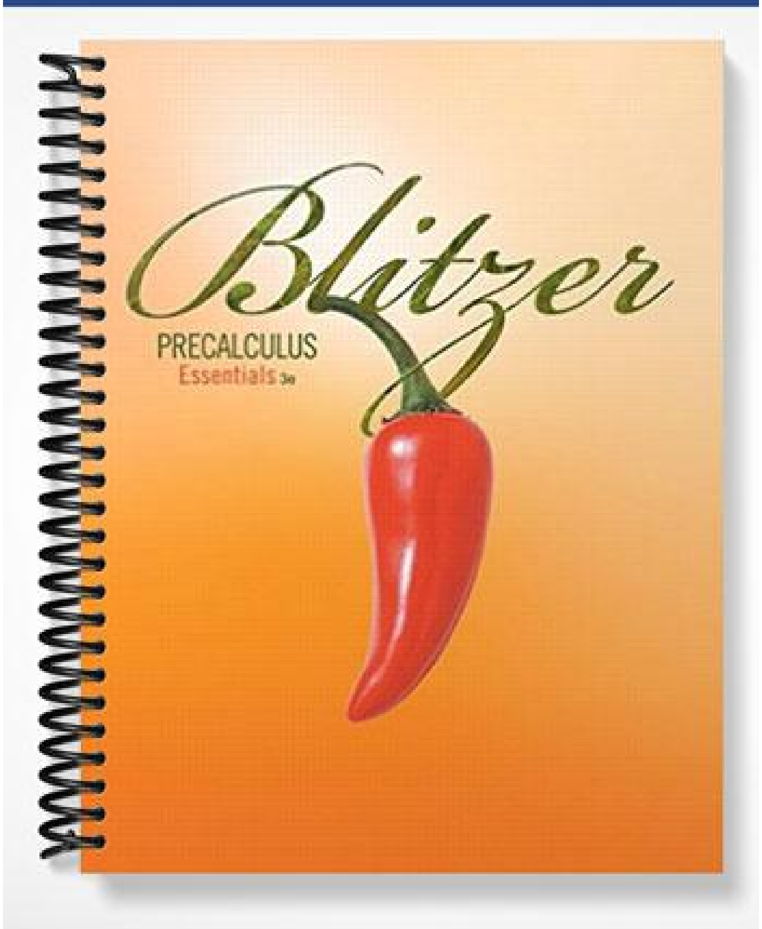
SOLUTIONS MANUAL



Chapter 2
Polynomial and Rational Functions

Section 2.1

Check Point Exercises

1. **a.**
$$\begin{aligned}&(5-2i)+(3+3i)\\&=5-2i+3+3i\\&=(5+3)+(-2+3)i\\&=8+i\end{aligned}$$

b.
$$\begin{aligned}&(2+6i)-(12-i)\\&=2+6i-12+i\\&=(2-12)+(6+1)i\\&=-10+7i\end{aligned}$$

2. **a.**
$$\begin{aligned}7i(2-9i)&=7i(2)-7i(9i)\\&=14i-63i^2\\&=14i-63(-1)\\&=63+14i\end{aligned}$$

b.
$$\begin{aligned}(5+4i)(6-7i)&=30-35i+24i-28i^2\\&=30-35i+24i-28(-1)\\&=30+28-35i+24i\\&=58-11i\end{aligned}$$

3.
$$\begin{aligned}\frac{5+4i}{4-i}&=\frac{5+4i}{4-i}\cdot\frac{4+i}{4+i}\\&=\frac{20+5i+16i+4i^2}{16+4i-4i-i^2}\\&=\frac{20+21i-4}{16+1}\\&=\frac{16+21i}{17}\\&=\frac{16}{17}+\frac{21}{17}i\end{aligned}$$

4. **a.**
$$\begin{aligned}\sqrt{-27}+\sqrt{-48}&=i\sqrt{27}+i\sqrt{48}\\&=i\sqrt{9\cdot 3}+i\sqrt{16\cdot 3}\\&=3i\sqrt{3}+4i\sqrt{3}\\&=7i\sqrt{3}\end{aligned}$$

b.
$$\begin{aligned}(-2+\sqrt{-3})^2&=(-2+i\sqrt{3})^2\\&=(-2)^2+2(-2)(i\sqrt{3})+(i\sqrt{3})^2\\&=4-4i\sqrt{3}+3i^2\\&=4-4i\sqrt{3}+3(-1)\\&=1-4i\sqrt{3}\end{aligned}$$

c.
$$\begin{aligned}\frac{-14+\sqrt{-12}}{2}&=\frac{-14+i\sqrt{12}}{2}\\&=\frac{-14+2i\sqrt{3}}{2}\\&=\frac{-14}{2}+\frac{2i\sqrt{3}}{2}\\&=-7+i\sqrt{3}\end{aligned}$$

5.
$$x^2-2x+2=0$$
$$a=1, b=-2, c=2$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(2)}}{2(1)}$$
$$x=\frac{2\pm\sqrt{4-8}}{2}$$
$$x=\frac{2\pm\sqrt{-4}}{2}$$
$$x=\frac{2\pm 2i}{2}$$
$$x=1\pm i$$
The solution set is $\{1+i, 1-i\}$.

Exercise Set 2.1

1. $(7 + 2i) + (1 - 4i) = 7 + 2i + 1 - 4i$
$= 7 + 1 + 2i - 4i$
$= 8 - 2i$

2. $(-2 + 6i) + (4 - i)$
$= -2 + 6i + 4 - i$
$= -2 + 4 + 6i - i$
$= 2 + 5i$

3. $(3 + 2i) - (5 - 7i) = 3 - 5 + 2i + 7i$
$= 3 + 2i - 5 + 7i$
$= -2 + 9i$

4. $(-7 + 5i) - (-9 - 11i) = -7 + 5i + 9 + 11i$
$= -7 + 9 + 5i + 11i$
$= 2 + 16i$

5. $6 - (-5 + 4i) - (-13 - i) = 6 + 5 - 4i + 13 + i$
$= 24 - 3i$

6. $7 - (-9 + 2i) - (-17 - i) = 7 + 9 - 2i + 17 + i$
$= 33 - i$

7. $8i - (14 - 9i) = 8i - 14 + 9i$
$= -14 + 8i + 9i$
$= -14 + 17i$

8. $15i - (12 - 11i) = 15i - 12 + 11i$
$= -12 + 15i + 11i$
$= -12 + 26i$

9. $-3i(7i - 5) = -21i^2 + 15i$
$= -21(-1) + 15i$
$= 21 + 15i$

10. $-8i\,(2i - 7) = -16i^2 + 56i = -16(-1) + 56i$
$= 9 - 25i^2 = 9 + 25 = 34 = 16 + 56i$

11. $(-5 + 4i)(3 + i) = -15 - 5i + 12i + 4i^2$
$= -15 + 7i - 4$
$= -19 + 7i$

12. $(-4 - 8i)(3 + i) = -12 - 4i - 24i - 8i^2$
$= -12 - 28i + 8$
$= -4 - 28i$

13. $(7 - 5i)(-2 - 3i) = -14 - 21i + 10i + 15i^2$
$= -14 - 15 - 11i$
$= -29 - 11i$

14. $(8 - 4i)(-3 + 9i) = -24 + 72i + 12i - 36i^2$
$= -24 + 36 + 84i$
$= 12 + 84i$

15. $(3 + 5i)(3 - 5i) = 9 - 15i + 15i - 25i^2$
$= 9 + 25$
$= 34$

16. $(2 + 7i)(2 - 7i) = 4 - 49i^2 = 4 + 49 = 53$

17. $(-5 + i)(-5 - i) = 25 + 5i - 5i - i^2$
$= 25 + 1$
$= 26$

18. $(-7 + i)(-7 - i) = 49 + 7i - 7i - i^2$
$= 49 + 1$
$= 50$

19. $(2 + 3i)^2 = 4 + 12i + 9i^2$
$= 4 + 12i - 9$
$= -5 + 12i$

20. $(5 - 2i)^2 = 25 - 20i + 4i^2$
$= 25 - 20i - 4$
$= 21 - 20i$

21. $\dfrac{2}{3 - i} = \dfrac{2}{3 - i} \cdot \dfrac{3 + i}{3 + i}$
$= \dfrac{2(3 + i)}{9 + 1}$
$= \dfrac{2(3 + i)}{10}$
$= \dfrac{3 + i}{5}$
$= \dfrac{3}{5} + \dfrac{1}{5}i$

22. $\dfrac{3}{4 + i} = \dfrac{3}{4 + i} \cdot \dfrac{4 - i}{4 - i}$
$= \dfrac{3(4 - i)}{16 - i^2}$
$= \dfrac{3(4 - i)}{17}$
$= \dfrac{12}{17} - \dfrac{3}{17}i$

23. $\frac{2i}{1+i}=\frac{2i}{1+i}\cdot\frac{1-i}{1-i}=\frac{2i-2i^2}{1+1}=\frac{2+2i}{2}=1+i$

24. $\frac{5i}{2-i}=\frac{5i}{2-i}\cdot\frac{2+i}{2+i}$
$=\frac{10i+5i^2}{4+1}$
$=\frac{-5+10i}{5}$
$=-1+2i$

25. $\frac{8i}{4-3i}=\frac{8i}{4-3i}\cdot\frac{4+3i}{4+3i}$
$=\frac{32i+24i^2}{16+9}$
$=\frac{-24+32i}{25}$
$=-\frac{24}{25}+\frac{32}{25}i$

26. $\frac{-6i}{3+2i}=\frac{-6i}{3+2i}\cdot\frac{3-2i}{3-2i}=\frac{-18i+12i^2}{9+4}$
$=\frac{-12-18i}{13}=-\frac{12}{13}-\frac{18}{13}i$

27. $\frac{2+3i}{2+i}=\frac{2+3i}{2+i}\cdot\frac{2-i}{2-i}$
$=\frac{4+4i-3i^2}{4+1}$
$=\frac{7+4i}{5}$
$=\frac{7}{5}+\frac{4}{5}i$

28. $\frac{3-4i}{4+3i}=\frac{3-4i}{4+3i}\cdot\frac{4-3i}{4-3i}$
$=\frac{12-25i+12i^2}{16+9}$
$=\frac{-25i}{25}$
$=-i$

29. $\sqrt{-64}-\sqrt{-25}=i\sqrt{64}-i\sqrt{25}$
$=8i-5i=3i$

30. $\sqrt{-81}-\sqrt{-144}=i\sqrt{81}-i\sqrt{144}=9i-12i$
$=-3i$

31. $5\sqrt{-16}+3\sqrt{-81}=5(4i)+3(9i)$
$=20i+27i=47i$

32. $5\sqrt{-8}+3\sqrt{-18}$
$=5i\sqrt{8}+3i\sqrt{18}=5i\sqrt{4\cdot 2}+3i\sqrt{9\cdot 2}$
$=10i\sqrt{2}+9i\sqrt{2}$
$=19i\sqrt{2}$

33. $\left(-2+\sqrt{-4}\right)^2=\left(-2+2i\right)^2$
$=4-8i+4i^2$
$=4-8i-4$
$=-8i$

34. $\left(-5-\sqrt{-9}\right)^2=(-5-i\sqrt{9})^2=\left(-5-3i\right)^2$
$=25+30i+9i^2$
$=25+30i-9$
$=16+30i$

35. $\left(-3-\sqrt{-7}\right)^2=\left(-3-i\sqrt{7}\right)^2$
$=9+6i\sqrt{7}+i^2(7)$
$=9-7+6i\sqrt{7}$
$=2+6i\sqrt{7}$

36. $\left(-2+\sqrt{-11}\right)^2=\left(-2+i\sqrt{11}\right)^2$
$=4-4i\sqrt{11}+i^2(11)$
$=4-11-4i\sqrt{11}$
$=-7-4i\sqrt{11}$

37. $\frac{-8+\sqrt{-32}}{24}=\frac{-8+i\sqrt{32}}{24}$
$=\frac{-8+i\sqrt{16\cdot 2}}{24}$
$=\frac{-8+4i\sqrt{2}}{24}$
$=-\frac{1}{3}+\frac{\sqrt{2}}{6}i$

38. $\frac{-12+\sqrt{-28}}{32}=\frac{-12+i\sqrt{28}}{32}=\frac{-12+i\sqrt{4\cdot 7}}{32}$
$=\frac{-12+2i\sqrt{7}}{32}=-\frac{3}{8}+\frac{\sqrt{7}}{16}i$

39. $\frac{-6-\sqrt{-12}}{48}=\frac{-6-i\sqrt{12}}{48}$
$=\frac{-6-i\sqrt{4\cdot 3}}{48}$
$=\frac{-6-2i\sqrt{3}}{48}$
$=-\frac{1}{8}-\frac{\sqrt{3}}{24}i$

40. $\frac{-15-\sqrt{-18}}{33}=\frac{-15-i\sqrt{18}}{33}=\frac{-15-i\sqrt{9\cdot 2}}{33}$
$=\frac{-15-3i\sqrt{2}}{33}=-\frac{5}{11}-\frac{\sqrt{2}}{11}i$

41. $\sqrt{-8}\left(\sqrt{-3}-\sqrt{5}\right)=i\sqrt{8}(i\sqrt{3}-\sqrt{5})$
$=2i\sqrt{2}\left(i\sqrt{3}-\sqrt{5}\right)$
$=-2\sqrt{6}-2i\sqrt{10}$

42. $\sqrt{-12}\left(\sqrt{-4}-\sqrt{2}\right)=i\sqrt{12}(i\sqrt{4}-\sqrt{2})$
$=2i\sqrt{3}\left(2i-\sqrt{2}\right)$
$=4i^2\sqrt{3}-2i\sqrt{6}$
$=-4\sqrt{3}-2i\sqrt{6}$

43. $\left(3\sqrt{-5}\right)\left(-4\sqrt{-12}\right)=\left(3i\sqrt{5}\right)\left(-8i\sqrt{3}\right)$
$=-24i^2\sqrt{15}$
$=24\sqrt{15}$

44. $\left(3\sqrt{-7}\right)\left(2\sqrt{-8}\right)$
$=(3i\sqrt{7})(2i\sqrt{8})=(3i\sqrt{7})(2i\sqrt{4\cdot 2})$
$=\left(3i\sqrt{7}\right)\left(4i\sqrt{2}\right)=12i^2\sqrt{14}=-12\sqrt{14}$

45. $x^2-6x+10=0$
$x=\frac{6\pm\sqrt{(-6)^2-4(1)(10)}}{2(1)}$
$x=\frac{6\pm\sqrt{36-40}}{2}$
$x=\frac{6\pm\sqrt{-4}}{2}$
$x=\frac{6\pm 2i}{2}$
$x=3\pm i$
The solution set is $\{3+i, 3-i\}$.

46. $x^2-2x+17=0$
$x=\frac{2\pm\sqrt{(-2)^2-4(1)(17)}}{2(1)}$
$x=\frac{2\pm\sqrt{4-68}}{2}$
$x=\frac{2\pm\sqrt{-64}}{2}$
$x=\frac{2\pm 8i}{2}$
$x=1\pm 4i$
The solution set is $\{1+4i, 1-4i\}$.

47. $4x^2+8x+13=0$
$x=\frac{-8\pm\sqrt{8^2-4(4)(13)}}{2(4)}$
$=\frac{-8\pm\sqrt{64-208}}{8}$
$=\frac{-8\pm\sqrt{-144}}{8}$
$=\frac{-8\pm 12i}{8}$
$=\frac{4(-2\pm 3i)}{8}$
$=\frac{-2\pm 3i}{2}$
$=-1\pm\frac{3}{2}i$

The solution set is $\left\{-1+\frac{3}{2}i, -1-\frac{3}{2}i\right\}$.

48. $2x^2 + 2x + 3 = 0$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(2)(3)}}{2(2)}$$
$$= \frac{-2 \pm \sqrt{4-24}}{4}$$
$$= \frac{-2 \pm \sqrt{-20}}{4}$$
$$= \frac{-2 \pm 2i\sqrt{5}}{4}$$
$$= \frac{2(-1 \pm i\sqrt{5})}{4}$$
$$= \frac{-1 \pm i\sqrt{5}}{2}$$
$$= -\frac{1}{2} \pm \frac{\sqrt{5}}{2} i$$

The solution set is $\left\{-\frac{1}{2} + \frac{\sqrt{5}}{2} i, -\frac{1}{2} - \frac{\sqrt{5}}{2} i\right\}$.

49. $3x^2 - 8x + 7 = 0$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(7)}}{2(3)}$$
$$= \frac{8 \pm \sqrt{64-84}}{6}$$
$$= \frac{8 \pm \sqrt{-20}}{6}$$
$$= \frac{8 \pm 2i\sqrt{5}}{6}$$
$$= \frac{2(4 \pm i\sqrt{5})}{6}$$
$$= \frac{4 \pm i\sqrt{5}}{3}$$
$$= \frac{4}{3} \pm \frac{\sqrt{5}}{3} i$$

The solution set is $\left\{\frac{4}{3} + \frac{\sqrt{5}}{3} i, \frac{4}{3} - \frac{\sqrt{5}}{3} i\right\}$.

50. $3x^2 - 4x + 6 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(6)}}{2(3)}$$
$$= \frac{4 \pm \sqrt{16-72}}{6}$$
$$= \frac{4 \pm \sqrt{-56}}{6}$$
$$= \frac{4 \pm 2i\sqrt{14}}{6}$$
$$= \frac{2(2 \pm i\sqrt{14})}{6}$$
$$= \frac{2 \pm i\sqrt{14}}{3}$$
$$= \frac{2}{3} \pm \frac{\sqrt{14}}{3} i$$

The solution set is $\left\{\frac{2}{3} + \frac{\sqrt{14}}{3} i, \frac{2}{3} - \frac{\sqrt{14}}{3} i\right\}$.

51. $(2-3i)(1-i) - (3-i)(3+i)$

$$= (2 - 2i - 3i + 3i^2) - (3^2 - i^2)$$
$$= 2 - 5i + 3i^2 - 9 + i^2$$
$$= -7 - 5i + 4i^2$$
$$= -7 - 5i + 4(-1)$$
$$= -11 - 5i$$

52. $(8+9i)(2-i) - (1-i)(1+i)$

$$= (16 - 8i + 18i - 9i^2) - (1^2 - i^2)$$
$$= 16 + 10i - 9i^2 - 1 + i^2$$
$$= 15 + 10i - 8i^2$$
$$= 15 + 10i - 8(-1)$$
$$= 23 + 10i$$

53. $(2+i)^2 - (3-i)^2$

$$= (4 + 4i + i^2) - (9 - 6i + i^2)$$
$$= 4 + 4i + i^2 - 9 + 6i - i^2$$
$$= -5 + 10i$$

54. $(4-i)^2-(1+2i)^2$
$=(16-8i+i^2)-(1+4i+4i^2)$
$=16-8i+i^2-1-4i-4i^2$
$=15-12i-3i^2$
$=15-12i-3(-1)$
$=18-12i$

55. $5\sqrt{-16}+3\sqrt{-81}$
$=5\sqrt{16}\sqrt{-1}+3\sqrt{81}\sqrt{-1}$
$=5\cdot 4i+3\cdot 9i$
$=20i+27i$
$=47i$ or $0+47i$

56. $5\sqrt{-8}+3\sqrt{-18}$
$=5\sqrt{4}\sqrt{2}\sqrt{-1}+3\sqrt{9}\sqrt{2}\sqrt{-1}$
$=5\cdot 2\sqrt{2}\,i+3\cdot 3\sqrt{2}\,i$
$=10i\sqrt{2}+9i\sqrt{2}$
$=(10+9)i\sqrt{2}$
$=19i\sqrt{2}$ or $0+19i\sqrt{2}$

57. $f(x)=x^2-2x+2$
$f(1+i)=(1+i)^2-2(1+i)+2$
$=1+2i+i^2-2-2i+2$
$=1+i^2$
$=1-1$
$=0$

58. $f(x)=x^2-2x+5$
$f(1-2i)=(1-2i)^2-2(1-2i)+5$
$=1-4i+4i^2-2+4i+5$
$=4+4i^2$
$=4-4$
$=0$

59. $f(x)=\dfrac{x^2+19}{2-x}$
$f(3i)=\dfrac{(3i)^2+19}{2-3i}$
$=\dfrac{9i^2+19}{2-3i}$
$=\dfrac{-9+19}{2-3i}$
$=\dfrac{10}{2-3i}$
$=\dfrac{10}{2-3i}\cdot\dfrac{2+3i}{2+3i}$
$=\dfrac{20+30i}{4-9i^2}$
$=\dfrac{20+30i}{4+9}$
$=\dfrac{20+30i}{13}$
$=\dfrac{20}{13}+\dfrac{30}{13}i$

60. $f(x)=\dfrac{x^2+11}{3-x}$
$f(4i)=\dfrac{(4i)^2+11}{3-4i}=\dfrac{16i^2+11}{3-4i}$
$=\dfrac{-16+11}{3-4i}$
$=\dfrac{-5}{3-4i}$
$=\dfrac{-5}{3-4i}\cdot\dfrac{3+4i}{3+4i}$
$=\dfrac{-15-20i}{9-16i^2}$
$=\dfrac{-15-20i}{9+16}$
$=\dfrac{-15-20i}{25}$
$=\dfrac{-15}{25}-\dfrac{20}{25}i$
$=-\dfrac{3}{5}-\dfrac{4}{5}i$

61. $E = IR = (4-5i)(3+7i)$
$= 12 + 28i - 15i - 35i^2$
$= 12 + 13i - 35(-1)$
$= 12 + 35 + 13i = 47 + 13i$
The voltage of the circuit is
(47 + 13i) volts.

62. $E = IR = (2-3i)(3+5i)$
$= 6 + 10i - 9i - 15i^2 = 6 + i - 15(-1)$
$= 6 + i + 15 = 21 + i$
The voltage of the circuit is $(21+i)$ volts.

63. Sum:
$(5 + i\sqrt{15}) + (5 - i\sqrt{15})$
$= 5 + i\sqrt{15} + 5 - i\sqrt{15}$
$= 5 + 5$
$= 10$
Product:
$(5 + i\sqrt{15})(5 - i\sqrt{15})$
$= 25 - 5i\sqrt{15} + 5i\sqrt{15} - 15i^2$
$= 25 + 15$
$= 40$

64. – 72. Answers may vary.

73. makes sense

74. does not make sense; Explanations will vary. Sample explanation: Imaginary numbers are not undefined.

75. does not make sense; Explanations will vary. Sample explanation: $i = \sqrt{-1}$; It is not a variable in this context.

76. makes sense

77. false; Changes to make the statement true will vary. A sample change is: All irrational numbers are complex numbers.

78. false; Changes to make the statement true will vary. A sample change is: (3 + 7i)(3 – 7i) = 9 + 49 = 58 which is a real9number.

79. false; Changes to make the statement true will vary. A sample change is:
$$\frac{7+3i}{5+3i} = \frac{7+3i}{5+3i} \cdot \frac{5-3i}{5-3i} = \frac{44-6i}{34} = \frac{22}{17} - \frac{3}{17}i$$

80. true

81.
$$\frac{4}{(2+i)(3-i)} = \frac{4}{6-2i+3i-i^2}$$
$$= \frac{4}{6+i+1}$$
$$= \frac{4}{7+i}$$
$$= \frac{4}{7+i} \cdot \frac{7-i}{7-i}$$
$$= \frac{28-4i}{49-i^2}$$
$$= \frac{28-4i}{49+1}$$
$$= \frac{28-4i}{50}$$
$$= \frac{28}{50} - \frac{4}{50}i$$
$$= \frac{14}{25} - \frac{2}{25}i$$

82.
$$\frac{1+i}{1+2i} + \frac{1-i}{1-2i}$$
$$= \frac{(1+i)(1-2i)}{(1+2i)(1-2i)} + \frac{(1-i)(1+2i)}{(1+2i)(1-2i)}$$
$$= \frac{(1+i)(1-2i) + (1-i)(1+2i)}{(1+2i)(1-2i)}$$
$$= \frac{1-2i+i-2i^2+1+2i-i-2i^2}{1-4i^2}$$
$$= \frac{1-2i+i+2+1+2i-i+2}{1+4}$$
$$= \frac{6}{5}$$
$$= \frac{6}{5} + 0i$$

83. $\dfrac{8}{1+\frac{2}{i}} = \dfrac{8}{\frac{i}{i}+\frac{2}{i}}$

$= \dfrac{8}{\frac{2+i}{i}}$

$= \dfrac{8i}{2+i}$

$= \dfrac{8i}{2+i}\cdot\dfrac{2-i}{2-i}$

$= \dfrac{16i-8i^2}{4-i^2}$

$= \dfrac{16i+8}{4+1}$

$= \dfrac{8+16i}{5}$

$= \dfrac{8}{5}+\dfrac{16}{5}i$

84. $0 = -2(x-3)^2+8$

$2(x-3)^2 = 8$

$(x-3)^2 = 4$

$x-3 = \pm\sqrt{4}$

$x = 3\pm 2$

$x = 1, 5$

85. $-x^2-2x+1=0$

$x^2+2x-1=0$

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \dfrac{-(-2)\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}$

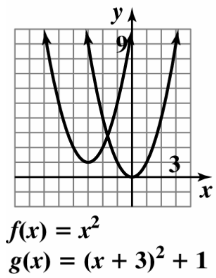
$= \dfrac{2\pm\sqrt{8}}{2}$

$= \dfrac{2\pm 2\sqrt{2}}{2}$

$= 1\pm\sqrt{2}$

The solution set is $\{1\pm\sqrt{2}\}$.

86. The graph of g is the graph of f shifted 1 unit up and 3 units to the left.



Section 2.2

Check Point Exercises

1. $f(x) = -(x-1)^2+4$

$f(x) = \overset{a=-1}{-}\left(x-\overset{h=1}{1}\right)^2+\overset{k=4}{4}$

Step 1: The parabola opens down because $a<0$.
Step 2: find the vertex: (1, 4)
Step 3: find the x-intercepts:

$0 = -(x-1)^2+4$

$(x-1)^2 = 4$

$x-1 = \pm 2$

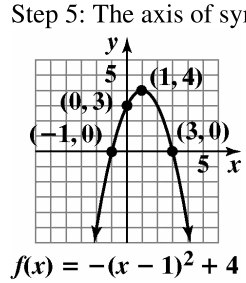
$x = 1\pm 2$

$x = 3$ or $x = -1$

Step 4: find the y-intercept:

$f(0) = -(0-1)^2+4 = 3$

Step 5: The axis of symmetry is $x = 1$.



2. $f(x) = (x-2)^2 + 1$

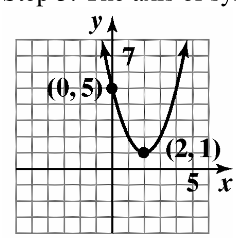
Step 1: The parabola opens up because $a > 0$.
Step 2: find the vertex: (2, 1)
Step 3: find the x-intercepts:

$$0 = (x-2)^2 + 1$$
$$(x-2)^2 = -1$$
$$x - 2 = \sqrt{-1}$$
$$x = 2 \pm i$$

The equation has no real roots, thus the parabola has no x-intercepts.
Step 4: find the y-intercept:

$$f(0) = (0-2)^2 + 1 = 5$$

Step 5: The axis of symmetry is $x = 2$.



$f(x) = (x-2)^2 + 1$

3. $f(x) = -x^2 + 4x + 1$

Step 1: The parabola opens down because $a < 0$.
Step 2: find the vertex:

$$x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$$
$$f(2) = -2^2 + 4(2) + 1 = 5$$

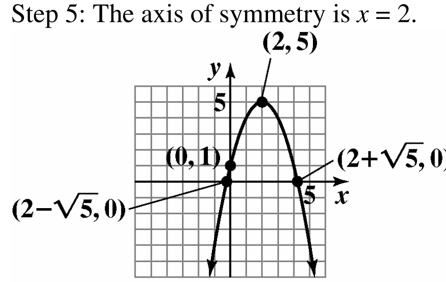
The vertex is (2, 5).
Step 3: find the x-intercepts:

$$0 = -x^2 + 4x + 1$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(1)}}{2(-1)}$$
$$x = \frac{-4 \pm \sqrt{20}}{-2}$$
$$x = 2 \pm \sqrt{5}$$

The x-intercepts are $x \approx -0.2$ and $x \approx -4.2$.
Step 4: find the y-intercept:

$$f(0) = -0^2 + 4(0) + 1 = 1$$

Step 5: The axis of symmetry is $x = 2$.



