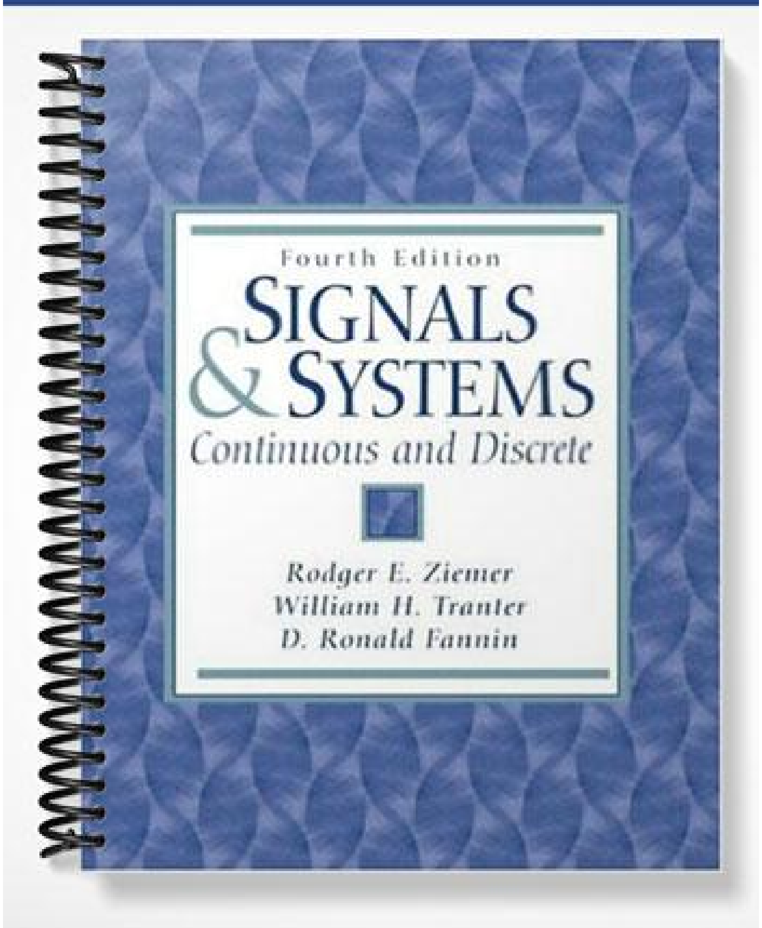
SOLUTIONS MANUAL



Solutions Manual

Fourth Edition

SIGNALS & SYSTEMS

Continuous and Discrete

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Manufacturing Buyer: Donna Sullivan

Printed in the United States of America

10 9 8 7 6 5 4 3 2

ISBN 0-13-673310-7

Prentice-Hall International (UK) Limited, *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall Canada, Inc., *London*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Simon & Schuster Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brazil, Ltda., *Rio de Janeiro*

PREFACE

This manual contains solutions to all end-of-chapter problems and all computer exercises contained in the Fourth Edition of *Signals and Systems: Continuous and Discrete*. The manual is divided into two separate parts. Part I contains solutions to the end-of-chapter problems and Part II contains solutions to the computer exercises.

All computer exercises are developed using MATLAB as are all end-of-chapter problems specifying the use of MATLAB. In several parts of this manual, especially in Chapters 8 and 9, MathCAD is used for a little variety in a few problems.

Thanks go to Carol Baker for her expert typing skills and for her help in assembling the final product.

While we have done our best to insure that the problem solutions contained herein are correct, it is inevitable that manuals such as this are never perfect. We apologize in advance for any frustration caused by such errors.

R.E.Z.
W.H.T.
D.R.F.

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PART I

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

CHAPTER 1

Problem 1-1

(a) Write the acceleration as

$$a(t) = \begin{cases} \alpha t, & t \le t_0 \\ 0, & t > t_0 \end{cases}$$

Thus the velocity and position are, respectively, given by

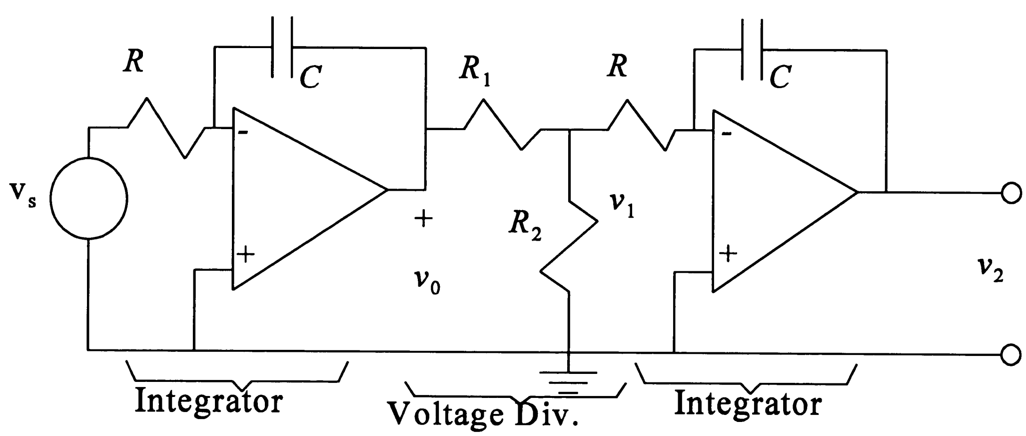
$$v(t) = \int_0^t a(\lambda)d\lambda = \begin{cases} \alpha t^2/2, & t \le t_0 \\ \alpha t_0^2/2, & t > t_0 \end{cases}$$

and

$$x(t) = \int_0^t v(\lambda)d\lambda = \begin{cases} \alpha t^3/6, & t \le t_0 \\ \alpha t_0^3/6 + \alpha t_0^2(t - t_0)/2, & t > t_0 \end{cases}$$

For $t_0 = 72$ s and $\alpha = 5/9$ m/s^2, we have $x(t) = (5/54)t^3$, $t \le 72$ s. At $t = t_0 = 72$ s (burnout), we have $x(t_0) = 35.56$ km.

(b) See the figure below for the integrator.



$$v_2(t) = -\frac{1}{RC}\int_0^t v_1(\lambda)d\lambda$$

Assume that $R_2 << R$. The input impedance to the op-amp integrator is therefore much larger than the output impedance of the previous stage, and

$$v_1(t) = \frac{R_2}{R_1 + R_2} v_0(t)$$

From Example 1-2,

$$v_0(t) = -\frac{1}{RC}\left(\frac{\beta t^2}{2}\right) = -\frac{\beta t^2}{2RC}$$

Therefore,

$$v_2(t) = -\frac{1}{RC}\int_0^t \frac{R_2}{R_1 + R_2}\left(-\frac{\beta}{2RC}\right)\lambda^2 d\lambda$$

Integrating and setting $t = t_0$, we obtain

$$v_2(t_0) = \frac{R_2}{R_1 + R_2}\left(\frac{\beta t_0^2}{2RC}\right)\left(\frac{t_0}{3RC}\right) = 10 \text{ V}$$

The second factor on the right is 10 V because of the maximum output limitation on the first integrator. Thus, we require that

$$\frac{R_2}{R_1 + R_2}\left(\frac{t_0}{3RC}\right) = 1$$

For example, from Example 1-2 we have $RC = 0.36$ s. With $t_0 = 72$ s and $R_1 = 10$ k ohms, we get $R_2 = 152$ ohms.

Problem 1-2

(a) Let $n = 0, 1, 2, 3, ..., N$. Then

$$\begin{aligned} v(T) &= v(0) + Ta(T) \quad \text{(a)} \\ v(2T) &= v(T) + Ta(2T) \quad \text{(b)} \\ &\dots \\ v(NT) &= v[(N-1)T] + Ta(NT) \quad \text{(c)} \end{aligned}$$

Substitute (a) into (b) and so on until (c is reached. This gives

$$v(NT) = v(0) + T\sum_{n=1}^{N} a(nT)$$

(b) Let $n = 0, 1, 2, 3, ..., N$. Then

$$\begin{aligned} v(T) &= v(0) + (T/2)[a(0) + a(T)] \quad \text{(a)} \\ v(2T) &= v(T) + (T/2)[a(T) + a(2T)] \quad \text{(b)} \\ &\dots \\ v(NT) &= v[(N-1)T] + (T/2)\{a[(N-1)T] + a(NT)\} \quad \text{(c)} \end{aligned}$$

Substitute (a) into (b) and so on until (c) is reached. The result is as given in the problem statement.

Problem 1-3

(a) A maximum departure of the weight from equilibrium of 1 cm requires a spring constant of

$$K = \frac{Ma_{\max}}{x_{\max}} = \frac{(0.002)(20)}{0.01} = 4 \text{ kg/s}^2$$

(b) For a minimum increment of 0.5 mm = 0.0005 m, we have

$$\Delta a_{\min} = \frac{K\Delta x_{\min}}{M} = \frac{4(0.0005)}{0.002} = 1 \text{ m/s}^2$$

(c) The velocity is given by

$$v_r(t) = \int_0^t a(\lambda)d\lambda = \int_0^t 20\,d\lambda = \begin{cases} 20t, & 0 \le 50 \text{ s} \\ 1000, & t > 50 \text{ s} \end{cases}$$

Problem 1-4

K is the same as in Example 1-1 because M, x_{max}, and a_{max} are the same. Also, Δa_{min} is the same. The velocity profile is

$$v_r(t) = \begin{cases} \int_0^t 20\,d\lambda = 20t, \; 0 \le t < 10 \\ 200, \; 10 \le t < 20 \\ 200 + \int_{20}^t 20\,d\lambda = 200 + 20(t-20), \; 20 \le t < 30 \\ 400, \; t > 30 \end{cases}$$

Problem 1-5

From (1-15) and using the $x(t)$ given in the problem, we have

$$\begin{aligned} s(t) &= \cos(\omega_0 t) + \alpha\beta\cos[\omega_0(t - 2\tau)] \\ &= [1 + \alpha\beta\cos(2\omega_0\tau)]\cos(\omega_0 t) + \alpha\beta\sin(2\omega_0\tau)\sin(\omega_0 t) \\ &= A(\tau)\cos[\omega_0 t - \theta(\tau)] \\ &= A(\tau)\cos\theta(\tau)\cos(\omega_0 t) + A(\tau)\sin\theta(\tau)\sin(\omega_0 t) \end{aligned}$$

