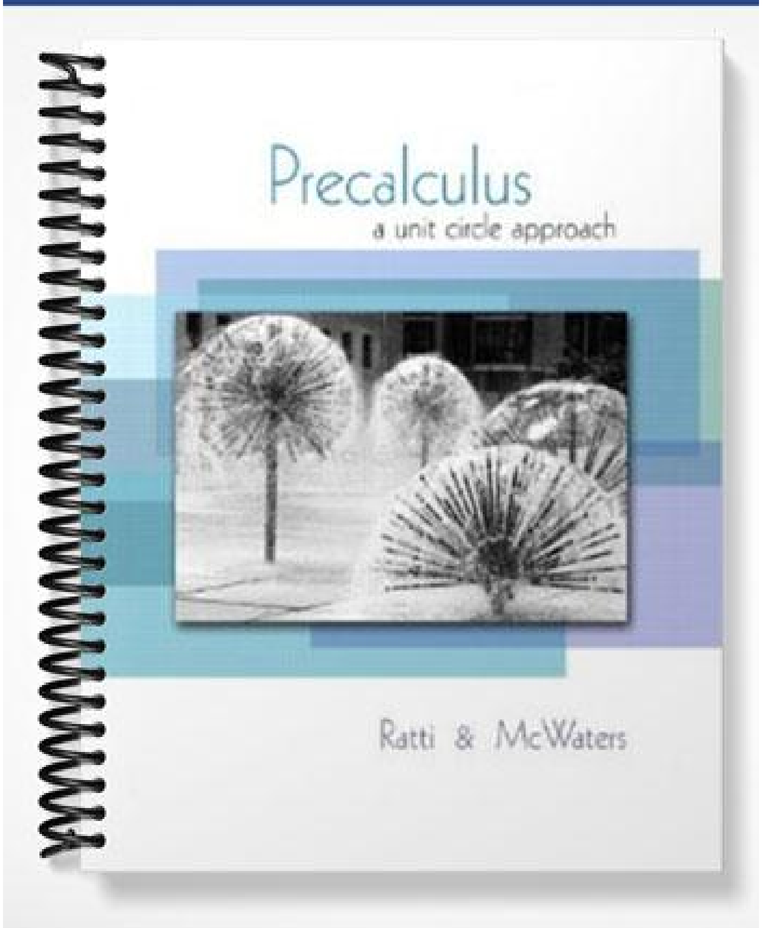
SOLUTIONS MANUAL

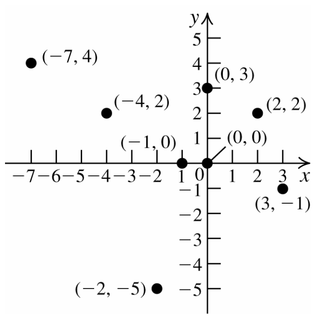


Chapter 2 Graphs and Functions

2.1 The Coordinate Plane

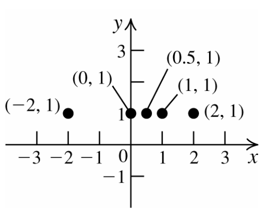
2.1 A Exercises: Basic Skills and Concepts

1.



2.a. Answers will vary. Sample answer: $(-2,0)$, $(-1,0),(0,0),(1,0),(2,0)$. The *y*-coordinate is 0.

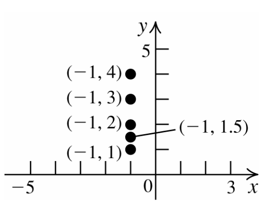
b.



The set of all points of the form $(x, 1)$ is a horizontal line that intersects the *y*-axis at 1.

3.a. If the *x*-coordinate of a point is 0, the point lies on the *y*-axis.

b.



The set of all points of the form $(–1, y)$ is a vertical line that intersects the *x*-axis at –1.

4.a. A vertical line that intersects the *x*-axis at –3.

b. A horizontal line that intersects the *y*-axis at 4.

5.a. $y > 0$ **b.** $y < 0$

c. $x < 0$ **d.** $x > 0$

6.a. Quadrant III **b.** Quadrant I

c. Quadrant IV **d.** Quadrant II

In Exercises 7–16, use the distance formula, $d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$ and the midpoint formula, $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

7.a. $d = \sqrt{(2-2)^2 + (5-1)^2} = \sqrt{4^2} = 4$

b. $M = \left(\frac{2+2}{2}, \frac{1+5}{2}\right) = (2,3)$

8.a. $d = \sqrt{(-2-3)^2 + (5-5)^2} = \sqrt{(-5)^2} = 5$

b. $M = \left(\frac{3+(-2)}{2}, \frac{5+5}{2}\right) = (0.5,5)$

9.a. $d = \sqrt{(2-(-1))^2 + (-3-(-5))^2}$
$= \sqrt{3^2 + 2^2} = \sqrt{13}$

b. $M = \left(\frac{-1+2}{2}, \frac{-5+(-3)}{2}\right) = (0.5,-4)$

10.a. $d = \sqrt{(-7-(-4))^2 + (-9-1)^2}$
$= \sqrt{(-3)^2 + (-10)^2} = \sqrt{109}$

b. $M = \left(\frac{-4+(-7)}{2}, \frac{1+(-9)}{2}\right) = (-5.5,-4)$

11.a. $d = \sqrt{(3-(-1))^2 + (-6.5-1.5)^2}$
$= \sqrt{4^2 + (-8)^2} = \sqrt{80} = 4\sqrt{5}$

b. $M = \left(\frac{-1+3}{2}, \frac{1.5+(-6.5)}{2}\right) = (1,-2.5)$

12.a. $d = \sqrt{(1-0.5)^2 + (-1-0.5)^2}$
$= \sqrt{(0.5)^2 + (-1.5)^2} = \sqrt{2.5} \approx 1.58$

b. $M = \left(\frac{0.5+1}{2}, \frac{0.5+(-1)}{2}\right) = (0.75,-0.25)$

13.a. $d = \sqrt{\left(\sqrt{2}-\sqrt{2}\right)^2 + (5-4)^2} = \sqrt{1^2} = 1$

b. $M = \left(\frac{\sqrt{2}+\sqrt{2}}{2}, \frac{4+5}{2}\right) = \left(\sqrt{2},4.5\right)$

14.a. $d = \sqrt{((v-w)-(v+w))^2 + (t-t)^2}$
$= \sqrt{(-2w)^2} = 2|w|$

b. $M = \left(\frac{(v+w)+(v-w)}{2}, \frac{t+t}{2}\right) = (v, t)$

15.a. $d = \sqrt{(t-k)^2 + (k-t)^2}$
$= \sqrt{\left(t^2 - 2tk + k^2\right) + \left(k^2 - 2kt + t^2\right)}$
$= \sqrt{2t^2 - 4tk + 2k^2} = \sqrt{2\left(t^2 - 2tk + k^2\right)}$
$= \sqrt{2(t-k)^2} = |t-k|\sqrt{2}$

b. $M = \left(\frac{t+k}{2}, \frac{t+k}{2}\right)$

16.a. $d = \sqrt{(-n-m)^2 + (-m-n)^2}$
$= \sqrt{\left(n^2 + 2mn + m^2\right) + \left(m^2 + 2mn + n^2\right)}$
$= \sqrt{2m^2 + 4mn + 2n^2}$
$= \sqrt{2\left(m^2 + 2mn + n^2\right)}$
$= \sqrt{2(m+n)^2} = |m+n|\sqrt{2}$

b. $M = \left(\frac{m+(-n)}{2}, \frac{n+(-m)}{2}\right)$
$= \left(\frac{m-n}{2}, \frac{n-m}{2}\right)$

17. $P = (-1, -2)$, $Q = (0, 0)$, $R = (1, 2)$
$d(P,Q) = \sqrt{\left(0-(-1)\right)^2 + \left(0-(-2)\right)^2} = \sqrt{5}$
$d(Q,R) = \sqrt{(1-0)^2 + (2-0)^2} = \sqrt{5}$
$d(P,R) = \sqrt{\left(1-(-1)\right)^2 + \left(2-(-2)\right)^2}$
$= \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$
Because *d*(*P*, *Q*) + *d*(*Q*, *R*) = *d*(*P*, *R*), the points are collinear.

18. $P = (-3, -4)$, $Q = (0, 0)$, $R = (3, 4)$
$d(P,Q) = \sqrt{\left(0-(-3)\right)^2 + \left(0-(-4)\right)^2} = \sqrt{25} = 5$
$d(Q,R) = \sqrt{(3-0)^2 + (4-0)^2} = \sqrt{25} = 5$
$d(P,R) = \sqrt{\left(3-(-3)\right)^2 + \left(4-(-4)\right)^2}$
$= \sqrt{6^2 + 8^2} = \sqrt{100} = 10$
Because *d*(*P*, *Q*) + *d*(*Q*, *R*) = *d*(*P*, *R*), the points are collinear.

19. $P = (4, -2)$, $Q = (1, 3)$, $R = (-2, 8)$
$d(P,Q) = \sqrt{(1-4)^2 + \left(3-(-2)\right)^2} = \sqrt{34}$
$d(Q,R) = \sqrt{(-2-1)^2 + (8-3)^2} = \sqrt{34}$
$d(P,R) = \sqrt{(-2-4)^2 + \left(8-(-2)\right)^2}$
$= \sqrt{(-6)^2 + 10^2} = \sqrt{136} = 2\sqrt{34}$
Because *d*(*P*, *Q*) + *d*(*Q*, *R*) = *d*(*P*, *R*), the points are collinear.

20. It is not possible to arrange the points in such a way so that d(P, Q) + d(Q, R) = d(P, R), so the points are not collinear.

21. $P = (-1, 4)$, $Q = (3, 0)$, $R = (11, -8)$
$d(P,Q) = \sqrt{\left(3-(-1)\right)^2 + (0-4)^2} = 4\sqrt{2}$
$d(Q,R) = \sqrt{(11-3)^2 + ((-8)-0)^2} = 8\sqrt{2}$
$d(P,R) = \sqrt{\left(11-(-1)\right)^2 + (-8-4)^2}$
$= \sqrt{(12)^2 + (-12)^2} = \sqrt{288} = 12\sqrt{2}$
Because *d*(*P*, *Q*) + *d*(*Q*, *R*) = *d*(*P*, *R*), the points are collinear.

22. It is not possible to arrange the points in such a way so that *d*(*P*, *Q*) + *d*(*Q*, *R*) = *d*(*P*, *R*), so the points are not collinear.

23. It is not possible to arrange the points in such a way so that *d*(*P*, *Q*) + *d*(*Q*, *R*) = *d*(*P*, *R*), so the points are not collinear.

24. $P = (1, 7)$, $Q = (-3, 7.5)$, $R = (-7, 8)$
$d(P,Q) = \sqrt{(-3-1)^2 + (7.5-7)^2} = \sqrt{16.25}$
$d(Q,R) = \sqrt{(-7-(-3))^2 + (8-7.5)^2}$
$= \sqrt{16.25}$
$d(P,R) = \sqrt{(-7-1)^2 + (8-7)^2}$
$= \sqrt{(-8)^2 + 1^2} = \sqrt{65} = 2\sqrt{16.25}$
Because *d*(*P*, *Q*) + *d*(*Q*, *R*) = *d*(*P*, *R*), the points are collinear.

25. $d(P,Q) = \sqrt{\left(-1-(-5)\right)^2 + (4-5)^2} = \sqrt{17}$
$d(Q,R) = \sqrt{\left(-4-(-1)\right)^2 + (1-4)^2} = 3\sqrt{2}$
$d(P,R) = \sqrt{\left(-4-(-5)\right)^2 + (1-5)^2} = \sqrt{17}$
The triangle is isosceles.

26. $d(P,Q) = \sqrt{(6-3)^2 + (6-2)^2} = 5$
$d(Q,R) = \sqrt{(-1-6)^2 + (5-6)^2} = 5\sqrt{2}$
$d(P,R) = \sqrt{(-1-3)^2 + (5-2)^2} = 5$
The triangle is isosceles.

27. $d(P,Q) = \sqrt{\left(0-(-4)\right)^2 + (7-8)^2} = \sqrt{17}$
$d(Q,R) = \sqrt{(-3-0)^2 + (5-7)^2} = \sqrt{13}$
$d(P,R) = \sqrt{\left(-3-(-4)\right)^2 + (5-8)^2} = \sqrt{10}$
The triangle is scalene.

28. $d(P,Q)=\sqrt{(3-(-1))^2+(-2-4)^2}=2\sqrt{13}$

$d(Q,R)=\sqrt{(7-3)^2+(5-(-2))^2}=\sqrt{65}$

$d(P,R)=\sqrt{(7-(-1))^2+(5-4)^2}=\sqrt{65}$

The triangle is isosceles.

29. $d(P,Q)=\sqrt{(-1-6)^2+(-1-6)^2}=7\sqrt{2}$

$d(Q,R)=\sqrt{(-5-(-1))^2+(3-(-1))^2}$
$=4\sqrt{2}$

$d(P,R)=\sqrt{(-5-6)^2+(3-6)^2}=\sqrt{130}$

The triangle is scalene.

30. $d(P,Q)=\sqrt{(9-0)^2+(-9-(-1))^2}=\sqrt{145}$

$d(Q,R)=\sqrt{(5-9)^2+(1-(-9))^2}=2\sqrt{29}$

$d(P,R)=\sqrt{(5-0)^2+(1-(-1))^2}=\sqrt{29}$

The triangle is scalene.

31. $d(P,Q)=\sqrt{(-1-1)^2+(4-1)^2}=\sqrt{13}$

$d(Q,R)=\sqrt{(5-(-1))^2+(8-4)^2}=2\sqrt{13}$

$d(P,R)=\sqrt{(5-1)^2+(8-1)^2}=\sqrt{65}$

The triangle is scalene.

32. $d(P,Q)=\sqrt{(4-(-4))^2+(5-4)^2}=\sqrt{65}$

$d(Q,R)=\sqrt{(0-4)^2+(-2-5)^2}=\sqrt{65}$

$d(P,R)=\sqrt{(0-(-4))^2+(-2-4)^2}=2\sqrt{13}$

The triangle is isosceles.

33. $d(P,Q)=\sqrt{(-1-1)^2+(1-(-1))^2}=2\sqrt{2}$

$$d(Q,R)=\sqrt{\left(-\sqrt{3}-(-1)\right)^2+\left(-\sqrt{3}-1\right)^2}$$
$$=\sqrt{\left(3-2\sqrt{3}+1\right)+\left(3+2\sqrt{3}+1\right)}$$
$$=\sqrt{8}=2\sqrt{2}$$

$$d(P,R)=\sqrt{\left(-\sqrt{3}-1\right)^2+\left(-\sqrt{3}-(-1)\right)^2}$$
$$=\sqrt{\left(3+2\sqrt{3}+1\right)+\left(3-2\sqrt{3}+1\right)}$$
$$=\sqrt{8}=2\sqrt{2}$$

The triangle is equilateral.

34. $d(P,Q)=\sqrt{(-1.5-(-0.5))^2+(1-(-1))^2}$
$=\sqrt{5}$

$$d(Q,R)=\sqrt{\left(\left(\sqrt{3}-1\right)-(-1.5)\right)^2+\left(\frac{\sqrt{3}}{2}-1\right)^2}$$
$$=\sqrt{\left(\left(\sqrt{3}-1\right)^2+3\left(\sqrt{3}-1\right)+2.25\right)+\left(\frac{3}{4}-\sqrt{3}+1\right)}$$
$$=\sqrt{\left(3-2\sqrt{3}+1+3\sqrt{3}-3+2.25\right)+\left(1.75-\sqrt{3}\right)}$$
$$=\sqrt{5}$$

$$d(P,R)=\sqrt{\left(\left(\sqrt{3}-1\right)-(-0.5)\right)^2+\left(\frac{\sqrt{3}}{2}-(-1)\right)^2}$$
$$=\sqrt{\left(\left(\sqrt{3}-1\right)^2+\left(\sqrt{3}-1\right)+0.25\right)+\left(\frac{3}{4}+\sqrt{3}+1\right)}$$
$$=\sqrt{\left(3-2\sqrt{3}+1+\sqrt{3}-1+0.25\right)+\left(1.75+\sqrt{3}\right)}$$
$$=\sqrt{5}$$

The triangle is equilateral.

35. First find the lengths of the sides:

$d(P,Q)=\sqrt{(-1-7)^2+(3-(-12))^2}=17$

$d(Q,R)=\sqrt{(14-(-1))^2+(11-3)^2}=17$

$d(R,S)=\sqrt{(22-14)^2+(-4-11)^2}=17$

$d(S,P)=\sqrt{(22-7)^2+(-4-(-12))^2}=17$

All the sides are equal, so the quadrilateral is either a square or a rhombus. Now find the length of the diagonals:

$d(P,R)=\sqrt{(14-7)^2+(11-(-12))^2}=17\sqrt{2}$ The

$d(Q,S)=\sqrt{(22-(-1))^2+(-4-3)^2}=17\sqrt{2}$

diagonals are equal, so the quadrilateral is a square.

36. First find the lengths of the sides:

$d(P,Q)=\sqrt{(9-8)^2+(-11-(-10))^2}=\sqrt{2}$

$d(Q,R)=\sqrt{(8-9)^2+(-12-(-11))^2}=\sqrt{2}$

$d(R,S)=\sqrt{(7-8)^2+(-11-(-12))^2}=\sqrt{2}$

$d(S,P)=\sqrt{(8-7)^2+(-10-(-11))^2}=\sqrt{2}$

All the sides are equal, so the quadrilateral is either a square or a rhombus. Now find the length of the diagonals:

$d(P,R)=\sqrt{(8-8)^2+(-12-(-10))^2}=2$
$d(Q,S)=\sqrt{(7-(-11))^2+(-11-(-10))^2}$
$=2$
The diagonals are equal, so the quadrilateral is a square.

37. $5=\sqrt{(x-2)^2+(2-(-1))^2}$
$=\sqrt{x^2-4x+4+9}\Rightarrow$
$5=\sqrt{x^2-4x+13}\Rightarrow 25=x^2-4x+13\Rightarrow$
$0=x^2-4x-12\Rightarrow 0=(x-6)(x+2)\Rightarrow$
$x=-2$ or $x=6$

38. $13=\sqrt{(2-(-10))^2+(y-(-3))^2}$
$=\sqrt{144+y^2+6y+9}$
$=\sqrt{y^2+6y+153}\Rightarrow$
$169=y^2+6y+153$
$0=y^2+6y-16\Rightarrow 0=(y+8)(y-2)\Rightarrow$
$y=-8$ or $y=2$

39. $P=(-5,2)$, $Q=(2,3)$, $R=(x,0)$ (R is on the x-axis, so the y-coordinate is 0).

$$d(P,R)=\sqrt{(x-(-5))^2+(0-2)^2}$$
$$d(Q,R)=\sqrt{(x-2)^2+(0-3)^2}$$
$$\sqrt{(x-(-5))^2+(0-2)^2}=\sqrt{(x-2)^2+(0-3)^2}$$
$$(x+5)^2+(0-2)^2=(x-2)^2+(0-3)^2$$
$$x^2+10x+25+4=x^2-4x+4+9$$
$$10x+29=-4x+13$$
$$14x=-16$$
$$x=-\frac{8}{7}$$

The coordinates of R are $\left(-\frac{8}{7},0\right)$.

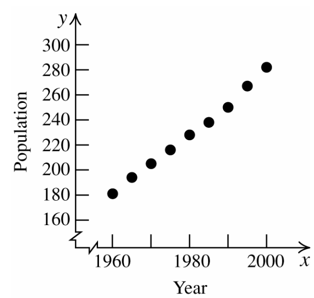
40. $P=(7,-4)$, $Q=(8,3)$, $R=(0,y)$ (R is on the y-axis, so the x-coordinate is 0).

$$d(P,R)=\sqrt{(0-7)^2+(y-(-4))^2}$$
$$d(Q,R)=\sqrt{(0-8)^2+(y-3)^2}$$
$$\sqrt{(0-7)^2+(y-(-4))^2}=\sqrt{(0-8)^2+(y-3)^2}$$
$$49+(y-(-4))^2=64+(y-3)^2$$
$$49+y^2+8y+16=64+y^2-6y+9$$
$$8y+65=-6y+73$$
$$14y=8$$
$$y=\frac{4}{7}$$

The coordinates of R are $\left(0,\frac{4}{7}\right)$.

2.1 B Exercises: Applying the Concepts

41.



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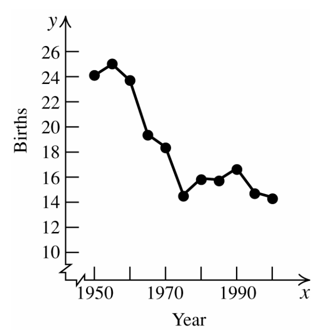

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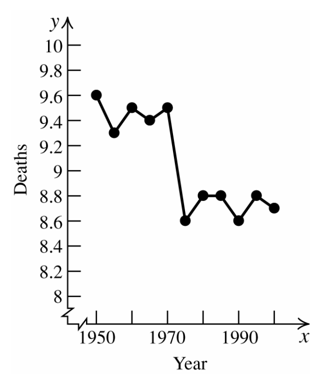

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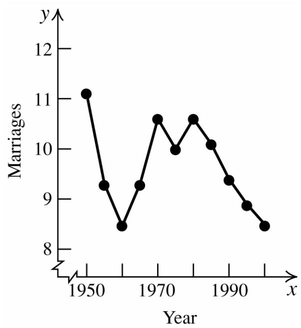

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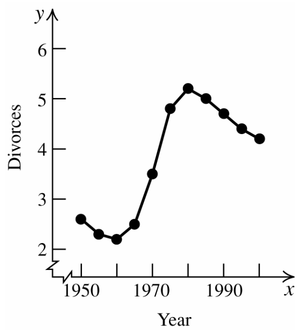


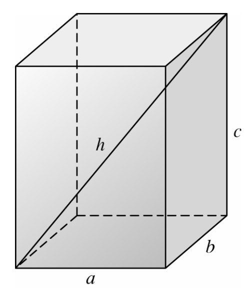
46. $M = \frac{22,000+18,000}{2} = 20,000$

47. 1999 is the midpoint of the initial range, so $M_{1999} = \frac{76+141}{2} = 108.5$. 1998 is the midpoint of the range [1997, 1999], so $M_{1998} = \frac{76+108.5}{2} = 92.25$. 2000 is the midpoint of the range [1999, 2001], so $M_{2000} = \frac{108.5+141}{2} = 124.75$. So, in 1998, 92.25 billion was spent; in 1999, 108.5 billion was spend; and, in 2000, 124.75 was spent.

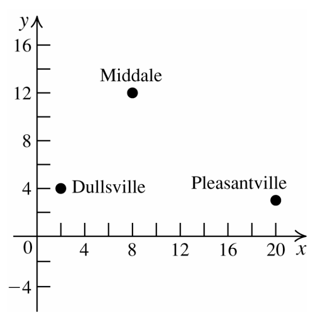
48. 1998 is the midpoint of the initial range, so $M_{1998} = \frac{320+400}{2} = 360$. 1996 is the midpoint of the range [1994, 1998], so $M_{1996} = \frac{320+360}{2} = 340$. 1995 is the midpoint of the range [1994, 1996], so $M_{1995} = \frac{320+340}{2} = 330$. Use similar reasoning to find the amounts for 1997, 1999, 2000, and 2001. Defense spending was as follows:

Year	Amount spent
1995	\$330 billion
1996	\$340 billion
1997	\$350 billion
1998	\$360 billion
1999	\$370 billion
2000	\$380 billion
2001	\$390 billion

49. Denote the diagonal connecting the endpoints of the edges *a* and *b* by *d*. Then *a*, *b*, and *d* form a right triangle. By the Pythagorean theorem, $a^2 + b^2 = d^2$. The edge *c* and the diagonals *d* and *h* also form a right triangle, so $c^2 + d^2 = h^2$. Substituting d^2 from the first equation, we obtain $a^2 + b^2 + c^2 = h^2$.



50.a.



b. $d(D,M) = \sqrt{(800-200)^2 + (1200-400)^2}$
$= 1000$

$d(M,P) = \sqrt{(2000-800)^2 + (300-1200)^2}$
$= 1500$

The distance traveled by the pilot = 1000 + 1500 = 2500 miles.

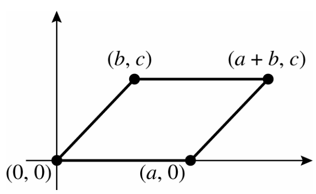
c. $d(D,P) = \sqrt{(2000-200)^2 + (300-400)^2}$
$= \sqrt{3,250,000} = 500\sqrt{13}$
≈ 1802.78 miles

51. First, find the initial length of the rope using the Pythagorean theorem: $c = \sqrt{24^2 + 10^2} = 26$. After t seconds, the length of the rope is $26 - 3t$. Now find the distance from the boat to the dock, x, using the Pythagorean theorem again and solving for x:

$$(26 - 3t)^2 = x^2 + 10^2$$
$$676 - 156t + 9t^2 = x^2 + 100$$
$$576 - 156t + 9t^2 = x^2$$
$$\sqrt{576 - 156t + 9t^2} = x$$

2.1 C Exercises: Beyond the Basics

52. The midpoint of the diagonal connecting (0, 0) and $(a + b, c)$ is $\left(\frac{a+b}{2}, \frac{c}{2}\right)$. The midpoint of the diagonal connecting $(a, 0)$ and (b, c) is also $\left(\frac{a+b}{2}, \frac{c}{2}\right)$. Because the midpoints of the two diagonals are the same, the diagonal bisect each other.



