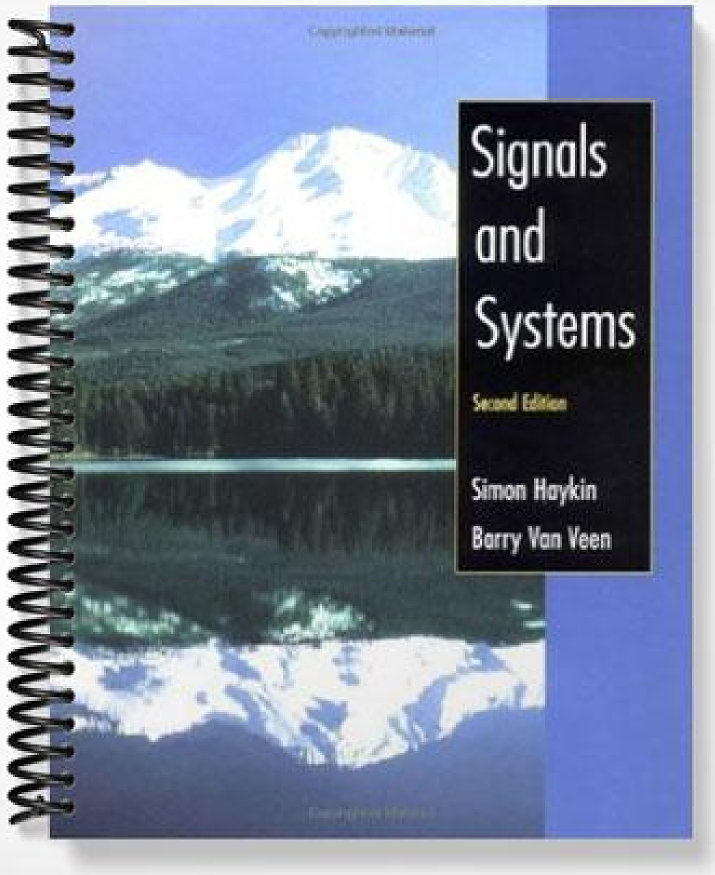
SOLUTIONS MANUAL



CHAPTER 1

1.1 to 1.41 - part of text

1.42 (a) Periodic:
Fundamental period = 0.5s

(b) Nonperiodic

(c) Periodic
Fundamental period = 3s

(d) Periodic
Fundamental period = 2 samples

(e) Nonperiodic

(f) Periodic:
Fundamental period = 10 samples

(g) Nonperiodic

(h) Nonperiodic

(i) Periodic:
Fundamental period = 1 sample

1.43
$$y(t) = \left(3\cos\left(200t + \frac{\pi}{6}\right)\right)^2$$
$$= 9\cos^2\left(200t + \frac{\pi}{6}\right)$$
$$= \frac{9}{2}\left[\cos\left(400t + \frac{\pi}{3}\right)1\right]$$

(a) DC component = $\frac{9}{2}$

(b) Sinusoidal component = $\frac{9}{2}\cos\left(400t + \frac{\pi}{3}\right)$

Amplitude = $\frac{9}{2}$

Fundamental frequency = $\frac{200}{\pi} Hz$

1.44 The RMS value of sinusoidal $x(t)$ is $A/\sqrt{2}$. Hence, the average power of $x(t)$ in a 1-ohm resistor is $(A/\sqrt{2})^2 = A^2/2$.

1.45 Let N denote the fundamental period of $x[N]$. which is defined by

$$N = \frac{2\pi}{\Omega}$$

The average power of $x[n]$ is therefore

$$\begin{aligned} P &= \frac{1}{N}\sum_{n=0}^{N-1} x^2[n] \\ &= \frac{1}{N}\sum_{n=0}^{N-1} A^2\cos^2\left(\frac{2\pi n}{N}+\phi\right) \\ &= \frac{A^2}{N}\sum_{n=0}^{N-1}\cos^2\left(\frac{2\pi n}{N}+\phi\right) \end{aligned}$$

1.46 The energy of the raised cosine pulse is

$$\begin{aligned} E &= \int_{-\pi/\omega}^{\pi/\omega}\frac{1}{4}(\cos(\omega t)+1)^2 dt \\ &= \frac{1}{2}\int_0^{\pi/\omega}(\cos^2(\omega t)+2\cos(\omega t)+1)dt \\ &= \frac{1}{2}\int_0^{\pi/\omega}\left(\frac{1}{2}\cos(2\omega t)+\frac{1}{2}+2\cos(\omega t)+1\right)dt \\ &= \frac{1}{2}\left(\frac{3}{2}\right)\left(\frac{\pi}{\omega}\right) = 3\pi/4\omega \end{aligned}$$

1.47 The signal $x(t)$ is even; its total energy is therefore

$$E = 2\int_0^5 x^2(t)dt$$

$$= 2\int_0^4 (1)^2 dt + 2\int_4^5 (5-t)^2 dt$$

$$= 2[t]_{t=0}^4 + 2\left[-\frac{1}{3}(5-t)^3\right]_{t=4}^5$$

$$= 8 + \frac{2}{3} = \frac{26}{3}$$

1.48 (a) The differentiator output is

$$y(t) = \begin{cases} 1 & \text{for } -5 < t < -4 \\ -1 & \text{for } 4 < t < 5 \\ 0 & \text{otherwise} \end{cases}$$

(b) The energy of $y(t)$ is

$$E = \int_{-5}^{-4} (1)^2 dt + \int_4^5 (-1)^2 dt$$

$$= 1 + 1 = 2$$

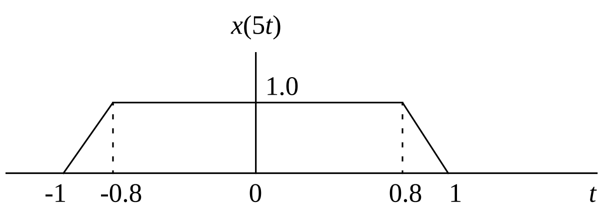
1.49 The output of the integrator is

$$y(t) = A\int_0^t \tau d\tau = At \qquad \text{for } 0 \le t \le T$$

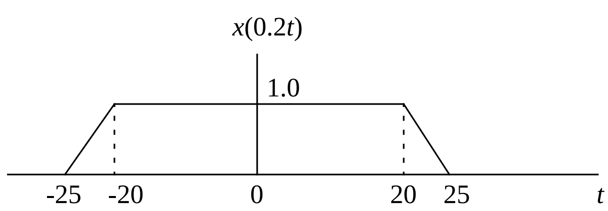
Hence the energy of $y(t)$ is

$$E = \int_0^T A^2 t^2 dt = \frac{A^2 T^3}{3}$$

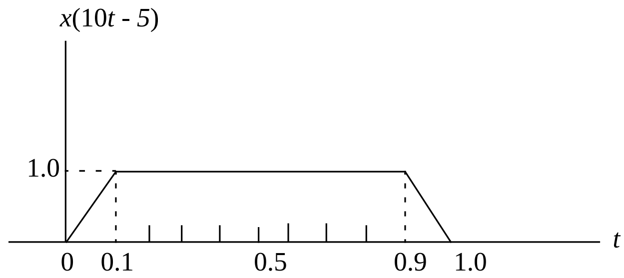
1.50 (a)



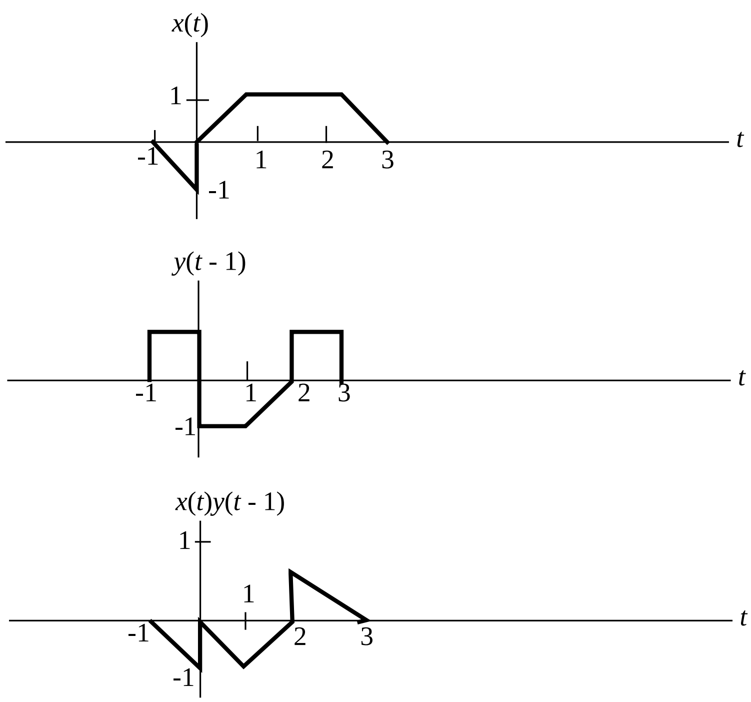
(b)



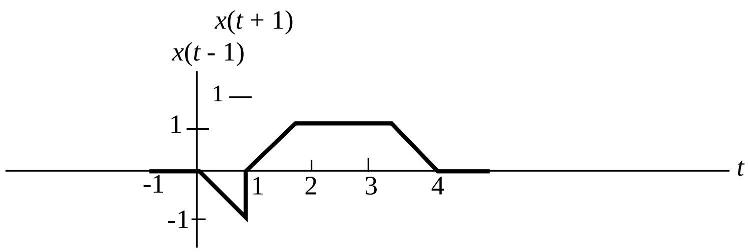
1.51

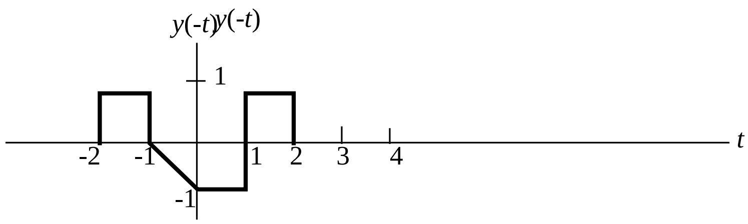


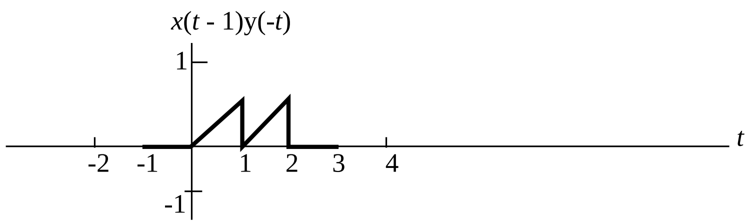
1.52 (a)



