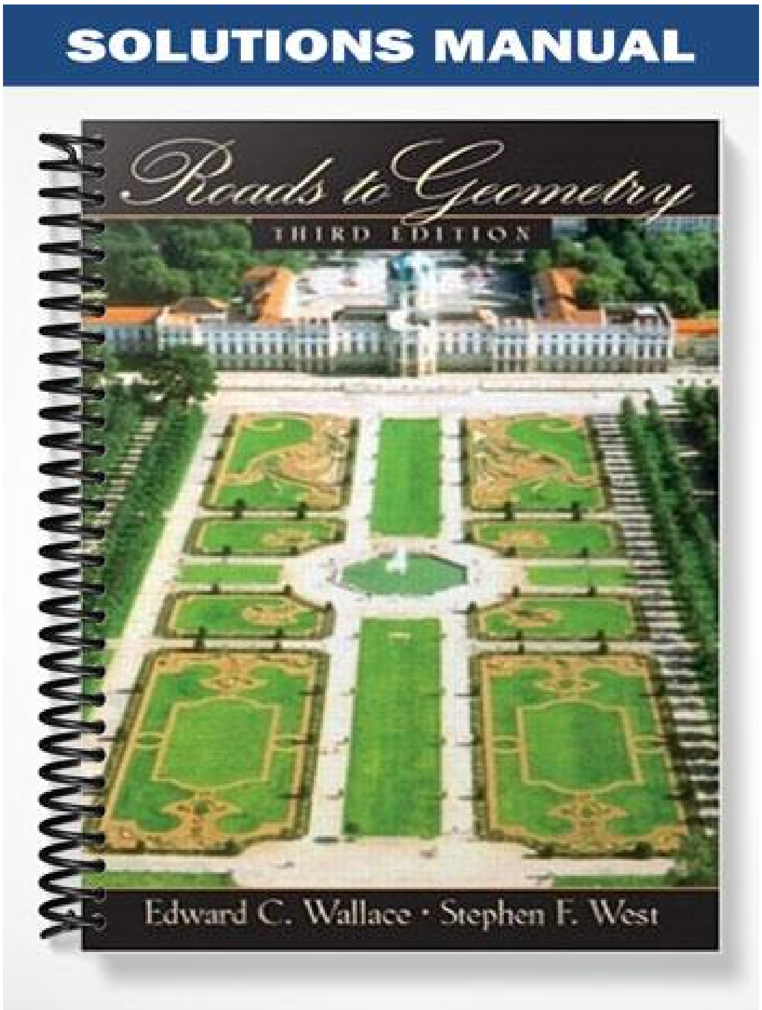
SOLUTIONS MANUAL
Roads to Geometry
THIRD EDITION
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Roads to Geometry

Instructor's Solutions Manual

Edward Wallace
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ROADS TO GEOMETRY
SUGGESTED SOLUTIONS
CHAPTER ONE

EXERCISE SET 1.1

1.1.1) (a) $A_{Square} = \frac{1}{4}(s+s)(s+s) = s_2$ (Correct)$A_{Rectangle} = \frac{1}{4}(a+a)(b+b) = ab$ (Correct).

(b) $A_{Trapezoid} = \frac{1}{2}(h)(b_1+b_2) = \frac{1}{2}(4)(6+12) = 36$ while$A = \frac{1}{4}(5+15)(6+12) = 45$A, which is too large.$A_{Trapezoid} = \frac{1}{2}(h)(b_1+b_2) = \frac{1}{2}(4)(10+12) = 52$ while $A = \frac{1}{4}(5+15)(10+16) = 65$, which is too large. $A_{Parallelogram} = ab = (4)(8) = 32$ while $A = \frac{1}{4}(5+5)(8+8) = 40$, which is too large.