53.a. If AB is one of the diagonals, then DC is the other diagonal, and both diagonals have the same midpoint. The midpoint of AB is $\left(\frac{2+5}{2}, \frac{3+4}{2}\right) = (3.5, 3.5)$. The midpoint of $DC = (3.5, 3.5) = \left(\frac{x+3}{2}, \frac{y+8}{2}\right)$. So we have $3.5 = \frac{x+3}{2} \Rightarrow x = 4$ and $3.5 = \frac{y+8}{2} \Rightarrow y = -1$. The coordinates of D are (4, –1).

b. If AC is one of the diagonals, then DB is the other diagonal, and both diagonals have the same midpoint. The midpoint of AC is $\left(\frac{2+3}{2}, \frac{3+8}{2}\right) = (2.5, 5.5)$. The midpoint of $DB = (2.5, 5.5) = \left(\frac{x+5}{2}, \frac{y+4}{2}\right)$. So we have $2.5 = \frac{x+5}{2} \Rightarrow x = 0$ and $5.5 = \frac{y+4}{2} \Rightarrow y = 7$. The coordinates of D are (0, 7).

c. If BC is one of the diagonals, then DA is the other diagonal, and both diagonals have the same midpoint. The midpoint of BC is $\left(\frac{5+3}{2}, \frac{4+8}{2}\right) = (4, 6)$. The midpoint of $DA = (4, 6) = \left(\frac{x+2}{2}, \frac{y+3}{2}\right)$. So we have $4 = \frac{x+2}{2} \Rightarrow x = 6$ and $6 = \frac{y+3}{2} \Rightarrow y = 9$. The coordinates of D are (6, 9).

54. The midpoint of the diagonal connecting (0, 0) and (x, y) is $\left(\frac{x}{2}, \frac{y}{2}\right)$. The midpoint of the diagonal connecting $(a, 0)$ and (b, c) is $\left(\frac{a+b}{2}, \frac{c}{2}\right)$. Because the diagonals bisect each other, the midpoints coincide. So $\frac{x}{2} = \frac{a+b}{2} \Rightarrow x = a + b$, and $\frac{y}{2} = \frac{c}{2} \Rightarrow y = c$. Therefore, the quadrilateral is a parallelogram.

55.a. The midpoint of the diagonal connecting (1, 2) and (5, 8) is $\left(\frac{1+5}{2}, \frac{2+8}{2}\right) = (3, 5)$. The midpoint of the diagonal connecting (–2, 6) and (8, 4) is $\left(\frac{-2+8}{2}, \frac{6+4}{2}\right) = (3, 5)$. Because the midpoints are the same, the figure is a parallelogram.

b. The midpoint of the diagonal connecting (3, 2) and (x, y) is $\left(\frac{3+x}{2}, \frac{2+y}{2}\right)$. The midpoint of the diagonal connecting (6, 3) and (6, 5) is (6, 4). So $\frac{3+x}{2} = 6 \Rightarrow x = 9$ and $\frac{2+y}{2} = 4 \Rightarrow y = 6$.

56. Let $P(0, 0)$, $Q(a, 0)$, $R(a + b, c)$, and $S(b, c)$ be the vertices of the parallelogram. $PQ = RS = \sqrt{(a-0)^2 + (0-0)^2} = a$. $QR = PS = \sqrt{((a+b)-a)^2 + (c-0)^2} = \sqrt{b^2 + c^2}$. The sum of the squares of the lengths of the sides $= 2(a^2 + b^2 + c^2)$. $d(P, R) = \sqrt{(a+b)^2 + c^2}$.

$d(Q,S)=\sqrt{(a-b)^2+(0-c)^2}$. The sum of the squares of the lengths of the diagonals is
$\left((a+b)^2+c^2\right)+\left((a-b)^2+c^2\right)=$
$a^2+2ab+b^2+c^2+a^2-2ab+b^2+c^2=$
$2a^2+2b^2+2c^2=2(a^2+b^2+c^2)$.

57. Let $P(0, 0)$, $Q(a, 0)$, and $R(0,b)$ be the vertices of the right triangle. The midpoint M of the hypotenuse is $\left(\frac{a}{2},\frac{b}{2}\right)$.

$$d(Q,M)=\sqrt{\left(a-\frac{a}{2}\right)^2+\left(0-\frac{b}{2}\right)^2}=\sqrt{\left(\frac{a}{2}\right)^2+\left(-\frac{b}{2}\right)^2}=\frac{\sqrt{a^2+b^2}}{2}$$

$$d(R,M)=\sqrt{\left(0-\frac{a}{2}\right)^2+\left(b-\frac{b}{2}\right)^2}=\sqrt{\left(-\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2}=\frac{\sqrt{a^2+b^2}}{2}$$

58. Let $P(0, 0)$, $Q(a, 0)$, $R(a, b)$, and $S(0,b)$ be the vertices of the rectangle. Then the midpoint, A, of PQ is $\left(\frac{a}{2},0\right)$, the midpoint, B, of QR is $\left(a,\frac{b}{2}\right)$, the midpoint, C, of RS is $\left(\frac{a}{2},b\right)$, and the midpoint, D, of SP is $\left(0,\frac{b}{2}\right)$.

$$d(A,B)=\sqrt{\left(\frac{a}{2}-a\right)^2+\left(\frac{b}{2}\right)^2}=\frac{\sqrt{a^2+b^2}}{2}.$$

$$d(B,C)=\sqrt{\left(a-\frac{a}{2}\right)^2+\left(\frac{b}{2}-b\right)^2}=\frac{\sqrt{a^2+b^2}}{2}.$$

$$d(C,D)=\sqrt{\left(\frac{a}{2}-0\right)^2+\left(b-\frac{b}{2}\right)^2}=\frac{\sqrt{a^2+b^2}}{2}.$$

$$d(A,D)=\sqrt{\left(\frac{a}{2}-0\right)^2+\left(0-\frac{b}{2}\right)^2}=\frac{\sqrt{a^2+b^2}}{2}.$$

All the sides are equal, so the figure is a rhombus.

59. Let $P(0, 0)$, $Q(a, 0)$, $R(b, c)$ be the vertices of the triangle. Then
$d(P,Q)=\sqrt{(a-0)^2+(0-0)^2}=a$,
$d(Q,R)=\sqrt{(b-a)^2+(c-0)^2}=\sqrt{(b-a)^2+c^2}$,
and $d(P,R)=\sqrt{(b-0)^2+(c-0)^2}=\sqrt{b^2+c^2}$.
The sum of the squares of the lengths of the sides is $a^2+(b-a)^2+c^2+b^2+c^2=$
$a^2+b^2-2ab+a^2+c^2+a^2+b^2=$
$2a^2+2b^2+2c^2-2ab=2(a^2+b^2+c^2-ab)$.

Then the midpoint, A, of PQ is $\left(\frac{a}{2},0\right)$, the midpoint, B, of QR is $\left(\frac{a+b}{2},\frac{c}{2}\right)$, and the midpoint, C, of PR is $\left(\frac{b}{2},\frac{c}{2}\right)$. So the lengths of the medians are:

$$d(A,R)=\sqrt{\left(b-\frac{a}{2}\right)^2+(c-0)^2}=\sqrt{\left(b-\frac{a}{2}\right)^2+c^2},$$

$$d(B,P)=\sqrt{\left(\frac{a+b}{2}-0\right)^2+\left(\frac{c}{2}-0\right)^2}=\sqrt{\left(\frac{a+b}{2}\right)^2+\left(\frac{c}{2}\right)^2},\text{ and}$$

$$d(C,Q)=\sqrt{\left(\frac{b}{2}-a\right)^2+\left(\frac{c}{2}-0\right)^2}=\sqrt{\left(\frac{b}{2}-a\right)^2+\left(\frac{c}{2}\right)^2}.$$ The sum of the squares of the lengths of the medians is

$$\left(b-\frac{a}{2}\right)^2+c^2+\left(\frac{a+b}{2}\right)^2+\left(\frac{c}{2}\right)^2+\left(\frac{b}{2}-a\right)^2+\left(\frac{c}{2}\right)^2$$
$$=b^2-ab+\frac{a^2}{4}+c^2+\frac{a^2+2ab+b^2}{4}+\frac{c^2}{4}+\frac{b^2}{4}-ab+a^2+\frac{c^2}{4}$$
$$=\frac{3a^2}{2}+\frac{3b^2}{2}+\frac{3c^2}{2}-\frac{3ab}{2}=\frac{3(a^2+b^2+c^2-ab)}{2}$$
$$=\frac{3}{4}\left(2(a^2+b^2+c^2-ab)\right).$$

60.a. $$d(A,C)=\sqrt{\left(x_1-\frac{2x_1+x_2}{3}\right)^2+\left(y_1-\frac{2y_1+y_2}{3}\right)^2}=\sqrt{\left(\frac{x_1-x_2}{3}\right)^2+\left(\frac{y_1-y_2}{3}\right)^2}$$

$$d(C,B) = \sqrt{\left(\frac{2x_1 + x_2}{3} - x_2\right)^2 + \left(\frac{2y_1 + y_2}{3} - y_2\right)^2}$$
$$= \sqrt{\left(\frac{2x_1 - 2x_2}{3}\right)^2 + \left(\frac{2y_1 - 2y_2}{3}\right)^2}$$

$$d(A,C) + d(C,B) = \frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{3} + \frac{\sqrt{(2x_1 - 2x_2)^2 + (2y_1 - 2y_2)^2}}{3}$$
$$= \frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{3} + \frac{2\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{3}$$
$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$d(A,B) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. So A, B, and C are collinear.

$$d(A,C) = \sqrt{\left(\frac{x_1 - x_2}{3}\right)^2 + \left(\frac{y_1 - y_2}{3}\right)^2}$$
$$= \frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{3}$$
$$= \frac{1}{3} d(A,B).$$

b. $$d(A,D) = \sqrt{\left(x_1 - \frac{x_1 + 2x_2}{3}\right)^2 + \left(y_1 - \frac{y_1 + 2y_2}{3}\right)^2}$$
$$= \sqrt{\left(\frac{2x_1 + 2x_2}{3}\right)^2 + \left(\frac{2y_1 + 2y_2}{3}\right)^2}$$

$$d(D,B) = \sqrt{\left(\frac{x_1 + 2x_2}{3} - x_2\right)^2 + \left(\frac{y_1 + 2y_2}{3} - y_2\right)^2}$$
$$= \sqrt{\left(\frac{x_1 - x_2}{3}\right)^2 + \left(\frac{y_1 - y_2}{3}\right)^2}$$

$$d(A,D) + d(D,B)$$
$$= \frac{\sqrt{(2x_1 + 2x_2)^2 + (2y_1 + 2y_2)^2}}{3} + \frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{3}$$
$$= \frac{2\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{3} + \frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{3}$$
$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$= d(A,B)$. So A, B, and C are collinear.

$$d(A,D) = \sqrt{\left(\frac{2x_1 + 2x_2}{3}\right)^2 + \left(\frac{2y_1 - 2y_2}{3}\right)^2}$$
$$= \frac{2\sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2}}{3}$$
$$= \frac{2}{3} d(A,B).$$

c. $$\frac{2x_1 + x_2}{3} = \frac{2(-1) + 4}{3} = \frac{2}{3}$$
$$\frac{2y_1 + y_2}{3} = \frac{2(2) + 1}{3} = \frac{5}{3}$$
$$\frac{x_1 + 2x_2}{3} = \frac{-1 + 2(4)}{3} = \frac{7}{3}$$
$$\frac{y_1 + 2y_2}{3} = \frac{2 + 2(1)}{3} = \frac{4}{3}$$

The points of trisection are $\left(\frac{2}{3}, \frac{5}{3}\right)$ and $\left(\frac{7}{3}, \frac{4}{3}\right)$.

2.1 Critical Thinking

61.a. the y-axis

b. the x-axis

62.a. the union of the x- and y- axes

b. the plane without the x- and y- axes

63.a. Quadrants I and III

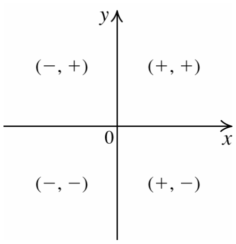
b. Quadrants II and IV

64.a. the origin

b. the plane without the origin

65. Let (x, y) be the point.

The point lies in	if
Quadrant I	$x > 0$ and $y > 0$
Quadrant II	$x < 0$ and $y > 0$
Quadrant III	$x < 0$ and $y < 0$
Quadrant IV	$x > 0$ and $y < 0$



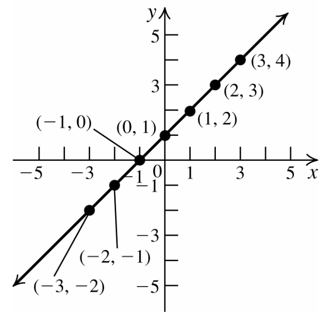
Section 2.2 Graphs of Equations

2.2 A Exercises: Basic Skills and Concepts

In exercises 1–6, to determine if a point lies on the graph of the equation, substitute the point's coordinates into the equation to see if the resulting statement is true.

1. on the graph: (–3, –4), (1, 0), (4, 3); not on the graph: (2, 3)
2. on the graph: (–1, 1), (1, 4), $\left(-\frac{5}{3}, 0\right)$; not on the graph: (0, 2)
3. on the graph: (3, 2), (0, 1), (8, 3); not on the graph: (8, –3)
4. on the graph: (1, 1), $\left(2, \frac{1}{2}\right)$; not on the graph: (0, 0), $\left(-3, \frac{1}{3}\right)$
5. on the graph: (1, 0), (0, –1); not on the graph: $\left(2, \sqrt{3}\right), \left(2, -\sqrt{3}\right)$
6. Each point is on the graph.
7. x-intercepts: –3, 0, 3; y-intercepts: –2, 0, 2
8. x-intercepts: –2, 4; y-intercept: –4

9.



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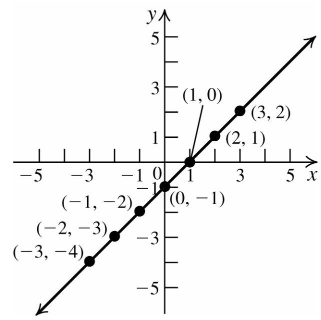

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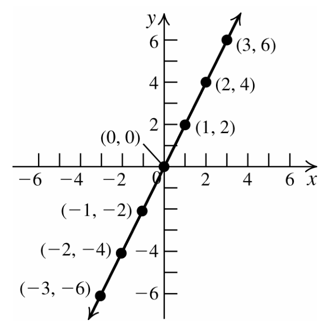

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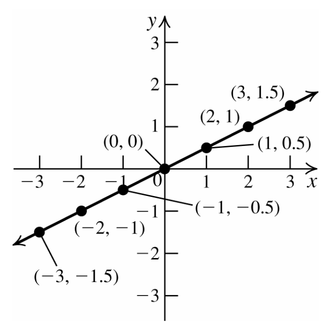

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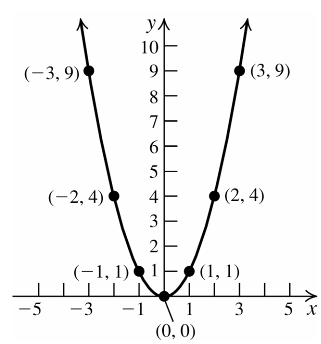

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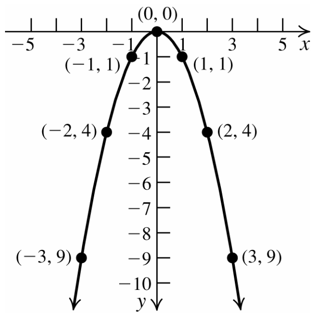

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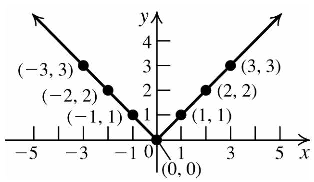

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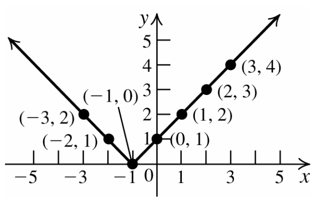

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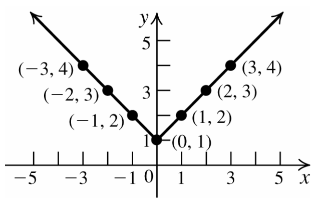

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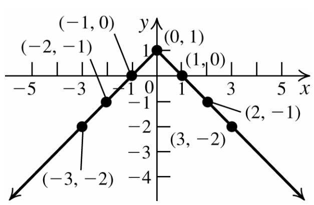

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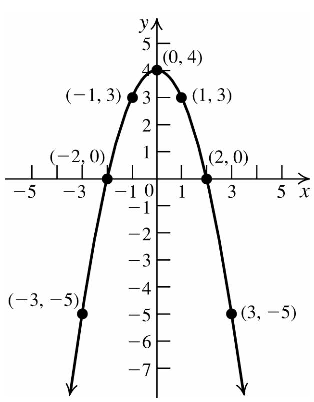

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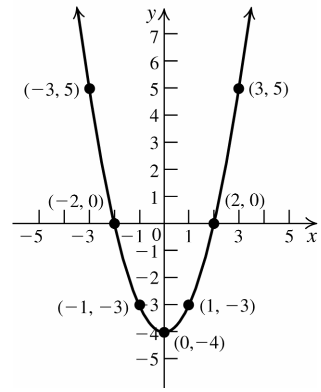

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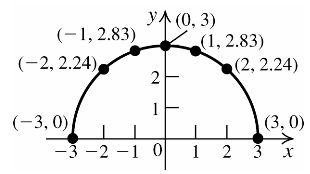

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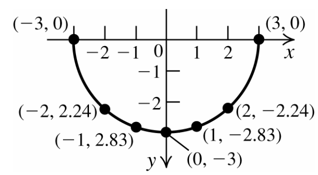

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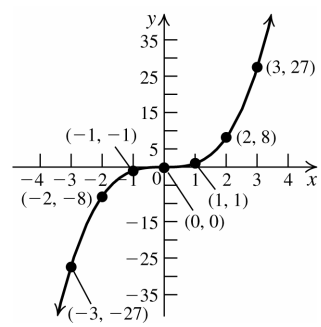

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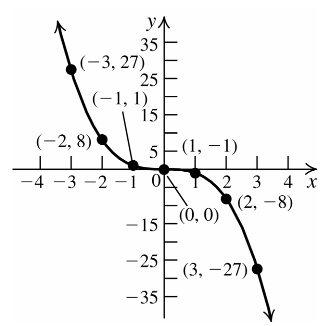

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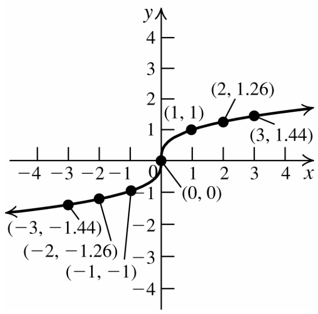

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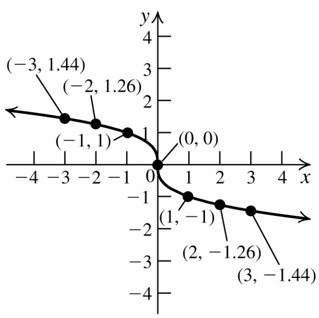

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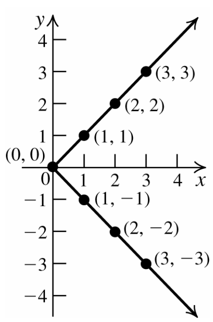

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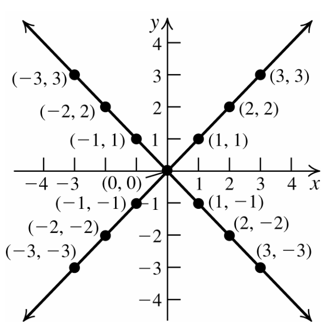

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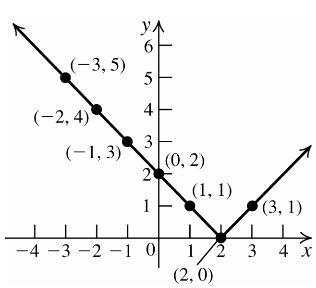

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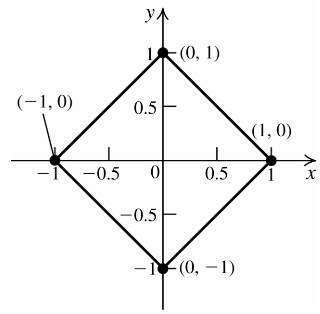


31. To find the x-intercept, let $y = 0$, and solve the equation for x: $3x + 4(0) = 12 \Rightarrow x = 4$. To find the y-intercept, let $x = 0$, and solve the equation for y: $3(0) + 4y = 12 \Rightarrow y = 3$. The x-intercept is 4; the y-intercept is 3.

32. To find the x-intercept, let $y = 0$, and solve the equation for x: $\frac{x}{5} + \frac{0}{3} = 1 \Rightarrow x = 5$. To find the y-intercept, let $x = 0$, and solve the equation for y: $\frac{0}{5} + \frac{y}{3} = 1 \Rightarrow y = 3$. The x-intercept is 5; the y-intercept is 3.

33. To find the x-intercept, let $y = 0$, and solve the equation for x: $2x + 3(0) = 5 \Rightarrow x = \frac{5}{2}$. To find the y-intercept, let $x = 0$, and solve the equation for y: $2(0) + 3y = 5 \Rightarrow y = \frac{5}{3}$. The x-intercept is $5/2$; the y-intercept is $5/3$.

34. To find the x-intercept, let $y = 0$, and solve the equation for x: $\frac{x}{2} - \frac{0}{3} = 1 \Rightarrow x = 2$. To find the y-intercept, let $x = 0$, and solve the equation for y: $\frac{0}{2} - \frac{y}{3} = 1 \Rightarrow x = -3$. The x-intercept is 2; the y-intercept is -3.

35. To find the x-intercept, let $y = 0$, and solve the equation for x: $0 = x^2 - 6x + 8 \Rightarrow x = 4$ or $x = 2$. To find the y-intercept, let $x = 0$, and solve the equation for y: $y = 0^2 - 6(0) + 8 \Rightarrow y = 8$. The x-intercepts are 2 and 4; the y-intercept is 8.

36. To find the x-intercept, let $y = 0$, and solve the equation for x: $x = 0^2 - 5(0) + 6 \Rightarrow x = 6$.

To find the y-intercept, let $x = 0$, and solve the equation for y: $0 = y^2 - 5y + 6 \Rightarrow y = 2$ or $y = 3$. The x-intercept is 6; the y-intercepts are 2 and 3.

37. To find the x-intercept, let $y = 0$, and solve the equation for x: $x^2 + 0^2 = 4 \Rightarrow x = \pm 2$. To find the y-intercept, let $x = 0$, and solve the equation for y: $0^2 + y^2 = 4 \Rightarrow y = \pm 2$. The x-intercepts are –2 and 2; the y-intercepts are –2 and 2.

38. To find the x-intercept, let $y = 0$, and solve the equation for x: $0 = \sqrt{9 - x^2} \Rightarrow x = \pm 3$. To find the y-intercept, let $x = 0$, and solve the equation for y: $y = \sqrt{9 - 0^2} \Rightarrow y = 3$. The x-intercepts are –3 and 3; the y-intercept is 3.

39. To find the x-intercept, let $y = 0$, and solve the equation for x: $0 = \sqrt{x^2 - 1} \Rightarrow x = \pm 1$. To find the y-intercept, let $x = 0$, and solve the equation for y: $y = \sqrt{0^2 - 1} \Rightarrow$ no solution. The x-intercepts are –1 and 1; there is no y-intercept.

40. To find the x-intercept, let $y = 0$, and solve the equation for x: $x(0) = 1 \Rightarrow$ no solution. To find the y-intercept, let $x = 0$, and solve the equation for y: $(0)y = 1 \Rightarrow$ no solution. There is no x-intercept 1; there is no y-intercept.

In exercises 41–50, to test for symmetry with respect to the x-axis, replace y with $-y$ to determine if $(x, -y)$ satisfies the equation. To test for symmetry with respect to the y-axis, replace x with $-x$ to determine if $(-x, y)$ satisfies the equation. To test for symmetry with respect to the origin, replace x with $-x$ and y with $-y$ to determine if $(-x, -y)$ satisfies the equation.

41. $-y = x^2 + 1$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis. $y = (-x)^2 + 1 \Rightarrow y = x^2 + 1$, so the equation is symmetric with respect to the y-axis. $-y = (-x)^2 + 1 \Rightarrow -y = x^2 + 1$, is not the same as the original equation, so the equation is not symmetric with respect to the origin.

42. $x = (-y)^2 + 1 \Rightarrow x = y^2 + 1$, so the equation is symmetric with respect to the x-axis. $-x = y^2 + 1$ is not the same as the original equation, so the equation is not symmetric with respect to the y-axis. $-x = (-y)^2 + 1 \Rightarrow -x = y^2 + 1$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

43. $-y = x^3 + x$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis. $y = (-x)^3 - x \Rightarrow y = -x^3 - x \Rightarrow y = -(x^3 + x)$ is not the same as the original equation, so the equation is not symmetric with respect to the y-axis. $-y = (-x)^3 - x \Rightarrow -y = -x^3 - x \Rightarrow -y = -(x^3 + x) \Rightarrow y = x^3 + x$, so the equation is symmetric with respect to the origin.

