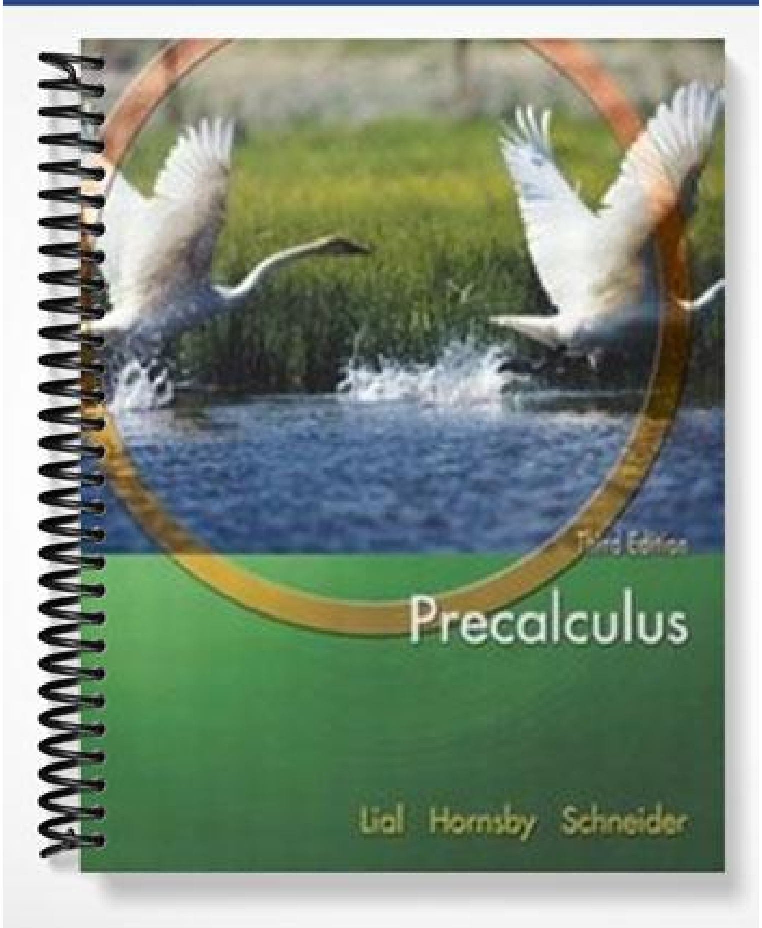
SOLUTIONS MANUAL



Chapter 2

GRAPHS AND FUNCTIONS

Section 2.1: Graphs of Equations

Connections (page 192)

1. Answers will vary.

2. Answers will vary.
Latitude and longitude values pinpoint distances north or south of the equator and east or west of the prime meridian. Similarly on a Cartesian coordinate system, *x*- and *y*- coordinates give distances and directions from the *y*-axis and *x*-axis, respectively.

Exercises

1. true; The origin has coordinates $(0,0)$. So, the distance from $(0,0)$ to (a,b) is $d=\sqrt{(a-0)^2+(b-0)^2}=\sqrt{a^2+b^2}$.

2. true; The midpoint has coordinates $\left(\frac{a+3a}{2},\frac{b+(-3b)}{2}\right)=\left(\frac{4a}{2},\frac{-2b}{2}\right)=(2a,-b)$.

3. false; The equation should be $x^2+y^2=4$ to satisfy these conditions.

4. Answers will vary.

5. any three of the following: $(2,-5),(-1,7),(3,-9),(5,-17),(6,-21)$

6. any three of the following: $(3,3),(-5,-21),(8,18),(4,6),(0,-6)$

7. any three of the following: $(1993,31),(1995,35),(1997,37),(1999,35),(2001,28)$

8. any three of the following: $(1997,87.8),(1998,90.0),(1999,83.7),(2000,88.5),(2001,84.3)$

9. $P(-5,-7)$, $Q(-13,1)$

(a) $d(P,Q)=\sqrt{[-13-(-5)]^2+[1-(-7)]^2}$
$=\sqrt{(-8)^2+8^2}=\sqrt{128}=8\sqrt{2}$

(b) The midpoint *M* of the segment joining points *P* and *Q* has coordinates
$\left(\frac{-5+(-13)}{2},\frac{-7+1}{2}\right)=\left(\frac{-18}{2},\frac{-6}{2}\right)$
$=(-9,-3)$.

10. $P(-4,3)$, $Q(2,-5)$

(a) $d(P,Q)=\sqrt{[2-(-4)]^2+(-5-3)^2}$
$=\sqrt{6^2+(-8)^2}=\sqrt{100}=10$

(b) The midpoint *M* of the segment joining points *P* and *Q* has coordinates
$\left(\frac{-4+2}{2},\frac{3+(-5)}{2}\right)=\left(\frac{-2}{2},\frac{-2}{2}\right)$
$=(-1,-1)$.

11. $P(8,2)$, $Q(3,5)$

(a) $d(P,Q)=\sqrt{(3-8)^2+(5-2)^2}=\sqrt{(-5)^2+3^2}=\sqrt{25+9}=\sqrt{34}$

(b) The midpoint *M* of the segment joining points *P* and *Q* has coordinates $\left(\frac{8+3}{2},\frac{2+5}{2}\right)=\left(\frac{11}{2},\frac{7}{2}\right)$.

12. $P(-6, -5)$, $Q(6, 10)$

(a) $d(P, Q) = \sqrt{[6-(-6)]^2 + [10-(-5)]^2}$
$= \sqrt{12^2 + 15^2} = \sqrt{144+225}$
$= \sqrt{369} = 3\sqrt{41}$

(b) The midpoint M of the segment joining points P and Q has coordinates $\left(\frac{-6+6}{2}, \frac{-5+10}{2}\right) = \left(\frac{0}{2}, \frac{5}{2}\right) = \left(0, \frac{5}{2}\right).$

13. $P(-8, 4)$, $Q(3, -5)$

(a) $d(P, Q) = \sqrt{\left[3-(-8)\right]^2 + (-5-4)^2}$
$= \sqrt{11^2 + (-9)^2} = \sqrt{121+81}$
$= \sqrt{202}$

(b) The midpoint M of the segment joining points P and Q has coordinates $\left(\frac{-8+3}{2}, \frac{4+(-5)}{2}\right) = \left(-\frac{5}{2}, -\frac{1}{2}\right).$

14. $P(6, -2)$, $Q(4, 6)$

(a) $d(P, Q) = \sqrt{(4-6)^2 + [6-(-2)]^2} = \sqrt{(-2)^2 + 8^2} = \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$

(b) The midpoint M of the segment joining points P and Q has coordinates $\left(\frac{6+4}{2}, \frac{-2+6}{2}\right) = \left(\frac{10}{2}, \frac{4}{2}\right) = (5, 2).$

15. $P(3\sqrt{2}, 4\sqrt{5})$, $Q(\sqrt{2}, -\sqrt{5})$

(a) $d(P, Q) = \sqrt{\left(\sqrt{2}-3\sqrt{2}\right)^2 + \left(-\sqrt{5}-4\sqrt{5}\right)^2} = \sqrt{\left(-2\sqrt{2}\right)^2 + \left(-5\sqrt{5}\right)^2} = \sqrt{8+125} = \sqrt{133}$

(b) The midpoint M of the segment joining points P and Q has coordinates $\left(\frac{3\sqrt{2}+\sqrt{2}}{2}, \frac{4\sqrt{5}+(-\sqrt{5})}{2}\right) = \left(\frac{4\sqrt{2}}{2}, \frac{3\sqrt{5}}{2}\right) = \left(2\sqrt{2}, \frac{3\sqrt{5}}{2}\right).$

16. $P(-\sqrt{7}, 8\sqrt{3}), Q(5\sqrt{7}, -\sqrt{3})$

(a) $d(P, Q) = \sqrt{[5\sqrt{7}-(-\sqrt{7})]^2 + (-\sqrt{3}-8\sqrt{3})^2} = \sqrt{(6\sqrt{7})^2 + (-9\sqrt{3})^2} = \sqrt{252+243} = \sqrt{495} = 3\sqrt{55}$

(b) The midpoint M of the segment joining points P and Q has coordinates $\left(\frac{-\sqrt{7}+5\sqrt{7}}{2}, \frac{8\sqrt{3}+(-\sqrt{3})}{2}\right) = \left(\frac{4\sqrt{7}}{2}, \frac{7\sqrt{3}}{2}\right) = \left(2\sqrt{7}, \frac{7\sqrt{3}}{2}\right).$

17. Label the points $A(-6, -4)$, $B(0, -2)$, and $C(-10, 8)$. Use the distance formula to find the length of each side of the triangle.

$d(A, B) = \sqrt{[0-(-6)]^2 + [-2-(-4)]^2} = \sqrt{6^2 + 2^2} = \sqrt{36+4} = \sqrt{40}$

$d(B, C) = \sqrt{(-10-0)^2 + [8-(-2)]^2} = \sqrt{(-10)^2 + 10^2} = \sqrt{100+100} = \sqrt{200}$

$d(A, C) = \sqrt{[-10-(-6)]^2 + [8-(-4)]^2} = \sqrt{(-4)^2 + 12^2} = \sqrt{16+144} = \sqrt{160}$

Since $\left(\sqrt{40}\right)^2 + \left(\sqrt{160}\right)^2 = \left(\sqrt{200}\right)^2$, triangle ABC is a right triangle.

18. Label the points $A(-2, -8)$, $B(0, -4)$, and $C(-4, -7)$. Use the distance formula to find the length of each side of the triangle.

$d(A, B) = \sqrt{[0-(-2)]^2 + [-4-(-8)]^2} = \sqrt{2^2 + 4^2} = \sqrt{4+16} = \sqrt{20}$

$d(B, C) = \sqrt{(-4-0)^2 + [-7-(-4)]^2} = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$d(A, C) = \sqrt{[-4-(-2)]^2 + [-7-(-8)]^2} = \sqrt{(-2)^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$

Since $(\sqrt{5})^2 + (\sqrt{20})^2 = 5+20 = 25 = 5^2$, triangle ABC is a right triangle.

19. Label the points $A(-4, 1)$, $B(1, 4)$, and $C(-6, -1)$.

$d(A,\ B) = \sqrt{[1-(-4)]^2 + (4-1)^2} = \sqrt{5^2 + 3^2} = \sqrt{25+9} = \sqrt{34}$

$d(B,\ C) = \sqrt{(-6-1)^2 + (-1-4)^2} = \sqrt{(-7)^2 + (-5)^2} = \sqrt{49+25} = \sqrt{74}$

$d(A,\ C) = \sqrt{[-6-(-4)]^2 + (-1-1)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$

Since $(\sqrt{8})^2 + (\sqrt{34})^2 \neq (\sqrt{74})^2$ because $8+34 = 42 \neq 74$, triangle ABC is not a right triangle.

20. Label the points $A(-2, -5)$, $B(1, 7)$, and $C(3, 15)$.

$d(A,\ B) = \sqrt{[1-(-2)]^2 + [7-(-5)]^2} = \sqrt{3^2 + 12^2} = \sqrt{9+144} = \sqrt{153}$

$d(B,\ C) = \sqrt{(3-1)^2 + (15-7)^2} = \sqrt{2^2 + 8^2} = \sqrt{4+64} = \sqrt{68}$

$d(A,\ C) = \sqrt{[3-(-2)]^2 + [15-(-5)]^2} = \sqrt{5^2 + 20^2} = \sqrt{25+400} = \sqrt{425}$

Since $(\sqrt{68})^2 + (\sqrt{153})^2 \neq \left(\sqrt{425}\right)^2$ because $68+153 = 221 \neq 425$, triangle ABC is not a right triangle.

21. Label the points $A(-4, 3)$, $B(2, 5)$, and $C(-1, -6)$.

$d(A,\ B) = \sqrt{\left[2-(-4)\right]^2 + (5-3)^2} = \sqrt{6^2 + 2^2} = \sqrt{36+4} = \sqrt{40}$

$d(B,\ C) = \sqrt{(-1-2)^2 + (-6-5)^2} = \sqrt{(-3)^2 + (-11)^2} = \sqrt{9+121} = \sqrt{130}$

$d(A,\ C) = \sqrt{\left[-1-(-4)\right]^2 + (-6-3)^2} = \sqrt{3^2 + (-9)^2} = \sqrt{9+81} = \sqrt{90}$

Since $\left(\sqrt{40}\right)^2 + \left(\sqrt{90}\right)^2 = \left(\sqrt{130}\right)^2$, triangle ABC is a right triangle.

22. Label the points $A(-7, 4)$, $B(6, -2)$, and $C(0, -15)$.

$d(A,\ B) = \sqrt{\left[6-(-7)\right]^2 + (-2-4)^2} = \sqrt{13^2 + (-6)^2} = \sqrt{169+36} = \sqrt{205}$

$d(B,\ C) = \sqrt{(0-6)^2 + \left[-15-(-2)\right]^2} = \sqrt{(-6)^2 + (-13)^2} = \sqrt{36+169} = \sqrt{205}$

$d(A,\ C) = \sqrt{\left[0-(-7)\right]^2 + (-15-4)^2} = \sqrt{7^2 + (-19)^2} = \sqrt{49+361} = \sqrt{410}$

Since $\left(\sqrt{205}\right)^2 + \left(\sqrt{205}\right)^2 = \left(\sqrt{410}\right)^2$, triangle ABC is a right triangle.

23. Label the given points $A(0, -7)$, $B(-3, 5)$, and $C(2, -15)$. Find the distance between each pair of points.

$d(A,\ B) = \sqrt{(-3-0)^2 + \left[5-(-7)\right]^2} = \sqrt{(-3)^2 + 12^2} = \sqrt{9+144} = \sqrt{153} = 3\sqrt{17}$

$d(B,\ C) = \sqrt{\left[2-(-3)\right]^2 + (-15-5)^2} = \sqrt{5^2 + (-20)^2} = \sqrt{25+400} = \sqrt{425} = 5\sqrt{17}$

$d(A,\ C) = \sqrt{(2-0)^2 + \left[-15-(-7)\right]^2} = \sqrt{2^2 + (-8)^2} = \sqrt{68} = 2\sqrt{17}$

Since $d(A,\ B) + d(A,\ C) = d(B,\ C)$ or $3\sqrt{17} + 2\sqrt{17} = 5\sqrt{17}$, the given points lie on a straight line.

24. Label the points $A(-1, 4)$, $B(-2, -1)$, and $C(1, 14)$. Apply the distance formula to each pair of points.

$$d(A,\ B)=\sqrt{\left[-2-(-1)\right]^2+(-1-4)^2}=\sqrt{(-1)^2+(-5)^2}=\sqrt{26}$$

$$d(B,\ C)=\sqrt{\left[1-(-2)\right]^2+\left[14-(-1)\right]^2}=\sqrt{3^2+15^2}=\sqrt{234}=3\sqrt{26}$$

$$d(A,\ C)=\sqrt{\left[1-(-1)\right]^2+(14-4)^2}=\sqrt{2^2+10^2}=\sqrt{104}=2\sqrt{26}$$

Because $\sqrt{26}+2\sqrt{26}=3\sqrt{26}$, the points are collinear.

25. Label the points $A(0, 9)$, $B(-3, -7)$, and $C(2, 19)$.

$$d(A,\ B)=\sqrt{(-3-0)^2+(-7-9)^2}=\sqrt{(-3)^2+(-16)^2}=\sqrt{9+256}=\sqrt{265}\approx 16.279$$

$$d(B,\ C)=\sqrt{\left[2-(-3)\right]^2+\left[19-(-7)\right]^2}=\sqrt{5^2+26^2}=\sqrt{25+676}=\sqrt{701}\approx 26.476$$

$$d(A,\ C)=\sqrt{(2-0)^2+(19-9)^2}=\sqrt{2^2+10^2}=\sqrt{4+100}=\sqrt{104}\approx 10.198$$

Since $d(A,\ B)+d(A,\ C)\neq d(B,\ C)$

or $\sqrt{265}+\sqrt{104}\neq\sqrt{701}$

$16.279+10.198\neq 26.476,$

$26.477\neq 26.476,$

the three given points are not collinear. (Note, however, that these points are very close to lying on a straight line and may appear to lie on a straight line when graphed.)

26. Label the points $A(-1, -3)$, $B(-5, 12)$, and $C(1, -11)$.

$$d(A,\ B)=\sqrt{\left[-5-(-1)\right]^2+\left[12-(-3)\right]^2}=\sqrt{(-4)^2+15^2}=\sqrt{16+225}=\sqrt{241}\approx 15.5242$$

$$d(B,\ C)=\sqrt{\left[1-(-5)\right]^2+(-11-12)^2}=\sqrt{6^2+(-23)^2}=\sqrt{36+529}=\sqrt{565}\approx 23.7697$$

$$d(A,\ C)=\sqrt{\left[1-(-1)\right]^2+\left[-11-(-3)\right]^2}=\sqrt{2^2+(-8)^2}=\sqrt{4+64}=\sqrt{68}\approx 8.2462$$

Since $d(A, B) + d(A, C) \neq d(B, C)$

or $\sqrt{241}+\sqrt{68}\neq\sqrt{565}$

$15.5242+8.2462\neq 23.7697$

$23.7704\neq 23.7697,$

the three given points are not collinear. (Note, however, that these points are very close to lying on a straight line and may appear to lie on a straight line when graphed.)

27. Label the points $A(-7, 4)$, $B(6,-2)$, and $C(-1,1)$.

$$d(A,\ B)=\sqrt{\left[6-(-7)\right]^2+(-2-4)^2}=\sqrt{13^2+(-6)^2}=\sqrt{169+36}=\sqrt{205}\approx 14.3178$$

$$d(B,\ C)=\sqrt{(-1-6)^2+\left[1-(-2)\right]^2}=\sqrt{(-7)^2+3^2}=\sqrt{49+9}=\sqrt{58}\approx 7.6158$$

$$d(A,\ C)=\sqrt{\left[-1-(-7)\right]^2+(1-4)^2}=\sqrt{6^2+(-3)^2}=\sqrt{36+9}=\sqrt{45}\approx 6.7082$$

Since $d(B, C) + d(A, C) \neq d(A, B)$

or $\sqrt{58}+\sqrt{45}\neq\sqrt{205}$

$7.6158+6.7082\neq 14.3178$

$14.3240\neq 14.3178,$

the three given points are not collinear. (Note, however, that these points are very close to lying on a straight line and may appear to lie on a straight line when graphed.)

28. Label the given points $A(-4,3)$, $B(2, 5)$, and $C(-1,4)$. Find the distance between each pair of points.

$$d(A, B) = \sqrt{[2-(-4)]^2 + (5-3)^2} = \sqrt{6^2 + 2^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$$

$$d(B, C) = \sqrt{(-1-2)^2 + (4-5)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$d(A, C) = \sqrt{[-1-(-4)]^2 + (4-3)^2} = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

Since $d(B, C) + d(A, C) = d(A, B)$ or $\sqrt{10} + \sqrt{10} = 2\sqrt{10}$, the given points lie on a straight line.

29. The points to use would be (1982, 79.1) and (2002, 69.3). Their midpoint is

$$\left(\frac{1982+2002}{2}, \frac{79.1+69.3}{2}\right) = \left(\frac{3984}{2}, \frac{148.4}{2}\right) = (1992, 74.2).$$

The estimate is 74.2%. This is 1.1% less than the actual percent of 75.3.

30. The year 1998 lies halfway between 1996 and 2000, so we must find the coordinates of the midpoint of the line segment joining (1996, 371) and (2000, 387). Their midpoint is

$$\left(\frac{1996+2000}{2}, \frac{371+387}{2}\right) = \left(\frac{3996}{2}, \frac{758}{2}\right) = (1998, 379).$$

The average payment in 1998 was $379.

31. The points to use would be (1990, 13359) and (2000, 17603). Their midpoint is

$$\left(\frac{1990+2000}{2}, \frac{13359+17603}{2}\right) = \left(\frac{3990}{2}, \frac{30962}{2}\right) = (1995, 15481).$$

In 1995, it was approximately $15,481.

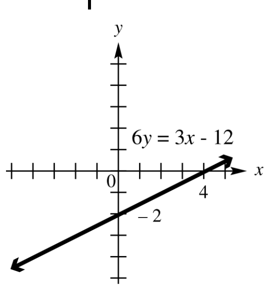
32. The midpoint between (1980, 4.5) and (1990, 5.2) is $\left(\frac{1980+1990}{2}, \frac{4.5+5.2}{2}\right) = (1985, 4.85).$

In 1985, the enrollment was 4.85 million. The midpoint between (1990, 5.2) and (2000, 5.8) is $\left(\frac{1990+2000}{2}, \frac{5.2+5.8}{2}\right) = (1995, 5.5).$ In 1995, the enrollment was 5.5 million.

33. (a)

x	***y***	
0	–2	y-intercept: $x = 0 \Rightarrow 6y = 3(0) - 12 \Rightarrow 6y = -12 \Rightarrow y = -2$
4	0	x-intercept: $y = 0 \Rightarrow 6(0) = 3x - 12 \Rightarrow 0 = 3x - 12 \Rightarrow 12 = 3x \Rightarrow 4 = x$
2	–1	additional point

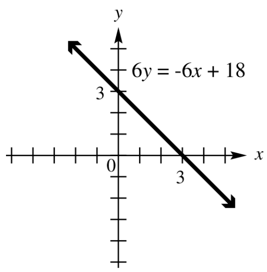
(b)



34. **(a)**

x	y	
0	3	y-intercept: $x=0 \Rightarrow 6y=-6(0)+18 \Rightarrow 6y=18 \Rightarrow y=3$
3	0	x-intercept: $y=0 \Rightarrow 6(0)=-6x+18 \Rightarrow 0=-6x+18 \Rightarrow 6x=18 \Rightarrow x=3$
1	2	additional point

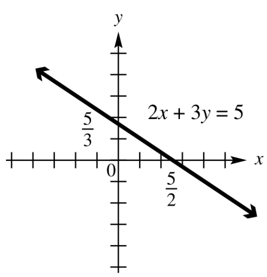
(b)



35. **(a)**

x	y	
0	$\frac{5}{3}$	y-intercept: $x=0 \Rightarrow 2(0)+3y=5 \Rightarrow 3y=5 \Rightarrow y=\frac{5}{3}$
$\frac{5}{2}$	0	x-intercept: $y=0 \Rightarrow 2x+3(0)=5 \Rightarrow 2x=5 \Rightarrow x=\frac{5}{2}$
4	-1	additional point

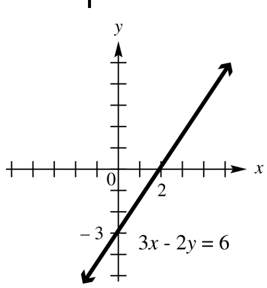
(b)



36. **(a)**

x	y	
0	-3	y-intercept: $x=0 \Rightarrow 3(0)-2y=6 \Rightarrow -2y=6 \Rightarrow y=-3$
2	0	x-intercept: $y=0 \Rightarrow 3x-2(0)=6 \Rightarrow 3x=6 \Rightarrow x=2$
4	3	additional point

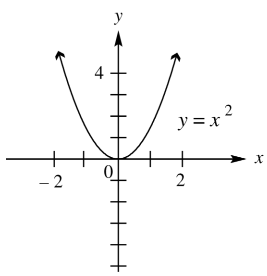
(b)



37. (a)

x	y	
0	0	x- and y-intercept: $0 = 0^2$
1	1	additional point
–2	4	additional point

(b)

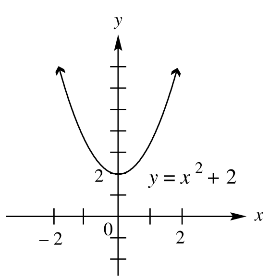


38. (a)

x	y	
0	2	y-intercept: $x = 0 \Rightarrow y = 0^2 + 2 \Rightarrow y = 0 + 2 \Rightarrow y = 2$
–1	3	additional point
2	6	additional point

no x-intercept: $y = 0 \Rightarrow 0 = x^2 + 2 \Rightarrow -2 = x^2 \Rightarrow \pm\sqrt{-2} = x$

(b)

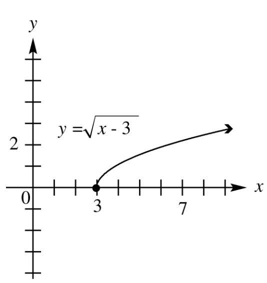


39. (a)

x	y	
3	0	x-intercept: $y = 0 \Rightarrow 0 = \sqrt{x-3} \Rightarrow 0 = x - 3 \Rightarrow 3 = x$
4	1	additional point
7	2	additional point

no y-intercept: $x = 0 \Rightarrow y = \sqrt{0-3} \Rightarrow y = \sqrt{-3}$

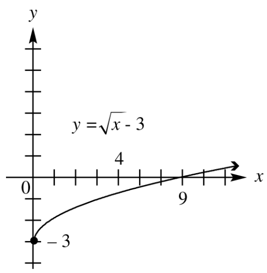
(b)



40. (a)

x	y	
0	−3	y-intercept: $x=0 \Rightarrow y=\sqrt{0}-3 \Rightarrow y=0-3 \Rightarrow y=-3$
4	−1	additional point
9	0	x-intercept: $y=0 \Rightarrow 0=\sqrt{x}-3 \Rightarrow 3=\sqrt{x} \Rightarrow 9=x$

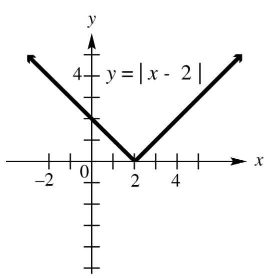
(b)



41. (a)

x	y	
0	2	y-intercept: $x=0 \Rightarrow y=\lvert 0-2\rvert \Rightarrow y=\lvert -2\rvert \Rightarrow y=2$
2	0	x-intercept: $y=0 \Rightarrow 0=\lvert x-2\rvert \Rightarrow 0=x-2 \Rightarrow 2=x$
−2	4	additional point
4	2	additional point

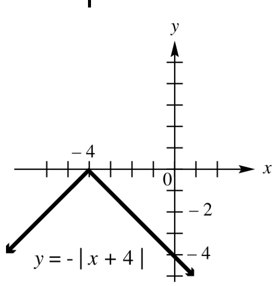
(b)



42. (a)

x	y	
−2	−2	additional point
−4	0	x-intercept: $y=0 \Rightarrow 0=-\lvert x+4\rvert \Rightarrow 0=\lvert x+4\rvert \Rightarrow 0=x+4 \Rightarrow -4=x$
0	−4	y-intercept: $x=0 \Rightarrow y=-\lvert 0+4\rvert \Rightarrow y=-\lvert 4\rvert \Rightarrow y=-4$

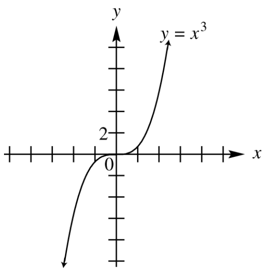
(b)



43. **(a)**

x	y	
0	0	x- and y-intercept: $0=0^3$
−1	−1	additional point
2	8	additional point

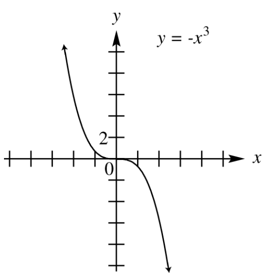
(b)



44. **(a)**

x	y	
0	0	x- and y-intercept: $0=-0^3$
1	−1	additional point
2	−8	additional point

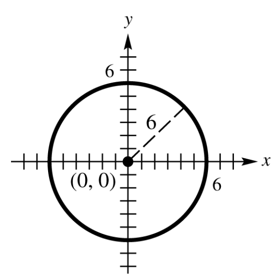
(b)



45. **(a)** Center (0, 0), radius 6

$$\sqrt{(x-0)^2+(y-0)^2}=6$$
$$(x-0)^2+(y-0)^2=6^2$$
$$x^2+y^2=36$$

(b)

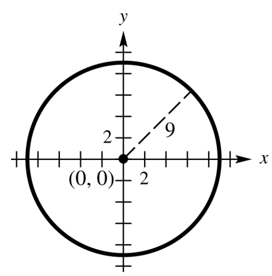


$x^2+y^2=36$

46. **(a)** Center (0, 0), radius 9

$$\sqrt{(x-0)^2+(y-0)^2}=9$$
$$(x-0)^2+(y-0)^2=9^2$$
$$x^2+y^2=81$$

(b)

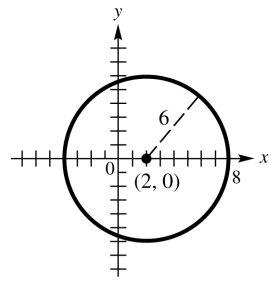


$x^2+y^2=81$

47. (a) Center (2, 0), radius 6

$$\sqrt{(x-2)^2+(y-0)^2}=6$$
$$(x-2)^2+(y-0)^2=6^2$$
$$(x-2)^2+y^2=36$$

(b)

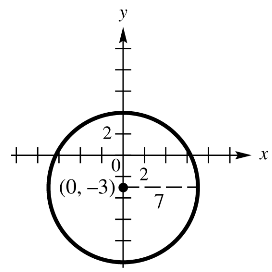


$(x-2)^2+y^2=36$

48. (a) Center (0, –3), radius 7

$$\sqrt{(x-0)^2+[y-(-3)]^2}=7$$
$$(x-0)^2+[y-(-3)]^2=7^2$$
$$x^2+(y+3)^2=49$$

(b)

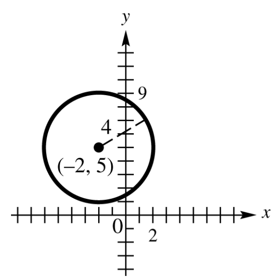


$x^2+(y+3)^2=49$

49. (a) Center (–2, 5), radius 4

$$\sqrt{[x-(-2)]^2+(y-5)^2}=4$$
$$[x-(-2)]^2+(y-5)^2=4^2$$
$$(x+2)^2+(y-5)^2=16$$

(b)

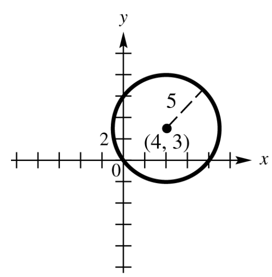


$(x+2)^2+(y-5)^2=16$

50. (a) Center (4, 3), radius 5

$$\sqrt{(x-4)^2+(y-3)^2}=5$$
$$(x-4)^2+(y-3)^2=5^2$$
$$(x-4)^2+(y-3)^2=25$$

(b)

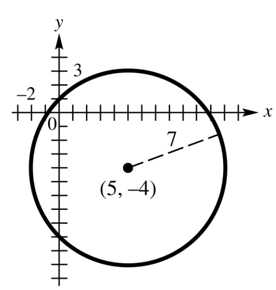


$(x-4)^2+(y-3)^2=25$

51. (a) Center (5, –4), radius 7

$$\sqrt{(x-5)^2+[y-(-4)]^2}=7$$
$$(x-5)^2+[y-(-4)]^2=7^2$$
$$(x-5)^2+(y+4)^2=49$$

(b)

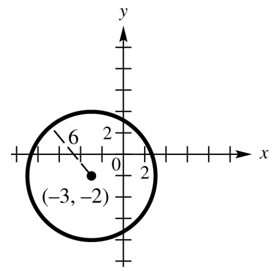


$(x-5)^2+(y+4)^2=49$

52. **(a)** Center (–3, –2), radius 6

$$\sqrt{\left[x-(-3)\right]^2+\left[y-(-2)\right]^2}=6$$
$$\left[x-(-3)\right]^2+\left[y-(-2)\right]^2=6^2$$
$$(x+3)^2+(y+2)^2=36$$

(b)



$(x + 3)^2 + (y + 2)^2 = 36$

53. The radius of this circle is the distance from the center $C(3, 2)$ to the x-axis. This distance is 2, so $r = 2$.

$$(x-3)^2+(y-2)^2=2^2 \Rightarrow (x-3)^2+(y-2)^2=4$$

54. The radius is the distance from the center $C(-4, 3)$ to the point $P(5, 8)$.

$$r=\sqrt{[5-(-4)]^2+(8-3)^2}=\sqrt{9^2+5^2}=\sqrt{106}$$

The equation of the circle is $[x-(-4)]^2+(y-3)^2=(\sqrt{106})^2 \Rightarrow (x+4)^2+(y-3)^3=106.$

55. Answers will vary.
If $m>0,$ this indicates the graph is a circle. If $m=0,$ this indicates the graph is a point If $m<0,$ this indicates the graph does not exist.

56. Since the center (–3, 5) is in quadrant II, choice B is the correct graph.

57. $x^2+6x+y^2+8y+9=0$
Complete the square on x and y separately.
$\left(x^2+6x\right)+\left(y^2+8y\right)=-9 \Rightarrow \left(x^2+6x+9\right)+\left(y^2+8y+16\right)=-9+9+16 \Rightarrow (x+3)^2+(y+4)^2=16$
Yes, it is a circle. The circle has its center at (–3, –4) and radius 4.

58. $x^2+8x+y^2-6y+16=0$
Complete the square on x and y separately.
$\left(x^2+8x\right)+\left(y^2-6y\right)=-16 \Rightarrow \left(x^2+8x+16\right)+\left(y^2-6y+9\right)=-16+16+9 \Rightarrow (x+4)^2+(y-3)^2=9$
Yes, it is a circle. The circle has its center at (–4, 3) and radius 3.

59. $x^2-4x+y^2+12y=-4 \Rightarrow \left(x^2-4x\right)+\left(y^2+12y\right)=-4$
$\left(x^2-4x+4\right)+\left(y^2+12y+36\right)=-4+4+36 \Rightarrow (x-2)^2+(y+6)^2=36$
Yes, it is a circle. The circle has its center at (2, –6) and radius 6.

60. $x^2-12x+y^2+10y=-25 \Rightarrow \left(x^2-12x\right)+\left(y^2+10y\right)=-25$
$\left(x^2-12x+36\right)+\left(y^2+10y+25\right)=-25+36+25 \Rightarrow (x-6)^2+(y+5)^2=36$
Yes, it is a circle. The circle has its center at (6, –5) and radius 6.

61. $4x^2+4x+4y^2-16y-19=0$
$4\left(x^2+x\right)+4\left(y^2-4y\right)=19 \Rightarrow 4\left(x^2+x+\frac{1}{4}\right)+4\left(y^2-4y+4\right)=19+4\left(\frac{1}{4}\right)+4(4)$
$4\left(x+\frac{1}{2}\right)^2+4(y-2)^2=36 \Rightarrow \left(x+\frac{1}{2}\right)^2+(y-2)^2=9$
Yes, it is a circle. The circle has its center at $\left(-\frac{1}{2}, 2\right)$ and radius 3.

62. $9x^2+12x+9y^2-18y-23=0 \Rightarrow 9\left(x^2+\frac{4}{3}x\right)+9\left(y^2-2y\right)=23$

$9\left(x^2+\frac{4}{3}x+\frac{4}{9}\right)+9\left(y^2-2y+1\right)=23+9\left(\frac{4}{9}\right)+9(1)$

$9\left(x+\frac{2}{3}\right)^2+9(y-1)^2=36 \Rightarrow \left(x+\frac{2}{3}\right)^2+(y-1)^2=4$

Yes, it is a circle. The circle has its center at $\left(-\frac{2}{3},\ 1\right)$ and radius 2.

63.
$$x^2+2x+y^2-6y+14=0$$
$$\left(x^2+2x\right)+\left(y^2-6y\right)=-14$$
$$\left(x^2+2x+1\right)+\left(y^2-6y+9\right)=-14+1+9$$
$$(x+1)^2+(y-3)^2=-4$$
No, it is not a circle.

64.
$$x^2+4x+y^2-8y+32=0$$
$$\left(x^2+4x\right)+\left(y^2-8y\right)=-32$$
$$\left(x^2+4x+4\right)+\left(y^2-8y+16\right)=-32+4+16$$
$$(x+2)^2+(y-4)^2=-12$$
No, it is not a circle.

65. The midpoint *M* has coordinates $\left(\frac{-1+5}{2},\frac{3+(-9)}{2}\right)=\left(\frac{4}{2},\frac{-6}{2}\right)=(2,-3).$

66. Use points $C(2, -3)$ and $P(-1, 3)$.
$$d(C,\ P)=\sqrt{(-1-2)^2+\left[3-(-3)\right]^2}$$
$$=\sqrt{(-3)^2+6^2}=\sqrt{9+36}$$
$$=\sqrt{45}=3\sqrt{5}$$
The radius is $3\sqrt{5}$.

67. Use points $C(2, -3)$ and $Q(5, -9)$.
$$d(C,\ Q)=\sqrt{(5-2)^2+\left[-9-(-3)\right]^2}$$
$$=\sqrt{3^2+(-6)^2}=\sqrt{9+36}$$
$$=\sqrt{45}=3\sqrt{5}$$
The radius is $3\sqrt{5}$.

68. Use the points $P(-1, 3)$ and $Q(5, -9)$. Since $d(P,\ Q)=\sqrt{\left[5-(-1)\right]^2+(-9-3)^2}=\sqrt{6^2+(-12)^2}$ $=\sqrt{36+144}=\sqrt{180}=6\sqrt{5},$ the radius is $\frac{1}{2}d(P,\ Q).$ Thus $r=\frac{1}{2}\left(6\sqrt{5}\right)=3\sqrt{5}.$

69. The center-radius form for this circle is $(x-2)^2+(y+3)^2=(3\sqrt{5})^2 \Rightarrow (x-2)^2+(y+3)^2=45.$

70. Label the endpoints of the diameter $P(3, -5)$ and $Q(-7, 3)$. The midpoint *M* of the segment joining *P* and *Q* has coordinates $\left(\frac{3+(-7)}{2},\frac{-5+3}{2}\right)=\left(\frac{-4}{2},\frac{-2}{2}\right)=(-2,-1).$ The center is $C(-2, -1)$.