$f(x) = -x^2 + 4x + 1$

4. $f(x) = 4x^2 - 16x + 1000$

a. $a = 4$. The parabola opens upward and has a minimum value.

b. $x = \frac{-b}{2a} = \frac{16}{8} = 2$

$f(2) = 4(2)^2 - 16(2) + 1000 = 984$

The minimum point is 984 at $x = 2$.

c. domain: $(-\infty, \infty)$ range: $[984, \infty)$

5. $y = -0.005x^2 + 2x + 5$

a. The information needed is found at the vertex.
x-coordinate of vertex

$$x = \frac{-b}{2a} = \frac{-2}{2(-0.005)} = 200$$

y-coordinate of vertex

$$y = -0.005(200)^2 + 2(200) + 5 = 205$$

The vertex is (200,205).
The maximum height of the arrow is 205 feet. This occurs 200 feet from its release.

b. The arrow will hit the ground when the height reaches 0.

$$y = -0.005x^2 + 2x + 5$$

$$0 = -0.005x^2 + 2x + 5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(-0.005)(5)}}{2(-0.005)}$$

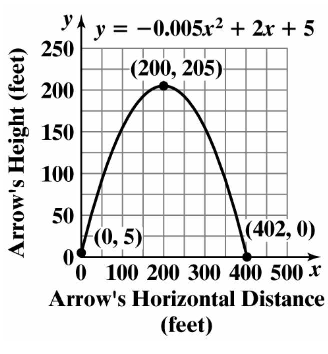
$x \approx -2$ or $x \approx 402$

The arrow travels 402 feet before hitting the ground.

c. The starting point occurs when $x = 0$. Find the corresponding y-coordinate.

$$y = -0.005(0)^2 + 2(0) + 5 = 5$$

Plot $(0,5)$, $(402,0)$, and $(200,205)$, and connect them with a smooth curve.



6. Let x = one of the numbers;
$x - 8$ = the other number.
The product is $f(x) = x(x-8) = x^2 - 8x$
The x-coordinate of the minimum is

$$x = -\frac{b}{2a} = -\frac{-8}{2(1)} = -\frac{-8}{2} = 4.$$

$$f(4) = (4)^2 - 8(4)$$
$$= 16 - 32 = -16$$

The vertex is $(4,-16)$.
The minimum product is -16. This occurs when the two number are 4 and $4 - 8 = -4$.

7. Maximize the area of a rectangle constructed with 120 feet of fencing.
Let x = the length of the rectangle. Let y = the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express y in terms of x.

$$2x + 2y = 120$$
$$2y = 120 - 2x$$
$$y = \frac{120 - 2x}{2} = 60 - x$$

We need to maximize $A = xy = x(60-x)$. Rewrite A as a function of x.

$$A(x) = x(60-x) = -x^2 + 60x$$

Since $a = -1$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{60}{2(-1)} = -\frac{60}{-2} = 30.$$

When the length x is 30, the width y is $y = 60 - x = 60 - 30 = 30$.
The dimensions of the rectangular region with maximum area are 30 feet by 30 feet. This gives an area of $30 \cdot 30 = 900$ square feet.

Exercise Set 2.2

1. vertex: (1, 1)
$h(x) = (x-1)^2 + 1$

2. vertex: (–1, 1)
$g(x) = (x+1)^2 + 1$

3. vertex: (1, –1)
$j(x) = (x-1)^2 - 1$

4. vertex: (–1, –1)
$f(x) = (x+1)^2 - 1$

5. The graph is $f(x) = x^2$ translated down one.
$h(x) = x^2 - 1$

6. The point (–1, 0) is on the graph and $f(-1) = 0$. $f(x) = x^2 + 2x + 1$

7. The point (1, 0) is on the graph and $g(1) = 0$. $g(x) = x^2 - 2x + 1$

8. The graph is $f(x) = -x^2$ translated down one.
$j(x) = -x^2 - 1$

9. $f(x) = 2(x-3)^2 + 1$
$h = 3, k = 1$
The vertex is at (3, 1).

10. $f(x) = -3(x-2)^2 + 12$
$h = 2, k = 12$
The vertex is at (2, 12).

11. $f(x) = -2(x+1)^2 + 5$
$h = -1, k = 5$
The vertex is at (–1, 5).

12. $f(x) = -2(x+4)^2 - 8$
$h = -4, k = -8$
The vertex is at (–4, –8).

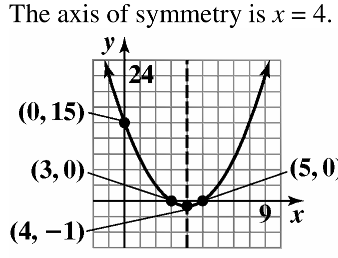
13. $f(x) = 2x^2 - 8x + 3$
$x = \frac{-b}{2a} = \frac{8}{4} = 2$
$f(2) = 2(2)^2 - 8(2) + 3$
$= 8 - 16 + 3 = -5$
The vertex is at (2, –5).

14. $f(x) = 3x^2 - 12x + 1$
$x = \frac{-b}{2a} = \frac{12}{6} = 2$
$f(2) = 3(2)^2 - 12(2) + 1$
$= 12 - 24 + 1 = -11$
The vertex is at (2, –11).

15. $f(x) = -x^2 - 2x + 8$
$x = \frac{-b}{2a} = \frac{2}{-2} = -1$
$f(-1) = -(-1)^2 - 2(-1) + 8$
$= -1 + 2 + 8 = 9$
The vertex is at (–1, 9).

16. $f(x) = -2x^2 + 8x - 1$
$x = \frac{-b}{2a} = \frac{-8}{-4} = 2$
$f(2) = -2(2)^2 + 8(2) - 1$
$= -8 + 16 - 1 = 7$
The vertex is at (2, 7).

17. $f(x) = (x-4)^2 - 1$
vertex: (4, –1)
x-intercepts:
$0 = (x-4)^2 - 1$
$1 = (x-4)^2$
$\pm 1 = x - 4$
$x = 3$ or $x = 5$
y-intercept:
$f(0) = (0-4)^2 - 1 = 15$
The axis of symmetry is $x = 4$.



$f(x) = (x-4)^2 - 1$

domain: $(-\infty, \infty)$
range: $[-1, \infty)$

18. $f(x)=(x-1)^2-2$

vertex: (1, –2)
x-intercepts:

$0=(x-1)^2-2$

$(x-1)^2=2$

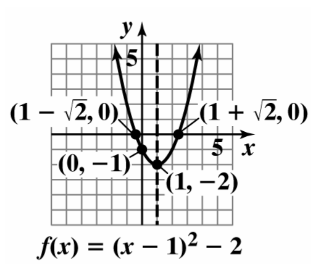
$x-1=\pm\sqrt{2}$

$x=1\pm\sqrt{2}$

y-intercept:

$f(0)=(0-1)^2-2\ =-1$

The axis of symmetry is $x = 1$.



domain: $(-\infty,\infty)$

range: $[-2,\infty)$

19. $f(x)=(x-1)^2+2$

vertex: (1, 2)
x-intercepts:

$0=(x-1)^2+2$

$(x-1)^2=-2$

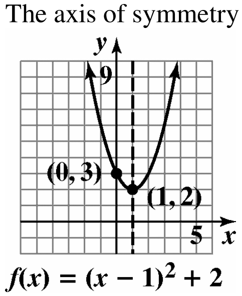
$x-1=\pm\sqrt{-2}$

$x=1\pm i\sqrt{2}$

No x-intercepts.
y-intercept:

$f(0)=(0-1)^2+2=3$

The axis of symmetry is $x = 1$.



domain: $(-\infty,\infty)$

range: $[2,\infty)$

20. $f(x)=(x-3)^2+2$

vertex: (3, 2)
x-intercepts:

$0=(x-3)^2+2$

$(x-3)^2=-2$

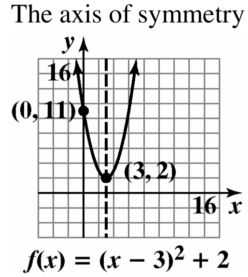
$x-3=\pm i\sqrt{2}$

$x=3\pm i\sqrt{2}$

No x-intercepts.
y-intercept:

$f(0)=(0-3)^2+2=11$

The axis of symmetry is $x = 3$.



domain: $(-\infty,\infty)$

range: $[2,\infty)$

21. $y-1=(x-3)^2$

$y=(x-3)^2+1$

vertex: (3, 1)
x-intercepts:

$0=(x-3)^2+1$

$(x-3)^2=-1$

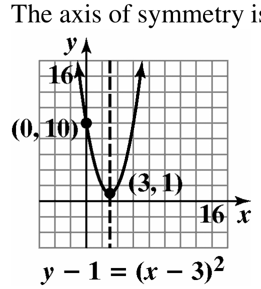
$x-3=\pm i$

$x=3\pm i$

No x-intercepts.
y-intercept: 10

$y=(0-3)^2+1=10$

The axis of symmetry is $x = 3$.



domain: $(-\infty,\infty)$

range: $[1,\infty)$

22. $y-3=(x-1)^2$

$y=(x-1)^2+3$

vertex: (1, 3)

x-intercepts:

$0=(x-1)^2+3$

$(x-1)^2=-3$

$x-1=\pm i\sqrt{3}$

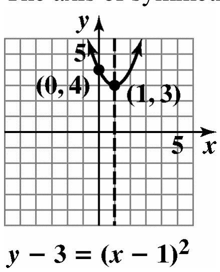
$x=1\pm i\sqrt{3}$

No x-intercepts

y-intercept:

$y=(0-1)^2+3=4$

The axis of symmetry is $x = 1$.



domain: $(-\infty,\infty)$

range: $[3,\infty)$

23. $f(x)=2(x+2)^2-1$

vertex: (–2, –1)

x-intercepts:

$0=2(x+2)^2-1$

$2(x+2)^2=1$

$(x+2)^2=\frac{1}{2}$

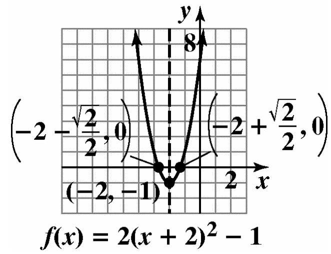
$x+2=\pm\frac{1}{\sqrt{2}}$

$x=-2\pm\frac{1}{\sqrt{2}}=-2\pm\frac{\sqrt{2}}{2}$

y-intercept:

$f(0)=2(0+2)^2-1=7$

The axis of symmetry is $x = –2$.



domain: $(-\infty,\infty)$

range: $[-1,\infty)$

24. $f(x)=\frac{5}{4}-\left(x-\frac{1}{2}\right)^2$

$f(x)=-\left(x-\frac{1}{2}\right)^2+\frac{5}{4}$

vertex: $\left(\frac{1}{2},\frac{5}{4}\right)$

x-intercepts:

$0=-\left(x-\frac{1}{2}\right)^2+\frac{5}{4}$

$\left(x-\frac{1}{2}\right)^2=\frac{5}{4}$

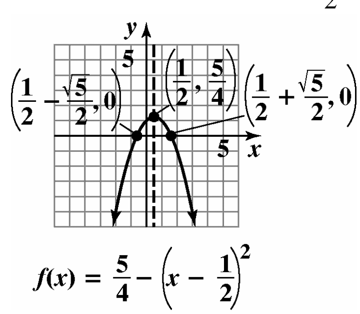
$x-\frac{1}{2}=\pm\frac{\sqrt{5}}{2}$

$x=\frac{1\pm\sqrt{5}}{2}$

y-intercept:

$f(0)=-\left(0-\frac{1}{2}\right)^2+\frac{5}{4}=1$

The axis of symmetry is $x=\frac{1}{2}$.



domain: $(-\infty,\infty)$

range: $\left(-\infty,\frac{5}{4}\right]$

25. $f(x) = 4 - (x-1)^2$

$f(x) = -(x-1)^2 + 4$

vertex: (1, 4)

x-intercepts:

$0 = -(x-1)^2 + 4$

$(x-1)^2 = 4$

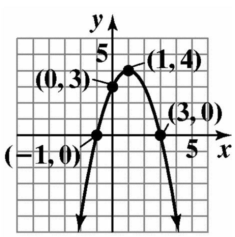
$x - 1 = \pm 2$

$x = -1$ or $x = 3$

y-intercept:

$f(x) = -(0-1)^2 + 4 = 3$

The axis of symmetry is $x = 1$.



$f(x) = 4 - (x - 1)^2$

domain: $(-\infty, \infty)$

range: $(-\infty, 4]$

26. $f(x) = 1 - (x-3)^2$

$f(x) = -(x-3)^2 + 1$

vertex: (3, 1)

x-intercepts:

$0 = -(x-3)^2 + 1$

$(x-3)^2 = 1$

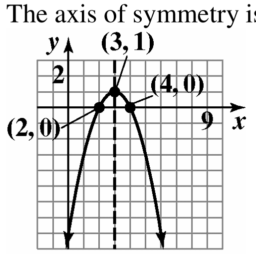
$x - 3 = \pm 1$

$x = 2$ or $x = 4$

y-intercept:

$f(0) = -(0-3)^2 + 1 = -8$

The axis of symmetry is $x = 3$.



$f(x) = 1 - (x - 3)^2$

domain: $(-\infty, \infty)$

range: $(-\infty, 1]$

27. $f(x) = x^2 - 2x - 3$

$f(x) = (x^2 - 2x + 1) - 3 - 1$

$f(x) = (x-1)^2 - 4$

vertex: (1, –4)

x-intercepts:

$0 = (x-1)^2 - 4$

$(x-1)^2 = 4$

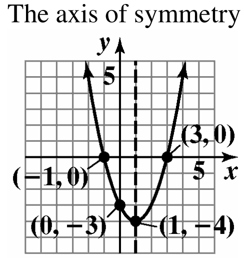
$x - 1 = \pm 2$

$x = -1$ or $x = 3$

y-intercept: –3

$f(0) = 0^2 - 2(0) - 3 = -3$

The axis of symmetry is $x = 1$.



$f(x) = x^2 + 3x - 10$

domain: $(-\infty, \infty)$

range: $[-4, \infty)$

28. $f(x) = x^2 - 2x - 15$

$f(x) = (x^2 - 2x + 1) - 15 - 1$

$f(x) = (x-1)^2 - 16$

vertex: (1, –16)

x-intercepts:

$0 = (x-1)^2 - 16$

$(x-1)^2 = 16$

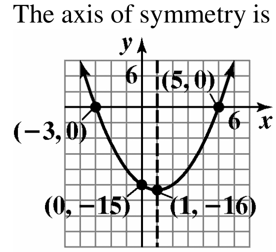
$x - 1 = \pm 4$

$x = -3$ or $x = 5$

y-intercept:

$f(0) = 0^2 - 2(0) - 15 = -15$

The axis of symmetry is $x = 1$.



$f(x) = x^2 - 2x - 15$

domain: $(-\infty, \infty)$

range: $[-16, \infty)$

29. $f(x) = x^2 + 3x - 10$

$f(x) = \left(x^2 + 3x + \frac{9}{4}\right) - 10 - \frac{9}{4}$

$f(x) = \left(x + \frac{3}{2}\right)^2 - \frac{49}{4}$

vertex: $\left(-\frac{3}{2}, -\frac{49}{4}\right)$

x-intercepts:

$0 = \left(x + \frac{3}{2}\right)^2 - \frac{49}{4}$

$\left(x + \frac{3}{2}\right)^2 = \frac{49}{4}$

$x + \frac{3}{2} = \pm\frac{7}{2}$

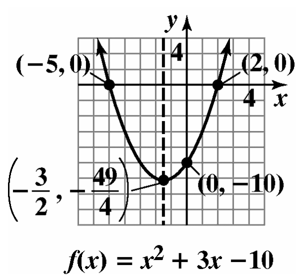
$x = -\frac{3}{2} \pm \frac{7}{2}$

$x = 2$ or $x = -5$

y-intercept:

$f(x) = 0^2 + 3(0) - 10 = -10$

The axis of symmetry is $x = -\frac{3}{2}$.



domain: $(-\infty, \infty)$

range: $\left[-\frac{49}{4}, \infty\right)$

30. $f(x) = 2x^2 - 7x - 4$

$f(x) = 2\left(x^2 - \frac{7}{2}x + \frac{49}{16}\right) - 4 - \frac{49}{8}$

$f(x) = 2\left(x - \frac{7}{4}\right)^2 - \frac{81}{8}$

vertex: $\left(\frac{7}{4}, -\frac{81}{8}\right)$

x-intercepts:

$0 = 2\left(x - \frac{7}{4}\right)^2 - \frac{81}{8}$

$2\left(x - \frac{7}{4}\right)^2 = \frac{81}{8}$

$\left(x - \frac{7}{4}\right)^2 = \frac{81}{16}$

$x - \frac{7}{4} = \pm\frac{9}{4}$

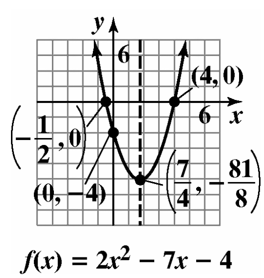
$x = \frac{7}{4} \pm \frac{9}{4}$

$x = -\frac{1}{2}$ or $x = 4$

y-intercept:

$f(0) = 2(0)^2 - 7(0) - 4 = -4$

The axis of symmetry is $x = \frac{7}{4}$.



domain: $(-\infty, \infty)$

range: $\left[-\frac{81}{8}, \infty\right)$

31. $f(x) = 2x - x^2 + 3$

$f(x) = -x^2 + 2x + 3$

$f(x) = -\left(x^2 - 2x + 1\right) + 3 + 1$

$f(x) = -\left(x - 1\right)^2 + 4$

vertex: (1, 4)

x-intercepts:

$0 = -\left(x-1\right)^2 + 4$

$\left(x-1\right)^2 = 4$

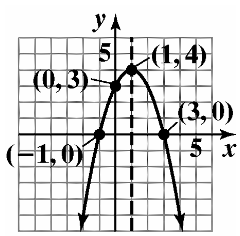
$x - 1 = \pm 2$

$x = -1$ or $x = 3$

y-intercept:

$f\left(0\right) = 2\left(0\right) - \left(0\right)^2 + 3 = 3$

The axis of symmetry is $x = 1$.



$f(x) = 2x - x^2 + 3$

domain: $\left(-\infty, \infty\right)$

range: $\left(-\infty, 4\right]$

32. $f(x) = 5 - 4x - x^2$

$f(x) = -x^2 - 4x + 5$

$f(x) = -\left(x^2 + 4x + 4\right) + 5 + 4$

$f(x) = -\left(x + 2\right)^2 + 9$

vertex: (–2, 9)

x-intercepts:

$0 = -\left(x+2\right)^2 + 9$

$\left(x+2\right)^2 = 9$

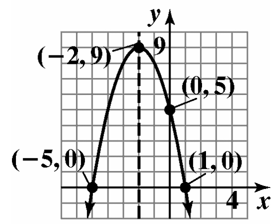
$x + 2 = \pm 3$

$x = -5, 1$

y-intercept:

$f(0) = 5 - 4\left(0\right) - \left(0\right)^2 = 5$

The axis of symmetry is $x = -2$.



$f(x) = 5 - 4x - x^2$

domain: $\left(-\infty, \infty\right)$

range: $\left(-\infty, 9\right]$

33. $f(x) = x^2 + 6x + 3$

$f(x) = (x^2 + 6x + 9) + 3 - 9$

$f(x) = \left(x + 3\right)^2 - 6$

vertex: $\left(-3, -6\right)$

x-intercepts:

$0 = \left(x+3\right)^2 - 6$

$\left(x+3\right)^2 = 6$

$x + 3 = \pm\sqrt{6}$

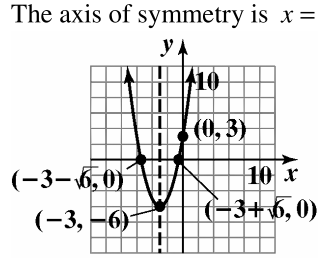
$x = -3 \pm \sqrt{6}$

y-intercept:

$f(0) = (0)^2 + 6(0) + 3$

$f(0) = 3$

The axis of symmetry is $x = -3$.



$f(x) = x^2 + 6x + 3$

domain: $\left(-\infty, \infty\right)$

range: $\left[-6, \infty\right)$

34. $f(x) = x^2 + 4x - 1$

$f(x) = (x^2 + 4x + 4) - 1 - 4$

$f(x) = (x+2)^2 - 5$

vertex: $(-2, -5)$

x-intercepts:

$0 = (x+2)^2 - 5$

$(x+2)^2 = 5$

$x + 2 = \pm\sqrt{5}$

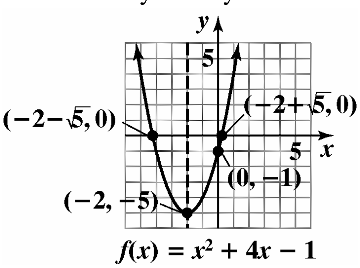
$x = -2 \pm \sqrt{5}$

y-intercept:

$f(0) = (0)^2 + 4(0) - 1$

$f(0) = -1$

The axis of symmetry is $x = -2$.



domain: $(-\infty, \infty)$

range: $[-5, \infty)$

35. $f(x) = 2x^2 + 4x - 3$

$f(x) = 2(x^2 + 2x \quad\ ) - 3$

$f(x) = 2(x^2 + 2x + 1) - 3 - 2$

$f(x) = 2(x+1)^2 - 5$

vertex: $(-1, -5)$

x-intercepts:

$0 = 2(x+1)^2 - 5$

$2(x+1)^2 = 5$

$(x+1)^2 = \frac{5}{2}$

$x + 1 = \pm\sqrt{\frac{5}{2}}$

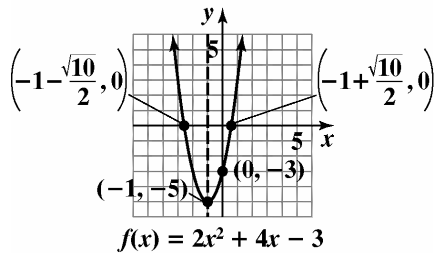
$x = -1 \pm \frac{\sqrt{10}}{2}$

y-intercept:

$f(0) = 2(0)^2 + 4(0) - 3$

$f(0) = -3$

The axis of symmetry is $x = -1$.



domain: $(-\infty, \infty)$

range: $[-5, \infty)$

36. $f(x) = 3x^2 - 2x - 4$

$f(x) = 3\left(x^2 - \frac{2}{3}x \quad\ \right) - 4$

$f(x) = 3\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) - 4 - \frac{1}{3}$

$f(x) = 3\left(x - \frac{1}{3}\right)^2 - \frac{13}{3}$

vertex: $\left(\frac{1}{3}, -\frac{13}{3}\right)$

x-intercepts:

$0 = 3\left(x - \frac{1}{3}\right)^2 - \frac{13}{3}$

$3\left(x - \frac{1}{3}\right)^2 = \frac{13}{3}$

$\left(x - \frac{1}{3}\right)^2 = \frac{13}{9}$

$x - \frac{1}{3} = \pm\sqrt{\frac{13}{9}}$

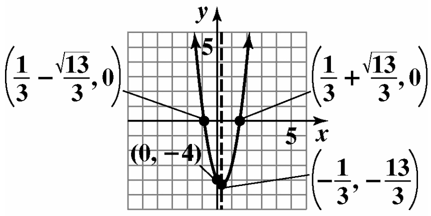
$x = \frac{1}{3} \pm \frac{\sqrt{13}}{3}$

y-intercept:

$f(0) = 3(0)^2 - 2(0) - 4$

$f(0) = -4$

The axis of symmetry is $x = \frac{1}{3}$.



***f*(*x*) = 3*x*² − 2*x* − 4**

domain: $(-\infty, \infty)$

range: $\left[-\frac{13}{3}, \infty\right)$

37. $f(x) = 2x - x^2 - 2$

$f(x) = -x^2 + 2x - 2$

$f(x) = -\left(x^2 - 2x + 1\right) - 2 + 1$

$f(x) = -\left(x-1\right)^2 - 1$

vertex: (1, –1)

x-intercepts:

$0 = -\left(x-1\right)^2 - 1$

$\left(x-1\right)^2 = -1$

$x - 1 = \pm i$

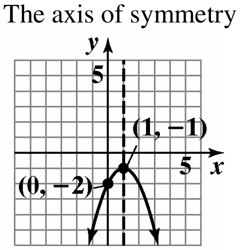
$x = 1 \pm i$

No *x*-intercepts.

y-intercept:

$f(0) = 2(0) - (0)^2 - 2 = -2$

The axis of symmetry is *x* = 1.



***f*(*x*) = 2*x* − *x*² − 2**

domain: $(-\infty, \infty)$

range: $(-\infty, -1]$

38. $f(x) = 6 - 4x + x^2$

$f(x) = x^2 - 4x + 6$

$f(x) = \left(x^2 - 4x + 4\right) + 6 - 4$

$f(x) = \left(x-2\right)^2 + 2$

vertex: (2, 2)

x-intercepts:

$0 = \left(x-2\right)^2 + 2$

$\left(x-2\right)^2 = -2$

$x - 2 = \pm i\sqrt{2}$

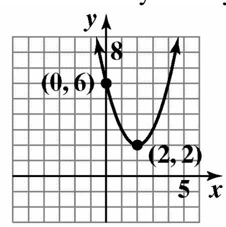
$x = 2 \pm i\sqrt{2}$

No *x*-intercepts

y-intercept:

$f(0) = 6 - 4(0) + (0)^2 = 6$

The axis of symmetry is *x* = 2.



