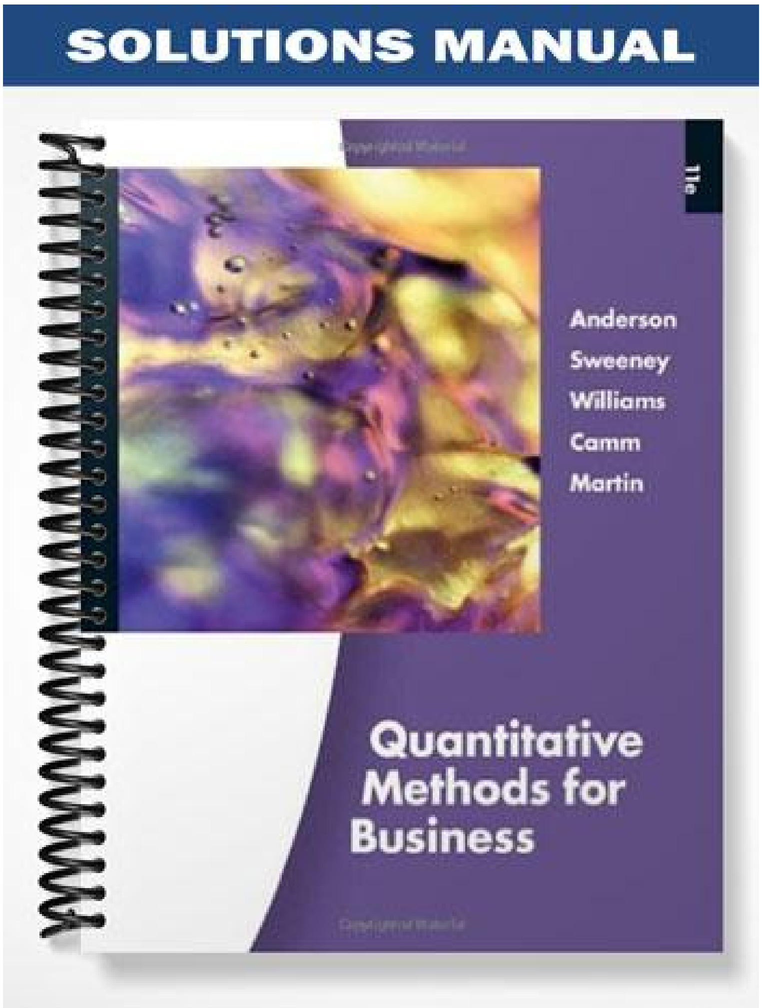
SOLUTIONS MANUAL
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Anderson
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Martin
Quantitative
Methods for
Business

Chapter 2
Introduction to Probability

Learning Objectives

1. Obtain an understanding of the role probability information plays in the decision making process.

2. Understand probability as a numerical measure of the likelihood of occurrence.

3. Be able to use the three methods (classical, relative frequency, and subjective) commonly used for assigning probabilities and understand when they should be used.

4. Be able to use the addition law and be able to compute the probabilities of events using conditional probability and the multiplication law.

5. Be able to use new information to revise initial (prior) probability estimates using Bayes' theorem.

6. Know the definition of the following terms:

experiment	addition law
sample space	mutually exclusive
event	conditional probability
complement	independent events
Venn Diagram	multiplication law
union of events	prior probability
intersection of events	posterior probability
Bayes' theorem	

Solutions:

1. a. Go to the x-ray department at 9:00 a.m. and record the number of persons waiting.

   b. The experimental outcomes (sample points) are the number of people waiting: 0, 1, 2, 3, and 4.

   Note: While it is theoretically possible for more than 4 people to be waiting, we use what has actually been observed to define the experimental outcomes.

   c.

Number Waiting	Probability
0	.10
1	.25
2	.30
3	.20
4	.15
Total:	1.00

   d. The relative frequency method was used.

2. a. Choose a person at random, have them taste the 4 blends and state their preference.

   b. Assign a probability of 1/4 to each blend. We use the classical method of equally likely outcomes here.

   c.

Blend	Probability
1	.20
2	.30
3	.35
4	.15
Total:	1.00

   The relative frequency method was used.

