview for students that have already seen this material in other courses. For students new to the material it is usually better to introduce shear and moment diagrams after an introduction to trusses (e.g. building on the equilibrium of sections) and before beams. Material properties are often best introduced just prior to or as part of beam analysis, when issues such as stress and strain are of paramount importance.

It is suggested that topics such as 'loadings' be treated at different levels of specificity at different points in the course. A general understanding and categorization of loadings can and should be presented quite early, but many detailed issues are better treated later in the course once students have a better understanding of what structures are all about. Once past a general introduction to the phenomenon of wind effects on structures, for example, more detailed treatments of wind loadings are often best done in connection with rigid frame structures (where structures are first analyzed for lateral loading conditions or at the end in connection with more general design issues).

Member design issues can obviously be talked about in connection with the treatment of specific elements, but here the instructor is encouraged to go beyond simple shaping and sizing concerns and deal with broader issues that students can immediately recognize as important, but which are not easily quantified. Again these issues are often best addressed using illustrations drawn from real buildings (for example, beam spacing, member hierarchies, etc.).

The kinds of very broad design observations contained in Chapter 13 and 14 on structural patterns can seemingly be introduced at any time, but are often best done as “summing up” of the course content and again in relation to illustrations drawn from the real world. By this time the student should have a reasonable understanding of how specific members work and can better appreciate the issues in the holistic design of structures.

Prerequisites: the bulk of the text assumes a working level in trigonometry on the part of the student. Some exposure to basic mechanics from a physics course is desirable, but not absolutely required since pertinent material is briefly reviewed in Chapter 2. Introductory calculus is also desirable, but again not absolutely required. While many fundamental relationships among variables, properties of cross-sections, or other measures can be exactly established only using calculus, most formal derivations are placed in the Appendices, and a more descriptive approach to establishing these relationships is taken in the main body of the text.

Introductory Qualitative Approach

At it’s most basic level, the book could also be used to support a purely qualitative approach to understanding structural behavior. This would be done by having students read only introductory level material in Part I (Chapter 1, Chapter 3 (Sections 3.1 and 3.2), the first two sections of each of the chapters in Part II (“Introduction” and “General Principles”), and the summary chapters in Part III (Chapter 14 and 15). This coverage would provide a completely nonmathematical view of the subject matter.

Prerequisites: No prerequisites in mathematics are required for this type of coverage.

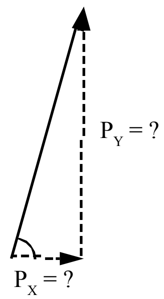
Approximate Techniques, Computer-Based Analysis and Design Techniques: Many parts of the book focus on approximate analyses, graphic approaches, and other techniques stemming from a seemingly older pre-computer view of structural analysis and design. This is done deliberately as a way of communicating to students the essentials of structural behavior. Computer-based techniques, on the other hand, have positively and radically altered the way the profession now operates, and the sixth edition clearly reflects the widespread use of structural analysis programs. Computer-based analysis programs are also a great asset for helping beginning students visualize the structural behavior of the kinds of complex assemblies that they will encounter in practice, and to get them excited about the profession they are entering. But the underlying algorithms (e.g., matrix-based approaches) are inherently difficult for novice students still struggling with how to understand basic structural behavior. For this reason approximate techniques are always presented first. Some sections contain a brief introduction to computer formulations, but more only to suggest the spirit of the underlying algorithms rather than to "teach" them. Many of the new problems introduced throughout the book are to be solved using a structural analysis program, again not so much with the expectation to derive final member sizing and code-compliant design, but to understand structural behavior.

A structural analysis program (Multiframe Academic 9.54) is included on the CD. Computer-based problem sets can be solved with this or any other structural analysis program that students may have access to. Programs should have the capability to generate moment diagrams, deflected shapes, etc. Use of these programs can be successfully introduced quite early, and before any attempt is made to teach the underlying algorithms. Indeed, today's students naturally expect access to such programs. A good approach for many areas of study is to have students do an assignment by hand using approximate methods, and then "check" results using a program. Special assignments, e.g. the influence of relative beam and column stiffnesses on moment distributions in statically indeterminate systems, directly depend on the use of structural analysis programs and can be very effectively used to enhance the learning experience.

