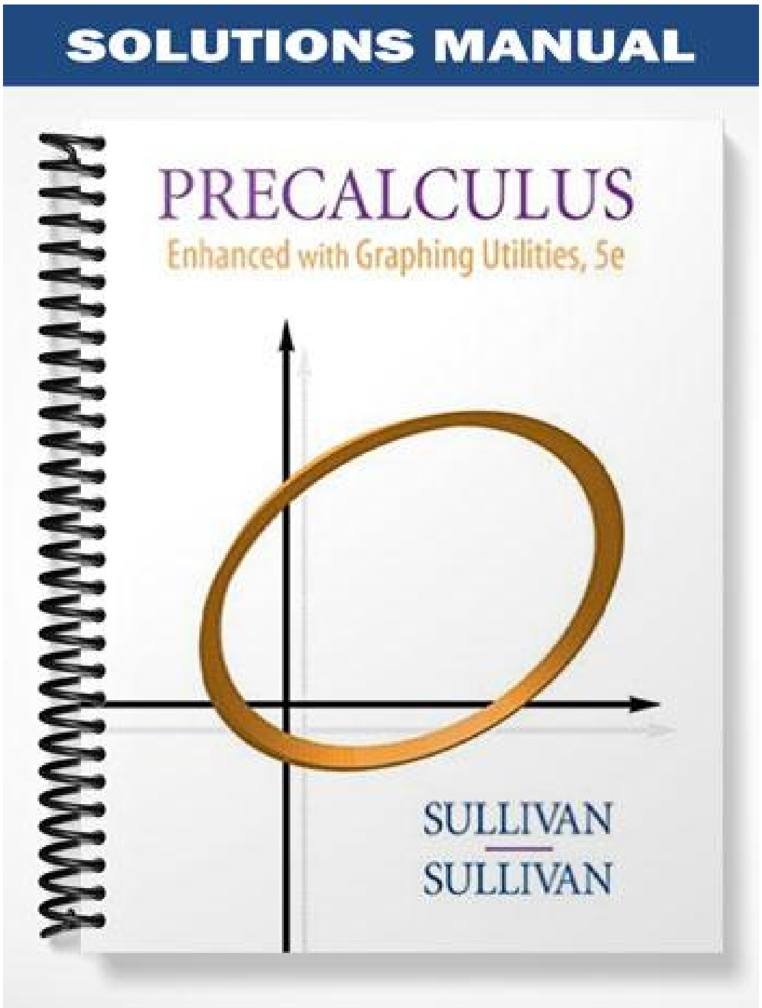

SOLUTIONS MANUAL
PRECALCULUS
Enhanced with Graphing Utilities, 5e
SULLIVAN
SULLIVAN

Chapter 2

Functions and Their Graphs

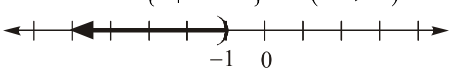
Section 2.1

1. $(-1,3)$

2. $$3(-2)^2-5(-2)+\frac{1}{(-2)}=3(4)-5(-2)-\frac{1}{2}$$
$$=12+10-\frac{1}{2}$$
$$=\frac{43}{2}\ \text{ or }\ 21\tfrac{1}{2}\ \text{ or }\ 21.5$$

3. We must not allow the denominator to be 0.
$x+4\neq 0 \Rightarrow x\neq -4$; Domain: $\{x \mid x\neq -4\}$.

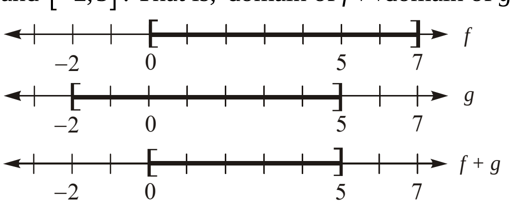
4. $3-2x>5$
$-2x>2$
$x<-1$
Solution set: $\{x \mid x<-1\}$ or $(-\infty,-1)$



5. independent; dependent

6. range

7. $[0,5]$
We need the intersection of the intervals $[0,7]$ and $[-2,5]$. That is, domain of f $\cap$ domain of g.



8. image

9. $(g-f)(x)$ or $g(x)-f(x)$

10. False; every function is a relation, but not every relation is a function. For example, the relation $x^2+y^2=1$ is not a function.

11. True

12. True

13. False; if the domain is not specified, we assume it is the largest set of real numbers for which the value of f is a real number.

14. False; the domain of $f(x)=\dfrac{x^2-4}{x}$ is $\{x \mid x\neq 0\}$.

15. Function
Domain: {Elvis, Colleen, Kaleigh, Marissa}
Range: {Jan. 8, Mar. 15, Sept. 17}

16. Not a function

17. Not a function

18. Function
Domain: {Less than 9th grade, 9th-12th grade, High School Graduate, Some College, College Graduate}
Range: {\$18,120, \$23,251, \$36,055, \$45,810, \$67,165}

19. Not a function

20. Function
Domain: {–2, –1, 3, 4}
Range: {3, 5, 7, 12}

21. Function
Domain: {1, 2, 3, 4}
Range: {3}

22. Function
Domain: {0, 1, 2, 3}
Range: {–2, 3, 7}

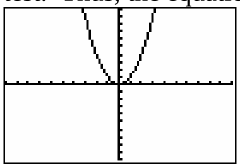
23. Not a function

24. Not a function

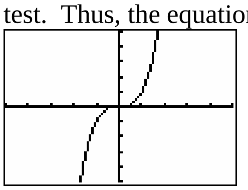
25. Function
Domain: {–2, –1, 0, 1}
Range: {0, 1, 4}

26. Function
Domain: {–2, –1, 0, 1}
Range: {3, 4, 16}

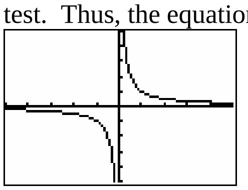
27. Graph $y = x^2$. The graph passes the vertical line test. Thus, the equation represents a function.



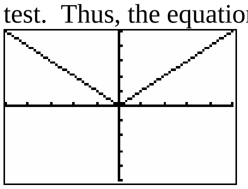
28. Graph $y = x^3$. The graph passes the vertical line test. Thus, the equation represents a function.



29. Graph $y = \frac{1}{x}$. The graph passes the vertical line test. Thus, the equation represents a function.



30. Graph $y = |x|$. The graph passes the vertical line test. Thus, the equation represents a function.



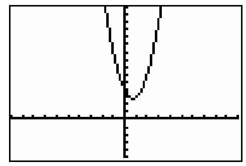
31. $y = \pm\sqrt{4 - x^2}$
For $x = 0, y = \pm 2$. Thus, (0, 2) and (0, –2) are on the graph. This is not a function, since a distinct *x*-value corresponds to two different *y*-values.

32. $y = \pm\sqrt{1 - 2x}$
For $x = 0, y = \pm 1$. Thus, (0, 1) and (0, –1) are on the graph. This is not a function, since a distinct *x*-value corresponds to two different *y*-values.

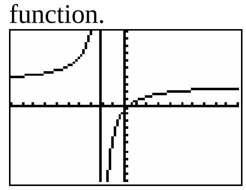
33. $x = y^2$
Solve for y: $y = \pm\sqrt{x}$
For $x = 1, y = \pm 1$. Thus, (1, 1) and (1, –1) are on the graph. This is not a function, since a distinct *x*-value corresponds to two different *y*-values.

34. $x + y^2 = 1$
Solve for y: $y = \pm\sqrt{1 - x}$
For $x = 0, y = \pm 1$. Thus, (0, 1) and (0, –1) are on the graph. This is not a function, since a distinct *x*-value corresponds to two different *y*-values.

35. Graph $y = 2x^2 - 3x + 4$. The graph passes the vertical line test. Thus, the equation represents a function.



36. Graph $y = \frac{3x - 1}{x + 2}$. The graph passes the vertical line test. Thus, the equation represents a function.



37. $2x^2 + 3y^2 = 1$
Solve for *y*:
$$\begin{aligned} 2x^2 + 3y^2 &= 1 \\ 3y^2 &= 1 - 2x^2 \\ y^2 &= \frac{1 - 2x^2}{3} \\ y &= \pm\sqrt{\frac{1 - 2x^2}{3}} \end{aligned}$$
For $x = 0, y = \pm\sqrt{\frac{1}{3}}$. Thus, $\left(0, \sqrt{\frac{1}{3}}\right)$ and $\left(0, -\sqrt{\frac{1}{3}}\right)$ are on the graph. This is not a function, since a distinct *x*-value corresponds to two different *y*-values.

38. $x^2 - 4y^2 = 1$

Solve for y: $x^2 - 4y^2 = 1$

$$4y^2 = x^2 - 1$$
$$y^2 = \frac{x^2 - 1}{4}$$
$$y = \frac{\pm\sqrt{x^2 - 1}}{2}$$

For $x = \sqrt{2}$, $y = \pm\frac{1}{2}$. Thus, $\left(\sqrt{2}, \frac{1}{2}\right)$ and $\left(\sqrt{2}, -\frac{1}{2}\right)$ are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

39. $f(x) = 3x^2 + 2x - 4$

a. $f(0) = 3(0)^2 + 2(0) - 4 = -4$

b. $f(1) = 3(1)^2 + 2(1) - 4 = 3 + 2 - 4 = 1$

c. $f(-1) = 3(-1)^2 + 2(-1) - 4 = 3 - 2 - 4 = -3$

d. $f(-x) = 3(-x)^2 + 2(-x) - 4 = 3x^2 - 2x - 4$

e. $-f(x) = -\left(3x^2 + 2x - 4\right) = -3x^2 - 2x + 4$

f.
$$\begin{aligned} f(x+1) &= 3(x+1)^2 + 2(x+1) - 4 \\ &= 3\left(x^2 + 2x + 1\right) + 2x + 2 - 4 \\ &= 3x^2 + 6x + 3 + 2x + 2 - 4 \\ &= 3x^2 + 8x + 1 \end{aligned}$$

g. $f(2x) = 3(2x)^2 + 2(2x) - 4 = 12x^2 + 4x - 4$

h.
$$\begin{aligned} f(x+h) &= 3(x+h)^2 + 2(x+h) - 4 \\ &= 3\left(x^2 + 2xh + h^2\right) + 2x + 2h - 4 \\ &= 3x^2 + 6xh + 3h^2 + 2x + 2h - 4 \end{aligned}$$

40. $f(x) = -2x^2 + x - 1$

a. $f(0) = -2(0)^2 + 0 - 1 = -1$

b. $f(1) = -2(1)^2 + 1 - 1 = -2$

c. $f(-1) = -2(-1)^2 + (-1) - 1 = -4$

d. $f(-x) = -2(-x)^2 + (-x) - 1 = -2x^2 - x - 1$

e. $-f(x) = -\left(-2x^2 + x - 1\right) = 2x^2 - x + 1$

f.
$$\begin{aligned} f(x+1) &= -2(x+1)^2 + (x+1) - 1 \\ &= -2\left(x^2 + 2x + 1\right) + x + 1 - 1 \\ &= -2x^2 - 4x - 2 + x \\ &= -2x^2 - 3x - 2 \end{aligned}$$

g. $f(2x) = -2(2x)^2 + (2x) - 1 = -8x^2 + 2x - 1$

h.
$$\begin{aligned} f(x+h) &= -2(x+h)^2 + (x+h) - 1 \\ &= -2\left(x^2 + 2xh + h^2\right) + x + h - 1 \\ &= -2x^2 - 4xh - 2h^2 + x + h - 1 \end{aligned}$$

41. $f(x) = \dfrac{x}{x^2 + 1}$

a. $f(0) = \dfrac{0}{0^2 + 1} = \dfrac{0}{1} = 0$

b. $f(1) = \dfrac{1}{1^2 + 1} = \dfrac{1}{2}$

c. $f(-1) = \dfrac{-1}{(-1)^2 + 1} = \dfrac{-1}{1+1} = -\dfrac{1}{2}$

d. $f(-x) = \dfrac{-x}{(-x)^2 + 1} = \dfrac{-x}{x^2 + 1}$

e. $-f(x) = -\left(\dfrac{x}{x^2 + 1}\right) = \dfrac{-x}{x^2 + 1}$

f.
$$\begin{aligned} f(x+1) &= \frac{x+1}{(x+1)^2 + 1} \\ &= \frac{x+1}{x^2 + 2x + 1 + 1} \\ &= \frac{x+1}{x^2 + 2x + 2} \end{aligned}$$

g. $f(2x) = \dfrac{2x}{(2x)^2 + 1} = \dfrac{2x}{4x^2 + 1}$

h. $f(x+h) = \dfrac{x+h}{(x+h)^2 + 1} = \dfrac{x+h}{x^2 + 2xh + h^2 + 1}$

42. $f(x)=\dfrac{x^2-1}{x+4}$

a. $f(0)=\dfrac{0^2-1}{0+4}=\dfrac{-1}{4}=-\dfrac{1}{4}$

b. $f(1)=\dfrac{1^2-1}{1+4}=\dfrac{0}{5}=0$

c. $f(-1)=\dfrac{(-1)^2-1}{-1+4}=\dfrac{0}{3}=0$

d. $f(-x)=\dfrac{(-x)^2-1}{-x+4}=\dfrac{x^2-1}{-x+4}$

e. $-f(x)=-\left(\dfrac{x^2-1}{x+4}\right)=\dfrac{-x^2+1}{x+4}$

f. $f(x+1)=\dfrac{(x+1)^2-1}{(x+1)+4}$

$=\dfrac{x^2+2x+1-1}{x+5}=\dfrac{x^2+2x}{x+5}$

g. $f(2x)=\dfrac{(2x)^2-1}{2x+4}=\dfrac{4x^2-1}{2x+4}$

h. $f(x+h)=\dfrac{(x+h)^2-1}{(x+h)+4}=\dfrac{x^2+2xh+h^2-1}{x+h+4}$

43. $f(x)=|x|+4$

a. $f(0)=|0|+4=0+4=4$

b. $f(1)=|1|+4=1+4=5$

c. $f(-1)=|-1|+4=1+4=5$

d. $f(-x)=|-x|+4=|x|+4$

e. $-f(x)=-(|x|+4)=-|x|-4$

f. $f(x+1)=|x+1|+4$

g. $f(2x)=|2x|+4=2|x|+4$

h. $f(x+h)=|x+h|+4$

44. $f(x)=\sqrt{x^2+x}$

a. $f(0)=\sqrt{0^2+0}=\sqrt{0}=0$

b. $f(1)=\sqrt{1^2+1}=\sqrt{2}$

c. $f(-1)=\sqrt{(-1)^2+(-1)}=\sqrt{1-1}=\sqrt{0}=0$

d. $f(-x)=\sqrt{(-x)^2+(-x)}=\sqrt{x^2-x}$

e. $-f(x)=-\left(\sqrt{x^2+x}\right)=-\sqrt{x^2+x}$

f. $f(x+1)=\sqrt{(x+1)^2+(x+1)}$

$=\sqrt{x^2+2x+1+x+1}$

$=\sqrt{x^2+3x+2}$

g. $f(2x)=\sqrt{(2x)^2+2x}=\sqrt{4x^2+2x}$

h. $f(x+h)=\sqrt{(x+h)^2+(x+h)}$

$=\sqrt{x^2+2xh+h^2+x+h}$

45. $f(x)=\dfrac{2x+1}{3x-5}$

a. $f(0)=\dfrac{2(0)+1}{3(0)-5}=\dfrac{0+1}{0-5}=-\dfrac{1}{5}$

b. $f(1)=\dfrac{2(1)+1}{3(1)-5}=\dfrac{2+1}{3-5}=\dfrac{3}{-2}=-\dfrac{3}{2}$

c. $f(-1)=\dfrac{2(-1)+1}{3(-1)-5}=\dfrac{-2+1}{-3-5}=\dfrac{-1}{-8}=\dfrac{1}{8}$

d. $f(-x)=\dfrac{2(-x)+1}{3(-x)-5}=\dfrac{-2x+1}{-3x-5}=\dfrac{2x-1}{3x+5}$

e. $-f(x)=-\left(\dfrac{2x+1}{3x-5}\right)=\dfrac{-2x-1}{3x-5}$

f. $f(x+1)=\dfrac{2(x+1)+1}{3(x+1)-5}=\dfrac{2x+2+1}{3x+3-5}=\dfrac{2x+3}{3x-2}$

g. $f(2x)=\dfrac{2(2x)+1}{3(2x)-5}=\dfrac{4x+1}{6x-5}$

h. $f(x+h)=\dfrac{2(x+h)+1}{3(x+h)-5}=\dfrac{2x+2h+1}{3x+3h-5}$

46. $f(x)=1-\dfrac{1}{(x+2)^2}$

a. $f(0)=1-\dfrac{1}{(0+2)^2}=1-\dfrac{1}{4}=\dfrac{3}{4}$

b. $f(1)=1-\dfrac{1}{(1+2)^2}=1-\dfrac{1}{9}=\dfrac{8}{9}$

c. $f(-1)=1-\dfrac{1}{(-1+2)^2}=1-\dfrac{1}{1}=0$

d. $f(-x)=1-\dfrac{1}{(-x+2)^2}=1-\dfrac{1}{(2-x)^2}$

e. $-f(x)=-\left(1-\dfrac{1}{(x+2)^2}\right)=\dfrac{1}{(x+2)^2}-1$

f. $f(x+1)=1-\dfrac{1}{(x+1+2)^2}=1-\dfrac{1}{(x+3)^2}$

g. $f(2x)=1-\dfrac{1}{(2x+2)^2}=1-\dfrac{1}{4(x+1)^2}$

h. $f(x+h)=1-\dfrac{1}{(x+h+2)^2}$

47. $f(x)=-5x+4$

Domain: $\{x\mid x \text{ is any real number}\}$

48. $f(x)=x^2+2$

Domain: $\{x\mid x \text{ is any real number}\}$

49. $f(x)=\dfrac{x}{x^2+1}$

Domain: $\{x\mid x \text{ is any real number}\}$

50. $f(x)=\dfrac{x^2}{x^2+1}$

Domain: $\{x\mid x \text{ is any real number}\}$

51. $g(x)=\dfrac{x}{x^2-16}$

$x^2-16\neq 0$

$x^2\neq 16 \Rightarrow x\neq \pm 4$

Domain: $\{x\mid x\neq -4,\ x\neq 4\}$

52. $h(x)=\dfrac{2x}{x^2-4}$

$x^2-4\neq 0$

$x^2\neq 4 \Rightarrow x\neq \pm 2$

Domain: $\{x\mid x\neq -2,\ x\neq 2\}$

53. $F(x)=\dfrac{x-2}{x^3+x}$

$x^3+x\neq 0$

$x(x^2+1)\neq 0$

$x\neq 0,\quad x^2\neq -1$

Domain: $\{x\mid x\neq 0\}$

54. $G(x)=\dfrac{x+4}{x^3-4x}$

$x^3-4x\neq 0$

$x(x^2-4)\neq 0$

$x\neq 0,\quad x^2\neq 4$

$x\neq 0,\quad x\neq \pm 2$

Domain: $\{x\mid x\neq -2,\ x\neq 0,\ x\neq 2\}$

55. $h(x)=\sqrt{3x-12}$

$3x-12\geq 0$

$3x\geq 12$

$x\geq 4$

Domain: $\{x\mid x\geq 4\}$

56. $G(x)=\sqrt{1-x}$

$1-x\geq 0$

$-x\geq -1$

$x\leq 1$

Domain: $\{x\mid x\leq 1\}$

57. $f(x)=\dfrac{4}{\sqrt{x-9}}$

$x-9>0$

$x>9$

Domain: $\{x\mid x>9\}$

58. $f(x)=\dfrac{x}{\sqrt{x-4}}$

$x-4>0$

$x>4$

Domain: $\{x \mid x>4\}$

59. $p(x)=\sqrt{\dfrac{2}{x-1}}=\dfrac{\sqrt{2}}{\sqrt{x-1}}$

$x-1>0$

$x>1$

Domain: $\{x \mid x>1\}$

60. $q(x)=\sqrt{-x-2}$

$-x-2\ge 0$

$-x\ge 2$

$x\le -2$

Domain: $\{x \mid x\le -2\}$

61. $f(x)=3x+4 \qquad g(x)=2x-3$

a. $(f+g)(x)=3x+4+2x-3=5x+1$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x)=(3x+4)-(2x-3)$

$=3x+4-2x+3$

$=x+7$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=(3x+4)(2x-3)$

$=6x^2-9x+8x-12$

$=6x^2-x-12$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{3x+4}{2x-3}$

$2x-3\ne 0 \Rightarrow 2x\ne 3 \Rightarrow x\ne \dfrac{3}{2}$

Domain: $\left\{x \,\middle|\, x\ne \dfrac{3}{2}\right\}$.

e. $(f+g)(3)=5(3)+1=15+1=16$

f. $(f-g)(4)=4+7=11$

g. $(f\cdot g)(2)=6(2)^2-2-12=24-2-12=10$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{3(1)+4}{2(1)-3}=\dfrac{3+4}{2-3}=\dfrac{7}{-1}=-7$

62. $f(x)=2x+1 \qquad g(x)=3x-2$

a. $(f+g)(x)=2x+1+3x-2=5x-1$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x)=(2x+1)-(3x-2)$

$=2x+1-3x+2$

$=-x+3$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=(2x+1)(3x-2)$

$=6x^2-4x+3x-2$

$=6x^2-x-2$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{2x+1}{3x-2}$

$3x-2\ne 0$

$3x\ne 2 \Rightarrow x\ne \dfrac{2}{3}$

Domain: $\left\{x \,\middle|\, x\ne \dfrac{2}{3}\right\}$.

e. $(f+g)(3)=5(3)-1=15-1=14$

f. $(f-g)(4)=-4+3=-1$

g. $(f\cdot g)(2)=6(2)^2-2-2$

$=6(4)-2-2$

$=24-2-2=20$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{2(1)+1}{3(1)-2}=\dfrac{2+1}{3-2}=\dfrac{3}{1}=3$

63. $f(x)=x-1 \qquad g(x)=2x^2$

a. $(f+g)(x)=x-1+2x^2=2x^2+x-1$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x)=(x-1)-(2x^2)$

$=x-1-2x^2$

$=-2x^2+x-1$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=(x-1)(2x^2)=2x^3-2x^2$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{x-1}{2x^2}$

Domain: $\{x \mid x\neq 0\}$.

e. $(f+g)(3)=2(3)^2+3-1$
$=2(9)+3-1$
$=18+3-1=20$

f. $(f-g)(4)=-2(4)^2+4-1$
$=-2(16)+4-1$
$=-32+4-1=-29$

g. $(f\cdot g)(2)=2(2)^3-2(2)^2$
$=2(8)-2(4)$
$=16-8=8$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{1-1}{2(1)^2}=\dfrac{0}{2(1)}=\dfrac{0}{2}=0$

64. $f(x)=2x^2+3 \qquad g(x)=4x^3+1$

a. $(f+g)(x)=2x^2+3+4x^3+1$
$=4x^3+2x^2+4$
Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x)=\left(2x^2+3\right)-\left(4x^3+1\right)$
$=2x^2+3-4x^3-1$
$=-4x^3+2x^2+2$
Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=\left(2x^2+3\right)\left(4x^3+1\right)$
$=8x^5+12x^3+2x^2+3$
Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{2x^2+3}{4x^3+1}$

$4x^3+1\neq 0$
$4x^3\neq -1$
$x^3\neq -\dfrac{1}{4}\Rightarrow x\neq\sqrt[3]{-\dfrac{1}{4}}=-\dfrac{\sqrt[3]{2}}{2}$

Domain: $\left\{x \,\middle|\, x\neq -\dfrac{\sqrt[3]{2}}{2}\right\}$.

e. $(f+g)(3)=4(3)^3+2(3)^2+4$
$=4(27)+2(9)+4$
$=108+18+4=130$

f. $(f-g)(4)=-4(4)^3+2(4)^2+2$
$=-4(64)+2(16)+2$
$=-256+32+2=-222$

g. $(f\cdot g)(2)=8(2)^5+12(2)^3+2(2)^2+3$
$=8(32)+12(8)+2(4)+3$
$=256+96+8+3=363$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{2(1)^2+3}{4(1)^3+1}=\dfrac{2(1)+3}{4(1)+1}=\dfrac{2+3}{4+1}=\dfrac{5}{5}=1$

65. $f(x)=\sqrt{x} \qquad g(x)=3x-5$

a. $(f+g)(x)=\sqrt{x}+3x-5$
Domain: $\{x \mid x\geq 0\}$.

b. $(f-g)(x)=\sqrt{x}-(3x-5)=\sqrt{x}-3x+5$
Domain: $\{x \mid x\geq 0\}$.

c. $(f\cdot g)(x)=\sqrt{x}(3x-5)=3x\sqrt{x}-5\sqrt{x}$
Domain: $\{x \mid x\geq 0\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{\sqrt{x}}{3x-5}$

$x\geq 0$ and $3x-5\neq 0$
$3x\neq 5\Rightarrow x\neq\dfrac{5}{3}$

Domain: $\left\{x \,\middle|\, x\geq 0 \text{ and } x\neq\dfrac{5}{3}\right\}$.

e. $(f+g)(3)=\sqrt{3}+3(3)-5$
$=\sqrt{3}+9-5=\sqrt{3}+4$

f. $(f-g)(4)=\sqrt{4}-3(4)+5$
$=2-12+5=-5$

g. $(f\cdot g)(2)=3(2)\sqrt{2}-5\sqrt{2}$
$=6\sqrt{2}-5\sqrt{2}=\sqrt{2}$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{\sqrt{1}}{3(1)-5}=\dfrac{1}{3-5}=\dfrac{1}{-2}=-\dfrac{1}{2}$

66. $f(x)=|x| \qquad g(x)=x$

a. $(f+g)(x)=|x|+x$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x)=|x|-x$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=|x|\cdot x=x|x|$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{|x|}{x}$

Domain: $\{x \mid x\neq 0\}$.

e. $(f+g)(3)=|3|+3=3+3=6$

f. $(f-g)(4)=|4|-4=4-4=0$

g. $(f\cdot g)(2)=2|2|=2\cdot 2=4$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{|1|}{1}=\dfrac{1}{1}=1$

67. $f(x)=1+\dfrac{1}{x} \qquad g(x)=\dfrac{1}{x}$

a. $(f+g)(x)=1+\dfrac{1}{x}+\dfrac{1}{x}=1+\dfrac{2}{x}$

Domain: $\{x \mid x\neq 0\}$.

b. $(f-g)(x)=1+\dfrac{1}{x}-\dfrac{1}{x}=1$

Domain: $\{x \mid x\neq 0\}$.

c. $(f\cdot g)(x)=\left(1+\dfrac{1}{x}\right)\dfrac{1}{x}=\dfrac{1}{x}+\dfrac{1}{x^2}$

Domain: $\{x \mid x\neq 0\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{1+\dfrac{1}{x}}{\dfrac{1}{x}}=\dfrac{\dfrac{x+1}{x}}{\dfrac{1}{x}}=\dfrac{x+1}{x}\cdot\dfrac{x}{1}=x+1$

Domain: $\{x \mid x\neq 0\}$.

e. $(f+g)(3)=1+\dfrac{2}{3}=\dfrac{5}{3}$

f. $(f-g)(4)=1$

g. $(f\cdot g)(2)=\dfrac{1}{2}+\dfrac{1}{(2)^2}=\dfrac{1}{2}+\dfrac{1}{4}=\dfrac{3}{4}$

h. $\left(\dfrac{f}{g}\right)(1)=1+1=2$

68. $f(x)=\sqrt{x-1} \qquad g(x)=\sqrt{4-x}$

a. $(f+g)(x)=\sqrt{x-1}+\sqrt{4-x}$

$x-1\ge 0$ and $4-x\ge 0$

$x\ge 1$ and $-x\ge -4$

$x\le 4$

Domain: $\{x \mid 1\le x\le 4\}$.

b. $(f-g)(x)=\sqrt{x-1}-\sqrt{4-x}$

$x-1\ge 0$ and $4-x\ge 0$

$x\ge 1$ and $-x\ge -4$

$x\le 4$

Domain: $\{x \mid 1\le x\le 4\}$.

c. $(f\cdot g)(x)=\left(\sqrt{x-1}\right)\left(\sqrt{4-x}\right)$

$=\sqrt{-x^2+5x-4}$

$x-1\ge 0$ and $4-x\ge 0$

$x\ge 1$ and $-x\ge -4$

$x\le 4$

Domain: $\{x \mid 1\le x\le 4\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{\sqrt{x-1}}{\sqrt{4-x}}=\sqrt{\dfrac{x-1}{4-x}}$

$x-1\ge 0$ and $4-x>0$

$x\ge 1$ and $-x>-4$

$x<4$

Domain: $\{x \mid 1\le x<4\}$.

e. $(f+g)(3)=\sqrt{3-1}+\sqrt{4-3}$

$=\sqrt{2}+\sqrt{1}=\sqrt{2}+1$

f. $(f-g)(4)=\sqrt{4-1}-\sqrt{4-4}$

$=\sqrt{3}-\sqrt{0}=\sqrt{3}-0=\sqrt{3}$

g. $(f\cdot g)(2)=\sqrt{-(2)^2+5(2)-4}$

$=\sqrt{-4+10-4}=\sqrt{2}$

h. $\left(\dfrac{f}{g}\right)(1)=\sqrt{\dfrac{1-1}{4-1}}=\sqrt{\dfrac{0}{3}}=\sqrt{0}=0$

69. $f(x)=\dfrac{2x+3}{3x-2} \qquad g(x)=\dfrac{4x}{3x-2}$

a. $(f+g)(x)=\dfrac{2x+3}{3x-2}+\dfrac{4x}{3x-2}$
$=\dfrac{2x+3+4x}{3x-2}=\dfrac{6x+3}{3x-2}$

$3x-2\neq 0$

$3x\neq 2 \Rightarrow x\neq \frac{2}{3}$

Domain: $\left\{x \middle| x\neq \frac{2}{3}\right\}$.

b. $(f-g)(x)=\dfrac{2x+3}{3x-2}-\dfrac{4x}{3x-2}$
$=\dfrac{2x+3-4x}{3x-2}=\dfrac{-2x+3}{3x-2}$

$3x-2\neq 0$

$3x\neq 2 \Rightarrow x\neq \dfrac{2}{3}$

Domain: $\left\{x \middle| x\neq \dfrac{2}{3}\right\}$.

c. $(f\cdot g)(x)=\left(\dfrac{2x+3}{3x-2}\right)\left(\dfrac{4x}{3x-2}\right)=\dfrac{8x^2+12x}{(3x-2)^2}$

$3x-2\neq 0$

$3x\neq 2 \Rightarrow x\neq \dfrac{2}{3}$

Domain: $\left\{x \middle| x\neq \dfrac{2}{3}\right\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{\frac{2x+3}{3x-2}}{\frac{4x}{3x-2}}=\dfrac{2x+3}{3x-2}\cdot\dfrac{3x-2}{4x}=\dfrac{2x+3}{4x}$

$3x-2\neq 0$ and $x\neq 0$

$3x\neq 2$

$x\neq \dfrac{2}{3}$

Domain: $\left\{x \middle| x\neq \dfrac{2}{3} \text{ and } x\neq 0\right\}$.

e. $(f+g)(3)=\dfrac{6(3)+3}{3(3)-2}=\dfrac{18+3}{9-2}=\dfrac{21}{7}=3$

f. $(f-g)(4)=\dfrac{-2(4)+3}{3(4)-2}=\dfrac{-8+3}{12-2}=\dfrac{-5}{10}=-\dfrac{1}{2}$

g. $(f\cdot g)(2)=\dfrac{8(2)^2+12(2)}{\left(3(2)-2\right)^2}$
$=\dfrac{8(4)+24}{\left(6-2\right)^2}=\dfrac{32+24}{\left(4\right)^2}=\dfrac{56}{16}=\dfrac{7}{2}$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{2(1)+3}{4(1)}=\dfrac{2+3}{4}=\dfrac{5}{4}$

70. $f(x)=\sqrt{x+1} \qquad g(x)=\dfrac{2}{x}$

a. $(f+g)(x)=\sqrt{x+1}+\dfrac{2}{x}$

$x+1\geq 0$ and $x\neq 0$

$x\geq -1$

Domain: $\{x \mid x\geq -1, \text{ and } x\neq 0\}$.

b. $(f-g)(x)=\sqrt{x+1}-\dfrac{2}{x}$

$x+1\geq 0$ and $x\neq 0$

$x\geq -1$

Domain: $\{x \mid x\geq -1, \text{ and } x\neq 0\}$.

c. $(f\cdot g)(x)=\sqrt{x+1}\cdot\dfrac{2}{x}=\dfrac{2\sqrt{x+1}}{x}$

$x+1\geq 0$ and $x\neq 0$

$x\geq -1$

Domain: $\{x \mid x\geq -1, \text{ and } x\neq 0\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{\sqrt{x+1}}{\frac{2}{x}}=\dfrac{x\sqrt{x+1}}{2}$

$x+1\geq 0$ and $x\neq 0$

$x\geq -1$

Domain: $\{x \mid x\geq -1, \text{ and } x\neq 0\}$.

e. $(f+g)(3)=\sqrt{3+1}+\dfrac{2}{3}=\sqrt{4}+\dfrac{2}{3}=2+\dfrac{2}{3}=\dfrac{8}{3}$

f. $(f-g)(4)=\sqrt{4+1}-\dfrac{2}{4}=\sqrt{5}-\dfrac{1}{2}$

g. $(f\cdot g)(2)=\dfrac{2\sqrt{2+1}}{2}=\dfrac{2\sqrt{3}}{2}=\sqrt{3}$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{1\sqrt{1+1}}{2}=\dfrac{\sqrt{2}}{2}$

71. $f(x)=3x+1 \qquad (f+g)(x)=6-\frac{1}{2}x$

$6-\frac{1}{2}x=3x+1+g(x)$

$5-\frac{7}{2}x=g(x)$

$g(x)=5-\frac{7}{2}x$

72. $f(x)=\frac{1}{x} \qquad \left(\frac{f}{g}\right)(x)=\frac{x+1}{x^2-x}$

$\frac{x+1}{x^2-x}=\frac{\frac{1}{x}}{g(x)}$

$g(x)=\frac{\frac{1}{x}}{\frac{x+1}{x^2-x}}=\frac{1}{x}\cdot\frac{x^2-x}{x+1}$

$=\frac{1}{x}\cdot\frac{x(x-1)}{x+1}=\frac{x-1}{x+1}$

73. $f(x)=4x+3$

$\frac{f(x+h)-f(x)}{h}=\frac{4(x+h)+3-(4x+3)}{h}$

$=\frac{4x+4h+3-4x-3}{h}$

$=\frac{4h}{h}=4$

74. $f(x)=-3x+1$

$\frac{f(x+h)-f(x)}{h}=\frac{-3(x+h)+1-(-3x+1)}{h}$

$=\frac{-3x-3h+1+3x-1}{h}$

$=\frac{-3h}{h}=-3$

75. $f(x)=x^2-x+4$

$\frac{f(x+h)-f(x)}{h}$

$=\frac{(x+h)^2-(x+h)+4-(x^2-x+4)}{h}$

$=\frac{x^2+2xh+h^2-x-h+4-x^2+x-4}{h}$

$=\frac{2xh+h^2-h}{h}$

$=2x+h-1$

76. $f(x)=x^2+5x-1$

$\frac{f(x+h)-f(x)}{h}$

$=\frac{(x+h)^2+5(x+h)-1-(x^2+5x-1)}{h}$

$=\frac{x^2+2xh+h^2+5x+5h-1-x^2-5x+1}{h}$

$=\frac{2xh+h^2+5h}{h}=2x+h+5$

77. $f(x)=3x^2-2x+6$

$\frac{f(x+h)-f(x)}{h}$

$=\frac{\left[3(x+h)^2-2(x+h)+6\right]-\left[3x^2-2x+6\right]}{h}$

$=\frac{3\left(x^2+2xh+h^2\right)-2x-2h+6-3x^2+2x-6}{h}$

$=\frac{3x^2+6xh+3h^2-2h-3x^2}{h}=\frac{6xh+3h^2-2h}{h}$

$=6x+3h-2$

78. $f(x)=4x^2+5x-7$

$\frac{f(x+h)-f(x)}{h}$

$=\frac{\left[4(x+h)^2+5(x+h)-7\right]-\left[4x^2+5x-7\right]}{h}$

$=\frac{4\left(x^2+2xh+h^2\right)+5x+5h-7-4x^2-5x+7}{h}$

$=\frac{4x^2+8xh+4h^2+5h-4x^2}{h}=\frac{8xh+4h^2+5h}{h}$

$=8x+4h+5$

79. $f(x)=x^3-2$

$\frac{f(x+h)-f(x)}{h}$

$=\frac{(x+h)^3-2-\left(x^3-2\right)}{h}$

$=\frac{x^3+3x^2h+3xh^2+h^3-2-x^3+2}{h}$

$=\frac{3x^2h+3xh^2+h^3}{h}=3x^2+3xh+h^2$

80. $f(x)=\dfrac{1}{x+3}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{x+h+3}-\frac{1}{x+3}}{h}=\frac{\frac{x+3-(x+3+h)}{(x+h+3)(x+3)}}{h}=\left(\frac{x+3-x-3-h}{(x+h+3)(x+3)}\right)\left(\frac{1}{h}\right)=\left(\frac{-h}{(x+h+3)(x+3)}\right)\left(\frac{1}{h}\right)=\frac{-1}{(x+h+3)(x+3)}$$

81. $f(x)=2x^3+Ax^2+4x-5$ and $f(2)=5$

$f(2)=2(2)^3+A(2)^2+4(2)-5$

$5=16+4A+8-5$

$5=4A+19$

$-14=4A$

$A=\dfrac{-14}{4}=-\dfrac{7}{2}$

82. $f(x)=3x^2-Bx+4$ and $f(-1)=12$:

$f(-1)=3(-1)^2-B(-1)+4$

$12=3+B+4$

$B=5$

83. $f(x)=\dfrac{3x+8}{2x-A}$ and $f(0)=2$

$f(0)=\dfrac{3(0)+8}{2(0)-A}$

$2=\dfrac{8}{-A}$

$-2A=8$

$A=-4$

84. $f(x)=\dfrac{2x-B}{3x+4}$ and $f(2)=\dfrac{1}{2}$

$f(2)=\dfrac{2(2)-B}{3(2)+4}$

$\dfrac{1}{2}=\dfrac{4-B}{10}$

$5=4-B$

$B=-1$

85. $f(x)=\dfrac{2x-A}{x-3}$ and $f(4)=0$

$f(4)=\dfrac{2(4)-A}{4-3}$

$0=\dfrac{8-A}{1}$

$0=8-A$

$A=8$

f is undefined when $x=3$.

86. $f(x)=\dfrac{x-B}{x-A}$, $f(2)=0$ and $f(1)$ is undefined

$1-A=0 \quad\Rightarrow\quad A=1$

$f(2)=\dfrac{2-B}{2-1}$

$0=\dfrac{2-B}{1}$

$0=2-B$

$B=2$

87. Let x represent the length of the rectangle. Then, $\dfrac{x}{2}$ represents the width of the rectangle since the length is twice the width. The function for the area is: $A(x)=x\cdot\dfrac{x}{2}=\dfrac{x^2}{2}=\dfrac{1}{2}x^2$

88. Let x represent the length of one of the two equal sides. The function for the area is:
$A(x)=\dfrac{1}{2}\cdot x\cdot x=\dfrac{1}{2}x^2$

89. Let x represent the number of hours worked. The function for the gross salary is:
$G(x)=10x$

90. Let x represent the number of items sold. The function for the gross salary is:
$G(x)=10x+100$

91. **a.** P is the dependent variable; a is the independent variable

b. $$\begin{aligned} P(20) &= 0.015(20)^2 - 4.962(20) + 290.580 \\ &= 6 - 99.24 + 290.580 \\ &= 197.34 \end{aligned}$$
In 2005 there are 197.34 million people who are 20 years of age or older.

c. $$\begin{aligned} P(0) &= 0.015(0)^2 - 4.962(0) + 290.580 \\ &= 290.580 \end{aligned}$$
In 2005 there are 290.580 million people.

92. **a.** N is the dependent variable; r is the independent variable

b. $$\begin{aligned} N(3) &= -1.44(3)^2 + 14.52(3) - 14.96 \\ &= -12.96 + 43.56 - 14.96 \\ &= 15.64 \end{aligned}$$
In 2005, there are 15.64 million housing units with 3 rooms.