44. $-y = 2x^3 - x$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis. $y = 2(-x)^3 - (-x) \Rightarrow y = -2x^3 + x \Rightarrow y = -2(x^3 - x)$ is not the same as the original equation, so the equation is not symmetric with respect to the y-axis. $-y = 2(-x)^3 - (-x) \Rightarrow -y = -2x^3 + x \Rightarrow -y = -2(x^3 - x) \Rightarrow y = 2x^3 - x$, so the equation is symmetric with respect to the origin.

45. $-y = 5x^4 + 2x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis. $y = 5(-x)^4 + 2(-x)^2 \Rightarrow -y = 5x^4 + 2x^2$, so the equation is symmetric with respect to the y-axis. $-y = 5(-x)^4 + 2(-x) \Rightarrow -y = 5x^4 + 2x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

46. $-y = -3x^6 + 2x^4 + x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis. $y = -3(-x)^6 + 2(-x)^4 + (-x)^2 \Rightarrow y = -3x^6 + 2x^4 + x^2$, so the equation is symmetric with respect to the y-axis.

$-y = -3(-x)^6 + 2(-x)^4 + (-x)^2 \Rightarrow$
$-y = -3x^6 + 2x^4 + x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

47. $-y = -3x^5 + 2x^3$ is not the same as the original equation, so the equation is not symmetric with respect to the *x*-axis. $y = -3(-x)^5 + 2(-x)^3 \Rightarrow$ $y = 3x^5 - 2x^3$ is not the same as the original equation, so the equation is not symmetric with respect to the *y*-axis.
$-y = -3(-x)^5 + 2(-x)^3 \Rightarrow -y = 3x^5 - 2x^3 \Rightarrow$
$-y = -(-3x^5 + 2x^3) \Rightarrow y = -3x^5 + 2x^3$, so the equation is symmetric with respect to the origin.

48. $-y = 2x^2 - |x|$ is not the same as the original equation, so the equation is not symmetric with respect to the *x*-axis. $y = 2(-x)^2 - |-x| \Rightarrow$ $y = 2x^2 - |x|$, so the equation is symmetric with respect to the *y*-axis. $-y = 2(-x)^2 - |-x| \Rightarrow$ $-y = 2x^2 - |x|$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

49. $x^2(-y)^2 + 2x(-y) = 1 \Rightarrow x^2y^2 - 2xy = 1$ is not the same as the original equation, so the equation is not symmetric with respect to the *x*-axis. $(-x)^2y^2 + 2(-x)y = 1 \Rightarrow x^2y^2 - 2xy = 1$ is not the same as the original equation, so the equation is not symmetric with respect to the *y*-axis. $(-x)^2(-y)^2 + 2(-x)(-y) = 1 \Rightarrow$ $x^2y^2 + 2xy = 1$, so the equation is symmetric with respect to the origin.

50. $x^2 + (-y)^2 = 1 \Rightarrow x^2 + y^2 = 1$, so the equation is not symmetric with respect to the *x*-axis. $(-x)^2 + y^2 = 1 \Rightarrow x^2 + y^2 = 1$, so the equation is not symmetric with respect to the *y*-axis. $(-x)^2 + (-y)^2 = 1 \Rightarrow x^2 + y^2 = 1$, so the equation is symmetric with respect to the origin.

For exercises 51–70, use the standard form of the equation of a circle, $(x-h)^2 + (y-k)^2 = r^2$.

51. Center (2, 3); radius = 6

52. Center (–1, 3); radius = 4

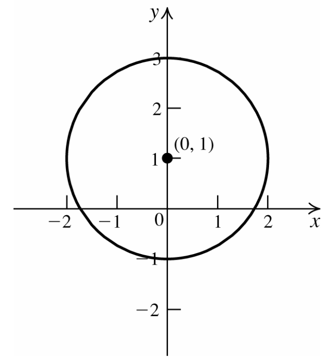
53. Center (–2, –3); radius = $\sqrt{11}$

54. Center $\left(\frac{1}{2}, -\frac{3}{2}\right)$; radius = $\frac{\sqrt{3}}{2}$

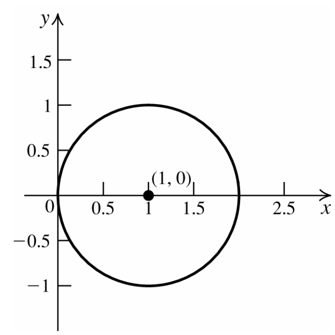
55. Center (*a*, –*b*); radius = $|r|$

56. Center (–*a*, –*b*); radius = $\sqrt{7}$

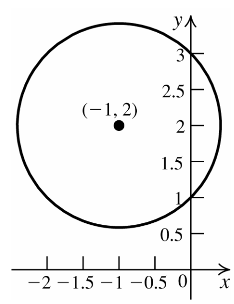
57. $x^2 + (y-1)^2 = 4$



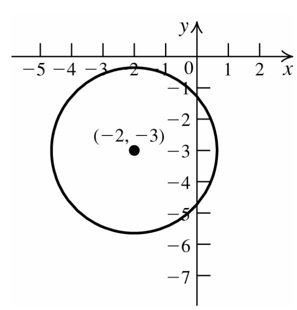
58. $(x-1)^2 + y^2 = 1$



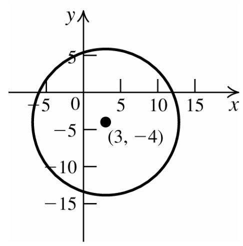
59. $(x+1)^2 + (y-2)^2 = 2$



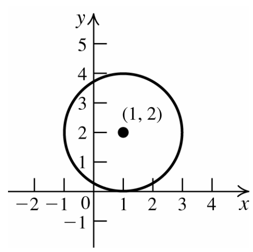
60. $(x+2)^2 + (y+3)^2 = 7$



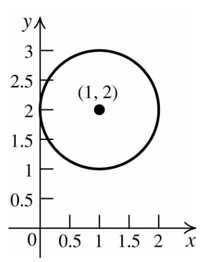
61. Find the radius by using the distance formula: $d=\sqrt{(-1-3)^2+(5-(-4))^2}=\sqrt{97}$. The equation of the circle is $(x-3)^2+(y+4)^2=97$.



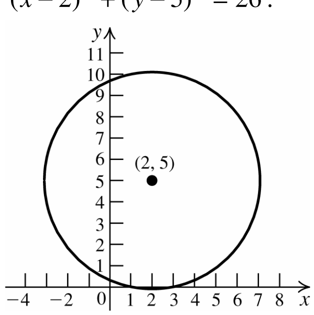
62. The circle touches the *x*-axis, so the radius is 2. The equation of the circle is $(x-1)^2+(y-2)^2=4$.



63. The circle touches the *y*-axis, so the radius is 1. The equation of the circle is $(x-1)^2+(y-2)^2=1$.



64. Find the diameter by using the distance formula: $d=\sqrt{(-3-7)^2+(6-4)^2}=\sqrt{104}=2\sqrt{26}$. So the radius is $\sqrt{26}$. Use the midpoint formula to find the center: $M=\left(\frac{7+(-3)}{2},\frac{4+6}{2}\right)=(2,5)$. The equation of the circle is $(x-2)^2+(y-5)^2=26$.



65. $x^2+y^2-2x-2y-4=0\Rightarrow$ $x^2-2x+y^2-2y=4$. Now complete the square: $x^2-2x+1+y^2-2y+1=4+1+1\Rightarrow$ $(x-1)^2+(y-1)^2=6$. This is a circle with center (1, 1) and radius $\sqrt{6}$.

66. $x^2+y^2-4x-2y-15=0\Rightarrow$ $x^2-4x+y^2-2y=15.$ Now complete the square: $x^2-4x+4+y^2-2y+1=15+4+1\Rightarrow$ $(x-2)^2+(y-1)^2=20.$ This is a circle with center (2, 1) and radius $2\sqrt{5}$.

67. $2x^2+2y^2+4y=0\Rightarrow 2(x^2+y^2+2y)=0\Rightarrow$ $x^2+y^2+2y=0.$ Now complete the square: $x^2+y^2+2y+1=0+1\Rightarrow x^2+(y+1)^2=1.$ This is a circle with center (0, –1) and radius 1.

68. $3x^2+3y^2+6x=0\Rightarrow 3(x^2+y^2+2x)=0\Rightarrow$ $x^2+2x+y^2=0.$ Now complete the square: $x^2+2x+1+y^2=0+1\Rightarrow(x+1)^2+y^2=1.$ This is a circle with center (–1, 0) and radius 1.

69. $x^2+y^2-x=0\Rightarrow x^2-x+y^2=0.$ Now complete the square:

$$x^2-x+\frac{1}{4}+y^2=0+\frac{1}{4}\Rightarrow\left(x-\frac{1}{2}\right)^2+y^2=\frac{1}{4}.$$

This is a circle with center $(1/2,0)$ and radius $1/2$.

70. $x^2+y^2+1=0\Rightarrow x^2+y^2=-1.$ The radius cannot be negative, so there is no graph.

2.2 B Exercises: Applying the Concepts

71. The distance from *P*(*x*, *y*) to the *x*-axis is $|x|$ while the distance from *P* to the *y*-axis is $|y|$. So the equation of the graph is $|x|=|y|$.

72. The distance from *P*(*x*, *y*) to (1, 2) is $\sqrt{(x-1)^2+(y-2)^2}$ while the distance from *P* to (3, –4) is $\sqrt{(x-3)^2+(y+4)^2}$.

So the equation of the graph is
$\sqrt{(x-1)^2+(y-2)^2} = \sqrt{(x-3)^2+(y+4)^2} \Rightarrow (x-1)^2+(y-2)^2 = (x-3)^2+(y+4)^2 \Rightarrow x^2-2x+1+y^2-4y+4 = x^2-6x+9+y^2+8y+16 \Rightarrow -2x-4y+5 = -6x+8y+25 \Rightarrow 4x-20=12y \Rightarrow y=\frac{1}{3}x-\frac{5}{3}.$

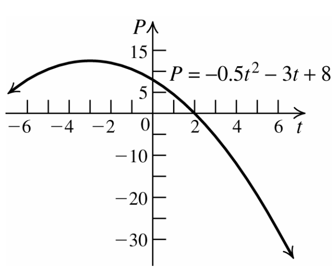
73. The distance from $P(x, y)$ to (2, 0) is $\sqrt{(x-2)^2+y^2}$ while the distance from P to the y-axis is $|x|$. So the equation of the graph is $\sqrt{(x-2)^2+y^2}=|x| \Rightarrow (x-2)^2+y^2=x^2 \Rightarrow .$ $x^2-4x+4+y^2=x^2 \Rightarrow y^2=4x-4 \Rightarrow \frac{y^2+4}{4}=\frac{y^2}{4}+1=x$

74. The distance from P to the point (0, 4) is $\sqrt{x^2+(y-4)^2}$ while the distance from P to the x-axis is $|y|$. So the equation of the graph is $\sqrt{x^2+(y-4)^2}=|y| \Rightarrow x^2+(y-4)^2=y^2 \Rightarrow x^2+y^2-8y+16=y^2 \Rightarrow x^2=8y-16 \Rightarrow \frac{x^2+16}{8}=\frac{x^2}{8}+2=y$

75.a. Since July 2004 is represented by $t = 0$, March 2004 is represented by $t = -4$. So the monthly profit for March is determined by $P=-0.5(-4)^2-3(-4)+8=\$12$ million.

b. Since July 2004 is represented by $t = 0$, October 2004 is represented by $t = 3$. So the monthly profit for October is determined by $P=-0.5(3)^2-3(3)+8=-\$5.5$ million.

c.



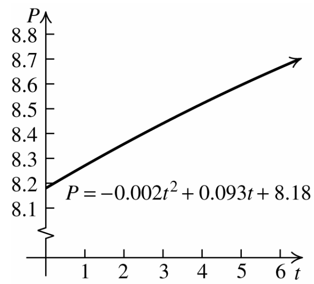
d. To find the t-intercept, set $P = 0$ and solve for t: $0=-0.5t^2-3t+8 \Rightarrow$

$$t=\frac{3\pm\sqrt{(-3)^2-4(-0.5)(8)}}{2(-0.5)}=\frac{3\pm\sqrt{25}}{-1}$$

$= 2$ or -8. The t-intercepts represent the months with no profit and no loss.

e. To find the P-intercept, set $t = 0$ and solve to P: $P=-0.5(0)^2-3(0)+8 \Rightarrow P=8$. The P-intercept represents the profit in July 2004.

76.a.



b. To find the t-intercept, set $P = 0$ and solve for t: $0=-0.002t^2+0.093t+8.18 \Rightarrow$

$$t=\frac{-0.093\pm\sqrt{0.093^2-4(-0.002)(8.18)}}{2(-0.002)}=\frac{-0.093\pm\sqrt{0.074089}}{-0.004}\approx -44.8 \text{ or } 91.3$$

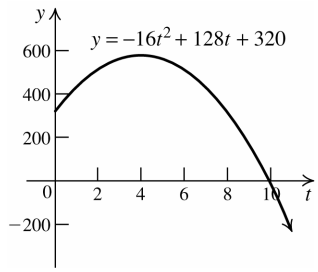
Because the domain is restricted to [0, 6], there is no t-intercept.

c. To find the P-intercept, set $t = 0$ and solve to P: $P=-0.002(0)^2+0.093(0)+8.18 \Rightarrow P=8.18$. The P-intercept represents the number of female college students in 1995.

77.a.

t	Height = $-16t^2+128t+320$
0	320 feet
1	432 feet
2	512 feet
3	560 feet
4	576 feet
5	560 feet
6	512 feet

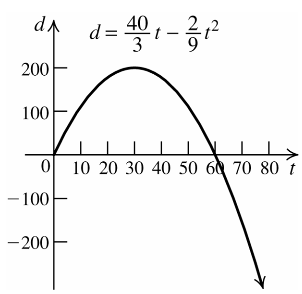
b.



c. $0 \le t \le 10$

d. To find the t-intercept, set $y = 0$ and solve for t: $0 = -16t^2 + 128t + 320 \Rightarrow$
$0 = -16(t^2 - 8t - 20) \Rightarrow 0 = (t-10)(t+2) \Rightarrow$
$t = 10$ or $t = -2$. The graph does not apply if $t < 0$, so the t-intercept is 10. This represents the time when the object hits the ground. To find the y-intercept, set $t = 0$ and solve for y: $y = -16(0)^2 + 128(0) + 320 \Rightarrow y = 320$. This represents the height of the building.

78.a.

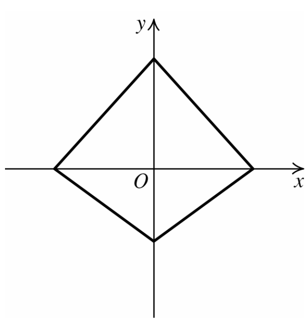


b. $0 \le t \le 60$

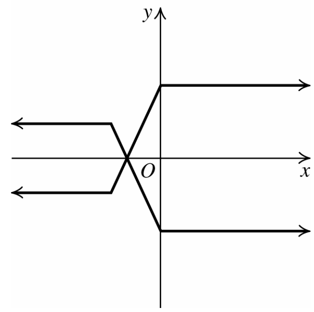
c. The total time of the experiment is 60 minutes or 1 hour.

2.2 C Exercises: Beyond the Basics

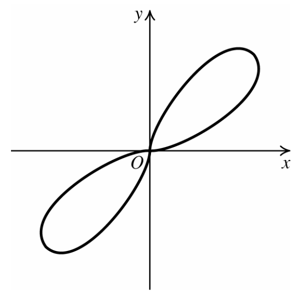
79.



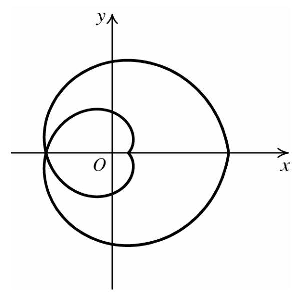
80.



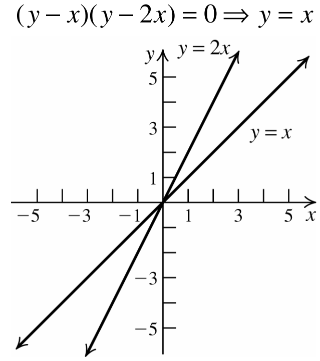
81.



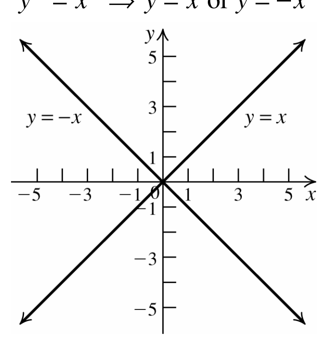
82.



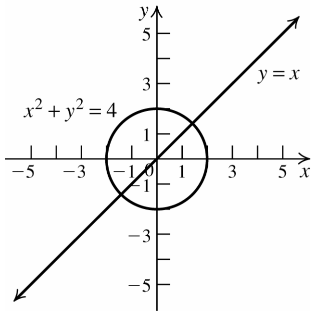
83. $(y - x)(y - 2x) = 0 \Rightarrow y = x$ or $y = 2x$



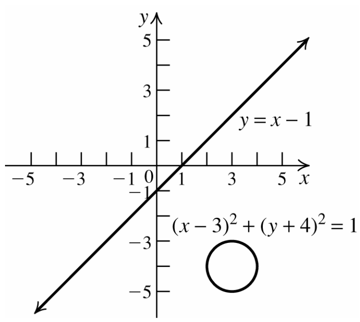
84. $y^2 = x^2 \Rightarrow y = x$ or $y = -x$



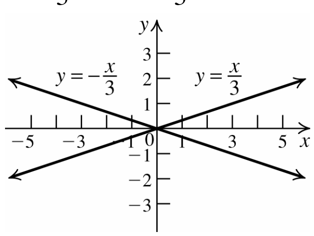
85. $(y-x)(x^2+y^2-4)=0 \Rightarrow y=x$ or $x^2+y^2=4$



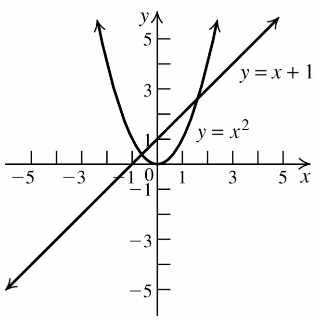
86. $(y-x-1)(x^2+y^2-6x+8y+24)=0 \Rightarrow$
$y=x+1$ or $x^2+y^2-6x+8y=-24 \Rightarrow$
$x^2-6x+9+y^2+8y+16=-24+9+16 \Rightarrow$
$(x-3)^2+(y+4)^2=1$



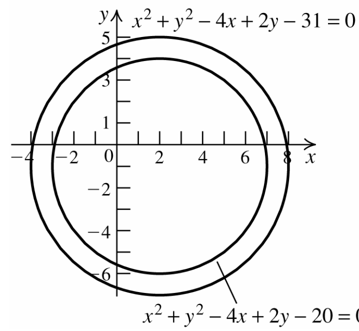
87. $x^2-9y^2=0 \Rightarrow (x-3y)(x+3y)=0 \Rightarrow$
$y=\frac{x}{3}$ or $y=-\frac{x}{3}$



88. $(y-x-1)(y-x^2)=0 \Rightarrow y=x+1$ or $y=x^2$

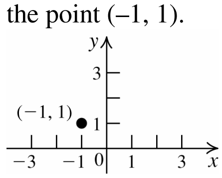


89. $x^2+y^2-4x+2y-20=0 \Rightarrow$
$x^2-4x+y^2+2y=20 \Rightarrow$
$x^2-4x+4+y^2+2y+1=20+4+1 \Rightarrow$
$(x-2)^2+(y+1)^2=25$
So this is the graph of a circle with center (2, –1) and radius 5. The area of this circle is 25π. $x^2+y^2-4x+2y-31=0 \Rightarrow$
$x^2-4x+y^2+2y=31 \Rightarrow$
$x^2-4x+4+y^2+2y+1=31+4+1 \Rightarrow$
$(x-2)^2+(y+1)^2=36$
So, this is the graph of a circle with center (2, –1) and radius 6. The area of this circle is 36π. Both circles have the same center, so the area of the region bounded by the two circles = $36\pi-25\pi=11\pi$.

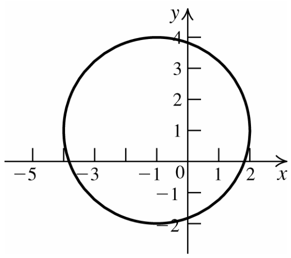


90.a. $x^2+y^2+2x-2y+3=0 \Rightarrow$
$x^2+2x+y^2-2y=-3 \Rightarrow$
$x^2+2x+1+y^2-2y+1=-3+1+1 \Rightarrow$
$(x+1)^2+(y-1)^2=-1$
This looks like the equation of a circle; however the radius cannot be negative, so there is no graph.

b. $x^2+y^2+2x-2y+2=0 \Rightarrow$
$x^2+2x+y^2-2y=-2 \Rightarrow$
$x^2+2x+1+y^2-2y+1=-2+1+1 \Rightarrow$
$(x+1)^2+(y-1)^2=0$
This looks like the equation of a circle; however the radius is 0, so it is the graph of the point (–1, 1).

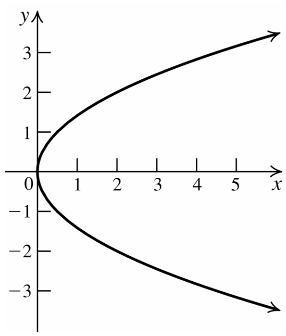


c. $x^2+y^2+2x-2y-7=0 \Rightarrow$
$x^2+2x+y^2-2y=7 \Rightarrow$
$x^2+2x+1+y^2-2y+1=7+1+1 \Rightarrow$
$(x+1)^2+(y-1)^2=9$
This is the equation of a circle with center (–1, 1) and radius 3.



2.2 Critical Thinking

91. The graph of $y^2=2x$ is the union of the graphs of $y=\sqrt{2x}$ and $y=-\sqrt{2x}$.

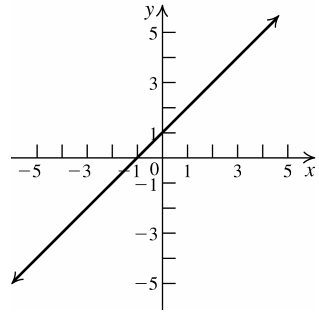


92. Let (*x*, *y*) be a point on the graph. Since the graph is symmetric with regard to the *x*-axis, then the point (*x*, –*y*) is also on the graph. Because the graph is symmetric with regard to the *y*-axis, the point (–*x*, *y*) is also on the graph. Therefore the point (–*x*, –*y*) is on the graph, and the graph is symmetric with respect to the origin. The graph of $y=x^3$ is an example of a graph that is symmetric with respect to the origin but is not symmetric with respect to the *x*- and *y*-axes.

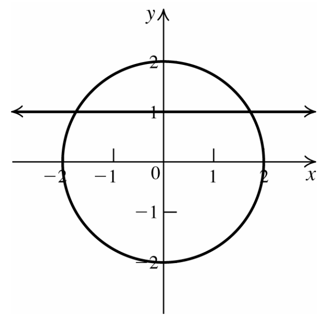
93. False. Setting *x* = 0 and solving for *y* gives the *y*-intercepts.

94. One equation is $y=-(x+2)(x-3)$.

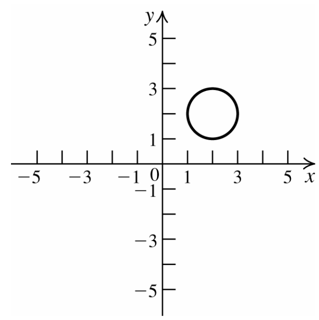
95.a.



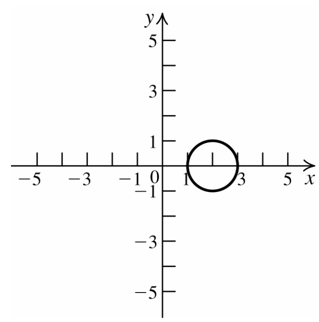
b.



c.



d.



Group Projects

1.a. First find the radius of the circle:
$d(A,B)=\sqrt{(6-0)^2+(8-1)^2}=\sqrt{85} \Rightarrow$
$r=\dfrac{\sqrt{85}}{2}$. The center of the circle is
$\left(\dfrac{6+0}{2},\dfrac{1+8}{2}\right)=\left(3,\dfrac{9}{2}\right)$.