***f*(*x*) = 6 − 4*x* + *x*²**

domain: $(-\infty, \infty)$

range: $[2, \infty)$

39. $f(x) = 3x^2 - 12x - 1$

a. $a = 3$. The parabola opens upward and has a minimum value.

b. $x = \frac{-b}{2a} = \frac{12}{6} = 2$

$f(2) = 3(2)^2 - 12(2) - 1$

$= 12 - 24 - 1 = -13$

The minimum is –13 at $x = 2$.

c. domain: $(-\infty, \infty)$ range: $[-13, \infty)$

40. $f(x) = 2x^2 - 8x - 3$

a. $a = 2$. The parabola opens upward and has a minimum value.

b. $x = \frac{-b}{2a} = \frac{8}{4} = 2$

$f(2) = 2(2)^2 - 8(2) - 3$

$= 8 - 16 - 3 = -11$

The minimum is –11 at $x = 2$.

c. domain: $(-\infty, \infty)$ range: $[-11, \infty)$

41. $f(x) = -4x^2 + 8x - 3$

a. a = -4. The parabola opens downward and has a maximum value.

b. $x = \frac{-b}{2a} = \frac{-8}{-8} = 1$

$f(1) = -4(1)^2 + 8(1) - 3$

$= -4 + 8 - 3 = 1$

The maximum is 1 at $x = 1$.

c. domain: $(-\infty, \infty)$ range: $(-\infty, 1]$

42. $f(x) = -2x^2 - 12x + 3$

a. $a = -2$. The parabola opens downward and has a maximum value.

b. $x = \frac{-b}{2a} = \frac{12}{-4} = -3$

$f(-3) = -2(-3)^2 - 12(-3) + 3$

$= -18 + 36 + 3 = 21$

The maximum is 21 at $x = -3$.

c. domain: $(-\infty, \infty)$ range: $(-\infty, 21]$

43. $f(x) = 5x^2 - 5x$

a. a = 5. The parabola opens upward and has a minimum value.

b. $x = \frac{-b}{2a} = \frac{5}{10} = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = 5\left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right)$

$= \frac{5}{4} - \frac{5}{2} = \frac{5}{4} - \frac{10}{4} = \frac{-5}{4}$

The minimum is $\frac{-5}{4}$ at $x = \frac{1}{2}$.

c. domain: $(-\infty, \infty)$ range: $\left[\frac{-5}{4}, \infty\right)$

44. $f(x) = 6x^2 - 6x$

a. a = 6. The parabola opens upward and has minimum value.

b. $x = \frac{-b}{2a} = \frac{6}{12} = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = 6\left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right)$

$= \frac{6}{4} - 3 = \frac{3}{2} - \frac{6}{2} = \frac{-3}{2}$

The minimum is $\frac{-3}{2}$ at $x = \frac{1}{2}$.

c. domain: $(-\infty, \infty)$ range: $\left[\frac{-3}{2}, \infty\right)$

45. Since the parabola opens up, the vertex $(-1, -2)$ is a minimum point.
domain: $(-\infty, \infty)$. range: $[-2, \infty)$

46. Since the parabola opens down, the vertex $(-3, -4)$ is a maximum point.
domain: $(-\infty, \infty)$. range: $(-\infty, -4]$

47. Since the parabola has a maximum, it opens down from the vertex $(10, -6)$.
domain: $(-\infty, \infty)$. range: $(-\infty, -6]$

48. Since the parabola has a minimum, it opens up from the vertex $(-6, 18)$.
domain: $(-\infty, \infty)$. range: $[18, \infty)$

49. $(h, k) = (5, 3)$

$f(x) = 2(x - h)^2 + k = 2(x - 5)^2 + 3$

50. $(h, k) = (7, 4)$

$f(x) = 2(x - h)^2 + k = 2(x - 7)^2 + 4$

51. $(h, k) = (-10, -5)$

$f(x) = 2(x - h)^2 + k$

$= 2[x - (-10)]^2 + (-5)$

$= 2(x + 10)^2 - 5$

52. $(h,k)=(-8,-6)$

$f(x)=2(x-h)^2+k$

$=2[x-(-8)]^2+(-6)$

$=2(x+8)^2-6$

53. Since the vertex is a maximum, the parabola opens down and $a=-3$.

$(h,k)=(-2,4)$

$f(x)=-3(x-h)^2+k$

$=-3[x-(-2)]^2+4$

$=-3(x+2)^2+4$

54. Since the vertex is a maximum, the parabola opens down and $a=-3$.

$(h,k)=(5,-7)$

$f(x)=-3(x-h)^2+k$

$=-3(x-5)^2+(-7)$

$=-3(x-5)^2-7$

55. Since the vertex is a minimum, the parabola opens up and $a=3$.

$(h,k)=(11,0)$

$f(x)=3(x-h)^2+k$

$=3(x-11)^2+0$

$=3(x-11)^2$

56. Since the vertex is a minimum, the parabola opens up and $a=3$.

$(h,k)=(9,0)$

$f(x)=3(x-h)^2+k$

$=3(x-9)^2+0$

$=3(x-9)^2$

57. **a.** $y=-0.01x^2+0.7x+6.1$

$a=-0.01,\ b=0.7,\ c=6.1$

x-coordinate of vertex

$=\frac{-b}{2a}=\frac{-0.7}{2(-0.01)}=35$

y-coordinate of vertex

$y=-0.01x^2+0.7x+6.1$

$y=-0.01(35)^2+0.7(35)+6.1=18.35$

The maximum height of the shot is about 18.35 feet. This occurs 35 feet from its point of release.

b. The ball will reach the maximum horizontal distance when its height returns to 0.

$y=-0.01x^2+0.7x+6.1$

$0=-0.01x^2+0.7x+6.1$

$a=-0.01,\ b=0.7,\ c=6.1$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-0.7\pm\sqrt{0.7^2-4(-0.01)(6.1)}}{2(-0.01)}$

$x\approx 77.8$ or $x\approx -7.8$

The maximum horizontal distance is 77.8 feet.

c. The initial height can be found at $x=0$.

$y=-0.01x^2+0.7x+6.1$

$y=-0.01(0)^2+0.7(0)+6.1=6.1$

The shot was released at a height of 6.1 feet.

58. **a.** $y=-0.04x^2+2.1x+6.1$

$a=-0.04,\ b=2.1,\ c=6.1$

x-coordinate of vertex

$=\frac{-b}{2a}=\frac{-2.1}{2(-0.04)}=26.25$

y-coordinate of vertex

$y=-0.04x^2+2.1x+6.1$

$y=-0.04(26.25)^2+2.1(26.25)+6.1\approx 33.7$

The maximum height of the shot is about 33.7 feet. This occurs 26.25 feet from its point of release.

b. The ball will reach the maximum horizontal distance when its height returns to 0.

$y=-0.04x^2+2.1x+6.1$

$0=-0.04x^2+2.1x+6.1$

$a=-0.04,\ b=2.1,\ c=6.1$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-2.1\pm\sqrt{2.1^2-4(-0.04)(6.1)}}{2(-0.04)}$

$x\approx 55.3$ or $x\approx -2.8$

The maximum horizontal distance is 55.3 feet.

c. The initial height can be found at $x = 0$.

$y = -0.04x^2 + 2.1x + 6.1$

$y = -0.04(0)^2 + 2.1(0) + 6.1 = 6.1$

The shot was released at a height of 6.1 feet.

59. $f(x) = 0.004x^2 - 0.094x + 2.6$

a. $f(25) = 0.004(25)^2 - 0.094(25) + 2.6$
$= 2.75$

According to the function, U.S. adult wine consumption in 2005 was 2.75 gallons per person. This underestimates the graph's value by 0.05 gallon.

b. $year = -\frac{b}{2a} = -\frac{-0.094}{2(0.004)} \approx 12$

Wine consumption was at a minimum about 12 years after 1980, or 1992.

$f(12) = 0.004(12)^2 - 0.094(12) + 2.6 \approx 2.048$

Wine consumption was about 2.048 gallons per U.S. adult in 1992.
This seems reasonable as compared to the values in the graph.

60. $f(x) = -0.03x^2 + 0.14x + 1.43$

a. $f(5) = -0.03(5)^2 + 0.14(5) + 1.43$
$= 1.38$

According to the function, 1.38 billion movie tickets were sold in 2005. This underestimates the graph's value by 0.03 billion.

b. $year = -\frac{b}{2a} = -\frac{0.14}{2(-0.03)} \approx 2$

Movie attendance was at a minimum about 2 years after 2000, or 2002.

$f(2) = -0.03(2)^2 + 0.14(2) + 1.43$
$= 1.59$

Movie attendance was about 1.59 billion in 2002.
This differs from the value in the graph by 0.04 billion.

61. Let x = one of the numbers;
$16 - x$ = the other number.

The product is $f(x) = x(16 - x)$
$= 16x - x^2 = -x^2 + 16x$

The x-coordinate of the maximum is

$x = -\frac{b}{2a} = -\frac{16}{2(-1)} = -\frac{16}{-2} = 8.$

$f(8) = -8^2 + 16(8) = -64 + 128 = 64$

The vertex is (8, 64). The maximum product is 64. This occurs when the two number are 8 and $16 - 8 = 8$.

62. Let x = one of the numbers
Let $20 - x$ = the other number

$P(x) = x(20 - x) = 20x - x^2 = -x^2 + 20x$

$x = -\frac{b}{2a} = -\frac{20}{2(-1)} = -\frac{20}{-2} = 10$

The other number is $20 - x = 20 - 10 = 10.$

The numbers which maximize the product are 10 and 10. The maximum product is $10 \cdot 10 = 100.$

63. Let x = one of the numbers;
$x - 16$ = the other number.

The product is $f(x) = x(x - 16) = x^2 - 16x$

The x-coordinate of the minimum is

$x = -\frac{b}{2a} = -\frac{-16}{2(1)} = -\frac{-16}{2} = 8.$

$f(8) = (8)^2 - 16(8)$
$= 64 - 128 = -64$

The vertex is $(8, -64)$. The minimum product is -64. This occurs when the two number are 8 and $8 - 16 = -8$.

64. Let x = the larger number. Then $x - 24$ is the smaller number. The product of these two numbers is given by

$P(x) = x(x - 24) = x^2 - 24x$

The product is minimized when

$x = -\frac{b}{2a} = -\frac{(-24)}{2(1)} = 12$

Since $12 - (-12) = 24$, the two numbers whose difference is 24 and whose product is minimized are 12 and -12.
The minimum product is
$P(12) = 12(12 - 24) = -144$.

65. Maximize the area of a rectangle constructed along a river with 600 feet of fencing.
Let x = the width of the rectangle;
$600-2x$ = the length of the rectangle
We need to maximize.

$$A(x) = x(600-2x)$$
$$= 600x - 2x^2 = -2x^2 + 600x$$

Since $a = -2$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{600}{2(-2)} = -\frac{600}{-4} = 150.$$

When the width is $x = 150$ feet, the length is $600 - 2(150) = 600 - 300 = 300$ feet.
The dimensions of the rectangular plot with maximum area are 150 feet by 300 feet. This gives an area of $150 \cdot 300 = 45,000$ square feet.

66. From the diagram, we have that x is the width of the rectangular plot and $200-2x$ is the length. Thus, the area of the plot is given by

$$A = l \cdot w = (200-2x)(x) = -2x^2 + 200x$$

Since the graph of this equation is a parabola that opens down, the area is maximized at the vertex.

$$x = -\frac{b}{2a} = -\frac{200}{2(-2)} = 50$$

$$A = -2(50)^2 + 200(50) = -5000 + 10,000$$
$$= 5000$$

The maximum area is 5000 square feet when the length is 100 feet and the width is 50 feet.

67. Maximize the area of a rectangle constructed with 50 yards of fencing.
Let x = the length of the rectangle. Let y = the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express y in terms of x.

$$2x + 2y = 50$$
$$2y = 50 - 2x$$
$$y = \frac{50-2x}{2} = 25 - x$$

We need to maximize $A = xy = x(25-x)$. Rewrite A as a function of x.

$$A(x) = x(25-x) = -x^2 + 25x$$

Since $a = -1$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{25}{2(-1)} = -\frac{25}{-2} = 12.5.$$

When the length x is 12.5, the width y is $y = 25 - x = 25 - 12.5 = 12.5$.
The dimensions of the rectangular region with maximum area are 12.5 yards by 12.5 yards. This gives an area of $12.5 \cdot 12.5 = 156.25$ square yards.

68. Let x = the length of the rectangle
Let y = the width of the rectangle

$$2x + 2y = 80$$
$$2y = 80 - 2x$$
$$y = \frac{80-2x}{2}$$
$$y = 40 - x$$

$$A(x) = x(40-x) = -x^2 + 40x$$

$$x = -\frac{b}{2a} = -\frac{40}{2(-1)} = -\frac{40}{-2} = 20.$$

When the length x is 20, the width y is $y = 40 - x = 40 - 20 = 20$.
The dimensions of the rectangular region with maximum area are 20 yards by 20 yards. This gives an area of $20 \cdot 20 = 400$ square yards.

69. Maximize the area of the playground with 600 feet of fencing.
Let x = the length of the rectangle. Let y = the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express y in terms of x.

$$2x+3y=600$$
$$3y=600-2x$$
$$y=\frac{600-2x}{3}$$
$$y=200-\frac{2}{3}x$$

We need to maximize $A=xy=x\left(200-\frac{2}{3}x\right)$.

Rewrite A as a function of x.

$$A(x)=x\left(200-\frac{2}{3}x\right)=-\frac{2}{3}x^2+200x$$

Since $a=-\frac{2}{3}$ is negative, we know the function opens downward and has a maximum at

$$x=-\frac{b}{2a}=-\frac{200}{2\left(-\frac{2}{3}\right)}=-\frac{200}{-\frac{4}{3}}=150.$$

When the length x is 150, the width y is

$$y=200-\frac{2}{3}x=200-\frac{2}{3}(150)=100.$$

The dimensions of the rectangular playground with maximum area are 150 feet by 100 feet. This gives an area of $150\cdot 100=15{,}000$ square feet.

70. Maximize the area of the playground with 400 feet of fencing.
Let x = the length of the rectangle. Let y = the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express y in terms of x.

$$2x+3y=400$$
$$3y=400-2x$$
$$y=\frac{400-2x}{3}$$
$$y=\frac{400}{3}-\frac{2}{3}x$$

We need to maximize $A=xy=x\left(\frac{400}{3}-\frac{2}{3}x\right)$.

Rewrite A as a function of x.

$$A(x)=x\left(\frac{400}{3}-\frac{2}{3}x\right)=-\frac{2}{3}x^2+\frac{400}{3}x$$

Since $a=-\frac{2}{3}$ is negative, we know the function opens downward and has a maximum at

$$x=-\frac{b}{2a}=-\frac{\frac{400}{3}}{2\left(-\frac{2}{3}\right)}=-\frac{\frac{400}{3}}{-\frac{4}{3}}=100.$$

When the length x is 100, the width y is

$$y=\frac{400}{3}-\frac{2}{3}x=\frac{400}{3}-\frac{2}{3}(100)=\frac{200}{3}=66\frac{2}{3}.$$

The dimensions of the rectangular playground with maximum area are 100 feet by $66\frac{2}{3}$ feet. This gives an area of $100\cdot 66\frac{2}{3}=6666\frac{2}{3}$ square feet.

71. Maximize the cross-sectional area of the gutter:

$$A(x)=x(20-2x)$$
$$=20x-2x^2=-2x^2+20x.$$

Since $a=-2$ is negative, we know the function opens downward and has a maximum at

$$x=-\frac{b}{2a}=-\frac{20}{2(-2)}=-\frac{20}{-4}=5.$$

When the height x is 5, the width is

$$20-2x=20-2(5)=20-10=10.$$
$$A(5)=-2(5)^2+20(5)$$
$$=-2(25)+100=-50+100=50$$

The maximum cross-sectional area is 50 square inches. This occurs when the gutter is 5 inches deep and 10 inches wide.

72.
$$A(x)=x(12-2x)=12x-2x^2$$
$$=-2x^2+12x$$
$$x=-\frac{b}{2a}=-\frac{12}{2(-2)}=-\frac{12}{-4}=3$$

When the height x is 3, the width is

$$12-2x=12-2(3)=12-6=6.$$
$$A(3)=-2(3)^2+12(3)=-2(9)+36$$
$$=-18+36=18$$

The maximum cross-sectional area is 18 square inches. This occurs when the gutter is 3 inches deep and 6 inches wide.

73. x = increase

$A = (50 + x)(8000 - 100x)$

$= 400,000 + 3000x - 100x^2$

$x = \frac{-b}{2a} = \frac{-3000}{2(-100)} = 15$

The maximum price is 50 + 15 = \$65.
The maximum revenue = 65(800 – 100·15) = \$422,500.

74. Maximize $A = (30 + x)(200 - 5x)$

$= 6000 + 50x - 5x^2$

$x = \frac{-(50)}{2(-5)} = 5$

Maximum rental = 30 + 5 = \$35
Maximum revenue = 35(200 – 5·5) = \$6125

75. x = increase

$A = (20 + x)(60 - 2x)$

$= 1200 + 20x - 2x^2$

$x = \frac{-b}{2a} = \frac{-20}{2(-2)} = 5$

The maximum number of trees is 20 + 5 = 25 trees.
The maximum yield is 60 – 2·5=50 pounds per tree, 50 x 25 = 1250 pounds.

76. Maximize $A = (30 + x)(50 - x)$

$= 1500 + 20x - x^2$

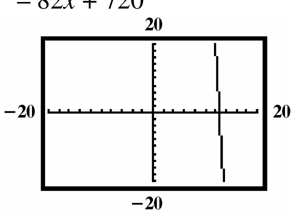
$x = \frac{-20}{2(-1)} = 10$

Maximum number of trees = 30 + 10 = 40 trees
Maximum yield = (30 + 10)(50 – 10) = 1600 pounds

77. – 83. Answers may vary.

84. $y = 2x^2 - 82x + 720$

a.



You can only see a little of the parabola.

b. $a = 2$; $b = -82$

$x = -\frac{b}{2a} = -\frac{-82}{4} = 20.5$

$y = 2(20.5)^2 - 82(20.5) + 720$

$= 840.5 - 1681 + 720$

$= -120.5$

vertex: (20.5, –120.5)

c. Ymax = 750

d. You can choose Xmin and Xmax so the x-value of the vertex is in the center of the graph. Choose Ymin to include the y-value of the vertex.

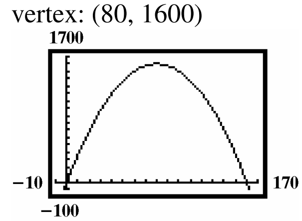
85. $y = -0.25x^2 + 40x$

$x = \frac{-b}{2a} = \frac{-40}{-0.5} = 80$

$y = -0.25(80)^2 + 40(80)$

$= 1600$

vertex: (80, 1600)



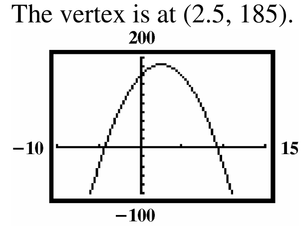
86. $y = -4x^2 + 20x + 160$

$x = \frac{-b}{2a} = \frac{-20}{-8} = 2.5$

$y = -4(2.5)^2 + 20(2.5) + 160$

$= -2.5 + 50 + 160 = 185$

The vertex is at (2.5, 185).



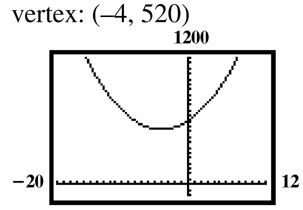
87. $y = 5x^2 + 40x + 600$

$x = \frac{-b}{2a} = \frac{-40}{10} = -4$

$y = 5(-4)^2 + 40(-4) + 600$

$= 80 - 160 + 600 = 520$

vertex: (–4, 520)



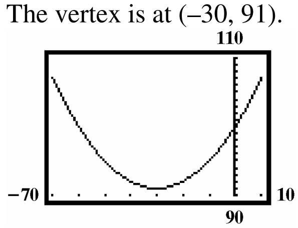
88. $y = 0.01x^2 + 0.6x + 100$

$x = \frac{-b}{2a} = \frac{-0.6}{0.02} = -30$

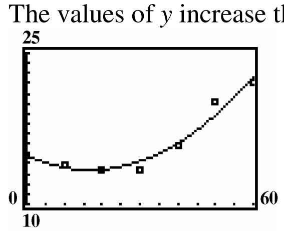
$y = 0.01(-30)^2 + 0.6(-30) + 100$

$= 9 - 18 + 100 = 91$

The vertex is at (–30, 91).



89. a. The values of y increase then decrease.



b. $y = 0.005x^2 - 0.170x + 14.817$

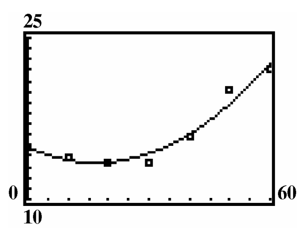
c. $x = \frac{-(-0.170)}{2(.005)} = 17; \quad 1940 + 17 = 1957$

$y = 0.005(17)^2 - 0.170(17) + 14.817$

≈ 13.372

The worst gas mileage was 13.372 mpg in 1957.

d.



90. does not make sense; Explanations will vary. Sample explanation: Some parabolas have the y-axis as the axis of symmetry.

91. makes sense

92. does not make sense; Explanations will vary. Sample explanation: If it is thrown vertically, its path will be a line segment.

93. does not make sense; Explanations will vary. Sample explanation: The football's path is better described by a quadratic model.

94. true

95. false; Changes to make the statement true will vary. A sample change is: The vertex is $(5,-1)$.

96. false; Changes to make the statement true will vary. A sample change is: The graph has no x–intercepts. To find x–intercepts, set $y = 0$ and solve for x.

$0 = -2(x+4)^2 - 8$

$2(x+4)^2 = -8$

$(x+4)^2 = -4$

Because the solutions to the equation are imaginary, we know that there are no x–intercepts.

97. false; Changes to make the statement true will vary. A sample change is: The x-coordinate of the maximum is $-\frac{b}{2a} = -\frac{1}{2(-1)} = -\frac{1}{-2} = \frac{1}{2}$ and the y–coordinate of the vertex of the parabola is

$f\left(-\frac{b}{2a}\right) = f\left(\frac{1}{2}\right) = \frac{5}{4}.$

The maximum y–value is $\frac{5}{4}$.

98. $f(x) = 3(x + 2)^2 - 5$; (–1, –2)
axis: $x = -2$
(–1, –2) is one unit right of (–2, –2). One unit left of (–2, –2) is (–3, –2).
point: (–3, –2)

99. Vertex (3, 2) Axis: $x = 3$
second point (0, 11)

100. We start with the form $f(x) = a(x-h)^2 + k$.

Since we know the vertex is $(h,k) = (-3,-4)$, we have $f(x) = a(x+3)^2 - 4$. We also know that the graph passes through the point $(1,4)$, which allows us to solve for a.

$4 = a(1+3)^2 - 4$

$8 = a(4)^2$

$8 = 16a$

$\frac{1}{2} = a$

Therefore, the function is $f(x) = \frac{1}{2}(x+3)^2 - 4$.

101. We know $(h,k)=(-3,-1)$, so the equation is of the form $f(x)=a(x-h)^2+k$

$$=a\left[x-(-3)\right]^2+(-1)$$
$$=a(x+3)^2-1$$

We use the point $(-2,-3)$ on the graph to determine the value of a: $f(x)=a(x+3)^2-1$

$$-3=a(-2+3)^2-1$$
$$-3=a(1)^2-1$$
$$-3=a-1$$
$$-2=a$$

Thus, the equation of the parabola is $f(x)=-2(x+3)^2-1$.

102. $2x+y-2=0$

$y=2-2x$

$d=\sqrt{x^2+(2-2x)^2}$

$d=\sqrt{x^2+4-8x+4x^2}$

$d=\sqrt{5x^2-8x+4}$

Minimize $5x^2-8x+4$

$x=\dfrac{-(-8)}{2(5)}=\dfrac{4}{5}$

$y=2-2\left(\dfrac{4}{5}\right)=\dfrac{2}{5}$

$\left(\dfrac{4}{5},\dfrac{2}{5}\right)$

103. $f(x)=(80+x)(300-3x)-10(300-3x)$

$=24000+60x-3x^2-3000+30x$

$=-3x^2+90x+21000$

$x=\dfrac{-b}{2a}=\dfrac{-90}{2(-3)}=\dfrac{3}{2}=15$

The maximum charge is 80 + 15 = \$95.00. the maximum profit is $-3(15)^2+9(15)+21000=$ \$21,675.

104. $440=2x+\pi y$

$440-2x=\pi y$

$\dfrac{440-2x}{\pi}=y$

Maximize $A=x\ \dfrac{440-2x}{\pi}\ =-\dfrac{2}{\pi}x^2+\dfrac{440}{\pi}x$

$x=\dfrac{-\frac{440}{\pi}}{2\ -\frac{2}{\pi}}=\dfrac{\frac{-440}{\pi}}{-\frac{4}{\pi}}=\dfrac{440}{4}=110$

$\dfrac{440-2(110)}{\pi}=\dfrac{220}{\pi}$

The dimensions are 110 yards by $\dfrac{220}{\pi}$ yards.

105. Answers may vary.

106. $x^3+3x^2-x-3=x^2(x+3)-1(x+3)$

$$=(x+3)(x^2-1)$$
$$=(x+3)(x+1)(x-1)$$

107. $f(x)=x^3-2x-5$

$f(2)=(2)^3-2(2)-5=-1$

$f(3)=(3)^3-2(3)-5=16$

The graph passes through (2, –1), which is below the *x*-axis, and (3, 16), which is above the *x*-axis. Since the graph of *f* is continuous, it must cross the *x*-axis somewhere between 2 and 3 to get from one of these points to the other.

108. $f(x)=x^4-2x^2+1$

$f(-x)=(-x)^4-2(-x)^2+1$

$\quad =x^4-2x^2+1$

Since $f(-x)=f(x)$, the function is even.

Thus, the graph is symmetric with respect to the *y*-axis.

Section 2.3

Check Point Exercises

1. Since n is even and $a_n > 0$, the graph rises to the left and to the right.

2. It is not necessary to multiply out the polynomial to determine its degree. We can find the degree of the polynomial by adding the degrees of each of its factors. $f(x) = 2\underbrace{x^3}_{\text{degree 3}}\underbrace{(x-1)}_{\text{degree 1}}\underbrace{(x+5)}_{\text{degree 1}}$ has degree $3+1+1=5$.
$f(x) = 2x^3(x-1)(x+5)$ is of odd degree with a positive leading coefficient. Thus its graph falls to the left and rises to the right.

3. Since n is odd and the leading coefficient is negative, the function falls to the right. Since the ratio cannot be negative, the model won't be appropriate.

4. The graph does not show the function's end behavior. Since $a_n > 0$ and n is odd, the graph should fall to the left.

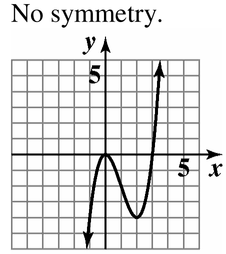
5. $f(x) = x^3 + 2x^2 - 4x - 8$
$0 = x^2(x+2) - 4(x+2)$
$0 = (x+2)(x^2-4)$
$0 = (x+2)^2(x-2)$
$x = 2$ or $x = -2$
The zeros are 2 and –2.

6. $f(x) = x^4 - 4x^2$
$x^4 - 4x^2 = 0$
$x^2(x^2-4) = 0$
$x^2(x+2)(x-2) = 0$
$x = 0$ or $x = -2$ or $x = 2$
The zeros are 0, –2, and 2.

7. $f(x) = -4\left(x+\frac{1}{2}\right)^2(x-5)^3$
$-4\left(x+\frac{1}{2}\right)^2(x-5)^3 = 0$
$x = -\frac{1}{2}$ or $x = 5$
The zeros are $-\frac{1}{2}$, with multiplicity 2, and 5, with multiplicity 3.
Because the multiplicity of $-\frac{1}{2}$ is even, the graph touches the x-axis and turns around at this zero. Because the multiplicity of 5 is odd, the graph crosses the x-axis at this zero.

8. $f(-3) = 3(-3)^3 - 10(-3) + 9 = -42$
$f(-2) = 3(-2)^3 - 10(-2) + 9 = 5$
The sign change shows there is a zero between –3 and –2.

9. $f(x) = x^3 - 3x^2$
Since $a_n > 0$ and n is odd, the graph falls to the left and rises to the right.
$x^3 - 3x^2 = 0$
$x^2(x-3) = 0$
$x = 0$ or $x = 3$
The x-intercepts are 0 and 3.
$f(0) = 0^3 - 3(0)^2 = 0$
The y-intercept is 0.
$f(-x) = (-x)^3 - 3(-x)^2 = -x^3 - 3x^2$
No symmetry.



$f(x) = x^3 - 3x^2$

Exercise Set 2.3

1. polynomial function;
degree: 3

2. polynomial function;
degree: 4

3. polynomial function;
degree: 5

4. polynomial function;
degree: 7

5. not a polynomial function

6. not a polynomial function

7. not a polynomial function

8. not a polynomial function

9. not a polynomial function

10. polynomial function;
degree: 2

11. polynomial function

12. Not a polynomial function because graph is not smooth.

13. Not a polynomial function because graph is not continuous.

14. polynomial function

15. (b)

16. (c)

17. (a)

18. (d)

19. $f(x) = 5x^3 + 7x^2 - x + 9$
Since $a_n > 0$ and n is odd, the graph of $f(x)$ falls to the left and rises to the right.

20. $f(x) = 11x^3 - 6x^2 + x + 3$
Since $a_n > 0$ and n is odd, the graph of $f(x)$ falls to the left and rises to the right.

21. $f(x) = 5x^4 + 7x^2 - x + 9$
Since $a_n > 0$ and n is even, the graph of $f(x)$ rises to the left and to the right.

22. $f(x) = 11x^4 - 6x^2 + x + 3$
Since $a_n > 0$ and n is even, the graph of $f(x)$ rises to the left and to the right.

23. $f(x) = -5x^4 + 7x^2 - x + 9$
Since $a_n < 0$ and n is even, the graph of $f(x)$ falls to the left and to the right.

24. $f(x) = -11x^4 - 6x^2 + x + 3$
Since $a_n < 0$ and n is even, the graph of $f(x)$ falls to the left and to the right.