Table of Contents

Chapter 2

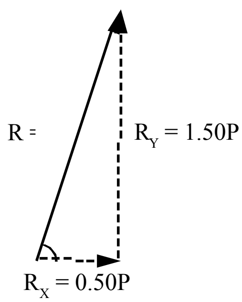
Question 2.1: A force of P defined by the angle $\theta_X = 75°$ to the horizontal acts through a point. What are the components of this force on the x and y axes?

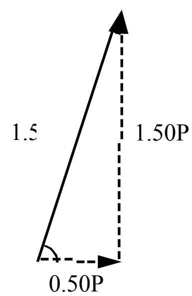


$$\begin{aligned} \cos 75° &= P_X/P \\ P * \cos 75° &= P_X \\ P_X &= 0.26P \end{aligned}$$

$$\begin{aligned} \sin 75° &= P_Y/P \\ P * \sin 75° &= P_Y \\ P_Y &= 0.97P \end{aligned}$$

Question 2.2: The components of a force on the x and y axes are 0.50P and 1.50P, respectively. What are the magnitude and direction of the resultant force?



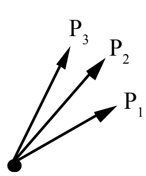


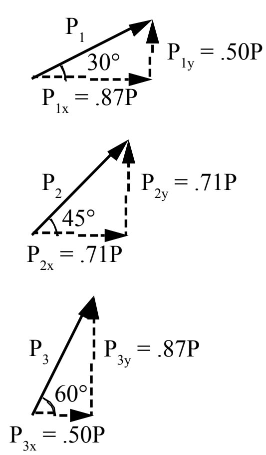
$$\begin{aligned} R &= \text{resultant force} \\ R^2 &= (0.50P)^2 + (1.50P)^2 \\ R &= 1.58P \end{aligned}$$

$$\begin{aligned} \tan\theta &= 1.50P/0.50P \\ \theta &= \tan^{-1} 1.50P/0.50P \\ \theta &= 71.6° \end{aligned}$$

Question 2.3: The following three forces act concurrently through a point: a force P acting to the right at $\theta_X = 30°$ to the horizontal, a force P acting to the right at $\theta_X = 45°$ to the horizontal, and a force P acting to the right at $\theta_X = 60°$ to the horizontal. Find the single resultant force that is equivalent to this three-force system.

Step 1: Find the horizontal and vertical components of each force and the net horizontal and vertical force.





$$\begin{aligned} P_{1x} &= P * \cos 30° \\ P_{1x} &= .87P \end{aligned}$$

$$\begin{aligned} P_{1y} &= P * \sin 30° \\ P_{1y} &= .50P \end{aligned}$$

$$\begin{aligned} P_{2x} &= P * \cos 45° \\ P_{2x} &= .71P \end{aligned}$$

$$\begin{aligned} P_{2y} &= P * \sin 45° \\ P_{2y} &= .71P \end{aligned}$$

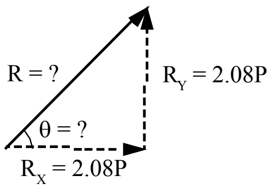
$$\begin{aligned} P_{3x} &= P * \cos 60° \\ P_{3x} &= .50P \end{aligned}$$

$$\begin{aligned} P_{3y} &= P * \sin 60° \\ P_{3y} &= .87P \end{aligned}$$

$$\begin{aligned} R_X = R_Y &= .71P + .50P + .87P \\ R_X = R_Y &= 2.08P \end{aligned}$$

Question 2.3 (continued):

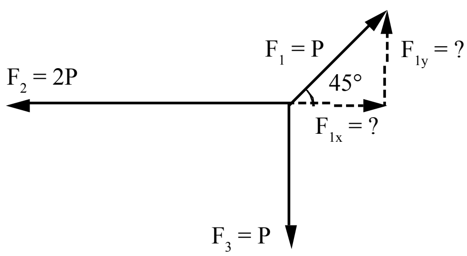
Step 2: Find the magnitude and direction of the resultant force.



$$
\begin{aligned}
R^2 &= (2.08P)^2 + (2.08P)^2 \\
R &= 2.93P \\
\tan\theta &= R_Y/R_X \\
\tan\theta &= 2.08P/2.08P \\
\theta &= \tan^{-1} 1 \\
\theta &= 45°
\end{aligned}
$$

Question 2.4: The following three forces act through a point: P at $\theta_x = 45°$, 2P at $\theta_x = 180°$, and P at $\theta_x = 270°$. Find the equivalent resultant force.