To find the radius, we can use points $C(-2, -1)$.and $P(3, -5)$

$$d(C,\ P)=\sqrt{\left[3-(-2)\right]^2+\left[-5-(-1)\right]^2}=\sqrt{5^2+(-4)^2}=\sqrt{25+16}=\sqrt{41}$$

We could also use points $C(-2, -1)$.and $Q(-7, 3)$.

$$d(C,\ Q)=\sqrt{\left[-7-(-2)\right]^2+\left[3-(-1)\right]^2}=\sqrt{(-5)^2+4^2}=\sqrt{25+16}=\sqrt{41}$$

We could also use points $P(3, -5)$ and $Q(-7, 3)$.

$$d(C,\ Q)=\sqrt{(-7-3)^2+\left[3-(-5)\right]^2}=\sqrt{(-10)^2+8^2}=\sqrt{100+64}=\sqrt{164}=2\sqrt{41}$$

$$\frac{1}{2}d(P,\ Q)=\frac{1}{2}\left(2\sqrt{41}\right)=\sqrt{41}$$

The center-radius form of the equation of the circle is

$[x-(-2)]^2+[y-(-1)]^2=(\sqrt{41})^2 \Rightarrow (x+2)^2+(y+1)^2=41.$

71. Midpoint (5, 8), endpoint (13, 10)

$$\frac{13+x}{2}=5 \quad \text{and} \quad \frac{10+y}{2}=8$$
$$13+x=10 \quad \text{and} \quad 10+y=16$$
$$x=-3 \quad \text{and} \quad y=6.$$

The other endpoint has coordinates (–3, 6).

72. Midpoint (–7, 6), endpoint (–9, 9)

$$\frac{-9+x}{2}=-7 \quad \text{and} \quad \frac{9+y}{2}=6$$
$$-9+x=-14 \quad \text{and} \quad 9+y=12$$
$$x=-5 \quad \text{and} \quad y=3.$$

The other endpoint has coordinates (–5, 3).

73. Midpoint (12, 6), endpoint (19, 16)

$$\frac{19+x}{2}=12 \quad \text{and} \quad \frac{16+y}{2}=6$$
$$19+x=24 \quad \text{and} \quad 16+y=12$$
$$x=5 \quad \text{and} \quad y=-4.$$

The other endpoint has coordinates (5, –4).

74. Midpoint (–9, 8) endpoint (–16, 9)

$$\frac{-16+x}{2}=-9 \quad \text{and} \quad \frac{9+y}{2}=8$$
$$-16+x=-18 \quad \text{and} \quad 9+y=16$$
$$x=-2 \quad \text{and} \quad y=7$$

The other endpoint has coordinates (–2, 7).

75. The midpoint M has coordinates $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

$$d(P,M)=\sqrt{\left(\frac{x_1+x_2}{2}-x_1\right)^2+\left(\frac{y_1+y_2}{2}-y_1\right)^2}=\sqrt{\left(\frac{x_1+x_2}{2}-\frac{2x_1}{2}\right)^2+\left(\frac{y_1+y_2}{2}-\frac{2y_1}{2}\right)^2}$$
$$=\sqrt{\left(\frac{x_2-x_1}{2}\right)^2+\left(\frac{y_2-y_1}{2}\right)^2}=\sqrt{\frac{(x_2-x_1)^2}{4}+\frac{(y_2-y_1)^2}{4}}=\sqrt{\frac{(x_2-x_1)^2+(y_2-y_1)^2}{4}}$$
$$=\tfrac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d(M,Q)=\sqrt{\left(x_2-\frac{x_1+x_2}{2}\right)^2+\left(y_2-\frac{y_1+y_2}{2}\right)^2}=\sqrt{\left(\frac{2x_2}{2}-\frac{x_1+x_2}{2}-\right)^2+\left(\frac{2y_2}{2}-\frac{y_1+y_2}{2}\right)^2}$$
$$=\sqrt{\left(\frac{x_2-x_1}{2}\right)^2+\left(\frac{y_2-y_1}{2}\right)^2}=\sqrt{\frac{(x_2-x_1)^2}{4}+\frac{(y_2-y_1)^2}{4}}=\sqrt{\frac{(x_2-x_1)^2+(y_2-y_1)^2}{4}}$$
$$=\tfrac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d(P,Q)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Since $\tfrac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}+\tfrac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, this shows $d(P,M)+d(M,Q)=d(P,Q)$ and $d(P,M)=d(M,Q)$.

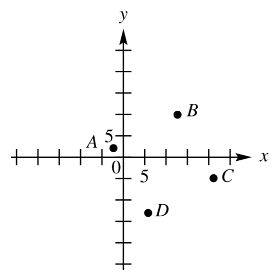
76. The distance formula, $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, can be written as $d=[(x_2-x_1)^2+(y_2-y_1)^2]^{1/2}$.

77. Points on the x-axis have y-coordinates equal to 0. The point on the x-axis will have the same x-coordinate as point (4, 3). Therefore, the line will intersect the x-axis at (4, 0).

78. Points on the y-axis have x-coordinates equal to 0. The point on the y-axis will have the same y-coordinate as point (4, 3). Therefore, the line will intersect the y-axis at (0, 3).

79. Since (a, b) is in the second quadrant, a is negative and b is positive.
Therefore, $(a, -b)$ will have a negative x-coordinate and a negative y-coordinate and will lie in quadrant III.
Also, $(-a, b)$ will have a positive x-coordinate and a positive y-coordinate and will lie in quadrant I.
Also, $(-a, -b)$ will have a positive x-coordinate and a negative y-coordinate and will lie in quadrant IV.
Finally, (b, a) will have a positive x-coordinate and a negative y-coordinate and will lie in quadrant IV.

80. Label the points $A(-2,2)$, $B(13,10)$, $C(21,-5)$, and $D(6,-13)$. To determine which points form sides of the quadrilateral (as opposed to diagonals), plot the points.



Use the distance formula to find the length of each side.

$$d(A, B) = \sqrt{[13-(-2)]^2 + (10-2)^2} = \sqrt{15^2 + 8^2} = \sqrt{225+64} = \sqrt{289} = 17$$

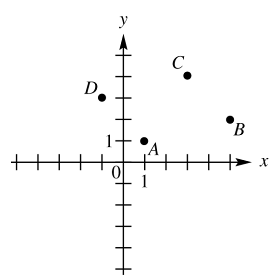
$$d(B, C) = \sqrt{(21-13)^2 + (-5-10)^2} = \sqrt{8^2 + (-15)^2} = \sqrt{64+225} = \sqrt{289} = 17$$

$$d(C, D) = \sqrt{(6-21)^2 + [-13-(-5)]^2} = \sqrt{(-15)^2 + (-8)^2} = \sqrt{225+64} = \sqrt{289} = 17$$

$$d(D, A) = \sqrt{(-2-6)^2 + [2-(-13)]^2} = \sqrt{(-8)^2 + 15^2} = \sqrt{64+225} = \sqrt{289} = 17$$

Since all sides have equal length, the four points form a rhombus.

81. To determine which points form sides of the quadrilateral (as opposed to diagonals), plot the points.



Use the distance formula to find the length of each side.

$$d(A, B) = \sqrt{(5-1)^2 + (2-1)^2} = \sqrt{4^2 + 1^2} = \sqrt{16+1} = \sqrt{17}$$

$$d(B, C) = \sqrt{(3-5)^2 + (4-2)^2} = \sqrt{(-2)^2 + 2^2} = \sqrt{4+4} = \sqrt{8}$$

$$d(C, D) = \sqrt{(-1-3)^2 + (3-4)^2} = \sqrt{(-4)^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17}$$

$$d(D, A) = \sqrt{[1-(-1)]^2 + (1-3)^2} = \sqrt{2^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$$

Since $d(A, B) = d(C, D)$ and $d(B, C) = d(D, A)$, the points are the vertices of a parallelogram. Since $d(A, B) \neq d(B, C)$, the points are not the vertices of a rhombus.

82. Since the center is in the third quadrant, the radius is $\sqrt{2}$, and the circle is tangent to both axes, the center must be at $(-\sqrt{2},-\sqrt{2})$. Using the center-radius of the equation of a circle, we have $\left[x-\left(-\sqrt{2}\right)\right]^2+\left[y-\left(-\sqrt{2}\right)\right]^2=\left(\sqrt{2}\right)^2 \Rightarrow \left(x+\sqrt{2}\right)^2+\left(y+\sqrt{2}\right)^2=2.$

83. Label the points $P(x, y)$ and $Q(1, 3)$.
If $d(P,Q)=4$, $\sqrt{(1-x)^2+(3-y)^2}=4 \Rightarrow (1-x)^2+(3-y)^2=16.$
If $x = y$, then we can either substitute x for y or y for x. Substituting x for y we solve the following.

$$\begin{aligned}(1-x)^2+(3-x)^2&=16\\ 1-2x+x^2+9-6x+x^2&=16\\ 2x^2-8x+10&=16\\ 2x^2-8x-6&=0\\ x^2-4x-3&=0\end{aligned}$$

To solve this equation, we can use the quadratic formula with $a = 1$, $b = -4$, and $c = -3$.

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-3)}}{2(1)}=\frac{4\pm\sqrt{16+12}}{2}=\frac{4\pm\sqrt{28}}{2}=\frac{4\pm2\sqrt{7}}{2}=2\pm\sqrt{7}$$

Since $x = y$, the points are $\left(2+\sqrt{7},2+\sqrt{7}\right)$ and $\left(2-\sqrt{7},2-\sqrt{7}\right)$.

84. Let $P(-2, 3)$ be a point which is 8 units from $Q(x, y)$. We have $d(P,Q)=\sqrt{(-2-x)^2+(3-y)^2}=8\Rightarrow$ $(-2-x)^2+(3-y)^2=64.$ Since $x + y = 0$, $x = -y$. We can either substitute $-x$ for y or $-y$ for x. Substituting $-x$ for y we solve the following.

$$\begin{aligned}(-2-x)^2+\left[3-(-x)\right]^2&=64\\ (-2-x)^2+(3+x)^2&=64\\ 4+4x+x^2+9+6x+x^2&=64\\ 2x^2+10x+13&=64\\ 2x^2+10x-51&=0\end{aligned}$$

To solve this equation, use the quadratic formula with $a = 2$, $b = 10$, and $c = -51$.

$$x=\frac{-10\pm\sqrt{10^2-4(2)(-51)}}{2(2)}=\frac{-10\pm\sqrt{100+408}}{4}$$

$$=\frac{-10\pm\sqrt{508}}{4}=\frac{-10\pm\sqrt{4(127)}}{4}=\frac{-10\pm2\sqrt{127}}{4}=\frac{-5\pm\sqrt{127}}{2}$$

Since $y=-x$ the points are $\left(\frac{-5-\sqrt{127}}{2},\frac{5+\sqrt{127}}{2}\right)$ and $\left(\frac{-5+\sqrt{127}}{2},\frac{5-\sqrt{127}}{2}\right)$.

85. Let $P(x, y)$ be a point whose distance from $A(1, 0)$ is $\sqrt{10}$ and whose distance from $B(5, 4)$ is $\sqrt{10}$.
$d(P, A) = \sqrt{10}$, so $\sqrt{(1-x)^2+(0-y)^2} = \sqrt{10} \Rightarrow (1-x)^2 + y^2 = 10.$
$d(P, B) = \sqrt{10}$, so $\sqrt{(5-x)^2+(4-y)^2} = \sqrt{10} \Rightarrow (5-x)^2+(4-y)^2 = 10.$
Thus,

$$\begin{aligned}(1-x)^2 + y^2 &= (5-x)^2 + (4-y)^2\\ 1-2x+x^2+y^2 &= 25-10x+x^2+16-8y+y^2\\ 1-2x &= 41-10x-8y\\ 8y &= 40-8x\\ y &= 5-x.\end{aligned}$$

Substitute $5 - x$ for y in the equation $(1-x)^2 + y^2 = 10$ and solve for x.

$$\begin{aligned}(1-x)^2+(5-x)^2 = 10 &\Rightarrow 1-2x+x^2+25-10x+x^2 = 10\\ 2x^2-12x+26 = 10 &\Rightarrow 2x^2-12x+16 = 0\\ x^2-6x+8 = 0 &\Rightarrow (x-2)(x-4) = 0\\ x-2 = 0 \quad &\text{or} \quad x-4 = 0\\ x = 2 \quad &\text{or} \quad x = 4\end{aligned}$$

To find the corresponding values of y use the equation $y = 5 - x$.
If $x = 2$, then $y = 5 - 2 = 3$.
If $x = 4$, then $y = 5 - 4 = 1$.
The points satisfying the conditions are (2, 3) and (4, 1).

86. The circle of smallest radius that contains the points $A(1, 4)$ and $B(-3, 2)$ within or on its boundary will be the circle having points A and B as endpoints of a diameter. The center will be M, the midpoint.

$$\left(\frac{1+(-3)}{2}, \frac{4+2}{2}\right) = \left(\frac{-2}{2}, \frac{6}{2}\right) = (-1, 3)$$

The radius will be the distance from M to either A or B.

$$d(M, A) = \sqrt{[1-(-1)]^2+(4-3)^2} = \sqrt{2^2+1^2} = \sqrt{4+1} = \sqrt{5}$$

An equation of the circle is $\left[x-(-1)\right]^2 + (y-3)^2 = \left(\sqrt{5}\right)^2 \Rightarrow (x+1)^2 + (y-3)^2 = 5.$

87. Label the points $A(3, y)$ and $B(-2, 9)$. If $d(A, B) = 12$,

$$\sqrt{(-2-3)^2+(9-y)^2} = 12 \Rightarrow \sqrt{(-5)^2+(9-y)^2} = 12 \Rightarrow (-5)^2+(9-y)^2 = 12^2$$
$$25+81-18y+y^2 = 144 \Rightarrow y^2-18y-38 = 0.$$

Solve this equation by using the quadratic formula with $a = 1$, $b = -18$, and $c = -38$.

$$y = \frac{-(-18)\pm\sqrt{(-18)^2-4(1)(-38)}}{2(1)} = \frac{18\pm\sqrt{324+152}}{2(1)}$$
$$= \frac{18\pm\sqrt{476}}{2} = \frac{18\pm\sqrt{4(119)}}{2} = \frac{18\pm2\sqrt{119}}{2} = 9\pm\sqrt{119}$$

The values of y are $9 + \sqrt{119}$ and $9-\sqrt{119}$.

88. For the points $A(4, 5)$ and $D(10, 14)$, the difference of the x-coordinates is $10 - 4 = 6$ and the difference of the y-coordinates is $14 - 5 = 9$. Dividing these differences by 3, we obtain 2 and 3, respectively. Adding 2 and 3 to the x and y coordinates of point A, respectively, we obtain $B(4 + 2, 5 + 3)$ or $B(6, 8)$. Adding 2 and 3 to the x and y coordinates of point B, respectively, we obtain $C(6 + 2, 8 + 3)$ or $C(8, 11)$. The desired points are $B(6, 8)$ and $C(8, 11)$.

We check these by showing that $d(A, B) = d(B, C) = d(C, D)$ and that $d(A, D) = d(A, B) + d(B, C) + d(C, D)$.

$$d(A, B) = \sqrt{(6-4)^2 + (8-5)^2} = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$$

$$d(B, C) = \sqrt{(8-6)^2 + (11-8)^2} = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$$

$$d(C, D) = \sqrt{(10-8)^2 + (14-11)^2} = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$$

$$d(A, D) = \sqrt{(10-4)^2 + (14-5)^2} = \sqrt{6^2 + 9^2} = \sqrt{36+81} = \sqrt{117} = \sqrt{9(13)} = 3\sqrt{13}$$

$d(A, B)$, $d(B, C)$, and $d(C, D)$ all have the same measure and $d(A, D) = d(A, B) + d(B, C) + d(C, D)$ since $3\sqrt{13} = \sqrt{13} + \sqrt{13} + \sqrt{13}$.

Section 2.2: Functions

1. Answers will vary.

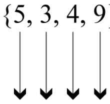
2. Answers will vary.

3. independent variable

4. One example is $\{(-3,4), (2,4), (2,6), (6,4)\}$.

5. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.

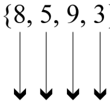
{5, 3, 4, 9} x-values



{1, 2, 9, 6} y-values

6. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.

{8, 5, 9, 3} x-values



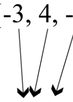
{0, 4, 3, 8} y-values

7. Two ordered pairs, namely $(2,4)$ and $(2,5)$, have the same x-value paired with different y-values, so the relation is not a function.

8. Two ordered pairs, namely $(9,-2)$ and $(9,2)$, have the same x-value paired with different y-values, so the relation is not a function.

9. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.

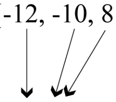
{-3, 4, -2} x-values



{1, 7} y-values

10. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.

{-12, -10, 8} x-values



{5, 3} y-values

11. Two sets of ordered pairs, namely $(1,1)$ and $(1,-1)$ as well as $(2,4)$ and $(2,-4)$, have the same x-value paired with different y-values, so the relation is not a function.

domain: $\{0,1,2\}$; range: $\{-4,-1,0,1,4\}$

12. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.

{2, 3, 4, 5} x-values

↓ ↓ ↓ ↓

{5, 7, 9, 11} y-values

domain: $\{2,3,4,5\}$; range: $\{5,7,9,11\}$

13. The relation is a function because for each different x-value there is exactly one y-value.

domain: $\{2,3,5,11,17\}$; range: $\{1,7,20\}$

14. Two ordered pairs, namely $(2,15)$ and $(2,19)$, have the same x-value paired with different y-values, so the relation is not a function.

domain: $\{1,2,3,5\}$; range: $\{10,15,19,27\}$

15. The relation is a function because for each different year there is exactly one number of rounds played in the U.S.

domain: $\{1997,1998,1999,2000\}$; range: $\{547,200,000,\ 528,500,000,\ 564,100,000,\ 587,100,000\}$

16. The relation is a function because for each different year there is exactly one number that represents the attendance at NCAA Women's College Basketball Games.

domain: $\{1998,1999,2000\}$; range: $\{7,387,000,\ 8,010,000,\ 8,698,000\}$

17. This graph represents a function. If you pass a vertical line through the graph, one x-value corresponds to only one y-value.

domain: $(-\infty,\infty)$; range: $(-\infty,\infty)$

18. This graph represents a function. If you pass a vertical line through the graph, one x-value corresponds to only one y-value.

domain: $(-\infty,\infty)$; range: $(-\infty,4]$

19. This graph does not represent a function. If you pass a vertical line through the graph, there are places where one value of x corresponds to two values of y.

domain: $[3,\infty)$; range: $(-\infty,\infty)$

20. This graph represents a function. If you pass a vertical line through the graph, one x-value corresponds to only one y-value.

domain: $(-\infty,\infty)$; range: $(-\infty,\infty)$

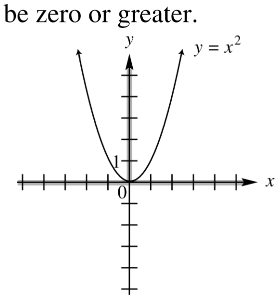
21. This graph does not represent a function. If you pass a vertical line through the graph, there are places where one value of x corresponds to two values of y.

domain: $[-4,4]$; range: $[-3,3]$

22. This graph represents a function. If you pass a vertical line through the graph, one x-value corresponds to only one y-value.

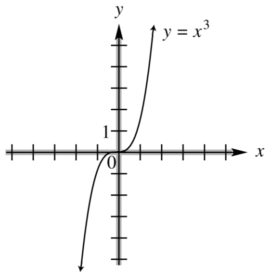
domain: $[-2,2]$; range: $[0,4]$

23. $y=x^2$ represents a function since y is always found by squaring x. Thus, each value of x corresponds to just one value of y. x can be any real number. Since the square of any real number is not negative, the range would be zero or greater.



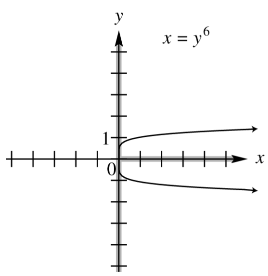
domain: $(-\infty,\infty)$; range: $[0,\infty)$

24. $y = x^3$ represents a function since y is always found by cubing x. Thus, each value of x corresponds to just one value of y. x can be any real number. Since the cube of any real number could be negative, positive, or zero, the range would be any real number.



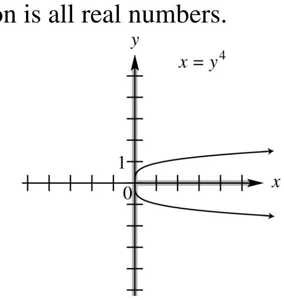
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

25. The ordered pairs $(1,1)$ and $(1,-1)$ both satisfy $x = y^6$. This equation does not represent a function. Because x is equal to the sixth power of y, the values of x are nonnegative. Any real number can be raised to the sixth power, so the range of the relation is all real numbers.



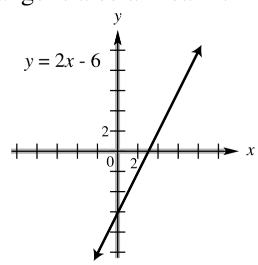
domain: $[0, \infty)$ range: $(-\infty, \infty)$

26. The ordered pairs $(1,1)$ and $(1,-1)$ both satisfy $x = y^4$. This equation does not represent a function. Because x is equal to the fourth power of y, the values of x are nonnegative. Any real number can be raised to the fourth power, so the range of the relation is all real numbers.



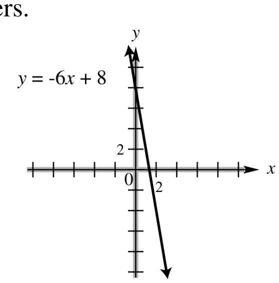
domain: $[0, \infty)$ range: $(-\infty, \infty)$

27. $y = 2x - 6$ represents a function since y is found by multiplying x by 2 and subtracting 6. Each value of x corresponds to just one value of y. x can be any real number, so the domain is all real numbers. Since y is twice x, less 6, y also may be any real number, and so the range is also all real numbers.



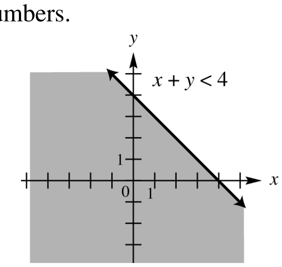
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

28. $y = -6x + 8$ represents a function since y is found by multiplying x by -6 and adding 8. Each value of x corresponds to just one value of y. x can be any real number, so the domain is all real numbers. Since y is -6 times x, plus 8, y also may be any real number, and so the range is also all real numbers.



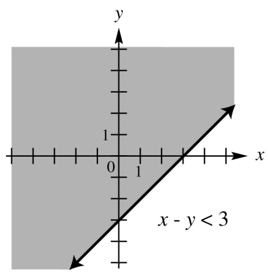
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

29. By definition, y is a function of x if every value of x leads to exactly one value of y. Substituting a particular value of x, say 1, into $x + y < 4$, corresponds to many values of y. The ordered pairs $(1,2)$, $(1,1)$, $(1,0)$, $(1,-1)$, and so on, all satisfy the inequality. This does not represent a function. Any number can be used for x or for y, so the domain and range of this relation are both all real numbers.



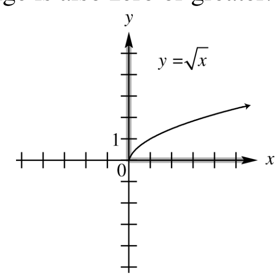
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

30. By definition, y is a function of x if every value of x leads to exactly one value of y. Substituting a particular value of x, say 1, into $x - y < 3$ corresponds to many values of y. The ordered pairs $(1,-1)$, $(1,0)$, $(1,1)$, $(1,2)$, and so on, all satisfy the inequality. This does not represent a function. Any number can be used for x or for y, so the domain and range of this relation are both all real numbers.



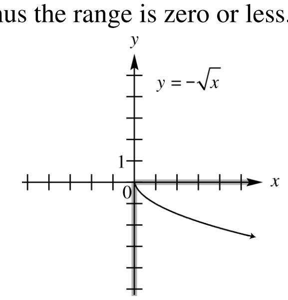
domain: $(-\infty,\infty)$; range: $(-\infty,\infty)$

31. For any choice of x in the domain of $y = \sqrt{x}$, there is exactly one corresponding value of y, so this equation defines a function. Since the quantity under the square root cannot be negative, we have $x \geq 0$. Because the radical is nonnegative, the range is also zero or greater.



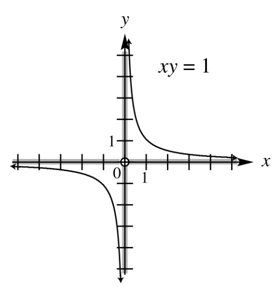
domain: $[0,\infty)$; range: $[0,\infty)$

32. For any choice of x in the domain of $y = -\sqrt{x}$, there is exactly one corresponding value of y, so this equation defines a function. Since the quantity under the square root cannot be negative, we have $x \geq 0$. The outcome of the radical is nonnegative, when you change the sign (by multiplying by -1), the range becomes nonpositive. Thus the range is zero or less.



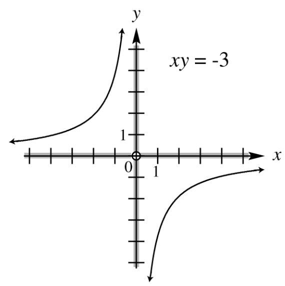
domain: $[0,\infty)$; range: $(-\infty,0]$

33. Since $xy = 1$ can be rewritten as $y = \frac{1}{x}$, we can see that y can be found by dividing x into 1. This process produces one value of y for each value of x in the domain, so this equation is a function. The domain includes all real numbers except those that make the denominator equal to zero, namely $x = 0$. Values of y can be negative or positive, but never zero. Therefore, the range will be all real numbers except zero.



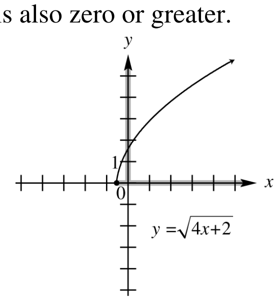
domain: $(-\infty,0)\cup(0,\infty)$; range: $(-\infty,0)\cup(0,\infty)$

34. Since $xy=-3$ can be rewritten as $y=\frac{-3}{x}$, we can see that y can be found by dividing x into -3. This process produces one value of y for each value of x in the domain, so this equation is a function. The domain includes all real numbers except those that make the denominator equal to zero, namely $x=0$. Values of y can be negative or positive, but never zero. Therefore, the range will be all real numbers except zero.



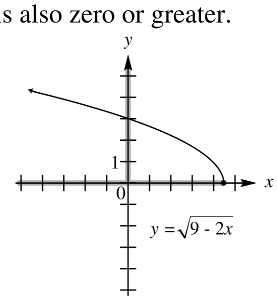
domain: $(-\infty,0)\cup(0,\infty)$; range: $(-\infty,0)\cup(0,\infty)$

35. For any choice of x in the domain of $y=\sqrt{4x+2}$ there is exactly one corresponding value of y, so this equation defines a function. Since the quantity under the square root cannot be negative, we have $4x+2\geq 0 \Rightarrow 4x\geq -2 \Rightarrow x\geq \frac{-2}{4}$ or $x\geq -\frac{1}{2}$. Because the radical is nonnegative, the range is also zero or greater.



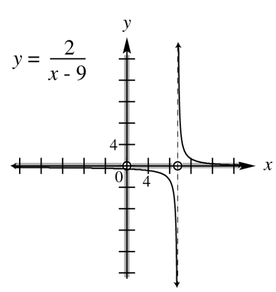
domain: $\left[-\frac{1}{2},\infty\right)$; range: $[0,\infty)$

36. For any choice of x in the domain of $y=\sqrt{9-2x}$ there is exactly one corresponding value of y, so this equation defines a function. Since the quantity under the square root cannot be negative, we have $9-2x\geq 0 \Rightarrow -2x\geq -9 \Rightarrow x\leq \frac{-9}{-2}$ or $x\leq \frac{9}{2}$. Because the radical is nonnegative, the range is also zero or greater.



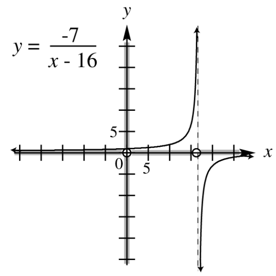
domain: $\left(-\infty,\frac{9}{2}\right]$; range: $[0,\infty)$

37. Given any value in the domain of $y=\frac{2}{x-9}$, we find y by subtracting 9, then dividing into 2. This process produces one value of y for each value of x in the domain, so this equation is a function. The domain includes all real numbers except those that make the denominator equal to zero, namely $x=9$. Values of y can be negative or positive, but never zero. Therefore, the range will be all real numbers except zero.



domain: $(-\infty,9)\cup(9,\infty)$; range: $(-\infty,0)\cup(0,\infty)$

38. Given any value in the domain of $y=\frac{-7}{x-16}$, we find y by subtracting 16, then dividing into -7. This process produces one value of y for each value of x in the domain, so this equation is a function. The domain includes all real numbers except those that make the denominator equal to zero, namely $x=16$. Values of y can be negative or positive, but never zero. Therefore, the range will be all real numbers except zero.



domain: $(-\infty,16)\cup(16,\infty)$; range: $(-\infty,0)\cup(0,\infty)$

39. B

40. Answers will vary.
An example is: The cost of gasoline depends on the number of gallons used; so cost is a function of number of gallons.

41. $f(x)=-3x+4$
$f(0)=-3\cdot 0+4=0+4=4$

42. $f(x)=-3x+4$
$f(-3)=-3(-3)+4=9+4=13$

43. $g(x)=-x^2+4x+1$
$g(-2)=-(-2)^2+4(-2)+1$
$=-4+(-8)+1=-11$

44. $g(x)=-x^2+4x+1$
$g(10)=-10^2+4\cdot 10+1$
$=-100+40+1=-59$

45. $f(x)=-3x+4$
$f(p)=-3p+4$

46. $g(x)=-x^2+4x+1$
$g(k)=-k^2+4k+1$

47. $f(x)=-3x+4$
$f(-x)=-3(-x)+4=3x+4$

48. $g(x)=-x^2+4x+1$
$g(-x)=-(-x)^2+4(-x)+1$
$=-x^2-4x+1$

49. $f(x)=-3x+4$
$f(x+2)=-3(x+2)+4$
$=-3x-6+4=-3x-2$

50. $f(x)=-3x+4$
$f(a+4)=-3(a+4)+4$
$=-3a-12+4=-3a-8$

51. $f(x)=-3x+4$
$f(2m-3)=-3(2m-3)+4$
$=-6m+9+4=-6m+13$

52. $f(x)=-3x+4$
$f(3t-2)=-3(3t-2)+4$
$=-9t+6+4=-9t+10$

53. **(a)** $f(2)=2$
(b) $f(-1)=3$

54. **(a)** $f(2)=5$
(b) $f(-1)=11$

55. **(a)** $f(2)=15$

(b) $f(-1)=10$

56. **(a)** $f(2)=1$

(b) $f(-1)=7$

57. **(a)** $f(2)=3$

(b) $f(-1)=-3$

58. **(a)** $f(2)=-3$

(b) $f(-1)=2$

59. **(a)**
$$x+3y=12$$
$$3y=-x+12$$
$$y=\frac{-x+12}{3}$$
$$y=-\tfrac{1}{3}x+4$$
$$f(x)=-\tfrac{1}{3}x+4$$

(b) $f(3)=-\frac{1}{3}(3)+4=-1+4=3$

60. **(a)**
$$x-4y=8$$
$$x=8+4y$$
$$x-8=4y$$
$$\frac{x-8}{4}=y$$
$$y=\tfrac{1}{4}x-2$$
$$f(x)=\tfrac{1}{4}x-2$$

(b) $f(3)=\frac{1}{4}(3)-2=\frac{3}{4}-2=\frac{3}{4}-\frac{8}{4}=-\frac{5}{4}$

61. **(a)**
$$y+2x^2=3$$
$$y=-2x^2+3$$
$$f(x)=-2x^2+3$$

(b)
$$f(3)=-2(3)^2+3=-2\cdot 9+3$$
$$=-18+3=-15$$

62. **(a)**
$$y-3x^2=2$$
$$y=3x^2+2$$
$$f(x)=3x^2+2$$

(b)
$$f(3)=3(3)^2+2=3\cdot 9+2$$
$$=27+2=29$$

63. **(a)**
$$4x-3y=8$$
$$4x=3y+8$$
$$4x-8=3y$$
$$\frac{4x-8}{3}=y$$
$$y=\tfrac{4}{3}x-\tfrac{8}{3}$$
$$f(x)=\tfrac{4}{3}x-\tfrac{8}{3}$$

(b) $f(3)=\frac{4}{3}(3)-\frac{8}{3}=\frac{12}{3}-\frac{8}{3}=\frac{4}{3}$

64. **(a)**
$$-2x+5y=9$$
$$5y=2x+9$$
$$y=\frac{2x+9}{5}$$
$$y=\tfrac{2}{5}x+\tfrac{9}{5}$$
$$f(x)=\tfrac{2}{5}x+\tfrac{9}{5}$$

(b) $f(3)=\frac{2}{5}(3)+\frac{9}{5}=\frac{6}{5}+\frac{9}{5}=\frac{15}{5}=3$

65. $f(3)=4$
Additional answers will vary.

66. Since $f(.2)=.2^2+3(.2)+1=.04+.6+1=1.64,$ the height of the rectangle is 1.64 units. The base measures $.3-.2=.1$ unit. Since the area of a rectangle is base times height, the area of this rectangle is $.1(1.64)=.164$ square unit.

67. $f(3)$ is the *y*-component of the coordinate, which is -4.

68. $f(-2)$ is the *y*-component of the coordinate, which is -3.

69. **(a)** $f(-2)=0$

(b) $f(0)=4$

(c) $f(1)=2$

(d) $f(4)=4$

70. **(a)** $f(-2)=5$ **(b)** $f(0)=0$

(c) $f(1)=2$ **(d)** $f(4)=4$

71. **(a)** $f(-2)=-3$ **(b)** $f(0)=-2$

(c) $f(1)=0$ **(d)** $f(4)=2$

72. **(a)** $f(-2)=3$ **(b)** $f(0)=3$

(c) $f(1)=3$ **(d)** $f(4)=3$

73. **(a)** $[4,\infty)$ **(b)** $(-\infty,-1]$

(c) $[-1,4]$

74. **(a)** $(-\infty,1]$ **(b)** $[4,\infty)$

(c) $[1,4]$

75. **(a)** $(-\infty,4]$ **(b)** $[4,\infty)$

(c) none

76. **(a)** none **(b)** $(-\infty,\infty)$

(c) none

77. **(a)** none

(b) $(-\infty,-2]$; $[3,\infty)$

(c) $(-2,3)$

78. **(a)** $(3,\infty)$ **(b)** $(-\infty,-3)$

(c) $(-3,3]$

79. **(a)** yes

(b) $[0,24]$

(c) When $t=8$, $y=1200$ from the graph. At 8 A.M., approximately 1200 megawatts is being used.

(d) at 17 hr or 5 P.M.; at 4 A.M.

(e) $f(12)=2000$; At 12 noon, electricity use is 2000 megawatts.

(f) increasing from 4 A.M. to 5 P.M.; decreasing from midnight to 4 A.M. and from 5 P.M. to midnight

80. **(a)** At t = 2, y = 240 from the graph. Therefore, at 2 seconds, the ball is 240 feet high.

(b) At y = 192, x = 1 and x = 5 from the graph. Therefore, at 1 second and at 5 seconds, the height will be 192 feet.

(c) The ball is going up from 0 to 3 seconds and down from 3 to 7 seconds.

(d) The coordinate of the highest point is (3, 256). Therefore, it reaches a maximum height of 256 feet at 3 seconds.

(e) At $x = 7$, $y = 0$. Therefore, at 7 seconds, the ball hits the ground.

81. **(a)** At t = 12 and t = 20, y = 55 from the graph. Therefore, after about 12 noon until about 8 P.M. the temperature was over 55°.

(b) At t = 5 and t = 22, y = 40 from the graph. Therefore, until about 6 A.M. and after 10 P.M. the temperature was below 40°.

(c) The temperature at noon in Bratenahl, Ohio was 55°. Since the temperature in Greenville is 7° higher, we are looking for the time at which Bratenahl, Ohio was $55°-7°$, or 48°. This occurred at approximately 10 A.M and 8:30 P.M.

82. **(a)** At t = 8, y = 24 from the graph. Therefore, there are 24 units of the drug in the bloodstream at 8 hours.

(b) Level increases between 0 and 2 hours and decreases between 2 and 12 hours.

(c) The coordinates of the highest point are (2, 64). Therefore, at 2 hours, the level of the drug in the bloodstream reaches its greatest value of 64 units.

(d) After the peak, $y = 16$ at $t = 10$.

10 hours – 2 hours = 8 hours after the peak. 8 additional hours are required for the level to drop to 16 units.

Section 2.3: Linear Functions

1 B; $f(x) = 2x + 6$ is a linear function with y-intercept 6.

2. H; $x = 3$ is a vertical line.

3. C; $f(x) = 7$ is a constant function.

4. G; $2x - y = -4$ or $y = 2x + 4$ is a linear equation with x-intercept -2 and y-intercept 4.

5. A; $f(x) = 2x$ is a linear function whose graph passes through the origin, (0, 0). $f(0) = 2(0) = 0$.

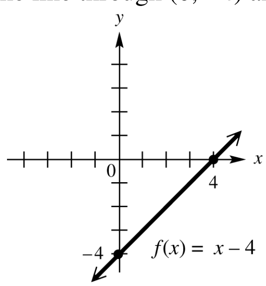
6. D; $f(x) = x^2$ is a function that is not linear.

7. $f(x) = x - 4$; Use the intercepts.

$$f(0) = 0 - 4 = -4: \ y\text{-intercept}$$

$$0 = x - 4 \Rightarrow x = 4: \ x\text{-intercept}$$

Graph the line through (0, –4) and (4, 0).



