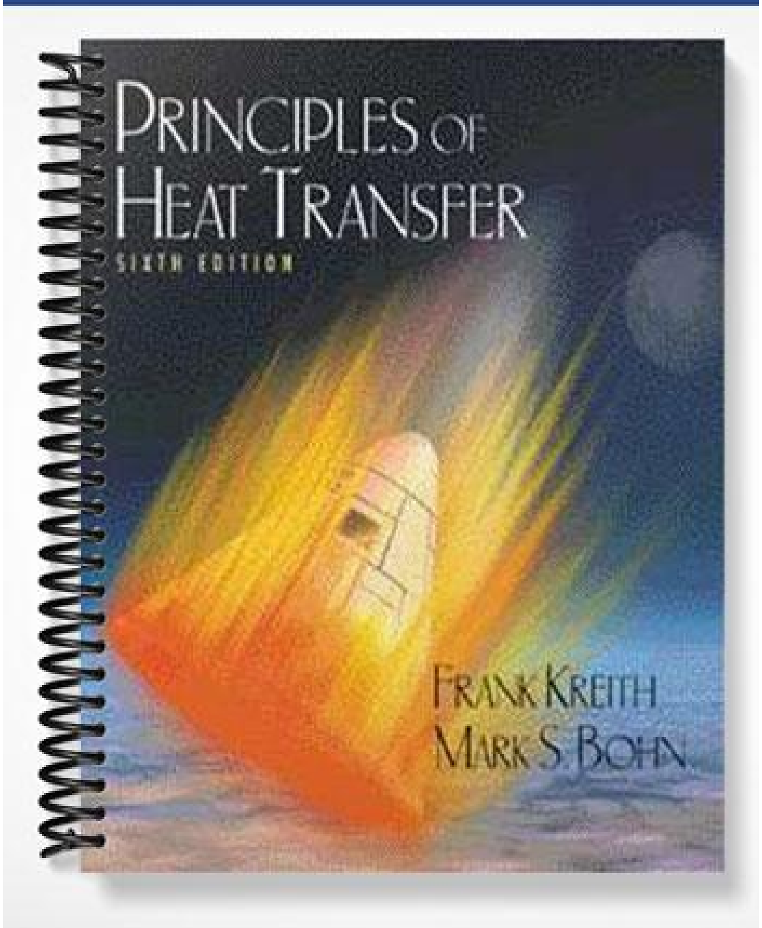
SOLUTIONS MANUAL



SIXTH EDITION

Solutions Manual to Accompany Principles of Heat Transfer

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PROBLEM 1.1

The outer surface of a 0.2m-thick concrete wall is kept at a temperature of –5°C, while the inner surface is kept at 20°C. The thermal conductivity of the concrete is 1.2 W / m K. Determine the heat loss through a wall 10 m long and 3 m high.

GIVEN

- 10 m long, 3 m high, and 0.2 m thick concrete wall
- Thermal conductivity of the concrete (k) = 1.2 W / m K
- Temperature of the inner surface (T_i) = 20°C
- Temperature of the outer surface (T_o) = -5°C

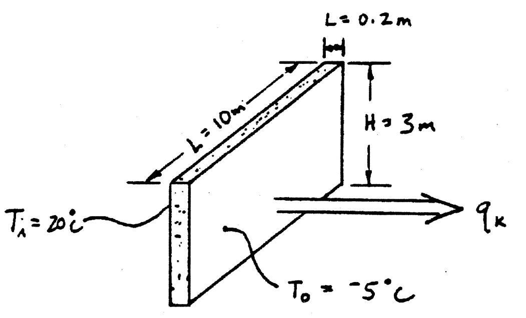
FIND

- The heat loss through the wall (q_k)

ASSUMPTIONS

- One dimensional heat flow
- The system has reached steady state

SKETCH



SOLUTION

The rate of heat loss through the wall is given by *equation (1.2):*

$$q_k = \frac{A\,k}{L}(\Delta T)$$

$$q_k = \frac{(10\text{ m})(3\text{ m})\left(1.2\ \frac{\text{W}}{\text{m K}}\right)}{0.2\text{ m}}\left(20°\text{C} - (-5°\text{C})\right)$$

$$q_k = 4500\text{ W}$$

COMMENTS

Since the inside surface temperature is higher than the outside temperature heat is transferred from the inside of the wall to the outside of the wall.