Step 1: Find the horizontal and vertical components of each force.

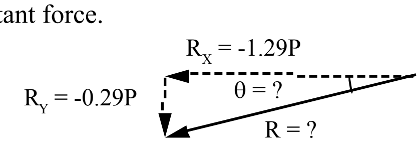


$$
\begin{aligned}
F_{1x} &= P * \cos 45° \\
F_{1x} &= .71P \\
F_{1y} &= P * \sin 45° \\
F_{1y} &= .71P \\
F_{2x} &= -2P \\
F_{2y} &= 0 \\
F_{3x} &= 0 \\
F_{3y} &= -P
\end{aligned}
$$

Step 2: Find the net horizontal and vertical force.

$$
\begin{aligned}
R_X &= .71P - 2P \\
R_X &= -1.29P \\
R_Y &= .71P - P \\
R_Y &= -0.29P
\end{aligned}
$$

Step 3: Find the magnitude and direction of the resultant force.

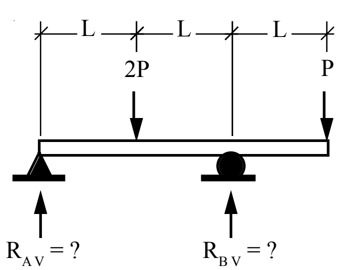


$$
\begin{aligned}
R^2 &= (-1.29P)^2 + (-0.29P)^2 \\
R &= 1.32P \\
\tan\theta &= -0.29P / -1.29P \\
\theta &= 12.7°
\end{aligned}
$$

Summary resultant force = R = 1.33P acting at 192.7°

Question 2.6: Determine the reactions for the structure shown in Figure 2.59(Q6).

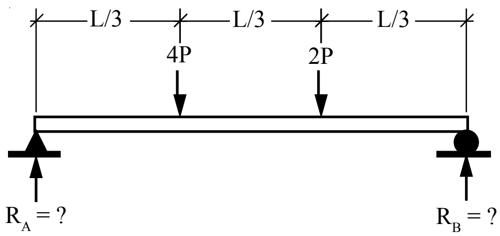
Sum rotational moments about point A. Assume that a counter-clockwise rotational effect is positive.



$$
\begin{aligned}
\sum M_A &= 0 \\
-(2P * L) + (R_B * 2L) - (P * 3L) &= 0 \\
R_B * 2L &= 2PL + 3PL \\
R_B &= 5PL/2L \\
R_B &= 5P/2 \uparrow
\end{aligned}
$$

$$
\begin{aligned}
\sum F_Y &= 0 \\
R_A + R_B - 2P - P &= 0 \\
R_A &= 3P - R_B \\
R_A &= 3P - 5P/2 \\
R_A &= P/2 \uparrow
\end{aligned}
$$

Question 2.8: Determine the reactions for the structure shown in Figure 2.59(Q8).

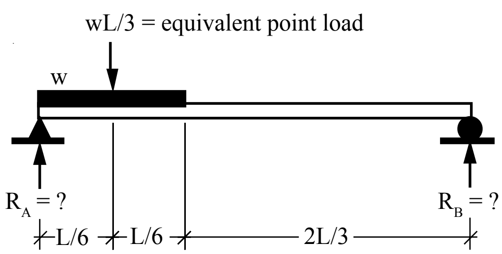


$$\sum M_A = 0$$
$$-(4P * L/3) - (2P * 2L/3) = 0$$
$$(R_B * L) = 0$$
$$R_B * L = (4P * L/3) + (2P * 2L/3)$$
$$R_B * L = 4PL/3 + 4PL/3$$
$$R_B = 8P/3 \uparrow$$

$$\sum F_Y = 0$$
$$R_A + R_B - 4P - 2P = 0$$
$$R_A = 6P - R_B$$
$$R_A = 6P - 8P/3$$
$$R_A = 10P/3 \uparrow$$

Question 2.10: Determine the reactions for the structure shown in Figure 2.59(Q10).

Sum moments about A. Assume that counter-clockwise moments are positive. Convert the uniform load *w* into an equivalent concentrated load for purposes of finding reactions.