25. $f(x) = 2(x-5)(x+4)^2$
x = 5 has multiplicity 1;
The graph crosses the x-axis.
$x = -4$ has multiplicity 2;
The graph touches the x-axis and turns around.

26. $f(x) = 3(x+5)(x+2)^2$
x = –5 has multiplicity 1;
The graph crosses the x-axis.
$x = -2$ has multiplicity 2;
The graph touches the x-axis and turns around.

27. $f(x) = 4(x-3)(x+6)^3$
x = 3 has multiplicity 1;
The graph crosses the x-axis.
$x = -6$ has multiplicity 3;
The graph crosses the x-axis.

28. $f(x) = -3\left(x + \frac{1}{2}\right)(x-4)^3$
$x = -\frac{1}{2}$ has multiplicity 1;
The graph crosses the x-axis.
$x = 4$ has multiplicity 3;
The graph crosses the x-axis.

29.
$$\begin{aligned} f(x) &= x^3 - 2x^2 + x \\ &= x\left(x^2 - 2x + 1\right) \\ &= x(x-1)^2 \end{aligned}$$
$x = 0$ has multiplicity 1;
The graph crosses the x-axis.
$x = 1$ has multiplicity 2;
The graph touches the x-axis and turns around.

30.
$$\begin{aligned} f(x) &= x^3 + 4x^2 + 4x \\ &= x\left(x^2 + 4x + 4\right) \\ &= x(x+2)^2 \end{aligned}$$
$x = 0$ has multiplicity 1;
The graph crosses the x-axis.
$x = -2$ has multiplicity 2;
The graph touches the x-axis and turns around.

31.
$$\begin{aligned} f(x) &= x^3 + 7x^2 - 4x - 28 \\ &= x^2(x+7) - 4(x+7) \\ &= \left(x^2 - 4\right)(x+7) \\ &= (x-2)(x+2)(x+7) \end{aligned}$$
$x = 2$, $x = -2$ and $x = -7$ have multiplicity 1;
The graph crosses the x-axis.

32. $f(x) = x^3 + 5x^2 - 9x - 45$
$= x^2(x+5) - 9(x+5)$
$= (x^2 - 9)(x+5)$
$= (x-3)(x+3)(x+5)$
$x = 3$, $x = -3$ and $x = -5$ have multiplicity 1;
The graph crosses the x-axis.

33. $f(x) = x^3 - x - 1$
$f(1) = -1$
$f(2) = 5$
The sign change shows there is a zero between the given values.

34. $f(x) = x^3 - 4x^2 + 2$
$f(0) = 2$
$f(1) = -1$
The sign change shows there is a zero between the given values.

35. $f(x) = 2x^4 - 4x^2 + 1$
$f(-1) = -1$
$f(0) = 1$
The sign change shows there is a zero between the given values.

36. $f(x) = x^4 + 6x^3 - 18x^2$
$f(2) = -8$
$f(3) = 81$
The sign change shows there is a zero between the given values.

37. $f(x) = x^3 + x^2 - 2x + 1$
$f(-3) = -11$
$f(-2) = 1$
The sign change shows there is a zero between the given values.

38. $f(x) = x^5 - x^3 - 1$
$f(1) = -1$
$f(2) = 23$
The sign change shows there is a zero between the given values.

39. $f(x) = 3x^3 - 10x + 9$
$f(-3) = -42$
$f(-2) = 5$
The sign change shows there is a zero between the given values.

40. $f(x) = 3x^3 - 8x^2 + x + 2$
$f(2) = -4$
$f(3) = 14$
The sign change shows there is a zero between the given values.

41. $f(x) = x^3 + 2x^2 - x - 2$

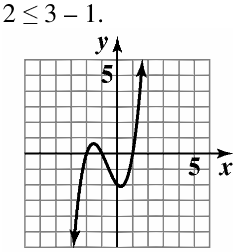
a. Since $a_n > 0$ and n is odd, $f(x)$ rises to the right and falls to the left.

b. $x^3 + 2x^2 - x - 2 = 0$
$x^2(x+2) - (x+2) = 0$
$(x+2)(x^2 - 1) = 0$
$(x+2)(x-1)(x+1) = 0$
$x = -2$, $x = 1$, $x = -1$
The zeros at –2, –1, and 1 have odd multiplicity so $f(x)$ crosses the x-axis at these points.

c. $f(0) = (0)^3 + 2(0)^2 - 0 - 2$
$= -2$
The y-intercept is –2.

d. $f(-x) = (-x) + 2(-x)^2 - (-x) - 2$
$= -x^3 + 2x^2 + x - 2$
$-f(x) = -x^3 - 2x^2 + x + 2$
The graph has neither origin symmetry nor y-axis symmetry.

e. The graph has 2 turning points and $2 \le 3 - 1$.



$y = x^3 + 2x^2 - x - 2$

42. $f(x) = x^3 + x^2 - 4x - 4$

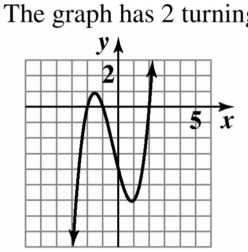
a. Since $a_n > 0$ and n is odd, $f(x)$ rises to the right and falls to the left.

b. $x^3 + x^2 - 4x - 4 = 0$
$x^2(x+1) - 4(x+1) = 0$
$(x+1)(x^2 - 4) = 0$
$(x + 1)(x - 2)(x + 2) = 0$
$x = -1$, or $x = 2$, or $x = -2$
The zeros at –2, –1 and 2 have odd multiplicity, so $f(x)$ crosses the x-axis at these points. The x-intercepts are –2, –1, and 2.

c. $f(0) = 0^3 + (0)^2 - 4(0) - 4 = -4$
The y-intercept is –4.

d. $f(-x) = -x^3 + x^2 + 4x - 4$
$-f(x) = -x^3 - x^2 + 4x + 4$
neither symmetry

e. The graph has 2 turning points and $2 \le 3 - 1$.



$y = x^3 + x^2 - 4x - 4$

43. $f(x) = x^4 - 9x^2$

a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

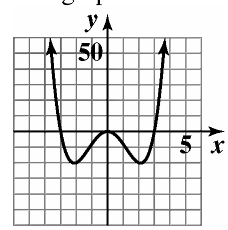
b. $x^4 - 9x^2 = 0$
$x^2(x^2 - 9) = 0$
$x^2(x-3)(x+3) = 0$
$x = 0, x = 3, x = -3$

The zeros at –3 and 3 have odd multiplicity, so $f(x)$ crosses the x-axis at these points. The root at 0 has even multiplicity, so $f(x)$ touches the x-axis at 0.

c. $f(0) = (0)^4 - 9(0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = x^4 - 9x^2$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = x^4 - 9x^2$

44. $f(x) = x^4 - x^2$

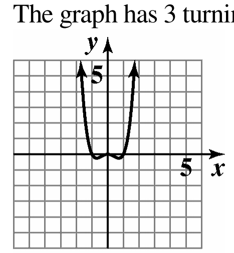
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x^4 - x^2 = 0$
$x^2(x^2 - 1) = 0$
$x^2(x-1)(x+1) = 0$
$x = 0, x = 1, x = -1$
f touches but does not cross the x-axis at 0.

c. $f(0) = (0)^4 - (0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = x^4 - x^2$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = x^4 - x^2$

45. $f(x) = -x^4 + 16x^2$

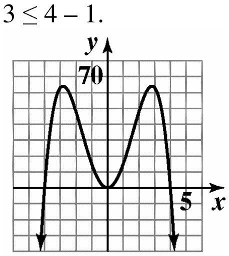
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b.
$$-x^4 + 16x^2 = 0$$
$$x^2(-x^2 + 16) = 0$$
$$x^2(4 - x)(4 + x) = 0$$
$x = 0$, $x = 4$, $x = -4$
The zeros at –4 and 4 have odd multiplicity, so $f(x)$ crosses the x-axis at these points. The root at 0 has even multiplicity, so $f(x)$ touches the x-axis at 0.

c. $f(0) = (0)^4 - 9(0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = -x^4 + 16x^2$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = -x^4 + 16x^2$

46. $f(x) = -x^4 + 4x^2$

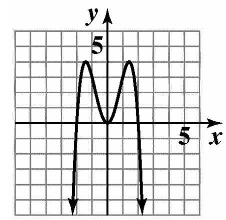
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b.
$$-x^4 + 4x^2 = 0$$
$$x^2(4 - x^2) = 0$$
$$x^2(2 - x)(2 + x) = 0$$
$x = 0$, $x = 2$, $x = -2$
The x-intercepts are –2,0, and 2. Since f has a double root at 0, it touches but does not cross the x-axis at 0.

c. $f(0) = -(0)^4 + 4(0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = -x^4 + 4x^2$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = -x^4 + 4x^2$

47. $f(x) = x^4 - 2x^3 + x^2$

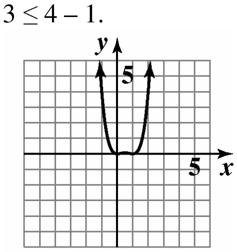
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b.
$$x^4 - 2x^3 + x^2 = 0$$
$$x^2(x^2 - 2x + 1) = 0$$
$$x^2(x - 1)(x - 1) = 0$$
$x = 0$, $x = 1$
The zeros at 1 and 0 have even multiplicity, so $f(x)$ touches the x-axis at 0 and 1.

c. $f(0) = (0)^4 - 2(0)^3 + (0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = x^4 + 2x^3 + x^2$
The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = x^4 - 2x^3 + x^2$

48. $f(x) = x^4 - 6x^3 + 9x^2$

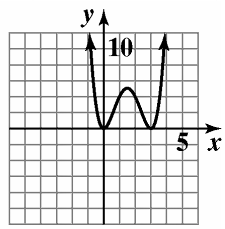
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b.
$$x^4 - 6x^3 + 9x^2 = 0$$
$$x^2(x^2 - 6x + 9) = 0$$
$$x^2(x - 3)^2 = 0$$
$x = 0$, $x = 3$
The zeros at 3 and 0 have even multiplicity, so $f(x)$ touches the x-axis at 3 and 0.

c. $f(0)=(0)^4-6(0)^3+9(0)^2=0$
The y-intercept is 0.

d. $f(-x)=x^4+6x^3+9x^2$
The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 4-1$.



$f(x) = x^4 - 6x^3 + 9x^2$

49. $f(x)=-2x^4+4x^3$

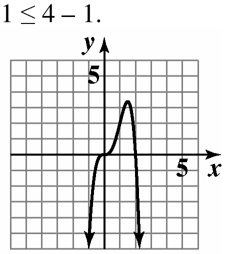
a. Since $a_n<0$ and n is even, $f(x)$ falls to the left and the right.

b. $-2x^4+4x^3=0$
$x^3(-2x+4)=0$
$x=0, x=2$
The zeros at 0 and 2 have odd multiplicity, so $f(x)$ crosses the x-axis at these points.

c. $f(0)=-2(0)^4+4(0)^3=0$
The y-intercept is 0.

d. $f(-x)=-2x^4-4x^3$
The graph has neither y-axis nor origin symmetry.

e. The graph has 1 turning point and $1 \le 4-1$.



$f(x) = -2x^4 + 4x^3$

50. $f(x)=-2x^4+2x^3$

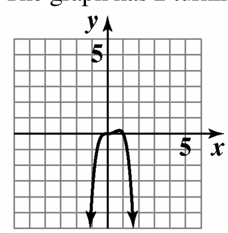
a. Since $a_n<0$ and n is even, $f(x)$ falls to the left and the right.

b. $-2x^4+2x^3=0$
$x^3(-2x+2)=0$
$x=0, x=1$
The zeros at 0 and 1 have odd multiplicity, so $f(x)$ crosses the x-axis at these points.

c. The y-intercept is 0.

d. $f(-x)=-2x^4-2x^3$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 4-1$.



$f(x) = -2x^4 + 2x^3$

51. $f(x)=6x^3-9x-x^5$

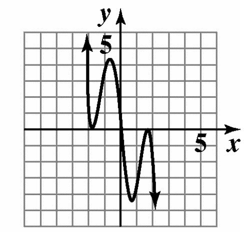
a. Since $a_n<0$ and n is odd, $f(x)$ rises to the left and falls to the right.

b. $-x^5+6x^3-9x=0$
$-x(x^4-6x^2+9)=0$
$-x(x^2-3)(x^2-3)=0$
$x=0,\ x=\pm\sqrt{3}$
The root at 0 has odd multiplicity so $f(x)$ crosses the x-axis at (0, 0). The zeros at $-\sqrt{3}$ and $\sqrt{3}$ have even multiplicity so $f(x)$ touches the x-axis at $\sqrt{3}$ and $-\sqrt{3}$.

c. $f(0)=-(0)^5+6(0)^3-9(0)=0$
The y-intercept is 0.

d. $f(-x)=x^5-6x^3+9x$
$f(-x)=-f(x)$
The graph has origin symmetry.

e. The graph has 4 turning point and $4 \le 5 - 1$.



$f(x) = 6x^3 - 9x - x^5$

52. $f(x) = 6x - x^3 - x^5$

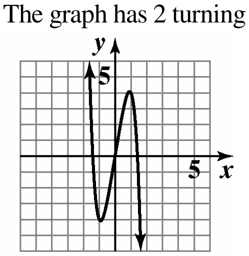
a. Since $a_n < 0$ and n is odd, $f(x)$ rises to the left and falls to the right.

b. $-x^5 - x^3 + 6x = 0$
$-x(x^4 + x^2 - 6) = 0$
$-x(x^2 + 3)(x^2 - 2) = 0$
$x = 0, x = \pm\sqrt{2}$
The zeros at $-\sqrt{2}$, 0, and $\sqrt{2}$ have odd multiplicity, so $f(x)$ crosses the x-axis at these points.

c. $f(0) = -(0)^5 - (0)^3 + 6(0) = 0$
The y-intercept is 0.

d. $f(-x) = x^5 + x^3 - 6x$
$f(-x) = -f(x)$
The graph has origin symmetry.

e. The graph has 2 turning points and $2 \le 5 - 1$.



$f(x) = 6x - x^3 - x^5$

53. $f(x) = 3x^2 - x^3$

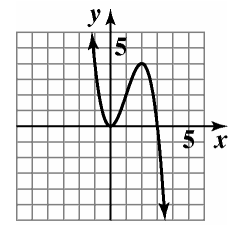
a. Since $a_n < 0$ and n is odd, $f(x)$ rises to the left and falls to the right.

b. $-x^3 + 3x^2 = 0$
$-x^2(x - 3) = 0$
$x = 0, x = 3$
The zero at 3 has odd multiplicity so $f(x)$ crosses the x-axis at that point. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0).

c. $f(0) = -(0)^3 + 3(0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = x^3 + 3x^2$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning point and $2 \le 3 - 1$.



$f(x) = 3x^2 - x^3$

54. $f(x) = \frac{1}{2} - \frac{1}{2}x^4$

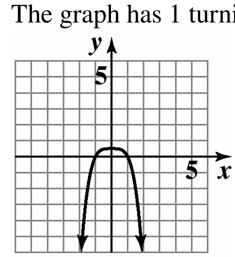
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-\frac{1}{2}x^4 + \frac{1}{2} = 0$
$-\frac{1}{2}(x^4 - 1) = 0$
$-\frac{1}{2}(x^2 + 1)(x^2 - 1) = 0$
$-\frac{1}{2}(x^2 + 1)(x - 1)(x + 1) = 0$
$x = \pm 1$
The zeros at –1 and 1 have odd multiplicity, so $f(x)$ crosses the x-axis at these points.

c. $f(0) = -\frac{1}{2}(0)^4 + \frac{1}{2} = \frac{1}{2}$
The y-intercept is $\frac{1}{2}$.

d. $f(-x) = \frac{1}{2} - \frac{1}{2}x^4$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 1 turning point and $1 \le 4 - 1$.



$f(x) = \frac{1}{2} - \frac{1}{2}x^4$

55. $f(x) = -3(x-1)^2(x^2-4)$

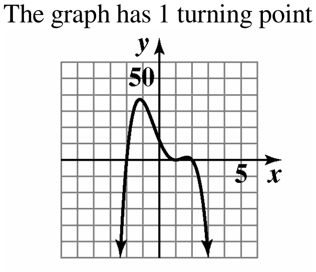
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-3(x-1)^2(x^2-4) = 0$
$x = 1, x = -2, x = 2$
The zeros at –2 and 2 have odd multiplicity, so $f(x)$ crosses the x-axis at these points. The root at 1 has even multiplicity, so $f(x)$ touches the x-axis at (1, 0).

c. $f(0) = -3(0-1)^2(0^2-4)^3$
$= -3(1)(-4) = 12$
The y-intercept is 12.

d. $f(-x) = -3(-x-1)^2(x^2-4)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 1 turning point and $1 \le 4 - 1$.



$f(x) = -3(x-1)^2(x^2-4)$

56. $f(x) = -2(x-4)^2(x^2-25)$

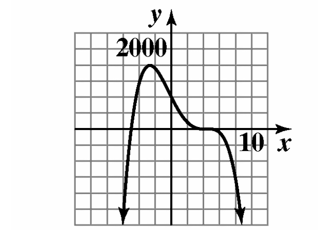
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-2(x-4)^2(x^2-25) = 0$
$x = 4, x = -5, x = 5$
The zeros at –5 and 5 have odd multiplicity so $f(x)$ crosses the x-axis at these points. The root at 4 has even multiplicity so $f(x)$ touches the x-axis at (4, 0).

c. $f(0) = -2(0-4)^2(0^2-25)$
$= -2(16)(-25)$
$= 800$
The y-intercept is 800.

d. $f(-x) = -2(-x-4)^2(x^2-2)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 1 turning point and $1 \le 4 - 1$.



$f(x) = -2(x-4)^2(x^2-25)$

57. $f(x) = x^2(x-1)^3(x+2)$

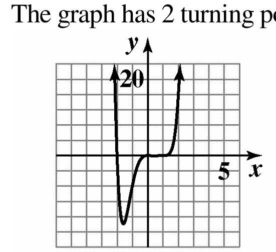
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x = 0, x = 1, x = -2$
The zeros at 1 and –2 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0).

c. $f(0) = 0^2(0-1)^3(0+2) = 0$
The y-intercept is 0.

d. $f(-x) = x^2(-x-1)^3(-x+2)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 6 - 1$.



$f(x) = x^2(x-1)^3(x+2)$

58. $f(x) = x^3(x+2)^2(x+1)$

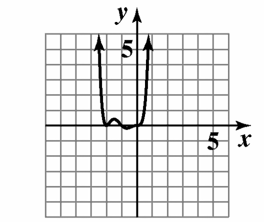
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x = 0, x = -2, x = -1$
The roots at 0 and –1 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at –2 has even multiplicity so $f(x)$ touches the axis at (–2, 0).

c. $f(0) = 0^3(0+2)^2(0+1) = 0$
The y-intercept is 0.

d. $f(-x) = -x^3(-x+2)^2(-x+1)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 6 - 1$.



$f(x) = x^3(x+2)^2(x+1)$

59. $f(x) = -x^2(x-1)(x+3)$

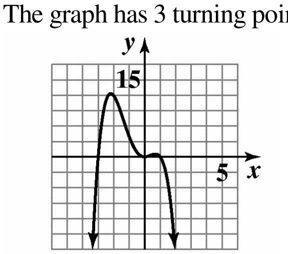
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $x = 0, x = 1, x = -3$
The zeros at 1 and –3 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0).

c. $f(0) = -0^2(0-1)(0+3) = 0$
The y-intercept is 0.

d. $f(-x) = -x^2(-x-1)(-x+3)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = -x^2(x-1)(x+3)$

60. $f(x) = -x^2(x+2)(x-2)$

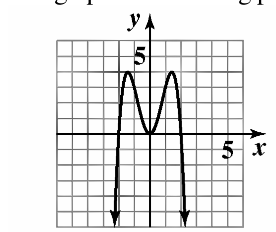
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $x = 0, x = 2, x = -2$
The zeros at 2 and –2 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0).

c. $f(0) = -0^2(0+2)(0-2) = 0$
The y-intercept is 0.

d. $f(-x) = -x^2(-x+2)(-x-2)$
$f(-x) = -x^2(-1)(x-2)(-1)(x+2)$
$f(-x) = -x^2(x+2)(x-2)$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = -x^2(x+2)(x-2)$

61. $f(x) = -2x^3(x-1)^2(x+5)$

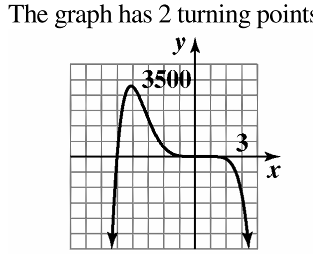
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $x = 0, x = 1, x = -5$
The roots at 0 and –5 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 1 has even multiplicity so $f(x)$ touches the axis at (1, 0).

c. $f(0) = -2(0)^3(0-1)^2(0+5) = 0$
The y-intercept is 0.

d. $f(-x) = 2x^3(-x-1)^2(-x+5)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 6 - 1$.



$f(x) = -2x^3(x-1)^2(x+5)$

62. $f(x) = -3x^3(x-1)^2(x+3)$

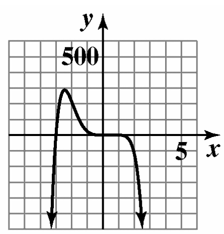
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $x = 0$, $x = 1$, $x = -3$
The roots at 0 and –3 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 1 has even multiplicity so $f(x)$ touches the axis at (1, 0).

c. $f(0) = -3(0)^3(0-1)^2(0+3) = 0$
The y-intercept is 0.

d. $f(-x) = 3x^3(-x-1)^2(-x+3)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 6 - 1$.



$f(x) = -3x^3(x-1)^2(x+3)$

63. $f(x) = (x-2)^2(x+4)(x-1)$

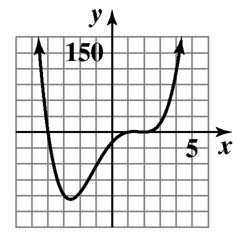
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and rises the right.

b. $x = 2$, $x = -4$, $x = 1$
The zeros at –4 and 1 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 2 has even multiplicity so $f(x)$ touches the axis at (2, 0).

c. $f(0) = (0-2)^2(0+4)(0-1) = -16$
The y-intercept is –16.

d. $f(-x) = (-x-2)^2(-x+4)(-x-1)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = (x-2)^2(x+4)(x-1)$

64. $f(x) = (x+3)(x+1)^3(x+4)$

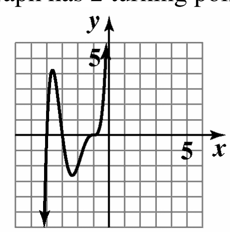
a. Since $a_n > 0$ and n is odd, $f(x)$ falls to the left and rises to the right.

b. $x = -3$, $x = -1$, $x = -4$
The zeros at all have odd multiplicity so $f(x)$ crosses the x-axis at these points.

c. $f(0) = (0+3)(0+1)^3(0+4) = 12$
The y-intercept is 12.

d. $f(-x) = (-x+3)(-x+1)^3(-x+4)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points



$f(x) = (x+3)(x+1)^3(x+4)$

65. a. The x-intercepts of the graph are -2, 1, and 4, so they are the zeros. Since the graph actually crosses the x-axis at all three places, all three have odd multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -2, 1, and 4 are the zeros, $x+2$, $x-1$, and $x-4$ are factors of the function. The lowest odd multiplicity is 1. From the end behavior, we can tell that the leading coefficient must be positive. Thus, the function is $f(x) = (x+2)(x-1)(x-4)$.

c. $f(0) = (0+2)(0-1)(0-4) = 8$

66. a. The *x*-intercepts of the graph are -3, 2, and 5, so they are the zeros. Since the graph actually crosses the *x*-axis at all three places, all three have odd multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -3, 2, and 5 are the zeros, $x+3$, $x-2$, and $x-5$ are factors of the function. The lowest odd multiplicity is 1. From the end behavior, we can tell that the leading coefficient must be positive. Thus, the function is $f(x)=(x+3)(x-2)(x-5)$.

c. $f(0)=(0+3)(0-2)(0-5)=30$

67. a. The *x*-intercepts of the graph are -1 and 3, so they are the zeros. Since the graph crosses the *x*-axis at -1, it has odd multiplicity. Since the graph touches the *x*-axis and turns around at 3, it has even multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -1 and 3 are the zeros, $x+1$ and $x-3$ are factors of the function. The lowest odd multiplicity is 1, and the lowest even multiplicity is 2. From the end behavior, we can tell that the leading coefficient must be positive. Thus, the function is $f(x)=(x+1)(x-3)^2$.

c. $f(0)=(0+1)(0-3)^2=9$

68. a. The *x*-intercepts of the graph are -2 and 1, so they are the zeros. Since the graph crosses the *x*-axis at -2, it has odd multiplicity. Since the graph touches the *x*-axis and turns around at 1, it has even multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -2 and 1 are the zeros, $x+2$ and $x-1$ are factors of the function. The lowest odd multiplicity is 1, and the lowest even multiplicity is 2. From the end behavior, we can tell that the leading coefficient must be positive. Thus, the function is $f(x)=(x+2)(x-1)^2$.

c. $f(0)=(0+2)(0-1)^2=2$

69. a. The *x*-intercepts of the graph are -3 and 2, so they are the zeros. Since the graph touches the *x*-axis and turns around at both -3 and 2, both have even multiplicity.

b. Since the graph has three turning points, the function must be at least of degree 4. Since -3 and 2 are the zeros, $x+3$ and $x-2$ are factors of the function. The lowest even multiplicity is 2. From the end behavior, we can tell that the leading coefficient must be negative. Thus, the function is $f(x)=-(x+3)^2(x-2)^2$.

c. $f(0)=-(0+3)^2(0-2)^2=-36$

70. a. The *x*-intercepts of the graph are -1 and 4, so they are the zeros. Since the graph touches the *x*-axis and turns around at both -1 and 4, both have even multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -1 and 4 are the zeros, $x+1$ and $x-4$ are factors of the function. The lowest even multiplicity is 2. From the end behavior, we can tell that the leading coefficient must be negative. Thus, the function is $f(x)=-(x+1)^2(x-4)^2$.

c. $f(0)=-(0+1)^2(0-4)^2=-16$

71. a. The *x*-intercepts of the graph are -2, -1, and 1, so they are the zeros. Since the graph crosses the *x*-axis at -1 and 1, they both have odd multiplicity. Since the graph touches the *x*-axis and turns around at -2, it has even multiplicity.

b. Since the graph has five turning points, the function must be at least of degree 6. Since -2, -1, and 1 are the zeros, $x+2$, $x+1$, and $x-1$ are factors of the function. The lowest even multiplicity is 2, and the lowest odd multiplicity is 1. However, to reach degree 6, one of the odd multiplicities must be 3. From the end behavior, we can tell that the leading coefficient must be positive. The function is $f(x)=(x+2)^2(x+1)(x-1)^3$.

c. $f(0)=(0+2)^2(0+1)(0-1)^3=-4$

72. a. The x-intercepts of the graph are -2, -1, and 1, so they are the zeros. Since the graph crosses the x-axis at -2 and 1, they both have odd multiplicity. Since the graph touches the x-axis and turns around at -1, it has even multiplicity.

b. Since the graph has five turning points, the function must be at least of degree 6. Since -2, -1, and 1 are the zeros, $x+2$, $x+1$, and $x-1$ are factors of the function. The lowest even multiplicity is 2, and the lowest odd multiplicity is 1. However, to reach degree 6, one of the odd multiplicities must be 3. From the end behavior, we can tell that the leading coefficient must be positive. The function is $f(x)=(x+2)(x+1)^2(x-1)^3$.

c. $f(0)=(0+2)(0+1)^2(0-1)^3=-2$

73. a.
$$f(x)=-3402x^2+42,203x+308,453$$
$$f(3)=-3402(3)^2+42,203(3)+308,453$$
$$=404,444$$
$$g(x)=2769x^3-28,324x^2+107,555x+261,931$$
$$g(3)=2769(3)^3-28,324(3)^2+107,555(3)+261,931$$
$$=404,443$$

Function *f* provides a better description of the actual number.

b. Since the degree of *f* is even and the leading coefficient is negative, the graph falls to the right. The function will not be a useful model over an extended period of time because it will eventually give negative values.

