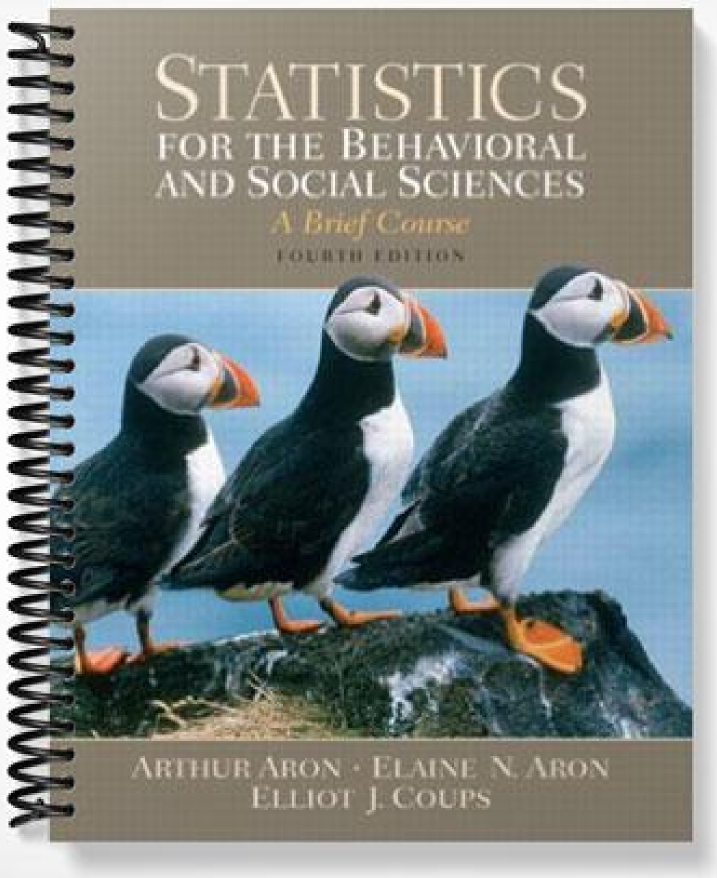
SOLUTIONS MANUAL



Chapter 2
The Mean, Variance, Standard Deviation, and *Z* Scores

Instructor's Summary of Chapter

Mean. The mean is the ordinary average—the sum of the scores divided by the number of scores. Expressed in symbols, $M = \sum X/N$. Less commonly used indicators of the typical or representative value of a distribution include the mode, the most common single value, and the median, the value of the middle case if you were to line up all the scores from highest to lowest.

Variation. The spread of the scores in a distribution can be described by the variance—the average of the squared deviation of each score from the mean: $SD^2 = \sum (X-M)^2 / N$. The standard deviation is the positive square root of the variance: $SD = \sqrt{SD^2}$. It can be best understood as an approximate measure of the average amount that scores differ from the mean.

Z Scores. A Z score is the number of standard deviations a raw score is from the mean of a distribution: $Z = (X-M)/SD$. Among other applications, Z scores permit comparisons of scores on different scales.

How the procedures of this chapter are described in research articles. Means and standard deviations (but not Z scores) are commonly reported in research articles; usually in tables.

Box 2-1. The Psychology of Statistics and the Tyranny of the Mean. Some psychologists—especially those associated with behaviorism, humanistic psychology, phenomenology, and qualitative methods—mistrust statistical processes because in the process of creating averages, knowledge about the individual case is lost. Many even holding these viewpoints, however, acknowledge that statistical analysis does play an important role, but argue that when studying any particular topic, careful study of individuals should always come first.

Lecture 2.1: The Mean

Materials

Lecture outline
Transparencies 1.5 and 2.1 through 2.4

Outline for Blackboard

I. Review/Last Assignment
II. Describing the Average
III. The Mean
IV. Formulas and Symbols: $M = \sum X / N$
V. The Median
VI. Review this Class

Instructor's Lecture Outline

I. Review

A. Idea of descriptive statistics and importance for their own right and as a foundation for the rest of the course.
B. Describing a distribution using frequency tables.
C. Describing a distribution graphically.