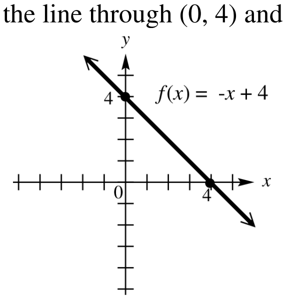
The domain and range are both $(-\infty, \infty)$.

8. $f(x) = -x + 4$; Use the intercepts.

$$f(0) = -0 + 4 = 4: \ y\text{-intercept}$$

$$0 = -x + 4 \Rightarrow x = 4: \ x\text{-intercept}$$

Graph the line through (0, 4) and (4, 0).



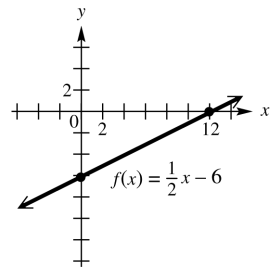
The domain and range are both $(-\infty, \infty)$.

9. $f(x) = \frac{1}{2}x - 6$; Use the intercepts.

$$f(0) = \tfrac{1}{2}(0) - 6 = -6: \ y\text{-intercept}$$

$$0 = \tfrac{1}{2}x - 6 \Rightarrow 6 = \tfrac{1}{2}x \Rightarrow x = 12: \ x\text{-intercept}$$

Graph the line through (0, –6) and (12, 0).



The domain and range are both $(-\infty, \infty)$.

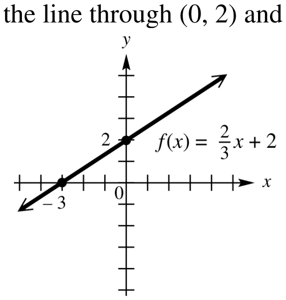
10. $f(x) = \frac{2}{3}x + 2$; Use the intercepts.

$$f(0) = \tfrac{2}{3}(0) + 2 = 2: \ y\text{-intercept}$$

$$0 = \tfrac{2}{3}x + 2 \Rightarrow -2 = \tfrac{2}{3}x$$

$$x = -3: \ x\text{-intercept}$$

Graph the line through (0, 2) and (–3, 0).



The domain and range are both $(-\infty, \infty)$.

11. $-4x + 3y = 9$; Use the intercepts.

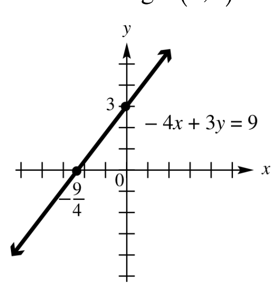
$$-4(0) + 3y = 9 \Rightarrow 3y = 9$$

$$y = 3: \ y\text{-intercept}$$

$$-4x + 3(0) = 9 \Rightarrow -4x = 9$$

$$x = -\tfrac{9}{4}: \ x\text{-intercept}$$

Graph the line through $(0, 3)$ and $\left(-\frac{9}{4}, 0\right)$.

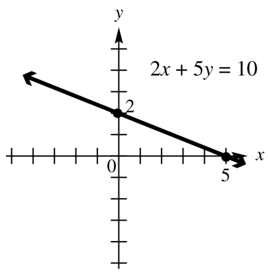


The domain and range are both $(-\infty, \infty)$.

12. $2x+5y=10$; Use the intercepts.

$$2(0)+5y=10 \Rightarrow 5y=10$$
$$y=2: y\text{-intercept}$$
$$2x+5(0)=10 \Rightarrow 2x=10$$
$$x=5: x\text{-intercept}$$

Graph the line through $(0,2)$ and $(5,0)$.



The domain and range are both $(-\infty,\infty)$.

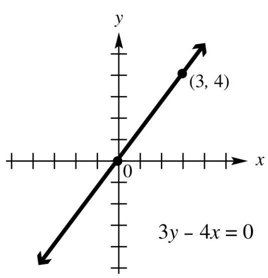
13. $3y-4x=0$; Use the intercepts.

$$3y-4(0)=0 \Rightarrow 3y=0$$
$$y=0: y\text{-intercept}$$
$$3(0)-4x=0 \Rightarrow -4x=0$$
$$x=0: x\text{-intercept}$$

The graph has just one intercept. Choose an additional value, say 3, for *x*.

$$3y-4(3)=0 \Rightarrow 3y-12=0$$
$$3y=12 \Rightarrow y=4$$

Graph the line through $(0,0)$ and $(3,4)$.



The domain and range are both $(-\infty,\infty)$.

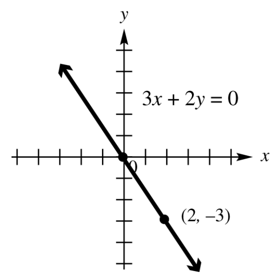
14. $3x+2y=0$; Use the intercepts.

$$3(0)+2y=0 \Rightarrow 2y=0$$
$$y=0: y\text{-intercept}$$
$$3x+2(0)=0 \Rightarrow 3x=0 \Rightarrow x=0: x\text{-intercept}$$

The graph has just one intercept. Choose an additional value, say 2, for *x*.

$$3(2)+2y=0 \Rightarrow 6+2y=0$$
$$2y=-6 \Rightarrow y=-3$$

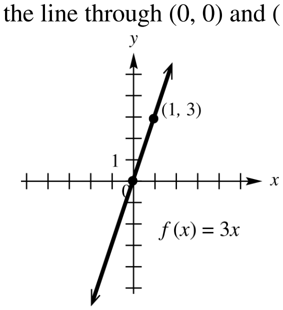
Graph the line through $(0,0)$ and $(2,-3)$.



The domain and range are both $(-\infty,\infty)$.

15 $f(x)=3x$

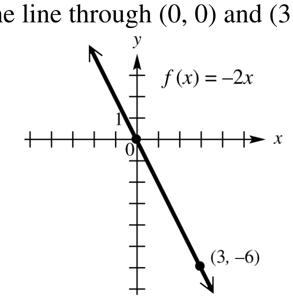
The *x*-intercept and the *y*-intercept are both zero. This gives us only one point, (0, 0). If $x = 1$, $y = 3(1) = 3$. Another point is (1, 3). Graph the line through (0, 0) and (1, 3).



The domain and range are both $(-\infty,\infty)$.

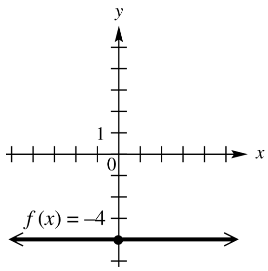
16. $y=-2x$

The *x*-intercept and the *y*-intercept are both zero. This gives us only one point, (0, 0).
If $x = 3$, $y = -2(3) = -6$.
Another point is (3, –6).
Graph the line through (0, 0) and (3, –6).



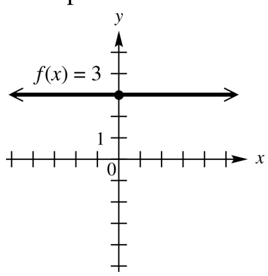
The domain and range are both $(-\infty,\infty)$.

17. $f(x) = -4$ is a constant function.
The graph of $f(x) = -4$ is a horizontal line with a y-intercept of -4.



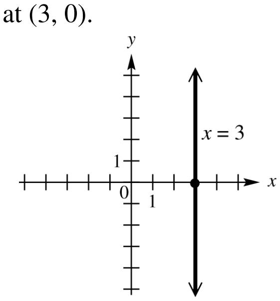
domain: $(-\infty, \infty)$; range: $\{-4\}$

18. $f(x) = 3$ is a constant function.
The graph of $f(x) = 3$ is a horizontal line with y-intercept of 3.



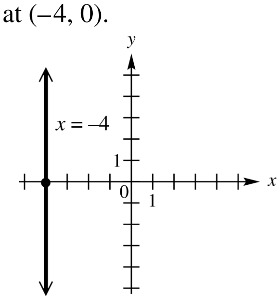
domain: $(-\infty, \infty)$; range: $\{3\}$

19. $x = 3$ is a vertical line, intersecting the x-axis at (3, 0).



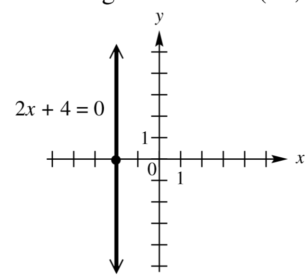
domain: $\{3\}$; range: $(-\infty, \infty)$

20. $x = -4$ is a vertical line intersecting the x-axis at (–4, 0).



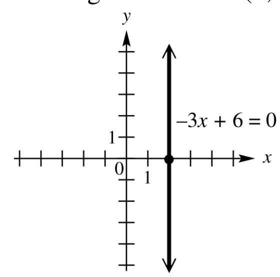
domain: $\{-4\}$; range: $(-\infty, \infty)$

21. $2x + 4 = 0 \Rightarrow 2x = -4 \Rightarrow x = -2$ is a vertical line intersecting the x-axis at (–2, 0).



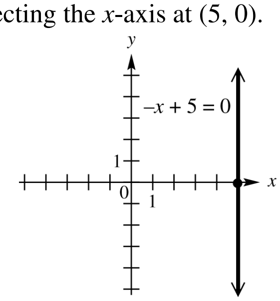
domain: $\{-2\}$; range: $(-\infty, \infty)$

22. $3x + 6 = 0 \Rightarrow -3x = -6 \Rightarrow x = 2$ is a vertical line intersecting the x-axis at (2, 0).



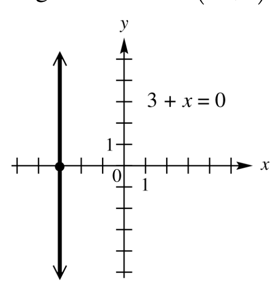
domain: $\{2\}$; range: $(-\infty, \infty)$

23. $-x + 5 = 0 \Rightarrow x = 5$ is a vertical line intersecting the x-axis at (5, 0).



domain: $\{5\}$; range: $(-\infty, \infty)$

24. $3 + x = 0 \Rightarrow x = -3$ is a vertical line intersecting the x-axis at $(-3, 0)$.



domain: $\{-3\}$; range: $(-\infty, \infty)$

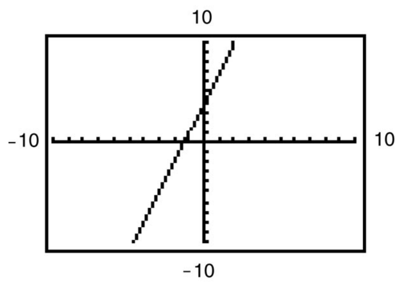
25. $y = 2$ is a horizontal line with y-intercept 2. Choice A resembles this.

26. $y = -2$ is a horizontal line with y-intercept -2. Choice C resembles this.

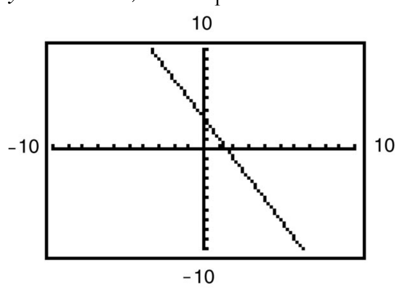
27. $x = 2$ is a vertical line with x-intercept 2. Choice D resembles this.

28. $x = -2$ is a vertical line with x-intercept -2. Choice B resembles this.

29. $y = 3x + 4$; Use $Y_1 = 3X + 4$.



30. $y = -2x + 3$; Use $Y_1 = -2X + 3$

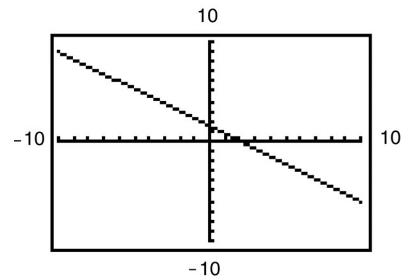


31. $3x + 4y = 6$; Solve for y.

$$3x + 4y = 6$$
$$4y = -3x + 6$$
$$y = -\tfrac{3}{4}x + \tfrac{3}{2}$$

Use $Y_1 = (-3/4)X + (3/2)$

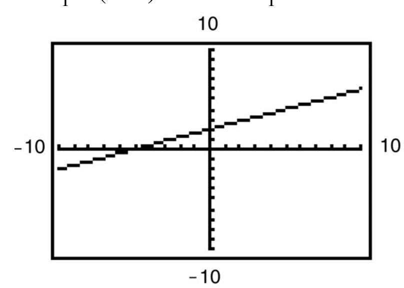
or $Y_1 = -3/4X + 3/2$.



32. $-2x + 5y = 10$; Solve for y.

$$-2x + 5y = 10$$
$$5y = 2x + 10$$
$$y = \tfrac{2}{5}x + 2$$

Use $Y_1 = (2/5)X + 2$ or $Y_1 = 2/5X + 2$



33. The rise is 2.5 feet while the run is 10 feet so the slope is $\frac{2.5}{10} = .25 = 25\% = \frac{1}{4}$. So A = .25, $C = \frac{2.5}{10}$, D = 25%, and $E = \frac{1}{4}$ are all expressions of the slope.

34. The pitch or slope is $\frac{1}{4}$. If the rise is 4 feet then $\frac{1}{4} = \frac{\text{rise}}{\text{run}} = \frac{4}{x}$ or $x = 16$ feet. So 16 feet in the horizontal direction corresponds to a rise of 4 feet.

35. through $(2, -1)$ and $(-3, -3)$

Let $x_1 = 2$, $y_1 = -1$, $x_2 = -3$, and $y_2 = -3$.

Then rise $= \Delta y = -3 - (-1) = -2$ and

run $= \Delta x = -3 - 2 = -5$.

The slope is $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{-2}{-5} = \frac{2}{5}$.

36. Through $(5, -3)$ and $(1, -7)$

Let $x_1 = 5$, $y_1 = -3$, $x_2 = 1$, and $y_2 = -7$.

Then rise $= \Delta y = -7 - (-3) = -4$ and

run $= \Delta x = 1 - 5 = -4$.

The slope is $m = \frac{\Delta y}{\Delta x} = \frac{-4}{-4} = 1$.

37. Through $(5, 9)$ and $(-2, 9)$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 9}{-2 - 5} = \frac{0}{-7} = 0$$

38. Through $(-2, 4)$ and $(6, 4)$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 4}{6 - (-2)} = \frac{0}{8} = 0$$

39. Horizontal, through $(3, -7)$

The slope of every horizontal line is zero, so $m = 0$.

40. Horizontal, through (–6, 5)
The slope of every horizontal line is zero, so $m = 0$.

41. Vertical, through (3, –7)
The slope of every vertical line is undefined; m is undefined.

42. Vertical, through (–6, 5)
The slope of every vertical line is undefined; m is undefined.

43. Both B and C can be used to find the slope. The form $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ is the form that is standardly used.

If you rename points 1 and 2, you will get the formula stated in choice B. Choice D is incorrect because it shows a change in x to a change in y, which is not how slope is defined. Choice A is incorrect because the y - values are subtracted in one way, and the x - values in the opposite way. This will result in the opposite (additive inverse) of the actual value of the slope of the line that passes between the two points.

44. Answers will vary.
No, the graph of a linear function cannot have an undefined slope. A line that has an undefined slope is vertical. With a vertical line, more than one y - value is associated with the x - value.

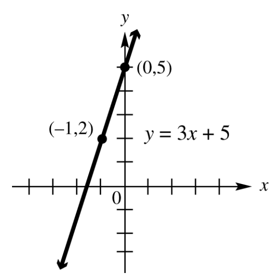
45. $y = 3x + 5$

Find two ordered pairs that are solutions to the equation.

If $x = 0$, then $y = 3(0) + 5 \Rightarrow y = 5$.

If $x = -1$ then $y = 3(-1) + 5 \Rightarrow y = -3 + 5 \Rightarrow y = 2$. Thus two ordered pairs are $(0,5)$ and $(-1,2)$.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{-1-0} = \frac{-3}{-1} = 3.$$



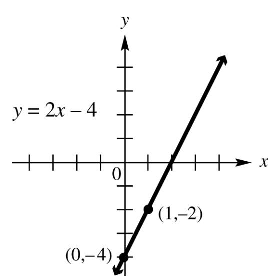
46. $y = 2x - 4$

Find two ordered pairs that are solutions to the equation.

If $x = 0$, then $y = 2(0) - 4 \Rightarrow y = -4$.

If $x = 1$, then $y = 2(1) - 4 \Rightarrow y = 2 - 4 \Rightarrow y = -2$. Thus two ordered pairs are $(0,-4)$ and $(1,-2)$.

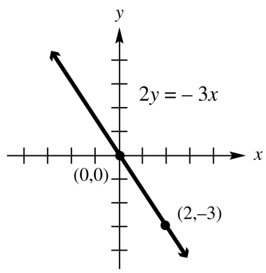
$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-4)}{1-0} = \frac{2}{1} = 2.$$



47. $2y = -3x$

Find two ordered pairs that are solutions to the equation.
If $x = 0$, then $2y = 0 \Rightarrow y = 0$.
If $y = -3$, then $2(-3) = -3x \Rightarrow -6 = -3x$ $\Rightarrow x = 2$. Thus two ordered pairs are $(0,0)$ and $(2,-3)$.

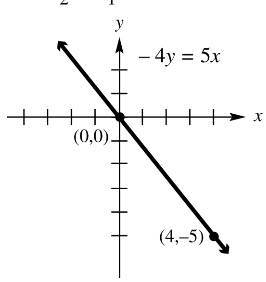
$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-0}{2-0} = -\frac{3}{2}.$$



48. $-4y = 5x$

Find two ordered pairs that are solutions to the equation.
If $x = 0$, then $-4y = 0 \Rightarrow y = 0$.
If $x = 4$, then $-4y = 5(4) \Rightarrow -4y = 20$ $\Rightarrow y = -5$. Thus two ordered pairs are $(0,0)$ and $(4,-5)$.

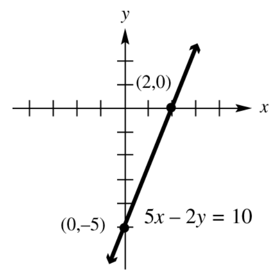
$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-0}{4-0} = -\frac{5}{4}.$$



49. $5x - 2y = 10$

Find two ordered pairs that are solutions to the equation.
If $x = 0$, then $5(0) - 2y = 10 \Rightarrow -2y = 10$ $\Rightarrow y = -5$.
If $y = 0$, then $5x - 2(0) = 10 \Rightarrow 5x = 10$ $\Rightarrow x = 2$. Thus two ordered pairs are $(0,-5)$ and $(2,0)$.

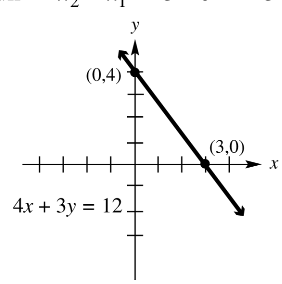
$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-5)}{2-0} = \frac{5}{2}.$$



50. $4x + 3y = 12$

Find two ordered pairs that are solutions to the equation.
If $x = 0$, then $4(0) + 3y = 12 \Rightarrow 3y = 12$ $\Rightarrow y = 4$.
If $y = 0$, then $4x + 3(0) = 12 \Rightarrow 4x = 12$ $\Rightarrow x = 3$. Thus two ordered pairs are $(0,4)$ and $(3,0)$.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-4}{3-0} = -\frac{4}{3}.$$



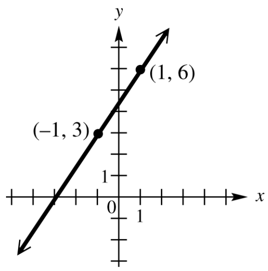
51. Answers will vary.

52. Answers will vary.

53. Through (–1, 3), $m = \frac{3}{2}$

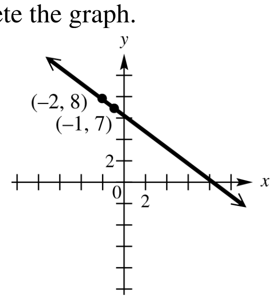
First locate the point (–1, 3).

Since the slope is $\frac{3}{2}$, a change of 2 units horizontally (2 units to the right) produces a change of 3 units vertically (3 units up). This gives a second point, (1, 6), which can be used to complete the graph.



54. Through (–2, 8), $m = –1$

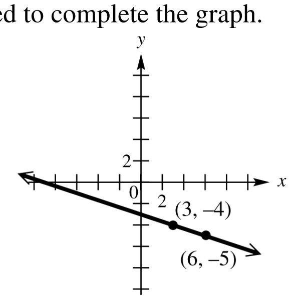
Since the slope is –1, a change of 1 unit horizontally (to the right) produces a change of –1 unit vertically (down 1). This gives a second point (–1, 7), which can be used to complete the graph.



55. Through (3, –4), $m = -\frac{1}{3}$

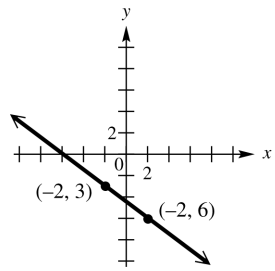
First locate the point (3, –4).

Since the slope is $-\frac{1}{3}$, a change of 3 units horizontally (3 units to the right) produces a change of –1 unit vertically (1 unit down). This gives a second point, (6, –5), which can be used to complete the graph.



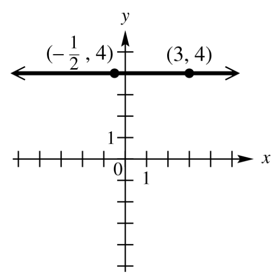
56. Through (–2, –3), $m = -\frac{3}{4}$

Since the slope is $-\frac{3}{4} = \frac{-3}{4}$, a change of 4 units horizontally (4 units to the right) produces a change of –3 units vertically (3 units down). This gives a second point (2, −6), which can be used to complete the graph.



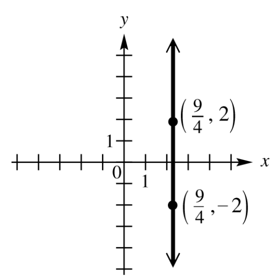
57. Through $\left(-\frac{1}{2}, 4\right)$, $m = 0$

The graph is the horizontal line through $\left(-\frac{1}{2}, 4\right)$.



58. Through $\left(\frac{9}{4}, 2\right)$, undefined slope

The slope is undefined, so the line is vertical, intersecting the *x*-axis at $\left(\frac{9}{4}, 0\right)$.



59. $m = \frac{1}{2}$ matches graph D because the line rises gradually as *x* increases.

60. $m = –2$ matches graph C because the line falls rapidly as *x* increases.

61. m = 0 matches graph A because horizontal lines have slopes of 0.

62. $m=-\frac{1}{2}$ matches graph F because the line falls gradually as x increases.

63. $m = 2$ matches graph E because the line rises rapidly as x increases.

64. m is undefined for graph B because vertical lines have undefined slopes.

65. The average rate of change is $m=\frac{\Delta y}{\Delta x}$ $\frac{20-4}{4-0}=\frac{-16}{4}=-\$4$ (thousand) per year; The value of the machine is decreasing \$4000 each year during these years.

66. The average rate of change is $m=\frac{\Delta y}{\Delta x}$ $=\frac{200-0}{4-0}=\frac{200}{4}=\$50$ per month; The amount saved is increasing \$50 each month during these months.

67. The average rate of change is $m=\frac{\Delta y}{\Delta x}$ $\frac{3-3}{4-0}=\frac{0}{4}=0\%$ per year; The percent of pay raise is not changing - it is 3% each year.

68. **(a)** The slope of –.0187 indicates that the average rate of change of the winning time for the 5000 m run is .0187 min less (faster). It is negative because the times are generally decreasing as time progresses.

(b) World War II (1939–1945) included the years 1940 and 1944.

(c) $y=-.0187(1996)+50.60\approx 13.27$ min

The times differ by $13.27-13.13=.14$ min

69. **(a)** Answers will vary.

(b) $m=\frac{12{,}057-2773}{1999-1950}=\frac{9284}{49}\approx 189.5$

This means that the average rate of change in the number of radio stations per year is an increase of about 189.5 stations.

70. **(a)** To find the change in subscribers, we need to subtract the number of subscribers in consecutive years.

Years	Change in subscribers (in thousands)
1994 – 1995	$33{,}786-24{,}134=9652$
1995 – 1996	$44{,}043-33{,}786=10{,}257$
1996 – 1997	$55{,}312-44{,}043=11{,}269$
1997 – 1998	$69{,}209-55{,}312=13{,}897$
1998 – 1999	$86{,}047-69{,}209=16{,}838$

(b) The change in successive years not the same. An approximately straight line could not be drawn through the points if they were plotted.

71. **(a)** $m=\frac{y_2-y_1}{x_2-x_1}=\frac{20.8-25.9}{1998-1995}=\frac{-5.1}{3}=-1.7$ million recipients per year

(b) The negative slope means the numbers of recipients *decreased* by 1.7 million each year.

72. **(a)** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{13.1 - 14.6}{1996 - 1912} = \frac{-1.5}{84} \approx -.0179$ min per year; The winning time decreased an average of .0179 min each event year from 1912 to 1996.

(b) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{13.6 - 14.6}{2000 - 1912} = \frac{-1}{88} \approx -.0114$ min per year; The winning time decreased an average of .0114 min each event year from 1912 to 2000.

73. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{16.7 - 15}{2000 - 1995} = \frac{1.7}{5} = .34$ per year; The percent of freshman listing business as their probable field of study increased an average of .34% per year from 1995 to 2000.

74. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{155 - 500}{2002 - 1997} = \frac{-345}{5} = -\$69$; The price decreased an average of \$69 each year from 1997 to 2002.

75. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15.5 - .349}{2002 - 1997} = \frac{15.151}{5} \approx 3.03$ million per year; Sales of DVD players increased an average of 3.03 million each year from 1997 to 2002.

76. The first two points are $A(0, -6)$ and $B(1, -3)$.
$m = \frac{-3 - (-6)}{1 - 0} = \frac{3}{1} = 3$

77. The second and third points are B(1, –3) and C(2, 0).
$m = \frac{0 - (-3)}{2 - 1} = \frac{3}{1} = 3$

78. If we use any two points on a line to find its slope, we find that the slope is <u>the same</u> in all cases.

79. The first two points are $A(0, -6)$ and $B(1, -3)$.
$d(A, B) = \sqrt{[-3 - (-6)]^2 + (1 - 0)^2} = \sqrt{3^2 + 1^2} = \sqrt{9 + 1} = \sqrt{10}$

80. The second and fourth points are $B(1, -3)$ and $D(3, 3)$.
$d(B, D) = \sqrt{[3 - (-3)]^2 + (3 - 1)^2} = \sqrt{6^2 + 2^2} = \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$

81. The first and fourth points are $A(0, -6)$ and $D(3, 3)$.
$d(A, D) = \sqrt{[3 - (-6)]^2 + (3 - 0)^2} = \sqrt{9^2 + 3^2} = \sqrt{81 + 9} = \sqrt{90} = 3\sqrt{10}$

82. $\sqrt{10} + 2\sqrt{10} = 3\sqrt{10}$; The sum is $3\sqrt{10}$, which is equal to the answer in Exercise 81.

83. If points A, B, and C lie on a line in that order, then the distance between A and B added to the distance between <u>B</u> and <u>C</u> is equal to the distance between <u>A</u> and <u>C</u>. (The order of the last two may be reversed.)

84. The midpoint of the segment joining $A(0, -6)$ and $G(6, 12)$ has coordinates $\left(\frac{0+6}{2}, \frac{-6+12}{2}\right) = \left(\frac{6}{2}, \frac{6}{2}\right)$ $= (3, 3)$. The midpoint is $M(3, 3)$, which is the same as the middle entry in the table.

85. The midpoint of the segment joining $E(4, 6)$ and $F(5, 9)$ has coordinates $\left(\frac{4+5}{2}, \frac{6+9}{2}\right) = \left(\frac{9}{2}, \frac{15}{2}\right)$ = (4.5, 7.5). If the x-value 4.5 were in the table, the corresponding y-value would be 7.5.

86. (a) $C(x) = 10x + 500$

(b) $R(x) = 35x$

(c) $P(x) = R(x) - C(x)$
$= 35x - (10x + 500)$
$= 35x - 10x - 500 = 25x - 500$

(d) $C(x) = R(x)$
$10x + 500 = 35x$
$500 = 25x$
$20 = x$
20 units; do not produce

87. (a) $C(x) = 11x + 180$

(b) $R(x) = 20x$

(c) $P(x) = R(x) - C(x)$
$= 20x - (11x + 180)$
$= 20x - 11x - 180 = 9x - 180$

(d) $C(x) = R(x)$
$11x + 180 = 20x$
$180 = 9x$
$20 = x$
20 units; produce

88. (a) $C(x) = 150x + 2700$

(b) $R(x) = 280x$

(c) $P(x) = R(x) - C(x)$
$= 280x - (150x + 2700)$
$= 280x - 150x - 2700$
$= 130x - 2700$

(d) $C(x) = R(x)$
$150x + 2700 = 280x$
$2700 = 130x$
$20.77 \approx x$ or 21 units
produce

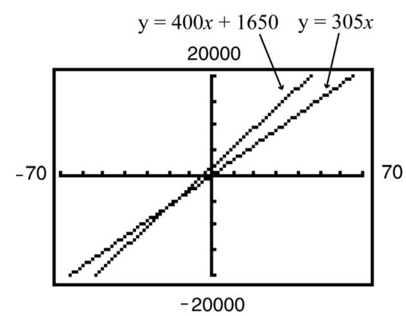
89. (a) $C(x) = 400x + 1650$

(b) $R(x) = 305x$

(c) $P(x) = R(x) - C(x)$
$= 305x - (400x + 1650)$
$= 305x - 400x - 1650$
$= -95x - 1650$

(d) $C(x) = R(x)$
$400x + 1650 = 305x$
$95x + 1650 = 0$
$95x = -1650$
$x \approx -17.37$ units

This result indicates a negative "break-even point," but the number of units produced must be a positive number. A calculator graph of the lines $Y_1 = 400X + 1650$ and $Y_2 = 305X$ on the same screen or solving the inequality $305x < 400x + 1650$ will show that $R(x) < C(x)$ for all positive values of x (in fact whenever x is greater than -17.4). Do not produce the product since it is impossible to make a profit.



90. (a) $C(x) = R(x) \Rightarrow 200x + 1000 = 240x \Rightarrow 1000 = 40x \Rightarrow 25 = x$; 25 units

(b) $C(25) = 200(25) + 1000 = \$6000$ which is the same as $R(25) = 240(25) = \$6000$

(c) $C(x) = R(x) \Rightarrow 220x + 1000 = 240x \Rightarrow 1000 = 20x \Rightarrow 50 = x$
The break-even point is 50 units instead of 25 units. The manager is not better off because twice as many units must be sold before beginning to show a profit.

Section 2.4: Equations of Lines; Curve Fitting

1. $y = \frac{1}{4}x + 2$ is graphed in D.

The slope is $\frac{1}{4}$ and the y-intercept is 2.

2. $4x + 3y = 12$ or $3y = -4x + 12$ or $y = -\frac{4}{3}x + 4$ is graphed in B.

The slope is $-\frac{4}{3}$ and the y-intercept is 4.

3. $y - (-1) = \frac{3}{2}(x - 1)$ is graphed in C.

The slope is $\frac{3}{2}$ and a point on the graph is $(1, -1)$.

4. $y = 4$ is graphed in A. $y = 4$ is a horizontal line with y-intercept 4.

5. Through (1, 3), $m = -2$
Write the equation in point-slope form.
$y - y_1 = m(x - x_1) \Rightarrow y - 3 = -2(x - 1)$
Then, change to standard form.
$$y - 3 = -2x + 2 \Rightarrow 2x + y = 5$$

6. Through (2, 4), m = –1
Write the equation in point-slope form.
$y - y_1 = m(x - x_1) \Rightarrow y - 4 = -1(x - 2)$
Then, change to standard form.
$$y - 4 = -x + 2 \Rightarrow x + y = 6$$

7. Through (–5, 4), $m = -\frac{3}{2}$

Write the equation in point-slope form.
$$y - 4 = -\frac{3}{2}[x - (-5)]$$
Change to standard form.
$$2(y - 4) = -3(x + 5)$$
$$2y - 8 = -3x - 15$$
$$3x + 2y = -7$$

8. Through $(-4, 3), m = \frac{3}{4}$

Write the equation in point-slope form.
$$y - 3 = \frac{3}{4}[x - (-4)]$$
Change to standard form.
$$4(y - 3) = 3(x + 4)$$
$$4y - 12 = 3x + 12$$
$$-3x + 4y = 24 \text{ or } 3x - 4y = -24$$

9. Through (–8, 4), undefined slope
Since undefined slope indicates a vertical line, the equation will have the form $x = a$. The equation of the line is $x = -8$.

10. Through (5, 1), $m = 0$
This is a horizontal line through (5, 1), so the equation is $y = 1$.

11. Through (–1, 3) and (3, 4)
First find m.
$$m = \frac{4 - 3}{3 - (-1)} = \frac{1}{4}$$
Use either point and the point-slope form.
$$y - 4 = \frac{1}{4}(x - 3)$$
Change to slope-intercept form.
$$4(y - 4) = x - 3$$
$$4y - 16 = x - 3$$
$$4y = x + 13$$
$$y = \frac{1}{4}x + \frac{13}{4}$$

12. Through (8, –1) and (4, 3)
First find m.
$$m = \frac{3 - (-1)}{4 - 8} = \frac{4}{-4} = -1$$
Use either point and the point-slope form.
$$y - 3 = -1(x - 4)$$
$$y - 3 = -x + 4$$
$$y = -x + 7$$

13. x-intercept 3, y-intercept –2
The line passes through (3, 0) and (0, –2). Use these points to find m.
$$m = \frac{-2 - 0}{0 - 3} = \frac{2}{3}$$
Using slope-intercept form we have $y = \frac{2}{3}x - 2$.

14. x-intercept –2, y-intercept 4
The line passes through the points (–2, 0) and (0, 4). Use these points to find m.
$$m = \frac{4 - 0}{0 - (-2)} = 2$$
Using slope-intercept form we have $y = 2x + 4$.

15. Vertical, through (–6, 4)
The equation of a vertical line has an equation of the form $x = a$. Since the line passes through (–6, 4), the equation is $x = -6$. (Since this slope of a vertical line is undefined, this equation cannot be written in slope-intercept form.)

16. Horizontal, through (2, 7)
The equation of a horizontal line has an equation of the form $y = b$. Since the line passes through (2, 7), the equation is $y = 7$.

17. $m = 5, b = 15$
Using slope-intercept form, we have $y = 5x + 15$.

18. $m = -2, b = 12$
Using slope-intercept form, we have $y = -2x + 12$.

19. $m = -\frac{2}{3}, b = -\frac{4}{5}$
Using slope-intercept form, we have $y = -\frac{2}{3}x - \frac{4}{5}$.

20. $m = -\frac{5}{8}, b = -\frac{1}{3}$
Using slope-intercept form, we have $y = -\frac{5}{8}x - \frac{1}{3}$.

21. slope 0, y-intercept $\frac{3}{2}$
These represent $m = 0$ and $b = \frac{3}{2}$. Using slope-intercept form we have $y = (0)x + \frac{3}{2} \Rightarrow y = \frac{3}{2}$.

22. Since this slope of a vertical line is undefined, this equation cannot be written in slope-intercept form. The equation of a vertical line has an equation of the form $x = a$. Since the line passes through the x - axis at $-\frac{5}{4}$, the equation is $x = -\frac{5}{4}$.

23. The line $x + 2 = 0$ has x-intercept <u>–2</u>. It <u>does not</u> have a y-intercept. The slope of this line is <u>undefined</u>. The line $4y = 2$ has y-intercept $\frac{1}{2}$. It <u>does not</u> have an x-intercept. The slope of this line is <u>0</u>.

24. **(a)** The graph of $y = 3x + 2$ has a positive slope and a positive y-intercept. These conditions match graph D.

(b) The graph of $y = -3x + 2$ has a negative slope and a positive y-intercept. These conditions match graph B.

(c) The graph of $y = 3x - 2$ has a positive slope and a negative y-intercept. These conditions match graph A.

(d) The graph of $y = -3x - 2$ has a negative slope and a negative y-intercept. These conditions match graph C.

25. **(a)** The graph of $y = 2x + 3$ has a positive slope and a positive y-intercept. These conditions match graph B.

(b) The graph of $y = -2x + 3$ has a negative slope and a positive y-intercept. These conditions match graph D.

(c) The graph of $y = 2x - 3$ has a positive slope and a negative y-intercept. These conditions match graph A.

(d) The graph of $y = -2x - 3$ has a negative slope and a negative y-intercept. These conditions match graph C.

26. **(a)** Use the first two points in the table, $A(-2, -11)$ and $B(-1, -8)$.

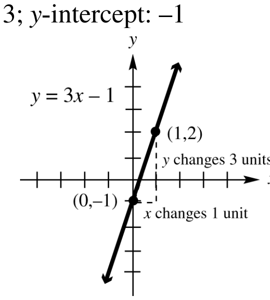
$$m = \frac{-8 - (-11)}{-1 - (-2)} = \frac{3}{1} = 3$$

(b) When $x = 0$, $y = -5$. The y-intercept is –5.