93. **a.** $$\begin{aligned} H(1) &= 20 - 4.9(1)^2 \\ &= 20 - 4.9 = 15.1 \text{ meters} \end{aligned}$$
$$\begin{aligned} H(1.1) &= 20 - 4.9(1.1)^2 \\ &= 20 - 4.9(1.21) \\ &= 20 - 5.929 = 14.071 \text{ meters} \end{aligned}$$
$$\begin{aligned} H(1.2) &= 20 - 4.9(1.2)^2 \\ &= 20 - 4.9(1.44) \\ &= 20 - 7.056 = 12.944 \text{ meters} \end{aligned}$$
$$\begin{aligned} H(1.3) &= 20 - 4.9(1.3)^2 \\ &= 20 - 4.9(1.69) \\ &= 20 - 8.281 = 11.719 \text{ meters} \end{aligned}$$

b. $H(x) = 15$:
$$\begin{aligned} 15 &= 20 - 4.9x^2 \\ -5 &= -4.9x^2 \\ x^2 &\approx 1.0204 \\ x &\approx 1.01 \text{ seconds} \end{aligned}$$
$H(x) = 10$:
$$\begin{aligned} 10 &= 20 - 4.9x^2 \\ -10 &= -4.9x^2 \\ x^2 &\approx 2.0408 \\ x &\approx 1.43 \text{ seconds} \end{aligned}$$
$H(x) = 5$:
$$\begin{aligned} 5 &= 20 - 4.9x^2 \\ -15 &= -4.9x^2 \\ x^2 &\approx 3.0612 \\ x &\approx 1.75 \text{ seconds} \end{aligned}$$

c. $H(x) = 0$
$$\begin{aligned} 0 &= 20 - 4.9x^2 \\ -20 &= -4.9x^2 \\ x^2 &\approx 4.0816 \\ x &\approx 2.02 \text{ seconds} \end{aligned}$$

94. **a.** $H(1) = 20 - 13(1)^2 = 20 - 13 = 7$ meters
$$\begin{aligned} H(1.1) &= 20 - 13(1.1)^2 = 20 - 13(1.21) \\ &= 20 - 15.73 = 4.27 \text{ meters} \end{aligned}$$
$$\begin{aligned} H(1.2) &= 20 - 13(1.2)^2 = 20 - 13(1.44) \\ &= 20 - 18.72 = 1.28 \text{ meters} \end{aligned}$$

b. $H(x) = 15$
$$\begin{aligned} 15 &= 20 - 13x^2 \\ -5 &= -13x^2 \\ x^2 &\approx 0.3846 \\ x &\approx 0.62 \text{ seconds} \end{aligned}$$
$H(x) = 10$
$$\begin{aligned} 10 &= 20 - 13x^2 \\ -10 &= -13x^2 \\ x^2 &\approx 0.7692 \\ x &\approx 0.88 \text{ seconds} \end{aligned}$$
$H(x) = 5$
$$\begin{aligned} 5 &= 20 - 13x^2 \\ -15 &= -13x^2 \\ x^2 &\approx 1.1538 \\ x &\approx 1.07 \text{ seconds} \end{aligned}$$

c. $H(x) = 0$
$$\begin{aligned} 0 &= 20 - 13x^2 \\ -20 &= -13x^2 \\ x^2 &\approx 1.5385 \\ x &\approx 1.24 \text{ seconds} \end{aligned}$$

95. $C(x)=100+\dfrac{x}{10}+\dfrac{36,000}{x}$

a. $C(500)=100+\dfrac{500}{10}+\dfrac{36,000}{500}$
$=100+50+72$
$=\$222$

b. $C(450)=100+\dfrac{450}{10}+\dfrac{36,000}{450}$
$=100+45+80$
$=\$225$

c. $C(600)=100+\dfrac{600}{10}+\dfrac{36,000}{600}$
$=100+60+60$
$=\$220$

d. $C(400)=100+\dfrac{400}{10}+\dfrac{36,000}{400}$
$=100+40+90$
$=\$230$

96. $A(x)=4x\sqrt{1-x^2}$

a. $A\left(\dfrac{1}{3}\right)=4\cdot\dfrac{1}{3}\sqrt{1-\left(\dfrac{1}{3}\right)^2}=\dfrac{4}{3}\sqrt{\dfrac{8}{9}}=\dfrac{4}{3}\cdot\dfrac{2\sqrt{2}}{3}$
$=\dfrac{8\sqrt{2}}{9}\approx 1.26\text{ ft}^2$

b. $A\left(\dfrac{1}{2}\right)=4\cdot\dfrac{1}{2}\sqrt{1-\left(\dfrac{1}{2}\right)^2}=2\sqrt{\dfrac{3}{4}}=2\cdot\dfrac{\sqrt{3}}{2}$
$=\sqrt{3}\approx 1.73\text{ ft}^2$

c. $A\left(\dfrac{2}{3}\right)=4\cdot\dfrac{2}{3}\sqrt{1-\left(\dfrac{2}{3}\right)^2}=\dfrac{8}{3}\sqrt{\dfrac{5}{9}}=\dfrac{8}{3}\cdot\dfrac{\sqrt{5}}{3}$
$=\dfrac{8\sqrt{5}}{9}\approx 1.99\text{ ft}^2$

97. $R(x)=\left(\dfrac{L}{P}\right)(x)=\dfrac{L(x)}{P(x)}$

98. $T(x)=(V+P)(x)=V(x)+P(x)$

99. $H(x)=(P\cdot I)(x)=P(x)\cdot I(x)$

100. $N(x)=(I-T)(x)=I(x)-T(x)$

101. a. $P(x)=R(x)-C(x)$
$=\left(-1.2x^2+220x\right)-\left(0.05x^3-2x^2+65x+500\right)$
$=-1.2x^2+220x-0.05x^3+2x^2-65x-500$
$=-0.05x^3+0.8x^2+155x-500$

b. $P(15)=-0.05(15)^3+0.8(15)^2+155(15)-500$
$=-168.75+180+2325-500$
$=\$1836.25$

c. When 15 hundred cell phones are sold, the profit is \$1836.25.

102. a. $P(x)=R(x)-C(x)$
$=30x-\left(0.1x^2+7x+400\right)$
$=30x-0.1x^2-7x-400$
$=-0.1x^2+23x-400$

b. $P(30)=-0.1(30)^2+23(30)-400$
$=-90+690-400$
$=\$200$

c. When 30 clocks are sold, the profit is \$200.

103. a. $h(x)=2x$
$h(a+b)=2(a+b)=2a+2b$
$=h(a)+h(b)$
$h(x)=2x$ has the property.

b. $g(x)=x^2$
$g(a+b)=(a+b)^2=a^2+2ab+b^2$
Since
$a^2+2ab+b^2\neq a^2+b^2=g(a)+g(b)$,
$g(x)=x^2$ does not have the property.

c. $F(x)=5x-2$
$F(a+b)=5(a+b)-2=5a+5b-2$
Since
$5a+5b-2\neq 5a-2+5b-2=F(a)+F(b)$,
$F(x)=5x-2$ does not have the property.

d. $G(x)=\dfrac{1}{x}$
$G(a+b)=\dfrac{1}{a+b}\neq\dfrac{1}{a}+\dfrac{1}{b}=G(a)+G(b)$
$G(x)=\dfrac{1}{x}$ does not have the property.

104. No. The domain of f is $\{x \mid x \text{ is any real number}\}$, but the domain of g is $\{x \mid x \neq -1\}$.

105. Answers will vary.

106. $f(x) = \sqrt{3x+5}$

107. *two less than the square of a number t.*

Section 2.2

1. $x^2 + 4y^2 = 16$

x-intercepts:

$$x^2 + 4(0)^2 = 16$$
$$x^2 = 16$$
$$x = \pm 4 \Rightarrow (-4,0),(4,0)$$

y-intercepts:

$$(0)^2 + 4y^2 = 16$$
$$4y^2 = 16$$
$$y^2 = 4$$
$$y = \pm 2 \Rightarrow (0,-2),(0,2)$$

2. False;
$$x = 2y - 2$$
$$-2 = 2y - 2$$
$$0 = 2y$$
$$0 = y$$
The point $(-2,0)$ is on the graph.

3. vertical

4. $f(5) = -3$

5. $(-2,7)$

6. False; it would fail the vertical line test.

7. False; e.g. $y = \dfrac{1}{x}$.

8. True

9. a. $f(0) = 3$ since $(0,3)$ is on the graph.
$f(-6) = -3$ since $(-6,-3)$ is on the graph.

b. $f(6) = 0$ since $(6, 0)$ is on the graph.
$f(11) = 1$ since $(11, 1)$ is on the graph.

c. $f(3)$ is positive since $f(3) \approx 3.7$.

d. $f(-4)$ is negative since $f(-4) \approx -1$.

e. $f(x) = 0$ when $x = -3$, $x = 6$, and $x = 10$.

f. $f(x) > 0$ when $-3 < x < 6$, and $10 < x \leq 11$.

g. The domain of f is $\{x \mid -6 \leq x \leq 11\}$ or $[-6, 11]$.

h. The range of f is $\{y \mid -3 \leq y \leq 4\}$ or $[-3, 4]$.

i. The x-intercepts are -3, 6, and 10.

j. The y-intercept is 3.

k. The line $y = \dfrac{1}{2}$ intersects the graph 3 times.

l. The line $x = 5$ intersects the graph 1 time.

m. $f(x) = 3$ when $x = 0$ and $x = 4$.

n. $f(x) = -2$ when $x = -5$ and $x = 8$.

10. a. $f(0) = 0$ since $(0,0)$ is on the graph.
$f(6) = 0$ since $(6,0)$ is on the graph.

b. $f(2) = -2$ since $(2, -2)$ is on the graph.
$f(-2) = 1$ since $(-2, 1)$ is on the graph.

c. $f(3)$ is negative since $f(3) \approx -1$.

d. $f(-1)$ is positive since $f(-1) \approx 1.0$.

e. $f(x) = 0$ when $x = 0$, $x = 4$, and $x = 6$.

f. $f(x) < 0$ when $0 < x < 4$.

g. The domain of f is $\{x \mid -4 \leq x \leq 6\}$ or $[-4, 6]$.

h. The range of f is $\{y \mid -2 \leq y \leq 3\}$ or $[-2, 3]$.

i. The x-intercepts are 0, 4, and 6.

j. The y-intercept is 0.

k. The line $y=-1$ intersects the graph 2 times.

l. The line $x=1$ intersects the graph 1 time.

m. $f(x)=3$ when $x=5$.

n. $f(x)=-2$ when $x=2$.

11. Not a function since vertical lines will intersect the graph in more than one point.

12. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y>0\}$

b. Intercepts: (0,1)

c. None

13. Function

a. Domain: $\{x \mid -\pi \le x \le \pi\}$;

Range: $\{y \mid -1 \le y \le 1\}$

b. Intercepts: $\left(-\frac{\pi}{2},0\right)$, $\left(\frac{\pi}{2},0\right)$, (0,1)

c. Symmetry about *y*-axis.

14. Function

a. Domain: $\{x \mid -\pi \le x \le \pi\}$;

Range: $\{y \mid -1 \le y \le 1\}$

b. Intercepts: $(-\pi, 0)$, $(\pi, 0)$, (0, 0)

c. Symmetry about the origin.

15. Not a function since vertical lines will intersect the graph in more than one point.

16. Not a function since vertical lines will intersect the graph in more than one point.

17. Function

a. Domain: $\{x \mid x>0\}$;

Range: $\{y \mid y \text{ is any real number}\}$

b. Intercepts: (1, 0)

c. None

18. Function

a. Domain: $\{x \mid 0 \le x \le 4\}$;

Range: $\{y \mid 0 \le y \le 3\}$

b. Intercepts: (0, 0)

c. None

19. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y \le 2\}$

b. Intercepts: (–3, 0), (3, 0), (0,2)

c. Symmetry about *y*-axis.

20. Function

a. Domain: $\{x \mid x \ge -3\}$;

Range: $\{y \mid y \ge 0\}$

b. Intercepts: (–3, 0), (2,0), (0,2)

c. None

21. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y \ge -3\}$

b. Intercepts: (1, 0), (3,0), (0,9)

c. None

22. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y \le 5\}$

b. Intercepts: (–1, 0), (2,0), (0,4)

c. None

23. $f(x)=2x^2-x-1$

a. $f(-1)=2(-1)^2-(-1)-1=2$

The point $(-1,2)$ is on the graph of *f*.

b. $f(-2)=2(-2)^2-(-2)-1=9$

The point $(-2,9)$ is on the graph of *f*.

c. Solve for x:

$$-1 = 2x^2 - x - 1$$
$$0 = 2x^2 - x$$
$$0 = x(2x-1) \Rightarrow x = 0, x = \frac{1}{2}$$

(0, –1) and $\left(\frac{1}{2}, -1\right)$ are on the graph of f.

d. The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

e. *x*-intercepts:

$$f(x) = 0 \Rightarrow 2x^2 - x - 1 = 0$$
$$(2x+1)(x-1) = 0 \Rightarrow x = -\frac{1}{2}, x = 1$$
$$\left(-\frac{1}{2}, 0\right) \text{ and } (1,0)$$

f. *y*-intercept:

$$f(0) = 2(0)^2 - 0 - 1 = -1 \Rightarrow (0,-1)$$

24. $f(x) = -3x^2 + 5x$

a. $f(-1) = -3(-1)^2 + 5(-1) = -8 \neq 2$

The point $(-1, 2)$ is not on the graph of *f*.

b. $f(-2) = -3(-2)^2 + 5(-2) = -22$

The point $(-2, -22)$ is on the graph of *f*.

c. Solve for x:

$$-2 = -3x^2 + 5x \Rightarrow 3x^2 - 5x - 2 = 0$$
$$(3x+1)(x-2) = 0 \Rightarrow x = -\frac{1}{3}, x = 2$$

(2, –2) and $\left(-\frac{1}{3}, -2\right)$ on the graph of f.

d. The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

e. *x*-intercepts:

$$f(x) = 0 \Rightarrow -3x^2 + 5x = 0$$
$$x(-3x+5) = 0 \Rightarrow x = 0, x = \frac{5}{3}$$
$$(0,0) \text{ and } \left(\frac{5}{3}, 0\right)$$

f. *y*-intercept:

$$f(0) = -3(0)^2 + 5(0) = 0 \Rightarrow (0,0)$$

25. $f(x) = \dfrac{x+2}{x-6}$

a. $f(3) = \dfrac{3+2}{3-6} = -\dfrac{5}{3} \neq 14$

The point $(3, 14)$ is not on the graph of *f*.

b. $f(4) = \dfrac{4+2}{4-6} = \dfrac{6}{-2} = -3$

The point $(4, -3)$ is on the graph of *f*.

c. Solve for x:

$$2 = \frac{x+2}{x-6}$$
$$2x - 12 = x + 2$$
$$x = 14$$

(14, 2) is a point on the graph of f.

d. The domain of *f* is $\{x \mid x \neq 6\}$.

e. *x*-intercepts:

$$f(x) = 0 \Rightarrow \frac{x+2}{x-6} = 0$$
$$x + 2 = 0 \Rightarrow x = -2 \Rightarrow (-2, 0)$$

f. *y*-intercept: $f(0) = \dfrac{0+2}{0-6} = -\dfrac{1}{3} \Rightarrow \left(0, -\dfrac{1}{3}\right)$

26. $f(x) = \dfrac{x^2+2}{x+4}$

a. $f(1) = \dfrac{1^2+2}{1+4} = \dfrac{3}{5}$

The point $\left(1, \dfrac{3}{5}\right)$ is on the graph of *f*.

b. $f(0) = \dfrac{0^2+2}{0+4} = \dfrac{2}{4} = \dfrac{1}{2}$

The point $\left(0, \dfrac{1}{2}\right)$ is on the graph of *f*.

c. Solve for x:

$$\frac{1}{2} = \frac{x^2+2}{x+4} \Rightarrow x + 4 = 2x^2 + 4$$
$$0 = 2x^2 - x$$
$$x(2x-1) = 0 \Rightarrow x = 0 \text{ or } x = \frac{1}{2}$$

$\left(0, \dfrac{1}{2}\right)$ and $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ are on the graph of f.

d. The domain of f is $\{x \mid x \neq -4\}$.

e. x-intercepts:

$$f(x)=0 \Rightarrow \frac{x^2+2}{x+4}=0 \Rightarrow x^2+2=0$$

This is impossible, so there are no x-intercepts.

f. y-intercept:

$$f(0)=\frac{0^2+2}{0+4}=\frac{2}{4}=\frac{1}{2} \Rightarrow \left(0,\frac{1}{2}\right)$$

27. $f(x)=\dfrac{2x^2}{x^4+1}$

a. $f(-1)=\dfrac{2(-1)^2}{(-1)^4+1}=\dfrac{2}{2}=1$

The point $(-1,1)$ is on the graph of f.

b. $f(2)=\dfrac{2(2)^2}{(2)^4+1}=\dfrac{8}{17}$

The point $\left(2,\dfrac{8}{17}\right)$ is on the graph of f.

c. Solve for x:

$$1=\frac{2x^2}{x^4+1}$$
$$x^4+1=2x^2$$
$$x^4-2x^2+1=0$$
$$(x^2-1)^2=0$$
$$x^2-1=0 \Rightarrow x=\pm 1$$

(1,1) and (–1,1) are on the graph of f.

d. The domain of f is $\{x \mid x \text{ is any real number}\}$.

e. x-intercept:

$$f(x)=0 \Rightarrow \frac{2x^2}{x^4+1}=0$$
$$2x^2=0 \Rightarrow x=0 \Rightarrow (0,0)$$

f. y-intercept:

$$f(0)=\frac{2(0)^2}{0^4+1}=\frac{0}{0+1}=0 \Rightarrow (0,0)$$

28. $f(x)=\dfrac{2x}{x-2}$

a. $f\left(\dfrac{1}{2}\right)=\dfrac{2\left(\frac{1}{2}\right)}{\frac{1}{2}-2}=\dfrac{1}{-\frac{3}{2}}=-\dfrac{2}{3}$

The point $\left(\dfrac{1}{2},-\dfrac{2}{3}\right)$ is on the graph of f.

b. $f(4)=\dfrac{2(4)}{4-2}=\dfrac{8}{2}=4$

The point $(4,4)$ is on the graph of f.

c. Solve for x:

$$1=\frac{2x}{x-2} \Rightarrow x-2=2x \Rightarrow -2=x$$

(–2,1) is a point on the graph of f.

d. The domain of f is $\{x \mid x \neq 2\}$.

e. x-intercept:

$$f(x)=0 \Rightarrow \frac{2x}{x-2}=0 \Rightarrow 2x=0$$
$$\Rightarrow x=0 \Rightarrow (0,0)$$

f. y-intercept: $f(0)=\dfrac{0}{0-2}=0 \Rightarrow (0,0)$

29. $h(x)=-\dfrac{44x^2}{v^2}+x+6$

a.
$$h(8)=-\frac{44(8)^2}{28^2}+(8)+6$$
$$=-\frac{2816}{784}+14$$
$$\approx 10.4 \text{ feet}$$

b.
$$h(12)=-\frac{44(12)^2}{28^2}+(12)+6$$
$$=-\frac{6336}{784}+18$$
$$\approx 9.9 \text{ feet}$$

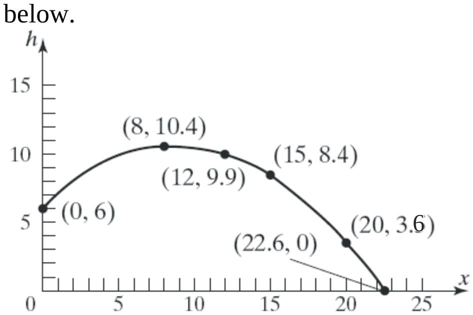
c. From part (a) we know the point $(8,10.4)$ is on the graph and from part (b) we know the point $(12,9.9)$ is on the graph. We could evaluate the function at several more values of *x* (e.g. $x=0$, $x=15$, and $x=20$) to obtain additional points.

$$h(0)=-\frac{44(0)^2}{28^2}+(0)+6=6$$

$$h(15)=-\frac{44(15)^2}{28^2}+(15)+6\approx 8.4$$

$$h(20)=-\frac{44(20)^2}{28^2}+(20)+6\approx 3.6$$

Some additional points are $(0,6)$, $(15,8.4)$ and $(20,3.6)$. The complete graph is given below.



d. $h(15)=-\frac{44(15)^2}{28^2}+(15)+6\approx 8.4$ feet

No; when the ball is 15 feet in front of the foul line, it will be below the hoop. Therefore it cannot go through the hoop.

In order for the ball to pass through the hoop, we need to have $h(15)=10$.

$$10=-\frac{44(15)^2}{v^2}+(15)+6$$

$$-11=-\frac{44(15)^2}{v^2}$$

$$v^2=4(225)$$

$$v^2=900$$

$$v=30 \text{ ft/sec}$$

The ball must be shot with an initial velocity of 30 feet per second in order to go through the hoop.

30. $h(x)=-\frac{136x^2}{v^2}+2.7x+3.5$

a. We want $h(15)=10$.

$$-\frac{136(15)^2}{v^2}+2.7(15)+3.5=10$$

$$-\frac{30,600}{v^2}=-34$$

$$v^2=900$$

$$v=30 \text{ ft/sec}$$

The ball needs to be thrown with an initial velocity of 30 feet per second.

b. $h(x)=-\frac{126x^2}{30^2}+2.7x+3.5$

which simplifies to

$h(x)=-\frac{34}{225}x^2+2.7x+3.5$

c. Using the velocity from part (b),

$h(9)=-\frac{34}{225}(9)^2+2.7(9)+3.5=15.56$ ft

The ball will be 15.56 feet above the floor when it has traveled 9 feet in front of the foul line.

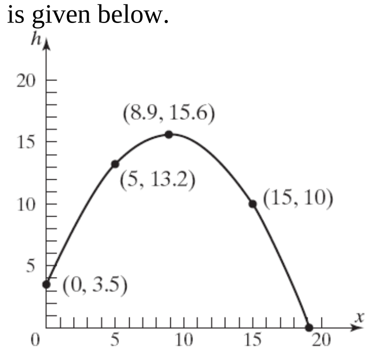
d. Select several values for *x* and use these to find the corresponding values for *h*. Use the results to form ordered pairs (x,h). Plot the points and connect with a smooth curve.

$$h(0)=-\frac{34}{225}(0)^2+2.7(0)+3.5=3.5 \text{ ft}$$

$$h(5)=-\frac{34}{225}(5)^2+2.7(5)+3.5\approx 13.2 \text{ ft}$$

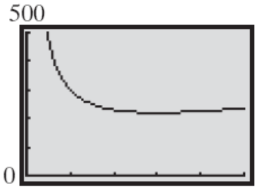
$$h(15)=-\frac{24}{225}(15)^2+2.7(15)+3.5\approx 10 \text{ ft}$$

Thus, some points on the graph are $(0,3.5)$, $(5,13.2)$, and $(15,10)$. The complete graph is given below.



31. $C(x) = 100 + \dfrac{x}{10} + \dfrac{36000}{x}$

a. Graphing:



b. TblStart = 0; ΔTbl = 50

X	Y_1
0	ERROR
50	825
100	470
150	355
200	300
250	269
300	250

Y_1=100+X/10+360…

c. The cost per passenger is minimized to about \$220 when the ground speed is roughly 600 miles per hour.

X	Y_1
450	225
500	222
550	220.45
600	220
650	220.38
700	221.43
750	223

X=600

32. $A(x) = 4x\sqrt{1-x^2}$

a. Domain of $A(x) = 4x\sqrt{1-x^2}$; we know that x must be greater than or equal to zero, since x represents a length. We also need $1-x^2 \ge 0$, since this expression occurs under a square root. In fact, to avoid Area = 0, we require $x > 0$ and $1-x^2 > 0$.

Solve: $1-x^2 > 0$

$(1+x)(1-x) > 0$

Case1: $1+x>0$ and $1-x>0$

$x>-1$ and $x<1$

(i.e. $-1<x<1$)

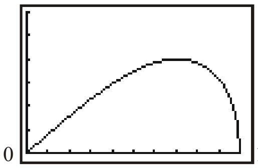
Case2: $1+x<0$ and $1-x<0$

$x<-1$ and $x>1$

(which is impossible)

Therefore the domain of A is $\{x \mid 0<x<1\}$.

b. Graphing $A(x) = 4x\sqrt{1-x^2}$



c. When $x = 0.7$ feet, the cross-sectional area is maximized at approximately 1.9996 square feet. Therefore, the length of the base of the beam should be 1.4 feet in order to maximize the cross-sectional area.

X	Y_1
.3	1.1447
.4	1.4664
.5	1.7321
.6	1.92
.7	1.9996
.8	1.92
.9	1.5692

X=.7

33. a. $(f+g)(2) = f(2)+g(2) = 2+1 = 3$

b. $(f+g)(4) = f(4)+g(4) = 1+(-3) = -2$

c. $(f-g)(6) = f(6)-g(6) = 0-1 = -1$

d. $(g-f)(6) = g(6)-f(6) = 1-0 = 1$

e. $(f\cdot g)(2) = f(2)\cdot g(2) = 2(1) = 2$

f. $\left(\dfrac{f}{g}\right)(4) = \dfrac{f(4)}{g(4)} = \dfrac{1}{-3} = -\dfrac{1}{3}$

34. a. $C(0) = 5000$

This represents the fixed overhead costs. That is, the company will incur costs of \$5000 per day even if no computers are manufactured.

b. $C(10) = 19{,}000$

It costs the company \$19,000 to produce 10 computers in a day.

c. $C(50) = 51{,}000$

It costs the company \$51,000 to produce 50 computers in a day.

d. The domain is $\{q \mid 0 \le q \le 100\}$. This indicates that production capacity is limited to 100 computers in a day.

e. The graph is curved down and rises slowly at first. As production increases, the graph becomes rises more quickly and changes to being curved up.

f. The inflection point is where the graph changes from being curved down to being curved up.

35. a. $C(0) = 80$

This represents the monthly fee. The plan costs \$80 per month even if no minutes are used.

b. $C(1000) = 80$

The monthly charge is \$80 if 1000 minutes are used. Since this is the same as the cost for 0 minutes, all these minutes are included in the base plan.

c. $C(2000) = 210$

The monthly charge is \$210 if 2000 minutes are used.

d. The domain is $\{m \mid 0 \le m \le 14,400\}$. The domain implies that there are at most 14,400 anytime minutes in a month.

e. The graph starts off flat (horizontal line), then increases at a constant rate (straight line with positive slope) after $m = 1000$.

36. Answers will vary. From a graph, the domain can be found by visually locating the x-values for which the graph is defined. The range can be found in a similar fashion by visually locating the *y*-values for which the function is defined.

If an equation is given, the domain can be found by locating any restricted values and removing them from the set of real numbers. The range can be found by using known properties of the graph of the equation, or estimated by means of a table of values.

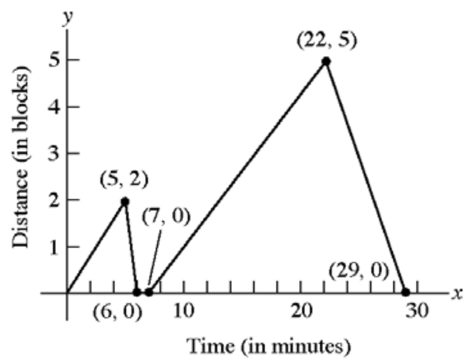
37. The graph of a function can have any number of *x*-intercepts. The graph of a function can have at most one *y*-intercept (otherwise the graph would fail the vertical line test).

38. Yes, the graph of a single point is the graph of a function since it would pass the vertical line test. The equation of such a function would be something like the following: $f(x) = 2$, where $x = 7$.

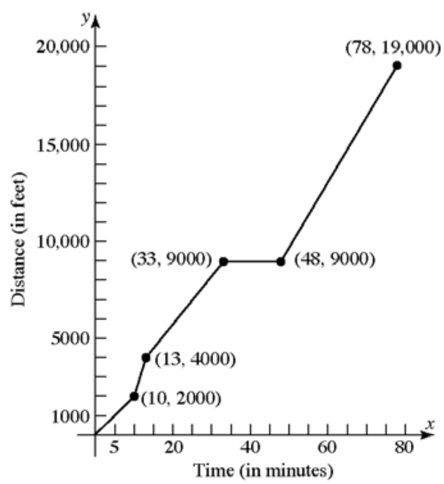
39. (a) III; (b) IV; (c) I; (d) V; (e) II

40. (a) II; (b) V; (c) IV; (d) III; (e) I

41.



42.



43. a. 2 hours elapsed; Kevin was between 0 and 3 miles from home.

b. 0.5 hours elapsed; Kevin was 3 miles from home.

c. 0.3 hours elapsed; Kevin was between 0 and 3 miles from home.

d. 0.2 hours elapsed; Kevin was at home.

e. 0.9 hours elapsed; Kevin was between 0 and 2.8 miles from home.

f. 0.3 hours elapsed; Kevin was 2.8 miles from home.

g. 1.1 hours elapsed; Kevin was between 0 and 2.8 miles from home.

h. The farthest distance Kevin is from home is 3 miles.

i. Kevin returned home 2 times.

44. **a.** Michael travels fastest between 7 and 7.4 minutes. That is, $(7, 7.4)$.

b. Michael's speed is zero between 4.2 and 6 minutes. That is, $(4.2, 6)$.

c. Between 0 and 2 minutes, Michael's speed increased from 0 to 30 miles/hour.

d. Between 4.2 and 6 minutes, Michael was stopped (i.e. his speed was 0 miles/hour).

e. Between 7 and 7.4 minutes, Michael was traveling at a steady rate of 50 miles/hour.

f. Michael's speed is constant between 2 and 4 minutes, between 4.2 and 6 minutes, between 7 and 7.4 minutes, and between 7.6 and 8 minutes. That is, on the intervals (2, 4), (4.2, 6), (7, 7.4), and (7.6, 8).

45. Answers (graphs) will vary. Points of the form (5, *y*) and of the form (*x*, 0) cannot be on the graph of the function.

46. The only such function is $f(x) = 0$ because it is the only function for which $f(x) = -f(x)$. Any other such graph would fail the vertical line test.

47. To draw a vertical line, we fix on an *x*-value. If a vertical line drawn at that *x*-value hits the graph in more than one location, this means that a single input, *x*, yields more than one output, *y*. Thus, the graph is not of a function.

Section 2.3

1. $2 < x < 5$

2. $\text{slope} = \dfrac{\Delta y}{\Delta x} = \dfrac{8-3}{3-(-2)} = \dfrac{5}{5} = 1$

3. *x*-axis: $y \to -y$

$(-y) = 5x^2 - 1$

$-y = 5x^2 - 1$

$y = -5x^2 + 1$ different

y-axis: $x \to -x$

$y = 5(-x)^2 - 1$

$y = 5x^2 - 1$ same

origin: $x \to -x$ and $y \to -y$

$(-y) = 5(-x)^2 - 1$

$-y = 5x^2 - 1$

$y = -5x^2 + 1$ different

The equation has symmetry with respect to the *y*-axis only.

4.
$$y - y_1 = m(x - x_1)$$
$$y - (-2) = 5(x - 3)$$
$$y + 2 = 5(x - 3)$$

5. $y = x^2 - 9$

x-intercepts:

$0 = x^2 - 9$

$x^2 = 9 \to x = \pm 3$

y-intercept:

$y = (0)^2 - 9 = -9$

The intercepts are $(-3, 0)$, $(3, 0)$, and $(0, -9)$.

6. increasing

7. even; odd

8. True

9. True

10. False; odd functions are symmetric with respect to the origin. Even functions are symmetric with respect to the *y*-axis.

11. Yes

12. No, it is increasing.

13. No, it only increases on (5, 10).

14. Yes

15. f is increasing on the intervals $(-8, -2)$, $(0, 2)$, $(5, \infty)$.

16. f is decreasing on the intervals: $(-\infty,-8)$, $(-2,0)$, $(2,5)$.

17. Yes. The local maximum at $x=2$ is 10.

18. No. There is a local minimum at $x=5$; the local minimum is 0.

19. f has local maxima at $x=-2$ and $x=2$. The local maxima are 6 and 10, respectively.

20. f has local minima at $x=-8$, $x=0$ and $x=5$. The local minima are –4, 0, and 0, respectively.

21. **a.** Intercepts: (–2, 0), (2, 0), and (0, 3).

b. Domain: $\{x \mid -4 \le x \le 4\}$ or $[-4, 4]$;

Range: $\{y \mid 0 \le y \le 3\}$ or $[0, 3]$.

c. Increasing: (–2, 0) and (2, 4);
Decreasing: (–4, –2) and (0, 2).

d. Since the graph is symmetric with respect to the *y*-axis, the function is <u>even</u>.

22. **a.** Intercepts: (–1, 0), (1, 0), and (0, 2).

b. Domain: $\{x \mid -3 \le x \le 3\}$ or $[-3, 3]$;

Range: $\{y \mid 0 \le y \le 3\}$ or $[0, 3]$.

c. Increasing: (–1, 0) and (1, 3);
Decreasing: (–3, –1) and (0, 1).

d. Since the graph is symmetric with respect to the *y*-axis, the function is <u>even</u>.

23. **a.** Intercepts: (0, 1).

b. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y > 0\}$ or $(0, \infty)$.

c. Increasing: $(-\infty, \infty)$; Decreasing: never.

d. Since the graph is not symmetric with respect to the *y*-axis or the origin, the function is <u>neither</u> even nor odd.

24. **a.** Intercepts: (1, 0).

b. Domain: $\{x \mid x > 0\}$ or $(0, \infty)$;

Range: $\{y \mid y \text{ is any real number}\}$.

c. Increasing: $(0, \infty)$; Decreasing: never.

d. Since the graph is not symmetric with respect to the *y*-axis or the origin, the function is <u>neither</u> even nor odd.

25. **a.** Intercepts: $(-\pi, 0)$, $(\pi, 0)$, and $(0, 0)$.

b. Domain: $\{x \mid -\pi \le x \le \pi\}$ or $[-\pi, \pi]$;

Range: $\{y \mid -1 \le y \le 1\}$ or $[-1, 1]$.

c. Increasing: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$;

Decreasing: $\left(-\pi, -\frac{\pi}{2}\right)$ and $\left(\frac{\pi}{2}, \pi\right)$.

d. Since the graph is symmetric with respect to the origin, the function is <u>odd</u>.

26. **a.** Intercepts: $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 0\right)$, and $(0, 1)$.

b. Domain: $\{x \mid -\pi \le x \le \pi\}$ or $[-\pi, \pi]$;

Range: $\{y \mid -1 \le y \le 1\}$ or $[-1, 1]$.

c. Increasing: $(-\pi, 0)$; Decreasing: $(0, \pi)$.

d. Since the graph is symmetric with respect to the *y*-axis, the function is <u>even</u>.

27. **a.** Intercepts: $\left(\frac{1}{3}, 0\right), \left(\frac{5}{2}, 0\right)$, and $\left(0, \frac{1}{2}\right)$.

b. Domain: $\{x \mid -3 \le x \le 3\}$ or $[-3, 3]$;

Range: $\{y \mid -1 \le y \le 2\}$ or $[-1, 2]$.

c. Increasing: $(2, 3)$; Decreasing: $(-1, 1)$;
Constant: $(-3, -1)$ and $(1, 2)$

d. Since the graph is not symmetric with respect to the *y*-axis or the origin, the function is <u>neither</u> even nor odd.

28. **a.** Intercepts: $(-2.3, 0), (3, 0)$, and $(0, 1)$.

b. Domain: $\{x \mid -3 \le x \le 3\}$ or $[-3, 3]$;

Range: $\{y \mid -2 \le y \le 2\}$ or $[-2, 2]$.

c. Increasing: $(-3, -2)$ and $(0, 2)$;
Decreasing: $(2, 3)$; Constant: $(-2, 0)$.

d. Since the graph is not symmetric with respect to the *y*-axis or the origin, the function is <u>neither</u> even nor odd.

29. **a.** f has a local maximum of 3 at $x=0$.

b. f has a local minimum of 0 at both $x=-2$ and $x=2$.

30. **a.** f has a local maximum of 2 at $x=0$.

b. f has a local minimum of 0 at both $x=-1$ and $x=1$.

31. **a.** f has a local maximum of 1 at $x=\dfrac{\pi}{2}$.

b. f has a local minimum of –1 at $x=-\dfrac{\pi}{2}$.

32. **a.** f has a local maximum of 1 at $x=0$.

b. f has a local minimum of –1 both at $x=-\pi$ and $x=\pi$.

33. $f(x)=4x^3+x$

$$\begin{aligned} f(-x)&=4(-x)^3+(-x)\\ &=-4x^3-x\\ &=-\left(4x^3+x\right)\\ &=-f(x)\end{aligned}$$

Therefore, f is odd.

34. $f(x)=2x^4-x^2$

$f(-x)=2(-x)^4-(-x)^2=2x^4-x^2=f(x)$

Therefore, f is even.

35. $g(x)=-3x^2-5$

$g(-x)=-3(-x)^2-5=-3x^2-5=g(x)$

Therefore, g is even.

36. $h(x)=3x^5+5x$

$$\begin{aligned} h(-x)&=3(-x)^5+5(-x)\\ &=-3x^5-5x\\ &=-\left(3x^5+5x\right)\\ &=-h(x)\end{aligned}$$

Therefore, h is odd.

37. $F(x)=\sqrt[3]{x}$

$F(-x)=\sqrt[3]{-x}=-\sqrt[3]{x}=-F(x)$

Therefore, F is odd.

38. $G(x)=\sqrt{x}$

$G(-x)=\sqrt{-x}$

G is neither even nor odd.

39. $f(x)=x+|x|$

$f(-x)=-x+|-x|=-x+|x|$

f is neither even nor odd.

40. $f(x)=\sqrt[3]{2x^2+1}$

$f(-x)=\sqrt[3]{2(-x)^2+1}=\sqrt[3]{2x^2+1}=f(x)$

Therefore, f is even.

41. $g(x)=\dfrac{1}{x^2}$

$g(-x)=\dfrac{1}{(-x)^2}=\dfrac{1}{x^2}=g(x)$

Therefore, g is even.

42. $h(x)=\dfrac{x}{x^2-1}$

$h(-x)=\dfrac{-x}{(-x)^2-1}=\dfrac{-x}{x^2-1}=-h(x)$

Therefore, h is odd.

43. $h(x)=\dfrac{-x^3}{3x^2-9}$

$h(-x)=\dfrac{-(-x)^3}{3(-x)^2-9}=\dfrac{x^3}{3x^2-9}=-h(x)$

Therefore, h is odd.

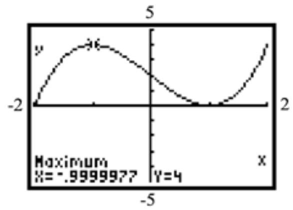
44. $F(x)=\dfrac{2x}{|x|}$

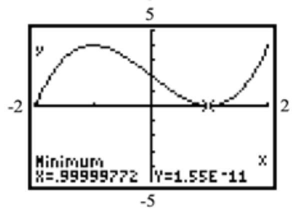
$F(-x)=\dfrac{2(-x)}{|-x|}=\dfrac{-2x}{|x|}=-F(x)$

Therefore, F is odd.