PROBLEM 1.2

The weight of the insulation in a spacecraft may be more important than the space required. Show analytically that the lightest insulation for a plane wall with a specified thermal resistance is that insulation which has the smallest product of density times thermal conductivity.

GIVEN

- Insulating a plane wall, the weight of insulation is most significant

FIND

- Show that lightest insulation for a given thermal resistance is that insulation which has the smallest product of density (ρ) times thermal conductivity (k)

ASSUMPTIONS

- One dimensional heat transfer through the wall
- Steady state conditions

SOLUTION:

The resistance of the wall (R_k), from *equation (1.13)* is:

$$R_k = \frac{L}{A\,k}$$

where: L = the thickness of the wall
A = the area of the wall

The weight of the wall (w) is

$$w = \rho\, A\, L$$

Solving this for L :

$$L = \frac{w}{\rho\, A}$$

Substituting this expression for L into the equation for the resistance:

$$R_k = \frac{w}{\rho\, k\, A^2}$$

$$\therefore\ w = \rho\, k\, R_k\, A^2$$

Therefore, when the product of ρ k for a given resistance is smallest, the weight is also smallest.

COMMENTS

Since ρ and k are physical properties of the insulation material they cannot be varied individually. Hence in this type of design different materials must be tried to minimize the weight.

PROBLEM 1.3

A furnace wall is to be constructed of brick having standard dimensions 9 by 4.5 by 3 in. Two kinds of material are available. One has a maximum usable temperature of 1900°F and a thermal conductivity of 1 Btu/h ft°F, and the other has a maximum temperature limit of 1600°F and a thermal conductivity of 0.5 Btu/h ft°F. The bricks cost the same and can be laid in any manner, but we wish to design the most economical wall for a furnace with a temperature on the hot side of 1900°F and on the cold side of 400°F. If the maximum amount of heat transfer permissible is 300 Btu/h for each square foot of area, determine the most economical arrangements for the available bricks.

GIVEN

- Furnace wall made of 9 × 4.5 × 3 inch bricks of two types:
- Type 1 bricks: Maximum useful temperature ($T_{1,max}$) = 1900°F
  Thermal conductivity (k_1) = 1.0 Btu/h ft°F
- Type 2 bricks: Maximum useful temperature ($T_{2,max}$) = 1600°F
  Thermal conductivity (k_2) = 0.5 Btu/h ft°F
- Bricks cost the same
- Wall hot side (T_{hot}) = 1900°F cold side (T_{cold}) = 400°F
- Maximum heat transfer permissible (q_{max}/A) = 300 Btu/h ft^2

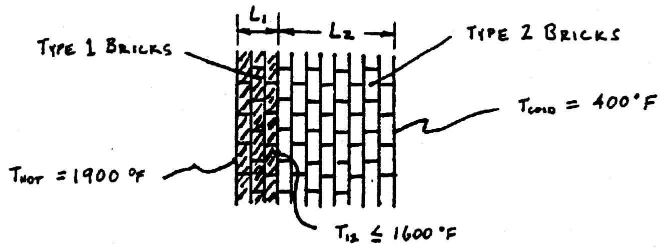
FIND

- The most economical arrangement for the bricks

ASSUMPTIONS

- One dimensional, steady state heat transfer conditions
- Constant thermal conductivities
- The contact resistance between the bricks is negligible

SKETCH



SOLUTION

Since the type 1 bricks have a higher thermal conductivity at the same cost as the type 2 bricks, the most economical wall would use as few type 1 bricks as possible. However, there must be a thick enough layer of type 1 bricks to keep the type 2 bricks at 1600°F or less.

For one dimensional conduction through the type 1 bricks (from *equation (1.2)*):

$$q_k = \frac{k\,A}{L}\Delta T$$

$$\frac{q_{max}}{A} = \frac{k_1}{L_1}\left(T_{hot} - T_{12}\right)$$

where L_1 = the minimum thickness of the type 1 bricks

Solving for L_1:

$$L_1 = \frac{k_1}{\left(\dfrac{q_{max}}{A}\right)}\left(T_{hot} - T_{12}\right)$$

$$L_1 = \frac{1.0\,\dfrac{\text{Btu}}{\text{h ft°F}}}{300\,\dfrac{\text{Btu}}{\text{h ft}^2}}\left(1900°\text{F} - 1600°\text{F}\right) = 1\text{ ft}$$

This thickness can be achieved with 4 layers of type 1 bricks using the 3 inch dimension.