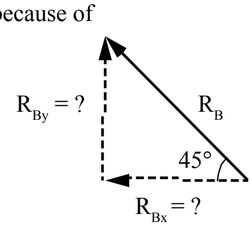
$$\sum M_A = 0$$
$$-(w * L/3 * L/6) + (R_B * L) = 0$$
$$R_B * L = w \times L/3 * L/6$$
$$R_B * L = wL^2/18$$
$$R_B = wL/18 \uparrow$$

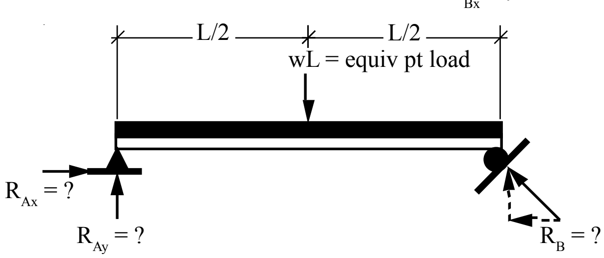
$$\sum F_Y = 0$$
$$R_A + R_B - wL/3 = 0$$
$$R_A = wL/3 - R_B$$
$$R_A = 6wL/18 - wL/18$$
$$R_A = 5wL/18 \uparrow$$

Question 2.12: Determine the reactions for the structure shown in Figure 2.59(Q12).

The angle of the roller on the right determines the direction of the reactive force at B which is then considered in terms of its components. The fact that the reaction at B is inclined means that the reaction at A must also be inclined (the horizontal components of the reactions must sum to zero because of equilibrium in the x direction).

An equivalent point load of *(w)(L)* is used to model the uniform load of *w* acting over the length of the beam.





$$\sum M_A = 0$$
$$(R_{By} * L) - (wL * L/2) = 0$$
$$R_{By} * L = wL * L/2$$
$$R_{By} = wL/2 \uparrow$$

$$\tan 45^\circ = R_{By} / R_{Bx}$$
$$1 = R_{By} / R_{Bx}$$
$$R_{Bx} = R_{By}$$
$$R_{Bx} = wL/2 \leftarrow$$

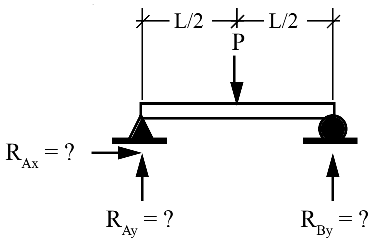
$$\sum F_Y = 0$$
$$R_{Ay} + R_{By} - wL = 0$$
$$R_{Ay} = wL - R_{By}$$
$$R_{Ay} = wL - wL/2$$
$$R_{Ay} = wL/2 \uparrow$$

$$\sum F_X = 0$$
$$R_{Ax} + R_{Bx} = 0$$
$$R_{Ax} = R_{B_x}$$
$$R_{Ax} = wL/2 \rightarrow$$

Question 2.13: Determine the reactions for the four beams shown in Figure 2.59(Q13).

Notice that the three inclined members are identical except for the type of end conditions present. Note how changing the support types radically alters the nature of the reactive forces.

Step 1: Figure 2.33(e)-1

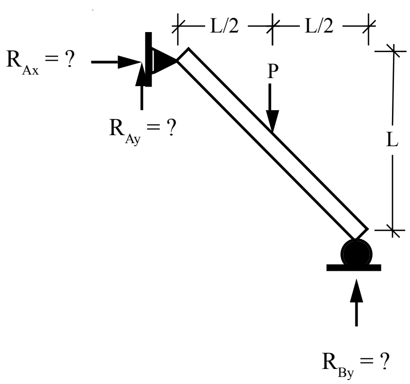


$$\sum M_A = 0$$
$$(R_{By} * L) - (P * L/2) = 0$$
$$R_{By} * L = PL/2$$
$$R_{By} = P/2 \uparrow$$

$$\sum F_Y = 0$$
$$R_{Ay} + R_{By} - P = 0$$
$$R_{Ay} = P - R_{By}$$
$$R_{Ay} = P - P/2$$
$$R_{Ay} = P/2 \uparrow$$