45. $f(x)=x^3-3x+2$ on the interval $(-2,2)$

Use MAXIMUM and MINIMUM on the graph of $y_1=x^3-3x+2$.





local maximum: 4 when $x=-1$

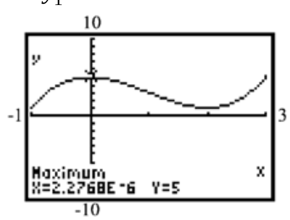
local minimum: 0 when $x=1$

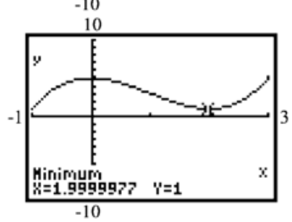
f is increasing on: $(-2,-1)$ and $(1,2)$;

f is decreasing on: $(-1,1)$

46. $f(x)=x^3-3x^2+5$ on the interval $(-1,3)$

Use MAXIMUM and MINIMUM on the graph of $y_1=x^3-3x^2+5$.





local maximum: 5 when $x=0$

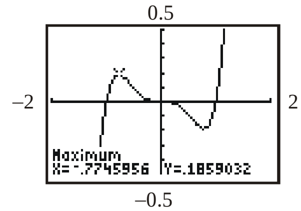
local minimum: 1 when $x=2$

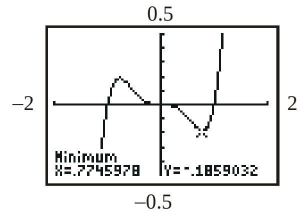
f is increasing on: $(-1,0)$ and $(2,3)$;

f is decreasing on: $(0,2)$

47. $f(x)=x^5-x^3$ on the interval $(-2,2)$

Use MAXIMUM and MINIMUM on the graph of $y_1=x^5-x^3$.





local maximum: 0.19 when $x=-0.77$

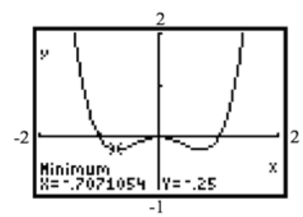
local minimum: -0.19 when $x=0.77$

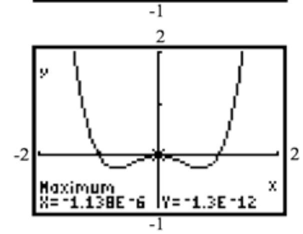
f is increasing on: $(-2,-0.77)$ and $(0.77,2)$;

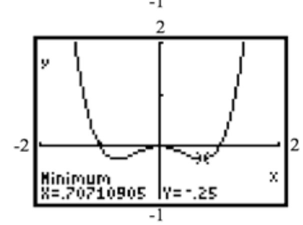
f is decreasing on: $(-0.77,0.77)$

48. $f(x)=x^4-x^2$ on the interval $(-2,2)$

Use MAXIMUM and MINIMUM on the graph of $y_1=x^4-x^2$.







local maximum: 0 when $x=0$

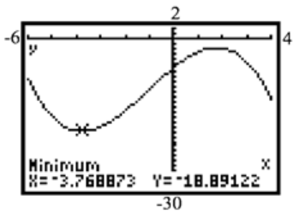
local minimum: -0.25 when $x=-0.71$ and $x=0.71$

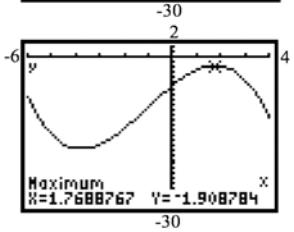
f is increasing on: $(-0.71,0)$ and $(0.71,2)$;

f is decreasing on: $(-2,-0.71)$ and $(0,0.71)$

49. $f(x) = -0.2x^3 - 0.6x^2 + 4x - 6$ on the interval $(-6, 4)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.2x^3 - 0.6x^2 + 4x - 6$.





local maximum: -1.91 when $x = 1.77$

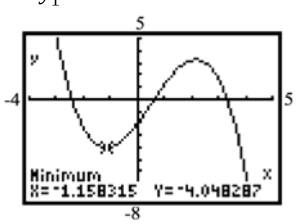
local minimum: -18.89 when $x = -3.77$

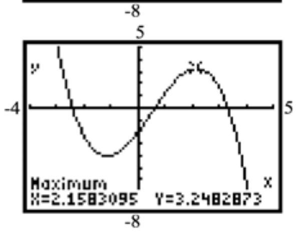
f is increasing on: $(-3.77, 1.77)$;

f is decreasing on: $(-6, -3.77)$ and $(1.77, 4)$

50. $f(x) = -0.4x^3 + 0.6x^2 + 3x - 2$ on the interval $(-4, 5)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.4x^3 + 0.6x^2 + 3x - 2$.





local maximum: 3.25 when $x = 2.16$

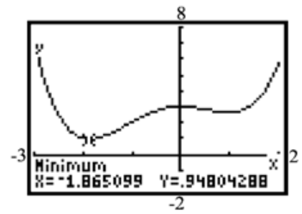
local minimum: -4.05 when $x = -1.16$

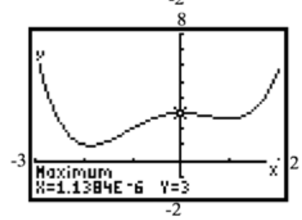
f is increasing on: $(-1.16, 2.16)$;

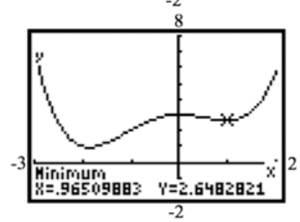
f is decreasing on: $(-4, -1.16)$ and $(2.16, 5)$

51. $f(x) = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3$ on the interval $(-3, 2)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3$.







local maximum: 3 when $x = 0$

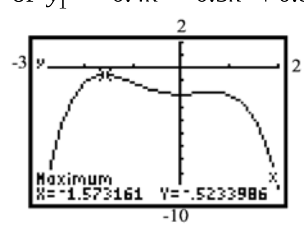
local minimum: 0.95 when $x = -1.87$; 2.65 when $x = 0.97$

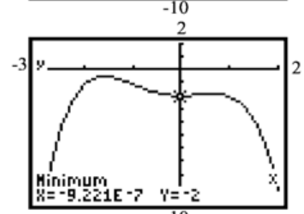
f is increasing on: $(-1.87, 0)$ and $(0.97, 2)$;

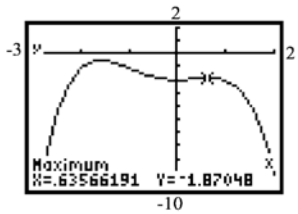
f is decreasing on: $(-3, -1.87)$ and $(0, 0.97)$

52. $f(x) = -0.4x^4 - 0.5x^3 + 0.8x^2 - 2$ on the interval $(-3, 2)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.4x^4 - 0.5x^3 + 0.8x^2 - 2$.







local maximum: -0.52 when $x = -1.57$; -1.87 when $x = 0.64$; local minimum at: $(0, -2)$

f is increasing on: $(-3, -1.57)$ and $(0, 0.64)$;

f is decreasing on: $(-1.57, 0)$ and $(0.64, 2)$

53. $f(x) = -2x^2 + 4$

a. Average rate of change of *f* from $x = 0$ to $x = 2$

$$\frac{f(2) - f(0)}{2 - 0} = \frac{\left(-2(2)^2 + 4\right) - \left(-2(0)^2 + 4\right)}{2}$$
$$= \frac{(-4) - (4)}{2} = \frac{-8}{2} = -4$$

b. Average rate of change of *f* from $x = 1$ to $x = 3$:

$$\frac{f(3) - f(1)}{3 - 1} = \frac{\left(-2(3)^2 + 4\right) - \left(-2(1)^2 + 4\right)}{2}$$
$$= \frac{(-14) - (2)}{2} = \frac{-16}{2} = -8$$

c. Average rate of change of *f* from $x = 1$ to $x = 4$:

$$\frac{f(4) - f(1)}{4 - 1} = \frac{\left(-2(4)^2 + 4\right) - \left(-2(1)^2 + 4\right)}{3}$$
$$= \frac{(-28) - (2)}{3} = \frac{-30}{3} = -10$$

54. $f(x) = -x^3 + 1$

a. Average rate of change of *f* from $x = 0$ to $x = 2$:

$$\frac{f(2) - f(0)}{2 - 0} = \frac{\left(-(2)^3 + 1\right) - \left(-(0)^3 + 1\right)}{2}$$
$$= \frac{-7 - 1}{2} = \frac{-8}{2} = -4$$

b. Average rate of change of *f* from $x = 1$ to $x = 3$:

$$\frac{f(3) - f(1)}{3 - 1} = \frac{\left(-(3)^3 + 1\right) - \left(-(1)^3 + 1\right)}{2}$$
$$= \frac{-26 - (0)}{2} = \frac{-26}{2} = -13$$

c. Average rate of change of *f* from $x = -1$ to $x = 1$:

$$\frac{f(1) - f(-1)}{1 - (-1)} = \frac{\left(-(1)^3 + 1\right) - \left(-(-1)^3 + 1\right)}{2}$$
$$= \frac{0 - 2}{2} = \frac{-2}{2} = -1$$

55. $g(x) = x^3 - 2x + 1$

a. Average rate of change of *g* from $x = -3$ to $x = -2$:

$$\frac{g(-2) - g(-3)}{-2 - (-3)}$$
$$= \frac{\left[(-2)^3 - 2(-2) + 1\right] - \left[(-3)^3 - 2(-3) + 1\right]}{1}$$
$$= \frac{(-3) - (-20)}{1} = \frac{17}{1} = 17$$

b. Average rate of change of *g* from $x = -1$ to $x = 1$:

$$\frac{g(1) - g(-1)}{1 - (-1)}$$
$$= \frac{\left[(1)^3 - 2(1) + 1\right] - \left[(-1)^3 - 2(-1) + 1\right]}{2}$$
$$= \frac{(0) - (2)}{2} = \frac{-2}{2} = -1$$

c. Average rate of change of *g* from $x = 1$ to $x = 3$:

$$\frac{g(3) - g(1)}{3 - 1}$$
$$= \frac{\left[(3)^3 - 2(3) + 1\right] - \left[(1)^3 - 2(1) + 1\right]}{2}$$
$$= \frac{(22) - (0)}{2} = \frac{22}{2} = 11$$

56. $h(x) = x^2 - 2x + 3$

a. Average rate of change of *h* from $x = -1$ to $x = 1$:

$$\frac{h(1) - h(-1)}{1 - (-1)}$$
$$= \frac{\left[(1)^2 - 2(1) + 3\right] - \left[(-1)^2 - 2(-1) + 3\right]}{2}$$
$$= \frac{(2) - (6)}{2} = \frac{-4}{2} = -2$$

b. Average rate of change of h from $x=0$ to $x=2$:

$$\frac{h(2)-h(0)}{2-0}$$
$$=\frac{\left[(2)^2-2(2)+3\right]-\left[(0)^2-2(0)+3\right]}{2}$$
$$=\frac{(3)-(3)}{2}=\frac{0}{2}=0$$

c. Average rate of change of h from $x=2$ to $x=5$:

$$\frac{h(5)-h(2)}{5-2}$$
$$=\frac{\left[(5)^2-2(5)+3\right]-\left[(2)^2-2(2)+3\right]}{3}$$
$$=\frac{(18)-(3)}{3}=\frac{15}{3}=5$$

57. $f(x)=5x-2$

a. Average rate of change of f from 1 to 3:

$$\frac{\Delta y}{\Delta x}=\frac{f(3)-f(1)}{3-1}=\frac{13-3}{3-1}=\frac{10}{2}=5$$

Thus, the average rate of change of f from 1 to 3 is 5.

b. From (a), the slope of the secant line joining $(1, f(1))$ and $(3, f(3))$ is 5. We use the point-slope form to find the equation of the secant line:

$$y-y_1=m_{\text{sec}}(x-x_1)$$
$$y-3=5(x-1)$$
$$y-3=5x-5$$
$$y=5x-2$$

58. $f(x)=-4x+1$

a. Average rate of change of f from 2 to 5:

$$\frac{\Delta y}{\Delta x}=\frac{f(5)-f(2)}{5-2}=\frac{-19-(-7)}{5-2}$$
$$=\frac{-12}{3}=-4$$

Therefore, the average rate of change of f from 2 to 5 is -4.

b. From (a), the slope of the secant line joining $(2, f(2))$ and $(5, f(5))$ is -4. We use the point-slope form to find the equation of the secant line:

$$y-y_1=m_{\text{sec}}(x-x_1)$$
$$y-(-7)=-4(x-2)$$
$$y+7=-4x+8$$
$$y=-4x+1$$

59. $g(x)=x^2-2$

a. Average rate of change of g from -2 to 1:

$$\frac{\Delta y}{\Delta x}=\frac{g(1)-g(-2)}{1-(-2)}=\frac{-1-2}{1-(-2)}=\frac{-3}{3}=-1$$

Therefore, the average rate of change of g from -2 to 1 is -1.

b. From (a), the slope of the secant line joining $(-2, g(-2))$ and $(1, g(1))$ is -1.We use the point-slope form to find the equation of the secant line:

$$y-y_1=m_{\text{sec}}(x-x_1)$$
$$y-2=-1(x-(-2))$$
$$y-2=-x-2$$
$$y=-x$$

60. $g(x)=x^2+1$

a. Average rate of change of g from -1 to 2:

$$\frac{\Delta y}{\Delta x}=\frac{g(2)-g(-1)}{2-(-1)}=\frac{5-2}{2-(-1)}=\frac{3}{3}=1$$

Therefore, the average rate of change of g from -1 to 2 is 1.

b. From (a), the slope of the secant line joining $(-1, g(-1))$ and $(2, g(2))$ is 1. We use the point-slope form to find the equation of the secant line:

$$y-y_1=m_{\text{sec}}(x-x_1)$$
$$y-2=1(x-(-1))$$
$$y-2=x+1$$
$$y=x+3$$

61. $h(x)=x^2-2x$

a. Average rate of change of h from 2 to 4:

$$\frac{\Delta y}{\Delta x}=\frac{h(4)-h(2)}{4-2}=\frac{8-0}{4-2}=\frac{8}{2}=4$$

Therefore, the average rate of change of h from 2 to 4 is 4.

b. From (a), the slope of the secant line joining $\left(2, h(2)\right)$ and $\left(4, h(4)\right)$ is 4. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 0 = 4(x - 2)$$
$$y = 4x - 8$$

62. $h(x) = -2x^2 + x$

a. Average rate of change from 0 to 3:

$$\frac{\Delta y}{\Delta x} = \frac{h(3) - h(0)}{3 - 0} = \frac{-15 - 0}{3 - 0}$$
$$= \frac{-15}{3} = -5$$

Therefore, the average rate of change of h from 0 to 3 is -5.

b. From (a), the slope of the secant line joining $\left(0, h(0)\right)$ and $\left(3, h(3)\right)$ is -5. We use the point-slope form to find the equation of the secant line:

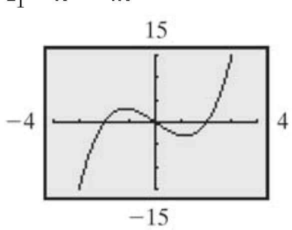
$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 0 = -5(x - 0)$$
$$y = -5x$$

63. a. $g(x) = x^3 - 4x$

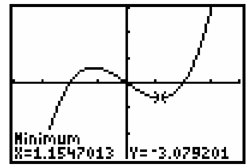
$$g(-x) = (-x)^3 - 4(-x)$$
$$= -x^3 + 4x$$
$$= -\left(x^3 - 4x\right)$$
$$= -g(x)$$

Since $g(-x) = -g(x)$, the function is odd.

b. $Y_1 = x^3 - 4x$



c. The local minimum is approximately -3.08 at $x \approx 1.15$.



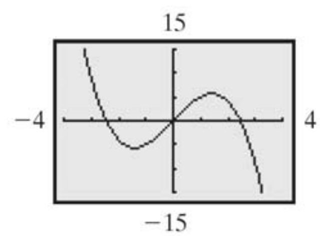
d. Since the function is odd, its graph has origin symmetry. The local maximum is approximately 3.08 at $x \approx -1.15$.

64. $f(x) = -x^3 + 6x$

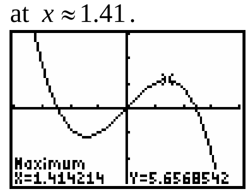
a.
$$f(-x) = -(-x)^3 + 6(-x)$$
$$= x^3 - 6x$$
$$= -\left(-x^3 + 6x\right)$$
$$= -f(x)$$

Since $f(-x) = -f(x)$, the function is odd.

b. $Y_1 = -x^3 + 6x$



c. The local maximum is approximately 5.66 at $x \approx 1.41$.



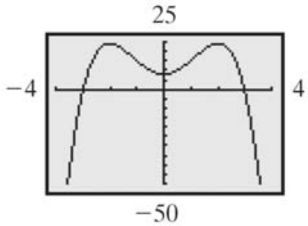
d. Since the function is odd, its graph has origin symmetry. The local minimum is approximately -5.66 at $x \approx -1.41$.

65. $F(x) = -x^4 + 8x^2 + 8$

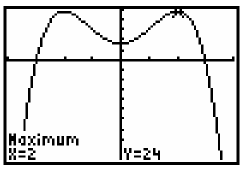
a.
$$F(-x) = -(-x)^4 + 8(-x)^2 + 8$$
$$= -x^4 + 8x + 8$$
$$= F(x)$$

Since $F(-x) = F(x)$, the function is even.

b. $Y_1 = -x^4 + 8x^2 + 8$



c. The local maximum is 24 at $x = 2$.



d. Since the function is even, its graph has y-axis symmetry. The local maximum in quadrant II is 24 at $x = -2$.

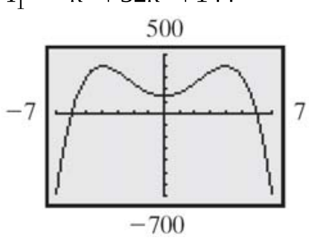
e. Because the graph has y-axis symmetry, the area under the graph between $x = 0$ and $x = 3$ bounded below by the x-axis is the same as the area under the graph between $x = -3$ and $x = 0$ bounded below the x-axis. Thus, the area is 47.4 square units.

66. $G(x) = -x^4 + 32x^2 + 144$

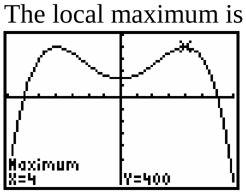
a.
$$\begin{aligned} G(-x) &= -(-x)^4 + 32(-x)^2 + 144 \\ &= -x^4 + 32x^2 + 144 \\ &= G(x) \end{aligned}$$

Since $G(-x) = G(x)$, the function is even.

b. $Y_1 = -x^4 + 32x^2 + 144$



c. The local maximum is 400 at $x = 4$.

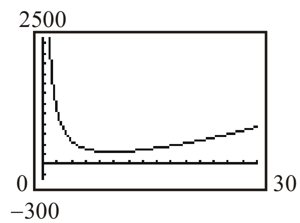


d. Since the function is even, its graph has y-axis symmetry. The local maximum in quadrant II is 400 at $x = -4$.

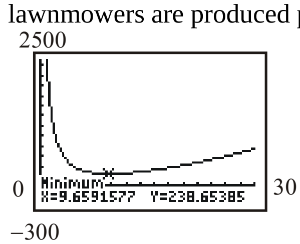
e. Because the graph has y-axis symmetry, the area under the graph between $x = 0$ and $x = 6$ bounded below by the x-axis is the same as the area under the graph between $x = -6$ and $x = 0$ bounded below the x-axis. Thus, the area is 1612.8 square units.

67. $\overline{C}(x) = 0.3x^2 + 21x - 251 + \dfrac{2500}{x}$

a. $y_1 = 0.3x^2 + 21x - 251 + \dfrac{2500}{x}$



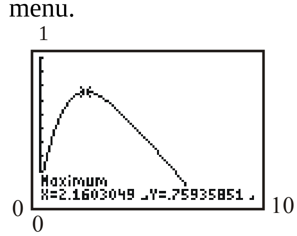
b. Use MINIMUM. Rounding to the nearest whole number, the average cost is minimized when approximately 10 lawnmowers are produced per hour.



c. The minimum average cost is approximately \$239 per mower.

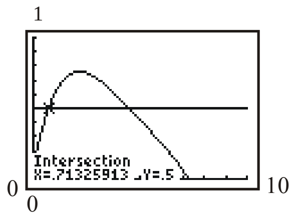
68. a. $C(t) = -.002t^4 + .039t^3 - .285t^2 + .766t + .085$

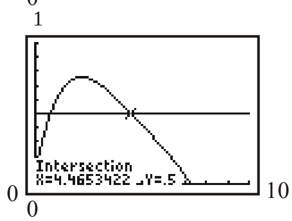
Graph the function on a graphing utility and use the Maximum option from the CALC menu.



The concentration will be highest after about 2.16 hours.

b. Enter the function in Y1 and 0.5 in Y2. Graph the two equations in the same window and use the Intersect option from the CALC menu.





After taking the medication, the woman can feed her child within the first 0.71 hours (about 42 minutes) or after 4.47 hours (about 4hours 28 minutes) have elapsed.

69. a.
$$\text{avg. rate of change} = \frac{P(2.5)-P(0)}{2.5-0} = \frac{0.18-0.09}{2.5-0} = \frac{0.09}{2.5} = 0.036 \text{ gram per hour}$$

On overage, the population is increasing at a rate of 0.036 gram per hour from 0 to 2.5 hours.

b.
$$\text{avg. rate of change} = \frac{P(6)-P(4.5)}{6-4.5} = \frac{0.50-0.35}{6-4.5} = \frac{0.15}{1.5} = 0.1 \text{ gram per hour}$$

On overage, the population is increasing at a rate of 0.1 gram per hour from 4.5 to 6 hours.

c. The average rate of change is increasing as time passes. This indicates that the population is increasing at an increasing rate.

70. a.
$$\text{avg. rate of change} = \frac{P(2000)-P(1998)}{2000-1998} = \frac{27.6-20.7}{2000-1998} = \frac{6.9}{2} = 3.45 \text{ percentage points per year}$$

On overage, the percentage of returns that are e-filed is increasing at a rate of 3.45 percentage points per year from 1998 to 2000.

b.
$$\text{avg. rate of change} = \frac{P(2003)-P(2001)}{2003-2001} = \frac{40.2-30.7}{2003-2001} = \frac{9.5}{2} = 4.75 \text{ percentage points per year}$$

On overage, the percentage of returns that are e-filed is increasing at a rate of 4.75 percentage points per year from 2001 to 2003.

c.
$$\text{avg. rate of change} = \frac{P(2006)-P(2004)}{2006-2004} = \frac{57.1-46.5}{2006-2004} = \frac{10.6}{2} = 5.3 \text{ percentage points per year}$$

On overage, the percentage of returns that are e-filed is increasing at a rate of 5.3 percentage points per year from 2004 to 2006.

d. The average rate of change is increasing as time passes. This indicates that the percentage of e-filers is increasing at an increasing rate.

71. $f(x) = x^2$

a. Average rate of change of *f* from $x = 0$ to $x = 1$:
$$\frac{f(1)-f(0)}{1-0} = \frac{1^2-0^2}{1} = \frac{1}{1} = 1$$

b. Average rate of change of *f* from $x=0$ to $x=0.5$:

$$\frac{f(0.5)-f(0)}{0.5-0}=\frac{(0.5)^2-0^2}{0.5}=\frac{0.25}{0.5}=0.5$$

c. Average rate of change of *f* from $x=0$ to $x=0.1$:

$$\frac{f(0.1)-f(0)}{0.1-0}=\frac{(0.1)^2-0^2}{0.1}=\frac{0.01}{0.1}=0.1$$

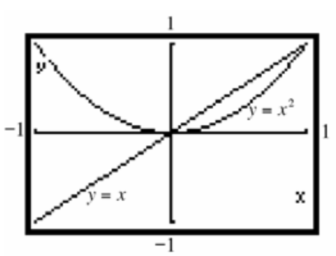
d. Average rate of change of *f* from $x=0$ to $x=0.01$:

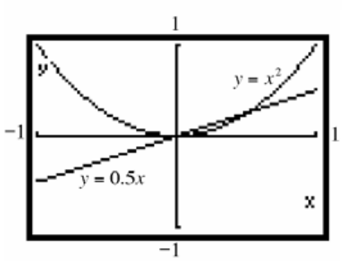
$$\frac{f(0.01)-f(0)}{0.01-0}=\frac{(0.01)^2-0^2}{0.01}$$
$$=\frac{0.0001}{0.01}=0.01$$

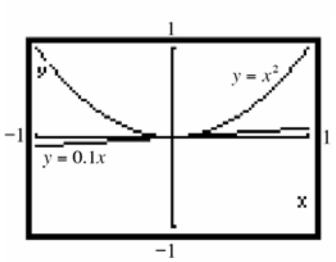
e. Average rate of change of *f* from $x=0$ to $x=0.001$:

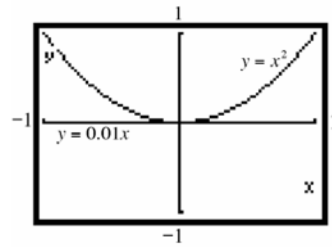
$$\frac{f(0.001)-f(0)}{0.001-0}=\frac{(0.001)^2-0^2}{0.001}$$
$$=\frac{0.000001}{0.001}=0.001$$

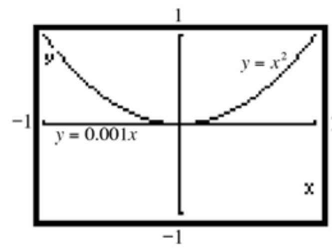
f. Graphing the secant lines:











g. The secant lines are beginning to look more and more like the tangent line to the graph of *f* at the point where $x=0$.

h. The slopes of the secant lines are getting smaller and smaller. They seem to be approaching the number zero.

72. $f(x)=x^2$

a. Average rate of change of *f* from $x=1$ to $x=2$:

$$\frac{f(2)-f(1)}{2-1}=\frac{2^2-1^2}{1}=\frac{3}{1}=3$$

b. Average rate of change of *f* from $x=1$ to $x=1.5$:

$$\frac{f(1.5)-f(1)}{1.5-1}=\frac{(1.5)^2-1^2}{0.5}=\frac{1.25}{0.5}=2.5$$

c. Average rate of change of *f* from $x=1$ to $x=1.1$:

$$\frac{f(1.1)-f(1)}{1.1-1}=\frac{(1.1)^2-1^2}{0.1}=\frac{0.21}{0.1}=2.1$$

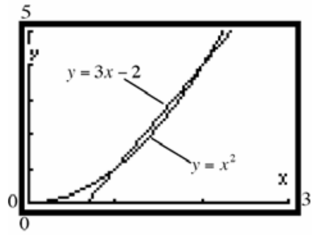
d. Average rate of change of *f* from $x=1$ to $x=1.01$:

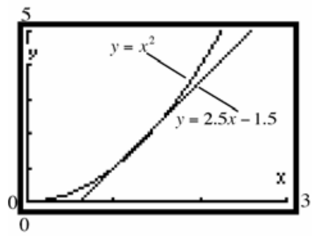
$$\frac{f(1.01)-f(1)}{1.01-1}=\frac{(1.01)^2-1^2}{0.01}=\frac{0.0201}{0.01}=2.01$$

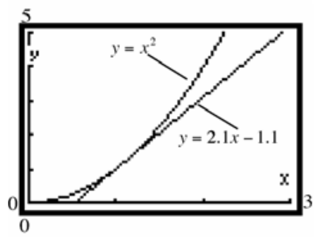
e. Average rate of change of *f* from $x=1$ to $x=1.001$:

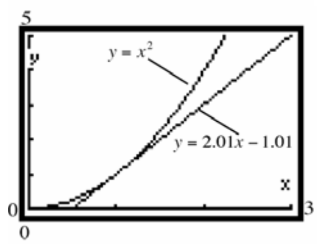
$$\frac{f(1.001)-f(1)}{1.001-1}=\frac{(1.001)^2-1^2}{0.001}$$
$$=\frac{0.002001}{0.001}=2.001$$

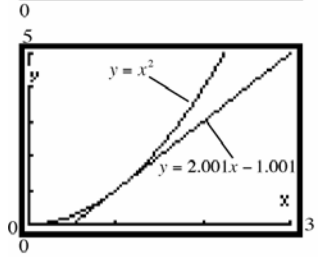
f. Graphing the secant lines:











g. The secant lines are beginning to look more and more like the tangent line to the graph of *f* at the point where $x = 1$.

h. The slopes of the secant lines are getting smaller and smaller. They seem to be approaching the number 2.

73. $f(x) = 2x + 5$

a. $m_{\text{sec}} = \dfrac{f(x+h) - f(x)}{h}$

$$= \frac{2(x+h)+5-2x-5}{h} = \frac{2h}{h} = 2$$

b. When $x = 1$:

$h = 0.5 \Rightarrow m_{\text{sec}} = 2$

$h = 0.1 \Rightarrow m_{\text{sec}} = 2$

$h = 0.01 \Rightarrow m_{\text{sec}} = 2$

as $h \to 0$, $m_{\text{sec}} \to 2$

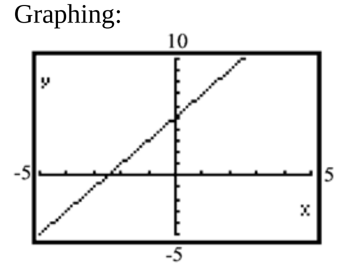
c. Using the point $(1, f(1)) = (1, 7)$ and slope, $m = 2$, we get the secant line:

$y - 7 = 2(x - 1)$

$y - 7 = 2x - 2$

$y = 2x + 5$

d. Graphing:



The graph and the secant line coincide.

74. $f(x) = -3x + 2$

a. $m_{\text{sec}} = \dfrac{f(x+h) - f(x)}{h}$

$$= \frac{-3(x+h)+2-(-3x+2)}{h} = \frac{-3h}{h} = -3$$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = -3$

$h = 0.1 \Rightarrow m_{\text{sec}} = -3$

$h = 0.01 \Rightarrow m_{\text{sec}} = -3$

as $h \to 0$, $m_{\text{sec}} \to -3$

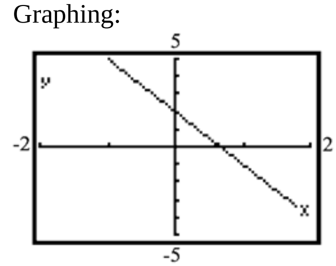
c. Using point $(1, f(1)) = (1, -1)$ and slope = -3, we get the secant line:

$y - (-1) = -3(x - 1)$

$y + 1 = -3x + 3$

$y = -3x + 2$

d. Graphing:



The graph and the secant line coincide.

75. $f(x) = x^2 + 2x$

a. $$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h} = \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h} = \frac{2xh + h^2 + 2h}{h} = 2x + h + 2$$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = 2 \cdot 1 + 0.5 + 2 = 4.5$

$h = 0.1 \Rightarrow m_{\text{sec}} = 2 \cdot 1 + 0.1 + 2 = 4.1$

$h = 0.01 \Rightarrow m_{\text{sec}} = 2 \cdot 1 + 0.01 + 2 = 4.01$

as $h \to 0$, $m_{\text{sec}} \to 2 \cdot 1 + 0 + 2 = 4$

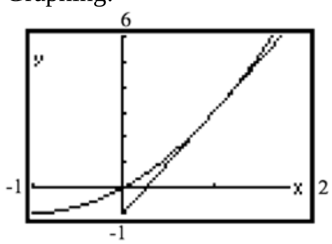
c. Using point $(1, f(1)) = (1, 3)$ and slope = 4.01, we get the secant line:

$y - 3 = 4.01(x - 1)$

$y - 3 = 4.01x - 4.01$

$y = 4.01x - 1.01$

d. Graphing:



76. $f(x) = 2x^2 + x$

a. $$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 + (x+h) - (2x^2 + x)}{h} = \frac{2(x^2 + 2xh + h^2) + x + h - 2x^2 - x}{h} = \frac{2x^2 + 4xh + 2h^2 + x + h - 2x^2 - x}{h} = \frac{4xh + 2h^2 + h}{h} = 4x + 2h + 1$$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.5) + 1 = 6$

$h = 0.1 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.1) + 1 = 5.2$

$h = 0.01 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.01) + 1 = 5.02$

as $h \to 0$, $m_{\text{sec}} \to 4 \cdot 1 + 2(0) + 1 = 5$

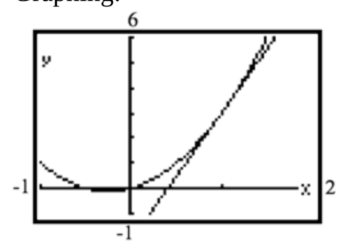
c. Using point $(1, f(1)) = (1, 3)$ and slope = 5.02, we get the secant line:

$y - 3 = 5.02(x - 1)$

$y - 3 = 5.02x - 5.02$

$y = 5.02x - 2.02$

d. Graphing:



77. $f(x) = 2x^2 - 3x + 1$

a. $$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h} = \frac{2(x^2 + 2xh + h^2) - 3x - 3h + 1 - 2x^2 + 3x - 1}{h} = \frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 1 - 2x^2 + 3x - 1}{h} = \frac{4xh + 2h^2 - 3h}{h} = 4x + 2h - 3$$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.5) - 3 = 2$

$h = 0.1 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.1) - 3 = 1.2$

$h = 0.01 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.01) - 3 = 1.02$

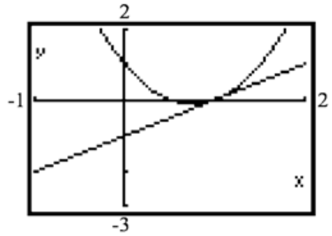
as $h \to 0$, $m_{\text{sec}} \to 4 \cdot 1 + 2(0) - 3 = 1$

c. Using point $(1, f(1)) = (1, 0)$ and slope = 1.02, we get the secant line:

$y - 0 = 1.02(x - 1)$

$y = 1.02x - 1.02$

d. Graphing:



78. $f(x) = -x^2 + 3x - 2$

a. $$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h}$$
$$= \frac{-(x+h)^2 + 3(x+h) - 2 - \left(-x^2 + 3x - 2\right)}{h}$$
$$= \frac{-(x^2 + 2xh + h^2) + 3x + 3h - 2 + x^2 - 3x + 2}{h}$$
$$= \frac{-x^2 - 2xh - h^2 + 3x + 3h - 2 + x^2 - 3x + 2}{h}$$
$$= \frac{-2xh - h^2 + 3h}{h}$$
$$= -2x - h + 3$$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = -2 \cdot 1 - 0.5 + 3 = 0.5$

$h = 0.1 \Rightarrow m_{\text{sec}} = -2 \cdot 1 - 0.1 + 3 = 0.9$

$h = 0.01 \Rightarrow m_{\text{sec}} = -2 \cdot 1 - 0.01 + 3 = 0.99$

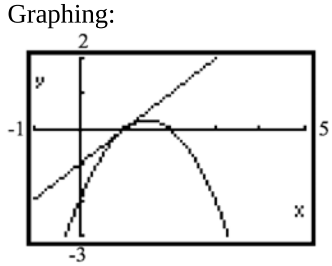
as $h \to 0$, $m_{\text{sec}} \to -2 \cdot 1 - 0 + 3 = 1$

c. Using point $\left(1, f(1)\right) = (1, 0)$ and slope = 0.99, we get the secant line:

$y - 0 = 0.99(x - 1)$

$y = 0.99x - 0.99$

d. Graphing:



79. $f(x) = \dfrac{1}{x}$

a. $$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h}$$
$$= \frac{\left(\frac{1}{x+h} - \frac{1}{x}\right)}{h} = \frac{\left(\frac{x - (x+h)}{(x+h)x}\right)}{h}$$
$$= \left(\frac{x - x - h}{(x+h)x}\right)\left(\frac{1}{h}\right) = \left(\frac{-h}{(x+h)x}\right)\left(\frac{1}{h}\right)$$
$$= -\frac{1}{(x+h)x}$$

b. When $x = 1$,

$$h = 0.5 \Rightarrow m_{\text{sec}} = -\frac{1}{(1+0.5)(1)} = -\frac{1}{1.5} = -\frac{2}{3} \approx -0.667$$

$$h = 0.1 \Rightarrow m_{\text{sec}} = -\frac{1}{(1+0.1)(1)} = -\frac{1}{1.1} = -\frac{10}{11} \approx -0.909$$

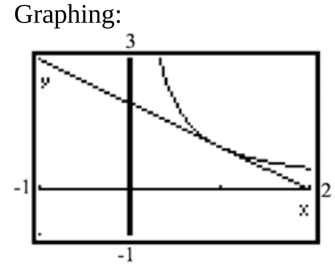
$$h = 0.01 \Rightarrow m_{\text{sec}} = -\frac{1}{(1+0.01)(1)} = -\frac{1}{1.01} = -\frac{100}{101} \approx -0.990$$

$$\text{as } h \to 0, \quad m_{\text{sec}} \to -\frac{1}{(1+0)(1)} = -\frac{1}{1} = -1$$

c. Using point $\left(1, f(1)\right) = (1, 1)$ and slope = $-\dfrac{100}{101}$, we get the secant line:

$$y - 1 = -\frac{100}{101}(x - 1)$$
$$y - 1 = -\frac{100}{101}x + \frac{100}{101}$$
$$y = -\frac{100}{101}x + \frac{201}{101}$$

d. Graphing:



80. $f(x) = \dfrac{1}{x^2}$

a. $$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h} = \frac{\left(\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}\right)}{h} = \frac{\left(\dfrac{x^2 - (x+h)^2}{(x+h)^2 x^2}\right)}{h} = \left(\frac{x^2 - \left(x^2 + 2xh + h^2\right)}{(x+h)^2 x^2}\right)\left(\frac{1}{h}\right) = \left(\frac{-2xh - h^2}{(x+h)^2 x^2}\right)\left(\frac{1}{h}\right) = \frac{-2x-h}{(x+h)^2 x^2} = \frac{-2x-h}{\left(x^2+2xh+h^2\right)x^2}$$

b. When $x = 1$,

$$h = 0.5 \Rightarrow m_{\text{sec}} = \frac{-2\cdot 1 - 0.5}{(1+0.5)^2 1^2} = -\frac{10}{9} \approx -1.1111$$

$$h = 0.1 \Rightarrow m_{\text{sec}} = \frac{-2\cdot 1 - 0.1}{(1+0.1)^2 1^2} = -\frac{210}{121} \approx -1.7355$$

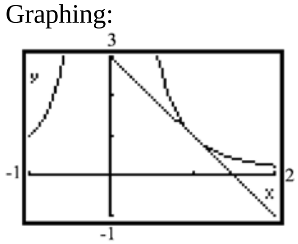
$$h = 0.01 \Rightarrow m_{\text{sec}} = \frac{-2\cdot 1 - 0.01}{(1+0.01)^2 1^2} = -\frac{20{,}100}{10{,}201} \approx -1.9704$$

as $h \to 0$, $m_{\text{sec}} \to \dfrac{-2\cdot 1 - 0}{(1+0)^2 1^2} = -2$

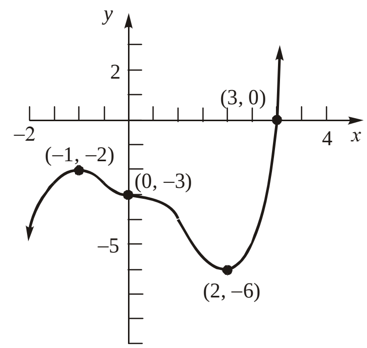
c. Using point $(1, f(1)) = (1,1)$ and slope $= -1.9704$, we get the secant line:

$$y - 1 = -1.9704(x-1)$$
$$y - 1 = -1.9704x + 1.9704$$
$$y = -1.9704x + 2.9704$$

d. Graphing:



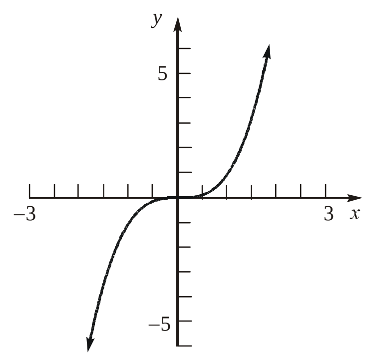
81. Answers will vary. One possibility follows:



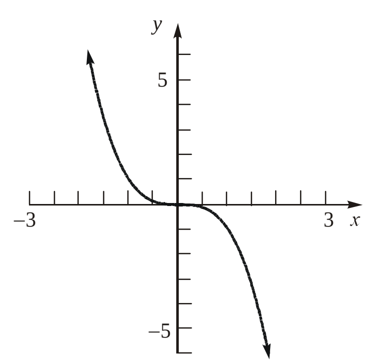
82. Answers will vary. See solution to Problem 81 for one possibility.

83. A function that is increasing on an interval can have at most one x-intercept on the interval. The graph of f could not "turn" and cross it again or it would start to decrease.

84. An increasing function is a function whose graph goes up as you read from left to right.

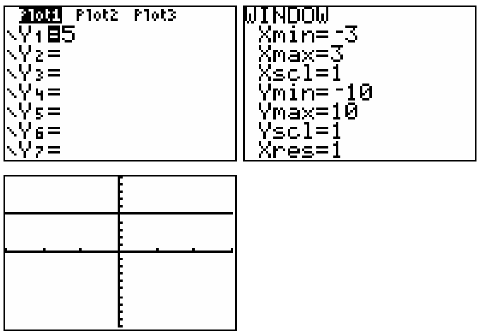


A decreasing function is a function whose graph goes down as you read from left to right.



85. To be an even function we need $f(-x)=f(x)$ and to be an odd function we need $f(-x)=-f(x)$. In order for a function be both even and odd, we would need $f(x)=-f(x)$. This is only possible if $f(x)=0$.

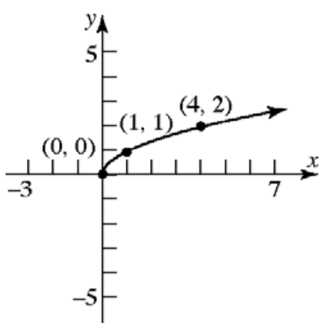
86. The graph of $y=5$ is a horizontal line.