Set coefficients of like sin/cos terms equal on each side of the identity to obtain

$$\begin{aligned} A(\tau)\cos\theta(\tau) &= 1 + \alpha\beta\cos(2\omega_0\tau) \\ A(\tau)\sin\theta(\tau) &= \alpha\beta\sin(2\omega_0\tau) \end{aligned}$$

Square and add to obtain

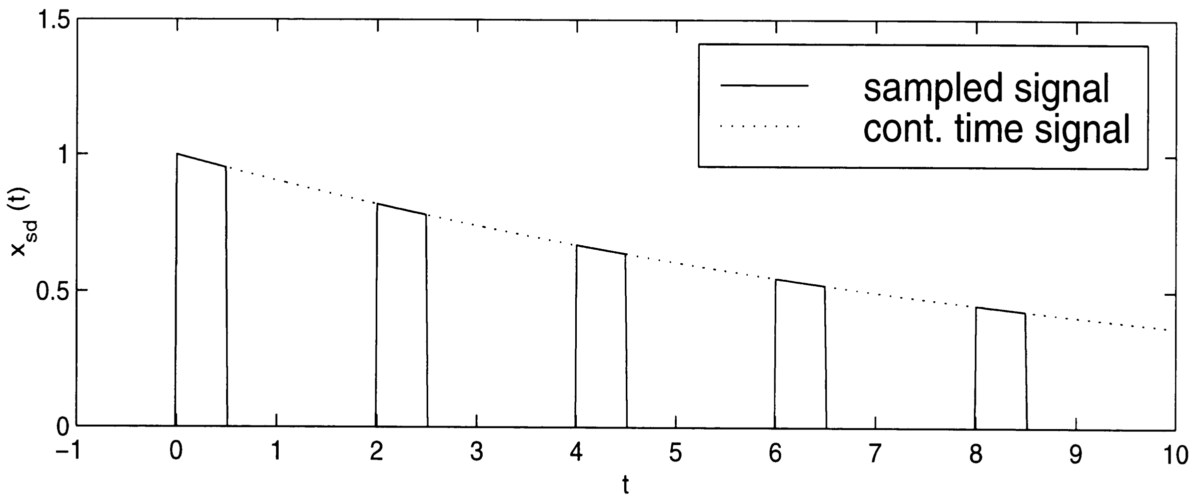
$$A(\tau) = \sqrt{1 + 2\alpha\beta\cos(2\omega_0\tau) + (\alpha\beta)^2}$$

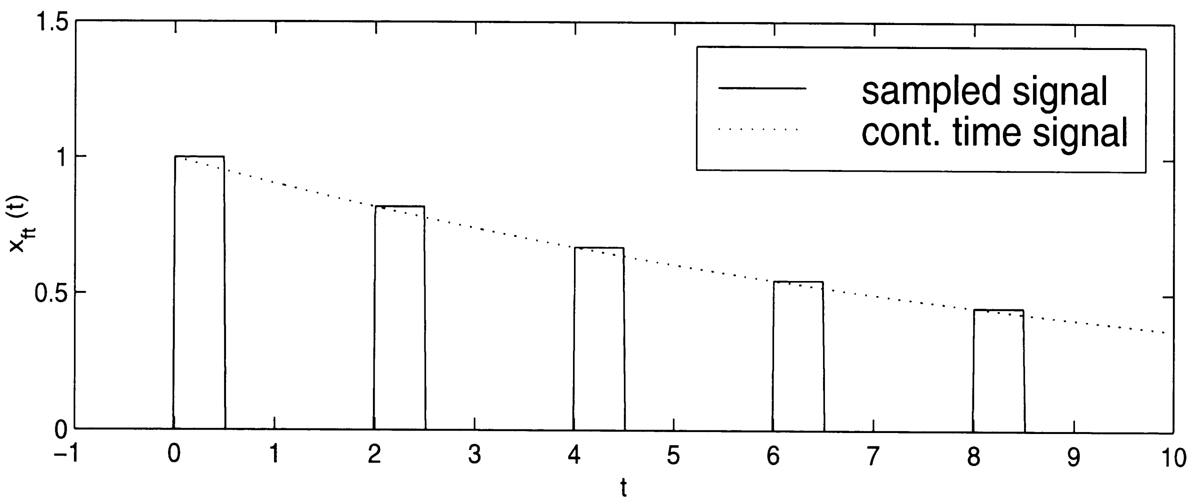
Divide the second equation by the first to obtain

$$\frac{\sin\theta(\tau)}{\cos\theta(\tau)} = \tan\theta(\tau) = \frac{\alpha\beta\sin(2\omega_0\tau)}{1 + \alpha\beta\cos(2\omega_0\tau)}$$

Problem 1-6

Sketches of the analog and sampled signals for both cases are shown below[(a) top and (b) bottom]:





Problem 1-7

(a) The impulse-sampled signal is

$$x_{\text{imp. samp}}(t) = \cos(2\pi t) \sum_{n=-\infty}^{\infty} \delta(t - 0.1n)$$
$$= \sum_{n=-\infty}^{\infty} \cos(2\pi t)\,\delta(t - 0.1n)$$
$$= \sum_{n=-\infty}^{\infty} \cos(0.2\pi n)\,\delta(t - 0.1n)$$

where property (1-59) for the unit impulse has been used to get the last result.
(b) The unit-pulse train sampled signal is

$$x_{\text{unit pulse samp}}(t) = \cos(2\pi t) \sum_{n=-\infty}^{\infty} \delta[t - 0.1n]$$
$$= \sum_{n=-\infty}^{\infty} \cos(2\pi t)\,\delta[t - 0.1n]$$
$$= \sum_{n=-\infty}^{\infty} \cos(0.2\pi n)\,\delta[t - 0.1n]$$

where the fact that the unit pulse is 1 for its argument 0 and 0 otherwise has been used.

Problem 1-8

(a) The signal can be developed in terms of equations as follows:

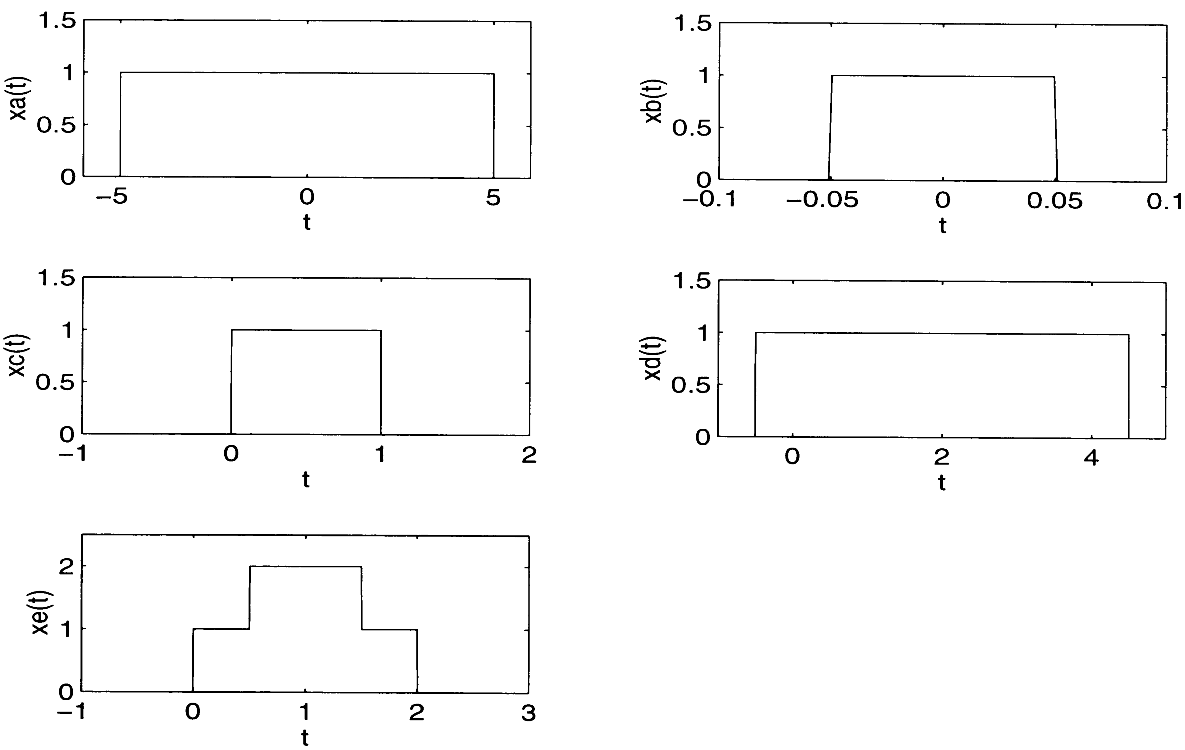
$$\Pi(0.1t) = \begin{cases} 1, & |0.1t| \le 1/2 \\ 0, & \text{otherwise} \end{cases}$$
$$= \begin{cases} 1, & |t| \le 10/2 = 5 \\ 0, & \text{otherwise} \end{cases}$$

This is a rectangular pulse of amplitude 1 between -5 and 5 and 0 otherwise. A sketch will be given at the end of the problem solution.
(b) Following a procedure similar to that of (a) one finds that this is a rectangular pulse of amplitude 1 between -0.05 and 0.05 and 0 otherwise. A sketch will be given at the end of the problem solution.
(c) This is a rectangular pulse of amplitude 1 between 0 and 1 and 0 otherwise. A sketch will be given at the end of the problem solution.
(d) This is a rectangular pulse of amplitude 1 between 0.5 and 4.5 and 0 otherwise. A sketch will be given at the end of the problem solution.

(e) The first term of this signal is a rectangular pulse of amplitude 1 between 0 and 2 and 0 otherwise. The second term is a rectangular pulse of amplitude 1 between 0.5 and 1.5 and 0 otherwise. Where both pulses are nonzero, the total amplitude is 2; where only one pulse is nonzero the amplitude is 1. A sketch is provided below.