$$\sum F_X = 0$$
$$R_{Ax} = 0$$

Step 2: Figure 2.33(e)-2

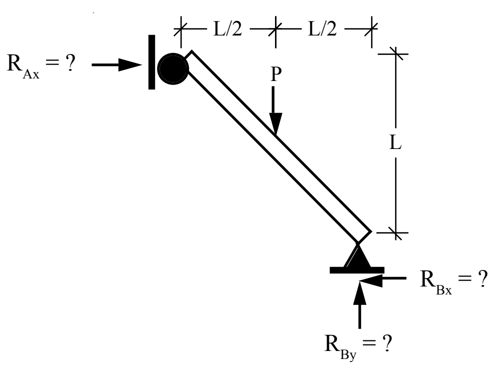


$$\sum M_A = 0$$
$$(R_{By} * L) - (P * L/2) = 0$$
$$R_{By} * L = PL/2$$
$$R_{By} = P/2 \uparrow$$

$$\sum F_Y = 0$$
$$R_{Ay} + R_{By} - P = 0$$
$$R_{Ay} = P - R_{By}$$
$$R_{Ay} = P - P/2$$
$$R_{Ay} = P/2 \uparrow$$

$$\sum F_X = 0$$
$$R_{Ax} = 0$$

Step 3: Figure 2.33(e)-3

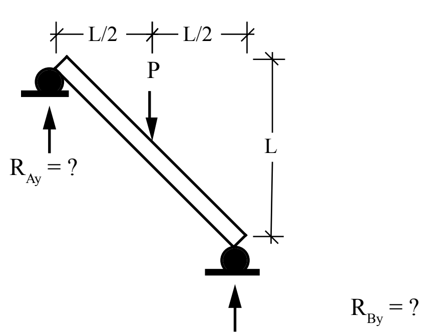


$$\sum M_B = 0$$
$$(P * L/2) - (R_{Ax} * h) = 0$$
$$(R_{Ax} * h) = PL/2$$
$$R_{Ax} = PL/2h \rightarrow$$

$$\sum F_X = 0$$
$$R_{Ax} + R_{Bx} = 0$$
$$R_{Bx} = -R_{Ax}$$
$$R_{Bx} = PL/2h \leftarrow$$

$$\sum F_Y = 0$$
$$R_{By} - P = 0$$
$$R_{By} = P \uparrow$$

Step 4: Figure 2.33(e)-4



$$R_{By} * L = PL/2$$
$$R_{By} = P/2 \uparrow$$

$$\sum F_Y = 0$$

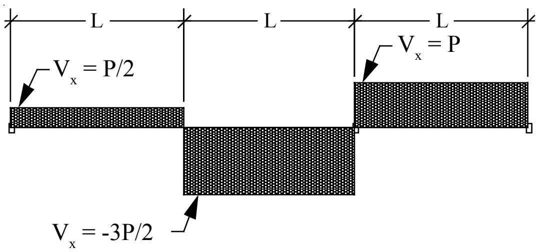
$$R_{Ay} = P - R_{By}$$
$$R_{Ay} = P - P/2$$
$$R_{Ay} = P/2 \uparrow$$

Question 2.15: Draw shear and moment diagrams for the beam analyzed in Question 2.6 [Figure 2.59 (Q6)]. What is the maximum shear force present? What is the maximum bending moment present?

Step 1: Find the reactions (see Question 2.6).

$R_A = P/2$ (upward)
$R_B = 5P/2$ (upward)

Step 2: Draw the shear diagram.

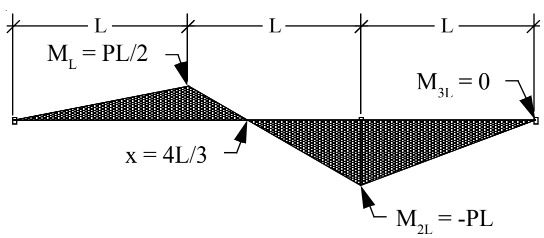


For $0 < x < L$:
$V_X = P/2$

For $L < x < 2L$:
$V_X = P/2 - 2P$
$V_X = -3P/2$

For $2L < x < 3L$:
$V_X = P/2 - 2P + 5P/2$
$V_X = P$

Step 3: Draw the moment diagram.



When the shear is positive, the slope to the moment diagram is positive and vice-versa. Also note that when the shear diagram passes through zero the bending moment values are critical. Since only concentrated loads are present, the moment diagram consists of linearly sloped lines only (uniform loadings produce curved lines). The point of zero moment on the bending moment diagram corresponds to a "point of inflection" (reverse curvature) on the deflected shape of the structure (see Section 2.4.4).