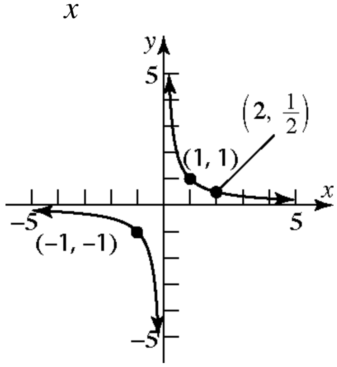
The local maximum is $y=5$ and it occurs at each x-value in the interval.

Section 2.4

1. $y=\sqrt{x}$



2. $y=\dfrac{1}{x}$



3. $y=x^3-8$

<u>y-intercept:</u>
Let $x=0$, then $y=(0)^3-8=-8$.

<u>x-intercept:</u>
Let $y=0$, then $0=x^3-8$

$$x^3=8$$
$$x=2$$

The intercepts are $(0,-8)$ and $(2,0)$.

4. $(-\infty,0)$

5. piecewise-defined

6. True

7. False; the cube root function is odd and increasing on the interval $(-\infty,\infty)$.

8. False; the domain and range of the reciprocal function are both the set of real numbers except for 0.

9. C

10. A

11. E

12. G

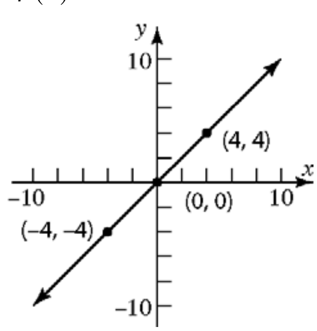
13. B

14. D

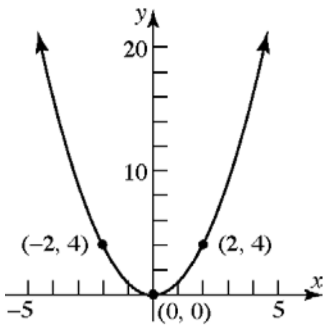
15. F

16. H

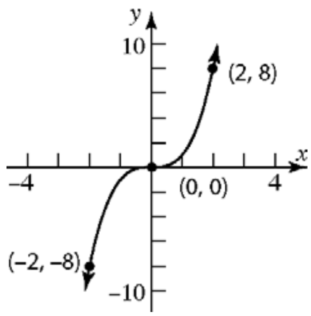
17. $f(x)=x$



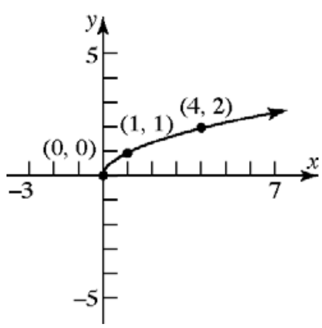
18. $f(x) = x^2$



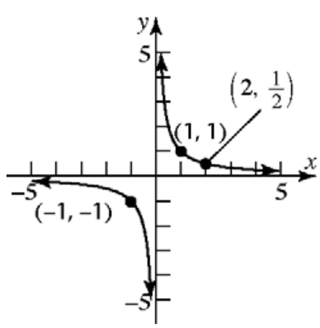
19. $f(x) = x^3$



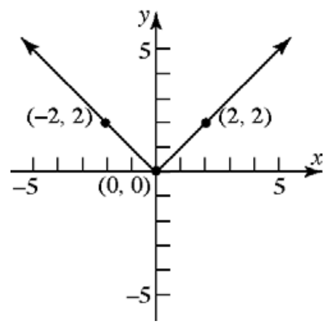
20. $f(x) = \sqrt{x}$



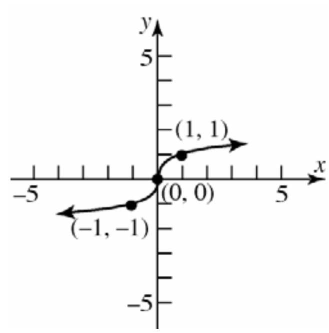
21. $f(x) = \dfrac{1}{x}$



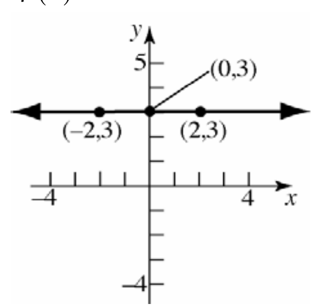
22. $f(x) = |x|$



23. $f(x) = \sqrt[3]{x}$



24. $f(x) = 3$



25. **a.** $f(-2) = (-2)^2 = 4$

b. $f(0) = 2$

c. $f(2) = 2(2) + 1 = 5$

26. **a.** $f(-2) = -3(-2) = 6$

b. $f(-1) = 0$

c. $f(0) = 2(0)^2 + 1 = 1$

27. **a.** $f(0) = 2(0) - 4 = -4$

b. $f(1) = 2(1) - 4 = -2$

c. $f(2) = 2(2) - 4 = 0$

d. $f(3) = (3)^3 - 2 = 25$

28. **a.** $f(-1) = (-1)^3 = -1$

b. $f(0) = (0)^3 = 0$

c. $f(1) = 3(1) + 2 = 5$

d. $f(3) = 3(3) + 2 = 11$

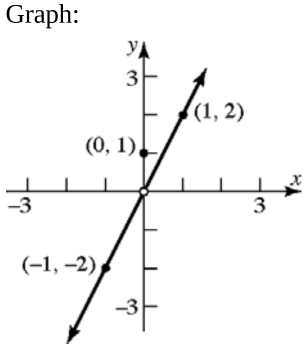
29. $f(x) = \begin{cases} 2x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercept: none
y-intercept:
$f(0) = 1$

The only intercept is $(0,1)$.

c. Graph:



d. Range: $\{y \mid y \neq 0\}$; $(-\infty, 0) \cup (0, \infty)$

e. The graph is not continuous. There is a jump at $x = 0$.

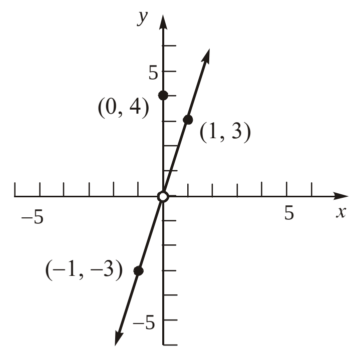
30. $f(x) = \begin{cases} 3x & \text{if } x \neq 0 \\ 4 & \text{if } x = 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercept: none
y-intercept: $f(0) = 4$

The only intercept is $(0,4)$.

c. Graph:



d. Range: $\{y \mid y \neq 0\}$; $(-\infty, 0) \cup (0, \infty)$

e. The graph is not continuous. There is a jump at $x = 0$.

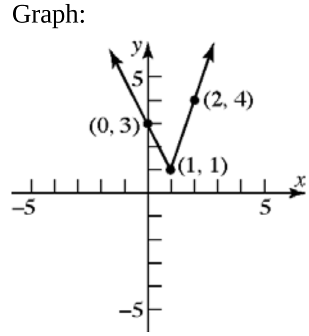
31. $f(x) = \begin{cases} -2x + 3 & \text{if } x < 1 \\ 3x - 2 & \text{if } x \geq 1 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercept: none
y-intercept: $f(0) = -2(0) + 3 = 3$

The only intercept is $(0,3)$.

c. Graph:



d. Range: $\{y \mid y \geq 1\}$; $[1, \infty)$

e. The graph is continuous. There are no holes or gaps.

32. $f(x)=\begin{cases} x+3 & \text{if } x<-2 \\ -2x-3 & \text{if } x\ge -2 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. $x+3=0$ $\quad$ $-2x-3=0$

$x=-3$ $\quad$ $-2x=3$

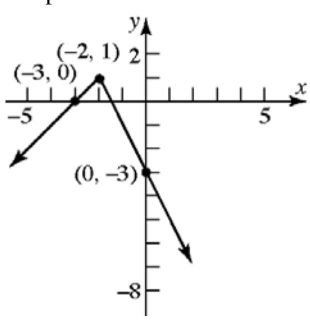
$x=-\frac{3}{2}$

x-intercepts: $-3, -\frac{3}{2}$

y-intercept: $f(0)=-2(0)-3=-3$

The intercepts are $(-3,0)$, $\left(-\frac{3}{2},0\right)$, and $(0,-3)$.

c. Graph:



d. Range: $\{y \mid y\le 1\}$; $(-\infty,1]$

e. The graph is continuous. There are no holes or gaps.

33. $f(x)=\begin{cases} x+3 & \text{if } -2\le x<1 \\ 5 & \text{if } x=1 \\ -x+2 & \text{if } x>1 \end{cases}$

a. Domain: $\{x \mid x\ge -2\}$; $[-2,\infty)$

b. $x+3=0$ $\quad$ $-x+2=0$

$x=-3$ $\quad$ $-x=-2$

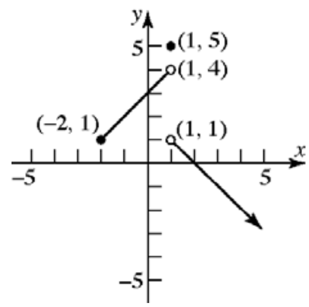
(not in domain) $\quad$ $x=2$

x-intercept: 2

y-intercept: $f(0)=0+3=3$

The intercepts are $(2,0)$ and $(0,3)$.

c. Graph:



d. Range: $\{y \mid y<4,\ y=5\}$; $(-\infty,4)\cup\{5\}$

e. The graph is not continuous. There is a jump at $x=1$.

34. $f(x)=\begin{cases} 2x+5 & \text{if } -3\le x<0 \\ -3 & \text{if } x=0 \\ -5x & \text{if } x>0 \end{cases}$

a. Domain: $\{x \mid x\ge -3\}$; $[-3,\infty)$

b. $2x+5=0$ $\quad$ $-5x=0$

$2x=-5$ $\quad$ $x=0$

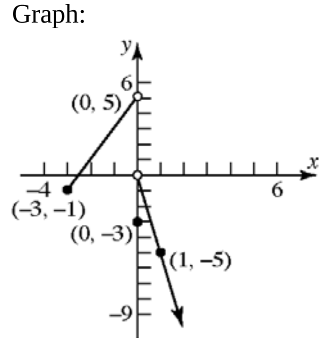
$x=-\frac{5}{2}$ $\quad$ (not in domain of piece)

x-intercept: $-\frac{5}{2}$

y-intercept: $f(0)=-3$

The intercepts are $\left(-\frac{5}{2},0\right)$ and $(0,-3)$.

c. Graph:



d. Range: $\{y \mid y<5\}$; $(-\infty,5)$

e. The graph is not continuous. There is a jump at $x=0$.

35. $f(x)=\begin{cases}1+x & \text{if } x<0\\ x^2 & \text{if } x\ge 0\end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. $1+x=0 \qquad x^2=0$

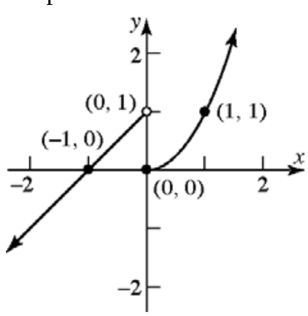
$x=-1 \qquad x=0$

x-intercepts: $-1, 0$

y-intercept: $f(0)=0^2=0$

The intercepts are $(-1,0)$ and $(0,0)$.

c. Graph:



d. Range: $\{y \mid y \text{ is any real number}\}$

e. The graph is not continuous. There is a jump at $x=0$.

36. $f(x)=\begin{cases}\dfrac{1}{x} & \text{if } x<0\\ \sqrt[3]{x} & \text{if } x\ge 0\end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. $\dfrac{1}{x}=0 \qquad \sqrt[3]{x}=0$

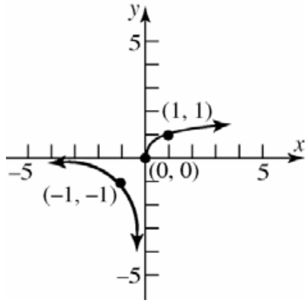
(no solution) $\qquad x=0$

x-intercept: 0

y-intercept: $f(0)=\sqrt[3]{0}=0$

The only intercept is $(0,0)$.

c. Graph:



d. Range: $\{y \mid y \text{ is any real number}\}$

e. The graph is not continuous. There is a break at $x=0$.

37. $f(x)=\begin{cases}|x| & \text{if } -2\le x<0\\ x^3 & \text{if } x>0\end{cases}$

a. Domain: $\{x \mid -2\le x<0 \text{ and } x>0\}$ or $\{x \mid x\ge -2, x\ne 0\}$; $[-2,0)\cup(0,\infty)$.

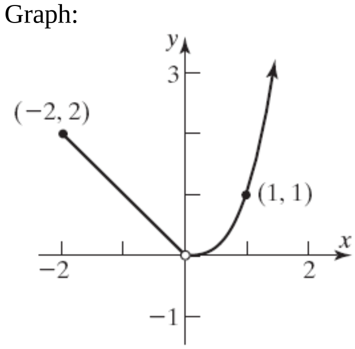
b. x-intercept: none

There are no x-intercepts since there are no values for x such that $f(x)=0$.

y-intercept:

There is no y-intercept since $x=0$ is not in the domain.

c. Graph:



d. Range: $\{y \mid y>0\}$; $(0,\infty)$

e. The graph is not continuous. There is a hole at $x=0$.

38. $f(x)=\begin{cases}2-x & \text{if } -3\le x<1\\ \sqrt{x} & \text{if } x>1\end{cases}$

a. Domain: $\{x \mid -3\le x<1 \text{ and } x>1\}$ or $\{x \mid x\ge -3, x\ne 1\}$; $[-3,1)\cup(1,\infty)$.

b. $2-x=0 \qquad \sqrt{x}=0$

$x=2 \qquad x=0$

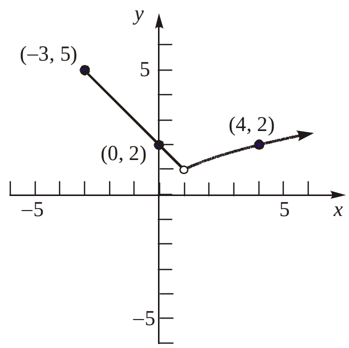
(not in domain of piece)

no x-intercepts

y-intercept: $f(0)=2-0=2$

The intercept is $(0,2)$.

c. Graph:



d. Range: $\{y \mid y > 1\}$; $(1, \infty)$

e. The graph is not continuous. There is a hole at $x = 1$.

39. $f(x) = 2\,\text{int}(x)$

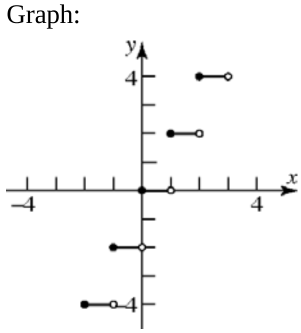
a. Domain: $\{x \mid x \text{ is any real number}\}$

b. *x*-intercepts:
All values for *x* such that $0 \le x < 1$.

y-intercept: $f(0) = 2\,\text{int}(0) = 0$

The intercepts are all ordered pairs $(x, 0)$ when $0 \le x < 1$.

c. Graph:



d. Range: $\{y \mid y \text{ is an even integer}\}$

e. The graph is not continuous. There is a jump at each integer value of *x*.

40. $f(x) = \text{int}(2x)$

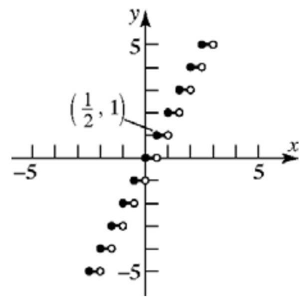
a. Domain: $\{x \mid x \text{ is any real number}\}$

b. *x*-intercepts:
All values for *x* such that $0 \le x < \frac{1}{2}$.

y-intercept: $f(0) = \text{int}(2(0)) = \text{int}(0) = 0$

The intercepts are all ordered pairs $(x, 0)$ when $0 \le x < \frac{1}{2}$.

c. Graph:



d. Range: $\{y \mid y \text{ is an integer}\}$

e. The graph is not continuous. There is a jump at each $x = \dfrac{k}{2}$, where *k* is an integer.

41. Answers may vary. One possibility follows:

$$f(x) = \begin{cases} -x & \text{if } -1 \le x \le 0 \\ \frac{1}{2}x & \text{if } 0 < x \le 2 \end{cases}$$

42. Answers may vary. One possibility follows:

$$f(x) = \begin{cases} x & \text{if } -1 \le x \le 0 \\ 1 & \text{if } 0 < x \le 2 \end{cases}$$

43. Answers may vary. One possibility follows:

$$f(x) = \begin{cases} -x & \text{if } x \le 0 \\ -x + 2 & \text{if } 0 < x \le 2 \end{cases}$$

44. Answers may vary. One possibility follows:

$$f(x) = \begin{cases} 2x + 2 & \text{if } -1 \le x \le 0 \\ x & \text{if } x > 0 \end{cases}$$

45. **a.** $f(1.2) = \text{int}(2(1.2)) = \text{int}(2.4) = 2$

b. $f(1.6) = \text{int}(2(1.6)) = \text{int}(3.2) = 3$

c. $f(-1.8) = \text{int}(2(-1.8)) = \text{int}(-3.6) = -4$

46. **a.** $f(1.2) = \text{int}\left(\dfrac{1.2}{2}\right) = \text{int}(0.6) = 0$

b. $f(1.6) = \text{int}\left(\dfrac{1.6}{2}\right) = \text{int}(0.8) = 0$

c. $f(-1.8) = \text{int}\left(\dfrac{-1.8}{2}\right) = \text{int}(-0.9) = -1$

47. $C = \begin{cases} 39.99 & \text{if } 0 < x \le 450 \\ 0.45x - 162.51 & \text{if } x > 450 \end{cases}$

a. $C(200) = \$39.99$

b. $C(465) = 0.45(465) - 162.51 = \$46.74$

c. $C(451) = 0.45(451) - 162.51 = \$40.44$

48. $F(x) = \begin{cases} 3 & \text{if } 0 < x \le 3 \\ 5\,\text{int}(x+1)+1 & \text{if } 3 < x < 9 \\ 50 & \text{if } 9 \le x \le 24 \end{cases}$

a. $F(2) = 3$
Parking for 2 hours costs \$3.

b. $F(7) = 5\,\text{int}(7+1)+1 = 41$
Parking for 7 hours costs \$41.

c. $F(15) = 50$
Parking for 15 hours costs \$50.

d. $24 \text{ min} \cdot \dfrac{1 \text{ hr}}{60 \text{ min}} = 0.4 \text{ hr}$
$F(8.4) = 5\,\text{int}(8.4+1)+1 = 5(9)+1 = 46$
Parking for 8 hours and 24 minutes costs \$46.

49. a. Charge for 50 therms:
$C = 9.45 + 0.8738(50) + 0.36375(50)$
$= \$71.33$

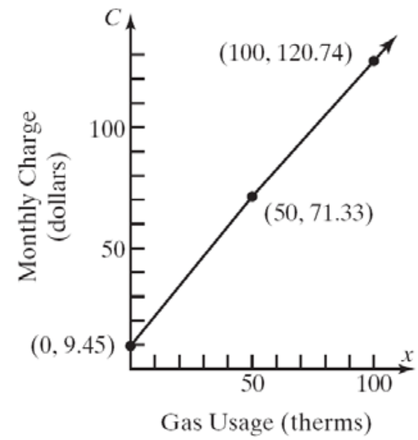
b. Charge for 500 therms:
$C = 9.45 + 0.36375(50) + 0.11445(450)$
$+ 0.8738(500)$
$= \$516.04$

c. For $0 \le x \le 50$:
$C = 9.45 + 0.36375x + 0.8738x$
$= 1.23755x + 9.45$

For $x > 50$:
$C = 9.45 + 0.36375(50) + 0.11445(x-50)$
$+ 0.8738x$
$= 9.45 + 18.1875 + 0.11445x - 5.7225$
$+ 0.8738x$
$= 0.98825x + 21.915$

The monthly charge function:
$C = \begin{cases} 1.23755x + 9.45 & \text{for } 0 \le x \le 50 \\ 0.98825x + 21.915 & \text{for } x > 50 \end{cases}$

d. Graph:



50. a. Charge for 40 therms:
$C = 8.85 + 0.1473(20) + 0.0579(20)$
$+ 0.67(40)$
$= \$39.75$

b. Charge for 202 therms:
$C = 8.85 + 0.1473(20) + 0.0579(30)$
$+ 0.0519(152) + 0.67(202)$
$= \$156.76$

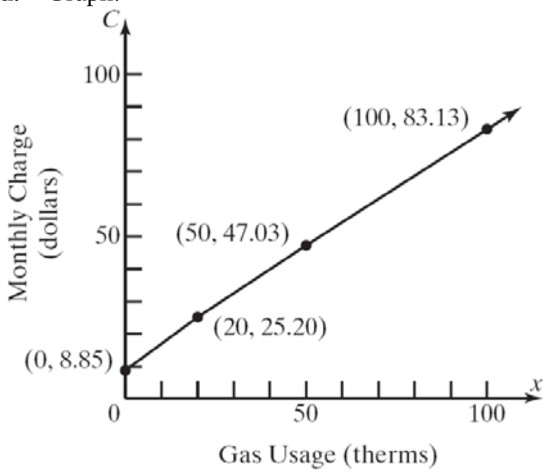
c. For $0 \le x \le 20$:
$C = 8.85 + 0.1473x + 0.67x$
$= 0.8173x + 8.85$

For $20 < x \le 50$:
$C = 8.85 + 0.1473(20) + 0.0579(x-20)$
$+ 0.67x$
$= 8.85 + 2.946 + 0.0579x - 1.158$
$+ 0.67x$
$= 0.7279x + 10.638$

For $x > 50$:
$C = 8.85 + 0.1473(20) + 0.0579(30)$
$+ 0.0519(x-50) + 0.67x$
$= 8.85 + 2.946 + 1.737 + 0.0519x - 2.595$
$+ 0.67x$
$= 0.7219x + 10.938$

The monthly charge function:
$C(x) = \begin{cases} 0.8173x + 8.85 & \text{if } 0 \le x \le 20 \\ 0.7279x + 10.638 & \text{if } 20 < x \le 50 \\ 0.7219x + 10.938 & \text{if } x > 50 \end{cases}$

d. Graph:



51. For schedule X:

$$f(x)=\begin{cases}0.10x & \text{if } 0<x\le 7825\\ 782.50+0.15(x-7825) & \text{if } 7825<x\le 31,850\\ 4386.25+0.25(x-31,850) & \text{if } 31,850<x\le 77,100\\ 15,698.75+0.28(x-77,100) & \text{if } 77,100<x\le 160,850\\ 39,148.75+0.33(x-160,850) & \text{if } 160,850<x\le 349,700\\ 101,469.25+0.35(x-349,700) & \text{if } x>349,700\end{cases}$$

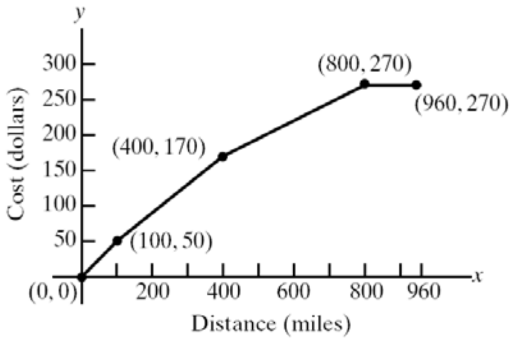
52. For Schedule $\text{Y}-1$:

$$f(x)=\begin{cases}0.10x & \text{if } 0<x\le 15,650\\ 1565.00+0.15(x-15,650) & \text{if } 15,650<x\le 63,700\\ 8772.50+0.25(x-63,700) & \text{if } 63,700<x\le 128,500\\ 24,972.50+0.28(x-128,500) & \text{if } 128,500<x\le 195,850\\ 43,830.50+0.33(x-195,850) & \text{if } 195,850<x\le 349,700\\ 94,601.00+0.35(x-349,700) & \text{if } x>349,700\end{cases}$$

53. a. Let x represent the number of miles and C be the cost of transportation.

$$C(x)=\begin{cases}0.50x & \text{if } 0\le x\le 100\\ 0.50(100)+0.40(x-100) & \text{if } 100<x\le 400\\ 0.50(100)+0.40(300)+0.25(x-400) & \text{if } 400<x\le 800\\ 0.50(100)+0.40(300)+0.25(400)+0(x-800) & \text{if } 800<x\le 960\end{cases}$$

$$C(x)=\begin{cases}0.50x & \text{if } 0\le x\le 100\\ 10+0.40x & \text{if } 100<x\le 400\\ 70+0.25x & \text{if } 400<x\le 800\\ 270 & \text{if } 800<x\le 960\end{cases}$$



b. For hauls between 100 and 400 miles the cost is: $C(x) = 10 + 0.40x$.

c. For hauls between 400 and 800 miles the cost is: $C(x) = 70 + 0.25x$.

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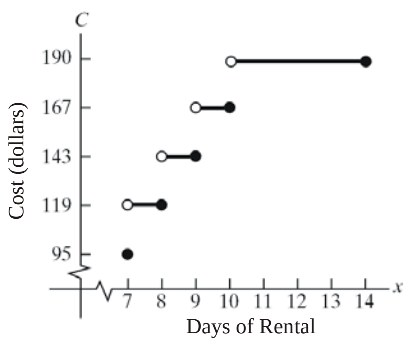
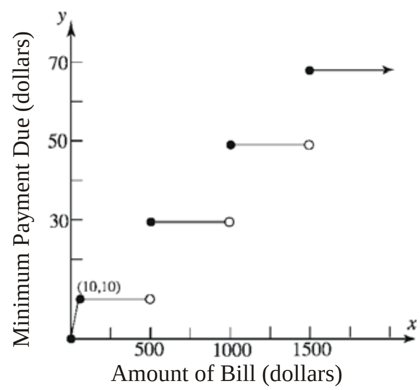
54. Let x = number of days car is used. The cost of renting is given by

$$C(x) = \begin{cases} 95 & \text{if } x = 7 \\ 119 & \text{if } 7 < x \le 8 \\ 143 & \text{if } 8 < x \le 9 \\ 167 & \text{if } 9 < x \le 10 \\ 190 & \text{if } 10 < x \le 14 \end{cases}$$



55. Let x = the amount of the bill in dollars. The minimum payment due is given by

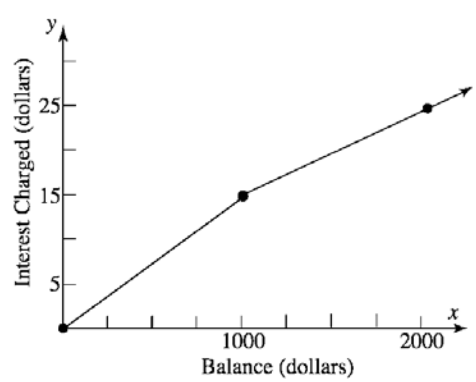
$$f(x) = \begin{cases} x & \text{if } 0 \le x < 10 \\ 10 & \text{if } 10 \le x < 500 \\ 30 & \text{if } 500 \le x < 1000 \\ 50 & \text{if } 1000 \le x < 1500 \\ 70 & \text{if } x \ge 1500 \end{cases}$$



56. Let x = the balance of the bill in dollars. The monthly interest charge is given by

$$g(x) = \begin{cases} 0.015x & \text{if } 0 \le x \le 1000 \\ 15 + 0.01(x - 1000) & \text{if } x > 1000 \end{cases}$$

$$= \begin{cases} 0.015x & \text{if } 0 \le x \le 1000 \\ 5 + 0.01x & \text{if } x > 1000 \end{cases}$$



57. **a.** $W = 10°C$

b. $W = 33 - \dfrac{(10.45 + 10\sqrt{5} - 5)(33-10)}{22.04} \approx 4°C$

c. $W = 33 - \dfrac{(10.45 + 10\sqrt{15} - 15)(33-10)}{22.04} \approx -3°C$

d. $W = 33 - 1.5958(33-10) = -4°C$

e. When $0 \le v < 1.79$, the wind speed is so small that there is no effect on the temperature.

f. When the wind speed exceeds 20, the wind chill depends only on the air temperature.

58. **a.** $W = -10°C$

b. $W = 33 - \dfrac{\left(10.45 + 10\sqrt{5} - 5\right)\left(33-(-10)\right)}{22.04}$
$\approx -21°C$

c. $W = 33 - \dfrac{\left(10.45 + 10\sqrt{15} - 15\right)\left(33-(-10)\right)}{22.04}$
$\approx -34°C$

d. $W = 33 - 1.5958\left(33-(-10)\right) = -36°C$

59. Let x = the number of ounces and $C(x)$ = the postage due.

For $0 < x \le 1$: $C(x) = \$0.80$

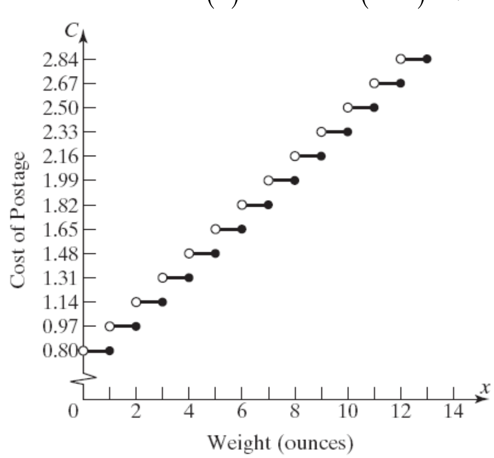
For $1 < x \le 2$: $C(x) = 0.80 + 0.17 = \$0.97$

For $2 < x \le 3$: $C(x) = 0.80 + 2(0.17) = \$1.14$

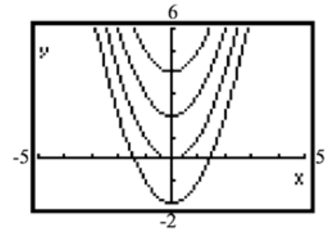
For $3 < x \le 4$: $C(x) = 0.80 + 3(0.17) = \$1.31$

$\vdots$

For $12 < x \le 13$: $C(x) = 0.80 + 12(0.17) = \$2.84$

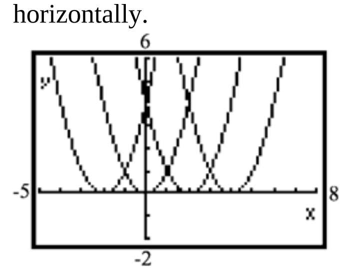


60. Each graph is that of $y = x^2$, but shifted vertically.



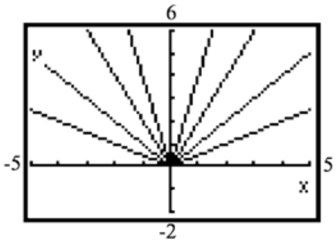
If $y = x^2 + k,\ k > 0$, the shift is up k units; if $y = x^2 - k,\ k > 0$, the shift is down k units. The graph of $y = x^2 - 4$ is the same as the graph of $y = x^2$, but shifted down 4 units. The graph of $y = x^2 + 5$ is the graph of $y = x^2$, but shifted up 5 units.

61. Each graph is that of $y = x^2$, but shifted horizontally.



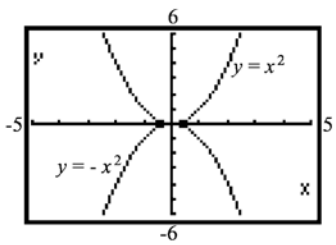
If $y = (x-k)^2,\ k > 0$, the shift is to the right k units; if $y = (x+k)^2,\ k > 0$, the shift is to the left k units. The graph of $y = (x+4)^2$ is the same as the graph of $y = x^2$, but shifted to the left 4 units. The graph of $y = (x-5)^2$ is the graph of $y = x^2$, but shifted to the right 5 units.

62. Each graph is that of $y = |x|$, but either compressed or stretched vertically.

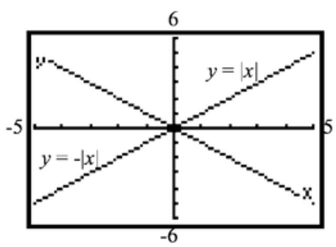


If $y = k|x|$ and $k > 1$, the graph is stretched vertically; if $y = k|x|$ and $0 < k < 1$, the graph is compressed vertically. The graph of $y = \frac{1}{4}|x|$ is the same as the graph of $y = |x|$, but compressed vertically. The graph of $y = 5|x|$ is the same as the graph of $y = |x|$, but stretched vertically.

63. The graph of $y = -x^2$ is the reflection of the graph of $y = x^2$ about the x-axis.

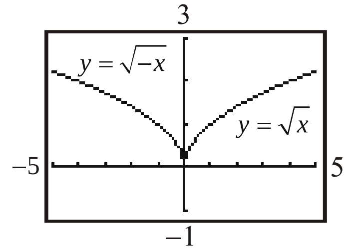


The graph of $y = -|x|$ is the reflection of the graph of $y = |x|$ about the x-axis.

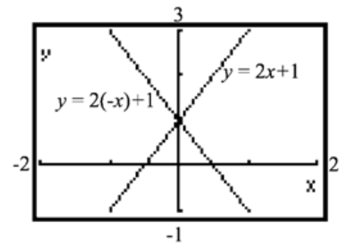


Multiplying a function by –1 causes the graph to be a reflection about the x-axis of the original function's graph.

64. The graph of $y = \sqrt{-x}$ is the reflection about the y-axis of the graph of $y = \sqrt{x}$.

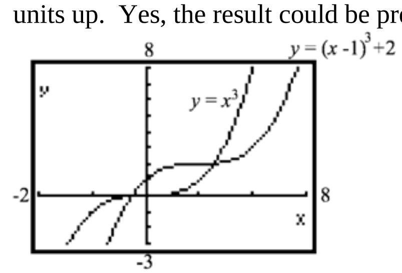


The same type of reflection occurs when graphing $y = 2x + 1$ and $y = 2(-x) + 1$.

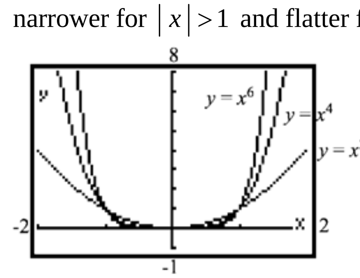


The graph of $y = f(-x)$ is the reflection about the y-axis of the graph of $y = f(x)$.

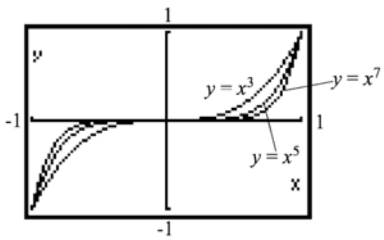
65. The graph of $y = (x-1)^3 + 2$ is a shifting of the graph of $y = x^3$ one unit to the right and two units up. Yes, the result could be predicted.



66. The graphs of $y = x^n$, n a positive even integer, are all U-shaped and open upward. All go through the points $(-1, 1)$, $(0, 0)$, and $(1, 1)$. As n increases, the graph of the function is narrower for $|x| > 1$ and flatter for $|x| < 1$.



67. The graphs of $y = x^n$, n a positive odd integer, all have the same general shape. All go through the points $(-1,-1)$, $(0, 0)$, and $(1,1)$. As n increases, the graph of the function increases at a greater rate for $|x|>1$ and is flatter around 0 for $|x|<1$.



68. $f(x)=\begin{cases}1 & \text{if } x \text{ is rational}\\ 0 & \text{if } x \text{ is irrational}\end{cases}$

Yes, it is a function.
Domain = $\{x | x \text{ is any real number}\}$ or $(-\infty,\infty)$
Range = {0, 1} or $\{y \mid y=0 \text{ or } y=1\}$
y-intercept: $x=0 \Rightarrow x$ is rational $\Rightarrow y=1$
So the y-intercept is $y=1$.

x-intercept: $y=0 \Rightarrow x$ is irrational
So the graph has infinitely many x-intercepts, namely, there is an x-intercept at each irrational value of x.

$f(-x)=1=f(x)$ when x is rational;
$f(-x)=0=f(x)$ when x is irrational.
Thus, f is even.

The graph of f consists of 2 infinite clusters of distinct points, extending horizontally in both directions. One cluster is located 1 unit above the x-axis, and the other is located along the x-axis.

69. For 0 < x < 1, the graph of $y = x^r$, r rational and $r>0$, flattens down toward the x-axis as r gets bigger. For x > 1, the graph of $y = x^r$ increases at a greater rate as r gets bigger.

Section 2.5

1. horizontal; right

2. y

3. -5, -2, and 2 (shift left three units)

4. True; the graph of $y=-f(x)$ is the reflection about the x-axis of the graph of $y=f(x)$.

5. False; to obtain the graph of $y=f(x+2)-3$ you shift the graph of $y=f(x)$ to the *left* 2 units and down 3 units.

6. True; to obtain the graph of $y=2f(x)$ we multiply the y-coordinates of the graph of $y=f(x)$ by 2. Since the y-coordinate of x-intercepts is 0 and $2\cdot 0=0$, multiplying by a constant does not change the x-intercepts.

7. b

8. e

9. h

10. d

11. i

12. a

13. l

14. c

15. f

16. j

17. g

18. k

19. $y=(x-4)^3$

20. $y=(x+4)^3$

21. $y=x^3+4$

22. $y=x^3-4$

23. $y=(-x)^3=-x^3$

24. $y=-x^3$

25. $y=4x^3$

26. $y=\left(\frac{1}{4}x\right)^3=\frac{1}{64}x^3$

27. (1) $y=\sqrt{x}+2$

(2) $y=-\left(\sqrt{x}+2\right)$

(3) $y=-\left(\sqrt{-x}+2\right)=-\sqrt{-x}-2$

28. (1) $y=-\sqrt{x}$

(2) $y=-\sqrt{x-3}$

(3) $y=-\sqrt{x-3}-2$

29. (1) $y=-\sqrt{x}$

(2) $y=-\sqrt{x}+2$

(3) $y=-\sqrt{x+3}+2$

30. (1) $y=\sqrt{x}+2$

(2) $y=\sqrt{-x}+2$

(3) $y=\sqrt{-(x+3)}+2=\sqrt{-x-3}+2$

31. (c); To go from $y=f(x)$ to $y=-f(x)$ we reflect about the *x*-axis. This means we change the sign of the *y*-coordinate for each point on the graph of $y=f(x)$. Thus, the point $(3, 6)$ would become $(3,-6)$.

32. (d); To go from $y=f(x)$ to $y=f(-x)$, we reflect each point on the graph of $y=f(x)$ about the *y*-axis. This means we change the sign of the *x*-coordinate for each point on the graph of $y=f(x)$. Thus, the point $(3,6)$ would become $(-3,6)$.

33. (c); To go from $y=f(x)$ to $y=2f(x)$, we stretch vertically by a factor of 2. Multiply the *y*-coordinate of each point on the graph of $y=f(x)$ by 2. Thus, the point $(1,3)$ would become $(1,6)$.

34. (c); To go from $y=f(x)$ to $y=f(2x)$, we compress horizontally by a factor of 2. Divide the *x*-coordinate of each point on the graph of $y=f(x)$ by 2. Thus, the point $(4,2)$ would become $(2,2)$.

35. a. The graph of $y=f(x+2)$ is the same as the graph of $y=f(x)$, but shifted 2 units to the left. Therefore, the *x*-intercepts are -7 and 1.

b. The graph of $y=f(x-2)$ is the same as the graph of $y=f(x)$, but shifted 2 units to the right. Therefore, the *x*-intercepts are -3 and 5.

c. The graph of $y=4f(x)$ is the same as the graph of $y=f(x)$, but stretched vertically by a factor of 4. Therefore, the *x*-intercepts are still -5 and 3 since the *y*-coordinate of each is 0.

d. The graph of $y=f(-x)$ is the same as the graph of $y=f(x)$, but reflected about the y-axis. Therefore, the *x*-intercepts are 5 and -3.

36. a. The graph of $y=f(x+4)$ is the same as the graph of $y=f(x)$, but shifted 4 units to the left. Therefore, the *x*-intercepts are -12 and -3.

b. The graph of $y=f(x-3)$ is the same as the graph of $y=f(x)$, but shifted 3 units to the right. Therefore, the *x*-intercepts are -5 and 4.

c. The graph of $y=2f(x)$ is the same as the graph of $y=f(x)$, but stretched vertically by a factor of 2. Therefore, the *x*-intercepts are still -8 and 1 since the *y*-coordinate of each is 0.

d. The graph of $y=f(-x)$ is the same as the graph of $y=f(x)$, but reflected about the *y*-axis. Therefore, the *x*-intercepts are 8 and -1.