The MATLAB program below uses the special function given in Section 1-6 (page 32) of the text to provide the plots.

```
%       Sketches for Problem 1-8
%
t = -6:0.0015:6;
xa = pls_fn(0.1*t);
xb = pls_fn(10*t);
xc = pls_fn(t - 0.5);
xd = pls_fn((t - 2)/5);
xe = pls_fn((t - 1)/2) + pls_fn(t - 1);
subplot(3,2,1),plot(t, xa,'-w'), axis([-6 6 0 1.5]),xlabel('t'),ylabel('xa(t)')
subplot(3,2,2),plot(t, xb,'-w'), axis([-.1 .1 0 1.5]),xlabel('t'),ylabel('xb(t)')
subplot(3,2,3),plot(t, xc,'-w'), axis([-1 2 0 1.5]),xlabel('t'),ylabel('xc(t)')
subplot(3,2,4),plot(t, xd,'-w'), axis([-1 5 0 1.5]),xlabel('t'),ylabel('xd(t)')
subplot(3,2,5),plot(t, xe,'-w'), axis([-1 3 0 2.5]),xlabel('t'),ylabel('xe(t)')
```



Problem 1-9

(a) $2\pi f_0 = 50\pi$, so $T_0 = 1/f_0 = 1/25 = 0.04$ s. (b) $2\pi f_0 = 60\pi$, so $T_0 = 1/f_0 = 1/30 = 0.0333$ s. (c) $2\pi f_0 = 70\pi$, so $T_0 = 1/f_0 = 1/35 = 0.0286$ s. (d) We have $50\pi = 2\pi m f_0$ and $60\pi = 2\pi n f_0$, where m and n are integers and f_0 is the largest constant that satisfies these equations. The largest f_0 is 5 Hz with $m = 5$ and $n = 6$. (e) We have $50\pi = 2\pi m f_0$ and $70\pi = 2\pi n f_0$, where m and n are integers and f_0 is the largest constant that satisfies these equations. The largest f_0 is 5 Hz with $m = 5$ and $n = 7$.

Problem 1-10

(a) $|A| = 4.2426$; angle(A) = 0.7854 radians; $B = 5.0 + j\,8.6603$, so Re(B) = 5 and Im(B) = 8.6603. (b) $A+B = 8.0 + j11.6603$. (c) $A - B = -2.0 - j5.6603$. (d) $A*B = -10.9808 + j40.9808$. (e) A/B = 0.4098 - j0.1098.

Problem 1-11

(a) $2\pi f_0 = 10\pi$, so $T_0 = 1/f_0 = 1/5 = 0.2$ s. (b) $2\pi f_0 = 17\pi$, so $T_0 = 1/f_0 = 1/8.5 = 0.1176$ s. (c) $2\pi f_0 = 19\pi$, so $T_0 = 1/f_0 = 1/9.5 = 0.1053$ s. (d) We have $10\pi = 2\pi m f_0$ and $17\pi = 2\pi n f_0$, where m and n are integers and f_0 is the largest constant that satisfies these equations. The largest f_0 is 0.5 Hz with $m = 10$ and $n = 17$. (e) We have $10\pi = 2\pi m f_0$ and $19\pi = 2\pi n f_0$, where m and n are integers and f_0 is the largest constant that satisfies these equations. The largest f_0 is 0.5 Hz with $m = 10$ and $n = 19$. (f) We have $17\pi = 2\pi m f_0$ and $19\pi = 2\pi n f_0$, where m and n are integers and f_0 is the largest constant that satisfies these equations. The largest f_0 is 0.5 Hz with $m = 17$ and $n = 19$.

Problem 1-12

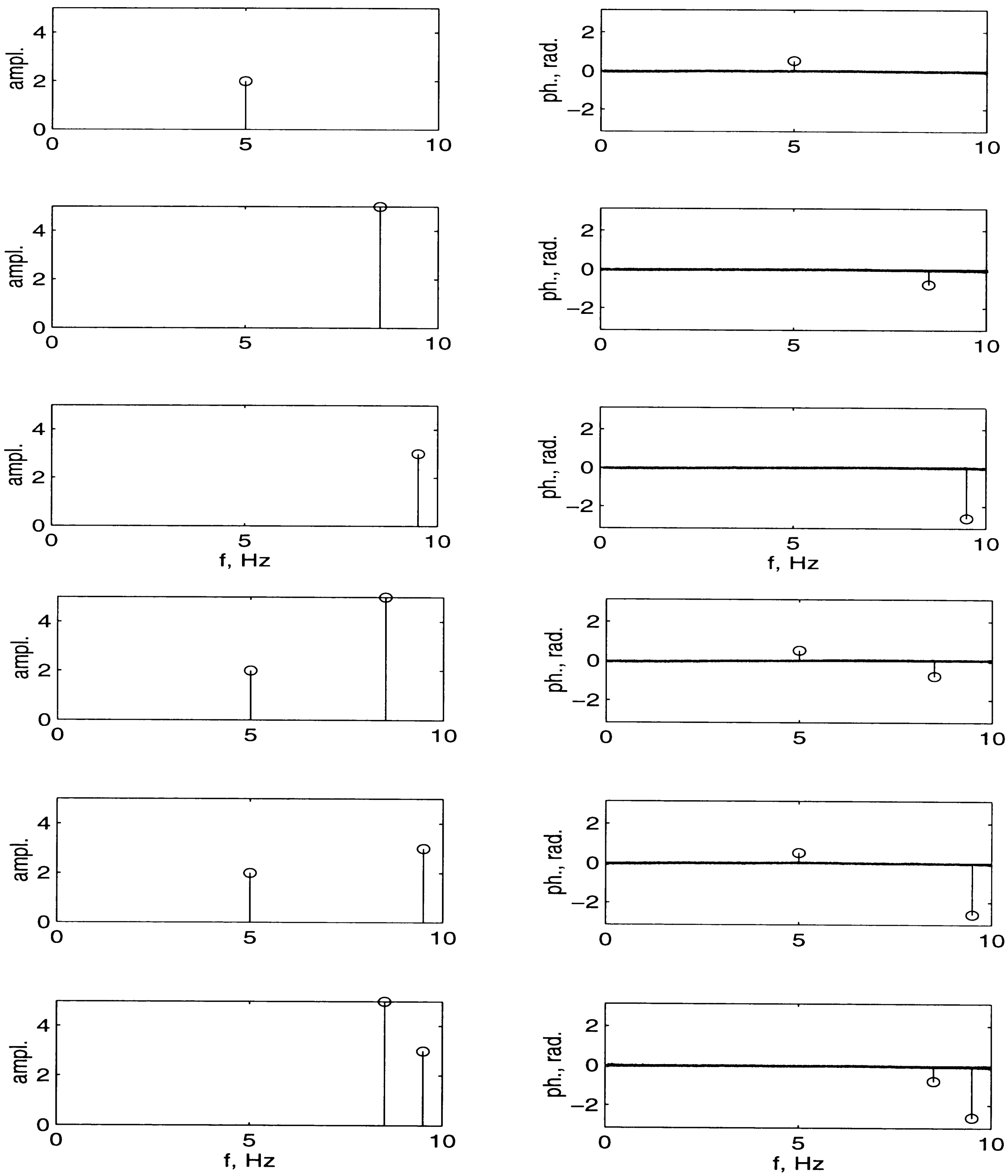
(a) Written as the real part of rotating phasors:

$$x_a(t) = \mathrm{Re}[2e^{j(10\pi t + \pi/6)}];\; x_b(t) = \mathrm{Re}[5e^{j(17\pi t - \pi/4)}]$$
$$x_c(t) = \mathrm{Re}[3e^{j(10\pi t - \pi/3 - \pi/2)}] = \mathrm{Re}[3e^{j(10\pi t - 5\pi/6)}]$$
$$x_d(t) = \mathrm{Re}[2e^{j(10\pi t + \pi/6)} + 5e^{j(17\pi t - \pi/4)}];\; x_e(t) = \mathrm{Re}[2e^{j(10\pi t + \pi/6)} + 3e^{j(10\pi t - 5\pi/6)}]$$
$$x_f(t) = \mathrm{Re}[5e^{j(17\pi t - \pi/4)} + 3e^{j(10\pi t - 5\pi/6)}]$$

(b) In terms of counter rotating phasors, the signals are:

$$x_a(t) = [e^{j(10\pi t + \pi/6)} + e^{-j(10\pi t + \pi/6)}];\; x_b(t) = [2.5e^{j(17\pi t - \pi/4)} + 2.5e^{j(17\pi t - \pi/4)}]$$
$$x_c(t) = [1.5e^{j(10\pi t - 5\pi/6)} + 1.5e^{-j(10\pi t - 5\pi/6)}]$$
$$x_d(t) = [e^{j(10\pi t + \pi/6)} + e^{-j(10\pi t + \pi/6)} + 2.5e^{j(17\pi t - \pi/4)} + 2.5e^{-j(17\pi t - \pi/4)}]$$
$$x_e(t) = [e^{j(10\pi t + \pi/6)} + e^{-j(10\pi t + \pi/6)} + 1.5e^{j(10\pi t - 5\pi/6)} + 1.5e^{-j(10\pi t - 5\pi/6)}]$$
$$x_f(t) = [2.5e^{j(17\pi t - \pi/4)} + 2.5e^{-j(17\pi t - \pi/4)} + 1.5e^{j(10\pi t - 5\pi/6)} + 1.5e^{-j(10\pi t - 5\pi/6)}]$$

(c) Single-sided spectra are plotted below. Double-sided amplitude spectra are obtained by halving the lines and taking mirror image; phase spectra are obtained by taking antisymmetric mirror image.