For $0 < x < L$:
$M_X = (P/2)x$

When $x = L$:
$M_L = (P/2)L$
$M_L = PL/2$

For $L < x < 2L$:
$M_X = (P/2)x - (2P)(x - L)$

When $M_X = 0$:
$0 = (P/2)x - (2P)(x - L)$
$0 = Px/2 - 2Px + 2PL$
$0 = -3Px/2 + 2PL$
$3Px/2 = 2PL$
$(2/3P)3Px/2 = 2PL(2/3P)$
$x = 4L/3$

When $x = 2L$:
$M_{2L} = (P/2)2L - (2P)(2L - L)$
$M_{2L} = PL - 2PL$
$M_{2L} = -PL$

For $2L < x < 3L$:
$M_X = (P/2)x - (2P)(x - L) + (5P/2)(x - 2L)$

Check: when $x = 3L$:
$M_{3L} = (P/2)3L - (2P)(3L - L) + (5P/2)(3L - 2L)$
$M_{3L} = 3PL/2 - (2P)(2L) + (5P/2)L$
$M_{3L} = 3PL/2 - 4PL + 5PL/2$
$M_{3L} = 8PL/2 - 4PL$
$M_{3L} = 0$

Summary

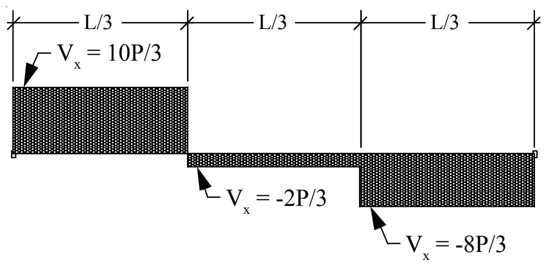
$V_{MAX} = -3P/2$
$M_{MAX} = -PL$

Question 2.17: Draw shear and moment diagrams for the beam analyzed in Question 2.8 [Figure 2.59]. What is the maximum shear force present? What is the maximum bending moment present?

Step 1: Find the reactions (see Question 2.8).

$R_A = 10P/3$ (upward)
$R_B = 8P/3$ (upward)

Step 2: Draw the shear diagram.

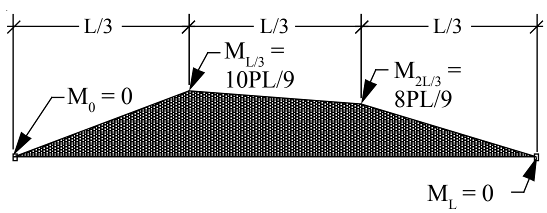


For $0 < x < L/3$:
$V_X = 10P/3$

For $L/3 < x < 2L/3$:
$V_X = 10P/3 - 4P$
$V_X = -2P/3$

For $2L/3 < x < L$:
$V_X = 10P/3 - 4P - 2P$
$V_X = -8P/3$

Step 3: Draw the moment diagram.



For $0 < x < L/3$:
$M_X = (10P/3)x$

When $x = L/3$:
$M_{L/3} = (10P/3)(L/3)$
$M_{L/3} = 10PL/9$

For $L/3 < x < 2L/3$:
$M_X = (10P/3)x - 4P(x - L/3)$

When $x = 2L/3$:
$M_{2L/3} = (10P/3)(2L/3) - 4P(L/3)$
$M_{2L/3} = 20PL/9 - 4PL/3$
$M_{2L/3} = 20PL/9 - 12PL/9$
$M_{2L/3} = 8PL/9$

For $2L/3 < x < L$:
$M_X = (10P/3)x - 4P(x - L/3) - 2P(x - 2L/3)$

Check: when $x = L$:
$M_L = (10P/3)L - 4P(L - L/3) - 2P(L - 2L/3)$
$M_L = 10PL/3 - 4P(2L/3) - 2P(L/3)$
$M_L = 10PL/3 - 8PL/3 - 2PL/3$
$M_L = 0$

Summary

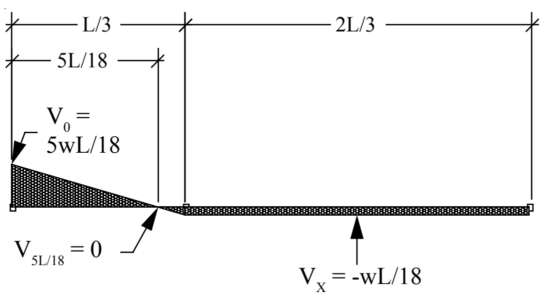
$V_{MAX} = +10P/3$
$M_{MAX} = +10PL/9$

Question 2.19: Draw shear and moment diagrams for the beam analyzed in Question 2.10 [Figure 2.59]. What is the maximum shear force present? What is the maximum bending moment present?