74. a.
$$f(x)=-3402x^2+42,203x+308,453$$
$$f(5)=-3402(5)^2+42,203(5)+308,453$$
$$=434,418$$
$$g(x)=2769x^3-28,324x^2+107,555x+261,931$$
$$g(5)=2769(5)^3-28,324(5)^2+107,555(5)+261,931$$
$$=437,731$$

Function *g* provides a better description of the actual number.

b. Since the degree of *g* is odd and the leading coefficient is negative, the graph rises to the right. Based on the end behavior, the function will be a useful model over an extended period of time.

75. a. The woman's heart rate was increasing from 1 through 4 minutes and from 8 through 10 minutes.

b. The woman's heart rate was decreasing from 4 through 8 minutes and from 10 through 12 minutes.

c. There were 3 turning points during the 12 minutes.

d. Since there were 3 turning points, a polynomial of degree 4 would provide the best fit.

e. The leading coefficient should be negative. The graph falls to the left and to the right.

f. The woman's heart rate reached a maximum of about 116 ± 1 beats per minute. This occurred after 10 minutes.

g. The woman's heart rate reached a minimum of about 64 ± 1 beats per minute. This occurred after 8 minutes.

76. a. The percentage of students with B+ averages or better was increasing from 1960 through 1975 and from 1985 through 2000.

b. The percentage of students with B+ averages or better was decreasing from 1975 through 1985 and from 2000 through 2005.

c. There were 3 turning points during the period shown.

d. Since there were 3 turning points, a polynomial of degree 4 would provide the best fit.

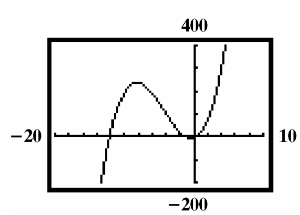
e. The leading coefficient should be negative. The graph falls to the left and to the right.

f. The percentage reached a maximum of about $69\pm1\%$ in 2000.

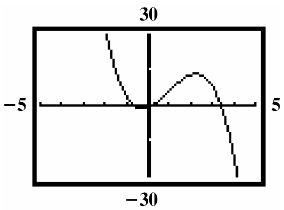
g. The percentage reached a minimum of about $18\pm1\%$ in 1960.

77. – 93. Answers may vary.

94.



95.



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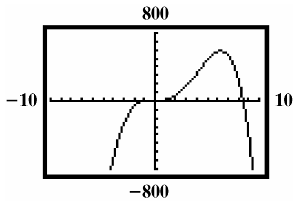
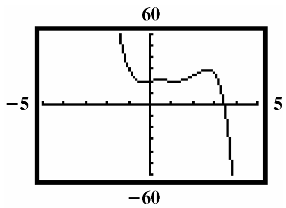
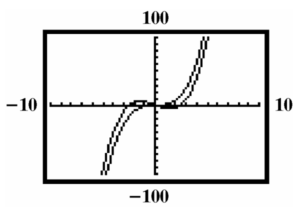


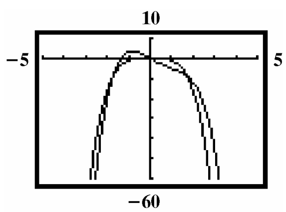
97.



98.



99.



100. makes sense

101. does not make sense; Explanations will vary. Sample explanation: Since $(x+2)$ is raised to an odd power, the graph crosses the x-axis at –2.

102. does not make sense; Explanations will vary. Sample explanation: A forth degree function has at most 3 turning points.

103. makes sense

104. false; Changes to make the statement true will vary. A sample change is: $f(x)$ falls to the left and rises to the right.

105. false; Changes to make the statement true will vary. A sample change is: Such a function falls to the right and will eventually have negative values.

106. true

107. false; Changes to make the statement true will vary. A sample change is: A function with origin symmetry either falls to the left and rises to the right, or rises to the left and falls to the right.

108. $f(x) = x^3 + x^2 - 12x$

109. $f(x) = x^3 - 2x^2$

110. $\dfrac{737}{21} = 35 + \dfrac{2}{21}$

111. $6x^3 - x^2 - 5x + 4$

112. $2x^3 - x^2 - 11x + 6 = (x-3)(2x^2 + 3x - 2)$
$= (x-3)(2x-1)(x+2)$

Section 2.4

Check Point Exercises

1.

$$\begin{array}{r} x+5 \\ x+9\overline{)\,x^2+14x+45} \\ \underline{x^2+9x} \\ 5x+45 \\ \underline{5x+45} \\ 0 \end{array}$$

The answer is $x + 5$.

2.

$$\begin{array}{r} 2x^2+3x-2 \\ x-3\overline{)\,2x^3-3x^2-11x+7} \\ \underline{2x^3-6x^2} \\ 3x^2-11x \\ \underline{3x^2-9x} \\ -2x+7 \\ \underline{-2x+6} \\ 1 \end{array}$$

The answer is $2x^2 + 3x - 2 + \dfrac{1}{x-3}$.

3.
$$\begin{array}{r} 2x^2+7x+14 \\ x^2-2x\overline{)2x^4+3x^3+0x^2-7x-10} \\ \underline{2x^4-4x^3} \\ 7x^3+0x^2 \\ \underline{7x^3-14x^2} \\ 14x^2-7x \\ \underline{14x^2-28x} \\ 21x-10 \end{array}$$

The answer is $2x^2+7x+14+\dfrac{21x-10}{x^2-2x}$.

4.

–2	1	0	–7	–6
		–2	4	6
	1	–2	–3	0

The answer is x^2-2x-3.

5.

–4	3	4	–5	3
		–12	32	–108
	3	–8	27	–105

$f(-4)=-105$

6.

–1	15	14	–3	–2
		–15	1	2
	15	–1	–2	0

$15x^2-x-2=0$

$(3x+1)(5x-2)=0$

$x=-\dfrac{1}{3}$ or $x=\dfrac{2}{5}$

The solution set is $\left\{-1,-\dfrac{1}{3},\dfrac{2}{5}\right\}$.

Exercise Set 2.4

1.
$$\begin{array}{r} x+3 \\ x+5\overline{)x^2+8x+15} \\ \underline{x^2+5x} \\ 3x+15 \\ \underline{3x+15} \\ 0 \end{array}$$

The answer is $x+3$.

2.
$$\begin{array}{r} x+5 \\ x-2\overline{)x^2+3x-10} \\ \underline{x^2-2x} \\ 5x-10 \\ \underline{5x-10} \\ 0 \end{array}$$

The answer is $x+5$.

3.
$$\begin{array}{r} x^2+3x+1 \\ x+2\overline{)x^3+5x^2+7x+2} \\ \underline{x^3+2x^2} \\ 3x^2+7x \\ \underline{3x^2+6x} \\ x+2 \\ \underline{x+2} \\ 0 \end{array}$$

The answer is x^2+3x+1.

4.
$$\begin{array}{r} x^2+x-2 \\ x-3\overline{)x^3-2x^2-5x+6} \\ \underline{x^3-3x^2} \\ x^2-5x \\ \underline{x^2-3x} \\ -2x+6 \\ \underline{-2x+6} \\ 0 \end{array}$$

The answer is x^2+x-2.

5.
$$\begin{array}{r} 2x^2+3x+5 \\ 3x-1\overline{)6x^3+7x^2+12x-5} \\ \underline{6x^3-2x^2} \\ 9x^2+12x \\ \underline{9x^2-3x} \\ 15x-5 \\ \underline{15x-5} \\ 0 \end{array}$$

The answer is $2x^2+3x+5$.

6.

$$\begin{array}{r} 2x^2+3x+5 \\ 3x+4\overline{)6x^3+17x^2+27x+20} \\ \underline{6x^3+8x^2} \qquad\qquad\quad \\ 9x^2+27x \qquad \\ \underline{9x^2+12x} \qquad \\ 15x+20 \\ \underline{15x+20} \\ 0 \end{array}$$

The answer is $2x^2+3x+5$.

7.

$$\begin{array}{r} 4x+3+\frac{2}{3x-2} \\ 3x-2\overline{)12x^2+x-4} \\ \underline{12x^2-8x} \qquad \\ 9x-4 \\ \underline{9x-6} \\ 2 \end{array}$$

The answer is $4x+3+\frac{2}{3x-2}$.

8.

$$\begin{array}{r} 2x-3+\frac{3}{2x-1} \\ 2x-1\overline{)4x^2-8x+6} \\ \underline{4x^2-2x} \qquad \\ -6x+6 \\ \underline{-6x+6} \\ 3 \end{array}$$

The answer is $2x-3+\frac{3}{2x-1}$.

9.

$$\begin{array}{r} 2x^2+x+6-\frac{38}{x+3} \\ x+3\overline{)2x^3+7x^2+9x-20} \\ \underline{2x^3+6x^2} \qquad\qquad \\ x^2+9x \qquad \\ \underline{x^2+3x} \qquad \\ 6x-20 \\ \underline{6x+18} \\ -38 \end{array}$$

The answer is $2x^2+x+6-\frac{38}{x+3}$.

10.

$$\begin{array}{r} 3x+7+\frac{26}{x-3} \\ x-3\overline{)3x^2-2x+5} \\ \underline{3x^2-9x} \qquad \\ 7x+5 \\ \underline{7x-21} \\ 26 \end{array}$$

The answer is $3x+7+\frac{26}{x-3}$.

11.

$$\begin{array}{r} 4x^3+16x^2+60x+246+\frac{984}{x-4} \\ x-4\overline{)4x^4-4x^2+6x} \qquad\qquad\qquad \\ \underline{4x^4-16x^3} \qquad\qquad\qquad\qquad \\ 16x^3-4x^2 \qquad\qquad\qquad \\ \underline{16x^3-64x^2} \qquad\qquad\qquad \\ 60x^2+\ 6x \qquad\qquad \\ \underline{60x^2-240x} \qquad\quad \\ 246x \qquad \\ \underline{246x-984} \\ 984 \end{array}$$

The answer is

$$4x^3+16x^2+60x+246+\frac{984}{x-4}.$$

12.

$$\begin{array}{r} x^3+3x^2+9x+27 \\ x-3\overline{)x^4 \qquad\qquad\qquad\qquad -81} \\ \underline{x^4-3x^3} \qquad\qquad\qquad\qquad \\ 3x^3 \qquad\qquad\qquad\quad \\ \underline{3x^2-9x^2} \qquad\qquad\quad \\ 9x^2 \qquad\qquad \\ \underline{9x^2-27x} \qquad \\ 27x-81 \\ \underline{27x-81} \\ 0 \end{array}$$

The answer is $x^3+3x^2+9x+27$.

13.

$$\begin{array}{r} 2x+5 \\ 3x^2-x-3\overline{)6x^3+13x^2-11x-15} \\ \underline{6x^3-2x^2-6x} \\ 15x^2-5x-15 \\ \underline{15x^2-5x-15} \\ 0 \end{array}$$

The answer is $2x+5$.

14.

$$\begin{array}{r} x^2+x-3 \\ x^2+x-2\overline{)x^4+2x^3-4x^2-5x-6} \\ \underline{x^4+x^3-2x^2} \\ x^3-2x^2-5x \\ \underline{x^3+x^2-2x} \\ -3x^2-3x-6 \\ \underline{-3x^2-3x+6} \\ -12 \end{array}$$

The answer is $x^2+x-3-\dfrac{12}{x^2+x-2}$.

15.

$$\begin{array}{r} 6x^2+3x-1 \\ 3x^2+1\overline{)18x^4+9x^3+3x^2} \\ \underline{18x^4+6x^2} \\ 9x^3-3x^2 \\ \underline{9x^3+3x} \\ -3x^2-3x \\ \underline{-3x^2-1} \\ -3x+1 \end{array}$$

The answer is $6x^2+3x-1-\dfrac{3x-1}{3x^2+1}$.

16.

$$\begin{array}{r} x^2-4x+1 \\ 2x^3+1\overline{)2x^5-8x^4+2x^3+x^2} \\ \underline{2x^5+x^2} \\ -8x^4+2x^3 \\ \underline{-8x^4-4x} \\ 2x^3+4x \\ \underline{2x^3+1} \\ 4x-1 \end{array}$$

The answer is $x^2-4x+1+\dfrac{4x-1}{2x^3+1}$.

17. $(2x^2+x-10)\div(x-2)$

$$\begin{array}{c|ccc} 2 & 2 & 1 & -10 \\ & & 4 & 10 \\ \hline & 2 & 5 & 0 \end{array}$$

The answer is $2x+5$.

18. $(x^2+x-2)\div(x-1)$

$$\begin{array}{c|ccc} 1 & 1 & 1 & -2 \\ & & 1 & 2 \\ \hline & 1 & 2 & 0 \end{array}$$

The answer is $x + 2$.

19. $(3x^2+7x-20)\div(x+5)$

$$\begin{array}{c|ccc} -5 & 3 & 7 & -20 \\ & & -15 & 40 \\ \hline & 3 & -8 & 20 \end{array}$$

The answer is $3x-8+\dfrac{20}{x+5}$.

20. $(5x^2-12x-8)\div(x+3)$

$$\begin{array}{c|ccc} -3 & 5 & 12 & -8 \\ & & -15 & 81 \\ \hline & 5 & -27 & 73 \end{array}$$

The answer is $5x-27+\dfrac{73}{x+3}$.

21. $(4x^3-3x^2+3x-1)\div(x-1)$

$$\begin{array}{c|cccc} 1 & 4 & -3 & 3 & -1 \\ & & 4 & 1 & 4 \\ \hline & 4 & 1 & 4 & 3 \end{array}$$

The answer is $4x^2+x+4+\dfrac{3}{x-1}$.

22. $(5x^3-6x^2+3x+11)\div(x-2)$

2	5	–6	3	11
		10	8	22
	5	4	11	33

The answer is $5x^2+4x+11+\dfrac{33}{x-2}$.

23. $(6x^5-2x^3+4x^2-3x+1)\div(x-2)$

2	6	0	–2	4	–3	1
		12	24	44	96	186
	6	12	22	48	93	187

The answer is

$6x^4+12x^3+22x^2+48x+93+\dfrac{187}{x-2}$.

24. $(x^5+4x^4-3x^2+2x+3)\div(x-3)$

3	1	4	0	–3	2	3
		3	21	63	180	546
	1	7	21	60	182	549

The answer is

$x^4+7x^3+21x^2+60x+182+\dfrac{549}{x-3}$.

25. $(x^2-5x-5x^3+x^4)\div(5+x)\Rightarrow$
$(x^4-5x^3+x^2-5x)\div(x+5)$

–5	1	–5	1	–5	0
		–5	50	–255	1300
	1	–10	51	–260	1300

The answer is

$x^3-10x^2+51x-260+\dfrac{1300}{x+5}$.

26. $(x^2-6x-6x^3+x^4)\div(6+x)\Rightarrow$
$(x^4-6x^3+x^2-6x)\div(x+6)$

–6	1	–6	1	–6	0
		–6	72	–438	2664
	1	–12	73	–444	2664

The answer is $x^3-12x^2+73x-444+\dfrac{2664}{x+6}$.

27. $\dfrac{x^5+x^3-2}{x-1}$

1	1	0	1	0	0	–2
		1	1	2	2	2
	1	1	2	2	2	0

The answer is $x^4+x^3+2x^2+2x+2$.

28. $\dfrac{x^7+x^5-10x^3+12}{x+2}$

–2	1	0	1	0	–10	0	0	12
		–2	4	–10	20	–20	40	–80
	1	–2	5	–10	10	–20	40	–68

The answer is $x^6-2x^5+5x^4-10x^3+10x^2$
$-20x+40-\dfrac{68}{x+2}$.

29. $\dfrac{x^4-256}{x-4}$

4	1	0	0	0	–256
		4	16	64	256
	1	4	16	64	0

The answer is $x^3+4x^2+16x+64$.

30. $\dfrac{x^7-128}{x-2}$

2	1	0	0	0	0	0	0	–128
		2	4	8	16	32	64	128
	1	2	4	8	16	32	64	0

The answer is
$x^6+2x^5+4x^4+8x^3+16x^2+32x+64$.

31. $\dfrac{2x^5-3x^4+x^3-x^2+2x-1}{x+2}$

–2	2	–3	1	–1	2	–1
		–4	14	–30	62	–128
	2	–7	15	–31	64	–129

The answer is

$2x^4-7x^3+15x^2-31x+64-\dfrac{129}{x+2}$.

32. $\dfrac{x^5-2x^4-x^3+3x^2-x+1}{x-2}$

2	1	–2	–1	3	–1	1
		2	0	–2	2	2
	1	0	–1	1	1	3

The answer is $x^4-x^2+x+1+\dfrac{3}{x-2}$.

33. $f(x)=2x^3-11x^2+7x-5$

4	2	–11	7	–5
		8	–12	–20
	2	–3	–5	–25

$f(4)=-25$

34.

3	1	–7	5	–6
		3	–12	–21
	1	–4	–7	–27

$f(3)=-27$

35. $f(x)=3x^3-7x^2-2x+5$

–3	3	–7	–2	5
		–9	48	–138
	3	–16	46	–133

$f(-3)=-133$

36.

–2	4	5	–6	–4
		–8	6	0
	4	–3	0	–4

$f(-2)=-4$

37. $f(x)=x^4+5x^3+5x^2-5x-6$

3	1	5	5	–5	–6
		3	24	87	246
	1	8	29	82	240

$f(3)=240$

38.

2	1	–5	5	5	–6
		2	–6	–2	6
	1	–3	–1	3	0

$f(2)=0$

39. $f(x)=2x^4-5x^3-x^2+3x+2$

$-\frac{1}{2}$	2	–5	–1	3	2
		–1	3	–1	–1
	2	–6	2	2	1

$f\left(-\frac{1}{2}\right)=1$

40.

$-\frac{2}{3}$	6	10	5	1	1
		–4	–4	$-\frac{2}{3}$	$-\frac{2}{9}$
	6	6	1	$\frac{1}{3}$	$\frac{7}{9}$

$f\left(-\frac{2}{3}\right)=\frac{7}{9}$

41. Dividend: x^3-4x^2+x+6
Divisor: $x+1$

–1	1	–4	1	6
		–1	5	–6
	1	–5	6	0

The quotient is x^2-5x+6.
$(x+1)(x^2-5x+6)=0$
$(x+1)(x-2)(x-3)=0$
$x=-1$, $x=2$, $x=3$
The solution set is $\{-1, 2, 3\}$.

42. Dividend: $x^3 - 2x^2 - x + 2$
Divisor: $x + 1$

-1	1	-2	-1	2
		-1	3	-2
	1	-3	2	0

The quotient is $x^2 - 3x + 2$.
$(x+1)(x^2 - 3x + 2) = 0$
$(x + 1)(x - 2)(x - 1) = 0$
$x = -1, x = 2, x = 1$
The solution set is $\{-1, 2, 1\}$.

43. $2x^3 - 5x^2 + x + 2 = 0$

2	2	-5	1	2
		4	-2	-2
	2	-1	-1	0

$(x-2)(2x^2 - x - 1) = 0$
$(x - 2)(2x + 1)(x - 1) = 0$
$x = 2,\ x = -\frac{1}{2},\ x = 1$
The solution set is $\left\{-\frac{1}{2}, 1, 2\right\}$.

44. $2x^3 - 3x^2 - 11x + 6 = 0$

-2	2	-3	-11	6
		-4	14	-6
	2	-7	3	0

$(x + 2)(2x^2 - 7x + 3) = 0$
$(x + 2)(2x - 1)(x - 3) = 0$
$x = -2,\ x = \frac{1}{2},\ x = 3$
The solution set is $\left\{-2, \frac{1}{2}, 3\right\}$.

45. $12x^3 + 16x^2 - 5x - 3 = 0$

$-\frac{3}{2}$	12	16	-5	-3
		-18	3	3
	12	-2	-2	0

$\left(x + \frac{3}{2}\right)(12x^2 - 2x - 2) = 0$
$\left(x + \frac{3}{2}\right)2\left(6x^2 - x - 1\right) = 0$
$\left(x + \frac{3}{2}\right)2(3x + 1)(2x - 1) = 0$
$x = -\frac{3}{2},\ x = -\frac{1}{3},\ x = \frac{1}{2}$
The solution set is $\left\{-\frac{3}{2}, -\frac{1}{3}, \frac{1}{2}\right\}$.

46. $3x^3 + 7x^2 - 22x - 8 = 0$

$-\frac{1}{3}$	3	7	-22	-8
		-1	-2	8
	3	6	-24	0

$\left(x + \frac{1}{3}\right)3x^2 + 6x - 24 = 0$
$\left(x + \frac{1}{3}\right)3(x + 4)(x - 2) = 0$
$x = -4, x = 2, x = -\frac{1}{3}$
The solution set is $\left\{-4, -\frac{1}{3}, 2\right\}$.

47. The graph indicates that 2 is a solution to the equation.

2	1	2	-5	-6
		2	8	6
	1	4	3	0

The remainder is 0, so 2 is a solution.
$x^3 + 2x^2 - 5x - 6 = 0$
$(x - 2)(x^2 + 4x + 3) = 0$
$(x - 2)(x + 3)(x + 1) = 0$
The solutions are 2, -3, and -1, or $\{-3, -1, 2\}$.

48. The graph indicates that -3 is a solution to the equation.

$\underline{-3}\rfloor$	2	1	−13	6
		−6	15	−6
	2	−5	2	0

The remainder is 0, so -3 is a solution.

$$2x^3+x^2-13x+6=0$$
$$(x+3)\left(2x^2-5x+2\right)=0$$
$$(x+3)(2x-1)(x-2)=0$$

The solutions are -3, $\frac{1}{2}$, and 2, or $\left\{-3,\frac{1}{2},2\right\}$.

49. The table indicates that 1 is a solution to the equation.

$\underline{1}\rfloor$	6	−11	6	−1
		6	−5	1
	6	−5	1	0

The remainder is 0, so 1 is a solution.

$$6x^3-11x^2+6x-1=0$$
$$(x-1)\left(6x^2-5x+1\right)=0$$
$$(x-1)(3x-1)(2x-1)=0$$

The solutions are 1, $\frac{1}{3}$, and $\frac{1}{2}$, or $\left\{\frac{1}{3},\frac{1}{2},1\right\}$.

50. The table indicates that 1 is a solution to the equation.

$\underline{1}\rfloor$	2	11	−7	−6
		2	13	−6
	2	13	6	0

The remainder is 0, so 1 is a solution.

$$2x^3+11x^2-7x-6=0$$
$$(x-1)\left(2x^2+13x+6\right)=0$$
$$(x-1)(2x+1)(x+6)=0$$

The solutions are 1, $-\frac{1}{2}$, and -6, or $\left\{-6,-\frac{1}{2},1\right\}$.

51. **a.** $14x^3-17x^2-16x-177=0$

$\underline{3}\rfloor$	14	−17	−16	−177
		42	75	177
	14	25	59	0

The remainder is 0 so 3 is a solution.

$$14x^3-17x^2-16x-177$$
$$=(x-3)\left(14x^2+25x+59\right)$$

b. $f(x)=14x^3-17x^2-16x+34$

We need to find x when $f(x)=211$.

$$f(x)=14x^3-17x^2-16x+34$$
$$211=14x^3-17x^2-16x+34$$
$$0=14x^3-17x^2-16x-177$$

This is the equation obtained in part **a.** One solution is 3. It can be used to find other solutions (if they exist).

$$14x^3-17x^2-16x-177=0$$
$$(x-3)\left(14x^2+25x+59\right)=0$$

The polynomial $14x^2+25x+59$ cannot be factored, so the only solution is $x = 3$. The female moth's abdominal width is 3 millimeters.

52. **a.**

$\underline{2}\rfloor$	2	14	0	−72
		4	36	72
	2	18	36	0

$$2h^3+14h^2-72=(h-2)\left(2h^2+18h+36\right)$$

b. $V=lwh$

$$72=(h+7)(2h)(h)$$
$$72=2h^3+14h^2$$
$$0=2h^3+14h^2-72$$
$$0=(h-2)\left(2h^2+18h+36\right)$$
$$0=(h-2)\left(2\left(h^2+9h+18\right)\right)$$
$$0=(h-2)\left(2(h+6)(h+3)\right)$$
$$0=2(h-2)(h+6)(h+3)$$
$$2(h-2)=0 \quad h+6=0 \quad h+3=0$$
$$h-2=0 \quad \cancel{h=-6} \quad \cancel{h=-3}$$
$$h=2$$

The height is 2 inches, the width is $2\cdot 2=4$ inches and the length is $2+7=9$ inches. The dimensions are 2 inches by 4 inches by 9 inches.

53. $A = l \cdot w$ so

$$l = \frac{A}{w} = \frac{0.5x^3 - 0.3x^2 + 0.22x + 0.06}{x + 0.2}$$

–0.2	0.5	–0.3	0.22	0.06
		–0.1	0.08	–0.06
	0.5	–0.4	0.3	0

Therefore, the length of the rectangle is $0.5x^2 - 0.4x + 0.3$ units.

54. $A = l \cdot w$ so,

$$l = \frac{A}{w} = \frac{8x^3 - 6x^2 - 5x + 3}{x + \frac{3}{4}}$$

$-\frac{3}{4}$	8	–6	–5	3
		–6	9	–3
	8	–12	4	0

Therefore, the length of the rectangle is $8x^2 - 12x + 4$ units.

55. **a.**

$$f(30) = \frac{80(30) - 8000}{30 - 110} = 70$$

(30, 70) At a 30% tax rate, the government tax revenue will be $70 ten billion.

b.

110	80	-8000
		8800
	80	800

$$f(x) = 80 + \frac{800}{x - 110}$$

$$f(30) = 80 + \frac{800}{80 - 110} = 70$$

(30, 70) same answer as in **a.**

c. $f(x)$ is not a polynomial function. It is a rational function because it is the quotient of two linear polynomials.

56. **a.** $f(40) = \dfrac{80(40) - 8000}{40 - 110} = 68.57$

(40, 68.57) At a 40% tax rate, the government's revenue is $68.57 ten billion.

b.

110	80	-8000
		8800
	80	800

$$f(x) = 80 + \frac{800}{x - 110}$$

$$f(40) = 80 + \frac{800}{40 - 110}$$
$$= 68.57$$

c. $f(x)$ is not a polynomial function. It is a rational function because it is the quotient of two linear polynomials.

57. – 65. Answers may vary.

66. does not make sense; Explanations will vary. Sample explanation: The division must account for the zero coefficients on the x^4, x^3, x^2 and x terms.

67. makes sense

68. does not make sense; Explanations will vary. Sample explanation: The remainder theorem provides an alternative method for evaluating a function at a given value.

69. does not make sense; Explanations will vary. Sample explanation: The zeros of *f* are the same as the solutions of $f(x) = 0$.

70. false; Changes to make the statement true will vary. A sample change is: The degree of the quotient is 3, since $\dfrac{x^6}{x^3} = x^3$.

71. true

72. true

73. false; Changes to make the statement true will vary. A sample change is: The divisor is a factor of the divided only if the remainder is the whole number 0.

74.

$$\begin{array}{r} 5x^2+2x-4 \\ 4x+3\overline{)20x^3+23x^2-10x+k} \\ \underline{20x^3+15x^2} \\ 8x^2-10 \\ \underline{8x^2+6x} \\ -16x+k \\ \underline{-16x-12} \end{array}$$

To get a remainder of zero, k must equal –12.
$k=-12$

75. $f(x)=d(x)\cdot q(x)+r(x)$

$2x^2-7x+9=d(x)(2x-3)+3$

$2x^2-7x+6=d(x)(2x-3)$

$\dfrac{2x^2-7x+6}{2x-3}=d(x)$

$$\begin{array}{r} x-2 \\ 2x-3\overline{)2x^2-7x+6} \\ \underline{2x^2-3x} \\ -4x+6 \\ \underline{-4x+6} \end{array}$$

The polynomial is $x-2$.