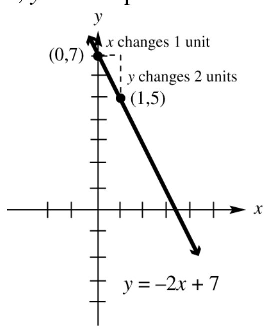
(c) Substitute 3 for m and –5 for b in the slope-intercept form.

$$y = mx + b \Rightarrow y = 3x - 5$$

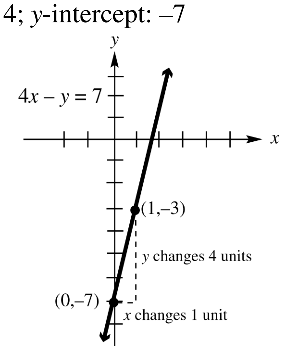
27. $y = 3x - 1$
This equation is in the slope-intercept form, $y = mx + b$.
slope: 3; y-intercept: –1



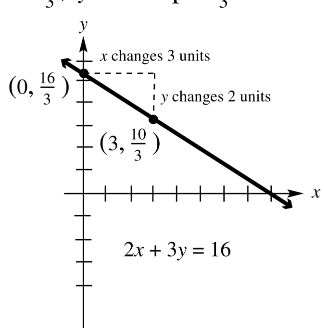
28. $y = -2x + 7$
slope: -2; y-intercept: 7



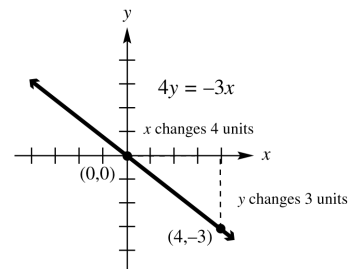
29. $4x - y = 7$
Solve for y to write the equation in slope-intercept form.
$-y = -4x + 7 \Rightarrow y = 4x - 7$
slope: 4; y-intercept: -7



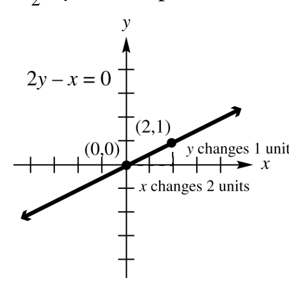
30. $2x + 3y = 16$
Solve the equation for y to write the equation in slope-intercept form.
$3y = -2x + 16 \Rightarrow y = -\frac{2}{3}x + \frac{16}{3}$
slope: $-\frac{2}{3}$; y-intercept: $\frac{16}{3}$



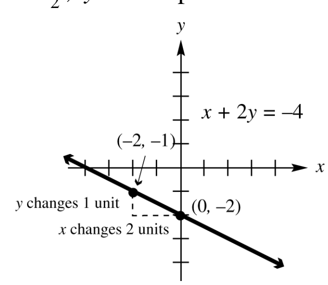
31. $4y = -3x$
$y = -\frac{3}{4}x$ or $y = -\frac{3}{4}x + 0$
slope: $-\frac{3}{4}$; y-intercept 0



32. $2y - x = 0$
$2y = x \Rightarrow y = \frac{1}{2}x$ or $y = \frac{1}{2}x + 0$
slope is $\frac{1}{2}$; y-intercept: 0



33. $x + 2y = -4$
Solve the equation for y to write the equation in slope-intercept form.
$2y = -x - 4 \Rightarrow y = -\frac{1}{2}x - 2$
slope: $-\frac{1}{2}$; y-intercept: -2

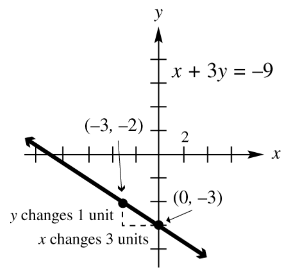


34. $x+3y=-9$

Solve the equation for y to write the equation in slope-intercept form.

$$3y=-x-9 \Rightarrow y=-\tfrac{1}{3}x-3$$

slope: $-\frac{1}{3}$; y-intercept: -3



35. **(a)** through (–1, 4), parallel to $x + 3y = 5$

First, find the slope of the line $x + 3y = 5$ by writing this equation in slope-intercept form.

$$x+3y=5 \Rightarrow 3y=-x+5 \Rightarrow y=-\tfrac{1}{3}x+\tfrac{5}{3}$$

The slope is $-\frac{1}{3}$. Since the lines are parallel, $-\frac{1}{3}$ is also the slope of the line whose equation is to be found. Substitute $m=-\frac{1}{3}$, $x_1=-1$, and $y_1=4$ into the point-slope form.

$$y-y_1=m(x-x_1) \Rightarrow y-4=-\tfrac{1}{3}[x-(-1)] \Rightarrow y-4=-\tfrac{1}{3}(x+1)$$
$$3(y-4)=-1(x+1) \Rightarrow 3y-12=-x-1 \Rightarrow x+3y=11$$

(b) Solve for y. $3y=-x+11 \Rightarrow y=-\frac{1}{3}x+\frac{11}{3}$

36. **(a)** through (3, –2), parallel to $2x - y = 5$

First, find the slope of the line $2x - y = 5$ by writing this equation in slope-intercept form.

$$2x-y=5 \Rightarrow -y=-2x+5 \Rightarrow y=2x-5$$

The slope is 2. Since the lines are parallel, 2 is also the slope of the line whose equation is to be found. Substitute $m=2$, $x_1=3$, and $y_1=-2$ into the point-slope form.

$$y-y_1=m(x-x_1) \Rightarrow y+2=2(x-3) \Rightarrow y+2=2x-6$$
$$-2x+y=-8 \text{ or } 2x-y=8$$

(b) Solve for y. $y = 2x - 8$

37. **(a)** through (1, 6), perpendicular to $3x + 5y = 1$

First, find the slope of the line $3x + 5y = 1$ by writing this equation in slope-intercept form.

$$3x+5y=1 \Rightarrow 5y=-3x+1 \Rightarrow y=-\tfrac{3}{5}x+\tfrac{1}{5}$$

This line has a slope of $-\frac{3}{5}$. The slope of any line perpendicular to this line is $\frac{5}{3}$, since $-\frac{3}{5}\left(\frac{5}{3}\right)=-1.$

Substitute $m=\frac{5}{3}$, $x_1=1$, and $y_1=6$ into the point-slope form.

$$y-6=\tfrac{5}{3}(x-1) \Rightarrow 3(y-6)=5(x-1) \Rightarrow 3y-18=5x-5$$
$$-13=5x-3y \text{ or } 5x-3y=-13$$

(b) Solve for y. $3y=5x+13 \Rightarrow y=\frac{5}{3}x+\frac{13}{3}$

38. **(a)** through (–2, 0), perpendicular to $8x-3y=7$

First, find the slope of the line $8x-3y=7$ by writing the equation in slope-intercept form.

$$8x-3y=7 \Rightarrow -3y=-8x+7 \Rightarrow y=\tfrac{8}{3}x-\tfrac{7}{3}$$

This line has a slope of $\tfrac{8}{3}$. The slope of any line perpendicular to this line is $-\tfrac{3}{8}$, since $\tfrac{8}{3}\left(-\tfrac{3}{8}\right)=-1.$

Substitute $m=-\tfrac{3}{8}$, $x_1=-2$, and $y_1=0$ into the point-slope form.

$$y-0=-\tfrac{3}{8}(x+2) \Rightarrow 8y=-3(x+2) \Rightarrow 8y=-3x-6 \Rightarrow 3x+8y=-6$$

(b) Solve for y. $8y=-3x-6 \Rightarrow y=-\tfrac{3}{8}x-\tfrac{6}{8} \Rightarrow y=-\tfrac{3}{8}x-\tfrac{3}{4}$

39. **(a)** through (4, 1), parallel to $y=-5$

Since $y=-5$ is a horizontal line, any line parallel to this line will be horizontal and have an equation of the form $y=b$. Since the line passes through (4, 1), the equation is $y=1$.

(b) The slope-intercept form is $y=1$.

40. **(a)** through $(-2,-2)$, parallel to $y=3$

Since $y=3$ is a horizontal line, any line parallel to this line will be horizontal and have an equation of the form $y=b$. Since the line passes through $(-2,-2)$, the equation is $y=-2$.

(b) The slope-intercept form is $y=-2$.

41. **(a)** through (–5, 6), perpendicular to $x=-2$.

Since $x=-2$ is a vertical line, any line perpendicular to this line will be horizontal and have an equation of the form $y=b$. Since the line passes through (–5, 6), the equation is $y=6$.

(b) The slope-intercept form is $y=6$.

42. **(a)** Through (4, −4), perpendicular to $x=4$

Since $x=4$ is a vertical line, any line perpendicular to this line will be horizontal and have an equation of the form $y=b$. Since the line passes through (4, −4), the equation is $y=-4$.

(b) The slope-intercept form is $y=-4$.

43. **(a)** Find the slope of the line $3y+2x=6$.

$$3y+2x=6 \Rightarrow 3y=-2x+6 \Rightarrow y=-\tfrac{2}{3}x+2$$

Thus, $m=-\tfrac{2}{3}$. A line parallel to $3y+2x=6$ also has slope $-\tfrac{2}{3}$.

$$\frac{2-(-1)}{k-4}=-\frac{2}{3}$$

Solve for k using the slope formula.

$$\frac{3}{k-4}=-\frac{2}{3} \Rightarrow 3(k-4)\left(\frac{3}{k-4}\right)=3(k-4)\left(-\frac{2}{3}\right) \Rightarrow 9=-2(k-4) \Rightarrow 9=-2k+8 \Rightarrow 2k=-1 \Rightarrow k=-\tfrac{1}{2}$$

(b) Find the slope of the line $2y-5x=1$.

$$2y-5x=1 \Rightarrow 2y=5x+1 \Rightarrow y=\tfrac{5}{2}x+\tfrac{1}{2}$$

Thus, $m=\tfrac{5}{2}$. A line perpendicular to $2y-5x=1$ will have slope $-\tfrac{2}{5}$, since $\tfrac{5}{2}\left(-\tfrac{2}{5}\right)=-1.$

Solve this equation for k.

$$\frac{3}{k-4}=-\frac{2}{5} \Rightarrow 5(k-4)\left(\frac{3}{k-4}\right)=5(k-4)\left(-\frac{2}{5}\right) \Rightarrow 15=-2(k-4) \Rightarrow 15=-2k+8 \Rightarrow 2k=-7 \Rightarrow k=-\tfrac{7}{2}$$

44. **(a)** Find the slope of the line $2x - 3y = 4$.

$$2x-3y=4 \Rightarrow -3y=-2x+4 \Rightarrow y=\tfrac{2}{3}x-\tfrac{4}{3}$$

Thus, $m=\frac{2}{3}$. A line parallel to $2x - 3y = 4$ also has slope $\frac{2}{3}$.

Solve for r using the slope formula.

$$\frac{r-6}{-4-2}=\frac{2}{3}\Rightarrow\frac{r-6}{-6}=\frac{2}{3}\Rightarrow -6\left(\frac{r-6}{-6}\right)=-6\left(\frac{2}{3}\right)\Rightarrow r-6=-4\Rightarrow r=2$$

(b) Find the slope of the line $x + 2y = 1$.

$$x+2y=1\Rightarrow 2y=-x+1\Rightarrow y=-\tfrac{1}{2}x+\tfrac{1}{2}$$

Thus, $m=-\frac{1}{2}$. A line perpendicular to the line $x + 2y = 1$ has slope 2, since $-\frac{1}{2}(2)=-1$. Solve for r using the slope formula.

$$\frac{r-6}{-4-2}=2\Rightarrow\frac{r-6}{-6}=2\Rightarrow r-6=-12\Rightarrow r=-6$$

45. (1970, 43.3), (1995, 58.9)

$$m=\frac{58.9-43.3}{1995-1970}=\frac{15.6}{25}=.624$$

Now use either point, say (1970, 43.3), and the point-slope form to find the equation.

$$y-43.3=.624(x-1970)$$
$$y-43.3=.624x-1229.28$$
$$y=.624x-1185.98$$

Let $x = 1996$.

$$y = .624\,(1996) - 1185.98 \approx 59.5$$

The percent of women in the civilian labor force is predicted to be 59.5%

This figure is very close to the actual figure.

46. (1975, 46.3), (2000, 60.0)

$$m=\frac{60.0-46.3}{2000-1975}=\frac{13.7}{25}=.548$$

Now use either point, say (2000, 60.0), and the point-slope form to find the equation.

$$y-60.0=.548(x-2000)$$
$$y-60=.548x-1096$$
$$y=.548x-1036$$

Let $x = 1996$.

$$y=.548(1996)-1036\approx 57.8$$

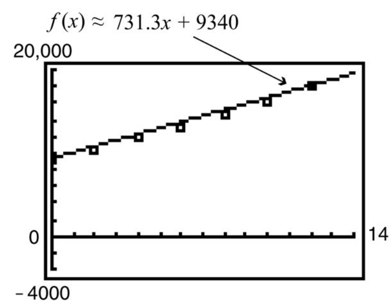
The percent of women in the civilian labor force is predicted to be 57.8%. This figure is reasonably close to the actual figure.

47. **(a)** (0, 9340), (12, 18,116)

$$m=\frac{18{,}116-9340}{12-0}=\frac{8776}{12}\approx 731.3$$

From the point (0, 9340), the value of b is 9340. Therefore we have the following.

$$f(x)\approx 731.3x+9340$$



The average tuition increase is about $731 per year for the period, because this is the slope of the line.

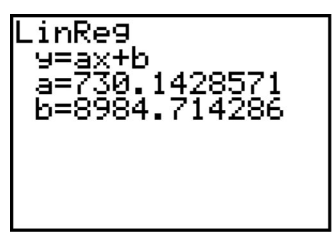
(b) 1995 corresponds to $x = 5$.

$$f(5)\approx 731.3(5)+9340=12{,}996.50$$

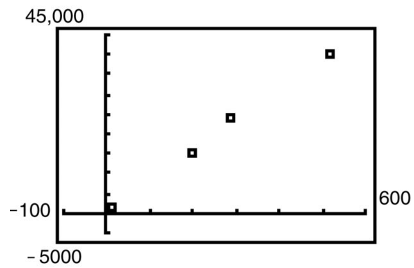
This is a fairly good approximation.

(c) From the calculator,

$$f(x)\approx 730.14x+8984.71$$



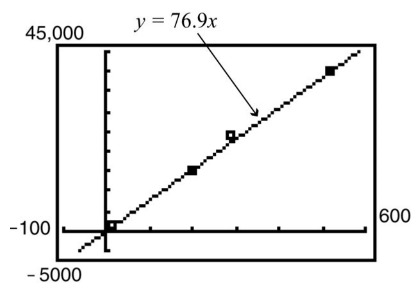
48. **(a)** There appears to be a linear relationship between the data. The farther the galaxy is from Earth, the faster it is receding.



(b) Using the points (520, 40,000) and (0, 0), we obtain

$$m = \frac{40,000 - 0}{520 - 0} = \frac{40,000}{520} \approx 76.9.$$

The equation of the line through these two points is $y = 76.9x$.



(c)
$$76.9x = 60,000$$
$$x = \frac{60,000}{76.9}$$
$$\approx 780$$

The galaxy Hydra is approximately 780 megaparsecs away.

(d)
$$A = \frac{9.5 \times 10^{11}}{m}$$
$$A = \frac{9.5 \times 10^{11}}{76.9} \approx 1.235 \times 10^{10}$$
$$\text{or } 12.35 \times 10^{9}$$

Using $m = 76.9$, we estimate that the age of the universe is approximately 12.35 billion years.

(e)
$$A = \frac{9.5 \times 10^{11}}{50} = 1.9 \times 10^{10} \text{ or } 19 \times 10^{9}$$
$$A = \frac{9.5 \times 10^{11}}{100} = 9.5 \times 10^{9}$$

The range for the age of the universe is between 9.5 billion and 19 billion years.

49. **(a)** The ordered pairs are (0, 32) and (100, 212).

The slope is $m = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}$.

Use $(x_1, y_1) = (0, 32)$ and $m = \frac{9}{5}$ in the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 32 = \tfrac{9}{5}(x - 0)$$
$$y - 32 = \tfrac{9}{5}x$$
$$y = \tfrac{9}{5}x + 32$$
$$\text{or } F = \tfrac{9}{5}C + 32$$

(b)
$$F = \tfrac{9}{5}C + 32$$
$$5F = 9(C + 32)$$
$$5F = 9C + 160$$
$$9C = 5F - 160$$
$$9C = 5(F - 32)$$
$$C = \tfrac{5}{9}(F - 32)$$

(c) If $F = C$,
$$F = \tfrac{5}{9}(F - 32)$$
$$9F = 5(F - 32)$$
$$9F = 5F - 160$$
$$4F = -160$$
$$F = -40.$$

$F = C$ when F is $-40°$.

50. **(a)** The ordered pairs are (0, 1) and (100, 3.92).
The slope is $m=\dfrac{3.92-1}{100-0}=\dfrac{2.92}{100}=.0292$ and $b=1$.
Using slope-intercept form we have $y=.0292x+1$ or $p(x)=.0292x+1$.

(b) Let $x = 60$.
$p(60) = .0292(60) + 1 = 2.752$
The pressure at 60 feet is approximately 2.75 atmospheres.

51. **(a)** Since we are wanting to find C as a function of I, use the points (6650, 3839) and (4215, 5494), where the first component represents the independent variable, I. First find the slope of the line.

$$m=\frac{5494-3839}{4215-6650}=\frac{1655}{-2435}\approx-.6797$$

Now use either point, say (6650, 3839), and the point-slope form to find the equation.

$$y-3839=-.6797(x-6650)$$
$$y-3839=-.6797x+4520.005$$
$$y\approx-.6797x+8359$$
$$\text{or } C=-.6797I+8359$$

(b) Since the slope is –.6797, the marginal propensity to consume is –.6797.

52. $2x+7-x=4x-2$

(a) $2x+7-x-(4x-2)=0$
$2x+7-x-4x+2=0$
$-3x+9=0$
$y=-3x+9$

(b) 3

(c) $2x+7-x=4x-2$
$x+7=4x-2$
$7=3x-2$
$9=3x$
$3=x$

Solution set: $\{3\}$

53. $7x-2x+4-5=3x+1$

(a) $7x-2x+4-5-3x-1=0$
$2x-2=0$
$y=2x-2$

(b) 1

(c) $7x-2x+4-5=3x+1$
$5x-1=3x+1$
$2x-1=1$
$2x=2$
$x=1$

Solution set: $\{1\}$

54. $3(2x+1)-2(x-2)=5$

(a) $3(2x+1)-2(x-2)-5=0$
$6x+3-2x+4-5=0$
$4x+2=0$
$y=4x+2$

(b) $-.5$

(c) $3(2x+1)-2(x-2)=5$
$6x+3-2x+4=5$
$4x+7=5\Rightarrow 4x=-2$
$x=-\frac{2}{4}=-\frac{1}{2}$

Solution set: $\{-\frac{1}{2}\}$ or $\{-.5\}$

55. $4x-3(4-2x)=2(x-3)+6x+2$

(a) $4x-3(4-2x)-2(x-3)-6x-2=0$
$4x-12+6x-2x+6-6x-2=0$
$2x-8=0$
$y=2x-8$

(b) 4

(c) $4x-3(4-2x)=2(x-3)+6x+2$
$4x-12+6x=2x-6+6x+2$
$10x-12=8x-4\Rightarrow 2x-12=-4$
$2x=8\Rightarrow x=4$

Solution set: $\{4\}$

56. D is the only possible answer, since the x-intercept occurs when $y = 0$, we can see from the graph that the value of the x-intercept exceeds 10.

57. (a)
$$\begin{aligned} -2(x-5) &= -x-2 \\ -2x+10 &= -x-2 \\ 10 &= x-2 \\ 12 &= x \end{aligned}$$
Solution set: $\{12\}$

(b) Answers will vary.
The largest value of x that is displayed in the standard viewing window is 10. As long as 12 is either a minimum or a maximum, or between the minimum and maximum, then the solution will be seen.

58. The Pythagorean Theorem and its converse assure us that in triangle *OPQ*, angle *POQ* is a right angle if and only if $[d(O,\,P)]^2 + [d(O,\,Q)]^2 = [d(P,\,Q)]^2$.

59. $d(O,\,P) = \sqrt{(x_1-0)^2 + (m_1x_1-0)^2}$
$= \sqrt{x_1{}^2 + m_1{}^2x_1{}^2}$

60. $d(O,\,Q) = \sqrt{(x_2-0)^2 + (m_2x_2-0)^2}$
$= \sqrt{x_2{}^2 + m_2{}^2x_2{}^2}$

61. $d(P,\,Q) = \sqrt{(x_2-x_1)^2 + (m_2x_2-m_1x_1)^2}$

62.
$$\begin{aligned} [d(O,\,P)]^2 + [d(O,\,Q)]^2 &= [d(P,\,Q)]^2 \\ \left[\sqrt{x_1{}^2+m_1{}^2x_1{}^2}\right]^2 + \left[\sqrt{x_2{}^2+m_2{}^2x_2{}^2}\right]^2 &= \left[\sqrt{(x_2-x_1)^2+(m_2x_2-m_1x_1)^2}\right]^2 \\ \left(x_1{}^2+m_1{}^2x_1{}^2\right)+\left(x_2{}^2+m_2{}^2x_2{}^2\right) &= (x_2-x_1)^2+(m_2x_2-m_1x_1)^2 \\ x_1{}^2+m_1{}^2x_1{}^2+x_2{}^2+m_2{}^2x_2{}^2 &= x_2{}^2-2x_2x_1+x_1{}^2+m_2{}^2x_2{}^2-2m_1m_2x_1x_2+m_1{}^2x_1{}^2 \\ 0 &= -2x_2x_1-2m_1m_2x_1x_2 \\ -2m_1m_2x_1x-2x_2x_1 &= 0 \end{aligned}$$

63. $-2m_1m_2x_1x_2 - 2x_1x_2 = 0$
$-2x_1x_2(m_1m_2+1) = 0$

64. $-2x_1x_2(m_1m_2+1) = 0$
Since $x_1 \neq 0$ and $x_2 \neq 0$, we have $m_1m_2+1=0$ implying that $m_1m_2 = -1$.

65. If two nonvertical lines are perpendicular, then the product of the slopes of these lines is –1.

66. To show that the line $y = x$ is the perpendicular bisector of the segment with endpoints (a,b) and (b,a), we must show that the line bisects the segment and is perpendicular to the segment. To show that it bisects the segment, we find the midpoint of the segment. The midpoint is $\left(\frac{a+b}{2}, \frac{b+a}{2}\right)$. Since $\frac{a+b}{2} = \frac{b+a}{2}$, we have a point that lies on the line $y = x$. Thus the line does bisect the segment.

In order to show that the line $y = x$ is perpendicular to the segment with endpoints (a,b) and (b,a), we must find the slope of the segment (the line has slope 1). Since $m = \dfrac{a-b}{b-a} = -1$ represents the slope of the segment, we have that the line $y = x$ and the segment are perpendicular since $1(-1) = -1$. Thus, $y = x$ is the perpendicular bisector of the segment with endpoints (a,b) and (b,a).

67. Label the points as follows: $A(-1,5)$, $B(2,-4)$, and $C(4,-10)$.

For A and B: $m=\frac{-4-5}{2-(-1)}=\frac{-9}{3}=-3$

For B and C, $m=\frac{-10-(-4)}{4-2}=\frac{-6}{2}=-3$

For A and C, $m=\frac{-10-5}{4-(-1)}=\frac{-15}{5}=-3$

Since all three slopes are the same, the points are collinear.

68. $A(0, -7)$, $B(-3, 5)$, $C(2, -15)$

For A and B, $m=\frac{5-(-7)}{-3-0}=\frac{12}{-3}=-4$

For B and C, $m=\frac{-15-5}{2-(-3)}=\frac{-20}{5}=-4$

For A and C, $m=\frac{-15-(-7)}{2-0}=\frac{-8}{2}=-4$

Since all three slopes are the same, the points are collinear.

69. $A(-1, 4)$, $B(-2, -1)$, $C(1, 14)$

For A and B, $m=\frac{-1-4}{-2-(-1)}=\frac{-5}{-1}=5$

For B and C, $m=\frac{14-(-1)}{1-(-2)}=\frac{15}{3}=5$

For A and C, $m=\frac{14-4}{1-(-1)}=\frac{10}{2}=5$

Since all three slopes are the same, the points are collinear.

70. $A(0, 9)$, $B(-3, -7)$, $C(2, 19)$

For A and B, $m=\frac{-7-9}{-3-0}=\frac{-16}{-3}=\frac{16}{3}$

For B and C, $m=\frac{19-(-7)}{2-(-3)}=\frac{26}{5}$

For A and C, $m=\frac{19-9}{2-0}=\frac{10}{2}=5$

Since all three slopes are not the same, the points are not collinear.

Summary Exercises on Graphs, Functions, and Equations

1. $P(3, 5)$, $Q(2, -3)$

(a)
$$d(P, Q)=\sqrt{(2-3)^2+(-3-5)^2}=\sqrt{(-1)^2+(-8)^2}=\sqrt{1+64}=\sqrt{65}$$

(b) The midpoint M of the segment joining points P and Q has coordinates $\left(\frac{3+2}{2},\frac{5+(-3)}{2}\right)=\left(\frac{5}{2},\frac{2}{2}\right)=\left(\frac{5}{2},1\right)$.

(c) First find m: $m=\frac{-3-5}{2-3}=\frac{-8}{-1}=8$

Use either point and the point-slope form.

$$y-5=8(x-3)$$

Change to slope-intercept form.

$$y-5=8x-24$$
$$y=8x-19$$

2. $P(-1, 0)$, $Q(4, -2)$

(a)
$$d(P, Q)=\sqrt{[4-(-1)]^2+(-2-0)^2}=\sqrt{5^2+(-2)^2}=\sqrt{25+4}=\sqrt{29}$$

(b) The midpoint M of the segment joining points P and Q has coordinates
$$\left(\frac{-1+4}{2},\frac{0+(-2)}{2}\right)=\left(\frac{3}{2},\frac{-2}{2}\right)=\left(\frac{3}{2},-1\right).$$

(c) First find m: $m=\frac{-2-0}{4-(-1)}=\frac{-2}{5}=-\frac{2}{5}$

Use either point and the point-slope form.

$$y-0=-\tfrac{2}{5}\left[x-(-1)\right]$$

Change to slope-intercept form.

$$5y=-2(x+1)$$
$$5y=-2x-2$$
$$y=-\tfrac{2}{5}x-\tfrac{2}{5}$$

3. $P(-2, 2)$, $Q(3, 2)$

(a) $d(P, Q) = \sqrt{[3-(-2)]^2 + (2-2)^2} = \sqrt{5^2 + 0^2} = \sqrt{25+0} = \sqrt{25} = 5$

(b) The midpoint M of the segment joining points P and Q has coordinates $\left(\frac{-2+3}{2}, \frac{2+2}{2}\right)$ $= \left(\frac{1}{2}, \frac{4}{2}\right) = \left(\frac{1}{2}, 2\right)$.

(c) First find m: $m = \frac{2-2}{3-(-2)} = \frac{0}{5} = 0$

All lines that have a slope of 0 are horizontal lines. The equation of a horizontal line has an equation of the form $y = b$. Since the line passes through (3, 2), the equation is $y = 2$.

4. $P\left(2\sqrt{2}, \sqrt{2}\right)$, $Q\left(\sqrt{2}, 3\sqrt{2}\right)$

(a) $d(P, Q) = \sqrt{\left(\sqrt{2} - 2\sqrt{2}\right)^2 + \left(3\sqrt{2} - \sqrt{2}\right)^2} = \sqrt{\left(-\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2} = \sqrt{2+8} = \sqrt{10}$

(b) The midpoint M of the segment joining points P and Q has coordinates $\left(\frac{2\sqrt{2}+\sqrt{2}}{2}, \frac{\sqrt{2}+3\sqrt{2}}{2}\right)$ $= \left(\frac{3\sqrt{2}}{2}, \frac{4\sqrt{2}}{2}\right) = \left(\frac{3\sqrt{2}}{2}, 2\sqrt{2}\right)$.

(c) First find m: $m = \frac{3\sqrt{2} - \sqrt{2}}{\sqrt{2} - 2\sqrt{2}} = \frac{2\sqrt{2}}{-\sqrt{2}} = -2$

Use either point and the point-slope form.

$$y - \sqrt{2} = -2\left(x - 2\sqrt{2}\right)$$

Change to slope-intercept form.

$$y - \sqrt{2} = -2x + 4\sqrt{2} \Rightarrow y = -2x + 5\sqrt{2}$$

5. $P(5, -1)$, $Q(5, 1)$

(a) $d(P, Q) = \sqrt{(5-5)^2 + [1-(-1)]^2}$
$= \sqrt{0^2 + 2^2} = \sqrt{0+4} = \sqrt{4} = 2$

(b) The midpoint M of the segment joining points P and Q has coordinates $\left(\frac{5+5}{2}, \frac{-1+1}{2}\right) = \left(\frac{10}{2}, \frac{0}{2}\right) = (5, 0)$.

(c) First find m.

$$m = \frac{1-(-1)}{5-5} = \frac{2}{0} = \text{undefined}$$

All lines that have an undefined slope are vertical lines. The equation of a vertical line has an equation of the form $x = a$. Since the line passes through (5, 1), the equation is $x = 5$. (Since this slope of a vertical line is undefined, this equation cannot be written in slope-intercept form.)

6. $P(1, 1)$, $Q(-3, -3)$

(a) $d(P, Q) = \sqrt{(-3-1)^2 + (-3-1)^2}$
$= \sqrt{(-4)^2 + (-4)^2}$
$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

(b) The midpoint M of the segment joining points P and Q has coordinates $\left(\frac{1+(-3)}{2}, \frac{1+(-3)}{2}\right) = \left(\frac{-2}{2}, \frac{-2}{2}\right)$
$= (-1, -1)$.

(c) First find m: $m = \frac{-3-1}{-3-1} = \frac{-4}{-4} = 1$

Use either point and the point-slope form.

$$y - 1 = 1(x - 1)$$

Change to slope-intercept form.

$$y - 1 = x - 1 \Rightarrow y = x$$

7. $P\left(2\sqrt{3}, 3\sqrt{5}\right),\ Q\left(6\sqrt{3}, 3\sqrt{5}\right)$

(a) $d(P, Q) = \sqrt{\left(6\sqrt{3} - 2\sqrt{3}\right)^2 + \left(3\sqrt{5} - 3\sqrt{5}\right)^2} = \sqrt{\left(4\sqrt{3}\right)^2 + 0^2} = \sqrt{48} = 4\sqrt{3}$

(b) The midpoint M of the segment joining points P and Q has coordinates
$$\left(\frac{2\sqrt{3} + 6\sqrt{3}}{2}, \frac{3\sqrt{5} + 3\sqrt{5}}{2}\right) = \left(\frac{8\sqrt{3}}{2}, \frac{6\sqrt{5}}{2}\right) = \left(4\sqrt{3},\ 3\sqrt{5}\right).$$

(c) First find m: $m = \dfrac{3\sqrt{5} - 3\sqrt{5}}{6\sqrt{3} - 2\sqrt{3}} = \dfrac{0}{4\sqrt{3}} = 0$

All lines that have a slope of 0 are horizontal lines. The equation of a horizontal line has an equation of the form $y = b$. Since the line passes through $\left(2\sqrt{3}, 3\sqrt{5}\right)$, the equation is $y = 3\sqrt{5}$.

8. $P(0, -4)$, $Q(3, 1)$

(a) $d(P, Q) = \sqrt{(3 - 0)^2 + [1 - (-4)]^2}$
$= \sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34}$

(b) The midpoint M of the segment joining points P and Q has coordinates
$$\left(\frac{0 + 3}{2}, \frac{-4 + 1}{2}\right) = \left(\frac{3}{2}, \frac{-3}{2}\right) = \left(\frac{3}{2}, -\frac{3}{2}\right).$$

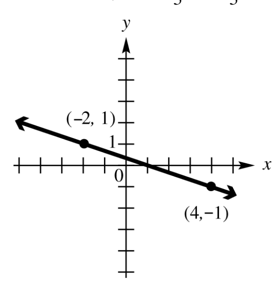
(c) First find m: $m = \dfrac{1 - (-4)}{3 - 0} = \dfrac{5}{3}$

Using slope-intercept form we have $y = \frac{5}{3}x - 4$.

9. Through $(-2, 1)$ and $(4, -1)$

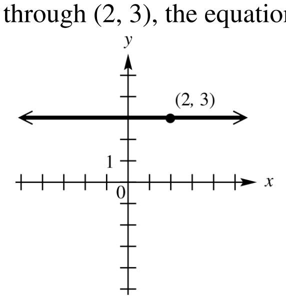
First find m: $m = \dfrac{-1 - 1}{4 - (-2)} = \dfrac{-2}{6} = -\dfrac{1}{3}$

Use either point and the point-slope form.
$$y - (-1) = -\tfrac{1}{3}(x - 4)$$
Change to slope-intercept form.
$$3(y + 1) = -(x - 4)$$
$$3y + 3 = -x + 4$$
$$3y = -x + 1$$
$$y = -\tfrac{1}{3}x + \tfrac{1}{3}$$

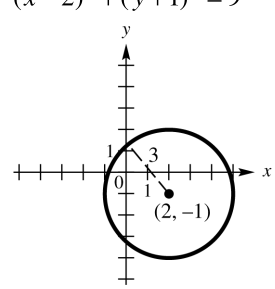


10. the horizontal line through (2, 3)

The equation of a horizontal line has an equation of the form $y = b$. Since the line passes through (2, 3), the equation is $y = 3$.



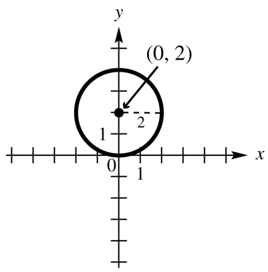
11. the circle with center $(2, -1)$ and radius 3
$$(x - 2)^2 + \left[y - (-1)\right]^2 = 3^2$$
$$(x - 2)^2 + (y + 1)^2 = 9$$



12. the circle with center (0, 2) and tangent to the x-axis

The distance from the center of the circle to the x-axis is 2, so $r = 2$.

$$(x-0)^2+(y-2)^2=2^2 \Rightarrow x^2+(y-2)^2=4$$



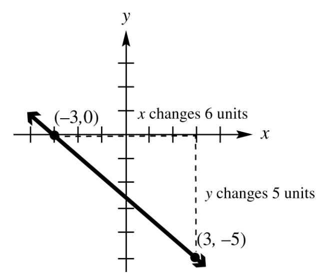
13. the line through $(3,-5)$ with slope $-\frac{5}{6}$

Write the equation in point-slope form.

$$y-(-5)=-\tfrac{5}{6}(x-3)$$

Change to standard form.

$$6(y+5)=-5(x-3)\Rightarrow 6y+30=-5x+15$$
$$6y=-5x-15\Rightarrow y=-\tfrac{5}{6}x-\tfrac{15}{6}$$
$$y=-\tfrac{5}{6}x-\tfrac{5}{2}$$



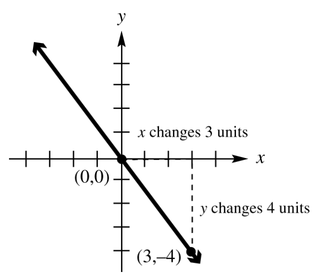
14. a line through the origin and perpendicular to the line $3x-4y=2$

First, find the slope of the line $3x-4y=2$ by writing this equation in slope-intercept form.

$$3x-4y=2\Rightarrow -4y=-3x+2\Rightarrow y=\tfrac{3}{4}x-\tfrac{2}{4}\Rightarrow y=\tfrac{3}{4}x-\tfrac{1}{2}$$

This line has a slope of $\frac{3}{4}$. The slope of any line perpendicular to this line is $-\frac{4}{3}$, since $-\frac{4}{3}\left(\frac{3}{4}\right)=-1$.

Using slope-intercept form we have $y=-\frac{4}{3}x+0$ or $y=-\frac{4}{3}x$.



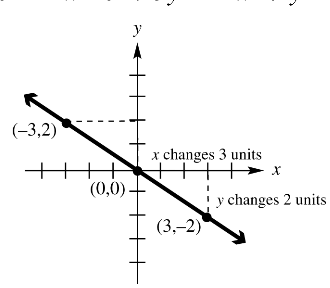
15. a line through (–3, 2) and parallel to the line $2x+3y=6$

First, find the slope of the line $2x+3y=6$ by writing this equation in slope-intercept form.

$$2x+3y=6\Rightarrow 3y=-2x+6\Rightarrow y=-\tfrac{2}{3}x+2$$

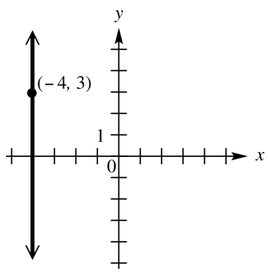
The slope is $-\frac{2}{3}$. Since the lines are parallel, $-\frac{2}{3}$ is also the slope of the line whose equation is to be found. Substitute $m=-\frac{2}{3}$, $x_1=-3$, and $y_1=2$ into the point-slope form.

$$y-y_1=m(x-x_1)\Rightarrow y-2=-\tfrac{2}{3}\left[x-(-3)\right]\Rightarrow 3(y-2)=-2(x+3)$$
$$3y-6=-2x-6\Rightarrow 3y=-2x\Rightarrow y=-\tfrac{2}{3}x$$



16. the vertical line through $(-4,3)$

The equation of a vertical line has an equation of the form $x = a$. Since the line passes through $(-4,3)$, the equation is $x = -4$.



17. $x^2-4x+y^2+2y=4$

Complete the square on x and y separately.

$$\left(x^2-4x\right)+\left(y^2+2y\right)=4$$
$$\left(x^2-4x+4\right)+\left(y^2+2y+1\right)=4+4+1$$
$$(x-2)^2+(y+1)^2=9$$

Yes, it is a circle. The circle has its center at $(2,-1)$ and radius 3.

18. $x^2+6x+y^2+10y+36=0$

Complete the square on x and y separately.

$\left(x^2+6x\right)+\left(y^2+10y\right)=-36\Rightarrow\left(x^2+6x+9\right)+\left(y^2+10y+25\right)=-36+9+25\Rightarrow(x+3)^2+(y+5)^2=-2$

No, it is not a circle.

19. $x^2-12x+y^2+20=0$

Complete the square on x and y separately.

$\left(x^2-12x\right)+y^2=-20\Rightarrow\left(x^2-12x+36\right)+y^2=-20+36\Rightarrow(x-6)^2+y^2=16$

Yes, it is a circle. The circle has its center at (6, 0) and radius 4.

20. $x^2+2x+y^2+16y=-61$

Complete the square on x and y separately.

