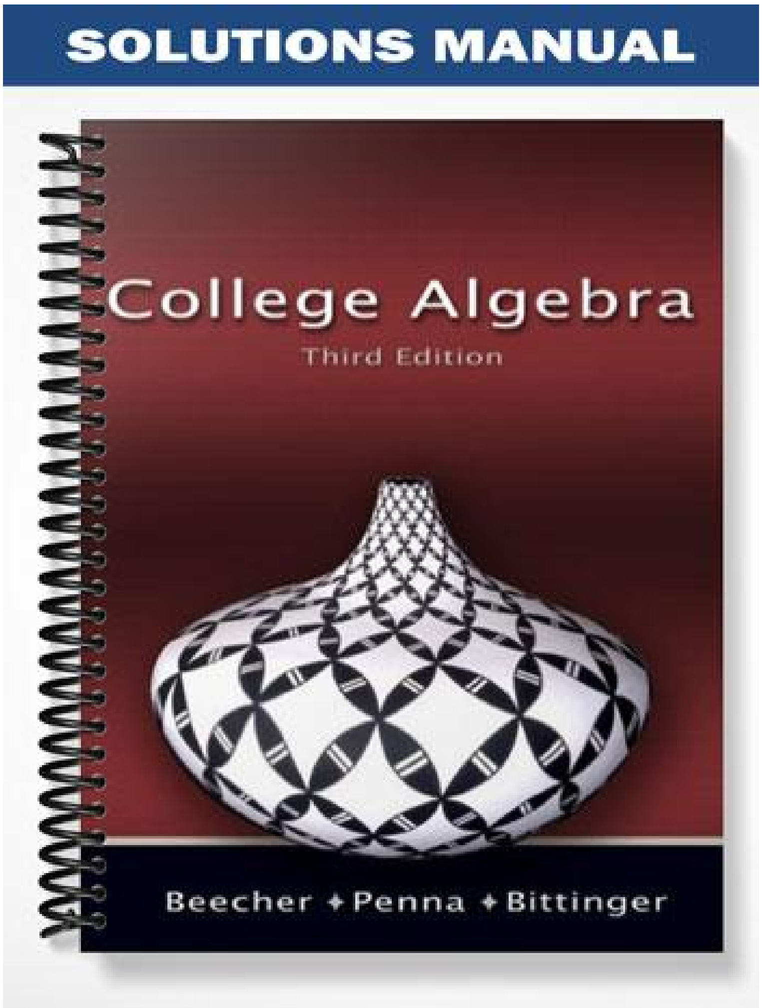

SOLUTIONS MANUAL
College Algebra
Third Edition
Beecher ✦ Penna ✦ Bittinger

Chapter 2

Functions, Equations, and Inequalities

Exercise Set 2.1

1. $4x + 5 = 21$

$4x = 16$ Subtracting 5 on both sides

$x = 4$ Dividing by 4 on both sides

The solution is 4.

2. $2y - 1 = 3$

$2y = 4$

$y = 2$

The solution is 2.

3. $4x + 3 = 0$

$4x = -3$ Subtracting 3 on both sides

$x = -\frac{3}{4}$ Dividing by 4 on both sides

The solution is $-\frac{3}{4}$.

4. $3x - 16 = 0$

$3x = 16$

$x = \frac{16}{3}$

The solution is $\frac{16}{3}$.

5. $3 - x = 12$

$-x = 9$ Subtracting 3 on both sides

$x = -9$ Multiplying (or dividing) by -1 on both sides

The solution is -9.

6. $4 - x = -5$

$-x = -9$

$x = 9$

The solution is 9.

7. $8 = 5x - 3$

$11 = 5x$ Adding 3 on both sides

$\frac{11}{5} = x$ Dividing by 5 on both sides

The solution is $\frac{11}{5}$.

8. $9 = 4x - 8$

$17 = 4x$

$\frac{17}{4} = x$

The solution is $\frac{17}{4}$.

9. $y + 1 = 2y - 7$

$1 = y - 7$ Subtracting y on both sides

$8 = y$ Adding 7 on both sides

The solution is 8.

10. $5 - 4x = x - 13$

$18 = 5x$

$\frac{18}{5} = x$

The solution is $\frac{18}{5}$.

11. $2x + 7 = x + 3$

$x + 7 = 3$ Subtracting x on both sides

$x = -4$ Subtracting 7 on both sides

The solution is -4.

12. $5x - 4 = 2x + 5$

$3x - 4 = 5$

$3x = 9$

$x = 3$

The solution is 3.

13. $3x - 5 = 2x + 1$

$x - 5 = 1$ Subtracting $2x$ on both sides

$x = 6$ Adding 5 on both sides

The solution is 6.

14. $4x + 3 = 2x - 7$

$2x = -10$

$x = -5$

The solution is -5.

15. $4x - 5 = 7x - 2$

$-5 = 3x - 2$ Subtracting $4x$ on both sides

$-3 = 3x$ Adding 2 on both sides

$-1 = x$ Dividing by 3 on both sides

The solution is -1.

16. $5x + 1 = 9x - 7$

$8 = 4x$

$2 = x$

The solution is 2.

17. $5x - 2 + 3x = 2x + 6 - 4x$

$8x - 2 = 6 - 2x$ Collecting like terms

$8x + 2x = 6 + 2$ Adding $2x$ and 2 on both sides

$10x = 8$ Collecting like terms

$x = \frac{8}{10}$ Dividing by 10 on both sides

$x = \frac{4}{5}$ Simplifying

The solution is $\frac{4}{5}$.

18. $5x - 17 - 2x = 6x - 1 - x$

$3x - 17 = 5x - 1$

$-2x = 16$

$x = -8$

The solution is -8.

19. $7(3x + 6) = 11 - (x + 2)$

$21x + 42 = 11 - x - 2$ Using the distributive property

$21x + 42 = 9 - x$ Collecting like terms

$21x + x = 9 - 42$ Adding x and subtracting 42 on both sides

$22x = -33$ Collecting like terms

$x = -\frac{33}{22}$ Dividing by 22 on both sides

$x = -\frac{3}{2}$ Simplifying

The solution is $-\frac{3}{2}$.

20. $4(5y + 3) = 3(2y - 5)$

$20y + 12 = 6y - 15$

$14y = -27$

$y = -\frac{27}{14}$

The solution is $-\frac{27}{14}$.

21. $3(x + 1) = 5 - 2(3x + 4)$

$3x + 3 = 5 - 6x - 8$ Removing parentheses

$3x + 3 = -6x - 3$ Collecting like terms

$9x + 3 = -3$ Adding $6x$

$9x = -6$ Subtracting 3

$x = -\frac{2}{3}$ Dividing by 9

The solution is $-\frac{2}{3}$.

22. $4(3x + 2) - 7 = 3(x - 2)$

$12x + 8 - 7 = 3x - 6$

$12x + 1 = 3x - 6$

$9x + 1 = -6$

$9x = -7$

$x = -\frac{7}{9}$

The solution is $-\frac{7}{9}$.

23. $2(x - 4) = 3 - 5(2x + 1)$

$2x - 8 = 3 - 10x - 5$ Using the distributive property

$2x - 8 = -10x - 2$ Collecting like terms

$12x = 6$ Adding $10x$ and 8 on both sides

$x = \frac{1}{2}$ Dividing by 12 on both sides

The solution is $\frac{1}{2}$.

24. $3(2x - 5) + 4 = 2(4x + 3)$

$6x - 15 + 4 = 8x + 6$

$6x - 11 = 8x + 6$

$-2x = 17$

$x = -\frac{17}{2}$

The solution is $-\frac{17}{2}$.

25. ***Familiarize***. Let $w =$ the wholesale sales of bottled water in 2001, in billions of dollars. An increase of 45% over this amount is $45\% \cdot w$, or $0.45w$.

Translate.

Wholesale sales in 2001	plus	increase in sales	is	Wholesale sales in 2005.
w	$+$	$0.45w$	$=$	10

Carry out. We solve the equation.

$w + 0.45w = 10$

$1.45w = 10$

$w = \frac{10}{1.45}$

$w \approx 6.9$

Check. 45% of 6.9 is 0.45(6.9) or about 3.1, and $6.9 + 3.1 = 10$. The answer checks.

State. Wholesale sales of bottled water in 2001 were about \$6.9 billion.

26. Let $d =$ the daily global demand for oil in 2005, in millions of barrels.

Solve: $d + 0.23d = 103$

$d \approx 84$ million barrels per day

27. ***Familiarize***. Let d = the average credit card debt per household in 1990.

Translate.

$$\underbrace{\text{Debt in 1990}}_{d} \quad \underset{+}{\text{plus}} \quad \underbrace{\text{additional debt}}_{6346} \quad \underset{=}{\text{is}} \quad \underbrace{\text{debt in 2004.}}_{9312}$$

Carry out. We solve the equation.

$$d + 6346 = 9312$$
$$d = 2966 \quad \text{Subtracting 6346}$$

Check. $\$2966 + \$6346 = \$9312$, so the answer checks.

State. The average credit card debt in 1990 was \$2966 per household.

28. Let n = the number of nesting pairs of bald eagles in the lower 48 states in 1963.

Solve: $n + 6649 = 7066$

$n = 417$ pairs of bald eagles

29. ***Familiarize***. Let d = the number of gigabytes of digital data stored in a typical household in 2004.

Translate.

$$\underbrace{\text{Data stored in 2004}}_{d} \quad \underset{+}{\text{plus}} \quad \underbrace{\text{additional data}}_{4024} \quad \underset{=}{\text{is}} \quad \underbrace{\text{data stored in 2010.}}_{4430}$$

Carry out. We solve the equation.

$$d + 4024 = 4430$$
$$d = 406 \quad \text{Subtracting 4024}$$

Check. $406 + 4024 = 4430$, so the answer checks.

State. In 2004, 406 GB of digital data were stored in a typical household.

30. Let s = the amount of a student's expenditure for books that goes to the college store.

Solve: $s = 0.232(501)$

$s \approx \$116.23$

31. ***Familiarize***. Let c = the average daily calorie requirement for many adults.

Translate.

$$\underbrace{\text{1560 calories}}_{1560} \quad \underset{=}{\text{is}} \quad \frac{3}{4} \quad \underset{\cdot}{\text{of}} \quad \underbrace{\text{the average daily requirement for many adults.}}_{c}$$

Carry out. We solve the equation.

$$1560 = \frac{3}{4} \cdot c$$
$$\frac{4}{3} \cdot 1560 = c \quad \text{Multiplying by } \frac{4}{3}$$
$$2080 = c$$

Check. $\frac{3}{4} \cdot 2080 = 1560$, so the answer checks.

State. The average daily calorie requirement for many adults is 2080 calories.

32. Let m = the number of calories in a Big Mac.

Solve: $m + (m + 20) = 1200$

$m = 590$, so a Big Mac has 590 calories and an order of Super-Size fries has $590 + 20$, or 610 calories.

33. ***Familiarize***. Let v = the number of ABC viewers, in millions. Then $v + 1.7$ = the number of CBS viewers and $v - 1.7$ = the number of NBC viewers.

Translate.

$$\underbrace{\text{ABC viewers}}_{v} \quad \underset{+}{\text{plus}} \quad \underbrace{\text{CBS viewers}}_{(v+1.7)} \quad \underset{+}{\text{plus}} \quad \underbrace{\text{NBC viewers}}_{(v-1.7)} \quad \underset{=}{\text{is}} \quad \underbrace{\text{total viewers.}}_{29.1}$$

Carry out.

$$v + (v + 1.7) + (v - 1.7) = 29.1$$
$$3v = 29.1$$
$$v = 9.7$$

Then $v+1.7 = 9.7+1.7 = 11.4$ and $v-1.7 = 9.7-1.7 = 8.0$.

Check. $9.7 + 11.4 + 8.0 = 29.1$, so the answer checks.

State. ABC had 9.7 million viewers, CBS had 11.4 million viewers, and NBC had 8.0 million viewers.

34. Let h = the number of households represented by the rating.

Solve: $h = 11.0(1,102,000)$

$h = 12,122,000$ households

35. ***Familiarize***. Let P = the amount Tamisha borrowed. We will use the formula $I = Prt$ to find the interest owed. For $r = 5\%$, or 0.05, and $t = 1$, we have $I = P(0.05)(1)$, or $0.05P$.

Translate.

$$\underbrace{\text{Amount borrowed}}_{P} \quad \underset{+}{\text{plus}} \quad \underset{0.05P}{\text{interest}} \quad \underset{=}{\text{is}} \quad \underset{1365}{\$1365.}$$

Carry out. We solve the equation.

$$P + 0.05P = 1365$$
$$1.05P = 1365 \quad \text{Adding}$$
$$P = 1300 \quad \text{Dividing by 1.05}$$

Check. The interest due on a loan of \$1300 for 1 year at a rate of 5% is \$1300(0.05)(1), or \$65, and \$1300 + \$65 = \$1365. The answer checks.

State. Tamisha borrowed \$1300.

36. Let P = the amount invested.

Solve: $P + 0.04P = \$1560$

$P = \$1500$

37. ***Familiarize***. Let s = Ryan's sales for the month. Then his commission is 8% of s, or $0.08s$.

Translate.

$$\underbrace{\text{Base salary}}_{1500} \quad \underset{+}{\text{plus}} \quad \underset{0.08s}{\text{commission}} \quad \underset{=}{\text{is}} \quad \underbrace{\text{total pay.}}_{2284}$$

Carry out. We solve the equation.

$$\begin{aligned} 1500 + 0.08s &= 2284 \\ 0.08s &= 784 \quad \text{Subtracting 1500} \\ s &= 9800 \end{aligned}$$

Check. 8% of \$9800, or 0.08(\$9800), is \$784 and \$1500 + \$784 = \$2284. The answer checks.

State. Ryan's sales for the month were \$9800.

38. Let s = the amount of sales for which the two choices will be equal.

Solve: $1800 = 1600 + 0.04s$

$s = \$5000$

39. ***Familiarize***. Let d = the number of miles Diego traveled in the cab.

Translate.

$$\underbrace{\text{Pickup fee}}_{1.75} \quad \underbrace{\text{plus}}_{+} \quad \underbrace{\text{cost per mile}}_{1.50} \quad \underbrace{\text{times}}_{\cdot} \quad \underbrace{\text{number of miles traveled}}_{d} \quad \underbrace{\text{is}}_{=} \quad \underbrace{\$19.75.}_{19.75}$$

Carry out. We solve the equation.

$$\begin{aligned} 1.75 + 1.50 \cdot d &= 19.75 \\ 1.5d &= 18 \quad \text{Subtracting 1.75} \\ d &= 12 \quad \text{Dividing by 1.5} \end{aligned}$$

Check. If Diego travels 12 mi, his fare is \$1.75 + \$1.50 · 12, or \$1.75 + \$18, or \$19.75. The answer checks.

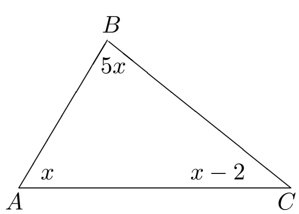
State. Diego traveled 12 mi in the cab.

40. Let w = Soledad's regular hourly wage. She worked 48 − 40, or 8 hr, of overtime.

Solve: $40w + 8(1.5w) = 442$

$w = \$8.50$

41. ***Familiarize***. We make a drawing.



We let x = the measure of angle A. Then $5x$ = the measure of angle B, and $x - 2$ = the measure of angle C. The sum of the angle measures is 180°.

Translate.

$$\underbrace{\text{Measure of angle A}}_{x} + \underbrace{\text{Measure of angle B}}_{5x} + \underbrace{\text{Measure of angle C}}_{x-2} = \underbrace{180.}_{180}$$

Carry out. We solve the equation.

$$\begin{aligned} x + 5x + x - 2 &= 180 \\ 7x - 2 &= 180 \\ 7x &= 182 \\ x &= 26 \end{aligned}$$

If $x = 26$, then $5x = 5 \cdot 26$, or 130, and $x - 2 = 26 - 2$, or 24.

Check. The measure of angle B, 130°, is five times the measure of angle A, 26°. The measure of angle C, 24°, is 2° less than the measure of angle A, 26°. The sum of the angle measures is 26° + 130° + 24°, or 180°. The answer checks.

State. The measure of angles A, B, and C are 26°, 130°, and 24°, respectively.

42. Let x = the measure of angle A.

Solve: $x + 2x + x + 20 = 180$

$x = 40°$, so the measure of angle A is 40°; the measure of angle B is $2 \cdot 40°$, or 80°; and the measure of angle C is $40° + 20°$, or 60°.

43. ***Familiarize***. Using the labels on the drawing in the text, we let w = the width of the test plot and $w + 25$ = the length, in meters. Recall that for a rectangle, Perimeter = 2 · length + 2 · width.

Translate.

$$\underbrace{\text{Perimeter}}_{322} = \underbrace{2 \cdot \text{ length}}_{2(w+25)} + \underbrace{2 \cdot \text{ width}}_{2 \cdot w}$$

Carry out. We solve the equation.

$$\begin{aligned} 322 &= 2(w + 25) + 2 \cdot w \\ 322 &= 2w + 50 + 2w \\ 322 &= 4w + 50 \\ 272 &= 4w \\ 68 &= w \end{aligned}$$

When $w = 68$, then $w + 25 = 68 + 25 = 93$.

Check. The length is 25 m more than the width: $93 = 68 + 25$. The perimeter is $2 \cdot 93 + 2 \cdot 68$, or $186 + 136$, or 322 m. The answer checks.

State. The length is 93 m; the width is 68 m.

44. Let w = the width of the garden.

Solve: $2 \cdot 2w + 2 \cdot w = 39$

$w = 6.5$, so the width is 6.5 m, and the length is 2(6.5), or 13 m.

45. ***Familiarize***. Let l = the length of the soccer field and $l - 35$ = the width, in yards.

Translate. We use the formula for the perimeter of a rectangle. We substitute 330 for P and $l - 35$ for w.

$$\begin{aligned} P &= 2l + 2w \\ 330 &= 2l + 2(l - 35) \end{aligned}$$

Carry out. We solve the equation.

$330 = 2l + 2(l - 35)$

$330 = 2l + 2l - 70$

$330 = 4l - 70$

$400 = 4l$

$100 = l$

If $l = 100$, then $l - 35 = 100 - 35 = 65$.

Check. The width, 65 yd, is 35 yd less than the length, 100 yd. Also, the perimeter is

$$2 \cdot 100 \text{ yd} + 2 \cdot 65 \text{ yd} = 200 \text{ yd} + 130 \text{ yd} = 330 \text{ yd}.$$

The answer checks.

State. The length of the field is 100 yd, and the width is 65 yd.

46. Let h = the height of the poster and $\frac{2}{3}h$ = the width, in inches.

Solve: $100 = 2 \cdot h + 2 \cdot \frac{2}{3}h$

$h = 30$, so the height is 30 in. and the width is $\frac{2}{3} \cdot 30$, or 20 in.

47. ***Familiarize***. Let w = the number of pounds of Kimiko's body weight that is water.

Translate.

$$\underbrace{50\%}\ \text{of}\ \underbrace{\text{body weight}}\ \text{is}\ \text{water.}$$

$$0.5 \times 135 = w$$

Carry out. We solve the equation.

$0.5 \times 135 = w$

$67.5 = w$

Check. Since 50% of 138 is 67.5, the answer checks.

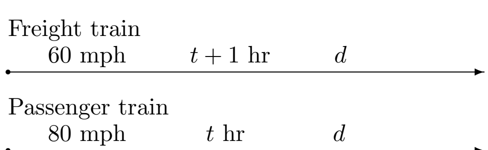
State. 67.5 lb of Kimiko's body weight is water.

48. Let w = the number of pounds of Emilio's body weight that is water.

Solve: $0.6 \times 186 = w$

$w = 111.6$ lb

49. ***Familiarize***. We make a drawing. Let t = the number of hours the passenger train travels before it overtakes the freight train. Then $t+1$ = the number of hours the freight train travels before it is overtaken by the passenger train. Also let d = the distance the trains travel.



We can also organize the information in a table.

$$d = r \cdot t$$

	Distance	Rate	Time
Freight train	d	60	$t+1$
Passenger train	d	80	t

Translate. Using the formula $d = rt$ in each row of the table, we get two equations.

$d = 60(t+1)$ and $d = 80t$.

Since the distances are the same, we have the equation

$60(t+1) = 80t$.

Carry out. We solve the equation.

$60(t+1) = 80t$

$60t + 60 = 80t$

$60 = 20t$

$3 = t$

When $t = 3$, then $t + 1 = 3 + 1 = 4$.

Check. In 4 hr the freight train travels $60 \cdot 4$, or 240 mi. In 3 hr the passenger train travels $80 \cdot 3$, or 240 mi. Since the distances are the same, the answer checks.

State. It will take the passenger train 3 hr to overtake the freight train.

50. Let t = the time the private airplane travels.

	Distance	Rate	Time
Private airplane	d	180	t
Jet	d	900	$t-2$

From the table we have the following equations:

$d = 180t$ and $d = 900(t-2)$

Solve: $180t = 900(t-2)$

$t = 2.5$

In 2.5 hr the private airplane travels 180(2.5), or 450 km. This is the distance from the airport at which it is overtaken by the jet.

51. ***Familiarize***. Let t = the number of hours it takes the kayak to travel 36 mi upstream. The kayak travels upstream at a rate of $12 - 4$, or 8 mph.

Translate. We use the formula $d = rt$.

$36 = 8 \cdot t$

Carry out. We solve the equation.

$36 = 8 \cdot t$

$4.5 = t$

Check. At a rate of 8 mph, in 4.5 hr the kayak travels 8(4.5), or 36 mi. The answer checks.

State. It takes the kayak 4.5 hr to travel 36 mi upstream.

52. Let t = the number of hours it will take Angelo to travel 20 km downstream. The kayak travels downstream at a rate of $14+2$, or 16 km/h.

Solve: $20 = 16t$

$t = 1.25$ hr

53. ***Familiarize***. Let t = the number of hours it will take the plane to travel 1050 mi into the wind. The speed into the headwind is $450-30$, or 420 mph.

Translate. We use the formula $d = rt$.

$$1050 = 420 \cdot t$$

Carry out. We solve the equation.

$$1050 = 420 \cdot t$$
$$2.5 = t$$

Check. At a rate of 420 mph, in 2.5 hr the plane travels 420(2.5), or 1050 mi. The answer checks.

State. It will take the plane 2.5 hr to travel 1050 mi into the wind.

54. Let t = the number of hours it will take the plane to travel 700 mi with the wind. The speed with the wind is $375+25$, or 400 mph.

Solve: $700 = 400t$

$t = 1.75$ hr

55. ***Familiarize***. Let x = the amount invested at 3% interest. Then $5000-x$ = the amount invested at 4%. We organize the information in a table, keeping in mind the simple interest formula, $I = Prt$.

	Amount invested	Interest rate	Time	Amount of interest
3% investment	x	3%, or 0.03	1 yr	$x(0.03)(1)$, or $0.03x$
4% investment	$5000-x$	4%, or 0.04	1 yr	$(5000-x)(0.04)(1)$, or $0.04(5000-x)$
Total	5000			176

Translate.

$$\underbrace{\text{Interest on 3\% investment}}_{0.03x} \ \underset{+}{\text{plus}} \ \underbrace{\text{interest on 4\% investment}}_{0.04(5000-x)} \ \underset{=}{\text{is}} \ \underset{176}{\$176.}$$

Carry out. We solve the equation.

$$0.03x + 0.04(5000-x) = 176$$
$$0.03x + 200 - 0.04x = 176$$
$$-0.01x + 200 = 176$$
$$-0.01x = -24$$
$$x = 2400$$

If $x = 2400$, then $5000 - x = 5000 - 2400 = 2600$.

Check. The interest on \$2400 at 3% for 1 yr is \$2400(0.03)(1) = \$72. The interest on \$2600 at 4% for 1 yr is \$2600(0.04)(1) = \$104. Since \$72 + \$104 = \$176, the answer checks.

State. \$2400 was invested at 3%, and \$2600 was invested at 4%.

56. Let x = the amount borrowed at 5%. Then $9000-x$ = the amount invested at 6%.

Solve: $0.05x + 0.06(9000-x) = 492$

$x = 4800$, so \$4800 was borrowed at 5% and \$9000 − \$4800 = \$4200 was borrowed at 6%.

57. ***Familiarize***. Let c = the calcium content of the cheese, in mg. Then $2c+4$ = the calcium content of the yogurt.

Translate.

$$\underbrace{\text{Calcium content of cheese}}_{c} \ \underset{+}{\text{plus}} \ \underbrace{\text{calcium content of yogurt}}_{(2c+4)} \ \underset{=}{\text{is}} \ \underbrace{\text{total calcium content.}}_{676}$$

Carry out. We solve the equation.

$$c + (2c+4) = 676$$
$$3c + 4 = 676$$
$$3c = 672$$
$$c = 224$$

Then $2c + 4 = 2 \cdot 224 + 4 = 448 + 4 = 452$.

Check. $224 + 452 = 676$, so the answer checks.

State. The cheese contains 224 mg of calcium, and the yogurt contains 452 mg.

58. Let p = the number of working pharmacists in the United States in 1975.

Solve: $224,500 = 1.84p$

$p \approx 122,011$ pharmacists

59. ***Familiarize***. Let s = the total holiday spending in 2004, in billions of dollars.

Translate.

$$\underbrace{\text{\$29.4 billion}}_{29.4} \ \underset{=}{\text{is}} \ \underset{0.07}{7\%} \ \underset{\cdot}{\text{of}} \ \underbrace{\text{total spending.}}_{s}$$

Carry out. We solve the equation.

$$29.4 = 0.07 \cdot s$$
$$420 = s \quad \text{Dividing by 0.07}$$

Check. 7% of 420 is $0.07 \cdot 420$, or 29.4, so the answer checks.

State. Total holiday spending in 2004 was \$420 billion.

60. Let b = the number of high-speed Internet customers in 2000, in millions.

Solve: $5b + 2.4 = 37.9$

$b = 7.1$ million customers

61. ***Familiarize***. Let n = the number of inches the volcano rises in a year. We will express one-half mile in inches:

$$\frac{1}{2}\text{ mi} \times \frac{5280\text{ ft}}{1\text{ mi}} \times \frac{12\text{ in.}}{1\text{ ft}} = 31,680\text{ in.}$$

Translate.

$$\underbrace{\text{50,000 yr}}_{\downarrow\, 50,000} \quad \underbrace{\text{times}}_{\downarrow\, \cdot} \quad \underbrace{\text{number of inches per year}}_{\downarrow\, n} \quad \underbrace{\text{is}}_{\downarrow\, =} \quad \underbrace{\text{31,680 in.}}_{\downarrow\, 31,680}$$

Carry out. We solve the equation.

$$50,000n = 31,680$$
$$n = 0.6336$$

Check. Rising at a rate of 0.6336 in. per year, in 50,000 yr the volcano will rise $50,000(0.6336)$, or 31,680 in. The answer checks.

State. On average, the volcano rises 0.6336 in. in a year.

62. Let x = the number of years it will take Horseshoe Falls to migrate one-fourth mile upstream. We will express one-fourth mile in feet:

$$\frac{1}{4}\text{ mi} \times \frac{5280\text{ ft}}{1\text{ mi}} = 1320\text{ ft}$$

Solve: $2x = 1320$

$x = 660$ yr

63.
$$x + 5 = 0 \qquad \text{Setting } f(x) = 0$$
$$x + 5 - 5 = 0 - 5 \qquad \text{Subtracting 5 on both sides}$$
$$x = -5$$

The zero of the function is -5.

64.
$$5x + 20 = 0$$
$$5x = -20$$
$$x = -4$$

65.
$$-x + 18 = 0 \qquad \text{Setting } f(x) = 0$$
$$-x + 18 + x = 0 + x \qquad \text{Adding } x \text{ on both sides}$$
$$18 = x$$

The zero of the function is 18.

66.
$$8 + x = 0$$
$$x = -8$$

67.
$$16 - x = 0 \qquad \text{Setting } f(x) = 0$$
$$16 - x + x = 0 + x \qquad \text{Adding } x \text{ on both sides}$$
$$16 = x$$

The zero of the function is 16.

68.
$$-2x + 7 = 0$$
$$-2x = -7$$
$$x = \frac{7}{2}$$

69.
$$x + 12 = 0 \qquad \text{Setting } f(x) = 0$$
$$x + 12 - 12 = 0 - 12 \qquad \text{Subtracting 12 on both sides}$$
$$x = -12$$

The zero of the function is -12.

70.
$$8x + 2 = 0$$
$$8x = -2$$
$$x = -\frac{1}{4}, \text{ or } -0.25$$

71.
$$-x + 6 = 0 \qquad \text{Setting } f(x) = 0$$
$$-x + 6 + x = 0 + x \qquad \text{Adding } x \text{ on both sides}$$
$$6 = x$$

The zero of the function is 6.

72.
$$4 + x = 0$$
$$x = -4$$

73.
$$20 - x = 0 \qquad \text{Setting } f(x) = 0$$
$$20 - x + x = 0 + x \qquad \text{Adding } x \text{ on both sides}$$
$$20 = x$$

The zero of the function is 20.

74.
$$-3x + 13 = 0$$
$$-3x = -13$$
$$x = \frac{13}{3}, \text{ or } 4.\overline{3}$$

75.
$$x - 6 = 0 \qquad \text{Setting } f(x) = 0$$
$$x = 6 \qquad \text{Adding 6 on both sides}$$

The zero of the function is 6.

76.
$$3x - 9 = 0$$
$$3x = 9$$
$$x = 3$$

77.
$$-x + 15 = 0 \qquad \text{Setting } f(x) = 0$$
$$15 = x \qquad \text{Adding } x \text{ on both sides}$$

The zero of the function is 15.

78.
$$4 - x = 0$$
$$4 = x$$

79. a) The graph crosses the x-axis at $(4, 0)$. This is the x-intercept.

b) The zero of the function is the first coordinate of the x-intercept. It is 4.

80. a) $(5, 0)$

b) 5

81. a) The graph crosses the x-axis at $(-2, 0)$. This is the x-intercept.

b) The zero of the function is the first coordinate of the x-intercept. It is -2.

82. a) $(2, 0)$

b) 2

83. a) The graph crosses the x-axis at $(-4, 0)$. This is the x-intercept.

b) The zero of the function is the first coordinate of the x-intercept. It is -4.

84. a) $(-2, 0)$

b) -2

85. $A = \frac{1}{2}bh$

$2A = bh$ Multiplying by 2 on both sides

$\frac{2A}{h} = b$ Dividing by h on both sides

86. $A = \pi r^2$

$\frac{A}{r^2} = \pi$

87. $P = 2l + 2w$

$P - 2l = 2w$ Subtracting $2l$ on both sides

$\frac{P - 2l}{2} = w$ Dividing by 2 on both sides

88. $A = P + Prt$

$A - P = Prt$

$\frac{A - P}{Pt} = r$

89. $A = \frac{1}{2}h(b_1 + b_2)$

$2A = h(b_1 + b_2)$ Multiplying by 2 on both sides

$\frac{2A}{b_1 + b_2} = h$ Dividing by $b_1 + b_2$ on both sides

90. $A = \frac{1}{2}h(b_1 + b_2)$

$\frac{2A}{h} = b_1 + b_2$

$\frac{2A}{h} - b_1 = b_2$, or

$\frac{2A - b_1h}{h} = b_2$

91. $V = \frac{4}{3}\pi r^3$

$3V = 4\pi r^3$ Multiplying by 3 on both sides

$\frac{3V}{4r^3} = \pi$ Dividing by $4r^3$ on both sides

92. $V = \frac{4}{3}\pi r^3$

$\frac{3V}{4\pi} = r^3$

93. $F = \frac{9}{5}C + 32$

$F - 32 = \frac{9}{5}C$ Subtracting 32 on both sides

$\frac{5}{9}(F - 32) = C$ Multiplying by $\frac{5}{9}$ on both sides

94. $Ax + By = C$

$By = C - Ax$

$y = \frac{C - Ax}{B}$

95. $Ax + By = C$

$Ax = C - By$ Subtracting By on both sides

$A = \frac{C - By}{x}$ Dividing by x on both sides

96. $2w + 2h + l = p$

$2w = p - 2h - l$

$w = \frac{p - 2h - l}{2}$

97. $2w + 2h + l = p$

$2h = p - 2w - l$ Subtracting $2w$ and l

$h = \frac{p - 2w - l}{2}$ Dividing by 2

98. $3x + 4y = 12$

$4y = 12 - 3x$

$y = \frac{12 - 3x}{4}$

99. $2x - 3y = 6$

$-3y = 6 - 2x$ Subtracting $2x$

$y = \frac{6 - 2x}{-3}$, or Dividing by -3

$\frac{2x - 6}{3}$

100. $T = \frac{3}{10}(I - 12,000)$

$\frac{10}{3}T = I - 12,000$

$\frac{10}{3}T + 12,000 = I$, or

$\frac{10T + 36,000}{3} = I$

101. $a = b + bcd$

$a = b(1 + cd)$ Factoring

$\frac{a}{1 + cd} = b$ Dividing by $1 + cd$

102. $q = p - np$

$q = p(1 - n)$

$\frac{q}{1 - n} = p$

103. $z = xy - xy^2$

$z = x(y - y^2)$ Factoring

$\frac{z}{y - y^2} = x$ Dividing by $y - y^2$

104. $st = t - 4$

$st - t = -4$

$t(s - 1) = -4$

$t = \frac{-4}{s - 1}$, or $\frac{4}{1 - s}$

105. Left to the student

106. Left to the student

107. The graph of $f(x) = mx + b$, $m \neq 0$, is a straight line that is not horizontal. The graph of such a line intersects the x-axis exactly once. Thus, the function has exactly one zero.

108. If a person wanted to convert several Fahrenheit temperatures to Celsius, it would be useful to solve the formula for C and then use the formula in that form.

109. First find the slope of the given line.

$$\begin{aligned} 3x + 4y &= 7 \\ 4y &= -3x + 7 \\ y &= -\frac{3}{4}x + \frac{7}{4} \end{aligned}$$

The slope is $-\frac{3}{4}$. Now write a slope-intersect equation of the line containing $(-1, 4)$ with slope $-\frac{3}{4}$.

$$\begin{aligned} y - 4 &= -\frac{3}{4}[x - (-1)] \\ y - 4 &= -\frac{3}{4}(x + 1) \\ y - 4 &= -\frac{3}{4}x - \frac{3}{4} \\ y &= -\frac{3}{4}x + \frac{13}{4} \end{aligned}$$

110. $m = \dfrac{4 - (-2)}{-5 - 3} = \dfrac{6}{-8} = -\dfrac{3}{4}$

$$\begin{aligned} y - 4 &= -\frac{3}{4}(x - (-5)) \\ y - 4 &= -\frac{3}{4}x - \frac{15}{4} \\ y &= -\frac{3}{4}x + \frac{1}{4} \end{aligned}$$

111. The domain of f is the set of all real numbers as is the domain of g, so the domain of $f + g$ is the set of all real numbers, or $(-\infty, \infty)$.

112. The domain of f is the set of all real numbers as is the domain of g. When $x = -2$, $g(x) = 0$, so the domain of f/g is $(-\infty, -2) \cup (-2, \infty)$.

113. $(f - g)(x) = f(x) - g(x) = 2x - 1 - (3x + 6) =$
$2x - 1 - 3x - 6 = -x - 7$

114. $fg(-1) = f(-1) \cdot g(-1) = [2(-1) - 1][3(-1) + 6] =$
$-3 \cdot 3 = -9$

115. $f(x) = 7 - \frac{3}{2}x = -\frac{3}{2}x + 7$

The function can be written in the form $y = mx + b$, so it is a linear function.

116. $f(x) = \dfrac{3}{2x} + 5$ cannot be written in the form $f(x) = mx + b$, so it is not a linear function.

117. $f(x) = x^2 + 1$ cannot be written in the form $f(x) = mx + b$, so it is not a linear function.

118. $f(x) = \frac{3}{4}x - (2.4)^2$ is in the form $f(x) = mx + b$, so it is a linear function.

119.

$$\begin{aligned} 2x - \{x - [3x - (6x + 5)]\} &= 4x - 1 \\ 2x - \{x - [3x - 6x - 5]\} &= 4x - 1 \\ 2x - \{x - [-3x - 5]\} &= 4x - 1 \\ 2x - \{x + 3x + 5\} &= 4x - 1 \\ 2x - \{4x + 5\} &= 4x - 1 \\ 2x - 4x - 5 &= 4x - 1 \\ -2x - 5 &= 4x - 1 \\ -6x - 5 &= -1 \\ -6x &= 4 \\ x &= -\frac{2}{3} \end{aligned}$$

The solution is $-\frac{2}{3}$.