Similarly, for one dimensional conduction through the type 2 bricks:

$$L_2 = \frac{k_2}{\left(\dfrac{q_{max}}{A}\right)}\left(T_{12} - T_{cold}\right)$$

$$L_2 = \frac{0.5\,\dfrac{\text{Btu}}{\text{h ft°F}}}{300\,\dfrac{\text{Btu}}{\text{h ft}^2}}\left(1600°\text{F} - 400°\text{F}\right) = 2\text{ ft}$$

This thickness can be achieved with 8 layers of type 2 brick using the 3 inch dimension.Therefore the most economical wall would be built using 4 layers of type 1 bricks and 8 layers of type 2 bricks with the three inch dimension of the bricks used as the thickness.

PROBLEM 1.4

To measure thermal conductivity, two similar 1-cm-thick specimens are placed in an apparatus shown in the accompanying sketch. Electric current is supplied to the 6-cm by 6-cm guarded heater, and a wattmeter shows that the power dissipation is 10 watts (W). Thermocouples attached to the warmer and to the cooler surfaces show temperatures of 322 and 300 K, respectively. Calculate the thermal conductivity of the material at the mean temperature in Btu/h ft°F and W/m K.

GIVEN

- Thermal conductivity measurement apparatus with two samples as shown
- Sample thickness (L) = 1 cm = .01 cm
- Area = 6 cm × 6 cm = 36 cm^2 = .0036 m^2
- Power dissipation rate of the heater (q_h) = 10 W
- Surface temperatures: T_{hot} = 322 K T_{cold} = 300 K

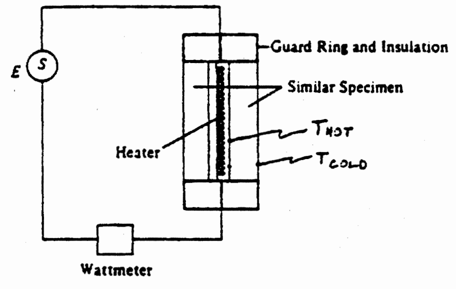
FIND

- The thermal conductivity of the sample at the mean temperature in Btu/h ft°F and W/m K

ASSUMPTIONS

- One dimensional, steady state conduction
- No heat loss from the edges of the apparatus

SKETCH



SOLUTION

By conservation of energy, the heat loss through the two specimens must equal the power dissipation of the heater. Therefore the heat transfer through one of the specimens is $q_h/2$.

For one dimensional, steady state conduction (from *equation (1.2)*):

$$q_k = \frac{k\,A}{L}\,\Delta T = \frac{q_h}{2}$$

Solving for the thermal conductivity:

$$k = \frac{\left(\frac{q_h}{2}\right)L}{A\,\Delta T}$$

$$k = \frac{(5\ W)(0.01\ m)}{(0.0036\ m^2)(322\ K - 300\ K)}$$

$$k = 0.63\ \frac{W}{m\,K}$$

Converting the thermal conductivity in the English system of units using the conversion factor found on the inside front cover of the text book:

$$k = 0.63\frac{W}{m\,K}\left(.057782\frac{\left(\frac{Btu}{h\,ft°F}\right)}{\left(\frac{W}{m\,K}\right)}\right)$$

$$k = 0.36\frac{Btu}{h\,ft°F}$$

COMMENTS

In the construction of the apparatus care must be taken to avoid edge losses so all the heat generated will be conducted through the two specimens.

PROBLEM 1.5

To determine the thermal conductivity of a structural material, a large 6-in-thick slab of the material was subjected to a uniform heat flux of 800 Btu/h ft², while thermocouples embedded in the wall 2 in. apart were read over a period of time. After the system had reached equilibrium, an operator recorded the readings of the thermocouples as shown below for two different environmental conditions:

Distance from the surface (in.)	Temperature (°F)
Test1	
0	100
2	150
4	206
6	270
Test 2	
0	200
2	265
4	335
6	406

From these data, determine an approximate expression for the thermal conductivity as a function of temperature between 100 and 400°F.