$\left(x^2+2x\right)+\left(y^2+16y\right)=-61\Rightarrow\left(x^2+2x+1\right)+\left(y^2+16y+64\right)=-61+1+64\Rightarrow(x+1)^2+(y+8)^2=4$

Yes, it is a circle. The circle has its center at $(-1,-8)$ and radius 2.

21. $x^2-2x+y^2+10=0$

Complete the square on x and y separately.

$$\left(x^2-2x\right)+y^2=-10$$
$$\left(x^2-2x+1\right)+y^2=-10+1$$
$$(x-1)^2+y^2=-9$$

No, it is not a circle.

22. $x^2+y^2-8y-9=0$

Complete the square on x and y separately.

$$x^2+\left(y^2-8y\right)=9$$
$$x^2+\left(y^2-8y+16\right)=9+16$$
$$x^2+(y-4)^2=25$$

Yes, it is a circle. The circle has its center at $(0,4)$ and radius 5.

23. **(a)** The equation can be rewritten as $-2y=-3x+1\Rightarrow y=\frac{3}{2}x-\frac{1}{2}$. x can be any real number, so the domain is all real numbers and the range is also all real numbers.

domain: $(-\infty,\infty)$; range: $(-\infty,\infty)$

(b) Each value of x corresponds to just one value of y. $3x-2y=1$ represents a function.

$$y=\tfrac{3}{2}x-\tfrac{1}{2}\Rightarrow f(x)=\tfrac{3}{2}x-\tfrac{1}{2}$$
$$f(-2)=\tfrac{3}{2}(-2)-\tfrac{1}{2}=\tfrac{-6}{2}-\tfrac{1}{2}=-\tfrac{7}{2}$$

24. **(a)** The equation can be rewritten as $y = -3x^2 - 1$. x can be any real number. Since the square of any real number is not negative, x^2 is never negative. Thus, $-3x^2$ is never positive, i.e. $(-\infty, 0]$. Taking the constant term into consideration, range would be $(-\infty, -1]$.
domain: $(-\infty, \infty)$; range: $(-\infty, -1]$

(b) Each value of x corresponds to just one value of y. $y + 3x^2 = -1$ represents a function.

$$y = -3x^2 - 1 \Rightarrow f(x) = -3x^2 - 1$$
$$f(-2) = -3(-2)^2 - 1 = -3(4) - 1 = -12 - 1 = -13$$

25. **(a)** The equation can be rewritten as $-4y = -x - 6 \Rightarrow y = \frac{1}{4}x + \frac{6}{4} \Rightarrow y = \frac{1}{4}x + \frac{3}{2}$. x can be any real number, so the domain is all real numbers and the range is also all real numbers.
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

(b) Each value of x corresponds to just one value of y. $x - 4y = -6$ represents a function.

$$y = \frac{1}{4}x + \frac{3}{2} \Rightarrow f(x) = \frac{1}{4}x + \frac{3}{2}$$
$$f(-2) = \frac{1}{4}(-2) + \frac{3}{2} = -\frac{1}{2} + \frac{3}{2} = \frac{2}{2} = 1$$

26. **(a)** The equation can be rewritten as $y^2 - 5 = x$. y can be any real number. Since the square of any real number is not negative, y^2 is never negative. Taking the constant term into consideration, domain would be $[-5, \infty)$.

domain: $[-5, \infty)$; range: $(-\infty, \infty)$

(b) Since $(-4, 1)$ and $(-4, -1)$ both satisfy the relation, $y^2 - x = 5$ does not represent a function.

27. **(a)** $(x+2)^2 + y^2 = 25$ is a circle centered at $(-2, 0)$ with a radius of 5. The domain will start 5 units to the left of –2 and end 5 units to the right of –2. The domain will be $[-2-5, -2+5] = [-7, 3]$. The range will start 5 units below 0 and end 5 units above 0. The range will be $[0-5, 0+5] = [-5, 5]$.

(b) Since $(-2, 5)$ and $(-2, -5)$ both satisfy the relation, $(x+2)^2 + y^2 = 25$ does not represent a function.

28. **(a)** The equation can be rewritten as $-2y = -x^2 + 3 \Rightarrow y = \frac{1}{2}x^2 - \frac{3}{2}$. x can be any real number. Since the square of any real number is not negative, $\frac{1}{2}x^2$ is never negative. Taking the constant term into consideration, range would be $\left[-\frac{3}{2}, \infty\right)$.
domain: $(-\infty, \infty)$; range: $\left[-\frac{3}{2}, \infty\right)$

(b) Each value of x corresponds to just one value of y. $x^2 - 2y = 3$ represents a function.
$y = \frac{1}{2}x^2 - \frac{3}{2} \Rightarrow f(x) = \frac{1}{2}x^2 - \frac{3}{2}$
$f(-2) = \frac{1}{2}(-2)^2 - \frac{3}{2} = \frac{1}{2}(4) - \frac{3}{2} = \frac{4}{2} - \frac{3}{2} = \frac{1}{2}$

Section 2.5: Graphs of Basic Functions

1. The function is continuous over the entire domain of real numbers $(-\infty,\infty)$.

2. The function is continuous over the entire domain of real numbers $(-\infty,\infty)$.

3. The function is continuous over the interval $[0,\infty)$.

4. The function is continuous over the interval $(-\infty,0]$.

5. The function has a point of discontinuity at $x=1$. It is continuous over the interval $(-\infty,1)$ and the interval $[1,\infty)$.

6. The function has a point of discontinuity at $x=1$. It is continuous over the interval $(-\infty,1)$ and the interval $(1.\infty)$.

7. The equation $y=x^2$ matches graph E. The domain is $(-\infty,\infty)$.

8. The equation of $y = |x|$ matches graph G. The function is increasing on $[0,\infty)$.

9. The equation $y=x^3$ matches graph A. The range is $(-\infty,\infty)$.

10. Graph C is not the graph of a function. Its equation is $x=y^2$.

11. Graph F is the graph of the identity function. Its equation is $y = x$.

12. The equation $y=[\![x]\!]$ matches graph B. $y=[\![1.5]\!]=1$

13. The equation $y=\sqrt[3]{x}$ matches graph H. No, there is no interval over which the function is decreasing.

14. The equation of $y=\sqrt{x}$ matches graph D. The domain is $[0,\infty)$.

15. The graph in B is discontinuous at many points. Assuming the graph continues, the range would be $\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$.

16. The graphs in E and G decrease over part of the domain and increase over part of the domain. They both decrease over $(-\infty,0]$ and increase over $[0,\infty)$.

17. $f(x)=\begin{cases}2x & \text{if } x\le -1\\ x-1 & \text{if } x>-1\end{cases}$

(a) $f(-5)=2(-5)=-10$
(b) $f(-1)=2(-1)=-2$
(c) $f(0) = 0 - 1 = -1$
(d) $f(3) = 3 - 1 = 2$

18. $f(x)=\begin{cases}x-2 & \text{if } x<3\\ 5-x & \text{if } x\ge 3\end{cases}$

(a) $f(-5) = -5 - 2 = -7$
(b) $f(-1) = -1 - 2 = -3$
(c) $f(0) = 0 - 2 = -2$
(d) $f(3) = 5 - 3 = 2$

19. $f(x)=\begin{cases}2+x & \text{if } x<-4\\ -x & \text{if } -4\le x\le 2\\ 3x & \text{if } x>2\end{cases}$

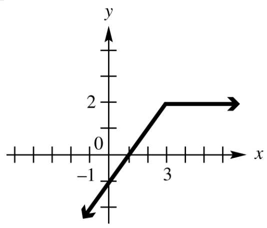
(a) $f(-5) = 2 + (-5) = -3$
(b) $f(-1) = -(-1) = 1$
(c) $f(0) = -0 = 0$
(d) $f(3)=3\cdot 3=9$

20. $\begin{cases}-2x & \text{if } x<-3\\ 3x-1 & \text{if } -3\le x\le 2\\ -4x & \text{if } x>2\end{cases}$

(a) $f(-5) = -2(-5) = 10$
(b) $f(-1) = 3(-1) - 1 = -3 - 1 = -4$
(c) $f(0) = 3(0) - 1 = 0 - 1 = -1$
(d) $f(3) = -4(3) = -12$

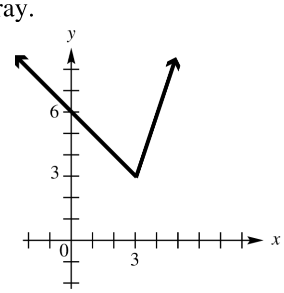
21. $f(x)=\begin{cases} x-1 \text{ if } x\le 3 \\ 2 \quad \text{ if } x>3 \end{cases}$

Draw the graph of $y = x - 1$ to the left of $x = 3$, including the endpoint at $x = 3$. Draw the graph of $y = 2$ to the right of $x = 3$, and note that the endpoint at $x = 3$ coincides with the endpoint of the other ray.



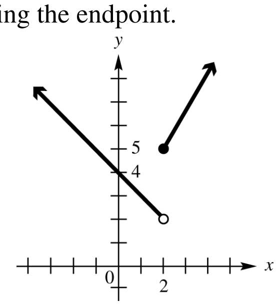
22. $f(x)=\begin{cases} 6-x \text{ if } x\le 3 \\ 3x-6 \text{ if } x>3 \end{cases}$

Graph the line $y = 6 - x$ to the left of $x = 3$, including the endpoint. Draw $y = 3x - 6$ to the right of $x = 3$, and note that the endpoint at $x = 3$ coincides with the endpoint of the other ray.



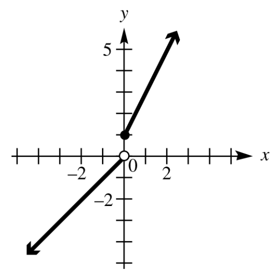
23. $f(x)=\begin{cases} 4-x \text{ if } x<2 \\ 1+2x \text{ if } x\ge 2 \end{cases}$

Draw the graph of $y=4-x$ to the left of $x = 2$, but do not include the endpoint. Draw the graph of $y=1+2x$ to the right of $x = 2$, including the endpoint.



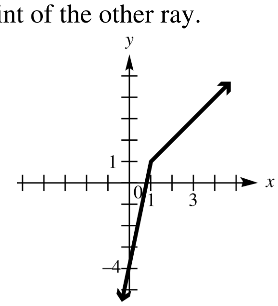
24. $f(x)=\begin{cases} 2x+1 \text{ if } x\ge 0 \\ x \quad \text{ if } x<0 \end{cases}$

Graph the line $y=2x+1$ to the right of $x = 0$, including the endpoint. Draw $y = x$ to the left of $x=0$, but do not include the endpoint.



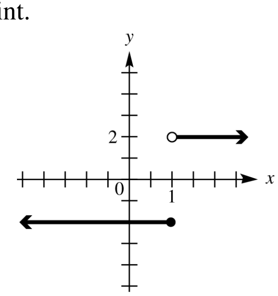
25. $f(x)=\begin{cases} 5x-4 \text{ if } x\le 1 \\ x \quad \text{ if } x>1 \end{cases}$

Graph the line $y=5x-4$ to the left of $x=1$, including the endpoint. Draw $y=x$ to the right of $x=1$, and note that the endpoint at $x=1$ coincides with the endpoint of the other ray.



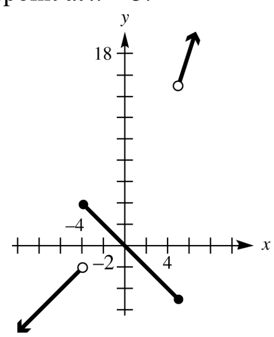
26. $f(x)=\begin{cases} -2 \text{ if } x\le 1 \\ 2 \quad \text{ if } x>1 \end{cases}$

Graph the line $y = -2$ to the left of $x=1$, including the endpoint. Draw $y = 2$ to the right of $x=1$, but do not include the endpoint.



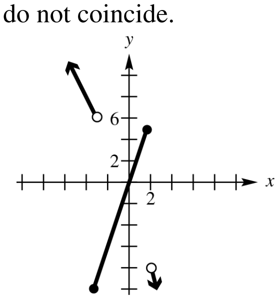
27. $f(x)=\begin{cases}2+x \text{ if } x<-4\\ -x \quad \text{if } -4\le x\le 5\\ 3x \quad \text{if } x>5\end{cases}$

Draw the graph of $y = 2 + x$ to the left of –4, but do not include the endpoint at $x = 4$. Draw the graph of $y = -x$ between –4 and 5, including both endpoints. Draw the graph of $y = 3x$ to the right of 5, but do not include the endpoint at $x = 5$.



28. $f(x)=\begin{cases}-2x \quad \text{if } x<-3\\ 3x-1 \text{ if } -3\le x\le 2\\ -4x \quad \text{if } x>2\end{cases}$

Graph the line $y = -2x$ to the left of $x = -3$, but do not include the endpoint. Draw $y = 3x - 1$ between $x = -3$ and $x = 2$, and include both endpoints. Draw $y = -4x$ to the right of $x = 2$, but do not include the endpoint. Notice that the endpoints of the pieces do not coincide.



29. The solid circle on the graph shows that the endpoint (0, –1) is part of the graph, while the open circle shows that the endpoint (0, 1) is not part of the graph. The graph is made up of parts of two horizontal lines. The function which fits this graph is

$f(x)=\begin{cases}-1 \text{ if } x\le 0\\ 1 \quad \text{if } x>0.\end{cases}$

domain: $(-\infty,\infty)$; range: {–1, 1}

30. We see that $y = 1$ for every value of x except $x = 0$, and that when $x = 0$, $y = 0$. We can write the function as $f(x)=\begin{cases}1 \text{ if } x\ne 0\\ 0 \text{ if } x=0.\end{cases}$

domain: $(-\infty,\infty)$; range: {0, 1}

31. The graph is made up of parts of two horizontal lines. The solid circle shows that the endpoint (0, 2) of the one on the left belongs to the graph, while the open circle shows that the endpoint (0, –1) of the one on the right does not belong to the graph. The function that fits this graph is

$f(x)=\begin{cases}2 \text{ if } x\le 0\\ -1 \text{ if } x>1.\end{cases}$

domain: $(-\infty,0]\cup(1,\infty)$; range: {–1, 2}

32. We see that $y = 1$ when $x\le -1$ and that $y = -1$ when $x > 2$. We can write the function as $f(x)=\begin{cases}1 \text{ if } x\le -1\\ -1 \text{ if } x>2.\end{cases}$

domain: $(-\infty,-1]\cup(2,\infty)$; range: {–1, 1}

33. $f(x) = [\![-x]\!]$

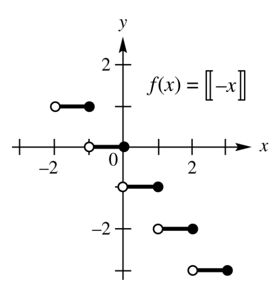
Plot points.

x	$-x$	$f(x) = [\![-x]\!]$
–2	2	2
–1.5	1.5	1
–1	1	1
–.5	.5	0
0	0	0
.5	–.5	–1
1	–1	–1
1.5	–1.5	–2
2	–2	–2

More generally, to get $y = 0$, we need $0\le -x<1 \Rightarrow 0\ge x>-1 \Rightarrow -1<x\le 0$.

To get $y = 1$, we need $1\le -x<2 \Rightarrow -1\ge x>-2 \Rightarrow -2<x\le -1$.

Follow this pattern to graph the step function.



domain: $(-\infty,\infty)$; range: {...,–2,–1,0,1,2,...}

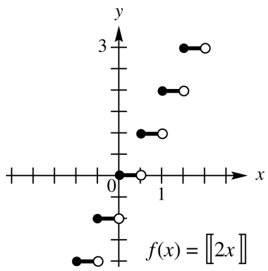
34. $f(x) = [\![2x]\!]$

To get $y = 0$, we need $0 \le 2x < 1 \Rightarrow 0 \le x < \frac{1}{2}$.

To get $y = 1$, we need $1 \le 2x < 2 \Rightarrow \frac{1}{2} \le x < 1$.

To get $y = 2$, we need $2 \le 2x < 3 \Rightarrow 1 \le x < \frac{3}{2}$.

Follow this pattern to graph the step function.



domain: $(-\infty, \infty)$; range: $\{..., -2, -1, 0, 1, 2, ...\}$

35. $g(x) = [\![2x - 1]\!]$

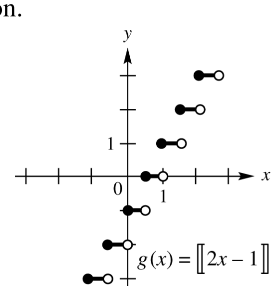
To get $y = 0$, we need

$0 \le 2x - 1 < 1 \Rightarrow 1 \le 2x < 2 \Rightarrow \frac{1}{2} \le x < 1$.

To get $y = 1$, we need

$1 \le 2x - 1 < 2 \Rightarrow 2 \le 2x < 3 \Rightarrow 1 \le x < \frac{3}{2}$.

Follow this pattern to graph the step function.



domain: $(-\infty, \infty)$; range: $\{..., 2, -1, 0, 1, 2, ...\}$

36. The function value is half the integer.

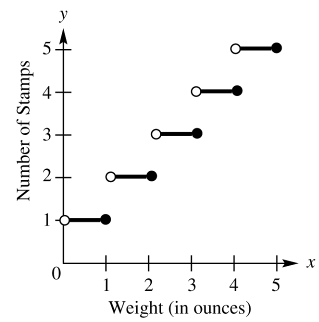
If $x = 2$, then $f(2) = \left[\!\!\left[\frac{1}{2}(2)\right]\!\!\right] = [\![1]\!] = 1$, if $x = 4$, then $f(4) = \left[\!\!\left[\frac{1}{2}(4)\right]\!\!\right] = [\![2]\!] = 2$, etc.

In general, if x is an even integer, it is of the form $2n$, where n is an integer.

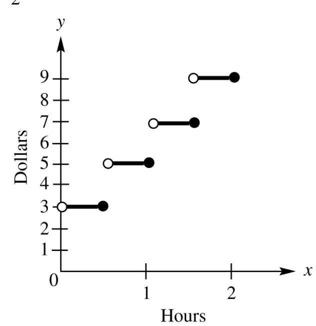
$$f(x) = f(2n) = \left[\!\!\left[\frac{1}{2}(2n)\right]\!\!\right] = [\![n]\!] = n$$

Since $x = 2n$, then $n = \frac{1}{2}x$.

37. The cost of mailing a letter that weights more than 1 ounce and less than 2 ounces is the same as the cost of a 2-ounce letter, and the cost of mailing a letter that weighs more than 2 ounces and less than 3 ounces is the same as the cost of a 3-ounce letter, etc.



38. The cost is the same for all cars parking between $\frac{1}{2}$ hour and 1-hour, between 1 hour and $1\frac{1}{2}$ hours, etc.



39. $f(x) = \begin{cases} x^2 - 4 \text{ if } x \ge 0 \\ -x + 5 \text{ if } x < 0 \end{cases}$

The graph is a line with negative slope for $x < 0$ and a parabola opening upward for $x \ge 0$. This matches graph B.

40. $g(x) = \begin{cases} |x - 4| & \text{if } x \ge -1 \\ -x^2 & \text{if } x < -1 \end{cases}$

The graph will be a parabola for $x < -1$ and a "V"-shaped absolute value graph for $x \ge -1$. This function matches graph A.

41. $f(x) = \begin{cases} 6 \text{ if } x \ge 0 \\ -6 \text{ if } x < 0 \end{cases}$

The graph is the horizontal line $y = -6$ for $x < 0$ and the horizontal line $y = 6$ for $x \ge 0$. This matches graph D.

42. $k(x)=\begin{cases}\sqrt{x} & \text{if } x\ge 0\\ -x^2 & \text{if } x<0\end{cases}$

The graph will be a parabola opening downward for $x < 0$ and a square root graph (part of a parabola opening to the right) for $x \ge 0$. This function matches graph C.

43. **(a)** For the interval $[3,6]$ we must consider the points $(3,73)$ and $(6,72)$.

First find m: $m=\dfrac{72-73}{6-3}=\dfrac{-1}{3}=-\dfrac{1}{3}$

Use either point and the point-slope form.

$$\begin{aligned}y-72&=-\tfrac{1}{3}(x-6)\\3(y-72)&=-(x-6)\\3y-216&=-x+6\\3y&=-x+222\\y&=-\tfrac{1}{3}x+74\end{aligned}$$

For the interval $(6,9]$ we must consider the points $(6,72)$ and $(9,69)$.

First find m: $m=\dfrac{69-72}{9-6}=\dfrac{-3}{3}=-1$

Use either point and the point-slope form.

$$\begin{aligned}y-72&=-1(x-6)\\y-72&=-x+6\\y&=-x+78\end{aligned}$$

(b) $f(x)=\begin{cases}-\frac{1}{3}x+74 & \text{if } 3\le x\le 6\\ -x+78 & \text{if } 6<x\le 9\end{cases}$

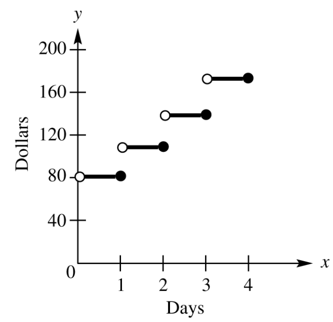
44. **(a) – (c)** If you rent the car for one day or any fraction of that day, the cost is the same. It would be the \$50 dropoff charge plus one day's rental. The answers to these three parts are the same $50+30=\$80$.

(d) $1\frac{5}{8}$ days is counted as a two-day rental. The cost would be $50+2(30)=50+60$ $=\$110$.

(e) 2.4 days is counted as a three-day rental. The cost would be $50+3(30)=$ $50+90=\$140$.

44. (continued)

(f)



45. $i(t)=\begin{cases}40t+100 & \text{if } 0\le t\le 3\\ 220 & \text{if } 3<t\le 8\\ -80t+860 & \text{if } 8<t\le 10\\ 60 & \text{if } 10<t\le 24\end{cases}$

(a) 6 A.M. is the starting time, so 7 A.M. corresponds to $t = 1$.
Use $i(t) = 40(t) + 100$.
Since $t = 1$,
$i(1)=40(1)+100=40+100=140.$

(b) 9 A.M. corresponds to $t = 3$.
$i(3)=40(3)+100=120+100=220$

(c) 10 A.M. corresponds to $t = 4$.
Use $i(t) = 220$.
$i(4) = 220$

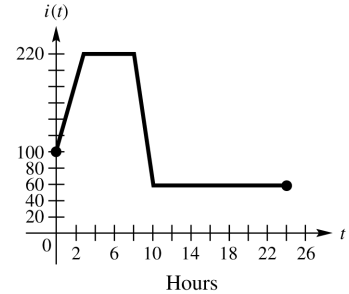
(d) Noon corresponds to $t = 6$.
Use $i(t) = 220$.
$i(6) = 220$

(e) 3 P.M. corresponds to $t = 9$.
Use $i(t)=-80t+860$.
$i(9)=-80(9)+860=-720+860=140$

(f) 5 P.M. corresponds to $t = 11$.
Use $i(t) = 60$.
$i(11) = 60$

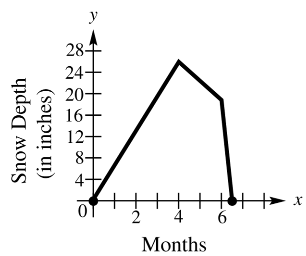
(g) Midnight corresponds to $t = 18$.
Use $i(t) = 60$.
$i(18) = 60$

(h)



46. **(a)** $f(x)=\begin{cases}6.5x & \text{if } 0\le x\le 4\\ -5.5x+48 & \text{if } 4<x\le 6\\ -30x+195 & \text{if } 6<x\le 6.5\end{cases}$

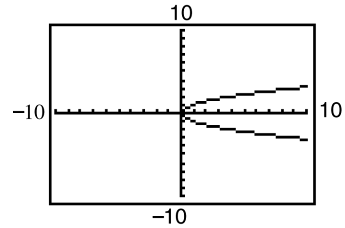
Draw a graph of $y = 6.5x$ between 0 and 4, including the endpoints. Draw the graph of $y = -5.5x + 48$ between 4 and 6, including the endpoint at 6 but not the one at 4. Draw the graph of $y = -30x + 195$, including the endpoint at 6.5 but not the one at 6. Notice that the endpoints of the three pieces coincide.



(b) From the graph, observe that the snow depth y reaches its deepest level (26 in.) when $x = 4$, $x = 4$ represents 4 months after the beginning of October, which is the beginning of February.

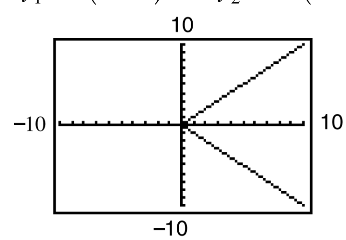
(c) From the graph, the snow depth y is nonzero when x is between 0 and 6.5. Snow begins at the beginning of October and ends 6.5 months later, in the middle of April.

47. Since $x = y^2$ implies $y = \pm\sqrt{x}$, we have $y_1 = \sqrt{x}$ and $y_2 = -\sqrt{x}$.



48. Since $x = |y| = \begin{cases} y & \text{if } y\ge 0\\ -y & \text{if } y<0\end{cases}$ we can consider this relation in two parts. In the top part of the relation, we have $x = y$ if $y \ge 0$. Since $x = y$, this would be equivalent to $y = x$ if $x \ge 0$. In the bottom part of the relation, we have $x = -y$ if $y < 0$. Since $x = -y$, this would imply $y = -x$. So our relation would be $y = -x$ if $-x < 0$ or $y = -x$ if $x > 0$.

$y_1 = x(x \ge 0)$ and $y_2 = -x(x > 0)$.



Section 2.6: Graphing Techniques

Connections (page 263)
Answers will vary.

Exercises

1. **(a)** B; $y = (x-7)^2$ is a shift of $y = x^2$, 7 units to the right.

(b) D; $y = x^2 - 7$ is a shift of $y = x^2$, 7 units downward.

(c) E; $y = 7x^2$ is a vertical stretch of $y = x^2$, by a factor of 7.

(d) A; $y = (x+7)^2$ is a shift of $y = x^2$, 7 units to the left.

(e) C; $y = x^2 + 7$ is a shift of $y = x^2$, 7 units upward.

2. **(a)** E; $y = 4\sqrt[3]{x}$ is a vertical stretch of $y = \sqrt[3]{x}$, by a factor of 4.

(b) C; $y = -\sqrt[3]{x}$ is a reflection of $y = \sqrt[3]{x}$, over the x-axis.

(c) D; $y = \sqrt[3]{-x}$ is a reflection of $y = \sqrt[3]{x}$, over the y-axis.

(d) A; $y = \sqrt[3]{x-4}$ is a shift of $y = \sqrt[3]{x}$, 4 units to the right.

(e) B; $y = \sqrt[3]{x} - 4$ is a shift of $y = \sqrt[3]{x}$, 4 units down.

3. **(a)** B; $y = x^2 + 2$ is a shift of $y = x^2$, 2 units upward.

(b) A; $y = x^2 - 2$ is a shift of $y = x^2$, 2 units downward.

(c) G; $y = (x+2)^2$ is a shift of $y = x^2$, 2 units to the left.

(d) C; $y = (x-2)^2$ is a shift of $y = x^2$, 2 units to the right.

(e) F; $y = 2x^2$ is a vertical stretch of $y = x^2$, by a factor of 2.

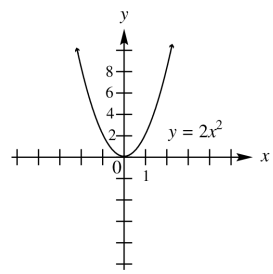
(f) D; $y = -x^2$ is a reflection of $y = x^2$, across the *x*-axis.

(g) H; $y = (x-2)^2 + 1$ is a shift of $y = x^2$, 2 units to the right and 1 unit upward.

(h) E; $y = (x+2)^2 + 1$ is a shift of $y = x^2$, 2 units to the left and 1 unit upward.

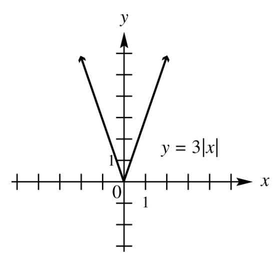
4. $y = 2x^2$

x	$y = x^2$	$y = 2x^2$
–2	4	8
–1	1	2
0	0	0
1	1	2
2	4	8



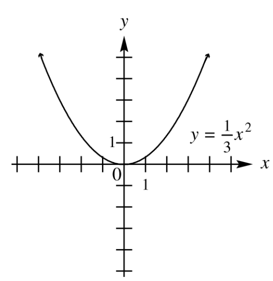
5. $y = 3|x|$

x	$y = \|x\|$	$y = 3\|x\|$
–2	2	6
–1	1	3
0	0	0
1	1	3
2	2	6



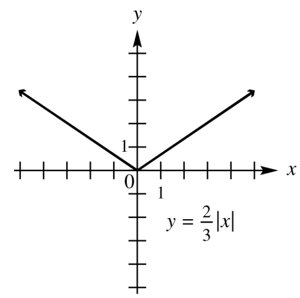
6. $y = \frac{1}{3}x^2$

x	$y = x^2$	$y = \frac{1}{3}x^2$
–3	9	3
–2	4	$\frac{4}{3}$
–1	1	$\frac{1}{3}$
0	0	0
1	1	$\frac{1}{3}$
2	4	$\frac{4}{3}$
3	9	3



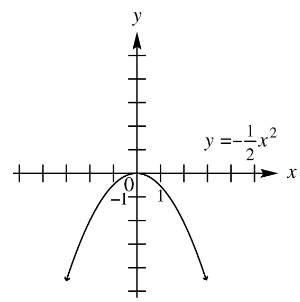
7. $y = \frac{2}{3}|x|$

x	$y = \lvert x \rvert$	$y = \frac{2}{3}\lvert x \rvert$
–3	3	2
–2	2	$\frac{4}{3}$
–1	1	$\frac{2}{3}$
0	0	0
1	1	$\frac{2}{3}$
2	2	$\frac{4}{3}$
3	3	2



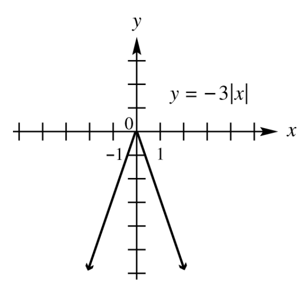
8. $y = -\frac{1}{2}x^2$

x	$y = x^2$	$y = -\frac{1}{2}x^2$
–3	9	$-\frac{9}{2}$
–2	4	–2
–1	1	$-\frac{1}{2}$
0	0	0
1	1	$-\frac{1}{2}$
2	4	–2
3	9	$-\frac{9}{2}$



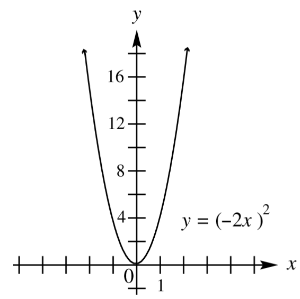
9. $y = -3|x|$

x	$y = \lvert x \rvert$	$y = -3\lvert x \rvert$
–2	2	–6
–1	1	–3
0	0	0
1	1	–3
2	2	–6



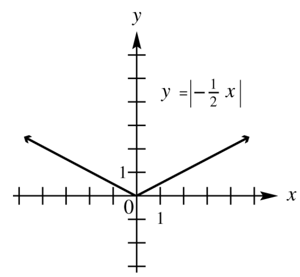
10. $y = (-2x)^2$

x	$y = x^2$	$y = (-2x)^2 = (-2)^2 x^2 = 4x^2$
–2	4	16
–1	1	4
0	0	0
1	1	4
2	4	16



11. $y = \left|-\frac{1}{2}x\right|$

x	$y = \lvert x \rvert$	$y = \left\lvert -\frac{1}{2}x \right\rvert = \left\lvert -\frac{1}{2} \right\rvert \lvert x \rvert = \frac{1}{2}\lvert x \rvert$
–4	4	2
–3	3	$\frac{3}{2}$
–2	2	1
–1	1	$\frac{1}{2}$
0	0	0
1	1	$\frac{1}{2}$
2	2	1
3	3	$\frac{3}{2}$
4	4	2



12. **(a)** $y = f(x+4)$ is a horizontal translation of *f*, 4 units to the left. The point that corresponds to $(8,12)$ on this translated function would be $(8-4,12) = (4,12)$.

(b) $y = f(x)+4$ is a vertical translation of *f*, 4 units up. The point that corresponds to $(8,12)$ on this translated function would be $(8,12+4) = (8,16)$.

13. **(a)** $y = \frac{1}{4}f(x)$ is a vertical shrinking of *f*, by a factor of $\frac{1}{4}$. The point that corresponds to $(8,12)$ on this translated function would be $\left(8, \frac{1}{4}\cdot 12\right) = (8,3)$.

(b) $y = 4f(x)$ is a vertical stretching of *f*, by a factor of 4. The point that corresponds to $(8,12)$ on this translated function would be $(8, 4\cdot 12) = (8,48)$.

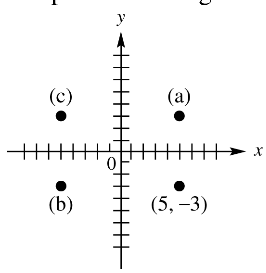
14. **(a)** The point that corresponds to $(8,12)$ when reflected across the *x*-axis would be $(8,-12)$.

(b) The point that corresponds to $(8,12)$ when reflected across the *y*-axis would be $(-8,12)$.

15. **(a)** The point that is symmetric to (5, –3) with respect to the x-axis is (5, 3).

(b) The point that is symmetric to (5, –3) with respect to the y-axis is (–5, –3).

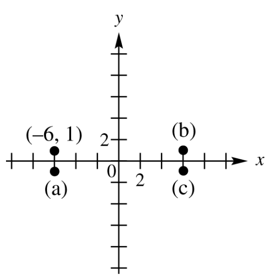
(c) The point that is symmetric to (5, –3) with respect to the origin is (–5, 3).



16. **(a)** The point that is symmetric to (–6, 1) with respect to the x-axis is (–6, –1).

(b) The point that is symmetric to (–6, 1) with respect to the y-axis is (6, 1).

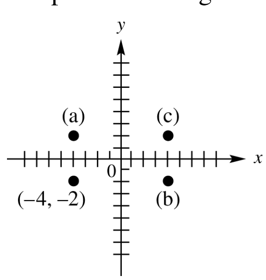
(c) The point that is symmetric to (–6, 1) with respect to the origin is (6, –1).



17. **(a)** The point that is symmetric to (–4, –2) with respect to the x-axis is (–4, 2).

(b) The point that is symmetric to (–4, –2) with respect to the y-axis is (4, –2).

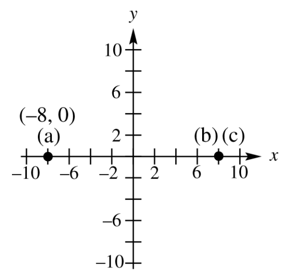
(c) The point that is symmetric to (–4, –2) with respect to the origin is (4, 2).



18. **(a)** The point that is symmetric to (–8, 0) with respect to the x-axis is (–8, 0), since this point lies on the x-axis.

(b) The point that is symmetric to the point (–8, 0) with respect to the y-axis is (8, 0).

(c) The point that is symmetric to the point (–8, 0) with respect to the origin is (8, 0).



19. $y = x^2 + 2$

Replace x with $-x$ to obtain

$y = (-x)^2 + 2 = x^2 + 2.$

The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Since y is a function of x, the graph cannot be symmetric with respect to the x-axis.

Replace x with $-x$ and y with $-y$ to obtain

$-y = (-x)^2 + 2 \Rightarrow -y = x^2 + 2 \Rightarrow y = -x^2 - 2.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. Therefore, the graph is symmetric with respect to the y-axis only.

20. $y = 2x^4 - 1$

Replace x with $-x$ to obtain

$y = 2(-x)^4 - 1 = 2x^4 - 1$.

The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Since y is a function of x, the graph cannot be symmetric with respect to the x-axis.

Replace x with $-x$ and y with $-y$ to obtain

$$-y = 2(-x)^4 - 1$$
$$-y = 2x^4 - 1$$
$$y = -2x^4 + 1.$$

The result is not the same as the original equation, so the graph is not symmetric with respect to the origin.

Therefore, the graph is symmetric with respect to the y-axis only.

21. $x^2 + y^2 = 10$

Replace x with $-x$ to obtain

$(-x)^2 + y^2 = 10$

$x^2 + y^2 = 10.$

The result is the same as the original equation, so the graph is symmetric with respect to the y-axis.

Replace y with $-y$ to obtain

$x^2 + (-y)^2 = 10$

$x^2 + y^2 = 10.$

The result is the same as the original equation, so the graph is symmetric with respect to the x-axis.

Since the graph is symmetric with respect to the x-axis and y-axis, it is also symmetric with respect to the origin.

22. $y^2 = \frac{-5}{x^2}$

Replace x with $-x$ to obtain

$y^2 = \frac{-5}{(-x)^2}$

$y^2 = \frac{-5}{x^2}.$

The result is the same as the original equation, so the graph is symmetric with respect to the y-axis.

Replace y with $-y$ to obtain

$(-y)^2 = \frac{-5}{x^2}$

$y^2 = \frac{-5}{x^2}.$

The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. Since the graph is symmetric with respect to the x-axis and y-axis, it is also symmetric with respect to the origin.

Therefore, the graph is symmetric with respect to the x-axis, the y-axis, and the origin.

23. $y = -3x^3$

Replace x with $-x$ to obtain

$y = -3(-x)^3$

$y = -3(-x^3)$

$y = 3x^3.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis.

Replace y with $-y$ to obtain

$-y = -3x^3$

$y = 3x^3.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the x-axis.

Replace x with $-x$ and y with $-y$ to obtain

$-y = -3(-x)^3$

$-y = -3(-x^3)$

$-y = 3x^3$

$y = -3x^3.$

The result is the same as the original equation, so the graph is symmetric with respect to the origin. Therefore, the graph is symmetric with respect to the origin only.