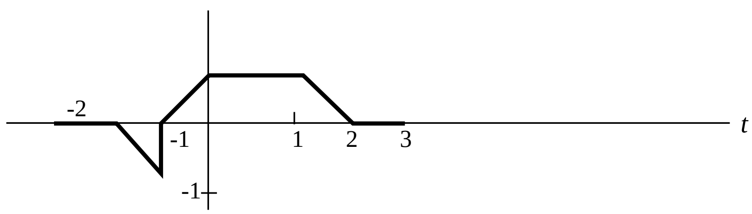
1.52 (b)

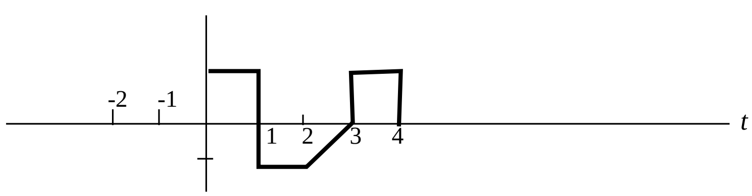

x(t + 1)
x(t - 1)
1
1
-1
-1
1
2
3
4
t

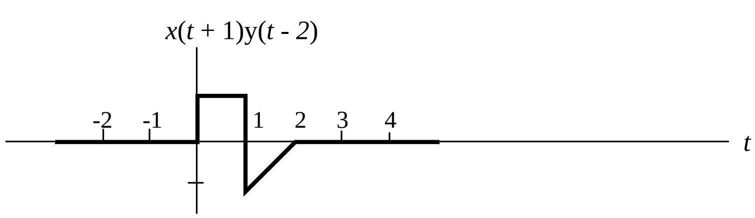

y(-t)y(-t)
1
-2
-1
1
2
3
4
-1
t


x(t - 1)y(-t)
1
-2
-1
1
2
3
4
-1
t

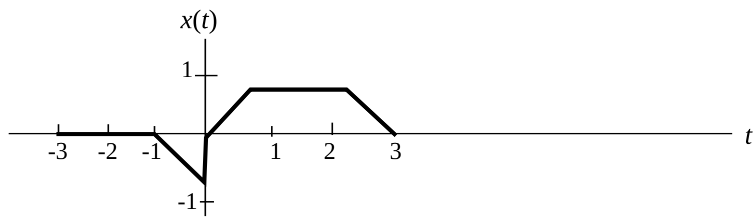
1.52 (c)

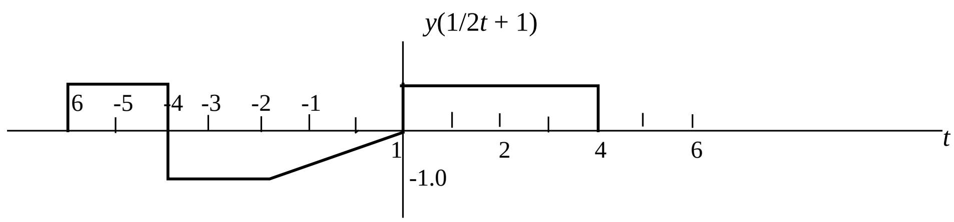

-2
-1
1
2
3
-1
t

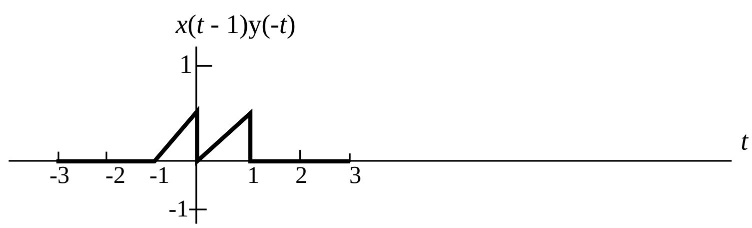

-2
-1
1
2
3
4
t


x(t + 1)y(t - 2)
-2
-1
1
2
3
4
t

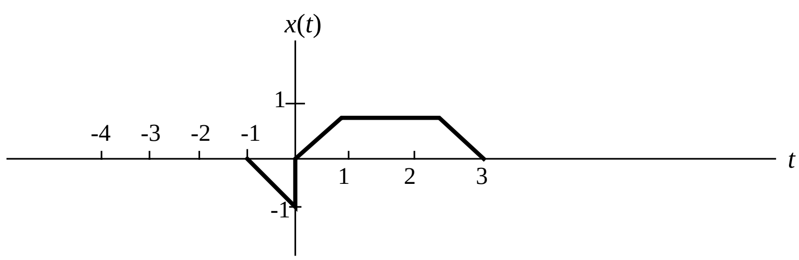
1.52 (d)

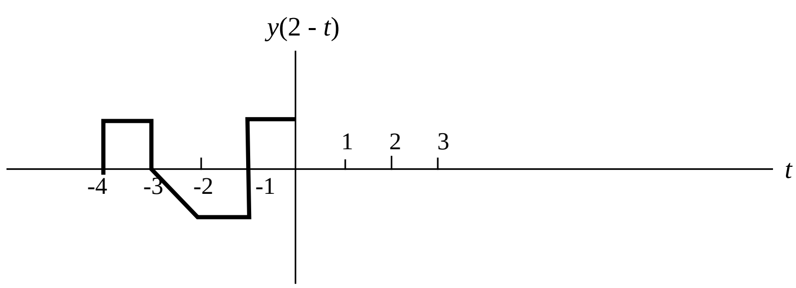

x(t)
1
-1
-3 -2 -1 1 2 3
t

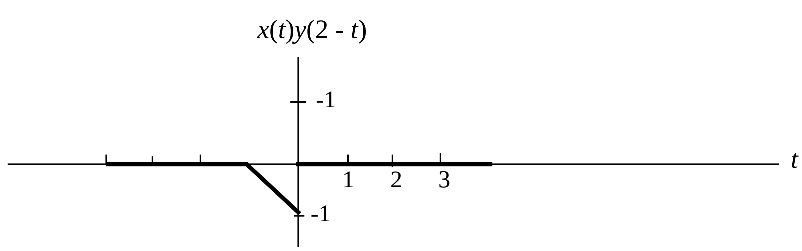

y(1/2t + 1)
6 -5 -4 -3 -2 -1
1 2 4 6
-1.0
t


x(t - 1)y(-t)
1
-1
-3 -2 -1 1 2 3
t

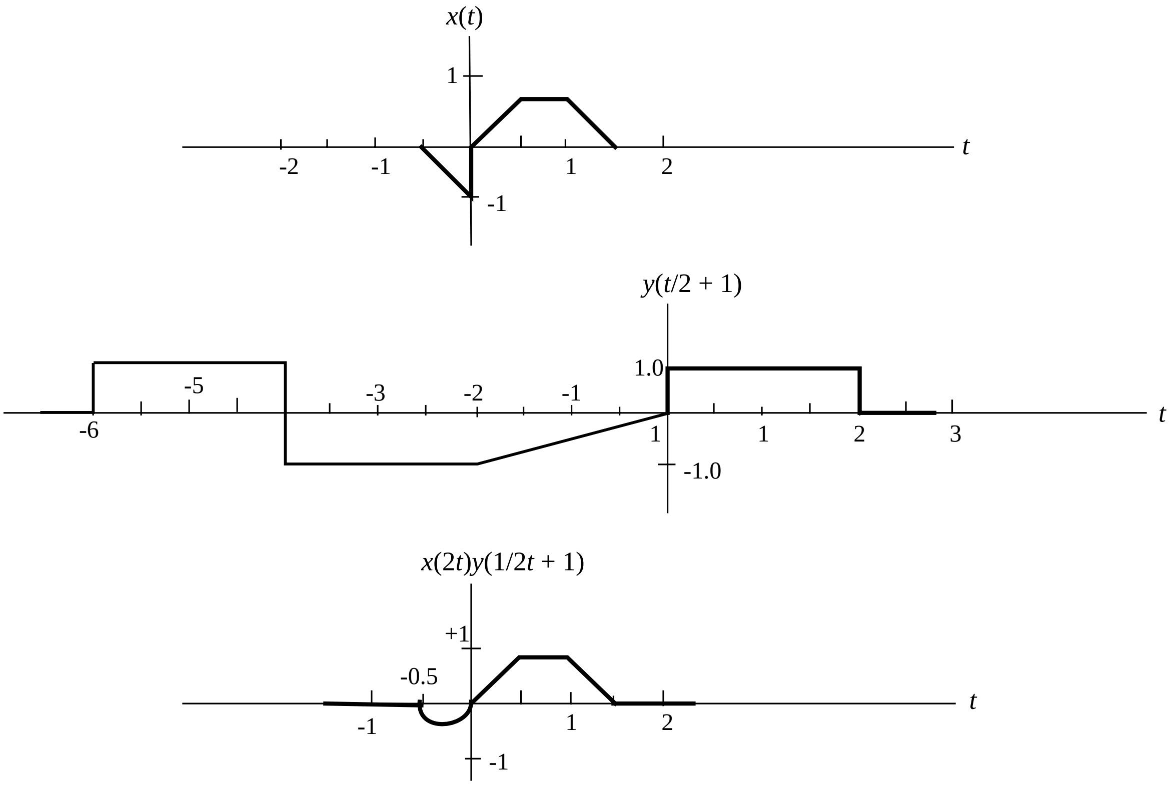
1.52 (e)