II. Describing the Average

A. Principle: Summarize a distribution of scores as a single number.
B. Show TRANSPARENCY 1.5 (stress-ratings example, from text).
C. Previously we summarized this group of numbers in a table and graph. Now we want to summarize it into a single number.

III. The Mean

A. Mean is the arithmetic average—sum of scores divided by number of scores.
B. Example calculation: Show TRANSPARENCY 2.1 top (stress-ratings mean computation).
C. Mean as balance point: Show TRANSPARENCY 2.1 bottom.
D. Show TRANSPARENCY 2.2 (example from class questionnaire).

IV. Formula and Symbols

A. Show TRANSPARENCY 2.1 top (stress-ratings mean computation) and discuss each symbol.
B. Emphasize value of symbols in statistics and importance of mastering them.

V. The Median

A. Problem with the mean is that it is highly influenced by extreme scores: Show TRANSPARENCY 2.3 (horn-honking and other examples) and discuss computation of mean with and without additional extreme score.

B. The median.

1. An alternative to the mean for describing the representative value of a group of scores.
2. The median is the middle score.
3. Computation:
   a. Organize scores from lowest to highest.
   b. Count to middle score.
   c. If an even number of scores, take the average of the middle two.
4. Show TRANSPARENCY 2.3 (horn-honking study and other examples) again and discuss:
   a. Computation of median.
   b. How median is not affected by extreme score in horn-honking study.
   c. How median is not affected by extreme scores as shown in example of 1-1-1-1-1-8-9-9-9-9-9 versus 7-7-7-7-7-8-50-50-50-50-50 (that is, both have same median but not same mean).
   d. Computation of median versus mean in feudal village example.
5. Show TRANSPARENCY 2.4 (steps for figuring median) and discuss.

VI. Review this Class: Use blackboard outline.

Lecture 2.2: The Variance and Standard Deviation

Materials

Lecture outline
Transparencies 2.1, 2.2, and 2.5 through 2.7

Outline for Blackboard

I. Review/Last Assignment
II. Variation
III. Deviation Scores and Squared Deviation Scores
IV. The Variance
V. Formulas and Symbols: $SD^2 = \sum (X\text{-}M)^2/N$
VI. The Standard Deviation: $SD = \sqrt{SD^2}$
VII. Review this Class

Instructor's Lecture Outline

I. Review

A. Idea of descriptive statistics and importance for their own right and as a foundation for the rest of the course.
B. Describing a distribution using frequency tables and graphs.
C. Describing the representative value of a group of scores as the mean: $M = \sum X/N$: Show TRANSPARENCY 2.1 (computation of mean for stress ratings) and discuss.

II. Variation

A. A distribution can be characterized by how much the scores in it vary from each other—they could all be bunched closely together or very spread out, or anywhere in between.
B. Knowing the mean and variation in a distribution gives a much more complete sense of how scores are distributed than the mean alone.
C. Examples of possible situations in which the mean SAT is 600 for entering students at a particular college.
   1. All have almost exactly 600—there is little variation.
   2. About half have SATs of 400 and half of 800—there is a very great deal of variation.
   3. About equal numbers having SATs of 500, 550, 600, 650, 700—a moderate amount of variation.
D. In general, the amount of variation and mean are independent of each other.
E. Show TRANSPARENCY 2.5 (distributions with various means and variances from the text) and discuss.

III. Deviation Scores and Squared Deviation Scores

A. One way of determining the variance numerically focuses on the extent to which scores differ from the mean.

B. A deviation score is the score minus the mean—if a score is 28 and the mean is 25, the deviation score is 3.
C. Over an entire distribution positive and negative deviation scores balance each other out.
D. For this and other more complicated reasons, we emphasize squared deviations.

IV. The Variance

A. A widely used measure of the variation in a group of scores.
B. It is the average of the squared deviation scores.
C. But is not the average amount that scores differ from the mean; it is the average amount of *squared* differences of scores from the mean. So it will typically be much larger than the average amount that scores differ from the mean.