3. Initially a probability of .20 would be assigned if selection is equally likely. Data does not appear to confirm the belief of equal consumer preference. For example using the relative frequency method we would assign a probability of 5 / 100 = .05 to the design 1 outcome, .15 to design 2, .30 to design 3, .40 to design 4, and .10 to design 5.

4. a. Use the relative frequency approach:

   P(California) = 1,434/2,374 = .60

   b. Number not from 4 states = 2,374 - 1,434 - 390 - 217 - 112 = 221

   P(Not from 4 States) = 221/2,374 = .09

   c. P(Not in Early Stages) = 1 - .22 = .78

   d. Estimate of number of Massachusetts companies in early stage of development - (.22)390 $\approx$ 86

e. If we assume the size of the awards did not differ by states, we can multiply the probability an award went to Colorado by the total venture funds disbursed to get an estimate.

Estimate of Colorado funds = (112/2374)($32.4) = $1.53 billion

Authors' Note: The actual amount going to Colorado was $1.74 billion.

5. a. No, the probabilities do not sum to one. They sum to 0.85.

   b. Owner must revise the probabilities so that they sum to 1.00.

6. a. $P(A) = P(150 - 199) + P(200 \text{ and over})$

$$= \frac{26}{100} + \frac{5}{100}$$

$$= 0.31$$

   b. $P(B) = P(\text{less than } 50) + P(50 - 99) + P(100 - 149)$

$$= 0.13 + 0.22 + 0.34$$

$$= 0.69$$

7. a. P(A) = .40, P(B) = .40, P(C) = .60

   b. $P(A \cup B) = P(E_1, E_2, E_3, E_4) = .80$. Yes $P(A \cup B) = P(A) + P(B)$.

   c. $A^c = \{E_3, E_4, E_5\}$ $C^c = \{E_1, E_4\}$ $P(A^c) = .60$ $P(C^c) = .40$

   d. $A \cup B^c = \{E_1, E_2, E_5\}$ $P(A \cup B^c) = .60$

   e. $P(B \cup C) = P(E_2, E_3, E_4, E_5) = .80$

8. Let Y = high one-year return
   M = high five-year return

   a. P(Y) = 15/30 = .50

   P(M) = 12/30 = .40

   $P(Y \cap M) = 6/30 = .20$

   b. $P(Y \cup M) = P(Y) + P(M) - P(Y \cap M)$

$$= .50 + .40 - .20 = .70$$

   c. $1 - P(Y \cup M) = 1 - .70 = .30$

9. Let E = event patient treated experienced eye relief.
S = event patient treated had skin rash clear up.

Given:

P (E) = 90 / 250 = 0.36

P (S) = 135 / 250 = 0.54

P (E $\cup$ S) = 45 / 250 = 0.18

$$P(E \cup S) = P(E) + P(S) - P(E \cap S) = 0.36 + 0.54 - 0.18 = 0.72$$

10. P(Defective and Minor) = 4/25

P(Defective and Major) = 2/25

P(Defective) = (4/25) + (2/25) = 6/25

P(Major Defect | Defective) = P(Defective and Major) / P(Defective) = (2/25)/(6/25) = 2/6 = 1/3.

11. a. Yes; the person cannot be in an automobile and a bus at the same time.

b. $P(B^c) = 1 - P(B) = 1 - 0.35 = 0.65$

12. a. $$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.40}{0.60} = 0.6667$$

b. $$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.40}{0.50} = 0.80$$

c. No because $P(A | B) \neq P(A)$

13. a.

	Reason for Applying			
	Quality	Cost/Convenience	Other	Total
Full Time	0.218	0.204	0.039	0.461
Part Time	0.208	0.307	0.024	0.539
Total	0.426	0.511	0.063	1.00

b. It is most likely a student will cite cost or convenience as the first reason: probability = 0.511. School quality is the first reason cited by the second largest number of students: probability = 0.426.

c. P (Quality | full time) = 0.218/0.461 = 0.473
d. P (Quality | part time) = 0.208/0.539 = 0.386

e. $P(B) = 0.426$ and $P(B | A) = 0.473$

Since $P(B) \neq P(B | A)$, the events are dependent.

14.

