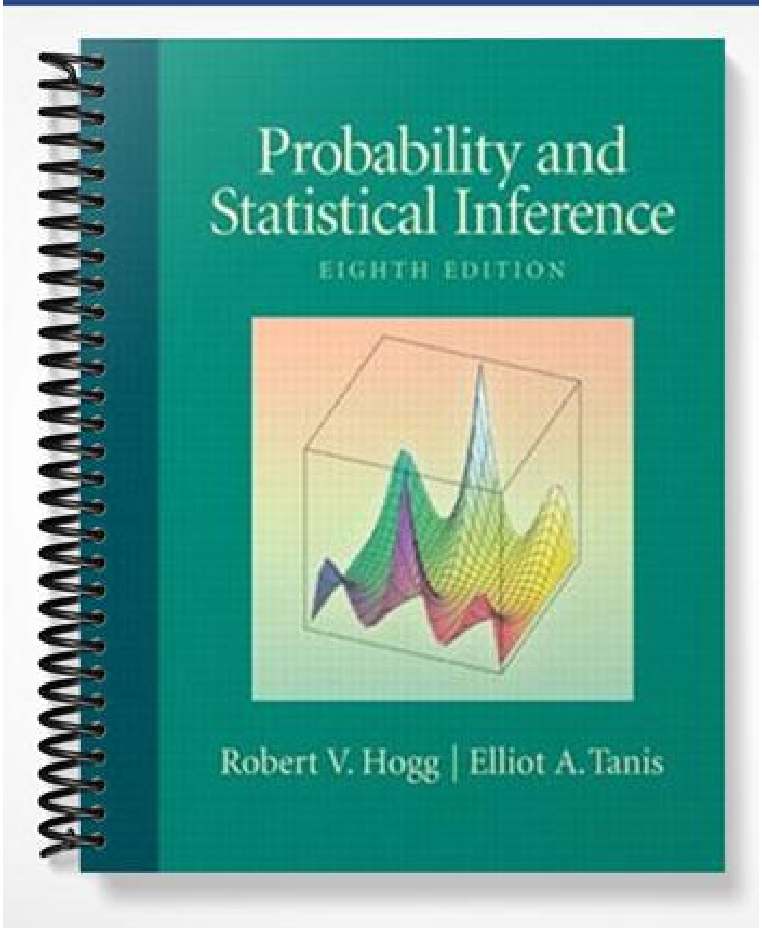
SOLUTIONS MANUAL



Instructor's Solutions Manual

Probability and Statistical Inference

Eighth Edition

Robert V. Hogg
University of Iowa

Elliot A. Tanis
Hope College

Prentice Hall
is an imprint of
PEARSON

Reproduced by Pearson Prentice Hall from electronic files supplied by the author.

Publishing as Pearson Prentice Hall, Upper Saddle River, NJ 07458.

ISBN-13: 978-0-321-58476-2

ISBN-10: 0-321-58476-7

Prentice Hall
is an imprint of

PEARSON

www.pearsonhighered.com

Contents

Preface

This solutions manual provides answers for the even-numbered exercises in *Probability and Statistical Inference*, 8th edition, by Robert V. Hogg and Elliot A. Tanis. Complete solutions are given for most of these exercises. You, the instructor, may decide how many of these answers you want to make available to your students. Note that the answers for the odd-numbered exercises are given in the textbook.

All of the figures in this manual were generated using *Maple*, a computer algebra system. Most of the figures were generated and many of the solutions, especially those involving data, were solved using procedures that were written by Zaven Karian from Denison University. We thank him for providing these. These procedures are available free of charge for your use. They are available on the CD-ROM in the textbook. Short descriptions of these procedures are provided in the "Maple Card" that is on the CD-ROM. Complete descriptions of these procedures are given in *Probability and Statistics: Explorations with MAPLE*, second edition, 1999, written by Zaven Karian and Elliot Tanis, published by Prentice Hall (ISBN 0-13-021536-8).

