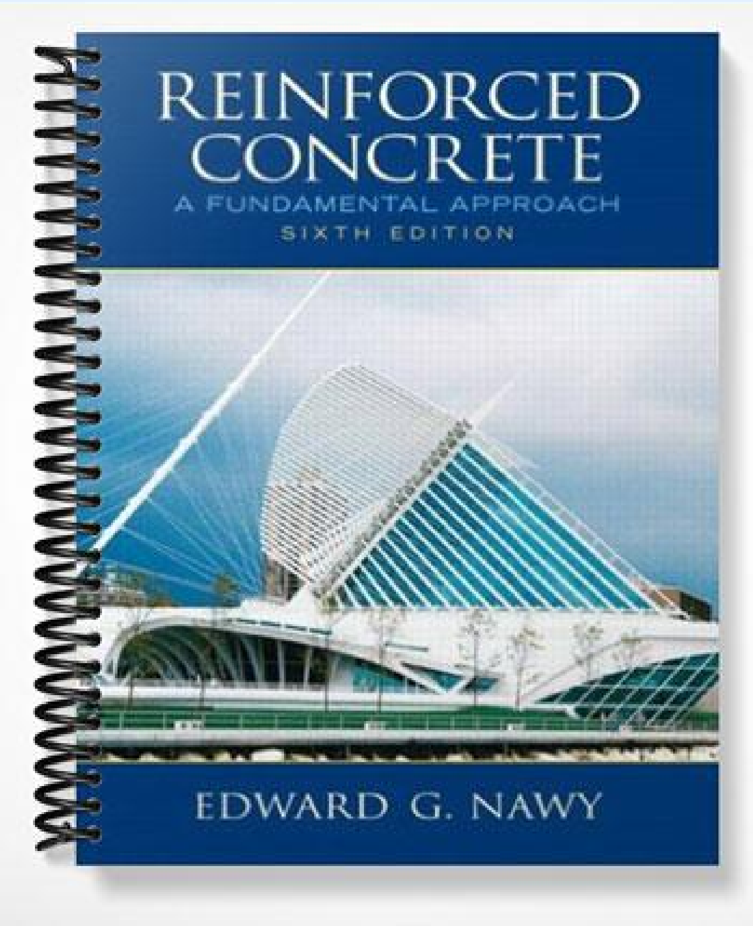
SOLUTIONS MANUAL



INSTRUCTOR'S SOLUTIONS MANUAL

REINFORCED CONCRETE

A FUNDAMENTAL APPROACH

SIXTH EDITION

EDWARD G. NAWY

PEARSON
Prentice Hall

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About the Cover: Shown is the Milwaukee Art Museum, Milwaukee, Wisconsin. Construction in reinforced and prestressed concrete in various shapes that include cantilevered canopies and a unique cable-stayed bridge. Rows of exposed situ-cast concrete arches form a galleria that overlooks Lake Michigan. Designed by Architect Santiago Calatrava and opened in 2002. Photo courtesy Professor Tarun Naik, University of Wisconsin at Milwaukee.

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CONTENTS

Please note that there are no solutions for Chapters 1 through 4. Solutions begin with Chapter 5.

5.1. For the beam cross-section shown in Fig. 5.33 determine whether the failure of the beam will be initiated by crushing of concrete or yielding of steel. Given:

$$f'_c = 4000 \text{ psi (27.6 MPa) for case (a)}, A_s = 10 \text{ in.}^2$$
$$f'_c = 7000 \text{ psi (48.3 MPa) for case (b)}, A_s = 5 \text{ in.}^2$$
$$f_y = 60{,}000 \text{ psi (414 MPa)}$$

Also determine whether the section satisfies ACI Code requirements.

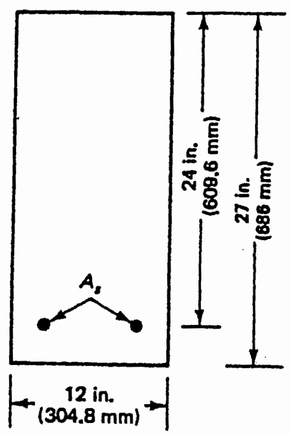


Figure 5.33

Solution:

a) $\beta_1 = 0.85$

$A_s = 10 \text{ in}^2$

$f_y = 60{,}000 \text{ psi}$

$f'_c = 4{,}000 \text{ psi}$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(10)(60{,}000)}{0.85(4{,}000)(12)} = 14.71 \text{ inches}$$

$$c = \frac{a}{\beta_1} = \frac{14.71}{0.85} = 17.31 \text{ inches}$$

$$\frac{c}{d_t} = \frac{17.31}{24} = 0.72 > 0.60$$

∴ Compression-controlled and concrete crushes before tension steel yields

b) $\beta_1 = 0.85 - 0.05\left(\frac{7{,}000 - 4{,}000}{1{,}000}\right) = 0.70$

$f'_c = 7{,}000$ psi

$A_s = 5\ in^2$

$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(5)(60{,}000)}{0.85(7{,}000)(12)} = 4.20$ inches

$c = \frac{a}{\beta_1} = \frac{4.2}{0.70} = 6.0$ inches

$\frac{c}{d_t} = \frac{6}{24} = 0.25 < 0.375$ ∴ Tension - controlled and steel yields before concrete crushes

$\rho = \frac{A_s}{bd} = \frac{5}{(12)(24)} = 0.017$ in/in.

$\rho_{min} = \max\left\{\frac{3\sqrt{7{,}000}}{60{,}000} = 0.0042,\ \frac{200}{60{,}000} = 0.0033\right\} = 0.0042$ in/in.

$0.017 > 0.0042$ ∴ <u>O.K</u> Satisfies ACI CODE

5.2. Calculate the nominal moment strength of the beam sections shown in Fig. 5.34. Given:

$f_c' = 5\,000$ psi (20.7 MPa) for case (a)

$f_c' = 6000$ psi (41.4 MPa) for case (b)

$f_y = 60{,}000$ psi (414 MPa)

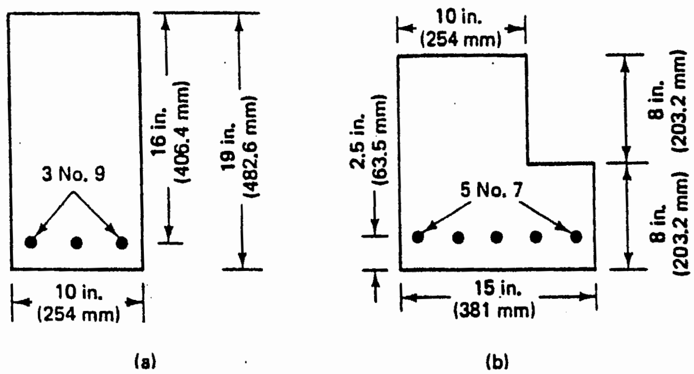


Figure 5.34

Solution:

a) $f_c' = 5{,}000$ psi

$\beta_1 = 0.80$

$A_s = 3$ No. 9 $= 3 \text{ in}^2$

$b = 10$ in.

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{(3)(60{,}000)}{0.85(5{,}000)(10)} = 4.24 \text{ inches}$$

$$c = \frac{a}{\beta_1} = \frac{4.24}{0.80} = 5.29 \text{ in.}$$

$$\frac{c}{d_t} = \frac{5.29}{16} = 0.33 < 0.375$$

∴ Tension-Controlled

$\phi = 0.90$

$$\rho = \frac{A_s}{bd} = \frac{3}{(10)(16)} = 0.019$$

$$\rho_{min} = \max\left\{\frac{3\sqrt{f_c'}}{f_y}, \frac{200}{f_y}\right\} = 0.0035 < 0.019 \qquad \underline{\underline{\text{O.K.}}}$$

$$M_n = A_s f_y (d - a/2) = (3)(60{,}000)\left(16 - \frac{4.24}{2}\right) = 2{,}498{,}824 \text{ in-lb}$$

$$M_u = \phi M_n = 0.90\,(2{,}498{,}824) = 2{,}248{,}941 \text{ in-lb.}$$

b) $f_c' = 6{,}000$ psi
$\beta_1 = 0.75$
$A_s = 5 \times 0.7 = 3 \text{ in}^2$
$b = 10$ in (assuming neutral axis is w/in top 8 inches)