So the equation of the circle is

$(x-3)^2+\left(y-\frac{9}{2}\right)^2=\frac{85}{4}$. To find the x-intercepts, set $y = 0$, and solve for x:

$(x-3)^2+\left(0-\frac{9}{2}\right)^2=\frac{85}{4}\Rightarrow(x-3)^2+\frac{81}{4}=\frac{85}{4}\Rightarrow$

$x^2-6x+9=1\Rightarrow x^2-6x+8=0$. The x-intercepts are the roots of this equation.

b. First find the radius of the circle:

$d(A,B)=\sqrt{(a-0)^2+(b-1)^2}=\sqrt{a^2+(b-1)^2}\Rightarrow$

$r=\frac{\sqrt{a^2+(b-1)^2}}{2}$. The center of the circle is $\left(\frac{a+0}{2},\frac{b+1}{2}\right)=\left(\frac{a}{2},\frac{b+1}{2}\right)$ So the equation of the circle is

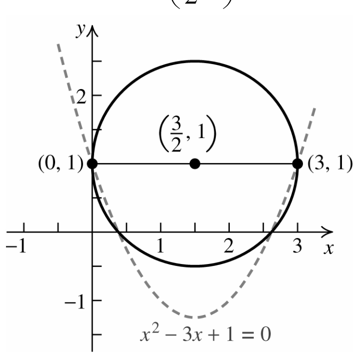
$\left(x-\frac{a}{2}\right)^2+\left(y-\frac{b+1}{2}\right)^2=\frac{a^2+(b-1)^2}{4}$.

To find the x-intercepts, set $y = 0$ and solve for x:

$$\left(x-\frac{a}{2}\right)^2+\left(0-\frac{b+1}{2}\right)^2=\frac{a^2+(b-1)^2}{4}$$
$$x^2-a+\frac{a^2}{4}+\frac{(b+1)^2}{4}=\frac{a^2+(b-1)^2}{4}$$
$$4x^2-4ax+a^2+b^2+2b+1=a^2+b^2-2b+1$$
$$4x^2-4ax+4b=0$$
$$x^2-ax+b=0$$

The x-intercepts are the roots of this equation.

c. $a = 3$ and $b = 1$. Approximate the roots of the equation by drawing a circle whose diameter has endpoints $A(0, 1)$ and $B(3, 1)$. The center of the circle is $\left(\frac{3}{2},1\right)$ and the radius is $\frac{3}{2}$.



2.a. First find the radius of the circle:

$d(A,B)=\sqrt{(10-1)^2+(7-0)^2}=\sqrt{130}\Rightarrow$

$r=\frac{\sqrt{130}}{2}$. The center of the circle is $\left(\frac{10+1}{2},\frac{7+0}{2}\right)=\left(\frac{11}{2},\frac{7}{2}\right)$. So the equation of the circle is

$\left(x-\frac{11}{2}\right)^2+\left(y-\frac{7}{2}\right)^2=\left(\frac{\sqrt{130}}{2}\right)^2$. To find the y-intercepts, set $x = 0$, and solve for y:

$\left(0-\frac{11}{2}\right)^2+\left(y-\frac{7}{2}\right)^2=\frac{130}{4}\Rightarrow\left(y-\frac{7}{2}\right)^2=\frac{9}{4}\Rightarrow$

$y^2-7y+\frac{49}{4}=\frac{9}{4}\Rightarrow y^2-7y+\frac{40}{4}=0\Rightarrow$

$y^2-7y+10=0$. The y-intercepts are the roots of this equation.

b. First find the radius of the circle:

$d(A,B)=\sqrt{(a-1)^2+(b-0)^2}=\sqrt{(a-1)^2+b^2}\Rightarrow$

$r=\frac{\sqrt{(a-1)^2+b^2}}{2}$. The center of the circle is $\left(\frac{a+1}{2},\frac{b+0}{2}\right)=\left(\frac{a+1}{2},\frac{b}{2}\right)$. So the equation of the circle is

$\left(x-\frac{a+1}{2}\right)^2+\left(y-\frac{b}{2}\right)^2=\frac{(a-1)^2+b^2}{4}$. To find the y-intercepts, set $x = 0$ and solve for y:

$$\left(0-\frac{a+1}{2}\right)^2+\left(y-\frac{b}{2}\right)^2=\frac{(a-1)^2+b^2}{4}$$
$$\frac{(a+1)^2}{4}+y^2-b+\frac{b^2}{4}=\frac{(a-1)^2+b^2}{4}$$
$$a^2+2a+1+4y^2-4by+b^2$$
$$=a^2-2a+1+b^2$$
$$4y^2-4by+4a=0$$
$$y^2-by+a=0$$

The y-intercepts are the roots of this equation.

2.3 Lines

2.3 A Exercises: Basic Skills and Concepts

1. $m=\frac{7-3}{4-1}=\frac{4}{3}$; the graph is rising.

2. $m = \frac{0-4}{2-0} = \frac{-4}{2} = -2$; the graph is falling.

3. $m = \frac{3-(-2)}{-2-3} = \frac{5}{-5} = -1$; the graph is falling.

4. $m = \frac{7-(-4)}{-3-(-3)} = \frac{11}{0} \Rightarrow$ slope is undefined; the graph is vertical.

5. $m = \frac{-3-(-2)}{2-3} = \frac{-1}{-1} = 1$; the graph is rising.

6. $m = \frac{-3.5-2}{3-0.5} = \frac{-5.5}{2.5} = -2.2$; the graph is falling.

7. $m = \frac{5-1}{\left(1+\sqrt{2}\right)-\sqrt{2}} = \frac{4}{1} = 4$; the graph is rising.

8. $m = \frac{3\sqrt{3}-0}{\left(1+\sqrt{3}\right)-\left(1-\sqrt{3}\right)} = \frac{3\sqrt{3}}{2\sqrt{3}} = \frac{3}{2}$; the graph is rising.

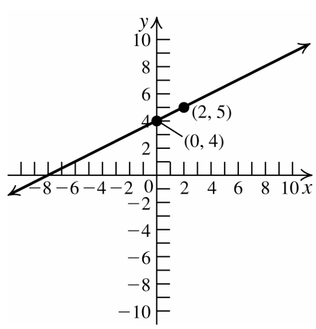
9.a. ℓ_3 **b.** ℓ_2 **c.** ℓ_4 **d.** ℓ_1

10. ℓ_1 has slope 1; ℓ_2 has slope 0; ℓ_3 has slope 2; ℓ_4 has slope $-\frac{4}{3}$.

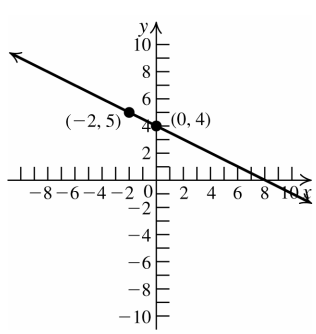
11.a. $y = 0$ **b.** $x = 0$

12.a. $y = -4$ **b.** $x = 5$

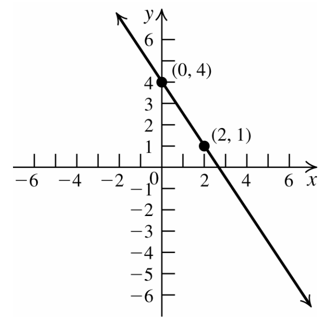
13. $y = \frac{1}{2}x + 4$



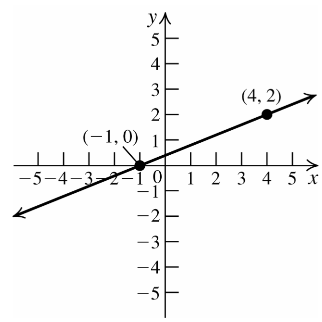
14. $y = -\frac{1}{2}x + 4$



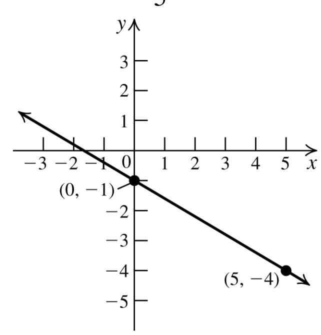
15. $y - 1 = -\frac{3}{2}(x-2) \Rightarrow y - 1 = -\frac{3}{2}x + 3 \Rightarrow$
$y = -\frac{3}{2}x + 4$



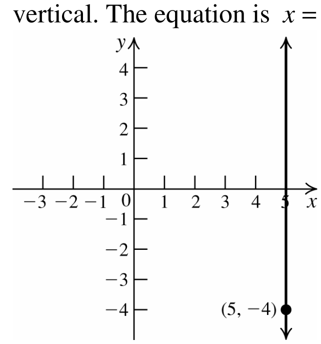
16. $y = \frac{2}{5}(x+1) \Rightarrow y = \frac{2}{5}x + \frac{2}{5}$



17. $y + 4 = -\frac{3}{5}(x-5) \Rightarrow y + 4 = -\frac{3}{5}x + 3 \Rightarrow$
$y = -\frac{3}{5}x - 1$



18. Because the slope is undefined, the graph is vertical. The equation is $x = 5$.



19. $m = \frac{0-1}{1-0} = -1$. The y-intercept is (0, 1), so the equation is $y = -x + 1$.

20. $m = \frac{3-1}{1-0} = 2$. The y-intercept is (0, 1), so the equation is $y = 2x + 1$.

21. $m = \frac{3-3}{3-(-1)} = 0$ Because the slope = 0, the line is horizontal. Its equation is $y = 3$.

22. $m = \frac{7-1}{2-(-5)} = \frac{6}{7}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.

$\frac{6}{7} = \frac{y-1}{x-(-5)} \Rightarrow y - 1 = \frac{6}{7}(x+5) \Rightarrow$

$y - 1 = \frac{6}{7}x + \frac{30}{7} \Rightarrow y = \frac{6}{7}x + \frac{37}{7}$

23. $m = \frac{1-(-1)}{1-(-2)} = \frac{2}{3}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.

$\frac{2}{3} = \frac{y-(-1)}{x-(-2)} \Rightarrow y + 1 = \frac{2}{3}(x+2) \Rightarrow$

$y + 1 = \frac{2}{3}x + \frac{4}{3} \Rightarrow y = \frac{2}{3}x + \frac{1}{3}$

24. $m = \frac{-9-(-3)}{6-(-1)} = -\frac{6}{7}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.

$-\frac{6}{7} = \frac{y-(-3)}{x-(-1)} \Rightarrow y + 3 = -\frac{6}{7}(x+1) \Rightarrow$

$y + 3 = -\frac{6}{7}x - \frac{6}{7} \Rightarrow y = -\frac{6}{7}x - \frac{27}{7}$

25. $m = \frac{2 - \frac{1}{4}}{0 - \frac{1}{2}} = \frac{\frac{7}{4}}{-\frac{1}{2}} = -\frac{7}{2}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.

$-\frac{7}{2} = \frac{y-2}{x-0} \Rightarrow y - 2 = -\frac{7}{2}x \Rightarrow y = -\frac{7}{2}x + 2$

26. $m = \frac{3-(-7)}{4-4} = \frac{10}{0} \Rightarrow$ the slope is undefined. So the graph is a vertical line. The equation is $x = 4$.

27. $x = 5$

28. $y = 1.5$

29. $y = 0$

30. $x = 0$

31. $y = 14$

32. $y = 2x + 5$

33. $y = -\frac{2}{3}x - 4$

34. $y = -6x - 3$

35. $m = \frac{4-0}{0-(-3)} = \frac{4}{3}$; $y = \frac{4}{3}x + 4$

36. $m = \frac{-2-0}{0-(-5)} = -\frac{2}{5}$; $y = -\frac{2}{5}x - 2$

37. $y = 7$

38. $x = 4$

39. $y = -5$

40. $x = -3$

41. Two lines are parallel if their slopes are equal. The lines are perpendicular if the slope of one is the negative reciprocal of the slope of the other.

a. $m_{\ell_2} = \frac{1-5}{3-1} = \frac{-4}{2} = -2 \Rightarrow \ell_2 \parallel \ell_1$

b. $m_{\ell_2} = \frac{4-3}{5-7} = -\frac{1}{2}$. The slope of ℓ_2 is neither equal to the slope of ℓ_1 nor the negative reciprocal of the slope of ℓ_1.

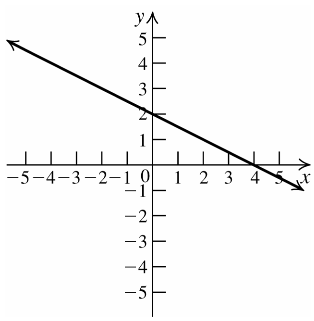
c. $m_{\ell_2} = \frac{4-3}{4-2} = \frac{1}{2} \Rightarrow \ell_2 \perp \ell_1$

42.a. $4 = \frac{y-3}{2-(-1)} \Rightarrow 12 = y - 3 \Rightarrow y = 15$

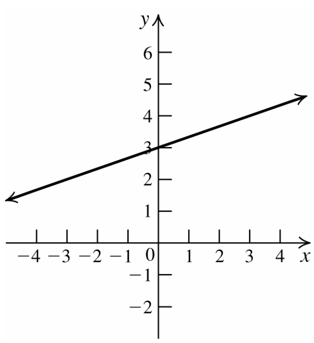
b. $4 = \frac{9-3}{x-(-1)} \Rightarrow 4(x+1) = 6 \Rightarrow x + 1 = \frac{3}{2} \Rightarrow$

$x = \frac{1}{2}$

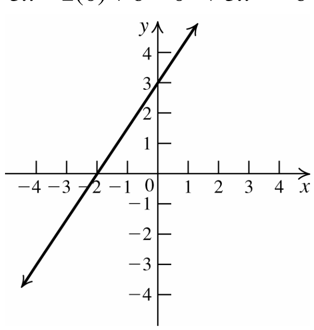
43. $x+2y-4=0 \Rightarrow 2y=-x+4 \Rightarrow y=-\frac{1}{2}x+2$.
The slope is $-1/2$, and the y-intercept is (0, 2). To find the x-intercept, set $y = 0$ and solve for x: $x+2(0)-4=0 \Rightarrow x=4$.



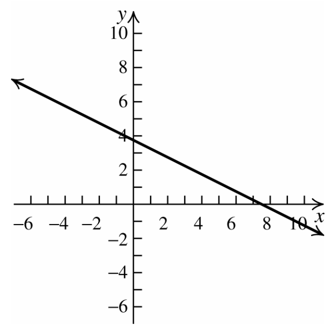
44. $x=3y-9 \Rightarrow x-9=3y \Rightarrow \frac{1}{3}x-3=y$. The slope is $1/3$, and the y-intercept is (0, –3). To find the x-intercept, set $y = 0$ and solve for x: $x=3(0)-9 \Rightarrow x=-9$.



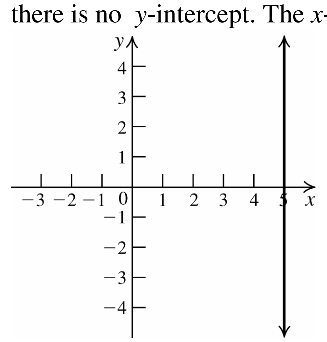
45. $3x-2y+6=0 \Rightarrow 3x+6=2y \Rightarrow \frac{3}{2}x+3=y$.
The slope is $3/2$, and the y-intercept is (0, 3). To find the x-intercept, set $y = 0$ and solve for x: $3x-2(0)+6=0 \Rightarrow 3x=-6 \Rightarrow x=-2$.



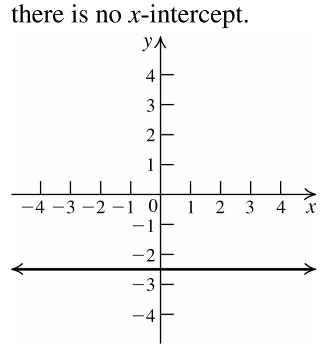
46. $2x=-4y+15 \Rightarrow 2x-15=-4y \Rightarrow$
$-\frac{1}{2}x+\frac{15}{4}=y$. The slope is $-1/2$, and the y-intercept is $15/4$. To find the x-intercept, set $y = 0$ and solve for x: $2x=-4(0)+15 \Rightarrow$
$x=11/2$.



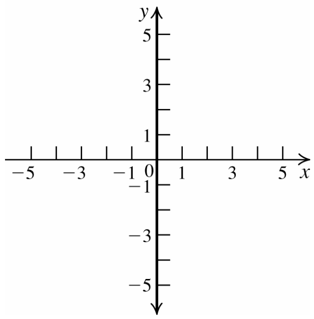
47. $x-5=0 \Rightarrow x=5$. The slope is undefined, and there is no y-intercept. The x-intercept is 5.



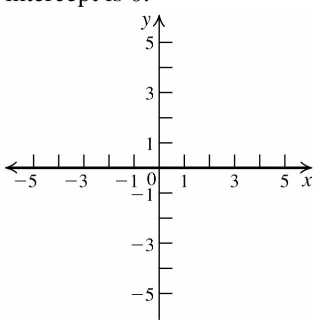
48. $2y+5=0 \Rightarrow y=-\frac{5}{2}$. The slope is 0, and the y-intercept is $-5/2$. This is a horizontal line, so there is no x-intercept.



49. $x=0$. The slope is undefined, and the y-intercepts are the y-axis. This is a vertical line whose x-intercept is 0.



50. $y = 0$. The slope is 0, and the x-intercepts are the x-axis. This is a horizontal line whose y-intercept is 0.



51. The slope of the line through $(a, 0)$ and $(0, b)$ is $\frac{b-0}{0-a} = -\frac{b}{a}$. The equation of the line can be written as $y - b = -\frac{b}{a}x \Rightarrow ay - ab = -bx \Rightarrow$

$ay + bx = ab \Rightarrow \frac{ay}{ab} + \frac{bx}{ab} = \frac{ab}{ab} \Rightarrow \frac{y}{b} + \frac{x}{a} = 1$

52. $\frac{x}{4} + \frac{y}{3} = 1$

53. $2x + 3y = 6 \Rightarrow \frac{2x}{6} + \frac{3y}{6} = \frac{6}{6} \Rightarrow \frac{x}{3} + \frac{y}{2} = 1$;
x-intercept = 3; y-intercept = 2

54. $3x - 4y + 12 = 0 \Rightarrow 3x - 4y = -12 \Rightarrow$
$\frac{3x}{-12} - \frac{4y}{-12} = \frac{-12}{-12} \Rightarrow -\frac{x}{4} + \frac{y}{3} = 1$;
x-intercept = –4; y-intercept = 3

55. Let the intercepts be $(a, 0)$ and $(0, a)$. Then the equation of the line is $\frac{x}{a} + \frac{y}{a} = 1$. Now substitute $x = 3$ and $y = -5$ into the equation to solve for a: $\frac{3}{a} - \frac{5}{a} = 1 \Rightarrow 3 - 5 = a \Rightarrow -2 = a$. So the equation of the line is $-\frac{x}{2} - \frac{y}{2} = 1 \Rightarrow$
$-x - y = 2 \Rightarrow -x - 2 = y$.

56. Let the intercepts be $(a, 0)$ and $(0, -a)$. Then the equation of the line is $\frac{x}{a} - \frac{y}{a} = 1$. Now substitute $x = -5$ and $y = -8$ into the equation to solve for a: $-\frac{5}{a} + \frac{8}{a} = 1 \Rightarrow -5 + 8 = a \Rightarrow 3 = a$.
So the equation of the line is $\frac{x}{3} - \frac{y}{3} = 1 \Rightarrow$
$x - y = 3 \Rightarrow x - 3 = y$.

57. $m = \frac{9-4}{7-2} = \frac{5}{5} = 1$. The equation of the line through (2, 4) and (7, 9) is $y - 4 = 1(x - 2) \Rightarrow$ $y = x + 2$. Check to see if (–1, 1) satisfies the equation by substituting $x = -1$ and $y = 1$:
$1 = -1 + 2 \Rightarrow 1 = 1$. So (–1, 1) lies on the line.

58. $m = \frac{-3-2}{2-7} = \frac{-5}{-5} = 1$. The equation of the line through (7, 2) and (2, –3) is $y - 2 = 1(x - 7) \Rightarrow$ $y = x - 5$. Check to see if (5, 1) satisfies the equation by substituting $x = 5$ and $y = 1$:
$1 = 5 - 5 \Rightarrow 1 \neq 0$. So (5, 1) does not lie on the line.

59. Both lines are vertical lines. The lines are parallel.

60. $x = 0$ is the equation of the y-axis. $y = 0$ is the equation of the x-axis. The lines are perpendicular.

61. The slope of $2x + 3y = 7$ is $-2/3$, while $y = 2$ is a horizontal line. The lines are neither parallel nor perpendicular.

62. The slope of $y = 3x + 1$ is 3. The slope of $6y + 2x = 0$ is $-1/3$. The lines are perpendicular.

63. The slope of $10x + 2y = 3$ is -5. The slope of $y + 1 = -5x$ is also -5, so the lines are parallel.

64. The slope of $4x + 3y = 1$ is $-4/3$, while the slope of $3 + y = 2x$ is 2. The lines are neither parallel nor perpendicular.

65. The slope of $3x + 8y = 7$ is $-3/8$, while the slope of $5x - 7y = 0$ is $5/7$. The lines are neither parallel nor perpendicular.

66. The slope of $x = 4y + 8$ is $1/4$. The slope of $y = -4x + 1$ is -4, so the lines are perpendicular.

67. The slope of $x + y = 1$ is -1. The lines are parallel, so they have the same slope. The equation of the line through (1, 1) with slope -1 is $y - 1 = -(x - 1) \Rightarrow y - 1 = -x + 1 \Rightarrow y = -x + 2$.

68. The slope of $y = 6x + 5$ is 6. The lines are parallel, so they have the same slope. The equation of the line with slope 6 and *y*-intercept -2 is $y = 6x - 2$.

69. The slope of $3x - 9y = 18$ is $1/3$. The lines are perpendicular, so the slope of the new line is -3. The equation of the line through (–2, 4) with slope -3 is $y - 4 = -3(x - (-2)) \Rightarrow y - 4 = -3x - 6 \Rightarrow y = -3x - 2$.

70. The slope of $-2x + y = 14$ is 2. The lines are perpendicular, so the slope of the new line is $-1/2$. The equation of the line through (0, 0) with slope $-1/2$ is $y - 0 = -\frac{1}{2}(x - 0) \Rightarrow y = -\frac{1}{2}x$.

71. The slope of the line $y = 6x + 5$ is 6. The lines are perpendicular, so the slope of the new line is $-1/6$. The equation of the line with slope $-1/6$ and *y*-intercept 4 is $y = -\frac{1}{6}x + 4$.

72. The slope of $-2x + 3y - 7 = 0$ is $2/3$. The lines are parallel, so they have the same slope. The equation of the line through (1, 0) with slope $2/3$ is $y - 0 = \frac{2}{3}(x - 1) \Rightarrow y = \frac{2}{3}x - \frac{2}{3}$.

2.3 B Exercises: Applying the Concepts

73. slope $= \frac{\text{rise}}{\text{run}} \Rightarrow \frac{4}{40} = \frac{1}{10}$

74. 4 miles = 21,120 feet. slope $= \frac{\text{rise}}{\text{run}} \Rightarrow \frac{2000}{21{,}120} = \frac{25}{264}$

75.a. x = the number of weeks; y = the amount of money in the account; $y = 7x + 130$

b. The slope is the amount of money deposited each week; the *y*-intercept is the initial deposit.

76.a. x = the number of sessions of golf; y = the yearly payment to the club; $y = 35x + 1000$

b. The slope is the charge per golf session; the *y*-intercept is the yearly membership fee.

77.a. x = the number of hours worked per week; y = the amount earned per week;

$$y = \begin{cases} 11x & x \le 40 \\ 16.5x - 220 & x > 40 \end{cases}$$

To compute the salary when $x > 40$, use the following steps: For 40 hours, Judy earns 40(11) = \$440. The number of overtime hours is x – 40. For those hours, she earns (1.5)(11)(x – 40) = 16.5x – 660. So her total wage is 440 + 16.50x – 660 = 16.5x – 220.

b. The slope is the hourly wage; the *y*-intercept is the wage for 0 hours of work.

78.a. x = the number of months owed to pay off the refrigerator; y = the amount owed; $y = -15x + 600$

b. The slope is the amount paid each month; the *y*-intercept is the initial amount owed.

79.a. x = the number of rupees; y = the number of dollars equal to x rupees; $y = 44x$

b. x = the number of dollars; y = the number of rupees equal to x dollars; $y = \frac{1}{44}x$

80.a. x = the number of years after 2004; y = the life expectancy of a female born in the year 2004 + x; $y = 0.27x + 82.3$

b. The slope is the rate of increase in life expectancy; the *y*-intercept is the life expectancy in 2004.

81.a. x = the number of TV sets; y = the cost of production for x TV sets; $y = 150x + 10{,}000$

The slope $= \frac{21{,}250 - 17{,}500}{75 - 50} = \frac{3750}{25} = 150$.
Find the *y*-intercept by solving $17{,}500 = 150(50) + b$ for b. The *y*-intercept = 10,000.

b. The slope is the marginal cost per TV set; the *y*-intercept is the fixed cost.

82.a. x = the demand for a product; y = the price per unit when the demand is x;
$y = -0.1x + 100$
The two points are (0, 100) and (1000, 0).
The slope = $\frac{100-0}{0-1000} = -\frac{1}{10} = -0.1$. The y-intercept is 100.

b. The slope is the rate of increase in the price; the y-intercept is the price per unit when the demand is 0.