24. $y = x^3 - x$

Replace x with $-x$ to obtain

$y = (-x)^3 - (-x)$

$y = -x^3 + x.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis.

Replace y with $-y$ to obtain

$-y = x^3 - x$

$y = -x^3 + x.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the x-axis.

Replace x with $-x$ and y with $-y$ to obtain

$-y = (-x)^3 - (-x)$

$-y = -x^3 + x$

$y = x^3 - x.$

The result is the same as the original equation, so the graph is symmetric with respect to the origin.

Therefore, the graph is symmetric with respect to the origin only.

25. $y = x^2 - x + 7$

Replace x with $-x$ to obtain

$y = (-x)^2 - (-x) + 7$

$y = x^2 + x + 7.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis.
Since y is a function of x, the graph cannot be symmetric with respect to the x-axis.
Replace x with $-x$ and y with $-y$ to obtain

$-y = (-x)^2 - (-x) + 7$

$-y = x^2 + x + 7$

$y = -x^2 - x - 7.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the origin.
Therefore, the graph has none of the listed symmetries.

26. $y = x + 12$

Replace x with $-x$ to obtain

$y = (-x) + 12$

$y = -x + 12.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis.
Since y is a function of x, the graph cannot be symmetric with respect to the x-axis.
Replace x with $-x$ and y with $-y$ to obtain

$-y = (-x) + 12$

$y = x - 12.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the origin.
Therefore, the graph has none of the listed symmetries.

27. $f(x) = -x^3 + 2x$

$f(-x) = -(-x)^3 + 2(-x)$

$= x^3 - 2x = -(-x^3 + 2x) = -f(x)$

The function is odd.

28. $f(x) = x^5 - 2x^3$

$f(-x) = (-x)^5 - 2(-x)^3$

$= -x^5 + 2x^3 = -(x^5 - 2x^3) = -f(x)$

The function is odd.

29. $f(x) = .5x^4 - 2x^2 + 1$

$f(-x) = .5(-x)^4 - 2(-x)^2 + 1$

$= .5x^4 - 2x^2 + 1 = f(x)$

The function is even.

30. $f(x) = .75x^2 + |x| + 1$

$f(-x) = .75(-x)^2 + |-x| + 1$

$= .75x^2 + |x| + 1 = f(x)$

The function is even.

31. $f(x) = x^3 - x + 3$

$f(x) = (-x)^3 - (-x) + 3$

$= -x^3 + x + 3 = -(x^3 - x - 3) \neq -f(x)$

The function is neither.

32. $f(x) = x^4 - 5x + 2$

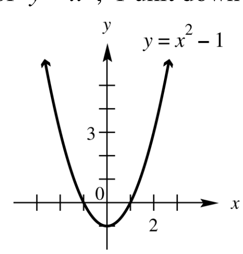
$f(-x) = (-x)^4 - 5(-x) + 2$

$= x^4 + 5x + 2 \neq f(x)$

The function is neither.

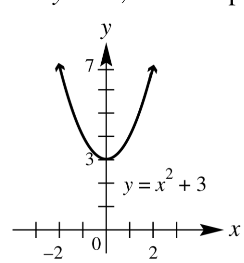
33. $y = x^2 - 1$

This graph may be obtained by translating the graph of $y = x^2$, 1 unit downward.



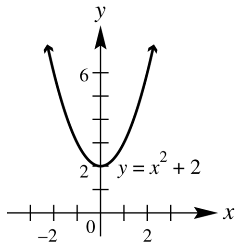
34. $y = x^2 + 3$

This graph may be obtained by translating the graph of $y = x^2$, 3 units upward.



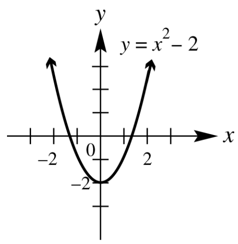
35. $y = x^2 + 2$

This graph may be obtained by translating the graph of $y = x^2$, 2 units upward.



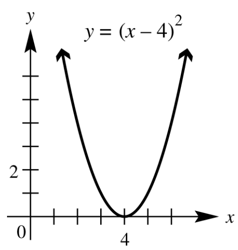
36. $y = x^2 - 2$

This graph may be obtained by translating the graph of $y = x^2$, 2 units downward.



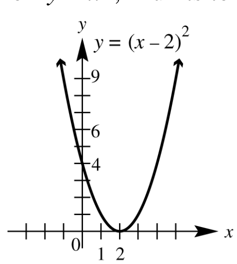
37. $y = (x - 4)^2$

This graph may be obtained by translating the graph of $y = x^2$, 4 units to the right.



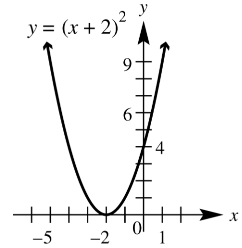
38. $y = (x - 2)^2$

This graph may be obtained by translating the graph of $y = x^2$, 2 units to the right.



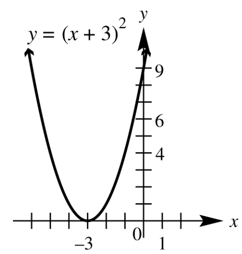
39. $y = (x + 2)^2$

This graph may be obtained by translating the graph of $y = x^2$, 2 units to the left.



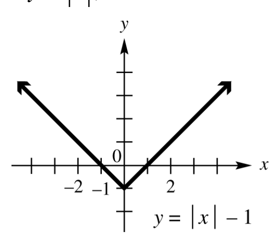
40. $y = (x + 3)^2$

This graph may be obtained by translating the graph of $y = x^2$, 3 units to the left.



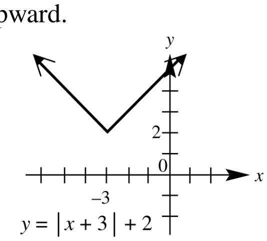
41. $y = |x| - 1$

The graph is obtained by translating the graph of $y = |x|$, 1 unit downward.



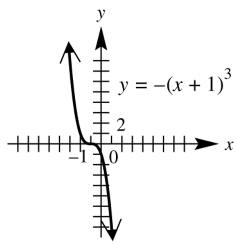
42. $y = |x + 3| + 2$

This graph may be obtained by translating the graph of $y = |x|$, 3 units to the left and 2 units upward.



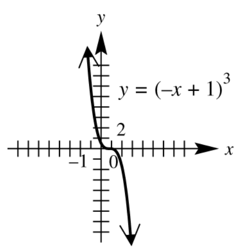
43. $y = -(x+1)^3$

This graph may be obtained by translating the graph of $y = x^3$, 1 unit to the left. It is then reflected about the x-axis.



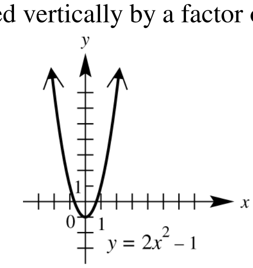
44. $y = (-x+1)^3$

If the given equation is written in the form $y = [-1(x-1)]^3 = (-1)^3(x-1)^3 = -(x-1)^3$, we see that this graph can be obtained by translating the graph of $y = x^3$, 1 unit to the right. It is then reflected about the y-axis. (We may also reflect the graph about the y-axis first and then translate it 1 unit to the right.)



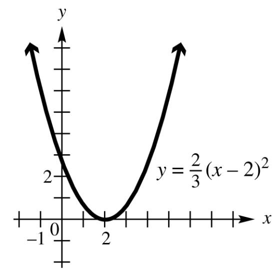
45. $y = 2x^2 - 1$

This graph may be obtained by translating the graph of $y = x^2$, 1 unit down. It is then stretched vertically by a factor of 2.



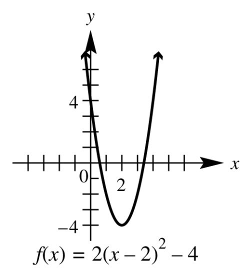
46. $y = \frac{2}{3}(x-2)^2$

This graph may be obtained by translating the graph of $y = x^2$, 2 units to the right. It is then shrunk vertically by a factor of $\frac{2}{3}$.



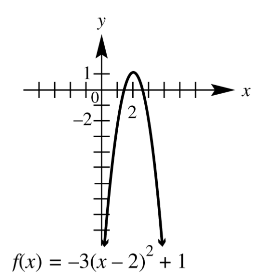
47. $f(x) = 2(x-2)^2 - 4$

This graph may be obtained by translating the graph of $y = x^2$, 2 units to the right and 4 units down. It is then stretched vertically by a factor of 2.

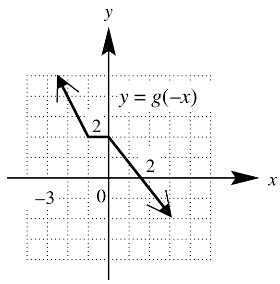


48. $f(x) = -3(x-2)^2 + 1$

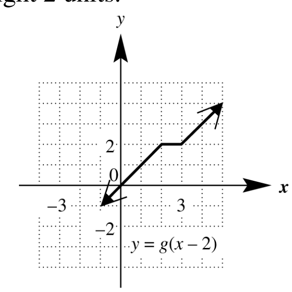
This graph may be obtained by translating the graph of $y = x^2$, 2 units to the right and 1 unit up. It is then stretched vertically by a factor of 3 and reflected over the x-axis.



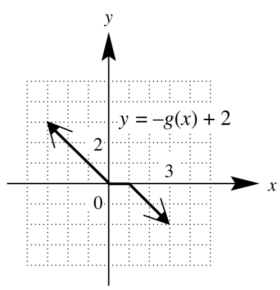
49. **(a)** $y = g(-x)$
The graph of $g(x)$ is reflected across the y-axis.



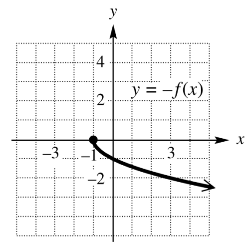
(b) $y = g(x - 2)$
The graph of $g(x)$ is translated to the right 2 units.



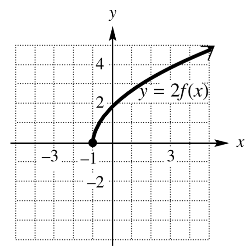
(c) $y = -g(x) + 2$
The graph of $g(x)$ is reflected across the x-axis and translated 2 units up.



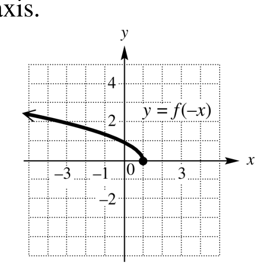
50. **(a)** $y = -f(x)$
The graph of $f(x)$ is reflected about the x-axis.



(b) $y = 2f(x)$
The graph of $f(x)$ is stretched vertically by a factor of 2.



(c) $y = f(-x)$
The graph of $f(x)$ is reflected across the y-axis.



51. It is the graph of $f(x) = |x|$ translated 1 unit to the left, reflected across the x-axis, and translated 3 units up. The equation is $y = -|x+1| + 3$.

52. It is the graph of $g(x) = \sqrt{x}$ translated 4 units to the left, reflected across the x-axis, and translated 2 units up. The equation is $y = -\sqrt{x+4} + 2$.

53. It is the graph of $g(x) = \sqrt{x}$ translated 4 units to the left, stretched vertically by a factor of 2, and translated 4 units down. The equation is $y = 2\sqrt{x+4} - 4$.

54. It is the graph of $f(x) = |x|$ translated 2 units to the right, shrunken vertically by a factor of $\frac{1}{2}$, and translated 1 unit down. The equation is $y = \frac{1}{2}|x-2| - 1$.

55. Since $f(3) = 6$, the point $(3, 6)$ is on the graph. Since the graph is symmetric with respect to the origin, the point $(-3, -6)$ is on the graph. Therefore, $f(-3) = -6$.

56. Since $f(3) = 6$, $(3, 6)$ is a point on the graph. Since the graph is symmetric with respect to the y-axis, $(-3, 6)$ is on the graph. Therefore, $f(-3) = 6$.

57. Since $f(3) = 6$, the point (3, 6) is on the graph. Since the graph is symmetric with respect to the line $x = 6$ and since the point (3, 6) is 3 units to the left of the line $x = 6$, the image point of (3, 6), 3 units to the right of the line $x = 6$, is (9, 6). Therefore, $f(9) = 6$.

58. Since $f(3) = 6$ and since $f(-x) = f(x)$, $f(-3) = f(3)$. Therefore, $f(-3) = 6$.

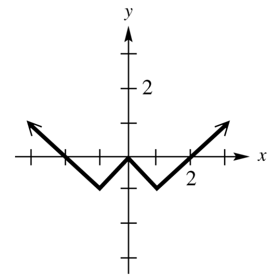
59. An odd function is a function whose graph is symmetric with respect to the origin. Since (3, 6) is on the graph, (–3, –6) must also be on the graph. Therefore, $f(-3) = -6$.

60. If f is an odd function, $f(-x) = -f(x)$. Since $f(3) = 6$ and $f(-x) = -f(x)$, $f(-3) = -f(3)$. Therefore, $f(-3) = -6$.

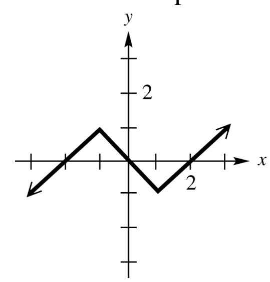
61. $f(x) = 2x + 5$
Translate the graph of $f(x)$ up 2 units to obtain the graph of
$t(x) = (2x+5)+2 = 2x+7$.
Now translate the graph of $t(x) = 2x + 7$ left 3 units to obtain the graph of $g(x) = 2(x+3)+7 = 2x+6+7 = 2x+13$.
(Note that if the original graph is first translated to the left 3 units and then up 2 units, the final result will be the same.)

62. $f(x) = 3 - x$
Translate the graph of $f(x)$ down 2 units to obtain the graph of
$t(x) = (3-x)-2 = -x+1$.
Now translate the graph of $t(x) = -x+1$ right 3 units to obtain the graph of $g(x) = -(x-3)+1 = -x+3+1 = -x+4$.
(Note that if the original graph is first translated to the right 3 units and then down 2 units, the final result will be the same.)

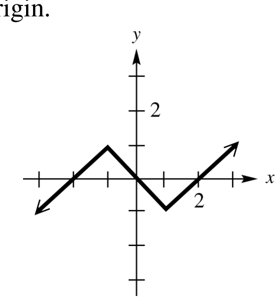
63. (a) Since $f(-x) = f(x)$, the graph is symmetric with respect to the y-axis.



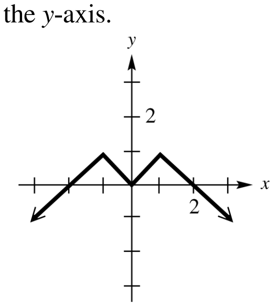
(b) Since $f(-x) = -f(x)$, the graph is symmetric with respect to the origin.



64. (a) $f(x)$ is odd. An odd function has a graph symmetric with respect to the origin. Reflect the left half of the graph in the origin.



(b) $f(x)$ is even. An even function has a graph symmetric with respect to the y-axis. Reflect the left half of the graph in the y-axis.



65. Answers will vary.
There are four possibilities for the constant, c.

i) $c > 0$ $|c| > 1$ The graph of $F(x)$ is stretched vertically by a factor of c.

ii) $c > 0$ $|c| < 1$ The graph of $F(x)$ is shrunk vertically by a factor of c.

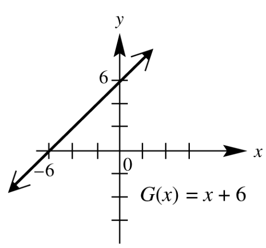
iii) $c < 0$ $|c| > 1$ The graph of $F(x)$ is stretched vertically by a factor of $-c$ and reflected over the x-axis.

iv) $c < 0$ $|c| < 1$ The graph of $F(x)$ is shrunk vertically by a factor of $-c$ and reflected over the x-axis.

66. The graph of $y = F(x+h)$ represents a horizontal shift of the graph of $y = F(x)$. If $h > 0$, it is a shift to the left h units. If $h < 0$, it is a shift to the left $-h$ units (h is negative).
The graph of $y = F(x) + h$ is not the same as the graph of $y = F(x+h)$. The graph of $y = F(x) + h$ represents a vertical shift of the graph of $y = F(x)$.

67. The graph of $F(x) = x^2 + 6$ is translated 6 units up from the graph of $f(x) = x^2$.

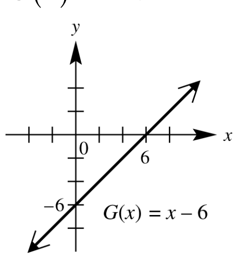
68. $G(x) = x + 6$



69. The graph of $G(x) = x + 6$ is translated 6 units up from the graph of $g(x) = x$.

70. The graph of $F(x) = (x-6)^2$ is translated 6 units to the right from the graph of $f(x) = x^2$.

71. $G(x) = x - 6$



72. The graph of $G(x) = x - 6$ is translated 6 units to the right from the graph of $g(x) = x$.

73. **(a)** Choose any value x. Find the corresponding value of y on both graphs and compare these values. For example, choose $x = 2$. From the graph, $f(2) = 1$ and $g(2) = 3$. For any value x, the y-value for $g(x)$ is 2 greater than the y-value for $f(x)$, so the graph of $g(x)$ is a vertical translation of the graph of $f(x)$ up 2 units. Therefore $g(x) = f(x) + 2$, that is $c = 2$.

(b) Choose any value y. Find the corresponding value of x on both graphs and compare these values. For example, choose $y = 3$. From the graph, $f(6) = 3$ and $g(2) = 3$. For any value y, the x-value for $g(x)$ is 4 less than the x-value for $f(x)$, so the graph of $g(x)$ is a horizontal translation of the graph of $f(x)$ to the left 4 units. Therefore $g(x) = f(x+4)$, that is $c = 4$.

Section 2.7: Function Operations and Composition

In Exercises 1–8, $f(x) = 5x^2 - 2x$ and $g(x) = 6x + 4$.

1. $(f+g)(3) = f(3) + g(3)$
$$= \left[5(3)^2 - 2(3)\right] + \left[6(3)+4\right]$$
$$= (45-6) + (18+4)$$
$$= 39 + 22 = 61$$

2. $(f-g)(-5) = f(-5) - g(-5)$
$$= [5(-5)^2 - 2(-5)] - [6(-5)+4]$$
$$= (125+10) - (-30+4)$$
$$= 135 - (-26) = 161$$

3. $(fg)(4) = f(4)\cdot g(4)$
$$= [5(4)^2 - 2(4)]\cdot[6(4)+4]$$
$$= [5(16) - 2(4)]\cdot[24+4]$$
$$= (80-8)\cdot(28) = 72(28) = 2016$$

4. $(fg)(-3) = f(-3)\cdot g(-3)$
$$= [5(-3)^2 - 2(-3)]\cdot[6(-3)+4]$$
$$= [5(9) - 2(-3)]\cdot[-18+4]$$
$$= (45+6)\cdot(-14)$$
$$= 51(-14) = -714$$

5. $\left(\frac{f}{g}\right)(-1) = \frac{f(-1)}{g(-1)} = \frac{5(-1)^2 - 2(-1)}{6(-1)+4}$
$$= \frac{5+2}{-6+4} = -\frac{7}{2}$$

6. $\left(\frac{f}{g}\right)(4) = \frac{f(4)}{g(4)} = \frac{5(4)^2 - 2(4)}{6(4)+4}$
$$= \frac{80-8}{24+4} = \frac{72}{28} = \frac{18}{7}$$

7. $(f-g)(m) = f(m) - g(m)$
$$= (5m^2 - 2m) - (6m+4)$$
$$= 5m^2 - 2m - 6m - 4$$
$$= 5m^2 - 8m - 4$$

8. $(f+g)(2k) = f(2k) + g(2k)$
$$= [5(2k)^2 - 2(2k)] + [6(2k)+4]$$
$$= [5(4)k^2 - 2(2k)] + [12k+4]$$
$$= (20k^2 - 4k) + (12k+4)$$
$$= 20k^2 + 8k + 4$$

9. $f(x) = 3x+4,\ g(x) = 2x-5$

i) $(f+g)(x) = f(x) + g(x) = (3x+4) + (2x-5) = 5x - 1$

ii) $(f-g)(x) = f(x) - g(x) = (3x+4) - (2x-5) = x + 9$

iii) $(fg)(x) = f(x)\cdot g(x) = (3x+4)(2x-5) = 6x^2 - 15x + 8x - 20 = 6x^2 - 7x - 20$

iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x+4}{2x-5}$

The domains of both *f* and *g* are the set of all real numbers, so the domains of *f* + *g*, *f* – *g*, and *fg* are all $(-\infty, \infty)$. The domain of $\frac{f}{g}$ is the set of all real numbers for which $g(x) \neq 0$. This is the set of all real numbers except $\frac{5}{2}$, which is written in interval notation as $\left(-\infty, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$.

10. $f(x) = 6 - 3x,\ g(x) = -4x + 1$

i) $(f+g)(x) = f(x) + g(x) = (6-3x) + (-4x+1) = -7x + 7$

ii) $(f-g)(x) = f(x) - g(x) = (6-3x) - (-4x+1) = x + 5$

iii) $(fg)(x) = f(x)\cdot g(x) = (6-3x)(-4x+1) = -24x + 6 + 12x^2 - 3x = 12x^2 - 27x + 6$

iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{6-3x}{-4x+1}$

The domains of both *f* and *g* are the set of all real numbers, so the domains of *f* + *g*, *f* – *g*, and *fg* are all $(-\infty, \infty)$. The domain of $\frac{f}{g}$ is the set of all real numbers for which $g(x) \neq 0$. This is the set of all real numbers except $\frac{1}{4}$, which is written in interval notation as $\left(-\infty, \frac{1}{4}\right) \cup \left(\frac{1}{4}, \infty\right)$.

11. $f(x) = 2x^2 - 3x, g(x) = x^2 - x + 3$

i) $(f+g)(x) = f(x) + g(x) = (2x^2 - 3x) + (x^2 - x + 3) = 3x^2 - 4x + 3$

ii) $(f-g)(x) = f(x) - g(x) = (2x^2 - 3x) - (x^2 - x + 3) = 2x^2 - 3x - x^2 + x - 3 = x^2 - 2x - 3$

iii) $(fg)(x) = f(x) \cdot g(x) = (2x^2 - 3x)(x^2 - x + 3)$

$$= 2x^4 - 2x^3 + 6x^2 - 3x^3 + 3x^2 - 9x = 2x^4 - 5x^3 + 9x^2 - 9x$$

iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x^2 - 3x}{x^2 - x + 3}$

The domains of both *f* and *g* are the set of all real numbers, so the domains of *f* + *g*, *f* − *g*, and *fg* are all $(-\infty, \infty)$. The domain of $\frac{f}{g}$ is the set of all real numbers for which $g(x) \neq 0$.

This is the set of all real numbers except $\frac{1}{4}$, which is written in interval notation as $\left(-\infty, \frac{1}{4}\right) \cup \left(\frac{1}{4}, \infty\right)$.

If $x^2 - x + 3 = 0$, then by the quadratic formula $x = \frac{1 \pm i\sqrt{11}}{2}$. The equation has no real solutions. There are no real numbers which make the denominator zero. Thus, the domain of $\frac{f}{g}$ is also $(-\infty, \infty)$.

12. $f(x) = 4x^2 + 2x - 3, g(x) = x^2 - 3x + 2$

i) $(f+g)(x) = f(x) + g(x) = (4x^2 + 2x - 3) + (x^2 - 3x + 2) = 5x^2 - x - 1$

ii) $(f-g)(x) = f(x) - g(x) = (4x^2 + 2x - 3) - (x^2 - 3x + 2) = 4x^2 + 2x - 3 - x^2 + 3x - 2 = 3x^2 + 5x - 5$

iii) $(fg)(x) = f(x) \cdot g(x) = (4x^2 + 2x - 3)(x^2 - 3x + 2)$

$$= 4x^4 - 12x^3 + 8x^2 + 2x^3 - 6x^2 + 4x - 3x^2 + 9x - 6 = 4x^4 - 10x^3 - x^2 + 13x - 6$$

iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{4x^2 + 2x - 3}{x^2 - 3x + 2}$

The domains of both *f* and *g* are the set of all real numbers, so the domains of *f* + *g*, *f* − *g*, and *fg* are all $(-\infty, \infty)$. The domain of $\frac{f}{g}$ is the set of all real numbers *x* such that $x^2 - 3x + 2 \neq 0$. Since $x^2 - 3x + 2 = (x-1)(x-2)$, the numbers which give this denominator a value of 0 are *x* = 1 and *x* = 2. Therefore, the domain of $\frac{f}{g}$ is the set of all real numbers except 1 and 2, which is written in interval notation as $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$.

13. $f(x) = \sqrt{4x - 1}, g(x) = \frac{1}{x}$

i) $(f+g)(x) = f(x) + g(x) = \sqrt{4x-1} + \frac{1}{x}$

ii) $(f-g)(x) = f(x) - g(x) = \sqrt{4x-1} - \frac{1}{x}$

iii) $(fg)(x) = f(x) \cdot g(x) = \sqrt{4x-1}\left(\frac{1}{x}\right) = \frac{\sqrt{4x-1}}{x}$

iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{4x-1}}{\frac{1}{x}} = x\sqrt{4x-1}$

Since $4x - 1 \geq 0 \Rightarrow 4x \geq 1 \Rightarrow x \geq \frac{1}{4}$, the domain of *f* is $\left[\frac{1}{4}, \infty\right)$. The domain of *g* is $(-\infty, 0) \cup (0, \infty)$. Considering the intersection of the domains of *f* and *g*, the domains of *f* + *g*, *f* − *g*, and *fg* are all $\left[\frac{1}{4}, \infty\right)$. Since $\frac{1}{x} \neq 0$ for any value of *x*, the domain of $\frac{f}{g}$ is also $\left[\frac{1}{4}, \infty\right)$.

14. $f(x)=\sqrt{5x-4},\ g(x)=-\frac{1}{x}$

i) $(f+g)(x)=f(x)+g(x)=\sqrt{5x-4}+\left(-\frac{1}{x}\right)=\sqrt{5x-4}-\frac{1}{x}$

ii) $(f-g)(x)=f(x)-g(x)=\sqrt{5x-4}-\left(-\frac{1}{x}\right)=\sqrt{5x-4}+\frac{1}{x}$

iii) $(fg)(x)=f(x)\cdot g(x)=\left(\sqrt{5x-4}\right)\left(-\frac{1}{x}\right)=-\frac{\sqrt{5x-4}}{x}$

iv) $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{\sqrt{5x-4}}{-\frac{1}{x}}=-x\sqrt{5x-4}$

Since $5x-4\geq 0\Rightarrow 5x\geq 4\Rightarrow x\geq\frac{4}{5}$, the domain of *f* is $\left[\frac{4}{5},\infty\right)$. The domain of *g* is $(-\infty,0)\cup(0,\infty)$. Considering the intersection of the domains of *f* and *g*, the domains of *f* + *g*, *f* – *g*, and *fg* are all $\left[\frac{4}{5},\infty\right)$. Since $-\frac{1}{x}\neq 0$ for any value of *x*, the domain of $\frac{f}{g}$ is also $\left[\frac{4}{5},\infty\right)$.

In the responses to Exercises 15 – 16, numerical answers may vary.

15. $G(1996)\approx 7.7$ and $B(1996)\approx 11.8$, thus $T(1996)=G(1996)+B(1996)=7.7+11.8=19.5$.

16. $G(1991)\approx 6.3$ and $B(1991)\approx 8.2$, thus $T(1991)=G(1991)+B(1991)=6.3+8.2=14.5$.

17. Looking at the graphs of the functions, the slopes of the line segments for the period 1991-1996 are much steeper than the slopes of the corresponding line segments for the period 1978-1991. Thus, the number of sodas increased more rapidly during the period 1991-1996.

18. Answers will vary.

In the responses to Exercises 19 – 20, numerical answers may vary.

19. $(T-S)(2000)=T(2000)-S(2000)=19-13=6$; It represents the dollars in billions spent for general science in 2000.

20. $(T-G)(2005)=T(2005)-G(2005)=23-8=15$; It represents the dollars in billions spent on space and other technologies in 2005.

21. In space and other technologies spending was almost static in the years 1995 – 2000.

22. In space and other technologies spending increased the most during the years 2000 – 2005.

23. **(a)** $(f+g)(2)=f(2)+g(2)$
$=4+(-2)=2$

(b) $(f-g)(1)=f(1)-g(1)=1-(-3)=4$

(c) $(fg)(0)=f(0)\cdot g(0)=0(-4)=0$

(d) $\left(\frac{f}{g}\right)(1)=\frac{f(1)}{g(1)}=\frac{1}{-3}=-\frac{1}{3}$

24. **(a)** $(f+g)(0)=f(0)+g(0)=0+2=2$

(b) $(f-g)(-1)=f(-1)-g(-1)$
$=-2-1=-3$

(c) $(fg)(1)=f(1)\cdot g(1)=2\cdot 1=2$

(d) $\left(\frac{f}{g}\right)(2)=\frac{f(2)}{g(2)}=\frac{4}{-2}=-2$

25. (a) $(f+g)(-1) = f(-1) + g(-1) = 0 + 3 = 3$

(b) $(f-g)(-2) = f(-2) - g(-2)$
$= -1 - 4 = -5$

(c) $(fg)(0) = f(0)\cdot g(0) = 1\cdot 2 = 2$

(d) $\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{3}{0} = \text{undefined}$

26. (a) $(f+g)(1) = f(1) + g(1) = -3 + 1 = -2$

(b) $(f-g)(0) = f(0) - g(0) = -2 - 0 = -2$

(c) $(fg)(-1) = f(-1)\cdot g(-1) = -3(-1) = 3$

(d) $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{-3}{1} = -3$

27. (a) $(f+g)(2) = f(2) + g(2)$
$= 7 + (-2) = 5$

(b) $(f-g)(4) = f(4) - g(4) = 10 - 5 = 5$

(c) $(fg)(-2) = f(-2)\cdot g(-2) = 0\cdot 6 = 0$

(d) $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{5}{0} = \text{undefined}$

28. (a) $(f+g)(2) = f(2) + g(2) = 5 + 4 = 9$

(b) $(f-g)(4) = f(4) - g(4) = 0 - 0 = 0$

(c) $(fg)(-2) = f(-2)\cdot g(-2) = -4\cdot 2 = -8$

(d) $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{8}{-1} = -8$

29.

x	$f(x)$	$g(x)$	$(f+g)(x)$	$(f-g)(x)$	$(fg)(x)$	$\left(\frac{f}{g}\right)(x)$
–2	0	6	$0+6=6$	$0-6=-6$	$0\cdot 6=0$	$\frac{0}{6}=0$
0	5	0	$5+0=5$	$5-0=5$	$5\cdot 0=0$	$\frac{5}{0}=$ undefined
2	7	–2	$7+(-2)=5$	$7-(-2)=9$	$7(-2)=-14$	$\frac{7}{-2}=-3.5$
4	10	5	$10+5=15$	$10-5=5$	$10\cdot 5=50$	$\frac{10}{5}=2$

30.

x	$f(x)$	$g(x)$	$(f+g)(x)$	$(f-g)(x)$	$(fg)(x)$	$\left(\frac{f}{g}\right)(x)$
–2	–4	2	$-4+2=-2$	$-4-2=-6$	$-4\cdot 2=-8$	$\frac{-4}{2}=-2$
0	8	–1	$8+(-1)=7$	$8-(-1)=9$	$8(-1)=-8$	$\frac{8}{-1}=-8$
2	5	4	$5+4=9$	$5-4=1$	$5\cdot 4=20$	$\frac{5}{4}=1.25$
4	0	0	$0+0=0$	$0-0=0$	$0\cdot 0=0$	$\frac{0}{0}=$ undefined

31. Answers will vary.
The difference quotient, $\frac{f(x+h)-f(x)}{h}$, represents the slope of the secant line which passes through points $(x, f(x))$ and $(x+h, f(x+h))$. The formula is derived by applying the rule that slope represents a change in *y* to a change in *x*.

32. Answers will vary.
The secant line *PQ* represents the line that is formed between points *P* and *Q*. This line exists when *h* is positive. The tangent line at point *P* is created when the difference in the *x* values between points *P* and *Q* (namely *h*) becomes zero.

33. $f(x) = 2 - x$

(a) $f(x+h) = 2-(x+h) = 2-x-h$

(b) $f(x+h) - f(x) = (2-x-h)-(2-x)$
$= 2-x-h-2+x = -h$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-h}{h} = -1$

34. $f(x) = 1 - x$

(a) $f(x+h) = 1-(x+h) = 1-x-h$

(b) $f(x+h) - f(x) = (1-x-h)-(1-x)$
$= 1-x-h-1+x = -h$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-h}{h} = -1$

35. $f(x) = 6x + 2$

(a) $f(x+h) = 6(x+h)+2 = 6x+6h+2$

(b) $f(x+h) - f(x)$
$= (6x+6h+2)-(6x+2)$
$= 6x+6h+2-6x-2 = 6h$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{6h}{h} = 6$

36. $f(x) = 4x + 11$

(a) $f(x+h) = 4(x+h)+11 = 4x+4h+11$

(b) $f(x+h) - f(x)$
$= (4x+4h+11)-(4x+11)$
$= 4x+4h+11-4x-11 = 4h$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{4h}{h} = 4$

37. $f(x) = -2x + 5$

(a) $f(x+h) = -2(x+h)+5$
$= -2x-2h+5$

(b) $f(x+h) - f(x)$
$= (-2x-2h+5)-(-2x+5)$
$= -2x-2h+5+2x-5$
$= -2h$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-2h}{h} = -2$

38. $f(x) = 1 - x^2$

(a) $f(x+h) = 1-(x+h)^2$
$= 1-(x^2+2xh+h^2)$
$= 1-x^2-2xh-h^2$

(b) $f(x+h) - f(x)$
$= (1-x^2-2xh-h^2)-(1-x^2)$
$= 1-x^2-2xh-h^2-1+x^2$
$= -2xh-h^2$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-2xh-h^2}{h}$
$= \dfrac{h(-2x-h)}{h}$
$= -2x-h$

39. $f(x) = x^2 - 4$

(a) $f(x+h) = (x+h)^2 - 4$
$= x^2+2xh+h^2-4$

(b) $f(x+h) - f(x)$
$= (x^2+2xh+h^2-4)-(x^2-4)$
$= x^2+2xh+h^2-4-x^2+4$
$= 2xh+h^2$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{2xh+h^2}{h}$
$= \dfrac{h(2x+h)}{h} = 2x+h$

40. $f(x) = 8 - 3x^2$

(a) $f(x+h) = 8-3(x+h)^2$
$= 8-3(x^2+2xh+h^2)$
$= 8-3x^2-6xh-3h^2$

(b) $f(x+h) - f(x)$
$= (8-3x^2-6xh-3h^2)-(8-3x^2)$
$= 8-3x^2-6xh-3h^2-8+3x^2$
$= -6xh-3h^2$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-6xh-3h^2}{h}$
$= \dfrac{h(-6x-3h)}{h}$
$= -6x-3h$

41. Since $g(x) = -x + 3,\ g(4) = -4 + 3 = -1.$
Therefore, $(f \circ g)(4) = f\left[g(4)\right] = f(-1) = 2(-1) - 3 = -2 - 3 = -5.$

42. Since $g(x) = -x + 3,\ g(2) = -2 + 3 = 1.$
Therefore, $(f \circ g)(2) = f\left[g(2)\right] = f(1) = 2(1) - 3 = 2 - 3 = -1.$

43. Since $g(x) = -x + 3,\ g(-2) = -(-2) + 3 = 5.$
Therefore, $(f \circ g)(-2) = f\left[g(-2)\right] = f(5) = 2(5) - 3 = 10 - 3 = 7.$

44. Since $f(x) = 2x - 3,\ f(3) = 2(3) - 3 = 6 - 3 = 3.$
Therefore, $(g \circ f)(3) = g\left[f(3)\right] = g(3) = -3 + 3 = 0.$

45. Since $f(x) = 2x - 3,\ f(0) = 2(0) - 3 = 0 - 3 = -3.$
Therefore, $(g \circ f)(0) = g\left[f(0)\right] = g(-3) = -(-3) + 3 = 3 + 3 = 6.$

46. Since $f(x) = 2x - 3,\ f(-2) = 2(-2) - 3 = -4 - 3 = -7.$
Therefore, $(g \circ f)(-2) = g\left[f(-2)\right] = g(-7) = -(-7) + 3 = 7 + 3 = 10.$

47. Since $f(x) = 2x - 3,\ f(2) = 2(2) - 3 = 4 - 3 = 1.$
Therefore, $(f \circ f)(2) = f\left[f(2)\right] = f(1) = 2(1) - 3 = 2 - 3 = -1.$

48. Since $g(x) = -x + 3,\ g(-2) = -(-2) + 3 = 5.$
Therefore, $(g \circ g)(-2) = g\left[g(-2)\right] = g(5) = -5 + 3 = -2.$

49. $(f \circ g)(2) = f[g(2)] = f(3) = 1$

50. $(f \circ g)(7) = f[g(7)] = f(6) = 9$

51. $(g \circ f)(3) = g[f(3)] = g(1) = 9$

52. $(g \circ f)(6) = g[f(6)] = g(9) = 12$

53. $(f \circ f)(4) = f[f(4)] = f(3) = 1$

54. $(g \circ g)(1) = g[g(1)] = g(9) = 12$

55. $(f \circ g)(1) = f[g(1)] = f(9)$
However, $f(9)$ cannot be determined from the table given.