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{(3)(60{,}000)}{0.85(6{,}000)(10)} = 3.53 \text{ in.}$$

$$c = a/\beta_1 = 4.71 \text{ in} < 8 \text{ in.} \quad \therefore\ b = 10 \text{ in.} \quad \underline{\underline{\text{O.K}}}$$

$$\frac{c}{d_t} = \frac{4.71}{13.5} = 0.35 < 0.375 \quad \therefore \text{ Tension-controlled}$$

$$\phi = 0.90$$

$$\rho = \frac{A_s}{bd} = 0.022 \qquad \rho_{min} = \max\left\{\frac{3\sqrt{f_c'}}{f_y}, \frac{200}{f_y}\right\} = 0.0039 < 0.022 \quad \underline{\underline{\text{O.K}}}$$

$$M_n = A_s f_y (d - a/2) = (3)(60{,}000)\left(13.5 - \frac{3.53}{2}\right) = 2{,}112{,}353 \text{ in-lb}$$

$$M_u = \phi M_n = 0.9\, M_n = 1{,}901{,}118 \text{ in-lb.}$$

5.3. Calculate the safe distributed load intensity that the beam shown in Fig. 5.35 can carry. Given:

$$f'_c = 4000 \text{ psi (27.6 MPa), normal-weight concrete}$$
$$f_y = 60{,}000 \text{ psi (414 MPa)}$$

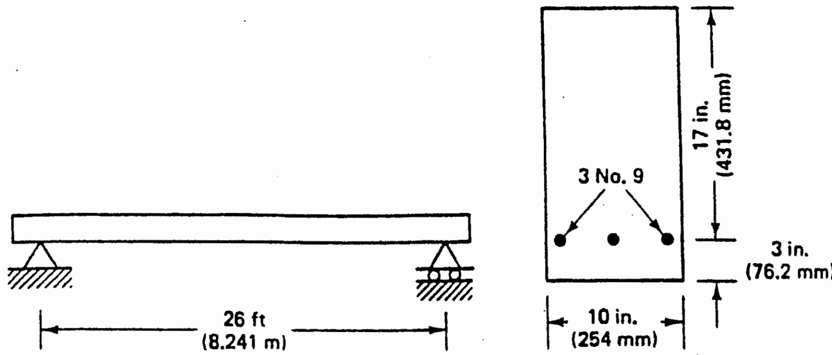


Figure 5.35

Solution:

Given

$b = 10$ in.

$h = 20$ in.

$d_t = 17$ in

$A_s = 3$ No.9 $= 3.0$ in^2

$f'_c = 4{,}000$ psi

$f_y = 60{,}000$ psi

$\beta_1 = 0.85$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3)(60{,}000)}{(0.85)(4000)(10)} = 5.29 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{5.29}{0.85} = 6.23 \text{ in.}$$

$$\frac{c}{d_t} = \frac{6.23}{17} = 0.37 < 0.375$$

$\therefore$ Tension-controlled $\phi = 0.90$

$$\rho = \frac{A_s}{bd} = \frac{(3)}{(10)(17)} = 0.018$$

$$\rho_{min} = \max\left\{\frac{3\sqrt{f'_c}}{f_y}, \frac{200}{f_y}\right\} = 0.0033 < 0.018$$

$\therefore$ Satisfies ACI CODE Requirement

$$M_n = A_s f_y (d - a/2) = (3)(60{,}000)(17 - \frac{5.29}{2}) = 2{,}583{,}529 \text{ in.-lb.}$$

$$\phi M_n = (0.9)(2{,}583{,}529) = 2{,}325{,}176 \text{ in-lb.}$$

<u>maximum applied moment</u>

$$M_u = \frac{w_u \ell^2}{8} = 2{,}325{,}176 \text{ in-lb}$$

$$w_u = \frac{(2{,}325{,}176)(8)}{(28 * 12)^2} = 191.1 \text{ lb/in} = 2293 \text{ lb/ft.}$$

$$w_u = 1.2\,DL + 1.6\,LL$$

$$DL = (150 \text{ lb/ft}^3)\frac{(10 * 20)}{144} = 208.3 \text{ lb/ft.}$$

$\therefore$

$$2293 = 1.2(208.3) + 1.6\,LL \quad \Rightarrow \quad LL = 1277 \text{ lb/ft.}$$

5.4. Design a one-way slab to carry a live load of 100 psf and an external dead load of 50 psf. The slab is simply supported over a span of 12 ft. Given:

f'_c = 4000 psi (27.6 MPa), normal-weight concrete
f_y = 60,000 psi (414 MPa)

Solution:

Design as a 1 ft. wide singly reinforced section.

Try, $\min h = \frac{L}{20} = \frac{(12)(12)}{20} = 7.2$ in.

Try $h = 8$ in, $d_t = 7$ in. ; $b = 12$ in.

$$\text{Self-weight} = (150)\frac{(8)(12)}{144} = 100 \text{ lb/ft.}$$

$$DL = \left(50 \frac{lb}{ft^2}\right)(1 \text{ ft}) = 50 \text{ lb/ft}$$

$$LL = \left(100 \frac{lb}{ft^2}\right)(1 \text{ ft}) = 100 \text{ lb/ft.}$$

$$\therefore \quad w_u = 1.2(100+50) + 1.6(100) = 340 \text{ lb/ft.}$$

$$M_u = \frac{w_u \ell^2}{8} = \frac{(340 \text{ lb/ft})(12 \text{ ft})^2}{8} = 6{,}120 \text{ ft-lb} = 73{,}440 \text{ in.-lb.}$$

Required nominal moment strength:

$$M_n = \frac{73{,}440}{0.90} = 81{,}600 \text{ in-lb.}$$

Assume $(d - a/2) \simeq 0.9\,d = 0.9(7) = 6.3$ in.

$$M_n = A_s f_y (d - a/2)$$

$$81{,}600 = A_s (60{,}000)(6.3)$$

$$A_s = 0.22 \ \text{in}^2/\text{12-in strip}$$

$$\rho_{min} = \max\left\{ \frac{3\sqrt{f'_c}}{f_y}, \frac{200}{f_y} \right\} = 0.0033 \qquad \therefore \min A_s = (0.0033)(12)(7) = 0.27 \ \text{in}^2/\text{12-in strip}$$

$\therefore$ try $A_s = 0.28 \ \text{in}^2/\text{12-in strip}$ (#4 bars at 8.5 in c-c)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(0.28)(60{,}000)}{0.85(4{,}000)(12)} = 0.41 \ \text{in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.41}{0.85} = 0.48 \ \text{in.}$$

$$\frac{c}{d_t} = \frac{0.48}{7} = 0.069 < 0.375 \qquad \therefore \text{Tension-controlled} \quad \phi = 0.90$$

Actual nominal moment strength.

$$M_n = A_s f_y (d - a/2) = (0.28)(60{,}000)\left(7 - \frac{0.41}{2}\right) = 114{,}156 \ \text{in-lb}$$

114,156 in-lb > Req'd 81,600 in-lb **O.K.**

Shrinkage and Temperature Reinforcement:

Req'd steel area $= 0.0018(12)(8) = 0.17 \ \text{in}^2/\text{12-in strip}$

maximum spacing $= \min\{5(8) = 40 \ \text{in}, 18 \ \text{in}\} = 18$ in.

$\therefore$ Use #4 bar @ 14 in. c-c ($A_s = 0.17 \ \text{in}^2/\text{12-in strip}$)

5.5. Design the simply supported beams shown in Fig. 5.36 as rectangular sections. Given:

$$f'_c = 5000 \text{ psi (34.5 MPa), normal-weight concrete}$$
$$f_y = 60{,}000 \text{ psi (414 MPa)}$$