	\$0-\$499	\$500-\$999	>=\$1000	
<2 yrs	120	240	90	450
>= 2 yrs	75	275	200	550
	195	515	290	1000

	\$0-\$499	\$500-\$999	>=\$1000	
<2 yrs	0.12	0.24	0.09	0.45
>= 2 yrs	0.075	0.275	0.2	0.55
	0.195	0.515	0.29	1.00

a. P(< 2 yrs) = .45

b. P(>= \$1000) = .29

c. P(2 accounts have > = \$1000) = (.29)(.29) = .0841

d. P(\$500-\$999 | >= 2 yrs) = P(\$500-\$999 and >= 2 yrs) / P(>=2yrs) = .275/.55 = .5

e. P(< 2 yrs and >=\$1000) = .09

f. P(>=2 yrs | \$500-\$999) = .275/.515 = .533981

15. a. Total sample size = 2000

Dividing each entry by 2000 provides the following joint probability table.

	Health Insurance		
Age	Yes	No	Total
18 to 34	.375	.085	.46
35 and over	.475	.065	.54
	.850	.150	1.00

Let A = 18 to 34 age group
B = 35 and over age group
Y = Insurance coverage
N = No insurance coverage

b. P(A) = .46
P(B) = .54

Of population age 18 and over

46% are ages 18 to 34
54% are ages 35 and over

c. P(N) = .15

d. $P(N|A) = \dfrac{P(N \cap A)}{P(A)} = \dfrac{.085}{.46} = .1848$

e. $P(N|B) = \dfrac{P(N \cap B)}{P(B)} = \dfrac{.065}{.54} = .1204$

f. $P(A|N) = \frac{P(A \cap N)}{P(N)} = \frac{.085}{.150} = .5677$

g. Probability of no health insurance coverage is .15. A higher probability exists for the younger population. Ages 18 to 34: .1848 or approximately 18.5% of the age group. Ages 35 and over: .1204 or approximately 12% of the age group. Of the no insurance group, more are in the 18 to 34 age group: .5677, or approximately 57% are ages 18 to 34.

16. a. $P(A \cap B) = P(A)P(B) = (0.55)(0.35) = 0.19$

b. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.90 - 0.19 = 0.71$

c. $1 - 0.71 = 0.29$

17. a.

	Satisfaction Score					
Occupation	Under 50	50-59	60-69	70-79	80-89	Total
Cabinetmaker	.000	.050	.100	.075	.025	.250
Lawyer	.150	.050	.025	.025	.000	.250
Physical Therapist	.000	.125	.050	.025	.050	.250
Systems Analyst	.050	.025	.100	.075	.000	.250
Total	.200	.250	.275	.200	.075	1.000

b. P(80s) = .075 (a marginal probability)

c. P(80s | PT) = .050/.250 = .20 (a conditional probability)

d. P(L) = .250 (a marginal probability)

e. $P(L \cap \text{Under } 50)$ = .150 (a joint probability)

f. P(Under 50 | L) = .150/.250 = .60 (a conditional probability)

g. P(70 or higher) = .275 (Sum of marginal probabilities)

18. a. $P(B) = 0.25$

$P(S|B) = 0.40$
$P(S \cap B) = 0.25(0.40) = 0.10$

b. $P(B|S) = \frac{P(S \cap B)}{P(S)} = \frac{0.10}{0.40} = 0.25$

c. B and S are independent. The program appears to have no effect.

19. Let: A = lost time accident in current year
B = lost time accident previous year

$\therefore$ Given: $P(B) = 0.06$, $P(A) = 0.05$, $P(A|B) = 0.15$

a. $P(A \cap B) = P(A|B)P(B) = 0.15(0.06) = 0.009$

b. $P(A \cup B) = P(A) + P(B) - P(A \mid B)$
$= 0.06 + 0.05 - 0.009 = 0.101$ or 10.1%

20. a. $P(B \cap A_1) = P(A_1)P(B \mid A_1) = (0.20)(0.50) = 0.10$

$P(B \cap A_2) = P(A_2)P(B \mid A_2) = (0.50)(0.40) = 0.20$

$P(B \cap A_3) = P(A_3)P(B \mid A_3) = (0.30)(0.30) = 0.09$

b. $P(A_2 \mid B) = \dfrac{0.20}{0.10 + 0.20 + 0.09} = 0.51$

c.