(c) $A_{Trapezoid} = \frac{1}{2}(h)(b1+b2) = \frac{1}{2}(h)(b+b+2x) = h(b+x)$ while $A = \frac{1}{4}(\sqrt{x^2+h^2}+\sqrt{x^2+h^2})(b+b+2x) = (\sqrt{x^2+h^2})(b+x)$. Now since $x > 0$, we have $\sqrt{x^2+h^2} > x$ and therefore$(\sqrt{x^2+h^2})(b+x) > h(b+x)$.Similarly for parallelograms.

1.1.2 (a) If a = 3, b = 4 then c = 5 while $c = b + a^2/2b = 4 + 9/8 = 51/8$.If a = 5, b = 12 then c = 13 while$c = b + a^2/2b = 12 + \frac{25}{24} = 13\frac{1}{24}$.If a = 12, b = 5 then c = 13 while $c = b + a^2/2b = 5 + \frac{144}{10} = \frac{192}{5}$.

(b) By the Pythagorean Theorem, $a^2 + b^2 = c^2$. If we consider$c = b + a^2/2b$, then $c^2 = b^2 + 2(b)(a^2/2b) + a^4/4b^2 = b^2 + a^2 + a^4/4b^2$. Now, since a>0 and b>0 then $a^4/4b^2 > 0$ and $c^2 > a^2 + b^2$.

1.1.3) The problem solver used this procedure to calculate the length of chord, CD, in a circle of circumference 60 (See Figure 1). The solver assumes a value of 3 for π, which produces a diameter of length 20, and then proceeds to use the Pythagorean Theorem on a triangle whose hypotenuse is 20, and whose legs are 16 and CD to find that $CD = 12$. Note: In the Figure, the right triangle ABC, whose hypotenuse is 10 and whose legs are 8 and x (one half of CD) is similar to the triangle used by the solver, thus justifying the use of the Pythagorean Theorem.

1.1.4) (See Figure 2). $V = \frac{1}{3}(6)(22 + (2)(4) + 42) = 56$.