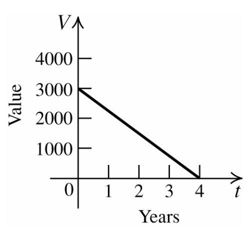
83.a. $v = -1400(2) + 14,000 = \$11,200$

b. $v = -1400(6) + 14,000 = \$5600$
To find when the tractor will have no value, set $v = 0$ and solve the equation for t:
$0 = -1400t + 14,000 \Rightarrow t = 10$

84.a. The amount depreciated per year is $3000/4 = \$750$.

b. $V = -750t + 3000$

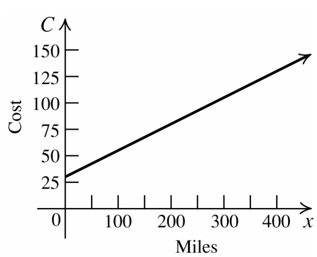
c.



85. $y = 1.5x + 1000$

86.a. $y = 0.25x + 30$

b.

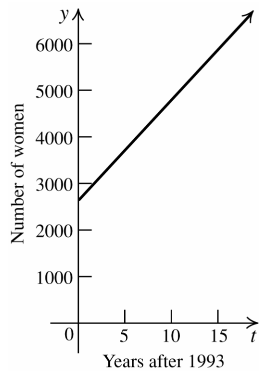


c. $y = 0.25(60) + 30 = \$45$

d. $47.75 = 0.25x + 30 \Rightarrow x = 71$ miles

87.a. The points are (0, 2638) and (10, 4796). So the slope is $\frac{4796-2638}{10} = 215.8$. The equation is $y - 2638 = 215.8(t-0) \Rightarrow$
$y = 215.8t + 2638$

b.



c. The year 2000 is represented by $t = 7$. So $y = 215.8(7) + 2638 \Rightarrow y = 4148.6$. Because there cannot be a fraction of a person, round up to 4149.

d. The year 2008 is represented by $t = 15$. So $y = 215.8(15) + 2638 \Rightarrow y = 5875$

88.a. The two points are (100, 212) and (0, 32). So the slope is $\frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$. The equation is
$F - 32 = \frac{9}{5}(C-0) \Rightarrow F = \frac{9}{5}C + 32$

b. One degree Celsius change in the temperature equals $9/5$ degrees change in degrees Fahrenheit.

c.

C	$F = \frac{9}{5}C + 32$
40°C	104°F
25° C	77° F
−5° C	23° F
−10° C	14° F

d. $100°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 37.78°\text{C}$

$90°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 36.22°\text{C}$

$-10°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = -23.33°\text{C}$

$-20°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = -28.89°\text{C}$

e. $97.6°F = \frac{9}{5}C + 32 \Rightarrow C = 34.44°C$;

$99.6°F = \frac{9}{5}C + 32 \Rightarrow C = 37.56°C$

f. Let x = °F = °C. Then $x = \frac{9}{5}x + 32 \Rightarrow$

$-\frac{4}{5}x = 32 \Rightarrow x = -40$. At –40°, °F = °C.

89.a. The two points are (4, 480) and (4.5, 400). So the slope is $\frac{400-480}{4.5-4} = -\frac{80}{0.5} = -160$. The equation is $q - 480 = -160(p-4) \Rightarrow$ $q = -160p + 1120$.

b. $q = -160(5) + 1120 = 320$

90.a. The two points are (50, 70) and (60, 100). So the slope is $\frac{100-70}{60-50} = \frac{30}{10} = 3$. The equation is $q - 100 = 3(p-60) \Rightarrow q = 3p - 80$.

b. $q = 3(62) - 80 = 106$

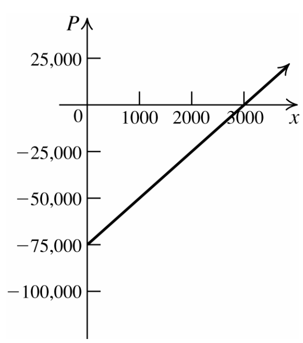
91. The change in the compound is 1 milligram in 2 years, so the slope is 0.5. Because the initial amount is 7 mg, the y-intercept is 7. So, the equation is $y = 0.5x + 7$. The year 2010 is represented by $x = 8$. In 2010, there will be $y = 0.5(8) + 7 \Rightarrow y = 11$ mg of the pollutant per 100 liters of water.

92. a. $R = 50x; C = 25x + 75{,}000$

$P = 50x - (25x + 75{,}000) \Rightarrow$

$P = 25x - 75{,}000$

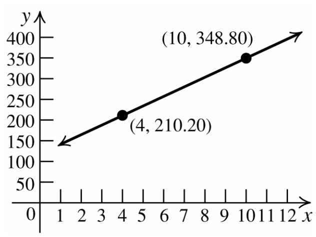
b.



c. The slope is 25. It represents the rate of increase in the profit per pair of shoes sold.

d. The y-intercept is –75,000. It represents the loss if 0 pairs of shoes are sold. The x-intercept is 3000. It represents the number of shoes that need to be sold to break even.

93.a. The two points are (4, 210.20) and (10, 348.80). So the slope is $\frac{348.80-210.20}{10-4} = \frac{138.6}{6} = 23.1$. The equation is $y - 348.8 = 23.1(x-10) \Rightarrow$ $y = 23.1x + 117.8$



b. The slope represents the cost of producing one modem. The y-intercept represents the fixed cost.

c. $y = 23.1(12) + 117.8 \Rightarrow y = \$395$

94.a. The two points are (5, 5.73) and (8, 6.27). The slope is $\frac{6.27-5.73}{8-5} = \frac{0.54}{3} = 0.18$. The equation is $y - 5.73 = 0.18(x-5) \Rightarrow$ $y = 0.18x + 4.83$.

b. The slope represents the monthly change in the number of viewers. The y-intercept represents the number of viewers when the show first started.

c. $y = 0.18(11) + 4.83 \Rightarrow y = 6.81$ million

95. The independent variable t represents the number of years after 1990, with $t = 0$ representing 1990. The two points are (0, 9.4) and (9, 11.2). So the slope is $\frac{11.2-9.4}{9} = 0.2$. The equation is $p - 9.4 = 0.2(t-0) \Rightarrow$ $p = 0.2t + 9.4$. The year 2010 is represented by $t = 20$. $p = 0.2(20) + 9.4 \Rightarrow p = 13.4\%$.

96. The two points are (0, 68.200) and (4, 73.215). So the slope is $\frac{73.215-68.200}{4} = \frac{5.015}{4} =$ 1.25375. The equation is $c = 1.25375t + 68.200$.

2.3 C Exercises: Beyond the Basics

97. $3 = \frac{c-3}{1-(-2)} \Rightarrow 9 = c - 3 \Rightarrow 12 = c$

98. First write the equation in slope-intercept form:

$$3x - cy - 2 = 0 \Rightarrow -cy = -3x + 2 \Rightarrow y = \frac{3}{c}x - \frac{2}{c}$$

Now solve for c by setting the y-intercept from the equation equal to –4: $-\frac{2}{c} = -4 \Rightarrow c = \frac{1}{2}$

99.a. Let $A = (0, 1)$, $B = (1, 3)$, $C = (-1, -1)$.

$$m_{AB} = \frac{3-1}{1-0} = 2; m_{BC} = \frac{-1-3}{-1-1} = \frac{-4}{-2} = 2$$

$m_{AC} = \frac{-1-1}{-1-0} = 2$. The slopes of the three segments are the same, so the points are collinear.

b. $d(A, B) = \sqrt{(3-1)^2 + (1-0)^2} = \sqrt{5}$

$d(B, C) = \sqrt{(-1-3)^2 + (-1-1)^2} = 2\sqrt{5}$

$d(A, C) = \sqrt{(-1-1)^2 + (-1-0)^2} = \sqrt{5}$

Because $d(B, C) = d(A, B) + d(A, C)$, the three points are collinear.

100.a. Let $A = (1, 2)$, $B = (-1, 4)$, $C = (2, 1)$.

$$m_{AB} = \frac{4-2}{-1-1} = -1; m_{BC} = \frac{2-(-1)}{1-4} = -1$$

$m_{AC} = \frac{1-2}{2-1} = -1$. The slopes of the three segments are the same, so the points are collinear.

b. $d(A, B) = \sqrt{(-1-1)^2 + (4-2)^2} = 2\sqrt{2}$

$d(B, C) = \sqrt{(2-(-1))^2 + (1-4)^2} = 3\sqrt{2}$

$d(A, C) = \sqrt{(2-1)^2 + (1-2)^2} = \sqrt{2}$

Because $d(B, C) = d(A, B) + d(A, C)$, the three points are collinear.

101.a. Let $A = (1, 2)$, $B = (0, -3)$, $C = (-1, -8)$.

$$m_{AB} = \frac{-3-2}{0-1} = 5; m_{BC} = \frac{-8-(-3)}{-1-0} = 5$$

$m_{AC} = \frac{-8-2}{-1-1} = 5$. The slopes of the three segments are the same, so the points are collinear.

b. $d(A, B) = \sqrt{(0-1)^2 + (-3-2)^2} = \sqrt{26}$

$d(B, C) = \sqrt{(-1-0)^2 + (-8-(-3))^2} = \sqrt{26}$

$d(A, C) = \sqrt{(-1-1)^2 + (-8-2)^2} = 2\sqrt{26}$

Because $d(A, C) = d(A, B) + d(B, C)$, the three points are collinear.

102.a. Let $A = (1, 0.5)$, $B = (2, 0)$, $C = (0.5, 0.75)$.

$$m_{AB} = \frac{0-0.5}{2-1} = -0.5; m_{BC} = \frac{0.75-0}{0.5-2} = -0.5$$

$m_{AC} = \frac{0.75-0.5}{0.5-1} = -0.5$. The slopes of the three segments are the same, so the points are collinear.

b. $d(A, B) = \sqrt{(1-2)^2 + \left(\frac{1}{2}-0\right)^2} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$

$d(B, C) = \sqrt{\left(\frac{1}{2}-2\right)^2 + \left(\frac{3}{4}-0\right)^2} = \sqrt{\frac{45}{16}} = \frac{3\sqrt{5}}{4}$

$d(A, C) = \sqrt{\left(\frac{1}{2}-1\right)^2 + \left(\frac{3}{4}-\frac{1}{2}\right)^2} = \sqrt{\frac{5}{16}} = \frac{\sqrt{5}}{4}$

Because $d(B, C) = d(A, B) + d(A, C)$, the three points are collinear.

103.a. $m_{AB} = \frac{4-1}{-1-1} = -\frac{3}{2}; m_{BC} = \frac{8-4}{5-(-1)} = \frac{2}{3}$.

The product of the slopes = –1, so $AB \perp BC$.

b. $d(A, B) = \sqrt{(-1-1)^2 + (4-1)^2} = \sqrt{13}$

$d(B, C) = \sqrt{(5-(-1))^2 + (8-4)^2} = \sqrt{52}$

$d(A, C) = \sqrt{(5-1)^2 + (8-1)^2} = \sqrt{65}$

$(d(A, B))^2 + (d(B, C))^2 = (d(A, C))^2$, so the triangle is a right triangle.

104. $m_{AB} = \frac{2-(-1)}{1-(-4)} = \frac{3}{5}; m_{BC} = \frac{1-2}{3-1} = -\frac{1}{2}$

$m_{CD} = \frac{-2-1}{-2-3} = \frac{3}{5}; m_{AD} = \frac{-2-(-1)}{-2-(-4)} = -\frac{1}{2}$

So, $AB \parallel CD$ and $BC \parallel AD$, and $ABCD$ is a parallelogram.

105. $m_{AB} = \frac{24-9}{-2-(-10)} = \frac{15}{8}; m_{CD} = \frac{16-1}{13-5} = \frac{15}{8}$

$m_{AC} = \frac{1-9}{5-(-10)} = -\frac{8}{15};$

$m_{BD} = \frac{16-24}{13-(-2)} = -\frac{8}{15}$

$AB \parallel CD$ and $AC \parallel BD$, so $ABDC$ is a parallelogram. Furthermore, $AB \perp AC$ and $CD \perp BD$, so $ABDC$ is a rectangle.

$d(A,B)=\sqrt{(-10-(-2))^2+(24-9)^2}=17$

$d(A,C)=\sqrt{(5-(-10))^2+(1-9)^2}=17$

So, $ABDC$ is a square.

106.a. The midpoint of AB is $\left(\frac{2+(-1)}{2},\frac{3+5}{2}\right)=\left(\frac{1}{2},4\right)$. The slope of AB is $\frac{5-3}{-1-2}=-\frac{2}{3}$, so the slope of the perpendicular bisector is $3/2$. The equation of the line through $\left(\frac{1}{2},4\right)$ with slope $\frac{3}{2}$ is $y-4=\frac{3}{2}\left(x-\frac{1}{2}\right)\Rightarrow$ $y=\frac{3}{2}x+\frac{13}{4}$.

b. The midpoint of AB is $\left(\frac{a+b}{2},\frac{a+b}{2}\right)$. The slope of AB is $\frac{b-a}{a-b}=-1$, so the slope of the perpendicular bisector is 1. The equation of the line through $\left(\frac{a+b}{2},\frac{a+b}{2}\right)$ with slope 1 is $y-\frac{a+b}{2}=1\left(x-\frac{a+b}{2}\right)\Rightarrow y=x.$

107. The equation of ℓ_1 is $y=m_1x+b_1$ and the equation of ℓ_2 is $y=m_2x+b_2$. Let (x_1,y_1) and (x_2,y_2) be on ℓ_1. If $\ell_1 \parallel \ell_2$, then the distance between them is b_1-b_2. In other words, $(x_1,y_1-(b_1-b_2))$ and $(x_2,y_2-(b_1-b_2))$ are on ℓ_2. So,

$y_1-(b_1-b_2)=m_2x_1+b_2\Rightarrow y_1-b_1=m_2x_1\Rightarrow$ $y_1=m_2x_1+b_1$. However, (x_1,y_1) lies on ℓ_1. So $y_1=m_2x_1+b_1=m_1x_1+b_1\Rightarrow m_2=m_1$.

108. $d(A,O)=\sqrt{x^2+(m_1x)^2}=\sqrt{x^2+m_1{}^2x^2}$.

$d(B,O)=\sqrt{x^2+(m_2x)^2}=\sqrt{x^2+m_2{}^2x^2}$.

$$d(A,B)=\sqrt{(x-x)^2+(m_2x-m_1x)^2}=\sqrt{(m_2x-m_1x)^2}.$$

Apply the Pythagorean theorem to obtain

$$\left(\sqrt{(m_2x-m_1x)^2}\right)^2=\left(\sqrt{x^2+m_1{}^2x^2}\right)^2+\left(\sqrt{x^2+m_2{}^2x^2}\right)^2$$

$$(m_2x-m_1x)^2=x^2+m_1{}^2x^2+x^2+m_2{}^2x^2$$

$$m_2{}^2x^2-2m_1m_2x^2+m_1{}^2x^2=x^2+m_1{}^2x^2+x^2+m_2{}^2x^2$$

$$x^2(m_2-m_1)^2=x^2(m_1{}^2+1)+x^2(m_2{}^2+1)$$

$$m_2{}^2-2m_1m_2+m_1{}^2=m_1{}^2+m_2{}^2+2$$

$$-2m_1m_2=2\Rightarrow m_1m_2=-1$$

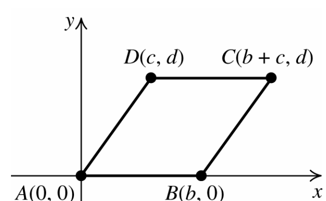
So, $\triangle AOB$ is a right triangle.

109. Let the quadrilateral $ABCD$ be such that $AB\cong CD$ and $AB\parallel CD$. Locate the points are as shown in the figure. Because $AB\parallel CD$, the y-coordinates of C and D are equal. Because $AB\cong CD$, the x-coordinates of the points are as shown in the figure. The slope of AD is d/c.

The slope of BC is $\frac{d-0}{b+c-b}=\frac{d}{c}$. So $AD\parallel BC$.

$d(A,D)=\sqrt{d^2+c^2}$.

$d(B,C)=\sqrt{d^2+((b+c)-b)^2}=\sqrt{d^2+c^2}$. So $AD\cong BC$.



110. Let $A(x_1,y_1),B(x_2,y_2),C(x_3,y_3)$ and $D(x_4,y_4)$ be the vertices of the quadrilateral. Then the midpoint M_1 of AB is $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$; the midpoint M_2 of BC is $\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2}\right)$; the midpoint M_3 of CD is $\left(\frac{x_3+x_4}{2},\frac{y_3+y_4}{2}\right)$; and the midpoint M_4 of AD is $\left(\frac{x_1+x_4}{2},\frac{y_1+y_4}{2}\right)$.

The slope of M_1M_2 is $\left(\frac{y_1+y_2}{2}-\frac{y_2+y_3}{2}\right)\Big/\left(\frac{x_1+x_2}{2}-\frac{x_2+x_3}{2}\right)=$ $\frac{y_1-y_3}{x_1-x_3}$. The slope of M_2M_3 is $\left(\frac{y_2+y_3}{2}-\frac{y_3+y_4}{2}\right)\Big/\left(\frac{x_2+x_3}{2}-\frac{x_3+x_4}{2}\right)=$ $\frac{y_2-y_4}{x_2-x_4}$. The slope of M_3M_4 is $\left(\frac{y_3+y_4}{2}-\frac{y_1+y_4}{2}\right)\Big/\left(\frac{x_3+x_4}{2}-\frac{x_1+x_4}{2}\right)=$ $\frac{y_3-y_1}{x_3-x_1}=\frac{y_1-y_3}{x_1-x_3}$. The slope of M_1M_4 is $\left(\frac{y_1+y_2}{2}-\frac{y_1+y_4}{2}\right)\Big/\left(\frac{x_1+x_2}{2}-\frac{x_1+x_4}{2}\right)=$ $\frac{y_2-y_4}{x_2-x_4}$. So $M_1M_2 \parallel M_3M_4$ and $M_2M_3 \parallel M_1M_4$, and $M_1M_2M_3M_4$ is a parallelogram.

111. Let (*x*, *y*) be the coordinates of point *B*. Then $d(A,B)=12.5=\sqrt{(x-2)^2+(y-2)^2} \Rightarrow$ $(x-2)^2+(y-2)^2=156.25$ and $m_{AB}=\frac{4}{3}=\frac{y-2}{x-2} \Rightarrow 4(x-2)=3(y-2) \Rightarrow$ $y=\frac{4}{3}x-\frac{2}{3}$. Substitute this into the first equation and solve for *x*:

$$(x-2)^2+\left(\left(\frac{4}{3}x-\frac{2}{3}\right)-2\right)^2=156.25$$
$$(x-2)^2+\left(\frac{4}{3}x-\frac{8}{3}\right)^2=156.25$$
$$x^2-4x+4+\frac{16}{9}x^2-\frac{64}{9}x+\frac{64}{9}=156.25$$
$$9x^2-36x+36+16x^2-64x+64=1406.25$$
$$25x^2-100x-1306.25=0$$

Solve this equation using the quadratic formula:

$$x=\frac{100\pm\sqrt{100^2-4(25)(-1306.25)}}{2(25)}$$
$$=\frac{100\pm\sqrt{10,000+130,625}}{50}$$
$$=\frac{100\pm\sqrt{140,625}}{50}=\frac{100\pm 375}{50}$$
$$=9.5 \text{ or } -5.5$$

Now find *y* by substituting the *x*-values into the slope formula: $\frac{4}{3}=\frac{y-2}{9.5-2} \Rightarrow y=12$ or $\frac{4}{3}=\frac{y-2}{-5.5-2} \Rightarrow y=-8$. So the coordinates of *B* are (9.5, 12) or (–5.5, –8).

112. Let (*x*, *y*) be a point on the circle with (x_1, y_1) and (x_2, y_2) as the endpoints of a diameter. Then the line that passes through (*x*, *y*) and (x_1, y_1) is perpendicular to the line that passes through (*x*, *y*) and (x_2, y_2), and their slopes are negative reciprocals. So $\frac{y-y_1}{x-x_1}=-\frac{x-x_2}{y-y_2} \Rightarrow$
$(y-y_1)(y-y_2)=-(x-x_1)(x-x_2) \Rightarrow$
$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0.$

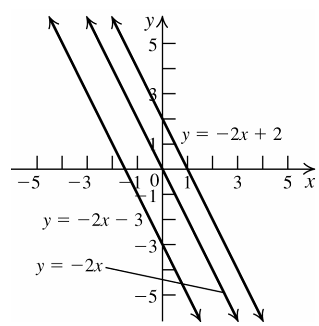
2.3 Critical Thinking

113. Let *A* = (1, –1), *B* = (–2, 5), *C* = (3, –5).
$M_{AB}=\frac{5-(-1)}{-2-1}=-2; M_{BC}=\frac{-5-5}{3-(-2)}=-2;$

$M_{AC}=\frac{-5-(-1)}{3-1}=-2$. The slopes are the same, so the points are collinear.

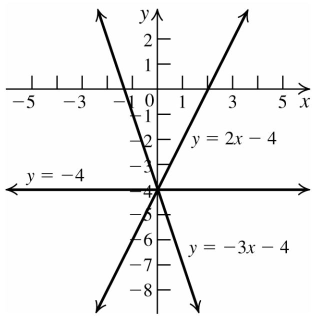
114. Let *A* = (–9, 6), *B* = (–2, 14), *C* = (–1, –1).
$M_{AB}=\frac{14-6}{-2-(-9)}=\frac{8}{7}; M_{BC}=\frac{-1-14}{-1-(-2)}=-15;$
$M_{AC}=\frac{-1-6}{-1-(-9)}=-\frac{7}{8}$. $M_{AB}M_{AC}=-1$, so the triangle is a right triangle.

115.a.



This is a family of lines parallel to the line $y=-2x$. They all have slope –2.

b.



This is a family of lines that passes through the point (0, –4). Their y-intercept is –4.

Section 2.4 Relations and Functions

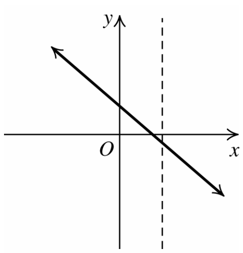
2.4 A Exercises: Basic Skills and Concepts

1. Domain: {–2, 0, 2}; range: {0, 2}; function
2. Domain: {3, 4, 5, 6}; range: {1}; function
3. Domain: {–2, 0, 1, 2}; range: {–2, 0, 1, 2}; function
4. Domain: {4, 9}; range: {–3, –2, 2, 3}; not a function
5. Domain: {a, b, c}; range: {d, e}; function
6. Domain: {a, b, c}; range: {d, e, f}; function
7. Domain: {a, b, c}; range: {1, 2}; function
8. Domain: {1, 2, 3}; range: {a, b, c, d}; not a function
9. Domain: {–2, –1, 1, 2, 3}; range: {–2, 1, 2}; function
10. Domain: {–2, –1, 1, 3, 4}; range: {–4, –2, 1, 2, 4}; not a function
11. Domain: {–3, –1, 0, 1, 2, 3}; range: {–8, –3, 0, 1}; function
12. Domain: {0, 3, 8}; range:{ –3, –2, –1, 1, 2}; not a function
13. $x+y=2 \Rightarrow y=-x+2$; a function
14. $x=y-1 \Rightarrow y=x+1$; a function
15. $y=\dfrac{1}{x}$; a function
16. $xy=-1 \Rightarrow y=-\dfrac{1}{x}$; a function
17. $y=|x-1|$; a function
18. $x=|y| \Rightarrow y=x \cup y=-x$; not a function
19. $y=\dfrac{1}{\sqrt{2x-5}}$; a function
20. $y=\dfrac{1}{\sqrt{x^2-1}}$; a function
21. $2-y=3x \Rightarrow y=2-3x$; a function
22. $3x-5y=15 \Rightarrow y=\dfrac{3}{5}x-3$; a function
23. $x^2+y=8 \Rightarrow y=-x^2+8$; a function
24. $x=y^2 \Rightarrow y=\sqrt{x} \cup y=-\sqrt{x}$; not a function
25. $x^2+y^3=5 \Rightarrow y=\sqrt[3]{5-x^2}$; a function
26. $x+y^3=8 \Rightarrow y=\sqrt[3]{8-x}$; a function
27. $(-\infty,\infty)$ **28.** $(-\infty,\infty)$
29. The denominator is not defined for x = 9. The domain is $(-\infty,9)\cup(9,\infty)$
30. The denominator is not defined for x = –9. The domain is $(-\infty,-9)\cup(-9,\infty)$
31. The denominator is not defined for x = –1 or x = 1. The domain is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$
32. The denominator is not defined for x = –2 or x = 2. The domain is $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$
33. The numerator is not defined for x < 3, and the denominator is not defined for x = –2. The domain is $[3,\infty)$
34. The numerator is not defined for x < –3, and the denominator is not defined for x = 1. The domain is $[-3,1)\cup(1,\infty)$
35. The denominator is not defined for x > 4. The domain is $(-\infty,4)$
36. The denominator is not defined for x > 2. The domain is $(-\infty,2)$
37. The denominator = 0 if x = –1 or x = –2. The domain is $(-\infty,-2)\cup(-2,-1)\cup(-1,\infty)$.
38. The denominator = 0 if x = –2 or x = –3. The domain is $(-\infty,-3)\cup(-3,-2)\cup(-2,\infty)$.

39. The denominator is not defined for $x = 0$. The domain is $(-\infty, 0) \cup (0, \infty)$

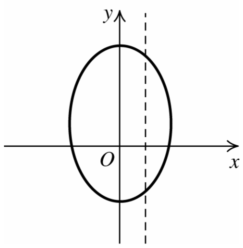
40. The denominator is defined for all values of x. The domain is $(-\infty, \infty)$.