Problem 1-13

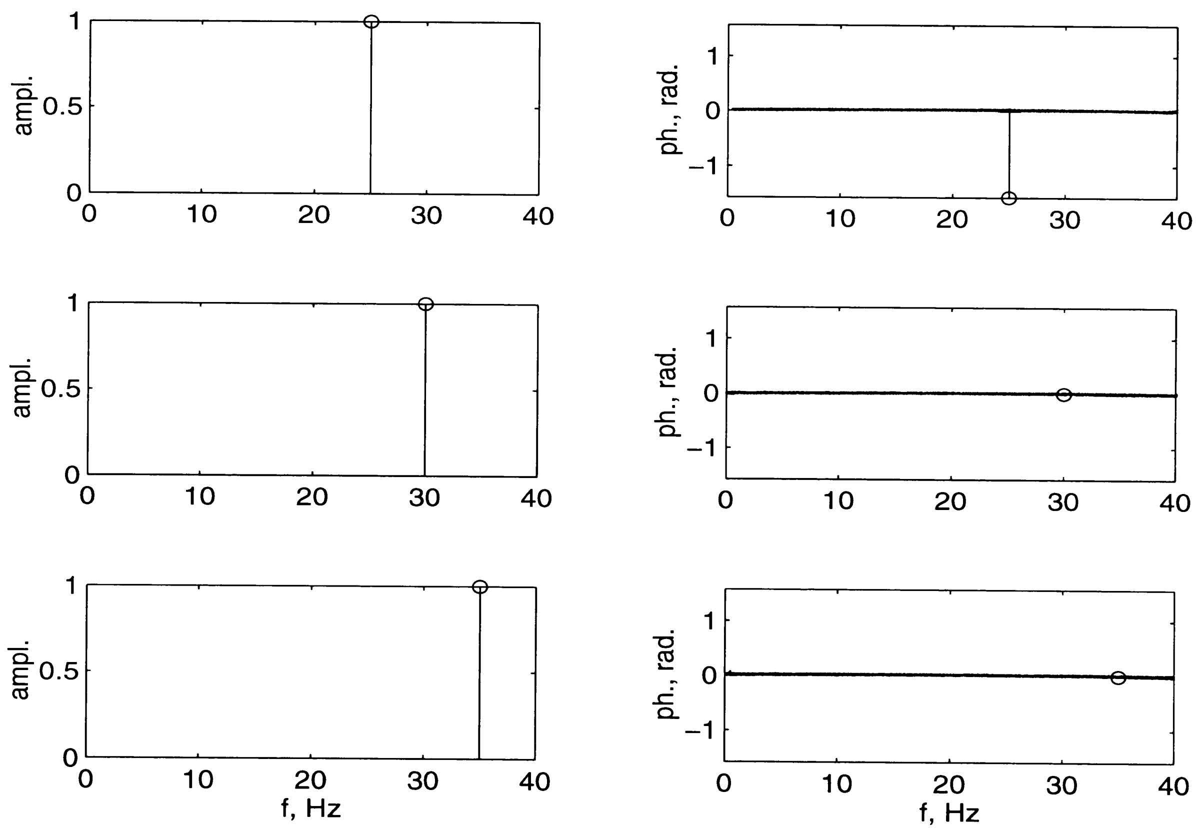
(a) Written as the real part of rotating phasors:

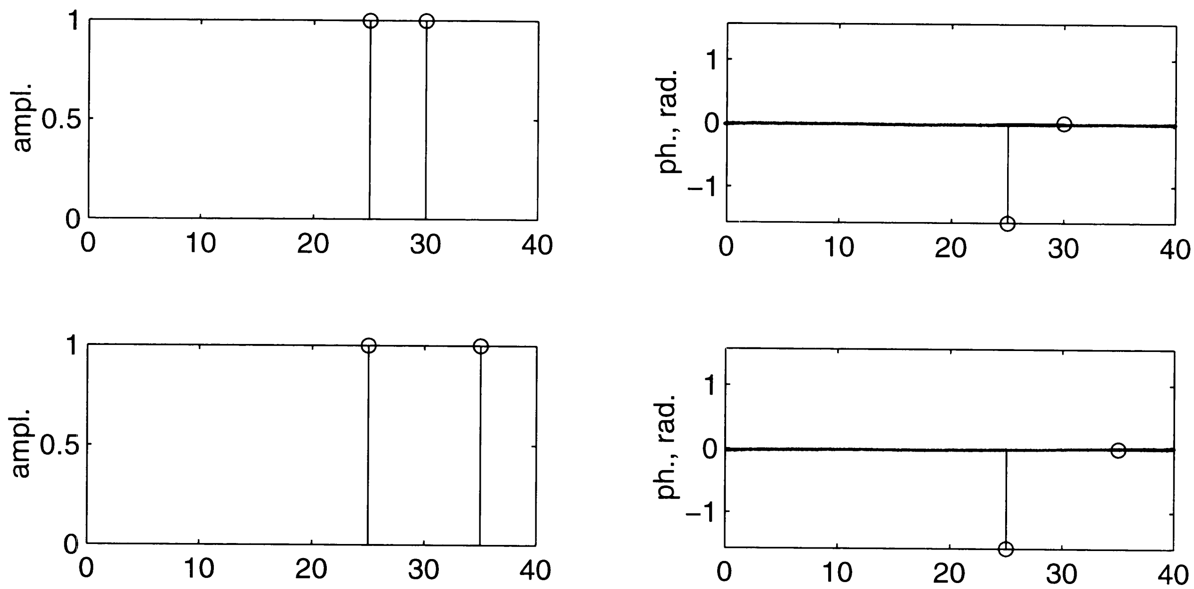
$$x_a(t) = \text{Re}[e^{j(50\pi t - \pi/2)}];\; x_b(t) = \text{Re}[e^{j60\pi t}];\; x_c(t) = \text{Re}[e^{j70\pi t}]$$
$$x_d(t) = \text{Re}[e^{j(50\pi t - \pi/2)} + e^{j60\pi t}];\; x_e(t) = \text{Re}[e^{j(50\pi t - \pi/2)} + e^{j70\pi t}]$$

(b) In terms of counter rotating phasors, the signals are:

$$x_a(t) = \text{Re}[0.5e^{j(50\pi t - \pi/2)} + 0.5e^{-j(50\pi t - \pi/2)}];\; x_b(t) = \text{Re}[0.5e^{j60\pi t} + 0.5e^{-j60\pi t}]$$
$$x_c(t) = \text{Re}[0.5e^{j70\pi t} + 0.5e^{-j70\pi t}]$$
$$x_d(t) = \text{Re}[0.5e^{j(50\pi t - \pi/2)} + 0.5e^{-j(50\pi t - \pi/2)} + 0.5e^{j60\pi t} + 0.5e^{-j60\pi t}]$$
$$x_e(t) = \text{Re}[0.5e^{j(50\pi t - \pi/2)} + 0.5e^{-j(50\pi t - \pi/2)} + 0.5e^{j70\pi t} + \; + 0.5e^{-j70\pi t}]$$

(c) The single-sided amplitude and phase spectra are shown below. See Prob. 1-12c for comments on obtaining double-sided spectra from single-sided spectra.

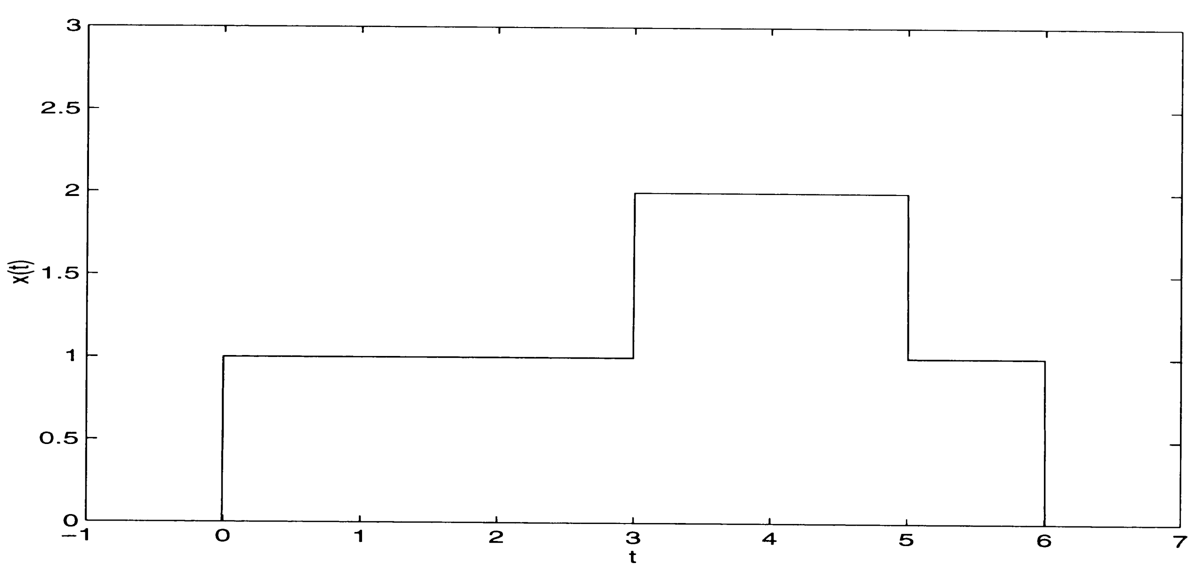




Problem 1-14

(a) A sketch is given below:

From the figure, it is evident that $x(t) = u(t) + u(t - 3) - u(t - 5) - u(t - 6)$.



(b) The derivative of $x(t)$ is $dx(t)/dt = \delta(t) + \delta(t - 3) - \delta(t - 5) - \delta(t - 6)$

Problem 1-15

Note that

$$\sin(\omega_0 t + \theta) = \frac{1}{2j} e^{j\theta} e^{j\omega_0 t} - \frac{1}{2j} e^{-j\theta} e^{-j\omega_0 t}$$
$$= \frac{1}{2} e^{j(\theta - \pi/2)} e^{j\omega_0 t} + \frac{1}{2} e^{-j(\theta - \pi/2)} e^{-j\omega_0 t}$$

Thus we conclude the following:

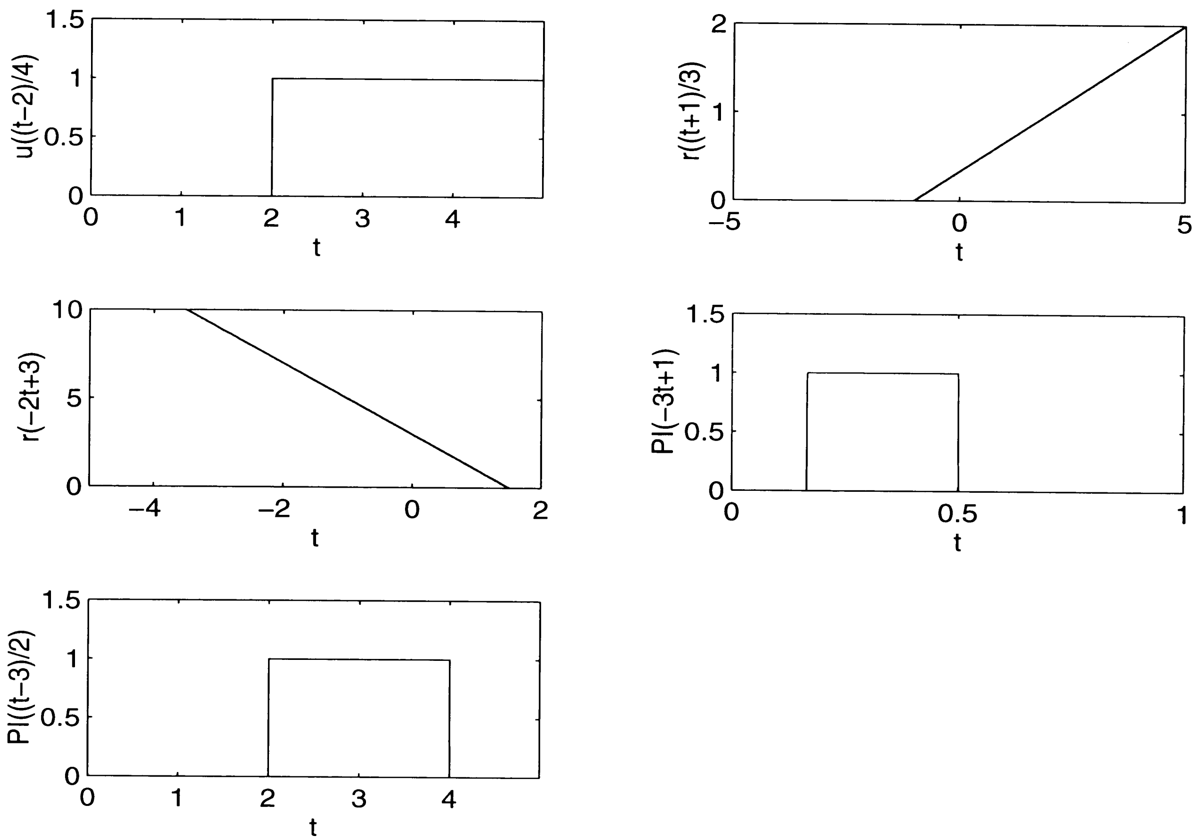
(1) The amplitude spectrum does not change;