V. Formulas and Symbols: $SD^2 = (X\text{-}M)^2/N$

A. Show TRANSPARENCY 2.1 (stress-ratings example) and discuss the meaning of each symbol in the computation of variance.
B. Show TRANSPARENCY 2.2 (scores from class questionnaire) and discuss the computation of the variance.

VI. The Standard Deviation: $SD = \sqrt{SD^2}$

A. The positive square root of the variance.
B. *Approximately* the average amount that scores differ from the mean in a particular distribution.
C. The most widely used descriptive statistic for describing the variation in a distribution.
D. Show TRANSPARENCY 2.1 (stress ratings example from text) and discuss computation of standard deviation.
E. Show TRANSPARENCY 2.2 (class questionnaire scores) focusing on computation of standard deviation at bottom.
F. Show TRANSPARENCY 2.6 (steps for figuring variance) and discuss.

VII. Review Class: Use blackboard outline and TRANSPARENCY 2.7 (review of mean, variance and standard deviation).

Lecture 2.3: *Z* Scores

Materials

Lecture outline
Transparencies 2.7 through 2.9

Outline for Blackboard

I. Review/Last Assignment
II. Describing a Score in Relation to the Distribution
III. Figuring the *Z* Score: *Z = (X-M)/SD*
IV. Changing *Z* Scores to Raw Scores: *X = (Z)(SD) + M*
V. Review this Class

Instructor's Lecture Outline

I. Review

A. Idea of descriptive statistics and importance for their own right and as a foundation for the rest of the course.
B. Describing a distribution using frequency tables and graphs.
C. Describing the representative value and variation in a group of scores: Show TRANSPARENCY 2.7 (review of mean, variance and standard deviation) and discuss.

II. Describing a Score in Relation to the Distribution

A. So far we have described distributions, now we turn to describing a single score's location in a distribution.
B. Knowing an individual's score gives little information without knowing where that score stands in relation to the entire distribution.
C. Knowing the mean of the distribution allows you to tell whether a score is above or below the average in that distribution.
D. Example: An individual has a score of 26 on a leadership test.
   1. What does that indicate? Is the person a particularly good leader? A particularly poor leader? About average?
   2. If the mean on this test is 20, then you know that the person is above average in leadership (compared to other people who have taken this test).
E. Knowing the standard deviation of the distribution allows you to tell how much above or below the average that score is in relation to the spread of scores in the distribution.
F. Leadership test example:
   1. Suppose the standard deviation is 3.
   2. The person with a score of 26 is two standard deviations above the mean of 20.
   3. Thus the person is about twice as much above the mean as the average score differs from the mean.
   4. Show TRANSPARENCY 2.8 (graphic illustration of this example) and discuss how the *SD* serves as a kind of unit of measure.

5. Note that in this example the overall distribution is a normal curve and that when this is the case this approach becomes especially useful, as the students will learn later.

G. Second example: Individual scores 84 on a test of planning ability.
   1. If mean is 90 and standard deviation is 12, this person's score is 1/2 standard deviation below the mean.
   2. Thus the person is below average, but not by a lot—about half as much as the average score differs from the mean.
   3. Show TRANSPARENCY 2.9 (graphic illustration of this example) and discuss.

H. The number of standard deviations a score is above or below the mean is called its *Z* score.

I. Show TRANSPARENCY 2.9 again and discuss relation of *Z* scores and raw scores—they are two different ways of measuring the same thing.

J. *Z* scores provide a helpful way to compare scores on measures that are on completely different scales. For example, if a person scored 26 on leadership and 84 on planning, we can say that the person scores much higher than average on leadership and slightly lower than average on planning.