x(t)
1
-1
-4 -3 -2 -1
1 2 3
t

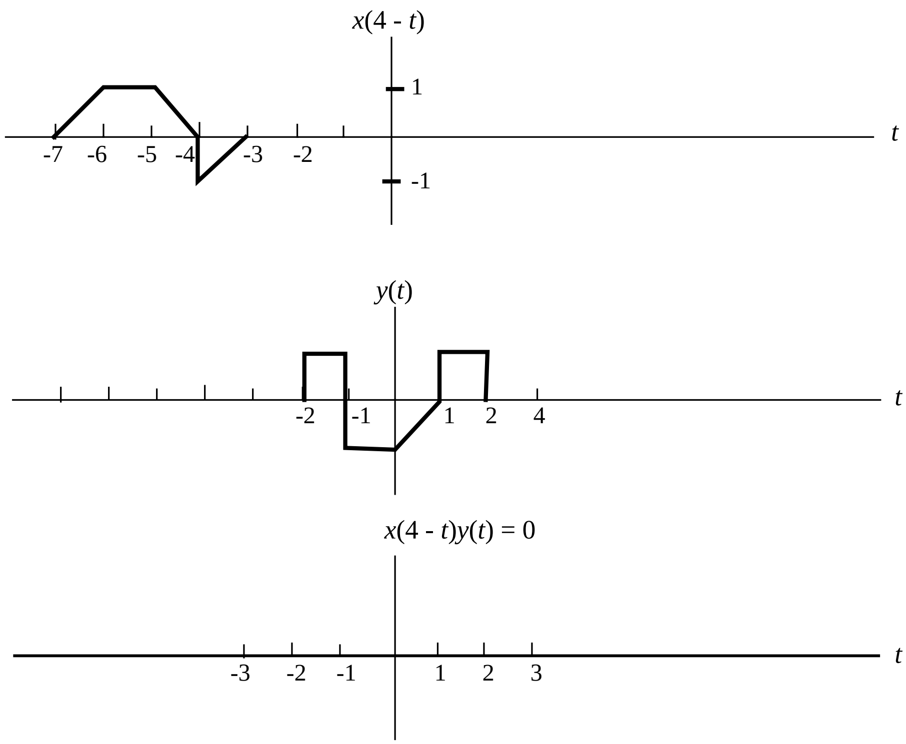

y(2 - t)
1 2 3
-4 -3 -2 -1
t


x(t)y(2 - t)
-1
1 2 3
-1
t

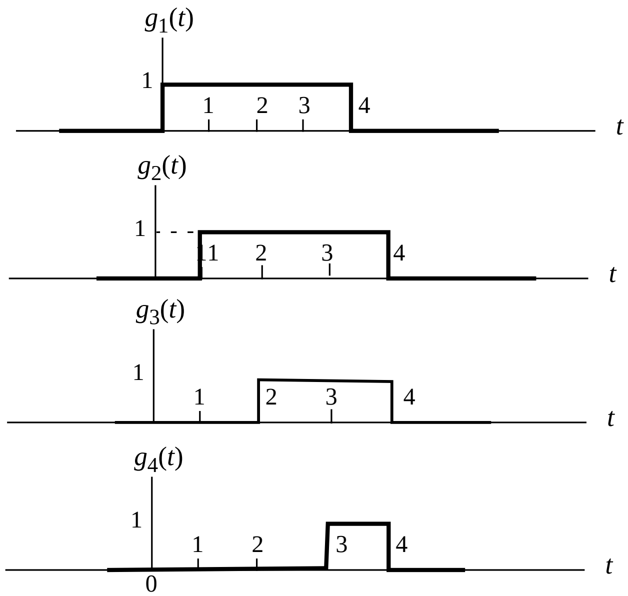
1.52 (f)


x(t)
1
-2
-1
1
2
-1
t
y(t/2 + 1)
1.0
-5
-3
-2
-1
-6
1
1
2
3
-1.0
t
x(2t)y(1/2t + 1)
+1
-0.5
-1
1
2
-1
t

1.52 (g)


x(4 - t)
1
-7
-6
-5
-4
-3
-2
-1
t
y(t)
-2
-1
1
2
4
t
x(4 - t)y(t) = 0
-3
-2
-1
1
2
3
t

1.53 We may represent $x(t)$ as the superposition of 4 rectangular pulses as follows:



To generate $g_1(t)$ from the prescribed $g(t)$, we let

$$g_1(t) = g(at - b)$$

where a and b are to be determined. The width of pulse $g(t)$ is 2, whereas the width of pulse $g_1(t)$ is 4. We therefore need to expand $g(t)$ by a factor of 2, which, in turn, requires that we choose

$$a = \frac{1}{2}$$

The mid-point of $g(t)$ is at $t = 0$, whereas the mid-point of $g_1(t)$ is at $t = 2$. Hence, we must choose b to satisfy the condition

$$a(2) - b = 0$$

or

$$b = 2a = 2\left(\frac{1}{2}\right) = 1$$

Hence, $g_1(t) = g\left(\frac{1}{2}t - 1\right)$

Proceeding in a similar manner, we find that

$$g_2(t) = g\left(\frac{2}{3}t - \frac{5}{3}\right)$$

$$g_3(t) = g(t - 3)$$

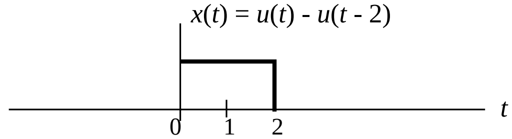
$$g_4(t) = g(2t - 7)$$

Accordingly, we may express the staircase signal $x(t)$ in terms of the rectangular pulse $g(t)$ as follows:

$$x(t) = g\left(\frac{1}{2}t-1\right)+g\left(\frac{2}{3}t-\frac{5}{3}\right)+g(t-3)+g(2t-7)$$

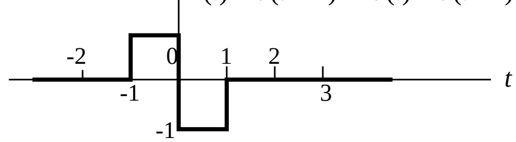
1.54 (a)

$x(t) = u(t) - u(t - 2)$



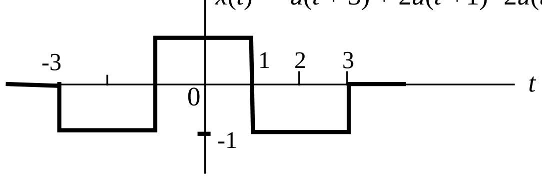
(b)

$x(t) = u(t + 1) - 2u(t) + u(t - 1)$



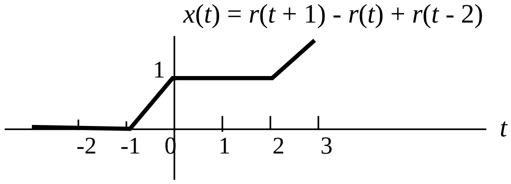
(c)

$x(t) = -u(t + 3) + 2u(t + 1) - 2u(t - 1) + u(t - 3)$



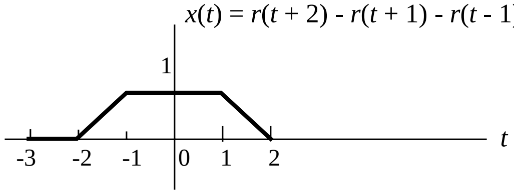
(d)

$x(t) = r(t + 1) - r(t) + r(t - 2)$

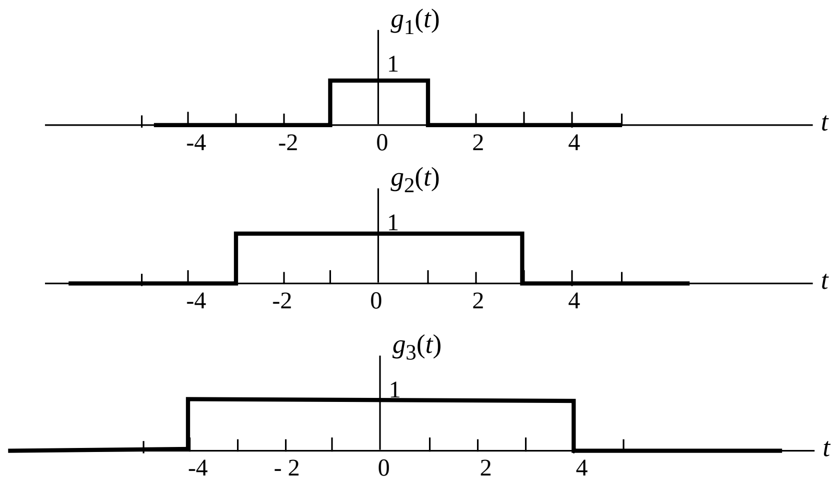


(e)

$x(t) = r(t + 2) - r(t + 1) - r(t - 1) + r(t - 2)$



1.55 We may generate $x(t)$ as the superposition of 3 rectangular pulses as follows:



All three pulses, $g_1(t)$, $g_2(t)$, and $g_3(t)$, are symmetrically positioned around the origin:

1. $g_1(t)$ is exactly the same as $g(t)$.
2. $g_2(t)$ is an expanded version of $g(t)$ by a factor of 3.
3. $g_3(t)$ is an expanded version of $g(t)$ by a factor of 4.