120.

$$\begin{aligned} 14 - 2[3 + 5(x - 1)] &= 3\{x - 4[1 + 6(2 - x)]\} \\ 14 - 2[3 + 5x - 5] &= 3\{x - 4[1 + 12 - 6x]\} \\ 14 - 2[5x - 2] &= 3\{x - 4[13 - 6x]\} \\ 14 - 10x + 4 &= 3\{x - 52 + 24x\} \\ 18 - 10x &= 3\{25x - 52\} \\ 18 - 10x &= 75x - 156 \\ 174 &= 85x \\ \frac{174}{85} &= x \end{aligned}$$

121. The size of the cup was reduced 8 oz − 6 oz, or 2 oz, and $\dfrac{2\text{ oz}}{8\text{ oz}} = 0.25$, so the size was reduced 25%. The price per ounce of the 8 oz cup was $\dfrac{89¢}{8\text{ oz}}$, or 11.25¢/oz. The price per ounce of the 6 oz cup is $\dfrac{71¢}{6\text{ oz}}$, or $11.8\overline{3}$¢/oz. Since the price per ounce was not reduced, it is clear that the price per ounce was not reduced by the same percent as the size of the cup. The price was increased by $11.8\overline{3} - 11.125$¢, or $0.708\overline{3}$¢ per ounce. This is an increase of $\dfrac{0.708\overline{3}¢}{11.8\overline{3}¢} \approx 0.064$, or about 6.4% per ounce.

122. The size of the container was reduced 100 oz − 80 oz, or 20 oz, and $\dfrac{20\text{ oz}}{100\text{ oz}} = 0.2$, so the size of the container was reduced 20%. The price per ounce of the 100-oz container was $\dfrac{\$6.99}{100\text{ oz}}$, or \$0.0699/oz. The price per ounce of the 80-oz container is $\dfrac{\$5.75}{80\text{ oz}}$, or \$0.071875. Since the price per ounce was not reduced, it is clear that the price per ounce was not reduced by the same percent as the size of the container. The price increased by \$0.071875 − \$0.0699, or \$0.001975. This is an increase of $\dfrac{\$0.001975}{\$0.0699} \approx 0.028$, or about 2.8% per ounce.

123. We use a proportion to determine the number of calories c burned running for 75 minutes, or 1.25 hr.

$$\begin{aligned} \frac{720}{1} &= \frac{c}{1.25} \\ 720(1.25) &= c \\ 900 &= c \end{aligned}$$

Next we use a proportion to determine how long the person would have to walk to use 900 calories. Let t represent this time, in hours. We express 90 min as 1.5 hr.

$$\frac{1.5}{480} = \frac{t}{900}$$
$$\frac{900(1.5)}{480} = t$$
$$2.8125 = t$$

Then, at a rate of 4 mph, the person would have to walk 4(2.8125), or 11.25 mi.

124. Let x = the number of copies of *The DaVinci Code* that were sold. Then $3570 - x$ = the number of copies of *Marley and Me* that were sold.

Solve: $\frac{x}{3570 - x} = \frac{10}{1.9}$

$x = 3000$, so 3000 copies of *The DaVinci Code* were sold and $3570 - x = 3570 - 3000 = 570$ copies of *Marley and Me* were sold.

Exercise Set 2.2

1. $\sqrt{-3} = \sqrt{-1 \cdot 3} = \sqrt{-1} \cdot \sqrt{3} = i\sqrt{3}$, or $\sqrt{3}i$

2. $\sqrt{-21} = \sqrt{-1 \cdot 21} = i\sqrt{21}$, or $\sqrt{21}i$

3. $\sqrt{-25} = \sqrt{-1 \cdot 25} = \sqrt{-1} \cdot \sqrt{25} = i \cdot 5 = 5i$

4. $\sqrt{-100} = \sqrt{-1 \cdot 100} = i \cdot 10 = 10i$

5. $-\sqrt{-33} = -\sqrt{-1 \cdot 33} = -\sqrt{-1} \cdot \sqrt{33} = -i\sqrt{33}$, or $-\sqrt{33}i$

6. $-\sqrt{-59} = -\sqrt{-1 \cdot 59} = -i\sqrt{59}$, or $-\sqrt{59}i$

7. $-\sqrt{-81} = -\sqrt{-1 \cdot 81} = -\sqrt{-1} \cdot \sqrt{81} = -i \cdot 9 = -9i$

8. $-\sqrt{-9} = -\sqrt{-1 \cdot 9} = -\sqrt{-1} \cdot \sqrt{9} = -i \cdot 3 = -3i$

9. $\sqrt{-98} = \sqrt{-1 \cdot 98} = \sqrt{-1} \cdot \sqrt{98} = i\sqrt{49 \cdot 2} =$
$i \cdot 7\sqrt{2} = 7i\sqrt{2}$, or $7\sqrt{2}i$

10. $\sqrt{-28} = \sqrt{-1 \cdot 28} = i\sqrt{28} = i\sqrt{4 \cdot 7} = 2i\sqrt{7}$, or $2\sqrt{7}i$

11. $(-5 + 3i) + (7 + 8i)$
$= (-5 + 7) + (3i + 8i)$ Collecting the real parts and the imaginary parts
$= 2 + (3 + 8)i$
$= 2 + 11i$

12. $(-6 - 5i) + (9 + 2i) = (-6 + 9) + (-5i + 2i) = 3 - 3i$

13. $(4 - 9i) + (1 - 3i)$
$= (4 + 1) + (-9i - 3i)$ Collecting the real parts and the imaginary parts
$= 5 + (-9 - 3)i$
$= 5 - 12i$

14. $(7 - 2i) + (4 - 5i) = (7 + 4) + (-2i - 5i) = 11 - 7i$

15. $(12 + 3i) + (-8 + 5i)$
$= (12 - 8) + (3i + 5i)$
$= 4 + 8i$

16. $(-11 + 4i) + (6 + 8i) = (-11 + 6) + (4i + 8i) = -5 + 12i$

17. $(-1 - i) + (-3 - i)$
$= (-1 - 3) + (-i - i)$
$= -4 - 2i$

18. $(-5 - i) + (6 + 2i) = (-5 + 6) + (-i + 2i) = 1 + i$

19. $(3 + \sqrt{-16}) + (2 + \sqrt{-25}) = (3 + 4i) + (2 + 5i)$
$= (3 + 2) + (4i + 5i)$
$= 5 + 9i$

20. $(7 - \sqrt{-36}) + (2 + \sqrt{-9}) = (7 - 6i) + (2 + 3i) =$
$(7 + 2) + (-6i + 3i) = 9 - 3i$

21. $(10 + 7i) - (5 + 3i)$
$= (10 - 5) + (7i - 3i)$ The 5 and the $3i$ are both being subtracted.
$= 5 + 4i$

22. $(-3 - 4i) - (8 - i) = (-3 - 8) + [-4i - (-i)] =$
$-11 - 3i$

23. $(13 + 9i) - (8 + 2i)$
$= (13 - 8) + (9i - 2i)$ The 8 and the $2i$ are both being subtracted.
$= 5 + 7i$

24. $(-7 + 12i) - (3 - 6i) = (-7 - 3) + [12i - (-6i)] =$
$-10 + 18i$

25. $(6 - 4i) - (-5 + i)$
$= [6 - (-5)] + (-4i - i)$
$= (6 + 5) + (-4i - i)$
$= 11 - 5i$

26. $(8 - 3i) - (9 - i) = (8 - 9) + [-3i - (-i)] = -1 - 2i$

27. $(-5 + 2i) - (-4 - 3i)$
$= [-5 - (-4)] + [2i - (-3i)]$
$= (-5 + 4) + (2i + 3i)$
$= -1 + 5i$

28. $(-6 + 7i) - (-5 - 2i) = [-6 - (-5)] + [7i - (-2i)] =$
$-1 + 9i$

29. $(4 - 9i) - (2 + 3i)$
$= (4 - 2) + (-9i - 3i)$
$= 2 - 12i$

30. $(10 - 4i) - (8 + 2i) = (10 - 8) + (-4i - 2i) =$
$2 - 6i$

31. $7i(2 - 5i)$
$= 14i - 35i^2$ Using the distributive law
$= 14i + 35$ $\quad i^2 = -1$
$= 35 + 14i$ Writing in the form $a + bi$

32. $3i(6+4i) = 18i + 12i^2 = 18i - 12 = -12 + 18i$

33. $-2i(-8+3i)$
$= 16i - 6i^2$ Using the distributive law
$= 16i + 6$ $i^2 = -1$
$= 6 + 16i$ Writing in the form $a + bi$

34. $-6i(-5+i) = 30i - 6i^2 = 30i + 6 = 6 + 30i$

35. $(1+3i)(1-4i)$
$= 1 - 4i + 3i - 12i^2$ Using FOIL
$= 1 - 4i + 3i - 12(-1)$ $i^2 = -1$
$= 1 - i + 12$
$= 13 - i$

36. $(1-2i)(1+3i) = 1 + 3i - 2i - 6i^2 = 1 + i + 6 = 7 + i$

37. $(2+3i)(2+5i)$
$= 4 + 10i + 6i + 15i^2$ Using FOIL
$= 4 + 10i + 6i - 15$ $i^2 = -1$
$= -11 + 16i$

38. $(3-5i)(8-2i) = 24 - 6i - 40i + 10i^2 = 24 - 6i - 40i - 10 = 14 - 46i$

39. $(-4+i)(3-2i)$
$= -12 + 8i + 3i - 2i^2$ Using FOIL
$= -12 + 8i + 3i + 2$ $i^2 = -1$
$= -10 + 11i$

40. $(5-2i)(-1+i) = -5 + 5i + 2i - 2i^2 = -5 + 5i + 2i + 2 = -3 + 7i$

41. $(8-3i)(-2-5i)$
$= -16 - 40i + 6i + 15i^2$
$= -16 - 40i + 6i - 15$ $i^2 = -1$
$= -31 - 34i$

42. $(7-4i)(-3-3i) = -21 - 21i + 12i + 12i^2 = -21 - 21i + 12i - 12 = -33 - 9i$

43. $(3+\sqrt{-16})(2+\sqrt{-25})$
$= (3+4i)(2+5i)$
$= 6 + 15i + 8i + 20i^2$
$= 6 + 15i + 8i - 20$ $i^2 = -1$
$= -14 + 23i$

44. $(7-\sqrt{-16})(2+\sqrt{-9}) = (7-4i)(2+3i) = 14 + 21i - 8i - 12i^2 = 14 + 21i - 8i + 12 = 26 + 13i$

45. $(5-4i)(5+4i) = 5^2 - (4i)^2$
$= 25 - 16i^2$
$= 25 + 16$ $i^2 = -1$
$= 41$

46. $(5+9i)(5-9i) = 25 - 81i^2 = 25 + 81 = 106$

47. $(3+2i)(3-2i)$
$= 9 - 6i + 6i - 4i^2$
$= 9 - 6i + 6i + 4$ $i^2 = -1$
$= 13$

48. $(8+i)(8-i) = 64 - 8i + 8i - i^2 = 64 - 8i + 8i + 1 = 65$

49. $(7-5i)(7+5i)$
$= 49 + 35i - 35i - 25i^2$
$= 49 + 35i - 35i + 25$ $i^2 = -1$
$= 74$

50. $(6-8i)(6+8i) = 36 + 48i - 48i - 64i^2 = 36 + 48i - 48i + 64 = 100$

51. $(4+2i)^2$
$= 16 + 2\cdot 4\cdot 2i + (2i)^2$ Recall $(A+B)^2 = A^2 + 2AB + B^2$
$= 16 + 16i + 4i^2$
$= 16 + 16i - 4$ $i^2 = -1$
$= 12 + 16i$

52. $(5-4i)^2 = 25 - 40i + 16i^2 = 25 - 40i - 16 = 9 - 40i$

53. $(-2+7i)^2$
$= (-2)^2 + 2(-2)(7i) + (7i)^2$ Recall $(A+B)^2 = A^2 + 2AB + B^2$
$= 4 - 28i + 49i^2$
$= 4 - 28i - 49$ $i^2 = -1$
$= -45 - 28i$

54. $(-3+2i)^2 = 9 - 12i + 4i^2 = 9 - 12i - 4 = 5 - 12i$

55. $(1-3i)^2$
$= 1^2 - 2\cdot 1\cdot (3i) + (3i)^2$
$= 1 - 6i + 9i^2$
$= 1 - 6i - 9$ $i^2 = -1$
$= -8 - 6i$

56. $(2-5i)^2 = 4 - 20i + 25i^2 = 4 - 20i - 25 = -21 - 20i$

57. $(-1-i)^2$
$= (-1)^2 - 2(-1)(i) + i^2$
$= 1 + 2i + i^2$
$= 1 + 2i - 1$ $i^2 = -1$
$= 2i$

58. $(-4-2i)^2 = 16 + 16i + 4i^2 = 16 + 16i - 4 = 12 + 16i$

59. $(3+4i)^2$
$= 9 + 2 \cdot 3 \cdot 4i + (4i)^2$
$= 9 + 24i + 16i^2$
$= 9 + 24i - 16$ $\quad i^2 = -1$
$= -7 + 24i$

60. $(6+5i)^2 = 36 + 60i + 25i^2 = 36 + 60i - 25 = 11 + 60i$

61. $\frac{3}{5-11i}$
$= \frac{3}{5-11i} \cdot \frac{5+11i}{5+11i}$ $\quad 5-11i$ is the conjugate of $5+11i$.
$= \frac{3(5+11i)}{(5-11i)(5+11i)}$
$= \frac{15+33i}{25-121i^2}$
$= \frac{15+33i}{25+121}$ $\quad i^2 = -1$
$= \frac{15+33i}{146}$
$= \frac{15}{146} + \frac{33}{146}i$ $\quad$ Writing in the form $a+bi$

62. $\frac{i}{2+i} = \frac{i}{2+i} \cdot \frac{2-i}{2-i}$
$= \frac{2i-i^2}{4-i^2}$
$= \frac{2i+1}{4+1}$
$= \frac{1}{5} + \frac{2}{5}i$

63. $\frac{5}{2+3i}$
$= \frac{5}{2+3i} \cdot \frac{2-3i}{2-3i}$ $\quad 2-3i$ is the conjugate of $2+3i$.
$= \frac{5(2-3i)}{(2+3i)(2-3i)}$
$= \frac{10-15i}{4-9i^2}$
$= \frac{10-15i}{4+9}$ $\quad i^2 = -1$
$= \frac{10-15i}{13}$
$= \frac{10}{13} - \frac{15}{13}i$ $\quad$ Writing in the form $a+bi$

64. $\frac{-3}{4-5i} = \frac{-3}{4-5i} \cdot \frac{4+5i}{4+5i}$
$= \frac{-12-15i}{16-25i^2}$
$= \frac{-12-15i}{16+25}$
$= -\frac{12}{41} - \frac{15}{41}i$

65. $\frac{4+i}{-3-2i}$
$= \frac{4+i}{-3-2i} \cdot \frac{-3+2i}{-3+2i}$ $\quad -3+2i$ is the conjugate of the divisor.
$= \frac{(4+i)(-3+2i)}{(-3-2i)(-3+2i)}$
$= \frac{-12+5i+2i^2}{9-4i^2}$
$= \frac{-12+5i-2}{9+4}$ $\quad i^2 = -1$
$= \frac{-14+5i}{13}$
$= -\frac{14}{13} + \frac{5}{13}i$ $\quad$ Writing in the form $a+bi$

66. $\frac{5-i}{-7+2i} = \frac{5-i}{-7+2i} \cdot \frac{-7-2i}{-7-2i}$
$= \frac{-35-3i+2i^2}{49-4i^2}$
$= \frac{-35-3i-2}{49+4}$
$= -\frac{37}{53} - \frac{3}{53}i$

67. $\frac{5-3i}{4+3i}$
$= \frac{5-3i}{4+3i} \cdot \frac{4-3i}{4-3i}$ $\quad 4-3i$ is the conjugate of $4+3i$.
$= \frac{(5-3i)(4-3i)}{(4+3i)(4-3i)}$
$= \frac{20-27i+9i^2}{16-9i^2}$
$= \frac{20-27i-9}{16+9}$ $\quad i^2 = -1$
$= \frac{11-27i}{25}$
$= \frac{11}{25} - \frac{27}{25}i$ $\quad$ Writing in the form $a+bi$

68. $\frac{6+5i}{3-4i} = \frac{6+5i}{3-4i} \cdot \frac{3+4i}{3+4i}$
$= \frac{18+39i+20i^2}{9-16i^2}$
$= \frac{18+39i-20}{9+16}$
$= -\frac{2}{25} + \frac{39}{25}i$

69. $\dfrac{2+\sqrt{3}i}{5-4i}$

$= \dfrac{2+\sqrt{3}i}{5-4i}\cdot\dfrac{5+4i}{5+4i}$ $\quad 5+4i$ is the conjugate of the divisor.

$= \dfrac{(2+\sqrt{3}i)(5+4i)}{(5-4i)(5+4i)}$

$= \dfrac{10+8i+5\sqrt{3}i+4\sqrt{3}i^2}{25-16i^2}$

$= \dfrac{10+8i+5\sqrt{3}i-4\sqrt{3}}{25+16}$ $\quad i^2=-1$

$= \dfrac{10-4\sqrt{3}+(8+5\sqrt{3})i}{41}$

$= \dfrac{10-4\sqrt{3}}{41}+\dfrac{8+5\sqrt{3}}{41}i$ $\quad$ Writing in the form $a+bi$

70. $\dfrac{\sqrt{5}+3i}{1-i} = \dfrac{\sqrt{5}+3i}{1-i}\cdot\dfrac{1+i}{1+i}$

$= \dfrac{\sqrt{5}+\sqrt{5}i+3i+3i^2}{1-i^2}$

$= \dfrac{\sqrt{5}+\sqrt{5}i+3i-3}{1+1}$

$= \dfrac{\sqrt{5}-3}{2}+\dfrac{\sqrt{5}+3}{2}i$

71. $\dfrac{1+i}{(1-i)^2}$

$= \dfrac{1+i}{1-2i+i^2}$

$= \dfrac{1+i}{1-2i-1}$ $\quad i^2=-1$

$= \dfrac{1+i}{-2i}$

$= \dfrac{1+i}{-2i}\cdot\dfrac{2i}{2i}$ $\quad 2i$ is the conjugate of $-2i$.

$= \dfrac{(1+i)(2i)}{(-2i)(2i)}$

$= \dfrac{2i+2i^2}{-4i^2}$

$= \dfrac{2i-2}{4}$ $\quad i^2=-1$

$= -\dfrac{2}{4}+\dfrac{2}{4}i$

$= -\dfrac{1}{2}+\dfrac{1}{2}i$

72. $\dfrac{1-i}{(1+i)^2} = \dfrac{1-i}{1+2i+i^2}$

$= \dfrac{1-i}{1+2i-1}$

$= \dfrac{1-i}{2i}$

$= \dfrac{1-i}{2i}\cdot\dfrac{-2i}{-2i}$

$= \dfrac{-2i+2i^2}{-4i^2}$

$= \dfrac{-2i-2}{4}$

$= -\dfrac{1}{2}-\dfrac{1}{2}i$

73. $\dfrac{4-2i}{1+i}+\dfrac{2-5i}{1+i}$

$= \dfrac{6-7i}{1+i}$ $\quad$ Adding

$= \dfrac{6-7i}{1+i}\cdot\dfrac{1-i}{1-i}$ $\quad 1-i$ is the conjugate of $1+i$.

$= \dfrac{(6-7i)(1-i)}{(1+i)(1-i)}$

$= \dfrac{6-13i+7i^2}{1-i^2}$

$= \dfrac{6-13i-7}{1+1}$ $\quad i^2=-1$

$= \dfrac{-1-13i}{2}$

$= -\dfrac{1}{2}-\dfrac{13}{2}i$

74. $\dfrac{3+2i}{1-i}+\dfrac{6+2i}{1-i} = \dfrac{9+4i}{1-i}$

$= \dfrac{9+4i}{1-i}\cdot\dfrac{1+i}{1+i}$

$= \dfrac{9+13i+4i^2}{1-i^2}$

$= \dfrac{9+13i-4}{1+1}$

$= \dfrac{5}{2}+\dfrac{13}{2}i$

75. $i^{11} = i^{10}\cdot i = (i^2)^5\cdot i = (-1)^5\cdot i = -1\cdot i = -i$

76. $i^7 = i^6\cdot i = (i^2)^3\cdot i = (-1)^3\cdot i = -1\cdot i = -i$

77. $i^{35} = i^{34}\cdot i = (i^2)^{17}\cdot i = (-1)^{17}\cdot i = -1\cdot i = -i$

78. $i^{24} = (i^2)^{12} = (-1)^{12} = 1$

79. $i^{64} = (i^2)^{32} = (-1)^{32} = 1$

80. $i^{42} = (i^2)^{21} = (-1)^{21} = -1$

81. $(-i)^{71} = (-1\cdot i)^{71} = (-1)^{71}\cdot i^{71} = -i^{70}\cdot i =$
$-(i^2)^{35}\cdot i = -(-1)^{35}\cdot i = -(-1)i = i$

82. $(-i)^6 = i^6 = (i^2)^3 = (-1)^3 = -1$

83. $(5i)^4 = 5^4 \cdot i^4 = 625(i^2)^2 = 625(-1)^2 = 625 \cdot 1 = 625$

84. $(2i)^5 = 32i^5 = 32 \cdot i^4 \cdot i = 32(i^2)^2 \cdot i = 32(-1)^2 \cdot i = 32 \cdot 1 \cdot i = 32i$

85. Left to the student

86. Left to the student

87. The sum of two imaginary numbers is not always an imaginary number. For example, $(2+i)+(3-i)=5$, a real number.

88. The product of two imaginary numbers is not always an imaginary number. For example, $i \cdot i = i^2 = -1$, a real number.

89. First find the slope of the given line.

$$\begin{aligned} 3x - 6y &= 7 \\ -6y &= -3x + 7 \\ y &= \frac{1}{2}x - \frac{7}{6} \end{aligned}$$

The slope is $\frac{1}{2}$. The slope of the desired line is the opposite of the reciprocal of $\frac{1}{2}$, or -2. Write a slope-intercept equation of the line containing $(3, -5)$ with slope -2.

$$\begin{aligned} y - (-5) &= -2(x - 3) \\ y + 5 &= -2x + 6 \\ y &= -2x + 1 \end{aligned}$$

90. The domain of f is the set of all real numbers as is the domain of g. Then the domain of $(f-g)(x)$ is the set of all real numbers, or $(-\infty, \infty)$.

91. The domain of f is the set of all real numbers as is the domain of g. When $x = -\frac{5}{3}$, $g(x) = 0$, so the domain of f/g is $\left(-\infty, -\frac{5}{3}\right) \cup \left(-\frac{5}{3}, \infty\right)$.

92. $(f-g)(x) = f(x) - g(x) = x^2 + 4 - (3x+5) = x^2 - 3x - 1$

93. $(f/g)(2) = \frac{f(2)}{g(2)} = \frac{2^2+4}{3 \cdot 2 + 5} = \frac{4+4}{6+5} = \frac{8}{11}$

94.

$$\begin{aligned} &\frac{f(x+h)-f(x)}{h} \\ &= \frac{(x+h)^2 - 3(x+h) + 4 - (x^2 - 3x + 4)}{h} \\ &= \frac{x^2 + 2xh + h^2 - 3x - 3h + 4 - x^2 + 3x - 4}{h} \\ &= \frac{2xh + h^2 - 3h}{h} \\ &= \frac{h(2x+h-3)}{h} \\ &= 2x + h - 3 \end{aligned}$$

95. $(a+bi)+(a-bi) = 2a$, a real number. Thus, the statement is true.

96. $(a+bi)+(c+di) = (a+c)+(b+d)i$. The conjugate of this sum is $(a+c)-(b+d)i = a+c-bi-di = (a-bi)+(c-di)$, the sum of the conjugates of the individual complex numbers. Thus, the statement is true.

97. $(a+bi)(c+di) = (ac-bd)+(ad+bc)i$. The conjugate of the product is $(ac-bd)-(ad+bc)i =$ $(a-bi)(c-di)$, the product of the conjugates of the individual complex numbers. Thus, the statement is true.

98. $\frac{1}{z} = \frac{1}{a+bi} \cdot \frac{a-bi}{a-bi} = \frac{a}{a^2+b^2} + \frac{-b}{a^2+b^2}i$

99. $z\overline{z} = (a+bi)(a-bi) = a^2 - b^2i^2 = a^2 + b^2$

100.

$$\begin{aligned} z + 6\overline{z} &= 7 \\ a + bi + 6(a - bi) &= 7 \\ a + bi + 6a - 6bi &= 7 \\ 7a - 5bi &= 7 \end{aligned}$$

Then $7a = 7$, so $a = 1$, and $-5b = 0$, so $b = 0$. Thus, $z = 1$.

Exercise Set 2.3

1. $(2x-3)(3x-2) = 0$

$2x - 3 = 0$ *or* $3x - 2 = 0$ Using the principle of zero products

$2x = 3$ *or* $3x = 2$

$x = \frac{3}{2}$ *or* $x = \frac{2}{3}$

The solutions are $\frac{3}{2}$ and $\frac{2}{3}$.

2. $(5x-2)(2x+3) = 0$

$x = \frac{2}{5}$ *or* $x = -\frac{3}{2}$

The solutions are $\frac{2}{5}$ and $-\frac{3}{2}$.

3. $x^2 - 8x - 20 = 0$

$(x-10)(x+2) = 0$ Factoring

$x - 10 = 0$ *or* $x + 2 = 0$ Using the principle of zero products

$x = 10$ *or* $x = -2$

The solutions are 10 and -2.

4. $x^2 + 6x + 8 = 0$

$(x+2)(x+4) = 0$

$x = -2$ *or* $x = -4$

The solutions are -2 and -4.

5. $3x^2 + x - 2 = 0$

$(3x-2)(x+1) = 0$ Factoring

$3x - 2 = 0$ *or* $x + 1 = 0$ Using the principle of zero products

$x = \frac{2}{3}$ *or* $x = -1$

The solutions are $\frac{2}{3}$ and -1.

6. $10x^2 - 16x + 6 = 0$

$2(5x-3)(x-1) = 0$

$x = \frac{3}{5}$ *or* $x = 1$

The solutions are $\frac{3}{5}$ and 1.

7. $4x^2 - 12 = 0$

$4x^2 = 12$

$x^2 = 3$

$x = \sqrt{3}$ *or* $x = -\sqrt{3}$ Using the principle of square roots

The solutions are $\sqrt{3}$ and $-\sqrt{3}$.

8. $6x^2 = 36$

$x^2 = 6$

$x = \sqrt{6}$ *or* $x = -\sqrt{6}$

The solutions are $\sqrt{6}$ and $-\sqrt{6}$.

9. $3x^2 = 21$

$x^2 = 7$

$x = \sqrt{7}$ *or* $x = -\sqrt{7}$ Using the principle of square roots

The solutions are $\sqrt{7}$ and $-\sqrt{7}$.

10. $2x^2 - 20 = 0$

$2x^2 = 20$

$x^2 = 10$

$x = \sqrt{10}$ *or* $x = -\sqrt{10}$

The solutions are $\sqrt{10}$ and $-\sqrt{10}$.

11. $5x^2 + 10 = 0$

$5x^2 = -10$

$x^2 = -2$

$x = \sqrt{2}i$ *or* $x = -\sqrt{2}i$

The solutions are $\sqrt{2}i$ and $-\sqrt{2}i$.

12. $4x^2 + 12 = 0$

$4x^2 = -12$

$x^2 = -3$

$x = \sqrt{3}i$ *or* $x = -\sqrt{3}i$

The solutions are $\sqrt{3}i$ and $-\sqrt{3}i$.

13. $2x^2 - 34 = 0$

$2x^2 = 34$

$x^2 = 17$

$x = \sqrt{17}$ *or* $x = -\sqrt{17}$

The solutions are $\sqrt{17}$ and $-\sqrt{17}$.

14. $3x^2 = 33$

$x^2 = 11$

$x = \sqrt{11}$ *or* $x = -\sqrt{11}$

The solutions are $\sqrt{11}$ and $-\sqrt{11}$.

15. $2x^2 = 6x$

$2x^2 - 6x = 0$ Subtracting $6x$ on both sides

$2x(x - 3) = 0$

$2x = 0$ *or* $x - 3 = 0$

$x = 0$ *or* $x = 3$

The solutions are 0 and 3.

16. $18x + 9x^2 = 0$

$9x(2 + x) = 0$

$x = 0$ *or* $x = -2$

The solutions are -2 and 0.

17. $3y^3 - 5y^2 - 2y = 0$

$y(3y^2 - 5y - 2) = 0$

$y(3y + 1)(y - 2) = 0$

$y = 0$ *or* $3y + 1 = 0$ *or* $y - 2 = 0$

$y = 0$ *or* $y = -\frac{1}{3}$ *or* $y = 2$

The solutions are $-\frac{1}{3}$, 0 and 2.

18. $3t^3 + 2t = 5t^2$

$3t^3 - 5t^2 + 2t = 0$

$t(t - 1)(3t - 2) = 0$

$t = 0$ *or* $t = 1$ *or* $t = \frac{2}{3}$

The solutions are 0, $\frac{2}{3}$, and 1.

19. $7x^3 + x^2 - 7x - 1 = 0$

$x^2(7x + 1) - (7x + 1) = 0$

$(x^2 - 1)(7x + 1) = 0$

$(x + 1)(x - 1)(7x + 1) = 0$

$x + 1 = 0$ *or* $x - 1 = 0$ *or* $7x + 1 = 0$

$x = -1$ *or* $x = 1$ *or* $x = -\frac{1}{7}$

The solutions are -1, $-\frac{1}{7}$, and 1.

20. $3x^3 + x^2 - 12x - 4 = 0$

$x^2(3x + 1) - 4(3x + 1) = 0$

$(3x + 1)(x^2 - 4) = 0$

$(3x + 1)(x + 2)(x - 2) = 0$

$x = -\frac{1}{3}$ *or* $x = -2$ *or* $x = 2$

The solutions are -2, $-\frac{1}{3}$, and 2.

21. a) The graph crosses the x-axis at $(-4, 0)$ and at $(2, 0)$. These are the x-intercepts.

b) The zeros of the function are the first coordinates of the x-intercepts of the graph. They are -4 and 2.

22. a) $(-1, 0)$, $(2, 0)$

b) -1, 2

23. a) The graph crosses the x-axis at $(-1, 0)$ and at $(3, 0)$. These are the x-intercepts.

b) The zeros of the function are the first coordinates of the x-intercepts of the graph. They are -1 and 3.

24. a) $(-3, 0)$, $(1, 0)$

b) -3, 1

25. a) The graph crosses the x-axis at $(-2,0)$ and at $(2,0)$. These are the x-intercepts.

b) The zeros of the function are the first coordinates of the x-intercepts of the graph. They are -2 and 2.

26. a) $(-1,0)$, $(1,0)$

b) -1, 1

27.
$$x^2+6x=7$$
$$x^2+6x+9=7+9 \quad \text{Completing the square: } \tfrac{1}{2}\cdot 6=3 \text{ and } 3^2=9$$
$$(x+3)^2=16 \quad \text{Factoring}$$
$$x+3=\pm 4 \quad \text{Using the principle of square roots}$$
$$x=-3\pm 4$$
$$x=-3-4 \quad or \quad x=-3+4$$
$$x=-7 \quad or \quad x=1$$
The solutions are -7 and 1.

28.
$$x^2+8x=-15$$
$$x^2+8x+16=-15+16 \quad \left(\tfrac{1}{2}\cdot 8=4 \text{ and } 4^2=16\right)$$
$$(x+4)^2=1$$
$$x+4=\pm 1$$
$$x=-4\pm 1$$
$$x=-4-1 \quad or \quad x=-4+1$$
$$x=-5 \quad or \quad x=-3$$
The solutions are -5 and -3.

29.
$$x^2=8x-9$$
$$x^2-8x=-9 \quad \text{Subtracting } 8x$$
$$x^2-8x+16=-9+16 \quad \text{Completing the square: } \tfrac{1}{2}(-8)=-4 \text{ and } (-4)^2=16$$
$$(x-4)^2=7 \quad \text{Factoring}$$
$$x-4=\pm\sqrt{7} \quad \text{Using the principle of square roots}$$
$$x=4\pm\sqrt{7}$$
The solutions are $4-\sqrt{7}$ and $4+\sqrt{7}$, or $4\pm\sqrt{7}$.

30.
$$x^2=22+10x$$
$$x^2-10x=22$$
$$x^2-10x+25=22+25 \quad \left(\tfrac{1}{2}(-10)=-5 \text{ and } (-5)^2=25\right)$$
$$(x-5)^2=47$$
$$x-5=\pm\sqrt{47}$$
$$x=5\pm\sqrt{47}$$
The solutions are $5-\sqrt{47}$ and $5+\sqrt{47}$, or $5\pm\sqrt{47}$.

31.
$$x^2+8x+25=0$$
$$x^2+8x=-25 \quad \text{Subtracting 25}$$
$$x^2+8x+16=-25+16 \quad \text{Completing the square: } \tfrac{1}{2}\cdot 8=4 \text{ and } 4^2=16$$
$$(x+4)^2=-9 \quad \text{Factoring}$$
$$x+4=\pm 3i \quad \text{Using the principle of square roots}$$
$$x=-4\pm 3i$$
The solutions are $-4-3i$ and $-4+3i$, or $-4\pm 3i$.

32.
$$x^2+6x+13=0$$
$$x^2+6x=-13$$
$$x^2+6x+9=-13+9 \quad \left(\tfrac{1}{2}\cdot 6=3 \text{ and } 3^2=9\right)$$
$$(x+3)^2=-4$$
$$x+3=\pm 2i$$
$$x=-3\pm 2i$$
The solution are $-3-2i$ and $-3+2i$, or $-3\pm 2i$.

33.
$$3x^2+5x-2=0$$
$$3x^2+5x=2 \quad \text{Adding 2}$$
$$x^2+\frac{5}{3}x=\frac{2}{3} \quad \text{Dividing by 3}$$
$$x^2+\frac{5}{3}x+\frac{25}{36}=\frac{2}{3}+\frac{25}{36} \quad \text{Completing the square: } \tfrac{1}{2}\cdot\tfrac{5}{3}=\tfrac{5}{6} \text{ and } \left(\tfrac{5}{6}\right)^2=\tfrac{25}{36}$$
$$\left(x+\frac{5}{6}\right)^2=\frac{49}{36} \quad \text{Factoring and simplifying}$$
$$x+\frac{5}{6}=\pm\frac{7}{6} \quad \text{Using the principle of square roots}$$
$$x=-\frac{5}{6}\pm\frac{7}{6}$$
$$x=-\frac{5}{6}-\frac{7}{6} \quad or \quad x=-\frac{5}{6}+\frac{7}{6}$$
$$x=-\frac{12}{6} \quad or \quad x=\frac{2}{6}$$
$$x=-2 \quad or \quad x=\frac{1}{3}$$
The solutions are -2 and $\frac{1}{3}$.

34. $2x^2 - 5x - 3 = 0$

$2x^2 - 5x = 3$

$x^2 - \frac{5}{2}x = \frac{3}{2}$

$x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{3}{2} + \frac{25}{16}$ $\left(\frac{1}{2}(-\frac{5}{2}) = -\frac{5}{4}\right.$ and $\left.(-\frac{5}{4})^2 = \frac{25}{16}\right)$

$\left(x - \frac{5}{4}\right)^2 = \frac{49}{16}$

$x - \frac{5}{4} = \pm\frac{7}{4}$

$x = \frac{5}{4} \pm \frac{7}{4}$

$x = \frac{5}{4} - \frac{7}{4}$ *or* $x = \frac{5}{4} + \frac{7}{4}$

$x = -\frac{1}{2}$ *or* $x = 3$

The solutions are $-\frac{1}{2}$ and 3.

35. $x^2 - 2x = 15$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$ Factoring

$x - 5 = 0$ *or* $x + 3 = 0$

$x = 5$ *or* $x = -3$

The solutions are 5 and -3.

36. $x^2 + 4x = 5$

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x + 5 = 0$ *or* $x - 1 = 0$

$x = -5$ *or* $x = 1$

The solutions are -5 and 1.

37. $5m^2 + 3m = 2$

$5m^2 + 3m - 2 = 0$

$(5m-2)(m+1) = 0$ Factoring

$5m - 2 = 0$ *or* $m + 1 = 0$

$m = \frac{2}{5}$ *or* $m = -1$

The solutions are $\frac{2}{5}$ and -1.

38. $2y^2 - 3y - 2 = 0$

$(2y+1)(y-2) = 0$

$2y + 1 = 0$ *or* $y - 2 = 0$

$y = -\frac{1}{2}$ *or* $y = 2$

The solutions are $-\frac{1}{2}$ and 2.

39. $3x^2 + 6 = 10x$

$3x^2 - 10x + 6 = 0$

We use the quadratic formula. Here $a = 3$, $b = -10$, and $c = 6$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4\cdot 3\cdot 6}}{2\cdot 3}$ Substituting

$= \frac{10 \pm \sqrt{28}}{6} = \frac{10 \pm 2\sqrt{7}}{6}$

$= \frac{2(5 \pm \sqrt{7})}{2\cdot 3} = \frac{5 \pm \sqrt{7}}{3}$

The solutions are $\frac{5 - \sqrt{7}}{3}$ and $\frac{5 + \sqrt{7}}{3}$, or $\frac{5 \pm \sqrt{7}}{3}$.