III. Figuring the *Z* Score: $Z = (X-M)/SD$

A. Show TRANSPARENCY 2.8 (leadership test) and discuss computation of *Z* score using the formula.

B. Show TRANSPARENCY 2.9 (planning test) and discuss computation of *Z* score using the formula.

IV. Changing *Z* Scores to Raw Scores: $X = (Z)(SD) + M$

A. Show TRANSPARENCY 2.8 (leadership test) and discuss conversion of *Z* scores to raw scores at bottom.

B. Show TRANSPARENCY 2.9 (planning test) and discuss conversion of *Z* scores to raw scores at bottom.

V. Review this Class

A. Go through blackboard outline.

B. Emphasize importance of knowing *Z* scores thoroughly as preparation for next topics.

TRANSPARENCY 2.1

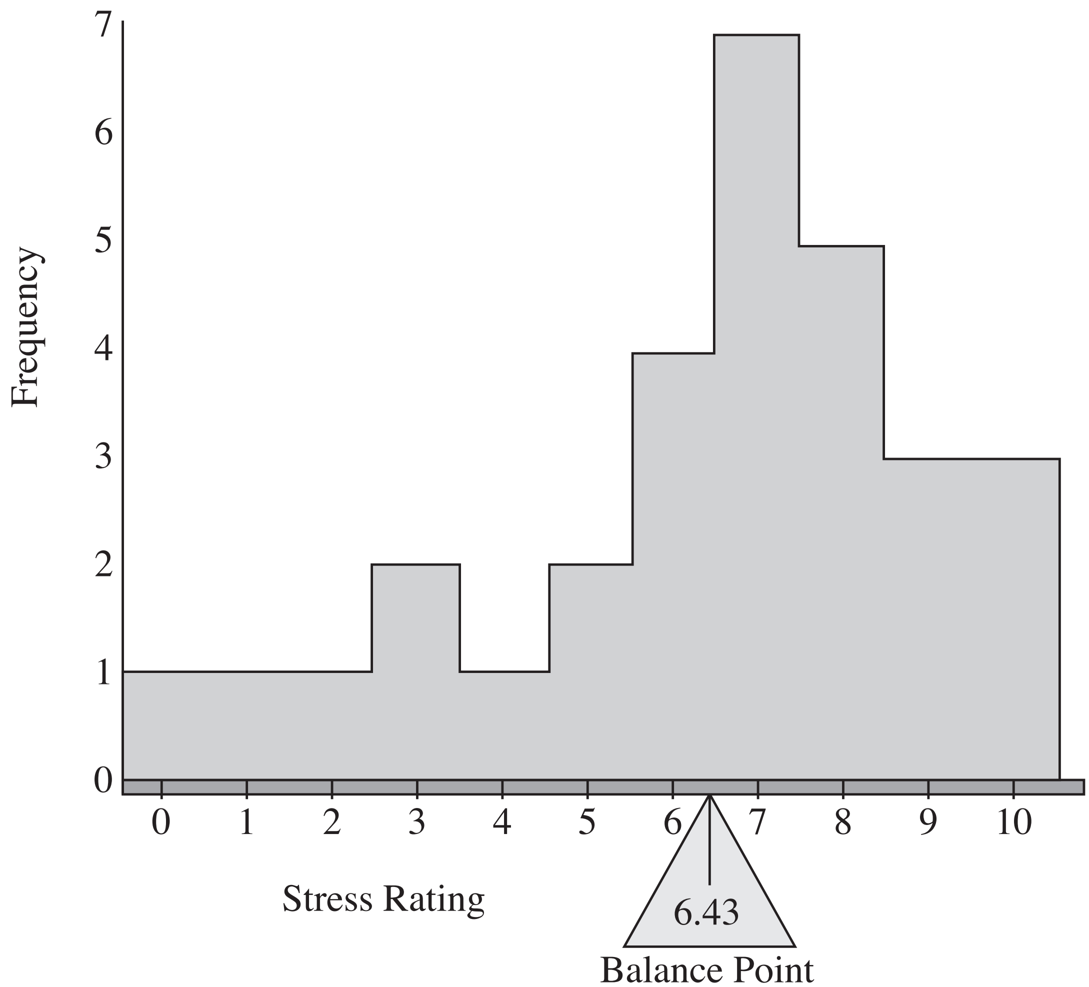
Ratings of stress of statistics class students.
(Data from Aron et al., 1995)

Sum of stress ratings = 193
Mean is 193 / 30 = 6.43

$$M = \frac{\sum X}{N} = \frac{193}{30} = 6.43$$

$SD^2 = \sum (X–M)^2 / N = 197.4 / 30$

$SD = \sqrt{SD^2} = \sqrt{6.58} = 2.57$



TRANSPARENCY 2.2

Figuring the mean, variance, and standard deviation.
(Scores from class questionnaire.)