Hence, it follows that

$$g_1(t) = g(t)$$

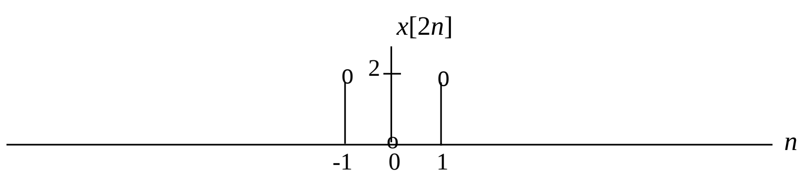
$$g_2(t) = g\left(\frac{1}{3}t\right)$$

$$g_3(t) = g\left(\frac{1}{4}t\right)$$

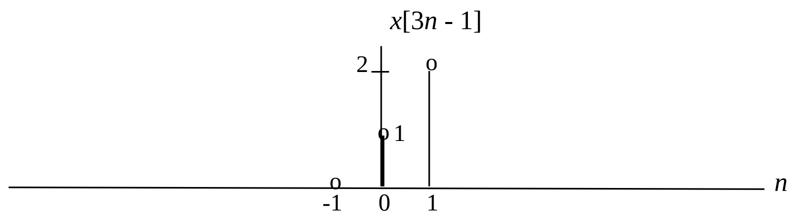
That is,

$$x(t) = g(t) + g\left(\frac{1}{3}t\right) + g\left(\frac{1}{4}t\right)$$

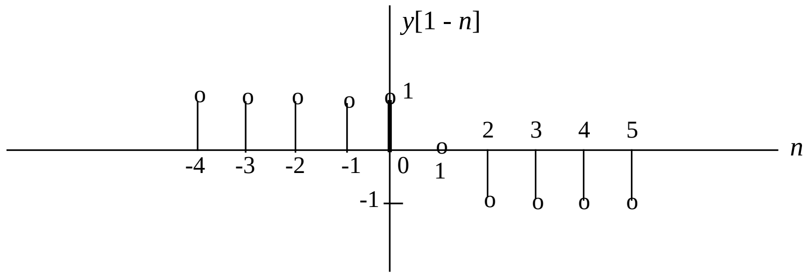
1.56 (a)



(b)

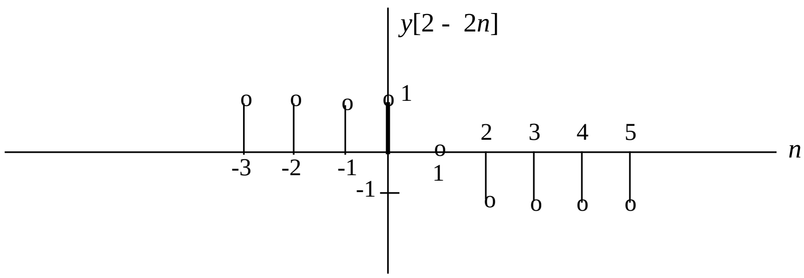


1.56 (c)



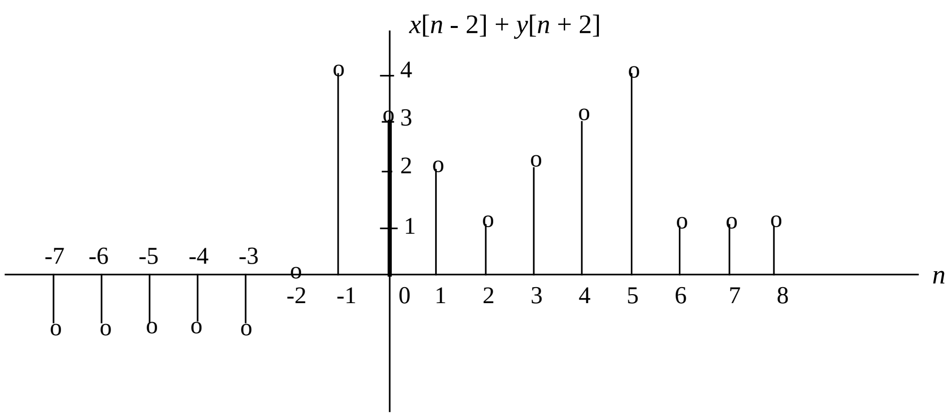
y[1 - n]
1
-1
n
-4 -3 -2 -1 0 1 2 3 4 5

(d)



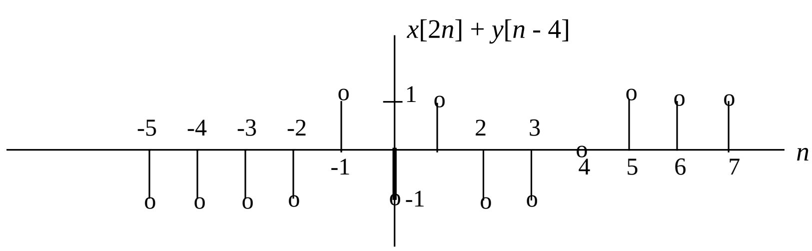
y[2 - 2n]
1
-1
n
-3 -2 -1 1 2 3 4 5

(e)



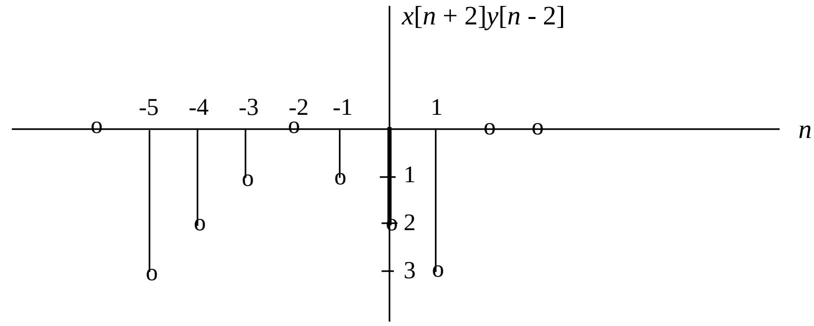
x[n - 2] + y[n + 2]
4
3
2
1
n
-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

(f)



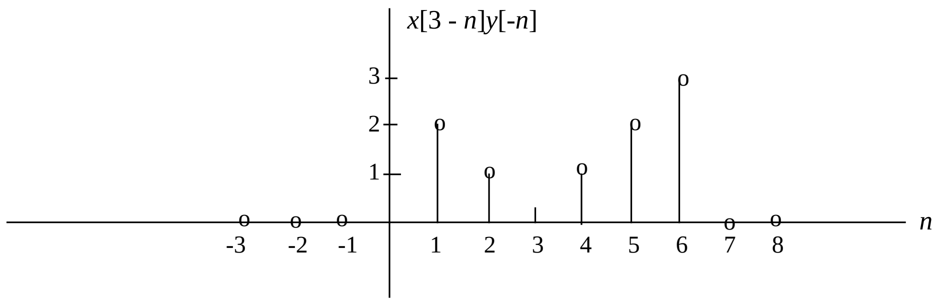
x[2n] + y[n - 4]
1
-1
n
-5 -4 -3 -2 -1 2 3 4 5 6 7

1.56 (g)



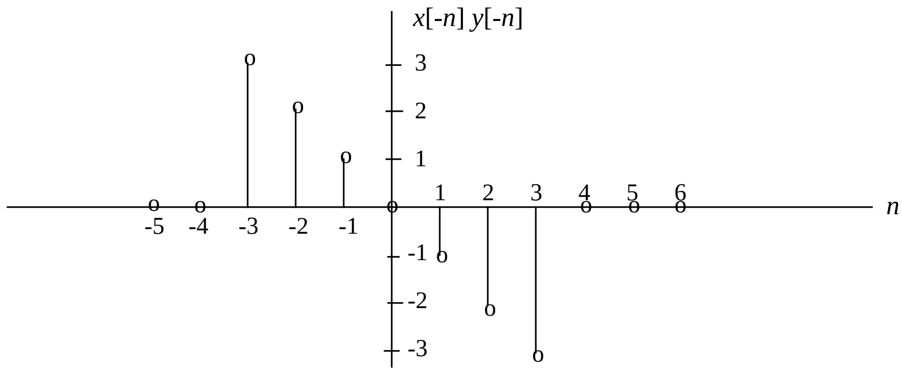
x[n + 2]y[n - 2]
-5
-4
-3
-2
-1
1
n
1
2
3

(h)



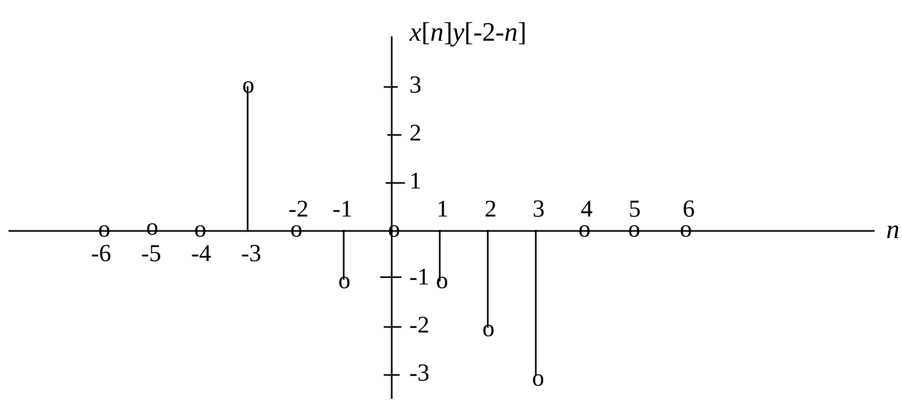
x[3 - n]y[-n]
3
2
1
-3
-2
-1
1
2
3
4
5
6
7
8
n

(i)