REMARK Note that *Probability and Statistics: Explorations with MAPLE*, second edition, written by Zaven Karian and Elliot Tanis, is available for download from Pearson Education's online catalog. It has been slightly revised and now contains references to several of the exercises in the 8th edition of *Probability and Statistical Inference*. ♦

Our hope is that this solutions manual will be helpful to each of you in your teaching. If you find an error or wish to make a suggestion, send these to Elliot Tanis at `tanis@hope.edu` and he will post corrections on his web page, http://www.math.hope.edu/tanis/.

R.V.H.
E.A.T.

Chapter 1

Probability

1.1 Basic Concepts

1.1-2 **(a)** $S = \{\text{bbb, gbb, bgb, bbg, bgg, gbg, ggb, ggg}\}$;

(b) $S = \{\text{female, male}\}$;

(c) $S = \{000, 001, 002, 003, \ldots, 999\}$.

1.1-4 **(a)**

Clutch size:	4	5	6	7	8	9	10	11	12	13	14
Frequency:	3	5	7	27	26	37	8	2	0	1	1

(b)

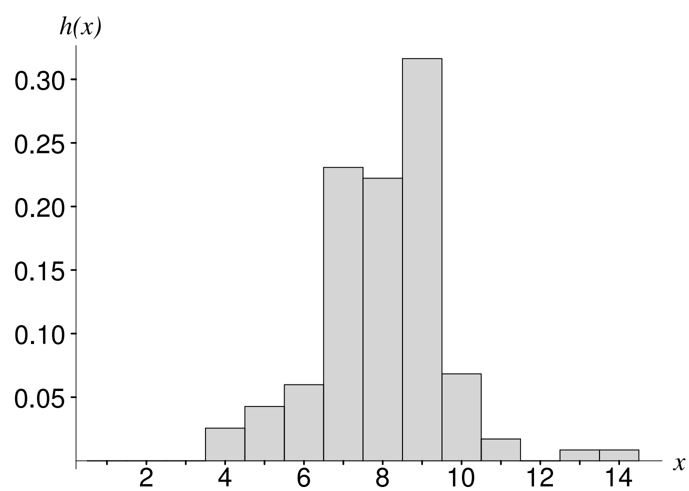


Figure 1.1–4: Clutch sizes for the common gallinule

(c) 9.

1.1-6 **(a)**

No. Boxes:	4	5	6	7	8	9	10	11	12	13	14	15	16	19	24
Frequency:	10	19	13	8	13	7	9	5	2	4	4	2	2	1	1

(b)

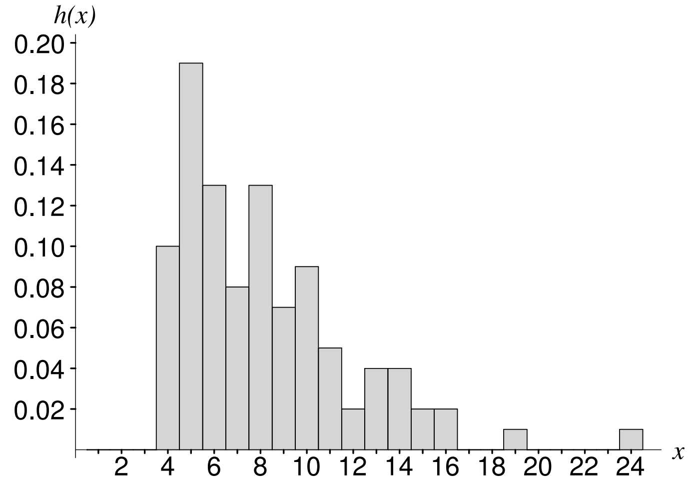


Figure 1.1–6: Number of boxes of cereal

1.1-8 **(a)** $f(1) = \dfrac{2}{10}, f(2) = \dfrac{3}{10},\ f(3) = \dfrac{3}{10}, f(4) = \dfrac{2}{10}$.