76.

$$\begin{array}{r} x^{2n}-x^n+1 \\ x^n+1\overline{)x^{3n}\qquad\quad +1} \\ \underline{x^{3n}+x^{2n}} \\ -x^{2n} \\ \underline{-x^{2n}-x^n} \\ x^n+1 \\ \underline{x^n+1} \\ 0 \end{array}$$

77. $2x-4=2(x-2)$
Use synthetic division to divide by $x-2$. Then divide the quotient by 2.

78. $x^4-4x^3-9x^2+16x+20=0$

5	1	–4	–9	16	20
		5	5	–20	–20
	1	1	–4	–4	0

The remainder is zero and 5 is a solution to the equation.

$x^4-4x^3-9x^2+16x+20$
$=(x-5)(x^3+x^2-4x-4)$

To solve the equation, we set it equal to zero and factor.

$$(x-5)(x^3+x^2-4x-4)=0$$
$$(x-5)(x^2(x+1)-4(x+1))=0$$
$$(x-5)(x+1)(x^2-4)=0$$
$$(x-5)(x+1)(x+2)(x-2)=0$$

Apply the zero product principle.

$x-5=0 \qquad x+1=0$
$x=5 \qquad x=-1$

$x+2=0 \qquad x-2=0$
$x=-2 \qquad x=2$

The solutions are –2, –1, 2 and 5 and the solution set is $\{-2,-1,2,5\}$.

79. $x^2+4x-1=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(4)\pm\sqrt{(4)^2-4(1)(-1)}}{2(1)}$$
$$x=\frac{-4\pm\sqrt{20}}{2}$$
$$x=\frac{-4\pm2\sqrt{5}}{2}$$
$$x=-2\pm\sqrt{5}$$

The solution set is $\{-2\pm\sqrt{5}\}$.

80. $x^2+4x+6=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(4)\pm\sqrt{(4)^2-4(1)(6)}}{2(1)}$$

$$x=\frac{-4\pm\sqrt{-8}}{2}$$

$$x=\frac{-4\pm 2i\sqrt{2}}{2}$$

$$x=-2\pm i\sqrt{2}$$

The solution set is $\left\{-2\pm i\sqrt{2}\right\}$.

81. $f(x)=a_n(x^4-3x^2-4)$

$$f(3)=-150$$

$$a_n\left((3)^4-3(3)^2-4\right)=-150$$

$$a_n\left(81-27-4\right)=-150$$

$$a_n\left(50\right)=-150$$

$$a_n=-3$$

Section 2.5

Check Point Exercises

1. $p:\pm1,\ \pm2,\ \pm3,\ \pm6$

$q:\pm1$

$\frac{p}{q}:\pm1,\ \pm2,\ \pm3,\ \pm6$

are the possible rational zeros.

2. $p:\pm1,\ \pm3$

$q:\pm1,\ \pm2,\ \pm4$

$\frac{p}{q}:\pm1,\ \pm3,\ \pm\frac{1}{2},\ \pm\frac{1}{4},\ \pm\frac{3}{2},\ \pm\frac{3}{4}$

are the possible rational zeros.

3. $\pm1,\ \pm2,\ \pm4,\ \pm5,\ \pm10,\ \pm20$ are possible rational zeros

1	1	8	11	–20
		1	9	20
	1	9	20	0

1 is a zero.

$x^2+9x+20=0$

$(x+4)(x+5)=0$

$x=-4$ or $x=-5$

The solution set is $\{1,-4,-5\}$.

4. $\pm1,\ \pm2$ are possible rational zeros

2	1	1	-5	–2
		2	6	2
	1	3	1	0

2 is a zero.

$x^2+3x+1=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-3\pm\sqrt{3^2-4(1)(1)}}{2(1)}$$

$$=\frac{-3\pm\sqrt{5}}{2}$$

The solution set is $\left\{2,\frac{-3+\sqrt{5}}{2},\frac{-3-\sqrt{5}}{2}\right\}$.

5. $\pm1,\ \pm13$ are possible rational zeros.

1	1	–6	22	–30	13
		1	–5	17	–13
	1	–5	17	–13	0

1 is a zero.

1	1	5	17	–13
		1	–4	13
	1	–4	13	0

1 is a double root.

$x^2-4x+13=0$

$$x=\frac{4\pm\sqrt{16-52}}{2}=\frac{4\pm\sqrt{-36}}{2}=2+3i$$

The solution set is$\{1,\ 2+3i,\ 2-3i\}$.

6. $(x+3)(x-i)(x+i)=(x+3)(x^2+1)$
$f(x)=a_n(x+3)(x^2+1)$
$f(1)=a_n(1+3)(1^2+1)=8a_n=8$
$a_n=1$
$f(x)=(x+3)(x^2+1)$ or x^3+3x^2+x+3

7. $f(x)=x^4-14x^3+71x^2-154x+120$
$f(-x)=x^4+14x^3+71x^2+154x+120$
Since $f(x)$ has 4 changes of sign, there are 4, 2, or 0 positive real zeros.
Since $f(-x)$ has no changes of sign, there are no negative real zeros.

Exercise Set 2.5

1. $f(x)=x^3+x^2-4x-4$
$p:\pm1, \pm2, \pm4$
$q:\pm1$
$\frac{p}{q}:\pm1, \pm2, \pm4$

2. $f(x)=x^3+3x^2-6x-8$
$p:\pm1, \pm2, \pm4, \pm8$
$q:\pm1$
$\frac{p}{q}:\pm1, \pm2, \pm4, \pm8$

3. $f(x)=3x^4-11x^3-x^2+19x+6$
p: ±1, ±2, ±3, ±6
q: ±1, ±3
$\frac{p}{q}:\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{3}, \pm\frac{2}{3}$

4. $f(x)=2x^4+3x^3-11x^2-9x+15$
p: ±1, ±3, ±5, ±15
q: ±1, ±2
$\frac{p}{q}:\pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

5. $f(x)=4x^4-x^3+5x^2-2x-6$
p: ±1, ±2, ±3, ±6
q: ±1, ±2, ±4
$\frac{p}{q}:\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{3}{2}, \pm\frac{3}{4}$

6. $f(x)=3x^4-11x^3-3x^2-6x+8$
p: ±1, ±2, ±4, ±8
q: ±1, ±3
$\frac{p}{q}:\pm1, \pm2, \pm4, \pm8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

7. $f(x)=x^5-x^4-7x^3+7x^2-12x-12$
$p:\pm1, \pm2, \pm3 \pm4 \pm6 \pm12$
$q:\pm1$
$\frac{p}{q}:\pm1, \pm2, \pm3 \pm4 \pm6 \pm12$

8. $f(x)=4x^5-8x^4-x+2$
$p:\pm1, \pm2$
$q:\pm1, \pm2, \pm4$
$\frac{p}{q}:\pm1, \pm2, \pm\frac{1}{2}, \pm\frac{1}{4}$

9. $f(x)=x^3+x^2-4x-4$

a. $p:\pm1, \pm2, \pm4$
$q:\pm1$
$\frac{p}{q}:\pm1, \pm2, \pm4$

b.

2	1	1	–4	–4
		2	6	4
	1	3	2	0

2 is a zero.
2, –2, –1 are rational zeros.

c. $x^3+x^2-4x-4=0$
$(x-2)(x^2+3x+2)=0$
$(x-2)(x+2)(x+1)=0$

$x-2=0 \quad x+2=0 \quad x+1=0$
$x=2, \; x=-2, \; x=-1$
The solution set is {2, –2, –1}.

10. a. $f(x) = x^3 - 2x - 11x + 12$

p: ±1, ±2, ±3, ±4, ±6, ±12

q: ±1

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

b.

4	1	–2	–11	12
		4	8	–12
	1	2	–3	0

4 is a zero.
4, –3, 1 are rational zeros.

c. $x^3 - 2x^2 - 11x + 12 = 0$

$(x-4)\ \left(x^2 + 2x - 3\right) = 0$

$(x-4)\ (x+3)(x-1) = 0$

$x = 4,\ \ x = -3,\ \ x = 1$

The solution set is {4, –3, 1}.

11. $f(x) = 2x^3 - 3x^2 - 11x + 6$

a. $p: \pm 1, \pm 2, \ \pm 3, \pm 6$

$q: \pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

b.

3	2	–3	–11	6
		6	9	–6
	2	3	–2	0

3 is a zero.
$3, \frac{1}{2}, -2$ are rational zeros.

c. $2x^3 - 3x^2 - 11x + 6 = 0$

$(x-3)(2x^2 + 3x - 2) = 0$

$(x-3)(2x-1)(x+2) = 0$

$x = 3,\ x = \frac{1}{2},\ x = -2$

The solution set is $\left\{3, \frac{1}{2}, -2\right\}$.

12. a. $f(x) = 2x^3 - 5x^2 + x + 2$

p: ±1, ±2

q: ±1, ±2

$\frac{p}{q}: \pm 1, \pm 2, \pm \frac{1}{2}$

b.

2	2	–5	1	2
		4	–2	–2
	2	–1	–1	0

2 is a zero.
$2, -\frac{1}{2}, 1$ are rational zeros.

c. $2x^3 - 5x^2 + x + 2 = 0$

$(x-2)\ \left(2x^2 - x - 1\right) = 0$

$(x-2)\ (2x+1)(x-1) = 0$

$x = 2,\ \ x = -\frac{1}{2},\ \ x = 1$

The solution set is $\left\{2, -\frac{1}{2}, 1\right\}$.

13. a. $f(x) = x^3 + 4x^2 - 3x - 6$

p: ±1, ±2, ±3, ±6

q: ±1

$\frac{p}{q}$: ±1, ±2, ±3, ±6

b.

–1	1	4	–3	–6
		–1	–3	6
	1	3	–6	0

–1 is a rational zero.

c. $x^2 + 3x - 6 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{33}}{2}$$

The solution set is $\left\{-1, \frac{-3+\sqrt{33}}{2}, \frac{-3-\sqrt{33}}{2}\right\}$.

14. **a.** $f(x) = 2x^3 + x^2 - 3x + 1$
p: ±1
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm \frac{1}{2}$

b.

$\frac{1}{2}$	2	1	–3	1
		1	1	–1
	2	2	–2	0

$\frac{1}{2}$ is a rational zero.

c. $2x^2 + 2x - 2 = 0$
$x^2 + x - 1 = 0$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{5}}{2}$$
The solution set is $\left\{\frac{1}{2}, \frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}\right\}$.

15. **a.** $f(x) = 2x^3 + 6x^2 + 5x + 2$
p: ±1, ±2
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm 2, \pm \frac{1}{2}$

b.

–2	2	6	5	2
		–4	–4	–2
	2	2	1	0

–2 is a rational zero.

c. $2x^2 + 2x + 1 = 0$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(1)}}{2(2)}$$
$$= \frac{-2 \pm \sqrt{-4}}{4}$$
$$= \frac{-2 \pm 2i}{4}$$
$$= \frac{-1 \pm i}{2}$$
The solution set is $\left\{-2, \frac{-1+i}{2}, \frac{-1-i}{2}\right\}$.

16. **a.** $f(x) = x^3 - 4x^2 + 8x - 5$
p: ±1, ±5
q: ±1
$\frac{p}{q}: \pm 1, \pm 5$

b.

1	1	–4	8	–5
		1	–3	5
	1	–3	5	0

1 is a rational zero.

c. $x^2 - 3x + 5 = 0$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(5)}}{2(1)}$$
$$= \frac{3 \pm \sqrt{-11}}{2}$$
$$= \frac{3 \pm i\sqrt{11}}{2}$$
The solution set is $\left\{1, \frac{3+i\sqrt{11}}{2}, \frac{3-i\sqrt{11}}{2}\right\}$.

17. $x^3 - 2x^2 - 11x + 12 = 0$

a. $p: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

b.
$$\begin{array}{r|rrrr} 4 & 1 & -2 & -11 & 12 \\ & & 4 & 8 & -12 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$
4 is a root.
–3, 1, 4 are rational roots.

c. $x^3 - 2x^2 - 11x + 12$
$(x-4)(x^2+2x-3)=0$
$(x-4)(x+3)(x-1)=0$
$x-4=0 \quad x+3=0 \quad x-1=0$
$x=4 \quad x=-3 \quad x=1$
The solution set is $\{-3, 1, 4\}$.

18. a. $x^3 - 2x^2 - 7x - 4 = 0$
p: ±1, ±2, ±4
q: ±1
$\frac{p}{q}: \pm 1, \pm 2, \pm 4$

b.
$$\begin{array}{r|rrrr} 4 & 1 & -2 & -7 & -4 \\ & & 4 & 8 & 4 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$
4 is a root.
–1, 4 are rational roots.

c. $x^3 + 2x^2 - 7x - 4 = 0$
$(x-4)\ \left(x^2+2x+1\right)=0$
$(x-4) \quad (x+1)^2$
$x=4, \quad x=-1$
The solution set is $\{4, -1\}$.

19. $x^3 - 10x - 12 = 0$

a. $p: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

b.
$$\begin{array}{r|rrrr} -2 & 1 & 0 & -10 & -12 \\ & & -2 & 4 & 12 \\ \hline & 1 & -2 & -6 & 0 \end{array}$$
–2 is a rational root.

c. $x^3 - 10x - 12 = 0$
$(x+2)(x^2-2x-6)=0$
$$x = \frac{2 \pm \sqrt{4+24}}{2} = \frac{2 \pm \sqrt{28}}{2}$$
$$= \frac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7}$$
The solution set is $\left\{-2, 1+\sqrt{7}, 1-\sqrt{7}\right\}$.

20. a. $x^3 - 5x^2 + 17x - 13 = 0$
p: ±1, ±13
q: ±1
$\frac{p}{q}: \pm 1, \pm 13$

b.
$$\begin{array}{r|rrrr} 1 & 1 & -5 & 17 & -13 \\ & & 1 & -4 & 13 \\ \hline & 1 & -4 & 13 & 0 \end{array}$$
1 is a rational root.

c. $x^3 - 5x^2 + 17x - 13 = 0$
$(x-1)\ \left(x^2-4x+13\right)=0$
$$x = \frac{4 + \sqrt{16-52}}{2} = \frac{4 \pm \sqrt{-36}}{2}$$
$$= \frac{4 \pm 6i}{2} = 2 \pm 3i$$
The solution set is $\{1,\ 2+3i,\ 2,-3i\}$.

21. $6x^3 + 25x^2 - 24x + 5 = 0$

a. $p: \pm 1, \pm 5$
$q: \pm 1, \pm 2, \pm 3, \pm 6$
$\frac{p}{q}: \pm 1, \pm 5, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$

b.

–5	6	25	–24	5
		–30	25	–5
	6	–5	1	0

–5 is a root.
$-5, \frac{1}{2}, \frac{1}{3}$ are rational roots.

c. $6x^3 + 25x^2 - 24x + 5 = 0$
$(x+5)(6x^2 - 5x + 1) = 0$
$(x+5)(2x-1)(3x-1) = 0$
$x+5=0 \quad 2x-1=0 \quad 3x-1=0$
$x = -5, \quad x = \frac{1}{2}, \quad x = \frac{1}{3}$
The solution set is $\left\{-5, \frac{1}{2}, \frac{1}{3}\right\}$.

22. a. $2x^3 - 5x^2 - 6x + 4 = 0$
p: ±1, ±2, ±4
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm 2 \pm 4 \pm \frac{1}{2}$

b.

$\frac{1}{2}$	2	–5	–6	4
		1	–2	–4
	2	–4	–8	0

$\frac{1}{2}$ is a rational root.

c. $2x^3 - 5x^2 - 6x + 4 = 0$
$(x - \frac{1}{2})\left(2x^2 - 4x - 8\right) = 0$
$2\left(x - \frac{1}{2}\right)\left(x^2 - 2x - 4\right) = 0$
$x = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$
The solution set is $\left\{\frac{1}{2}, 1+\sqrt{5}, 1-\sqrt{5}\right\}$.

23. $x^4 - 2x^3 - 5x^2 + 8x + 4 = 0$

a. $p: \pm 1, \pm 2, \pm 4$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4$

b.

2	1	–2	–5	8	4
		2	0	–10	–4
	1	0	–5	–2	0

2 is a root.
$-2, 2$ are rational roots.

c. $x^4 - 2x^3 - 5x^2 + 8x + 4 = 0$
$(x-2)(x^3 - 5x - 2) = 0$

–2	1	0	–5	–2
		–2	4	2
	1	–2	–1	0

–2 is a zero of $x^3 - 5x - 2 = 0$.

$(x-2)(x+2)\left(x^2 - 2x - 1\right) = 0$
$x = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$
$= 1 \pm \sqrt{2}$
The solution set is
$\left\{-2, 2, 1+\sqrt{2}, 1-\sqrt{2}\right\}$.

24. a. $x^4 - 2x^2 - 16x - 15 = 0$
p: ±1, ±3, ±5, ±15
q: ±1
$\frac{p}{q}: \pm 1, \pm 3 \pm 5 \pm 15$

b.

3	1	0	–2	–16	–15
		3	9	21	15
	1	3	7	5	0

3 is a root.
$-1, 3$ are rational roots.

c. $x^4 - 2x^2 - 16x - 15 = 0$

$(x-3)\left(x^3 + 3x^2 + 7x + 5\right) = 0$

–1	1	3	7	5
		–1	–2	–5
	1	2	5	0

–1 is a root of $x^3 + 3x^2 + 7x + 5$

$(x-3)\ (x+1)\ \left(x^2 + 2x + 5\right)$

$x = \dfrac{-2 \pm \sqrt{4-20}}{2} = \dfrac{-2 \pm \sqrt{-16}}{2}$

$= \dfrac{-2 \pm 4i}{2} = -1 \pm 2i$

The solution set is $\{3, -1, -1+2i, -1-2i\}$.

25. $(x-1)\ (x+5i)\ (x-5i)$

$= (x-1)\ \left(x^2 + 25\right)$

$= x^3 + 25x - x^2 - 25$

$= x^3 - x^2 + 25x - 25$

$f(x) = a_n\left(x^3 - x^2 + 25x - 25\right)$

$f(-1) = a_n(-1-1-25-25)$

$-104 = a_n(-52)$

$a_n = 2$

$f(x) = 2\left(x^3 - x^2 + 25x - 25\right)$

$f(x) = 2x^3 - 2x^2 + 50x - 50$

26. $(x-4)\ (x+2i)\ (x-2i)$

$= (x-4)\ \left(x^2 + 4\right)$

$= x^3 - 4x^2 + 4x - 16$

$f(x) = a_n\left(x^3 - 4x^2 + 4x - 16\right)$

$f(-1) = a_n(-1-4-4-16)$

$-50 = a_n(-25)$

$a_n = 2$

$f(x) = 2\left(x^3 - 4x^2 + 4x - 16\right)$

$f(x) = 2x^3 - 8x^2 + 8x - 32$

27. $(x+5)(x-4-3i)(x-4+3i)$

$= (x+5)\left(x^2 - 4x + 3ix - 4x + 16 - 12i\right.$

$\left.-3ix + 12i - 9i^2\right)$

$= (x+5)\left(x^2 - 8x + 25\right)$

$= \left(x^3 - 8x^2 + 25x + 5x^2 - 40x + 125\right)$

$= x^3 - 3x^2 - 15x + 125$

$f(x) = a_n(x^3 - 3x^2 - 15x + 125)$

$f(2) = a_n\left(2^3 - 3(2)^2 - 15(2) + 125\right)$

$91 = a_n(91)$

$a_n = 1$

$f(x) = 1\left(x^3 - 3x^2 - 15x + 125\right)$

$f(x) = x^3 - 3x^2 - 15x + 125$

28. $(x-6)\ (x+5+2i)\ (x+5-2i)$

$= (x-6)\ \left(x^2 + 5x - 2ix + 5x + 25 - 10i + 2ix + 10i - 4i^2\right)$

$= (x-6)\ \left(x^2 + 10x + 29\right)$

$= x^3 + 10x^2 + 29x - 6x^2 - 60x - 174$

$= x^3 + 4x^2 - 31x - 174$

$f(x) = a_n\left(x^3 + 4x^2 - 31x - 174\right)$

$f(2) = a_n(8 + 16 - 62 - 174)$

$-636 = a_n(-212)$

$a_n = 3$

$f(x) = 3\left(x^3 + 4x^2 - 31x - 174\right)$

$f(x) = 3x^3 + 12x^2 - 93x - 522$

29. $(x-i)(x+i)(x-3i)(x+3i)$

$= \left(x^2 - i^2\right)\left(x^2 - 9i^2\right)$

$= \left(x^2 + 1\right)\left(x^2 + 9\right)$

$= x^4 + 10x^2 + 9$

$f(x) = a_n(x^4 + 10x^2 + 9)$

$f(-1) = a_n((-1)^4 + 10(-1)^2 + 9)$

$20 = a_n(20)$

$a_n = 1$

$f(x) = x^4 + 10x^2 + 9$

30. $(x+2)\left(x+\frac{1}{2}\right)(x-i)(x+i)$

$=\left(x^2+\frac{5}{2}x+1\right)(x^2+1)$

$=x^4+x^2+\frac{5}{2}x^3+\frac{5}{2}x+x^2+1$

$=x^4+\frac{5}{2}x^3+2x^2+\frac{5}{2}x+1$

$f(x)=a_n\left(x^4+\frac{5}{2}x^3+2x^2+\frac{5}{2}x+1\right)$

$f(1)=a_n\left[(1)^4+\frac{5}{2}(1)^3+2(1)^2+\frac{5}{2}(1)+1\right]$

$18=a_n(9)$

$a_n=2$

$f(x)=2\left(x^4+\frac{5}{2}x^3+2x^2+\frac{5}{2}x+1\right)$

$f(x)=2x^4+5x^3+4x^2+5x+2$

31. $(x+2)\ (x-5)\ (x-3+2i)\ (x-3-2i)$

$=(x^2-3x-10)(x^2-3x-2ix-3x+9+6i+2ix-6i-4i^2)$

$=(x^2-3x-10)\ (x^2-6x+13)$

$=x^4-6x+13x^2-3x^3+18x^2-39x-10x^2+60x-130$

$=x^4-9x^3+21x^2+21x-130$

$f(x)=a_n\left(x^4-9x^3+21x^2+21x-130\right)$

$f(1)=a_n(1-9+21+21-130)$

$-96=a_n(-96)$

$a_n=1$

$f(x)=x^4-9x^3+21x^2+21x-130$

32. $(x+4)\ (3x-1)\ (x-2+3i)\ (x-2-3i)$

$=(3x^2+11x-4)\ (x^2-2x-3ix-2x+4+6i+3ix-6i-9i^2)$

$=(3x^2+11x-4)\ (x^2-4x+13)$

$=3x^4-12x^3+39x^2+11x^3-44x^2+143x-4x^2+16x-52$

$=3x^4-x^3-9x^2+159x-52$

$f(x)=a_n\left(3x^4-x^3-9x^2+159x-52\right)$

$f(1)=a_n(3-1-9+159-52)$

$100=a_n(100)$

$a_n=1$

$f(x)=3x^4-x^3-9x^2+159x-52$

33. $f(x)=x^3+2x^2+5x+4$
Since $f(x)$ has no sign variations,
no positive real roots exist.
$f(-x)=-x^3+2x^2-5x+4$
Since $f(-x)$ has 3 sign variations,
3 or 1 negative real roots exist.

34. $f(x)=x^3+7x^2+x+7$
Since $f(x)$ has no sign variations no positive real roots exist.
$f(-x)=-x^3+7x^2-x+7$
Since $f(-x)$ has 3 sign variations, 3 or 1 negative real roots exist.

35. $f(x)=5x^3-3x^2+3x-1$
Since $f(x)$ has 3 sign variations, 3 or 1 positive real roots exist.
$f(-x)=-5x^3-3x^2-3x-1$
Since $f(-x)$ has no sign variations, no negative real roots exist.

36. $f(x)=-2x^3+x^2-x+7$
Since $f(x)$ has 3 sign variations,
3 or 1 positive real roots exist.
$f(-x)=2x^3+x^2+x+7$
Since $f(-x)$ has no sign variations,
no negative real roots exist.

37. $f(x)=2x^4-5x^3-x^2-6x+4$
Since $f(x)$ has 2 sign variations, 2 or 0 positive real roots exist.
$f(-x)=2x^4+5x^3-x^2+6x+4$
Since $f(-x)$ has 2 sign variations, 2 or 0 negative real roots exist.

38. $f(x)=4x^4-x^3+5x^2-2x-6$
Since $f(x)$ has 3 sign variations, 3 or 1 positive real roots exist.
$f(-x)=4x^4+x^3+5x^2+2x-6$
Since $f(x)$ has 1 sign variations, 1 negative real roots exist.

39. $f(x) = x^3 - 4x^2 - 7x + 10$

$p: \pm 1, \pm 2, \pm 5, \pm 10$

$q: \pm 1$

$\frac{p}{q}: \pm 1, \pm 2, \pm 5, \pm 10$

Since $f(x)$ has 2 sign variations, 0 or 2 positive real zeros exist.

$f(-x) = -x^3 - 4x^2 + 7x + 10$

Since $f(-x)$ has 1 sign variation, exactly one negative real zeros exists.

–2	1	–4	–7	10
		–2	12	–10
	1	–6	5	0

–2 is a zero.

$$f(x) = (x+2)(x^2 - 6x + 5)$$
$$= (x+2)(x-5)(x-1)$$

$x = -2, x = 5, x = 1$

The solution set is{–2, 5, 1}.

40. $f(x) = x^3 + 12x^2 + 2x + 10$

p: ±1, ±2, ±5, ±10

q: ±1,

$\frac{p}{q}: \pm 1, \pm 2 \pm 5 \pm 10$

Since $f(x)$ has no sign variations, no positive zeros exist.

$f(-x) = -x^3 + 12x^2 - 21x + 10$

Since $f(-x)$ has 3 sign variations, 3 or 1 negative zeros exist.

–1	1	12	21	10
		–1	–11	–10
	1	11	10	0

–1 is a zero.

$$f(x) = (x+1)(x^2 + 11x + 10)$$
$$= (x+1)(x+10)(x+1)$$
$$x = -1, x = -10$$

The solution set is {–1, –10}.

41. $2x^3 - x^2 - 9x - 4 = 0$

$p: \pm 1, \pm 2, \pm 4$

$q: \pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm 2, \pm 4 \pm \frac{1}{2}$

1 positive real root exists.

$f(-x) = -2x^3 - x^2 + 9x - 4$ 2 or no negative real roots exist.

$-\frac{1}{2}$	2	–1	–9	–4
		–1	1	4
	2	–2	–8	0

$-\frac{1}{2}$ is a root.

$$\left(x + \frac{1}{2}\right)(2x^2 - 2x - 8) = 0$$
$$2\left(x + \frac{1}{2}\right)(x^2 - x - 4) = 0$$
$$x = \frac{1 \pm \sqrt{1+16}}{2} = \frac{1 \pm \sqrt{17}}{2}$$

The solution set is $\left\{-\frac{1}{2}, \frac{1+\sqrt{17}}{2}, \frac{1-\sqrt{17}}{2}\right\}$.

42. $3x^3 - 8x^2 - 8x + 8 = 0$

p: ±1, ±2, ±4, ±8

q: ±1, ±3

$\frac{p}{q}: \pm 1, \pm 2 \pm 4 \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$

Since $f(x)$ has 2 sign variations, 2 or no positive real roots exist.

$$f(-x) = -3x^3 - 8x^2 + 8x + 8$$

Since $f(-x)$ has 1 sign changes, exactly 1 negative real zero exists.

$\frac{2}{3}$	3	–8	–8	8
		2	–4	–8
	3	–6	–12	0

$\frac{2}{3}$ is a zero.

$$f(x)=\left(x-\frac{2}{3}\right)\left(3x^2-6x-12\right)$$

$$x=\frac{6\pm\sqrt{36+144}}{6}=\frac{6\pm 6\sqrt{5}}{6}$$

$$=1\pm\sqrt{5}$$

The solution set is $\left\{\frac{2}{3},1+\sqrt{5},1-\sqrt{5}\right\}$.

43. $f(x)=x^4-2x^3+x^2+12x+8$

$p:\pm1,\ \pm2,\ \pm4,\ \pm8$

$q:\pm1$

$\frac{p}{q}:\pm1,\ \pm2,\ \pm4,\ \pm8$

Since $f(x)$ has 2 sign changes, 0 or 2 positive roots exist.