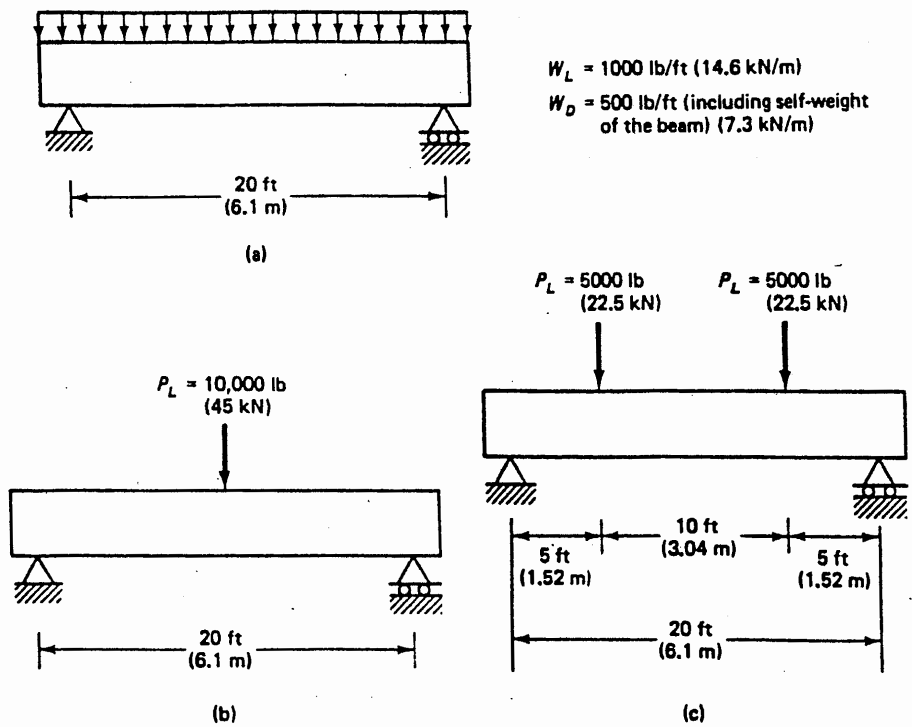


Figure 5.36

Solution:

a) $w_u = 1.2DL + 1.6LL = 1.2(500) + 1.6(1{,}000) = 2200$ lb/ft.

$$M_u = \frac{w_u l^2}{8} = \frac{(2200)(20)^2}{8} \times 12 = 1{,}320{,}000 \text{ in-lb}$$

Required nominal moment strength:

$$M_n = \frac{1{,}320{,}000}{0.90} = 1{,}466{,}667 \text{ in-lb}$$

$$\min h = \frac{l}{16} = \frac{20(12)}{16} = 15 \text{ in.}$$

Try $h = 18$ in $\quad b = 9$ in $\quad d = 14$ in.

Assume $c/d_t = 0.30$

$$c = 0.30(14) = 4.20 \text{ in.}$$
$$a = \beta_1 c = (0.80)(4.20) = 3.36 \text{ in.}$$

$$0.85 f'_c b a = A_s f_y \qquad A_s = \frac{0.85(5{,}000)(9)(3.36)}{60{,}000} = 2.14 \text{ in}^2$$

try $A_s = 2.37$ in^2 (3 #7 bars)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(2.37)(60{,}000)}{0.85(5{,}000)(9)} = 3.72 \text{ in.}$$

$$c = \frac{a}{\beta_1} = 4.65 \text{ in.} \qquad c/d_t = \frac{4.65}{14} = 0.33 < 0.375$$

$\therefore$ Tension-controlled

$\phi = 0.90$

$$\rho = \frac{A_s}{bd} = 0.019 \qquad \rho_{min} = \max\left\{\frac{3\sqrt{f'_c}}{f_y}, \frac{200}{f_y}\right\} = 0.0035$$

$0.019 > 0.0035 \quad \therefore$ Satisfies ACI CODE

$$M_n = A_s f_y (d - a/2) = (2.37)(60{,}000)\left(14 - \frac{3.72}{2}\right) = 1{,}726{,}475 \text{ in-lb.}$$

1,726,475 in-lb $>$ 1,466,667 in-lb $\qquad$ <u>O.K.</u>

b) Try $h = 15$ in. $d = 13$ in. $b = 8$ in.

$$\text{Self-weight} = \frac{(150)(8)(15)}{144} = 125 \text{ lb/ft.}$$

$$M_u \text{ due to dead load} = \frac{(1.2)(125)(20)^2}{8} = 7500 \text{ ft-lb} = 90{,}000 \text{ in-lb}$$

$$M_u \text{ due to live load} = \frac{(1.6)(10{,}000)(10)}{2} = 80{,}000 \text{ ft-lb} = 960{,}000 \text{ in-lb}$$

Required nominal moment strength:

$$M_n = \frac{90{,}000 + 960{,}000}{0.90} = 1{,}166{,}667 \text{ in-lb.}$$

Assume $c/d_t = 0.30$

$$c = (0.30)(13) = 3.90 \text{ in.}$$
$$a = \beta_1 c = (0.80)(3.90) = 3.12 \text{ in.}$$

$C = T$ $\quad 0.85 f_c' ba = A_s f_y \quad A_s = \frac{0.85(5{,}000)(8)(3.12)}{60{,}000} = 1.77 \text{ in}^2$

Try 2 No. 9 bars $A_s = 2.0 \text{ in}^2$

$$a = \frac{(2)(60{,}000)}{0.85(5000)(8)} = 3.53 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{3.53}{0.80} = 4.41 \text{ in.}$$

$\frac{c}{d_t} = \frac{4.41}{13} = 0.34 < 0.375$ ∴ Tension-Controlled

$\phi = 0.90$

$\rho = \frac{A_s}{bd} = \frac{2}{(8)(13)} = 0.019$

$\rho_{min} = \max\left\{\frac{3\sqrt{f'_c}}{f_y}, \frac{200}{f_y}\right\} = 0.0035$ $\quad$ 0.019 ∴ Satisfies ACI CODE Requirements

Actual moment strength:

$M_n = A_s f_y (d - a/2) = (2)(60{,}000)(13 - \frac{3.53}{2}) = 1{,}348{,}235$ in-lb

$1{,}348{,}235 > 1{,}666{,}667$ ∴ O.K.

c) Use same dimensions as part (b).

M_u due to live load $= (1.6)(5{,}000)(5) = 40{,}000$ ft-lb
$= 480{,}000$ in-lb

Required nominal moment strength:

$M_n = \frac{90{,}000 + 480{,}000}{0.90} = 633{,}333$ in-lb

$1{,}348{,}235 > 633{,}333$ ∴ O.K.

Use 2 #9 bars ($A_s = 2.0\ in^2$), h = 15 in d = 13 in b = 8 in.

Other combinations of height and A_s are possible. But if h < 15 in., a check on deflection will be required.

5.6. Check whether the sections shown in Fig. 5.37 satisfy ACI 318 Code requirements for maximum and minimum reinforcement. Given:

$$f'_c = 5000 \text{ psi (34.5 MPa), normal-weight concrete}$$
$$f_y = 60{,}000 \text{ psi (414 MPa)}$$

The compression fibers in all the figures are the top fibers of the sections.

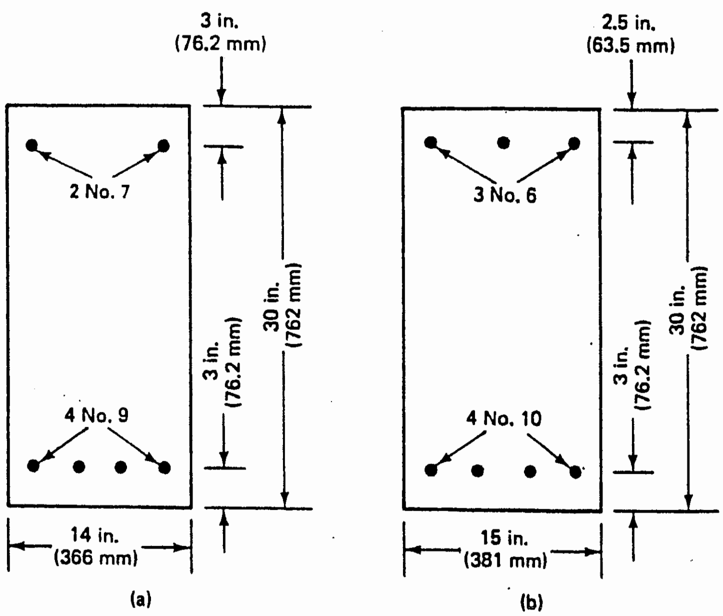


Figure 5.37

Solution:

a) Given

$b = 14$ in. $\quad A_s = 4 \text{ No.9} = 4.0 \text{ in}^2 \quad f'_c = 5{,}000$ psi

$h = 30$ in $\quad A'_s = 2 \text{ No.7} = 1.2 \text{ in}^2 \quad f_y = 60{,}000$ psi

$d_t = 27$ in $\quad \beta_1 = 0.80$

$d' = 3$ in.

Assume compression steel has yielded:

$$a = \frac{(A_s - A'_s) f_y}{0.85 f'_c b} = \frac{(4 - 1.2)(60{,}000)}{0.85(5{,}000)(14)} = 2.82 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{2.82}{0.80} = 3.53 \text{ in.}$$

$$\epsilon_s' = 0.003\left(\frac{c-d'}{c}\right) = 0.003\left(\frac{3.53-3}{3.53}\right) = 0.00045 \text{ in/in}$$

$$\epsilon_y = \frac{f_y}{29\times10^6} = \frac{60{,}000}{29\times10^6} = 0.002 \text{ in./in.} > 0.00045 \text{ in./in.}$$

∴ compression steel has not yielded and $f_s' < f_y$.