"Would you prefer to live out in the country
with not many people around?"

1	2	3	4	5	6	7
Not at All			Moderately			Extremely

7, 5, 1, 4, 7, 7, 5, 1, 3, 4, 3, 4, 6, 2,
4, 4, 2, 3, 4, 1, 4, 2, 3, 2, 1, 3, 2, 3,
4, 6, . , 4, 2, 6, 4, 2, 1, 1, 4, 6, 2, 5,
6, 1, 3, 2, 2, 4, 4, 4, 5, 4, 4, 7, 5, 5,
5, 1, 6, 2, 1, 2, 6, 6, 3, 4, 5, 1, 1, 5,
2, 3, 5, 7, 4, 2, 4, 5, 4, 4, 7, 4, 1, 5,
3, 4, 4, 4, 3, 5, 2, 2, 3, 4, 7, 1, 4, 7,
1, 3, 3, 4, 5, 6, 4, 1, 2, 7, 1, 4, 2

RESPONDENT	RATING		M		X-M	$(X\text{-}M)^2$
1	7	-	3.65	=	3.35	11.22
2	5	-	3.65	=	1.35	1.82
3	1	-	3.65	=	-2.65	7.02
4	4	-	3.65	=	.35	.12
.	.		.		.	.
.	.		.		.	.
109	4	-	3.65	=	.35	.12
110	2	-	3.65	=	-1.65	2.72
	===					========
	$\sum X = 401$					$\sum(X\text{-}M)^2 = 350.34$

$M = \sum X/N = 401/110 = 3.65$

$$SD^2 = \sum(X\text{-}M)^2/N = 350.34/110 = 3.18$$

$$SD = \sqrt{SD^2} = \sqrt{3.18} = 1.78$$

TRANSPARENCY 2.3

Measures of representative values of a group of scores.
(Fictional data based on Kenrick & MacFarlane, 1986.)

Seconds honking at stalled car.= (N = 29):

3.5, 2.0, 0, 5.0, .5, 1.0, 4.0, 3.5
3.0, 1.5, 1.5, 2.0, 2.5, 3.0, 3.0
3.5, 4.5, 2.0, 2.5, 4.5, 4.0, 3.5
3.0, 2.5, 2.5, 3.5, 3.5, 4.0, 3.0

$\sum X = 82.5 \quad N = 29 M = \sum X / N = 82.5 / 29 = 2.85$

With one additional case of 13 seconds:

$\sum X = 82.5+13=95.5 \quad N=29+1=30 \quad M=\sum X/N=95.5/30 = 3.18$

1	0	
2	.5	
3	1.0	
4	1.5	
5	1.5	
6	2.0	
7	2.0	
8	2.0	
9	2.5	
10	2.5	
11	2.5	
12	2.5	
13	3.0	
14	3.0	
15	3.0	<—— MEDIAN
16	3.0	
17	3.0	
18	3.5	
19	3.5	
20	3.5	
21	3.5	
22	3.5	
23	3.5	
24	4.0	
25	4.0	
26	4.0	
27	4.5	
28	4.5	
29	5.0	

Feudal Village Example

50 Families @ $100/Yr = 5,000

1 Family @ $100,000/Yr = 100,000

Village total = $105,000

$\sum X = 105{,}000 \quad N = 51$

$M = \sum X = 105{,}000/51 = \$2{,}059$

Median = $100   Mode = $100

Median examples:

1–1–1–1–1–8–9–9–9–9–9 —> Median = 8

7–7–7–7–8–50–50–50–50–50 —> Median = 8

$\sum(X–M)^2$: $(3.5–2.85)^2 + (2.0–2.85)^2 + \ldots + (3.5–2.85)^2 = 39.98$

$SD^2 = \sum(X–M)^2/N = 39.98 / 29 = 1.38 \qquad SD = \sqrt{SD^2} = \sqrt{1.38} = 1.17$