40. $3t^2 + 8t + 3 = 0$

$t = \frac{-8 \pm \sqrt{8^2 - 4\cdot 3\cdot 3}}{2\cdot 3}$

$= \frac{-8 \pm \sqrt{28}}{6} = \frac{-8 \pm 2\sqrt{7}}{6}$

$= \frac{2(-4 \pm \sqrt{7})}{2\cdot 3} = \frac{-4 \pm \sqrt{7}}{3}$

The solutions are $\frac{-4 - \sqrt{7}}{3}$ and $\frac{-4 + \sqrt{7}}{3}$, or $\frac{-4 \pm \sqrt{7}}{3}$.

41. $x^2 + x + 2 = 0$

We use the quadratic formula. Here $a = 1$, $b = 1$, and $c = 2$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-1 \pm \sqrt{1^2 - 4\cdot 1\cdot 2}}{2\cdot 1}$ Substituting

$= \frac{-1 \pm \sqrt{-7}}{2}$

$= \frac{-1 \pm \sqrt{7}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{7}}{2}i$

The solutions are $-\frac{1}{2} - \frac{\sqrt{7}}{2}i$ and $-\frac{1}{2} + \frac{\sqrt{7}}{2}i$, or $-\frac{1}{2} \pm \frac{\sqrt{7}}{2}i$.

42. $x^2 + 1 = x$

$x^2 - x + 1 = 0$

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4\cdot 1\cdot 1}}{2\cdot 1}$

$= \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm \sqrt{3}i}{2}$

$= \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

The solutions are $\frac{1}{2} - \frac{\sqrt{3}}{2}i$ and $\frac{1}{2} + \frac{\sqrt{3}}{2}i$, or $\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$.

43. $5t^2 - 8t = 3$

$5t^2 - 8t - 3 = 0$

We use the quadratic formula. Here $a = 5$, $b = -8$, and $c = -3$.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 5(-3)}}{2 \cdot 5} = \frac{8 \pm \sqrt{124}}{10} = \frac{8 \pm 2\sqrt{31}}{10} = \frac{2(4 \pm \sqrt{31})}{2 \cdot 5} = \frac{4 \pm \sqrt{31}}{5}$$

The solutions are $\frac{4 - \sqrt{31}}{5}$ and $\frac{4 + \sqrt{31}}{5}$, or $\frac{4 \pm \sqrt{31}}{5}$.

44. $5x^2 + 2 = x$

$5x^2 - x + 2 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 5 \cdot 2}}{2 \cdot 5} = \frac{1 \pm \sqrt{-39}}{10} = \frac{1 \pm \sqrt{39}i}{10} = \frac{1}{10} \pm \frac{\sqrt{39}}{10}i$$

The solutions are $\frac{1}{10} - \frac{\sqrt{39}}{10}i$ and $\frac{1}{10} + \frac{\sqrt{39}}{10}i$, or $\frac{1}{10} \pm \frac{\sqrt{39}}{10}i$.

45. $3x^2 + 4 = 5x$

$3x^2 - 5x + 4 = 0$

We use the quadratic formula. Here $a = 3$, $b = -5$, and $c = 4$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 3 \cdot 4}}{2 \cdot 3} = \frac{5 \pm \sqrt{-23}}{6} = \frac{5 \pm \sqrt{23}i}{6} = \frac{5}{6} \pm \frac{\sqrt{23}}{6}i$$

The solutions are $\frac{5}{6} - \frac{\sqrt{23}}{6}i$ and $\frac{5}{6} + \frac{\sqrt{23}}{6}i$, or $\frac{5}{6} \pm \frac{\sqrt{23}}{6}i$.

46. $2t^2 - 5t = 1$

$2t^2 - 5t - 1 = 0$

$$t = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 2(-1)}}{2 \cdot 2} = \frac{5 \pm \sqrt{33}}{4}$$

The solutions are $\frac{5 - \sqrt{33}}{4}$ and $\frac{5 + \sqrt{33}}{4}$, or $\frac{5 \pm \sqrt{33}}{4}$.

47. $x^2 - 8x + 5 = 0$

We use the quadratic formula. Here $a = 1$, $b = -8$, and $c = 5$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1} = \frac{8 \pm \sqrt{44}}{2} = \frac{8 \pm 2\sqrt{11}}{2} = \frac{2(4 \pm \sqrt{11})}{2} = 4 \pm \sqrt{11}$$

The solutions are $4 - \sqrt{11}$ and $4 + \sqrt{11}$, or $4 \pm \sqrt{11}$.

48. $x^2 - 6x + 3 = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{6 \pm \sqrt{24}}{2} = \frac{6 \pm 2\sqrt{6}}{2} = \frac{2(3 \pm \sqrt{6})}{2} = 3 \pm \sqrt{6}$$

The solutions are $3 - \sqrt{6}$ and $3 + \sqrt{6}$, or $3 \pm \sqrt{6}$.

49. $3x^2 + x = 5$

$3x^2 + x - 5 = 0$

We use the quadratic formula. We have $a = 3$, $b = 1$, and $c = -5$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 3 \cdot (-5)}}{2 \cdot 3} = \frac{-1 \pm \sqrt{61}}{6}$$

The solutions are $\frac{-1 - \sqrt{61}}{6}$ and $\frac{-1 + \sqrt{61}}{6}$, or $\frac{-1 \pm \sqrt{61}}{6}$.

50. $5x^2 + 3x = 1$

$5x^2 + 3x - 1 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 5 \cdot (-1)}}{2 \cdot 5} = \frac{-3 \pm \sqrt{29}}{10}$$

The solutions are $\frac{-3 - \sqrt{29}}{10}$ and $\frac{-3 + \sqrt{29}}{10}$, or $\frac{-3 \pm \sqrt{29}}{10}$.

51. $2x^2+1=5x$

$2x^2-5x+1=0$

We use the quadratic formula. We have $a=2$, $b=-5$, and $c=1$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-5)\pm\sqrt{(-5)^2-4\cdot 2\cdot 1}}{2\cdot 2}=\frac{5\pm\sqrt{17}}{4}$$

The solutions are $\frac{5-\sqrt{17}}{4}$ and $\frac{5+\sqrt{17}}{4}$, or $\frac{5\pm\sqrt{17}}{4}$.

52. $4x^2+3=x$

$4x^2-x+3=0$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot 4\cdot 3}}{2\cdot 4}$$
$$=\frac{1\pm\sqrt{-47}}{8}=\frac{1\pm\sqrt{47}i}{8}=\frac{1}{8}\pm\frac{\sqrt{47}}{8}i$$

The solutions are $\frac{1}{8}-\frac{\sqrt{47}}{8}i$ and $\frac{1}{8}+\frac{\sqrt{47}}{8}i$, or $\frac{1}{8}\pm\frac{\sqrt{47}}{8}i$.

53. $5x^2+2x=-2$

$5x^2+2x+2=0$

We use the quadratic formula. We have $a=5$, $b=2$, and $c=2$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-2\pm\sqrt{2^2-4\cdot 5\cdot 2}}{2\cdot 5}$$
$$=\frac{-2\pm\sqrt{-36}}{10}=\frac{-2\pm 6i}{10}$$
$$=\frac{2(-1\pm 3i)}{2\cdot 5}=\frac{-1\pm 3i}{5}$$
$$=-\frac{1}{5}\pm\frac{3}{5}i$$

The solutions are $-\frac{1}{5}-\frac{3}{5}i$ and $-\frac{1}{5}+\frac{3}{5}i$, or $-\frac{1}{5}\pm\frac{3}{5}i$.

54. $3x^2+3x=-4$

$3x^2+3x+4=0$

$$x=\frac{-3\pm\sqrt{3^2-4\cdot 3\cdot 4}}{2\cdot 3}$$
$$=\frac{-3\pm\sqrt{-39}}{6}=\frac{-3\pm\sqrt{39}i}{6}$$
$$=-\frac{1}{2}\pm\frac{\sqrt{39}}{6}i$$

The solutions are $-\frac{1}{2}-\frac{\sqrt{39}}{6}i$ and $-\frac{1}{2}+\frac{\sqrt{39}}{6}i$ or $-\frac{1}{2}\pm\frac{\sqrt{39}}{6}i$.

55. $4x^2=8x+5$

$4x^2-8x-5=0$

$a=4,\ b=-8,\ c=-5$

$b^2-4ac=(-8)^2-4\cdot 4(-5)=144$

Since $b^2-4ac>0$, there are two different real-number solutions.

56. $4x^2-12x+9=0$

$b^2-4ac=(-12)^2-4\cdot 4\cdot 9=0$

There is one real-number solution.

57. $x^2+3x+4=0$

$a=1,\ b=3,\ c=4$

$b^2-4ac=3^2-4\cdot 1\cdot 4=-7$

Since $b^2-4ac<0$, there are two different imaginary-number solutions.

58. $x^2-2x+4=0$

$b^2-4ac=(-2)^2-4\cdot 1\cdot 4=-12<0$

There are two different imaginary-number solutions.

59. $5t^2-7t=0$

$a=5,\ b=-7,\ c=0$

$b^2-4ac=(-7)^2-4\cdot 5\cdot 0=49$

Since $b^2-4ac>0$, there are two different real-number solutions.

60. $5t^2-4t=11$

$5t^2-4t-11=0$

$b^2-4ac=(-4)^2-4\cdot 5(-11)=236>0$

There are two different real-number solutions.

61. $x^2+6x+5=0$ Setting $f(x)=0$

$(x+5)(x+1)=0$ Factoring

$x+5=0$ *or* $x+1=0$

$x=-5$ *or* $x=-1$

The zeros of the function are -5 and -1.

62. $x^2-x-2=0$

$(x+1)(x-2)=0$

$x+1=0$ *or* $x-2=0$

$x=-1$ *or* $x=2$

The zeros of the function are -1 and 2.

63. $x^2-3x-3=0$

$a=1$, $b=-3$, $c=-3$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-3)\pm\sqrt{(-3)^2-4\cdot 1\cdot(-3)}}{2\cdot 1}$$
$$=\frac{3\pm\sqrt{9+12}}{2}$$
$$=\frac{3\pm\sqrt{21}}{2}$$

The zeros of the function are $\frac{3-\sqrt{21}}{2}$ and $\frac{3+\sqrt{21}}{2}$, or $\frac{3\pm\sqrt{21}}{2}$.

64. $3x^2+8x+2=0$

$$x=\frac{-8\pm\sqrt{8^2-4\cdot3\cdot2}}{2\cdot3}$$
$$=\frac{-8\pm\sqrt{40}}{6}=\frac{-8\pm2\sqrt{10}}{6}$$
$$=\frac{-4\pm\sqrt{10}}{3}$$

The zeros of the function are $\frac{-4-\sqrt{10}}{3}$ and $\frac{-4+\sqrt{10}}{3}$, or $\frac{-4\pm\sqrt{10}}{3}$.

65. $x^2-5x+1=0$

$a=1,\ b=-5,\ c=1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-5)\pm\sqrt{(-5)^2-4\cdot1\cdot1}}{2\cdot1}$$
$$=\frac{5\pm\sqrt{25-4}}{2}$$
$$=\frac{5\pm\sqrt{21}}{2}$$

The zeros of the function are $\frac{5-\sqrt{21}}{2}$ and $\frac{5+\sqrt{21}}{2}$, or $\frac{5\pm\sqrt{21}}{2}$.

66. $x^2-3x-7=0$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4\cdot1\cdot(-7)}}{2\cdot1}$$
$$=\frac{3\pm\sqrt{37}}{2}$$

The zeros of the function are $\frac{3-\sqrt{37}}{2}$ and $\frac{3+\sqrt{37}}{2}$, or $\frac{3\pm\sqrt{37}}{2}$.

67. $x^2+2x-5=0$

$a=1,\ b=2,\ c=-5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-2\pm\sqrt{2^2-4\cdot1\cdot(-5)}}{2\cdot1}$$
$$=\frac{-2\pm\sqrt{4+20}}{2}=\frac{-2\pm\sqrt{24}}{2}$$
$$=\frac{-2\pm2\sqrt{6}}{2}=-1\pm\sqrt{6}$$

The zeros of the function are $-1+\sqrt{6}$ and $-1-\sqrt{6}$, or $-1\pm\sqrt{6}$.

68. $x^2-x-4=0$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot1\cdot(-4)}}{2\cdot1}$$
$$=\frac{1\pm\sqrt{17}}{2}$$

The zeros of the function are $\frac{1+\sqrt{17}}{2}$ or $\frac{1-\sqrt{17}}{2}$, or $\frac{1\pm\sqrt{17}}{2}$.

69. $2x^2-x+4=0$

$a=2,\ b=-1,\ c=4$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot2\cdot4}}{2\cdot2}$$
$$=\frac{1\pm\sqrt{-31}}{4}=\frac{1\pm\sqrt{31}i}{4}$$
$$=\frac{1}{4}\pm\frac{\sqrt{31}}{4}i$$

The zeros of the function are $\frac{1}{4}-\frac{\sqrt{31}}{4}i$ and $\frac{1}{4}+\frac{\sqrt{31}}{4}i$, or $\frac{1}{4}\pm\frac{\sqrt{31}}{4}i$.

70. $2x^2+3x+2=0$

$$x=\frac{-3\pm\sqrt{3^2-4\cdot2\cdot2}}{2\cdot2}$$
$$=\frac{-3\pm\sqrt{-7}}{4}=\frac{-3\pm\sqrt{7}i}{4}$$
$$=-\frac{3}{4}\pm\frac{\sqrt{7}}{4}i$$

The zeros of the function are $-\frac{3}{4}-\frac{\sqrt{7}}{4}i$ and $-\frac{3}{4}+\frac{\sqrt{7}}{4}i$, or $-\frac{3}{4}\pm\frac{\sqrt{7}}{4}i$.

71. $3x^2-x-1=0$

$a=3,\ b=-1,\ c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot3\cdot(-1)}}{2\cdot3}$$
$$=\frac{1\pm\sqrt{13}}{6}$$

The zeros of the function are $\frac{1-\sqrt{13}}{6}$ and $\frac{1+\sqrt{13}}{6}$, or $\frac{1\pm\sqrt{13}}{6}$.

72. $3x^2+5x+1=0$

$$x=\frac{-5\pm\sqrt{5^2-4\cdot3\cdot1}}{2\cdot3}$$
$$=\frac{-5\pm\sqrt{13}}{6}$$

The zeros of the function are $\frac{-5-\sqrt{13}}{6}$ and $\frac{-5+\sqrt{13}}{6}$, or $\frac{-5\pm\sqrt{13}}{6}$.

73. $5x^2 - 2x - 1 = 0$

$a = 5,\ b = -2,\ c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 5 \cdot (-1)}}{2 \cdot 5}$$
$$= \frac{2 \pm \sqrt{24}}{10} = \frac{2 \pm 2\sqrt{6}}{10}$$
$$= \frac{2(1 \pm \sqrt{6})}{2 \cdot 5} = \frac{1 \pm \sqrt{6}}{5}$$

The zeros of the function are $\frac{1 - \sqrt{6}}{5}$ and $\frac{1 + \sqrt{6}}{5}$, or $\frac{1 \pm \sqrt{6}}{5}$.

74. $4x^2 - 4x - 5 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 4 \cdot (-5)}}{2 \cdot 4}$$
$$= \frac{4 \pm \sqrt{96}}{8} = \frac{4 \pm 4\sqrt{6}}{8}$$
$$= \frac{4(1 \pm \sqrt{6})}{4 \cdot 2} = \frac{1 \pm \sqrt{6}}{2}$$

The zeros of the function are $\frac{1 - \sqrt{6}}{2}$ and $\frac{1 + \sqrt{6}}{2}$, or $\frac{1 \pm \sqrt{6}}{2}$.

75. $4x^2 + 3x - 3 = 0$

$a = 4,\ b = 3,\ c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{3^2 - 4 \cdot 4 \cdot (-3)}}{2 \cdot 4}$$
$$= \frac{-3 \pm \sqrt{57}}{8}$$

The zeros of the function are $\frac{-3 - \sqrt{57}}{8}$ and $\frac{-3 + \sqrt{57}}{8}$, or $\frac{-3 \pm \sqrt{57}}{8}$.

76. $x^2 + 6x - 3 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1}$$
$$= \frac{-6 \pm \sqrt{48}}{2} = \frac{-6 \pm 4\sqrt{3}}{2}$$
$$= \frac{2(-3 \pm 2\sqrt{3})}{2} = -3 \pm 2\sqrt{3}$$

The zeros of the function are $-3 - 2\sqrt{3}$ and $-3 + 2\sqrt{3}$, or $-3 \pm 2\sqrt{3}$.

77. $x^4 - 3x^2 + 2 = 0$

Let $u = x^2$.

$u^2 - 3u + 2 = 0$ Substituting u for x^2

$(u - 1)(u - 2) = 0$

$u - 1 = 0$ *or* $u - 2 = 0$

$u = 1$ *or* $u = 2$

Now substitute x^2 for u and solve for x.

$x^2 = 1$ *or* $x^2 = 2$

$x = \pm 1$ *or* $x = \pm\sqrt{2}$

The solutions are -1, 1, $-\sqrt{2}$, and $\sqrt{2}$.

78. $x^4 + 3 = 4x^2$

$x^4 - 4x^2 + 3 = 0$

Let $u = x^2$.

$u^2 - 4u + 3 = 0$ Substituting u for x^2

$(u - 1)(u - 3) = 0$

$u - 1 = 0$ *or* $u - 3 = 0$

$u = 1$ *or* $u = 3$

Substitute x^2 for u and solve for x.

$x^2 = 1$ *or* $x^2 = 3$

$x = \pm 1$ *or* $x = \pm\sqrt{3}$

The solutions are -1, 1, $-\sqrt{3}$, and $\sqrt{3}$.

79. $x^4 + 3x^2 = 10$

$x^4 + 3x^2 - 10 = 0$

Let $u = x^2$.

$u^2 + 3u - 10 = 0$ Substituting u for x^2

$(u + 5)(u - 2) = 0$

$u + 5 = 0$ *or* $u - 2 = 0$

$u = -5$ *or* $u = 2$

Now substitute x^2 for u and solve for x.

$x^2 = -5$ *or* $x^2 = 2$

$x = \pm\sqrt{5}i$ *or* $x = \pm\sqrt{2}$

The solutions are $-\sqrt{5}i$, $\sqrt{5}i$, $-\sqrt{2}$, and $\sqrt{2}$.

80. $x^4 - 8x^2 = 9$

$x^4 - 8x^2 - 9 = 0$

Let $u = x^2$.

$u^2 - 8u - 9 = 0$ Substituting u for x^2

$(u - 9)(u + 1) = 0$

$u - 9 = 0$ *or* $u + 1 = 0$

$u = 9$ *or* $u = -1$

Now substitute x^2 for u and solve for x.

$x^2 = 9$ *or* $x^2 = -1$

$x = \pm 3$ *or* $x = \pm i$

The solutions are -3, 3, i, and $-i$.

81. $y^4 + 4y^2 - 5 = 0$

Let $u = y^2$.

$u^2 + 4u - 5 = 0$ Substituting u for y^2

$(u - 1)(u + 5) = 0$

$u - 1 = 0$ *or* $u + 5 = 0$

$u = 1$ *or* $u = -5$

Substitute y^2 for u and solve for y.

$y^2 = 1 \quad or \quad y^2 = -5$

$y = \pm 1 \quad or \quad y = \pm\sqrt{5}i$

The solutions are -1, 1, $-\sqrt{5}i$, and $\sqrt{5}i$.

82. $y^4 - 15y^2 - 16 = 0$

Let $u = y^2$.

$u^2 - 15u - 16 = 0$ Substituting u for y^2

$(u-16)(u+1) = 0$

$u - 16 = 0 \quad or \quad u + 1 = 0$

$u = 16 \quad or \quad u = -1$

Now substitute y^2 for u and solve for y.

$y^2 = 16 \quad or \quad y^2 = -1$

$y = \pm 4 \quad or \quad y = \pm i$

The solutions are -4, 4, $-i$ and i.

83. $x - 3\sqrt{x} - 4 = 0$

Let $u = \sqrt{x}$.

$u^2 - 3u - 4 = 0$ Substituting u for $\sqrt{x}$

$(u+1)(u-4) = 0$

$u + 1 = 0 \quad or \quad u - 4 = 0$

$u = -1 \quad or \quad u = 4$

Now substitute $\sqrt{x}$ for u and solve for x.

$\sqrt{x} = -1 \quad or \quad \sqrt{x} = 4$

No solution $\quad x = 16$

Note that $\sqrt{x}$ must be nonnegative, so $\sqrt{x} = -1$ has no solution. The number 16 checks and is the solution. The solution is 16.

84. $2x - 9\sqrt{x} + 4 = 0$

Let $u = \sqrt{x}$.

$2u^2 - 9u + 4 = 0$ Substituting u for $\sqrt{x}$

$(2u-1)(u-4) = 0$

$2u - 1 = 0 \quad or \quad u - 4 = 0$

$u = \frac{1}{2} \quad or \quad u = 4$

Substitute $\sqrt{x}$ for u and solve for u.

$\sqrt{x} = \frac{1}{2} \quad or \quad \sqrt{x} = 4$

$x = \frac{1}{4} \quad or \quad x = 16$

Both numbers check. The solutions are $\frac{1}{4}$ and 16.

85. $m^{2/3} - 2m^{1/3} - 8 = 0$

Let $u = m^{1/3}$.

$u^2 - 2u - 8 = 0$ Substituting u for $m^{1/3}$

$(u+2)(u-4) = 0$

$u + 2 = 0 \quad or \quad u - 4 = 0$

$u = -2 \quad or \quad u = 4$

Now substitute $m^{1/3}$ for u and solve for m.

$m^{1/3} = -2 \quad or \quad m^{1/3} = 4$

$(m^{1/3})^3 = (-2)^3 \quad or \quad (m^{1/3})^3 = 4^3$ Using the principle of powers

$m = -8 \quad or \quad m = 64$

The solutions are -8 and 64.

86. $t^{2/3} + t^{1/3} - 6 = 0$

Let $u = t^{1/3}$.

$u^2 + u - 6 = 0$

$(u+3)(u-2) = 0$

$u + 3 = 0 \quad or \quad u - 2 = 0$

$u = -3 \quad or \quad u = 2$

Substitute $t^{1/3}$ for u and solve for t.

$t^{1/3} = -3 \quad or \quad t^{1/3} = 2$

$t = -27 \quad or \quad t = 8$

The solutions are -27 and 8.

87. $x^{1/2} - 3x^{1/4} + 2 = 0$

Let $u = x^{1/4}$.

$u^2 - 3u + 2 = 0$ Substituting u for $x^{1/4}$

$(u-1)(u-2) = 0$

$u - 1 = 0 \quad or \quad u - 2 = 0$

$u = 1 \quad or \quad u = 2$

Now substitute $x^{1/4}$ for u and solve for x.

$x^{1/4} = 1 \quad or \quad x^{1/4} = 2$

$(x^{1/4})^4 = 1^4 \quad or \quad (x^{1/4})^4 = 2^4$

$x = 1 \quad or \quad x = 16$

The solutions are 1 and 16.

88. $x^{1/2} - 4x^{1/4} = -3$

$x^{1/2} - 4x^{1/4} + 3 = 0$

Let $u = x^{1/4}$.

$u^2 - 4u + 3 = 0$

$(u-1)(u-3) = 0$

$u - 1 = 0 \quad or \quad u - 3 = 0$

$u = 1 \quad or \quad u = 3$

Substitute $x^{1/4}$ for u and solve for x.

$x^{1/4} = 1 \quad or \quad x^{1/4} = 3$

$x = 1 \quad or \quad x = 81$

The solutions are 1 and 81.

89. $(2x-3)^2 - 5(2x-3) + 6 = 0$

Let $u = 2x - 3$.

$u^2 - 5u + 6 = 0$ Substituting u for $2x - 3$

$(u-2)(u-3) = 0$

$u - 2 = 0 \quad or \quad u - 3 = 0$

$u = 2 \quad or \quad u = 3$

Now substitute $2x - 3$ for u and solve for x.

$2x-3=2$ *or* $2x-3=3$

$2x=5$ *or* $2x=6$

$x=\frac{5}{2}$ *or* $x=3$

The solutions are $\frac{5}{2}$ and 3.

90. $(3x+2)^2+7(3x+2)-8=0$

Let $u=3x+2$.

$u^2+7u-8=0$ Substituting u for $3x+2$

$(u+8)(u-1)=0$

$u+8=0$ *or* $u-1=0$

$u=-8$ *or* $u=1$

Substitute $3x+2$ for u and solve for x.

$3x+2=-8$ *or* $3x+2=1$

$3x=-10$ *or* $3x=-1$

$x=-\frac{10}{3}$ *or* $x=-\frac{1}{3}$

The solutions are $-\frac{10}{3}$ and $-\frac{1}{3}$.

91. $(2t^2+t)^2-4(2t^2+t)+3=0$

Let $u=2t^2+t$.

$u^2-4u+3=0$ Substituting u for $2t^2+t$

$(u-1)(u-3)=0$

$u-1=0$ *or* $u-3=0$

$u=1$ *or* $u=3$

Now substitute $2t^2+t$ for u and solve for t.

$2t^2+t=1$ *or* $2t^2+t=3$

$2t^2+t-1=0$ *or* $2t^2+t-3=0$

$(2t-1)(t+1)=0$ *or* $(2t+3)(t-1)=0$

$2t-1=0$ *or* $t+1=0$ *or* $2t+3=0$ *or* $t-1=0$

$t=\frac{1}{2}$ *or* $t=-1$ *or* $t=-\frac{3}{2}$ *or* $t=1$

The solutions are $\frac{1}{2}$, -1, $-\frac{3}{2}$ and 1.

92. $12=(m^2-5m)^2+(m^2-5m)$

$0=(m^2-5m)^2+(m^2-5m)-12$

Let $u=m^2-5m$.

$0=u^2+u-12$ Substituting u for m^2-5m

$0=(u+4)(u-3)$

$u+4=0$ *or* $u-3=0$

$u=-4$ *or* $u=3$

Substitute m^2-5m for u and solve for m.

$m^2-5m=-4$ *or* $m^2-5m=3$

$m^2-5m+4=0$ *or* $m^2-5m-3=0$

$(m-1)(m-4)=0$ *or*

$$m=\frac{-(-5)\pm\sqrt{(-5)^2-4\cdot 1(-3)}}{2\cdot 1}$$

$m=1$ *or* $m=4$ *or* $m=\frac{5\pm\sqrt{37}}{2}$

The solutions are 1, 4, $\frac{5-\sqrt{37}}{2}$, and $\frac{5+\sqrt{37}}{2}$, or 1, 4, and $\frac{5\pm\sqrt{37}}{2}$.

93. ***Familiarize and Translate***. We will use the formula $s=16t^2$, substituting 2120 for s.

$2120=16t^2$

Carry out. We solve the equation.

$2120=16t^2$

$132.5=t^2$ Dividing by 16 on both sides

$11.5\approx t$ Taking the square root on both sides

Check. When $t=11.5$, $s=16(11.5)^2=2116\approx 2120$. The answer checks.

State. It would take an object about 11.5 sec to reach the ground.

94. Solve: $2063=16t^2$

$t\approx 11.4$ sec

95. Substitute 9.7 for $w(x)$ and solve for x.

$-0.01x^2+0.27x+8.60=9.7$

$-0.01x^2+0.27x-1.1=0$

$a=-0.01,\ b=0.27,\ c=-1.1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-0.27\pm\sqrt{(0.27)^2-4(-0.01)(-1.1)}}{2(-0.01)}$$

$$=\frac{-0.27\pm\sqrt{0.0289}}{-0.02}$$

$x\approx 5$ *or* $x\approx 22$

There were 9.7 million self-employed workers in the United States 5 years after 1980 and also 22 years after 1980, or in 1985 and in 2002.

96. Solve: $-0.01x^2+0.27x+8.60=9.1$

$x\approx 2$ or $x\approx 25$, so there were or will be 9.1 self-employed workers in the United States 2 years after 1980, or in 1982, and 25 years after 1980, or in 2005.

97. ***Familiarize***. Let $w=$ the width of the rug. Then $w+1=$ the length.

Translate. We use the Pythagorean equation.

$w^2+(w+1)^2=5^2$

Carry out. We solve the equation.

$w^2+(w+1)^2=5^2$

$w^2+w^2+2w+1=25$

$2w^2+2w+1=25$

$2w^2+2w-24=0$

$2(w+4)(w-3)=0$

$w+4=0$ *or* $w-3=0$

$w=-4$ *or* $w=3$

Since the width cannot be negative, we consider only 3. When $w=3$, $w+1=3+1=4$.

Check. The length, 4 ft, is 1 ft more than the width, 3 ft. The length of a diagonal of a rectangle with width 3 ft and length 4 ft is $\sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$. The answer checks.

State. The length is 4 ft, and the width is 3 ft.

98. Let x = the length of the longer leg.

Solve: $x^2 + (x-7)^2 = 13^2$

$x = -5$ *or* $x = 12$

Only 12 has meaning in the original problem. The length of one leg is 12 cm, and the length of the other leg is $12-7$, or 5 cm.

99. ***Familiarize***. Let n = the smaller number. Then $n+5$ = the larger number.

Translate.

$$\underbrace{\text{The product of the numbers}}_{\downarrow\; n(n+5)} \quad \underset{\downarrow\; =}{\text{is}} \quad \underset{\downarrow\; 36}{36.}$$

Carry out.

$$\begin{aligned} n(n+5) &= 36 \\ n^2+5n &= 36 \\ n^2+5n-36 &= 0 \\ (n+9)(n-4) &= 0 \end{aligned}$$

$$n+9=0 \quad \textit{or} \quad n-4=0$$
$$n=-9 \quad \textit{or} \quad n=4$$

If $n=-9$, then $n+5=-9+5=-4$. If $n=4$, then $n+5=4+5=9$.

Check. The number -4 is 5 more than -9 and $(-4)(-9)=36$, so the pair -9 and -4 check. The number 9 is 5 more than 4 and $9\cdot 4 = 36$, so the pair 4 and 9 also check.

State. The numbers are -9 and -4 or 4 and 9.

100. Let n = the larger number.

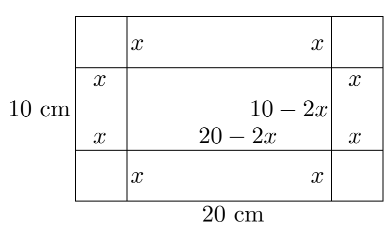
Solve: $n(n-6)=72$

$n=-6$ *or* $n=12$

When $n=-6$, then $n-6=-6-6=-12$, so one pair of numbers is -6 and -12. When $n=12$, then $n-6=12-6=6$, so the other pair of numbers is 6 and 12.

101. ***Familiarize***. We add labels to the drawing in the text.

We let x represent the length of a side of the square in each corner. Then the length and width of the resulting base are represented by $20-2x$ and $10-2x$, respectively. Recall that for a rectangle, Area = length × width.



Translate.

$$\underbrace{\text{The area of the base}}_{(20-2x)(10-2x)} \quad \underset{=}{\text{is}} \quad \underbrace{96\ \text{cm}^2.}_{96}$$

Carry out. We solve the equation.

$$\begin{aligned} 200-60x+4x^2 &= 96 \\ 4x^2-60x+104 &= 0 \\ x^2-15x+26 &= 0 \\ (x-13)(x-2) &= 0 \end{aligned}$$

$$x-13=0 \quad \textit{or} \quad x-2=0$$
$$x=13 \quad \textit{or} \quad x=2$$

Check. When $x=13$, both $20-2x$ and $10-2x$ are negative numbers, so we only consider $x=2$. When $x=2$, then $20-2x=20-2\cdot 2=16$ and $10-2x=10-2\cdot 2=6$, and the area of the base is $16\cdot 6$, or 96 cm^2. The answer checks.

State. The length of the sides of the squares is 2 cm.

102. Let w = the width of the frame.

Solve: $(32-2w)(28-2w)=192$

$w=8$ or $w=22$

Only 8 has meaning in the original problem. The width of the frame is 8 cm.

103. ***Familiarize***. We have $P=2l+2w$, or $28=2l+2w$. Solving for w, we have

$$\begin{aligned} 28 &= 2l+2w \\ 14 &= l+w \quad \text{Dividing by 2} \\ 14-l &= w. \end{aligned}$$

Then we have l = the length of the rug and $14-l$ = the width, in feet. Recall that the area of a rectangle is the product of the length and the width.

Translate.

$$\underbrace{\text{The area}}_{\downarrow\; l(14-l)} \quad \underset{\downarrow\; =}{\text{is}} \quad \underbrace{48\ \text{ft}^2.}_{\downarrow\; 48}$$

Carry out. We solve the equation.

$$\begin{aligned} l(14-l) &= 48 \\ 14l-l^2 &= 48 \\ 0 &= l^2-14l+48 \\ 0 &= (l-6)(l-8) \end{aligned}$$

$$l-6=0 \quad \textit{or} \quad l-8=0$$
$$l=6 \quad \textit{or} \quad l=8$$

If $l=6$, then $14-l=14-6=8$.

If $l=8$, then $14-l=14-8=6$.

In either case, the dimensions are 8 ft by 6 ft. Since we usually consider the length to be greater than the width, we let 8 ft = the length and 6 ft = the width.

Check. The perimeter is $2\cdot 8$ ft$+2\cdot 6$ ft $=16$ ft $+12$ ft $=28$ ft. The answer checks.

State. The length of the rug is 8 ft, and the width is 6 ft.

104. We have $170 = 2l + 2w$, so $w = 85 - l$.

Solve: $l(85 - l) = 1750$

$l = 35$ *or* $l = 50$

Choosing the larger number to be the length, we find that the length of the petting area is 50 m, and the width is 35 m.

105. $f(x) = 4 - 5x = -5x + 4$

The function can be written in the form $y = mx + b$, so it is a linear function.

106. $f(x) = 4 - 5x^2 = -5x^2 + 4$

The function can be written in the form $f(x) = ax^2 + bx + c$, $a \neq 0$, so it is a quadratic function.

107. $f(x) = 7x^2$

The function is in the form $f(x) = ax^2 + bx + c$, $a \neq 0$, so it is a quadratic function.

108. $f(x) = 23x + 6$

The function is in the form $f(x) = mx + b$, so it is a linear function.

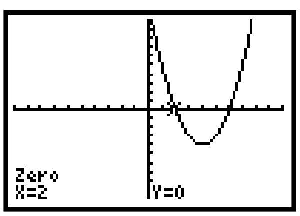
109. $f(x) = 1.2x - (3.6)^2$

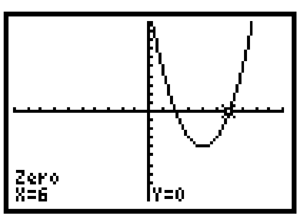
The function is in the form $f(x) = mx + b$, so it is a linear function.

110. $f(x) = 2 - x - x^2 = -x^2 - x + 2$

The function can be written in the form $f(x) = ax^2 + bx + c$, $a \neq 0$, so it is a quadratic function.

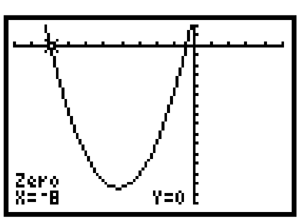
111. Graph $y = x^2 - 8x + 12$ and use the Zero feature twice.

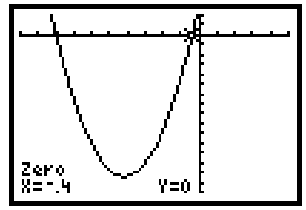




The solutions are 2 and 6.

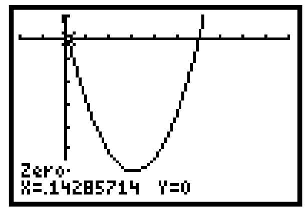
112. Graph $y = 5x^2 + 42x + 16$ and use the Zero feature twice.

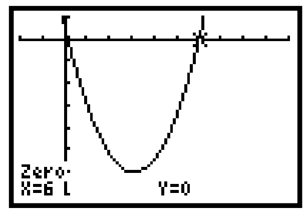




The solutions are -8 and -0.4.

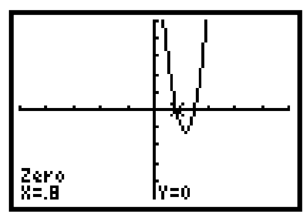
113. Graph $y = 7x^2 - 43x + 6$ and use the Zero feature twice.

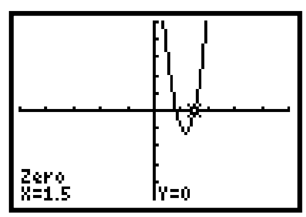




One solution is approximately 0.143 and the other is 6.