Events	$P(A_i)$	$P(B \mid A_i)$	$P(A_i \cap B)$	$P(A_i \mid B)$
A_1	0.20	0.50	0.10	0.26
A_2	0.50	0.40	0.20	0.51
A_3	0.30	0.30	0.09	0.23
	1.00		0.39	1.00

21. S_1 = successful, S_2 = not successful and B = request received for additional information.

a. $P(S_1) = 0.50$

b. $P(B \mid S_1) = 0.75$

c. $P(S_1 \mid B) = \dfrac{(0.50)(0.75)}{(0.50)(0.75) + (0.50)(0.40)} = \dfrac{0.375}{0.575} = 0.65$

22. a. Let F = female. Using past history as a guide, P(F) = .40

b. Let D = Dillard's

$$P(F \mid D) = \frac{.40(3/4)}{.40(3/4) + .60(1/4)} = \frac{.30}{.30 + .15} = .67$$

The revised (posterior) probability that the visitor is female is .67.

We should display the offer that appeals to female visitors.

23. a. P(Oil) = 0.50 + 0.20 = 0.70

b. Let S = Soil test results

Events	$P(A_i)$	$P(S \mid A_i)$	$P(A_i \cap S)$	$P(A_i \mid S)$
High Quality (A_1)	0.50	0.20	0.10	0.31
Medium Quality (A_2)	0.20	0.80	0.16	0.50
No Oil (A_3)	0.30	0.20	0.06	0.19
	1.00		P(S) = 0.32	1.00

P(Oil) = 0.81 which is good; however, probabilities now favor medium quality rather than high quality oil.

24. Let: S = small car
S^c = other type of vehicle
F = accident leads to fatality for vehicle occupant

We have P(S) = .18, so P(S^c) = .82. Also P(F | S) = .128 and P(F | S^c) = .05. Using the tabular form of Bayes Theorem provides:

Events	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Posterior Probabilities
S	.18	.128	.023	.36
S^c	.82	.050	.041	.64
	1.00		.064	1.00

From the posterior probability column, we have P(S | F) = .36. So, if an accident leads to a fatality, the probability a small car was involved is .36.

25.

Events	$P(A_i)$	$P(D \mid A_i)$	$P(A_i \cap D)$	$P(A_i \mid D)$
Supplier A	0.60	0.0025	0.0015	0.23
Supplier B	0.30	0.0100	0.0030	0.46
Supplier C	0.10	0.0200	0.0020	0.31
	1.00		P(D) = 0.0065	1.00

a. P(D) = 0.0065

b. B is the most likely supplier if a defect is found.

26. a.

Events	$P(D_i)$	$P(S_1 \mid D_i)$	$P(D_i \cap S_1)$	$P(D_i \mid S_1)$
D_1	.60	.15	.090	.2195
D_2	.40	.80	.320	.7805
	1.00		$P(S_1)$ = .410	1.0000

$P(D_1 \mid S_1) = .2195$

$P(D_2 \mid S_1) = .7805$

b.

Events	$P(D_i)$	$P(S_2 \mid D_i)$	$P(D_i \cap S_2)$	$P(D_i \mid S_2)$
D_1	.60	.10	.060	.500
D_2	.40	.15	.060	.500
	1.00		$P(S_2)$ = .120	1.000

$P(D_1 \mid S_2) = .50$

$P(D_2 \mid S_2) = .50$

c.

Events	$P(D_i)$	$P(S_3 \mid D_i)$	$P(D_i \cap S_3)$	$P(D_i \mid S_3)$
D_1	.60	.15	.090	.8824
D_2	.40	.03	.012	.1176
	1.00		$P(S_3) = .102$	1.0000

$P(D_1 \mid S_3) = .8824$

$P(D_2 \mid S_3) = .1176$

d. Use the posterior probabilities from part (a) as the prior probabilities here.

Events	$P(D_i)$	$P(S_2 \mid D_i)$	$P(D_i \cap S_2)$	$P(D_i \mid S_2)$
D_1	.2195	.10	.0220	.1582
D_2	.7805	.15	.1171	.8418
	1.0000		.1391	1.0000

$P(D_1 \mid S_1 \text{ and } S_2) = .1582$

$P(D_2 \mid S_1 \text{ and } S_2) = .8418$