GIVEN

- Thermal conductivity test on a large, 6-in.-thick slab
- Thermocouples are embedded in the wall 2 in. apart
- Heat flux (q/A) = 800 Btu/h ft^2
- Two equilibrium conditions were recorded (shown above)

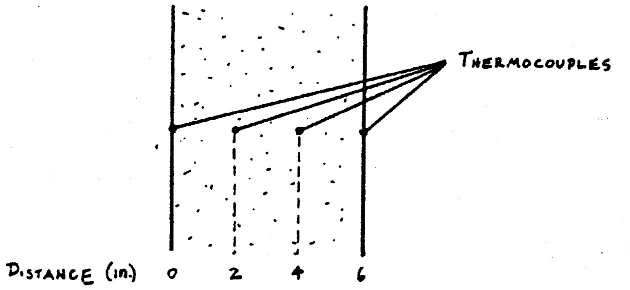
FIND

- An approximate expression for thermal conductivity as a function of temperature between 100 and 400°F

ASSUMPTIONS

- One dimensional conduction

SKETCH



SOLUTION

The thermal conductivity can be calculated for each pair of adjacent thermocouples using the equation for one dimensional conduction (*equation (1.2))*:

$$q = k\,A\,\frac{\Delta T}{L}$$

Solving for thermal conductivity:

$$k = \frac{q}{A}\,\frac{L}{\Delta T}$$

This will yield a thermal conductivity for each pair of adjacent thermocouples which will then be assigned to the average temperature for that pair of thermocouples. As an example, for the first pair of thermocouples in Test 1, the thermal conductivity (k_o) is:

$$k_o = \left(800\,\frac{Btu}{h\,ft^2}\right)\left(\frac{\frac{2}{12}\,ft}{150°F - 100°F}\right) = 2.67\,\frac{Btu}{h\,ft°F}$$

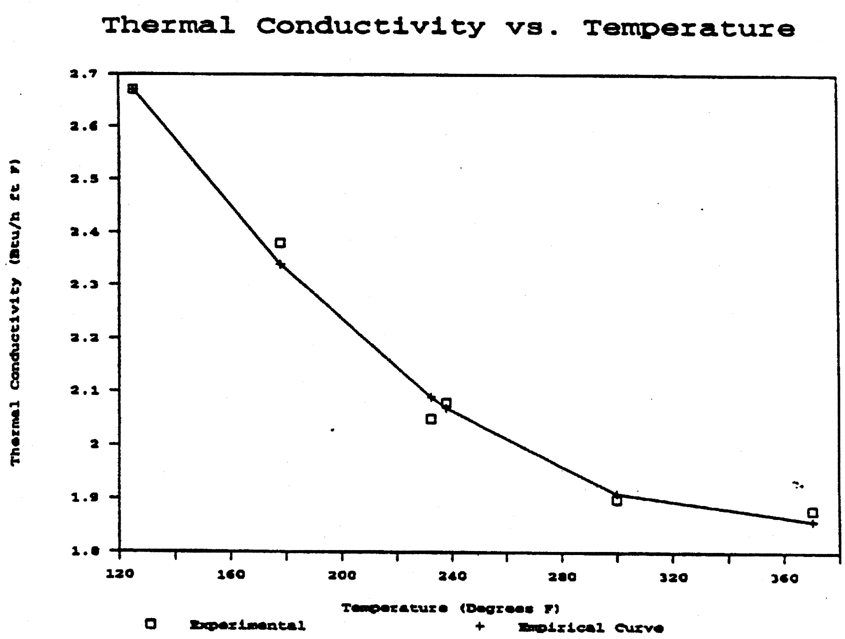
The average temperature for this pair of thermocouples is:

$$T_{ave.} = \frac{100°F + 150°F}{2} = 125°F$$

Thermal conductivities and average temperatures for the rest of the data can be calculated in a similar manner:

n	Temperature (°F)	Thermal conductivity (Btu/h ft°F)
1	125	2.67
2	178	2.38
3	238	2.08
4	232.5	2.05
5	300	1.90
6	370.5	1.88

These points are displayed graphically on the following page.



We will use the best fit quadratic function to represent the relationship between thermal conductivity and temperature:

$$k\,(T) = a + b\,T + c\,T^2$$

The constants a, b, and c can be found using a least squares fit.