$f(-x)=(-x)^4-2(-x)^3+(-x)^2-12x+8$

$=x^4+2x^3+x^2-12x+8$

Since $f(-x)$ has 2 sign changes, 0 or 2 negative roots exist.

–1	1	–2	1	12	8
		–1	4	–4	–8
	1	–3	4	8	0

–1	1	–3	4	8
		–1	4	–8
	1	–4	8	0

$0=x^2-4x+8$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(8)}}{2(1)}$$

$$x=\frac{4\pm\sqrt{16-32}}{2}$$

$$x=\frac{4\pm\sqrt{-16}}{2}$$

$$x=\frac{4\pm4i}{2}$$

$x=2\pm2i$

The solution set is $\{-1, -1, 2+2i, 2-2i\}$.

44. $f(x)=x^4-4x^3-x^2+14x+10$

p: ±1, ±2, ±5, ±10

q: ±1

$\frac{p}{q}$: ±1, ±2, ±5, ±10

–1	1	-4	-1	14	10
		–1	5	-4	-10
	1	-5	4	10	0

–1	1	-5	4	10
		–1	6	–10
	1	-6	10	0

$f(x)=(x-1)(x-1)(x^2-6x+10)$

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(10)}}{2(1)}\qquad x=1$$

$$x=\frac{6\pm\sqrt{36-40}}{2}$$

$$x=\frac{6\pm\sqrt{-4}}{2}$$

$$x=\frac{6\pm2i}{2}$$

$x=3\pm i$

The solution set is $\{-1, 3-i, 3+i\}$

45. $x^4-3x^3-20x^2-24x-8=0$

$p:\pm1,\pm2,\pm4,\pm8$

$q:\pm1$

$\frac{p}{q}:\pm1,\pm2,\pm4\ \pm8$

1 positive real root exists.

3 or 1 negative real roots exist.

–1	1	–3	–20	–24	–8
		–1	4	16	8
	1	–4	–16	–8	0

$(x+1)\left(x^3-4x^2-16x-8\right)=0$

–2	1	–4	–16	–8
		–2	12	8
	1	–6	–4	0

$(x+1)(x+2)\left(x^2-6x-4\right)=0$

$x=\dfrac{6\pm\sqrt{36+16}}{2}=\dfrac{6\pm\sqrt{52}}{2}$

$=\dfrac{6\pm 2\sqrt{13}}{2}=\dfrac{3\pm\sqrt{13}}{2}$

The solution set is

$\left\{-1,\,-2,\,3\pm\sqrt{13},\,3-\sqrt{13}\right\}$.

46. $x^4-x^3+2x^2-4x-8=0$

p: ±1, ±2, ±4, ±8

q: ±1

$\dfrac{p}{q}:\pm1,\pm2\pm4\pm8$

1 negative real root exists.

–1	1	–1	2	–4	–8
		–1	2	–4	8
	1	–2	4	–8	0

$(x+1)\left(x^3-2x^2+4x-8\right)$

2	1	–2	4	–8
		2	0	8
	1	0	4	0

$(x+1)\quad (x-2)\quad \left(x^2+4\right)$

$x+1=0\quad x-2=0\quad x^2+4=0$

$x=-1\quad x=2\quad x^2=-4$

$x=\pm 2i$

The solution set is $\{-1,\,2,\,2i,\,-2i\}$.

47. $f(x)=3x^4-11x^3-x^2+19x+6$

$p:\pm1,\,\pm2,\,\pm3,\,\pm6$

$q:\pm1,\,\pm3$

$\dfrac{p}{q}:\pm1,\,\pm2,\,\pm3,\,\pm6,\,\pm\dfrac{1}{3},\,\pm\dfrac{2}{3}$

2 or no positive real zeros exists.

$f(-x)=3x^4+11x^3-x^2-19x+6$

2 or no negative real zeros exist.

–1	3	–11	–1	19	6
		–3	14	–13	–6
	3	–14	13	6	0

$f(x)=(x+1)\left(3x^3-14x^2+13x+6\right)$

2	3	–14	13	6
		6	–16	–6
	3	–8	–3	0

$f(x)=(x+1)(x-2)\left(3x^2-8x-3\right)$

$=(x+1)(x-2)(3x+1)(x-3)$

$x=-1,\ x=2\ x=-\dfrac{1}{3},\ x=3$

The solution set is $\left\{-1,\,2,\,-\dfrac{1}{3},\,3\right\}$.

48. $f(x)=2x^4+3x^3-11x^2-9x+15$

p: ±1, ±3, ±5, ±15

q: ±1, ±2

$\dfrac{p}{q}:\pm1,\pm3,\pm5,\pm15,\pm\dfrac{1}{2},\pm\dfrac{3}{2},\pm\dfrac{5}{2},\pm\dfrac{15}{2}$

2 or no positive real zeros exist.

$f(-x)=2x^4-3x^3-11x^2+9x+15$

2 or no negative real zeros exist.

1	2	3	–11	–9	15
		2	5	–6	–15
	2	5	–6	–15	0

$f(x)=(x-1)\left(2x^3+5x^2-6x-15\right)$

$-\frac{5}{2}$	2	5	–6	–15
		–5	0	15
	2	0	–6	0

$$f(x) = (x-1)\left(x+\frac{5}{2}\right)\left(2x^2-6\right)$$
$$= 2(x-1)\left(x+\frac{5}{2}\right)\left(x^2-3\right)$$

$x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

$x = 1,\ x = -\frac{5}{2},\ x = \sqrt{3},\ x = -\sqrt{3}$

The solution set is $\left\{1, -\frac{5}{2}, \sqrt{3}, -\sqrt{3}\right\}$.

49. $4x^4 - x^3 + 5x^2 - 2x - 6 = 0$

$p: \pm 1, \pm 2, \pm 3, \pm 6$

$q: \pm 1, \pm 2, \pm 4$

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

3 or 1 positive real roots exists.
1 negative real root exists.

1	4	–1	5	–2	–6
		4	3	8	6
	4	3	8	6	0

$(x-1)(4x^3 + 3x^2 + 8x + 6) = 0$

$4x^3 + 3x^2 + 8x + 6 = 0$ has no positive real roots.

$-\frac{3}{4}$	4	3	8	6
		–3	0	–6
	4	0	8	0

$$(x-1)\left(x+\frac{3}{4}\right)\left(4x^2+8\right) = 0$$
$$4(x-1)\left(x+\frac{3}{4}\right)\left(x^2+2\right) = 0$$

$x^2 + 2 = 0$

$x^2 = -2$

$x = \pm i\sqrt{2}$

The solution set is $\left\{1, -\frac{3}{4}, i\sqrt{2}, -i\sqrt{2}\right\}$.

50. $3x^4 - 11x^3 - 3x^2 - 6x + 8 = 0$

p: ±1, ±2, ±4, ±8

q: ±1, ±3

$\frac{p}{q}: \pm 1, \pm 2 \pm 4 \pm 8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

2 or no positive real roots exist.

$f(-x) = 3x^4 + 11x^3 - 3x^2 + 6x + 8$ 2 or no negative real roots exist.

4	3	–11	–3	–6	8
		12	4	4	–8
	3	1	1	–2	0

$(x-4)\left(3x^3 + x^2 + x - 2\right) = 0$

Another positive real root must exist.

$\frac{2}{3}$	3	1	1	–2
		2	2	2
	3	3	3	0

$$(x-4)\left(x-\frac{2}{3}\right)\left(3x^2+3x+3\right) = 0$$
$$3(x-4)\left(x-\frac{2}{3}\right)\left(x^2+x+1\right) = 0$$
$$x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

The solution set is $\left\{4, \frac{2}{3}, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}\right\}$.

51. $2x^5 + 7x^4 - 18x^2 - 8x + 8 = 0$

$p: \pm 1, \pm 2, \pm 4, \pm 8$

$q: \pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{2}$

2 or no positive real roots exists.
3 or 1 negative real root exist.

–2	2	7	0	–18	–8	8
		–4	–6	12	12	–8
	2	3	–6	–6	4	0

$(x+2)(2x^4 + 3x^3 - 6x^2 - 6x + 4) = 0$

$4x^3 + 3x^2 + 8x + 6 = 0$ has no positive real roots.

–2	2	3	–6	–6	4
		–4	2	8	–4
	2	–1	–4	2	0

$(x+2)^2(2x^3-x^2-4x+2)$

$\frac{1}{2}$	2	–1	–4	2
		1	0	2
	2	0	–4	0

$(x+2)^2\left(x-\frac{1}{2}\right)\left(2x^2-4\right)=0$

$2(x+2)^2\left(x-\frac{1}{2}\right)\left(x^2-2\right)=0$

$x^2-2=0$

$x^2=2$

$x=\pm\sqrt{2}$

The solution set is $\left\{-2, \frac{1}{2}, \sqrt{2}, -\sqrt{2}\right\}$.

52. $4x^5+12x^4-41x^3-99x^2+10x+24=0$

p: ±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24

q: ±1, ±2, ±4

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24, \pm\frac{1}{2}, \pm\frac{3}{2},$

$\pm\frac{1}{4}, \pm\frac{3}{4}$

2 or no positive real roots exist.

$f(-x)=-4x^5+12x^4+41x^3-99x^2-10x+24$

3 or 1 negative real roots exist.

3	4	12	–41	–99	10	24
		12	72	93	–18	–24
	4	24	31	–6	–8	0

$(x-3)\left(4x^4+24x^3+31x^2-6x-8\right)=0$

–2	4	24	31	–6	–8
		–8	–32	2	8
	4	16	–1	–4	0

$(x-3)(x+2)\left(4x^3+16x^2-x-4\right)=0$

–4	4	16	–1	4
		–16	0	4
	4	0	–1	0

$(x-3)(x+2)(x+4)\left(4x^2-1\right)=0$

$4x^2-1=0$

$4x^2=1$

$x^2=\frac{1}{4}$

$x=\pm\frac{1}{2}$

The solution set is $\left\{3, -2, -4, \frac{1}{2}, -\frac{1}{2}\right\}$.

53. $f(x)=-x^3+x^2+16x-16$

a. From the graph provided, we can see that -4 is an x-intercept and is thus a zero of the function. We verify this below:

–4	–1	1	16	–16
		4	–20	16
	–1	5	–4	0

Thus, $-x^3+x^2+16x-16=0$

$(x+4)\left(-x^2+5x-4\right)=0$

$-(x+4)\left(x^2-5x+4\right)=0$

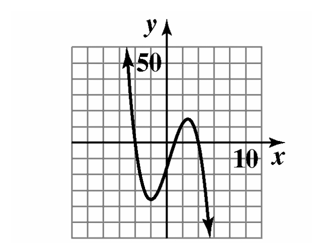
$-(x+4)(x-1)(x-4)=0$

$x+4=0$ or $x-1=0$ or $x-4=0$

$x=-4$ $\quad x=1$ $\quad x=4$

The zeros are -4, 1, and 4.

b.



***f*(*x*) = −*x*³ + *x*² + 16*x* − 16**

54. $f(x) = -x^3 + 3x^2 - 4$

a. From the graph provided, we can see that -1 is an x-intercept and is thus a zero of the function. We verify this below:

$$\begin{array}{r|rrrr} -1 & -1 & 3 & 0 & -4 \\ & & 1 & -4 & 4 \\ \hline & -1 & 4 & -4 & 0 \end{array}$$

Thus, $-x^3 + 3x^2 - 4 = 0$

$$(x+1)(-x^2+4x-4) = 0$$
$$-(x+1)(x^2-4x+4) = 0$$
$$-(x+1)(x-2)^2 = 0$$

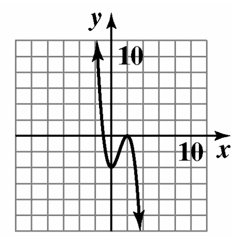
$x+1=0$ or $(x-2)^2 = 0$

$x=-1$ $\quad x-2=0$

$x=2$

The zeros are -1 and 2.

b.



***f*(*x*) = −*x*³ + 3*x*² − 4**

55. $f(x) = 4x^3 - 8x^2 - 3x + 9$

a. From the graph provided, we can see that -1 is an x-intercept and is thus a zero of the function. We verify this below:

$$\begin{array}{r|rrrr} -1 & 4 & -8 & -3 & 9 \\ & & -4 & 12 & -9 \\ \hline & 4 & -12 & 9 & 0 \end{array}$$

Thus, $4x^3 - 8x^2 - 3x + 9 = 0$

$$(x+1)(4x^2-12x+9) = 0$$
$$(x+1)(2x-3)^2 = 0$$

$x+1=0$ or $(2x-3)^2=0$

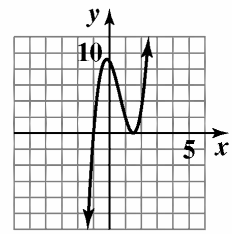
$x=-1$ $\quad 2x-3=0$

$2x=3$

$x=\frac{3}{2}$

The zeros are -1 and $\frac{3}{2}$.

b.



***f*(*x*) = 4*x*³ − 8*x*² − 3*x* + 9**

56. $f(x) = 3x^3 + 2x^2 + 2x - 1$

a. From the graph provided, we can see that $\frac{1}{3}$ is an x-intercept and is thus a zero of the function. We verify this below:

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & 2 & 2 & -1 \\ & & 1 & 1 & 1 \\ \hline & 3 & 3 & 3 & 0 \end{array}$$

Thus, $3x^3 + 2x^2 + 2x - 1 = 0$

$$\left(x-\frac{1}{3}\right)(3x^2+3x+3) = 0$$
$$3\left(x-\frac{1}{3}\right)(x^2+x+1) = 0$$

Note that x^2+x+1 will not factor, so we use the quadratic formula:

$x - \frac{1}{3} = 0$ or $x^2+x+1=0$

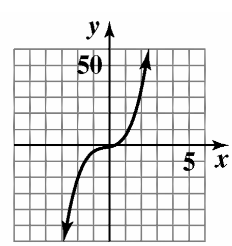
$a=1 \quad b=1 \quad c=1$

$x = \frac{1}{3}$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{-3}}{2}$$
$$= \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

The zeros are $\frac{1}{3}$ and $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$.

b.



***f*(*x*) = 3*x*³ + 2*x*² + 2*x* − 1**

57. $f(x) = 2x^4 - 3x^3 - 7x^2 - 8x + 6$

a. From the graph provided, we can see that $\frac{1}{2}$ is an x-intercept and is thus a zero of the function. We verify this below:

$\frac{1}{2}$	2	−3	−7	−8	6
		1	−1	−4	−6
	2	−2	−8	−12	0

Thus, $2x^4 - 3x^3 - 7x^2 - 8x + 6 = 0$

$$\left(x - \frac{1}{2}\right)\left(2x^3 - 2x^2 - 8x - 12\right) = 0$$

$$2\left(x - \frac{1}{2}\right)\left(x^3 - x^2 - 4x - 6\right) = 0$$

To factor $x^3 - x^2 - 4x - 6$, we use the Rational Zero Theorem to determine possible rational zeros.

Factors of the constant term -6:
$\pm 1, \ \pm 2, \ \pm 3, \ \pm 6$

Factors of the leading coefficient 1: ± 1

The possible rational zeros are:

$$\frac{\text{Factors of } -6}{\text{Factors of } 1} = \frac{\pm 1, \ \pm 2, \ \pm 3, \ \pm 6}{\pm 1}$$
$$= \pm 1, \ \pm 2, \ \pm 3, \ \pm 6$$

We test values from above until we find a zero. One possibility is shown next:

Test 3:

3	1	−1	−4	−6
		3	6	6
	1	2	2	0

The remainder is 0, so 3 is a zero of *f*.

$$2x^4 - 3x^3 - 7x^2 - 8x + 6 = 0$$

$$\left(x - \frac{1}{2}\right)\left(2x^3 - 2x^2 - 8x - 12\right) = 0$$

$$2\left(x - \frac{1}{2}\right)\left(x^3 - x^2 - 4x - 6\right) = 0$$

$$2\left(x - \frac{1}{2}\right)(x - 3)\left(x^2 + 2x + 2\right) = 0$$

Note that $x^2 + x + 1$ will not factor, so we use the quadratic formula:

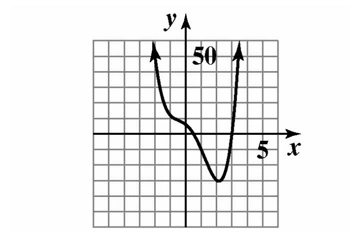
$a = 1 \quad b = 2 \quad c = 2$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i$$

The zeros are $\frac{1}{2}$, 3, and $-1 \pm i$.

b.



***f*(*x*) = 2*x*⁴ − 3*x*³ − 7*x*² − 8*x* + 6**

58. $f(x) = 2x^4 + 2x^3 - 22x^2 - 18x + 36$

a. From the graph provided, we can see that 1 and 3 are x-intercepts and are thus zeros of the function. We verify this below:

1	2	2	−22	−18	36
		2	4	−18	−36
	2	4	−18	−36	0

Thus, $2x^4 + 2x^3 - 22x^2 - 18x + 36$

$$= (x - 1)\left(2x^3 + 4x^2 - 18x - 36\right)$$

3	2	4	−18	−36
		6	30	36
	2	10	12	0

Thus, $2x^4 + 2x^3 - 22x^2 - 18x + 36 = 0$

$$(x - 1)(x - 3)\left(2x^2 + 10x + 12\right) = 0$$

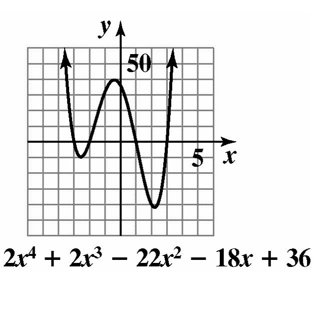
$$2(x - 1)(x - 3)\left(x^2 + 5x + 6\right) = 0$$

$$2(x - 1)(x - 3)(x + 3)(x + 2) = 0$$

$$x = 1, \quad x = 3, \quad x = -3, \quad x = -2$$

The zeros are -3, -2, 1, and 3.

b.



***f*(*x*) = 2*x*⁴ + 2*x*³ − 22*x*² − 18*x* + 36**

59. $f(x) = 3x^5 + 2x^4 - 15x^3 - 10x^2 + 12x + 8$

a. From the graph provided, we can see that 1 and 2 are x-intercepts and are thus zeros of the function. We verify this below:

1	3	2	−15	−10	12	8
		3	5	−10	−20	−8
	3	5	−10	−20	−8	0

Thus, $3x^5 + 2x^4 - 15x^3 - 10x^2 + 12x + 8$

$= (x-1)(3x^4 + 5x^3 - 10x^2 - 20x - 8)$

2	3	5	−10	−20	−8
		6	22	24	8
	3	11	12	4	0

Thus, $3x^5 + 2x^4 - 15x^3 - 10x^2 + 12x + 8$

$= (x-1)(3x^4 + 5x^3 - 10x^2 - 20x - 8)$

$= (x-1)(x-2)(3x^3 + 11x^2 + 12x + 4)$

To factor $3x^3 + 11x^2 + 12x + 4$, we use the Rational Zero Theorem to determine possible rational zeros.

Factors of the constant term 4: $\pm1, \pm2, \pm4$
Factors of the leading coefficient 3: $\pm1, \pm3$

The possible rational zeros are:

$$\frac{\text{Factors of } 4}{\text{Factors of } 3} = \frac{\pm1, \pm2, \pm4}{\pm1, \pm3}$$

$$= \pm1, \pm2, \pm4, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$$

We test values from above until we find a zero. One possibility is shown next:

Test -1:

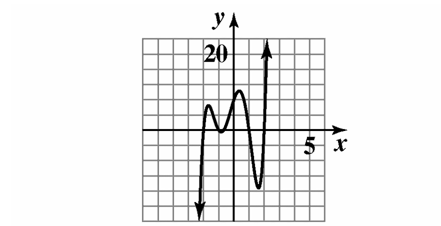
−1	3	11	12	4
		−3	−8	−4
	3	8	4	0

The remainder is 0, so -1 is a zero of *f*. We can now finish the factoring:

$$3x^5 + 2x^4 - 15x^3 - 10x^2 + 12x + 8 = 0$$
$$(x-1)(3x^4 + 5x^3 - 10x^2 - 20x - 8) = 0$$
$$(x-1)(x-2)(3x^3 + 11x^2 + 12x + 4) = 0$$
$$(x-1)(x-2)(x+1)(3x^2 + 8x + 4) = 0$$
$$(x-1)(x-2)(x+1)(3x+2)(x+2) = 0$$
$$x = 1,\ x = 2,\ x = -1,\ x = -\frac{2}{3},\ x = -2$$

The zeros are -2, -1, $-\frac{2}{3}$, 1 and 2.

b.



***f*(*x*) = 3*x*⁵ + 2*x*⁴ − 15*x*³ − 10*x*² + 12*x* + 8**

60. $f(x) = -5x^4 + 4x^3 - 19x^2 + 16x + 4$

a. From the graph provided, we can see that 1 is an x-intercept and is thus a zero of the function. We verify this below:

1	−5	4	−19	16	4
		−5	−1	−20	−4
	−5	−1	−20	−4	0

Thus, $-5x^4 + 4x^3 - 19x^2 + 16x + 4 = 0$

$$(x-1)(-5x^3 - x^2 - 20x - 4) = 0$$
$$-(x-1)(5x^3 + x^2 + 20x + 4) = 0$$

To factor $5x^3 + x^2 + 20x + 4$, we use the Rational Zero Theorem to determine possible rational zeros.

Factors of the constant term 4: $\pm1, \pm2, \pm4$
Factors of the leading coefficient 5: $\pm1, \pm5$

The possible rational zeros are:

$$\frac{\text{Factors of } 4}{\text{Factors of } 5} = \frac{\pm1, \pm2, \pm4}{\pm1, \pm5}$$

$$= \pm1, \pm2, \pm4, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{4}{5}$$

We test values from above until we find a zero. One possibility is shown next:

Test $-\frac{1}{5}$:

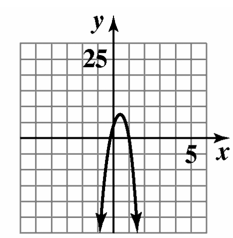
$$\begin{array}{r|rrrr} -\frac{1}{5} & 5 & 1 & 20 & 4 \\ & & -1 & 0 & -4 \\ \hline & 5 & 0 & 20 & 0 \end{array}$$

The remainder is 0, so $-\frac{1}{5}$ is a zero of *f*.

$$-5x^4+4x^3-19x^2+16x+4=0$$
$$(x-1)(-5x^3-x^2-20x-4)=0$$
$$-(x-1)(5x^3+x^2+20x+4)=0$$
$$-(x-1)\left(x+\frac{1}{5}\right)(5x^2+20)=0$$
$$-5(x-1)\left(x+\frac{1}{5}\right)(x^2+4)=0$$
$$-5(x-1)\left(x+\frac{1}{5}\right)(x+2i)(x-2i)=0$$
$$x=1,\ x=-\frac{1}{5},\ x=-2i,\ x=2i$$

The zeros are $-\frac{1}{5}$, 1, and $\pm 2i$.

b.



***f*(*x*) = −5*x*⁴ + 4*x*³ − 19*x*² + 16*x* + 4**

61. $V(x)=x(x+10)(30-2x)$

$$2000=x(x+10)(30-2x)$$
$$2000=-2x^3+10x^2+300x$$
$$2x^3-10x^2-300x+2000=0$$
$$x^3-5x^2-150x+1000=0$$

Find the roots.

$$\begin{array}{r|rrrr} 10 & 1 & -5 & -150 & 1000 \\ & & 10 & 50 & -1000 \\ \hline & 1 & 5 & -100 & 0 \end{array}$$

Use the remaining quadratic to find the other 2 roots.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(5)\pm\sqrt{(5)^2-4(1)(-100)}}{2(1)}$$
$$x\approx -12.8,\ 7.8$$

Since the depth must be positive, reject the negative value.

The depth can be 10 inches or 7.8 inches to obtain a volume of 2000 cubic inches.

62. $V(x)=x(x+10)(30-2x)$

$$1500=x(x+10)(30-2x)$$
$$1500=-2x^3+10x^2+300x$$
$$2x^3-10x^2-300x+1500=0$$
$$x^3-5x^2-150x+750=0$$

Find the roots.

$$\begin{array}{r|rrrr} 5 & 1 & -5 & -150 & 750 \\ & & 5 & 0 & -750 \\ \hline & 1 & 0 & -150 & 0 \end{array}$$

Use the remaining quadratic to find the other 2 roots.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(0)\pm\sqrt{(0)^2-4(1)(-150)}}{2(1)}$$
$$x\approx -12.2,\ 12.2$$

Since the depth must be positive, reject the negative value.

The depth can be 5 inches or 12.2 inches to obtain a volume of 1500 cubic inches.

63. **a.** The answers correspond to the points (7.8, 2000) and (10, 2000).

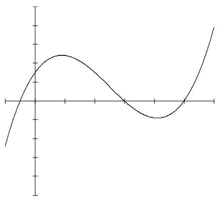
b. The range is (0, 15).

64. **a.** The answers correspond to the points (5, 1500) and (12.2, 1500).

b. The range is (0, 15).

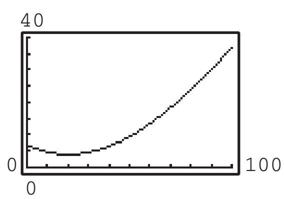
65. – 71. Answers may vary.

72. $2x^3 - 15x^2 + 22x + 15 = 0$
p: ±1, ±3, ±5, ±15
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$



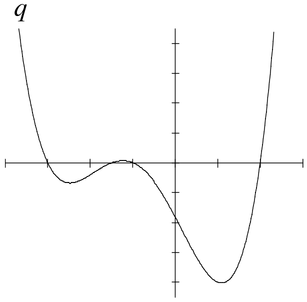
From the graph we see that the solutions are $-\frac{1}{2}$, 3 and 5.

73. $6x^3 - 19x^2 + 16x - 4 = 0$
p: ±1, ±2, ±4
q: ±1, ±2, ±3, ±6
$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{1}{6}$



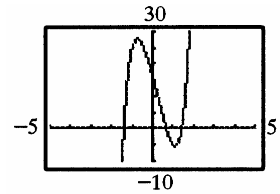
From the graph, we see that the solutions are $\frac{1}{2}, \frac{2}{3}$ and 2.

74. $2x^4 + 7x^3 - 4x^2 - 27x - 18 = 0$
$p: \quad \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$
$q: \quad \pm 1, \pm 2$
$\frac{p}{q}: \quad \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$



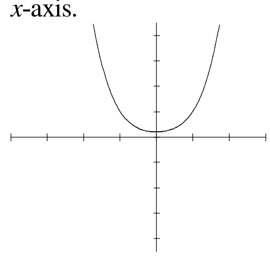
From the graph we see the solutions are $-3, \ -\frac{3}{2}, \ -1, \ 2$.

75. $4x^4 + 4x^3 + 7x^2 - x - 2 = 0$
p: ±1, ±2
q: ±1, ±2, ±4
$\frac{p}{q}: \pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{4}$



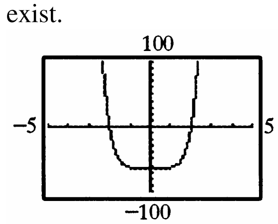
From the graph, we see that the solutions are $-\frac{1}{2}$ and $\frac{1}{2}$.

76. $f(x) = 3x^4 + 5x^2 + 2$
Since $f(x)$ has no sign variations, it has no positive real roots.
$f(-x) = 3x^4 + 5x^2 + 2$
Since $f(-x)$ has no sign variations, no negative roots exist.
The polynomial's graph doesn't intersect the x-axis.



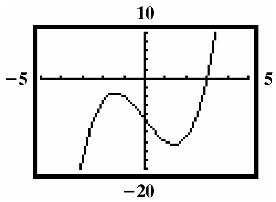
From the graph, we see that there are no real solutions.