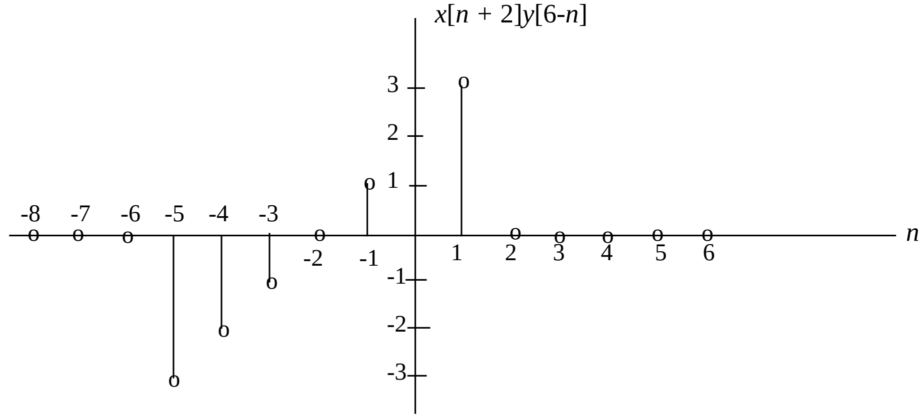
x[-n] y[-n]
3
2
1
1
2
3
4
5
6
-5
-4
-3
-2
-1
n
-1
-2
-3

(j)



x[n]y[-2-n]
3
2
1
-2
-1
1
2
3
4
5
6
n
-6
-5
-4
-3
-1
-2
-3

1.56 (k)



1.57 (a) Periodic
Fundamental period = 15 samples

(b) Periodic
Fundamental period = 30 samples

(c) Nonperiodic

(d) Periodic
Fundamental period = 2 samples

(e) Nonperiodic

(f) Nonperiodic

(g) Periodic
Fundamental period = 2π seconds

(h) Nonperiodic

(i) Periodic
Fundamental period = 15 samples

1.58 The fundamental period of the sinusoidal signal $x[n]$ is $N = 10$. Hence the angular frequency of $x[n]$ is

$$\Omega = \frac{2\pi m}{N} \qquad m\text{: integer}$$

The smallest value of Ω is attained with $m = 1$. Hence,

$$\Omega = \frac{2\pi}{10} = \frac{\pi}{5} \quad \text{radians/cycle}$$

1.59 The amplitude of complex signal $x(t)$ is defined by

$$\begin{aligned}\sqrt{x_R^2(t) + x_I^2(t)} &= \sqrt{A^2\cos^2(\omega t + \phi) + A^2\sin^2(\omega t + \phi)} \\ &= A\sqrt{\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)} \\ &= A\end{aligned}$$

1.60 Real part of $x(t)$ is

$$\mathrm{Re}\{x(t)\} = Ae^{\alpha t}\cos(\omega t)$$

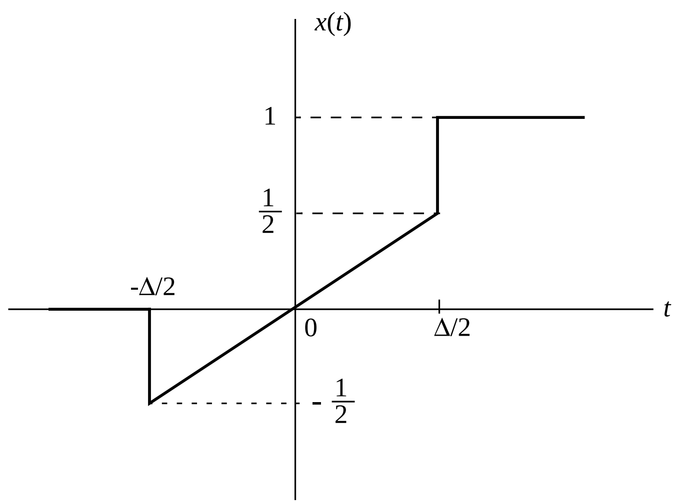
Imaginary part of $x(t)$ is

$$\mathrm{Im}\{x(t)\} = Ae^{\alpha t}\sin(\omega t)$$

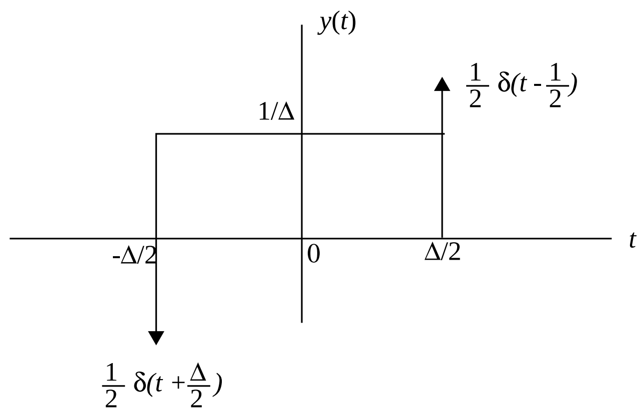
1.61 We are given

$$x(t) = \begin{cases} \dfrac{t}{\Delta} & \text{for } -\dfrac{\Delta}{2} \le t \le \dfrac{\Delta}{2} \\ 1 & \text{for } t \ge \dfrac{\Delta}{2} \\ 2 & \text{for } t < -\dfrac{\Delta}{2} \end{cases}$$

The waveform of $x(t)$ is as follows



The output of a differentiator in response to $x(t)$ has the corresponding waveform:



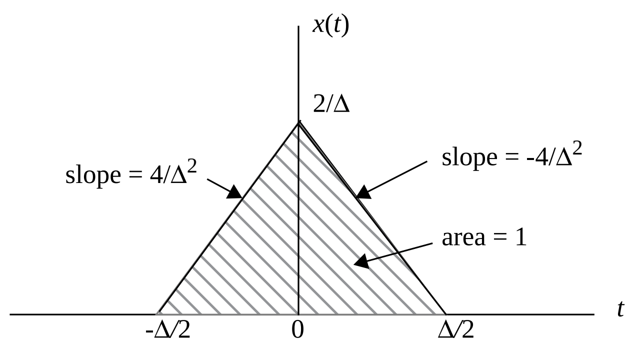
$y(t)$ consists of the following components:

1. Rectangular pulse of duration Δ and amplitude $1/\Delta$ centred on the origin; the area under this pulse is unity.
2. An impulse of strength 1/2 at $t = \Delta/2$.
3. An impulse of strength -1/2 at $t = -\Delta/2$.

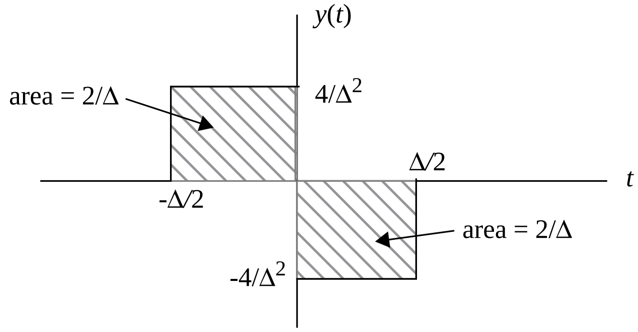
As the duration Δ is permitted to approach zero, the impulses $(1/2)\delta(t-\Delta/2)$ and $-(1/2)\delta(t+\Delta/2)$ coincide and therefore cancel each other. At the same time, the rectangular pulse of unit area (i.e., component 1) approaches a unit impulse at $t = 0$. We may thus state that in the limit:

$$\lim_{\Delta \to 0} y(t) = \lim_{\Delta \to 0} \frac{d}{dt}x(t)$$
$$= \delta(t)$$

1.62 We are given a triangular pulse of total duration Δ and unit area, which is symmetrical about the origin:



(a) Applying $x(t)$ to a differentiator, we get an output $y(t)$ depicted as follows:



(b) As the triangular pulse duration Δ approaches zero, the differentiator output approaches the combination of two impulse functions described as follows:

- An impulse of positive infinite strength at $t = 0^-$.
- An impulse of negative infinite strength at $t = 0^+$.

(c) The total area under the differentiator output $y(t)$ is equal to $(2/\Delta) + (-2/\Delta) = 0$.

In light of the results presented in parts (a), (b), and (c) of this problem, we may now make the following statement:

When the unit impulse $\delta(t)$ is differentiated with respect to time t, the resulting output consists of a pair of impulses located at $t = 0^-$ and $t = 0^+$, whose respective strengths are $+\infty$ and $-\infty$.

1.63 From Fig. P.1.63 we observe the following:

$$x_1(t) = x_2(t) = x_3(t) = x(t)$$

$$x_4(t) = y_3(t)$$

Hence, we may write

$$y_1(t) = x(t)x(t-1) \tag{1}$$

$$y_2(t) = |x(t)| \tag{2}$$

$$y_4(t) = \cos(y_3(t)) = \cos(1 + 2x(t)) \tag{3}$$

The overall system output is

$$y(t) = y_1(t) + y_2(t) - y_4(t) \tag{4}$$

Substituting Eqs. (1) to (3) into (4):

$$y(t) = x(t)x(t-1) + |x(t)| - \cos(1 + 2x(t)) \tag{5}$$

Equation (5) describes the operator H that defines the output $y(t)$ in terms of the input $x(t)$.