<u>First Trial</u>

$$f_s' = (0.00045)(29\times10^6) = 13{,}050 \text{ psi}$$

$$a = \frac{A_s f_y - A_s' f_s'}{0.85 f_c' b} = \frac{(4)(60{,}000) - (1.2)(13{,}050)}{0.85(5{,}000)(14)} = 3.77 \text{ inches}$$

$$c = \frac{a}{\beta_1} = \frac{3.77}{0.80} = 4.71 \text{ in.}$$

$$\epsilon_s' = 0.003\left(\frac{4.71-3}{4.71}\right) = 0.00109 \text{ in/in.}$$

<u>Second Trial</u>

$$f_s' = (0.00109)(29{,}000{,}000) = 31{,}622 \text{ psi}$$

$$a = \frac{(4)(60{,}000) - (1.2)(31{,}622)}{0.85(5{,}000)(14)} = 3.40 \text{ inches}$$

$$c = \frac{a}{\beta_1} = \frac{3.40}{0.80} = 4.24 \text{ in.}$$

$$\epsilon_s' = 0.003\left(\frac{4.24-3}{4.24}\right) = 0.00088 \text{ in/in.}$$

After several trials: $f_s' = 27{,}666$ psi $a = 3.48$ in. $c = 4.34$ in.

$$\epsilon_s' = 0.003\left(\frac{4.34-3}{4.34}\right) = 0.0009 \text{ in./in.}$$

$$\epsilon_t = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{27-4.34}{4.34}\right) = 0.0156 \text{ in./in.}$$

$0.0156 > 0.002$ in/in ∴ Tension steel has yielded

and

0.0156 in/in > 0.005 in/in ∴ Tension-controlled

$\phi = 0.90$

$$\rho = \frac{A_s}{bd} = \frac{4}{(14)(27)} = 0.011$$

$$\rho_{min} = \max\left\{\frac{3\sqrt{f'_c}}{f_y}, \frac{200}{f_y}\right\} = 0.0035 < 0.011$$ ∴ satisfies ACI CODE

b) Given

$b = 15$ in.

$h = 30$ in.

$d_t = 27$ in.

$d' = 2.5$ in.

$A_s = 4$ No. 10 $= 5.08$ in²

$A'_s = 3$ No. 6 $= 1.32$ in²

$f'_c = 5{,}000$ psi

$f_y = 60{,}000$ psi

$\beta_1 = 0.80$

Assume compression steel has yielded:

$$a = \frac{(5.08-1.32)}{0.85(5{,}000)(15)} = 3.54 \text{ in.}$$

$$c = \frac{3.54}{0.80} = 4.42 \text{ in.}$$

$$\epsilon'_s = 0.003\left(\frac{4.42-2.5}{4.42}\right) = 0.00130 \text{ in/in.} < 0.002 \text{ in./in.}$$

∴ Compression steel has not yielded

First Trial

$$f'_s = (0.00130)(29\times10^4) = 37{,}831 \text{ psi}$$

$$a = \frac{(5.08)(60{,}000) - (1.32)(37{,}831)}{0.85(5{,}000)(15)} = 4.0 \text{ in.}$$

$$c = \frac{4.0}{0.80} = 5.0 \text{ in.}$$

$$\epsilon_s' = 0.003\left(\frac{5.0 - 2.5}{5.0}\right) = 0.00150 \text{ in/in.}$$

Second Trial

$$f_s' = (0.0015)(29 \times 10^6) = 43{,}477 \text{ psi}$$

$$a = \frac{(5.08)(60{,}000) - (1.32)(43{,}477)}{0.85(5{,}000)(15)} = 3.88 \text{ in.}$$

$$c = \frac{3.88}{0.80} = 4.85 \text{ in.}$$

$$\epsilon_s' = 0.003\left(\frac{4.85 - 2.5}{4.85}\right) = 0.00145 \text{ in/in.}$$

After several trials: $f_s' = 42{,}477$ psi $\quad a = 3.90$ in. $\quad c = 4.88$ in.

$$\epsilon_s' = 0.003\left(\frac{4.88 - 2.5}{4.88}\right) = 0.0015 \text{ in/in.}$$

$$\epsilon_t = 0.003\left(\frac{27 - 4.88}{4.88}\right) = 0.0136 \text{ in/in} > 0.002 \text{ in/in}$$

$\therefore$ Tension steel has yielded

0.0136 in/in > 0.005 in/in $\therefore$ Tension-controlled

$$\rho = \frac{5.08}{(15)(27)} = 0.013 > 0.0035$$ $\therefore$ Satisfies ACI CODE

5.7. Compute the stresses in the compression steel, f'_s, for the cross sections shown in Fig. 5.38. Also compute the nominal moment strength for the section in part (b). Given:

f'_c = 6000 psi (41.4 MPa), normal-weight concrete
f_y = 60,000 psi (414 MPa)

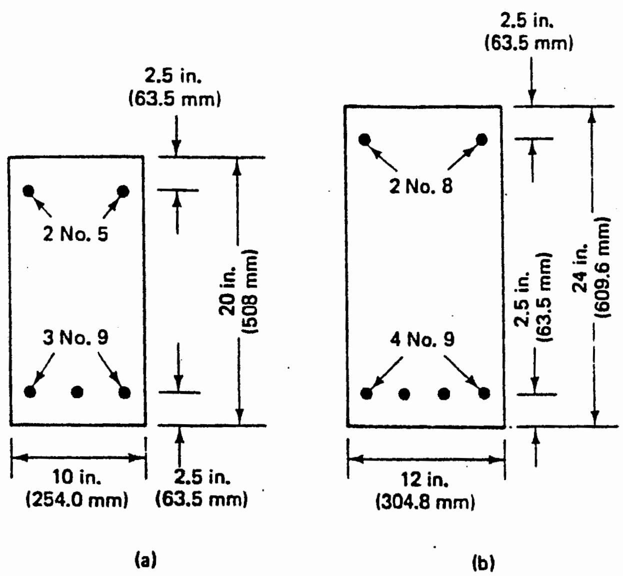


Figure 5.38

Solution :

a) Given

b= 10 in. A_s= 3 No.9= 3.0 in² f'_c= 6,000 psi
h= 20 in. A_s'= 2 No.5 = 0.62 in² f_y= 60,000 psi
d_t= 17.5 in. β_1= 0.75
d'= 2.5 in.

- Assume compression steel has yielded.

$$a = \frac{(3.0-0.62)(60,000)}{0.85(6,000)(10)} = 2.80 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{2.80}{0.75} = 3.73 \text{ in.}$$

$$\epsilon_s' = 0.003\left(\frac{c-d'}{c}\right) = 0.003\left(\frac{3.73-2.5}{3.73}\right) = 0.00099 \text{ in/in.}$$

$$\epsilon_y = \frac{f_y}{E_s} = \frac{60{,}000}{29\times10^6} = 0.002 \text{ in/in.} > 0.00099 \text{ in/in.}$$

∴ Compression steel has not yielded.

First Trial

$$f_s' = \epsilon_s' E_s = (0.00099)(29\times10^6) = 28{,}741 \text{ psi}$$

$$a = \frac{A_s f_y - A_s' f_s'}{0.85 f_c' b} = \frac{(3)(60{,}000) - (0.62)(28{,}741)}{0.85(6{,}000)(10)} = 3.18 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{3.18}{0.75} = 4.24 \text{ in.}$$

$$\epsilon_s' = 0.003\left(\frac{4.24-2.5}{4.24}\right) = 0.00123 \text{ in/in.}$$

Second Trial

$$f_s' = (0.00123)(29\times10^6) = 35{,}703 \text{ psi}$$

$$a = \frac{(3)(60{,}000) - (0.62)(35{,}703)}{0.85(6{,}000)(10)} = 3.10 \text{ in.}$$

$$c = \frac{3.10}{0.75} = 4.13 \text{ in.}$$

$$\epsilon_s' = 0.003\left(\frac{4.13-2.5}{4.13}\right) = 0.00118 \text{ in/in.}$$

After several trials: $f_s' = 34{,}589$ psi $a = 3.11$ in. $c = 4.15$ in.

$$\epsilon_s' = 0.003\left(\frac{4.15-2.5}{4.15}\right) = 0.0012 \text{ in/in.}$$

b) Given

$b = 12$ in, $h = 24$ in, $d_t = 21.5$ in, $d' = 2.5$ in.