Step 1: Find the reactions (see Question 2.7).

R_A = 5wL/18 (upward)
R_B = wL/18 (upward)

Step 2: Draw the shear diagram.



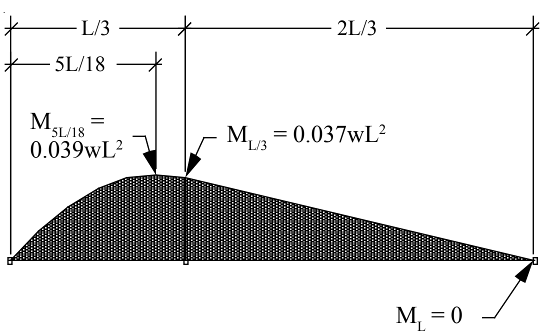
For 0 < x < L/3:
V_X = 5wL/18 - wx

When x = 0:
V_X = 5wL/18 - wx
V_X = 5wL/18

When V_X = 0:
0 = 5wL/18 - wx
wx = 5wL/18
x = 5L/18

For L/3 < x < L:
V_X = 5wL/18 - w * L/3
V_X = 5wL/18 - 6wL/18
V_X = -wL/18

Step 3: Draw the moment diagram.



Check: when x = L:
M_L = (5wL/18)L
- (wL/3)(L - L/6)
M_L = $5wL^2/18$
- (wL/3)(5L/6)
M_L = $5wL^2/18 - 5wL^2/18$
M_L = 0

For 0 < x < L/3:
M_X = (5wL/18)x - wx(x/2)
M_X = $5wxL/18 - wx^2/2$

When x = 5L/18 (V_x = 0):
$M_{5L/18}$ = (5wL/18)(5L/18)
- $w(5L/18)^2/2$
$M_{5L/18}$ = $25wL^2/324 - 25wL^2/648$
$M_{5L/18}$ = $25wL^2/648$
$M_{5L/18}$ = $0.039\ wL^2$

When x = L/3:
$M_{L/3}$ = $(5wL/18)(L/3) - w(L/3)^2/2$
$M_{L/3}$ = $5wL^2/54 - wL^2/18$
$M_{L/3}$ = $5wL^2/54 - 3wL^2/54$
$M_{L/3}$ = $2wL^2/54$
$M_{L/3}$ = $wL^2/27$
$M_{L/3}$ = $0.037wL^2$

For L/3 < x < L:
M_x = (5wL/18)x - (wL/3)(x - L/6)

Summary

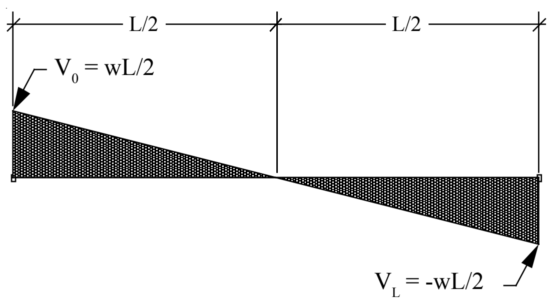
V_{MAX} = +5wL/18
M_{MAX} = $+25wL^2/648$

Question 2.21: Draw shear and moment diagrams for the beam analyzed in Question 2.12 [Figure 2.59]. What is the maximum shear force present? What is the maximum bending moment present?

Step 1: Find the reactions (see Question 2.12).

R_{Ax} = wL/2 (to the right)
R_{Ay} = wL/2 (upward)
R_{Bx} = wL/2 (to the left)
R_{By} = wL/2 (upward)

Step 2: Draw the shear diagram.



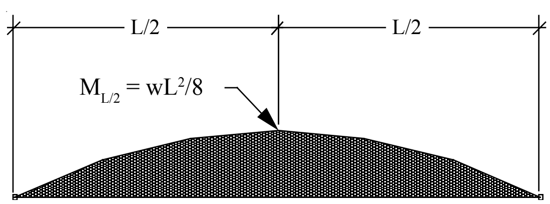
For 0 > x > L:
$V_X = wL/2 - wx$

When x = 0:
$V_0 = wL/2$

When x = L:
$V_L = wL/2 - wL$
$V_L = -wL/2$

When V_X = 0:
$0 = wL/2 - wx$
$wx = wL/2$
$x = L/2$

Step 3: Draw the moment diagram.