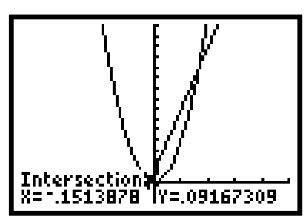
114. Graph $y = 10x^2 - 23x + 12$ and use the Zero feature twice.

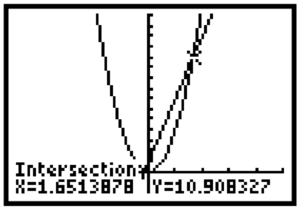




The solutions are 0.8 and 1.5.

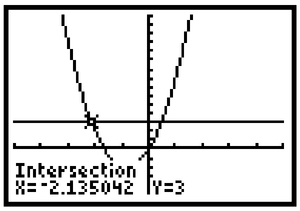
115. Graph $y_1 = 6x + 1$ and $y_2 = 4x^2$ and use the Intersect feature twice.

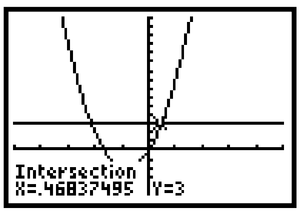




The solutions are approximately -0.151 and 1.651.

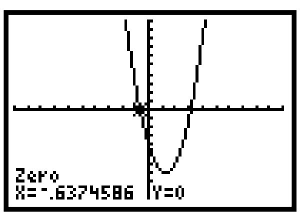
116. Graph $y_1 = 3x^2 + 5x$ and $y_2 = 3$ and use the Intersect feature twice.

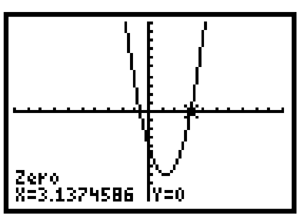




The solutions are approximately -2.135 and 0.468.

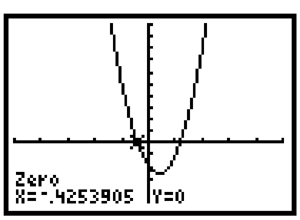
117. Graph $y = 2x^2 - 5x - 4$ and use the Zero feature twice.

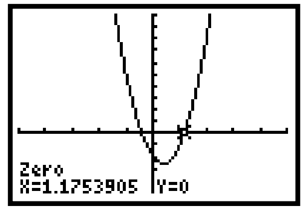




The zeros are approximately -0.637 and 3.137.

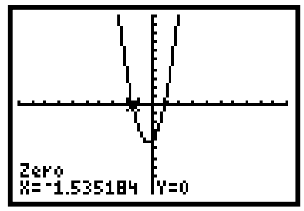
118. Graph $y = 4x^2 - 3x - 2$ and use the Zero feature twice.

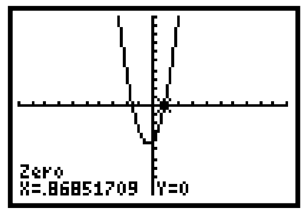




The zeros are approximately -0.425 and 1.175.

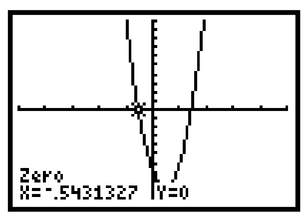
119. Graph $y = 3x^2 + 2x - 4$ and use the Zero feature twice.

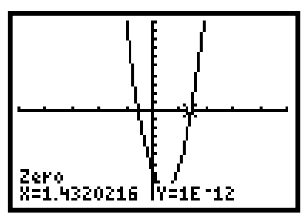




The zeros are approximately -1.535 and 0.869.

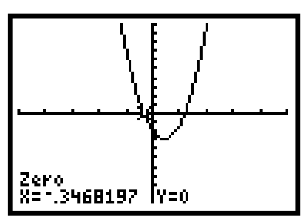
120. Graph $y = 9x^2 - 8x - 7$ and use the Zero feature twice.

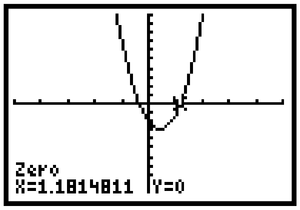




The zeros are approximately -0.543 and 1.432.

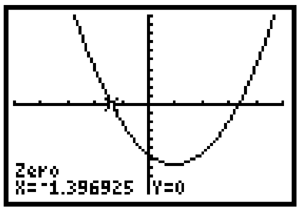
121. Graph $y = 5.02x^2 - 4.19x - 2.057$ and use the Zero feature twice.

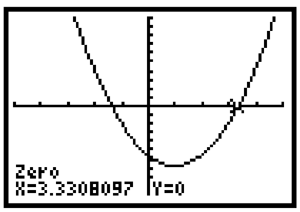




The zeros are approximately -0.347 and 1.181.

122. Graph $y = 1.21x^2 - 2.34x - 5.63$ and use the Zero feature twice.





The zeros are approximately -1.397 and 3.331.

123. No; consider the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. If $b^2 - 4ac = 0$, then $x = \dfrac{-b}{2a}$, so there is one real zero. If $b^2 - 4ac > 0$, then $\sqrt{b^2 - 4ac}$ is a real number and there are two real zeros. If $b^2 - 4ac < 0$, then $\sqrt{b^2 - 4ac}$ is an imaginary number and there are two imaginary zeros. Thus, a quadratic function cannot have one real zero and one imaginary zero.

124. Use the discriminant. If $b^2 - 4ac < 0$, there are no x-intercepts. If $b^2 - 4ac = 0$, there is one x-intercept. If $b^2 - 4ac > 0$, there are two x-intercepts.

125. $1998 - 1980 = 18$, so we substitute 18 for x.
$a(18) = 9096(18) + 387,725 = 551,453$ associate's degrees

126. $2010 - 1980 = 30$

$a(30) = 9096(30) + 387,725 = 660,605$ associate's degrees

127. Test for symmetry with respect to the x-axis:

$3x^2 + 4y^2 = 5$ Original equation
$3x^2 + 4(-y)^2 = 5$ Replacing y by $-y$
$3x^2 + 4y^2 = 5$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$3x^2 + 4y^2 = 5$ Original equation
$3(-x)^2 + 4y^2 = 5$ Replacing x by $-x$
$3x^2 + 4y^2 = 5$ Simplifying

The last equation is equivalent to the original equation, so the equation is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$3x^2 + 4y^2 = 5$ Original equation
$3(-x)^2 + 4(-y)^2 = 5$ Replacing x by $-x$ and y by $-y$
$3x^2 + 4y^2 = 5$ Simplifying

The last equation is equivalent to the original equation, so the equation is symmetric with respect to the origin.

128. Test for symmetry with respect to the x-axis:

$y^3 = 6x^2$ Original equation
$(-y)^3 = 6x^2$ Replacing y by $-y$
$-y^3 = 6x$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$y^3 = 6x^2$ Original equation
$y^3 = 6(-x)^2$ Replacing x by $-x$
$y^3 = 6x^2$ Simplifying

The last equation is equivalent to the original equation, so the equation is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$y^3 = 6x^2$ Original equation
$(-y)^3 = 6(-x)^2$ Replacing x by $-x$ y by $-y$
$-y^3 = 6x^2$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

129. $f(x) = 2x^3 - x$

$f(-x) = 2(-x)^3 - (-x) = -2x^3 + x$

$-f(x) = -2x^3 + x$

$f(x) \neq f(-x)$ so f is not even

$f(-x) = -f(x)$, so f is odd.

130. $f(x) = 4x^2 + 2x - 3$

$f(-x) = 4(-x)^2 + 2(-x) - 3 = 4x^2 - 2x - 3$

$-f(x) = -4x^2 - 2x + 3$

$f(x) \neq f(-x)$ so f is not even

$f(-x) \neq -f(x)$, so f is not odd.

Thus $f(x) = 4x^2 + 2x - 3$ is neither even nor odd.

131. a) $kx^2 - 17x + 33 = 0$

$k(3)^2 - 17(3) + 33 = 0$ Substituting 3 for x

$9k - 51 + 33 = 0$

$9k = 18$

$k = 2$

b) $2x^2 - 17x + 33 = 0$ Substituting 2 for k

$(2x-11)(x-3) = 0$

$2x - 11 = 0$ *or* $x - 3 = 0$

$x = \frac{11}{2}$ *or* $x = 3$

The other solution is $\frac{11}{2}$.

132. a) $kx^2 - 2x + k = 0$

$k(-3)^2 - 2(-3) + k = 0$ Substituting -3 for x

$9k + 6 + k = 0$

$10k = -6$

$k = -\frac{3}{5}$

b) $-\frac{3}{5}x^2 - 2x - \frac{3}{5} = 0$ Substituting $-\frac{3}{5}$ for k

$3x^2 + 10x + 3 = 0$ Multiplying by -5

$(3x+1)(x+3) = 0$

$3x + 1 = 0$ *or* $x + 3 = 0$

$3x = -1$ *or* $x = -3$

$x = -\frac{1}{3}$ *or* $x = -3$

The other solution is $-\frac{1}{3}$.

133. a) $(1+i)^2 - k(1+i) + 2 = 0$ Substituting $1+i$ for x

$1 + 2i - 1 - k - ki + 2 = 0$

$2 + 2i = k + ki$

$2(1+i) = k(1+i)$

$2 = k$

b) $x^2 - 2x + 2 = 0$ Substituting 2 for k

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4\cdot 1\cdot 2}}{2\cdot 1}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2} = 1 \pm i$$

The other solution is $1 - i$.

134. a) $x^2 - (6+3i)x + k = 0$

$3^2 - (6+3i)\cdot 3 + k = 0$ Substituting 3 for x

$9 - 18 - 9i + k = 0$

$k = 9 + 9i$

b) $x^2 - (6+3i)x + 9 + 9i = 0$

$$x = \frac{-[-(6+3i)] \pm \sqrt{[-(6+3i)]^2 - 4(1)(9+9i)}}{2\cdot 1}$$

$$x = \frac{6 + 3i \pm \sqrt{36 + 36i - 9 - 36 - 36i}}{2}$$

$$x = \frac{6 + 3i \pm \sqrt{-9}}{2} = \frac{6 + 3i \pm 3i}{2}$$

$x = \frac{6+3i+3i}{2}$ *or* $x = \frac{6+3i-3i}{2}$

$x = \frac{6+6i}{2}$ *or* $x = \frac{6}{2}$

$x = 3 + 3i$ *or* $x = 3$

The other solution is $3 + 3i$.

135. $(x-2)^3 = x^3 - 2$

$x^3 - 6x^2 + 12x - 8 = x^3 - 2$

$0 = 6x^2 - 12x + 6$

$0 = 6(x^2 - 2x + 1)$

$0 = 6(x-1)(x-1)$

$x - 1 = 0$ *or* $x - 1 = 0$

$x = 1$ *or* $x = 1$

The solution is 1.

136. $(x+1)^3 = (x-1)^3 + 26$

$x^3 + 3x^2 + 3x + 1 = x^3 - 3x^2 + 3x - 1 + 26$

$x^3 + 3x^2 + 3x + 1 = x^3 - 3x^2 + 3x + 25$

$6x^2 - 24 = 0$

$6(x^2 - 4) = 0$

$6(x+2)(x-2) = 0$

$x + 2 = 0$ *or* $x - 2 = 0$

$x = -2$ *or* $x = 2$

The solutions are -2 and 2.

137. $(6x^3 + 7x^2 - 3x)(x^2 - 7) = 0$

$x(6x^2 + 7x - 3)(x^2 - 7) = 0$

$x(3x-1)(2x+3)(x^2-7) = 0$

$x=0$ *or* $3x - 1 = 0$ *or* $2x + 3 = 0$ *or* $x^2 - 7 = 0$

$x=0$ *or* $x = \frac{1}{3}$ *or* $x = -\frac{3}{2}$ *or* $x = \sqrt{7}$ *or* $x = -\sqrt{7}$

The exact solutions are $-\sqrt{7}$, $-\frac{3}{2}$, 0, $\frac{1}{3}$, and $\sqrt{7}$.

138. $\left(x - \frac{1}{5}\right)\left(x^2 - \frac{1}{4}\right) + \left(x - \frac{1}{5}\right)\left(x^2 + \frac{1}{8}\right) = 0$

$\left(x - \frac{1}{5}\right)\left(2x^2 - \frac{1}{8}\right) = 0$

$\left(x - \frac{1}{5}\right)(2)\left(x + \frac{1}{4}\right)\left(x - \frac{1}{4}\right) = 0$

$x = \frac{1}{5}$ *or* $x = -\frac{1}{4}$ *or* $x = \frac{1}{4}$

The solutions are $-\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{4}$.

139. $x^2 + x - \sqrt{2} = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1(-\sqrt{2})}}{2 \cdot 1} = \frac{-1 \pm \sqrt{1 + 4\sqrt{2}}}{2}$$

The solutions are $\dfrac{-1 \pm \sqrt{1 + 4\sqrt{2}}}{2}$.

140. $x^2 + \sqrt{5}x - \sqrt{3} = 0$

Use the quadratic formula. Here $a = 1$, $b = \sqrt{5}$, and $c = -\sqrt{3}$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\sqrt{5} \pm \sqrt{(\sqrt{5})^2 - 4 \cdot 1(-\sqrt{3})}}{2 \cdot 1}$$

$$= \frac{-\sqrt{5} \pm \sqrt{5 + 4\sqrt{3}}}{2}$$

The solutions are $\dfrac{-\sqrt{5} \pm \sqrt{5 + 4\sqrt{3}}}{2}$.

141.

$$2t^2 + (t-4)^2 = 5t(t-4) + 24$$
$$2t^2 + t^2 - 8t + 16 = 5t^2 - 20t + 24$$
$$0 = 2t^2 - 12t + 8$$
$$0 = t^2 - 6t + 4 \quad \text{Dividing by 2}$$

Use the quadratic formula.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$$

$$= \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm 2\sqrt{5}}{2}$$

$$= \frac{2(3 \pm \sqrt{5})}{2} = 3 \pm \sqrt{5}$$

The solutions are $3 \pm \sqrt{5}$.

142.

$$9t(t+2) - 3t(t-2) = 2(t+4)(t+6)$$
$$9t^2 + 18t - 3t^2 + 6t = 2t^2 + 20t + 48$$
$$4t^2 + 4t - 48 = 0$$
$$4(t+4)(t-3) = 0$$
$$t + 4 = 0 \quad \textit{or} \quad t - 3 = 0$$
$$t = -4 \quad \textit{or} \quad t = 3$$

The solutions are -4 and 3.

143. $\sqrt{x-3} - \sqrt[4]{x-3} = 2$

Substitute u for $\sqrt[4]{x-3}$.

$$u^2 - u - 2 = 0$$
$$(u-2)(u+1) = 0$$
$$u - 2 = 0 \quad \textit{or} \quad u + 1 = 0$$
$$u = 2 \quad \textit{or} \quad u = -1$$

Substitute $\sqrt[4]{x-3}$ for u and solve for x.

$$\sqrt[4]{x-3} = 2 \quad \textit{or} \quad \sqrt[4]{x-3} = 1$$
$$x - 3 = 16 \quad \text{No solution}$$
$$x = 19$$

The value checks. The solution is 19.

144. $x^6 - 28x^3 + 27 = 0$

Substitute u for x^3.

$$u^2 - 28u + 27 = 0$$
$$(u-27)(u-1) = 0$$
$$u = 27 \quad \text{or} \quad u = 1$$

Substitute x^3 for u and solve for x.

$$x^3 = 27 \quad \textit{or} \quad x^3 = 1$$
$$x^3 - 27 = 0 \quad \textit{or} \quad x^3 - 1 = 0$$
$$(x-3)(x^2+3x+9) = 0 \quad \textit{or} \quad (x-1)(x^2+x+1) = 0$$

Using the principle of zero products and, where necessary, the quadratic formula, we find that the solutions are $3, -\dfrac{3}{2} \pm \dfrac{3\sqrt{3}}{2}i, 1$, and $-\dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2}i$.

145.

$$\left(y + \frac{2}{y}\right)^2 + 3y + \frac{6}{y} = 4$$
$$\left(y + \frac{2}{y}\right)^2 + 3\left(y + \frac{2}{y}\right) - 4 = 0$$

Substitute u for $y + \dfrac{2}{y}$.

$$u^2 + 3u - 4 = 0$$
$$(u+4)(u-1) = 0$$
$$u = -4 \quad \textit{or} \quad u = 1$$

Substitute $y + \dfrac{2}{y}$ for u and solve for y.

$$y + \frac{2}{y} = -4 \quad \textit{or} \quad y + \frac{2}{y} = 1$$
$$y^2 + 2 = -4y \quad \textit{or} \quad y^2 + 2 = y$$
$$y^2 + 4y + 2 = 0 \quad \textit{or} \quad y^2 - y + 2 = 0$$
$$y = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} \quad \textit{or}$$
$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1}$$
$$y = \frac{-4 \pm \sqrt{8}}{2} \quad \textit{or} \quad y = \frac{1 \pm \sqrt{-7}}{2}$$
$$y = \frac{-4 \pm 2\sqrt{2}}{2} \quad \textit{or} \quad y = \frac{1 \pm \sqrt{7}i}{2}$$
$$y = -2 \pm \sqrt{2} \quad \textit{or} \quad y = \frac{1}{2} \pm \frac{\sqrt{7}}{2}i$$

The solutions are $-2 \pm \sqrt{2}$ and $\dfrac{1}{2} \pm \dfrac{\sqrt{7}}{2}i$.

146.
$$x^2+3x+1-\sqrt{x^2+3x+1}=8$$
$$x^2+3x+1-\sqrt{x^2+3x+1}-8=0$$
$$u^2-u-8=0$$
$$u=\frac{1+\sqrt{33}}{2} \text{ or } u=\frac{1-\sqrt{33}}{2}$$
$$\sqrt{x^2+3x+1}=\frac{1+\sqrt{33}}{2} \text{ or } \sqrt{x^2+3x+1}=\frac{1-\sqrt{33}}{2}$$
$$x^2+3x+1=\frac{34+2\sqrt{33}}{4} \text{ or } x^2+3x+1=\frac{34-2\sqrt{33}}{4}$$
$$x^2+3x+\frac{-15-\sqrt{33}}{2}=0 \text{ or } x^2+3x+\frac{-15+\sqrt{33}}{2}=0$$
$$x=\frac{-3\pm\sqrt{39+2\sqrt{33}}}{2} \text{ or } x=\frac{-3\pm\sqrt{39-2\sqrt{33}}}{2}$$

Only $\dfrac{-3\pm\sqrt{39+2\sqrt{33}}}{2}$ checks. The solutions are $\dfrac{-3\pm\sqrt{39+2\sqrt{33}}}{2}$.

147. $\frac{1}{2}at+v_0t+x_0=0$

Use the quadratic formula. Here $a=\frac{1}{2}a$, $b=v_0$, and $c=x_0$.
$$t=\frac{-v_0\pm\sqrt{(v_0)^2-4\cdot\frac{1}{2}a\cdot x_0}}{2\cdot\frac{1}{2}a}$$
$$t=\frac{-v_0\pm\sqrt{v_0^2-2ax_0}}{a}$$

Exercise Set 2.4

1. a) The minimum function value occurs at the vertex, so the vertex is $\left(-\frac{1}{2},-\frac{9}{4}\right)$.

b) The axis of symmetry is a vertical line through the vertex. It is $x=-\frac{1}{2}$.

c) The minimum value of the function is $-\frac{9}{4}$.

2. a) $\left(-\frac{1}{2},\frac{25}{4}\right)$

b) Axis of symmetry: $x=-\frac{1}{2}$

c) Maximum: $\frac{25}{4}$

3.
$$\begin{aligned} f(x) &= x^2-8x+12 && \text{16 completes the square for } x^2-8x.\\ &= x^2-8x+16-16+12 && \text{Adding } 16-16 \text{ on the right side}\\ &= (x^2-8x+16)-16+12 &&\\ &= (x-4)^2-4 && \text{Factoring and simplifying}\\ &= (x-4)^2+(-4) && \text{Writing in the form } f(x)=a(x-h)^2+k \end{aligned}$$

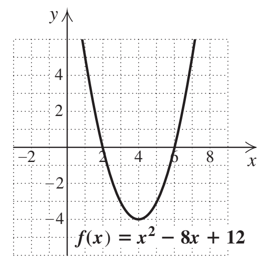
a) Vertex: $(4,-4)$

b) Axis of symmetry: $x=4$

c) Minimum value: -4

d) We plot the vertex and find several points on either side of it. Then we plot these points and connect them with a smooth curve.

x	$f(x)$
4	-4
2	0
1	5
5	-3
6	0



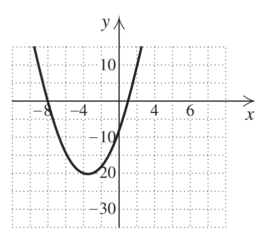
4.
$$\begin{aligned} g(x) &= x^2+7x-8\\ &= x^2+7x+\frac{49}{4}-\frac{49}{4}-8 && \left(\frac{1}{2}\cdot 7=\frac{7}{2} \text{ and } \left(\frac{7}{2}\right)^2=\frac{49}{4}\right)\\ &= \left(x+\frac{7}{2}\right)^2-\frac{81}{4}\\ &= \left[x-\left(-\frac{7}{2}\right)\right]^2+\left(-\frac{81}{4}\right) \end{aligned}$$

a) Vertex: $\left(-\frac{7}{2},-\frac{81}{4}\right)$

b) Axis of symmetry: $x=-\frac{7}{2}$

c) Minimum value: $-\frac{81}{4}$

d)



5. $f(x) = x^2 - 7x + 12$ $\quad \frac{49}{4}$ completes the square for $x^2 - 7x$.

$= x^2 - 7x + \frac{49}{4} - \frac{49}{4} + 12$ Adding $\frac{49}{4} - \frac{49}{4}$ on the right side

$= \left(x^2 - 7x + \frac{49}{4}\right) - \frac{49}{4} + 12$

$= \left(x - \frac{7}{2}\right)^2 - \frac{1}{4}$ Factoring and simplifying

$= \left(x - \frac{7}{2}\right)^2 + \left(-\frac{1}{4}\right)$ Writing in the form $f(x) = a(x-h)^2 + k$

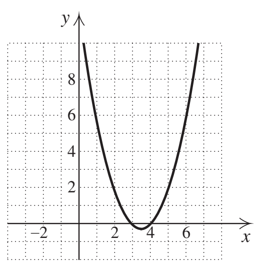
a) Vertex: $\left(\frac{7}{2}, -\frac{1}{4}\right)$

b) Axis of symmetry: $x = \frac{7}{2}$

c) Minimum value: $-\frac{1}{4}$

d) We plot the vertex and find several points on either side of it. Then we plot these points and connect them with a smooth curve.

x	$f(x)$
$\frac{7}{2}$	$-\frac{1}{4}$
4	0
5	2
3	0
1	6



$f(x) = x^2 - 7x + 12$

6. $g(x) = x^2 - 5x + 6$

$= x^2 - 5x + \frac{25}{4} - \frac{25}{4} + 6$ $\left(\frac{1}{2}(-5) = -\frac{5}{2}\right.$ and $\left.\left(-\frac{5}{2}\right)^2 = \frac{25}{4}\right)$

$= \left(x - \frac{5}{2}\right)^2 - \frac{1}{4}$

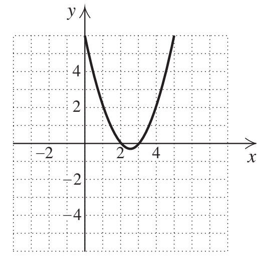
$= \left(x - \frac{5}{2}\right)^2 + \left(-\frac{1}{4}\right)$

a) Vertex: $\left(\frac{5}{2}, -\frac{1}{4}\right)$

b) Axis of symmetry: $x = \frac{5}{2}$

c) Minimum value: $-\frac{1}{4}$

d)



$g(x) = x^2 - 5x + 6$

7. $f(x) = x^2 + 4x + 5$ $\quad$ 4 completes the square for $x^2 + 4x$

$= x^2 + 4x + 4 - 4 + 5$ Adding $4 - 4$ on the right side

$= (x+2)^2 + 1$ Factoring and simplifying

$= [x - (-2)]^2 + 1$ Writing in the form $f(x) = a(x-h)^2 + k$

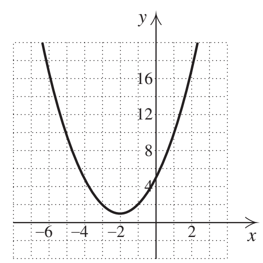
a) Vertex: $(-2, 1)$

b) Axis of symmetry: $x = -2$

c) Minimum value: 1

d) We plot the vertex and find several points on either side of it. Then we plot these points and connect them with a smooth curve.

x	$f(x)$
−2	1
−1	2
0	5
−3	2
−4	5



$f(x) = x^2 + 4x + 5$

8. $f(x) = x^2 + 2x + 6$

$= x^2 + 2x + 1 - 1 + 6$ $\left(\frac{1}{2} \cdot 2 = 1 \text{ and } 1^2 = 1\right)$

$= (x+1)^2 + 5$

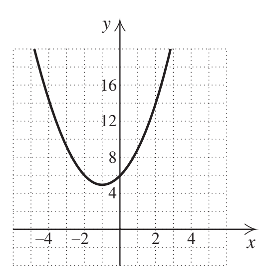
$= [x - (-1)]^2 + 5$

a) Vertex: $(-1, 5)$

b) Axis of symmetry: $x = -1$

c) Minimum value: 5

d)



$f(x) = x^2 + 2x + 6$

9. $g(x) = \dfrac{x^2}{2} + 4x + 6$

$= \dfrac{1}{2}(x^2 + 8x) + 6$ Factoring $\dfrac{1}{2}$ out of the first two terms

$= \dfrac{1}{2}(x^2+8x+16-16)+6$ Adding $16 - 16$ inside the parentheses

$= \dfrac{1}{2}(x^2+8x+16)-\dfrac{1}{2}\cdot 16+6$ Removing -16 from within the parentheses

$= \dfrac{1}{2}(x+4)^2 - 2$ Factoring and simplifying

$= \dfrac{1}{2}[x-(-4)]^2 + (-2)$

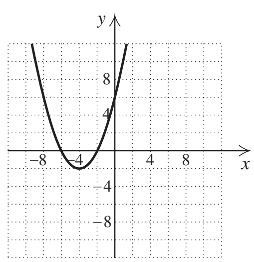
a) Vertex: $(-4, -2)$

b) Axis of symmetry: $x = -4$

c) Minimum value: -2

d) We plot the vertex and find several points on either side of it. Then we plot these points and connect them with a smooth curve.

x	$g(x)$
-4	-2
-2	0
0	6
-6	0
-8	6



$g(x) = \dfrac{x^2}{2} + 4x + 6$

10. $g(x) = \dfrac{x^2}{3} - 2x + 1$

$= \dfrac{1}{3}(x^2 - 6x) + 1$

$= \dfrac{1}{3}(x^2 - 6x + 9 - 9) + 1$

$= \dfrac{1}{3}(x^2 - 6x + 9) - \dfrac{1}{3} \cdot 9 + 1$

$= \dfrac{1}{3}(x-3)^2 - 2$

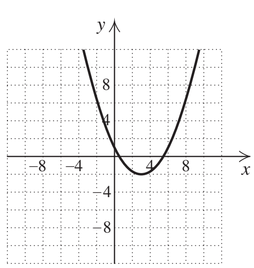
$= \dfrac{1}{3}(x-3) + (-2)$

a) Vertex: $(3, -2)$

b) Axis of symmetry: $x = 3$

c) Minimum value: -2

d)



$g(x) = \dfrac{x^2}{3} - 2x + 1$

11. $g(x) = 2x^2 + 6x + 8$

$= 2(x^2 + 3x) + 8$ Factoring 2 out of the first two terms

$= 2\left(x^2 + 3x + \dfrac{9}{4} - \dfrac{9}{4}\right) + 8$ Adding $\dfrac{9}{4} - \dfrac{9}{4}$ inside the parentheses

$= 2\left(x^2 + 3x + \dfrac{9}{4}\right) - 2\cdot\dfrac{9}{4} + 8$ Removing $-\dfrac{9}{4}$ from within the parentheses

$= 2\left(x + \dfrac{3}{2}\right)^2 + \dfrac{7}{2}$ Factoring and simplifying

$= 2\left[x - \left(-\dfrac{3}{2}\right)\right]^2 + \dfrac{7}{2}$

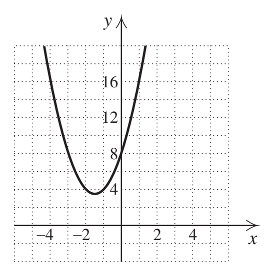
a) Vertex: $\left(-\dfrac{3}{2}, \dfrac{7}{2}\right)$

b) Axis of symmetry: $x = -\dfrac{3}{2}$

c) Minimum value: $\dfrac{7}{2}$

d) We plot the vertex and find several points on either side of it. Then we plot these points and connect them with a smooth curve.

x	$f(x)$
$-\dfrac{3}{2}$	$\dfrac{7}{2}$
-1	4
0	8
-2	4
-3	8



$g(x) = 2x^2 + 6x + 8$

12. $f(x) = 2x^2 - 10x + 14$

$= 2(x^2 - 5x) + 14$

$= 2\left(x^2 - 5x + \dfrac{25}{4} - \dfrac{25}{4}\right) + 14$

$= 2\left(x^2 - 5x + \dfrac{25}{4}\right) - 2\cdot\dfrac{25}{4} + 14$

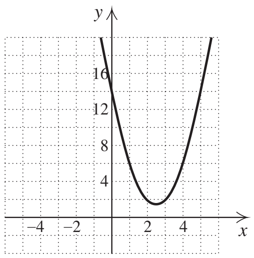
$= 2\left(x - \dfrac{5}{2}\right)^2 + \dfrac{3}{2}$

a) Vertex: $\left(\dfrac{5}{2}, \dfrac{3}{2}\right)$

b) Axis of symmetry: $x = \dfrac{5}{2}$

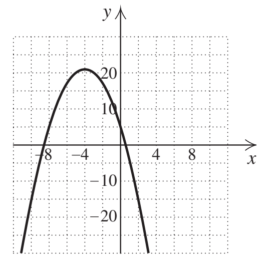
c) Minimum value: $\dfrac{3}{2}$

d)



$f(x) = 2x^2 - 10x + 14$

d)



$f(x) = -x^2 - 8x + 5$

13. $f(x) = -x^2 - 6x + 3$

$= -(x^2 + 6x) + 3$ 9 completes the square for $x^2 + 6x$.

$= -(x^2 + 6x + 9 - 9) + 3$

$= -(x+3)^2 - (-9) + 3$ Removing -9 from the parentheses

$= -(x+3)^2 + 9 + 3$

$= -[x - (-3)]^2 + 12$

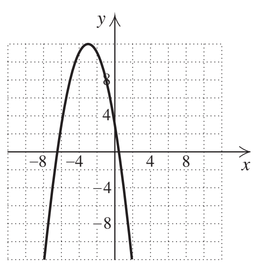
a) Vertex: $(-3, 12)$

b) Axis of symmetry: $x = -3$

c) Maximum value: 12

d) We plot the vertex and find several points on either side of it. Then we plot these points and connect them with a smooth curve.

x	$f(x)$
-3	12
0	3
1	-4
-6	3
-7	-4



$f(x) = -x^2 - 6x + 3$

14. $f(x) = -x^2 - 8x + 5$

$= -(x^2 + 8x) + 5$

$= -(x^2 + 8x + 16 - 16) + 5$

$\left(\frac{1}{2} \cdot 8 = 4 \textit{ and } 4^2 = 16\right)$

$= -(x^2 + 8x + 16) - (-16) + 5$

$= -(x^2 + 8x + 16) + 21$

$= -[x - (-4)]^2 + 21$

a) Vertex: $(-4, 21)$

b) Axis of symmetry: $x = -4$

c) Maximum value: 21

15. $g(x) = -2x^2 + 2x + 1$

$= -2(x^2 - x) + 1$ Factoring -2 out of the first two terms

$= -2\left(x^2 - x + \frac{1}{4} - \frac{1}{4}\right) + 1$ Adding $\frac{1}{4} - \frac{1}{4}$ inside the parentheses

$= -2\left(x^2 - x + \frac{1}{4}\right) - 2\left(-\frac{1}{4}\right) + 1$

Removing $-\frac{1}{4}$ from within the parentheses

$= -2\left(x - \frac{1}{2}\right)^2 + \frac{3}{2}$

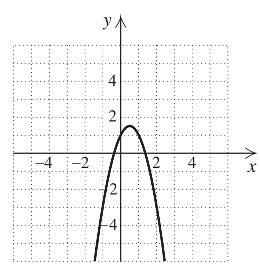
a) Vertex: $\left(\frac{1}{2}, \frac{3}{2}\right)$

b) Axis of symmetry: $x = \frac{1}{2}$

c) Maximum value: $\frac{3}{2}$

d) We plot the vertex and find several points on either side of it. Then we plot these points and connect them with a smooth curve.

x	$f(x)$
$\frac{1}{2}$	$\frac{3}{2}$
1	1
2	-3
0	1
-1	-3



$g(x) = -2x^2 + 2x + 1$

16. $f(x) = -3x^2 - 3x + 1$

$= -3(x^2 + x) + 1$

$= -3\left(x^2 + x + \frac{1}{4} - \frac{1}{4}\right) + 1$

$= -3\left(x^2 + x + \frac{1}{4}\right) - 3\left(-\frac{1}{4}\right) + 1$

$= -3\left(x + \frac{1}{2}\right)^2 + \frac{7}{4}$

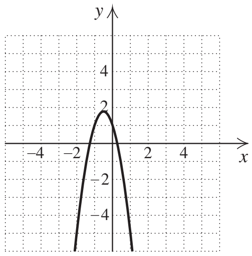
$= -3\left[x - \left(-\frac{1}{2}\right)\right]^2 + \frac{7}{4}$

a) Vertex: $\left(-\frac{1}{2}, \frac{7}{4}\right)$

b) Axis of symmetry: $x = -\frac{1}{2}$

c) Maximum value: $\frac{7}{4}$

d)



$f(x) = -3x^2 - 3x + 1$

17. The graph of $y = (x+3)^2$ has vertex $(-3, 0)$ and opens up. It is graph (f).

18. The graph of $y = -(x-4)^2 + 3$ has vertex $(4, 3)$ and opens down. It is graph (e).

19. The graph of $y = 2(x-4)^2 - 1$ has vertex $(4, -1)$ and opens up. It is graph (b).

20. The graph of $y = x^2 - 3$ has vertex $(0, -3)$ and opens up. It is graph (g).

21. The graph of $y = -\frac{1}{2}(x+3)^2 + 4$ has vertex $(-3, 4)$ and opens down. It is graph (h).

22. The graph of $y = (x-3)^2$ has vertex $(3, 0)$ and opens up. It is graph (a).

23. The graph of $y = -(x+3)^2 + 4$ has vertex $(-3, 4)$ and opens down. It is graph (c).

24. The graph of $y = 2(x-1)^2 - 4$ has vertex $(1, -4)$ and opens up. It is graph (d).

25. $f(x) = x^2 - 6x + 5$

a) The x-coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{-6}{2 \cdot 1} = 3.$$

Since $f(3) = 3^2 - 6 \cdot 3 + 5 = -4$, the vertex is $(3, -4)$.

b) Since $a = 1 > 0$, the graph opens up so the second coordinate of the vertex, -4, is the minimum value of the function.

c) The range is $[-4, \infty)$.

d) Since the graph opens up, function values decrease to the left of the vertex and increase to the right of the vertex. Thus, $f(x)$ is increasing on $(3, \infty)$ and decreasing on $(-\infty, 3)$.

26. $f(x) = x^2 + 4x - 5$

a) $-\frac{b}{2a} = -\frac{4}{2 \cdot 1} = -2$

$f(-2) = (-2)^2 + 4(-2) - 5 = -9$

The vertex is $(-2, -9)$.

b) Since $a = 1 > 0$, the graph opens up. The minimum value of $f(x)$ is -9.

c) Range: $[-9, \infty)$

d) Increasing: $(-2, \infty)$; decreasing: $(-\infty, -2)$

27. $f(x) = 2x^2 + 4x - 16$

a) The x-coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{4}{2 \cdot 2} = -1.$$

Since $f(-1) = 2(-1)^2 + 4(-1) - 16 = -18$, the vertex is $(-1, -18)$.

b) Since $a = 2 > 0$, the graph opens up so the second coordinate of the vertex, -18, is the minimum value of the function.

c) The range is $[-18, \infty)$.

d) Since the graph opens up, function values decrease to the left of the vertex and increase to the right of the vertex. Thus, $f(x)$ is increasing on $(-1, \infty)$ and decreasing on $(-\infty, -1)$.