1.1-10 This is an experiment.

1.1-12 **(a)** $50/204 = 0.245$; $93/329 = 0.283$;

(b) $124/355 = 0.349$; $21/58 = 0.362$;

(c) $174/559 = 0.311$; $114/387 = 0.295$;

(d) Although James' batting average is higher that Hrbek's on both grass and artificial turf, Hrbek's is higher over all. Note the different numbers of at bats on grass and artificial turf and how this affects the batting averages.

1.2 Properties of Probability

1.2-2 Sketch a figure and fill in the probabilities of each of the disjoint sets.

Let $A = \{\text{insure more than one car}\}$, $P(A) = 0.85$.

Let $B = \{\text{insure a sports car}\}$, $P(B) = 0.23$.

Let $C = \{\text{insure exactly one car}\}$, $P(C) = 0.15$.

It is also given that $P(A \cap B) = 0.17$. Since $P(A \cap C) = 0$, it follows that $P(A \cap B \cap C') = 0.17$. Thus $P(A' \cap B \cap C') = 0.06$ and $P(A' \cap B' \cap C) = 0.09$.

1.2-4 **(a)** $S = \{$HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, HTHT, THTH, THHT, HTTT, THTT, TTHT, TTTH, TTTT$\}$;

(b) **(i)** 5/16, **(ii)** 0, **(iii)** 11/16, **(iv)** 4/16, **(v)** 4/16, **(vi)** 9/16, **(vii)** 4/16.

1.2-6 **(a)** 1/6;

(b) $P(B) = 1 - P(B') = 1 - P(A) = 5/6$;

(c) $P(A \cup B) = P(S) = 1$.

1.2-8 **(a)** $P(A \cup B) = 0.4 + 0.5 - 0.3 = 0.6$;

(b)
$$\begin{aligned} A &= (A \cap B') \cup (A \cap B) \\ P(A) &= P(A \cap B') + P(A \cap B) \\ 0.4 &= P(A \cap B') + 0.3 \\ P(A \cap B) &= 0.1; \end{aligned}$$

(c) $P(A' \cup B') = P[(A \cap B)'] = 1 - P(A \cap B) = 1 - 0.3 = 0.7$.

1.2-10 Let $A =$ {lab work done}, $B =$ {referral to a specialist},
$P(A) = 0.41,\ P(B) = 0.53,\ P([A \cup B]') = 0.21$.

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ 0.79 &= 0.41 + 0.53 - P(A \cap B) \\ P(A \cap B) &= 0.41 + 0.53 - 0.79 \;=\; 0.15. \end{aligned}$$

1.2-12
$$\begin{aligned} A \cup B \cup C &= A \cup (B \cup C) \\ P(A \cup B \cup C) &= P(A) + P(B \cup C) - P[A \cap (B \cup C)] \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P[(A \cap B) \cup (A \cap C)] \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) \\ &\quad + P(A \cap B \cap C). \end{aligned}$$

1.2-14 **(a)** $1/3$; **(b)** $2/3$; **(c)** 0; **(d)** $1/2$.

1.2-16 **(a)** $S = \{(1,2),(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),(3,4),(3,5),(4,5)\}$;
(b) **(i)** $1/10$; **(ii)** $5/10$.

1.2-18 $P(A) = \dfrac{2[r - r(\sqrt{3}/2)]}{2r} = 1 - \dfrac{\sqrt{3}}{2}$.

1.2-20 Note that the respective probabilities are $p_0,\ p_1 = p_0/4,\ p_2 = p_0/4^2, \ldots$.

$$\begin{aligned} \sum_{k=0}^{\infty} \frac{p_0}{4^k} &= 1 \\ \frac{p_0}{1 - 1/4} &= 1 \\ p_0 &= \frac{3}{4} \end{aligned}$$

$$1 - p_0 - p_1 = 1 - \frac{15}{16} = \frac{1}{16}.$$

1.3 Methods of Enumeration

1.3-2 $(4)(3)(2) = 24$.