$A_s = 4$ No.9 $= 4.0\ in^2$, $A_s' = 2$ No.8 $= 1.58\ in^2$

$f_c' = 6{,}000$ psi, $f_y = 60{,}000$ psi, $\beta_1 = 0.75$

assume compression steel has yielded.

$$a = \frac{(4-1.58)(60{,}000)}{0.85(6{,}000)(12)} = 2.37 \text{ in.}$$

$$c = \frac{a}{0.75} = \frac{2.37}{0.75} = 3.16 \text{ in.}$$

$$\epsilon_s' = 0.003\left(\frac{3.16-2.5}{3.16}\right) = 0.00063 \text{ in/in.} < 0.002 \text{ in/in.}$$

∴ compression steel has not yielded.

First Trial

$$f_s' = \epsilon_s' E_s = (0.00063)(29 \times 10^6) = 18{,}245 \text{ psi}$$

$$a = \frac{(4)(60{,}000) - (1.58)(18{,}245)}{0.85(6{,}000)(12)} = 3.45 \text{ in.}$$

$$c = \frac{3.45}{0.75} = 4.60 \text{ in.}$$

$$\epsilon_s' = 0.003\left(\frac{4.60-2.5}{4.60}\right) = 0.00137 \text{ in/in.}$$

Second Trial

$$f_s' = (0.00137)(29 \times 10^6) = 39{,}725 \text{ psi}$$

$$a = \frac{(4)(60{,}000) - (1.58)(39{,}725)}{0.85(6{,}000)(12)} = 2.90 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{2.90}{0.75} = 3.86 \text{ in.}$$

$$\varepsilon_s' = 0.003\left(\frac{3.86-2.5}{3.86}\right) = 0.00106 \text{ in/in.}$$

After several trials: $f_s' = 34{,}878$ psi $a = 3.02$ in $c = 4.03$ in.

$$\varepsilon_s' = 0.003\left(\frac{4.03-2.5}{4.03}\right) = 0.0011 \text{ in/in.}$$

$$\varepsilon_t = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{21.5-4.03}{4.03}\right) = 0.0130 \text{ in/in} > 0.002 \text{ in/in}$$

∴ Tension steel has yielded.

0.0130 in/in > 0.005 in/in ∴ Tension-controlled

$\phi = 0.90$

$$\rho = \frac{A_s}{bd} = \frac{4}{(12)(21.5)} = 0.016$$

$$\rho_{min} = \max\left\{\frac{3\sqrt{f_c'}}{f_y}, \frac{200}{f_y}\right\} = 0.0039 < 0.016$$

∴ satisfies ACI Code

$$M_n = (A_s f_y - A_s' f_s')(d - a/2) + A_s' f_s' (d-d')$$
$$= [(4)(60{,}000) - (1.58)(34{,}878)]\left(21.5 - \frac{3.02}{2}\right) + (1.58)(34{,}878)(21.5-2.5)$$

$$M_n = 4{,}742{,}940 \text{ in-lb}$$

$$\phi M_n = (0.90)(4{,}742{,}940) = 4{,}268{,}646 \text{ in-lb.}$$

5.8. Calculate the ultimate moment capacity of the beam sections of Problem 5.2. Assume two No. 6 bars for compression reinforcement.

Solution:

a) $b = 10$ in. $A_s = 3\,No.9 = 3\ in^2$ $f'_c = 5{,}000$ psi

$h = 19$ in. $A'_s = 2\,No.6 = 0.88\,in^2$ $f_y = 60{,}000$ psi

$d_t = 16$ in $\beta_1 = 0.80$

$d' = 2.5$ in

assume compression steel has yielded

$$a = \frac{(A_s - A'_s) f_y}{0.85 f'_c b} = \frac{(3-0.88)(60{,}000)}{0.85(5{,}000)(10)} = 2.99 \text{ in}$$

$$c = a/\beta_1 = 2.99/0.80 = 3.74 \text{ in.}$$

$$\epsilon'_s = 0.003\left(\frac{c-d'}{c}\right) = 0.003\left(\frac{3.74-2.5}{3.74}\right) = 0.0010 \text{ in./in.}$$

$$\epsilon_y = \frac{f_y}{E_s} = \frac{60{,}000}{29 \times 10^6} = 0.002 \text{ in/in} > 0.0010$$ ∴ compression steel has not yielded.

First Trial

$$f'_s = \epsilon'_s E_s = (0.0010)(29 \times 10^6) = 28{,}863 \text{ psi}$$

$$a = \frac{A_s f_y - A'_s f'_s}{0.85 f'_c b} = 3.64 \text{ in.}$$

$$c = a/\beta_1 = 4.55 \text{ in.}$$

$$\epsilon'_s = 0.003\left(\frac{c-d'}{c}\right) = 0.00135 \text{ in/in.}$$

Second Trial

$$f_s' = \epsilon_s' E_s = (0.00135)(29\times10^6) = 39{,}167 \text{ psi}$$

$a = 3.42$ in. $\quad c = 4.28$ in $\quad \epsilon_s' = 0.00125$ in./in.

after several trials: $f_s' = 37{,}086$ psi $\quad a = 3.47$ in $\quad c = 4.33$ in.

$$\epsilon_s' = 0.003\left(\frac{4.33-2.5}{4.33}\right) = 0.0013$$

$$\epsilon_t = 0.003\left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{16-4.33}{4.33}\right) = 0.0081 \text{ in/in} > 0.005 \text{ in/in}$$

$\therefore$ Tension Controlled

$\phi = 0.90$

$$M_n = (A_s f_y - A_s' f_s')(d - a/2) + A_s' f_s'(d-d')$$

$$= [(3)(60{,}000) - (0.88)(37{,}086)]\left(16 - \frac{3.47}{2}\right) + (0.88)(37{,}086)(16-2.5)$$

$$= 2{,}542{,}926 \text{ in-lb}$$

$$M_u = \phi M_n = 0.90 M_n = 2{,}288{,}633 \text{ in-lb}$$

b) $b = 10$ in. (assuming neutral axis is w/in top 8in.)

$d_t = 13.5$ in

$d' = 2.5$ in

$A_s = 5$ No.7 $= 3$ in^2

$A_s' = 2$ No.6 $= 0.88$ in^2

$f_c' = 6{,}000$ psi

$f_y = 60{,}000$ psi

$\beta_1 = 0.75$

assume compression steel has yielded.

$$a = \frac{(A_s - A_s')f_y}{0.85 f_c' b} = 2.49 \text{ in.}$$

$$c = a/\beta_1 = 3.33 \text{ in.}$$

$$\epsilon_s' = 0.003\left(\frac{c-d'}{c}\right) = 0.00074 \text{ in/in} < 0.002 \text{ in/in}$$

∴ Compression steel has not yielded.

<u>First</u> <u>Trial</u>

$$f_s' = \epsilon_s' E_s = (0.00074)(29\times10^6) = 21{,}596 \text{ psi}$$

$$a = \frac{(A_s f_y - A_s' f_s')}{0.85 f_c' b} = 3.16 \text{ in.}$$

$$c = a/\beta_1 = 4.21 \text{ in.}$$

$$\epsilon_s' = 0.003\left(\frac{c-d'}{c}\right) = 0.00122 \text{ in/in.}$$

<u>Second</u> <u>Trial</u>

$$f_s' = \epsilon_s' E_s = 35{,}325 \text{ psi}$$

$a = 2.92$ in. $\quad c = 3.89$ in. $\quad \epsilon_s' = 0.00107$ in/in.

After several trials: $f_s' = 32{,}484$ psi $\quad a = 2.97$ in. $\quad c = 3.96$ in

$$\epsilon_t = 0.003\left(\frac{d-c}{c}\right) = 0.0072 \text{ in/in} > 0.005 \text{ in/in}$$

∴ Tension-controlled

$\phi = 0.90$

note: $c = 3.96$ in < 8 in.