28. $f(x) = \frac{1}{2}x^2 - 3x + \frac{5}{2}$

a) $-\frac{b}{2a} = -\frac{-3}{2 \cdot \frac{1}{2}} = 3$

$f(3) = \frac{1}{2} \cdot 3^2 - 3 \cdot 3 + \frac{5}{2} = -2$

The vertex is $(3, -2)$.

b) Since $a = \frac{1}{2} > 0$, the graph opens up. The minimum value of $f(x)$ is -2.

c) Range: $[-2, \infty)$

d) Increasing: $(3, \infty)$; decreasing: $(-\infty, 3)$

29. $f(x) = -\frac{1}{2}x^2 + 5x - 8$

a) The x-coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{5}{2\left(-\frac{1}{2}\right)} = 5.$$

Since $f(5) = -\frac{1}{2} \cdot 5^2 + 5 \cdot 5 - 8 = \frac{9}{2}$, the vertex is $\left(5, \frac{9}{2}\right)$.

b) Since $a = -\frac{1}{2} < 0$, the graph opens down so the second coordinate of the vertex, $\frac{9}{2}$, is the maximum value of the function.

c) The range is $\left(-\infty, \frac{9}{2}\right]$.

d) Since the graph opens down, function values increase to the left of the vertex and decrease to the right of the vertex. Thus, $f(x)$ is increasing on $(-\infty, 5)$ and decreasing on $(5, \infty)$.

30. $f(x) = -2x^2 - 24x - 64$

a) $-\frac{b}{2a} = -\frac{-24}{2(-2)} = -6.$

$f(-6) = -2(-6)^2 - 24(-6) - 64 = 8$

The vertex is $(-6, 8)$.

b) Since $a = -2 < 0$, the graph opens down. The maximum value of $f(x)$ is 8.

c) Range: $(-\infty, 8]$

d) Increasing: $(-\infty, -6)$; decreasing: $(-6, \infty)$

31. $f(x) = 3x^2 + 6x + 5$

a) The x-coordinate of the vertex is

$-\frac{b}{2a} = -\frac{6}{2 \cdot 3} = -1.$

Since $f(-1) = 3(-1)^2 + 6(-1) + 5 = 2$, the vertex is $(-1, 2)$.

b) Since $a = 3 > 0$, the graph opens up so the second coordinate of the vertex, 2, is the minimum value of the function.

c) The range is $[2, \infty)$.

d) Since the graph opens up, function values decrease to the left of the vertex and increase to the right of the vertex. Thus, $f(x)$ is increasing on $(-1, \infty)$ and decreasing on $(-\infty, -1)$.

32. $f(x) = -3x^2 + 24x - 49$

a) $-\frac{b}{2a} = -\frac{24}{2(-3)} = 4.$

$f(4) = -3 \cdot 4^2 + 24 \cdot 4 - 49 = -1$

The vertex is $(4, -1)$.

b) Since $a = -3 < 0$, the graph opens down. The maximum value of $f(x)$ is -1.

c) Range: $(-\infty, -1]$

d) Increasing: $(-\infty, 4)$; decreasing: $(4, \infty)$

33. $g(x) = -4x^2 - 12x + 9$

a) The x-coordinate of the vertex is

$-\frac{b}{2a} = -\frac{-12}{2(-4)} = -\frac{3}{2}.$

Since $g\left(-\frac{3}{2}\right) = -4\left(-\frac{3}{2}\right)^2 - 12\left(-\frac{3}{2}\right) + 9 = 18$, the vertex is $\left(-\frac{3}{2}, 18\right)$.

b) Since $a = -4 < 0$, the graph opens down so the second coordinate of the vertex, 18, is the maximum value of the function.

c) The range is $(-\infty, 18]$.

d) Since the graph opens down, function values increase to the left of the vertex and decrease to the right of the vertex. Thus, $g(x)$ is increasing on $\left(-\infty, -\frac{3}{2}\right)$ and decreasing on $\left(-\frac{3}{2}, \infty\right)$.

34. $g(x) = 2x^2 - 6x + 5$

a) $-\frac{b}{2a} = -\frac{-6}{2 \cdot 2} = \frac{3}{2}$

$g\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^2 - 6\left(\frac{3}{2}\right) + 5 = \frac{1}{2}$

The vertex is $\left(\frac{3}{2}, \frac{1}{2}\right)$.

b) Since $a = 2 > 0$, the graph opens up. The minimum value of $g(x)$ is $\frac{1}{2}$.

c) Range: $\left[\frac{1}{2}, \infty\right)$

d) Increasing: $\left(\frac{3}{2}, \infty\right)$; decreasing: $\left(-\infty, \frac{3}{2}\right)$

35. ***Familiarize and Translate***. The function $s(t) = -16t^2 + 20t + 6$ is given in the statement of the problem.

Carry out. The function $s(t)$ is quadratic and the coefficient of t^2 is negative, so $s(t)$ has a maximum value. It occurs at the vertex of the graph of the function. We find the first coordinate of the vertex. This is the time at which the ball reaches its maximum height.

$t = -\frac{b}{2a} = -\frac{20}{2(-16)} = 0.625$

The second coordinate of the vertex gives the maximum height.

$s(0.625) = -16(0.625)^2 + 20(0.625) + 6 = 12.25$

Check. Completing the square, we write the function in the form $s(t) = -16(t - 0.625)^2 + 12.25$. We see that the coordinates of the vertex are $(0.625, 12.25)$, so the answer checks.

State. The ball reaches its maximum height after 0.625 seconds. The maximum height is 12.25 ft.

36. Find the first coordinate of the vertex:

$t = -\frac{60}{2(-16)} = 1.875$

Then $s(1.875) = -16(1.875)^2 + 60(1.875) + 30 = 86.25$. Thus the maximum height is reached after 1.875 sec. The maximum height is 86.25 ft.

37. ***Familiarize and Translate***. The function $s(t) = -16t^2 + 120t + 80$ is given in the statement of the problem.

Carry out. The function $s(t)$ is quadratic and the coefficient of t^2 is negative, so $s(t)$ has a maximum value. It occurs at the vertex of the graph of the function. We find the first coordinate of the vertex. This is the time at which the rocket reaches its maximum height.

$t = -\frac{b}{2a} = -\frac{120}{2(-16)} = 3.75$

The second coordinate of the vertex gives the maximum height.

$s(3.75) = -16(3.75)^2 + 120(3.75) + 80 = 305$

Check. Completing the square, we write the function in the form $s(t) = -16(t - 3.75)^2 + 305$. We see that the coordinates of the vertex are $(3.75, 305)$, so the answer checks.

State. The rocket reaches its maximum height after 3.75 seconds. The maximum height is 305 ft.

38. Find the first coordinate of the vertex:

$$t = -\frac{150}{2(-16)} = 4.6875$$

Then $s(4.6875) = -16(4.6875)^2 + 150(4.6875) + 40 = 391.5625$. Thus the maximum height is reached after 4.6875 sec. The maximum height is 391.5625 ft.

39. ***Familiarize***. Using the label in the text, we let $x =$ the height of the file. Then the length $= 10$ and the width $= 18 - 2x$.

Translate. Since the volume of a rectangular solid is length $\times$ width $\times$ height we have

$$V(x) = 10(18 - 2x)x, \text{ or } -20x^2 + 180x.$$

Carry out. Since $V(x)$ is a quadratic function with $a = -20 < 0$, the maximum function value occurs at the vertex of the graph of the function. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{180}{2(-20)} = 4.5.$$

Check. When $x = 4.5$, then $18 - 2x = 9$ and $V(x) = 10 \cdot 9(4.5)$, or 405. As a partial check, we can find $V(x)$ for a value of x less than 4.5 and for a value of x greater than 4.5. For instance, $V(4.4) = 404.8$ and $V(4.6) = 404.8$. Since both of these values are less than 405, our result appears to be correct.

State. The file should be 4.5 in. tall in order to maximize the volume.

40. Let $w =$ the width of the garden. Then the length $= 32 - 2w$ and the area is given by $A(w) = (32 - 2w)w$, or $-2w^2 + 32w$. The maximum function value occurs at the vertex of the graph of $A(w)$. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{32}{2(-2)} = 8.$$

When $w = 8$, then $32 - 2w = 16$ and the area is $16 \cdot 8$, or 128 ft^2. A garden with dimensions 8 ft by 16 ft yields this area.

41. ***Familiarize***. Let $b =$ the length of the base of the triangle. Then the height $= 20 - b$.

Translate. Since the area of a triangle is $\frac{1}{2} \times$ base$\times$height, we have

$$A(b) = \frac{1}{2}b(20 - b), \text{ or } -\frac{1}{2}b^2 + 10b.$$

Carry out. Since $A(b)$ is a quadratic function with $a = -\frac{1}{2} < 0$, the maximum function value occurs at the vertex of the graph of the function. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{10}{2\left(-\frac{1}{2}\right)} = 10.$$

When $b = 10$, then $20 - b = 20 - 10 = 10$, and the area is $\frac{1}{2} \cdot 10 \cdot 10 = 50$ cm^2.

Check. As a partial check, we can find $A(b)$ for a value of b less than 10 and for a value of b greater than 10. For instance, $V(9.9) = 49.995$ and $V(10.1) = 49.995$. Since both of these values are less than 50, our result appears to be correct.

State. The area is a maximum when the base and the height are both 10 cm.

42. Let $b =$ the length of the base. Then $69 - b =$ the height and $A(b) = b(69-b)$, or $-b^2+69b$. The maximum function value occurs at the vertex of the graph of $A(b)$. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{69}{2(-1)} = 34.5.$$

When $b = 34.5$, then $69-b = 34.5$. The area is a maximum when the base and height are both 34.5 cm.

43. $C(x) = 0.1x^2 - 0.7x + 2.425$

Since $C(x)$ is a quadratic function with $a = 0.1 > 0$, a minimum function value occurs at the vertex of the graph of $C(x)$. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{-0.7}{2(0.1)} = 3.5.$$

Thus, 3.5 hundred, or 350 bicycles should be built to minimize the average cost per bicycle.

44. $P(x) = R(x) - C(x)$

$P(x) = 5x - (0.001x^2 + 1.2x + 60)$

$P(x) = -0.001x^2 + 3.8x - 60$

Since $P(x)$ is a quadratic function with $a = -0.001 < 0$, a maximum function value occurs at the vertex of the graph of the function. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{3.8}{2(-0.001)} = 1900.$$

$P(1900) = -0.001(1900)^2 + 3.8(1900) - 60 = 3550$

Thus, the maximum profit is \$3550. It occurs when 1900 units are sold.

45. $P(x) = R(x) - C(x)$

$P(x) = (50x - 0.5x^2) - (10x + 3)$

$P(x) = -0.5x^2 + 40x - 3$

Since $P(x)$ is a quadratic function with $a = -0.5 < 0$, a maximum function value occurs at the vertex of the graph of the function. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{40}{2(-0.5)} = 40.$$

$P(40) = -0.5(40)^2 + 40 \cdot 40 - 3 = 797$

Thus, the maximum profit is \$797. It occurs when 40 units are sold.

46. $P(x) = R(x) - C(x)$

$P(x) = 20x - 0.1x^2 - (4x + 2)$

$P(x) = -0.1x^2 + 16x - 2$

Since $P(x)$ is a quadratic function with $a = -0.1 < 0$, a maximum function value occurs at the vertex of the graph of the function. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{16}{2(-0.1)} = 80.$$

$P(80) = -0.1(80)^2 + 16(80) - 2 = 638$

Thus, the maximum profit is \$638. It occurs when 80 units are sold.

47. ***Familiarize***. We let s = the height of the elevator shaft, t_1 = the time it takes the screwdriver to reach the bottom of the shaft, and t_2 = the time it takes the sound to reach the top of the shaft.

Translate. We know that $t_1 + t_2 = 5$. Using the information in Example 4 we also know that

$$s = 16t_1^2, \quad \textit{or} \quad t_1 = \frac{\sqrt{s}}{4} \text{ and}$$

$$s = 1100t_2, \quad \textit{or} \quad t_2 = \frac{s}{1100}.$$

Then $\dfrac{\sqrt{s}}{4} + \dfrac{s}{1100} = 5$.

Carry out. We solve the last equation above.

$$\frac{\sqrt{s}}{4} + \frac{s}{1100} = 5$$

$$275\sqrt{s} + s = 5500 \quad \text{Multiplying by 1100}$$

$$s + 275\sqrt{s} - 5500 = 0$$

Let $u = \sqrt{s}$ and substitute.

$u^2 + 275u - 5500 = 0$

$$u = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{We only want the positive solution.}$$

$$= \frac{-275 + \sqrt{275^2 - 4 \cdot 1(-5500)}}{2 \cdot 1}$$

$$= \frac{-275 + \sqrt{97,625}}{2} \approx 18.725$$

Since $u \approx 18.725$, we have $\sqrt{s} = 18.725$, so $s \approx 350.6$.

Check. If $s \approx 350.6$, then $t_1 = \dfrac{\sqrt{s}}{4} = \dfrac{\sqrt{350.6}}{4} \approx 4.68$ and $t_2 = \dfrac{s}{1100} = \dfrac{350.6}{1100} \approx 0.32$, so $t_1 + t_2 = 4.68 + 0.32 = 5$.

The result checks.

State. The elevator shaft is about 350.6 ft tall.

48. Let s = the height of the cliff, t_1 = the time it takes the balloon to hit the ground, and t_2 = the time it takes for the sound to reach the top of the cliff. Then we have

$t_1 + t_2 = 3$,

$$s = 16t_1^2, \quad \textit{or} \quad t_1 = \frac{\sqrt{s}}{4}, \text{ and}$$

$$s = 1100t_2, \quad \textit{or} \quad t_2 = \frac{s}{1100}, \text{ so}$$

$$\frac{\sqrt{s}}{4} + \frac{s}{1100} = 3.$$

Solving the last equation, we find that $s \approx 132.7$ ft.

49. ***Familiarize***. Using the labels on the drawing in the text, we let x = the width of each corral and $240 - 3x$ = the total length of the corrals.

Translate. Since the area of a rectangle is length × width, we have

$$A(x) = (240 - 3x)x = -3x^2 + 240x.$$

Carry out. Since $A(x)$ is a quadratic function with $a = -3 < 0$, the maximum function value occurs at the vertex of the graph of $A(x)$. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{240}{2(-3)} = 40.$$

$A(40) = -3(40)^2 + 240(40) = 4800$

Check. As a partial check we can find $A(x)$ for a value of x less than 40 and for a value of x greater than 40. For instance, $A(39.9) = 4799.97$ and $A(40.1) = 4799.97$. Since both of these values are less than 4800, our result appears to be correct.

State. The largest total area that can be enclosed is 4800 yd^2.

50. $\dfrac{1}{2} \cdot 2\pi x + 2x + 2y = 24$, so $y = 12 - \dfrac{\pi x}{2} - x$.

$$A(x) = \frac{1}{2} \cdot \pi x^2 + 2x\left(12 - \frac{\pi x}{2} - x\right)$$

$$A(x) = \frac{\pi x^2}{2} + 24x - \pi x^2 - 2x^2$$

$$A(x) = 24x - \frac{\pi x^2}{2} - 2x^2, \text{ or } 24x - \left(\frac{\pi}{2} + 2\right)x^2$$

Since $A(x)$ is a quadratic function with $a = -\left(\dfrac{\pi}{2} + 2\right) < 0$, the maximum function value occurs at the vertex of the graph of $A(x)$. The first coordinate of the vertex is

$$\frac{-b}{2a} = -\frac{24}{2\left[-\left(\frac{\pi}{2} + 2\right)\right]} = \frac{24}{\pi + 4}.$$

When $x = \dfrac{24}{\pi + 4}$, then $y = \dfrac{24}{\pi + 4}$. Thus, the maximum amount of light will enter when the dimensions of the rectangular part of the window are $2x$ by y, or $\dfrac{48}{\pi + 4}$ ft by $\dfrac{24}{\pi + 4}$ ft, or approximately 6.72 ft by 3.36 ft.

51. Left to the student

52. Left to the student

53. Left to the student

54. Left to the student

55. Answers will vary. The problem could be similar to Examples 5 and 6 or Exercises 35 through 50.

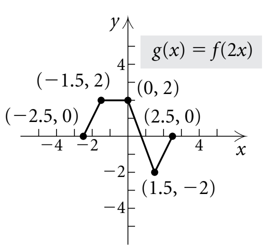
56. Completing the square was used in Section 2.3 to solve quadratic equations. It was used again in this section to write quadratic functions in the form $f(x) = a(x-h)^2+k$.

57. The x-intercepts of $g(x)$ are also $(x_1, 0)$ and $(x_2, 0)$. This is true because $f(x)$ and $g(x)$ have the same zeros. Consider $g(x) = 0$, or $-ax^2 - bx - c = 0$. Multiplying by -1 on both sides, we get an equivalent equation $ax^2+bx+c = 0$, or $f(x) = 0$.

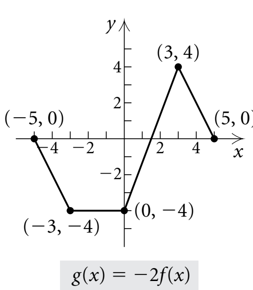
58.
$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{3(x+h)-7-(3x-7)}{h}\\ &= \frac{3x+3h-7-3x+7}{h}\\ &= \frac{3h}{h} = 3\end{aligned}$$

59. $f(x) = 2x^2 - x + 4$
$$\begin{aligned}f(x+h) &= 2(x+h)^2 - (x+h) + 4\\ &= 2(x^2+2xh+h^2) - (x+h) + 4\\ &= 2x^2 + 4xh + 2h^2 - x - h - 4\end{aligned}$$
$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{2x^2+4xh+2h^2-x-h-4-(2x^2-x+4)}{h}$$
$$= \frac{2x^2+4xh+2h^2-x-h-4-2x^2+x-4}{h}$$
$$= \frac{4xh+2h^2-h}{h} = \frac{h(4x+2h-1)}{h}$$
$$= 4x+2h-1$$

60.



61. The graph of $f(x)$ is stretched vertically and reflected across the x-axis.



62. $f(x) = -4x^2 + bx + 3$

The x-coordinate of the vertex of $f(x)$ is $-\dfrac{b}{2(-4)}$, or $\dfrac{b}{8}$.

Now we find b such that $f\left(\dfrac{b}{8}\right) = 50$.
$$\begin{aligned}-4\left(\frac{b}{8}\right)^2 + b\cdot\frac{b}{8} + 3 &= 50\\ -\frac{b^2}{16} + \frac{b^2}{8} + 3 &= 50\\ \frac{b^2}{16} &= 47\\ b^2 &= 16\cdot 47\\ b &= \pm\sqrt{16\cdot 47}\\ b &= \pm 4\sqrt{47}\end{aligned}$$

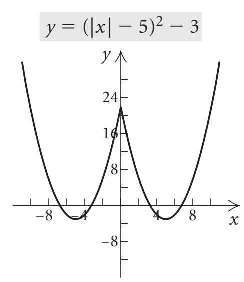
63. $f(x) = -0.2x^2 - 3x + c$

The x-coordinate of the vertex of $f(x)$ is $-\dfrac{b}{2a} = -\dfrac{-3}{2(-0.2)} = -7.5$. Now we find c such that $f(-7.5) = -225$.
$$\begin{aligned}-0.2(-7.5)^2 - 3(-7.5) + c &= -225\\ -11.25 + 22.5 + c &= -225\\ c &= -236.25\end{aligned}$$

64. $f(x) = a(x-h)^2 + k$

$1 = a(-3-4)^2 - 5$, so $a = \dfrac{6}{49}$. Then $f(x) = \dfrac{6}{49}(x-4)^2 - 5$.

65.



66. First we find the radius r of a circle with circumference x:
$$\begin{aligned}2\pi r &= x\\ r &= \frac{x}{2\pi}\end{aligned}$$
Then we find the length s of a side of a square with perimeter $24 - x$:
$$\begin{aligned}4s &= 24 - x\\ s &= \frac{24-x}{4}\end{aligned}$$

Then S = area of circle + area of square

$$S = \pi r^2 + s^2$$

$$S(x) = \pi\left(\frac{x}{2\pi}\right)^2 + \left(\frac{24-x}{4}\right)^2$$

$$S(x) = \left(\frac{1}{4\pi}+\frac{1}{16}\right)x^2 - 3x + 36$$

Since $S(x)$ is a quadratic function with $a = \frac{1}{4\pi}+\frac{1}{16} > 0$, the minimum function value occurs at the vertex of the graph of $S(x)$. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{-3}{2\left(\frac{1}{4\pi}+\frac{1}{16}\right)} = \frac{24\pi}{4+\pi}.$$

Then the string should be cut so that one piece is $\frac{24\pi}{4+\pi}$ in., or about 10.56 in. The other piece will be $24 - \frac{24\pi}{4+\pi}$, or $\frac{96}{4+\pi}$ in., or about 13.44 in.

Exercise Set 2.5

1. $\frac{1}{4}+\frac{1}{5} = \frac{1}{t}$, LCD is $20t$

$$20t\left(\frac{1}{4}+\frac{1}{5}\right) = 20t\cdot\frac{1}{t}$$

$$20t\cdot\frac{1}{4}+20t\cdot\frac{1}{5} = 20t\cdot\frac{1}{t}$$

$$5t+4t = 20$$

$$9t = 20$$

$$t = \frac{20}{9}$$

Check:

$\frac{1}{4}+\frac{1}{5} = \frac{1}{t}$		
$\frac{1}{4}+\frac{1}{5}$?	$\frac{1}{\frac{20}{9}}$	
$\frac{5}{20}+\frac{4}{20}$	$1\cdot\frac{9}{20}$	
$\frac{9}{20}$	$\frac{9}{20}$	TRUE

The solution is $\frac{20}{9}$.

2. $\frac{1}{3}-\frac{5}{6} = \frac{1}{x}$, LCD is $6x$

$2x-5x = 6$ Multiplying by $6x$

$$-3x = 6$$

$$x = -2$$

-2 checks. The solution is -2.

3. $\frac{x+2}{4}-\frac{x-1}{5} = 15$, LCD is 20

$$20\left(\frac{x+2}{4}-\frac{x-1}{5}\right) = 20\cdot 15$$

$$5(x+2)-4(x-1) = 300$$

$$5x+10-4x+4 = 300$$

$$x+14 = 300$$

$$x = 286$$

The solution is 286.

4. $\frac{t+1}{3}-\frac{t-1}{2} = 1$, LCD is 6

$2t+2-3t+3 = 6$ Multiplying by 6

$$-t = 1$$

$$t = -1$$

The solution is -1.

5. $\frac{1}{2}+\frac{2}{x} = \frac{1}{3}+\frac{3}{x}$, LCD is $6x$

$$6x\left(\frac{1}{2}+\frac{2}{x}\right) = 6x\left(\frac{1}{3}+\frac{3}{x}\right)$$

$$3x+12 = 2x+18$$

$$3x-2x = 18-12$$

$$x = 6$$

Check:

$\frac{1}{2}+\frac{2}{x} = \frac{1}{3}+\frac{3}{x}$		
$\frac{1}{2}+\frac{2}{6}$?	$\frac{1}{3}+\frac{3}{6}$	
$\frac{1}{2}+\frac{1}{3}$	$\frac{1}{3}+\frac{1}{2}$	TRUE

The solution is 6.

6. $\frac{1}{t}+\frac{1}{2t}+\frac{1}{3t} = 5$, LCD is $6t$

$6+3+2 = 30t$ Multiplying by $6t$

$$11 = 30t$$

$$\frac{11}{30} = t$$

$\frac{11}{30}$ checks. The solution is $\frac{11}{30}$.

7. $\frac{5}{3x+2} = \frac{3}{2x}$, LCD is $2x(3x+2)$

$$2x(3x+2)\cdot\frac{5}{3x+2} = 2x(3x+2)\cdot\frac{3}{2x}$$

$$2x\cdot 5 = 3(3x+2)$$

$$10x = 9x+6$$

$$x = 6$$

6 checks, so the solution is 6.

8. $\dfrac{2}{x-1} = \dfrac{3}{x+2}$, LCD is $(x-1)(x+2)$

$$2(x+2) = 3(x-1)$$
$$2x+4 = 3x-3$$
$$7 = x$$

The answer checks. The solution is 7.

9. $x + \dfrac{6}{x} = 5$, LCD is x

$$x\left(x+\frac{6}{x}\right) = x \cdot 5$$
$$x^2+6 = 5x$$
$$x^2-5x+6 = 0$$
$$(x-2)(x-3) = 0$$
$$x-2=0 \quad or \quad x-3=0$$
$$x=2 \quad or \quad x=3$$

Both numbers check. The solutions are 2 and 3.

10. $x - \dfrac{12}{x} = 1$, LCD is x

$$x^2-12 = x$$
$$x^2-x-12 = 0$$
$$(x-4)(x+3) = 0$$
$$x=4 \quad or \quad x=-3$$

Both numbers check. The solutions are 4 and -3.

11. $\dfrac{6}{y+3} + \dfrac{2}{y} = \dfrac{5y-3}{y^2-9}$

$\dfrac{6}{y+3} + \dfrac{2}{y} = \dfrac{5y-3}{(y+3)(y-3)}$, LCD is $y(y+3)(y-3)$

$$y(y+3)(y-3)\left(\frac{6}{y+3}+\frac{2}{y}\right) = y(y+3)(y-3)\cdot\frac{5y-3}{(y+3)(y-3)}$$
$$6y(y-3)+2(y+3)(y-3) = y(5y-3)$$
$$6y^2-18y+2(y^2-9) = 5y^2-3y$$
$$6y^2-18y+2y^2-18 = 5y^2-3y$$
$$8y^2-18y-18 = 5y^2-3y$$
$$3y^2-15y-18 = 0$$
$$y^2-5y-6 = 0$$
$$(y-6)(y+1) = 0$$
$$y-6=0 \quad or \quad y+1=0$$
$$y=6 \quad or \quad y=-1$$

Both numbers check. The solutions are 6 and -1.

12. $\dfrac{3}{m+2} + \dfrac{2}{m} = \dfrac{4m-4}{m^2-4}$

$\dfrac{3}{m+2} + \dfrac{2}{m} = \dfrac{4m-4}{(m+2)(m-2)}$, LCD is $m(m+2)(m-2)$

$$3m^2-6m+2m^2-8 = 4m^2-4m$$

Multiplying by $m(m+2)(m-2)$

$$m^2-2m-8 = 0$$
$$(m-4)(m+2) = 0$$
$$m=4 \quad \text{or} \quad m=-2$$

Only 4 checks. The solution is 4.

13. $\dfrac{2x}{x-1} = \dfrac{5}{x-3}$, LCD is $(x-1)(x-3)$

$$(x-1)(x-3)\cdot\frac{2x}{x-1} = (x-1)(x-3)\cdot\frac{5}{x-3}$$
$$2x(x-3) = 5(x-1)$$
$$2x^2-6x = 5x-5$$
$$2x^2-11x+5 = 0$$
$$(2x-1)(x-5) = 0$$
$$2x-1=0 \quad or \quad x-5=0$$
$$2x=1 \quad or \quad x=5$$
$$x=\frac{1}{2} \quad or \quad x=5$$

Both numbers check. The solutions are $\dfrac{1}{2}$ and 5.

14. $\dfrac{2x}{x+7} = \dfrac{5}{x+1}$, LCD is $(x+7)(x+1)$

$$2x(x+1) = 5(x+7)$$
$$2x^2+2x = 5x+35$$
$$2x^2-3x-35 = 0$$
$$(2x+7)(x-5) = 0$$
$$x=-\frac{7}{2} \quad or \quad x=5$$

Both numbers check. The solutions are $-\dfrac{7}{2}$ and 5.

15. $\dfrac{2}{x+5} + \dfrac{1}{x-5} = \dfrac{16}{x^2-25}$

$\dfrac{2}{x+5} + \dfrac{1}{x-5} = \dfrac{16}{(x+5)(x-5)}$, LCD is $(x+5)(x-5)$

$$(x+5)(x-5)\left(\frac{2}{x+5}+\frac{1}{x-5}\right) = (x+5)(x-5)\cdot\frac{16}{(x+5)(x-5)}$$
$$2(x-5)+x+5 = 16$$
$$2x-10+x+5 = 16$$
$$3x-5 = 16$$
$$3x = 21$$
$$x = 7$$

7 checks, so the solution is 7.

16. $\dfrac{2}{x^2-9} + \dfrac{5}{x-3} = \dfrac{3}{x+3}$

$\dfrac{2}{(x+3)(x-3)} + \dfrac{5}{x-3} = \dfrac{3}{x+3}$, LCD is $(x+3)(x-3)$

$$2+5(x+3) = 3(x-3)$$
$$2+5x+15 = 3x-9$$
$$5x+17 = 3x-9$$
$$2x = -26$$
$$x = -13$$

The answer checks. The solution is -13.

17. $$\frac{3x}{x+2}+\frac{6}{x}=\frac{12}{x^2+2x}$$
$$\frac{3x}{x+2}+\frac{6}{x}=\frac{12}{x(x+2)},\ \text{LCD is } x(x+2)$$
$$x(x+2)\left(\frac{3x}{x+2}+\frac{6}{x}\right)=x(x+2)\cdot\frac{12}{x(x+2)}$$
$$3x\cdot x+6(x+2)=12$$
$$3x^2+6x+12=12$$
$$3x^2+6x=0$$
$$3x(x+2)=0$$
$$3x=0 \quad or \quad x+2=0$$
$$x=0 \quad or \quad x=-2$$
Neither 0 nor -2 checks, so the equation has no solution.

18. $$\frac{3y+5}{y^2+5y}+\frac{y+4}{y+5}=\frac{y+1}{y}$$
$$\frac{3y+5}{y(y+5)}+\frac{y+4}{y+5}=\frac{y+1}{y},\quad \text{LCD is } y(y+5)$$
$$3y+5+y^2+4y=y^2+6y+5$$
Multiplying by $y(y+5)$
$$y=0$$
0 does not check. There is no solution.

19. $$\frac{1}{5x+20}-\frac{1}{x^2-16}=\frac{3}{x-4}$$
$$\frac{1}{5(x+4)}-\frac{1}{(x+4)(x-4)}=\frac{3}{x-4},$$
LCD is $5(x+4)(x-4)$
$$5(x+4)(x-4)\left(\frac{1}{5(x+4)}-\frac{1}{(x+4)(x-4)}\right)=5(x+4)(x-4)\cdot\frac{3}{x-4}$$
$$x-4-5=15(x+4)$$
$$x-9=15x+60$$
$$-14x-9=60$$
$$-14x=69$$
$$x=-\frac{69}{14}$$
$-\frac{69}{14}$ checks, so the solution is $-\frac{69}{14}$.

20. $$\frac{1}{4x+12}-\frac{1}{x^2-9}=\frac{5}{x-3}$$
$$\frac{1}{4(x+3)}-\frac{1}{(x+3)(x-3)}=\frac{5}{x-3},$$
LCD is $4(x+3)(x-3)$
$$x-3-4=20x+60$$
$$-19x=67$$
$$x=-\frac{67}{19}$$
$-\frac{67}{19}$ checks. The solution is $-\frac{67}{19}$.

21. $$\frac{2}{5x+5}-\frac{3}{x^2-1}=\frac{4}{x-1}$$
$$\frac{2}{5(x+1)}-\frac{3}{(x+1)(x-1)}=\frac{4}{x-1},$$
LCD is $5(x+1)(x-1)$
$$5(x+1)(x-1)\left(\frac{2}{5(x+1)}-\frac{3}{(x+1)(x-1)}\right)=5(x+1)(x-1)\cdot\frac{4}{x-1}$$
$$2(x-1)-5\cdot 3=20(x+1)$$
$$2x-2-15=20x+20$$
$$2x-17=20x+20$$
$$-18x-17=20$$
$$-18x=37$$
$$x=-\frac{37}{18}$$
$-\frac{37}{18}$ checks, so the solution is $-\frac{37}{18}$.

22. $$\frac{1}{3x+6}-\frac{1}{x^2-4}=\frac{3}{x-2}$$
$$\frac{1}{3(x+2)}-\frac{1}{(x+2)(x-2)}=\frac{3}{x-2},$$
LCD is $3(x+2)(x-2)$
$$x-2-3=9x+18$$
$$x-5=9x+18$$
$$-8x=23$$
$$x=-\frac{23}{8}$$
$-\frac{23}{8}$ checks. The solution is $-\frac{23}{8}$.

23. $$\frac{8}{x^2-2x+4}=\frac{x}{x+2}+\frac{24}{x^3+8},$$
LCD is $(x+2)(x^2-2x+4)$
$$(x+2)(x^2-2x+4)\cdot\frac{8}{x^2-2x+4}=$$
$$(x+2)(x^2-2x+4)\left(\frac{x}{x+2}+\frac{24}{(x+2)(x^2-2x+4)}\right)$$
$$8(x+2)=x(x^2-2x+4)+24$$
$$8x+16=x^3-2x^2+4x+24$$
$$0=x^3-2x^2-4x+8$$
$$0=x^2(x-2)-4(x-2)$$
$$0=(x-2)(x^2-4)$$
$$0=(x-2)(x+2)(x-2)$$
$$x-2=0 \quad \text{or} \quad x+2=0 \quad \text{or} \quad x-2=0$$
$$x=2 \quad \text{or} \quad x=-2 \quad \text{or} \quad x=2$$
Only 2 checks. The solution is 2.

24. $\dfrac{18}{x^2-3x+9} - \dfrac{x}{x+3} = \dfrac{81}{x^3+27}$,

LCD is $(x+3)(x^2-3x+9)$

$18x+54-x^3+3x^2-9x = 81$ Multiplying by $(x+3)(x^2-3x+9)$

$$-x^3+3x^2+9x-27 = 0$$
$$-x^2(x-3)+9(x-3) = 0$$
$$(x-3)(9-x^2) = 0$$
$$(x-3)(3+x)(3-x) = 0$$
$$x = 3 \quad \text{or} \quad x = -3$$

Only 3 checks. The solution is 3.

25. $$\frac{x}{x-4} - \frac{4}{x+4} = \frac{32}{x^2-16}$$

$$\frac{x}{x-4} - \frac{4}{x+4} = \frac{32}{(x+4)(x-4)},$$

LCD is $(x+4)(x-4)$

$$(x+4)(x-4)\left(\frac{x}{x-4} - \frac{4}{x+4}\right) = (x+4)(x-4)\cdot\frac{32}{(x+4)(x-4)}$$
$$x(x+4)-4(x-4) = 32$$
$$x^2+4x-4x+16 = 32$$
$$x^2+16 = 32$$
$$x^2 = 16$$
$$x = \pm 4$$

Neither 4 nor -4 checks, so the equation has no solution.

26. $$\frac{x}{x-1} - \frac{1}{x+1} = \frac{2}{x^2-1}$$

$$\frac{x}{x-1} - \frac{1}{x+1} = \frac{2}{(x+1)(x-1)},$$

LCD is $(x+1)(x-1)$

$$x^2+x-x+1 = 2$$
$$x^2 = 1$$
$$x = \pm 1$$

Neither 1 nor -1 checks. There is no solution.

27. $$\frac{1}{x-6} - \frac{1}{x} = \frac{6}{x^2-6x}$$

$$\frac{1}{x-6} - \frac{1}{x} = \frac{6}{x(x-6)}, \text{ LCD is } x(x-6)$$

$$x(x-6)\left(\frac{1}{x-6} - \frac{1}{x}\right) = x(x-6)\cdot\frac{6}{x(x-6)}$$
$$x-(x-6) = 6$$
$$x-x+6 = 6$$
$$6 = 6$$

We get an equation that is true for all real numbers. Note, however, that when $x = 6$ or $x = 0$, division by 0 occurs in the original equation. Thus, the solution set is $\{x|x$ is a real number *and* $x \neq 6$ *and* $x \neq 0\}$, or $(-\infty,0)\cup(0,6)\cup(6,\infty)$.

28. $$\frac{1}{x-15} - \frac{1}{x} = \frac{15}{x^2-15x}$$

$$\frac{1}{x-15} - \frac{1}{x} = \frac{15}{x(x-15)}, \text{ LCD is } x(x-15)$$

$$x-(x-15) = 15$$
$$x-x+15 = 15$$
$$15 = 15$$

We get an equation that is true for all real numbers. Note, however, that when $x = 0$ or $x = 15$, division by 0 occurs in the original equation. Thus, the solution set is $\{x|x$ is a real number *and* $x \neq 0$ *and* $x \neq 15\}$, or $(-\infty,0)\cup(0,15)\cup(15,\infty)$.

29. $$\sqrt{3x-4} = 1$$
$$(\sqrt{3x-4})^2 = 1^2$$
$$3x-4 = 1$$
$$3x = 5$$
$$x = \frac{5}{3}$$

Check:

$\sqrt{3x-4}$	$= 1$	
$\sqrt{3\cdot\frac{5}{3}-4}$	? 1	
$\sqrt{5-4}$		
$\sqrt{1}$		
1	1	TRUE

The solution is $\frac{5}{3}$.