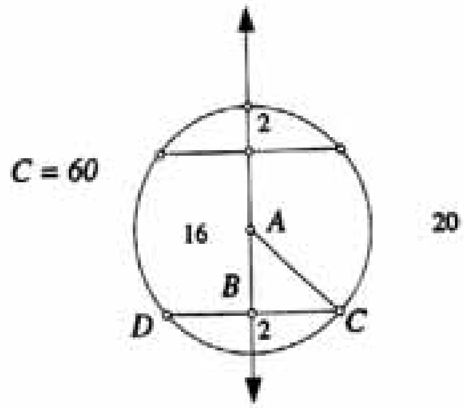

Figure 1: Exercise 1.1.3

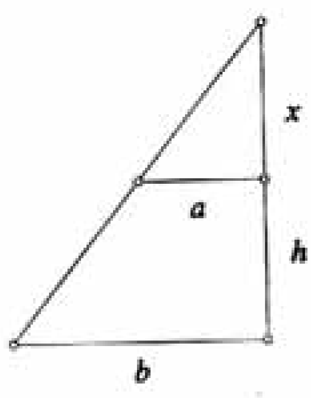

Figure 2: Exercise 1.1.4

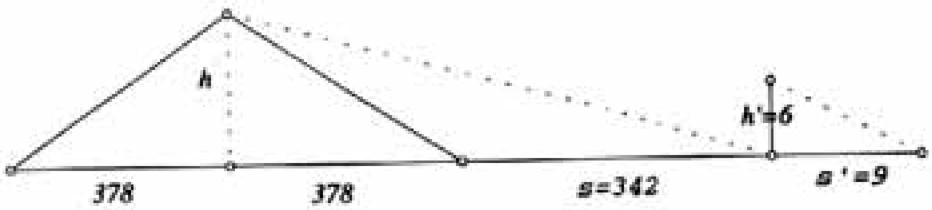

Figure 3: Exercise 1.1.6

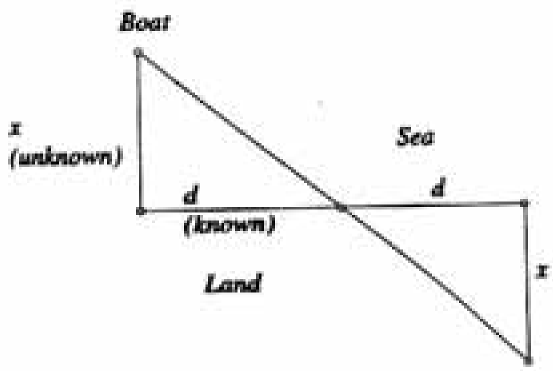

Figure 4: Exercise 1.1.7

From similar triangles we have $x/x+h = a/b$ and $x = ah/b-a$.
$V_{Frustum} = \frac{1}{3}(b^2)(x+h) - \frac{1}{3}(a^2)(x) = \frac{1}{3}(b^2x + b^2h - a^2x)$
$= \frac{1}{3}[(b+a)(ah) + b^2h] = \frac{1}{3}h[a^2+ab+b^2]$

1.1.5) $A_{Circle} = \pi(d/2)^2 = \pi d^2/4$, $A_{Square} = [(8/9)d]^2 = (64/81)d^2$.
Now if we let $(64/81)d^2 = \pi d^2/4$, then $\pi/4 = 64/81$, thus $\pi = 256/81$.

1.1.6) (See Figure 3). Using similar triangles, $h = \frac{(378+s)h'}{s'} = \frac{2}{3}(720) = 480$.

1.1.7) (See Figure 4).

1.1.8) (See Figure 5).

Using proportions, we find that $360°/(7°12') = x/5000$ and therefore $x =$ $(5000)(360)/(7°12')$ stades $= 250000$ stades or 26500 miles.

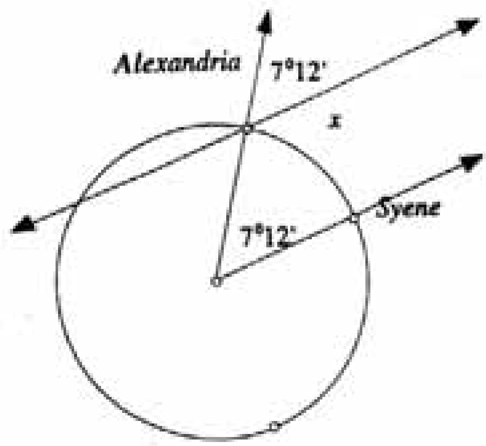

Figure 5: Exercise 1.1.8

1.1.9)(See Figure 6.)

$$A_{semicircle R} = \frac{1}{2}\pi(\frac{1}{2}R)^2 = \frac{1}{8}\pi R^2$$

$$A_{semicircle r} = \frac{1}{2}\pi(\frac{1}{2}r)^2 = \frac{1}{8}\pi r^2$$

and

$$A_{semicircle s} = \frac{1}{2}\pi(\frac{1}{2}s)^2 = \frac{1}{8}\pi s^2$$

therefore

$$A_{arbelos} = \frac{1}{8}\pi R^2 - (\frac{1}{8}\pi r^2 + \frac{1}{8}\pi s^2) = \frac{1}{8}\pi\left[R^2 - (r^2 + s^2\right].$$

1.1.10) Let r = the radius of c_1, therefore $AC = r$ and $A_{\Delta ABC} = \frac{1}{2}r^2$.(See Figure 7). Now,

$$A_{lune} = A_{semicircle c_2} - \left[\frac{1}{2}A_{semicircle c_1} - A_{\Delta ABC}\right].$$

Since $AC = r$, $AB = r\sqrt{2}$ then

$$A_{semicircle c_2} = \frac{1}{2}\pi(\frac{1}{2}r\sqrt{2})^2 = \frac{1}{4}\pi r^2.$$