41.



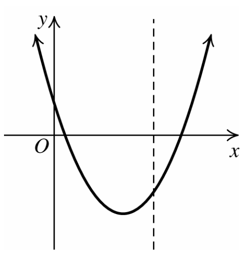
A function

42.



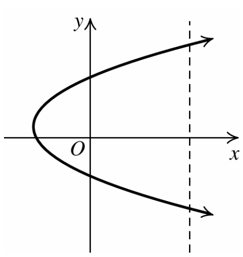
Not a function

43.



A function

44.



Not a function

45. $f(3) = 5; f(5) = 7; f(-1) = 1; f(-4) = -2$

46. $g(-2) = 5; g(1) = -4; g(3) = 0; g(4) = 5$

47. $h(-2) = -5; h(-1) = 4; h(0) = 3; h(1) = 4$

48. $f(-1) = 4; f(0) = 0; f(1) = -4$

49. $f(0) = 0^2 - 3(0) + 1 = 1; g(0) = \dfrac{2}{\sqrt{0}} \Rightarrow$

$g(0)$ is undefined; $h(0) = \sqrt{2-0} = \sqrt{2}$;

$f(a) = a^2 - 3a + 1; f(-x) = (-x)^2 - 3(-x) + 1 =$

$x^2 + 3x + 1$

50. $f(1) = 1^2 - 3(1) + 1 = -1; g(1) = \dfrac{2}{\sqrt{1}} = 2;$

$h(1) = \sqrt{2-1} = 1; g(a) = \dfrac{2}{\sqrt{a}};$

$g(x^2) = \dfrac{2}{\sqrt{x^2}} = \dfrac{2}{|x|}$

51. $f(-1) = (-1)^2 - 3(-1) + 1 = 5;$

$g(-1) = \dfrac{2}{\sqrt{-1}} \Rightarrow g(-1)$ is undefined;

$h(-1) = \sqrt{2-(-1)} = \sqrt{3}; h(c) = \sqrt{2-c};$

$h(-x) = \sqrt{2-(-x)} = \sqrt{2+x}$

52. $f(4) = 4^2 - 3(4) + 1 = 5; g(4) = \dfrac{2}{\sqrt{4}} = 1;$

$h(4) = \sqrt{2-4} = \sqrt{-2} \Rightarrow h(4)$ is undefined;

$g(2+k) = \dfrac{2}{\sqrt{2+k}};$

$f(a+k) = (a+k)^2 - 3(a+k) + 1$

$= a^2 + 2ak + k^2 - 3a - 3k + 1$

53.a. $f(0) = \dfrac{2(0)}{\sqrt{4-0^2}} = 0$

b. $f(1) = \dfrac{2(1)}{\sqrt{4-1^2}} = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

c. $f(2) = \dfrac{2(2)}{\sqrt{4-2^2}} = \dfrac{4}{0} \Rightarrow f(2)$ is undefined

d. $f(-2) = \dfrac{2(-2)}{\sqrt{4-(-2)^2}} = \dfrac{-4}{0} \Rightarrow f(-2)$ is undefined

e. $f(-x) = \dfrac{2(-x)}{\sqrt{4-(-x)^2}} = \dfrac{-2x}{\sqrt{4-x^2}}$

54.a. $g(0) = 2(0) + \sqrt{0^2 - 4} \Rightarrow g(0)$ is undefined

b. $g(1) = 2(1) + \sqrt{1^2 - 4} \Rightarrow g(1)$ is undefined

c. $g(2) = 2(2) + \sqrt{2^2 - 4} = 4$

d. $g(-3) = 2(-3) + \sqrt{(-3)^2 - 4} = -6 + \sqrt{5}$

e. $g(-x) = 2(-x) + \sqrt{(-x)^2 - 4}$
$= -2x + \sqrt{x^2 - 4}$

55.a. $f(x+h) = x + h$

b. $f(x+h) - f(x) = x + h - x = h$

c. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h}{h} = 1$

56.a. $f(x+h) = 3(x+h) + 2 = 3x + 3h + 2$

b. $f(x+h) - f(x) = 3x + 3h + 2 - (3x + 2) = 3h$

c. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{3h}{h} = 3$

57.a. $f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$

b. $f(x+h) - f(x) = x^2 + 2xh + h^2 - x^2$
$= 2xh + h^2$

c. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2xh + h^2}{h} = 2x + h$

58.a. $f(x+h) = (x+h)^2 - (x+h)$
$= x^2 + 2xh + h^2 - x - h$
$= x^2 + 2xh - x + h^2 - h$

b. $f(x+h) - f(x)$
$= x^2 + 2xh - x + h^2 - h - (x^2 - x)$
$= 2xh + h^2 - h$

c. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2xh + h^2 - h}{h} = 2x + h - 1$

59.a. $f(x+h) = 2(x+h)^2 + 3(x+h)$
$= 2x^2 + 4xh + 2h^2 + 3x + 3h$
$= 2x^2 + 4xh + 3x + 2h^2 + 3h$

b. $f(x+h) - f(x)$
$= 2x^2 + 4xh + 3x + 2h^2 + 3h - (2x^2 + 3x)$
$= 4xh + 2h^2 + 3h$

c. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{4xh + 2h^2 + 3h}{h}$
$= 4x + 2h + 3$

60.a. $f(x+h) = 3(x+h)^2 - 2(x+h) + 5$
$= 3x^2 + 6xh + 3h^2 - 2x - 2h + 5$
$= 3x^2 + 6xh - 2x + 3h^2 - 2h + 5$

b. $f(x+h) - f(x) = 3x^2 + 6xh - 2x + 3h^2$
$- 2h + 5 - (3x^2 - 2x + 5)$
$= 6xh + 3h^2 - 2h$

c. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{6xh + 3h^2 - 2h}{h}$
$= 6x + 3h - 2$

61.a. $f(x+h) = 4$

b. $f(x+h) - f(x) = 4 - 4 = 0$

c. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{0}{h} = 0$

62.a. $f(x+h) = -3$

b. $f(x+h) - f(x) = -3 - (-3) = 0$

c. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{0}{h} = 0$

63.a. $f(x+h) = \dfrac{1}{x+h}$

b. $f(x+h) - f(x) = \dfrac{1}{x+h} - \dfrac{1}{x}$
$= \dfrac{x}{x(x+h)} - \dfrac{x+h}{x(x+h)}$
$= -\dfrac{h}{x(x+h)}$

c. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-\frac{h}{x(x+h)}}{h} = -\dfrac{1}{x(x+h)}$

64.a. $f(x+h) = -\dfrac{1}{x+h}$

b. $f(x+h) - f(x) = -\dfrac{1}{x+h} - \left(-\dfrac{1}{x}\right)$
$= -\dfrac{x}{x(x+h)} + \dfrac{x+h}{x(x+h)}$
$= \dfrac{h}{x(x+h)}$

c. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\frac{h}{x(x+h)}}{h} = \dfrac{1}{x(x+h)}$

65. $h(x) = 7$, so solve the equation $7 = x^2 - x + 1$.
$x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0 \Rightarrow x = -2$ or $x = 3$.

66. $H(x) = 7$, so solve the equation $7 = x^2 + x + 8$.

$x^2 + x + 1 = 0 \Rightarrow x = \dfrac{-1 \pm \sqrt{1 - 4(1)(1)}}{2(1)} \Rightarrow$

$x = \dfrac{-1 \pm \sqrt{-3}}{2} \Rightarrow$ there is no real solution.

2.4 B Exercises: Applying the Concepts

67. a function, because there is only one high temperature per day.

68. a function because there is only one cost of a first-class stamp on January 1 each year.

69. not a function because there are several states that begin with N (i.e., New York, New Jersey, New Mexico, Nevada, North Carolina, North Dakota); there are also several states that begin with T and S.

70. not a function because people with the same name may have the same birthday.

71. a function because there is only one winner in each year.

72. not a function because New England won three times.

73. $A(x) = x^2; A(4) = 16;$ $A(4)$ represents the area of a tile with side 4.

74. $V(x) = x^3; V(3) = 27$ in.^{3}; $V(3)$ represents the volume of a cube with edge 3.

75. It is a function. $S(x) = 6x^2; S(3) = 54$

76. $f(x) = \dfrac{x}{39.37}; f(59) \approx 1.5$ meters

77.a. $C(x) = 210x + 10{,}500$

b. $C(50) = 210(50) + 10{,}500 = \$21{,}000$

c. average cost = $\$21{,}000/50 = \$420$

d. $\dfrac{210x + 10{,}500}{x} = 315$

$210x + 10{,}500 = 315x$

$10{,}500 = 105x \Rightarrow x = 100$

78.a. $C(x) = 4x + 20{,}000$

b. $C(12{,}000) = 4(12{,}000) + 20{,}000 = \$68{,}000$

c. average cost = $68{,}000/12{,}000 = \$5.67$

d. $\dfrac{4x + 20{,}000}{x} = 4.5 \Rightarrow 4x + 20{,}000 = 4.5x \Rightarrow$

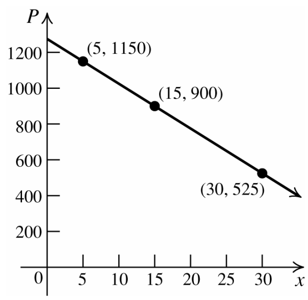
$20{,}000 = 0.5x \Rightarrow x = 40{,}000$

79.a. $p(5) = 1275 - 25(5) = 1150$. If 5000 TVs can be sold, the price per TV is \$1150.

$p(15) = 1275 - 25(15) = 900$. If 15,000 TVs can be sold, the price per TV is \$900.

$p(30) = 1275 - 25(30) = 525$. If 30,000 TVs can be sold, the price per TV is \$525.

b.



c. $650 = 1275 - 25x \Rightarrow -625 = -25x \Rightarrow x = 25$

80.a. $R(x) = (1275 - 25x)x = 1275x - 25x^2$

domain [0, 30]

b. $R(1) = 1275(1) - 25(1^2) = 1250$

$R(5) = 1275(5) - 25(5^2) = 5750$

$R(10) = 1275(10) - 25(10^2) = 10{,}250$

$R(15) = 1275(15) - 25(15^2) = 13{,}500$

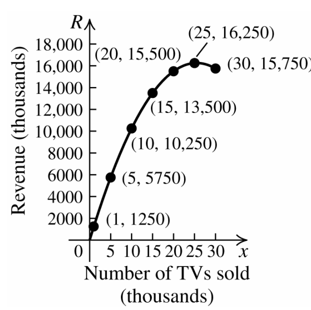
$R(20) = 1275(20) - 25(20^2) = 15{,}500$

$R(25) = 1275(25) - 25(25^2) = 16{,}250$

$R(30) = 1275(30) - 25(30^2) = 15{,}750$

This is the amount of revenue for the given number of TVs sold.

c.



d. $4700 = 1275x - 25x^2 \Rightarrow x^2 - 51x + 188 = 0 \Rightarrow$

$\dfrac{51 \pm \sqrt{51^2 - 4(1)(188)}}{2(1)} = x \Rightarrow x = 4$ or $x = 47$

47 is not in the domain, so 4000 TVs must be sold in order to generate revenue of 4.7 million dollars.

81.a. $C(x) = 5.5x + 75{,}000$

b. $R(x) = 0.6(15)x = 9x$

c. $P(x) = R(x) - C(x) = 9x - (5.5x + 75{,}000)$
$= 3.5x - 75{,}000$

d. The break-even point is when the profit is zero: $3.5x - 75{,}000 = 0 \Rightarrow x = 21{,}429$

e. $P(46{,}000) = 3.5(46{,}000) - 75{,}000$
$= \$86{,}000$

82.a. $C(x) = 0.5x + 500{,}000; R(x) = 5x$. The break-even point is when the profit is zero (when the revenue equals the cost):
$5x = 0.5x + 500{,}000 \Rightarrow 4.5x = 500{,}000 \Rightarrow$
$x = 111{,}111.11$. Because a fraction of a CD cannot be sold, round up to 111,112.

b. $P(x) = R(x) - C(x)$
$750{,}000 = 5x - (0.5x + 500{,}000)$
$1{,}250{,}000 = 4.5x \Rightarrow x = 277{,}778$

83.a. The domain is [0, 8].

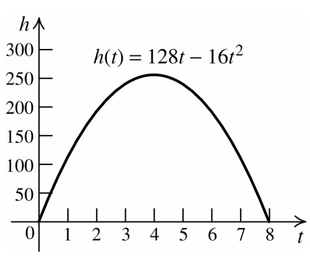
b. $h(2) = 128(2) - 16(2^2) = 192$

$h(4) = 128(4) - 16(4^2) = 256$

$h(6) = 128(6) - 16(6^2) = 192$

c. $0 = 128t - 16t^2 \Rightarrow 0 = 16t(8 - t) \Rightarrow$
$t = 0 \cup t = 8$. It will take 8 seconds for the stone to hit the ground.

d.

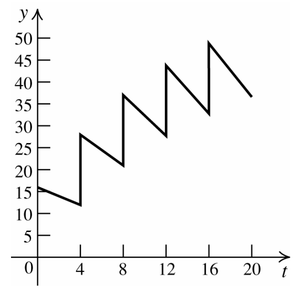


84.a. average rate of change =
$\dfrac{121.8 - 111.1}{2} = 5.35$

b. average rate of change = $\dfrac{182.1 - 97.1}{13} = 6.54$

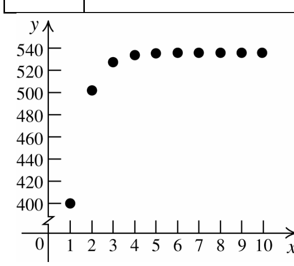
c. Based on (a), 111.1 + 5.35 = 116.45 thousand = $116,450. Based on (b) 111.1+ 6.54 = 117.64 thousand = $117,640

85. After 4 hours, there are (0.75)(16) = 12 ml of the drug. After 8 hours, there are (0.75)(12 + 16) = 21 ml. After 12 hours, there are (0.75)(21 + 16) = 27.75 ml. After 16 hours, there are (0.75)(27.75 + 16) = 32.81 ml. After 20 hours, there are
(0.75)(32.81 + 16) = 36.61 ml.



86.

Day	Maximum Concentration
1	(0.8)(500) = 400 mg
2	(0.25)(400) + (0.8)(500) = 500 mg
3	(0.25)(500) + (0.8)(500) = 525 mg
4	(0.25)(525) + (0.8)(500) = 531.25 mg
5	(0.25)(531.25) + (0.8)(500) = 532.81 mg
6	(0.25)(532.81) + (0.8)(500) = 533.20 mg
7	(0.25)(533.20) + (0.8)(500) = 533.30 mg
8	(0.25)(533.30) + (0.8)(500) = 533.30 mg
9	(0.25)(533.30) + (0.8)(500) = 533.30 mg
10	(0.25)(533.30) + (0.8)(500) = 533.30 mg



2.4 C Exercises: Beyond the Basics

87.a. $f(x) = |x|$ **b.** $f(x) = 0$

c. $f(x) = x^3$

d. $f(x) = \sqrt{-x^2}$ (Note: the point is the origin.)

e. $f(x) = 1$

f. A vertical line is not a function.

88.a. $f(x) \neq g(x)$ because the domain of *f* is $[0, \infty)$ while the domain of *g* is $(-\infty, \infty)$.

b. $f(x)=g(x)$ because $\left(\sqrt[3]{x}\right)^3=x$ for every real number x.

89. $3x-5y=15\Rightarrow y=\dfrac{3}{5}x-3\Rightarrow f(x)=\dfrac{3}{5}x-3$
Domain: $(-\infty,\infty)$; $f(4)=-3/5$.

90. $x=\dfrac{y}{y-1}\Rightarrow xy-x=y\Rightarrow -x=y-xy\Rightarrow$
$-x=y(1-x)\Rightarrow -\dfrac{x}{1-x}=y\Rightarrow \dfrac{x}{x-1}=y\Rightarrow$
$f(x)=\dfrac{x}{x-1}$; Domain: $(-\infty,1)\cup(1,\infty)$.
$f(4)=4/3$.

91. $x=\dfrac{2}{y-4}\Rightarrow xy-4x=2\Rightarrow xy=2+4x\Rightarrow$
$y=\dfrac{4x+2}{x}\Rightarrow f(x)=\dfrac{4x+2}{x}$; Domain: $(-\infty,0)\cup(0,\infty)$. $f(4)=9/2$.

92. $xy-3=2y\Rightarrow 2y-xy=-3\Rightarrow$
$y(2-x)=-3\Rightarrow y=-\dfrac{3}{2-x}\Rightarrow f(x)=\dfrac{3}{x-2}$
Domain: $(-\infty,2)\cup(2,\infty)$. $f(4)=3/2$.

93. $\left(x^2+1\right)y+x=2\Rightarrow y=\dfrac{2-x}{x^2+1}\Rightarrow$
$f(x)=\dfrac{2-x}{x^2+1}$; Domain: $(-\infty,\infty)$; $f(4)=-2/17$.

94. $yx^2-\sqrt{x}=-2y\Rightarrow yx^2+2y=\sqrt{x}\Rightarrow$
$y\left(x^2+2\right)=\sqrt{x}\Rightarrow y=\dfrac{\sqrt{x}}{x^2+2}\Rightarrow f(x)=\dfrac{\sqrt{x}}{x^2+2}$
Domain: $[0,\infty)$; $f(4)=1/9$.

95. $f(x)\neq g(x)$ because they have different domains.

96. $f(x)\neq g(x)$ because they have different domains.

97. $f(x)\neq g(x)$ because they have different domains. $g(x)$ is not defined for $x=-1$, while $f(x)$ is defined for all real numbers.

98. $f(x)\neq g(x)$ because they have different domains. $g(x)$ is not defined for $x=3$, while $f(x)$ is not defined for $x=3$ or $x=-2$.

99. $f(x)=g(x)$ because
$\dfrac{x^2-4}{x-2}=\dfrac{(x-2)(x+2)}{x-2}=x+2$ and $x=2$ is not in the given domain.

100. $f(x)=g(x)$ because
$\dfrac{x-2}{x^2-6x+8}=\dfrac{x-2}{(x-2)(x-4)}=\dfrac{1}{x-4}$ and $x=2$ is not in the given domain.

101. $f(2)=15=a(2^2)+2a-3\Rightarrow 15=6a-3\Rightarrow$
$a=3$.

102. $g(6)=28=6^2+6b+b^2\Rightarrow b^2+6b+8=0\Rightarrow$
$(b-2)(b-4)=0\Rightarrow b=2\cup b=4$.

103. $h(6)=0=\dfrac{3(6)+2a}{2(6)-b}\Rightarrow 0=18+2a\Rightarrow a=-9$
$h(3)$ is undefined $\Rightarrow \dfrac{3(3)+2(-9)}{2(3)-b}$ has a zero in the denominator. So $6-b=0\Rightarrow b=6$.

104. $f(x)=2x-3\Rightarrow f\left(x^2\right)=2x^2-3$
$\left(f(x)\right)^2=(2x-3)^2=4x^2-12x+9$

105. $g(x)=x^2-\dfrac{1}{x^2}\Rightarrow g\left(\dfrac{1}{x}\right)=\dfrac{1}{x^2}-\dfrac{1}{\frac{1}{x^2}}=\dfrac{1}{x^2}-x^2$
$g(x)+g\left(\dfrac{1}{x}\right)=\left(x^2-\dfrac{1}{x^2}\right)+\left(\dfrac{1}{x^2}-x^2\right)=0$

106. $f(x)=\dfrac{x-1}{x+1}\Rightarrow f\left(\dfrac{x-1}{x+1}\right)=\dfrac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}$
$=\dfrac{\frac{(x-1)-(x+1)}{x+1}}{\frac{(x-1)+(x+1)}{x+1}}=-\dfrac{2}{2x}=-\dfrac{1}{x}$

107. $f(x)=\dfrac{x+3}{4x-5}\Rightarrow f(t)=\dfrac{\frac{3+5x}{4x-1}+3}{4\left(\frac{3+5x}{4x-1}\right)-5}$
$=\dfrac{\frac{(3+5x)+3(4x-1)}{4x-1}}{\frac{(12+20x)-(5(4x-1))}{4x-1}}$
$=\dfrac{(3+5x)+(12x-3)}{(12+20x)-(20x+5)}=\dfrac{17x}{17}=x$

2.4 Critical Thinking

108. Answers may vary. Sample answers are given

a. $\sqrt{x-2}$ **b.** $\dfrac{1}{\sqrt{x-2}}$

c. $\sqrt{2-x}$ **d.** $\dfrac{1}{\sqrt{2-x}}$

109.a. $ax^2+bx+c=0$

b. $y=c$

c. The equation will have no x-intercepts if $b^2-4ac<0$.

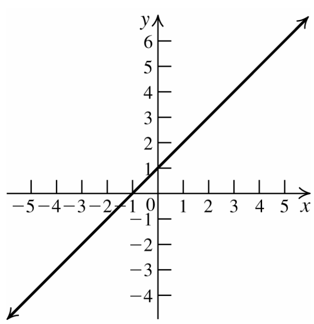
d. It is not possible for the equation to have no y-intercepts because $y=f(x)$.

2.5 A Library of Functions

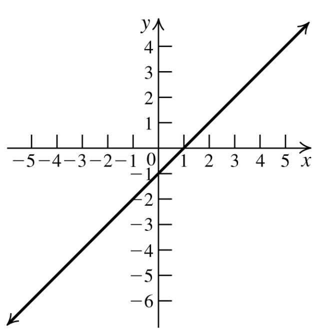
2.5 A Exercises: Basic Skills and Concepts

In exercises 1-10, first find the slope of the line using the two points given. Then substitute the coordinates of one of the points into the point-slope form of the equation to solve for b.

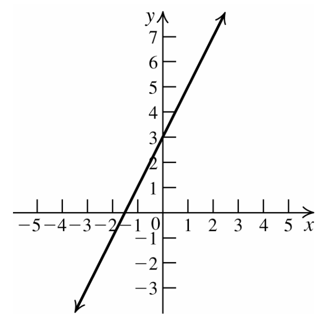
1. The two points are (0, 1) and (–1, 0).
$m=\dfrac{0-1}{-1-0}=1.$ $1=1(0)+b\Rightarrow b=1$.
$f(x)=x+1$



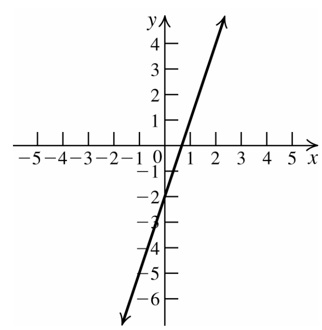
2. The two points are (1, 0) and (2, 1).
$m=\dfrac{1-0}{2-1}=1.$ $0=1+b\Rightarrow b=-1$.
$f(x)=x-1$



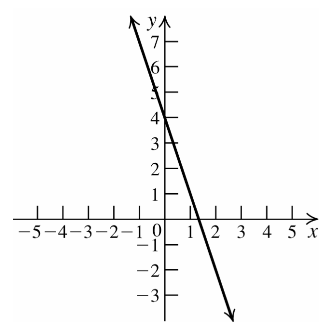
3. The two points are (–1, 1) and (2, 7).
$m=\dfrac{7-1}{2-(-1)}=2.$ $0=1+b\Rightarrow b=-1$.
$f(x)=2x+3$



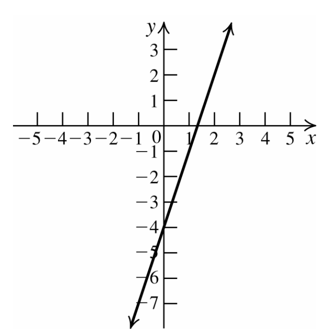
4. The two points are (–1, –5) and (2, 4).
$m=\dfrac{4-(-5)}{2-(-1)}=3.$ $4=3(2)+b\Rightarrow b=-2.$
$f(x)=3x-2.$



5. The two points are (1, 1) and (2, –2).
$m=\dfrac{-2-1}{2-1}=-3.$ $1=-3(1)+b\Rightarrow b=4.$
$f(x)=-3x+4.$



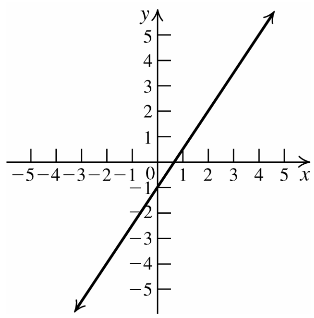
6. The two points are (1, –1) and (3, 5).
$m=\dfrac{5-(-1)}{3-1}=3.$ $-1=3(1)+b\Rightarrow b=-4.$
$f(x)=3x-4.$



7. The two points are (–2, 2) and (2, 4).

$m = \dfrac{4-2}{2-(-2)} = \dfrac{1}{2}.\ \ 4 = \dfrac{1}{2}(2) + b \Rightarrow b = 3.$

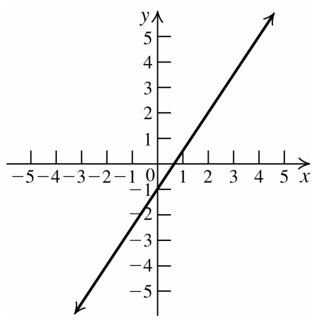
$f(x) = \dfrac{1}{2}x + 3.$



8. The two points are (2, 2) and (4, 5).

$m = \dfrac{5-2}{4-2} = \dfrac{3}{2}.\ \ 2 = \dfrac{3}{2}(2) + b \Rightarrow b = -1.$

$f(x) = \dfrac{3}{2}x - 1.$

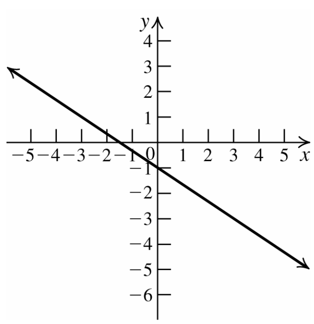


9. The two points are (0, –1) and (3, –3).

$m = \dfrac{-3-(-1)}{3-0} = -\dfrac{2}{3}.$

$-1 = -\dfrac{2}{3}(0) + b \Rightarrow b = -1.$

$f(x) = -\dfrac{2}{3}x - 1.$

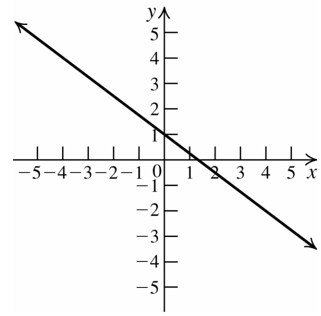


10. The two points are (1, 1/4) and (4, –2).

$m = \dfrac{-2-1/4}{4-1} = \dfrac{-9/4}{3} = -\dfrac{3}{4}.$

$-2 = -\dfrac{3}{4}(4) + b \Rightarrow b = 1.$

$f(x) = -\dfrac{3}{4}x + 1.$



11.a. Domain: [–2, 6]; range: [–3, 2]

b. To find the intercepts, first determine the equation of the line containing (–2, –3) and (2, 2): $m = \dfrac{2-(-3)}{2-(-2)} = \dfrac{5}{4}.$

$2 = \dfrac{5}{4}(2) + b \Rightarrow -\dfrac{1}{2} = b.\ \ f(x) = \dfrac{5}{4}x - \dfrac{1}{2}.$

Set $f(x) = 0$ and solve for *x* to find the *x*-intercept: $\dfrac{5}{4}x - \dfrac{1}{2} = 0 \Rightarrow x = \dfrac{2}{5}$. The *x*-intercept is $\dfrac{2}{5}$ and the *y*-intercept is $-\dfrac{1}{2}$.

c. $f(x)$ is increasing on (–2, 2); $f(x)$ is constant on (2, 6).

d. $f(x)$ even $\Rightarrow f(-x) = f(x).\ \ f(2) = 2$ $\neq f(-2) = -3$, so $f(x)$ is not even. $f(x)$ odd $\Rightarrow f(-x) = -f(x).\ \ f(-2) = -3$ $\neq -f(2) = -2$, so $f(x)$ is not odd.