1.64

	Memoryless	Stable	Causal	Linear	Time-invariant
(a)	✓	✓	✓	x	✓
(b)	✓	✓	✓	✓	✓
(c)	✓	✓	✓	x	✓
(d)	x	✓	✓	✓	✓
(e)	x	✓	x	✓	✓
(f)	x	✓	✓	✓	✓
(g)	✓	✓	x	x	✓
(h)	x	✓	✓	✓	✓
(i)	x	✓	x	✓	✓
(j)	✓	✓	✓	✓	✓
(k)	✓	✓	✓	✓	✓
(l)	✓	✓	✓	x	✓

1.65 We are given

$$y[n] = a_0 x[n] + a_1 x[n-1] + a_2 x[n-2] + a_3 x[n-3] \qquad (1)$$

Let

$$S^k\{x(n)\} = x(n-k)$$

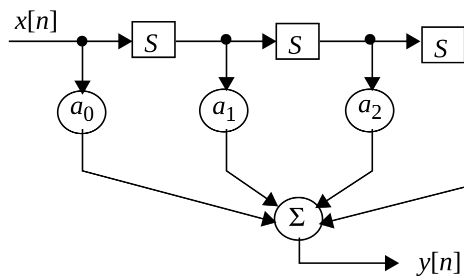
We may then rewrite Eq. (1) in the equivalent form

$$\begin{aligned} y[n] &= a_0 x[n] + a_1 S^1\{x[n]\} + a_2 S^2\{x[n]\} + a_3 S^3\{x[n]\} \\ &= (a_0 + a_1 S^1 + a_2 S^2 + a_3 S^3)\{x[n]\} \\ &= H\{x[n]\} \end{aligned}$$

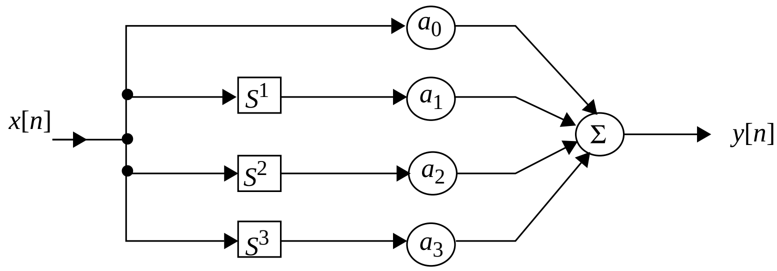
where

$$H = a_0 + a_1 S^1 + a_2 S^2 + a_3 S^3$$

(a) Cascade implementation of operator H:



(b) Parallel implementation of operator H:



1.66 Using the given input-output relation:

$$y[n] = a_0x[n] + a_1x[n-1] + a_2x[n-2] + a_3x[n-3]$$

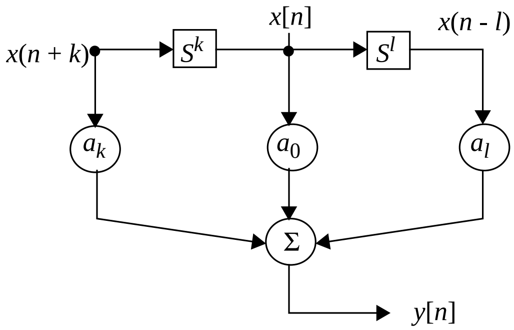
we may write

$$\begin{aligned}|y[n]| &= |a_0x[n] + a_1x[n-1] + a_2x[n-2] + a_3x[n-3]| \\ &\le |a_0x[n]| + |a_1x[n-1]| + |a_2x[n-2]| + |a_3x[n-3]| \\ &\le |a_0|M_x + |a_1|M_x + |a_2|M_x + |a_3|M_x \\ &= (|a_0| + |a_1| + |a_2| + |a_3|)M_x\end{aligned}$$

where $M_x = |x(n)|$. Hence, provided that M_x is finite, the absolute value of the output will always be finite. This assumes that the coefficients a_0, a_1, a_2, a_3 have finite values of their own. It follows therefore that the system described by the operator H of Problem 1.65 is stable.

1.67 The memory of the discrete-time described in Problem 1.65 extends 3 time units into the past.

1.68 It is indeed possible for a noncausal system to possess memory. Consider, for example, the system illustrated below:



That is, with $S^l\{x[n]\} = x[n - l]$, we have the input-output relation

$$y[n] = a_0x[n] + a_kx[n+k] + a_lx[n-l]$$

This system is noncausal by virtue of the term $a_kx[n + k]$. The system has memory by virtue of the term $a_lx[n - l]$.

1.69 (a) The operator H relating the output $y[n]$ to the input $x[n]$ is

$$H = 1 + S^1 + S^2$$

where

$$S^k\{x[n]\} = x[n-k] \qquad \text{for integer } k$$

(b) The inverse operator H^{inv} is correspondingly defined by

$$H^{\text{inv}} = \frac{1}{1 + S^1 + S^2}$$

Cascade implementation of the operator H is described in Fig. 1. Correspondingly, feedback implementation of the inverse operator H^{inv} is described in Fig. 2

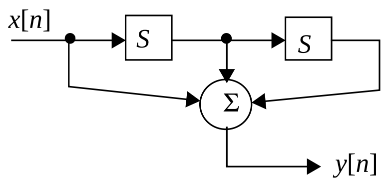


Fig. 1
Operator H

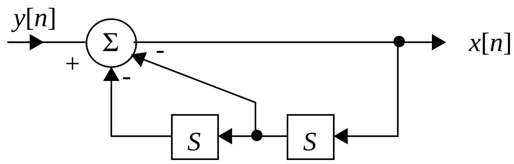


Fig. 2
Inverse Operator H^{inv}

Figure 2 follows directly from the relation:

$$x[n] = y[n] - x[n-1] - x[n-2]$$

1.70 For the discrete-time system (i.e., the operator H) described in Problem 1.65 to be time-invariant, the following relation must hold

$$S^{n_0}H = HS^{n_0} \qquad \text{for integer } n_0 \tag{1}$$

where

$$S^{n_0}\{x[n]\} = x[n-n_0]$$

and

$$H = 1 + S^1 + S^2$$

We first note that

$$\begin{aligned} S^{n_0}H &= S^{n_0}(1 + S^1 + S^2) \\ &= S^{n_0} + S^{n_0+1} + S^{n_0+2} \end{aligned} \tag{2}$$

Next we note that

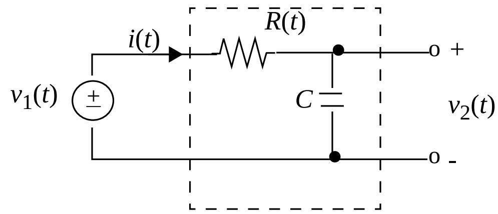
$$HS^{n_0} = (1 + S^1 + S^2)S^{n_0}$$

$$= S^{n_0} + S^{1+n_0} + S^{2+n_0} \qquad (3)$$

From Eqs. (2) and (3) we immediately see that Eq. (1) is indeed satisfied. Hence, the system described in Problem 1.65 is time-invariant.

1.71 (a) It is indeed possible for a time-variant system to be linear.

(b) Consider, for example, the resistance-capacitance circuit where the resistive component is time variant, as described here:



This circuit, consisting of the series combination of the resistor $R(t)$ and capacitor C, is time variant because of $R(t)$.

The input of the circuit, $v_1(t)$, is defined in terms of the output $v_2(t)$ by

$$v_1(t) = R(t)C\frac{dv_2(t)}{dt} + v_2(t)$$

Doubling the input $v_1(t)$ results in doubling the output $v_2(t)$. Hence, the property of homogeneity is satisfied.

Moreover, if

$$v_1(t) = \sum_{k=1}^{N} v_{1,k}(t)$$

then

$$v_2(t) = \sum_{k=1}^{N} v_{2,k}(t)$$

where

$$v_{1,k}(t) = R(t)C\frac{dv_{2,k}(t)}{dt} + v_{2,k}(t), \qquad k = 1,2,...,N$$

Hence, the property of superposition is also satisfied.

We therefore conclude that the time-varying circuit of Fig. P1.71 is indeed linear.

1.72 We are given the pth power law device:

$$y(t) = x^p(t) \qquad (1)$$

Let $y_1(t)$ and $y_2(t)$ be the outputs of this system produced by the inputs $x_1(t)$ and $x_2(t)$, respectively. Let $x(t) = x_1(t) + x_2(t)$, and let $y(t)$ be the corresponding output. We then note that

$$y(t) = (x_1(t) + x_2(t))^p \neq y_1(t) + y_2(t) \qquad \text{for } p \neq 0, 1$$

Hence the system described by Eq. (1) is nonlinear.

1.73 Consider a discrete-time system described by the operator H_1:

$$H_1: \quad y[n] = a_0 x[n] + a_k x[n-k]$$

This system is both linear and time invariant. Consider another discrete-time system described by the operator H_2:

$$H_2: \quad y[n] = b_0 x[n] + b_k x[n+k]$$

which is also both linear and time invariant. The system H_1 is causal, but the second system H_2 is noncausal.

1.74 The system configuration shown in Fig. 1.56(a) is simpler than the system configuration shown in Fig. 1.56(b). They both involve the same number of multipliers and summer. however, Fig. 1.56(b) requires N replicas of the operator H, whereas Fig. 1.56(a) requires a single operator H for its implementation.

1.75 (a) All three systems

- have memory because of an integrating action performed on the input,
- are causal because (in each case) the output does not appear before the input, and
- are time-invariant.

(b) H_1 is noncausal because the output appears before the input. The input-output relation of H_1 is representative of a differentiating action, which by itself is memoryless. However, the duration of the output is twice as long as that of the input. This suggests that H_1 may consist of a differentiator in parallel with a storage device, followed by a combiner. On this basis, H_1 may be viewed as a time-invariant system with memory.

System H_2 is causal because the output does not appear before the input. The duration of the output is longer than that of the input. This suggests that H_2 must have memory. It is time-invariant.

System H_3 is noncausal because the output appears before the input. Part of the output, extending from t = -1 to t = +1, is due to a differentiating action performed on the input; this action is memoryless. The rectangular pulse, appearing in the output from t = +1 to t = +3, may be due to a pulse generator that is triggered by the termination of the input. On this basis, H_3 would have to be viewed as time-varying.

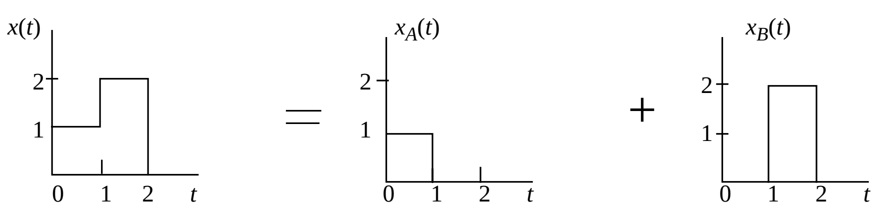
Finally, the output of H_4 is exactly the same as the input, except for an attenuation by a factor of 1/2. Hence, H_4 is a causal, memoryless, and time-invariant system.