(2) The phase spectrum has a $-\pi/2$ radian phase shift with respect to the cosine-convention phase spectrum. This destroys the odd symmetry present in the phase spectrum using the real part convention.

Problem 1-16

A MATLAB script is provided below to show all the plots:

```
%       Plots for Problem 1-16
%
t = -5:.001:5;
xa = stp_fn((t-2)/4);
xb = rmp_fn((t+1)/3);
xc = rmp_fn(-2*t+3);
xd = pls_fn(-3*t+1);
xe = pls_fn((t-3)/2);
subplot(3,2,1),plot(t,xa,'-w'),xlabel('t'),ylabel('u((t-2)/4)'),...
        axis([0 5 0 1.5])
subplot(3,2,2),plot(t,xb,'-w'),xlabel('t'),ylabel('r((t+1)/3)')
subplot(3,2,3),plot(t,xc,'-w'),xlabel('t'),ylabel('r(-2t+3)'),...
        axis([-5 2 0 10])
subplot(3,2,4),plot(t,xd,'-w'),xlabel('t'),ylabel('PI(-3t+1)'),...
        axis([0 1 0 1.5])
subplot(3,2,5),plot(t,xe,'-w'),xlabel('t'),ylabel('PI((t-3)/2)'),...
        axis([0 5 0 1.5])
```



Problem 1-17

From (1-37)

$$u_{-3}(t) = \begin{cases} t^2/2, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Therefore, from (1-35a) with $i = -3$,

$$u_{-4}(t) = \int_{-\infty}^{t} \frac{1}{2}\lambda^2 u(\lambda)\,d\lambda = \begin{cases} t^3/(2\times3), & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

In general,

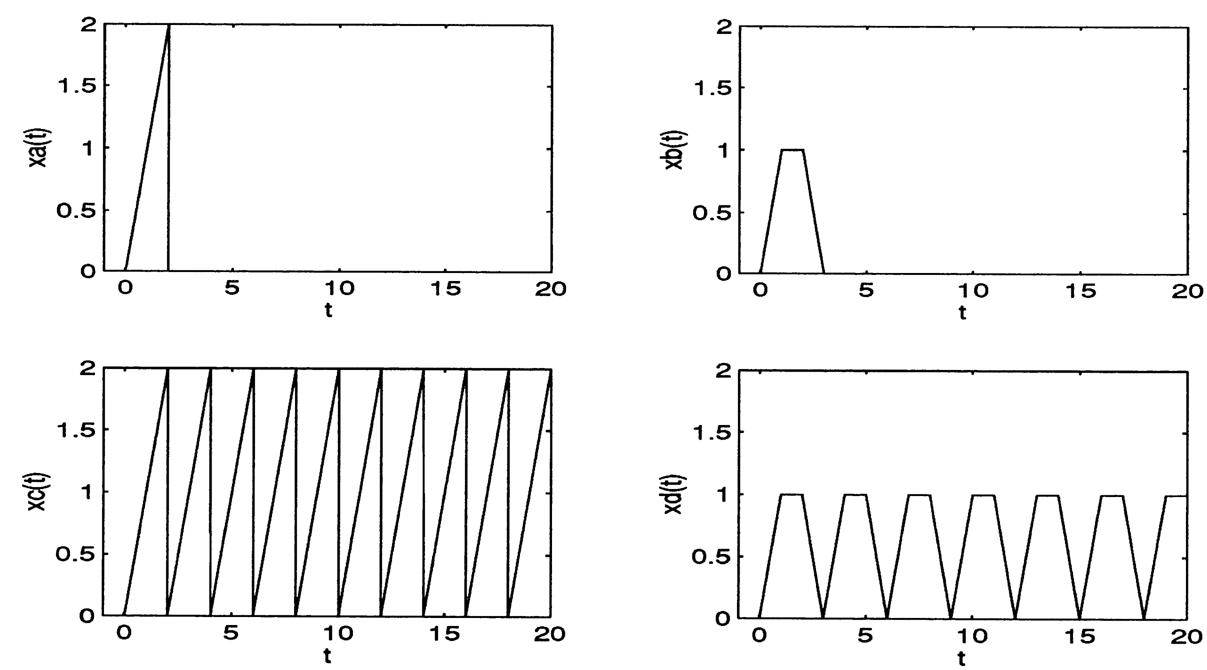
$$u_{-n}(t) = \begin{cases} t^{n-1}/(n-1)!, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Problem 1-18

A MATLAB script is given below for making the plots. Note that use was made of functions to plot the repetitive signals in (c) and (d).

```
%       Plots for Problem 1-18
%
clf
t = -1:.005:20;
xa = xa_fn(t);
xb = xb_fn(t);
xc = xa;
xd = xb;
for n = 1:10
        xc = xc + xa_fn(t - 2*n);
        xd = xd + xb_fn(t - 3*n);
end
subplot(2,2,1),plot(t,xa),xlabel('t'),ylabel('xa(t)'),...
        axis([-1 20 0 2])
subplot(2,2,2),plot(t,xb),xlabel('t'),ylabel('xb(t)'),...
        axis([-1 20 0 2])
subplot(2,2,3),plot(t,xc),xlabel('t'),ylabel('xc(t)'),...
        axis([-1 20 0 2])
subplot(2,2,4),plot(t,xd),xlabel('t'),ylabel('xd(t)'),...
        axis([-1 20 0 2])
```

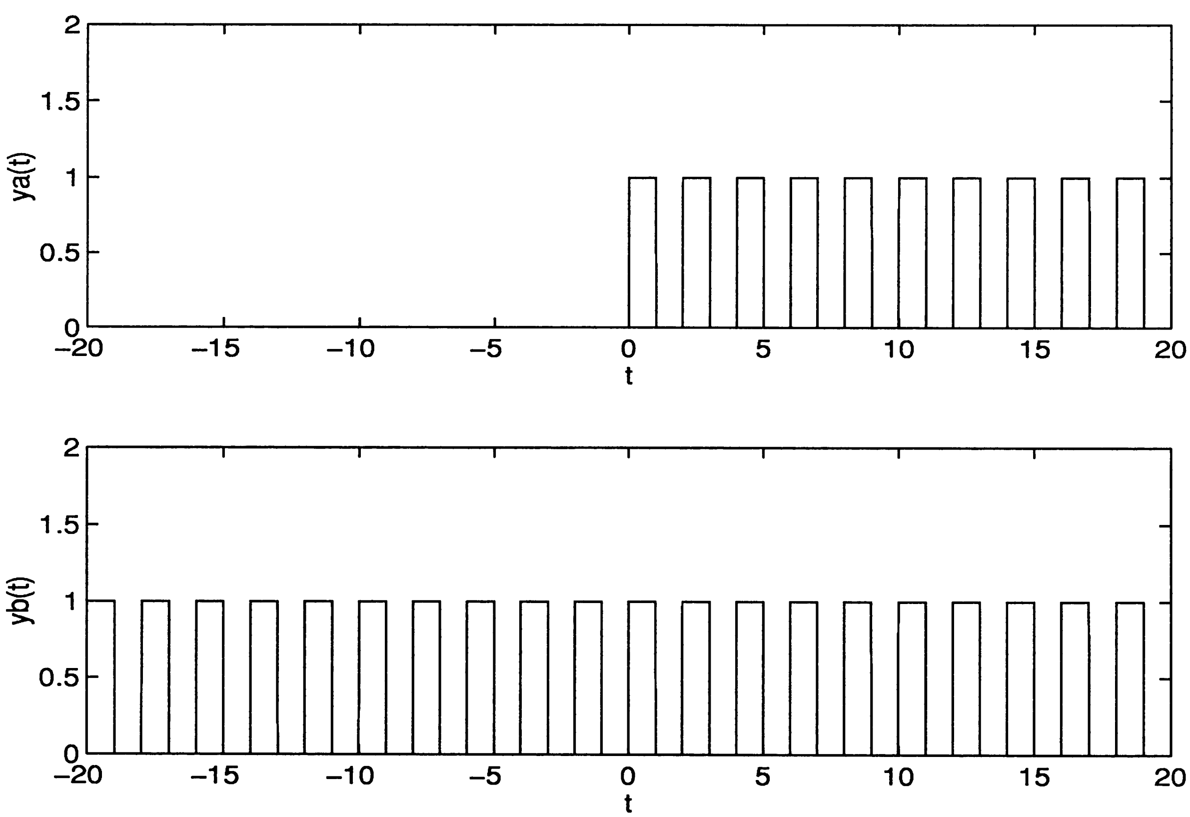
```
%       Function to compute xa for problem 1-18
%

function x = xa_fn(t)
x = rmp_fn(t).*stp_fn(2 - t);
%       Function to compute xb for problem 1-18
%

function x = xb_fn(t)
x = rmp_fn(t)-rmp_fn(t-1)-rmp_fn(t-2)+rmp_fn(t-3);
```