12.a. Domain: [–3, 0) ∪ (0, 3]; range: {–1, 1}

b. There is no *x*-intercept. There is no *y*-intercept.

c. $f(x)$ is constant on (–3, 0) and (0, 3).

d. $f(x)$ even $\Rightarrow f(-x) = f(x).\ \ f(3) = 1$ $\neq f(-3) = -1$, so $f(x)$ is not even. $f(x)$ odd $\Rightarrow f(-x) = -f(x).\ \ f(-3) = -1$ $= -f(3) = -1$, so $f(x)$ is odd.

13.a. Domain: [–2, 4]; range: [–1, 2]

b. The *x*-intercepts are –2 and 0. The *y*-intercept is 0.

c. $f(x)$ is decreasing on (–2, –1) and (2, 4); $f(x)$ is increasing on (–1, 2).

d. $f(x)$ even $\Rightarrow$ $f(-x) = f(x)$. $f(2) = 2$ $\neq f(-2) = 0$, so $f(x)$ is not even. $f(x)$ odd $\Rightarrow$ $f(-x) = -f(x)$. $f(-2) = 0$ $\neq -f(2) = -2$, so $f(x)$ is not odd.

14.a. Domain: [–3, 5]; range: [–2, 3]

b. The x-intercept is 0. The y-intercept is 0.

c. $f(x)$ is increasing on (–3, 5).

d. $f(x)$ even $\Rightarrow$ $f(-x) = f(x)$. $f(5) = 3$ $\neq f(-5)$ which is undefined, , so $f(x)$ is not even. $f(x)$ odd $\Rightarrow$ $f(-x) = -f(x)$. By the same reasoning $f(-5)$ is undefined while $-f(5) = -3$. So $f(x)$ is not odd.

15.a. Domain: $(0, \infty)$; range: $(0, \infty)$

b. There is no x-intercept nor is there a y-intercept.

c. $f(x)$ is decreasing on $(0, \infty)$.

d. Because the function is defined for only positive values, $f(x) \neq f(-x)$ and $-f(x) \neq f(-x)$. The function is neither even nor odd.

16.a. Domain: $(-\infty, 3) \cup (3, \infty)$; range: $(-\infty, 3) \cup (3, \infty)$

b. The x-intercept is 2. The y-intercept is 2.

c. $f(x)$ is decreasing on $(-\infty, 3) \cup (3, \infty)$.

d. $f(x)$ even $\Rightarrow$ $f(-x) = f(x)$. $f(2) = 0$ $\neq f(-2) \approx 2.5$, so $f(x)$ is not even. $f(x)$ odd $\Rightarrow$ $f(-x) = -f(x)$. $f(-2) \approx 2.5$ $\neq -f(2) = 0$, so $f(x)$ is not odd.

17.a. Domain: $(-\infty, \infty)$; range: $(0, \infty)$

b. There is no x-intercept. The y-intercept is 1.

c. $f(x)$ is increasing on $(-\infty, \infty)$.

d. $f(x)$ even $\Rightarrow$ $f(-x) = f(x)$. $f(0) = 1$ $\neq f(-1)$, so $f(x)$ is not even. $f(x)$ odd $\Rightarrow$ $f(-x) = -f(x)$. $f(-1) \approx 0.8 \neq -f(1) = 0$, so $f(x)$ is not odd.

18.a. Domain: $(0, \infty)$; range: $(-\infty, \infty)$

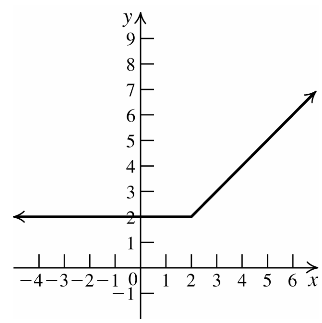
b. The x-intercept is 1.5. There is no y-intercept.

c. $f(x)$ is increasing on $(0, \infty)$.

d. $f(x)$ even $\Rightarrow$ $f(-x) = f(x)$. $f(1.5) = 0$ $\neq f(-1.5)$, so $f(x)$ is not even. $f(x)$ odd $\Rightarrow$ $f(-x) = -f(x)$. $f(-1.5)$ is not defined, so $f(x)$ is not odd.

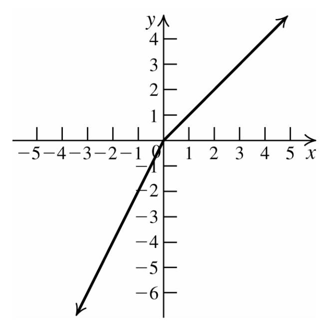
19.a. $f(1) = 2; f(2) = 2; f(3) = 3$

b.



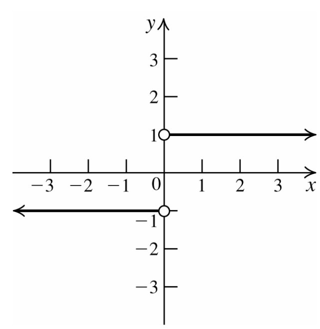
20.a. $g(-1) = -2; g(0) = 0; g(1) = 1$

b.



21.a. $f(-15) = -1; f(12) = 1$

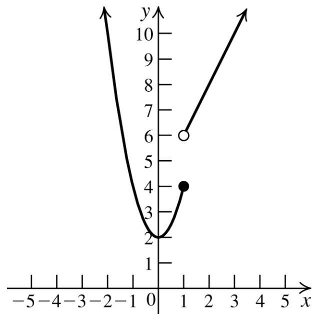
b.



c. Domain: $(-\infty, 0) \cup (0, \infty)$; range: $\{-1, 1\}$

22.a. $g(-3) = 20; g(1) = 4; g(3) = 10$

b.



c. Domain: $(-\infty, \infty)$; range: $[2, \infty)$

23. $f(x)$ is constant on $(-\infty, \infty)$.

24. $f(x)$ is constant on $(-\infty, \infty)$.

25. $f(x)$ is increasing on $(-\infty, \infty)$.

26. $f(x)$ is decreasing on $(-\infty, \infty)$.

27. $f(x)$ is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.

28. $f(x)$ is decreasing on $(-\infty, \infty)$.

For exercises 29–42, $f(-x) = f(x) \Rightarrow f(x)$ is even and $f(-x) = -f(x) \Rightarrow f(x)$ is odd.

For exercises 29–32, use the function values given in the table to determine if the function is even, odd, or neither.

29. even **30.** neither

31. neither **32.** odd

33. $f(-x) = 2(-x)^4 + 4 = 2x^4 + 4 = f(x) \Rightarrow$
$f(x)$ is even.

34. $f(-x) = 3(-x)^4 - 5 = 3x^4 - 5 = f(x) \Rightarrow$
$f(x)$ is even.

35. $f(-x) = 5(-x)^3 - 3(-x) = -5x^3 + 3x$
$= -(5x^3 - 3x) = -f(x) \Rightarrow$
$f(x)$ is odd.

36. $f(-x) = 2(-x)^3 + 4(-x) = -2x^3 - 4x$
$= -(2x^3 + 4x) = -f(x) \Rightarrow$
$f(x)$ is odd.

37. $f(-x) = \dfrac{1}{(-x)^2 + 4} = \dfrac{1}{x^2 + 4} = f(x) \Rightarrow$
$f(x)$ is even.

38. $g(-x) = \dfrac{-x}{(-x)^2 + 1} = -\dfrac{x}{x^2 + 1} \Rightarrow g(x)$ is odd.

39. $f(-x) = \dfrac{(-x)^3}{(-x)^2 + 1} = -\dfrac{x^3}{x^2 + 1} = -f(x) \Rightarrow$
$f(x)$ is odd.

40. $g(-x) = \dfrac{(-x)^4 + 3}{2(-x)^3 - 3(-x)} = \dfrac{x^4 + 3x}{-2x^3 + 3x}$
$= -\dfrac{x^4 + 3x}{2x^3 - 3x} = -f(x) \Rightarrow f(x)$ is odd.

41. $f(-x) = \dfrac{(-x)^2 - 2(-x)}{5(-x)^4 + 4(-x)^2 + 7} = \dfrac{x^2 + 2x}{5x^4 + 4x^2 + 7}$
$\neq -f(x) \neq f(x) \Rightarrow f(x)$ is neither even nor odd.

42. $g(-x) = \dfrac{(-x)^2 + 7}{3(-x)^4 + 16(-x)^2 + 9}$
$= \dfrac{x^2 + 7}{3x^4 + 16x^2 + 9} = g(x) \Rightarrow$
$g(x)$ is even.

43.a. $f(2x) = 3 - 2(2x) = 3 - 4x$

b. $2f(x) = 2(3 - 2x) = 6 - 4x$

c. $f(-x) = 3 - 2(-x) = 3 + 2x$

d. $-f(x) = -(3 - 2x) = 2x - 3$

e. $f\left(\dfrac{1}{x}\right) = 3 - 2\left(\dfrac{1}{x}\right) = 3 - \dfrac{2}{x}$

f. $\dfrac{1}{f(x)} = \dfrac{1}{3 - 2x}$

44.a. $f(2x) = 5(2x) + 1 = 10x + 1$

b. $2f(x) = 2(5x + 1) = 10x + 2$

c. $f(-x) = 5(-x) + 1 = 1 - 5x$

d. $-f(x) = -(5x + 1) = -5x - 1$

e. $f\left(\dfrac{1}{x}\right) = 5\left(\dfrac{1}{x}\right) + 1 = \dfrac{5}{x} + 1$

f. $\dfrac{1}{f(x)} = \dfrac{1}{5x + 1}$

45.a. $f(2x) = (2x)^2 - 2(2x) = 4x^2 - 4x$

b. $2f(x) = 2(x^2 - 2x) = 2x^2 - 4x$

c. $f(-x) = (-x)^2 - 2(-x) = x^2 + 2x$

d. $-f(x) = -\left(x^2 - 2x\right) = 2x - x^2$

e. $f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^2 - 2\left(\frac{1}{x}\right) = \frac{1}{x^2} - \frac{2}{x}$

f. $\frac{1}{f(x)} = \frac{1}{x^2 - 2x}$

46.a. $f(2x) = 2x - 4(2x^2) = 2x - 16x^2$

b. $2f(x) = 2(x - 4x^2)2x - 8x^2$

c. $f(-x) = -x - 4(-x)^2 = -x - 4x^2$

d. $-f(x) = -(x - 4x^2) = 4x^2 - x$

e. $f\left(\frac{1}{x}\right) = \frac{1}{x} - 4\left(\frac{1}{x}\right)^2 = \frac{1}{x} - \frac{4}{x^2}$

f. $\frac{1}{f(x)} = \frac{1}{x - 4x^2}$

47.a. $f(2x) = 1 - (2x)^3 = 1 - 8x^3$

b. $2f(x) = 2(1 - x^3) = 2 - 2x^3$

c. $f(-x) = 1 - (-x)^3 = 1 + x^3$

d. $-f(x) = -(1 - x^3) = x^3 - 1$

e. $f\left(\frac{1}{x}\right) = 1 - \left(\frac{1}{x}\right)^3 = 1 - \frac{1}{x^3}$

f. $\frac{1}{f(x)} = \frac{1}{1 - x^3}$

48.a. $f(2x) = (2x)^3 + 2x = 8x^3 + 2x$

b. $2f(x) = 2(x^3 + x) = 2x^3 + 2x$

c. $f(-x) = (-x)^3 + (-x) = -x^3 - x$

d. $-f(x) = -(x^3 + x) = -x^3 - x$

e. $f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 + \frac{1}{x} = \frac{1}{x^3} + \frac{1}{x}$

f. $\frac{1}{f(x)} = \frac{1}{x^3 + x}$

49.a. $f(2x) = \frac{1}{2x}$ **b.** $2f(x) = \frac{2}{x}$

c. $f(-x) = -\frac{1}{x}$ **d.** $-f(x) = -\frac{1}{x}$

e. $f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = x$ **f.** $\frac{1}{f(x)} = \frac{1}{\frac{1}{x}} = x$

50.a. $f(2x) = \frac{1}{|2x|}$ **b.** $2f(x) = \frac{2}{|x|}$

c. $f(-x) = \frac{1}{|-x|} = \frac{1}{|x|}$ **d.** $-f(x) = -\frac{1}{|x|}$

e. $f\left(\frac{1}{x}\right) = \frac{1}{\left|\frac{1}{x}\right|} = |x|$ **f.** $\frac{1}{f(x)} = \frac{1}{\frac{1}{|x|}} = |x|$

2.5 B Exercises: Applying the Concepts

51.a. $f(x) = \frac{x}{33.81}$; domain: $[0, \infty)$; range: $[0, \infty)$.

b. $f(3) = \frac{3}{33.81} = 0.0887$. This means that 3 oz = 0.0887 liters.

c. $f(12) = \frac{12}{33.81} = 0.3549$ liters.

52.a. $B(0) = -1.8(0) + 212 = 212$. The y-intercept is 212. This means that water boils at 212°F at sea level. $0 = -1.8h + 212 \Rightarrow h = 117.78$. The h-intercept is 117.78. This means that water boils at 0°F at 117,780 feet above sea level.

b. Domain: closed interval from 0 to the end of the atmosphere, in feet.

c. $98.6 = -1.8h + 212 \Rightarrow h = 63.$ Water boils at 98.6°F at 63,000 feet. It is dangerous because 98.6°F is the temperature of human blood.

53.a. $P(0) = \frac{1}{33}(0) + 1 = 1.$ The y-intercept is 1. This means that the pressure at sea level (d = 0) is 1 atm. $0 = \frac{1}{33}d + 1 \Rightarrow d = -33.$ d can't be negative, so there is no d-intercept.

b. $P(0) = 1; P(10) = \frac{1}{33}(10) + 1 \approx 1.3;$
$P(33) = \frac{1}{33}(33) + 1 = 2;$
$P(100) = \frac{1}{33}(100) + 1 \approx 4.03.$

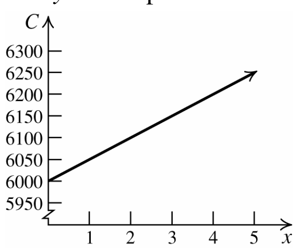
c. $5 = \frac{1}{33}d + 1 \Rightarrow d = 132$ feet

54.a. $V(90) = 1055 + 1.1(90) = 1154$ ft/sec

b. $1100 = 1055 + 1.1T \Rightarrow T = 40.91°\text{F}$

55.a. $C(x) = 50x + 6000$

b. The y-intercept is the fixed cost.



c. $11{,}500 = 50x + 6000 \Rightarrow 110$

56.a. The rate of change (slope) is 100. Find the y-intercept by using the point (10, 750): $750 = 100(10) + b \Rightarrow b = -250.$ The equation is $f(p) = 100p - 250.$

b. $f(15) = 100(15) - 250 = 1250$ units.

c. $1750 = 100p - 250 \Rightarrow p = \$20.$

57.a. $R = 900 - 30x$

b. $R(6) = 900 - 30(6) = \$720$

c. $600 = 900 - 30x \Rightarrow x = 10$ days after the first of the month.

58.a. Let $t = 0$ represent the year 2002. The rate of change (slope) is $\frac{1150 - 1120}{2} = 15.$ The y-intercept is 1120, so the equation is $f(t) = 15t + 1120.$

b. $f(8) = 15(8) + 1120 = 1240$

c. $1300 = 15t + 1120 \Rightarrow t = 12.$ 2002 + 12 = 2014.

59. The rate of change (slope) is $\frac{100 - 40}{20 - 80} = -1.$ Use the point (20, 100) to find the equation of the line: $100 = -20 + b \Rightarrow b = 120.$ The equation of the line is $y = -x + 120.$ Now solve $50 = -x + 120 \Rightarrow x = 70.$

60.a. $y = \frac{2}{25}(5)(60) = 24$ mg

b. $60 = \frac{2}{25}(60)a \Rightarrow a = 12.5$ years old

61.a. The rate of change (slope) is $\frac{50 - 30}{420 - 150} = \frac{2}{27}.$ The equation of the line is
$y - 30 = \frac{2}{27}(x - 150) \Rightarrow y = \frac{2}{27}(x - 150) + 30.$

b. $y = \frac{2}{27}(350 - 150) + 30 \Rightarrow y = \frac{1210}{27}$

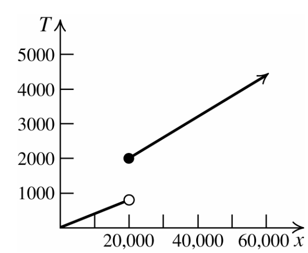
c. $45 = \frac{2}{27}(x - 150) + 30 \Rightarrow x = 352.5 \text{ mg/m}^3$

62.a. The rate of change is $\frac{1}{3}.$ The y-intercept is $\frac{47}{12}$, so the equation is $y = L(S) = \frac{1}{3}S + \frac{47}{12}.$

b. $L(4) = \frac{1}{3}(4) + \frac{47}{12} = 5.25$ in.

c. $\frac{61}{10} = \frac{1}{3}x + \frac{47}{12} \Rightarrow x \approx 7$

63.a.



b. **(i)** $T(12{,}000) = 0.04(12{,}000) = \$480$
(ii) $T(20{,}000) = 800 + 0.06(20{,}000) = \$2000$
(iii) $T(50{,}000) = 800 + 0.06(50{,}000) = \$3800$

c. **(i)** $600 = 0.04x \Rightarrow x = \$15{,}000$

(ii) $1200 = 0.04x \Rightarrow x = \$30,000$, which is outside of the domain. Try $1200 = 800 + 0.06x \Rightarrow x \approx \$6667$, which is also outside of the domain. So, \$1200 is not possible as a tax liability.

(iii) $2300 = 800 + 0.06x \Rightarrow x = \$25,000$

64.a. If $7300 < x \le 29,700$, $f(x) = 730 + 0.15(x - 7300) = 0.15x - 365$
If $29,700 < x \le 71,950$, $f(x) = 4090 + 0.25(x - 29,700) = 0.25x - 3335$
If $71,950 < x \le 150,150$, $f(x) = 14,652.50 + 0.28(x - 71,950) = 0.28x - 5493.5$
If $150,150 < x \le 326,450$, $f(x) = 36,548.50 + 0.33(x - 150,150) = 0.33x - 13,001$
If $326,450 > x$, $f(x) = 94,727.50 + 0.35(x - 326,450) = 0.35x - 19,530$

Write the equation as:

$$f(x) = \begin{cases} 0.1x & \text{if } 0 < x \le 7300 \\ 0.15x - 365 & \text{if } 7300 < x \le 29,700 \\ 0.25x - 3335 & \text{if } 29,700 < x \le 71,950 \\ 0.28x - 5493.50 & \text{if } 71,950 < x \le 150,150 \\ 0.33x - 13,001 & \text{if } 150,150 < x \le 326,450 \\ 0.35x - 19,530 & \text{if } x > 326,450 \end{cases}$$

b. **(i)** $f(35,000) = 0.25(35,000) - 3335 = \$5415$

(ii) $f(100,000) = 0.28(100,000) - 5493.50 = \$22,506.50$

(iii) $f(500,000) = 0.35(500,000) - 19,530 = \$155,470$

c. **(i)** $3500 = 0.15x - 365 \Rightarrow x = \$25,766.67$

(ii) $12,700 = 0.25x - 3335 = \$64,140$

(iii) $35,000 = 0.28x - 5493.50 \Rightarrow x = \$144,619.64$

2.5 C Exercises: Beyond the Basics

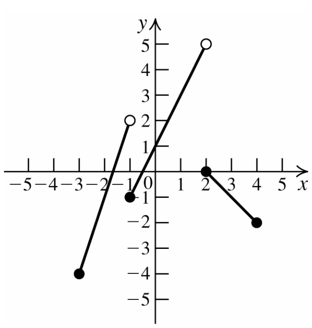
65.a. **(i)** $f(-2) = 3(-2) + 5 = -1$

(ii) $f(-1) = 2(-1) + 1 = -1$

(iii) $f(3) = 2 - 3 = -1$

b. Try the first rule: $2 = 3x + 5 \Rightarrow x = -1$, which is not in the domain for that rule. Now try the second rule: $2 = 2x + 1 \Rightarrow x = \frac{1}{2}$, which is in the domain for that rule.

c.



66.a. **(i)** $g(-2) = 3$

(ii) $g\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right) - 3 = -\frac{3}{2}$

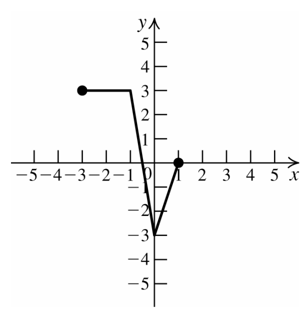
(iii) $g(0) = 3(0) - 3 = -3$

(iv) $g(-1) = -6(-1) - 3 = 3$

b. **(i)** $0 = -6x - 3 \Rightarrow x = -\frac{1}{2}$ or $0 = 3x - 3 \Rightarrow x = 1$

(ii)
$$2g(x) + 3 = 0$$
$$2(-6x - 3) + 3 = 0$$
$$-12x - 6 + 3 = 0 \Rightarrow x = -\frac{1}{4} \text{ or}$$
$$2(3x - 3) + 3 = 0$$
$$6x - 6 + 3 = 0 \Rightarrow x = \frac{1}{2}$$

c.



67.a. Domain: $(-\infty, \infty)$; range: $[0,1)$

b. The function is increasing on $(n, n + 1)$ for every integer n.

c. $f(-x) = -x - [\![x]\!] \ne -f(x) \ne f(x)$, so the function is neither even nor odd.

68.a. Domain: $(-\infty,0)\cup[1,\infty)$; range: $\left\{\frac{1}{n}: n\neq 0, n \text{ an integer}\right\}$

b. The function is constant on $(n, n+1)$ for every nonzero integer n.

c. $f(-x)=\frac{1}{[\![-x]\!]}\neq -f(x)\neq f(x)$, so the function is neither even nor odd.