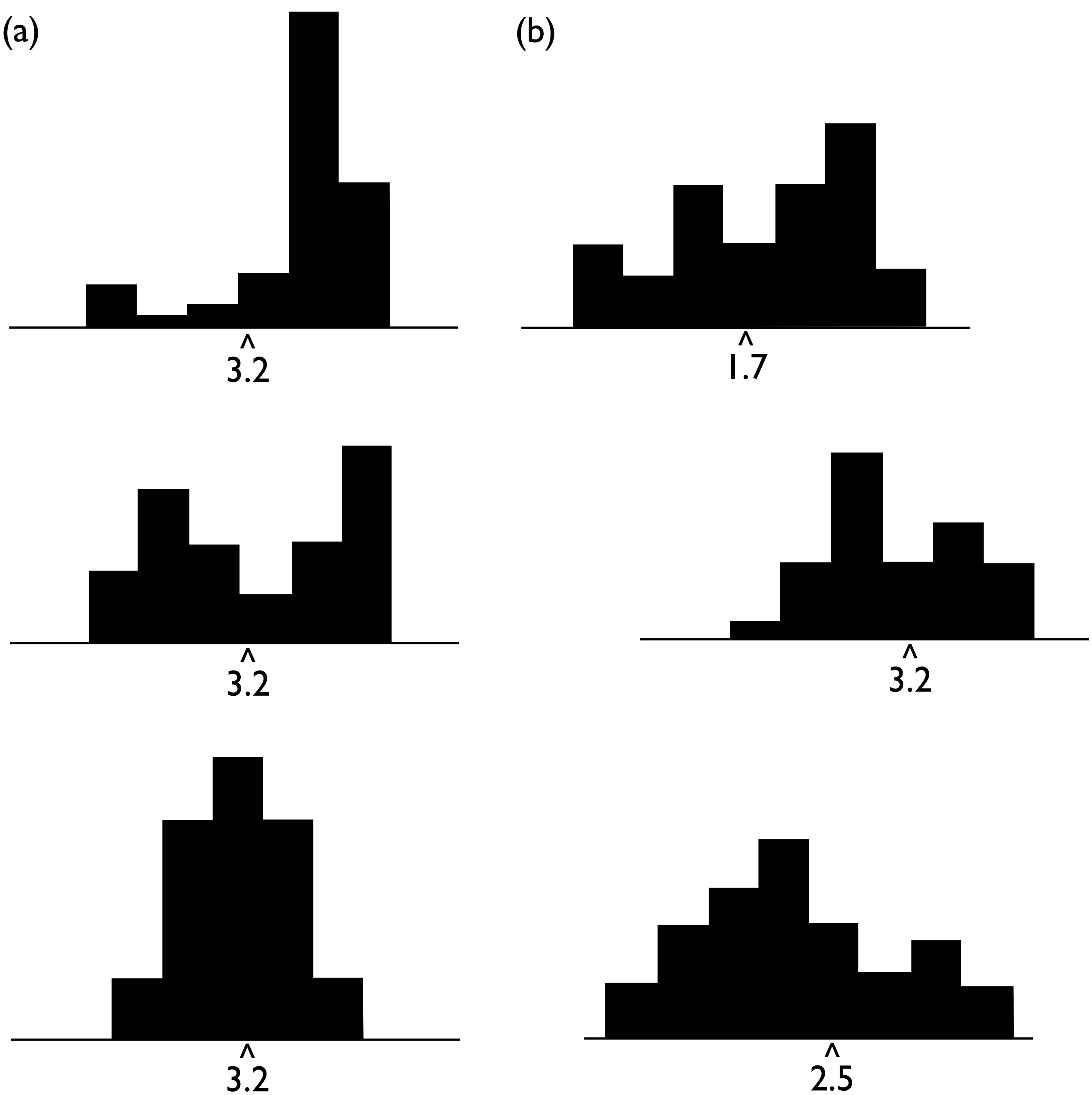
Steps for Figuring the Median

❶ **Line up all the scores from highest to lowest.**

❷ **Figure how many scores there are to the middle score, by adding 1 to the number of scores and dividing by 2.**

❸ **Count up to the middle score or scores.** If you have one middle score, this is the median. If you have two middle scores, the median is the average (the mean) of these two scores.