30. $$\sqrt{4x+1} = 3$$
$$4x+1 = 9$$
$$4x = 8$$
$$x = 2$$

The answer checks. The solution is 2.

31. $$\sqrt{2x-5} = 2$$
$$(\sqrt{2x-5})^2 = 2^2$$
$$2x-5 = 4$$
$$2x = 9$$
$$x = \frac{9}{2}$$

Check:

$\sqrt{2x-5}$	$= 2$	
$\sqrt{2\cdot\frac{9}{2}-5}$	? 2	
$\sqrt{9-5}$		
$\sqrt{4}$		
2	2	TRUE

The solution is $\frac{9}{2}$.

32. $\sqrt{3x+2} = 6$

$3x + 2 = 36$

$3x = 34$

$x = \frac{34}{3}$

The answer checks. The solution is $\frac{34}{3}$.

33. $\sqrt{7-x} = 2$

$(\sqrt{7-x})^2 = 2^2$

$7 - x = 4$

$-x = -3$

$x = 3$

Check:

$\sqrt{7-x} = 2$		
$\sqrt{7-3}$?	2	
$\sqrt{4}$		
2	2	TRUE

The solution is 3.

34. $\sqrt{5-x} = 1$

$5 - x = 1$

$4 = x$

The answer checks. The solution is 4.

35. $\sqrt{1-2x} = 3$

$(\sqrt{1-2x})^2 = 3^2$

$1 - 2x = 9$

$-2x = 8$

$x = -4$

Check:

$\sqrt{1-2x} = 3$		
$\sqrt{1-2(-4)}$?	3	
$\sqrt{1+8}$		
$\sqrt{9}$		
3	3	TRUE

The solution is -4.

36. $\sqrt{2-7x} = 2$

$2 - 7x = 4$

$-7x = 2$

$x = -\frac{2}{7}$

The answer checks. The solution is $-\frac{2}{7}$.

37. $\sqrt[3]{5x-2} = -3$

$(\sqrt[3]{5x-2})^3 = (-3)^3$

$5x - 2 = -27$

$5x = -25$

$x = -5$

Check:

$\sqrt[3]{5x-2} = -3$		
$\sqrt[3]{5(-5)-2}$?	-3	
$\sqrt[3]{-25-2}$		
$\sqrt[3]{-27}$		
-3	-3	TRUE

The solution is -5.

38. $\sqrt[3]{2x+1} = -5$

$2x + 1 = -125$

$2x = -126$

$x = -63$

The answer checks. The solution is -63.

39. $\sqrt[4]{x^2-1} = 1$

$(\sqrt[4]{x^2-1})^4 = 1^4$

$x^2 - 1 = 1$

$x^2 = 2$

$x = \pm\sqrt{2}$

Check:

$\sqrt[4]{x^2-1} = 1$		
$\sqrt[4]{(\pm\sqrt{2})^2-1}$?	1	
$\sqrt[4]{2-1}$		
$\sqrt[4]{1}$		
1	1	TRUE

The solutions are $\pm\sqrt{2}$.

40. $\sqrt[5]{3x+4} = 2$

$3x + 4 = 32$

$3x = 28$

$x = \frac{28}{3}$

The answer checks. The solution is $\frac{28}{3}$.

41. $\sqrt{y-1} + 4 = 0$

$\sqrt{y-1} = -4$

The principal square root is never negative. Thus, there is no solution.

If we do not observe the above fact, we can continue and reach the same answer.

$(\sqrt{y-1})^2 = (-4)^2$

$y - 1 = 16$

$y = 17$

Check:

$\sqrt{y-1} + 4 = 0$		
$\sqrt{17-1} + 4$?	0	
$\sqrt{16} + 4$		
$4 + 4$		
8	0	FALSE

Since 17 does not check, there is no solution.

42. $\sqrt{m+1}-5=8$

$\sqrt{m+1}=13$

$m+1=169$

$m=168$

The answer checks. The solution is 168.

43. $\sqrt{b+3}-2=1$

$\sqrt{b+3}=3$

$(\sqrt{b+3})^2=3^2$

$b+3=9$

$b=6$

Check:

$\sqrt{b+3}-2=1$		
$\sqrt{6+3}-2$	? 1	
$\sqrt{9}-2$		
$3-2$		
1	1	TRUE

The solution is 6.

44. $\sqrt{x-4}+1=5$

$\sqrt{x-4}=4$

$x-4=16$

$x=20$

The answer checks. The solution is 20.

45. $\sqrt{z+2}+3=4$

$\sqrt{z+2}=1$

$(\sqrt{z+2})^2=1^2$

$z+2=1$

$z=-1$

Check:

$\sqrt{z+2}+3=4$		
$\sqrt{-1+2}+3$	? 4	
$\sqrt{1}+3$		
$1+3$		
4	4	TRUE

The solution is -1.

46. $\sqrt{y-5}-2=3$

$\sqrt{y-5}=5$

$y-5=25$

$y=30$

The answer checks. The solution is 30.

47. $\sqrt{2x+1}-3=3$

$\sqrt{2x+1}=6$

$(\sqrt{2x+1})^2=6^2$

$2x+1=36$

$2x=35$

$x=\frac{35}{2}$

Check:

$\sqrt{2x+1}-3=3$		
$\sqrt{2\cdot\frac{35}{2}+1}-3$	? 3	
$\sqrt{35+1}-3$		
$\sqrt{36}-3$		
$6-3$		
3	3	TRUE

The solution is $\frac{35}{2}$.

48. $\sqrt{3x-1}+2=7$

$\sqrt{3x-1}=5$

$3x-1=25$

$3x=26$

$x=\frac{26}{3}$

The answer checks. The solution is $\frac{26}{3}$.

49. $\sqrt{2-x}-4=6$

$\sqrt{2-x}=10$

$(\sqrt{2-x})^2=10^2$

$2-x=100$

$-x=98$

$x=-98$

Check:

$\sqrt{2-x}-4=6$		
$\sqrt{2-(-98)}-4$	? 6	
$\sqrt{100}-4$		
$10-4$		
6	6	TRUE

The solution is -98.

50. $\sqrt{5-x}+2=8$

$\sqrt{5-x}=6$

$5-x=36$

$-x=31$

$x=-31$

The answer checks. The solution is -31.

51. $\sqrt[3]{6x+9}+8=5$

$\sqrt[3]{6x+9}=-3$

$(\sqrt[3]{6x+9})^3=(-3)^3$

$6x+9=-27$

$6x=-36$

$x=-6$

Check:

$$\begin{array}{r|l} \sqrt[3]{6x+9}+8 = 5 & \\ \hline \sqrt[3]{6(-6)+9}+8 \ ? & 5 \\ \sqrt[3]{-27}+8 & \\ -3+8 & \\ 5 & 5 \quad \text{TRUE} \end{array}$$

The solution is -6.

52. $\sqrt[5]{2x-3}-1 = 1$

$$\begin{aligned} \sqrt[5]{2x-3} &= 2 \\ 2x-3 &= 32 \\ 2x &= 35 \\ x &= \frac{35}{2} \end{aligned}$$

The answer checks. The solution is $\frac{35}{2}$.

53. $\sqrt{x+4}+2 = x$

$$\begin{aligned} \sqrt{x+4} &= x-2 \\ (\sqrt{x+4})^2 &= (x-2)^2 \\ x+4 &= x^2-4x+4 \\ 0 &= x^2-5x \\ 0 &= x(x-5) \end{aligned}$$

$x = 0$ *or* $x-5 = 0$

$x = 0$ *or* $x = 5$

Check:

For 0:

$$\begin{array}{r|l} \sqrt{x+4}+2 = x & \\ \hline \sqrt{0+4}+2 \ ? & 0 \\ 2+2 & \\ 4 & 0 \quad \text{FALSE} \end{array}$$

For 5:

$$\begin{array}{r|l} \sqrt{x+4}+2 = x & \\ \hline \sqrt{5+4}+2 \ ? & 5 \\ \sqrt{9}+2 & \\ 3+2 & \\ 5 & 5 \quad \text{TRUE} \end{array}$$

The number 5 checks but 0 does not. The solution is 5.

54. $\sqrt{x+1}+1 = x$

$$\begin{aligned} \sqrt{x+1} &= x-1 \\ x+1 &= x^2-2x+1 \\ 0 &= x^2-3x \\ 0 &= x(x-3) \end{aligned}$$

$x = 0$ *or* $x = 3$

Only 3 checks. The solution is 3.

55. $\sqrt{x-3}+5 = x$

$$\begin{aligned} \sqrt{x-3} &= x-5 \\ (\sqrt{x-3})^2 &= (x-5)^2 \\ x-3 &= x^2-10x+25 \\ 0 &= x^2-11x+28 \\ 0 &= (x-4)(x-7) \end{aligned}$$

$x-4 = 0$ *or* $x-7 = 0$

$x = 4$ *or* $x = 7$

Check:

For 4:

$$\begin{array}{r|l} \sqrt{x-3}+5 = x & \\ \hline \sqrt{4-3}+5 \ ? & 4 \\ \sqrt{1}+5 & \\ 1+5 & \\ 6 & 4 \quad \text{FALSE} \end{array}$$

For 7:

$$\begin{array}{r|l} \sqrt{x-3}+5 = x & \\ \hline \sqrt{7-3}+5 \ ? & 7 \\ \sqrt{4}+5 & \\ 2+5 & \\ 7 & 7 \quad \text{TRUE} \end{array}$$

The number 7 checks but 4 does not. The solution is 7.

56. $\sqrt{x+3}-1 = x$

$$\begin{aligned} \sqrt{x+3} &= x+1 \\ x+3 &= x^2+2x+1 \\ 0 &= x^2+x-2 \\ 0 &= (x+2)(x-1) \end{aligned}$$

$x = -2$ *or* $x = 1$

Only 1 checks. The solution is 1.

57.

$$\begin{aligned} \sqrt{x+7} &= x+1 \\ (\sqrt{x+7})^2 &= (x+1)^2 \\ x+7 &= x^2+2x+1 \\ 0 &= x^2+x-6 \\ 0 &= (x+3)(x-2) \end{aligned}$$

$x+3 = 0$ *or* $x-2 = 0$

$x = -3$ *or* $x = 2$

Check:

For -3:

$$\begin{array}{r|l} \sqrt{x+7} = & x+1 \\ \hline \sqrt{-3+7} \ ? & -3+1 \\ \sqrt{4} & -2 \\ 2 & -2 \quad \text{FALSE} \end{array}$$

For 2:

$$\begin{array}{c|c} \sqrt{x+7} = x+1 & \\ \hline \sqrt{2+7} \ ? & 2+1 \\ \sqrt{9} & 3 \\ 3 & 3 \quad \text{TRUE} \end{array}$$

The number 2 checks but -3 does not. The solution is 2.

58.
$$\begin{aligned} \sqrt{6x+7} &= x+2 \\ 6x+7 &= x^2+4x+4 \\ 0 &= x^2-2x-3 \\ 0 &= (x-3)(x+1) \end{aligned}$$

$x=3$ *or* $x=-1$

Both values check. The solutions are 3 and -1.

59.
$$\begin{aligned} \sqrt{3x+3} &= x+1 \\ (\sqrt{3x+3})^2 &= (x+1)^2 \\ 3x+3 &= x^2+2x+1 \\ 0 &= x^2-x-2 \\ 0 &= (x-2)(x+1) \end{aligned}$$

$x-2=0$ *or* $x+1=0$

$x=2$ *or* $x=-1$

Check:

For 2:

$$\begin{array}{c|c} \sqrt{3x+3} = x+1 & \\ \hline \sqrt{3\cdot 2+3} \ ? & 2+1 \\ \sqrt{9} & 3 \\ 3 & 3 \quad \text{TRUE} \end{array}$$

For -1:

$$\begin{array}{c|c} \sqrt{3x+3} = x+1 & \\ \hline \sqrt{3(-1)+3} \ ? & -1+1 \\ \sqrt{0} & 0 \\ 0 & 0 \quad \text{TRUE} \end{array}$$

Both numbers check. The solutions are 2 and -1.

60.
$$\begin{aligned} \sqrt{2x+5} &= x-5 \\ 2x+5 &= x^2-10x+25 \\ 0 &= x^2-12x+20 \\ 0 &= (x-2)(x-10) \end{aligned}$$

$x=2$ *or* $x=10$

Only 10 checks. The solution is 10.

61.
$$\begin{aligned} \sqrt{5x+1} &= x-1 \\ (\sqrt{5x+1})^2 &= (x-1)^2 \\ 5x+1 &= x^2-2x+1 \\ 0 &= x^2-7x \\ 0 &= x(x-7) \end{aligned}$$

$x=0$ *or* $x-7=0$

$x=0$ *or* $x=7$

Check:

For 0:

$$\begin{array}{c|c} \sqrt{5x+1} = x-1 & \\ \hline \sqrt{5\cdot 0+1} \ ? & 0-1 \\ \sqrt{1} & -1 \\ 1 & -1 \quad \text{FALSE} \end{array}$$

For 7:

$$\begin{array}{c|c} \sqrt{5x+1} = x-1 & \\ \hline \sqrt{5\cdot 7+1} \ ? & 7-1 \\ \sqrt{36} & 6 \\ 6 & 6 \quad \text{TRUE} \end{array}$$

The number 7 checks but 0 does not. The solution is 7.

62.
$$\begin{aligned} \sqrt{7x+4} &= x+2 \\ 7x+4 &= x^2+4x+4 \\ 0 &= x^2-3x \\ 0 &= x(x-3) \end{aligned}$$

$x=0$ *or* $x=3$

Both numbers check. The solutions are 0 and 3.

63.
$$\begin{aligned} \sqrt{x-3}+\sqrt{x+2} &= 5 \\ \sqrt{x+2} &= 5-\sqrt{x-3} \\ (\sqrt{x+2})^2 &= (5-\sqrt{x-3})^2 \\ x+2 &= 25-10\sqrt{x-3}+(x-3) \\ x+2 &= 22-10\sqrt{x-3}+x \\ 10\sqrt{x-3} &= 20 \\ \sqrt{x-3} &= 2 \\ (\sqrt{x-3})^2 &= 2^2 \\ x-3 &= 4 \\ x &= 7 \end{aligned}$$

Check:

$$\begin{array}{c|c} \sqrt{x-3}+\sqrt{x+2} = 5 & \\ \hline \sqrt{7-3}+\sqrt{7+2} \ ? & 5 \\ \sqrt{4}+\sqrt{9} & \\ 2+3 & \\ 5 & 5 \quad \text{TRUE} \end{array}$$

The solution is 7.

64.
$$\begin{aligned} \sqrt{x}-\sqrt{x-5} &= 1 \\ \sqrt{x} &= \sqrt{x-5}+1 \\ x &= x-5+2\sqrt{x-5}+1 \\ 4 &= 2\sqrt{x-5} \\ 2 &= \sqrt{x-5} \\ 4 &= x-5 \\ 9 &= x \end{aligned}$$

The answer checks. The solution is 9.

65. $\sqrt{3x-5}+\sqrt{2x+3}+1=0$

$\sqrt{3x-5}+\sqrt{2x+3}=-1$

The principal square root is never negative. Thus the sum of two principal square roots cannot equal -1. There is no solution.

66.
$$\begin{aligned}\sqrt{2m-3} &= \sqrt{m+7}-2\\ 2m-3 &= m+7-4\sqrt{m+7}+4\\ m-14 &= -4\sqrt{m+7}\\ m^2-28m+196 &= 16m+112\\ m^2-44m+84 &= 0\\ (m-2)(m-42) &= 0\end{aligned}$$

$m=2$ *or* $m=42$

Only 2 checks. The solution is 2.

67.
$$\begin{aligned}\sqrt{x}-\sqrt{3x-3} &= 1\\ \sqrt{x} &= \sqrt{3x-3}+1\\ (\sqrt{x})^2 &= (\sqrt{3x-3}+1)^2\\ x &= (3x-3)+2\sqrt{3x-3}+1\\ 2-2x &= 2\sqrt{3x-3}\\ 1-x &= \sqrt{3x-3}\\ (1-x)^2 &= (\sqrt{3x-3})^2\\ 1-2x+x^2 &= 3x-3\\ x^2-5x+4 &= 0\\ (x-4)(x-1) &= 0\end{aligned}$$

$x=4$ *or* $x=1$

The number 4 does not check, but 1 does. The solution is 1.

68.
$$\begin{aligned}\sqrt{2x+1}-\sqrt{x} &= 1\\ \sqrt{2x+1} &= \sqrt{x}+1\\ 2x+1 &= x+2\sqrt{x}+1\\ x &= 2\sqrt{x}\\ x^2 &= 4x\\ x^2-4x &= 0\\ x(x-4) &= 0\end{aligned}$$

$x=0$ *or* $x=4$

Both values check. The solutions are 0 and 4.

69.
$$\begin{aligned}\sqrt{2y-5}-\sqrt{y-3} &= 1\\ \sqrt{2y-5} &= \sqrt{y-3}+1\\ (\sqrt{2y-5})^2 &= (\sqrt{y-3}+1)^2\\ 2y-5 &= (y-3)+2\sqrt{y-3}+1\\ y-3 &= 2\sqrt{y-3}\\ (y-3)^2 &= (2\sqrt{y-3})^2\\ y^2-6y+9 &= 4(y-3)\\ y^2-6y+9 &= 4y-12\\ y^2-10y+21 &= 0\\ (y-7)(y-3) &= 0\end{aligned}$$

$y=7$ *or* $y=3$

Both numbers check. The solutions are 7 and 3.

70.
$$\begin{aligned}\sqrt{4p+5}+\sqrt{p+5} &= 3\\ \sqrt{4p+5} &= 3-\sqrt{p+5}\\ 4p+5 &= 9-6\sqrt{p+5}+p+5\\ 3p-9 &= -6\sqrt{p+5}\\ p-3 &= -2\sqrt{p+5}\\ p^2-6p+9 &= 4p+20\\ p^2-10p-11 &= 0\\ (p-11)(p+1) &= 0\end{aligned}$$

$p=11$ *or* $p=-1$

Only -1 checks. The solution is -1.

71.
$$\begin{aligned}\sqrt{y+4}-\sqrt{y-1} &= 1\\ \sqrt{y+4} &= \sqrt{y-1}+1\\ (\sqrt{y+4})^2 &= (\sqrt{y-1}+1)^2\\ y+4 &= y-1+2\sqrt{y-1}+1\\ 4 &= 2\sqrt{y-1}\\ 2 &= \sqrt{y-1} \quad \text{Dividing by 2}\\ 2^2 &= (\sqrt{y-1})^2\\ 4 &= y-1\\ 5 &= y\end{aligned}$$

The answer checks. The solution is 5.

72.
$$\begin{aligned}\sqrt{y+7}+\sqrt{y+16} &= 9\\ \sqrt{y+7} &= 9-\sqrt{y+16}\\ y+7 &= 81-18\sqrt{y+16}+y+16\\ -90 &= -18\sqrt{y+16}\\ 5 &= \sqrt{y+16}\\ 25 &= y+16\\ 9 &= y\end{aligned}$$

The answer checks. The solution is 9.

73.
$$\begin{aligned}\sqrt{x+5}+\sqrt{x+2} &= 3\\ \sqrt{x+5} &= 3-\sqrt{x+2}\\ (\sqrt{x+5})^2 &= (3-\sqrt{x+2})^2\\ x+5 &= 9-6\sqrt{x+2}+x+2\\ -6 &= -6\sqrt{x+2}\\ 1 &= \sqrt{x+2} \quad \text{Dividing by } -6\\ 1^2 &= (\sqrt{x+2})^2\\ 1 &= x+2\\ -1 &= x\end{aligned}$$

The answer checks. The solution is -1.

74.
$$\begin{aligned}\sqrt{6x+6} &= 5+\sqrt{21-4x}\\ 6x+6 &= 25+10\sqrt{21-4x}+21-4x\\ 10x-40 &= 10\sqrt{21-4x}\\ x-4 &= \sqrt{21-4x}\\ x^2-8x+16 &= 21-4x\\ x^2-4x-5 &= 0\\ (x-5)(x+1) &= 0\end{aligned}$$

$x = 5 \quad or \quad x = -1$

Only 5 checks. The solution is 5.

75. $x^{1/3} = -2$

$(x^{1/3})^3 = (-2)^3 \qquad (x^{1/3} = \sqrt[3]{x})$

$x = -8$

The value checks. The solution is -8.

76. $t^{1/5} = 2$

$t = 32$

The value checks. The solution is 32.

77. $t^{1/4} = 3$

$(t^{1/4})^4 = 3^4 \qquad (t^{1/4} = \sqrt[4]{t})$

$t = 81$

The value checks. The solution is 81.

78. $m^{1/2} = -7$

The principal square root is never negative. There is no solution.

79. $|x| = 7$

The solutions are those numbers whose distance from 0 on a number line is 7. They are -7 and 7. That is,

$x = -7 \quad or \quad x = 7.$

The solutions are -7 and 7.

80. $|x| = 4.5$

$x = -4.5 \quad or \quad x = 4.5$

The solutions are -4.5 and 4.5.

81. $|x| = -10.7$

The absolute value of a number is nonnegative. Thus, the equation has no solution.

82. $|x| = -\frac{3}{5}$

The absolute value of a number is nonnegative. Thus, there is no solution.

83. $|x - 1| = 4$

$x - 1 = -4 \quad or \quad x - 1 = 4$

$x = -3 \quad or \quad x = 5$

The solutions are -3 and 5.

84. $|x - 7| = 5$

$x - 7 = -5 \quad or \quad x - 7 = 5$

$x = 2 \quad or \quad x = 12$

The solutions are 2 and 12.

85. $|3x| = 1$

$3x = -1 \quad or \quad 3x = 1$

$x = -\frac{1}{3} \quad or \quad x = \frac{1}{3}$

The solutions are $-\frac{1}{3}$ and $\frac{1}{3}$.

86. $|5x| = 4$

$5x = -4 \quad or \quad 5x = 4$

$x = -\frac{4}{5} \quad or \quad x = \frac{4}{5}$

The solutions are $-\frac{4}{5}$ and $\frac{4}{5}$.

87. $|x| = 0$

The distance of 0 from 0 on a number line is 0. That is,

$x = 0.$

The solution is 0.

88. $|6x| = 0$

$6x = 0$

$x = 0$

The solution is 0.

89. $|3x + 2| = 1$

$3x + 2 = -1 \quad or \quad 3x + 2 = 1$

$3x = -3 \quad or \quad 3x = -1$

$x = -1 \quad or \quad x = -\frac{1}{3}$

The solutions are -1 and $-\frac{1}{3}$.

90. $|7x - 4| = 8$

$7x - 4 = -8 \quad or \quad 7x - 4 = 8$

$7x = -4 \quad or \quad 7x = 12$

$x = -\frac{4}{7} \quad or \quad x = \frac{12}{7}$

The solutions are $-\frac{4}{7}$ and $\frac{12}{7}$.

91. $\left|\frac{1}{2}x - 5\right| = 17$

$\frac{1}{2}x - 5 = -17 \quad or \quad \frac{1}{2}x - 5 = 17$

$\frac{1}{2}x = -12 \quad or \quad \frac{1}{2}x = 22$

$x = -24 \quad or \quad x = 44$

The solutions are -24 and 44.

92. $\left|\frac{1}{3}x - 4\right| = 13$

$\frac{1}{3}x - 4 = -13 \quad or \quad \frac{1}{3}x - 4 = 13$

$\frac{1}{3}x = -9 \quad or \quad \frac{1}{3}x = 17$

$x = -27 \quad or \quad x = 51$

The solutions are -27 and 51.

93. $|x - 1| + 3 = 6$

$|x - 1| = 3$

$x - 1 = -3 \quad or \quad x - 1 = 3$

$x = -2 \quad or \quad x = 4$

The solutions are -2 and 4.

94. $|x+2|-5=9$

$|x+2|=14$

$x+2=-14$ *or* $x+2=14$

$x=-16$ *or* $x=12$

The solutions are -16 and 12.

95. $|x+3|-2=8$

$|x+3|=10$

$x+3=-10$ *or* $x+3=10$

$x=-13$ *or* $x=7$

The solutions are -13 and 7.

96. $|x-4|+3=9$

$|x-4|=6$

$x-4=-6$ *or* $x-4=6$

$x=-2$ *or* $x=10$

The solutions are -2 and 10.

97. $|3x+1|-4=-1$

$|3x+1|=3$

$3x+1=-3$ *or* $3x+1=3$

$3x=-4$ *or* $3x=2$

$x=-\frac{4}{3}$ *or* $x=\frac{2}{3}$

The solutions are $-\frac{4}{3}$ and $\frac{2}{3}$.

98. $|2x-1|-5=-3$

$|2x-1|=2$

$2x-1=-2$ *or* $2x-1=2$

$2x=-1$ *or* $2x=3$

$x=-\frac{1}{2}$ *or* $x=\frac{3}{2}$

The solutions are $-\frac{1}{2}$ and $\frac{3}{2}$.

99. $|4x-3|+1=7$

$|4x-3|=6$

$4x-3=-6$ *or* $4x-3=6$

$4x=-3$ *or* $4x=9$

$x=-\frac{3}{4}$ *or* $x=\frac{9}{4}$

The solutions are $-\frac{3}{4}$ and $\frac{9}{4}$.

100. $|5x+4|+2=5$

$|5x+4|=3$

$5x+4=-3$ *or* $5x+4=3$

$5x=-7$ *or* $5x=-1$

$x=-\frac{7}{5}$ *or* $x=-\frac{1}{5}$

The solutions are $-\frac{7}{5}$ and $-\frac{1}{5}$.

101. $12-|x+6|=5$

$-|x+6|=-7$

$|x+6|=7$ Multiplying by -1

$x+6=-7$ *or* $x+6=7$

$x=-13$ *or* $x=1$

The solutions are -13 and 1.

102. $9-|x-2|=7$

$2=|x-2|$

$x-2=-2$ *or* $x-2=2$

$x=0$ *or* $x=4$

The solutions are 0 and 4.

103. $\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$

$P_1V_1T_2=P_2V_2T_1$ Multiplying by T_1T_2 on both sides

$\frac{P_1V_1T_2}{P_2V_2}=T_1$ Dividing by P_2V_2 on both sides

104. $\frac{1}{F}=\frac{1}{m}+\frac{1}{p}$

$mp=Fp+Fm$

$mp=F(p+m)$

$\frac{mp}{p+m}=F$

105. $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$

$RR_1R_2\cdot\frac{1}{R}=RR_1R_2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

Multiplying by RR_1R_2 on both sides

$R_1R_2=RR_2+RR_1$

$R_1R_2-RR_2=RR_1$ Subtracting RR_2 on both sides

$R_2(R_1-R)=RR_1$ Factoring

$R_2=\frac{RR_1}{R_1-R}$ Dividing by R_1-R on both sides

106. $A=P(1+i)^2$

$\frac{A}{P}=(1+i)^2$

$\sqrt{\frac{A}{P}}=1+i$

$\sqrt{\frac{A}{P}}-1=i$

107.
$$\frac{1}{F} = \frac{1}{m} + \frac{1}{p}$$
$$Fmp \cdot \frac{1}{F} = Fmp\left(\frac{1}{m} + \frac{1}{p}\right) \quad \text{Multiplying by } Fmp \text{ on both sides}$$
$$mp = Fp + Fm$$
$$mp - Fp = Fm \quad \text{Subtracting } Fp \text{ on both sides}$$
$$p(m - F) = Fm \quad \text{Factoring}$$
$$p = \frac{Fm}{m - F} \quad \text{Dividing by } m - F \text{ on both sides}$$

108. Left to the student

109. Left to the student

110. When both sides of an equation are multiplied by the LCD, the resulting equation might not be equivalent to the original equation. One or more of the possible solutions of the resulting equation might make a denominator of the original equation 0.

111. When both sides of an equation are raised to an even power, the resulting equation might not be equivalent to the original equation. For example, the solution set of $x = -2$ is $\{-2\}$, but the solution set of $x^2 = (-2)^2$, or $x^2 = 4$, is $\{-2, 2\}$.

112.
$$-3x + 9 = 0$$
$$-3x = -9$$
$$x = 3$$

The zero of the function is 3.

113.
$$15 - 2x = 0 \quad \text{Setting } f(x) = 0$$
$$15 = 2x$$
$$\frac{15}{2} = x, \text{ or}$$
$$7.5 = x$$

The zero of the function is $\frac{15}{2}$, or 7.5.

114. Let d = the number of acres Disneyland occupies.

Solve: $(d + 11) + d = 181$

$d = 85$, so Disneyland occupies 85 acres and the Mall of America occupies $85 + 11$, or 96 acres.

115. ***Familiarize***. Let p = the number of prescriptions for sleeping pills filled in 2000, in millions. Then the number of prescriptions filled in 2005 was $p + 60\% \cdot p$, or $p + 0.6p$, or $1.6p$.

Translate.

$$\underbrace{\text{Number of prescriptions filled in 2005}}_{\downarrow\; 1.6p} \quad \underset{\downarrow\; =}{\text{was}} \quad \underbrace{\text{42 million.}}_{\downarrow\; 42}$$

Carry out. We solve the equation.
$$1.6p = 42$$
$$p = 26.25$$

Check. 60% of 26.25 is 0.6(26.25), or 15.75, and $26.25 + 15.75 = 42$. The answer checks.

State. 26.25 million prescriptions for sleeping pills were filled in 2000.

116.
$$\frac{x+3}{x+2} - \frac{x+4}{x+3} = \frac{x+5}{x+4} - \frac{x+6}{x+5},$$
LCD is $(x + 2)(x + 3)(x + 4)(x + 5)$
$$x^4 + 15x^3 + 83x^2 + 201x + 180 - x^4 - 15x^3 -$$
$$82x^2 - 192x - 160 = x^4 + 15x^3 + 81x^2 + 185x +$$
$$150 - x^4 - 15x^3 - 80x^2 - 180x - 144$$
$$x^2 + 9x + 20 = x^2 + 5x + 6$$
$$4x = -14$$
$$x = -\frac{7}{2}$$

The number $-\frac{7}{2}$ checks. The solution is $-\frac{7}{2}$.

117.
$$(x - 3)^{2/3} = 2$$
$$[(x - 3)^{2/3}]^3 = 2^3$$
$$(x - 3)^2 = 8$$
$$x^2 - 6x + 9 = 8$$
$$x^2 - 6x + 1 = 0$$
$a = 1$, $b = -6$, $c = 1$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$
$$= \frac{6 \pm \sqrt{32}}{2} = \frac{6 \pm 4\sqrt{2}}{2}$$
$$= \frac{2(3 \pm 2\sqrt{2})}{2} = 3 \pm 2\sqrt{2}$$

Both values check. The solutions are $3 \pm 2\sqrt{2}$.

118.
$$\sqrt{15 + \sqrt{2x + 80}} = 5$$
$$\left(\sqrt{15 + \sqrt{2x + 80}}\right)^2 = 5^2$$
$$15 + \sqrt{2x + 80} = 25$$
$$\sqrt{2x + 80} = 10$$
$$(\sqrt{2x + 80})^2 = 10^2$$
$$2x + 80 = 100$$
$$2x = 20$$
$$x = 10$$

This number checks. The solution is 10.

119.
$$\sqrt{x + 5} + 1 = \frac{6}{\sqrt{x + 5}}, \text{ LCD is } \sqrt{x + 5}$$
$$x + 5 + \sqrt{x + 5} = 6 \quad \text{Multiplying by } \sqrt{x + 5}$$
$$\sqrt{x + 5} = 1 - x$$
$$x + 5 = 1 - 2x + x^2$$
$$0 = x^2 - 3x - 4$$
$$0 = (x - 4)(x + 1)$$
$x = 4$ *or* $x = -1$

Only -1 checks. The solution set is -1.

120.
$$\begin{aligned} x^{2/3} &= x \\ (x^{2/3})^3 &= x^3 \\ x^2 &= x^3 \\ 0 &= x^3 - x^2 \\ 0 &= x^2(x-1) \\ x^2 = 0 \quad &or \quad x - 1 = 0 \\ x = 0 \quad &or \quad x = 1 \end{aligned}$$

Both numbers check. The solutions are 0 and 1.

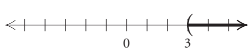
Exercise Set 2.6

1.
$$\begin{aligned} x + 6 &< 5x - 6 \\ 6 + 6 &< 5x - x \quad \text{Subtracting } x \text{ and adding 6 on both sides} \\ 12 &< 4x \\ \frac{12}{4} &< x \quad \text{Dividing by 4 on both sides} \\ 3 &< x \end{aligned}$$

This inequality could also be solved as follows:

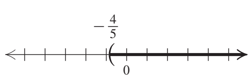
$$\begin{aligned} x + 6 &< 5x - 6 \\ x - 5x &< -6 - 6 \quad \text{Subtracting } 5x \text{ and 6 on both sides} \\ -4x &< -12 \\ x &> \frac{-12}{-4} \quad \text{Dividing by } -4 \text{ on both sides and reversing the inequality symbol} \\ x &> 3 \end{aligned}$$

The solution set is $\{x|x > 3\}$, or $(3, \infty)$. The graph is shown below.



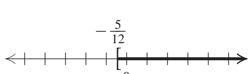
2.
$$\begin{aligned} 3 - x &< 4x + 7 \\ -5x &< 4 \\ x &> -\frac{4}{5} \end{aligned}$$

The solution set is $\left\{x\middle|x > -\frac{4}{5}\right\}$, or $\left(-\frac{4}{5}, \infty\right)$. The graph is shown below.



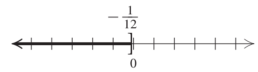
3.
$$\begin{aligned} 3x - 3 + 2x &\geq 1 - 7x - 9 \\ 5x - 3 &\geq -7x - 8 \quad \text{Collecting like terms} \\ 5x + 7x &\geq -8 + 3 \quad \text{Adding } 7x \text{ and 3 on both sides} \\ 12x &\geq -5 \\ x &\geq -\frac{5}{12} \quad \text{Dividing by 12 on both sides} \end{aligned}$$

The solution set is $\left\{x\middle|x \geq -\frac{5}{12}\right\}$, or $\left[-\frac{5}{12}, \infty\right)$. The graph is shown below.



4.
$$\begin{aligned} 5y - 5 + y &\leq 2 - 6y - 8 \\ 6y - 5 &\leq -6y - 6 \\ 12y &\leq -1 \\ y &\leq -\frac{1}{12} \end{aligned}$$

The solution set is $\left\{y\middle|y \leq -\frac{1}{12}\right\}$, or $\left(-\infty, -\frac{1}{12}\right]$. The graph is shown below.

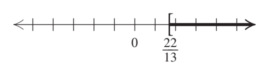


5.
$$\begin{aligned} 14 - 5y &\leq 8y - 8 \\ 14 + 8 &\leq 8y + 5y \\ 22 &\leq 13y \\ \frac{22}{13} &\leq y \end{aligned}$$

This inequality could also be solved as follows:

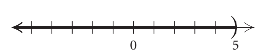
$$\begin{aligned} 14 - 5y &\leq 8y - 8 \\ -5y - 8y &\leq -8 - 14 \\ -13y &\leq -22 \\ y &\geq \frac{22}{13} \quad \text{Dividing by } -13 \text{ on both sides and reversing the inequality symbol} \end{aligned}$$

The solution set is $\left\{y\middle|y \geq \frac{22}{13}\right\}$, or $\left[\frac{22}{13}, \infty\right)$. The graph is shown below.



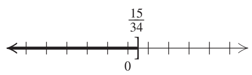
6.
$$\begin{aligned} 8x - 7 &< 6x + 3 \\ 2x &< 10 \\ x &< 5 \end{aligned}$$

The solution set is $\{x|x < 5\}$, or $(-\infty, 5)$. The graph is shown below.



7.
$$\begin{aligned} -\frac{3}{4}x &\geq -\frac{5}{8} + \frac{2}{3}x \\ \frac{5}{8} &\geq \frac{3}{4}x + \frac{2}{3}x \\ \frac{5}{8} &\geq \frac{9}{12}x + \frac{8}{12}x \\ \frac{5}{8} &\geq \frac{17}{12}x \\ \frac{12}{17} \cdot \frac{5}{8} &\geq \frac{12}{17} \cdot \frac{17}{12}x \\ \frac{15}{34} &\geq x \end{aligned}$$

The solution set is $\left\{x\middle|x \leq \frac{15}{34}\right\}$, or $\left(-\infty, \frac{15}{34}\right]$. The graph is shown below.

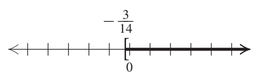


8. $-\dfrac{5}{6}x \le \dfrac{3}{4} + \dfrac{8}{3}x$

$-\dfrac{21}{6}x \le \dfrac{3}{4}$

$x \ge -\dfrac{3}{14}$

The solution set is $\left\{x\middle|x \ge -\dfrac{3}{14}\right\}$, or $\left[-\dfrac{3}{14}, \infty\right)$. The graph is shown below.