Let the experimental thermal conductivity at data point n be designated as k_n. A least squares fit of the data can be obtained as follows:

The sum of the squares of the errors is:

$$S = \sum_N [k_n - k(T_n)]^2$$

$$S = \sum k_n^2 - 2a\sum k_n + N{\cdot}a^2 + 2ab\sum T_n - 2b\sum k_n T_n + 2ac\sum T_n^2 + b^2\sum T_n^2 - 2c\sum k_n T_n^2 + 2bc\sum T_n^3 + c^2\sum T_n^4$$

By setting the derivatives of S (with respect to a, b, and c) equal to zero, the following equations result:

$$Na + \sum T_n b + \sum T_n^2 c = \sum k_n$$
$$\sum T_n a + \sum T_n^2 b + \sum T_n^3 c = \sum k_n T_n$$
$$\sum T_n^2 a + \sum T_n^3 b + \sum T_n^4 c = \sum k_n T_n^2$$

For this problem:

$$\sum T_n = 1444$$
$$\sum T_n^2 = 3.853 \times 10^5$$
$$\sum T_n^3 = 1.115 \times 10^8$$
$$\sum T_n^4 = 3.432 \times 10^{10}$$
$$\sum k_n = 12.96$$
$$\sum k_n T_n = 2996$$
$$\sum k_n T_n^2 = 7.748 \times 10^5$$

Solving for a, b, and c: $a = 3.76$
$b = -0.0106$
$c = 1.476 \times 10^{-5}$

Therefore the expression for thermal conductivity as a function of temperature between 100 and 400° F is:

$$\mathbf{k(T) = 3.76 - 0.0106\,T + 1.476 \times 10^{-5}\,T^2}$$

This empirical expression for the thermal conductivity as a function of temperature is plotted with the thermal conductivities derived from the experimental data in the above graph.

COMMENTS

Note that the derived empirical expression is only valid within the temperature range of the experimental data.

PROBLEM 1.6

A square silicone chip 7 mm by 7 mm in size and 0.5 mm thick is mounted on a plastic substrate with its front surface cooled by a synthetic liquid flowing over it. Electronic circuits in the back of the chip generate heat at a rate of 5 watts that have to be transferred through the chip. Estimate the steady state temperature difference between the front and back surfaces of the chip. The thermal conductivity of silicone is 150 W/m K.

GIVEN

- A 0.007 m by 0.007 m silicone chip
- Thickness of the chip (L) = 0.5 mm = 0.0005 m
- Heat generated at the back of the chip ($\dot{q}_G$) = 5 W
- The thermal conductivity of silicon (k) = 150 W/m K

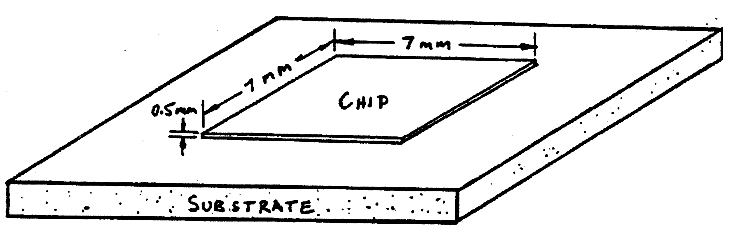
FIND

- The steady state temperature difference (ΔT)

ASSUMPTIONS

- One dimensional conduction (edge effects are negligible)
- The thermal conductivity is constant
- The heat lost through the plastic substrate is negligible

SKETCH



SOLUTION

For steady state the rate of heat loss through the chip, given by *equation (1.2),* must equal the rate of heat generation:

$$q_k = \frac{A\,k}{L}(\Delta T) = \dot{q}_G$$

Solving this for the temperature difference:

$$\Delta T = \frac{L\,\dot{q}_G}{k\,A}$$

$$\Delta T = \frac{(0.0005)\,(5\text{ W})}{(150\text{ W/m K})\,(0.007\text{ m})\,(0.007\text{ m})}$$

$$\Delta T = 0.34°C$$

PROBLEM 1.7

A warehouse is to be designed for keeping perishable foods cool prior to transportation to grocery stores. The warehouse has an effective surface area of 20,000 ft² exposed to an ambient air temperature of 90°F. The warehouse wall insulation (k = 0.1 Btu/h ft°F) is 3 in. thick. Determine the rate at which heat must be removed (Btu/h) from the warehouse to maintain the food at 40°F.