56. $(g \circ (f \circ g))(7) = g(f(g(7)))$
$= g(f(6)) = g(9) = 12$

57. $f(x) = -6x + 9,\ g(x) = 5x + 7$
$(f \circ g)(x) = f[g(x)]$
$= f(5x+7) = -6(5x+7) + 9$
$= -30x - 42 + 9 = -30x - 33$
$(g \circ f)(x) = g[f(x)]$
$= g(-6x+9) = 5(-6x+9) + 7$
$= -30x + 45 + 7 = -30x + 52$

58. $f(x) = 8x + 12,\ g(x) = 3x - 1$
$(f \circ g)(x) = f[g(x)] = f(3x-1)$
$= 8(3x-1) + 12$
$= 24x - 8 + 12 = 24x + 4$
$(g \circ f)(x) = g[f(x)]$
$= g(8x+12) = 3(8x+12) - 1$
$= 24x + 36 - 1 = 24x + 35$

59. $f(x)=4x^2+2x+8,\ g(x)=x+5$

$$\begin{aligned}(f\circ g)(x)&=f[g(x)]=f(x+5)\\&=4(x+5)^2+2(x+5)+8\\&=4(x^2+10x+25)+2x+10+8\\&=4x^2+40x+100+2x+18\\&=4x^2+42x+118\end{aligned}$$

$$\begin{aligned}(g\circ f)(x)&=g[f(x)]=g(4x^2+2x+8)\\&=(4x^2+2x+8)+5\\&=4x^2+2x+13\end{aligned}$$

60 $f(x)=5x+3,\ g(x)=-x^2+4x+3$

$$\begin{aligned}(f\circ g)(x)&=f[g(x)]=f(-x^2+4x+3)\\&=5(-x^2+4x+3)+3\\&=-5x^2+20x+15+3\\&=-5x^2+20x+18\end{aligned}$$

$$\begin{aligned}(g\circ f)(x)]&=g[f(x)]=g(5x+3)\\&=-(5x+3)^2+4(5x+3)+3\\&=-(25x^2+30x+9)+20x+12+3\\&=-25x^2-30x-9+20x+15\\&=-25x^2-10x+6\end{aligned}$$

61. $f(x)=\dfrac{2}{x^4},\ g(x)=2-x$

$$(f\circ g)(x)=f[g(x)]=f(2-x)=\frac{2}{(2-x)^4}$$

Domain: $(-\infty,2)\cup(2,\infty)$

$$(g\circ f)(x)=g[f(x)]=g\left(\frac{2}{x^4}\right)=2-\frac{2}{x^4}$$

Domain: $(-\infty,0)\cup(0,\infty)$

62. $f(x)=\dfrac{1}{x},\ g(x)=x^2$

$$(f\circ g)(x)=f[g(x)]=f(x^2)=\frac{1}{x^2}$$

Domain: $(-\infty,0)\cup(0,\infty)$

$$(g\circ f)(x)=g\left(\frac{1}{x}\right)=\left(\frac{1}{x}\right)^2=\frac{1}{x^2}$$

Domain: $(-\infty,0)\cup(0,\infty)$

63. $f(x)=9x^2-11x,\ g(x)=2\sqrt{x+2}$

$$\begin{aligned}(f\circ g)(x)&=f[g(x)]=f(2\sqrt{x+2})\\&=9(2\sqrt{x+2})^2-11(2\sqrt{x+2})\\&=9[4(x+2)]-22\sqrt{x+2}\\&=9\left(4x+8\right)-22\sqrt{x+2}\\&=36x+72-22\sqrt{x+2}\end{aligned}$$

$$\begin{aligned}(g\circ f)(x)&=g[f(x)]=g(9x^2-11x)\\&=2\sqrt{(9x^2-11x)+2}\\&=2\sqrt{9x^2-11x+2}\end{aligned}$$

64. $f(x)=\sqrt{x+2},\ g(x)=8x^2-6$

$$\begin{aligned}(f\circ g)(x)&=f[g(x)]=f(8x^2-6)\\&=\sqrt{(8x^2-6)+2}=\sqrt{8x^2-4}\\&\text{or }\sqrt{4\left(2x^2-1\right)}\text{ or }2\sqrt{2x^2-1}\end{aligned}$$

$$\begin{aligned}(g\circ f)(x)&=g[f(x)]=g\left(\sqrt{x+2}\right)\\&=8\left(\sqrt{x+2}\right)^2-6=8\left(x+2\right)-6\\&=8x+16-6=8x+10\end{aligned}$$

65. $g\left[f(2)\right]=g(1)=2$ and $g\left[f(3)\right]=g(2)=5$

Since $g\left[f(1)\right]=7$ and $f(1)=3,\ g(3)=7.$

x	$f(x)$	$g(x)$	$g\left[f(x)\right]$
1	3	2	7
2	1	5	**2**
3	2	**7**	**5**

66. Since $f(x)$ is odd, $f(-1)=-f(1)=-(-2)=2$. Since $g(x)$ is even, $g(1)=g(-1)=2$ and $g(2)=g(-2)=0$. Since $(f\circ g)(-1)=1$, $f\left[g(-1)\right]=1$ and $f(2)=1$. Since $f(x)$ is odd, $f(-2)=-f(2)=-1$. Thus, $(f\circ g)(-2)=f\left[g(-2)\right]=f(0)=0$ and $(f\circ g)(1)=f\left[g(1)\right]=f(2)=1$ and $(f\circ g)(2)=f\left[g(2)\right]=f(0)=0$.

x	–2	–1	0	1	2
$f(x)$	**–1**	**2**	0	–2	**1**
$g(x)$	0	2	1	**2**	**0**
$(f\circ g)(x)$	**0**	1	–2	**1**	**0**

67. Answers will vary.
In general, composition of functions is not commutative.

68. Answers will vary.
To find $f\circ g$, the function g must be substituted into the function f.
$(f\circ g)(x)=f\left[g(x)\right]=2\left(x^2+3\right)-5=2x^2+6-5=2x^2+1$

69. $(f\circ g)(x)=f\left[g(x)\right]=4\left[\frac{1}{4}(x-2)\right]+2=\left(4\cdot\frac{1}{4}\right)(x-2)+2=(x-2)+2=x-2+2=x$

$(g\circ f)(x)=g\left[f(x)\right]=\frac{1}{4}\left[(4x+2)-2\right]=\frac{1}{4}(4x+2-2)=\frac{1}{4}(4x)=x$

70. $(f\circ g)(x)=f\left[g(x)\right]=-3\left(-\frac{1}{3}x\right)=\left[-3\left(-\frac{1}{3}\right)\right]x=x$

$(g\circ f)(x)=g\left[f(x)\right]=-\frac{1}{3}(-3x)=\left[-\frac{1}{3}(-3)\right]x=x$

71. $(f\circ g)(x)=f\left[g(x)\right]=\sqrt[3]{5\left(\frac{1}{5}x^3-\frac{4}{5}\right)+4}=\sqrt[3]{x^3-4+4}=\sqrt[3]{x^3}=x$

$(g\circ f)(x)=g\left[f(x)\right]=\frac{1}{5}\left(\sqrt[3]{5x+4}\right)^3-\frac{4}{5}=\frac{1}{5}(5x+4)-\frac{4}{5}=\frac{5x}{5}+\frac{4}{5}-\frac{4}{5}=\frac{5x}{5}=x$

72. $(f\circ g)(x)=f\left[g(x)\right]=\sqrt[3]{\left(x^3-1\right)+1}=\sqrt[3]{x^3-1+1}=\sqrt[3]{x^3}=x$

$(g\circ f)(x)=g\left[f(x)\right]=\left(\sqrt[3]{x^3+1}\right)^3-1=x^3+1-1=x$

In Exercises 73–78, we give only one of many possible ways.

73. $h(x)=(6x-2)^2$

Let $g(x)=6x-2$ and $f(x)=x^2$.

$(f\circ g)(x)=f(6x-2)=(6x-2)^2=h(x)$

74. $h(x)=(11x^2+12x)^2$

Let $g(x)=11x^2+12x$ and $f(x)=x^2$.

$(f\circ g)(x)=f(11x^2+12x)$
$=(11x^2+12x)^2=h(x)$

75. $h(x)=\sqrt{x^2-1}$

Let $g(x)=x^2-1$ and $f(x)=\sqrt{x}$.

$(f\circ g)(x)=f(x^2-1)=\sqrt{x^2-1}=h(x).$

76. $h(x)=(2x-3)^3$

Let $g(x)=2x-3$ and $f(x)=x^3$.

$(f\circ g)(x)=f(2x-3)=(2x-3)^3=h(x)$

77. $h(x)=\sqrt{6x}+12$

Let $g(x)=6x$ and $f(x)=\sqrt{x}+12$.

$(f\circ g)(x)=f(6x)=\sqrt{6x}+12=h(x)$

78. $h(x)=\sqrt[3]{2x+3}-4$

Let $g(x)=2x+3$ and $f(x)=\sqrt[3]{x}-4$.

$(f\circ g)(x)=f(2x+3)=\sqrt[3]{2x+3}-4=h(x)$

79. $f(x)=12x$, $g(x)=5280x$

$(f\circ g)(x)=f[g(x)]=f(5280x)$

$=12(5280x)=63,360x$

The function $f\circ g$ computes the number of inches in x miles.

80. **(a)** $x=4s\Rightarrow \frac{x}{4}=s\Rightarrow s=\frac{x}{4}$

(b) $y=s^2=\left(\frac{x}{4}\right)^2=\frac{x^2}{16}$

(c) $y=\dfrac{6^2}{16}=\dfrac{36}{16}=2.25$ square units

81. $A(x)=\dfrac{\sqrt{3}}{4}x^2$

(a) $A(2x)=\dfrac{\sqrt{3}}{4}(2x)^2=\dfrac{\sqrt{3}}{4}(4x^2)=\sqrt{3}x^2$

(b) $A(16)=A(2\cdot 8)=\sqrt{3}(8)^2$

$=64\sqrt{3}$ square units

82. **(a)** $y_1=.04x$

(b) $y_2=.025(x+500)$

(c) y_1+y_2 represents the total annual interest.

(d) $(y_1+y_2)(250)=y_1(250)+y_2(250)$

$=.04(250)+.025(250+500)=10+.025(750)=10+18.75=\$28.75$

83. **(a)** $r(t)=4t$ and $A(r)=\pi r^2$

$(A\circ r)(t)=A[r(t)]$

$=A(4t)=\pi(4t)^2=16\pi t^2$

(b) $(A\circ r)(t)$ defines the area of the leak in terms of the time t, in minutes.

(c) $A(3)=16\pi(3)^2=144\pi\text{ ft}^2$

84. **(a)** $(A\circ r)(t)=A[r(t)]$

$=A(2t)=\pi(2t)^2=4\pi t^2$

(b) It defines the area of the circular layer in terms of the time t, in hours.

(c) $(A\circ r)(4)=4\pi(4)^2=64\pi\text{ mi}^2$

85. Let x = the number of people less than 100 people that attend.

(a) x people fewer than 100 attend, so $100-x$ people do attend $N(x)=100-x$

(b) The cost per person starts at \$20 and increases by \$5 for each of the x people that do not attend. The total increase is \$5x, and the cost per person increases to \$20 + \$5x. Thus, $G(x)=20+5x$.

(c) $C(x)=N(x)\cdot G(x)=(100-x)(20+5x)$

(d) If 80 people attend, $x=100-80=20$.

$C(20)=(100-20)\left[20+5(20)\right]=(80)(20+100)=(80)(120)=\$9600$

86. If the area of a square is x^2 square inches, each side must have a length of x inches. If 3 inches is added to one dimension and 1 inch is subtracted from the other, the new dimensions will be $x+3$ and $x-1$. Thus, the area of the resulting rectangle is $A(x)=(x+3)(x-1)$.

Chapter 2: Review Exercises

1. $P(3, -1)$, $Q(-4, 5)$

$$d(P, Q) = \sqrt{(-4-3)^2 + [5-(-1)]^2} = \sqrt{(-7)^2 + 6^2} = \sqrt{49+36} = \sqrt{85}$$

Midpoint: $\left(\dfrac{3+(-4)}{2}, \dfrac{-1+5}{2}\right) = \left(\dfrac{-1}{2}, \dfrac{4}{2}\right) = \left(-\dfrac{1}{2}, 2\right)$

2. $M(-8, 2)$, $N(3, -7)$

$$d(M, N) = \sqrt{[3-(-8)]^2 + (-7-2)^2} = \sqrt{11^2 + (-9)^2} = \sqrt{121+81} = \sqrt{202}$$

Midpoint: $\left(\dfrac{-8+3}{2}, \dfrac{2+(-7)}{2}\right) = \left(-\dfrac{5}{2}, -\dfrac{5}{2}\right)$

3. $A(-6, 3)$, $B(-6, 8)$

$$d(A, B) = \sqrt{[-6-(-6)]^2 + (8-3)^2} = \sqrt{0+5^2} = \sqrt{25} = 5$$

Midpoint: $\left(\dfrac{-6+(-6)}{2}, \dfrac{3+8}{2}\right) = \left(\dfrac{-12}{2}, \dfrac{11}{2}\right) = \left(-6, \dfrac{11}{2}\right)$

4. Label the points $A(5, 7)$, $B(3, 9)$, and $C(6, 8)$.

$$d(A, B) = \sqrt{(3-5)^2 + (9-7)^2} = \sqrt{(-2)^2 + 2^2} = \sqrt{4+4} = \sqrt{8}$$

$$d(A, C) = \sqrt{(6-5)^2 + (8-7)^2} = \sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$$

$$d(B, C) = \sqrt{(6-3)^2 + (8-9)^2} = \sqrt{3^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

Since $\left(\sqrt{8}\right)^2 + \left(\sqrt{2}\right)^2 = \left(\sqrt{10}\right)^2$, triangle ABC is a right triangle.

5. Label the points $A(-1, 2)$, $B(-10, 5)$, and $C(-4, k)$.

$$d(A, B) = \sqrt{[-1-(-10)]^2 + (2-5)^2} = \sqrt{9^2 + (-3)^2} = \sqrt{81+9} = \sqrt{90}$$

$$d(A, C) = \sqrt{[-4-(-1)]^2 + (k-2)^2} = \sqrt{9+(k-2)^2}$$

$$d(B, C) = \sqrt{[-10-(-4)]^2 + (5-k)^2} = \sqrt{36+(k-5)^2}$$

If segment AB is the hypotenuse, then $\left(\sqrt{90}\right)^2 = \left[\sqrt{9+(k-2)^2}\right]^2 + \left[\sqrt{36+(k-5)^2}\right]^2$.

$$\left(\sqrt{90}\right)^2 = \left[\sqrt{9+(k-2)^2}\right]^2 + \left[\sqrt{36+(k-5)^2}\right]^2 \Rightarrow 90 = 9 + (k-2)^2 + 36 + (k-5)^2$$

$$90 = 9 + k^2 - 4k + 4 + 36 + k^2 - 10k + 25 \Rightarrow 0 = 2k^2 - 14k - 16$$

$$0 = k^2 - 7k - 8 \Rightarrow 0 = (k-8)(k+1) \Rightarrow k = 8 \text{ or } k = -1$$

Another approach is if segment AB is the hypotenuse, the product of the slopes of lines AC and BC is -1 since the product of slopes of perpendicular lines is -1.

$$\left(\frac{k-2}{-4-(-1)}\right)\cdot\left(\frac{k-5}{-4-(-10)}\right) = -1 \Rightarrow \left(\frac{k-2}{-3}\right)\cdot\left(\frac{k-5}{6}\right) = -1 \Rightarrow \frac{(k-2)(k-5)}{-18} = -1$$

$$\frac{k^2 - 5k - 2k + 10}{-18} = -1 \Rightarrow k^2 - 7k + 10 = 18 \Rightarrow k^2 - 7k - 8 = 0 \Rightarrow (k+8)(k-1) = 0 \Rightarrow k = 8 \text{ or } k = -1$$

Continued on next page

5. (continued)

We will use the second approach for investigating the other two sides of the triangle.

If segment AC is the hypotenuse, the product of the slopes of lines AB and BC is –1 since the product of slopes of perpendicular lines is –1.

$$\left(\frac{5-2}{-10-(-1)}\right)\cdot\left(\frac{k-5}{-4-(-10)}\right)=-1\Rightarrow\left(\frac{3}{-9}\right)\cdot\left(\frac{k-5}{6}\right)=-1\Rightarrow\frac{k-5}{-18}=-1\Rightarrow k-5=18\Rightarrow k=23$$

If segment BC is the hypotenuse, the product of the slopes of lines AB and AC is –1.

$$\left(\frac{3}{-9}\right)\cdot\left(\frac{k-2}{-4-(-1)}\right)=-1\Rightarrow\left(\frac{-1}{3}\right)\cdot\left(\frac{k-2}{-3}\right)=-1\Rightarrow\frac{k-2}{9}=-1\Rightarrow k-2=-9\Rightarrow k=-7$$

The possible values of k are –7, 23, 8, and –1.

6. $P(-2,-5)$, $Q(1, 7)$, $R(3, 15)$

$$d(P, Q)=\sqrt{(-2-1)^2+(-5-7)^2}=\sqrt{(-3)^2+(-12)^2}=\sqrt{9+144}=\sqrt{153}=3\sqrt{17}$$

$$d(Q, R)=\sqrt{(3-1)^2+(15-7)^2}=\sqrt{2^2+8^2}=\sqrt{4+64}=\sqrt{68}=2\sqrt{17}$$

$$d(P, R)=\sqrt{(-2-3)^2+(-5-15)^2}=\sqrt{(-5)^2+(-20)^2}=\sqrt{25+400}=\sqrt{425}=5\sqrt{17}$$

Since $d(P, Q)+d(Q, R)=3\sqrt{17}+2\sqrt{17}=5\sqrt{17}=d(P, R),$ these three points are collinear.

7. Center (–2, 3), radius 15

$$(x-h)^2+(y-k)^2=r^2$$
$$[x-(-2)]^2+(y-3)^2=15^2$$
$$(x+2)^2+(y-3)^2=225$$

8. Center $(\sqrt{5},-\sqrt{7})$, radius $\sqrt{3}$

$$(x-h)^2+(y-k)^2=r^2$$
$$\left(x-\sqrt{5}\right)^2+\left[y-\left(-\sqrt{7}\right)\right]^2=\left(\sqrt{3}\right)^2$$
$$\left(x-\sqrt{5}\right)^2+\left(y+\sqrt{7}\right)^2=3$$

9. Center (–8, 1), passing through (0, 16)

The radius is the distance from the center to any point on the circle. The distance between (–8, 1) and (0, 16) is $r=\sqrt{(-8-0)^2+(1-16)^2}=\sqrt{8^2+15^2}=\sqrt{64+225}=\sqrt{289}=17.$

The equation of the circle is $[x-(-8)]^2+(y-1)^2=17^2\Rightarrow(x+8)^2+(y-1)^2=289.$

10. Center (3, –6), tangent to the x-axis

The point (3, –6) is 6 units directly below the x-axis. Any segment joining a circle's center to a point on the circle must be a radius, so in this case the length of the radius is 6 units.

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-3)^2+[y-(-6)]^2=6^2$$
$$(x-3)^2+(y+6)^2=36$$

11. $x^2-4x+y^2+6y+12=0$

Complete the square on x and y to put the equation in center-radius form.

$$\left(x^2-4x\right)+\left(y^2+6y\right)=-12$$
$$\left(x^2-4x+4\right)+\left(y^2+6y+9\right)=-12+4+9$$
$$\left(x-2\right)^2+\left(y+3\right)^2=1$$

The circle has center (2, –3) and radius 1.

12. $x^2-6x+y^2-10y+30=0$

Complete the square on x and y to put the equation in center-radius form.

$$(x^2-6x+9)+(y^2-10y+25)=-30+9+25$$
$$(x-3)^2+(y-5)^2=4$$

The circle has center (3, 5) and radius 2.

13.
$$2x^2+14x+2y^2+6y+2=0$$
$$x^2+7x+y^2+3y+1=0$$
$$\left(x^2+7x\right)+\left(y^2+3y\right)=-1$$
$$\left(x^2+7x+\tfrac{49}{4}\right)+\left(y^2+3y+\tfrac{9}{4}\right)=-1+\tfrac{49}{4}+\tfrac{9}{4}$$
$$\left(x+\tfrac{7}{2}\right)^2+\left(y+\tfrac{3}{2}\right)^2=-\tfrac{4}{4}+\tfrac{49}{4}+\tfrac{9}{4}$$
$$\left(x+\tfrac{7}{2}\right)^2+\left(y+\tfrac{3}{2}\right)^2=\tfrac{54}{4}$$

The circle has center $\left(-\frac{7}{2},-\frac{3}{2}\right)$ and radius $\sqrt{\frac{54}{4}}=\frac{\sqrt{54}}{\sqrt{4}}=\frac{\sqrt{9\cdot 6}}{\sqrt{4}}=\frac{3\sqrt{6}}{2}$.

14.
$$3x^2+33x+3y^2-15y=0$$
$$x^2+11x+y^2-5y=0$$
$$\left(x^2+11x\right)+\left(y^2-5y\right)=0$$
$$\left(x^2+11x+\tfrac{121}{4}\right)+\left(y^2-5y+\tfrac{25}{4}\right)=0+\tfrac{121}{4}+\tfrac{25}{4}$$
$$\left(x+\tfrac{11}{2}\right)^2+\left(y-\tfrac{5}{2}\right)^2=\tfrac{146}{4}$$

The circle has center $\left(-\frac{11}{2},\frac{5}{2}\right)$ and radius $\frac{\sqrt{146}}{2}$.

15. Find all possible values of *x* so that the distance between (*x*, –9) and (3, –5) is 6.
$$\sqrt{(3-x)^2+(-5+9)^2}=6$$
$$\sqrt{9-6x+x^2+16}=6$$
$$\sqrt{x^2-6x+25}=6$$
$$x^2-6x+25=36$$
$$x^2-6x-11=0$$

Apply the quadratic formula where $a=1, b=-6,$ and $c=-11$.
$$x=\frac{6\pm\sqrt{36-4(1)(-11)}}{2}=\frac{6\pm\sqrt{36+44}}{2}=\frac{6\pm\sqrt{80}}{2}=\frac{6\pm4\sqrt{5}}{2}=\frac{2(3\pm2\sqrt{5})}{2}$$
$$x=3+2\sqrt{5} \text{ or } x=3-2\sqrt{5}$$

16. Let *P*(6, *y*) be a point that is 4 units from *Q*(1, 3). Use the distance formula with *d*(*P*, *Q*) = 4.
$$4=\sqrt{(6-1)^2+(y-3)^2}$$
$$4=\sqrt{25+y^2-6y+9}$$
$$16=y^2-6y+34$$
$$0=y^2-6y+18$$

The only solutions to this equation are $y = 3 \pm 3i$. Since the equation has no real solution, no such points exist.

17. The graph of $(x-4)^2+(y+5)^2=0$ is the single point (4, –5). (One could think of it as a circle with center (4, –5) and radius zero.)

18. This is not the graph of a function because a vertical line can intersect it in two points.
domain: $(-\infty,\infty)$; range: $[0,\infty)$

19. This is not the graph of a function because a vertical line can intersect it in two points.
domain: $[-6,6]$; range: $[-6,6]$

20. This is the graph of a function. No vertical line will intersect the graph in more than one point.
domain: $(-\infty, -2]\cup[2, \infty)$; range: $[0,\infty)$

21. This is not the graph of a function because a vertical line can intersect it in two points.
domain: $(-\infty,\infty)$; range: $(-\infty, -1]\cup[1, \infty)$

22. This is the graph of a function. No vertical line will intersect the graph in more than one point.
domain: $(-\infty,\infty)$; range: $(-\infty,\infty)$

23. This is not the graph of a function because a vertical line can intersect it in two points.
domain: $[0,\infty)$; range: $(-\infty,\infty)$

24. The equation $x=\frac{1}{2}y^2$ does not define *y* as a function of *x*. For some values of *x*, there will be more than one value of *y*. For example, ordered pairs (8, 4) and (8, −4) satisfy the relation. Thus, the relation would not be a function.

25. $y=3-x^2$
Each value of *x* corresponds to exactly one value of *y*, so this equation defines a function.

26. The equation $y=-\dfrac{8}{x}$ defines *y* as a function of *x* because for every *x* in the domain, which is $(-\infty, 0)\cup(0, \infty)$, there will be exactly one value of *y*.

27. $y=\sqrt{x-7}$
For each value of *x* in the domain, which is $[7,\infty)$, there is exactly one value of *y*. Thus, this equation defines a function.

28. In the function $y=-4+|x|$, we may use any real number for *x*. The domain is $(-\infty,\infty)$.

29. $y=\dfrac{8+x}{8-x}$
x can be any real number except 8, since this will give a denominator of zero. Thus, the domain is $(-\infty, 8)\cup(8, \infty)$.

30. In the function $y=-\sqrt{\dfrac{5}{x^2+9}}$, we must have $\dfrac{5}{x^2+9}\geq 0$. Since x^2+9 is positive for any value of *x*, the quotient of 5 and x^2+9 will always be positive. So, the domain is $(-\infty,\infty)$.

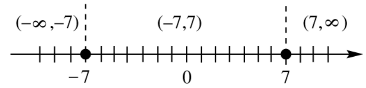
31. $y = \sqrt{49 - x^2}$

In the function $y = \sqrt{49 - x^2}$, we must have $49 - x^2 \geq 0$.

Step 1: Find the values of x that satisfy $49 - x^2 = 0$.

$$49 - x^2 = 0$$
$$(7 + x)(7 - x) = 0$$
$$7 + x = 0 \quad \text{or} \quad 7 - x = 0$$
$$x = -7 \quad \text{or} \quad 7 = x$$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $49 - x^2 \geq 0$.

Interval	Test Value	Is $49 - x^2 \geq 0$ True or False?
$(-\infty, -7)$	-8	$49 - (-8)^2 \geq 0$? $-15 \geq 0$ False
$(-7, 7)$	0	$49 - 0^2 \geq 0$? $49 \geq 0$ True
$(7, \infty)$	8	$49 - 8^2 \geq 0$? $-15 \geq 0$ False

Solving this inequality, we obtain a solution interval of [–7, 7].

32. **(a)** As x is getting larger on the interval $[2, \infty)$, the value of y is increasing.

(b) As x is getting larger on the interval $(-\infty, -2]$, the value of y is decreasing.

33. We need to consider the solid dot. Thus, $f(0) = 0$.

34. $f(x) = -2x^2 + 3x - 6 \Rightarrow f(0) = -2 \cdot 0^2 + 3 \cdot 0 - 6 = -2 \cdot 0 + 3 \cdot 0 - 6 = 0 + 0 - 6 = -6$

35. $f(x) = -2x^2 + 3x - 6 \Rightarrow f(2.1) = -2 \cdot 2.1^2 + 3 \cdot 2.1 - 6 = -2 \cdot 4.41 + 3 \cdot 2.1 - 6 = -8.82 + 6.3 - 6 = -8.52$

36. $f(x) = -2x^2 + 3x - 6 \Rightarrow f\left(-\frac{1}{2}\right) = -2\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) - 6 = -2\left(\frac{1}{4}\right) + 3\left(-\frac{1}{2}\right) - 6 = -\frac{1}{2} - \frac{3}{2} - 6 = -8$

37. $f(x) = -2x^2 + 3x - 6 \Rightarrow f(k) = -2k^2 + 3k - 6$

38. $3x + 7y = 14$

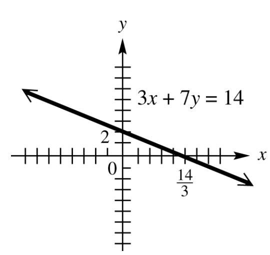
$$7y = -3x + 14$$
$$y = -\frac{3}{7}x + 2$$

The graph is the line with slope of $-\frac{3}{7}$ and y-intercept 2.

It may also be graphed using intercepts. To do this, locate the x-intercept.

x-intercept: $y = 0$

$3x + 7(0) = 14 \Rightarrow 3x = 14 \Rightarrow x = \frac{14}{3}$



39. $2x - 5y = 5$

$-5y = -2x + 5$

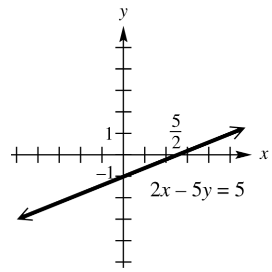
$y = \frac{2}{5}x - 1$

The graph is the line with slope $\frac{2}{5}$ and y-intercept -1.

It may also be graphed using intercepts. To do this, locate the x-intercept.

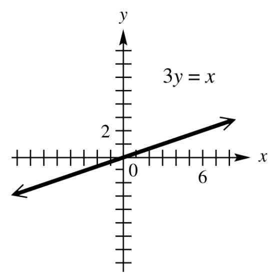
x-intercept: $y = 0$

$2x - 5(0) = 5 \Rightarrow 2x = 5 \Rightarrow x = \frac{5}{2}$



40. $3y = x \Rightarrow y = \frac{1}{3}x$

The graph is the line with slope $\frac{1}{3}$ and y-intercept 0, which means that it passes through the origin. Use another point such as (6, 2) to complete the graph.



41. $2x + 5y = 20$

$5y = -2x + 20$

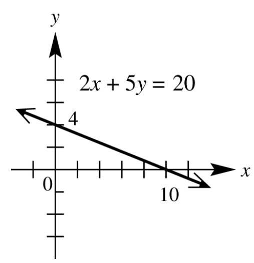
$y = -\frac{2}{5}x + 4$

The graph is the line with slope of $-\frac{2}{5}$ and y-intercept 4.

It may also be graphed using intercepts. To do this, locate the x-intercept.

x-intercept: $y = 0$

$2x + 5(0) = 20 \Rightarrow 2x = 20 \Rightarrow x = 10$



42. $x - 4y = 8$

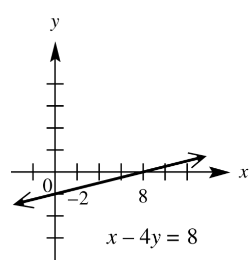
$-4y = -x + 8$

$y = \frac{1}{4}x - 2$

The graph is the line with slope $\frac{1}{4}$ and y-intercept -2.

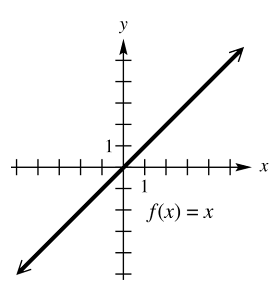
It may also be graphed using intercepts. To do this, locate the x-intercept.

x-intercept: $y = 0$ $\quad x - 4(0) = 8 \Rightarrow x = 8$

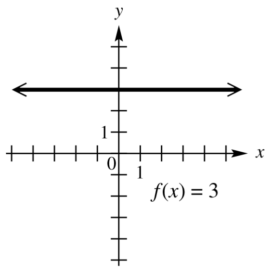


43. $f(x) = x$

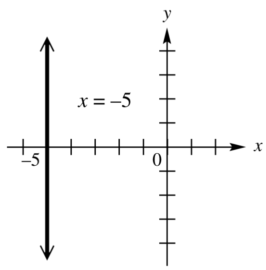
The graph is the line with slope 1 and y-intercept 0, which means that it passes through the origin. Use another point such as (1, 1) to complete the graph.



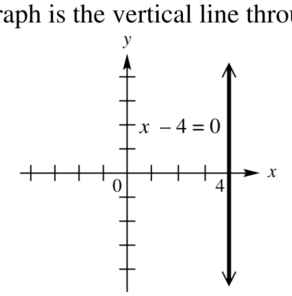
44. $f(x) = 3$
The graph is the horizontal line through (0, 3).



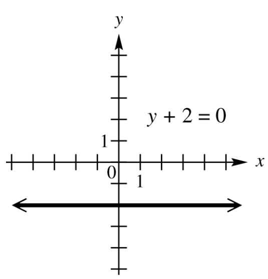
45. $x = -5$
The graph is the vertical line through (–5, 0).



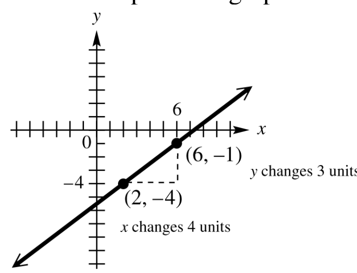
46. $x - 4 = 0 \Rightarrow x = 4$
The graph is the vertical line through (4, 0).



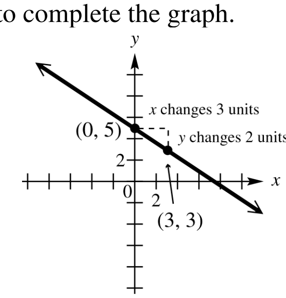
47. $y + 2 = 0 \Rightarrow y = -2$
The graph is the horizontal line through $(0, -2)$.



48. Line through (2, −4), $m = \frac{3}{4}$
First locate the point (2, −4).
Since the slope is $\frac{3}{4}$, a change of 4 units horizontally(4 units to the right) produces a change of 3 units vertically (3 units up). This gives a second point, (6, –1), which can be used to complete the graph.



49. Line through (0, 5), $m = -\frac{2}{3}$
Note that $m = -\frac{2}{3} = \frac{-2}{3}$.
Begin by locating the point (0, 5). Since the slope is $\frac{-2}{3}$, a change of 3 units horizontally (3 units to the right) produces a change of –2 units vertically (2 units down). This gives a second point, (3, 3), which can be used to complete the graph.



50. through (8, 7) and $\left(\frac{1}{2}, -2\right)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-7}{\frac{1}{2}-8} = \frac{-9}{-\frac{15}{2}}$$
$$= -9\left(-\frac{2}{15}\right) = \frac{18}{15} = \frac{6}{5}$$

51. through (2, –2) and (3, –4)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-(-2)}{3-2} = \frac{-2}{1} = -2$$

52. through (5, 6) and (5, –2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-6}{5-5} = \frac{-8}{0}$$

The slope is undefined.

53. through (0, –7) and (3, –7)

$$m = \frac{-7-(-7)}{3-0} = \frac{0}{3} = 0$$

54. $9x - 4y = 2$.
Solve for y to put the equation in slope-intercept form.

$$-4y = -9x + 2 \Rightarrow y = \tfrac{9}{4}x - \tfrac{1}{2}$$

Thus, the slope is $\frac{9}{4}$.

55. $11x + 2y = 3$
Solve for y to put the equation in slope-intercept form.

$$2y = -11x + 3 \Rightarrow y = -\tfrac{11}{2}x + \tfrac{3}{2}$$

Thus, the slope is $-\frac{11}{2}$.

56. $x - 5y = 0$.
Solve for y to put the equation in slope-intercept form.

$$-5y = -x \Rightarrow y = \tfrac{1}{5}x$$

Thus, the slope is $\frac{1}{5}$.

57. $x - 2 = 0 \Rightarrow x = 2$
The graph is a vertical line, through (2, 0). The slope is undefined.

58. **(a)** This is the graph of a function since no vertical line intersects the graph in more than one point.

(b) The lowest point on the graph occurs in December, so the most jobs lost occurred in December. The highest point on the graph occurs in January, so the most jobs gained occurred in January.

(c) The number of jobs lost in December is approximately 6000. The number of jobs gained in January is approximately 2000.

(d) It shows a slight downward trend.

59. Initially, the car is at home. After traveling for 30 mph for 1 hr, the car is 30 mi away from home. During the second hour the car travels 20 mph until it is 50 mi away. During the third hour the car travels toward home at 30 mph until it is 20 mi away. During the fourth hour the car travels away from home at 40 mph until it is 60 mi away from home. During the last hour, the car travels 60 mi at 60 mph until it arrived home.

60. We need to find the slope of a line that passes between points $(1970,\ 10{,}000)$ and $(1999, 49{,}000)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{49{,}000 - 10{,}000}{1999 - 1970} = \frac{39{,}000}{29} \approx \$1345 \text{ per year}$$

61. **(a)** We need to first find the slope of a line that passes between points $(6,\ 12.6)$ and $(11, 30.7)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30.7 - 12.6}{11 - 6} = \frac{18.1}{5} = 3.62$$

Now use the point-slope form with $(x_1,\ y_1) = (6, 12.6)$ and $m = 3.62$. (The other point, (11,30.7), could also have been used.)

$$y - 12.6 = 3.62(x - 6) \Rightarrow y - 12.6 = 3.62x - 21.72 \Rightarrow y = 3.62x - 9.12$$

The slope, 3.62, indicates that the number of e-filing taxpayers increased by 3.62% each year from 1996 to 2001.

(b) For 2005, we evaluate the function for $x = 15$.

$$y = 3.62(15) - 9.12 = 54.3 - 9.12 = 45.18$$

45.18% of the tax returns are predicted to be filed electronically.

62. through (–2, 4) and (1, 3)
First find the slope.

$$m = \frac{3-4}{1-(-2)} = \frac{-1}{3}$$

Now use the point-slope form with $(x_1, y_1) = (1, 3)$ and $m = -\frac{1}{3}$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3 &= -\tfrac{1}{3}(x-1) \\ 3(y-3) &= -1(x-1) \\ 3y - 9 &= -x + 1 \\ 3y &= -x + 10 \\ y &= -\tfrac{1}{3}x + \tfrac{10}{3} \end{aligned}$$

63. through (3, –5) with slope –2
Use the point-slope form.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-5) &= -2(x-3) \\ y + 5 &= -2(x-3) \\ y + 5 &= -2x + 6 \\ y &= -2x + 1 \end{aligned}$$

64. x-intercept –3, y-intercept 5
Two points of the line are (–3, 0) and (0, 5). First, find the slope.

$$m = \frac{5-0}{0+3} = \frac{5}{3}$$

The slope is $\frac{5}{3}$ and the y-intercept is 5.

Write the equation in slope-intercept form.

$$y = \tfrac{5}{3}x + 5$$

65. through (2, –1) parallel to $3x - y = 1$
Find the slope of $3x - y = 1$.

$$\begin{aligned} 3x - y &= 1 \\ -y &= -3x + 1 \\ y &= 3x - 1 \end{aligned}$$

The slope of this line is 3. Since parallel lines have the same slope, 3 is also the slope of the line whose equation is to be found.