Problem 1-19

(a) Note that for $n = 0$ the summand can be written as $\Pi(t - 1/2)$. The MATLAB script below provides the plots for parts (a) and (c). The signal in part (a) is not periodic because it starts at $t = 0$. The signal of part (c) is periodic because it starts at $t = -\infty$.

```
%	Plots for Problem 1-19
%
clg
t = -20:.005:20;
ya = pls_fn(t - 0.5);
yb = pls_fn(t - 0.5);
for n = 1:10
	ya = ya + pls_fn(t-.5-2*n);
	yb = yb + pls_fn(t-.5-2*n)+ pls_fn(t-.5+2*n);
end
subplot(2,1,1),plot(t, ya, '-w'),xlabel('t'),ylabel('ya(t)'),...
	axis([-20 20 0 2])
subplot(2,1,2),plot(t, yb, '-w'),xlabel('t'),ylabel('ya(t)'),...
	axis([-20 20 0 2])
```



Problem 1-20

Representations for the signals are given below (others may be possible):

$$x_a(t) = \sum_{n=0}^{\infty} r(t-3n)u(2-t-3n)$$

$$x_b(t) = \sum_{n=0}^{\infty} u(t-4n)u(2-t-4n)$$

$$x_c(t) = \sum_{n=0}^{\infty} 2\delta(t-2.5n)$$

$$x_d(t) = \sum_{n=0}^{\infty} \frac{2}{3}u(t-3n)r(3-t-3n)$$

Problem 1-21

One possible representation for each (these follow from the results of Prob. 1-17) is:

$$x_1(t) = 2u_{-3}(t-1)u(2-t) + u(t-2)u(4-t) + 2u_{-3}(5-t)u(t-4)$$

$$x_2(t) = \frac{3}{2}u_{-4}(t)u(2-t) + \frac{4}{9}u_{-3}(5-t)u(t-2)$$

Problem 1-22

One possible representation for each is

$$x_1(t) = u(t) + r(t-1) - 2r(t-2) + r(t-3) - u(t-4)$$

$$x_2(t) = u(t) - 2u(t-1) + 2u(t-2) - u(t-3)$$

$$x_3(t) = r(t) - r(t-1) - r(t-3) + r(t-4)$$

$$x_4(t) = r(t) - 2u(t-1) - r(t-2)$$

Problem 1-23

(a) First note that the integral of the function is 1, no matter what the value for ϵ:

$$I = \int_{-\infty}^{\infty} \delta_\epsilon(t)dt = \int_0^{\infty} \frac{1}{\epsilon}e^{-t/\epsilon}dt = -e^{-t/\epsilon}\Big|_0^{\infty} = 1$$

Second, note that the pulse becomes infinitely narrow and infinitely high as $\epsilon \to \infty$.

(b) Use the integral

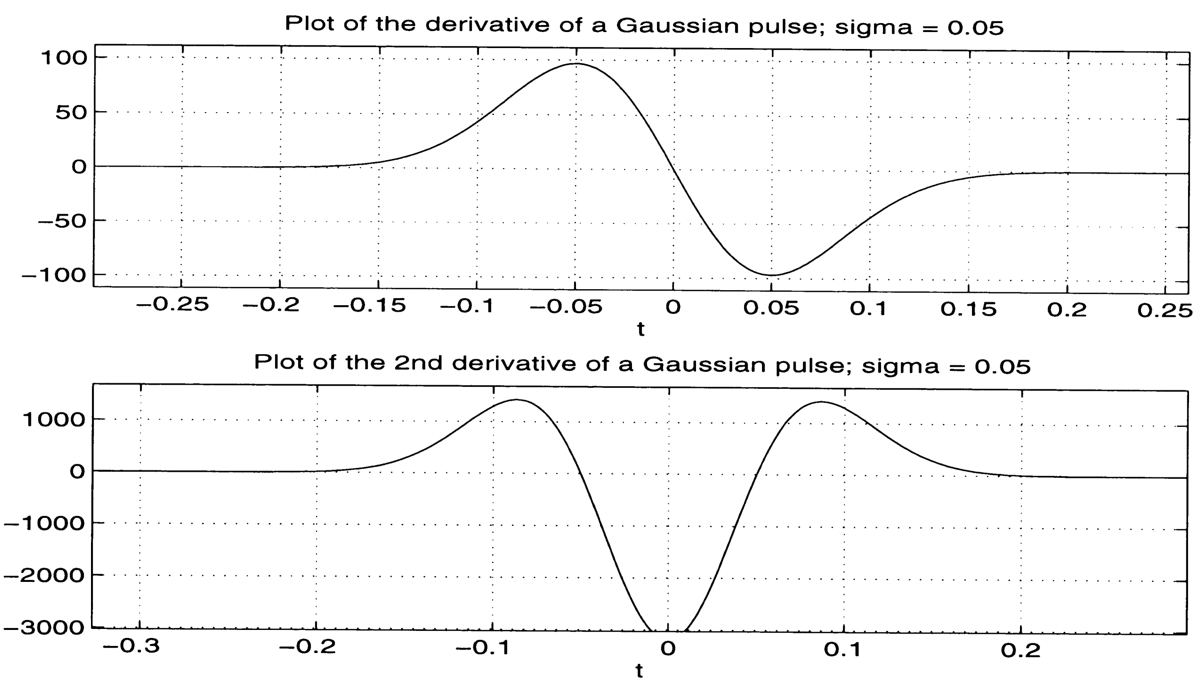
$$\int_{-\infty}^{\infty} e^{-\alpha t^2} dt = \sqrt{\frac{\pi}{\alpha}}$$

to show that the area under the given function is 1. Then note that as $\sigma \to 0$, the function becomes infinitely narrow and infinitely high. Thus the properties of a delta function are satisfied.

Problem 1-24

A MATLAB script using symbolic operations is given below for plotting the desired functions:

```
%       Plots for Problem 1-24
%
format short
sigma = 0.05;
y = 'exp(-t^2/(2*0.05^2))/sqrt(2*pi*0.05^2)';
y_prime = diff(y)
y_dbl_prime = diff(y_prime)
subplot(2,1,1),ezplot(y_prime),...
        title(['Plot of the derivative of a Gaussian pulse; sigma = ',num2str(sigma)])
subplot(2,1,2),ezplot(y_dbl_prime),...
        title(['Plot of the 2nd derivative of a Gaussian pulse; sigma = ',num2str(sigma)])
```



Problem 1-25

Using the stated rules, the first derivative is

$$\frac{dh(t)}{dt} = e^{-\alpha t}\frac{du(t)}{dt} - \alpha e^{-\alpha t}u(t) = 1\times\delta(t) - \alpha e^{-\alpha t}u(t)$$

The second derivative is

$$\frac{dh^2(t)}{dt^2} = \frac{d\delta(t)}{dt} + \alpha^2 e^{-\alpha t}u(t) - \alpha e^{-\alpha t}\frac{du(t)}{dt} = \frac{d\delta(t)}{dt} + \alpha^2 e^{-\alpha t}u(t) - \alpha\,\delta(t)$$

Problem 1-26

(a) The integral is zero because the delta function is outside the range of integration.
(b) The integral evaluates as follows:

$$\int_0^5 \cos(2\pi t)\,\delta(t-2)\,dt = \cos(4\pi) = 1$$

(c) This integral can be evaluated as

$$\int_0^5 \cos(2\pi t)\,\delta(t-0.5)\,dt = \cos(\pi) = -1$$

(d) The value of this integral is 0:

$$\int_{-\infty}^{\infty} (t-2)^2\,\delta(t-2)\,dt = (2-2)^2 = 0$$

(e) This integral evaluates to

$$\int_{-\infty}^{\infty} t^2\,\delta(t-2)\,dt = 2^2 = 4$$

Problem 1-27

(a) Using (1-66), this integral becomes

$$\int_{-\infty}^{\infty} e^{3t}\ddot{\delta}(t-2)\,dt = (-1)^2 \frac{d^2}{dt^2} e^{3t}\Big|_{t=2} = 9e^6$$

(b) Again applying (1-66), we have

$$\int_{0}^{10} \cos(2\pi t)\,\dddot{\delta}(t-0.5)\,dt = (-1)^3 \frac{d^3}{dt^3}\cos(2\pi t)\Big|_{t=0.5} = -(2\pi)^3 \sin(2\pi t)\big|_{t=0.5} = 0$$

(c) Using (1-66) we get

$$\int_{-\infty}^{\infty} \left[e^{-3t} + \cos(2\pi t)\right]\dot{\delta}(t)\,dt = (-1)\frac{d}{dt}\left[e^{-3t} + \cos(2\pi t)\right]_{t=0} = -\left[-3e^{-3t} - 2\pi\sin(2\pi t)\right]_{t=0} = 3$$

Problem 1-28

Match coefficients of like derivatives of $\delta(t)$ on either side of the given equations:
(a) In this case, we obtain

$$10 = 3 + C_3 \text{ or } C_3 = 7;\ C_1 = 5;\ 2 + C_2 = 6 \text{ or } C_2 = 4$$

(b) The resulting equations are

$$3 + C_1 = 0 \text{ or } C_1 = -3;\ C_2 = 0;\ C_3 = 0;\ C_4 = 0;\ C_5 = 0$$

Problem 1-29

(a) (1) This plots to a triangle 2 units high, centered on $t = 0$, and going from $t = -2$ to 2.
(2) This is a rectangle of unit height starting at $t = 0$ and ending at $t = 10$.
(3) This is a step of height 2 starting at $t = 0$ with an impulse of unit area at $t = 2$ superimposed.
(4) This an impulse of area 2 at $t = 2$.