1.3-4 **(a)** $(4)(5)(2) = 40$; **(b)** $(2)(2)(2) = 8$.

1.3-6 **(a)** $4\dbinom{6}{3} = 80$;

(b) $4(2^6) = 256$;

(c) $\dfrac{(4-1+3)!}{(4-1)!3!} = 20$.

1.3-8 ${}_9P_4 = \dfrac{9!}{5!} = 3024$.

1.3-10 $S = \{$ HHH, HHCH, HCHH, CHHH, HHCCH, HCHCH, CHHCH, HCCHH, CHCHH, CCHHH, CCC, CCHC, CHCC, HCCC, CCHHC, CHCHC, HCCHC, CHHCC, HCHCC, HHCCC $\}$ so there are 20 possibilities.

1.3-12 $3 \cdot 3 \cdot 2^{12} = 36,864$.

1.3-14
$$\begin{aligned}\binom{n-1}{r} + \binom{n-1}{r-1} &= \frac{(n-1)!}{r!(n-1-r)!} + \frac{(n-1)!}{(r-1)!(n-r)!} \\ &= \frac{(n-r)(n-1)! + r(n-1)!}{r!(n-r)!} = \frac{n!}{r!(n-r)!} = \binom{n}{r}.\end{aligned}$$

1.3-16
$$\begin{aligned}0 &= (1-1)^n = \sum_{r=0}^{n} \binom{n}{r} (-1)^r (1)^{n-r} = \sum_{r=0}^{n} (-1)^r \binom{n}{r}. \\ 2^n &= (1+1)^n = \sum_{r=0}^{n} \binom{n}{r} (1)^r (1)^{n-r} = \sum_{r=0}^{n} \binom{n}{r}.\end{aligned}$$

1.3-18
$$\begin{aligned}\binom{n}{n_1, n_2, \ldots, n_s} &= \binom{n}{n_1}\binom{n-n_1}{n_2}\binom{n-n_1-n_2}{n_3} \cdots \binom{n-n_1-\cdots-n_{s-1}}{n_s} \\ &= \frac{n!}{n_1!(n-n_1)!} \cdot \frac{(n-n_1)!}{n_2!(n-n_1-n_2)!} \\ &\qquad \cdot \frac{(n-n_1-n_2)!}{n_3!(n-n_1-n_2-n_3)!} \cdots \frac{(n-n_1-n_2-\cdots-n_{s-1})!}{n_s!0!} \\ &= \frac{n!}{n_1!n_2!\ldots n_s!}.\end{aligned}$$

1.3-20 **(a)** $\dfrac{\binom{19}{3}\binom{52-19}{6}}{\binom{52}{9}} = \dfrac{102,486}{351,325} = 0.2917;$

(b) $\dfrac{\binom{19}{3}\binom{10}{2}\binom{7}{1}\binom{3}{0}\binom{5}{1}\binom{2}{0}\binom{6}{2}}{\binom{52}{9}} = \dfrac{7,695}{1,236,664} = 0.00622.$

1.3-22 $\binom{45}{36} = 886{,}163{,}135.$

1.4 Conditional Probability

1.4-2 **(a)** $\dfrac{1041}{1456}$;

(b) $\dfrac{392}{633}$;

(c) $\dfrac{649}{823}$.

(d) The proportion of women who favor a gun law is greater than the proportion of men who favor a gun law.

1.4-4 **(a)** $P(\text{HH}) = \dfrac{13}{52} \cdot \dfrac{12}{51} = \dfrac{1}{17};$

(b) $P(\text{HC}) = \dfrac{13}{52} \cdot \dfrac{13}{51} = \dfrac{13}{204};$

(c) $P(\text{Non-Ace Heart, Ace}) + P(\text{Ace of Hearts, Non-Heart Ace})$

$$= \frac{12}{52} \cdot \frac{4}{51} + \frac{1}{52} \cdot \frac{3}{51} = \frac{51}{52 \cdot 51} = \frac{1}{52}.$$

1.4-6 Let $A = \{3 \text{ or } 4 \text{ kings}\}$, $B = \{2, 3, \text{ or } 4 \text{ kings}\}$.