∴ b = 10 in. <u>ok</u>

$$M_n = (A_s f_y - A_s' f_s')(d - a/2) + A_s' f_s' (d - d')$$

$$= [(3)(60,000) - (0.88)(32,484)]\left(13.5 - \frac{2.97}{2}\right)$$

$$+ (0.88)(32,484)(13.5 - 2.5)$$

$$= 2,133,768 \text{ in-lb}$$

$$M_u = \phi M_n = 1,920,391 \text{ in-lb}$$

5.9. Solve Problem 5.3 if two No. 6 bars are added as compression reinforcement.

Solution :

Given

$b = 10$ in. $\quad A_s = 3$ No. 9 $= 3.0$ in^2 $\quad f'_c = 4{,}000$ psi

$h = 20$ in. $\quad A'_s = 2$ No. 6 $= 0.88$ in^2 $\quad f_y = 60{,}000$ psi

$d_t = 17$ in. $\quad \beta_1 = 0.85$

$d' = 2.5$ in.

assume compression steel has yielded :

$$a = \frac{(A_s - A'_s) f_y}{0.85 f'_c b} = \frac{(3-0.88)(60{,}000)}{0.85(4{,}000)(10)} = 3.74 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{3.74}{0.85} = 4.40 \text{ in.}$$

$$\epsilon'_s = 0.003\left(\frac{c-d'}{c}\right) = 0.003\left(\frac{4.40-2.5}{4.40}\right) = 0.00130 \text{ in/in.} < 0.002 \text{ in/in.}$$

$\therefore$ Compression steel has not yielded.

First Trial

$$f'_s = \epsilon'_s E_s = (0.0013)(29 \times 10^6) = 37{,}584 \text{ psi}$$

$$a = \frac{(3)(60{,}000) - (0.88)(37{,}584)}{0.85(4{,}000)(10)} = 4.32 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{4.32}{0.85} = 5.08 \text{ in.}$$

$$\epsilon'_s = 0.003\left(\frac{5.08-2.5}{5.08}\right) = 0.00152 \text{ in/in.}$$

Second Trial

$$f_s' = (0.00152)(29 \times 10^6) = 44,218 \text{ psi}$$

$$a = \frac{(3)(60,000) - (0.88)(44,218)}{0.85(4,000)(10)} = 4.15 \text{ in.}$$

$$c = \frac{4.15}{0.85} = 4.88 \text{ in.}$$

$$\epsilon_s' = 0.003\left(\frac{4.88 - 2.5}{4.88}\right) = 0.00146 \text{ in/in.}$$

After several trials: $f_s' = 42,935$ psi $a = 4.18$ in. $c = 4.92$ in.

$$\epsilon_s' = 0.003\left(\frac{4.92 - 2.5}{4.92}\right) = 0.0015 \text{ in/in.}$$

$$\epsilon_t = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{17 - 4.92}{4.92}\right) = 0.0074 \text{ in/in.} > 0.002 \text{ in/in}$$

∴ Tension steel has yielded

0.0074 > 0.005 ∴ Tension-controlled

$\phi = 0.90$

$$\rho = \frac{A_s}{bd} = \frac{3}{(10)(17)} = 0.018$$

$$\rho_{min} = \max\left\{\frac{3\sqrt{f_c'}}{f_y}, \frac{200}{f_y}\right\} = 0.0033 < 0.018$$ ∴ Satisfies ACI code

$$M_n = (A_s f_y - A_s' f_s')(d - a/2) + A_s' f_s'(d - d')$$
$$= [(3)(60,000) - (0.88)(42,935)]\left(17 - \frac{4.18}{2}\right) + (0.88)(42,935)(17 - 2.5)$$

$$M_n = 2,668,105 \text{ in-lb.}$$

$$M_u = \frac{M_n}{\phi = 0.90} = 2,401,295 \text{ in-lb.} = \frac{w_u l^2}{8}$$

$$w_u = \frac{(2,401,295)(8)}{(26 * 12)^2} = 197.3 \text{ lb/in}$$
$$= 2368 \text{ lb/ft.}$$

From Problem 5.3, Self-weight = 208.3 lb/ft.

$w_u = 1.2\,DL + 1.6\,LL$

$2368 = 1.2\,(208.3) + 1.6\,LL$

$LL = 1323$ lb/ft.

5.10. At failure, determine whether the precast sections shown in Fig. 5.39 will act similarly to rectangular sections or as flanged sections. Given:

f'_c = 4000 psi (27.6 MPa), normal-weight concrete
f_y = 60,000 psi (414 MPa)

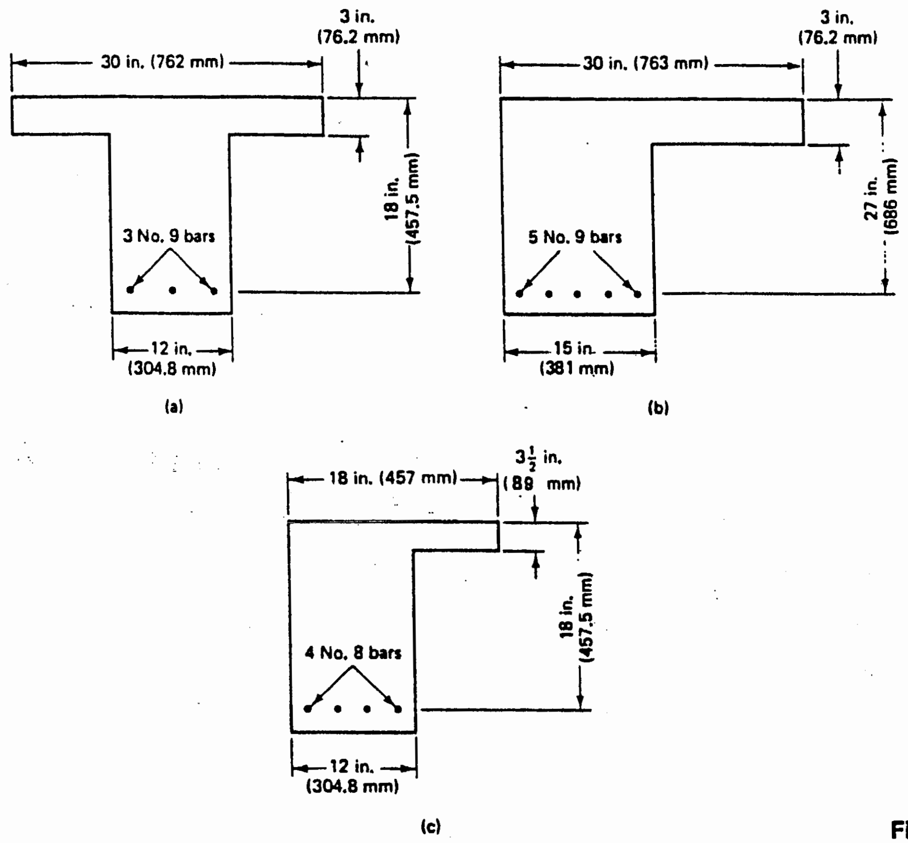


Figure 5.39

Solution :

a) Given

b_w = 12 in.
b = 30 in.
h_f = 3 in.
d_t = 18 in.

A_s = 3(1.0) = 3.0 in^2
f'_c = 4,000 psi
f_y = 60,000 psi
β_1 = 0.85

- assume as a rectangular section:

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3)(60{,}000)}{(0.85)(4{,}000)(30)} = 1.76 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{1.76}{0.85} = 2.08 \text{ in.} < h_f = 3 \text{ in}$$

∴ Treat as rectangular section.

b) <u>Given</u>

$b_w = 15$ in

$b = 30$ in

$h_f = 3.0$ in

$d_t = 27$ in

$A_s = 5\#9.9 = 5.0 \text{ in}^2$

$f'_c = 4{,}000$ psi

$f_y = 60{,}000$ psi

$\beta_1 = 0.85$

assume as a rectangular section:

$$a = \frac{(5)(60{,}000)}{0.85(4{,}000)(30)} = 2.94 \text{ in.} < h_f = 3 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{2.94}{0.85} = 3.46 \text{ in.} > h_f = 3 \text{ in}$$

∴ Could Treat as rectangular section or as a L-section

c) <u>Given</u>

$b_w = 12$ in

$b = 18$ in

$h_f = 3.5$ in

$d_t = 18$ in

$A_s = 4\#0.8 = 3.16 \text{ in}^2$

$f'_c = 4{,}000$ psi

$f_y = 60{,}000$ psi

$\beta_1 = 0.85$

assume as a rectangular section:

$$a = \frac{(3.16)(60{,}000)}{0.85(4{,}000)(18)} = 3.10 \text{ in} < h_f = 3.5 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{3.10}{0.85} = 3.64 \text{ in} > h_f = 3.5 \text{ in}$$

∴ Could treat as rectangular section or as a L-section.