Now use the point-slope form with $(x_1, y_1) = (2, -1)$ and $m = 3$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-1) &= 3(x-2) \\ y + 1 &= 3x - 6 \\ y &= 3x - 7 \end{aligned}$$

66. through (0, 5), perpendicular to $8x + 5y = 3$
Find the slope of $8x + 5y = 3$.

$$8x + 5y = 3 \Rightarrow 5y = -8x + 3 \Rightarrow y = -\tfrac{8}{5}x + \tfrac{3}{5}$$

The slope of this line is $-\frac{8}{5}$. The slope of any line perpendicular to this line is $\frac{5}{8}$, since $-\frac{8}{5}\left(\frac{5}{8}\right) = -1$.

The equation in slope-intercept form with slope $\frac{5}{8}$ and y-intercept 5 is $y = \frac{5}{8}x + 5$.

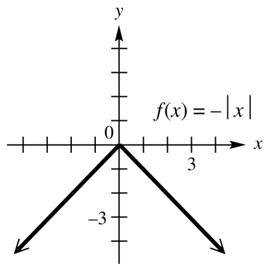
67. through (2, –10), perpendicular to a line with an undefined slope
A line with an undefined slope is a vertical line. Any line perpendicular to a vertical line is a horizontal line, with an equation of the form $y = b$. Since the line passes through (2, –10), the equation of the line is $y = -10$.

68. through (3, –5), parallel to $y = 4$
This will be a horizontal line through (3, –5). Since y has the same value at all points on the line, the equation is $y = -5$.

69. through (–7, 4), perpendicular to $y = 8$
The line $y = 8$ is a horizontal line, so any line perpendicular to it will be a vertical line. Since x has the same value at all points on the line, the equation is $x = -7$.

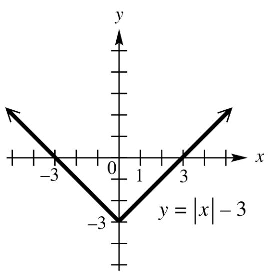
70. $f(x) = -|x|$

The graph of $f(x) = -|x|$ is the reflection of the graph of $y = |x|$ about the x-axis.



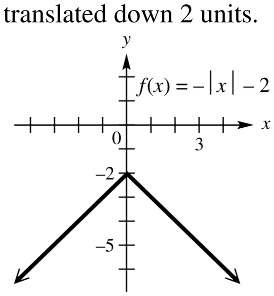
71. $f(x) = |x| - 3$

The graph is the same as that of $y = |x|$, except that it is translated 3 units downward.



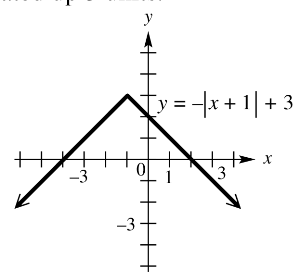
72. $f(x) = -|x| - 2$

The graph of $f(x) = -|x| - 2$ is the reflection of the graph of $y = |x|$ about the x-axis, translated down 2 units.



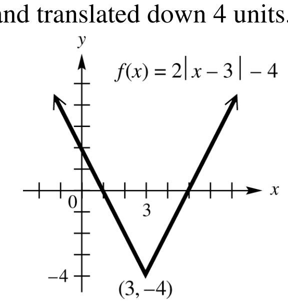
73. $f(x) = -|x+1| + 3 = -|x-(-1)| + 3$

The graph of $f(x) = -|x+1| + 3$ is a translation of the graph of $y = |x|$ to the left 1 unit, reflected over the x-axis and translated up 3 units.



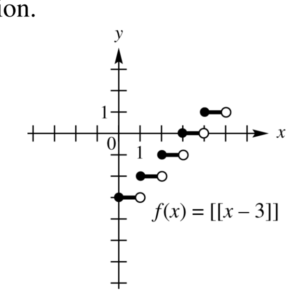
74. $f(x) = 2|x-3| - 4$

The graph of $f(x) = 2|x-3| - 4$ is a translation of the graph of $y = |x|$ to the right 3 units, stretched vertically by a factor of 2, and translated down 4 units.



75. $f(x) = [\![x-3]\!]$

To get $y = 0$, we need $0 \le x - 3 < 1 \Rightarrow$ $3 \le x < 4$.
To get $y = 1$, we need $1 \le x - 3 < 2 \Rightarrow$ $4 \le x < 5$.
Follow this pattern to graph the step function.

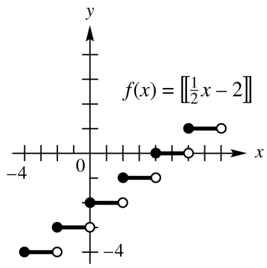


76. $f(x)=[\![\frac{1}{2}x-2]\!]$

For y to be 0, we need $0\le\frac{1}{2}x-2<1\Rightarrow$

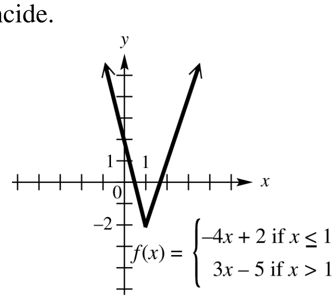
$2\le\frac{1}{2}x<3\Rightarrow 4\le x<6.$

Follow this pattern to graph the step function.



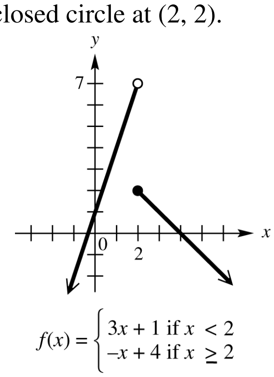
77. $f(x)=\begin{cases}-4x+2 & \text{if } x\le 1\\ 3x-5 & \text{if } x>1\end{cases}$

Draw the graph of $y = -4x + 2$ to the left of $x = 1$, including the endpoint at $x = 1$. Draw the graph of $y = 3x - 5$ to the right of $x = 1$, but do not include the endpoint at $x = 1$. Observe that the endpoints of the two pieces coincide.



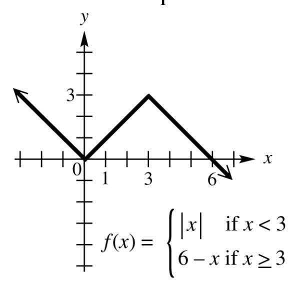
78. $f(x)=\begin{cases}3x+1 & \text{if } x<2\\ -x+4 & \text{if } x\ge 2\end{cases}$

Graph the line $y = 3x + 1$ to the left of $x = 2$, and graph the line $y = -x + 4$ to the right of $x = 2$. The graph has an open circle at (2, 7) and a closed circle at (2, 2).



79. $f(x)=\begin{cases}|x| & \text{if } x<3\\ 6-x & \text{if } x\ge 3\end{cases}$

Draw the graph of $y=|x|$ to the left of $x = 3$, but do not include the endpoint. Draw the graph of $y = 6 - x$ to the right of $x = 3$, including the endpoint. Observe that the endpoints of the two pieces coincide.



80. The graph of a nonzero function cannot be symmetric with respect to the x-axis.
Such a graph would fail the vertical line test, so the statement is true.

81. The graph of an even function is symmetric with respect to the y-axis.
This statement is true.

82. The graph of an odd function is symmetric with respect to the origin.
This statement is true.

83. If (a, b) is on the graph of an even function, so is $(a, -b)$.
The statement is false. For example, $f(x)=x^2$ is even, and (2, 4) is on the graph but (2, −4) is not.

84. If (a, b) is on the graph of an odd function, so is $(-a, b)$.
This statement is false. For example, $f(x)=x^3$ is odd, and (2, 8) is on the graph but (−2, 8) is not.

85. The constant function $f(x)=0$ is both even and odd.
Since $f(-x)=0=f(x)$, the function is even. Also since $f(-x)=0=-0=-f(x)$, the function is odd.
This statement is true.

86. $3y^2 - 5x^2 = 15$

Replace *x* with –*x* to obtain $3y^2 - 5(-x)^2 = 15 \Rightarrow 3y^2 - 5x^2 = 15$.
The result is the same as the original equation, so the graph is symmetric with respect to the *y*-axis.

Replace *y* with –*y* to obtain $3(-y)^2 - 5x^2 = 15 \Rightarrow 3y^2 - 5x^2 = 15$.
The result is the same as the original equation, so the graph is symmetric with respect to the *x*-axis.

Since the graph is symmetric with respect to the *y*-axis and *x*-axis, it must also be symmetric with respect to the origin.

87. $x + y^2 = 8$

Replace *x* with –*x* to obtain $(-x) + y^2 = 8$.
The result is not the same as the original equation, so the graph is not symmetric with respect to the *y*-axis.

Replace *y* with –*y* to obtain $x + (-y)^2 = 8 \Rightarrow x + y^2 = 8$.
The result is the same as the original equation, so the graph is symmetric with respect to the *x*-axis.

Replace *x* with –*x* and *y* with –*y* to obtain $(-x) + (-y)^2 = 8 \Rightarrow (-x) + y^2 = 8$.
The result is not the same as the original equation, so the graph is not symmetric with respect to the origin.

The graph is symmetric with respect to the *x*-axis only.

88. $y^3 = x + 1$

Replace *x* with –*x* to obtain $y^3 = -x + 1$.
The result is not the same as the original equation, so the graph is not symmetric with respect to the *y*-axis.

Replace *y* with –*y* to obtain $(-y)^3 = x + 1 \Rightarrow -y^3 = x + 1 \Rightarrow y^3 = -x - 1$
The result is not the same as the original equation, so the graph is not symmetric with respect to the *x*-axis.

Replace *x* with –*x* and *y* with –*y* to obtain $(-y)^3 = (-x) + 1 \Rightarrow -y^3 = -x + 1 \Rightarrow y^3 = x - 1$.
The result is not the same as the original equation, so the graph is not symmetric with respect to the origin.

Therefore, the graph has none of the listed symmetries.

89. $x^2 = y^3$

Replace *x* with –*x* to obtain $(-x)^2 = y^3 \Rightarrow x^2 = y^3$.
The result is the same as the original equation, so the graph is symmetric with respect to the *y*-axis.

Replace *y* with –*y* to obtain $x^2 = (-y)^3 \Rightarrow x^2 = -y^3$.
The result is not the same as the original equation, so the graph is not symmetric with respect to the *x*-axis.

Replace *x* with –*x* and *y* with –*y* to obtain $(-x)^2 = (-y)^3 \Rightarrow x^2 = -y^3$.
The result is not the same as the original equation, so the graph is not symmetric with respect to the origin.

Therefore, the graph is symmetric with respect to the *y*-axis only.

90. $|y| = -x$

Replace x with $-x$ to obtain $|y| = -(-x) \Rightarrow |y| = x$.

The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis.

Replace y with $-y$ to obtain $|-y| = -x \Rightarrow |y| = -x$.

The result is the same as the original equation, so the graph is symmetric with respect to the x-axis.

Replace x with $-x$ and y with $-y$ to obtain $|-y| = -(-x) \Rightarrow |y| = x$.

The result is not the same as the original equation, so the graph is not symmetric with respect to the origin.

Therefore, the graph is symmetric with respect to the x-axis only.

91. $|x+2| = |y-3|$

Replace x with $-x$ to obtain $|-x+2| = |y-3|$

Since $|-x+2| \neq |x+2|$, the graph is not symmetric with respect to the y-axis.

Replace y with $-y$ to obtain $|x+2| = |-y-3|$.

Since $|-y-3| \neq |y-3|$, the graph is not symmetric with respect to the x-axis.

Replace x with $-x$ and y with $-y$ to obtain $|-x+2| = |-y-3|$.

This equation is not equivalent to the original one, so the graph is not symmetric with respect to the origin.

Therefore, the graph has none of the listed symmetries.

92. $|x| = |y|$

Replace x with $-x$ to obtain $|-x| = |y| \Rightarrow |x| = |y|$.

The result is the same as the original equation, so the graph is symmetric with respect to the y-axis.

Replace y with $-y$ to obtain $|x| = |-y| \Rightarrow |x| = |y|$.

The result is the same as s the original equation, so the graph is symmetric with respect to the x-axis.

Since the graph is symmetric with respect to the x-axis and with respect to the y-axis, it must also by symmetric with respect to the origin.

93. To obtain the graph of $g(x) = -|x|$, reflect the graph of $f(x) = |x|$ across the x-axis.

94. To obtain the graph of $h(x) = |x| - 2$, translate the graph of $f(x) = |x|$ down 2 units.

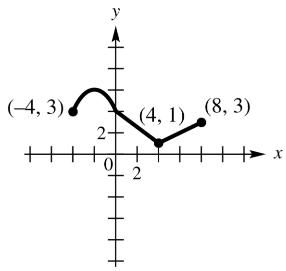
95. To obtain the graph of $k(x) = 2|x-4|$, translate the graph of $f(x) = |x|$ to the right 4 units and stretch vertically by a factor of 2.

96. If the graph of $f(x) = 3x - 4$ is reflected about the x-axis, we obtain a graph whose equation is $y = -(3x-4) = -3x+4$.

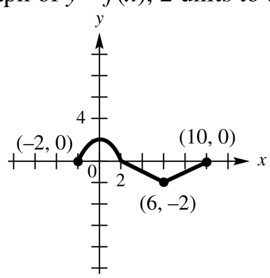
97. If the graph of $f(x) = 3x - 4$ is reflected about the y-axis, we obtain a graph whose equation is $y = f(-x) = 3(-x) - 4 = -3x - 4$.

98. If the graph of $f(x) = 3x - 4$ is reflected about the origin, every point (x, y) will be replaced by the point $(-x, -y)$. The equation for the graph will change from $y = 3x - 4$ to $-y = 3(-x) - 4 \Rightarrow -y = -3x - 4 \Rightarrow y = 3x + 4$.

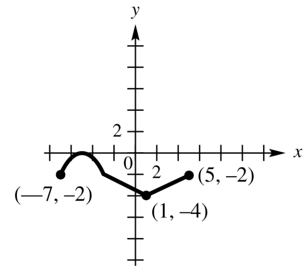
99. **(a)** To graph $y = f(x) + 3$, translate the graph of $y = f(x)$, 3 units up.



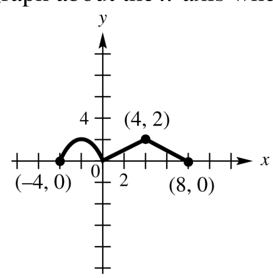
(b) To graph $y = f(x - 2)$, translate the graph of $y = f(x)$, 2 units to the right.



(c) To graph $y = f(x + 3) - 2$, translate the graph of $y = f(x)$, 3 units to the left and 2 units down.



(d) To graph $y = |f(x)|$, keep the graph of $y = f(x)$ as it is where $y \geq 0$ and reflect the graph about the x-axis where $y < 0$.



For Exercises 100–108, $f(x) = 3x^2 - 4$ and $g(x) = x^2 - 3x - 4$.

100. $(f + g)(x) = f(x) + g(x) = (3x^2 - 4) + (x^2 - 3x - 4) = 4x^2 - 3x - 8$

101.
$$\begin{aligned}(fg)(x) &= f(x) \cdot g(x)\\ &= (3x^2 - 4)(x^2 - 3x - 4)\\ &= 3x^4 - 9x^3 - 12x^2 - 4x^2 + 12x + 16\\ &= 3x^4 - 9x^3 - 16x^2 + 12x + 16\end{aligned}$$

102.
$$\begin{aligned}(f - g)(4) &= f(4) - g(4)\\ &= (3 \cdot 4^2 - 4) - (4^2 - 3 \cdot 4 - 4)\\ &= (3 \cdot 16 - 4) - (16 - 3 \cdot 4 - 4)\\ &= (48 - 4) - (16 - 12 - 4)\\ &= 44 - 0 = 44\end{aligned}$$

103.
$$\begin{aligned}(f + g)(-4) &= f(-4) + g(-4) = [3(-4)^2 - 4] + [(-4)^2 - 3(-4) - 4]\\ &= [3(16) - 4] + [16 - 3(-4) - 4] = [48 - 4] + [16 + 12 - 4] = 44 + 24 = 68\end{aligned}$$

104.
$$\begin{aligned}(f + g)(2k) &= f(2k) + g(2k) = [3(2k)^2 - 4] + [(2k)^2 - 3(2k) - 4]\\ &= [3(4)k^2 - 4] + [4k^2 - 3(2k) - 4] = (12k^2 - 4) + (4k^2 - 6k - 4) = 16k^2 - 6k - 8\end{aligned}$$

105. $$\left(\frac{f}{g}\right)(3) = \frac{f(3)}{g(3)} = \frac{3 \cdot 3^2 - 4}{3^2 - 3 \cdot 3 - 4} = \frac{3 \cdot 9 - 4}{9 - 3 \cdot 3 - 4} = \frac{27 - 4}{9 - 9 - 4} = \frac{23}{-4} = -\frac{23}{4}$$

106. $\left(\dfrac{f}{g}\right)(-1)=\dfrac{3(-1)^2-4}{(-1)^2-3(-1)-4}=\dfrac{3(1)-4}{1-3(-1)-4}=\dfrac{3-4}{1+3-4}=\dfrac{-1}{0}=\text{undefined}$

107. The domain of $(fg)(x)$ is the intersection of the domain of $f(x)$ and the domain of $g(x)$. Both have domain $(-\infty,\infty)$, so the domain of $(fg)(x)$ is $(-\infty,\infty)$.

108. $\left(\dfrac{f}{g}\right)(x)=\dfrac{3x^2-4}{x^2-3x-4}=\dfrac{3x^2-4}{(x+1)(x-4)}$

Since both $f(x)$ and $g(x)$ have domain $(-\infty,\infty)$, we are concerned about values of x that make $g(x)=0$. Thus, the expression is undefined if $(x+1)(x-4)=0$, that is, if $x=-1$ or $x=4$. Thus, the domain is the set of all real numbers except $x=-1$ and $x=4$, or $(-\infty,-1)\cup(-1,4)\cup(4,\infty)$.

109. $f(x)=\dfrac{1}{x},\ g(x)=x^2+1$

Since $(f\circ g)(x)=f[g(x)]$ and $(f\circ g)(x)=\dfrac{1}{x^2+1}$, choices (C) and (D) are not equal to $(f\circ g)(x)$.

110. $f(x)=2x+9$

$f(x+h)=2(x+h)+9=2x+2h+9$

$f(x+h)-f(x)=(2x+2h+9)-(2x+9)=2x+2h+9-2x-9=2h$

Thus, $\dfrac{f(x+h)-f(x)}{h}=\dfrac{2h}{h}=2.$

111. $f(x)=x^2-5x+3$

$f(x+h)=(x+h)^2-5(x+h)+3=x^2+2xh+h^2-5x-5h+3$

$f(x+h)-f(x)=(x^2+2xh+h^2-5x-5h+3)-(x^2-5x+3)$

$=x^2+2xh+h^2-5x-5h+3-x^2+5x-3=2xh+h^2-5h$

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{2xh+h^2-5h}{h}=\dfrac{h(2x+h-5)}{h}=2x+h-5$

For Exercises 112–116, $f(x)=\sqrt{x-2}$ and $g(x)=x^2$.

112. $(f\circ g)(x)=f[g(x)]=f(x^2)=\sqrt{x^2-2}$

113. $(g\circ f)(x)=g[f(x)]=g\left(\sqrt{x-2}\right)=\left(\sqrt{x-2}\right)^2=x-2$

114. Since $g(x)=x^2$, $g(-6)=(-6)^2=36$.

Therefore, $(f\circ g)(2)=f\left[g(-6)\right]=f(36)=\sqrt{36-2}=\sqrt{34}$.

115. Since $f(x)=\sqrt{x-2}$, $f(3)=\sqrt{3-2}=\sqrt{1}=1$.

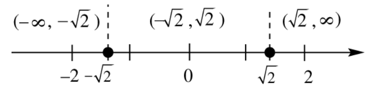
Therefore, $(g\circ f)(3)=g\left[f(3)\right]=g(1)=1^2=1$.

116. To find the domain of $f \circ g$, we must consider the domain of g as well as the composed function, $f \circ g$. Since $(f \circ g)(x) = f\left[g(x)\right] = \sqrt{x^2 - 2}$ we need to determine when $x^2 - 2 \ge 0$.

Step 1: Find the values of x that satisfy $x^2 - 2 = 0$.

$$x^2 = 2 \Rightarrow x = \pm\sqrt{2}$$

Step 2: The two numbers divide a number line into three regions.



Step 3 Choose a test value to see if it satisfies the inequality, $x^2 - 2 \ge 0$.

Interval	Test Value	Is $x^2 - 2 \ge 0$ True or False?
$\left(-\infty, -\sqrt{2}\right)$	–2	$(-2)^2 - 2 \ge 0$? $2 \ge 0$ True
$\left(-\sqrt{2}, \sqrt{2}\right)$	0	$0^2 - 2 \ge 0$? $-2 \ge 0$ False
$\left(\sqrt{2}, \infty\right)$	2	$2^2 - 2 \ge 0$? $2 \ge 0$ True

The domain of $f \circ g$ will be $\left(-\infty, -\sqrt{2}\right] \cup \left[\sqrt{2}, \infty\right)$.

117. $(f+g)(1) = f(1) + g(1) = 7 + 1 = 8$

118. $(f-g)(3) = f(3) - g(3) = 9 - 9 = 0$

119. $(fg)(-1) = f(-1) \cdot g(-1) = 3(-2) = -6$

120. $\left(\dfrac{f}{g}\right)(0) = \dfrac{f(0)}{g(0)} = \dfrac{5}{0} =$ undefined

121. $(g \circ f)(-2) = g[f(-2)] = g(1) = 2$

122. $(f \circ g)(3) = f[g(3)] = f(-2) = 1$

123. $(f \circ g)(2) = f[g(2)] = f(2) = 1$

124. $(g \circ f)(3) = g[f(3)] = g(4) = 8$

125. Let x = number of yards.
$f(x) = 36x$, where $f(x)$ is the number of inches.
$g(x) = 1760x$, where $g(x)$ is the number of miles.
Then $(g \circ f)(x) = g\left[f(x)\right] = 1760(36x) = 63,360x$.
There are $63,360x$ inches in x miles.

126. Use the definition for the perimeter of a rectangle.
P = length + width + length + width
$P(x) = 2x + x + 2x + x$
$P(x) = 6x$
This is a linear function.

127. If $V(r) = \frac{4}{3}\pi r^3$ and if the radius is increased by 3 inches, then the amount of volume gained is given by $V_g(r) = V(r+3) - V(r) = \frac{4}{3}\pi(r+3)^3 - \frac{4}{3}\pi r^3$.

128. **(a)** $V = \pi r^2 h$

If d is the diameter of its top, then $h = d$ and $r = \frac{d}{2}$. So, $V(d) = \pi\left(\frac{d}{2}\right)^2(d) = \pi\left(\frac{d^2}{4}\right)(d) = \frac{\pi d^3}{4}$.

(b) $S = 2\pi r^2 + 2\pi rh \Rightarrow S(d) = 2\pi\left(\frac{d}{2}\right)^2 + 2\pi\left(\frac{d}{2}\right)(d) = \frac{\pi d^2}{2} + \pi d^2 = \frac{\pi d^2}{2} + \frac{2\pi d^2}{2} = \frac{3\pi d^2}{2}$

Chapter 2: Test

1. **(a)** The domain of $f(x)=\sqrt{x}+3$ occurs when $x \ge 0$. In interval notation, this correlates to the interval in D, $[0,\infty)$.

(b) The range of $f(x)=\sqrt{x-3}$ is all real numbers greater than or equal to 0. In interval notation, this correlates to the interval in D, $[0,\infty)$.

(c) The domain of $f(x)=x^2-3$ is all real numbers. In interval notation, this correlates to the interval in C, $(-\infty,\infty)$.

(d) The range of $f(x)=x^2+3$ is all real numbers greater than or equal to 3. In interval notation, this correlates to the interval in B, $[3,\infty)$.

(e) The domain of $f(x)=\sqrt[3]{x-3}$ is all real numbers. In interval notation, this correlates to the interval in C, $(-\infty,\infty)$.

(f) The range of $f(x)=\sqrt[3]{x}+3$ is all real numbers. In interval notation, this correlates to the interval in C, $(-\infty,\infty)$.

(g) The domain of $f(x)=|x|-3$ is all real numbers. In interval notation, this correlates to the interval in C, $(-\infty,\infty)$.

(h) The range of $f(x)=|x+3|$ is all real numbers greater than or equal to 0. In interval notation, this correlates to the interval in D, $[0,\infty)$.

(i) The domain of $x=y^2$ is $x \ge 0$ since when you square any value of *y*, the outcome will be nonnegative. In interval notation, this correlates to the interval in D, $[0,\infty)$.

(j) The range of $x=y^2$ is all real numbers. In interval notation, this correlates to the interval in C, $(-\infty,\infty)$.

2. Consider the points $(-2,1)$ and $(3,4)$.

$$m=\frac{4-1}{3-(-2)}=\frac{3}{5}$$

3. We label the points $A(-2,1)$ and $B(3,4)$.

$$\begin{aligned} d(A,B)&=\sqrt{[3-(-2)]^2+(4-1)^2}\\ &=\sqrt{5^2+3^2}\\ &=\sqrt{25+9}\\ &=\sqrt{34} \end{aligned}$$

4. The midpoint has coordinates

$$\left(\frac{-2+3}{2},\frac{1+4}{2}\right)=\left(\frac{1}{2},\frac{5}{2}\right).$$

5. Use the point-slope form with $(x_1, y_1)=(-2,1)$ and $m=\frac{3}{5}$.

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-1&=\tfrac{3}{5}[x-(-2)]\\ y-1&=\tfrac{3}{5}(x+2)\\ 5(y-1)&=3(x+2)\\ 5y-5&=3x+6\\ 5y&=3x+11\\ -3x+5y&=11\\ 3x-5y&=-11 \end{aligned}$$

6. Solve $3x-5y=-11$ for *y*.

$$\begin{aligned} 3x-5y&=-11\\ -5y&=-3x-11\\ y&=\tfrac{3}{5}x+\tfrac{11}{5} \end{aligned}$$

Therefore, the linear function is $f(x)=\frac{3}{5}x+\frac{11}{5}$.

7. **(a)** This is not the graph of a function because some vertical lines intersect it in more than one point. The domain of the relation is [0, 4]. The range is [−4, 4].

(b) This is the graph of a function because no vertical line intersects the graph in more than one point. The domain of the function is $(-\infty, -1) \cup (-1, \infty)$. The range is $(-\infty, 0) \cup (0, \infty)$. As x is getting larger on the intervals $(-\infty, -1)$ and $(-1, \infty)$, the value of y is decreasing, so the function is decreasing on these intervals. (The function is never increasing or constant.)

8. Point A has coordinates (5, –3).

(a) The equation of a vertical line through A is $x = 5$.

(b) The equation of a horizontal line through A is $y = -3$.

9. The slope of the graph of $y = -3x + 2$ is –3.

(a) A line parallel to the graph of $y = -3x + 2$ has a slope of –3. Use the point-slope form with $(x_1, y_1) = (2,3)$ and $m = -3$.

$$y - y_1 = m(x - x_1)$$
$$y - 3 = -3(x - 2)$$
$$y - 3 = -3x + 6$$
$$y = -3x + 9$$

(b) A line perpendicular to the graph of $y = -3x + 2$ has a slope of $\frac{1}{3}$ since $-3\left(\frac{1}{3}\right) = -1$.

$$y - 3 = \tfrac{1}{3}(x - 2)$$
$$3(y - 3) = x - 2$$
$$3y - 9 = x - 2$$
$$3y = x + 7$$
$$y = \tfrac{1}{3}x + \tfrac{7}{3}$$

10. **(a)** $(-\infty, -3)$

(b) $(4, \infty)$

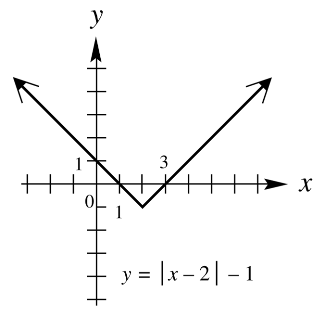
(c) $[-3, 4]$

(d) $(-\infty, -3)$; $[-3, 4]$; $(4, \infty)$

(e) $(-\infty, \infty)$

(f) $(-\infty, 2)$

11. To graph $y = |x - 2| - 1$, we translate the graph of $y = |x|$, 2 units to the right and 1 unit down.

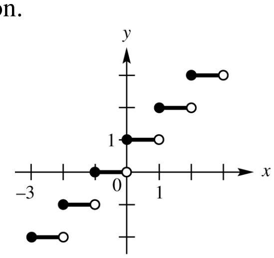


12. $f(x) = [\![x + 1]\!]$

To get $y = 0$, we need $0 \le x + 1 < 1 \Rightarrow -1 \le x < 0$.

To get $y = 1$, we need $1 \le x + 1 < 2 \Rightarrow 0 \le x < 1$.

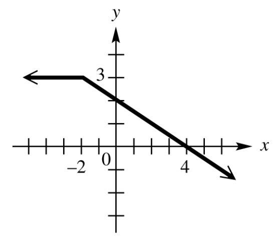
Follow this pattern to graph the step function.



$f(x) = [\![x + 1]\!]$

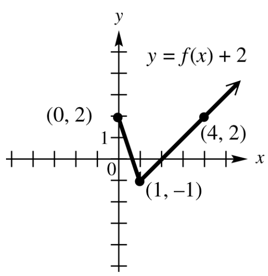
13. $f(x)=\begin{cases}3 & \text{if } x<-2\\ 2-\frac{1}{2}x & \text{if } x\ge -2\end{cases}$

For values of x with $x < -2$, we graph the horizontal line $y = 3$. For values of x with $x \ge -2$, we graph the line with a slope of $-\frac{1}{2}$ and a y-intercept of 2. Two points on this line are $(-2, 3)$ and $(0, 2)$.

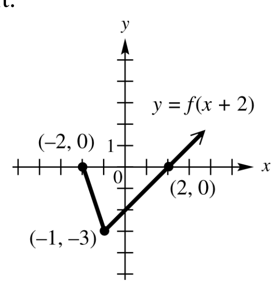


$f(x)=\begin{cases}3 & \text{if } x<-2\\ 2-\frac{1}{2}x & \text{if } x\ge -2\end{cases}$

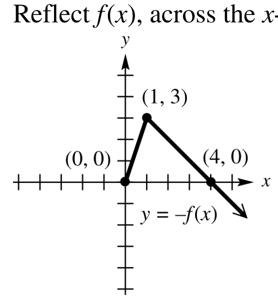
14. **(a)** Shift $f(x)$, 2 units vertically upward.



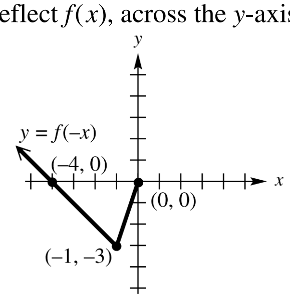
(b) Shift $f(x)$, 2 units horizontally to the left.



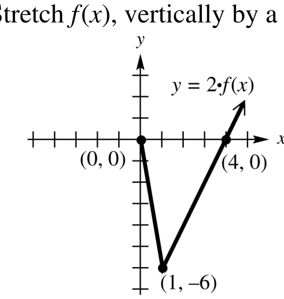
(c) Reflect $f(x)$, across the x-axis.



(d) Reflect $f(x)$, across the y-axis.



(e) Stretch $f(x)$, vertically by a factor of 2.



15. Answers will vary.

Starting with $y=\sqrt{x}$, we shift it to the left 2 units and stretch it vertically by a factor of 2. The graph is then reflected over the x-axis and then shifted down 3 units.

16. $3x^2 - y^2 = 3$

(a) Replace *y* with –*y* to obtain $3x^2 - (-y)^2 = 3 \Rightarrow 3x^2 - y^2 = 3.$
The result is the same as the original equation, so the graph is symmetric with respect to the *x*-axis.

(b) Replace *x* with –*x* to obtain $3(-x)^2 - y^2 = 3 \Rightarrow 3x^2 - y^2 = 3.$
The result is the same as the original equation, so the graph is symmetric with respect to the *y*-axis.

(c) Since the graph is symmetric with respect to the *x*-axis and with respect to the *y*-axis, it must also be symmetric with respect to the origin.

17. $f(x) = 2x^2 - 3x + 2,\ g(x) = -2x + 1$

(a) $(f - g)(x) = f(x) - g(x) = \left(2x^2 - 3x + 2\right) - \left(-2x + 1\right) = 2x^2 - 3x + 2 + 2x - 1 = 2x^2 - x + 1$

(b) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x^2 - 3x + 2}{-2x + 1}$

(c) We must determine which values solve the equation $-2x + 1 = 0.$

$$-2x + 1 = 0 \Rightarrow -2x = -1 \Rightarrow x = \tfrac{1}{2}$$

Thus, $\frac{1}{2}$ is excluded from the domain, and the domain is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$. .

(d) $f(x) = 2x^2 - 3x + 2$

$$f\left(x + h\right) = 2\left(x + h\right)^2 - 3\left(x + h\right) + 2$$
$$= 2\left(x^2 + 2xh + h^2\right) - 3x - 3h + 2 = 2x^2 + 4xh + 2h^2 - 3x - 3h + 2$$

$$f(x + h) - f(x) = (2x^2 + 4xh + 2h^2 - 3x - 3h + 2) - (2x^2 - 3x + 2)$$
$$= 2x^2 + 4xh + 2h^2 - 3x - 3h + 2 - 2x^2 + 3x - 2 = 4xh + 2h^2 - 3h$$

$$\frac{f(x + h) - f(x)}{h} = \frac{4xh + 2h^2 - 3h}{h} = \frac{h(4x + 2h - 3)}{h} = 4x + 2h - 3$$

18. (a) $(f + g)(1) = f(1) + g(1) = (2 \cdot 1^2 - 3 \cdot 1 + 2) + (-2 \cdot 1 + 1)$
$= (2 \cdot 1 - 3 \cdot 1 + 2) + (-2 \cdot 1 + 1) = (2 - 3 + 2) + (-2 + 1) = 1 + (-1) = 0$

(b) $(fg)(2) = f(2) \cdot g(2) = (2 \cdot 2^2 - 3 \cdot 2 + 2) \cdot (-2 \cdot 2 + 1)$
$= (2 \cdot 4 - 3 \cdot 2 + 2) \cdot (-2 \cdot 2 + 1) = (8 - 6 + 2) \cdot (-4 + 1) = 4(-3) = -12$

(c) Since $g\left(x\right) = -2x + 1,\ g\left(0\right) = -2\left(0\right) + 1 = 0 + 1 = 1.$
Therefore, $\left(f \circ g\right)\left(0\right) = f\left[g\left(0\right)\right] = f\left(1\right) = 2 \cdot 1^2 - 3 \cdot 1 + 2 = 2 \cdot 1 - 3 \cdot 1 + 2 = 2 - 3 + 2 = 1.$

19. $f(x) = .4[\![x]\!] + .75 \Rightarrow f(5.5) = .4[\![5.5]\!] + .75 = .4(5) + .75 = 2 + .75 = \$2.75$

20. **(a)** $C(x) = 3300 + 4.50x$

(b) $R(x) = 10.50x$

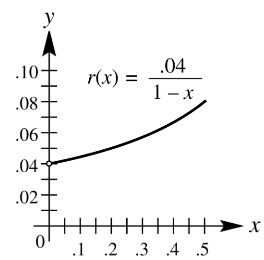
(c) $P(x) = R(x) - C(x) = 10.50x - (3300 + 4.50x) = 6.00x - 3300$

(d)
$$\begin{aligned} P(x) &> 0 \\ 6.00x - 3300 &> 0 \\ 6.00x &> 3300 \\ x &> 550 \end{aligned}$$

He must produce and sell 551 items before he earns a profit.

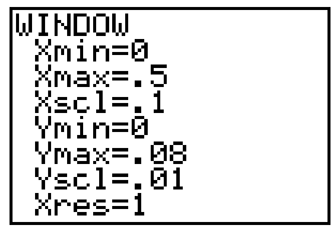
Chapter 2: Quantitative Reasoning

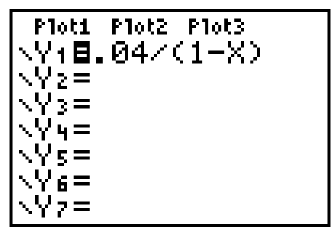
1.

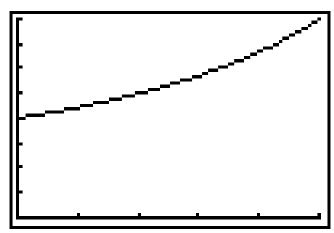


x	$r(x) = \frac{.04}{1-x}$
.1	$r(.1) = \frac{.04}{1-.1} = \frac{.04}{.9} \approx .044$
.2	$r(.2) = \frac{.04}{1-.2} = \frac{.04}{.8} = .05$
.3	$r(.3) = \frac{.04}{1-.3} = \frac{.04}{.7} \approx .057$
.4	$r(.4) = \frac{.04}{1-.4} = \frac{.04}{.6} \approx .067$
.5	$r(.5) = \frac{.04}{1-.5} = \frac{.04}{.5} = .08$

Using the graphing calculator, we have the following screens.







This is not a linear function because it cannot be written in the form $y = ax + b$. The $1 - x$ in the denominator prevents this. Also, when you look at the graph, it doesn't appear to form a line.

2. Evaluate $r(x) = \frac{.04}{1-x}$, where $x = .31$.

$$\begin{aligned} r(.31) &= \frac{.04}{1-.31} \\ &= \frac{.04}{.69} \\ &\approx .058 \text{ or } 5.8\% \end{aligned}$$

3. Solve $r(x) = \frac{.04}{1-x}$, where $r(x) = .0626$.

$$\begin{aligned} .0626 &= \frac{.04}{1-x} \\ .0626(1-x) &= .04 \\ .0626 - .0626x &= .04 \\ -.0626x &= -.0226 \\ x &= \tfrac{-.0226}{-.0626} \approx .36 \text{ or } 36\% \end{aligned}$$