TRANSPARENCY 2.5



Steps for Figuring the Variance

❶ **Subtract the mean from each score.**

❷ **Square each of these deviation scores.**

❸ **Add up the squared deviation scores.**

❹ **Divide the sum of squared deviations by the number of scores.**

TRANSPARENCY 2.7

Review of Mean, Variance, and Standard Deviation

A. Principle: Describe (these are descriptive statistics) by reducing a group of numbers—a distribution—to some simple terms.

B. The mean (arithmetic average):

RULE: Add up the numbers and divide by the number of numbers.

FORMULA AND SYMBOLS: $M = \sum X/N$

Mean is most widely used, and generally the best indicator of the typical score.

C. Variation: A single number that describes how much variation there is in a group of numbers—that is, how spread out or narrow a distribution is.

1. VARIANCE is average of squared deviations from the mean.

RULE:
1. Subtract the mean from each score to get deviation score.
2. Square each deviation score.
3. Add up all the squared deviations scores.
4. Divide by number of scores to get average of squared deviation scores.

FORMULA AND SYMBOLS: $SD^2 = \sum(X-M)^2 / N$

2. STANDARD DEVIATION is the positive square root of variance.

INTUITIVE INTERPRETATION: Standard deviation is roughly the average amount each score differs from the mean.

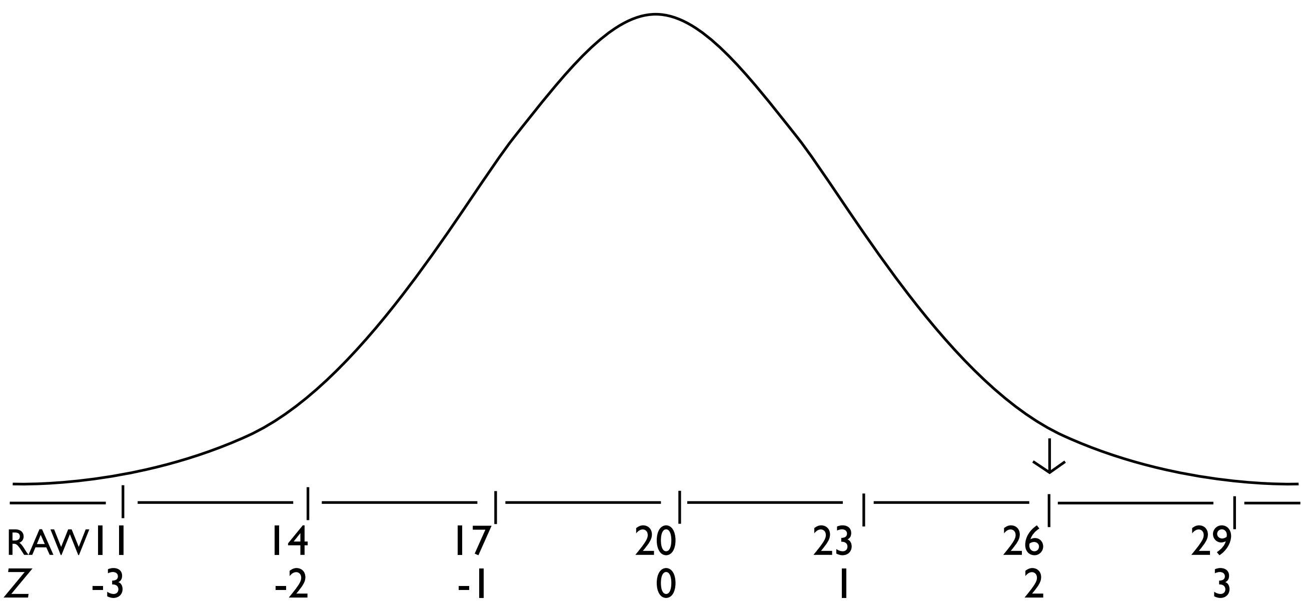
RULE: Compute variance and take the square root.