(b) One possible representation is

$$x(t) = r(t+4) - r(t+2) + u(t) - 3r(t-4) + 3r(t-5)$$

Problem 1-30

A possible representation is

$$x(t) = 2u(t) - u(t-2) + u(t-4) - r(t-6) + r(t-8)$$

Problem 1-31

(a) Using the sifting property of the delta function, we get

$$\int_{-\infty}^{\infty} t^3 \delta(t-3)\,dt = t^3\big|_{t=3} = 27$$

(b) Using (1-66), we get

$$\int_{-\infty}^{\infty} [3t + \cos(2\pi t)]\,\dot{\delta}(t-5)\,dt = (-1)\frac{d}{dt}[3t + \cos(2\pi t)]_{t=5} = -[3 - 2\pi\sin(2\pi t)]_{t=5} = -3$$

(c) From (1-66) we have

$$\int_{-\infty}^{\infty} (1 + t^2)\,\dot{\delta}(t-1.5)\,dt = (-1)\frac{d}{dt}(1 + t^2)\big|_{t=1.5} = -(2t)\big|_{t=1.5} = -3$$

Problem 1-32

(a) A possible representation is

$$x_a(t) = A[2u(t) - 2u(t-T) + u(t-2T) - u(t-3T)]$$

(b) One representation of this signal is

$$x_b(t) = r(t) - 2r(t-1) + 2r(t-3) - r(t-4)]$$

(c) One way of writing this signal is

$$x_c(t) = r(t-1) - 2r(t-2) + r(t-3) + 0.5[u(t-1.5) - u(t-2.5)]$$

Problem 1-33

(a) This is a decaying exponential starting at $t = 0$. Its energy is

$$E = \int_0^\infty e^{-20t}dt = \left.\frac{e^{-20t}}{-20}\right|_0^\infty = \frac{1}{20}\ \mathrm{J}$$

(b) This is a rectangular pulse starting at $t = 0$ and ending at $t = 15$. Its energy is

$$E = \int_{-\infty}^{\infty}[u(t) - u(t-15)]^2 dt = \int_0^{15} 1^2 dt = 15\ \mathrm{J}$$

(c) This is a cosine burst starting at $t = 0$ and ending at $t = 2$. It contains 10 cycles. Its energy is calculated as

$$E = \int_{-\infty}^{\infty}\cos^2(10\pi t)\,[u(t)\,u(2-t)]^2 dt = \int_0^2 \cos^2(10\pi t)\,dt = \int_0^2\left[\frac{1}{2} + \frac{1}{2}\cos(10\pi t)\right]dt = 1\ \mathrm{J}$$

(d) This is a triangle going from $t = 0$ and ending at $t = 2$ of unit height. For $0 \le t \le 1$ its equation is just t. The integral of t^2 from 0 to 1 can be doubled to yield the total energy with the result

$$E = 2\int_0^1 t^2 dt = 2\left.\frac{t^3}{3}\right|_0^t = \frac{2}{3}\ \mathrm{J}$$

Problem 1-34

(a) Note that $x_1(t)$ is symmetrical about $t = 2$. Therefore

$$E_1 = 2\left[\int_0^2 1^2 dt + \int_1^2 t^2 dt\right] = 2\left[t\,\Big|_0^2 + \frac{t^3}{3}\Big|_1^2\right] = \frac{20}{3} \text{ J}$$

(b) Note that $x_2^2(t) = 1$ for t between 0 and 3, and is 0 otherwise. Therefore

$$E_2 = \int_0^3 1^2 dt = 3 \text{ J}$$

(c) Note that $x_3(t)$ is symmetrical about $t = 2$ which allows the energy to be calculated as

$$E_3 = 2\left[\int_0^1 t^2 dt + \int_1^2 1^2 dt\right] = 2\left[\frac{t^3}{3}\Big|_0^1 + t\,\Big|_1^2\right] = \frac{8}{3} \text{ J}$$

(d) Note that $x_4^2(t)$ is symmetrical about $t = 1$ which allows the energy of $x_4(t)$ to be calculated as

$$E_4 = 2\int_0^1 t^2 dt = 2\frac{t^3}{3}\Big|_0^1 = \frac{2}{3} \text{ J}$$

Problem 1-35

Only (a) and (b) are energy signals. For (a)

$$E_a = \int_0^2 t^2 dt = \frac{t^3}{3}\Big|_0^2 = \frac{8}{3} \text{ J}$$

For (b), we note that it is symmetric about $t = 1.5$. It is a ramp from 0 to 1 and constant from 1 to 1.5, which yields

$$E_b = 2\left[\int_0^1 t^2 dt + \int_1^{1.5} 1^2 dt\right] = 2\left[\frac{t^3}{3}\Big|_0^1 + t\,\Big|_1^{1.5}\right] = 2[1/3 + 1.5 - 1] = 5/3 \text{ J}$$

The other functions are semi-infinite in extent, so their squares will integrate to infinity.

Problem 1-36

The average powers of (a) - (c) are ½ W; for (d) and (e), the powers are 1 W. These are obtained by squaring the amplitudes of the separate frequency components, dividing by 2 to get power, and adding. This is permissible since the sinusoids have frequencies that are integer multiples of a fundamental frequency.

Problem 1-37

(a) $P = 2^2/2 = 2$ W; (b) $P = 5^2/2 = 12.5$ W; (c) $P = 3^2/2 = 4.5$ W; (d) $P = 2^2/2 + 5^2/2 = 14.5$ W; (e) $P = 2^2/2 + 3^2/2 = 6.5$ W; (f) $P = 5^2/2 + 3^2/2 = 17$ W.

Problem 1-38

(a) Power:

$$P_a = \lim_{T \to \infty} \frac{1}{2T}\left[\int_0^1 1^2\,dt + \int_1^2 6^2\,dt + \int_2^T 4^2\,dt\right] = 0 + 0 + \lim_{T \to \infty} \frac{16(T-2)}{2T} = 8 \text{ W}$$

(b) Energy:

$$E_b = \int_0^1 1^2\,dt + \int_1^2 6^2\,dt = 37 \text{ J}$$

(c) Energy:

$$E_c = \int_0^\infty e^{-10t}\,dt = -\frac{e^{-10t}}{10}\Bigg|_0^\infty = \frac{1}{10} \text{ J}$$

(d) Power:

$$P_d = \lim_{T \to \infty} \frac{1}{2T}\int_0^T \left[e^{-5t} + 1\right]^2 dt = \lim_{T \to \infty} \frac{1}{2T}\int_0^T \left[e^{-10t} + 2e^{-5t} + 1\right] dt$$

$$= \lim_{T \to \infty} \frac{1}{2T}\left[-\frac{e^{-10t}}{10} - \frac{2e^{-5t}}{5} + t\right]_0^T = \frac{1}{2} \text{ W}$$

(e) Power: similarly to (d), it can be shown that $P_e = ½$ W. (f) Neither: it can be shown that both the power and energy are infinite. (g) Power: $P_g = ½$ W. (h) Neither: $E_h = \infty$ and $P_h = 0$.

Problem 1-39

(a) Yes. The frequencies of its separate components are commensurable: $f_1 = 3\times 1$ Hz and $f_2 = 5\times 1$ Hz. Therefore, the fundamental frequency is 1 Hz and the period is 1 s.
(b) Its amplitude spectrum consists of a line of height 2 at 3 Hz and a line of height 4 at 5 Hz. Its phase spectrum consists of a line of height $-\pi/3$ at 3 Hz and a line of height $-\pi/2$ at 5 Hz.
(c) Written as the sum of counter-rotating phasors, the signal is

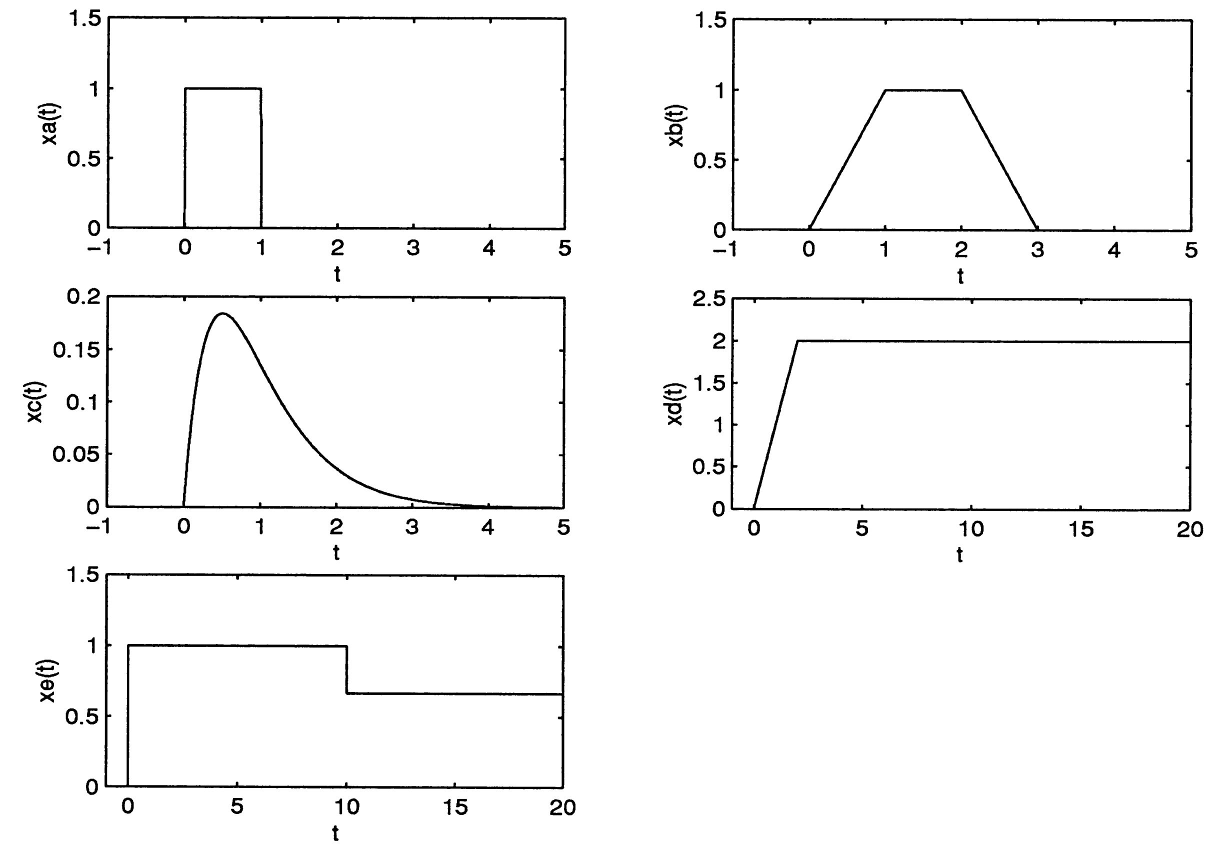
$$x(t) = e^{-j\pi/3} e^{j6\pi t} + e^{j\pi/3} e^{-j6\pi t} + 2e^{-j\pi/2} e^{j10\pi t} + 2e^{j\pi/2} e^{-j10\pi t}$$

(d) See (b): for the amplitude spectrum, halve the lines and take the mirror image about $f = 0$; for the phase spectrum, take the antisymmetric image about $f = 0$.
(e) It is clear that it is a power signal because it is the sum of sinusoids whose frequencies are harmonics of a fundamental frequency. The total power is $2^2/2 + 4^2/2 = 10$ W.