$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} = \frac{N(A)}{N(B)} \\ &= \frac{\binom{4}{3}\binom{48}{10} + \binom{4}{4}\binom{48}{9}}{\binom{4}{2}\binom{48}{11} + \binom{4}{3}\binom{48}{10} + \binom{4}{4}\binom{48}{9}} = 0.170. \end{aligned}$$

1.4-8 Let H ={died from heart disease}; P ={at least one parent had heart disease}.

$$P(H \mid P') = \frac{N(H \cap P')}{N(P')} = \frac{110}{648}.$$

1.4-10 **(a)** $\dfrac{3}{20} \cdot \dfrac{2}{19} \cdot \dfrac{1}{18} = \dfrac{1}{1140};$

(b) $\dfrac{\binom{3}{2}\binom{17}{1}}{\binom{20}{3}} \cdot \dfrac{1}{17} = \dfrac{1}{760};$

(c) $\displaystyle\sum_{k=1}^{9} \frac{\binom{3}{2}\binom{17}{2k-2}}{\binom{20}{2k}} \cdot \frac{1}{20-2k} = \frac{35}{76} = 0.4605.$

(d) Draw second. The probability of winning in $1 - 0.4605 = 0.5395$.

1.4-12 $\dfrac{\binom{2}{0}\binom{8}{5}}{\binom{10}{5}} \cdot \dfrac{2}{5} + \dfrac{\binom{2}{1}\binom{8}{4}}{\binom{10}{5}} \cdot \dfrac{1}{5} = \dfrac{1}{5}.$

1.4-14 **(a)** $P(A) = \dfrac{52}{52} \cdot \dfrac{51}{52} \cdot \dfrac{50}{52} \cdot \dfrac{49}{52} \cdot \dfrac{48}{52} \cdot \dfrac{47}{52} = \dfrac{8,808,975}{11,881,376} = 0.74141;$

(b) $P(A') = 1 - P(A) = 0.25859.$

1.4-16 **(a)** It doesn't matter because $P(B_1) = \dfrac{1}{18},\ P(B_5) = \dfrac{1}{18},\ P(B_{18}) = \dfrac{1}{18};$

(b) $P(B) = \dfrac{2}{18} = \dfrac{1}{9}$ on each draw.

1.4-18 $\dfrac{3}{5} \cdot \dfrac{5}{8} + \dfrac{2}{5} \cdot \dfrac{4}{8} = \dfrac{23}{40}.$

1.4-20 **(a)** $P(A_1) = 30/100$;

(b) $P(A_3 \cap B_2) = 9/100$;

(c) $P(A_2 \cup B_3) = 41/100 + 28/100 - 9/100 = 60/100$;

(d) $P(A_1 \,|\, B_2) = 11/41$;

(e) $P(B_1 \,|\, A_3) = 13/29$.

1.5 Independent Events

1.5-2 **(a)**
$$\begin{aligned} P(A \cap B) &= P(A)P(B) = (0.3)(0.6) = 0.18; \\ P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.3 + 0.6 - 0.18 \\ &= 0.72. \end{aligned}$$

(b) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0}{0.6} = 0.$

1.5-4 Proof of **(b)**:
$$\begin{aligned} P(A' \cap B) &= P(B)P(A'|B) \\ &= P(B)[1 - P(A|B)] \\ &= P(B)[1 - P(A)] \\ &= P(B)P(A'). \end{aligned}$$