77. $f(x) = x^5 - x^4 + x^3 - x^2 + x - 8$
$f(x)$ has 5 sign variations, so either 5, 3, or 1 positive real roots exist.
$f(-x) = -x^5 - x^4 - x^3 - x^2 - x - 8$
$f(-x)$ has no sign variations, so no negative real roots exist.



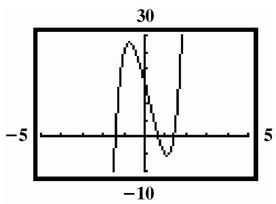
78. Odd functions must have at least one real zero. Even functions do not.

79. $f(x) = x^3 - 6x - 9$



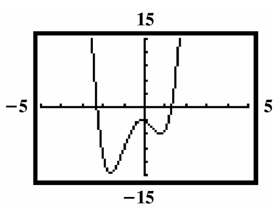
1 real zero
2 nonreal complex zeros

80. $f(x) = 3x^5 - 2x^4 + 6x^3 - 4x^2 - 24x + 16$

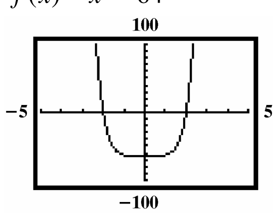


3 real zeros
2 nonreal complex zeros

81. $f(x) = 3x^4 + 4x^3 - 7x^2 - 2x - 3$



82. $f(x) = x^6 - 64$



2 real zeros
4 nonreal complex zeros

83. makes sense

84. does not make sense; Explanations will vary. Sample explanation: The quadratic formula is can be applied only of equations of degree 2.

85. makes sense

86. makes sense

87. false; Changes to make the statement true will vary. A sample change is: The equation has 0 sign variations, so no positive roots exist.

88. false; Changes to make the statement true will vary. A sample change is: Descartes' Rule gives the maximum possible number of real roots.

89. true

90. false; Changes to make the statement true will vary. A sample change is: Polynonials of degree *n* have at most *n* distinct solutions.

91. $(2x+1)(x+5)(x+2) - 3x(x+5) = 208$

$(2x^2 + 11x + 5)(x+2) - 3x^2 - 15x = 208$

$2x^3 + 4x^2 + 11x^2 + 22x + 5x$

$+10 - 3x^2 - 15x = 208$

$2x^3 + 15x^2 + 27x - 3x^2 - 15x - 198 = 0$

$2x^3 + 12x^2 + 12x - 198 = 0$

$2(x^3 + 6x^2 + 6x - 99) = 0$

3	1	6	6	–99
		3	27	99
	1	9	33	0

$x^2 + 9x + 33 = 0$

$b^2 - 4ac = -51$

$x = 3$ in.

92. Answers will vary

93. Because the polynomial has two obvious changes of direction; the smallest degree is 3.

94. Because the polynomial has no obvious changes of direction but the graph is obviously not linear, the smallest degree is 3.

95. Because the polynomial has two obvious changes of direction and two roots have multiplicity 2, the smallest degree is 5.

96. Two roots appear twice, the smallest degree is 5.

97. Answers may vary.

98. The function is undefined at $x = 1$ and $x = 2$.

99. The equation of the vertical asymptote is $x = 1$.

100. The equation of the horizontal asymptote is $y = 0$.

Mid-Chapter 2 Check Point

1. $(6-2i)-(7-i)=6-2i-7+i=-1-i$

2. $3i(2+i)=6i+3i^2=-3+6i$

3. $(1+i)(4-3i)=4-3i+4i-3i^2$
$=4+i+3=7+i$

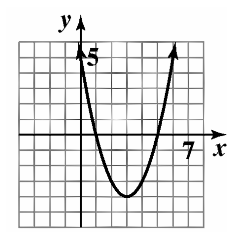
4. $\frac{1+i}{1-i}=\frac{1+i}{1-i}\cdot\frac{1+i}{1+i}=\frac{1+i+i+i^2}{1-i^2}$
$=\frac{1+2i-1}{1+1}$
$=\frac{2i}{2}$
$=i$

5. $\sqrt{-75}-\sqrt{-12}=5i\sqrt{3}-2i\sqrt{3}=3i\sqrt{3}$

6. $\left(2-\sqrt{-3}\right)^2=\left(2-i\sqrt{3}\right)^2$
$=4-4i\sqrt{3}+3i^2$
$=4-4i\sqrt{3}-3$
$=1-4i\sqrt{3}$

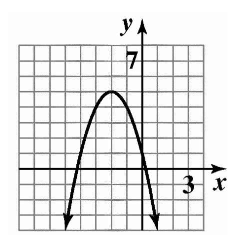
7. $x(2x-3)=-4$
$2x^2-3x=-4$
$2x^2-3x+4=0$
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(4)}}{2(2)}$
$x=\frac{3\pm\sqrt{-23}}{4}$
$x=\frac{3}{4}\pm\frac{\sqrt{23}}{4}i$

8. $f(x)=(x-3)^2-4$
The parabola opens up because $a>0$.
The vertex is (3, –4).
x-intercepts:
$0=(x-3)^2-4$
$(x-3)^2=4$
$x-3=\pm\sqrt{4}$
$x=3\pm 2$
The equation has x-intercepts at $x=1$ and $x=5$.
y-intercept:
$f(0)=(0-3)^2-4=5$
domain: $(-\infty,\infty)$ range: $[-4,\infty)$



***f*(*x*) = (*x* − 3)² − 4**

9. $f(x)=5-(x+2)^2$
The parabola opens down because $a<0$.
The vertex is (–2, 5).
x-intercepts:
$0=5-(x+2)^2$
$(x+2)^2=5$
$x+2=\pm\sqrt{5}$
$x=-2\pm\sqrt{5}$
y-intercept:
$f(0)=5-(0+2)^2=1$
domain: $(-\infty,\infty)$ range: $(-\infty,5]$



***f*(*x*) = 5 − (*x* + 2)²**

10. $f(x) = -x^2 - 4x + 5$

The parabola opens down because $a < 0$.

vertex: $x = -\frac{b}{2a} = -\frac{-4}{2(-1)} = -2$

$f(-2) = -(-2)^2 - 4(-2) + 5 = 9$

The vertex is (–2, 9).

x-intercepts:

$0 = -x^2 - 4x + 5$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-1)(5)}}{2(-1)}$

$x = \frac{4 \pm \sqrt{36}}{-2}$

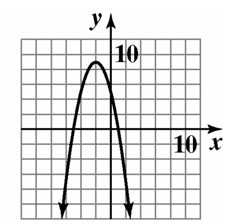
$x = -2 \pm 3$

The *x*-intercepts are $x = 1$ and $x = -5$.

y-intercept:

$f(0) = -0^2 - 4(0) + 5 = 5$

domain: $(-\infty, \infty)$ range: $(-\infty, 9]$



***f*(*x*) = −*x*² − 4*x* + 5**

11. $f(x) = 3x^2 - 6x + 1$

The parabola opens up because $a > 0$.

vertex: $x = -\frac{b}{2a} = -\frac{-6}{2(3)} = 1$

$f(1) = 3(1)^2 - 6(1) + 1 = -2$

The vertex is (1, –2).

x-intercepts:

$0 = 3x^2 - 6x + 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(1)}}{2(3)}$

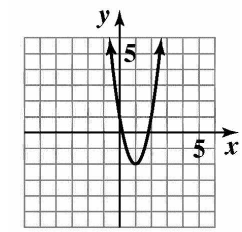
$x = \frac{6 \pm \sqrt{24}}{6}$

$x = \frac{3 \pm \sqrt{6}}{3}$

y-intercept:

$f(0) = 3(0)^2 - 6(0) + 1 = 1$

domain: $(-\infty, \infty)$ range: $[-2, \infty)$



***f*(*x*) = 3*x*² − 6*x* + 1**

12. $f(x) = (x-2)^2(x+1)^3$

$(x-2)^2(x+1)^3 = 0$

Apply the zero-product principle:

$(x-2)^2 = 0$ or $(x+1)^3 = 0$

$x - 2 = 0$ $\quad$ $x + 1 = 0$

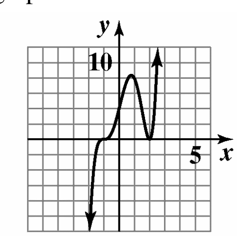
$x = 2$ $\quad$ $x = -1$

The zeros are -1 and 2.

The graph of *f* crosses the *x*-axis at -1, since the zero has multiplicity 3. The graph touches the *x*-axis and turns around at 2 since the zero has multiplicity 2.

Since *f* is an odd-degree polynomial, degree 5, and since the leading coefficient, 1, is positive, the graph falls to the left and rises to the right.

Plot additional points as necessary and construct the graph.



***f*(*x*) = (*x* − 2)² (*x* + 1)³**

13. $f(x) = -(x-2)^2(x+1)^2$

$-(x-2)^2(x+1)^2 = 0$

Apply the zero-product principle:

$(x-2)^2 = 0$ or $(x+1)^2 = 0$

$x-2=0 \qquad x+1=0$

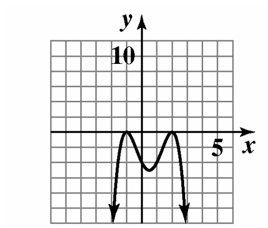
$x=2 \qquad x=-1$

The zeros are -1 and 2.

The graph touches the *x*-axis and turns around both at -1 and 2 since both zeros have multiplicity 2.

Since *f* is an even-degree polynomial, degree 4, and since the leading coefficient, -1, is negative, the graph falls to the left and falls to the right.

Plot additional points as necessary and construct the graph.



$f(x) = -(x-2)^2(x+1)^2$

14. $f(x) = x^3 - x^2 - 4x + 4$

$x^3 - x^2 - 4x + 4 = 0$

$x^2(x-1) - 4(x-1) = 0$

$(x^2-4)(x-1) = 0$

$(x+2)(x-2)(x-1) = 0$

Apply the zero-product principle:

$x+2=0$ or $x-2=0$ or $x-1=0$

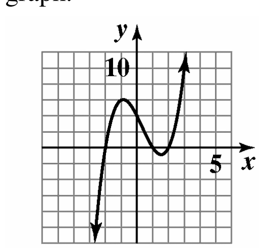
$x=-2 \qquad x=2 \qquad x=1$

The zeros are -2, 1, and 2.

The graph of *f* crosses the *x*-axis at all three zeros, -2, 1, and 2, since all have multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, 1, is positive, the graph falls to the left and rises to the right.

Plot additional points as necessary and construct the graph.



$f(x) = x^3 - x^2 - 4x + 4$

15. $f(x) = x^4 - 5x^2 + 4$

$x^4 - 5x^2 + 4 = 0$

$(x^2-4)(x^2-1) = 0$

$(x+2)(x-2)(x+1)(x-1) = 0$

Apply the zero-product principle,

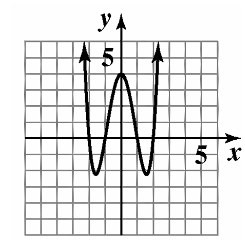
$x=-2,\ x=2,\ x=-1,\ x=1$

The zeros are -2, -1, 1, and 2.

The graph crosses the *x*-axis at all four zeros, -2, -1, 1, and 2., since all have multiplicity 1.

Since *f* is an even-degree polynomial, degree 4, and since the leading coefficient, 1, is positive, the graph rises to the left and rises to the right.

Plot additional points as necessary and construct the graph.



$f(x) = x^4 - 5x^2 + 4$

16. $f(x) = -(x+1)^6$

$-(x+1)^6 = 0$

$(x+1)^6 = 0$

$x+1=0$

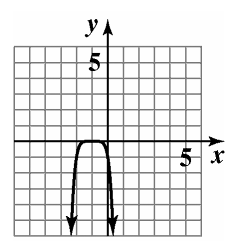
$x=-1$

The zero is are -1.

The graph touches the *x*-axis and turns around at -1 since the zero has multiplicity 6.

Since *f* is an even-degree polynomial, degree 6, and since the leading coefficient, -1, is negative, the graph falls to the left and falls to the right.

Plot additional points as necessary and construct the graph.



$f(x) = -(x+1)^6$

17. $f(x) = -6x^3 + 7x^2 - 1$

To find the zeros, we use the Rational Zero Theorem:
List all factors of the constant term -1: ± 1
List all factors of the leading coefficient -6:
$\pm 1, \pm 2, \pm 3, \pm 6$

The possible rational zeros are:

$$\frac{\text{Factors of } -1}{\text{Factors of } -6} = \frac{\pm 1}{\pm 1, \pm 2, \pm 3, \pm 6}$$

$$= \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}$$

We test values from the above list until we find a zero. One is shown next:

Test 1:

1	–6	7	0	–1
		–6	1	1
	–6	1	1	0

The remainder is 0, so 1 is a zero. Thus,

$$-6x^3 + 7x^2 - 1 = 0$$
$$(x-1)(-6x^2 + x + 1) = 0$$
$$-(x-1)(6x^2 - x - 1) = 0$$
$$-(x-1)(3x+1)(2x-1) = 0$$

Apply the zero-product property:

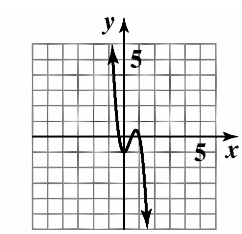
$x = 1, \quad x = -\frac{1}{3}, \quad x = \frac{1}{2}$

The zeros are $-\frac{1}{3}$, $\frac{1}{2}$, and 1.

The graph of *f* crosses the *x*-axis at all three zeros, $-\frac{1}{3}$, $\frac{1}{2}$, and 1, since all have multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, -6, is negative, the graph rises to the left and falls to the right.

Plot additional points as necessary and construct the graph.



***f*(*x*) = −6*x*³ + 7*x*² − 1**

18. $f(x) = 2x^3 - 2x$

$$2x^3 - 2x = 0$$
$$2x(x^2 - 1) = 0$$
$$2x(x+1)(x-1) = 0$$

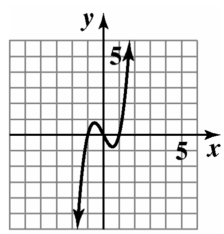
Apply the zero-product principle:

$x = 0, \quad x = -1, \quad x = 1$

The zeros are -1, 0, and 1.

The graph of *f* crosses the *x*-axis at all three zeros, -1, 0, and 1, since all have multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, 2, is positive, the graph falls to the left and rises to the right.
Plot additional points as necessary and construct the graph.



f*(*x*) = 2*x*³ − 2*x

19. $f(x) = x^3 - 2x^2 + 26x$

$$x^3 - 2x^2 + 26x = 0$$
$$x(x^2 - 2x + 26) = 0$$

Note that $x^2 - 2x + 26$ does not factor, so we use the quadratic formula:

$x = 0$ or $x^2 - 2x + 26 = 0$

$$a = 1, \quad b = -2, \quad c = 26$$

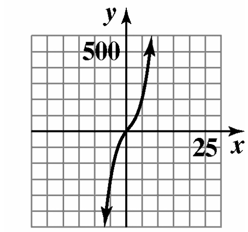
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(26)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-100}}{2} = \frac{2 \pm 10i}{2} = 1 \pm 5i$$

The zeros are 0 and $1 \pm 5i$.
The graph of *f* crosses the *x*-axis at 0 (the only real zero), since it has multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, 1, is positive, the graph falls to the left and rises to the right.

Plot additional points as necessary and construct the graph.



$f(x) = x^3 - 2x^2 + 26x$

20. $f(x) = -x^3 + 5x^2 - 5x - 3$

To find the zeros, we use the Rational Zero Theorem:
List all factors of the constant term -3: $\pm 1, \pm 3$
List all factors of the leading coefficient -1: ± 1

The possible rational zeros are:

$$\frac{\text{Factors of } -3}{\text{Factors of } -1} = \frac{\pm 1, \ \pm 3}{\pm 1} = \pm 1, \ \pm 3$$

We test values from the previous list until we find a zero. One is shown next:

Test 3:

3	−1	5	−5	−3
		−3	6	3
	−1	2	1	0

The remainder is 0, so 3 is a zero. Thus,

$$-x^3 + 5x^2 - 5x - 3 = 0$$
$$(x-3)(-x^2 + 2x + 1) = 0$$
$$-(x-3)(x^2 - 2x - 1) = 0$$

Note that $x^2 - 2x - 1$ does not factor, so we use the quadratic formula:

$x - 3 = 0$ or $x^2 - 2x - 1 = 0$

$x = 3$ $\quad a = 1, \ b = -2, \ c = -1$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

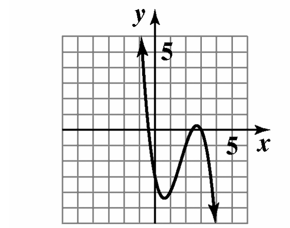
$$= \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

The zeros are 3 and $1 \pm \sqrt{2}$.

The graph of *f* crosses the *x*-axis at all three zeros, 3 and $1 \pm \sqrt{2}$, since all have multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, -1, is negative, the graph rises to the left and falls to the right.

Plot additional points as necessary and construct the graph.



$f(x) = -x^3 + 5x^2 - 5x - 3$

21. $x^3 - 3x + 2 = 0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term 2: $\pm 1, \ \pm 2$
Factors of the leading coefficient 1: ± 1

The possible rational zeros are:

$$\frac{\text{Factors of } 2}{\text{Factors of } 1} = \frac{\pm 1, \ \pm 2}{\pm 1} = \pm 1, \ \pm 2$$

We test values from above until we find a root. One is shown next:

Test 1:

1	1	0	−3	2
		1	1	−2
	1	1	−2	0

The remainder is 0, so 1 is a root of the equation. Thus,

$$x^3 - 3x + 2 = 0$$
$$(x-1)(x^2 + x - 2) = 0$$
$$(x-1)(x+2)(x-1) = 0$$
$$(x-1)^2(x+2) = 0$$

Apply the zero-product property:

$(x-1)^2 = 0$ or $x + 2 = 0$

$x - 1 = 0$ $\quad x = -2$

$x = 1$

The solutions are -2 and 1, and the solution set is $\{-2, 1\}$.

22. $6x^3-11x^2+6x-1=0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term -1: ±1

Factors of the leading coefficient 6: $\pm1,\ \pm2,\ \pm3,\ \pm6$

The possible rational zeros are:

$$\frac{\text{Factors of } -1}{\text{Factors of } 6}=\frac{\pm1}{\pm1,\ \pm2,\ \pm3,\ \pm6}$$

$$=\pm1,\ \pm\frac{1}{2},\ \pm\frac{1}{3},\ \pm\frac{1}{6}$$

We test values from above until we find a root. One is shown next:

Test 1:

1	6	−11	6	−1
		6	−5	1
	6	−5	1	0

The remainder is 0, so 1 is a root of the equation. Thus,

$$6x^3-11x^2+6x-1=0$$
$$(x-1)(6x^2-5x+1)=0$$
$$(x-1)(3x-1)(2x-1)=0$$

Apply the zero-product property:

$x-1=0$ or $3x-1=0$ or $2x-1=0$

$x=1$ $\quad x=\frac{1}{3}$ $\quad x=\frac{1}{2}$

The solutions are $\frac{1}{3}$, $\frac{1}{2}$ and 1, and the solution set is $\left\{\frac{1}{3},\ \frac{1}{2},\ 1\right\}$.

23. $(2x+1)(3x-2)^3(2x-7)=0$

Apply the zero-product property:

$2x+1=0$ or $(3x-2)^3=0$ or $2x-7=0$

$x=-\frac{1}{2}$ $\quad 3x-2=0$ $\quad x=\frac{7}{2}$ T

$x=\frac{2}{3}$

he solutions are $-\frac{1}{2}$, $\frac{2}{3}$ and $\frac{7}{2}$, and the solution set is $\left\{-\frac{1}{2},\ \frac{2}{3},\ \frac{7}{2}\right\}$.

24. $2x^3+5x^2-200x-500=0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term -500:

$\pm1,\ \pm2,\ \pm4,\ \pm5,\ \pm10,\ \pm20,\ \pm25,$

$\pm50,\ \pm100,\ \pm125,\ \pm250,\ \pm500$

Factors of the leading coefficient 2: $\pm1,\ \pm2$

The possible rational zeros are:

$$\frac{\text{Factors of } 500}{\text{Factors of } 2}=\pm1,\ \pm2,\ \pm4,\ \pm5,$$

$$\pm10,\ \pm20,\ \pm25,\ \pm50,\ \pm100,\ \pm125,$$

$$\pm250,\ \pm500,\ \pm\frac{1}{2},\ \pm\frac{5}{2},\ \pm\frac{25}{2},\ \pm\frac{125}{2}$$

We test values from above until we find a root. One is shown next:

Test 10:

10	2	5	−200	−500
		20	250	500
	2	25	50	0

The remainder is 0, so 10 is a root of the equation. Thus,

$$2x^3+5x^2-200x-500=0$$
$$(x-10)(2x^2+25x+50)=0$$
$$(x-10)(2x+5)(x+10)=0$$

Apply the zero-product property:

$x-10=0$ or $2x+5=0$ or $x+10=0$

$x=10$ $\quad x=-\frac{5}{2}$ $\quad x=-10$

The solutions are $-10,\ -\frac{5}{2},$ and 10, and the solution set is $\left\{-10,\ -\frac{5}{2},\ 10\right\}$.

25. $x^4 - x^3 - 11x^2 = x + 12$

$x^4 - x^3 - 11x^2 - x - 12 = 0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term -12:
$\pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ \pm 6,\ \pm 12$

Factors of the leading coefficient 1: ± 1

The possible rational zeros are:

$$\frac{\text{Factors of } -12}{\text{Factors of 1}} = \frac{\pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ \pm 6,\ \pm 12}{\pm 1}$$
$$= \pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ \pm 6,\ \pm 12$$

We test values from this list we find a root. One possibility is shown next:

Test -3:

-3	1	-1	-11	-1	-12
		-3	12	-3	12
	1	-4	1	-4	0

The remainder is 0, so -3 is a root of the equation. Using the Factor Theorem, we know that $x-1$ is a factor. Thus,

$$x^4 - x^3 - 11x^2 - x - 12 = 0$$
$$(x+3)(x^3 - 4x^2 + x - 4) = 0$$
$$(x+3)\left[x^2(x-4) + 1(x-4)\right] = 0$$
$$(x+3)(x-4)(x^2+1) = 0$$

As this point we know that -3 and 4 are roots of the equation. Note that x^2+1 does not factor, so we use the square-root principle: $x^2 + 1 = 0$

$$x^2 = -1$$
$$x = \pm\sqrt{-1} = \pm i$$

The roots are -3, 4, and $\pm i$, and the solution set is $\{-3,\ 4,\ \pm i\}$.

26. $2x^4 + x^3 - 17x^2 - 4x + 6 = 0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term 6: $\pm 1,\ \pm 2,\ \pm 3,\ \pm 6$

Factors of the leading coefficient 4: $\pm 1,\ \pm 2$

The possible rational roots are:

$$\frac{\text{Factors of 6}}{\text{Factors of 2}} = \frac{\pm 1,\ \pm 2,\ \pm 3,\ \pm 6}{\pm 1,\ \pm 2}$$
$$= \pm 1,\ \pm 2,\ \pm 3,\ \pm 6,\ \pm\frac{1}{2},\ \pm\frac{3}{2}$$

We test values from above until we find a root. One possibility is shown next:

Test -3:

-3	2	1	-17	-4	6
		-6	15	6	-6
	2	-5	-2	2	0

The remainder is 0, so -3 is a root. Using the Factor Theorem, we know that $x+3$ is a factor of the polynomial. Thus,

$$2x^4 + x^3 - 17x^2 - 4x + 6 = 0$$
$$(x+3)(2x^3 - 5x^2 - 2x + 2) = 0$$

To solve the equation above, we need to factor $2x^3 - 5x^2 - 2x + 2$. We continue testing potential roots:

Test $\frac{1}{2}$:

$\frac{1}{2}$	2	-5	-2	2
		1	-2	-2
	2	-4	-4	0

The remainder is 0, so $\frac{1}{2}$ is a zero and $x - \frac{1}{2}$ is a factor.

Summarizing our findings so far, we have

$$2x^4 + x^3 - 17x^2 - 4x + 6 = 0$$
$$(x+3)(2x^3 - 5x^2 - 2x + 2) = 0$$
$$(x+3)\left(x - \frac{1}{2}\right)(2x^2 - 4x - 4) = 0$$
$$2(x+3)\left(x - \frac{1}{2}\right)(x^2 - 2x - 2) = 0$$

At this point, we know that -3 and $\frac{1}{2}$ are roots of the equation. Note that $x^2 - 2x - 2$ does not factor, so we use the quadratic formula:

$x^2 - 2x - 2 = 0$

$a = 1,\ \ b = -2,\ \ c = -2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

The solutions are -3, $\frac{1}{2}$, and $1 \pm \sqrt{3}$, and the solution set is $\left\{-3,\ \frac{1}{2},\ 1 \pm \sqrt{3}\right\}$.

27. $P(x) = -x^2 + 150x - 4425$

Since $a = -1$ is negative, we know the function opens down and has a maximum at

$$x = -\frac{b}{2a} = -\frac{150}{2(-1)} = -\frac{150}{-2} = 75.$$

$$P(75) = -75^2 + 150(75) - 4425$$
$$= -5625 + 11,250 - 4425 = 1200$$

The company will maximize its profit by manufacturing and selling 75 cabinets per day. The maximum daily profit is \$1200.

28. Let x = one of the numbers;
$-18 - x$ = the other number

The product is $f(x) = x(-18 - x) = -x^2 - 18x$

The x-coordinate of the maximum is

$$x = -\frac{b}{2a} = -\frac{-18}{2(-1)} = -\frac{-18}{-2} = -9.$$

$$f(-9) = -9\left[-18 - (-9)\right]$$
$$= -9(-18 + 9) = -9(-9) = 81$$

The vertex is $(-9, 81)$. The maximum product is 81. This occurs when the two number are -9 and $-18 - (-9) = -9$.

29. Let x = height of triangle;
$40 - 2x$ = base of triangle

$$A = \frac{1}{2}bh = \frac{1}{2}x(40 - 2x)$$

$$A(x) = 20x - x^2$$

The height at which the triangle will have maximum area is $x = -\frac{b}{2a} = -\frac{20}{2(-1)} = 10.$

$$A(10) = 20(10) - (10)^2 = 100$$

The maximum area is 100 squares inches.

30.

$$\begin{array}{r} 2x^2 - x - 3 \\ 3x^2 - 1 \enclose{longdiv}{6x^4 - 3x^3 - 11x^2 + 2x + 4} \\ \underline{6x^4 \qquad - 2x^2} \\ -3x^3 - 9x^2 + 2x \\ \underline{-3x^3 \qquad + x} \\ -9x^2 + x + 4 \\ \underline{-9x^2 \qquad + 3} \\ x + 1 \end{array}$$

$$2x^2 - x - 3 + \frac{x+1}{3x^2 - 1}$$

31. $\left(2x^4 - 13x^3 + 17x^2 + 18x - 24\right) \div (x - 4)$

4	2	–13	17	18	–24
		8	-20	–12	24
	2	–5	–3	6	0

The quotient is $2x^3 - 5x^2 - 3x + 6$.

32. $(x-1)(x-i)(x+i) = (x-1)(x^2+1)$

$f(x) = a_n(x-1)(x^2+1)$

$f(-1) = a_n(-1-1)\left((-1)^2+1\right) = -4a_n = 8$

$a_n = -2$

$f(x) = -2(x-1)(x^2+1)$ or $-2x^3 + 2x^2 - 2x + 2$

33. $(x-2)(x-2)(x-3i)(x+3i)$

$= (x-2)(x-2)(x^2+9)$

$f(x) = a_n(x-2)(x-2)(x^2+9)$

$f(0) = a_n(0-2)(0-2)(0^2+9)$

$36 = 36a_n$

$a_n = 1$

$f(x) = 1(x-2)(x-2)(x^2+9)$

$f(x) = x^4 - 4x^3 + 13x^2 - 36x + 36$

34. $f(x) = x^3 - x - 5$

$f(1) = 1^3 - 1 - 5 = -5$

$f(2) = 2^3 - 2 - 5 = 1$

Yes, the function must have a real zero between 1 and 2 because $f(1)$ and $f(2)$ have opposite signs.