5.11. Check whether the sections of Problem 5.10 satisfy ACI Code requirements.

a) From Problem 5.10; treat beam as a rectangular section.

$$a = \frac{(3)(60{,}000)}{0.85(4{,}000)(30)} = 1.76 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{1.76}{0.85} = 2.08 \text{ in.}$$

$$\frac{c}{d_t} = \frac{2.08}{18} = 0.1155 < 0.375$$ ∴ Tension-controlled $\phi = 0.90$

$$\rho = \frac{A_s}{bd} = \frac{3}{(30)(18)} = 0.0056$$

$$\rho_{min} = \max\left\{\frac{3\sqrt{f'_c}}{f_y}, \frac{200}{f_y}\right\} = 0.003 < 0.0056$$ ∴ satisfies ACI code

b) treat as a rectangular section.

$a = 2.94$ in. and $c = 3.46$ in (From Problem 5.10)

$$\frac{c}{d_t} = \frac{3.46}{27} = 0.128 < 0.375$$ ∴ Tension-controlled $\phi = 0.90$

$$\rho = \frac{A_s}{bd} = \frac{5}{(30)(27)} = 0.0062 > 0.003$$ ∴ Satisfies ACI code

c) treat as a rectangular section; $a = 3.10$ in $c = 3.64$ in.

$$\frac{c}{d_t} = \frac{3.64}{18} = 0.202 < 0.375$$ ∴ Tension-controlled $\phi = 0.90$

$$\rho = \frac{A_s}{bd} = \frac{3.16}{(18)(18)} = 0.0098 > 0.003$$ ∴ Satisfies ACI Code

5.12. Calculate the nominal moment strength of the sections shown for Problem 5.10:

Solution :

a) From Problem 5.11 ; $a = 1.76$ in., $A_s = 3.0$ in^2 , $d_t = 18$ in.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (3)(60{,}000)\left(18 - \frac{1.76}{2}\right) = 3{,}081{,}176 \text{ in-lb.}$$

$$M_u = \frac{M_n}{\phi = 0.90} = 2{,}773{,}059 \text{ in-lb.}$$

b) $a = 2.94$ in , $A_s = 5.0$ in^2 , $d_t = 27$ in.

$$M_n = (5)(60{,}000)\left(27 - \frac{2.94}{2}\right) = 7{,}658{,}824 \text{ in-lb.}$$

$$M_u = \frac{M_n}{\phi = 0.90} = 6{,}892{,}941 \text{ in-lb.}$$

c) $a = 3.10$ in , $A_s = 3.16$ in^2 , $d_t = 18$ in.

$$M_n = (3.16)(60{,}000)\left(18 - \frac{3.10}{2}\right) = 3{,}119{,}106 \text{ in-lb.}$$

$$M_u = \frac{M_n}{\phi = 0.90} = 2{,}807{,}195 \text{ in-lb.}$$

5.13. Repeat Problem 5.5 using a T-section instead of a rectangular section. Use a flange thickness of 3 in. (76.2 mm) and a flange width of 30 in. (762 mm).

Solution:

a) From Problem 5.5a ; Required $M_n = 1{,}466{,}667$ in-lb.

$$\min h = \frac{\ell}{16} = \frac{20(12)}{16} = 15 \text{ in.}$$

Try $h = 15$ in $\quad d_t = 13$ in $\quad b_w = 12$ in $\quad b = 30$ in $\quad h_f = 3.0$ in.

assume as a rectangular section.
assume $(d - a/2) = jd = 0.9\,d \qquad jd \approx 0.9\,(13) = 11.7$ in.

$$M_n = A_s f_y (d - a/2) \qquad 1{,}466{,}667 = A_s (60{,}000)(11.7)$$
$$A_s = 2.09 \text{ in}^2$$

Try 3 No. 8 bars $A_s = 2.37$ in^2 (diameter = 1.0 in)

actual $d_t = 15 - 2.0 - \tfrac{1}{2}(1.0) = 12.5$ in.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(2.37)(60{,}000)}{0.85(5{,}000)(30)} = 1.12 \text{ in} < 3.0 \text{ in}$$ ∴ treat as a rectangular section

$$\rho = \frac{A_s}{bd} = \frac{2.37}{(30)(12.5)} = 0.0063$$

$$\rho_{min} = \max\left\{\frac{3\sqrt{f'_c}}{f_y}, \frac{200}{f_y}\right\} = 0.0035 < 0.0063$$ ∴ satisfies ACI code

$$c = \frac{a}{\beta_1} = \frac{1.12}{0.80} = 1.39 \text{ in} \quad ; \quad \frac{c}{d_t} = \frac{1.39}{12.5} = 0.1112 < 0.375$$ ∴ Tension-Controlled

$\phi = 0.90$

Actual $M_n = A_s f_y (d - a/2) = (2.37)(60,000)(12.5 - \frac{1.12}{2})$

$= 1,698,203 \text{ in-lb} > 1,466,667 \text{ in-lb.}$ O.K.

b) From Problem 5.5b,

Try $h = 15$ in. $d = 13$ in. $b_w = 8$ in $b = 30$ in $h_f = 3.0$ in.

Required nominal moment strength $= 1,166,667$ in-lb.

Assume as a rectangular section.
Assume $jd \approx 0.9d = 0.9(13) = 11.7$ in.

$M_n = A_s f_y jd \qquad A_s = \frac{1,166,667}{(60,000)(11.7)} = 1.66 \text{ in}^2$

Try 2 No. 9 bars ($A_s = 2.0$ in^2) ; diameter = 1.1 inch

actual $d_t = 15 - 2 - \frac{1}{2}(1.1) \approx 12.5$ in.

$a = \frac{(2)(60,000)}{0.85(5,000)(30)} = 0.94 \text{ in.} < h_f = 3.0 \text{ in}$ ∴ Treat as rectangular section.

$c = \frac{0.94}{0.80} = 1.18$ in ; $\frac{c}{d_t} = \frac{1.18}{12.5} = 0.094 < 0.375$ ∴ Tension-Controlled $\phi = 0.90$

$\rho = \frac{A_s}{bd} = 0.0053 > 0.003$ ∴ Satisfies ACI Code

Actual $M_n = (2)(60,000)(12.5 - \frac{0.94}{2}) = 1,443,600$ in-lb.
$> 1,166,667$ in-lb. O.K.

c) From Problem 5.5C,

Try $h = 15$ in $d = 13$ in $b_w = 8$ in $b = 30$ in $h_f = 3.0$ in.

Required $M_n = 633{,}333$ in-lb

assume as a rectangular section.
assume $jd \approx 0.9d = 0.9(13) = 11.7$ in.

$$A_s = \frac{633{,}333}{(60{,}000)(11.7)} = 0.91 \text{ in}^2$$

Try 2 No. 9 bars – $A_s = 2.0$ in²

Same as part b; treat as rectangular section with

$M_n = 1{,}443{,}600$ in-lb $>$ $633{,}333$ in-lb. O.K.

5.14. Using the details of Problem 5.4, design a reinforced concrete T-beam for the slab floor system shown in Fig. 5.40. The floor area is 30 ft × 60 ft (9.14 m × 18.29 m) with an effective T-beam span of 30 ft (9.14 m).

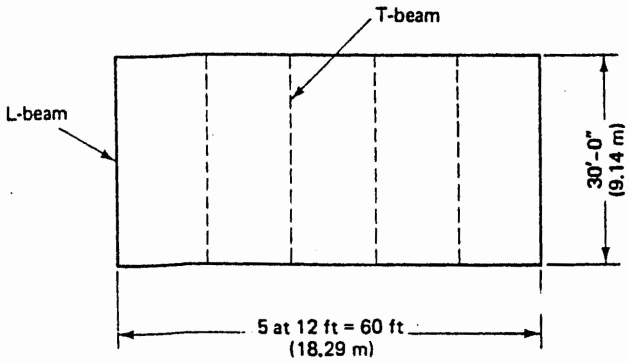


Figure 5.40 Plan of one-way slab floor system.