Proof of **(c)**:
$$\begin{aligned} P(A' \cap B') &= P[(A \cup B)'] \\ &= 1 - P(A \cup B) \\ &= 1 - P(A) - P(B) + P(A \cap B) \\ &= 1 - P(A) - P(B) + P(A)P(B) \\ &= [1 - P(A)][1 - P(B)] \\ &= P(A')P(B'). \end{aligned}$$

1.5-6
$$\begin{aligned} P[A \cap (B \cap C)] &= P[A \cap B \cap C] \\ &= P(A)P(B)P(C) \\ &= P(A)P(B \cap C). \end{aligned}$$

$$\begin{aligned} P[A \cap (B \cup C)] &= P[(A \cap B) \cup (A \cap C)] \\ &= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \\ &= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C) \\ &= P(A)[P(B) + P(C) - P(B \cap C)] \\ &= P(A)P(B \cup C). \end{aligned}$$

$$\begin{aligned} P[A' \cap (B \cap C')] &= P(A' \cap C' \cap B) \\ &= P(B)[P(A' \cap C') \,|\, B] \\ &= P(B)[1 - P(A \cup C \,|\, B)] \\ &= P(B)[1 - P(A \cup C)] \\ &= P(B)P[(A \cup C)'] \\ &= P(B)P(A' \cap C') \\ &= P(B)P(A')P(C') \\ &= P(A')P(B)P(C') \\ &= P(A')P(B \cap C') \end{aligned}$$

$$\begin{aligned} P[A' \cap B' \cap C'] &= P[(A \cup B \cup C)'] \\ &= 1 - P(A \cup B \cup C) \\ &= 1 - P(A) - P(B) - P(C) + P(A)P(B) + P(A)P(C) + \\ &\qquad P(B)P(C) - PA)P(B)P(C) \\ &= [1 - P(A)][1 - P(B)][1 - P(C)] \\ &= P(A')P(B')P(C'). \end{aligned}$$

1.5-8 $\frac{1}{6}\cdot\frac{2}{6}\cdot\frac{3}{6}+\frac{1}{6}\cdot\frac{4}{6}\cdot\frac{3}{6}+\frac{5}{6}\cdot\frac{2}{6}\cdot\frac{3}{6}=\frac{2}{9}$.

1.5-10 **(a)** $\frac{3}{4}\cdot\frac{3}{4}=\frac{9}{16}$;

(b) $\frac{1}{4}\cdot\frac{3}{4}+\frac{3}{4}\cdot\frac{2}{4}=\frac{9}{16}$;

(c) $\frac{2}{4}\cdot\frac{1}{4}+\frac{2}{4}\cdot\frac{4}{4}=\frac{10}{16}$.

1.5-12 **(a)** $\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$;

(b) $\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$;

(c) $\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$;

(d) $\frac{5!}{3!\,2!}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$.

1.5-14 **(a)** $1-(0.4)^3=1-0.064=0.936$;

(b) $1-(0.4)^8=1-0.00065536=0.99934464$.

1.5-16 **(a)** $\sum_{k=0}^{\infty}\frac{1}{5}\left(\frac{4}{5}\right)^{2k}=\frac{5}{9}$;

(b) $\frac{1}{5}+\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{1}{3}+\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}\cdot\frac{1}{1}=\frac{3}{5}$.

1.5-18 **(a)** 7; **(b)** $(1/2)^7$; **(c)** 63; **(d)** No! $(1/2)^{63}=1/9{,}223{,}372{,}036{,}854{,}775{,}808$.

1.5-20

n	3	6	9	12	15
(a)	0.7037	0.6651	0.6536	0.6480	0.6447
(b)	0.6667	0.6319	0.6321	0.6321	0.6321

(c) Very little when $n > 15$, sampling with replacement

Very little when $n > 10$, sampling without replacement.

(d) Convergence is faster when sampling with replacement.