37. a. The graph of $y=f(x+2)$ is the same as the graph of $y=f(x)$, but shifted 2 units to the left. Therefore, the graph of $f(x+2)$ is increasing on the interval $(-3,3)$.

b. The graph of $y = f(x-5)$ is the same as the graph of $y = f(x)$, but shifted 5 units to the right. Therefore, the graph of $f(x-5)$ is increasing on the interval $(4,10)$.

c. The graph of $y = -f(x)$ is the same as the graph of $y = f(x)$, but reflected about the x-axis. Therefore, we can say that the graph of $y = -f(x)$ must be *decreasing* on the interval $(-1,5)$.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, we can say that the graph of $y = f(-x)$ must be *decreasing* on the interval $(-5,1)$.

38. **a.** The graph of $y = f(x+2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the left. Therefore, the graph of $f(x+2)$ is decreasing on the interval $(-4,5)$.

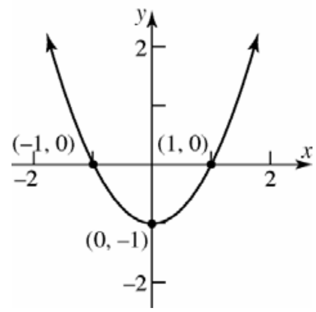
b. The graph of $y = f(x-5)$ is the same as the graph of $y = f(x)$, but shifted 5 units to the right. Therefore, the graph of $f(x-5)$ is decreasing on the interval $(3,12)$.

c. The graph of $y = -f(x)$ is the same as the graph of $y = f(x)$, but reflected about the x-axis. Therefore, we can say that the graph of $y = -f(x)$ must be *increasing* on the interval $(-2,7)$.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, we can say that the graph of $y = f(-x)$ must be *increasing* on the interval $(-7,2)$.

39. $f(x) = x^2 - 1$

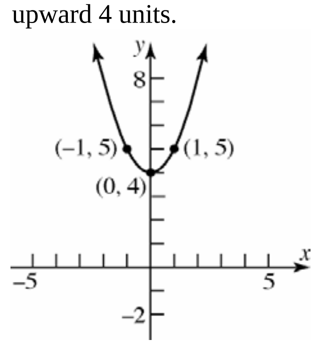
Using the graph of $y = x^2$, vertically shift downward 1 unit.



The domain is $(-\infty,\infty)$ and the range is $[-1,\infty)$.

40. $f(x) = x^2 + 4$

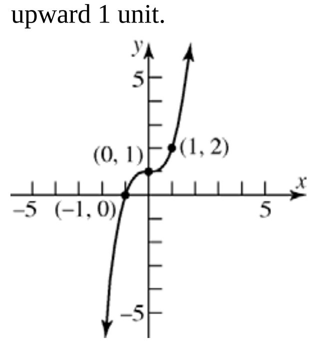
Using the graph of $y = x^2$, vertically shift upward 4 units.



The domain is $(-\infty,\infty)$ and the range is $[4,\infty)$.

41. $g(x) = x^3 + 1$

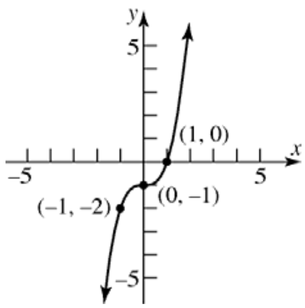
Using the graph of $y = x^3$, vertically shift upward 1 unit.



The domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.

42. $g(x) = x^3 - 1$

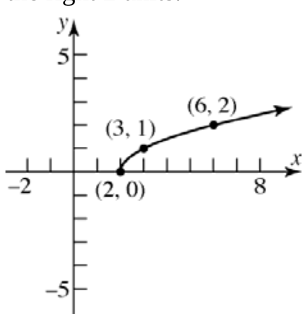
Using the graph of $y = x^3$, vertically shift downward 1 unit.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

43. $h(x) = \sqrt{x-2}$

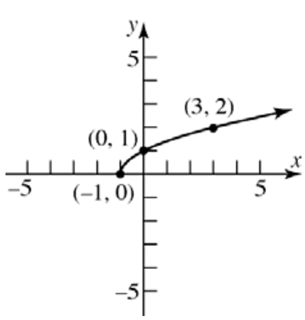
Using the graph of $y = \sqrt{x}$, horizontally shift to the right 2 units.



The domain is $[2, \infty)$ and the range is $[0, \infty)$.

44. $h(x) = \sqrt{x+1}$

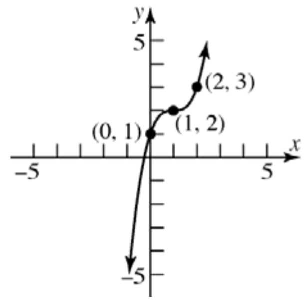
Using the graph of $y = \sqrt{x}$, horizontally shift to the left 1 unit.



The domain is $[-1, \infty)$ and the range is $[0, \infty)$.

45. $f(x) = (x-1)^3 + 2$

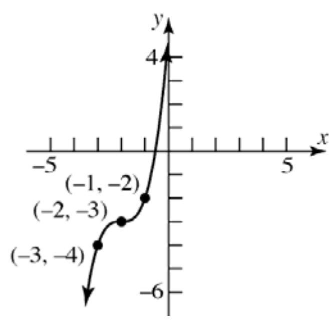
Using the graph of $y = x^3$, horizontally shift to the right 1 unit $\left[y = (x-1)^3\right]$, then vertically shift up 2 units $\left[y = (x-1)^3 + 2\right]$.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

46. $f(x) = (x+2)^3 - 3$

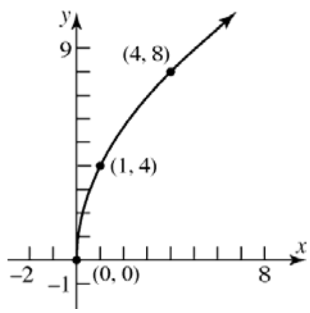
Using the graph of $y = x^3$, horizontally shift to the left 2 units $\left[y = (x+2)^3\right]$, then vertically shift down 3 units $\left[y = (x+2)^3 - 3\right]$.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

47. $g(x)=4\sqrt{x}$

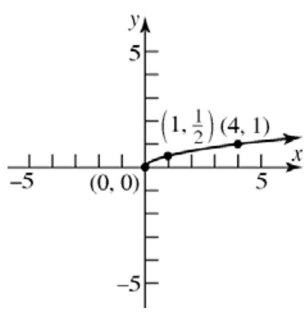
Using the graph of $y=\sqrt{x}$, vertically stretch by a factor of 4.



The domain is $[0,\infty)$ and the range is $[0,\infty)$.

48. $g(x)=\frac{1}{2}\sqrt{x}$

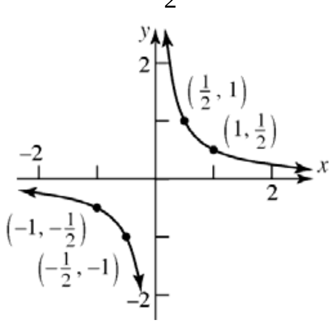
Using the graph of $y=\sqrt{x}$, vertically compress by a factor of $\frac{1}{2}$.



The domain is $[0,\infty)$ and the range is $[0,\infty)$.

49. $h(x)=\frac{1}{2x}=\left(\frac{1}{2}\right)\left(\frac{1}{x}\right)$

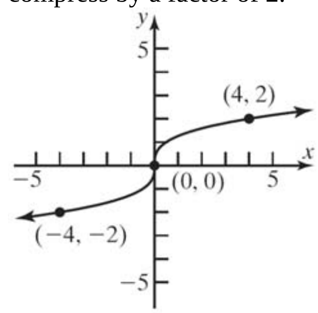
Using the graph of $y=\frac{1}{x}$, vertically compress by a factor of $\frac{1}{2}$.



The domain is $(-\infty,0)\cup(0,\infty)$ and the range is $(-\infty,0)\cup(0,\infty)$.

50. $h(x)=\sqrt[3]{2x}$

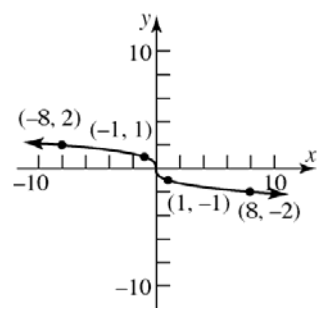
Using the graph of $y=\sqrt[3]{x}$, horizontally compress by a factor of 2.



The domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.

51. $f(x)=-\sqrt[3]{x}$

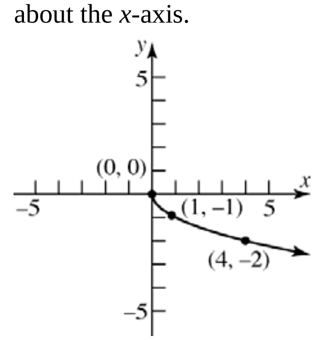
Using the graph of $y=\sqrt[3]{x}$, reflect the graph about the *x*-axis.



The domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.

52. $f(x)=-\sqrt{x}$

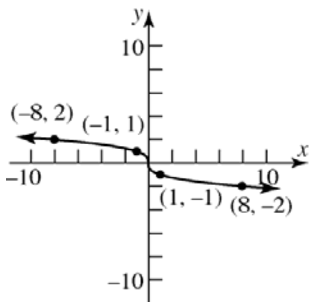
Using the graph of $y=\sqrt{x}$, reflect the graph about the *x*-axis.



The domain is $[0,\infty)$ and the range is $(-\infty,0]$.

53. $g(x)=\sqrt[3]{-x}$

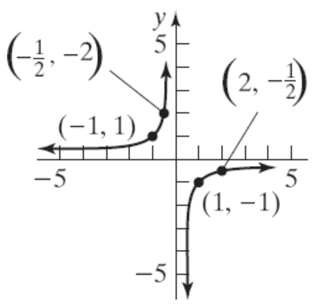
Using the graph of $y=\sqrt[3]{x}$, reflect the graph about the y-axis.



The domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.

54. $g(x)=\dfrac{1}{-x}$

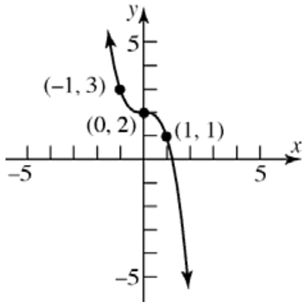
Using the graph of $y=\dfrac{1}{x}$, reflect the graph about the y-axis.



The domain is $(-\infty,0)\cup(0,\infty)$ and the range is $(-\infty,0)\cup(0,\infty)$.

55. $h(x)=-x^3+2$

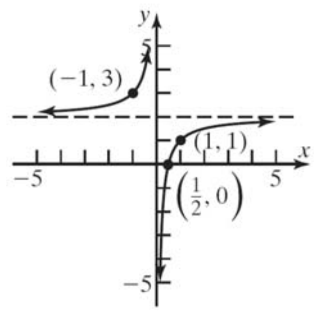
Using the graph of $y=x^3$, reflect the graph about the x-axis $\left[y=-x^3\right]$, then shift vertically upward 2 units $\left[y=-x^3+2\right]$.



The domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.

56. $h(x)=\dfrac{1}{-x}+2$

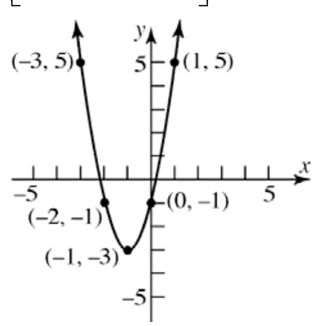
Using the graph of $y=\dfrac{1}{x}$, reflect the graph about the y-axis $\left[y=\dfrac{1}{-x}\right]$, then shift vertically upward 2 units $\left[y=\dfrac{1}{-x}+2\right]$.



The domain is $(-\infty,0)\cup(0,\infty)$ and the range is $(-\infty,2)\cup(2,\infty)$.

57. $f(x)=2(x+1)^2-3$

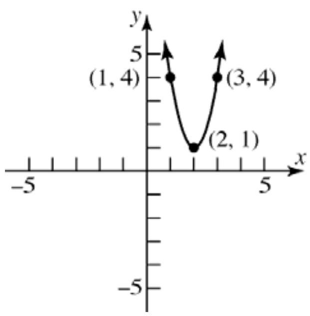
Using the graph of $y=x^2$, horizontally shift to the left 1 unit $\left[y=(x+1)^2\right]$, vertically stretch by a factor of 2 $\left[y=2(x+1)^2\right]$, and then vertically shift downward 3 units $\left[y=2(x+1)^2-3\right]$.



The domain is $(-\infty,\infty)$ and the range is $[-3,\infty)$.

58. $f(x) = 3(x-2)^2 + 1$

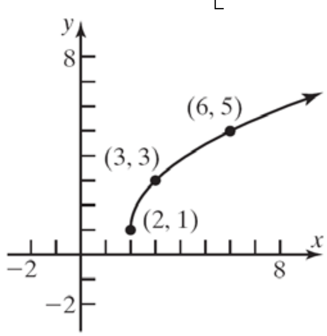
Using the graph of $y = x^2$, horizontally shift to the right 2 units $\left[y = (x-2)^2 \right]$, vertically stretch by a factor of 3 $\left[y = 3(x-2)^2 \right]$, and then vertically shift upward 1 unit $\left[y = 3(x-2)^2 + 1 \right]$.



The domain is $(-\infty, \infty)$ and the range is $[1, \infty)$.

59. $g(x) = 2\sqrt{x-2} + 1$

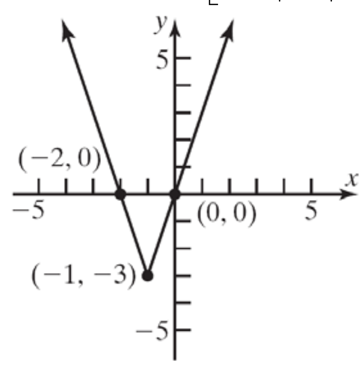
Using the graph of $y = \sqrt{x}$, horizontally shift to the right 2 units $\left[y = \sqrt{x-2} \right]$, vertically stretch by a factor of 2 $\left[y = 2\sqrt{x-2} \right]$, and vertically shift upward 1 unit $\left[y = 2\sqrt{x-2} + 1 \right]$.



The domain is $[2, \infty)$ and the range is $[1, \infty)$.

60. $g(x) = 3|x+1| - 3$

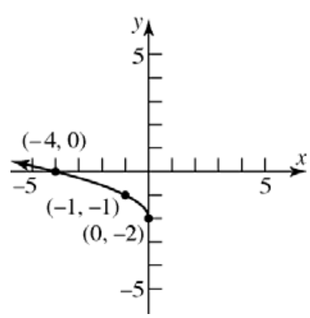
Using the graph of $y = |x|$, horizontally shift to the left 1 unit $\left[y = |x+1| \right]$, vertically stretch by a factor of 3 $\left[y = 3|x+1| \right]$, and vertically shift downward 3 units $\left[y = 3|x+1| - 3 \right]$.



The domain is $(-\infty, \infty)$ and the range is $[-3, \infty)$.

61. $h(x) = \sqrt{-x} - 2$

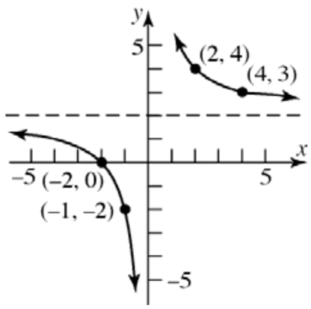
Using the graph of $y = \sqrt{x}$, reflect the graph about the y-axis $\left[y = \sqrt{-x} \right]$ and vertically shift downward 2 units $\left[y = \sqrt{-x} - 2 \right]$.



The domain is $(-\infty, 0]$ and the range is $[-2, \infty)$.

62. $h(x)=\dfrac{4}{x}+2=4\left(\dfrac{1}{x}\right)+2$

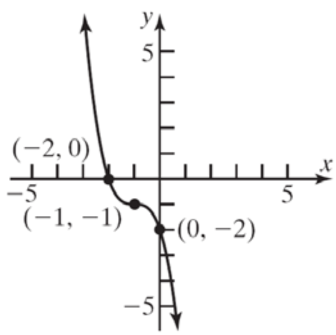
Stretch the graph of $y=\dfrac{1}{x}$ vertically by a factor of 4 $\left[y=4\cdot\dfrac{1}{x}=\dfrac{4}{x}\right]$ and vertically shift upward 2 units $\left[y=\dfrac{4}{x}+2\right]$.



The domain is $(-\infty,0)\cup(0,\infty)$ and the range is $(-\infty,2)\cup(2,\infty)$.

63. $f(x)=-(x+1)^3-1$

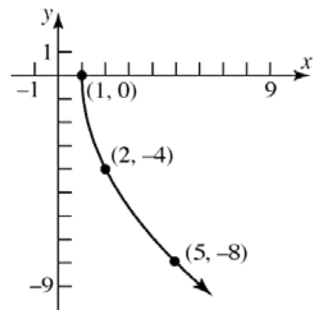
Using the graph of $y=x^3$, horizontally shift to the left 1 unit $\left[y=(x+1)^3\right]$, reflect the graph about the x-axis $\left[y=-(x+1)^3\right]$, and vertically shift downward 1 unit $\left[y=-(x+1)^3-1\right]$.



The domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.

64. $f(x)=-4\sqrt{x-1}$

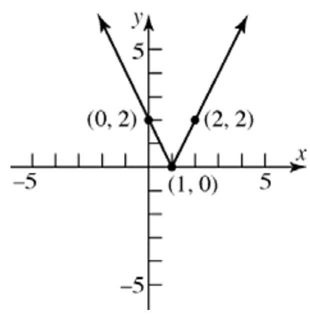
Using the graph of $y=\sqrt{x}$, horizontally shift to the right 1 unit $\left[y=\sqrt{x-1}\right]$, reflect the graph about the x-axis $\left[y=-\sqrt{x-1}\right]$, and stretch vertically by a factor of 4 $\left[y=-4\sqrt{x-1}\right]$.



The domain is $[1,\infty)$ and the range is $(-\infty,0]$.

65. $g(x)=2|1-x|=2|-(-1+x)|=2|x-1|$

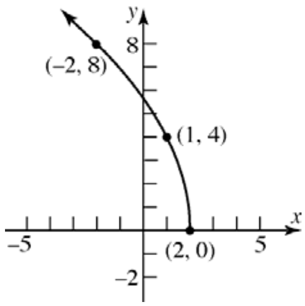
Using the graph of $y=|x|$, horizontally shift to the right 1 unit $\left[y=|x-1|\right]$, and vertically stretch by a factor or 2 $\left[y=2|x-1|\right]$.



The domain is $(-\infty,\infty)$ and the range is $[0,\infty)$.

66. $g(x) = 4\sqrt{2-x} = 4\sqrt{-(x-2)}$

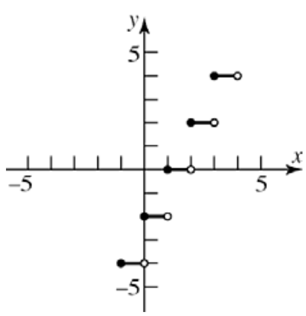
Using the graph of $y = \sqrt{x}$, reflect the graph about the y-axis $\left[y = \sqrt{-x}\right]$, horizontally shift to the right 2 units $\left[y = \sqrt{-(x-2)}\right]$, and vertically stretch by a factor of 4 $\left[y = 4\sqrt{-(x-2)}\right]$.



The domain is $(-\infty, 2]$ and the range is $[0, \infty)$.

67. $h(x) = 2\,\text{int}(x-1)$

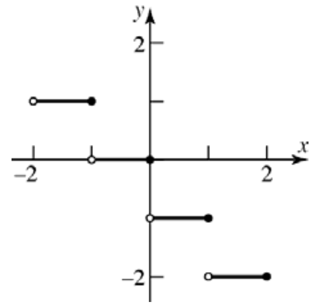
Using the graph of $y = \text{int}(x)$, horizontally shift to the right 1 unit $[y = \text{int}(x-1)]$, and vertically stretch by a factor of 2 $[y = 2\,\text{int}(x-1)]$.



The domain is $(-\infty, \infty)$ and the range is $\{y \mid y \text{ is an even integer}\}$.

68. $h(x) = \text{int}(-x)$

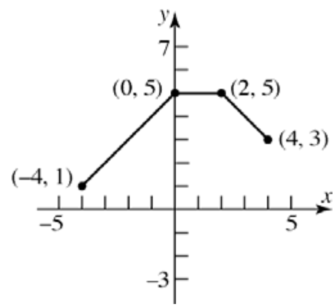
Reflect the graph of $y = \text{int}(x)$ about the y-axis.



The domain is $(-\infty, \infty)$ and the range is $\{y \mid y \text{ is an integer}\}$.

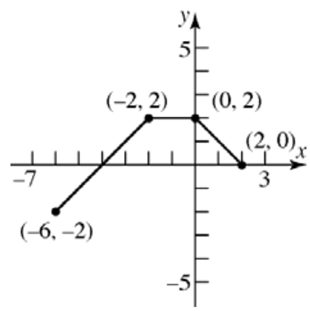
69. **a.** $F(x) = f(x) + 3$

Shift up 3 units.



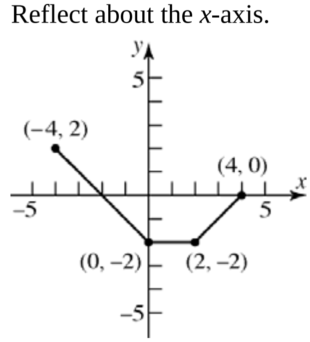
b. $G(x) = f(x+2)$

Shift left 2 units.



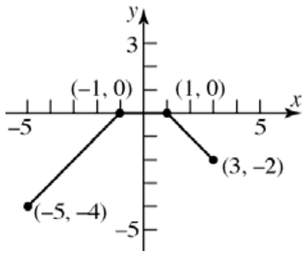
c. $P(x) = -f(x)$

Reflect about the x-axis.



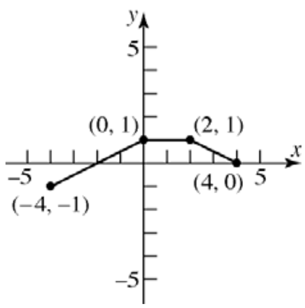
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



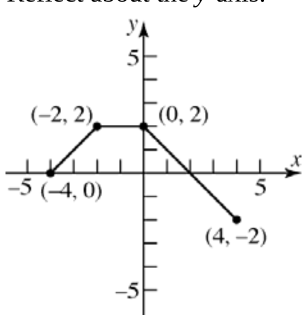
e. $Q(x) = \dfrac{1}{2} f(x)$

Compress vertically by a factor of $\dfrac{1}{2}$.



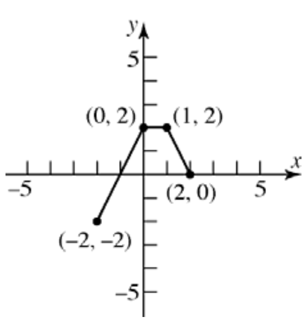
f. $g(x) = f(-x)$

Reflect about the *y*-axis.



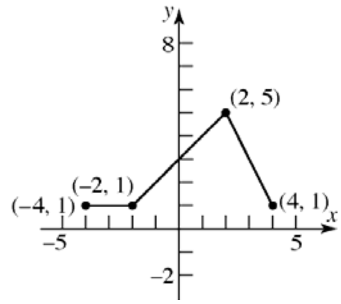
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\dfrac{1}{2}$.



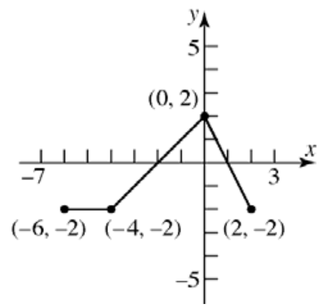
70. a. $F(x) = f(x) + 3$

Shift up 3 units.



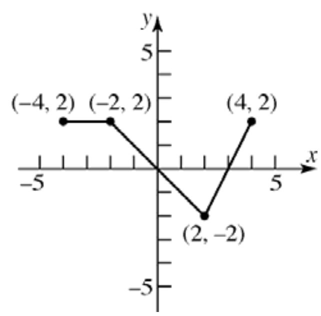
b. $G(x) = f(x+2)$

Shift left 2 units.



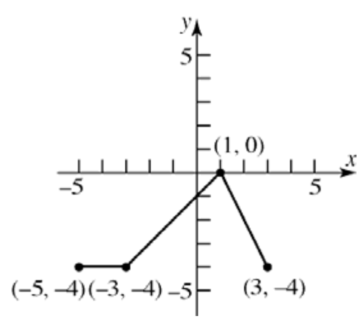
c. $P(x) = -f(x)$

Reflect about the *x*-axis.



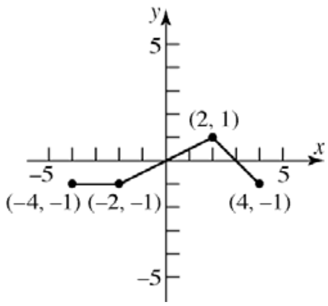
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



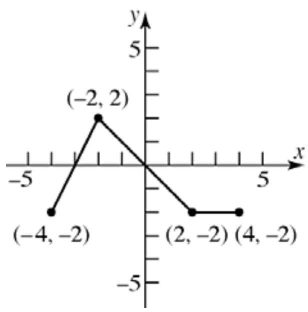
e. $Q(x) = \frac{1}{2} f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



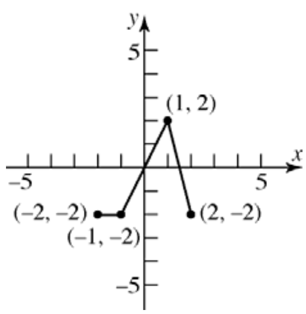
f. $g(x) = f(-x)$

Reflect about the y-axis.



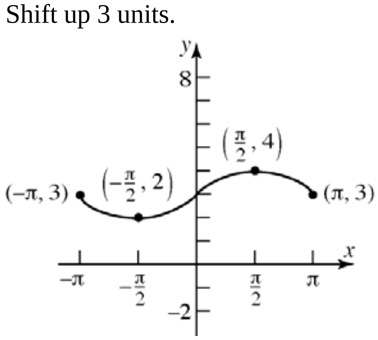
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



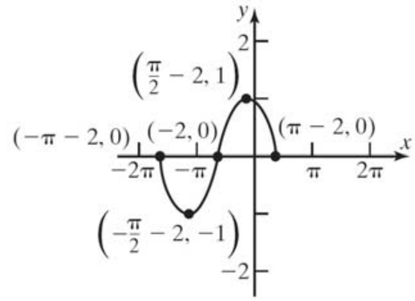
71. a. $F(x) = f(x) + 3$

Shift up 3 units.



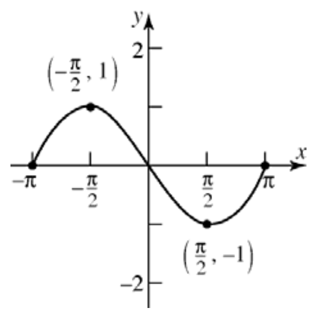
b. $G(x) = f(x+2)$

Shift left 2 units.



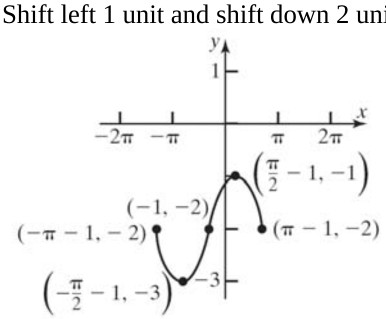
c. $P(x) = -f(x)$

Reflect about the x-axis.



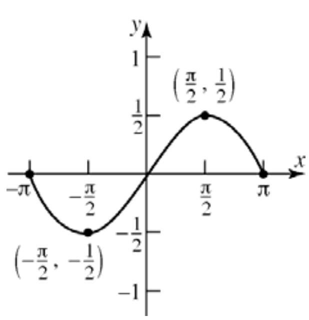
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



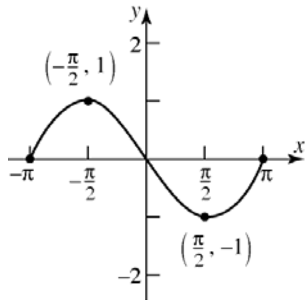
e. $Q(x) = \frac{1}{2} f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



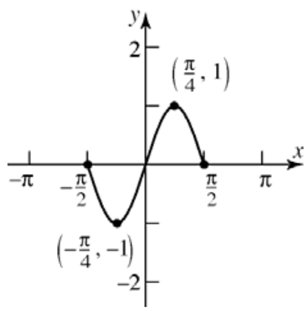
f. $g(x) = f(-x)$

Reflect about the y-axis.



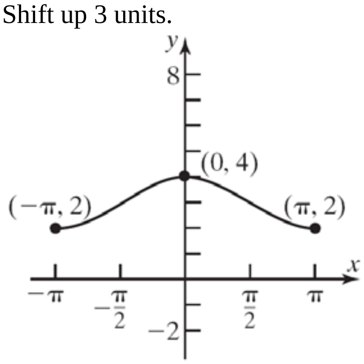
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



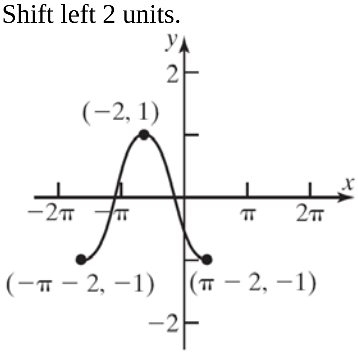
72. a. $F(x) = f(x) + 3$

Shift up 3 units.



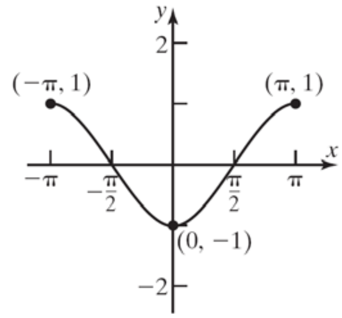
b. $G(x) = f(x + 2)$

Shift left 2 units.



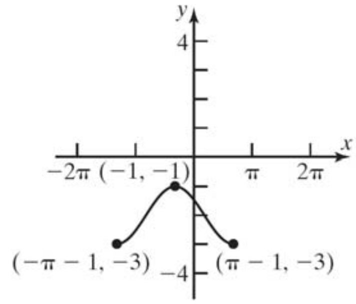
c. $P(x) = -f(x)$

Reflect about the x-axis.



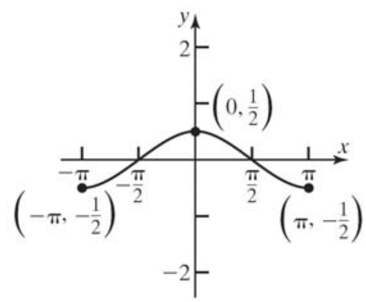
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



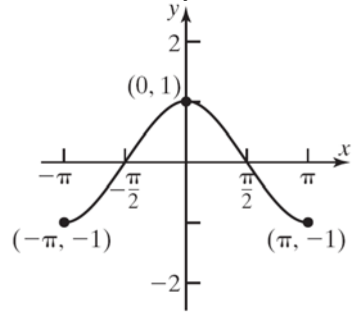
e. $Q(x) = \frac{1}{2} f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



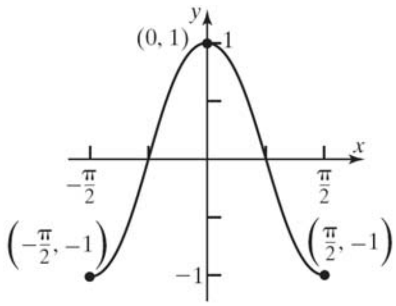
f. $g(x) = f(-x)$

Reflect about the y-axis.

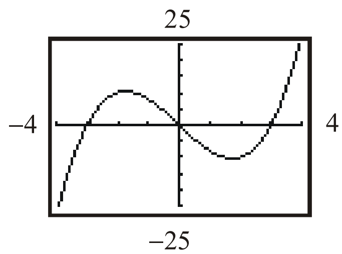


g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



73. a. $f(x) = x^3 - 9x$, $-4 < x < 4$



b. $0 = x^3 - 9x$

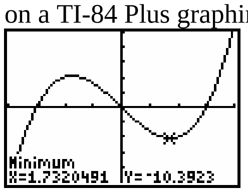
$0 = x\left(x^2 - 9\right)$

$0 = x(x-3)(x+3)$

$x = 0,\ x = 3,\ x = -3$

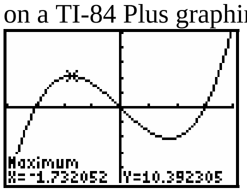
The x-intercepts are -3, 0, and 3.

c. The local minimum can be found by using the Minimum feature from the CALC menu on a TI-84 Plus graphing calculator.



The local minimum is approximately -10.39 when $x \approx 1.73$.

The local maximum can be found by using the Maximum feature from the CALC menu on a TI-84 Plus graphing calculator.



The local maximum is approximately 10.39 when $x \approx -1.73$.

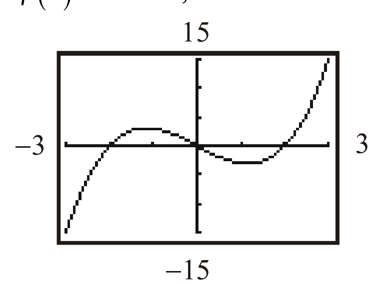
d. From the graph above, we see that *f* is initially increasing, decreasing between the two extrema, and increasing again at the end. Thus, *f* is increasing on the interval $(-4, -1.73)$ and on the interval $(1.73, 4)$. It is decreasing on the interval $(-1.73, 1.73)$.

e. $y = f(x+2)$ involves a shift to the left 2 units so we subtract 2 from each x-value. Therefore, we have the following:
x-intercepts: -5, -2, and 1
local minimum: -10.39 when $x \approx -0.27$
local maximum: 10.39 when $x \approx -3.73$
increasing on $(-6, -3.73)$ and $(-0.27, 2)$;
decreasing on $(-3.73, -0.27)$.

f. $y = 2f(x)$ involves a vertical stretch by a factor of 2 so we multiply each y-value by 2. Therefore, we have the following:
x-intercepts: -3, 0, and 3
local minimum: -20.78 when $x \approx 1.73$
local maximum: 20.78 when $x \approx -1.73$
increasing on $(-4, -1.73)$ and $(1.73, 4)$;
decreasing on $(-1.73, 1.73)$.

g. $y = f(-x)$ involves a reflection about the y-axis. Therefore, we have the following:
x-intercepts: -3, 0, and 3
local minimum: -10.39 when $x \approx -1.73$
local maximum: 10.39 when $x \approx 1.73$
increasing on $(-1.73, 1.73)$;
decreasing on $(-4, -1.73)$ and $(1.73, 4)$.

74. a. $f(x) = x^3 - 9x$, $-3 < x < 3$



b. $0 = x^3 - 4x$

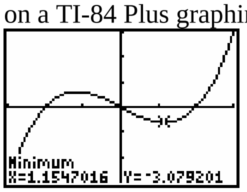
$0 = x\left(x^2 - 4\right)$

$0 = x\left(x-2\right)\left(x+2\right)$

$x = 0,\ x = 2,\ x = -2$

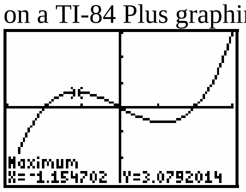
The x-intercepts are -2, 0, and 2.

c. The local minimum can be found by using the Minimum feature from the CALC menu on a TI-84 Plus graphing calculator.



The local minimum is approximately -3.08 when $x \approx 1.15$.

The local maximum can be found by using the Maximum feature from the CALC menu on a TI-84 Plus graphing calculator.



The local maximum is approximately 3.08 when $x \approx -1.15$.

d. From the graph above, we see that *f* is initially increasing, decreasing between the two extrema, and increasing again at the end. Thus, *f* is increasing on the interval $\left(-3,-1.15\right)$ and on the interval $\left(1.15,3\right)$. It is decreasing on the interval $\left(-1.15,1.15\right)$.

e. $y = f\left(x-4\right)$ involves a shift to the right 4 units so we add 4 to each x-value. Therefore, we have the following:
x-intercepts: 2, 4, and 6
local minimum: -3.08 when $x \approx 5.15$
local maximum: 3.08 when $x \approx 2.85$
increasing on $\left(1,2.85\right)$ and $\left(5.15,7\right)$;
decreasing on $\left(2.85,5.15\right)$.

f. $y = f\left(2x\right)$ involves a horizontal compression by a factor of 2 so we divide each x-value by 2. Therefore, we have the following:
x-intercepts: -1, 0, and 1
local minimum: -3.08 when $x \approx 0.575$
local maximum: 3.08 when $x \approx -0.575$
increasing on $\left(-\frac{3}{2},-0.575\right)$ and $\left(0.575,\frac{3}{2}\right)$;
decreasing on $\left(-0.575,0.575\right)$.

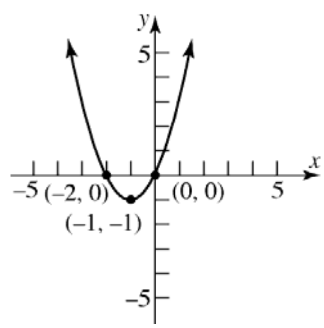
g. $y = -f\left(x\right)$ involves a reflection about the x-axis. Therefore, we have the following:
x-intercepts: -2, 0, and 2
local minimum: -3.08 when $x \approx -1.15$
local maximum: 3.08 when $x \approx 1.15$
increasing on $\left(-1.15,1.15\right)$;
decreasing on $\left(-3,-1.15\right)$ and $\left(1.15,3\right)$.

75. $f(x) = x^2 + 2x$

$f(x) = (x^2 + 2x + 1) - 1$

$f(x) = (x+1)^2 - 1$

Using $f(x) = x^2$, shift left 1 unit and shift down 1 unit.

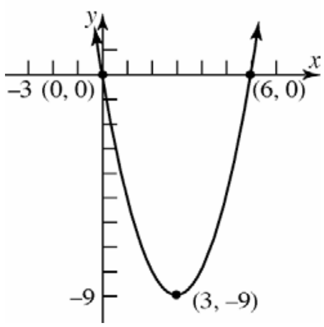


76. $f(x) = x^2 - 6x$

$f(x) = (x^2 - 6x + 9) - 9$

$f(x) = (x-3)^2 - 9$

Using $f(x) = x^2$, shift right 3 units and shift down 9 units.

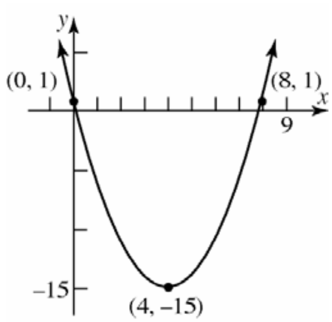


77. $f(x) = x^2 - 8x + 1$

$f(x) = \left(x^2 - 8x + 16\right) + 1 - 16$

$f(x) = \left(x-4\right)^2 - 15$

Using $f(x) = x^2$, shift right 4 units and shift down 15 units.

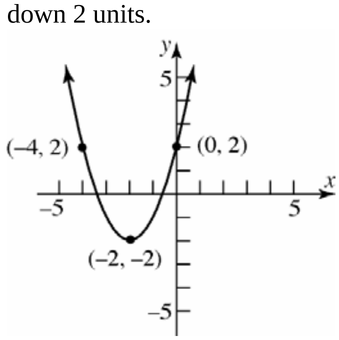


78. $f(x) = x^2 + 4x + 2$

$f(x) = \left(x^2 + 4x + 4\right) + 2 - 4$

$f(x) = \left(x+2\right)^2 - 2$

Using $f(x) = x^2$, shift left 2 units and shift down 2 units.

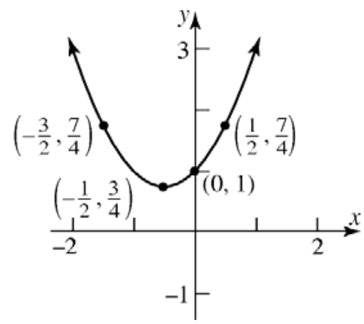


79. $f(x) = x^2 + x + 1$

$f(x) = \left(x^2 + x + \frac{1}{4}\right) + 1 - \frac{1}{4}$

$f(x) = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

Using $f(x) = x^2$, shift left $\frac{1}{2}$ unit and shift up $\frac{3}{4}$ unit.

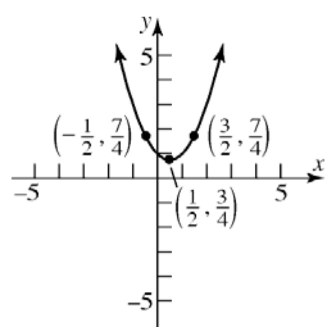


80. $f(x) = x^2 - x + 1$

$f(x) = \left(x^2 - x + \frac{1}{4}\right) + 1 - \frac{1}{4}$

$f(x) = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$

Using $f(x) = x^2$, shift right $\frac{1}{2}$ unit and shift up $\frac{3}{4}$ unit.