FORMULA AND SYMBOLS: $SD = \sqrt{SD^2}$

TRANSPARENCY 2.8

Z score examples for leadership test. (Fictional data.)

$M = 20 \qquad SD = 3$



Leadership Score = 26: $Z = X\text{-}M \,/\, SD = 26–20 \,/\, 3 = 6 \,/\, 3 = 2$

Z Score to Raw Score Formula $X = (Z)(SD) + M$

Z Score = 0 $X = (0)\,(3) + 20 = 0 + 20 = 20$

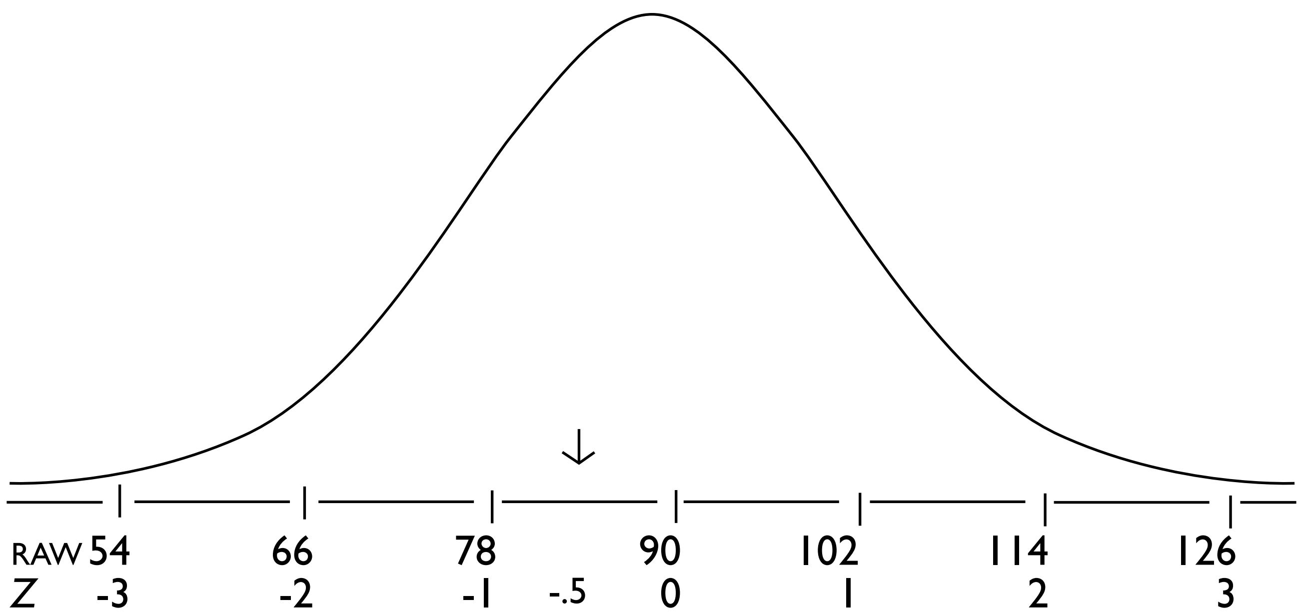
1: $X = (1)\,(3) + 20 = 3 + 20 = 23$

–1: $X = (–1)\,(3) + 20 = –3 + 20 = 17$

TRANSPARENCY 2.9

Z score examples for planning test. (Fictional data.)

$M = 90 \qquad SD = 12$



IF $X = 84$: $\quad Z = X–M \;/\; SD = 84–90 \;/\; 12 = –6 \;/\; 12 = –.5$

Z Score to Raw Score Formula $\;X = (Z)(SD) + M$

Z Score = $–.5$: $X = (–.5)\,(12) + 90 = –6 + 90 = 84$

2.0: $X = (2.0)\,(12) + 90 = 24 + 90 = 114$

1.2: $X = (1.2)\,(12) + 90 = 14.4 + 90 = 104.4$