1.6 Bayes's Theorem

1.6-2 **(a)**
$$\begin{aligned}P(G) &= P(A\cap G)+P(B\cap G)\\ &= P(A)P(G\,|\,A)+P(B)P(G\,|\,B)\\ &= (0.40)(0.85)+(0.60)(0.75)=0.79;\end{aligned}$$

(b)
$$\begin{aligned}P(A\,|\,G) &= \frac{P(A\cap G)}{P(G)}\\ &= \frac{(0.40)(0.85)}{0.79}=0.43.\end{aligned}$$

1.6-4 Let event B denote an accident and let A_1 be the event that age of the driver is 16–25. Then

$$\begin{aligned} P(A_1 \mid B) &= \frac{(0.1)(0.05)}{(0.1)(0.05) + (0.55)(0.02) + (0.20)(0.03) + (0.15)(0.04)} \\ &= \frac{50}{50 + 110 + 60 + 60} = \frac{50}{280} = 0.179. \end{aligned}$$

1.6-6 Let B be the event that the policyholder dies. Let A_1, A_2, A_3 be the events that the deceased is standard, preferred and ultra-preferred, respectively. Then

$$\begin{aligned} P(A_1 \mid B) &= \frac{(0.60)(0.01)}{(0.60)(0.01) + (0.30)(0.008) + (0.10)(0.007)} \\ &= \frac{60}{60 + 24 + 7} = \frac{60}{91} = 0.659; \\ P(A_2 \mid B) &= \frac{24}{91} = 0.264; \\ P(A_3 \mid B) &= \frac{7}{91} = 0.077. \end{aligned}$$

1.6-8 Let A be the event that the DVD player is under warranty.

$$\begin{aligned} P(B_1 \mid A) &= \frac{(0.40)(0.10)}{(0.40)(0.10) + (0.30)(0.05) + (0.20)(0.03) + (0.10)(0.02)} \\ &= \frac{40}{40 + 15 + 6 + 2} = \frac{40}{63} = 0.635; \\ P(B_2 \mid A) &= \frac{15}{63} = 0.238; \\ P(B_3 \mid A) &= \frac{6}{63} = 0.095; \\ P(B_4 \mid A) &= \frac{2}{63} = 0.032. \end{aligned}$$

1.6-10 **(a)** $P(AD) = (0.02)(0.92) + (0.98)(0.05) = 0.0184 + 0.0490 = 0.0674;$

(b) $P(N \mid AD) = \dfrac{0.0490}{0.0674} = 0.727;\ P(A \mid AD) = \dfrac{0.0184}{0.0674} = 0.273;$

(c) $P(N \mid ND) = \dfrac{(0.98)(0.95)}{(0.02)(0.08) + (0.98)(0.95)} = \dfrac{9310}{16 + 9310} = 0.998;\ P(A \mid ND) = 0.002.$

(d) Yes, particularly those in part (b).

1.6-12 Let $D = \{\text{has the disease}\}$, $DP = \{\text{detects presence of disease}\}$. Then

$$\begin{aligned} P(D \mid DP) &= \frac{P(D \cap DP)}{P(DP)} \\ &= \frac{P(D) \cdot P(DP \mid D)}{P(D) \cdot P(DP \mid D) + P(D') \cdot P(DP \mid D')} \\ &= \frac{(0.005)(0.90)}{(0.005)(0.90) + (0.995)(0.02)} \\ &= \frac{0.0045}{0.0045 + 0.199} = \frac{0.0045}{0.0244} = 0.1844. \end{aligned}$$

1.6-14 Let $D = \{\text{defective roll}\}$ Then

$$\begin{aligned}
P(I \mid D) &= \frac{P(I \cap D)}{P(D)} \\
&= \frac{P(I) \cdot P(D \mid I)}{P(I) \cdot P(D \mid I) + P(II) \cdot P(D \mid II)} \\
&= \frac{(0.60)(0.03)}{(0.60)(0.03) + (0.40)(0.01)} \\
&= \frac{0.018}{0.018 + 0.004} = \frac{0.018}{0.022} = 0.818.
\end{aligned}$$