For 0 > x > L:
$M_X = (wL/2)x - wx(x/2)$
$M_X = wxL/2 - wx^2/2$

When x = 0:
$M_X = 0$

When x = L/2:
$M_X = (wL/2)(L/2) - w(L/2)(L/4)$
$M_X = wL^2/4 - wL^2/8$
$M_X = wL^2/8$

Check: when x = L:
$M_L = (wL/2)L - wL^2/2$
$M_L = wL^2/2 - wL^2/2$
$M_L = 0$

Summary

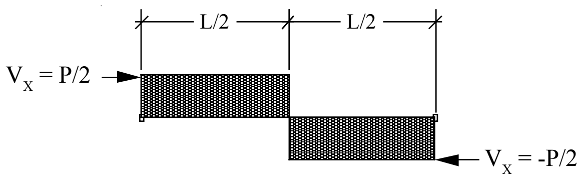
$V_{MAX} = \pm wL/2$
$M_{MAX} = +wL^2/8$

Question 2.22: Draw shear and moment diagrams for the four beams in Question 13 [Figure 2.59]. For the inclined members, the shear and moment diagrams should be drawn with respect to the longitudinal axes of the members. Transverse components of the applied and reactive forces should thus be considered in determining shears and moments. Compare the maximum moments developed in all four beams.

Beam 2.59(Q13a)
Step 1: Find the reactions (see Question 2.13).

$R_{Ax} = 0$
$R_{Ay} = P/2$ (upward)
$R_{By} = P/2$ (upward)

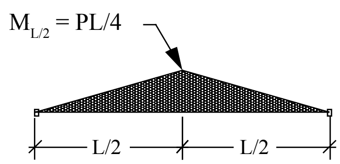
Step 2: Draw the shear diagram.



For $0 < x < L/2$:
$V_X = P/2$

For $L/2 < x < L$:
$V_X = P/2 - P$
$V_X = -P/2$

Step 3: Draw the moment diagram.



For $0 < x < L/2$:
$M_X = (P/2)x$

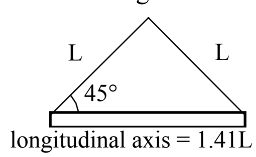
When $x = L/2$:
$M_X = P/2 * L/2$
$M_X = PL/4$

For $L/2 < x < L$:
$M_X = P/2(x) - P(x - L/2)$

Beam 2.59(Q13b)
Step 1: Find the reactions (see Question 2.13).

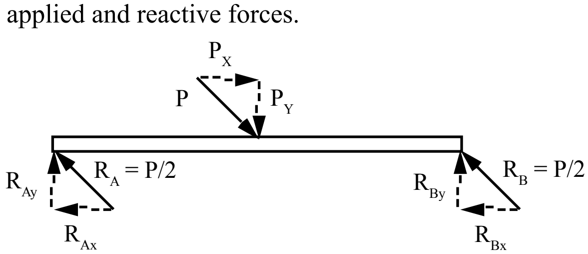
$R_{Ax} = 0$
$R_{Ay} = P/2$ (upward)
$R_{By} = P/2$ (upward)

Step 2: Calculate the longitudinal axis of the member.



$\cos 45° = L/\text{longitudinal axis}$
$\text{longitudinal axis} = L/\cos 45°$
$\text{longitudinal axis} = 1.41L$

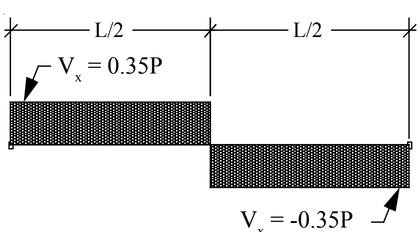
Step 3: Calculate the transverse components of applied and reactive forces.



$P_Y = P * \sin 45°$
$P_Y = 0.71\ P$

$R_{Ay} = R_{By} = P/2 * \sin 45°$
$R_{Ay} = R_{By} = 0.35\ P$

Step 4: Draw the shear diagram.



For $0 < x < .71L$:
$V_X = 0.35P$

For $.71L < x < 1.41L$:
$V_X = 0.35P - 0.71P$
$V_X = -0.35P$