9. $4x(x-2) < 2(2x-1)(x-3)$

$4x(x-2) < 2(2x^2-7x+3)$

$4x^2-8x < 4x^2-14x+6$

$-8x < -14x+6$

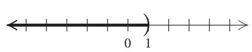
$-8x+14x < 6$

$6x < 6$

$x < \dfrac{6}{6}$

$x < 1$

The solution set is $\{x|x < 1\}$, or $(-\infty, 1)$. The graph is shown below.



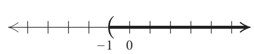
10. $(x+1)(x+2) > x(x+1)$

$x^2+3x+2 > x^2+x$

$2x > -2$

$x > -1$

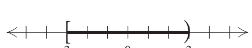
The solution set is $\{x|x > -1\}$, or $(-1, \infty)$. The graph is shown below.



11. $-2 \le x+1 < 4$

$-3 \le x < 3$ Subtracting 1

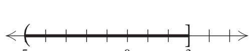
The solution set is $[-3, 3)$. The graph is shown below.



12. $-3 < x+2 \le 5$

$-5 < x \le 3$

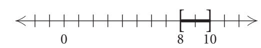
$(-5, 3]$



13. $5 \le x-3 \le 7$

$8 \le x \le 10$ Adding 3

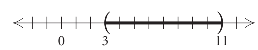
The solution set is $[8, 10]$. The graph is shown below.



14. $-1 < x-4 < 7$

$3 < x < 11$

$(3, 11)$



15. $-3 \le x+4 \le 3$

$-7 \le x \le -1$ Subtracting 4

The solution set is $[-7, -1]$. The graph is shown below.

16. $-5 < x+2 < 15$

$-7 < x < 13$

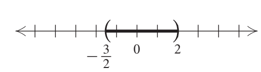
$(-7, 13)$

17. $-2 < 2x+1 < 5$

$-3 < 2x < 4$ Adding -1

$-\dfrac{3}{2} < x < 2$ Multiplying by $\dfrac{1}{2}$

The solution set is $\left(-\dfrac{3}{2}, 2\right)$. The graph is shown below.

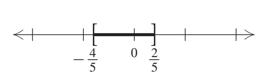


18. $-3 \le 5x+1 \le 3$

$-4 \le 5x \le 2$

$-\dfrac{4}{5} \le x \le \dfrac{2}{5}$

$\left[-\dfrac{4}{5}, \dfrac{2}{5}\right]$



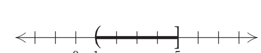
19. $-4 \le 6-2x < 4$

$-10 \le -2x < -2$ Adding -6

$5 \ge x > 1$ Multiplying by $-\dfrac{1}{2}$

or $1 < x \le 5$

The solution set is $(1, 5]$. The graph is shown below.

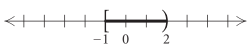


20. $-3 < 1 - 2x \leq 3$

$-4 < -2x \leq 2$

$2 > x \geq -1$

$[-1, 2)$



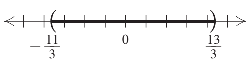
21. $-5 < \frac{1}{2}(3x+1) < 7$

$-10 < 3x + 1 < 14$ Multiplying by 2

$-11 < 3x < 13$ Adding -1

$-\frac{11}{3} < x < \frac{13}{3}$ Multiplying by $\frac{1}{3}$

The solution set is $\left(-\frac{11}{3}, \frac{13}{3}\right)$. The graph is shown below.

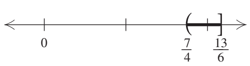


22. $\frac{2}{3} \leq -\frac{4}{5}(x-3) < 1$

$-\frac{5}{6} \geq x - 3 > -\frac{5}{4}$

$\frac{13}{6} \geq x > \frac{7}{4}$

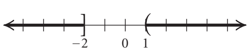
$\left(\frac{7}{4}, \frac{13}{6}\right]$



23. $3x \leq -6$ *or* $x - 1 > 0$

$x \leq -2$ *or* $x > 1$

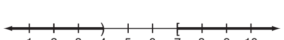
The solution set is $(-\infty, -2] \cup (1, \infty)$. The graph is shown below.



24. $2x < 8$ *or* $x + 3 \geq 10$

$x < 4$ *or* $x \geq 7$

$(-\infty, 4) \cup [7, \infty)$

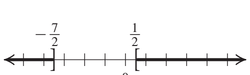


25. $2x + 3 \leq -4$ *or* $2x + 3 \geq 4$

$2x \leq -7$ *or* $2x \geq 1$

$x \leq -\frac{7}{2}$ *or* $x \geq \frac{1}{2}$

The solution set is $\left(-\infty, -\frac{7}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$. The graph is shown below.

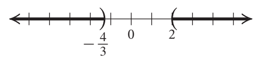


26. $3x - 1 < -5$ *or* $3x - 1 > 5$

$3x < -4$ *or* $3x > 6$

$x < -\frac{4}{3}$ *or* $x > 2$

$\left(-\infty, -\frac{4}{3}\right) \cup (2, \infty)$



27. $2x - 20 < -0.8$ *or* $2x - 20 > 0.8$

$2x < 19.2$ *or* $2x > 20.8$

$x < 9.6$ *or* $x > 10.4$

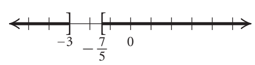
The solution set is $(-\infty, 9.6) \cup (10.4, \infty)$. The graph is shown below.

28. $5x + 11 \leq -4$ *or* $5x + 11 \geq 4$

$5x \leq -15$ *or* $5x \geq -7$

$x \leq -3$ *or* $x \geq -\frac{7}{5}$

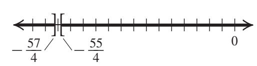
$(-\infty, -3] \cup \left[-\frac{7}{5}, \infty\right)$



29. $x + 14 \leq -\frac{1}{4}$ *or* $x + 14 \geq \frac{1}{4}$

$x \leq -\frac{57}{4}$ *or* $x \geq -\frac{55}{4}$

The solution set is $\left(-\infty, -\frac{57}{4}\right] \cup \left[-\frac{55}{4}, \infty\right)$. The graph is shown below.



30. $x - 9 < -\frac{1}{2}$ *or* $x - 9 > \frac{1}{2}$

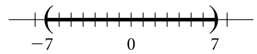
$x < \frac{17}{2}$ *or* $x > \frac{19}{2}$

$\left(-\infty, \frac{17}{2}\right) \cup \left(\frac{19}{2}, \infty\right)$

31. $|x| < 7$

To solve we look for all numbers x whose distance from 0 is less than 7. These are the numbers between -7 and 7. That is, $-7 < x < 7$. The solution set and its graph are as follows:

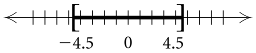
$(-7, 7)$



32. $|x| \leq 4.5$

$-4.5 \leq x \leq 4.5$

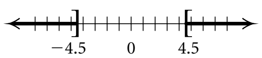
The solution set is $[-4.5, 4.5]$.



33. $|x| \geq 4.5$

To solve we look for all numbers x whose distance from 0 is greater than or equal to 4.5. That is, $x \leq -4.5$ or $x \geq 4.5$. The solution set and its graph are as follows.

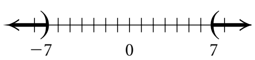
$\{x | x \leq -4.5 \text{ or } x \geq 4.5\}$, or $(-\infty, -4.5] \cup [4.5, \infty)$



34. $|x| > 7$

$x < -7$ or $x > 7$

The solution set is $(-\infty, -7) \cup (7, \infty)$.

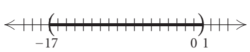


35. $|x + 8| < 9$

$-9 < x + 8 < 9$

$-17 < x < 1$ Subtracting 8

The solution set is $(-17, 1)$. The graph is shown below.

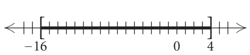


36. $|x + 6| \leq 10$

$-10 \leq x + 6 \leq 10$

$-16 \leq x \leq 4$

The solution set is $[-16, 4]$.



37. $|x + 8| \geq 9$

$x + 8 \leq -9$ *or* $x + 8 \geq 9$

$x \leq -17$ *or* $x \geq 1$ Subtracting 8

The solution set is $(-\infty, -17] \cup [1, \infty)$. The graph is shown below.

38. $|x + 6| > 10$

$x + 6 < -10$ *or* $x + 6 > 10$

$x < -16$ *or* $x > 4$

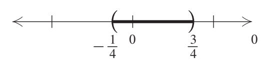
The solution set is $(-\infty, -16) \cup (4, \infty)$.

39. $\left|x - \frac{1}{4}\right| < \frac{1}{2}$

$-\frac{1}{2} < x - \frac{1}{4} < \frac{1}{2}$

$-\frac{1}{4} < x < \frac{3}{4}$ Adding $\frac{1}{4}$

The solution set is $\left(-\frac{1}{4}, \frac{3}{4}\right)$. The graph is shown below.

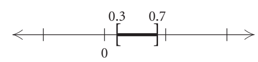


40. $|x - 0.5| \leq 0.2$

$-0.2 \leq x - 0.5 \leq 0.2$

$0.3 \leq x \leq 0.7$

The solution set is [0.3,0.7].

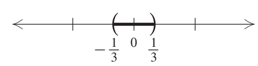


41. $|3x| < 1$

$-1 < 3x < 1$

$-\frac{1}{3} < x < \frac{1}{3}$ Dividing by 3

The solution set is $\left(-\frac{1}{3}, \frac{1}{3}\right)$. The graph is shown below.

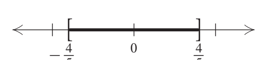


42. $|5x| \leq 4$

$-4 \leq 5x \leq 4$

$-\frac{4}{5} \leq x \leq \frac{4}{5}$

The solution set is $\left[-\frac{4}{5}, \frac{4}{5}\right]$.



43. $|2x + 3| \leq 9$

$-9 \leq 2x + 3 \leq 9$

$-12 \leq 2x \leq 6$ Subtracting 3

$-6 \leq x \leq 3$ Dividing by 2

The solution set is $[-6, 3]$. The graph is shown below.

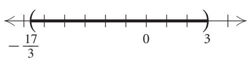
44. $|3x+4| < 13$

$-13 < 3x+4 < 13$

$-17 < 3x < 9$

$-\dfrac{17}{3} < x < 3$

The solution set is $\left(-\dfrac{17}{3}, 3\right)$.

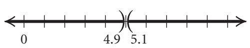


45. $|x-5| > 0.1$

$x - 5 < -0.1$ *or* $x - 5 > 0.1$

$x < 4.9$ *or* $x > 5.1$ Adding 5

The solution set is $(-\infty, 4.9) \cup (5.1, \infty)$. The graph is shown below.

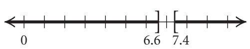


46. $|x-7| \geq 0.4$

$x - 7 \leq -0.4$ *or* $x - 7 \geq 0.4$

$x \leq 6.6$ *or* $x \geq 7.4$

The solution set is $(-\infty, 6.6] \cup [7.4, \infty)$.



47. $|6-4x| \leq 8$

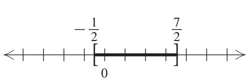
$-8 \leq 6 - 4x \leq 8$

$-14 \leq -4x \leq 2$ Subtracting 6

$\dfrac{14}{4} \geq x \geq -\dfrac{2}{4}$ Dividing by -4 and reversing the inequality symbols

$\dfrac{7}{2} \geq x \geq -\dfrac{1}{2}$ Simplifying

The solution set is $\left[-\dfrac{1}{2}, \dfrac{7}{2}\right]$. The graph is shown below.



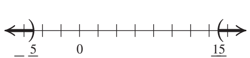
48. $|5-2x| > 10$

$5 - 2x < -10$ *or* $5 - 2x > 10$

$-2x < -15$ *or* $-2x > 5$

$x > \dfrac{15}{2}$ *or* $x < -\dfrac{5}{2}$

The solution set is $\left(-\infty, -\dfrac{5}{2}\right) \cup \left(\dfrac{15}{2}, \infty\right)$.

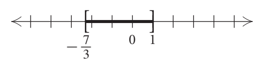


49. $\left|x + \dfrac{2}{3}\right| \leq \dfrac{5}{3}$

$-\dfrac{5}{3} \leq x + \dfrac{2}{3} \leq \dfrac{5}{3}$

$-\dfrac{7}{3} \leq x \leq 1$ Subtracting $\dfrac{2}{3}$

The solution set is $\left[-\dfrac{7}{3}, 1\right]$. The graph is shown below.

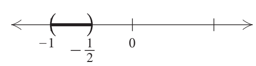


50. $\left|x + \dfrac{3}{4}\right| < \dfrac{1}{4}$

$-\dfrac{1}{4} < x + \dfrac{3}{4} < \dfrac{1}{4}$

$-1 < x < -\dfrac{1}{2}$

The solution set is $\left(-1, -\dfrac{1}{2}\right)$.



51. $\left|\dfrac{2x+1}{3}\right| > 5$

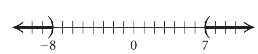
$\dfrac{2x+1}{3} < -5$ *or* $\dfrac{2x+1}{3} > 5$

$2x + 1 < -15$ *or* $2x + 1 > 15$ Multiplying by 3

$2x < -16$ *or* $2x > 14$ Subtracting 1

$x < -8$ *or* $x > 7$ Dividing by 2

The solution set is $\{x | x < -8 \text{ or } x > 7\}$, or $(-\infty, -8) \cup (7, \infty)$. The graph is shown below.



52. $\left|\dfrac{2x-1}{3}\right| \geq \dfrac{5}{6}$

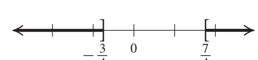
$\dfrac{2x-1}{3} \leq -\dfrac{5}{6}$ *or* $\dfrac{2x-1}{3} \geq \dfrac{5}{6}$

$2x - 1 \leq -\dfrac{5}{2}$ *or* $2x - 1 \geq \dfrac{5}{2}$

$2x \leq -\dfrac{3}{2}$ *or* $2x \geq \dfrac{7}{2}$

$x \leq -\dfrac{3}{4}$ *or* $x \geq \dfrac{7}{4}$

The solution set is $\left(-\infty, -\dfrac{3}{4}\right] \cup \left[\dfrac{7}{4}, \infty\right)$.



53. $|2x-4| < -5$

Since $|2x-4| \geq 0$ for all x, there is no x such that $|2x-4|$ would be less than -5. There is no solution.

54. $|3x+5| < 0$

$|3x+5| \geq 0$ for all x, so there is no solution.

55. ***Familiarize and Translate***. Spending is given by the equation $y = 12.7x + 15.2$. We want to know when the spending will be more than \$66 billion, so we have

$$12.7x + 15.2 > 66.$$

Carry out. We solve the inequality.

$$\begin{aligned} 12.7x + 15.2 &> 66 \\ 12.7x &> 50.8 \\ x &> 4 \end{aligned}$$

Check. When $x = 4$, the spending is $12.7(4) + 15.2 = 66$. As a partial check, we could try a value of x less than 4 and one greater than 4. When $x = 3.9$, we have $y = 12.7(3.9) + 15.2 = 64.73 < 66$; when $x = 4.1$, we have $y = 12.7(4.1) + 15.2 = 67.27 > 66$. Since $y = 66$ when $x = 4$ and $y > 66$ when $x = 4.1 > 4$, the answer is probably correct.

State. The spending will be more than \$66 billion more than 4 yr after 2002.

56. Solve: $5x + 5 \geq 20$

$x \geq 3$, so 3 or more y after 2002, or in 2005 and later, there will be at least 20 million homes with devices installed that receive and manage broadband TV and Internet content.

57. ***Familiarize***. Let t = the number of hours worked. Then Acme Movers charge $100 + 30t$ and Hank's Movers charge $55t$.

Translate.

$$\underbrace{\text{Hank's charge}}_{55t} \quad \underbrace{\text{is less than}}_{<} \quad \underbrace{\text{Acme's charge.}}_{100+30t}$$

Carry out. We solve the inequality.

$$\begin{aligned} 55t &< 100 + 30t \\ 25t &< 100 \\ t &< 4 \end{aligned}$$

Check. When $t = 4$, Hank's Movers charge $55 \cdot 4$, or \$220 and Acme Movers charge $100 + 30 \cdot 4 = 100 + 120 = \$220$, so the charges are the same. As a partial check, we find the charges for a value of $t < 4$. When $t = 3.5$, Hank's Movers charge $55(3.5) = \$192.50$ and Acme Movers charge $100 + 30(3.5) = 100 + 105 = \$205$. Since Hank's charge is less than Acme's, the answer is probably correct.

State. For times less than 4 hr it costs less to hire Hank's Movers.

58. Let x = the amount invested at 4%. Then $12,000 - x$ = the amount invested at 6%.

Solve: $0.04x + 0.06(12,000 - x) \geq 650$

$x \leq 3500$, so at most \$3500 can be invested at 4%.

59. ***Familiarize***. Let x = the amount invested at 4%. Then $7500 - x$ = the amount invested at 5%. Using the simple-interest formula, $I = Prt$, we see that in one year the 4% investment earns $0.04x$ and the 5% investment earns $0.05(7500 - x)$.

Translate.

$$\underbrace{\text{Interest at 4\%}}_{0.04x} \quad \underbrace{\text{plus}}_{+} \quad \underbrace{\text{interest at 5\%}}_{0.05(7500-x)} \quad \underbrace{\text{is at least}}_{\geq} \quad \underbrace{\$325.}_{325}$$

Carry out. We solve the inequality.

$$\begin{aligned} 0.04x + 0.05(7500 - x) &\geq 325 \\ 0.04x + 375 - 0.05x &\geq 325 \\ -0.01x + 375 &\geq 325 \\ -0.01x &\geq -50 \\ x &\leq 5000 \end{aligned}$$

Check. When \$5000 is invested at 4%, then \$7500−\$5000, or \$2500, is invested at 5%. In one year the 4% investment earns 0.04(\$5000), or \$200, in simple interest and the 5% investment earns 0.05(\$2500), or \$125, so the total interest is \$200 + \$125, or \$325. As a partial check, we determine the total interest when an amount greater than \$5000 is invested at 4%. Suppose \$5001 is invested at 4%. Then \$2499 is invested at 5%, and the total interest is 0.04(\$5001) + 0.05(\$2499), or \$324.99. Since this amount is less than \$325, the answer is probably correct.

State. The most that can be invested at 4% is \$5000.

60. Let c = the number of checks written per month.

Solve: $0.20c < 6 + 0.05c$

$c < 40$, so the Smart Checking plan will cost less than the Consumer Checking plan when fewer than 40 checks are written per month.

61. ***Familiarize***. Let c = the number of checks written per month. Then the No Frills plan costs $0.35c$ per month and the Simple Checking plan costs $5 + 0.10c$ per month.

Translate.

$$\underbrace{\text{Simple Checking cost}}_{5+0.10c} \quad \underbrace{\text{is less than}}_{<} \quad \underbrace{\text{No Frills cost.}}_{0.35c}$$

Carry out. We solve the inequality.

$$\begin{aligned} 5 + 0.10c &< 0.35c \\ 5 &< 0.25c \\ 20 &< c \end{aligned}$$

Check. When 20 checks are written the No Frills plan costs 0.35(20), or \$7 per month and the Simple Checking plan costs 5 + 0.10(20), or \$7, so the costs are the same. As a partial check, we compare the cost for some number of checks greater than 20. When 21 checks are written, the No Frills plan costs 0.35(21), or \$7.35 and the Simple Checking plan costs 5+0.10(21), or \$7.10. Since the Simple Checking plan costs less than the No Frills plan, the answer is probably correct.

State. The Simple Checking plan costs less when more than 20 checks are written per month.

62. Let s = the monthly sales.

Solve: $750 + 0.1s > 1000 + 0.08(s - 2000)$

$s > 4500$, so Plan A is better for monthly sales greater than \$4500.

63. ***Familiarize***. Let s = the monthly sales. Then the amount of sales in excess of \$8000 is $s - 8000$.

Translate.

$$\underbrace{\text{Income from plan B}}_{1200+0.15(s-8000)} \quad \underbrace{\text{is greater than}}_{>} \quad \underbrace{\text{income from plan A.}}_{900+0.1s}$$

Carry out. We solve the inequality.

$$\begin{aligned} 1200 + 0.15(s - 8000) &> 900 + 0.1s \\ 1200 + 0.15s - 1200 &> 900 + 0.1s \\ 0.15s &> 900 + 0.1s \\ 0.05s &> 900 \\ s &> 18,000 \end{aligned}$$

Check. For sales of \$18,000 the income from plan A is $\$900+0.1(\$18,000)$, or \$2700, and the income from plan B is $1200 + 0.15(18,000 - 8000)$, or \$2700 so the incomes are the same. As a partial check we can compare the incomes for an amount of sales greater than \$18,000. For sales of \$18,001, for example, the income from plan A is $\$900 + 0.1(\$18,001)$, or \$2700.10, and the income from plan B is $\$1200 + 0.15(\$18,001 - \$8000)$, or \$2700.15. Since plan B is better than plan A in this case, the answer is probably correct.

State. Plan B is better than plan A for monthly sales greater than \$18,000.

64. Solve: $200 + 12n > 20n$

$n < 25$

65. Left to the student

66. Left to the student

67. Absolute value is nonnegative.

68. $|x| \geq 0 > p$ for any real number x.

69. y-intercept

70. distance formula

71. relation

72. function

73. horizontal line

74. parallel

75. decreasing

76. symmetric with respect to the y-axis

77. $2x \leq 5 - 7x < 7 + x$

$$\begin{aligned} 2x \leq 5 - 7x \quad &\textit{and} \quad 5 - 7x < 7 + x \\ 9x \leq 5 \quad &\textit{and} \quad -8x < 2 \\ x \leq \frac{5}{9} \quad &\textit{and} \quad x > -\frac{1}{4} \end{aligned}$$

The solution set is $\left(-\frac{1}{4}, \frac{5}{9}\right]$.

78. $x \leq 3x - 2 \leq 2 - x$

$$\begin{aligned} x \leq 3x - 2 \quad &\textit{and} \quad 3x - 2 \leq 2 - x \\ -2x \leq -2 \quad &\textit{and} \quad 4x \leq 4 \\ x \geq 1 \quad &\textit{and} \quad x \leq 1 \end{aligned}$$

The solution is 1.

79. $|3x - 1| > 5x - 2$

$$\begin{aligned} 3x - 1 < -(5x - 2) \quad &\textit{or} \quad 3x - 1 > 5x - 2 \\ 3x - 1 < -5x + 2 \quad &\textit{or} \quad 1 > 2x \\ 8x < 3 \quad &\textit{or} \quad \frac{1}{2} > x \\ x < \frac{3}{8} \quad &\textit{or} \quad \frac{1}{2} > x \end{aligned}$$

The solution set is $\left(-\infty, \frac{3}{8}\right) \cup \left(-\infty, \frac{1}{2}\right)$. This is equivalent to $\left(-\infty, \frac{1}{2}\right)$.

80. $|x + 2| \leq |x - 5|$

Divide the set of real numbers into three intervals:

$(-\infty, -2)$, $[-2, 5)$, and $[5, \infty,)$.

Find the solution set of $|x + 2| \leq |x - 5|$ in each interval. Then find the union of the three solution sets.

If $x < -2$, then $|x + 2| = -(x + 2)$ and $|x - 5| = -(x - 5)$.

$$\begin{aligned} \text{Solve: } x < -2 \quad &\textit{and} \quad -(x + 2) \leq -(x - 5) \\ x < -2 \quad &\textit{and} \quad -x - 2 \leq -x + 5 \\ x < -2 \quad &\textit{and} \quad -2 \leq 5 \end{aligned}$$

The solution set for this interval is $(-\infty, -2)$.

If $-2 \leq x < 5$, then $|x + 2| = x + 2$ and $|x - 5| = -(x - 5)$.

$$\begin{aligned} \text{Solve: } -2 \leq x < 5 \quad &\textit{and} \quad x + 2 \leq -(x - 5) \\ -2 \leq x < 5 \quad &\textit{and} \quad x + 2 \leq -x + 5 \\ -2 \leq x < 5 \quad &\textit{and} \quad 2x \leq 3 \\ -2 \leq x < 5 \quad &\textit{and} \quad x \leq \frac{3}{2} \end{aligned}$$

The solution set for this interval is $\left[-2, \frac{3}{2}\right]$.

If $x \geq 5$, then $|x + 2| = x + 2$ and $|x - 5| = x - 5$.

$$\begin{aligned} \text{Solve: } x \geq 5 \quad &\textit{and} \quad x + 2 \leq x - 5 \\ x \geq 5 \quad &\textit{and} \quad 2 \leq -5 \end{aligned}$$

The solution set for this interval is $\emptyset$.

The union of the above three solution set is $\left(-\infty, \frac{3}{2}\right]$. This is the solution set of $|x + 2| \leq |x - 5|$.

81. $|p - 4| + |p + 4| < 8$

If $p < -4$, then $|p - 4| = -(p - 4)$ and $|p + 4| = -(p + 4)$.

$$\begin{aligned} \text{Solve: } -(p - 4) + [-(p + 4)] &< 8 \\ -p + 4 - p - 4 &< 8 \\ -2p &< 8 \\ p &> -4 \end{aligned}$$

Since this is false for all values of p in the interval $(-\infty, -4)$ there is no solution in this interval.

If $p \geq -4$, then $|p+4| = p+4$.

Solve: $|p-4| + p + 4 < 8$

$$|p-4| < 4-p$$

$$p-4 > -(4-p) \text{ and } p-4 < 4-p$$
$$p-4 > p-4 \quad \text{and} \quad 2p < 8$$
$$-4 > -4 \quad \text{and} \quad p < 4$$

Since $-4 > -4$ is false for all values of p, there is no solution in the interval $[-4, \infty)$.

Thus, $|p-4| + |p+4| < 8$ has no solution.

82. $|x| + |x+1| < 10$

If $x < -1$, then $|x| = -x$ and $|x+1| = -(x+1)$ and we have:

$$x < -1 \text{ and } -x + [-(x+1)] < 10$$
$$x < -1 \text{ and } -x - x - 1 < 10$$
$$x < -1 \text{ and } -2x - 1 < 10$$
$$x < -1 \text{ and } -2x < 11$$
$$x < -1 \text{ and } x > -\frac{11}{2}$$

The solution set for this interval is $\left(-\frac{11}{2}, -1\right)$.

If $-1 \leq x < 0$, then $|x| = -x$ and $|x+1| = x+1$ and we have:

$$-1 \leq x \text{ and } -x + x + 1 < 10$$
$$-1 \leq x \text{ and } 1 < 10$$

The solution set for this interval is $[-1, 0]$.

If $x \geq 0$, then $|x| = x$ and $|x+1| = x+1$ and we have:

$$x \geq 0 \text{ and } x + x + 1 < 10$$
$$x \geq 0 \text{ and } 2x + 1 < 10$$
$$x \geq 0 \text{ and } 2x < 9$$
$$x \geq 0 \text{ and } x < \frac{9}{2}$$

The solution set for this interval is $\left[0, \frac{9}{2}\right)$.

The union of the three solution sets above is $\left(-\frac{11}{2}, \frac{9}{2}\right)$. This is the solution set of $|x| + |x+1| < 10$.

83. $|x-3| + |2x+5| > 6$

Divide the set of real numbers into three intervals: $\left(-\infty, -\frac{5}{2}\right)$, $\left[-\frac{5}{2}, 3\right)$, and $[3, \infty)$.

Find the solution set of $|x-3| + |2x+5| > 6$ in each interval. Then find the union of the three solution sets.

If $x < -\frac{5}{2}$, then $|x-3| = -(x-3)$ and $|2x+5| = -(2x+5)$.

Solve: $x < -\frac{5}{2}$ *and* $-(x-3) + [-(2x+5)] > 6$

$$x < -\frac{5}{2} \text{ and } -x + 3 - 2x - 5 > 6$$
$$x < -\frac{5}{2} \text{ and } -3x > 8$$
$$x < -\frac{5}{2} \text{ and } x < -\frac{8}{3}$$

The solution set in this interval is $\left(-\infty, -\frac{8}{3}\right)$.

If $-\frac{5}{2} \leq x < 3$, then $|x-3| = -(x-3)$ and $|2x+5| = 2x+5$.

Solve: $-\frac{5}{2} \leq x < 3$ *and* $-(x-3) + 2x + 5 > 6$

$$-\frac{5}{2} \leq x < 3 \text{ and } -x + 3 + 2x + 5 > 6$$
$$-\frac{5}{2} \leq x < 3 \text{ and } x > -2$$

The solution set in this interval is $(-2, 3)$.

If $x \geq 3$, then $|x-3| = x-3$ and $|2x+5| = 2x+5$.

Solve: $x \geq 3$ *and* $x - 3 + 2x + 5 > 6$

$$x \geq 3 \text{ and } 3x > 4$$
$$x \geq 3 \text{ and } x > \frac{4}{3}$$

The solution set in this interval is $[3, \infty)$.

The union of the above solution sets is $\left(-\infty, -\frac{8}{3}\right) \cup (-2, \infty)$. This is the solution set of $|x-3| + |2x+5| > 6$.

Chapter 2 Review Exercises

1. The statement is false. We find the zeros of a function $f(x)$ by finding the values of x for which $f(x) = 0$.

2. The statement is true. See page 186 in the text.

3. The statement is true. See page 205 in the text.

4. The statement is true. See page 217 in the text.

5. The statement is false. For example, $3^2 = (-3)^2$, but $3 \neq -3$.

6. If $a < b$ and $c < 0$ are true, then $ac > bc$ is true. Thus, the statement is false.

7.
$$4y - 5 = 1$$
$$4y = 6$$
$$y = \frac{3}{2}$$

The solution is $\frac{3}{2}$.

8.
$$3x - 4 = 5x + 8$$
$$-12 = 2x$$
$$-6 = x$$

9. $5(3x+1)=2(x-4)$

$15x+5=2x-8$

$13x=-13$

$x=-1$

The solution is -1.

10. $2(n-3)=3(n+5)$

$2n-6=3n+15$

$-21=n$

11. $(2y+5)(3y-1)=0$

$2y+5=0$ *or* $3y-1=0$

$2y=-5$ *or* $3y=1$

$y=-\frac{5}{2}$ *or* $y=\frac{1}{3}$

The solutions are $-\frac{5}{2}$ and $\frac{1}{3}$.

12. $x^2+4x-5=0$

$(x+5)(x-1)=0$

$x=-5$ *or* $x=1$

13. $3x^2+2x=8$

$3x^2+2x-8=0$

$(x+2)(3x-4)=0$

$x+2=0$ *or* $3x-4=0$

$x=-2$ *or* $3x=4$

$x=-2$ *or* $x=\frac{4}{3}$

The solutions are -2 and $\frac{4}{3}$.

14. $5x^2=15$

$x^2=3$

$x=-\sqrt{3}$ *or* $x=\sqrt{3}$

15. $x^2-10=0$

$x^2=10$

$x=-\sqrt{10}$ *or* $x=\sqrt{10}$

The solutions are $-\sqrt{10}$ and $\sqrt{10}$.

16. $6x-18=0$

$6x=18$

$x=3$

17. $x-4=0$

$x=4$

The zero of the function is 4.

18. $2-10x=0$

$-10x=-2$

$x=\frac{1}{5}$

19. $8-2x=0$

$-2x=-8$

$x=4$

The zero of the function is 4.

20. $x^2-2x+1=0$

$(x-1)^2=0$

$x-1=0$

$x=1$

21. $x^2+2x-15=0$

$(x+5)(x-3)=0$

$x+5=0$ *or* $x-3=0$

$x=-5$ *or* $x=3$

The zeros of the function are -5 and 3.

22. $2x^2-x-5=0$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot2\cdot(-5)}}{2\cdot2}$$

$$=\frac{1\pm\sqrt{41}}{4}$$

23. $3x^2+2x-3=0$

$a=3,\ b=2,\ c=-3$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-2\pm\sqrt{2^2-4\cdot3\cdot(-3)}}{2\cdot3}$$

$$=\frac{-2\pm\sqrt{40}}{6}=\frac{-2\pm2\sqrt{10}}{6}=\frac{-1\pm\sqrt{10}}{3}$$

The zeros of the function are $\frac{-1\pm\sqrt{10}}{3}$.

24. $\frac{5}{2x+3}+\frac{1}{x-6}=0$, LCD is $(2x+3)(x-6)$

$5(x-6)+2x+3=0$

$5x-30+2x+3=0$

$7x-27=0$

$7x=27$

$x=\frac{27}{7}$

This number checks.

25. $\frac{3}{8x+1}+\frac{8}{2x+5}=1$

LCD is $(8x+1)(2x+5)$

$$(8x+1)(2x+5)\left(\frac{3}{8x+1}+\frac{8}{2x+5}\right)=(8x+1)(2x+5)\cdot1$$

$3(2x+5)+8(8x+1)=(8x+1)(2x+5)$

$6x+15+64x+8=16x^2+42x+5$

$70x+23=16x^2+42x+5$

$0=16x^2-28x-18$

$0=2(8x^2-14x-9)$

$0=2(2x+1)(4x-9)$

$$2x+1=0 \quad or \quad 4x-9=0$$
$$2x=-1 \quad or \quad 4x=9$$
$$x=-\frac{1}{2} \quad or \quad x=\frac{9}{4}$$

Both numbers check. The solutions are $-\frac{1}{2}$ and $\frac{9}{4}$.

26.
$$\sqrt{5x+1}-1=\sqrt{3x}$$
$$5x+1-2\sqrt{5x+1}+1=3x$$
$$-2\sqrt{5x+1}=-2x-2$$
$$\sqrt{5x+1}=x+1$$
$$5x+1=x^2+2x+1$$
$$0=x^2-3x$$
$$0=x(x-3)$$

$x=0$ *or* $x=3$

Both numbers check.

27.
$$\sqrt{x-1}-\sqrt{x-4}=1$$
$$\sqrt{x-1}=\sqrt{x-4}+1$$
$$(\sqrt{x-1})^2=(\sqrt{x-4}+1)^2$$
$$x-1=x-4+2\sqrt{x-4}+1$$
$$x-1=x-3+2\sqrt{x-4}$$
$$2=2\sqrt{x-4}$$
$$1=\sqrt{x-4} \quad \text{Dividing by 2}$$
$$1^2=(\sqrt{x-4})^2$$
$$1=x-4$$
$$5=x$$

This number checks. The solution is 5.

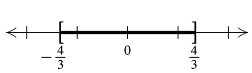
28. $|x-4|=3$
$$x-4=-3 \quad or \quad x-4=3$$
$$x=1 \quad or \quad x=7$$

The solutions are 1 and 7.

29. $|2y+7|=9$
$$2y+7=-9 \quad or \quad 2y+7=9$$
$$2y=-16 \quad or \quad 2y=2$$
$$y=-8 \quad or \quad y=1$$

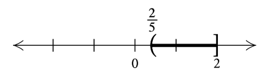
The solutions are -8 and 1.

30.
$$-3\le 3x+1\le 5$$
$$-4\le 3x\le 4$$
$$-\frac{4}{3}\le x\le\frac{4}{3}$$
$$\left[-\frac{4}{3},\frac{4}{3}\right]$$



31.
$$-2<5x-4\le 6$$
$$2<5x\le 10 \quad \text{Adding 4}$$
$$\frac{2}{5}<x\le 2 \quad \text{Dividing by 5}$$

The solution set is $\left(\frac{2}{5},2\right]$.

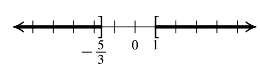


32.
$$2x<-1 \quad or \quad x+3>0$$
$$x<-\frac{1}{2} \quad or \quad x>-3$$

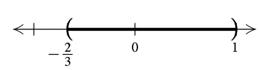
Every real number is less than $-\frac{1}{2}$ or greater than -3, so the solution set is the set of all real numbers, or $(-\infty,\infty)$.

33.
$$3x+7\le 2 \quad or \quad 2x+3\ge 5$$
$$3x\le -5 \quad or \quad 2x\ge 2$$
$$x\le -\frac{5}{3} \quad or \quad x\ge 1$$

The solution set is $\left(-\infty,-\frac{5}{3}\right]\cup[1,\infty)$.

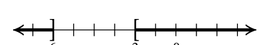


34.
$$|6x-1|<5$$
$$-5<6x-1<5$$
$$-4<6x<6$$
$$-\frac{2}{3}<x<1$$
$$\left(-\frac{2}{3},1\right)$$



35. $|x+4|\ge 2$
$$x+4\le -2 \quad or \quad x+4\ge 2$$
$$x\le -6 \quad or \quad x\ge -2$$

The solution is $(-\infty,-6]\cup[-2,\infty)$.