Problem 1-40

(a) through (c) are energy signals; (d) and (e) are power signals. By applying the definitions of energy and power, (1-75) and (1-76), respectively, the energies are $E_a = 1$ J, $E_b = 5/3$ J, $E_c = 1/32$ J, $P_d = 2$ W, and $P_e = 2/9$ W. The MATLAB script given below plots these signals:

```
%      Plots for Problem 1-40
%
t = -1:.005:20;
xa = stp_fn(t) - stp_fn(t-1);
xb = rmp_fn(t) - rmp_fn(t-1) - rmp_fn(t-2) + rmp_fn(t-3);
xc = t.*exp(-2*t).*stp_fn(t);
xd = rmp_fn(t) - rmp_fn(t-2);
xe = stp_fn(t) - (1/3)*stp_fn(t-10);
subplot(3,2,1),plot(t,xa,'-w'),xlabel('t'),ylabel('xa(t)'),...
      axis([-1 5 0 1.5])
subplot(3,2,2),plot(t,xb),xlabel('t'),ylabel('xb(t)'),...
      axis([-1 5 0 1.5])
subplot(3,2,3),plot(t,xc),xlabel('t'),ylabel('xc(t)'),...
      axis([-1 5 0 .2])
subplot(3,2,4),plot(t,xd),xlabel('t'),ylabel('xd(t)'),...
      axis([-1 20 0 2.5])
subplot(3,2,5),plot(t,xe),xlabel('t'),ylabel('xe(t)'),...
      axis([-1 20 0 1.5])
```



Problem 1-41

(a) Only (1) is periodic; $f_1 = 2.5$ Hz $= 0.5m$ and $f_2 = 3$ Hz $= 0.5n$ where the integers m and n are 5 and 6, respectively. The fundamental frequency is 0.5 Hz and the period is 2 s.
(b) Signals (1) and (2) are power signals. Their powers are both 1 W.
(c) Only signal (3) is an energy signal; its energy is 1/20 J. Signal (4) is neither energy nor power.

Problem 1-42

By definition, the average power is

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

For a periodic signal $x(t) = x(t + T_0)$, and the integral can be broken into segments one period long plus the end pieces that are less than a period. Because of periodicity, these integrals are equal with the exception of the end pieces. Thus, we can write the integral as

$$\int_{-T}^{T} |x(t)|^2 dt = 2N \int_{t_0}^{t_0 + T_0} |x(t)|^2 dt + \epsilon_{-N} + \epsilon_N$$

where the latter two terms represent the integrals over the end intervals. Also, $2T = 2NT_0 + \Delta t_1 + \Delta t_2$. The latter two time segments are the lengths of the end intervals which are less than a period. Thus, the power expression becomes

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T \to \infty} \frac{1}{2T} \left[\frac{2N \int_{t_0}^{t_0 + T_0} |x(t)|^2 dt + \epsilon_{-N} + \epsilon_N}{2NT_0 + \Delta t_1 + \Delta t_2} \right] = \frac{1}{T_0} \int_{t_0}^{t_0 + T_0} |x(t)|^2 dt$$

Problem 1-43

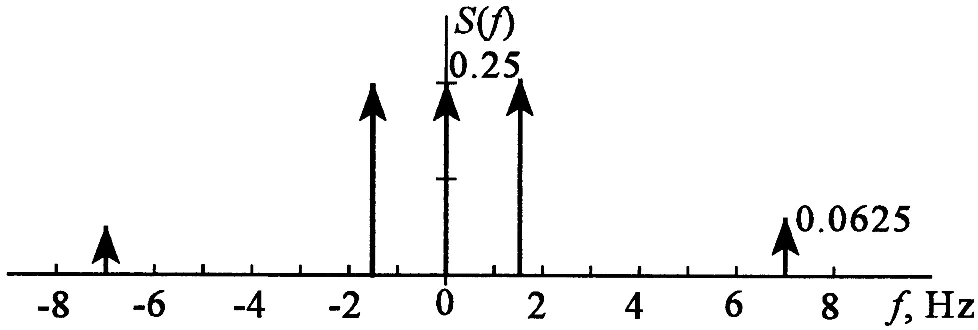
Use the trigonometric identity for $\sin^2(x) = ½ - ½ \cos(2x)$ to write the signal as

$$x(t) = \frac{1}{2} - \frac{1}{2}\cos(14\pi t - \pi/3) + \cos(3\pi t - \pi/3) = \frac{1}{2} + \frac{1}{2}\cos(14\pi t - \pi/3 + \pi) + \cos(3\pi t - \pi/3)$$
$$= \frac{1}{2} + \frac{1}{2}\cos(14\pi t + 2\pi/3) + \cos(3\pi t - \pi/3)$$

(a) Its single-sided amplitude spectrum consists of a line of height ½ at $f = 0$, a line of height 1 at $f = 1.5$ Hz, and a line of height ½ at $f = 7$ Hz. Its single-sided phase spectrum consists of no line at $f = 0$, a line of height $-\pi/3$ at $f = 1.5$ Hz, and a line of height $2\pi/3$ at $f = 7$ Hz.
(b) To get the double-sided amplitude spectrum, halve the lines in the single-sided spectrum and take its mirror image about $f = 0$. To get the double-sided phase spectrum, take the antisymmetric image of the single-sided spectrum about $f = 0$.

Problem 1-44

The signal has frequency components at 0, 1.5, and 7 Hz of amplitudes ½, 1, and ½, respectively. The power at dc is $(½)^2 = 0.25$ W which is placed at the single frequency $f = 0$ Hz. The power at the other frequencies is split between the positive and corresponding negative frequency. Thus, at f = 1.5 Hz we have $(1)^2/4 = 0.25$ W (one 2 in the denominator is from computing power in a sinusoid and the other 2 is from splitting it between positive and negative frequencies) and similarly at f = -1.5 Hz. At f = 7 Hz, we have a power of $(1/2)^2/4 = 0.0625$ W with a similar power at f = -7 Hz. All these are represented by impulses of the appropriate weights, so the plot is as shown below:



Problem 1-45

(a) Following the solution to Problem 1-44, we have spectral components at f = 10, 15, and 20 Hz of amplitudes 16, 6, and 4, respectively. The power in these components gets split between positive and negative frequencies. Thus, and f = 10 Hz we have a power of $(16)^2/4 = 64$ W with a corresponding power at f = -10 Hz. At f = 15 Hz we have a power of $(6)^2/4 = 9$ W with a corresponding power at f = -15 Hz. Finally, at f = 20 Hz we have a power of $(4)^2/4 = 4$ W with a corresponding power at f = -20 Hz. Mathematically, this can be expressed as

$$S_x(f) = 64[\delta(f-10) + \delta(f+10)] + 9[\delta(f-15) + \delta(f+15)] + 4[\delta(f-20) + \delta(f+20)]$$

(b) The power contained between 12 and 22 Hz is

$$P[12 \le |f| \le 22 \text{ Hz}] = \int_{-22}^{-12} S_x(f)\,df + \int_{12}^{22} S_x(f)\,df = 2\int_{12}^{22} S_x(f)\,df = 26 \text{ W}$$

CHAPTER 2

Problem 2-1

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Problem 2-2

(a) First order; (b) first order (differentiate once to get rid of the integral on y); (c) zero order; (d) first order; (e) second order.

Problem 2-3

(a), (b), (c), and (e) are fixed; (d) is not because of the time-varying coefficient, t^2.

Problem 2-4

Only (c) and (d) are nonlinear. Superposition will not hold in (e) because of the term +10. As an example to show linearity, consider (d):

$$\frac{dy_1(t)}{dt} + t^2 y_1(t) = \int_{-\infty}^{t} x_1(\lambda)\,d\lambda$$

$$\frac{dy_2(t)}{dt} + t^2 y_2(t) = \int_{-\infty}^{t} x_2(\lambda)\,d\lambda$$

Multiply the first equation by a constant, say a, and the second equation by another constant, say b; add to obtain:

$$\frac{d[ay_1(t) + by_2(t)]}{dt} + t^2[ay_1(t) + by_2(t)] = \int_{-\infty}^{t} [ax_1(\lambda) + bx_2(\lambda)]\,d\lambda$$

This is of the same form as the original equation.

Problem 2-5

Noncausal. Consider $t = 0.25$, which gives $y(0.25) = x(0.5)$; i.e., the output depends on a future value of the input.

Problem 2-6

(a) Nonlinear. The proof is similar to Example 2-4 in the text. (b) Noncausal because of the +2 in the argument of x. Consider $t = 0$; the output at time 0 depends on the value of the input at time 2, or a future value.

Problem 2-7

(a) Linear. Consider the responses to two arbitrary inputs:

$$y_1(t) = x_1(t^2)$$
$$y_2(t) = x_2(t^2)$$

Multiply first by a and the second by b and add to get

$$ay_1(t) + by_2(t) = ax_1(t^2) + bx_2(t^2)$$

That is, for the input $ax_1(t) + bx_2(t)$, we replace t by t^2 to get the new output which is the right-hand side of the above equation.
(b) Time varying. Consider the response to the delayed input:

$$y_a(t) = x(t^2 - \tau)$$

Now consider the delayed output due to the undelayed input:

$$y(t - \tau) = x[(t - \tau)^2]$$

Clearly the two are not the same.
(c) Noncausal. Consider $t = 2$ which gives $y(2) = x(4)$; i.e., the output depends on a future value of the input.
(d) Not zero memory. This follows from (c) where it was found that the output does not depend only on values of the input at the present time only.

Problem 2-8

(a) Consider two inputs and the corresponding outputs:

$$y_1(t) = x_1(t) + \alpha x_1(t - \tau_0)$$
$$y_2(t) = x_2(t) + \alpha x_2(t - \tau_0)$$

Multiply the top equation by a and the bottom by b (two constants); add to get

$$ay_1(t) + by_2(t) = ax_1(t) + bx_2(t) + \alpha[ax_2(t - \tau_0) + bx_1(t - \tau_0)]$$

This is of the same form as the original input/output relationship, so linarity is proved.
(b) The only way for the system to be zero memory is for τ_0 to be 0.
(c) It is causal only if $\tau_0 \geq 0$, for in that case the system doesn't respond before the input is applied.
(d) See the MATLAB plots below ($\alpha = 0.5$ and 1.5 in that order):