1.76 H_1 is representative of an integrator, and therefore has memory. It is causal because the output does not appear before the input. It is time-invariant.

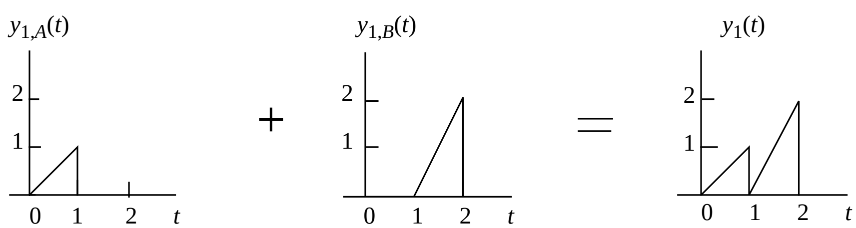
H_2 is noncausal because the output appears at $t = 0$, one time unit before the delayed input at $t = +1$. It has memory because of the integrating action performed on the input. But, how do we explain the constant level of +1 at the front end of the output, extending from $t = 0$ to $t = +1$? Since the system is noncausal, and therefore operating in a non real-time fashion, this constant level of duration 1 time unit may be inserted into the output by artificial means. On this basis, H_2 may be viewed as time-varying.

H_3 is causal because the output does not appear before the input. It has memory because of the integrating action performed on the input from $t = 1$ to $t = 2$. The constant level appearing at the back end of the output, from $t = 2$ to $t = 3$, may be explained by the presence of a strong device connected in parallel with the integrator. On this basis, H_3 is time-invariant.

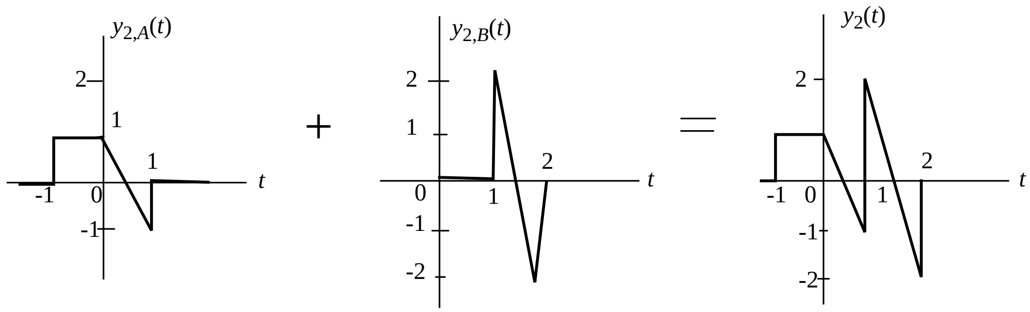
Consider next the input $x(t)$ depicted in Fig. P1.76b. This input may be decomposed into the sum of two rectangular pulses, as shown here:



Response of H_1 to $x(t)$:

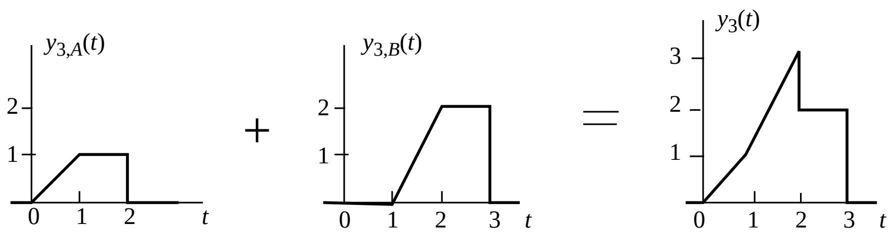


Response of H_2 to $x(t)$:

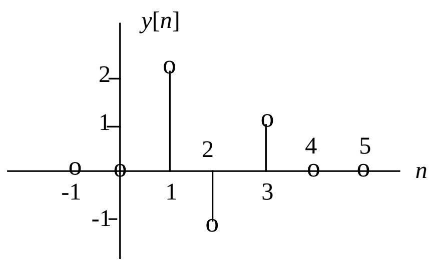


The rectangular pulse of unit amplitude and unit duration at the front end of $y_2(t)$ is inserted in an off-line manner by artificial means

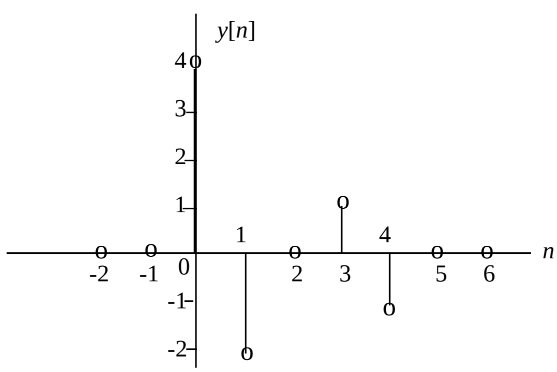
Response of H_3 to $x(t)$:



1.77 (a) The response of the LTI discrete-time system to the input $\delta[n-1]$ is as follows:



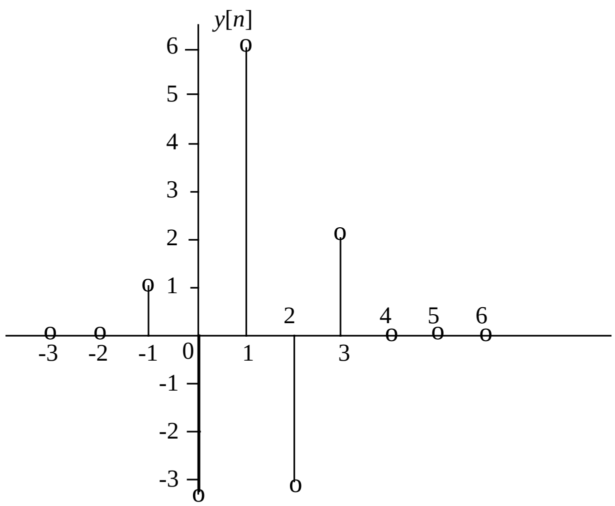
(b) The response of the system to the input $2\delta[n] - \delta[n-2]$ is as follows



(c) The input given in Fig. P1.77b may be decomposed into the sum of 3 impulse functions: $\delta[n+1]$, $-\delta[n]$, and $2\delta[n-1]$. The response of the system to these three components is given in the following table:

Time n	$\delta[n+1]$	$-\delta[n]$	$2\delta[n-1]$	Total response
-1	+2			+1
0	-1	-2		-3
1	+1	+1	+4	+6
2		-1	-2	-3
3			+2	2

Thus, the total response $y[n]$ of the system is as shown here:



Advanced Problems

1.78 (a) The energy of the signal x(t) is defined by

$$E = \int_{-\infty}^{\infty} x^2(t)dt$$

Substituting

$$x(t) = x_e(t) + x_o(t)$$

into this formula yields

$$E = \int_{-\infty}^{\infty} [x_e(t) + x_o(t)]^2 dt$$

$$= \int_{-\infty}^{\infty} [x_e^2(t) + x_o^2(t) + 2x_e(t)x_o(t)]^2 dt$$

$$= \int_{-\infty}^{\infty} x_e^2(t)dt + \int_{-\infty}^{\infty} x_o^2(t)dt + 2\int_{-\infty}^{\infty} x_e(t)x_o(t)dt \qquad (1)$$

With $x_e(t)$ even and $x_o(t)$ odd, it follows that the product $x_e(t)x_o(t)$ is odd, as shown by

$$\begin{aligned} x_e(-t)x_o(-t) &= x_e(t)[-x_o(t)] \\ &= -x_e(t)x_o(t) \end{aligned}$$

Hence,

$$\begin{aligned} \int_{-\infty}^{\infty}(x_e(t)x_o(t))dt &= \int_{-\infty}^{0} x_e(t)x_o(t)dt + \int_{0}^{\infty} x_e(t)x_o(t)dt \\ &= \int_{-\infty}^{0}(-x_e(t)x_o(t))(-dt) + \int_{0}^{\infty}(x_e(t)x_o(t))dt \\ &= -\int_{0}^{\infty} x_e(t)x_o(t)dt + \int_{0}^{\infty} x_e(t)x_o(t)dt \\ &= 0 \end{aligned}$$

Accordingly, Eq. (1) reduces to

$$E = \int_{-\infty}^{\infty} x_e^2(t)dt + \int_{-\infty}^{\infty} x_o^2(t)dt$$

(b) For a discrete-time signal $x[n]$, $-\infty \le n \le \infty$, we may similarly write

$$\begin{aligned} E &= \sum_{n=-\infty}^{\infty} x^2[n] \\ &= \sum_{n=-\infty}^{\infty} [x_e[n] + x_o[n]]^2 \\ &= \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]^2 + 2\sum_{n=-\infty}^{\infty} x_e[n]x_o[n] \end{aligned} \qquad (2)$$

With

$$x_e[-n]x_o[-n] = -x_e[n]x_o[n]$$

it follows that

$$\begin{aligned} \sum_{n=-\infty}^{\infty} x_e[n]x_o[n] &= \sum_{n=-\infty}^{0} x_e[n]x_o[n] + \sum_{n=-0}^{\infty} x_e[n]x_o[n] \\ &= \sum_{n=-\infty}^{0} x_e[-n]x_o[-n] + \sum_{n=-0}^{\infty} x_e[n]x_o[n] \\ &= -\sum_{n=\infty}^{0} x_e[n]x_o[n] + \sum_{n=0}^{\infty} x_e[n]x_o[n] \\ &= 0 \end{aligned}$$

Accordingly, Eq. (2) reduces to

$$E = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]$$

1.79 (a) From Fig. P1.79,

$$i(t) = i_1(t) + i_2(t) \tag{1}$$

$$L\frac{di_1(t)}{dt} + Ri_1(t) = \frac{1}{C}\int_{-\infty}^{t} i_2(\tau)d\tau \tag{2}$$

Differentiating Eq. (2) with respect to time t:

$$L\frac{d^2 i_1(t)}{dt^2} + R\frac{di_1(t)}{dt} = \frac{1}{C}i_2(t) \tag{3}$$

Eliminating i_2(t) between Eqs. (1) and (2):

$$L\frac{d^2 i_1(t)}{dt^2} + R\frac{di_1(t)}{dt} = \frac{1}{C}[i(t) - i_1(t)]$$

Rearranging terms:

$$\frac{d^2 i_1(t)}{dt^2} + \frac{R}{L}\frac{di_1(t)}{dt} + \frac{1}{LC}i_1(t) = \frac{1}{LC}i(t) \tag{4}$$

(b) Comparing Eqs. (4) with Eq. (1.108) for the MEMS as presented in the text, we may derive the following analogy:

MEMS of Fig. 1.64	LRC circuit of Fig. P1.79
$y(t)$	$i_1(t)$
ω_n	$1/\sqrt{LC}$
Q	$\frac{\omega_n L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$
$x(t)$	$\frac{1}{LC}i(t)$

1.80 (a) As the pulse duration Δ approaches zero, the area under the pulse $x_\Delta(t)$ remains equal to unity, and the amplitude of the pulse approaches infinity.