Solution:

LL = 100 psf   DL = 50 psf

Slab Design:

$\ell = 12$ ft $\ell_n = 11$ ft $= 132$ in.

$$h_{min} = \frac{\ell_n}{28} = \frac{11(12)}{28} = 4.71 \text{ in.}$$

Try h = 5 in, d = 4 in, b = 12 in.

$$\text{Self-weight of slab} = \frac{150(5)(12)}{144} = 62.5 \text{ lb/ft}$$

$$w_u = 1.2(62.5 + 50) + 1.6(100) = 295 \text{ lb/ft}$$

$$-M_u = \frac{w_u l_n^2}{11} = \frac{(295)(11)^2}{11} \times 12 = 38{,}940 \text{ in-lb}$$

$$\text{Required } -M_n = \frac{M_u}{\phi = 0.90} = 43{,}267 \text{ in-lb}$$

$$M_n = A_s f_y (d - a/2) \qquad \text{Assume } (d - a/2) \approx 0.9d$$

$$A_s = \frac{43{,}267}{60{,}000\,(0.9)(4)} = 0.20 \text{ in}^2/12\text{-in strip}$$

$$\text{minimum reinforcement ratio} = \max\left\{\frac{6\sqrt{f_c'}}{f_y}, \frac{200}{f_y}\right\} = 0.0033$$

$$\text{minimum } A_s = 0.0033(12)(4) = 0.16 \text{ in}^2/12\text{-in strip}$$

∴ Try #4 bars at 12-in c-c $\quad A_s = 0.20 \text{ in}^2$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{(0.20)(60{,}000)}{0.85(4{,}000)(12)} = 0.29 \text{ in.}$$

$$c = a/\beta_1 = 0.29/0.85 = 0.35 \text{ in.}$$

$$\epsilon_t = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{4-0.35}{0.35}\right) = 0.03168 \text{ in/in} > 0.005 \text{ in/in}$$

∴ Tension-controlled
$\phi = 0.90$

$$M_n = (0.20)(60{,}000)\left(4 - \frac{0.29}{2}\right) = 46{,}235 \text{ in-lb} > 43{,}267 \text{ in-lb} \quad \underline{\underline{\text{O.K}}}$$

$$+M_u = \frac{w_u l_n^2}{16} = \frac{(295)(11)^2}{16} \times 12 = 26{,}771 \text{ in-lb}$$

$$\text{Required } +M_n = \frac{M_u}{\phi = 0.90} = 29{,}746 \text{ in-lb}$$

$$A_s = \frac{29{,}746}{60{,}000\,(0.9)(4)} = 0.14 \text{ in}^2 / \text{12-in strip}$$

minimum $A_s = 0.16 \text{ in}^2/\text{12-in strip}$

Try #3 bar at 8.5 in c-c $\quad A_s = 0.16 \text{ in}^2$

$$a = \frac{(0.16)(60{,}000)}{0.85(4000)(12)} = 0.24 \text{ in}$$

$$c = 0.24/0.85 = 0.28 \text{ in}$$

$$\epsilon_t = 0.003\left(\frac{4-0.28}{0.28}\right) = 0.04035 \text{ in/in} > 0.005 \text{ in/in}$$

∴ Tension- controlled

$\phi = 0.90$

$$M_n = (0.16)(60{,}000)\left(4 - \frac{0.24}{2}\right) = 37{,}271 \text{ in-lb} >$$

29,746 in-lb **OK**

Temperature and Shrinkage reinforcement:

$A_s = 0.0018 bh = 0.11 \text{ in}^2$

maximum spacing = $5h$ = 25 in

Use No. 3 bars at 12 in c-c

Hence, No. 4 bars at 12-in c-c for $-M_n$
No. 3 bars at 8.5-in c-c for $+M_n$
No. 3 bars at 12-in c-c for temp.

Beam Design:

$L = 30$ ft $\quad l_n = 11$ ft

$$\text{minimum } h = \frac{l_n}{16} = \frac{(29)(12)}{16} = 21.75 \text{ in.}$$

Try $h = 22$ in $\quad d = 19.5$ in. $\quad b_w = 10$ in.

$$b = \min\{16h_f + b_w,\ l_n + b_w,\ 1/4\, L\}$$
$$= \min\{16(5)+10 = 90,\ 11(12)+10 = 142,\ 1/4(30)(12) = 90\}$$
$$= 90 \text{ inches}$$

$$\text{Beam self-weight} = 150\,\frac{(22-5)(10)}{144} = 177.1 \text{ lb/ft}$$

Slab self-weight = 62.5 lb/ft

$$w_u = 1.2(62.5 + 177.1) + 1.2(50)(12) + 1.6(100)(12) = 2928 \text{ lb/ft}$$

$$M_u = \frac{(2928)(30)^2}{8} \times 12 = 3{,}952{,}125 \text{ in-lb}$$

Required $M_n = \frac{M_u}{\phi = 0.90} = 4{,}391{,}250$ in-lb

$$A_s = \frac{4{,}391{,}250}{60{,}000\,(0.9)(19.5)} = 4.17 \text{ in}^2$$

Try 6 No. 8 bars, $A_s = 4.74 \text{ in}^2$

actual $d_t = 22 - 1.5 - 0.5 - 1 = 19$ in.

<u>assume $a < h_f$</u>:

$$a = \frac{4.74\,(60{,}000)}{0.85\,(4{,}000)(\ \ )} = 0.93 \text{ in} < 5 \text{in}$$

∴ consider as a Rectangular section

$$\rho = \frac{A_s}{bd} = 0.025$$

$$\min \rho = \max\left\{\frac{3\sqrt{f_c'}}{f_y}, \frac{200}{f_y}\right\} = 0.0033 < 0.025 \quad \underline{\underline{\text{O.K}}}$$

$$c = \frac{a}{\beta_1} = 1.09 \text{ in}$$

$$\frac{c}{d_t} = \frac{1.09}{19} = 0.057 < 0.375$$ ∴ Tension-controlled

<u>O.K</u>

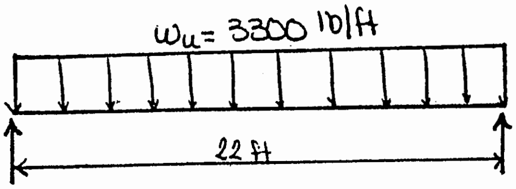
$$M_n = A_s f_y (d - a/2) = (4.74)(60{,}000)\left(19 - \frac{0.93}{2}\right)$$

$$= 5{,}271{,}438 \text{ in-lb} > 4{,}391{,}250 \text{ in-lb} \quad \underline{\underline{\text{O.K}}}$$

6.1. A simply supported beam has a clear span $l_n = 22$ ft (6.70 m) and is subjected to an external uniform service dead load $W_D = 900$ lb per ft (17.5 kN/m) and live load $w_L = 1200$ lb per ft (13.1 kN/m). Determine the maximum factored vertical shear V_u at the critical section. Also determine the nominal shear resistance V_n by both the short method and by the more refined method of taking the contribution of the flexural steel into account. Design the size and spacing of the diagonal tension reinforcement. Given:

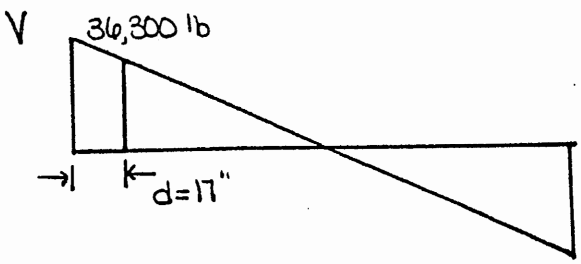
b_w = 12 in. (305 mm)
d = 17 in. (432 mm)
h = 20 in. (508 mm)
A_s = 6.0 in.^{2} (3780 mm^2)
f'_c = 4000 psi (27.6 MPa), normal-weight concrete
f_y = 60.000 psi (413.7 MPa)

Assume that no torsion exists.



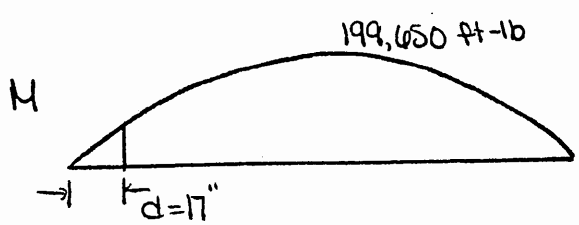
$$\text{Self-weight} = \frac{12 \times 20}{144} \times 150 = 250 \text{ lb/ft}$$

$$w_u = 1.2(250 + 900) + 1.6(1200)$$
$$= 3300 \text{ lb/ft}$$



$$V_u \text{ (at support)} = \frac{(3300)(22)}{2}$$
$$= 36{,}300 \text{ lb.}$$

$$V_u \text{ (at d)} = 36{,}300 - 3300\,(17/12)$$
$$= 31{,}625 \text{ lb.}$$



$$M_u \text{ (at d)} = 36{,}300\,(17/12) - \frac{3300(17/12)^2}{2}$$

$$= 48{,}113 \text{ ft-lb}$$
$$= 577{,}363 \text{ in-lb.}$$