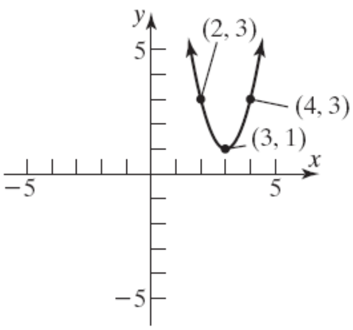
81. $f(x) = 2x^2 - 12x + 19$

$= 2(x^2 - 6x) + 19$

$= 2(x^2 - 6x + 9) + 19 - 18$

$= 2(x-3)^2 + 1$

Using $f(x) = x^2$, shift right 3 units, vertically stretch by a factor of 2, and then shift up 1 unit.



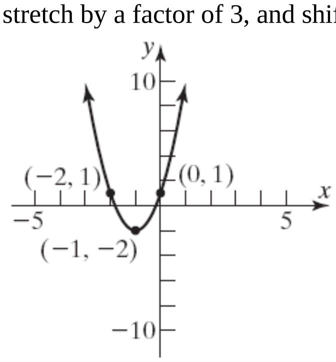
82. $f(x) = 3x^2 + 6x + 1$

$= 3(x^2 + 2x) + 1$

$= 3(x^2 + 2x + 1) + 1 - 3$

$= 3(x+1)^2 - 2$

Using $f(x) = x^2$, shift left 1 unit, vertically stretch by a factor of 3, and shift down 2 units.



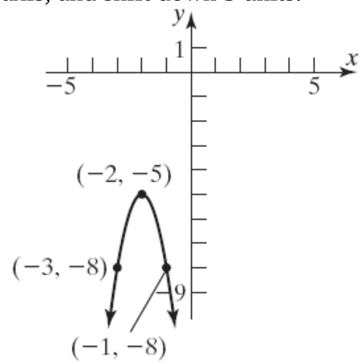
83. $f(x) = -3x^2 - 12x - 17$

$= -3(x^2 + 4x) - 17$

$= -3(x^2 + 4x + 4) - 17 + 12$

$= -3(x+2)^2 - 5$

Using $f(x) = x^2$, shift left 2 units, stretch vertically by a factor of 3, reflect about the x-axis, and shift down 5 units.



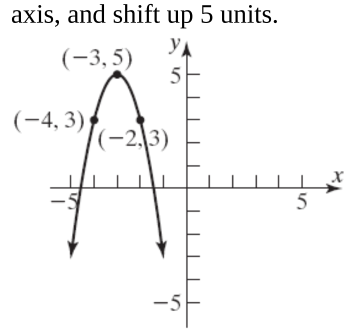
84. $f(x) = -2x^2 - 12x - 13$

$= -2(x^2 + 6x) - 13$

$= -2(x^2 + 6x + 9) - 13 + 18$

$= -2(x+3)^2 + 5$

Using $f(x) = x^2$, shift left 3 units, stretch vertically by a factor of 2, reflect about the x-axis, and shift up 5 units.

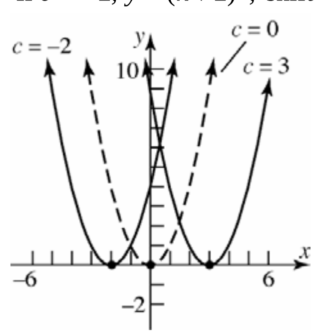


85. $y = (x-c)^2$

If $c = 0$, $y = x^2$.

If $c = 3$, $y = (x-3)^2$; shift right 3 units.

If $c = -2$, $y = (x+2)^2$; shift left 2 units.

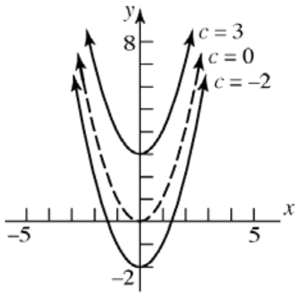


86. $y = x^2 + c$

If $c = 0$, $y = x^2$.

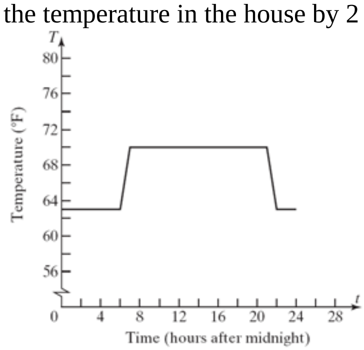
If $c = 3$, $y = x^2 + 3$; shift up 3 units.

If $c = -2$, $y = x^2 - 2$; shift down 2 units.

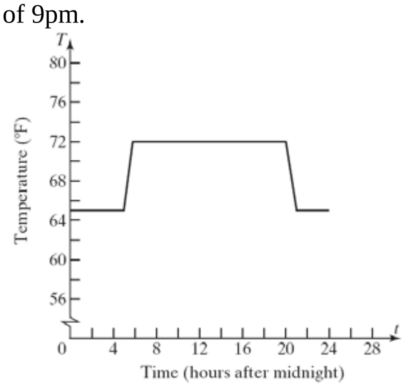


87. **a.** From the graph, the thermostat is set at $72°F$ during the daytime hours. The thermostat appears to be set at $65°F$ overnight.

b. To graph $y = T(t) - 2$, the graph of $T(t)$ is shifted down 2 units. This change will lower the temperature in the house by 2 degrees.



c. To graph $y = T(t+1)$, the graph of $T(t)$ should be shifted left one unit. This change will cause the program to switch between the daytime temperature and overnight temperature one hour sooner. The home will begin warming up at 5am instead of 6am and will begin cooling down at 8pm instead of 9pm.



88. **a.** $R(0) = 170.7(0)^2 + 1373(0) + 1080 = 1080$

The estimated worldwide music revenue for 2005 is $1080 million.

$$R(3) = 170.7(3)^2 + 1373(3) + 1080$$
$$= 6735.3$$

The estimated worldwide music revenue for 2008 is $6735.3 million.

$$R(5) = 170.7(5)^2 + 1373(5) + 1080$$
$$= 12,212.5$$

The estimated worldwide music revenue for 2010 is $12,212.5 million.

b.
$$r(x) = R(x-5)$$
$$= 170.7(x-5)^2 + 1373(x-5) + 1080$$
$$= 170.7(x^2 - 10x + 25) + 1373(x-5)$$
$$+1080$$
$$= 170.7x^2 - 1707x + 4267.5 + 1373x$$
$$-6865 + 1080$$
$$= 170.7x^2 - 334x - 1517.5$$

c. The graph of $r(x)$ is the graph of $R(x)$ shifted 5 units to the left. Thus, $r(x)$ represents the estimated worldwide music revenue, x years after 2000.

$r(5) = 170.7(5)^2 - 334(5) - 1517.5 = 1080$

The estimated worldwide music revenue for 2005 is $1080 million.

$$r(8) = 170.7(8)^2 - 334(8) - 1517.5$$
$$= 6735.3$$

The estimated worldwide music revenue for 2008 is $6735.3 million.

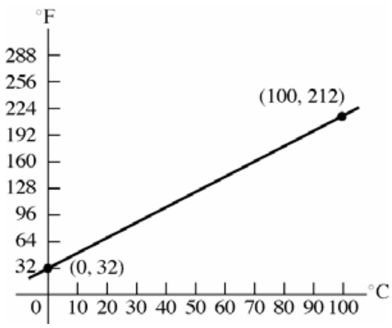
$$r(10) = 170.7(10)^2 - 334(10) - 1517.5$$
$$= 12,212.5$$

The estimated worldwide music revenue for 2010 is $12,212.5 million.

d. In $r(x)$, x represents the number of years after 2000 (see the previous part).

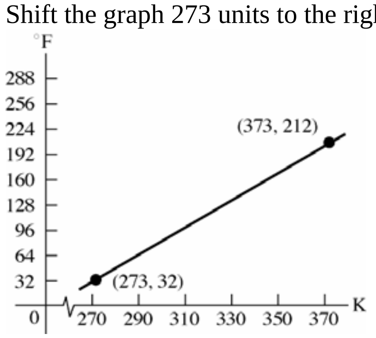
e. Answers will vary. One advantage might be that it is easier to determine what value should be substituted for x when using $r(x)$ instead of $R(x)$ to estimate worldwide music revenue.

89. $F = \frac{9}{5}C + 32$

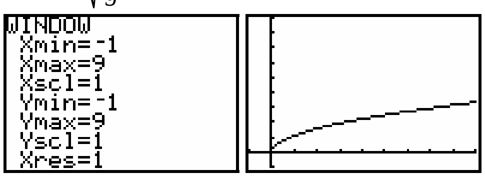


$F = \frac{9}{5}(K - 273) + 32$

Shift the graph 273 units to the right.

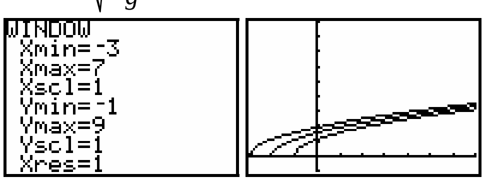


90. **a.** $T = 2\pi\sqrt{\frac{l}{g}}$



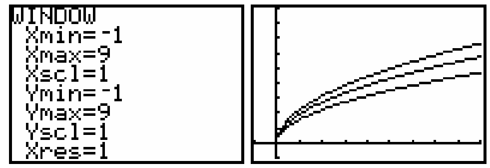
b. $T_1 = 2\pi\sqrt{\frac{l+1}{g}}$; $T_2 = 2\pi\sqrt{\frac{l+2}{g}}$;

$T_3 = 2\pi\sqrt{\frac{l+3}{g}}$



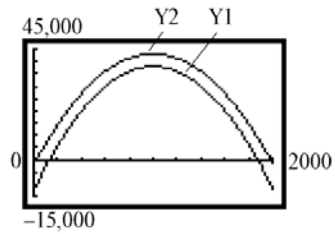
c. As the length of the pendulum increases, the period increases.

d. $T_1 = 2\pi\sqrt{\frac{2l}{g}}$; $T_2 = 2\pi\sqrt{\frac{3l}{g}}$; $T_3 = 2\pi\sqrt{\frac{4l}{g}}$



e. If the length of the pendulum is multiplied by k, the period is multiplied by $\sqrt{k}$.

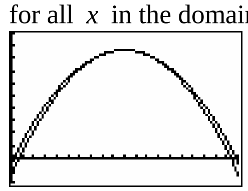
91. **a.** $p(x) = -0.05x^2 + 100x - 2000$



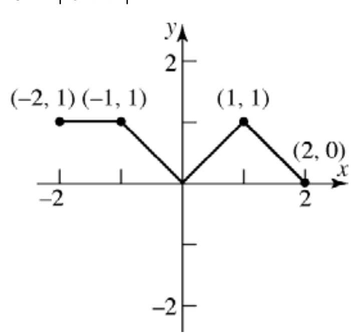
b. Select the 10% tax since the profits are higher.

c. The graph of Y1 is obtained by shifting the graph of $p(x)$ vertically down 10,000 units. The graph of Y2 is obtained by multiplying the *y*-coordinate of the graph of $p(x)$ by 0.9. Thus, Y2 is the graph of $p(x)$ vertically compressed by a factor of 0.9.

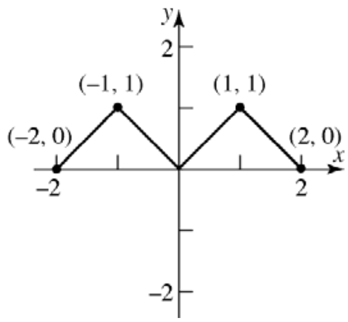
d. Select the 10% tax since the graph of $Y1 = 0.9p(x) \geq Y2 = -0.05x^2 + 100x - 6800$ for all x in the domain.



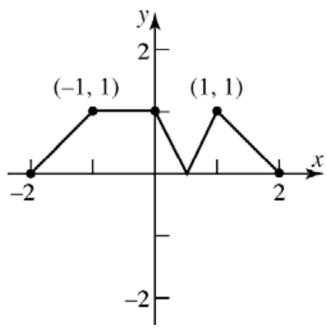
92. **a.** $y = |f(x)|$



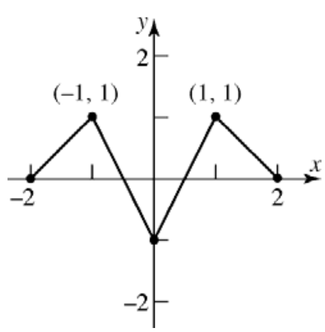
b. $y = f\left(|x|\right)$



93. a. To graph $y = |f(x)|$, the part of the graph for *f* that lies in quadrants III or IV is reflected about the *x*-axis.



b. To graph $y = f\left(|x|\right)$, the part of the graph for *f* that lies in quadrants II or III is replaced by the reflection of the part in quadrants I and IV reflected about the *y*-axis.



94. a. The graph of $y = f\left(x+3\right)-5$ is the graph of $y = f\left(x\right)$ but shifted left 3 units and down 5 units. Thus, the point $\left(1,3\right)$ becomes the point $\left(-2,-2\right)$.

b. The graph of $y = -2f\left(x-2\right)+1$ is the graph of $y = f\left(x\right)$ but shifted right 2 units, stretched vertically by a factor of 2, reflected about the *x*-axis, and shifted up 1 unit. Thus, the point $\left(1,3\right)$ becomes the point $\left(3,-5\right)$.

c. The graph of $y = f\left(2x+3\right)$ is the graph of $y = f\left(x\right)$ but shifted left 3 units and horizontally compressed by a factor of 2. Thus, the point $\left(1,3\right)$ becomes the point $\left(-1,3\right)$.

95. a. The graph of $y = f\left(x+1\right)-3$ is the graph of $y = f\left(x\right)$ but shifted left 1 unit and down 3 units. Thus, the point $\left(-3,5\right)$ becomes the point $\left(-4,2\right)$.

b. The graph of $y = -3f\left(x-4\right)+3$ is the graph of $y = f\left(x\right)$ but shifted right 4 units, stretched vertically by a factor of 3, reflected about the *x*-axis, and shifted up 3 units. Thus, the point $\left(-3,5\right)$ becomes the point $\left(1,-12\right)$.

c. The graph of $y = f\left(3x+9\right)$ is the graph of $y = f\left(x\right)$ but shifted left 9 units and horizontally compressed by a factor of 3. Thus, the point $\left(-3,5\right)$ becomes the point $\left(-4,5\right)$.

96. The graph of $y = 4f(x)$ is a vertical stretch of the graph of *f* by a factor of 4, while the graph of $y = f(4x)$ is a horizontal compression of the graph of *f* by a factor of $\frac{1}{4}$.

97. The graph of $y = \sqrt{-x}$ is the graph of $y = \sqrt{x}$ but reflected horizontally over the *y*-axis. By symmetry, the area under $y = \sqrt{x}$ bounded below by the *x*-axis, on the left by *x* = 0, and on the right by *x* = 4 will be equal to the area under the curve $y = \sqrt{-x}$ bounded below by the *x*-axis, on the left by $x = -4$, and on the right by *x* = 0. Thus, the area is $\frac{16}{3}$ square units.

Section 2.6

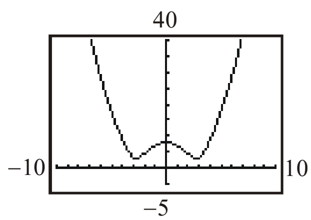
1. a. The distance d from P to the origin is $d=\sqrt{x^2+y^2}$. Since P is a point on the graph of $y=x^2-8$, we have:

$$d(x)=\sqrt{x^2+(x^2-8)^2}=\sqrt{x^4-15x^2+64}$$

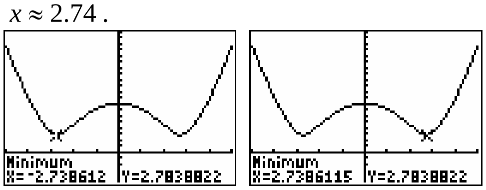
b. $d(0)=\sqrt{0^4-15(0)^2+64}=\sqrt{64}=8$

c. $d(1)=\sqrt{(1)^4-15(1)^2+64}$
$=\sqrt{1-15+64}=\sqrt{50}=5\sqrt{2}\approx 7.07$

d.



e. d is smallest when $x\approx -2.74$ or when $x\approx 2.74$.



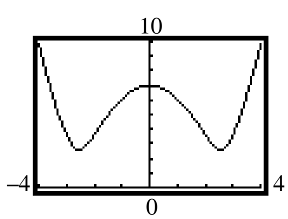
2. a. The distance d from P to $(0,-1)$ is $d=\sqrt{x^2+(y+1)^2}$. Since P is a point on the graph of $y=x^2-8$, we have:

$$d(x)=\sqrt{x^2+(x^2-8+1)^2}$$
$$=\sqrt{x^2+\left(x^2-7\right)^2}=\sqrt{x^4-13x^2+49}$$

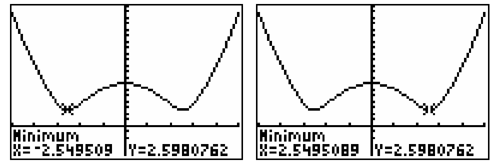
b. $d(0)=\sqrt{0^4-13(0)^2+49}=\sqrt{49}=7$

c. $d(-1)=\sqrt{(-1)^4-13(-1)^2+49}=\sqrt{37}\approx 6.08$

d.



e. d is smallest when $x\approx -2.55$ or when $x\approx 2.55$.

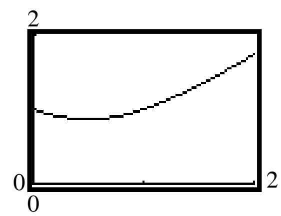


3. a. The distance d from P to the point (1, 0) is $d=\sqrt{(x-1)^2+y^2}$. Since P is a point on the graph of $y=\sqrt{x}$, we have:

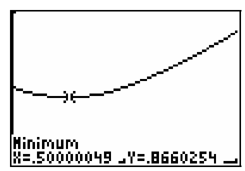
$$d(x)=\sqrt{(x-1)^2+\left(\sqrt{x}\right)^2}=\sqrt{x^2-x+1}$$

where $x\geq 0$.

b.



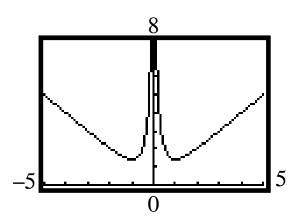
c. d is smallest when $x=\frac{1}{2}$.



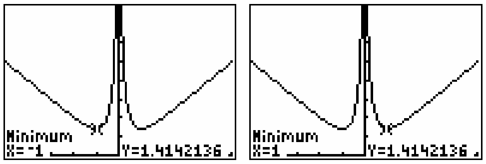
4. a. The distance d from P to the origin is $d=\sqrt{x^2+y^2}$. Since P is a point on the graph of $y=\dfrac{1}{x}$, we have:

$$d(x)=\sqrt{x^2+\left(\frac{1}{x}\right)^2}=\sqrt{x^2+\frac{1}{x^2}}=\sqrt{\frac{x^4+1}{x^2}}$$
$$=\frac{\sqrt{x^2+1}}{|x|}$$

b.



c. d is smallest when $x=-1$ or $x=1$.



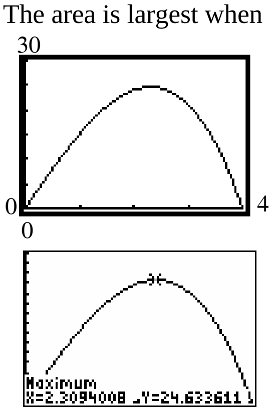
5. By definition, a triangle has area $A=\frac{1}{2}b\,h$, $b=$ base, $h=$ height. From the figure, we know that $b=x$ and $h=y$. Expressing the area of the triangle as a function of x, we have:
$A(x)=\frac{1}{2}xy=\frac{1}{2}x\left(x^3\right)=\frac{1}{2}x^4$.

6. By definition, a triangle has area $A=\frac{1}{2}b\,h$, b=base, $h=$ height. Because one vertex of the triangle is at the origin and the other is on the x-axis, we know that $b=x$ and $h=y$. Expressing the area of the triangle as a function of x, we have:
$A(x)=\frac{1}{2}xy=\frac{1}{2}x\left(9-x^2\right)=\frac{9}{2}x-\frac{1}{2}x^3$.

7. a. $A(x)=xy=x\left(16-x^2\right)=-x^3+16x$

b. Domain: $\{x \mid 0<x<4\}$

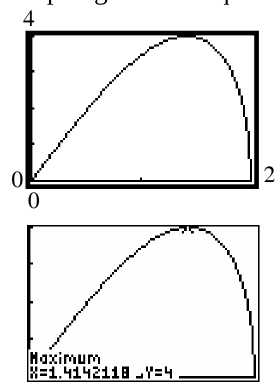
c. The area is largest when $x\approx 2.31$.



8. a. $A(x)=2xy=2x\sqrt{4-x^2}$

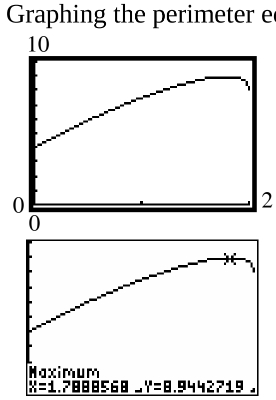
b. $p(x)=2(2x)+2(y)=4x+2\sqrt{4-x^2}$

c. Graphing the area equation:



The area is largest when $x\approx 1.41$.

d. Graphing the perimeter equation:

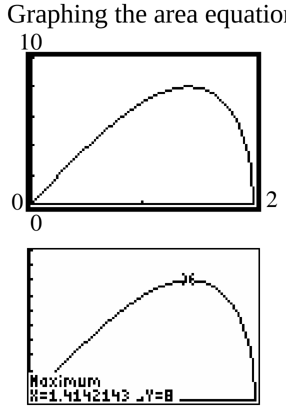


The perimeter is largest when $x\approx 1.79$.

9. a. In Quadrant I, $x^2+y^2=4 \to y=\sqrt{4-x^2}$
$A(x)=(2x)(2y)=4x\sqrt{4-x^2}$

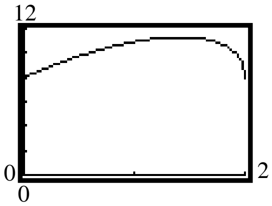
b. $p(x)=2(2x)+2(2y)=4x+4\sqrt{4-x^2}$

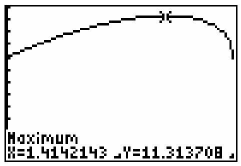
c. Graphing the area equation:



The area is largest when $x\approx 1.41$.

d. Graphing the perimeter equation:





The perimeter is largest when $x \approx 1.41$.

10. a. $A(r) = (2r)(2r) = 4r^2$

b. $p(r) = 4(2r) = 8r$

11. a. $C =$ circumference, $A =$ total area,

$r =$ radius, $x =$ side of square

$C = 2\pi r = 10 - 4x \quad \Rightarrow \quad r = \frac{5-2x}{\pi}$

Total Area $=$ area$_{\text{square}}$ $+$ area$_{\text{circle}}$ $= x^2 + \pi r^2$

$$A(x) = x^2 + \pi\left(\frac{5-2x}{\pi}\right)^2 = x^2 + \frac{25 - 20x + 4x^2}{\pi}$$

b. Since the lengths must be positive, we have:

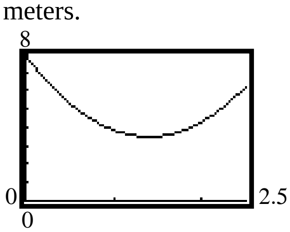
$10 - 4x > 0$ and $x > 0$

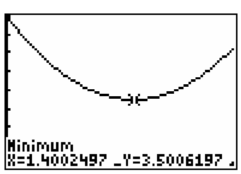
$-4x > -10$ and $x > 0$

$x < 2.5$ and $x > 0$

Domain: $\{x \mid 0 < x < 2.5\}$

c. The total area is smallest when $x \approx 1.40$ meters.





12. a. $C =$ circumference, $A =$ total area,

$r =$ radius, $x =$ side of equilateral triangle

$$C = 2\pi r = 10 - 3x \Rightarrow r = \frac{10 - 3x}{2\pi}$$

The height of the equilateral triangle is $\frac{\sqrt{3}}{2}x$.

Total Area $=$ area$_{\text{triangle}}$ $+$ area$_{\text{circle}}$

$$= \frac{1}{2}x\left(\frac{\sqrt{3}}{2}x\right) + \pi r^2$$

$$A(x) = \frac{\sqrt{3}}{4}x^2 + \pi\left(\frac{10 - 3x}{2\pi}\right)^2$$

$$= \frac{\sqrt{3}}{4}x^2 + \frac{100 - 60x + 9x^2}{4\pi}$$

b. Since the lengths must be positive, we have:

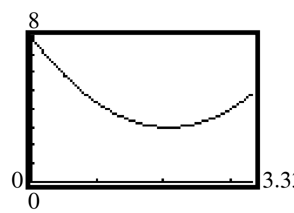
$10 - 3x > 0$ and $x > 0$

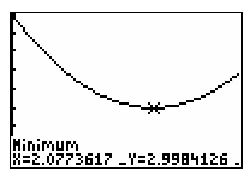
$-3x > -10$ and $x > 0$

$x < \frac{10}{3}$ and $x > 0$

Domain: $\left\{x \,\middle|\, 0 < x < \frac{10}{3}\right\}$

c. The area is smallest when $x \approx 2.08$ meters.





13. a. Since the wire of length x is bent into a circle, the circumference is x. Therefore, $C(x) = x$.

b. Since $C = x = 2\pi r$, $r = \frac{x}{2\pi}$.

$$A(x) = \pi r^2 = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}.$$

14. a. Since the wire of length x is bent into a square, the perimeter is x. Therefore, $p(x) = x$.

b. Since $P = x = 4s$, $s = \frac{1}{4}x$, we have

$$A(x) = s^2 = \left(\frac{1}{4}x\right)^2 = \frac{1}{16}x^2.$$

15. a. A = area, r = radius; diameter = $2r$

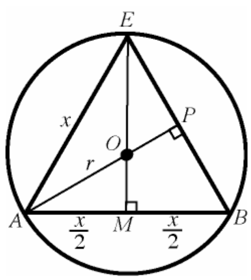
$$A(r) = (2r)(r) = 2r^2$$

b. p = perimeter

$$p(r) = 2(2r) + 2r = 6r$$

16. C = circumference, r = radius;

x = length of a side of the triangle



Since ΔABC is equilateral, $EM = \frac{\sqrt{3}x}{2}$.

Therefore, $OM = \frac{\sqrt{3}x}{2} - OE = \frac{\sqrt{3}x}{2} - r$

In ΔOAM, $r^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{\sqrt{3}x}{2} - r\right)^2$

$$r^2 = \frac{x^2}{4} + \frac{3}{4}x^2 - \sqrt{3}\,rx + r^2$$

$$\sqrt{3}\,rx = x^2$$

$$r = \frac{x}{\sqrt{3}}$$

Therefore, the circumference of the circle is

$$C(x) = 2\pi r = 2\pi\left(\frac{x}{\sqrt{3}}\right) = \frac{2\pi\sqrt{3}}{3}x$$

17. Area of the equilateral triangle

$$A = \frac{1}{2}x \cdot \frac{\sqrt{3}}{2}x = \frac{\sqrt{3}}{4}x^2$$

From problem 16, we have $r^2 = \frac{x^2}{3}$.

Area inside the circle, but outside the triangle:

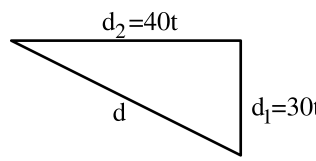
$$A(x) = \pi r^2 - \frac{\sqrt{3}}{4}x^2$$

$$= \pi\frac{x^2}{3} - \frac{\sqrt{3}}{4}x^2 = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)x^2$$

18. $d^2 = d_1^{\,2} + d_2^{\,2}$

$$d^2 = (30t)^2 + (40t)^2$$

$$d(t) = \sqrt{900t^2 + 1600t^2} = \sqrt{2500t^2} = 50t$$



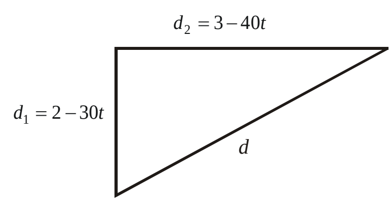
19. a. $d^2 = d_1^{\,2} + d_2^{\,2}$

$$d^2 = (2-30t)^2 + (3-40t)^2$$

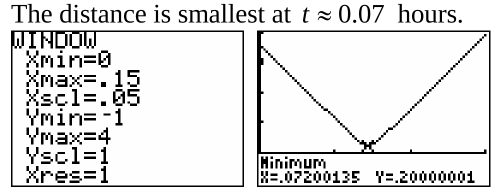
$$d(t) = \sqrt{(2-30t)^2 + (3-40t)^2}$$

$$= \sqrt{4 - 120t + 900t^2 + 9 - 240t + 1600t^2}$$

$$= \sqrt{2500t^2 - 360t + 13}$$



b. The distance is smallest at $t \approx 0.07$ hours.



20. $r =$ radius of cylinder, $h =$ height of cylinder, $V =$ volume of cylinder

$$r^2 + \left(\frac{h}{2}\right)^2 = R^2 \Rightarrow r^2 + \frac{h^2}{4} = R^2 \Rightarrow r^2 = R^2 - \frac{h^2}{4}$$

$$V = \pi r^2 h$$

$$V(h) = \pi\left(R^2 - \frac{h^2}{4}\right)h = \pi h\left(R^2 - \frac{h^2}{4}\right)$$

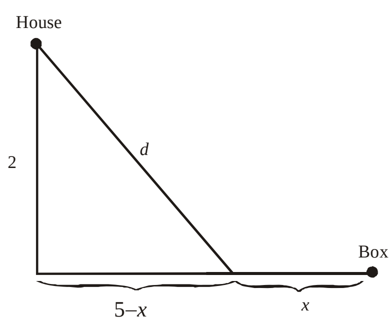
21. $r =$ radius of cylinder, $h =$ height of cylinder, $V =$ volume of cylinder

By similar triangles: $\dfrac{H}{R} = \dfrac{H-h}{r}$

$$Hr = R(H-h)$$
$$Hr = RH - Rh$$
$$Rh = RH - Hr$$
$$h = \frac{RH - Hr}{R} = \frac{H(R-r)}{R}$$

$$V = \pi r^2 h = \pi r^2\left(\frac{H(R-r)}{R}\right) = \frac{\pi H(R-r)r^2}{R}$$

22. a. The total cost of installing the cable along the road is $500x$. If cable is installed x miles along the road, there are $5-x$ miles between the road to the house and where the cable ends along the road.



$$d = \sqrt{(5-x)^2 + 2^2}$$
$$= \sqrt{25 - 10x + x^2 + 4} = \sqrt{x^2 - 10x + 29}$$

The total cost of installing the cable is:

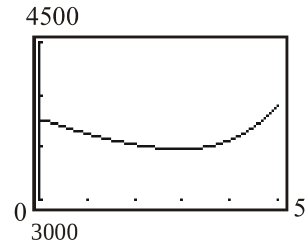
$$C(x) = 500x + 700\sqrt{x^2 - 10x + 29}$$

Domain: $\{x \mid 0 \le x \le 5\}$

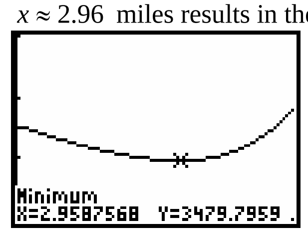
b. $C(1) = 500(1) + 700\sqrt{1^2 - 10(1) + 29}$
$= 500 + 700\sqrt{20} = \$3630.50$

c. $C(3) = 500(3) + 700\sqrt{3^2 - 10(3) + 29}$
$= 1500 + 700\sqrt{8} = \$3479.90$

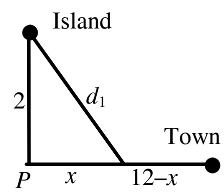
d.



e. Using MINIMUM, the graph indicates that $x \approx 2.96$ miles results in the least cost.



23. a. The time on the boat is given by $\dfrac{d_1}{3}$. The time on land is given by $\dfrac{12-x}{5}$.



$$d_1 = \sqrt{x^2 + 2^2} = \sqrt{x^2 + 4}$$

The total time for the trip is:

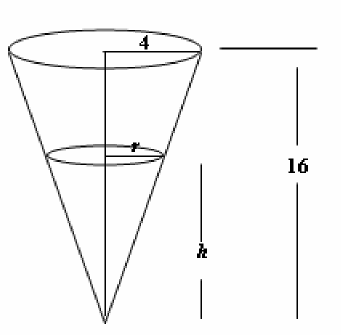
$$T(x) = \frac{12-x}{5} + \frac{d_1}{3} = \frac{12-x}{5} + \frac{\sqrt{x^2+4}}{3}$$

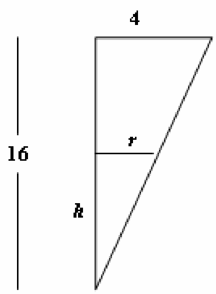
b. Domain: $\{x \mid 0 \le x \le 12\}$

c. $T(4) = \dfrac{12-4}{5} + \dfrac{\sqrt{4^2+4}}{3}$
$= \dfrac{8}{5} + \dfrac{\sqrt{20}}{3} \approx 3.09$ hours

d. $T(8) = \dfrac{12-8}{5} + \dfrac{\sqrt{8^2+4}}{3}$
$= \dfrac{4}{5} + \dfrac{\sqrt{68}}{3} \approx 3.55$ hours

24. Consider the diagrams shown below.





There is a pair of similar triangles in the diagram. Since the smaller triangle is similar to the larger triangle, we have the proportion

$$\frac{r}{h}=\frac{4}{16}\Rightarrow\frac{r}{h}=\frac{1}{4}\Rightarrow r=\frac{1}{4}h$$

Substituting into the volume formula for the conical portion of water gives

$$V(h)=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi\left(\frac{1}{4}h\right)^2 h=\frac{1}{48}\pi h^3.$$

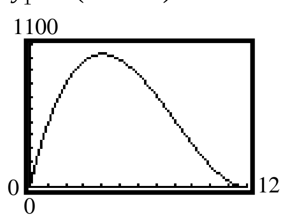
25. a. length = $24-2x$; width = $24-2x$; height = x

$$V(x)=x(24-2x)(24-2x)=x(24-2x)^2$$

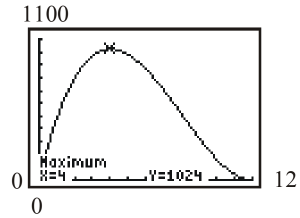
b. $V(3)=3(24-2(3))^2=3(18)^2$
$=3(324)=972$ cu.in.

c. $V(10)=10(24-2(10))^2=10(4)^2$
$=10(16)=160$ cu.in.

d. $y_1=x(24-2x)^2$



Use MAXIMUM.



The volume is largest when $x=4$ inches.

26. a. Let $A=$ amount of material, $x=$ length of the base, $h=$ height, and $V=$ volume.

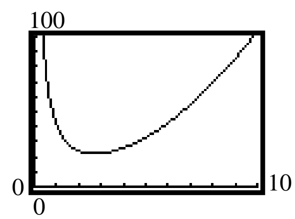
$$V=x^2h=10\Rightarrow h=\frac{10}{x^2}$$

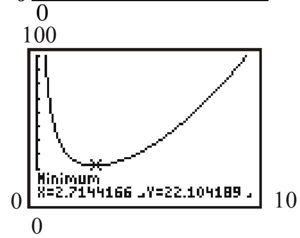
$$\begin{aligned}\text{Total Area } A&=\left(\text{Area}_{\text{base}}\right)+(4)\left(\text{Area}_{\text{side}}\right)\\&=x^2+4xh\\&=x^2+4x\left(\frac{10}{x^2}\right)\\&=x^2+\frac{40}{x}\\A(x)&=x^2+\frac{40}{x}\end{aligned}$$

b. $A(1)=1^2+\frac{40}{1}=1+40=41\text{ ft}^2$

c. $A(2)=2^2+\frac{40}{2}=4+20=24\text{ ft}^2$

d. $y_1=x^2+\frac{40}{x}$





The amount of material is least when $x=2.71$ ft.