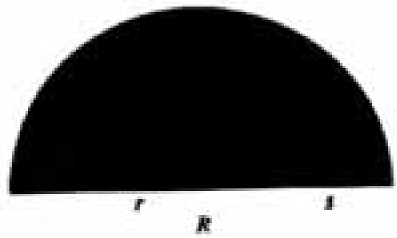

Figure 6: Exercise 1.1.9

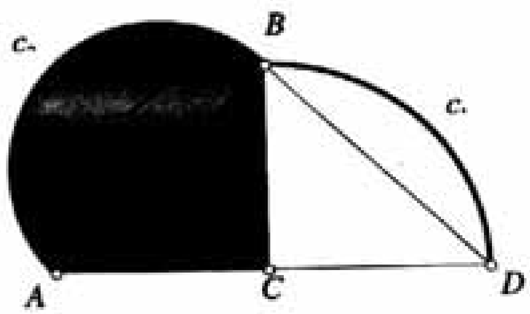

Figure 7: Exercise 1.1.10

Now $A_{semicircle}c_1 = \frac{1}{2}\pi r^2$ therefore

$$A_{lune} = \frac{1}{4}\pi r^2 = \left[\frac{1}{4}\pi r^2 - \frac{1}{2}r^2\right] = \frac{1}{2}r^2 = A_{\Delta ABC}$$

EXERCISE SET 1.2

The Axiomatic Method

1.2.1) By Axiom #1 there are exactly three Fe's in the system. The assumption of the existence of a Fo which contains three distinct Fe's immediately contradicts Axiom #3.

1.2.2) Fe-Fo Theorem #2 implies the existence of exactly three Fo's. choose two of those Fo's. By Theorem #1 they must share exactly one Fe and by Theorem #3 they each contain exactly two Fe's. Therefore two distinct Fo's contain exactly three Fe's which by Axiom #1 is all the Fe's of the system.

1.2.3) Suppose that there exists a set of two distinct Fe's such that at least one Fo contains neither of them. By Axiom #2 these two Fe's belong to exactly one Fo. Now we have two Fo's which do not share a Fe, contradicting Axiom #4.

1.2.4) By Theorem #2 there exists exactly three Fo's. If they all contain the same Fe and they each contain a distinct second Fe (which they must by Theorem #3) then the system must contain at least four Fe's which contradicts Axiom #1.

1.2.5) Using Axioms #1 and 2 and simple combinatorics, we find that we have at least 10 y's. Now, suppose that there exists a distinct eleventh y. By Axiom #3 it must on exactly two x's. This fact contradicts either Axiom #1 or Axiom #2.

1.2.6) Suppose that there exists two distinct y's which share more that one x, with the simplest case being that they share two x's. But these two y's now contradict Axiom #2.

1.2.7) Suppose that all x's lie on the same y and look for a contradiction.

1.2.8) By Axiom #1 there exists exactly five x's. If we choose one specific x, that x paired with each of the other four x's must determine exactly one y, that is, there exists at least four y's on that x. Now suppose that there exists a fifth distinct y on the x, it must contain a second distinct x, which cannot be any of the other x's.

1.2.9) Consider y_1, which must, by Axiom #3, be on exactly two x's, say x_2 and x_3. Let x_1 not be on y_1. Now by Problem #1.2.8, there are exactly four y's on x_1. by Axiom #2 one of those y's must be on x_2 and one on x_3, and therefore the other two must contain no x's that are on y_1.

Models

1.2.10) See Figure 1.2.1.

1.2.11) AXIOM #2 - The Mathematics and Physics Books are not on a shelf.
AXIOM #4 - One book cannot be physically on two distinct horizontal shelves.