36.
$$V=lwh$$
$$\frac{V}{lw}=h$$

37.
$$M=n+0.3s$$
$$M-n=0.3s$$
$$\frac{M-n}{0.3}=s$$

38.
$$v = \sqrt{2gh}$$
$$v^2 = 2gh$$
$$\frac{v^2}{2g} = h$$

39.
$$\frac{1}{a} + \frac{1}{b} = \frac{1}{t},\ \text{LCD is } abt$$
$$abt\left(\frac{1}{a} + \frac{1}{b}\right) = abt \cdot \frac{1}{t}$$
$$bt + at = ab$$
$$t(b+a) = ab$$
$$t = \frac{ab}{a+b}$$

40. $-\sqrt{-40} = -\sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{10} = -2\sqrt{10}i$

41.
$$\sqrt{-12} \cdot \sqrt{-20} = \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{3} \cdot \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{5}$$
$$= 2i\sqrt{3} \cdot 2i\sqrt{5}$$
$$= 4i^2\sqrt{3 \cdot 5}$$
$$= -4\sqrt{15}$$

42. $\dfrac{\sqrt{-49}}{-\sqrt{-64}} = \dfrac{7i}{-8i} = -\dfrac{7}{8}$

43.
$$(6+2i)(-4-3i) = -24 - 18i - 8i - 6i^2$$
$$= -24 - 26i + 6$$
$$= -18 - 26i$$

44.
$$\frac{2-3i}{1-3i} = \frac{2-3i}{1-3i} \cdot \frac{1+3i}{1+3i}$$
$$= \frac{2+3i-9i^2}{1-9i^2}$$
$$= \frac{2+3i+9}{1+9}$$
$$= \frac{11+3i}{10}$$
$$= \frac{11}{10} + \frac{3}{10}i$$

45.
$$(3-5i) - (2-i) = (3-2) + [-5i - (-i)]$$
$$= 1 - 4i$$

46.
$$(6+2i) + (-4-3i) = (6-4) + (2i-3i)$$
$$= 2 - i$$

47. $i^{23} = (i^2)^{11} \cdot i = (-1)^{11} \cdot i = -1 \cdot i = -i$

48. $(-3i)^{28} = (-3)^{28} \cdot i^{28} = 3^{28} \cdot (i^2)^{14} =$
$3^{28} \cdot (-1)^{14} = 3^{28}$

49.
$$x^2 - 3x = 18$$
$$x^2 - 3x + \frac{9}{4} = 18 + \frac{9}{4} \quad \left(\frac{1}{2}(-3) = -\frac{3}{2} \text{ and } \left(-\frac{3}{2}\right)^2 = \frac{9}{4}\right)$$
$$\left(x - \frac{3}{2}\right)^2 = \frac{81}{4}$$
$$x - \frac{3}{2} = \pm\frac{9}{2}$$
$$x = \frac{3}{2} \pm \frac{9}{2}$$
$$x = \frac{3}{2} - \frac{9}{2} \quad \textit{or} \quad x = \frac{3}{2} + \frac{9}{2}$$
$$x = -3 \quad \textit{or} \quad x = 6$$
The solutions are -3 and 6.

50.
$$3x^2 - 12x - 6 = 0$$
$$3x^2 - 12x = 6$$
$$x^2 - 4x = 2$$
$$x^2 - 4x + 4 = 2 + 4 \quad \left(\frac{1}{2}(-4) = -2 \text{ and } (-2)^2 = 4\right)$$
$$(x-2)^2 = 6$$
$$x - 2 = \pm\sqrt{6}$$
$$x = 2 \pm \sqrt{6}$$

51.
$$3x^2 + 10x = 8$$
$$3x^2 + 10x - 8 = 0$$
$$(x+4)(3x-2) = 0$$
$$x + 4 = 0 \quad \textit{or} \quad 3x - 2 = 0$$
$$x = -4 \quad \textit{or} \quad 3x = 2$$
$$x = -4 \quad \textit{or} \quad x = \frac{2}{3}$$
The solutions are -4 and $\dfrac{2}{3}$.

52. $r^2 - 2r + 10 = 0$
$$r = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 10}}{2 \cdot 1}$$
$$= \frac{2 \pm \sqrt{-36}}{2} = \frac{2 \pm 6i}{2}$$
$$= 1 \pm 3i$$

53.
$$x^2 = 10 + 3x$$
$$x^2 - 3x - 10 = 0$$
$$(x+2)(x-5) = 0$$
$$x + 2 = 0 \quad \textit{or} \quad x - 5 = 0$$
$$x = -2 \quad \textit{or} \quad x = 5$$
The solutions are -2 and 5.

54.
$$x = 2\sqrt{x} - 1$$
$$x - 2\sqrt{x} + 1 = 0$$
Let $u = \sqrt{x}$.
$$u^2 - 2u + 1 = 0$$
$$(u-1)^2 = 0$$
$$u - 1 = 0$$
$$u = 1$$
Substitute $\sqrt{x}$ for u and solve for x.
$$\sqrt{x} = 1$$
$$x = 1$$

55. $y^4 - 3y^2 + 1 = 0$
Let $u = y^2$.
$$u^2 - 3u + 1 = 0$$
$$u = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{3 \pm \sqrt{5}}{2}$$

Substitute y^2 for u and solve for y.

$$y^2 = \frac{3 \pm \sqrt{5}}{2}$$
$$y = \pm\sqrt{\frac{3 \pm \sqrt{5}}{2}}$$

The solutions are $\pm\sqrt{\dfrac{3 \pm \sqrt{5}}{2}}$.

56. $(x^2 - 1)^2 - (x^2 - 1) - 2 = 0$

Let $u = x^2 - 1$.

$$u^2 - u - 2 = 0$$
$$(u + 1)(u - 2) = 0$$
$$u + 1 = 0 \quad or \quad u - 2 = 0$$
$$u = -1 \quad or \quad u = 2$$

Substitute $x^2 - 1$ for u and solve for x.

$$x^2 - 1 = -1 \quad or \quad x^2 - 1 = 2$$
$$x^2 = 0 \quad or \quad x^2 = 3$$
$$x = 0 \quad or \quad x = \pm\sqrt{3}$$

57. $(p - 3)(3p + 2)(p + 2) = 0$

$$p - 3 = 0 \quad or \quad 3p + 2 = 0 \quad or \quad p + 2 = 0$$
$$p = 3 \quad or \quad 3p = -2 \quad or \quad p = -2$$
$$p = 3 \quad or \quad p = -\frac{2}{3} \quad or \quad p = -2$$

The solutions are -2, $-\dfrac{2}{3}$ and 3.

58.
$$x^3 + 5x^2 - 4x - 20 = 0$$
$$x^2(x + 5) - 4(x + 5) = 0$$
$$(x + 5)(x^2 - 4) = 0$$
$$(x + 5)(x + 2)(x - 2) = 0$$
$$x + 5 = 0 \quad or \quad x + 2 = 0 \quad or \quad x - 2 = 0$$
$$x = -5 \quad or \quad x = -2 \quad or \quad x = 2$$

59.
$$f(x) = -4x^2 + 3x - 1$$
$$= -4\left(x^2 - \frac{3}{4}x\right) - 1$$
$$= -4\left(x^2 - \frac{3}{4}x + \frac{9}{64} - \frac{9}{64}\right) - 1$$
$$= -4\left(x^2 - \frac{3}{4}x + \frac{9}{64}\right) - 4\left(-\frac{9}{64}\right) - 1$$
$$= -4\left(x^2 - \frac{3}{4}x + \frac{9}{64}\right) + \frac{9}{16} - 1$$
$$= -4\left(x - \frac{3}{8}\right)^2 - \frac{7}{16}$$

a) Vertex: $\left(\dfrac{3}{8}, -\dfrac{7}{16}\right)$

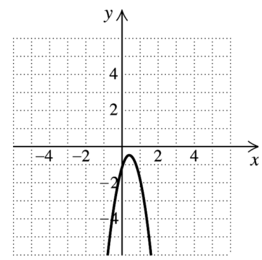
b) Axis of symmetry: $x = \dfrac{3}{8}$

c) Maximum value: $-\dfrac{7}{16}$

d) Range: $\left(-\infty, -\dfrac{7}{16}\right]$

e) Since the graph opens down, function values increase to the left of the vertex and decrease to the right of the vertex. Thus, $f(x)$ is increasing on $\left(-\infty, \dfrac{3}{8}\right)$ and decreasing on $\left(\dfrac{3}{8}, \infty\right)$.

f)



$f(x) = -4x^2 + 3x - 1$

60.
$$f(x) = 5x^2 - 10x + 3$$
$$= 5(x^2 - 2x) + 3$$
$$= 5(x^2 - 2x + 1 - 1) + 3$$
$$= 5(x^2 - 2x + 1) - 5 \cdot 1 + 3$$
$$= 5(x - 1)^2 - 2$$

a) Vertex: $(1, -2)$

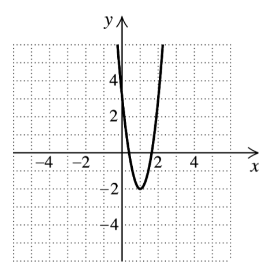
b) Axis of symmetry: $x = 1$

c) Minimum value: -2

d) Range: $[-2, \infty)$

e) Since the graph opens up, function values decrease to the left of the vertex and increase to the right of the vertex. Thus, $f(x)$ is increasing on $(1, \infty)$ and decreasing on $(-\infty, 1)$.

f)



$f(x) = 5x^2 - 10x + 3$

61. The graph of $y = (x - 2)^2$ has vertex $(2, 0)$ and opens up. It is graph (d).

62. The graph of $y = (x + 3)^2 - 4$ has vertex $(-3, -4)$ and opens up. It is graph (c).

63. The graph of $y = -2(x + 3)^2 + 4$ has vertex $(-3, 4)$ and opens down. It is graph (b).

64. The graph of $y = -\dfrac{1}{2}(x - 2)^2 + 5$ has vertex $(2, 5)$ and opens down. It is graph (a).

65. ***Familiarize***. Using the labels in the textbook, the legs of the right triangle are represented by x and $x + 10$.

Translate. We use the Pythagorean theorem.

$x^2 + (x+10)^2 = 50^2$

Carry out. We solve the equation.

$$\begin{aligned} x^2 + (x+10)^2 &= 50^2 \\ x^2 + x^2 + 20x + 100 &= 2500 \\ 2x^2 + 20x - 2400 &= 0 \\ 2(x^2 + 10x - 1200) &= 0 \\ 2(x+40)(x-30) &= 0 \end{aligned}$$

$$x + 40 = 0 \quad \textit{or} \quad x - 30 = 0$$
$$x = -40 \quad \textit{or} \quad x = 30$$

Check. Since the length cannot be negative, we need to check only 30. If $x = 30$, then $x+10 = 30+10 = 40$. Since $30^2 + 40^2 = 900 + 1600 = 2500 = 50^2$, the answer checks.

State. The lengths of the legs are 30 ft and 40 ft.

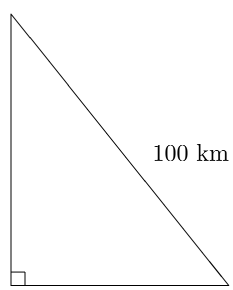
66. Let r = the speed of the boat in still water.

Solve: $\dfrac{8}{r-2} + \dfrac{8}{r+2} = 3$

$r = -\dfrac{2}{3}$ *or* $r = 6$

Only 6 has meaning in the original problem. The speed of the boat in still water is 6 mph.

67. ***Familiarize***. Let r = the speed of the second train, in km/h. After 1 hr the first train has traveled 60 km, and the second train has traveled r km, and they are 100 km apart. We make a drawing.



Translate. We use the Pythagorean theorem.

$60^2 + r^2 = 100^2$

Carry out. We solve the equation.

$$\begin{aligned} 60^2 + r^2 &= 100^2 \\ 3600 + r^2 &= 10,000 \\ r^2 &= 6400 \\ r &= \pm 80 \end{aligned}$$

Check. Since the speed cannot be negative, we need to check only 80. We see that $60^2 + 80^2 = 3600 + 6400 = 10,000 = 100^2$, so the answer checks.

State. The second train is traveling 80 km/h.

68. Let w = the width of the sidewalk.

Solve: $(80 - 2w)(60 - 2w) = \dfrac{2}{3} \cdot 80 \cdot 60$

$w = 35 \pm 5\sqrt{33}$

If $w = 35 + 5\sqrt{33} \approx 64$, both the new length, $80 - 2w$, and the new width, $60 - 2w$, would be negative, so $35 + 5\sqrt{33}$ cannot be a solution.

The other number, $35 - 5\sqrt{33}$ ft ≈ 6.3 ft, checks in the original problem.

69. ***Familiarize***. Let l = the length of the toy corral, in ft. Then the width is $\dfrac{24 - 2l}{2}$, or $12 - l$. The height of the corral is 2 ft.

Translate. We use the formula for the volume of a rectangular solid, $V = lwh$.

$$\begin{aligned} V(l) &= l(12 - l)(2) \\ &= 24l - 2l^2 \\ &= -2l^2 + 24l \end{aligned}$$

Carry out. Since $V(l)$ is a quadratic function with $a = -2 < 0$, the maximum function value occurs at the vertex of the graph of the function. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{24}{2(-2)} = 6$$

When $l = 6$, then $12 - l = 12 - 6 = 6$.

Check. The volume of a corral with length 6 ft, width 6 ft, and height 2 ft is $6 \cdot 6 \cdot 2$, or 72 ft^3. As a partial check, we can find $V(l)$ for a value of l less than 6 and for a value of l greater than 6. For instance, $V(5.9) = 71.98$ and $V(6.1) = 71.98$. Since both of these values are less than 72, our result appears to be correct.

State. The dimensions of the corral should be 6 ft by 6 ft.

70. Using the labels in the textbook, let x = the length of the sides of the squares, in cm.

Solve: $(20 - 2x)(10 - 2x) = 90$

$x = \dfrac{15 \pm \sqrt{115}}{2}$

If $x = \dfrac{15 + \sqrt{115}}{2} \approx 12.9$, both the length of the base, $20 - 2x$, and the width $10 - 2x$, would be negative, so $\dfrac{15 + \sqrt{115}}{2}$ cannot be a solution.

The other number, $\dfrac{15 - \sqrt{115}}{2}$ cm ≈ 2.1 cm, checks in the original problem.

71. ***Familiarize and Translate***. The number of faculty, in thousands, is given by the equation $y = 6x + 121$. We want to know when this number will exceed 325 thousand, so we have

$6x + 121 > 325$.

Carry out. We solve the inequality.

$$\begin{aligned} 6x + 121 &> 325 \\ 6x &> 204 \\ x &> 34 \end{aligned}$$

Check. When $x = 34$. the number of faculty is $6 \cdot 34 + 121 = 325$. As a partial check, we could try a value of x less than 34 and one greater than 34. When $x = 33.9$ we have $y = 6(33.9) + 121 = 324.4 < 325$; when $x = 34.1$

we have $y = 6(34.1) + 121 = 325.6 > 325$. Since $y = 325$ when $x = 34$ and $y > 325$ when $x > 34$, the answer is probably correct.

State. There will be more than 325 thousand faculty members more than 34 years after 1970, or in years after 2004.

72. Solve: $\frac{5}{9}(F-32) < 45$

$F < 113$, so Celsius temperature is lower than 45° for Fahrenheit temperatures less than 113°.

73.
$$2x^2 - 5x + 1 = 0$$
$$a = 2,\ b = -5,\ c = 1$$
$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\ &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4\cdot 2\cdot 1}}{2\cdot 2} = \frac{5 \pm \sqrt{25-8}}{4} \\ &= \frac{5 \pm \sqrt{17}}{4} \end{aligned}$$

Answer B is correct.

74.
$$\begin{aligned} \sqrt{4x+1} + \sqrt{2x} &= 1 \\ \sqrt{4x+1} &= 1 - \sqrt{2x} \\ (\sqrt{4x+1})^2 &= (1-\sqrt{2x})^2 \\ 4x+1 &= 1 - 2\sqrt{2x} + 2x \\ 2x &= -2\sqrt{2x} \\ x &= -\sqrt{2x} \\ x^2 &= (-\sqrt{2x})^2 \\ x^2 &= 2x \\ x^2 - 2x &= 0 \\ x(x-2) &= 0 \end{aligned}$$
$$x = 0 \quad or \quad x = 2$$

Only 0 checks, so answer B is correct.

75. Left to the student

76. Left to the student

77. Left to the student

78. Left to the student

79. If an equation contains no fractions, using the addition principle before using the multiplication principle eliminates the need to add or subtract fractions.

80. You can conclude that $|a_1| = |a_2|$ since these constants determine how wide the parabolas are. Nothing can be concluded about the h's and the k's.

81.
$$\begin{aligned} \sqrt{\sqrt{\sqrt{x}}} &= 2 \\ \left(\sqrt{\sqrt{\sqrt{x}}}\right)^2 &= 2^2 \\ \sqrt{\sqrt{x}} &= 4 \\ \left(\sqrt{\sqrt{x}}\right)^2 &= 4^2 \\ \sqrt{x} &= 16 \\ (\sqrt{x})^2 &= 16^2 \\ x &= 256 \end{aligned}$$

The answer checks. The solution is 256.

82.
$$\begin{aligned} (x-1)^{2/3} &= 4 \\ (x-1)^2 &= 4^3 \\ x-1 &= \pm\sqrt{64} \\ x-1 &= \pm 8 \end{aligned}$$
$$x - 1 = -8 \quad or \quad x - 1 = 8$$
$$x = -7 \quad or \quad x = 9$$

Both numbers check.

83.
$$\begin{aligned} (t-4)^{4/5} &= 3 \\ \left[(t-4)^{4/5}\right]^5 &= 3^5 \\ (t-4)^4 &= 243 \\ t-4 &= \pm\sqrt[4]{243} \\ t &= 4 \pm \sqrt[4]{243} \end{aligned}$$

The exact solutions are $4 + \sqrt[4]{243}$ and $4 - \sqrt[4]{243}$. The approximate solutions are 7.948 and 0.052.

84. $\sqrt{x+2} + \sqrt[4]{x+2} - 2 = 0$

Let $u = \sqrt[4]{x+2}$, so $u^2 = (\sqrt[4]{x+2})^2 = \sqrt{x+2}$.
$$\begin{aligned} u^2 + u - 2 &= 0 \\ (u+2)(u-1) &= 0 \end{aligned}$$
$$u = -2 \quad or \quad u = 1$$

Substitute $\sqrt[4]{x+2}$ for u and solve for x.

$\sqrt[4]{x+2} = -2$ *or* $\sqrt[4]{x+2} = 1$

No real solution $\qquad x + 2 = 1$

$x = -1$

This number checks.

85.
$$\begin{aligned} (2y-2)^2 + y - 1 &= 5 \\ 4y^2 - 8y + 4 + y - 1 &= 5 \\ 4y^2 - 7y + 3 &= 5 \\ 4y^2 - 7y - 2 &= 0 \\ (4y+1)(y-2) &= 0 \end{aligned}$$
$$4y + 1 = 0 \quad or \quad y - 2 = 0$$
$$4y = -1 \quad or \quad y = 2$$
$$y = -\frac{1}{4} \quad or \quad y = 2$$

The solutions are $-\frac{1}{4}$ and 2.

86. The maximum value occurs at the vertex. The first coordinate of the vertex is $-\frac{b}{2a} = -\frac{b}{2(-3)} = \frac{b}{6}$ and $f\left(\frac{b}{6}\right) = 2$.

$$-3\left(\frac{b}{6}\right)^2 + b\left(\frac{b}{6}\right) - 1 = 2$$
$$-\frac{b^2}{12} + \frac{b^2}{6} - 1 = 2$$
$$-b^2 + 2b^2 - 12 = 24$$
$$b^2 = 36$$
$$b = \pm 6$$

87. ***Familiarize***. When principal P is deposited in an account at interest rate r, compounded annually, the amount A to which it grows in t years is given by $A = P(1+r)^t$. In 2 years the \$3500 deposit had grown to \$3500$(1+r)^2$. In one year the \$4000 deposit had grown to \$4000$(1+r)$.

Translate. The amount in the account at the end of 2 years was \$8518.35, so we have

$$3500(1+r)^2 + 4000(1+r) = 8518.35.$$

Carry out. We solve the equation. Let $u = 1 + r$.

$$3500u^2 + 4000u = 8518.35$$
$$3500u^2 + 4000u - 8518.35 = 0$$

Using the quadratic formula, we find that $u = 1.09$ or $u \approx -2.23$. Substitute $1 + r$ for u and solve for r.

$$1 + r = 1.09 \quad or \quad 1 + r = -2.23$$
$$r = 0.09 \quad or \quad r = -3.23$$

Check. Since the interest rate cannot be negative, we need to check only 0.09. At 9%, the \$3500 deposit would grow to \$3500$(1+0.09)^2$, or \$4158.35. The \$4000 deposit would grow to \$4000(1+0.09), or \$4360. Since \$4158.35+\$4360 = \$8518.35, the answer checks.

State. The interest rate was 9%.

Chapter 2 Test

1. $6x + 7 = 1$
$$6x = -6$$
$$x = -1$$
The solution is -1.

2. $3y - 4 = 5y + 6$
$$-4 = 2y + 6$$
$$-10 = 2y$$
$$-5 = y$$
The solution is -5.

3. $2(4x+1) = 8 - 3(x-5)$
$$8x + 2 = 8 - 3x + 15$$
$$8x + 2 = 23 - 3x$$
$$11x + 2 = 23$$
$$11x = 21$$
$$x = \frac{21}{11}$$
The solution is $\frac{21}{11}$.

4. $(2x-1)(x+5) = 0$
$$2x - 1 = 0 \quad or \quad x + 5 = 0$$
$$2x = 1 \quad or \quad x = -5$$
$$x = \frac{1}{2} \quad or \quad x = -5$$
The solutions are $\frac{1}{2}$ and -5.

5. $6x^2 - 36 = 0$
$$6x^2 = 36$$
$$x^2 = 6$$
$$x = -\sqrt{6} \quad or \quad x = \sqrt{6}$$
The solutions are $-\sqrt{6}$ and $\sqrt{6}$.

6. $x^2 + 4 = 0$
$$x^2 = -4$$
$$x = \pm\sqrt{-4}$$
$$x = -2i \quad or \quad x = 2i$$
The solutions are $-2i$ and $2i$.

7. $x^2 - 2x - 3 = 0$
$$(x+1)(x-3) = 0$$
$$x + 1 = 0 \quad or \quad x - 3 = 0$$
$$x = -1 \quad or \quad x = 3$$
The solutions are -1 and 3.

8. $x^2 - 5x + 3 = 0$

$a = 1$, $b = -5$, $c = 3$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$$
$$= \frac{5 \pm \sqrt{13}}{2}$$
The solutions are $\frac{5+\sqrt{13}}{2}$ and $\frac{5-\sqrt{13}}{2}$.

9. $2t^2 - 3t + 4 = 0$

$a = 2$, $b = -3$, $c = 4$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 2 \cdot 4}}{2 \cdot 2}$$
$$= \frac{3 \pm \sqrt{-23}}{4} = \frac{3 \pm i\sqrt{23}}{4}$$
$$= \frac{3}{4} \pm \frac{\sqrt{23}}{4}i$$
The solutions are $\frac{3}{4} + \frac{\sqrt{23}}{4}i$ and $\frac{3}{4} - \frac{\sqrt{23}}{4}i$.

10. $x + 5\sqrt{x} - 36 = 0$

Let $u = \sqrt{x}$.
$$u^2 + 5u - 36 = 0$$
$$(u+9)(u-4) = 0$$

$u + 9 = 0$ *or* $u - 4 = 0$

$u = -9$ *or* $u = 4$

Substitute $\sqrt{x}$ for u and solve for x.

$\sqrt{x} = -9$ *or* $\sqrt{x} = 4$

No solution $\quad x = 16$

The number 16 checks. It is the solution.

11. $\dfrac{3}{3x+4} + \dfrac{2}{x-1} = 2$, LCD is $(3x+4)(x-1)$

$$\begin{aligned}(3x+4)(x-1)\left(\frac{3}{3x+4}+\frac{2}{x-1}\right) &= (3x+4)(x-1)(2)\\ 3(x-1)+2(3x+4) &= 2(3x^2+x-4)\\ 3x-3+6x+8 &= 6x^2+2x-8\\ 9x+5 &= 6x^2+2x-8\\ 0 &= 6x^2-7x-13\\ 0 &= (x+1)(6x-13)\end{aligned}$$

$x + 1 = 0$ *or* $6x - 13 = 0$

$x = -1$ *or* $6x = 13$

$x = -1$ *or* $x = \dfrac{13}{6}$

Both numbers check. The solutions are -1 and $\dfrac{13}{6}$.

12.
$$\begin{aligned}\sqrt{x+4}-2 &= 1\\ \sqrt{x+4} &= 3\\ (\sqrt{x+4})^2 &= 3^2\\ x+4 &= 9\\ x &= 5\end{aligned}$$

This number checks. The solution is 5.

13.
$$\begin{aligned}\sqrt{x+4}-\sqrt{x-4} &= 2\\ \sqrt{x+4} &= \sqrt{x-4}+2\\ (\sqrt{x+4})^2 &= (\sqrt{x-4}+2)^2\\ x+4 &= x-4+4\sqrt{x-4}+4\\ 4 &= 4\sqrt{x-4}\\ 1 &= \sqrt{x-4}\\ 1^2 &= (\sqrt{x-4})^2\\ 1 &= x-4\\ 5 &= x\end{aligned}$$

This number checks. The solution is 5.

14. $|4y - 3| = 5$

$4y - 3 = -5$ *or* $4y - 3 = 5$

$4y = -2$ *or* $4y = 8$

$y = -\dfrac{1}{2}$ *or* $y = 2$

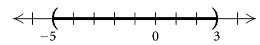
The solutions are $-\dfrac{1}{2}$ and 2.

15. $-7 < 2x + 3 < 9$

$-10 < 2x < 6$ Subtracting 3

$-5 < x < 3$ Dividing by 2

The solution set is $(-5, 3)$.

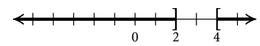


16. $2x - 1 \le 3$ *or* $5x + 6 \ge 26$

$2x \le 4$ *or* $5x \ge 20$

$x \le 2$ *or* $x \ge 4$

The solution set is $(-\infty, 2] \cup [4, \infty)$.

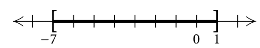


17. $|x + 3| \le 4$

$-4 \le x + 3 \le 4$

$-7 \le x \le 1$

The solution set is $[-7, 1]$.

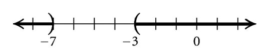


18. $|x + 5| > 2$

$x + 5 < -2$ *or* $x + 5 > 2$

$x < -7$ *or* $x > -3$

The solution set is $(-\infty, -7) \cup (-3, \infty)$.



19. $V = \dfrac{2}{3}\pi r^2 h$

$\dfrac{3V}{2} = \pi r^2 h$ Multiplying by $\dfrac{3}{2}$

$\dfrac{3V}{2\pi r^2} = h$ Dividing by πr^2

20.
$$\begin{aligned}R &= \sqrt{3np}\\ R^2 &= (\sqrt{3np})^2\\ R^2 &= 3np\\ \frac{R^2}{3p} &= n\end{aligned}$$

21. $x^2 + 4x = 1$

$x^2 + 4x + 4 = 1 + 4 \quad \left(\dfrac{1}{2}(4) = 2 \text{ and } 2^2 = 4\right)$

$(x + 2)^2 = 5$

$x + 2 = \pm\sqrt{5}$

$x = -2 \pm \sqrt{5}$

The solutions are $-2 + \sqrt{5}$ and $-2 - \sqrt{5}$.

22. ***Familiarize***. Let l = the length, in meters. Then $\dfrac{3}{4}l$ = the width. Recall that the formula for the perimeter P of a rectangle with length l and width w is $P = 2l + 2w$.

Translate.

$$\underbrace{\text{The perimeter}}_{\downarrow} \quad \text{is} \quad \underbrace{210 \text{ m.}}_{\downarrow}$$

$$2l + 2\cdot\frac{3}{4}l \quad = \quad 210$$

Carry out. We solve the equation.

$$2l + 2 \cdot \frac{3}{4}l = 210$$
$$2l + \frac{3}{2}l = 210$$
$$\frac{7}{2}l = 210$$
$$l = 60$$

If $l = 60$, then $\frac{3}{4}l = \frac{3}{4} \cdot 60 = 45$.

Check. The width, 45 m, is three-fourths of the length, 60 m. Also, $2 \cdot 60 \text{ m} + 2 \cdot 45 \text{ m} = 210 \text{ m}$, so the answer checks.

State. The length is 60 m and the width is 45 m.

23. ***Familiarize***. Let c = the speed of the current, in km/h. The boat travels downstream at a speed of $12 + c$ and upstream at a speed of $12 - c$. Using the formula $d = rt$ in the form $t = \frac{d}{r}$, we see that the travel time downstream is $\frac{45}{12+c}$ and the time upstream is $\frac{45}{12-c}$.

Translate.

$$\underbrace{\text{Total travel time}}\ \text{is}\ \underbrace{8\text{ hr.}}$$
$$\frac{45}{12+c} + \frac{45}{12-c} = 8$$

Carry out. We solve the equation. First we multiply both sides by the LCD, $(12 + c)(12 - c)$.

$$\frac{45}{12+c} + \frac{45}{12-c} = 8$$
$$(12+c)(12-c)\left(\frac{45}{12+c} + \frac{45}{12-c}\right) = (12+c)(12-c)(8)$$
$$45(12 - c) + 45(12 + c) = 8(144 - c^2)$$
$$540 - 45c + 540 + 45c = 1152 - 8c^2$$
$$1080 = 1152 - 8c^2$$
$$0 = 72 - 8c^2$$
$$0 = 8(9 - c^2)$$
$$0 = 8(3 + c)(3 - c)$$

$3 + c = 0$ *or* $3 - c = 0$

$c = -3$ *or* $3 = c$

Check. Since the speed of the current cannot be negative, we need to check only 3. If the speed of the current is 3 km/h, then the boat's speed downstream is $12 + 3$, or 15 km/h, and the speed upstream is $12 - 3$, or 9 km/h. At 15 km/h, it takes the boat $\frac{45}{15}$, or 3 hr, to travel 45 km downstream. At 9 km/h, it takes the boat $\frac{45}{9}$, or 5 hr, to travel 45 km upstream. The total travel time is 3 hr + 5 hr, or 8 hr, so the answer checks.

State. The speed of the current is 3 km/h.

24. ***Familiarize***. Let p = the wholesale price of the juice.

Translate. We express 25¢ as \$0.25.

Wholesale price	plus	50% of wholesale price	plus	\$0.25	is	\$2.95.
↓	↓	↓	↓	↓	↓	↓
p	$+$	$0.5p$	$+$	0.25	$=$	2.95

Carry out. We solve the equation.

$$p + 0.5p + 0.25 = 2.95$$
$$1.5p + 0.25 = 2.95$$
$$1.5p = 2.7$$
$$p = 1.8$$

Check. 50% of \$1.80 is \$0.90 and \$1.80 + \$0.90 + \$0.25 = \$2.95, so the answer checks.

State. The wholesale price of a bottle of juice is \$1.80.

25. $\sqrt{-43} = \sqrt{-1} \cdot \sqrt{43} = i\sqrt{43}$, or $\sqrt{43}i$

26. $-\sqrt{-25} = -\sqrt{-1} \cdot \sqrt{25} = -5i$

27. $(5 - 2i) - (2 + 3i) = (5 - 2) + (-2i - 3i)$
$= 3 - 5i$

28. $(3 + 4i)(2 - i) = 6 - 3i + 8i - 4i^2$
$= 6 + 5i + 4 \quad (i^2 = -1)$
$= 10 + 5i$

29.
$$\frac{1-i}{6+2i} = \frac{1-i}{6+2i} \cdot \frac{6-2i}{6-2i}$$
$$= \frac{6 - 2i - 6i + 2i^2}{36 - 4i^2}$$
$$= \frac{6 - 8i - 2}{36 + 4}$$
$$= \frac{4 - 8i}{40}$$
$$= \frac{4}{40} - \frac{8}{40}i$$
$$= \frac{1}{10} - \frac{1}{5}i$$

30. $i^{33} = (i^2)^{16} \cdot i = (-1)^{16} \cdot i = 1 \cdot i = i$

31.
$$3x + 9 = 0$$
$$3x = -9$$
$$x = -3$$

The zero of the function is -3.

32.
$$4x^2 - 11x - 3 = 0$$
$$(4x + 1)(x - 3) = 0$$

$4x + 1 = 0$ *or* $x - 3 = 0$

$4x = -1$ *or* $x = 3$

$x = -\frac{1}{4}$ *or* $x = 3$

The zeros of the functions are $-\frac{1}{4}$ and 3.

33. $2x^2 - x - 7 = 0$

$a = 2,\ b = -1,\ c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 2 \cdot (-7)}}{2 \cdot 2}$$

$$= \frac{1 \pm \sqrt{57}}{4}$$

The solutions are $\frac{1+\sqrt{57}}{4}$ and $\frac{1-\sqrt{57}}{4}$.

34.
$$\begin{aligned} f(x) &= -x^2 + 2x + 8 \\ &= -(x^2 - 2x) + 8 \\ &= -(x^2 - 2x + 1 - 1) + 8 \\ &= -(x^2 - 2x + 1) - (-1) + 8 \\ &= -(x^2 - 2x + 1) + 1 + 8 \\ &= -(x-1)^2 + 9 \end{aligned}$$

a) Vertex: $(1, 9)$

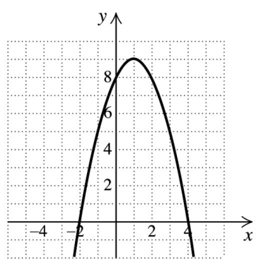
b) Axis of symmetry: $x = 1$

c) Maximum value: 9

d) Range: $(-\infty, 9]$

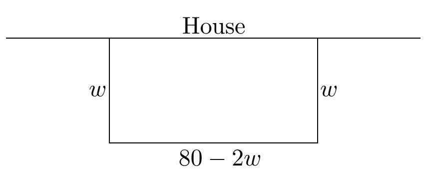
e) The graph opens down, so function values increase to the left of the vertex and decrease to the right of the vertex. Thus, $f(x)$ is increasing on $(-\infty, 1)$ and decreasing on $(1, \infty)$.

f)



$f(x) = -x^2 + 2x + 8$

35. ***Familiarize***. We make a drawing, letting $w =$ the width of the rectangle, in ft. This leaves $80 - w - w$, or $80 - 2w$ ft of fencing for the length.



Translating. The area of a rectangle is given by length times width.

$$\begin{aligned} A(w) &= (80 - 2w)w \\ &= 80w - 2w^2,\ \text{or}\ -2w^2 + 80w \end{aligned}$$

Carry out. This is a quadratic function with $a < 0$, so it has a maximum value that occurs at the vertex of the graph of the function. The first coordinate of the vertex is

$$w = -\frac{b}{2a} = -\frac{80}{2(-2)} = 20.$$

If $w = 20$, then $80 - 2w = 80 - 2 \cdot 20 = 40$.

Check. The area of a rectangle with length 40 ft and width 20 ft is $40 \cdot 20$, or 800 ft^2. As a partial check, we can find $A(w)$ for a value of w less than 20 and for a value of w greater than 20. For instance, $A(19.9) = 799.98$ and $A(20.1) = 799.98$. Since both of these values are less than 800, the result appears to be correct.

State. The dimensions for which the area is a maximum are 20 ft by 40 ft.

36. ***Familiarize***. Let $t =$ the number of hours a move requires. Then Morgan Movers charges $90 + 25t$ to make a move and McKinley Movers charges $40t$.

Translate.

$$\underbrace{\text{Morgan Movers' charge}}_{90 + 25t} \quad \underbrace{\text{is less than}}_{<} \quad \underbrace{\text{McKinley Movers' charge.}}_{40t}$$

Carry out. We solve the inequality.

$$\begin{aligned} 90 + 25t &< 40t \\ 90 &< 15t \\ 6 &< t \end{aligned}$$

Check. For $t = 6$, Morgan Movers charge $90 + 25 \cdot 6$, or \$240, and McKinley Movers charge $40 \cdot 6$, or \$240, so the charge is the same for 6 hours. As a partial check, we can find the charges for a value of t greater than 6. For instance, for 6.5 hr Morgan Movers charge $90 + 25(6.5)$, or \$252.50, and McKinley Movers charge $40(6.5)$, or \$260. Since Morgan Movers cost less for a value of t greater than 6, the answer is probably correct.

State. It costs less to hire Morgan Movers when a move takes more than 6 hr.

37. The maximum value occurs at the vertex. The first coordinate of the vertex is $-\frac{b}{2a} = -\frac{(-4)}{2a} = \frac{2}{a}$ and $f\left(\frac{2}{a}\right) = 12$.

Then we have:

$$\begin{aligned} a\left(\frac{2}{a}\right)^2 - 4\left(\frac{2}{a}\right) + 3 &= 12 \\ a \cdot \frac{4}{a^2} - \frac{8}{a} + 3 &= 12 \\ \frac{4}{a} - \frac{8}{a} + 3 &= 12 \\ -\frac{4}{a} + 3 &= 12 \\ -\frac{4}{a} &= 9 \\ -4 &= 9a \\ -\frac{4}{9} &= a \end{aligned}$$