Chapter 2 Review Exercises

1. This relation represents a function.
Domain = {–1, 2, 4}; Range = {0, 3}.

2. This relation does not represent a function, since 4 is paired with two different values.

3. $f(x)=\dfrac{3x}{x^2-1}$

a. $f(2)=\dfrac{3(2)}{(2)^2-1}=\dfrac{6}{4-1}=\dfrac{6}{3}=2$

b. $f(-2)=\dfrac{3(-2)}{(-2)^2-1}=\dfrac{-6}{4-1}=\dfrac{-6}{3}=-2$

c. $f(-x)=\dfrac{3(-x)}{(-x)^2-1}=\dfrac{-3x}{x^2-1}$

d. $-f(x)=-\left(\dfrac{3x}{x^2-1}\right)=\dfrac{-3x}{x^2-1}$

e. $f(x-2)=\dfrac{3(x-2)}{(x-2)^2-1}$
$$=\frac{3x-6}{x^2-4x+4-1}=\frac{3\left(x-2\right)}{x^2-4x+3}$$

f. $f(2x)=\dfrac{3(2x)}{(2x)^2-1}=\dfrac{6x}{4x^2-1}$

4. $f(x)=\dfrac{x^2}{x+1}$

a. $f(2)=\dfrac{2^2}{2+1}=\dfrac{4}{3}$

b. $f(-2)=\dfrac{(-2)^2}{-2+1}=\dfrac{4}{-1}=-4$

c. $f(-x)=\dfrac{(-x)^2}{-x+1}=\dfrac{x^2}{-x+1}$

d. $-f(x)=-\dfrac{x^2}{x+1}=\dfrac{-x^2}{x+1}$

e. $f(x-2)=\dfrac{(x-2)^2}{(x-2)+1}=\dfrac{\left(x-2\right)^2}{x-1}$

f. $f(2x)=\dfrac{(2x)^2}{(2x)+1}=\dfrac{4x^2}{2x+1}$

5. $f(x)=\sqrt{x^2-4}$

a. $f(2)=\sqrt{2^2-4}=\sqrt{4-4}=\sqrt{0}=0$

b. $f(-2)=\sqrt{(-2)^2-4}=\sqrt{4-4}=\sqrt{0}=0$

c. $f(-x)=\sqrt{(-x)^2-4}=\sqrt{x^2-4}$

d. $-f(x)=-\sqrt{x^2-4}$

e. $f(x-2)=\sqrt{(x-2)^2-4}$
$$=\sqrt{x^2-4x+4-4}$$
$$=\sqrt{x^2-4x}$$

f. $f(2x)=\sqrt{(2x)^2-4}=\sqrt{4x^2-4}$
$$=\sqrt{4\left(x^2-1\right)}=2\sqrt{x^2-1}$$

6. $f(x)=\left|x^2-4\right|$

a. $f(2)=\left|2^2-4\right|=|4-4|=|0|=0$

b. $f(-2)=\left|(-2)^2-4\right|=|4-4|=|0|=0$

c. $f(-x)=\left|(-x)^2-4\right|=\left|x^2-4\right|$

d. $-f(x)=-\left|x^2-4\right|$

e. $f(x-2)=\left|(x-2)^2-4\right|$
$$=\left|x^2-4x+4-4\right|$$
$$=\left|x^2-4x\right|$$

f. $f(2x)=\left|(2x)^2-4\right|=\left|4x^2-4\right|$
$$=\left|4\left(x^2-1\right)\right|=4\left|x^2-1\right|$$

7. $f(x)=\dfrac{x^2-4}{x^2}$

a. $f(2)=\dfrac{2^2-4}{2^2}=\dfrac{4-4}{4}=\dfrac{0}{4}=0$

b. $f(-2)=\dfrac{(-2)^2-4}{(-2)^2}=\dfrac{4-4}{4}=\dfrac{0}{4}=0$

c. $f(-x)=\dfrac{(-x)^2-4}{(-x)^2}=\dfrac{x^2-4}{x^2}$

d. $-f(x)=-\left(\dfrac{x^2-4}{x^2}\right)=\dfrac{4-x^2}{x^2}=-\dfrac{x^2-4}{x^2}$

e. $f(x-2)=\dfrac{(x-2)^2-4}{(x-2)^2}=\dfrac{x^2-4x+4-4}{(x-2)^2}$

$=\dfrac{x^2-4x}{(x-2)^2}=\dfrac{x(x-4)}{(x-2)^2}$

f. $f(2x)=\dfrac{(2x)^2-4}{(2x)^2}=\dfrac{4x^2-4}{4x^2}$

$=\dfrac{4\left(x^2-1\right)}{4x^2}=\dfrac{x^2-1}{x^2}$

8. $f(x)=\dfrac{x^3}{x^2-9}$

a. $f(2)=\dfrac{2^3}{2^2-9}=\dfrac{8}{4-9}=\dfrac{8}{-5}=-\dfrac{8}{5}$

b. $f(2)=\dfrac{(-2)^3}{(-2)^2-9}=\dfrac{-8}{4-9}=\dfrac{-8}{-5}=\dfrac{8}{5}$

c. $f(-x)=\dfrac{(-x)^3}{(-x)^2-9}=\dfrac{-x^3}{x^2-9}$

d. $-f(x)=-\dfrac{x^3}{x^2-9}=\dfrac{-x^3}{x^2-9}$

e. $f(x-2)=\dfrac{(x-2)^3}{(x-2)^2-9}$

$=\dfrac{(x-2)^3}{x^2-4x+4-9}$

$=\dfrac{(x-2)^3}{x^2-4x-5}$

f. $f(2x)=\dfrac{(2x)^3}{(2x)^2-9}=\dfrac{8x^3}{4x^2-9}$

9. $f(x)=\dfrac{x}{x^2-9}$

The denominator cannot be zero:

$x^2-9\neq 0$

$(x+3)(x-3)\neq 0$

$x\neq -3$ or 3

Domain: $\{x \mid x\neq -3, x\neq 3\}$

10. $f(x)=\dfrac{3x^2}{x-2}$

The denominator cannot be zero:

$x-2\neq 0$

$x\neq 2$

Domain: $\{x \mid x\neq 2\}$

11. $f(x)=\sqrt{2-x}$

The radicand must be non-negative:

$2-x\geq 0$

$x\leq 2$

Domain: $\{x \mid x\leq 2\}$ or $(-\infty,2]$

12. $f(x)=\sqrt{x+2}$

The radicand must be non-negative:

$x+2\geq 0$

$x\geq -2$

Domain: $\{x \mid x\geq -2\}$ or $[-2,\infty)$

13. $f(x)=\dfrac{\sqrt{x}}{|x|}$

The radicand must be non-negative and the denominator cannot be zero: $x>0$

Domain: $\{x \mid x>0\}$ or $(0,\infty)$

14. $g(x)=\dfrac{|x|}{x}$

The denominator cannot be zero:

$x\neq 0$

Domain: $\{x \mid x\neq 0\}$

15. $f(x)=\dfrac{x}{x^2+2x-3}$

The denominator cannot be zero:

$x^2+2x-3\neq 0$

$(x+3)(x-1)\neq 0$

$x\neq -3$ or 1

Domain: $\{x \mid x\neq -3,\ x\neq 1\}$

16. $F(x)=\dfrac{1}{x^2-3x-4}$

The denominator cannot be zero:

$x^2-3x-4\neq 0$

$(x+1)(x-4)\neq 0$

$x\neq -1$ or 4

Domain: $\{x \mid x\neq -1,\ x\neq 4\}$

17. $f(x)=2-x \quad g(x)=3x+1$

$(f+g)(x)=f(x)+g(x)$
$\quad=2-x+3x+1=2x+3$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f-g)(x)=f(x)-g(x)$
$\quad=2-x-(3x+1)$
$\quad=2-x-3x-1$
$\quad=-4x+1$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f\cdot g)(x)=f(x)\cdot g(x)$
$\quad=(2-x)(3x+1)$
$\quad=6x+2-3x^2-x$
$\quad=-3x^2+5x+2$

Domain: $\{x \mid x \text{ is any real number}\}$

$\left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)}=\dfrac{2-x}{3x+1}$

$3x+1\neq 0$

$3x\neq -1\Rightarrow x\neq -\dfrac{1}{3}$

Domain: $\left\{x \,\middle|\, x\neq -\dfrac{1}{3}\right\}$

18. $f(x)=2x-1 \qquad g(x)=2x+1$

$(f+g)(x)=f(x)+g(x)$
$\quad=2x-1+2x+1$
$\quad=4x$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f-g)(x)=f(x)-g(x)$
$\quad=2x-1-(2x+1)$
$\quad=2x-1-2x-1$
$\quad=-2$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f\cdot g)(x)=f(x)\cdot g(x)$
$\quad=(2x-1)(2x+1)$
$\quad=4x^2-1$

Domain: $\{x \mid x \text{ is any real number}\}$

$\left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)}=\dfrac{2x-1}{2x+1}$

$2x+1\neq 0\Rightarrow 2x\neq -1\Rightarrow x\neq -\dfrac{1}{2}$

Domain: $\left\{x \,\middle|\, x\neq -\dfrac{1}{2}\right\}$

19. $f(x)=3x^2+x+1 \qquad g(x)=3x$

$(f+g)(x)=f(x)+g(x)$
$\quad=3x^2+x+1+3x$
$\quad=3x^2+4x+1$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f-g)(x)=f(x)-g(x)$
$\quad=3x^2+x+1-3x$
$\quad=3x^2-2x+1$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f\cdot g)(x)=f(x)\cdot g(x)$
$\quad=(3x^2+x+1)(3x)$
$\quad=9x^3+3x^2+3x$

Domain: $\{x \mid x \text{ is any real number}\}$

$\left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)}=\dfrac{3x^2+x+1}{3x}$

$3x\neq 0\Rightarrow x\neq 0$

Domain: $\{x \mid x\neq 0\}$

20. $f(x)=3x \qquad g(x)=1+x+x^2$

$$(f+g)(x)=f(x)+g(x) = 3x+1+x+x^2 = x^2+4x+1$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$(f-g)(x)=f(x)-g(x) = 3x-\left(1+x+x^2\right) = -x^2+2x-1$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$(f\cdot g)(x)=f(x)\cdot g(x) = (3x)\left(1+x+x^2\right) = 3x+3x^2+3x^3$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{3x}{1+x+x^2}$$

$1+x+x^2 \neq 0$

$x^2+x+1\neq 0$

Since the discriminant is $1^2-4(1)(1)=-3<0$,

x^2+x+1 will never equal 0.

Domain: $\{x \mid x \text{ is any real number}\}$

21. $f(x)=\dfrac{x+1}{x-1} \qquad g(x)=\dfrac{1}{x}$

$$(f+g)(x)=f(x)+g(x) = \frac{x+1}{x-1}+\frac{1}{x}=\frac{x(x+1)+1(x-1)}{x(x-1)} = \frac{x^2+x+x-1}{x(x-1)}=\frac{x^2+2x-1}{x(x-1)}$$

Domain: $\{x \mid x\neq 0, x\neq 1\}$

$$(f-g)(x)=f(x)-g(x) = \frac{x+1}{x-1}-\frac{1}{x}=\frac{x(x+1)-1(x-1)}{x(x-1)} = \frac{x^2+x-x+1}{x(x-1)}=\frac{x^2+1}{x(x-1)}$$

Domain: $\{x \mid x\neq 0, x\neq 1\}$

$$(f\cdot g)(x)=f(x)\cdot g(x)=\left(\frac{x+1}{x-1}\right)\left(\frac{1}{x}\right)=\frac{x+1}{x(x-1)}$$

Domain: $\{x \mid x\neq 0, x\neq 1\}$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{\frac{x+1}{x-1}}{\frac{1}{x}}=\left(\frac{x+1}{x-1}\right)\left(\frac{x}{1}\right)=\frac{x(x+1)}{x-1}$$

Domain: $\{x \mid x\neq 0, x\neq 1\}$

22. $f(x)=\dfrac{1}{x-3} \qquad g(x)=\dfrac{3}{x}$

$$(f+g)(x)=f(x)+g(x) = \frac{1}{x-3}+\frac{3}{x}=\frac{x+3(x-3)}{x(x-3)} = \frac{x+3x-9}{x(x-3)}=\frac{4x-9}{x(x-3)}$$

Domain: $\{x \mid x\neq 0, x\neq 3\}$

$$(f-g)(x)=f(x)-g(x)=\frac{1}{x-3}-\frac{3}{x} = \frac{x-3(x-3)}{x(x-3)}=\frac{x-3x+9}{x(x-3)} = \frac{-2x+9}{x(x-3)}$$

Domain: $\{x \mid x\neq 0, x\neq 3\}$

$$(f\cdot g)(x)=f(x)\cdot g(x)=\left(\frac{1}{x-3}\right)\left(\frac{3}{x}\right)=\frac{3}{x(x-3)}$$

Domain: $\{x \mid x\neq 0, x\neq 3\}$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{\frac{1}{x-3}}{\frac{3}{x}} = \left(\frac{1}{x-3}\right)\left(\frac{x}{3}\right) = \frac{x}{3(x-3)}$$

Domain: $\{x \mid x\neq 0, x\neq 3\}$

23. $f(x) = -2x^2 + x + 1$

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{-2(x+h)^2 + (x+h) + 1 - (-2x^2 + x + 1)}{h}$$
$$= \frac{-2(x^2 + 2xh + h^2) + x + h + 1 + 2x^2 - x - 1}{h}$$
$$= \frac{-2x^2 - 4xh - 2h^2 + x + h + 1 + 2x^2 - x - 1}{h}$$
$$= \frac{-4xh - 2h^2 + h}{h} = \frac{h(-4x - 2h + 1)}{h}$$
$$= -4x - 2h + 1$$

24. $f(x) = 3x^2 - 2x + 4$

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{3(x+h)^2 - 2(x+h) + 4 - (3x^2 - 2x + 4)}{h}$$
$$= \frac{3(x^2 + 2xh + h^2) - 2x - 2h + 4 - 3x^2 + 2x - 4}{h}$$
$$= \frac{3x^2 + 6xh + 3h^2 - 2x - 2h + 4 - 3x^2 + 2x - 4}{h}$$
$$= \frac{6xh + 3h^2 - 2h}{h} = \frac{h(6x + 3h - 2)}{h}$$
$$= 6x + 3h - 2$$

25. **a.** Domain: $\{x \mid -4 \le x \le 3\}$; $[-4, 3]$

Range: $\{y \mid -3 \le y \le 3\}$; $[-3, 3]$

b. Intercept: $(0, 0)$

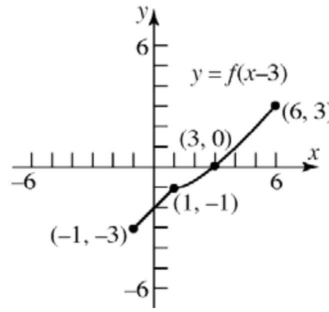
c. $f(-2) = -1$

d. $f(x) = -3$ when $x = -4$

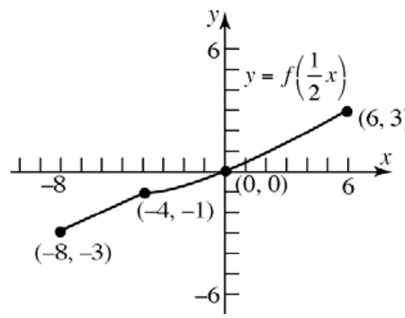
e. $f(x) > 0$ when $0 < x \le 3$

$\{x \mid 0 < x \le 3\}$

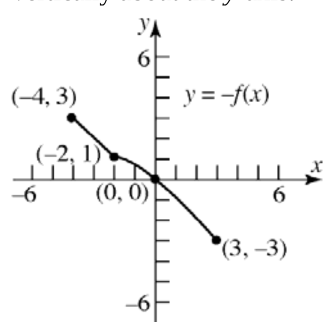
f. To graph $y = f(x-3)$, shift the graph of f horizontally 3 units to the right.



g. To graph $y = f\left(\frac{1}{2}x\right)$, stretch the graph of f horizontally by a factor of 2.



h. To graph $y = -f(x)$, reflect the graph of f vertically about the y-axis.



26. **a.** Domain: $\{x \mid -5 \le x \le 4\}$; $[-5, 4]$

Range: $\{y \mid -3 \le y \le 1\}$; $[-3, 1]$

b. $g(-1) = 1$

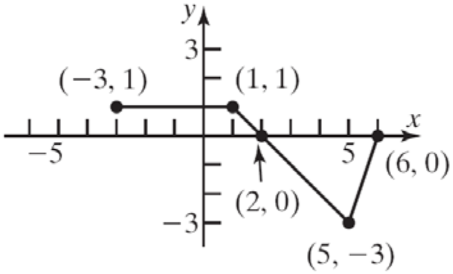
c. Intercepts: $(0, 0)$, $(4, 0)$

d. $g(x) = -3$ when $x = 3$

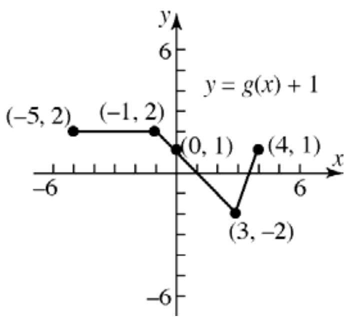
e. $g(x) > 0$ when $-5 \le x < 0$

$\{x \mid -5 \le x < 0\}$

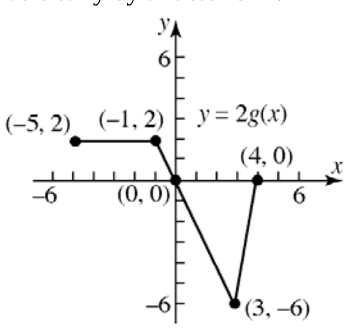
f. To graph $y = g(x-2)$, shift the graph of g horizontally 2 units to the right.



g. To graph $y = g(x)+1$, shift the graph of g vertically up 1 unit.



h. To graph $y = 2g(x)$, stretch the graph of g vertically by a factor of 2.



27. a. Domain: $\{x \mid -4 \le x \le 4\}$; $[-4, 4]$

Range: $\{y \mid -3 \le y \le 1\}$; $[-3, 1]$

b. Increasing: $(-4, -1)$ and $(3, 4)$;

Decreasing: $(-1, 3)$

c. Local minimum is -3 when $x = 3$;
Local maximum is 1 when $x = -1$.
Note that $x = 4$ and $x = -4$ do not yield local extrema because there is no open interval that contains either value.

d. The graph is not symmetric with respect to the x-axis, the y-axis or the origin.

e. The function is neither even nor odd.

f. x-intercepts: $-2, 0, 4$
y-intercept: 0

28. a. Domain: $\{x \mid x \text{ is any real number}\}$

Range: $\{y \mid y \text{ is any real number}\}$

b. Increasing: $(-\infty, -2)$ and $(2, \infty)$;

Decreasing: $(-2, 2)$

c. Local minimum is -1 at $x = 2$;
Local maximum is 1 at $x = -2$

d. The graph is symmetric with respect to the origin.

e. The function is odd.

f. x-intercepts: $-3, 0, 3$;
y-intercept: 0

29. $f(x) = x^3 - 4x$

$$f(-x) = (-x)^3 - 4(-x) = -x^3 + 4x$$
$$= -\left(x^3 - 4x\right) = -f(x)$$

f is odd.

30. $g(x) = \dfrac{4 + x^2}{1 + x^4}$

$$g(-x) = \frac{4 + (-x)^2}{1 + (-x)^4} = \frac{4 + x^2}{1 + x^4} = g(x)$$

g is even.

31. $h(x) = \dfrac{1}{x^4} + \dfrac{1}{x^2} + 1$

$$h(-x) = \frac{1}{(-x)^4} + \frac{1}{(-x)^2} + 1 = \frac{1}{x^4} + \frac{1}{x^2} + 1 = h(x)$$

h is even.

32. $F(x) = \sqrt{1 - x^3}$

$$F(-x) = \sqrt{1 - (-x)^3} = \sqrt{1 + x^3} \ne F(x) \text{ or } -F(x)$$

F is neither even nor odd.

33. $G(x) = 1 - x + x^3$

$G(-x) = 1 - (-x) + (-x)^3$

$= 1 + x - x^3 \neq -G(x) \text{ or } G(x)$

G is neither even nor odd.

34. $H(x) = 1 + x + x^2$

$H(-x) = 1 + (-x) + (-x)^2$

$= 1 - x + x^2 \neq -H(x) \text{ or } H(x)$

H is neither even nor odd.

35. $f(x) = \dfrac{x}{1+x^2}$

$f(-x) = \dfrac{-x}{1+(-x)^2} = \dfrac{-x}{1+x^2} = -f(x)$

f is odd.

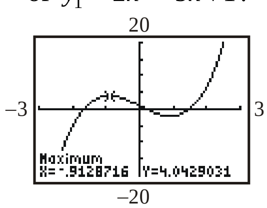
36. $g(x) = \dfrac{1+x^2}{x^3}$

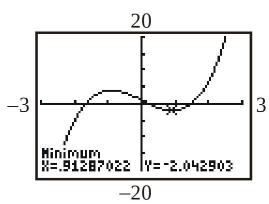
$g(-x) = \dfrac{1+(-x)^2}{(-x)^3} = \dfrac{1+x^2}{-x^3} = -\dfrac{1+x^2}{x^3} = -g(x)$

g is odd.

37. $f(x) = 2x^3 - 5x + 1$ on the interval $(-3, 3)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = 2x^3 - 5x + 1$.





local maximum: 4.04 when $x \approx -0.91$

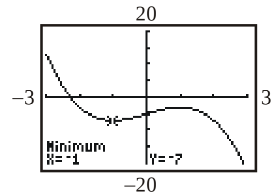
local minimum: -2.04 when $x = 0.91$

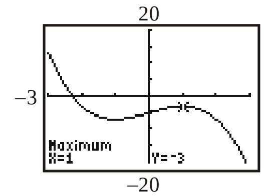
f is increasing on: $(-3, -0.91)$ and $(0.91, 3)$;

f is decreasing on: $(-0.91, 0.91)$.

38. $f(x) = -x^3 + 3x - 5$ on the interval $(-3, 3)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = -x^3 + 3x - 5$.





local maximum: -3 when $x = 1$

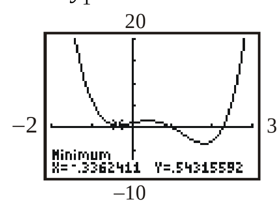
local minimum: -7 when $x = -1$

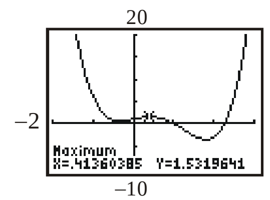
f is increasing on: $(-1, 1)$;

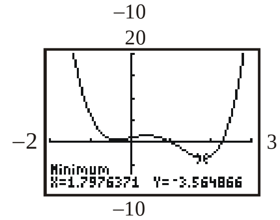
f is decreasing on: $(-3, -1)$ and $(1, 3)$.

39. $f(x) = 2x^4 - 5x^3 + 2x + 1$ on the interval $(-2, 3)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = 2x^4 - 5x^3 + 2x + 1$.







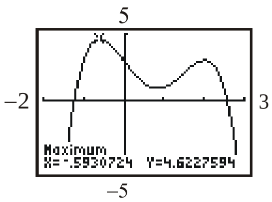
local maximum: 1.53 when $x = 0.41$

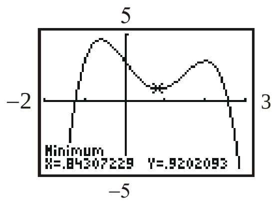
local minima: 0.54 when $x = -0.34$, -3.56 when $x = 1.80$

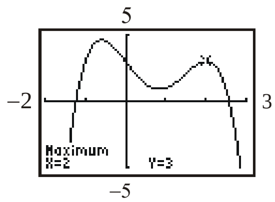
f is increasing on: $(-0.34, 0.41)$ and $(1.80, 3)$;

f is decreasing on: $(-2, -0.34)$ and $(0.41, 1.80)$.

40. $f(x) = -x^4 + 3x^3 - 4x + 3$ on the interval $(-2, 3)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = -x^4 + 3x^3 - 4x + 3$.







local maximum: 4.62 when $x = -0.59$, 3 when $x = 2$
local minimum: 0.92 when $x = 0.84$
f is increasing on: $(-2, -0.59)$ and $(0.84, 2)$;
f is decreasing on: $(-0.59, 0.84)$ and $(2, 3)$.

41. $f(x) = 8x^2 - x$

a. $$\frac{f(2) - f(1)}{2 - 1} = \frac{8(2)^2 - 2 - [8(1)^2 - 1]}{1} = 32 - 2 - (7) = 23$$

b. $$\frac{f(1) - f(0)}{1 - 0} = \frac{8(1)^2 - 1 - [8(0)^2 - 0]}{1} = 8 - 1 - (0) = 7$$

c. $$\frac{f(4) - f(2)}{4 - 2} = \frac{8(4)^2 - 4 - [8(2)^2 - 2]}{2} = \frac{128 - 4 - (30)}{2} = \frac{94}{2} = 47$$

42. $f(x) = 2x^3 + x$

a. $$\frac{f(2) - f(1)}{2 - 1} = \frac{2(2)^3 + 2 - \left(2(1)^3 + 1\right)}{1} = 16 + 2 - (3) = 15$$

b. $$\frac{f(1) - f(0)}{1 - 0} = \frac{2(1)^3 + 1 - \left(2(0)^3 + 0\right)}{1} = 2 + 1 - (0) = 3$$

c. $$\frac{f(4) - f(2)}{4 - 2} = \frac{2(4)^3 + 4 - (2(2)^3 + 2)}{2} = \frac{128 + 4 - (18)}{2} = \frac{114}{2} = 57$$

43. $f(x) = 2 - 5x$
$$\frac{f(3) - f(2)}{3 - 2} = \frac{\left[2 - 5(3)\right] - \left[2 - 5(2)\right]}{3 - 2} = \frac{(2 - 15) - (2 - 10)}{1} = -13 - (-8) = -5$$

44. $f(x) = 2x^2 + 7$
$$\frac{f(3) - f(2)}{3 - 2} = \frac{\left[2(3)^2 + 7\right] - \left[2(2)^2 + 7\right]}{3 - 2} = \frac{(18 + 7) - (8 + 7)}{1} = 25 - 15 = 10$$

45. $f(x) = 3x - 4x^2$
$$\frac{f(3) - f(2)}{3 - 2} = \frac{\left[3(3) - 4(3)^2\right] - \left[3(2) - 4(2)^2\right]}{3 - 2} = \frac{(9 - 36) - (6 - 16)}{1} = -27 + 10 = -17$$

46. $f(x) = x^2 - 3x + 2$
$$\frac{f(3) - f(2)}{3 - 2} = \frac{\left[(3)^2 - 3(3) + 2\right] - \left[(2)^2 - 3(2) + 2\right]}{3 - 2} = \frac{(9 - 9 + 2) - (4 - 6 + 2)}{1} = 2 - 0 = 2$$

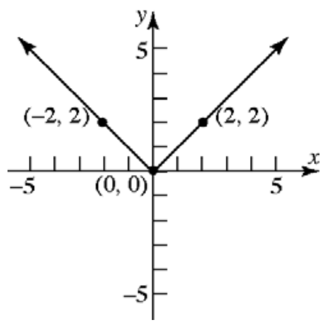
47. The graph does not pass the Vertical Line Test and is therefore not a function.

48. The graph passes the Vertical Line Test and is therefore a function.

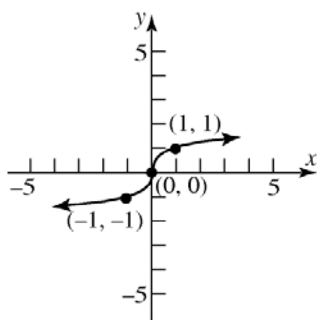
49. The graph passes the Vertical Line Test and is therefore a function.

50. The graph passes the Vertical Line Test and is therefore a function.

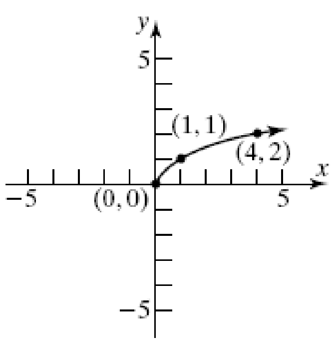
51. $f(x) = |x|$



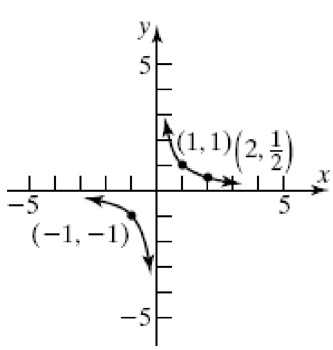
52. $f(x) = \sqrt[3]{x}$



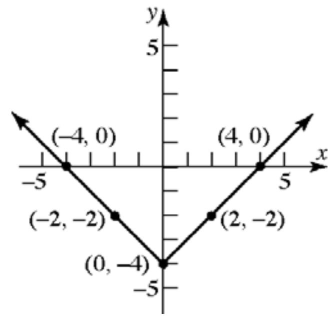
53. $f(x) = \sqrt{x}$



54. $f(x) = \dfrac{1}{x}$

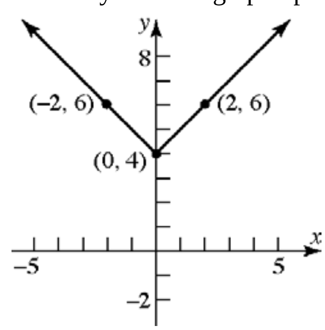


55. $F(x) = |x| - 4$. Using the graph of $y = |x|$, vertically shift the graph downward 4 units.



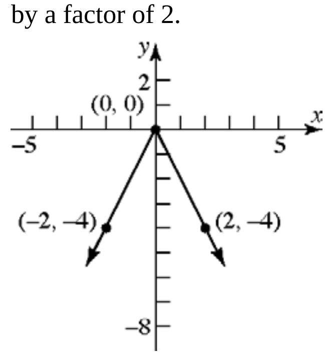
Intercepts: (–4,0), (4,0), (0,–4)
Domain: $\{x \mid x \text{ is any real number}\}$
Range: $\{y \mid y \geq -4\}$ or $[-4, \infty)$

56. $f(x) = |x| + 4$. Using the graph of $y = |x|$, vertically shift the graph upward 4 units.



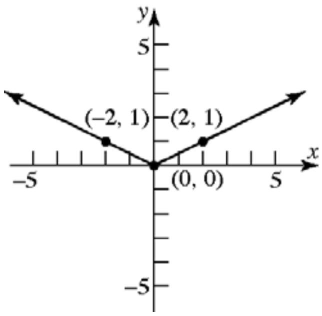
Intercepts: (0, 4)
Domain: $\{x \mid x \text{ is any real number}\}$
Range: $\{y \mid y \geq 4\}$ or $[4, \infty)$

57. $g(x) = -2|x|$. Reflect the graph of $y = |x|$ about the *x*-axis and vertically stretch the graph by a factor of 2.



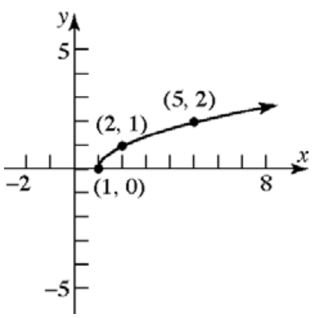
Intercepts: (0, 0)
Domain: $\{x \mid x \text{ is any real number}\}$
Range: $\{y \mid y \leq 0\}$ or $(-\infty, 0]$

58. $g(x)=\frac{1}{2}|x|$. Using the graph of $y=|x|$, vertically shrink the graph by a factor of $\frac{1}{2}$.



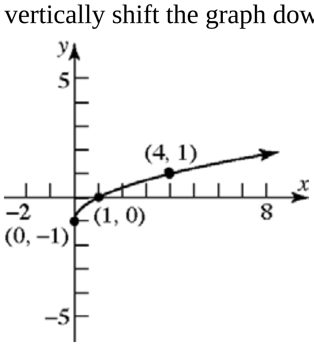
Intercepts: (0, 0)
Domain: $\{x\,|\,x \text{ is any real number}\}$
Range: $\{y\,|\,y \ge 0\}$ or $[0, \infty)$

59. $h(x)=\sqrt{x-1}$. Using the graph of $y=\sqrt{x}$, horizontally shift the graph to the right 1 unit.



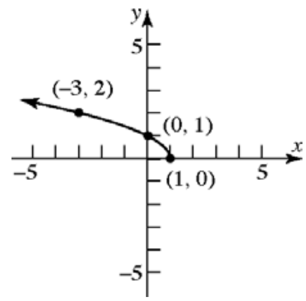
Intercept: (1, 0)
Domain: $\{x\,|\,x \ge 1\}$ or $[1, \infty)$
Range: $\{y\,|\,y \ge 0\}$ or $[0, \infty)$

60. $h(x)=\sqrt{x}-1$. Using the graph of $y=\sqrt{x}$, vertically shift the graph downward 1 unit.



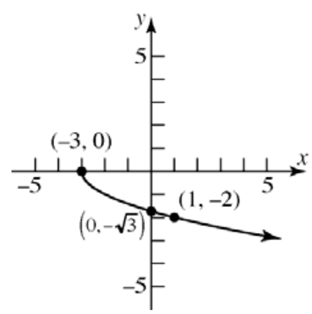
Intercepts: (1, 0), (0, –1)
Domain: $\{x\,|\,x \ge 0\}$ or $[0, \infty)$
Range: $\{y\,|\,y \ge -1\}$ or $[-1, \infty)$

61. $f(x)=\sqrt{1-x}=\sqrt{-(x-1)}$. Reflect the graph of $y=\sqrt{x}$ about the *y*-axis and horizontally shift the graph to the right 1 unit.



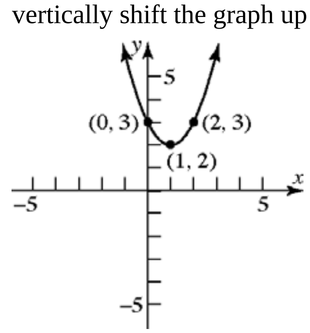
Intercepts: (1, 0), (0, 1)
Domain: $\{x\,|\,x \le 1\}$ or $(-\infty, 1]$
Range: $\{y\,|\,y \ge 0\}$ or $[0, \infty)$

62. $f(x)=-\sqrt{x+3}$. Using the graph of $y=\sqrt{x}$, horizontally shift the graph to the left 3 units, and reflect on the *x*-axis.



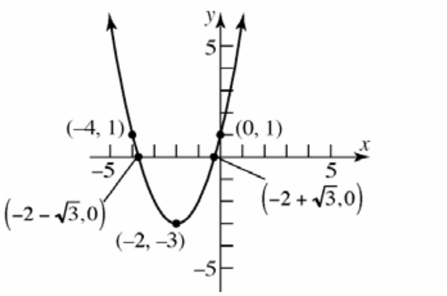
Intercepts: (–3, 0), $\left(0, -\sqrt{3}\right)$
Domain: $\{x\,|\,x \ge -3\}$ or $[-3, \infty)$
Range: $\{y\,|\,y \le 0\}$ or $(-\infty, 0]$

63. $h(x)=(x-1)^2+2$. Using the graph of $y=x^2$, horizontally shift the graph to the right 1 unit and vertically shift the graph up 2 units.



Intercepts: (0, 3)
Domain: $\{x\,|\,x \text{ is any real number}\}$
Range: $\{y\,|\,y \ge 2\}$ or $[2, \infty)$

64. $h(x)=(x+2)^2-3$. Using the graph of $y=x^2$, horizontally shift the graph to the left 2 units and vertically shift the graph down 3 units.

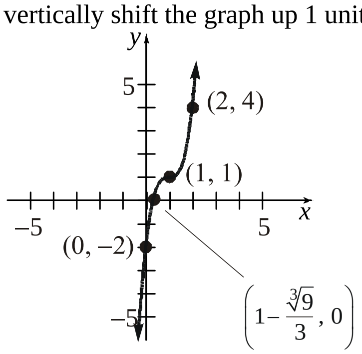


Intercepts: (0, 1), $\left(-2+\sqrt{3},0\right)$, $\left(-2-\sqrt{3},0\right)$

Domain: $\{x \mid x \text{ is any real number}\}$

Range: $\{y \mid y \geq -3\}$ or $[-3,\infty)$

65. $g(x)=3(x-1)^3+1$. Using the graph of $y=x^3$, horizontally shift the graph to the right 1 unit vertically stretch the graph by a factor of 3, and vertically shift the graph up 1 unit.



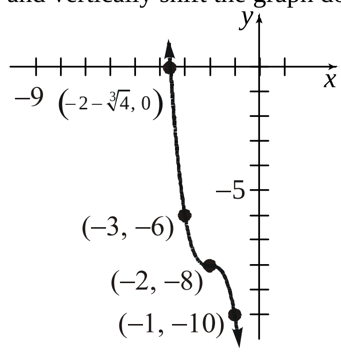
Intercepts: (0,–2), $\left(1-\dfrac{\sqrt[3]{9}}{3},0\right)\approx(0.3,0)$

Domain: $\{x \mid x \text{ is any real number}\}$

Range: $\{y \mid y \text{ is any real number}\}$

66. $g(x)=-2(x+2)^3-8$

Using the graph of $y=x^3$, horizontally shift the graph to the left 2 units, vertically stretch the graph by a factor of 2, reflect about the x-axis, and vertically shift the graph down 8 units.



Intercepts: (0,–24), $\left(-2-\sqrt[3]{4},0\right)\approx(-3.6,0)$

Domain: $\{x \mid x \text{ is any real number}\}$

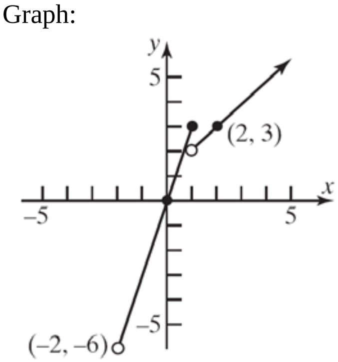
Range: $\{y \mid y \text{ is any real number}\}$

67. $f(x)=\begin{cases}3x & \text{if } -2<x\leq 1\\ x+1 & \text{if } x>1\end{cases}$

a. Domain: $\{x \mid x>-2\}$ or $(-2,\infty)$

b. Intercept: $(0,0)$

c. Graph:



d. Range: $\{y \mid y>-6\}$ or $(-6,\infty)$

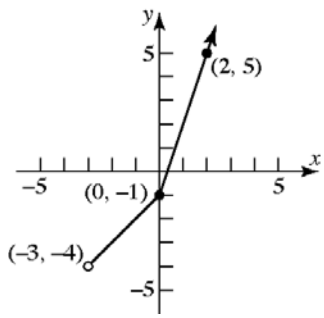
e. There is a jump in the graph at $x=1$. Therefore, the function is not continuous.

68. $f(x)=\begin{cases} x-1 & \text{if } -3<x<0 \\ 3x-1 & \text{if } x\ge 0 \end{cases}$

a. Domain: $\{x \mid x>-3\}$ or $(-3,\infty)$

b. Intercepts: $\left(\frac{1}{3},0\right)$, $(0,-1)$

c. Graph:



d. Range: $\{y \mid y>-4\}$ or $(-4,\infty)$

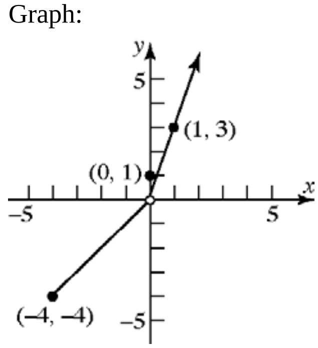
e. There are no holes, gaps, or jumps over the domain of the function. Therefore, the function is continuous over its domain.

69. $f(x)=\begin{cases} x & \text{if } -4\le x<0 \\ 1 & \text{if } x=0 \\ 3x & \text{if } x>0 \end{cases}$

a. Domain: $\{x \mid x\ge -4\}$ or $[-4,\infty)$

b. Intercept: (0, 1)

c. Graph:



d. Range: $\{y \mid y\ge -4,\ y\ne 0\}$

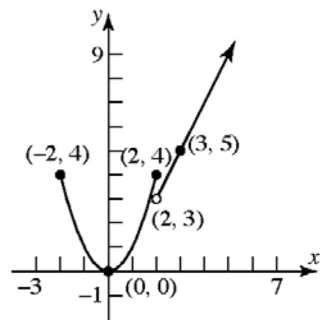
e. There is a jump at $x=0$. Therefore, the function is not continuous.

70. $f(x)=\begin{cases} x^2 & \text{if } -2\le x\le 2 \\ 2x-1 & \text{if } x>2 \end{cases}$

a. Domain: $\{x \mid x\ge -2\}$ or $[-2,\infty)$

b. Intercept: (0, 0)

c. Graph:



d. Range: $\{y \mid y\ge 0\}$ or $[0,\infty)$

e. There is a jump at $x=2$. Therefore, the function is not continuous.

71. $f(x)=\dfrac{Ax+5}{6x-2}$ and $f(1)=4$

$$\frac{A(1)+5}{6(1)-2}=4$$
$$\frac{A+5}{4}=4$$
$$A+5=16$$
$$A=11$$

72. $g(x)=\dfrac{A}{x}+\dfrac{8}{x^2}$ and $g(-1)=0$

$$\frac{A}{-1}+\frac{8}{(-1)^2}=0$$
$$-A+8=0$$
$$A=8$$

73. a. The printed region is a rectangle. Its area is given by

$A = (\text{length})(\text{width}) = (11 - 2x)(8.5 - 2x)$

$A(x) = (11 - 2x)(8.5 - 2x)$

b. For the domain of $A(x) = (11 - 2x)(8.5 - 2x)$ recall that the dimensions of a rectangle must be non-negative.

$x \geq 0$ and $11 - 2x > 0$ and $8.5 - 2x > 0$

$-2x > -11 \qquad -2x > 8.5$

$x < 5.5 \qquad x < 4.25$

The domain is given by $0 \leq x < 4.25$.

The range of $A(x) = (11 - 2x)(8.5 - 2x)$ is given by $A(4.25) < A \leq A(0) \Rightarrow$ $0 < A \leq 93.5$.

c. $A(1) = (11 - 2(1))(8.5 - 2(1))$

$= 9 \cdot 6.5 = 58.5 \text{ in}^2$

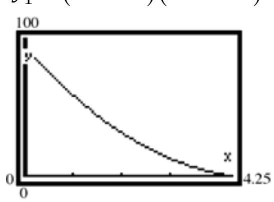
$A(1.2) = (11 - 2(1.2))(8.5 - 2(1.2))$

$= 8.6 \cdot 6.1 = 52.46 \text{ in}^2$

$A(1.5) = (11 - 2(1.5))(8.5 - 2(1.5))$

$= 8 \cdot 5.5 = 44 \text{ in}^2$

d. $y_1 = (11 - 2x)(8.5 - 2x)$



e. Using TRACE,

$A \approx 70$ when $x \approx 0.643$ inches

$A \approx 50$ when $x \approx 1.28$ inches

74. a. $x^2 h = 10 \Rightarrow h = \dfrac{10}{x^2}$

$A(x) = 2x^2 + 4xh$

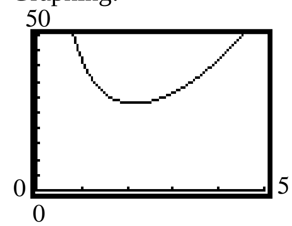
$= 2x^2 + 4x\left(\dfrac{10}{x^2}\right)$

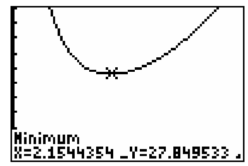
$= 2x^2 + \dfrac{40}{x}$

b. $A(1) = 2 \cdot 1^2 + \dfrac{40}{1} = 2 + 40 = 42 \text{ ft}^2$

c. $A(2) = 2 \cdot 2^2 + \dfrac{40}{2} = 8 + 20 = 28 \text{ ft}^2$

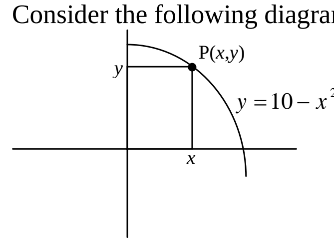
d. Graphing:





The area is smallest when $x \approx 2.15$ feet.

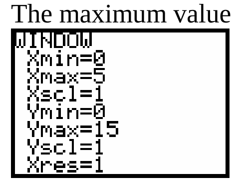
75. a. Consider the following diagram:

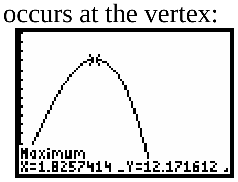


The area of the rectangle is $A = xy$. Thus, the area function for the rectangle is:

$A(x) = x(10 - x^2) = -x^3 + 10x$

b. The maximum value occurs at the vertex:





The maximum area is roughly:

$A(1.83) = -(1.83)^3 + 10(1.83)$

≈ 12.17 square units

Chapter 2 Test

1. a. $\{(2,5),(4,6),(6,7),(8,8)\}$

This relation is a function because there are no ordered pairs that have the same first element and different second elements.