1.2.12) The one-to-one correspondence is indicated. We must only verify that the relationships between the "Fe's" and the "Fo's" in the models are preserved under that one-to-one correspondence. For example: Bob is the only person on both the Entertainment and Refreshment Committees and P is the only letter in both {P,Q} and {P,R} and P is mapped to Bob, {P,Q} is mapped to Entertainment Committee, and {P,R} is mapped to the Refreshment Committee.

1.2.13) $\{x, z\} \leftrightarrow P \qquad \{P,Q\} \leftrightarrow z$
$\{y, z\} \leftrightarrow Q \qquad \{Q,R\} \leftrightarrow y$
$\{x, y\} \leftrightarrow R \qquad \{P,R\} \leftrightarrow x$

1.2.14) (a) Many answers.
(b) Not possible.

1.2.15) Let the x's be elements of the set, $S = \{A, B, C, D, E\}$ and the y's be the two element subset of S, $\{A, B\}$, $\{A, C\}$, $\{A, D\}$, $\{A, E\}$, $\{B, C\}$, $\{B, D\}$, $\{B, E\}$, $\{C, D\}$, $\{C, E\}$, and $\{D, E\}$. Others similarly.

1.2.16) (a) AXIOM #1, If a and $b \epsilon Z$ and $a < b$ then $a \neq b$, and AXIOM #2, If a,b, and $c \epsilon Z$ and $a < b$ and $b < c$ then $a < c$.
(b) Yes.
(c) Yes since $a > b \leftrightarrow b < a$.
(d) Yes.
(e) No, since Z cannot be placed in a one-to-one correspondence with R.

Properties of Axiomatic Systems

1.2.17) Many answers.

1.2.18) Many answers.

1.2.19 Abstract, since Z and R do not have physical representations.

1.2.20) It is impossible to collect a physical set containing a infinite number of objects.

1.2.21) (a) Independence of Axiom #2 - Consider the following model: Let the Fe's be elements of the set, $S = \{P, Q, R\}$ and the Fo's be the subsets, $\{P, Q\}$ and $\{P, R\}$. Clearly Axioms #1, 3, and 4 are true, but since Q and R are not on a Fo, Axiom #2 is false.

(b) Independence of Axiom #3 - Consider the following model: Let the Fe's be elements of the set, $S = \{P, Q, R\}$ and the subset $\{P, Q, R\}$ be the only Fo. Axioms #1 and 2 are obviously true and Axiom #4 is true vacuously. Axiom #3 is clearly false.

(c) Independence of Axiom #4 - Consider the following model: Let the Fe's be elements of the set, $S = \{P, Q, R\}$ and the Fo's be the subsets, $\{P\}$, $\{P, Q\}$, $\{P, R\}$, and $\{Q, R\}$. Axioms #1,2,and 3 are true and since $\{P\}$ and $\{Q, R\}$ do not share a Fe, Axiom #4 is false.

1.2.24) Independence of Axiom #1 - Consider the following model:Let the x's be elements of the set, $S = \{A, B, C, D\}$ and the y's be the subsets, $\{A, B\}$, $\{A, C\}$, $\{A, D\}$, $\{B, C\}$,$\{B, D\}$, and $\{C, D\}$.

Independence of Axiom #2 - Consider the following model:Let the x's be elements of the set, $S = \{A, B, C, D, E\}$ and the y's be the subsets, $\{A, B\}$, $\{A, C\}$, $\{A, D\}$, and $\{A, E\}$.

Independence of Axiom #3 - Consider the following model:Let the x's be elements of thes et, $S = \{A, B, C, D, E\}$and the y's be the subsets, $\{A, B\}$, $\{A, C\}$, $\{A, D\}$, $\{A, E\}$, and $\{B, C, D, E\}$.

1.2.25) (a) Consider the following model: Let $S = \{p \mid p$ is a person living on the planet Earth$\}$, and let R be "*is the same age as*".

(b) Consider the following model: Let $S = \{t \mid t$ is a triangle$\}$ and let R be "*is congruent to*".

(c) Yes