(b) The limiting form of the pulse $x_\Delta(t)$ *violates* the even-function property of the unit impulse:

$$\delta(-t) = \delta(t)$$

1.81 The output $y(t)$ is related to the input $x(t)$ as

$$y(t) = H\{x(t)\} \tag{1}$$

Let T_0 denote the fundamental period of $x(t)$, assumed to be periodic. Then, by definition,

$$x(t) = x(t + T_0) \tag{2}$$

Substituting $t + T_0$ for t into Eq. (1) and then using Eq. (2), we may write

$$\begin{aligned} y(t+T_0) &= H\{x(t+T_0)\} \\ &= H\{x(t)\} \\ &= y(t) \end{aligned} \tag{3}$$

Hence, the output $y(t)$ is also periodic with the same period T_0.

1.82 (a) For $0 \le t < \infty$, we have

$$x_\Delta(t) = \frac{1}{\Delta}e^{-t/\tau}$$

At $t = \Delta/2$, we have

$$\begin{aligned} A &= x_\Delta(\Delta/2) \\ &= \frac{1}{\Delta}e^{-\Delta/(2\tau)} \end{aligned}$$

Since $x_\Delta(t)$ is even, then

$$A = x_\Delta(\Delta/2) = x_\Delta(-\Delta/2) = \frac{1}{\Delta}e^{-\Delta/(2\tau)}$$

(b) The area under the pulse $x_\Delta(t)$ must equal unity for

$$\delta(t) = \lim_{\Delta \to 0} x_\Delta(t)$$

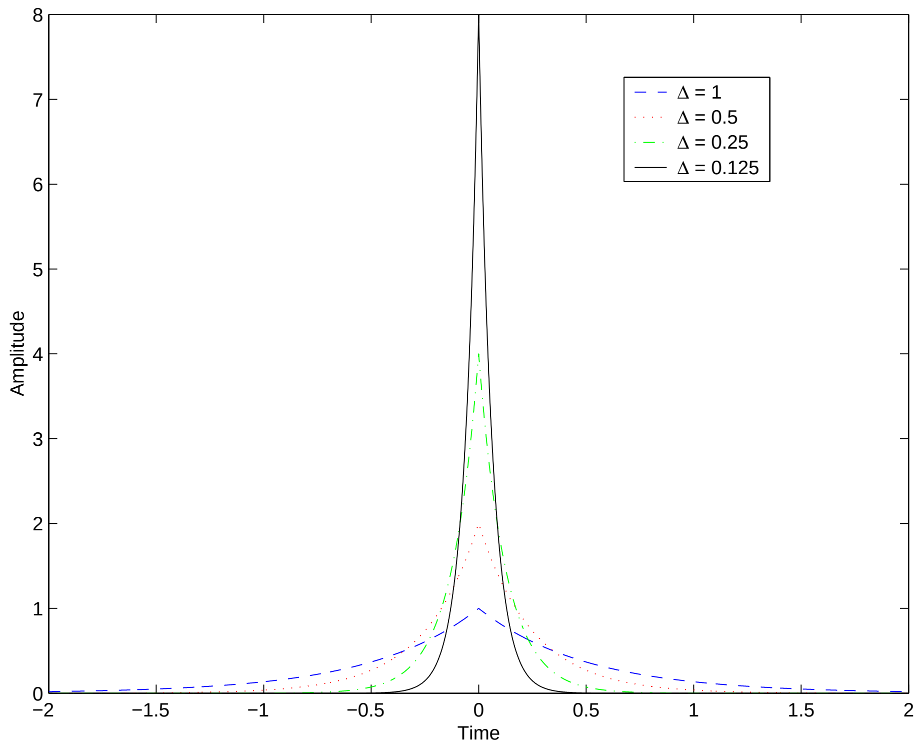
The area under $x_\Delta(t)$ is

$$\begin{aligned}\int_{-\infty}^{\infty} x_\Delta(t)dt &= 2\int_0^\infty x_\Delta(t)dt \\ &= 2\int_0^\infty \frac{1}{\Delta}e^{-t/\tau}dt \\ &= \frac{2}{\Delta}(-\tau)e^{-t/\tau}\Big|_0^\infty \\ &= \frac{2\tau}{\Delta}\end{aligned}$$

For this area to equal unity, we require

$$\tau = \frac{\Delta}{2}$$

(c)



1.83 (a) Let the integral of a continuous-time signal $x(t)$, $-\infty < t < \infty$, be defined by

$$\begin{aligned}y(t) &= \int_{-\infty}^{t} x(\tau)d\tau \\ &= \int_{-\infty}^{0} x(t)dt + \int_0^t x(\tau)d\tau\end{aligned}$$

The definite integral $\int_{-\infty}^{0} x(t)dt$, representing the *initial condition*, is a constant.

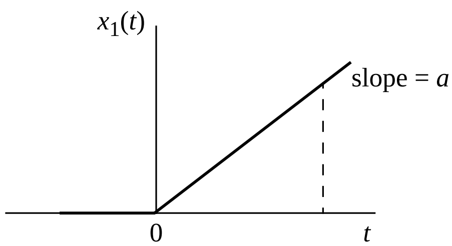
With differentiation as the operation of interest, we may also write

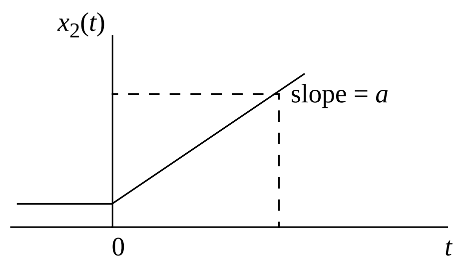
$$x(t) = \frac{dy(t)}{dt}$$

Clearly, the value of $x(t)$ is unaffected by the value assumed by the initial condition

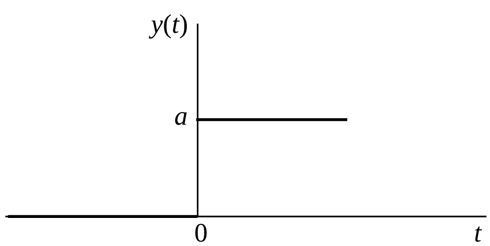
$$\int_{-\infty}^{0} x(t)dt$$

It would therefore be wrong to say that differentiation and integration are the inverse of each other. To illustrate the meaning of this statement, consider the two following two waveforms that differ from each other by a constant value for $-\infty < t < \infty$:





Yet, $y(t) = \frac{dx_1(t)}{dt} = \frac{dx_2(t)}{dt}$, as illustrated below:



(b) For Fig. P1.83(a):

$$y(t) + \frac{R}{L}\int_{-\infty}^{t} y(\tau)d\tau = x(t)$$

For R/L large, we approximately have

$$\frac{R}{L}\int_{-\infty}^{t} y(\tau)d\tau \approx x(t)$$

Equivalently, we have a differentiator described by

$$y(t) \approx \frac{L}{R}\,\frac{dx(t)}{dt}, \qquad \frac{R}{L} \text{ large}$$

For Fig. P1.83(b):

$$y(t) + \frac{L}{R}\frac{dy(t)}{dt} = x(t)$$

For R/L small, we approximately have

$$\frac{L}{R}\frac{dy(t)}{dt} \approx x(t)$$

Equivalently, we have an integrator described by

$$y(t) \approx \frac{R}{L}\int_{-\infty}^{t} x(\tau)d\tau \qquad \frac{R}{L}\text{ small}$$

(c) Consider the following two scenarios describing the *LR* circuits of Fig. P1.83

- The input x(t) consists of a voltage source with an average value equal to zero.
- The input x(t) includes a dc component E (exemplified by a battery).

These are two different input conditions. Yet for large R/L, the differentiator of Fig. P1.83(a) produces the same output. On the other hand, for small R/L the integrator of Fig. P1.83(b) produces different outputs. Clearly, on this basis it would be wrong to say that these two *LR* circuits are the inverse of each other.

1.84 (a) The output $y(t)$ is defined by

$$y(t) = A_0\cos(\omega_0 t + \phi)x(t) \qquad (1)$$

This input-output relation satisfies the following two conditions:

- Homogeneity: If the input $x(t)$ is scaled by an arbitrary factor a, the output $y(t)$ will be scaled by the same factor.
- Superposition: If the input $x(t)$ consists of two additive components $x_1(t)$ and $x_2(t)$, then

  $$y(t) = y_1(t) + y_2(t)$$

  where

  $$y(t) = A_0\cos(\omega_0 t + \phi)x_k(t), \qquad k = 1,2$$

Hence, the system of Fig. P1.84 is linear.

(b) For the impulse input

$$x(t) = \delta(t),$$

Eq. (1) yields the corresponding output

$$y'(t) = A_0\cos(\omega_0 t + \phi)\delta(t)$$