Domain: $\{2,4,6,8\}$

Range: $\{5,6,7,8\}$

b. $\{(1,3),(4,-2),(-3,5),(1,7)\}$

This relation is not a function because there are two ordered pairs that have the same first element but different second elements.

c. This relation is not a function because the graph fails the vertical line test.

d. This relation is a function because it passes the vertical line test.

Domain: $\{x \mid x \text{ is any real number}\}$

Range: $\{y \mid y \geq 2\}$ or $[2, \infty)$

2. $f(x)=\sqrt{4-5x}$

The function tells us to take the square root of $4-5x$. Only nonnegative numbers have real square roots so we need $4-5x \geq 0$.

$$4-5x \geq 0$$
$$4-5x-4 \geq 0-4$$
$$-5x \geq -4$$
$$\frac{-5x}{-5} \leq \frac{-4}{-5}$$
$$x \leq \frac{4}{5}$$

Domain: $\left\{x \middle| x \leq \frac{4}{5}\right\}$ or $\left(-\infty, \frac{4}{5}\right]$

$$f(-1)=\sqrt{4-5(-1)}=\sqrt{4+5}=\sqrt{9}=3$$

3. $g(x)=\dfrac{x+2}{|x+2|}$

The function tells us to divide $x+2$ by $|x+2|$. Division by 0 is undefined, so the denominator can never equal 0. This means that $x \neq -2$.

Domain: $\{x \mid x \neq -2\}$

$$g(-1)=\frac{(-1)+2}{|(-1)+2|}=\frac{1}{|1|}=1$$

4. $h(x)=\dfrac{x-4}{x^2+5x-36}$

The function tells us to divide $x-4$ by $x^2+5x-36$. Since division by 0 is not defined, we need to exclude any values which make the denominator 0.

$$x^2+5x-36=0$$
$$(x+9)(x-4)=0$$
$$x=-9 \text{ or } x=4$$

Domain: $\{x \mid x \neq -9, x \neq 4\}$

(note: there is a common factor of $x-4$ but we must determine the domain prior to simplifying)

$$h(-1)=\frac{(-1)-4}{(-1)^2+5(-1)-36}=\frac{-5}{-40}=\frac{1}{8}$$

5. a. To find the domain, note that all the points on the graph will have an *x*-coordinate between -5 and 5, inclusive. To find the range, note that all the points on the graph will have a *y*-coordinate between -3 and 3, inclusive.

Domain: $\{x \mid -5 \leq x \leq 5\}$ or $[-5, 5]$

Range: $\{y \mid -3 \leq y \leq 3\}$ or $[-3, 3]$

b. The intercepts are $(0,2)$, $(-2,0)$, and $(2,0)$.

x-intercepts: -2, 2

y-intercept: 2

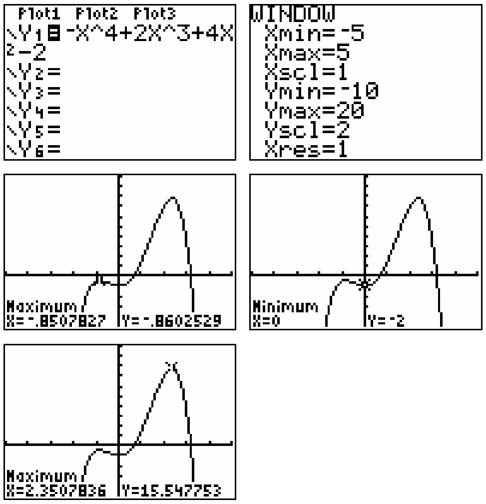
c. $f(1)$ is the value of the function when $x=1$. According to the graph, $f(1)=3$.

d. Since $(-5,-3)$ and $(3,-3)$ are the only points on the graph for which $y=f(x)=-3$, we have $f(x)=-3$ when $x=-5$ and $x=3$.

e. To solve $f(x)<0$, we want to find *x*-values such that the graph is below the x-axis. The graph is below the x-axis for values in the domain that are less than -2 and greater than 2. Therefore, the solution set is $\{x \mid -5 \leq x < -2 \text{ or } 2 < x \leq 5\}$. In interval notation we would write the solution set as $[-5,-2) \cup (2,5]$.

6. $f(x) = -x^4 + 2x^3 + 4x^2 - 2$

We set Xmin = -5 and Xmax = 5. The standard Ymin and Ymax will not be good enough to see the whole picture so some adjustment must be made.



We see that the graph has a local maximum of -0.86 (rounded to two places) when $x = -0.85$ and another local maximum of 15.55 when $x = 2.35$. There is a local minimum of -2 when $x = 0$. Thus, we have

Local maxima: $f(-0.85) \approx -0.86$

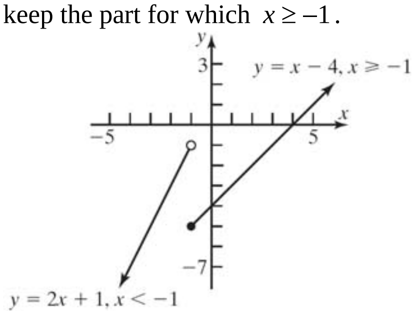
$$f(2.35) \approx 15.55$$

Local minima: $f(0) = -2$

The function is increasing on the intervals $(-5, -0.85)$ and $(0, 2.35)$ and decreasing on the intervals $(-0.85, 0)$ and $(2.35, 5)$.

7. a. $f(x) = \begin{cases} 2x+1 & x < -1 \\ x-4 & x \geq -1 \end{cases}$

To graph the function, we graph each "piece". First we graph the line $y = 2x+1$ but only keep the part for which $x < -1$. Then we plot the line $y = x-4$ but only keep the part for which $x \geq -1$.



b. To find the intercepts, notice that the only piece that hits either axis is $y = x-4$.

$y = x-4$	$y = x-4$
$y = 0-4$	$0 = x-4$
$y = -4$	$4 = x$

The intercepts are $(0, -4)$ and $(4, 0)$.

c. To find $g(-5)$ we first note that $x = -5$ so we must use the first "piece" because $-5 < -1$.

$g(-5) = 2(-5)+1 = -10+1 = -9$

d. To find $g(2)$ we first note that $x = 2$ so we must use the second "piece" because $2 \geq -1$.

$g(2) = 2-4 = -2$

8. The average rate of change from 3 to 4 is given by

$$\frac{\Delta y}{\Delta x} = \frac{f(4) - f(3)}{4-3}$$
$$= \frac{\left(3(4)^2 - 2(4) + 4\right) - \left(3(3)^2 - 2(3) + 4\right)}{4-3}$$
$$= \frac{44-25}{4-3} = \frac{19}{1} = 19$$

9. a. $f - g = \left(2x^2+1\right) - (3x-2)$

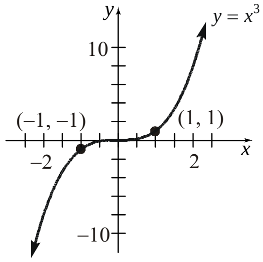
$$= 2x^2 + 1 - 3x + 2 = 2x^2 - 3x + 3$$

b. $f \cdot g = \left(2x^2+1\right)(3x-2) = 6x^3 - 4x^2 + 3x - 2$

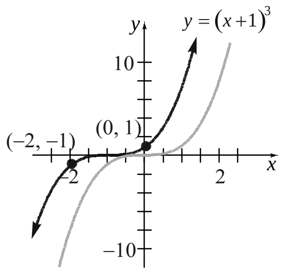
c. $f(x+h) - f(x)$

$$= \left(2(x+h)^2 + 1\right) - \left(2x^2+1\right)$$
$$= \left(2\left(x^2 + 2xh + h^2\right) + 1\right) - \left(2x^2+1\right)$$
$$= 2x^2 + 4xh + 2h^2 + 1 - 2x^2 - 1$$
$$= 4xh + 2h^2$$

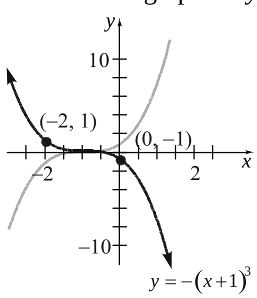
10. **a.** The basic function is $y = x^3$ so we start with the graph of this function.



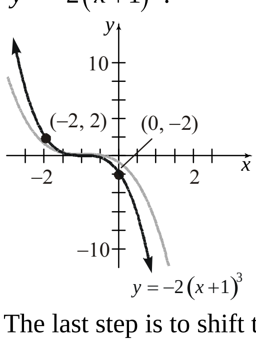
Next we shift this graph 1 unit to the left to obtain the graph of $y = (x+1)^3$.



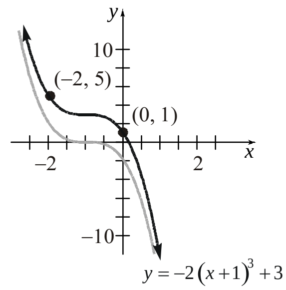
Next we reflect this graph about the x-axis to obtain the graph of $y = -(x+1)^3$.



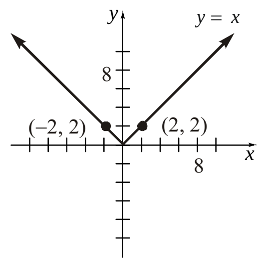
Next we stretch this graph vertically by a factor of 2 to obtain the graph of $y = -2(x+1)^3$.



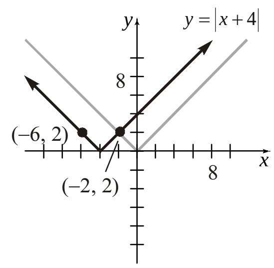
The last step is to shift this graph up 3 units to obtain the graph of $y = -2(x+1)^3 + 3$.



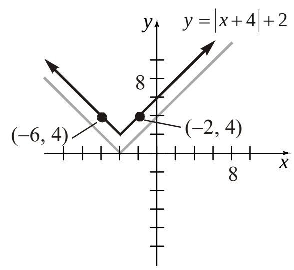
b. The basic function is $y = |x|$ so we start with the graph of this function.



Next we shift this graph 4 units to the left to obtain the graph of $y = |x+4|$.

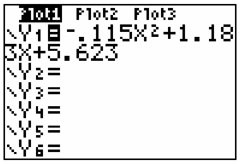


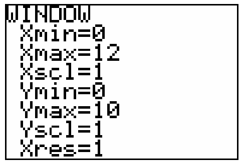
Next we shift this graph up 2 units to obtain the graph of $y = |x+4| + 2$.

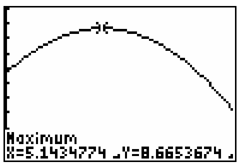


11. a. $r(x) = -0.115x^2 + 1.183x + 5.623$

For the years 1992 to 2004, we have values of x between 0 and 12. Therefore, we can let Xmin = 0 and Xmax = 12. Since r is the interest rate as a percent, we can try letting Ymin = 0 and Ymax = 10.



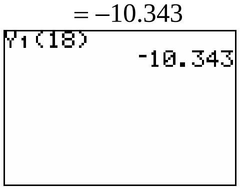




The highest rate during this period appears to be 8.67%, occurring in 1997 ($x \approx 5$).

b. For 2010, we have $x = 2010 - 1992 = 18$.

$$r(18) = -0.115(18)^2 + 1.183(18) + 5.623$$
$$= -10.343$$



The model predicts that the interest rate will be -10.343%. This is not a reasonable value since it implies that the bank would be paying interest to the borrower.

12. a. Let x = width of the rink in feet. Then the length of the rectangular portion is given by $2x - 20$. The radius of the semicircular portions is half the width, or $r = \frac{x}{2}$.

To find the volume, we first find the area of the surface and multiply by the thickness of the ice. The two semicircles can be combined to form a complete circle, so the area is given by

$$A = l \cdot w + \pi r^2$$
$$= (2x - 20)(x) + \pi\left(\frac{x}{2}\right)^2$$
$$= 2x^2 - 20x + \frac{\pi x^2}{4}$$

We have expressed our measures in feet so we need to convert the thickness to feet as well.

$$0.75 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = \frac{0.75}{12} \text{ ft} = \frac{1}{16} \text{ ft}$$

Now we multiply this by the area to obtain the volume. That is,

$$V(x) = \frac{1}{16}\left(2x^2 - 20x + \frac{\pi x^2}{4}\right)$$
$$V(x) = \frac{x^2}{8} - \frac{5x}{4} + \frac{\pi x^2}{64}$$

b. If the rink is 90 feet wide, then we have $x = 90$.

$$V(90) = \frac{90^2}{8} - \frac{5(90)}{4} + \frac{\pi(90)^2}{64} \approx 1297.61$$

The volume of ice is roughly 1297.61 ft^3.

Chapter 2 Cumulative Review

1. $-5x + 4 = 0$
$$-5x = -4$$
$$x = \frac{-4}{-5} = \frac{4}{5}$$
The solution set is $\left\{\frac{4}{5}\right\}$.

2. $x^2 - 7x + 12 = 0$
$$(x-4)(x-3) = 0 \Rightarrow x = 4, x = 3$$
The solution set is $\{3, 4\}$.

3. $3x^2 - 5x - 2 = 0$
$$(3x+1)(x-2) = 0 \Rightarrow x = -\frac{1}{3}, x = 2$$
The solution set is $\left\{-\frac{1}{3}, 2\right\}$.

4. $4x^2 + 4x + 1 = 0$
$$(2x+1)(2x+1) = 0 \Rightarrow x = -\frac{1}{2}$$
The solution set is $\left\{-\frac{1}{2}\right\}$.

5. $4x^2 - 2x + 4 = 0 \Rightarrow 2x^2 - x + 2 = 0$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(2)}}{2(2)}$$
$$= \frac{1 \pm \sqrt{1-16}}{4} = \frac{1 \pm \sqrt{-15}}{4}$$
no real solution

6. $\sqrt[3]{1-x}=2$

$$\left(\sqrt[3]{1-x}\right)^3=2^3$$
$$1-x=8$$
$$-x=7$$
$$x=-7$$

The solution set is $\{-7\}$.

7. $\sqrt[5]{1-x}=2$

$$\left(\sqrt[5]{1-x}\right)^5=2^5$$
$$1-x=32$$
$$-x=31$$
$$x=-31$$

The solution set is $\{-31\}$.

8. $|2-3x|=1$

$$2-3x=1 \quad \text{or} \quad 2-3x=-1$$
$$-3x=-1 \quad \text{or} \quad -3x=-3$$
$$x=\frac{1}{3} \quad \text{or} \quad x=1$$

The solution set is $\left\{\frac{1}{3},1\right\}$.

9. $4x^2-2x+4=0 \Rightarrow 2x^2-x+2=0$

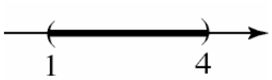
$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(2)(2)}}{2(2)}$$
$$=\frac{1\pm\sqrt{1-16}}{4}=\frac{1\pm\sqrt{-15}}{4}=\frac{1\pm\sqrt{15}i}{4}$$

The solution set is $\left\{\frac{1-\sqrt{15}i}{4},\frac{1+\sqrt{15}i}{4}\right\}$.

10. $-2<3x-5<7$

$$3<3x<12$$
$$\frac{3}{3}<x<\frac{12}{3}$$
$$1<x<4$$

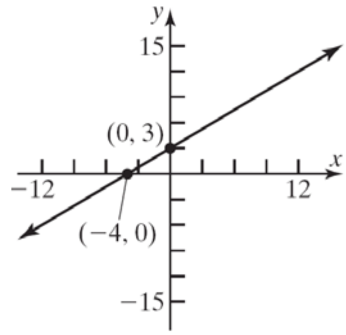
$\{x \mid 1<x<4\}$ or $(1,4)$



11. $-3x+4y=12 \Rightarrow 4y=3x+12$

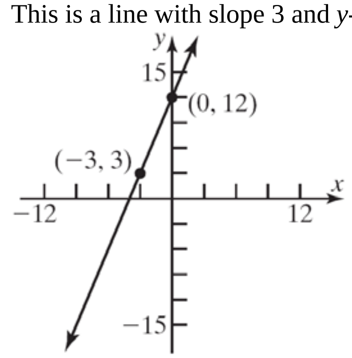
$$y=\frac{3}{4}x+3$$

This is a line with slope $\frac{3}{4}$ and y-intercept (0, 3).



12. $y=3x+12$

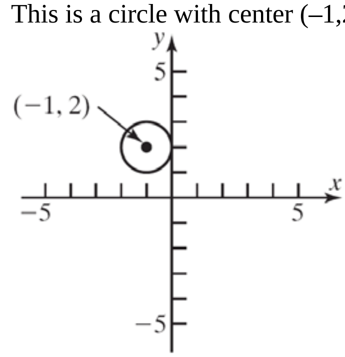
This is a line with slope 3 and y-intercept (0, 12).



13. $x^2+y^2+2x-4y+4=0$

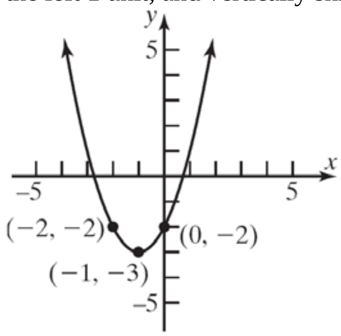
$$x^2+2x+y^2-4y=-4$$
$$(x^2+2x+1)+(y^2-4y+4)=-4+1+4$$
$$(x+1)^2+(y-2)^2=1$$
$$(x+1)^2+(y-2)^2=1^2$$

This is a circle with center (–1,2) and radius 1.



14. $y=(x+1)^2-3$

Using the graph of $y=x^2$, horizontally shift to the left 1 unit, and vertically shift down 3 units.



15. a. Domain: $\{x \mid -4 \le x \le 4\}$
Range: $\{y \mid -1 \le y \le 3\}$

b. Intercepts: $(-1,0)$, $(0,-1)$, $(1,0)$
x-intercepts: $-1, 1$
y-intercept: -1

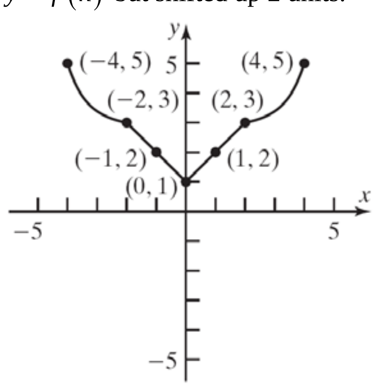
c. The graph is symmetric with respect to the *y*-axis.

d. When $x=2$, the function takes on a value of 1. Therefore, $f(2)=1$.

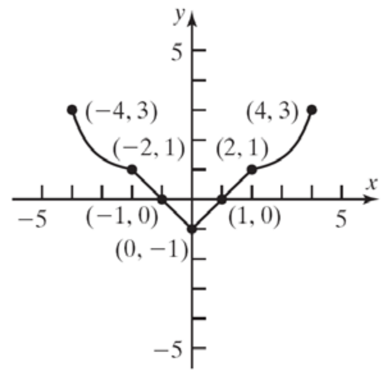
e. The function takes on the value 3 at $x=-4$ and $x=4$.

f. $f(x)<0$ means that the graph lies below the x-axis. This happens for *x* values between -1 and 1. Thus, the solution set is $\{x \mid -1<x<1\}$.

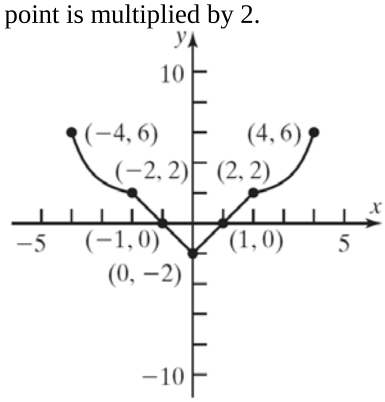
g. The graph of $y=f(x)+2$ is the graph of $y=f(x)$ but shifted up 2 units.



h. The graph of $y=f(-x)$ is the graph of $y=f(x)$ but reflected about the y-axis.



i. The graph of $y=2f(x)$ is the graph of $y=f(x)$ but stretched vertically by a factor of 2. That is, the coordinate of each point is multiplied by 2.



j. Since the graph is symmetric about the y-axis, the function is even.

k. The function is increasing on the open interval $(0,4)$.

l. The function is decreasing on the open interval $(-4,0)$.

m. There is a local minimum of -1 at $x=0$. There are no local maxima.

n. $\dfrac{f(4)-f(1)}{4-1}=\dfrac{3-0}{3}=\dfrac{3}{3}=1$
The average rate of change of the function from 1 to 4 is 1.

16. $d\left(P,Q\right)=\sqrt{\left(-1-4\right)^2+\left(3-\left(-2\right)\right)^2}$
$=\sqrt{\left(-5\right)^2+\left(5\right)^2}$
$=\sqrt{25+25}$
$=\sqrt{50}=5\sqrt{2}$

17. $y=x^3-3x+1$

(a) $\left(-2,-1\right)$
$\left(-2\right)^3-\left(3\right)\left(-2\right)+1=-8+6+1=-1$
$\left(-2,-1\right)$ is on the graph.

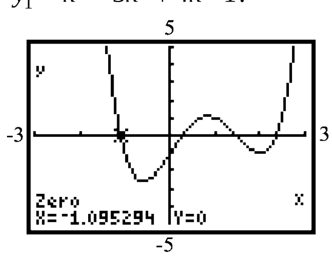
(b) $\left(2,3\right)$
$\left(2\right)^3-\left(3\right)\left(2\right)+1=8-6+1=3$
$\left(2,3\right)$ is on the graph.

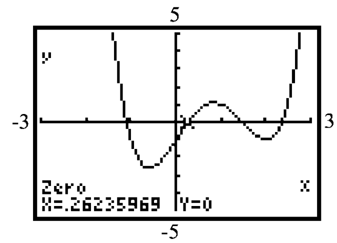
(c) $\left(3,1\right)$
$\left(3\right)^3-\left(3\right)\left(3\right)+1=27-9+1=19\neq 1$
$\left(3,1\right)$ is not on the graph.

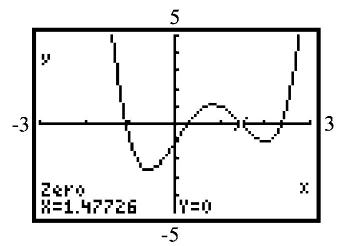
18. $y=3x^2+14x-5$
x-intercept(s): solve $3x^2+14x-5=0$
$\left(3x-1\right)\left(x+5\right)=0\Rightarrow x=\frac{1}{3}, x=-5$

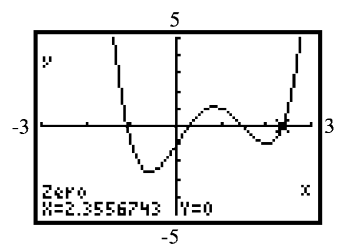
y-intercept: let $x=0$
$y=0^2+14\left(0\right)-5=-5$

19. Use ZERO (or ROOT) on the graph of $y_1=x^4-3x^3+4x-1$.



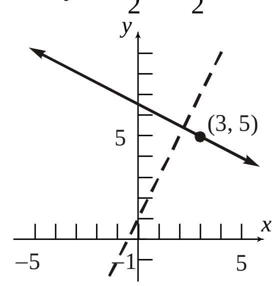






The solution set is $\left\{-1.10, 0.26, 1.48, 2.36\right\}$.

20. Perpendicular to $y=2x+1$;
Slope of perpendicular = $-\frac{1}{2}$; Containing $\left(3,5\right)$
$y-y_1=m(x-x_1)$
$y-5=-\frac{1}{2}(x-3)$
$y-5=-\frac{1}{2}x+\frac{3}{2}$
$y=-\frac{1}{2}x+\frac{13}{2}$



21. Yes, each x corresponds to exactly 1 y.

22. $f(x)=x^2-4x+1$

a. $f(2)=(2)^2-4(2)+1=4-8+1=-3$

b.
$$\begin{aligned} f(x)+f(2) &= x^2-4x+1+(2)^2-4(2)+1 \\ &= x^2-4x+1+4-8+1 \\ &= x^2-4x-2 \end{aligned}$$

c. $f(-x)=(-x)^2-4(-x)+1=x^2+4x+1$

d. $-f(x)=-(x^2-4x+1)=-x^2+4x-1$

e.
$$\begin{aligned} f(x+2) &= (x+2)^2-4(x+2)+1 \\ &= x^2+4x+4-4x-8+1 \\ &= x^2-3 \end{aligned}$$

f. $\dfrac{f(x+h)-f(x)}{h}, h\neq 0$

$$\begin{aligned} &\frac{f(x+h)-f(x)}{h} \\ &= \frac{(x+h)^2-4(x+h)+1-(x^2-4x+1)}{h} \\ &= \frac{x^2+2xh+h^2-4x-4h+1-x^2+4x-1}{h} \\ &= \frac{2xh+h^2-4h}{h}=\frac{h(2x+h-4)}{h} \\ &= 2x+h-4 \end{aligned}$$

23. $h(z)=\dfrac{3z-1}{z^2-6z-7}$

The denominator cannot be zero:

$$\begin{aligned} z^2-6z-7 &\neq 0 \\ (z+1)(z-7) &\neq 0 \\ z &\neq -1 \text{ or } 7 \end{aligned}$$

Domain: $\{z \mid z\neq -1, z\neq 7\}$

24. Yes, since the graph passes the Vertical Line Test.

25. $f(x)=\dfrac{x}{x+4}$

a. $f(1)=\dfrac{1}{1+4}=\dfrac{1}{5}\neq\dfrac{1}{4}$

$\left(1,\dfrac{1}{4}\right)$ is not on the graph of f

b. $f(-2)=\dfrac{-2}{-2+4}=\dfrac{-2}{2}=-1$

$(-2,-1)$ is on the graph of f

c. Solve for x:

$$\begin{aligned} \frac{x}{x+4} &= 2 \\ x &= 2(x+4) \\ x &= 2x+8 \\ -8 &= x \end{aligned}$$

$(-8,2)$ is on the graph of f.

26.
$$\begin{aligned} &6x^2(x^2+1)^{1/2}+2(x^2+1)^{3/2} \\ &= 2(x^2+1)^{1/2}\left(3x^2+(x^2+1)^1\right) \\ &= 2(x^2+1)^{1/2}(3x^2+x^2+1) \\ &= 2(x^2+1)^{1/2}(4x^2+1) \end{aligned}$$

27.
$$\begin{aligned} &2(x+5)^{1/2}+x(x+5)^{-1/2} \\ &= (x+5)^{-1/2}\left(2(x+5)^1+x\right) \\ &= (x+5)^{-1/2}(2x+10+x) \\ &= (x+5)^{-1/2}(3x+10) \\ &= \frac{3x+10}{(x+5)^{1/2}}=\frac{(3x+10)(x+5)^{1/2}}{x+5} \end{aligned}$$

Chapter 2 Projects

Project I

1. Plan A1: Total cost = \$39.99 x 24 = \$959.76
   Plan A2: Total cost = \$59.99 x 24 = \$1439.76
   Plan B1: Total cost = \$39.99 x 24 = \$959.76
   Plan B2: Total cost = \$49.99 x 24 = \$ 1199.76
   Plan C1: Total cost = \$59.99 x 24 = \$1439.76
   Plan C2: Total cost = \$69.99 x 24 = \$1679.76

2. <u>400 Anytime; 200 MTM; 4500 NW</u>
   All plans allow for 4500 night and weekend minutes free and at least 400 Anytime minutes. Although company B does not offer MTM minutes, the combined Anytime and MTM minutes does not exceed the base amount. Thus, the cost of usage is just the monthly fee.

   A1: \$39.99
   A2: \$59.99
   B1: \$39.99
   B2: \$49.99
   C1: \$59.99
   C2: \$69.99
   The best plan here is either plan A1 or B1 at \$39.99.

   <u>400 Anytime; 200 MTM; 5500 NW</u>
   The only plan that changes price from above when the night and weekend minutes increase to 5500 is A1. It only has 5000 free night and weekend minutes.

   A1: \$39.99 + 0.45(450) = \$242.49

   The best plan is B1. (Note: plan A1 only charges for 450 minutes because there were still 50 Anytime minutes remaining)

   <u>500 Anytime; 1000MTM; 2000 NW</u>
   All plans allow for at least 2000 night and weekend minutes free and at least 400 Anytime minutes. Company B does not offer MTM minutes

   A1: \$39.99 + \$0.45(50) = \$62.49
   A2: \$59.99
   B1: \$39.99 + \$0.40(900) = \$399.99
   B2: \$49.99 + \$0.40(500) = \$249.99
   C1: \$59.99
   C2: \$69.99
   The best plan here is either plan A2 or C1 at \$59.99.

3. For 850 minutes,
   A1: \$39.99 + 0.45(400) = \$219.99
   A2: \$59.99
   B1: \$39.99 + 0.40(250) = \$139.99
   B2: \$49.99
   C1: \$59.99 + 5(300/50) = \$89.99
   C2: \$69.99 + 5(50/50) = \$74.99
   The best priced plan is B2 at \$49.99.

   For 1050 minutes:
   A1: \$39.99 + 0.45(600) = \$309.99
   A2: \$59.99 + 0.40(150) = \$119.99
   B1: \$39.99 + 0.40(450) = \$219.99
   B2: \$49.99 + 0.40(50) = \$69.99
   C1: \$59.99 + 5(500/50) = \$109.99
   C2: \$69.99 + 5(250/50) = \$94.99
   The best priced plan is B2 at \$69.99.

4. $$\begin{matrix}\text{Monthly}\\ \text{cost}\end{matrix} = \begin{matrix}\text{Base}\\ \text{Price}\end{matrix} + \begin{pmatrix}\text{charge per}\\ \text{minute}\end{pmatrix}\begin{pmatrix}\text{\# of min. over}\\ \text{those included}\end{pmatrix}$$

   $$\text{A1: } C(x) = \begin{cases} 39.99 & 0 \le x \le 450 \\ 0.45x - 162.51 & x > 450 \end{cases}$$

   $$\text{A2: } C(x) = \begin{cases} 59.99 & 0 \le x \le 900 \\ 0.40x - 300.01 & x > 900 \end{cases}$$

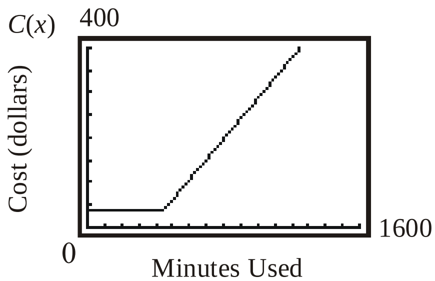
   $$\text{B1: } C(x) = \begin{cases} 39.99 & 0 \le x \le 600 \\ 0.40x - 200.01 & x > 600 \end{cases}$$

   $$\text{B2: } C(x) = \begin{cases} 49.99 & 0 \le x \le 1000 \\ 0.40x - 350.01 & x > 1000 \end{cases}$$

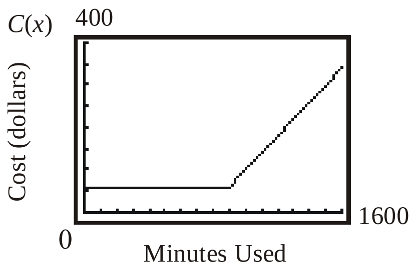
   C1:
   $$C(x) = \begin{cases} 59.99 & 0 \le x \le 550 \\ 59.99 + 5\{\text{int}\left[\left(x - 550\right)/50\right] + 1\} & 550 < x < 1050 \\ 0.10x + 4.99 & x \ge 1050 \end{cases}$$

   C2:
   $$C(x) = \begin{cases} 69.99 & 0 \le x \le 800 \\ 69.99 + 5\{\text{int}\left[\left(x - 800\right)/50\right] + 1\} & 800 < x < 1300 \\ 0.10x - 10.01 & x \ge 1300 \end{cases}$$

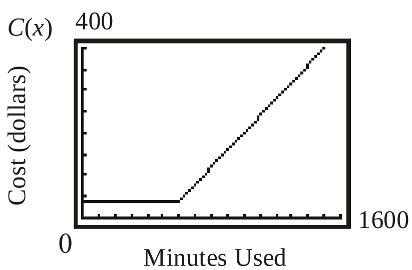
5. Graph for plan A1:



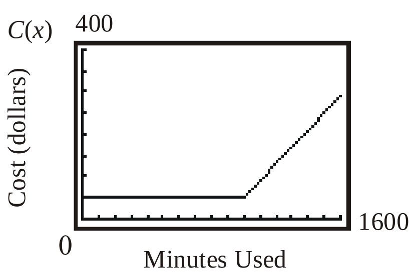
Graph for plan A2:



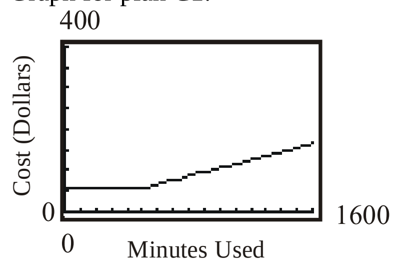
Graph for plan B1:



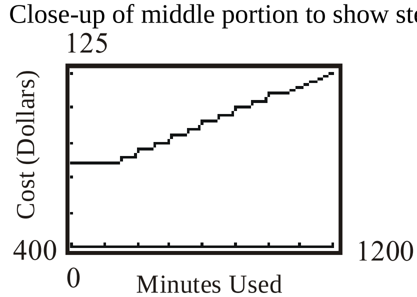
Graph for plan B2:



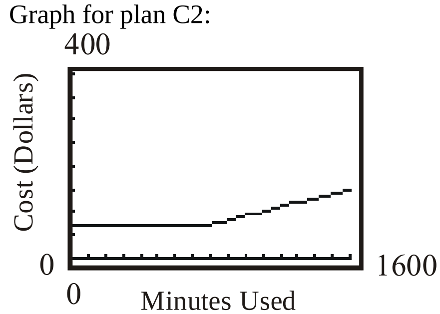
Graph for plan C1:



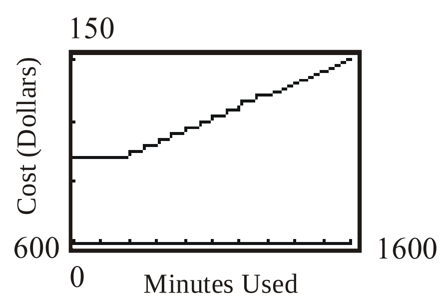
Close-up of middle portion to show steps:



Graph for plan C2:



Close-up of middle portion to show steps:



6. A1: $\dfrac{\$39.99}{450\,\text{min}} = \$0.089/\text{min}$

A2: $\dfrac{\$59.99}{900\,\text{min}} = \$0.067/\text{min}$

A2 is the better plan.

B1: $\dfrac{\$39.99}{600\,\text{min}} = \$0.067/\text{min}$

B2: $\dfrac{\$49.99}{1000\,\text{min}} = \$0.050/\text{min}$

B2 is the better plan.

C1: $\dfrac{\$59.99}{550\,\text{min}} = \$0.109/\text{min}$

C2: $\dfrac{\$69.99}{800\,\text{min}} = \$0.087/\text{min}$

C2 is the better plan.

7. Out of A2, B2, and C2, the best plan to choose is B2 since its $/min rate is best.

8. Answers will vary.

Project II

1. Silver: $C(x) = 20 + 0.16(x - 200) = 0.16x - 12$

$$C(x) = \begin{cases} 20 & 0 \le x \le 200 \\ 0.16x - 12 & x > 200 \end{cases}$$

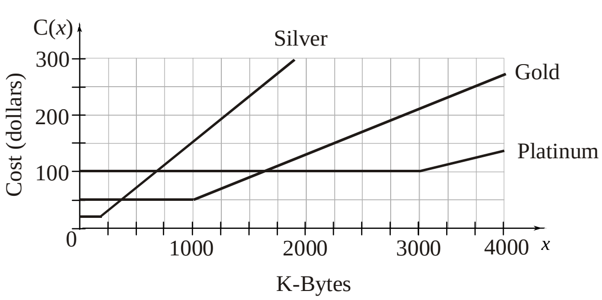
Gold: $C(x) = 50 + 0.08(x - 1000) = 0.08x - 30$

$$C(x) = \begin{cases} 50.00 & 0 \le x \le 1000 \\ 0.08x - 30 & x > 1000 \end{cases}$$

Platinum: $C(x) = 100 + 0.04(x - 3000)$

$= 0.04x - 20$

$$C(x) = \begin{cases} 100.00 & 0 \le x \le 3000 \\ 0.04x - 20 & x > 3000 \end{cases}$$



3. Let y = #K-bytes of service over the plan minimum.

Silver: $20 + 0.16y \le 50$

$0.16y \le 30$

$y \le 187.5$

Silver is the best up to $187.5 + 200 = 387.5$ K-bytes of service.

Gold: $50 + 0.08y \le 100$

$0.08y \le 50$

$y \le 625$

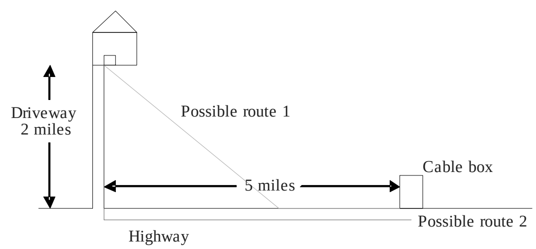
Gold is the best from 387.5 K-bytes to $625 + 1000 = 1625$ K-bytes of service.

Platinum: Platinum will be the best if more than 1625 K-bytes is needed.

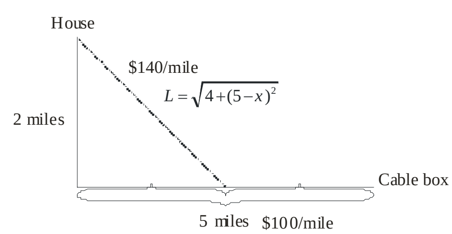
4. Answers will vary.

Project III

1.



2.



$C(x) = 100x + 140L$

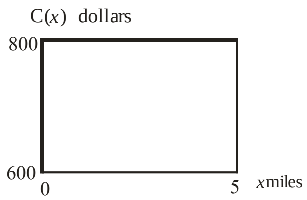
$C(x) = 100x + 140\sqrt{4 + (5 - x)^2}$

3.

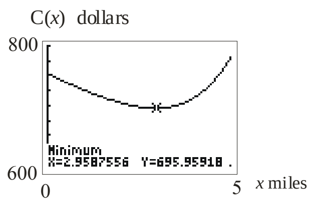
x	$C(x)$
0	$100(0) + 140\sqrt{4+25} \approx \$753.92$
1	$100(1) + 140\sqrt{4+16} \approx \$726.10$
2	$100(2) + 140\sqrt{4+9} \approx \$704.78$
3	$100(3) + 140\sqrt{4+4} \approx \$695.98$
4	$100(4) + 140\sqrt{4+1} \approx \$713.05$
5	$100(5) + 140\sqrt{4+0} = \$780.00$

The choice where the cable goes 3 miles down the road then cutting up to the house seems to yield the lowest cost.

4. Since all of the costs are less than \$800, there would be a profit made with any of the plans.



Using the MINIMUM function on a graphing calculator, the minimum occurs at $x \approx 2.96$.



The minimum cost occurs when the cable runs for 2.96 mile along the road.

6. $C(4.5) = 100(4.5) + 140\sqrt{4+(5-4.5)^2}$

$\approx \$738.62$

The cost for the Steven's cable would be \$738.62.

7. 5000(738.62) = \$3,693,100 State legislated
5000(695.96) = \$3,479,800 cheapest cost
It will cost the company \$213,300 more.

Project IV

1. $A = \pi r^2$

2. $r = 2.2t$

3. $r = 2.2(2) = 4.4$ ft
$r = 2.2(2.5) = 5.5$ ft

4. $A = \pi(4.4)^2 = 60.82 \text{ ft}^2$
$A = \pi(5.5)^2 = 95.03 \text{ ft}^2$

5. $A = \pi(2.2t)^2 = 4.84\pi t^2$

6. $A = 4.84\pi(2)^2 = 60.82 \text{ ft}^2$
$A = 4.84\pi(2.5)^2 = 95.03 \text{ ft}^2$

7. $\dfrac{A(2.5)-A(2)}{2.5-2} = \dfrac{95.03-60.82}{0.5} = 68.42$ ft/hr

8. $\dfrac{A(3.5)-A(3)}{3.5-3} = \dfrac{186.27-136.85}{0.5} = 98.84$ ft/hr

9. The average rate of change is increasing.

10. 150 yds = 450 ft
$r = 2.2t$
$t = \dfrac{450}{2.2} = 204.5$ hours

11. 6 miles = 31680 ft
Therefore, we need a radius of 15,840 ft.
$t = \dfrac{15{,}840}{2.2} = 7200$ hours