GIVEN

- Cooled warehouse
- Effective area (A) = 20,000 ft^2
- Temperatures: Outside air = 90°F   Food inside = 40°F
- Thickness of wall insulation (L) = 3 in. = .25 ft
- Thermal conductivity of insulation (k) = 0.1 Btu/h ft°F

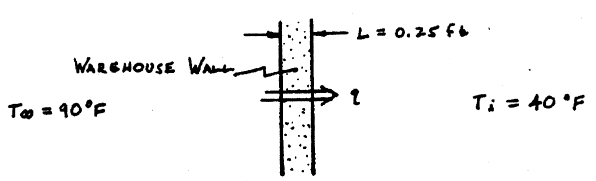
FIND

- Rate at which heat must be removed (q)

ASSUMPTIONS

- One dimensional, steady state heat flow
- The food and the air inside the warehouse are at the same temperature
- The thermal resistance of the wall is approximately equal to the thermal resistance of the wall insulation alone

SKETCH



SOLUTION

The rate at which heat must be removed is equal to the rate at which heat flows into the warehouse. There will be convective resistance to heat flow on the inside and outside of the wall. To estimate the upper limit of the rate at which heat must be removed these convective resistances will be neglected. Therefore the inside and outside wall surfaces are assumed to be at the same temperature as the air inside and outside of the wall. Then the heat flow, from *equation (1.2),* is:

$$q = \frac{k\,A}{L}\Delta T$$

$$q = \frac{\left(0.1\frac{\text{Btu}}{\text{h ft°F}}\right)(20{,}000\ \text{ft}^2)}{0.25\ \text{ft}}(90°\text{F} - 40°\text{F})$$

$$q = 400{,}000\ \frac{Btu}{h}$$

PROBLEM 1.8

With increasing emphasis on energy conservation, the heat loss from buildings has become a major concern. For a small tract house the typical exterior surface areas and R-factors (area × thermal resistance) are listed below:

Element	*Area (m^2)*	*R-Factors = Area × Thermal Resistance (m^2 K/W)*
Walls	*150*	*2.0*
Ceiling	*120*	*2.8*
Floor	*120*	*2.0*
Windows	*20*	*0.1*
Doors	*5*	*0.5*

(a) Calculate the rate of heat loss from the house when the interior temperature is 22°C and the exterior is –5°C.

(b) Suggest ways and means to reduce the heat loss and show quantitatively the effect of doubling the wall insulation and the substitution of double glazed windows (thermal resistance = 0.2 m^2 K/W) for the single glazed type in the table above.

GIVEN

- Small house
- Areas and thermal resistances shown in the table above
- Interior temperature = 22 °C
- Exterior temperature = –5 °C

FIND

(a) Heat loss from the house (q_a)

(b) Heat loss from the house with doubled wall insulation and double glazed windows (q_b). Suggest improvements.

ASSUMPTIONS

- All heat transfer can be treated as one dimensional
- Steady state has been reached
- The temperatures given are wall surface temperatures
- Infiltration is negligible
- The exterior temperature of the floor is the same as the rest of the house

SOLUTION

(a) The rate of heat transfer through each element of the house is given by *equations (1.33) and (1.34):*

$$q = \frac{\Delta T}{R_{th}}$$

The total rate of heat loss from the house is simply the sum of the loss through each element:

$$q = \Delta T\left(\frac{1}{\left(\frac{AR}{A}\right)_{wall}} + \frac{1}{\left(\frac{AR}{A}\right)_{ceiling}} + \frac{1}{\left(\frac{AR}{A}\right)_{floor}} + \frac{1}{\left(\frac{AR}{A}\right)_{windows}} + \frac{1}{\left(\frac{AR}{A}\right)_{doors}}\right)$$

$$q = (22°C - -5°C)\left(\frac{1}{\left(\frac{2.0\frac{m^2K}{W}}{150m^2}\right)} + \frac{1}{\left(\frac{2.8\frac{m^2K}{W}}{120m^2}\right)} + \frac{1}{\left(\frac{2.0\frac{m^2K}{W}}{120m^2}\right)} + \frac{1}{\left(\frac{0.1\frac{m^2K}{W}}{20m^2}\right)} + \frac{1}{\left(\frac{0.5\frac{m^2K}{W}}{5m^2}\right)}\right)$$

$$q = (22°C - -5°C)(75 + 42.8 + 60 + 200 + 10)\frac{W}{K}$$

$$q = 10{,}500\ W$$

(b) Doubling the resistance of the walls and windows and recalculating the total heat loss:

$$q = (22°C - -5°C)\left(\frac{1}{\left(\frac{4.0\frac{m^2K}{W}}{150m^2}\right)} + \frac{1}{\left(\frac{2.8\frac{m^2K}{W}}{120m^2}\right)} + \frac{1}{\left(\frac{2.0\frac{m^2K}{W}}{120m^2}\right)} + \frac{1}{\left(\frac{0.2\frac{m^2K}{W}}{20m^2}\right)} + \frac{1}{\left(\frac{0.5\frac{m^2K}{W}}{5m^2}\right)}\right)$$

$$q = (22°C - -5°C)(37.5 + 42.8 + 60 + 100 + 10)\frac{W}{K}$$

$$q = 6800\ W$$

Doubling the wall and window insulation led to a 35% reduction in the total rate of heat loss.

COMMENTS

Notice that the single glazed windows account for slightly over half of the total heat lost in case (a) and that the majority of the heat loss reduction in case (b) is due to the double glazed windows. Therefore double glazed windows are strongly suggested.

PROBLEM 1.9

Heat is transferred at a rate of 0.1 kW through glass wool insulation (density = 100 kg/m³) of 5 cm thickness and 2 m² area. If the hot surface is at 70°C, determine the temperature of the cooler surface.

GIVEN

- Glass wool insulation with a density (ρ) = 100 kg/m^3
- Thickness (L) = 5 cm = .05 m
- Area (A) = 2 m^2
- Temperature of the hot surface (T_h) = 70°C
- Rate of heat transfer (q_k)= 0.1 kW = 100 W

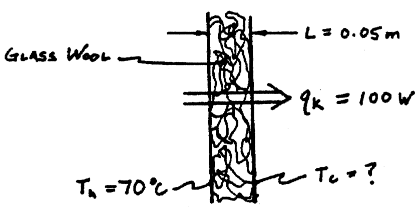
FIND

- The temperature of the cooler surface (T_c)

ASSUMPTIONS

- One dimensional, steady state conduction
- Constant thermal conductivity

SKETCH



PROPERTIES AND CONSTANTS

From *Appendix 2, Table 11:*
The thermal conductivity of glass wool at 20°C (k) = 0.036 W/m K

SOLUTION

For one dimensional, steady state conduction, the rate of heat transfer, from *equation (1.2),* is:

$$q_k = \frac{A\,k}{L}(T_h - T_c)$$

Solving this for T_c:

$$T_c = T_h - \frac{q_k L}{A\,k}$$

$$T_c = 70°\text{C} - \frac{(100\text{ W})(0.05\text{ m})}{(2\text{ m}^2)\left(0.036\frac{\text{W}}{\text{m K}}\right)}$$

$$T_c = 0.6°\text{C}$$

PROBLEM 1.10

A heat flux meter at the outer (cold) wall of a concrete building indicates that the heat loss through a wall of 10 cm thickness is 20 W/m². If a thermocouple at the inner surface of the wall indicates a temperature of 22°C while another at the outer surface shows 6°C , calculate the thermal conductivity of the concrete and compare your result with the value in Appendix 2, Table 11.

GIVEN

- Concrete wall
- Thickness (L) = 10 cm = 0.1 m
- Heat loss (q/A) = 20 W/m^2
- Surface temperatures: Inner (T_i) = 22°C
  Outer (T_o) = 6°C

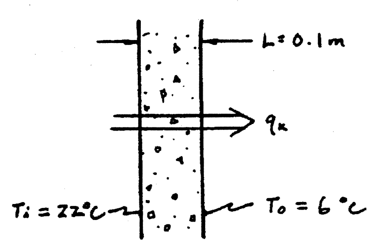
FIND

- The thermal conductivity (k) and compare it to the tabulated value

ASSUMPTIONS

- One dimensional heat flow through the wall
- Steady state conditions exist

SKETCH



SOLUTION:

The rate of heat transfer for steady state, one dimensional conduction, from *equation (1.2),* is:

$$q_k = \frac{kA}{L}(T_{hot} - T_{cold})$$

Solving for the thermal conductivity:

$$k = \left(\frac{q_k}{A}\right)\frac{L}{(T_i - T_o)}$$

$$k = \left(20\,\frac{W}{m^2}\right)\left(\frac{0.1\,m^2}{22°C - 6°C}\right) = 0.125\,\frac{W}{m\,K}$$

This result is very close to the tabulated value in *Appendix 2, Table 11* where the thermal conductivity of concrete is given as 0.128 W/m K.