69. $|f(x)-f(-x)|=\left|\frac{|x|}{x}-\frac{|-x|}{-x}\right|=\left|\frac{|x|}{x}+\frac{|x|}{x}\right| = \begin{cases}|1+1| \\ |-1+(-1)|\end{cases}=2$

70.a. **(i)** $WCI(2)=40$

(ii) $WCI(16)=91.4+(91.4-40)\bullet (0.0203(16)-0.304\sqrt{16}-0.474) = 21.23$

(iii) $WCI(50)=1.6(40)-55=9$

b. **(i)** $-58=91.4+(91.4-T)\bullet (0.0203(36)-0.304\sqrt{36}-0.474$

$-58=91.4+(91.4-T)(-1.5672)$

$-58=91.4-143.24+1.56872T$

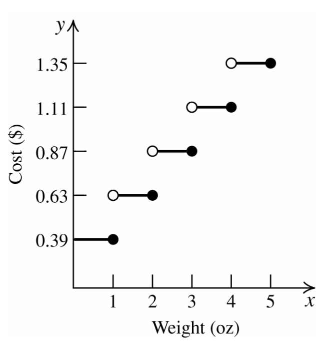
$-58=-51.84+1.56872T \Rightarrow T\approx -4°\text{F}$

(ii) $-10=1.6T-55 \Rightarrow T\approx 28°\text{F}$

2.5 Critical Thinking

71.a. $C(x)=24(f(x)-1)+39$

b.



c. Domain: $(0,\infty)$; range: $\{24n+39: n \text{ a nonnegative integer}\}$

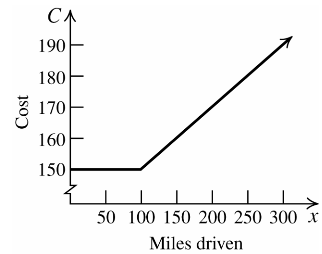
72. $C(x)=2[\![x]\!]+4$

c. $g(x)$: domain: $(-\infty,\infty)$, range: $[1,\infty)$

$h(x)$: domain: $(-\infty,\infty)$, range: $[-2,\infty)$

73.a. $C(x)=\begin{cases}150 & \text{if } x<100 \\ 0.2[\![x-99]\!]+150 & \text{if } x\geq 100\end{cases}$

b.



c. $190=0.2[\![x-99]\!]+150$

$40=0.2[\![x-99]\!]\Rightarrow 200=[\![x-99]\!]\Rightarrow$

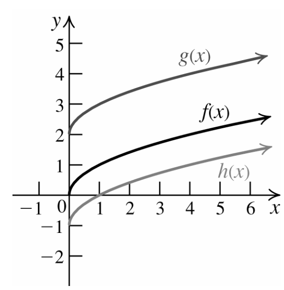
$x=300$ miles

Section 2.6 Transformations of Functions

2.6 A Exercises: Basic Skills and Concepts

1.a. The graph of g is the graph of f shifted two units up. The graph of h is the graph of f shifted one unit down.

b.

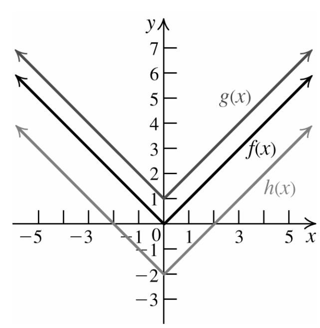


c. $g(x)$: domain: $[0,\infty)$, range: $[2,\infty)$

$h(x)$: domain: $[0,\infty)$, range: $[-1,\infty)$

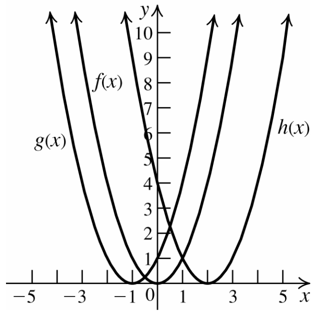
2.a. The graph of g is the graph of f shifted one unit up. The graph of h is the graph of f shifted two units down.

b.



3.a. The graph of g is the graph of f shifted one unit to the left. The graph of h is the graph of f shifted two units to the right.

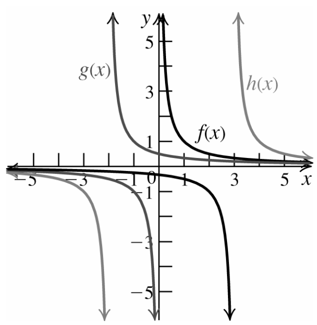
b.



c. $g(x)$: domain: $(-\infty, \infty)$, range: $[0, \infty)$
$h(x)$: domain: $(-\infty, \infty)$, range: $[0, \infty)$

4.a. The graph of g is the graph of f shifted two units to the left. The graph of h is the graph of f shifted three units to the right.

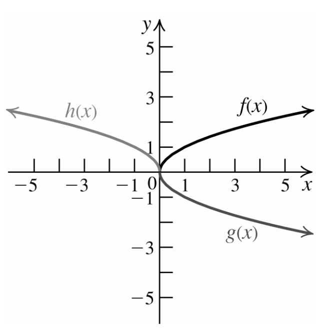
b.



c. $g(x)$: domain: $(-\infty, -2) \cup (-2, \infty)$, range: $(-\infty, 0) \cup (0, \infty)$
$h(x)$: domain: $(-\infty, 3) \cup (3, \infty)$, range: $(-\infty, 0) \cup (0, \infty)$

5.a. The graph of g is the graph of f reflected across the x-axis. The graph of h is the graph of f reflected across the y-axis.

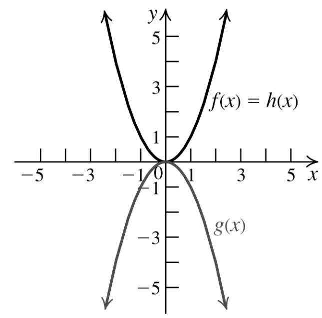
b.



c. $g(x)$: domain: $[0, \infty)$, range: $(-\infty, 0]$
$h(x)$: domain: $(-\infty, 0]$, range: $[0, \infty)$

6.a. The graph of g is the graph of f reflected across the x-axis. The graph of h is the same as the graph of f.

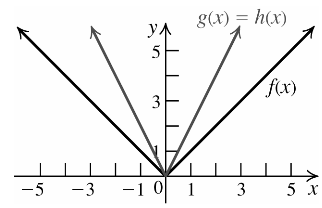
b.



c. $g(x)$: domain: $(-\infty, \infty)$, range: $(-\infty, 0]$
$h(x)$: domain: $(-\infty, \infty)$, range: $[0, \infty)$

7.a. The graph of g is the graph of f vertically stretched by a factor of 2. $2|x| = |2x| \Rightarrow$ the graph of h is the same as the graph of g.

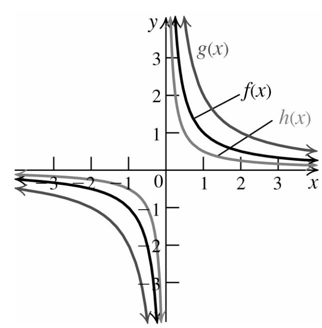
b.



c. $g(x)$: domain: $(-\infty, \infty)$, range: $[0, \infty)$
$h(x)$: domain: $(-\infty, \infty)$, range: $[0, \infty)$

8.a. The graph of g is the graph of f vertically stretched by a factor of 2. The graph of h is the graph of f horizontally compressed by a factor of 1/2. The two graphs are the same.

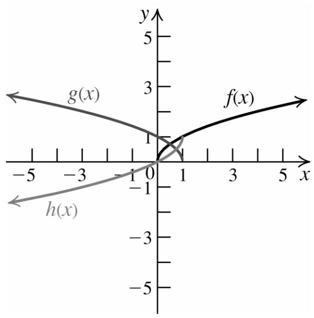
b.



c. $g(x)$: domain: $(-\infty, 0) \cup (0, \infty)$, range: $(-\infty, 0) \cup (0, \infty)$
$h(x)$: domain: $(-\infty, 0) \cup (0, \infty)$, range: $(-\infty, 0) \cup (0, \infty)$

9.a. The graph of g is the graph of f shifted one unit to the left and reflected across the y-axis. The graph of h is the graph of f shifted one unit to the left, reflected across the y-axis, reflected across the x-axis, and then shifted up one unit.

b.

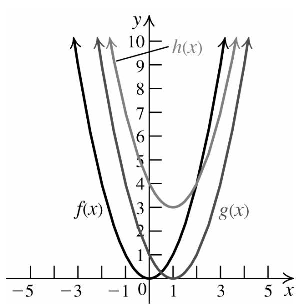


c. $g(x)$: domain: $(-\infty, 1]$, range: $[0, \infty)$

$h(x)$: domain: $(-\infty, 1]$, range: $(-\infty, 1]$

10.a. The graph of g is the graph of f shifted one unit to the right. The graph of h is the graph of f shifted one unit to the right and then three units up.

b.

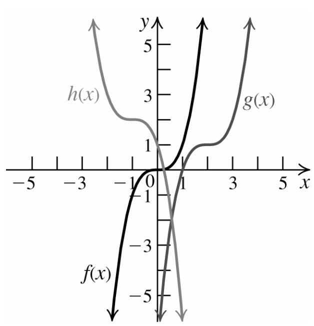


c. $g(x)$: domain: $(-\infty, \infty)$, range: $[0, \infty)$

$h(x)$: domain: $(-\infty, \infty)$, range: $[3, \infty)$

11.a. The graph of g is the graph of f shifted two units to the right and one unit up. The graph of h is the graph of f shifted one unit to the left, reflected across the x-axis, and then two units up.

b.

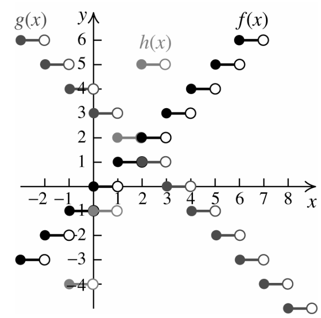


c. $g(x)$: domain: $(-\infty, \infty)$, range: $(-\infty, \infty)$

$h(x)$: domain: $(-\infty, \infty)$, range: $(-\infty, \infty)$

12.a. The graph of g is the graph of f shifted one unit to the right, reflected across the x-axis, and then shifted two units up. The graph of h is the graph of f vertically stretched by a factor of three and then shifted one unit down.

b.



c. $g(x)$: domain: $(-\infty, \infty)$, range:

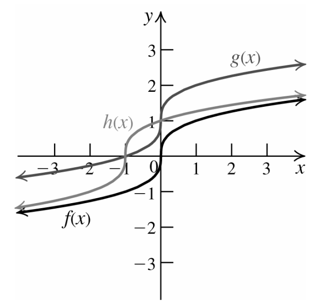
$\{n : n \text{ is an integer}\}$

$h(x)$: domain: $(-\infty, \infty)$, range:

$\{n : n = 3k - 1, k \text{ an integer}\}$

13.a. The graph of g is the graph of f shifted one unit up. The graph of h is the graph of f shifted one unit to the left.

b.

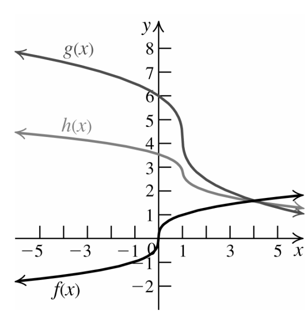


c. $g(x)$: domain: $(-\infty, \infty)$, range: $(-\infty, \infty)$

$h(x)$: domain: $(-\infty, \infty)$, range: $(-\infty, \infty)$

14.a. The graph of g is the graph of f shifted one unit left, reflected across the y-axis, vertically stretched by a factor of 2, and then shifted 4 units up. The graph of h is the graph of f shifted one unit to the right, reflected across the x-axis, and then shifted three units up.

b.



c. $g(x)$: domain: $(-\infty,\infty)$, range: $(-\infty,\infty)$
$h(x)$: domain: $(-\infty,\infty)$, range: $(-\infty,\infty)$

15. e **16.** c **17.** g **18.** h

19. i **20.** a **21.** b **22.** k

23. l **24.** f **25.** d **26.** j

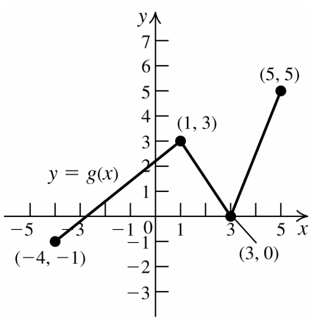
27. x^3+2 **28.** $\sqrt{x+3}$

29. $-|x|$ **30.** $\sqrt{-x}$

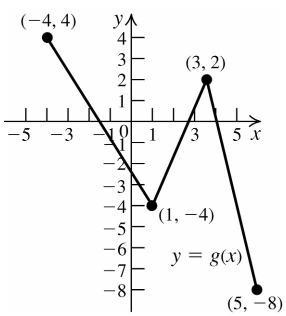
31. $(x-3)^2+2$ **32.** $-\sqrt{x+3}-2$

33. $3(-x+4)^3+2$ **34.** $-2|x-4|-3$

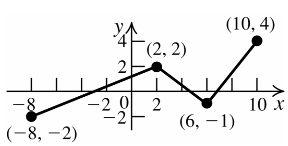
35.



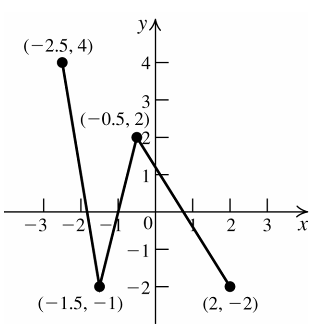
36.



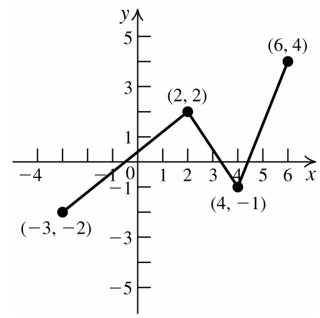
37.



38.



39.



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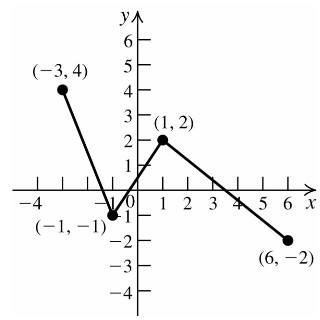
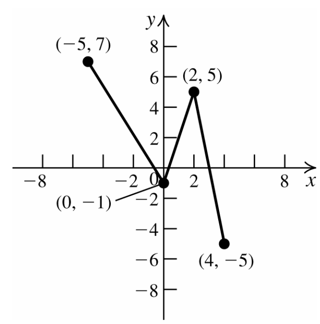
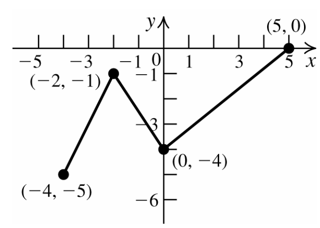


41.



42.



2.6 B Exercises: Applying the Concepts

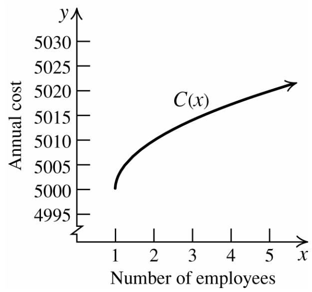
43. $g(x)=f(x)+800$

44. $h(x)=1.05f(x)$

45. $p(x)=1.02(x+500)$

46. $p(x)=\begin{cases}1.1x & \text{if } x<30{,}000\\ 1.02x & \text{if } x\geq 30{,}000\end{cases}$

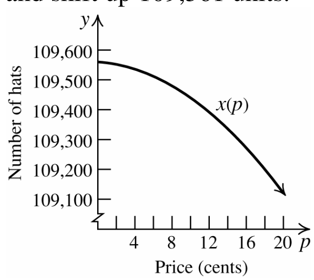
47.a. Shift one unit right, stretch vertically by a factor of 10, and shift 5000 units up.



b. $C(400) = 5000 + 10\sqrt{400-1} = \$5199.75$

48. $1.1C(x) = 1.1\left(5000 + 10\sqrt{x-1}\right)$
$= 5500 + 11\sqrt{x-1}$

49.a. Shift one unit left, reflect across the *x*-axis, and shift up 109,561 units.



b. $69,160 = 109,561 - (p+1)^2$
$40,401 = (p+1)^2$
$201 = p + 1 \Rightarrow p = 200¢ = \$2.00$

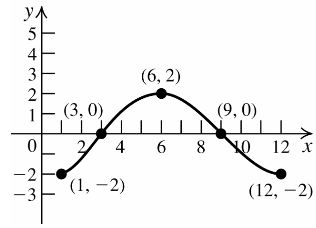
c. $0 = 109,561 - (p+1)^2$
$109,561 = (p+1)^2$
$331 = p + 1 \Rightarrow p = 330¢ = \$3.30$

50. Write *R*(*p*) in the form $-3(p-h)^2 + k$:

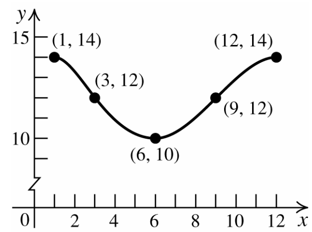
$R(p) = -3p^2 + 600p = -3(p^2 - 200p)$
Complete the square
$= -3(p^2 - 200p + 10,000) + 30,000$
$= -3(p - 100) + 30,000$

To graph this, shift *R*(*p*) 100 units to the right, stretch by a factor of 3, reflect across the *x*-axis, and shift by 30,000 units up.

51. The first coordinate gives the month; the second coordinate gives the hours of daylight. From March to September, there is daylight more than half of the day each day. From September to March, more than half of the day is dark each day.

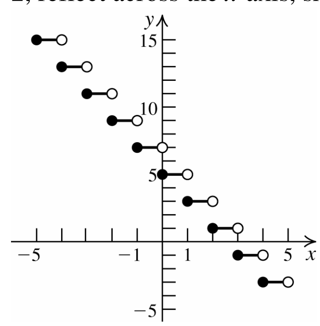


52. The graph shows the number of hours of darkness.

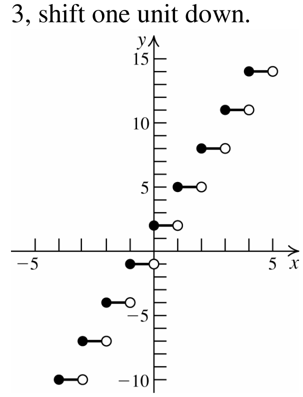


2.6 C Exercises: Beyond the Basics

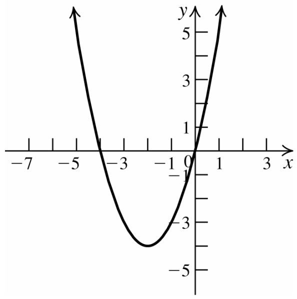
53. Shift one unit to the right, stretch by a factor of 2, reflect across the *x*-axis, shift three units up.



54. Shift two units to the left, stretch by a factor of 3, shift one unit down.

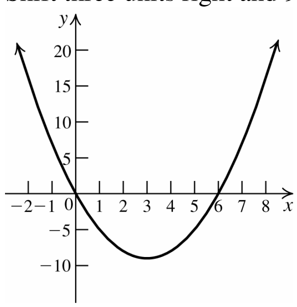


55. Shift two units left and 4 units down.



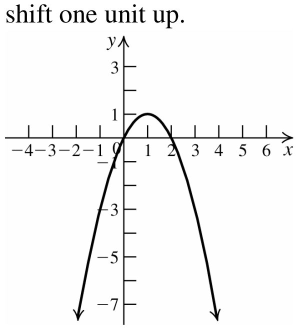
56. $f(x) = x^2 - 6x = \left(x^2 - 6x + 9\right) - 9$
$= (x-3)^2 - 9$

Shift three units right and 9 units down.



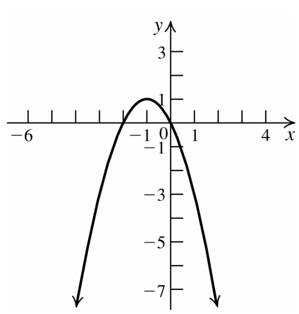
57. $f(x) = -x^2 + 2x = -\left(x^2 - 2x + 1\right) + 1$
$= -(x-1)^2 + 1$

Shift one unit right, reflect across the *x*-axis, shift one unit up.



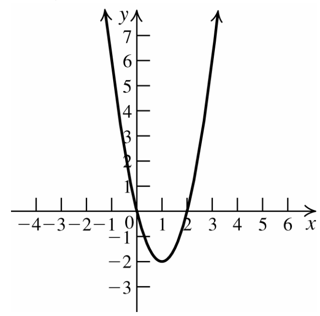
58. $f(x) = -x^2 - 2x = -\left(x^2 + 2x + 1\right) + 1$
$= -(x+1)^2 + 1$

Shift one unit left, reflect across the axis, shift one unit up.*x*



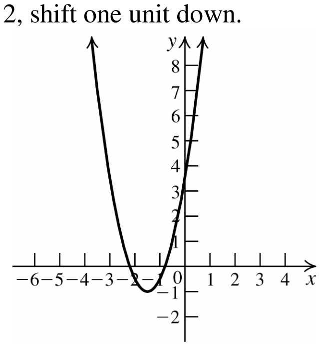
59. $f(x) = 2x^2 - 4x = 2\left(x^2 - 2x + 1\right) - 2$
$= 2(x-1)^2 - 2$

Shift one unit right, stretch vertically by a factor of 2, shift two units down.



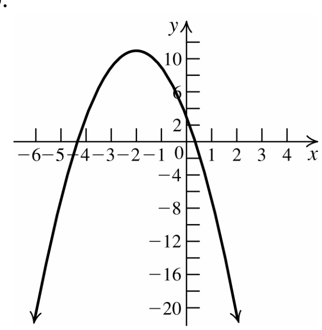
60. $f(x) = 2x^2 + 6x + 3.5$
$= 2\left(x^2 + 3x + 1.75 + 0.5\right) - 1$
$= 2(x+1.5)^2 - 1$

Shift 1.5 units left, stretch vertically by a factor of 2, shift one unit down.



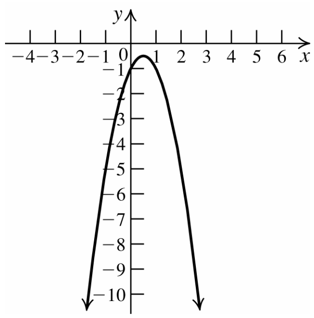
61. $f(x) = -2x^2 - 8x + 3 = -2(x^2 + 4x - 1.5)$
$= -2\left(x^2 + 4x - 1.5 + 5.5\right) + 11$
$= -2(x+2)^2 + 11$

Shift two units left, stretch vertically by a factor of 2, reflect across the *x*-axis, shift eleven units up.

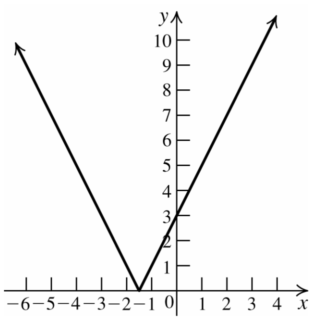


62. $f(x) = -2x^2 + 2x - 1 = -2(x^2 - x + 0.5)$
$= -2\left(x^2 - x + 0.5 - 0.25\right) - 0.5$
$= -2\left(x^2 - x + 0.25\right) - 0.5$
$= -2(x - 0.5)^2 - 0.5$

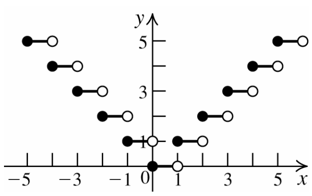
Shift 0.5 unit right, stretch vertically by a factor of 2, reflect across the x-axis, shift 0.5 unit down.



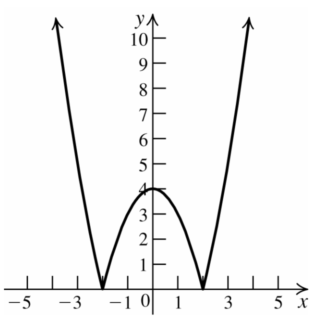
63.



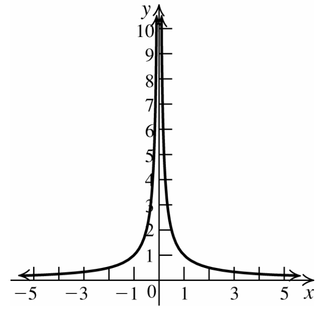
64.



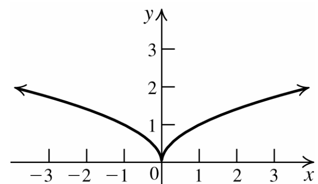
65.



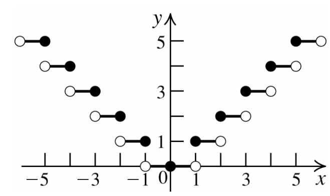
66.



67.



68.



2.6 Critical Thinking

69.a. $g(x) = h(x-3) + 3$. The graph of g is the graph of h shifted three units to the right and three units up.

b. $g(x) = h(x-1) - 1$. The graph of g is the graph of h shifted one unit to the right and one unit down.

c. $g(x) = 2h\left(\frac{1}{2}x\right)$. The graph of g is the graph of h stretched horizontally and vertically by a factor of 2.

d. $g(x) = -3h\left(-\frac{1}{3}x\right)$. The graph of g is the graph of h stretched horizontally by a factor of 3, reflected across the y-axis, stretched vertically by a factor of 3, and reflected across the x-axis.

70. $y = f(x) = f\left(-\frac{1}{4}(-4x)\right)$. Stretch the graph of $y = f(-4x)$ horizontally by a factor of 4 and reflect it across the y-axis.

2.7 Combining Functions; Composite Functions

2.7 A Exercises: Basic Skills and Concepts

1.a. $(f+g)(-1) = f(-1) + g(-1) = 2(-1) + -(-1) = -2 + 1 = -1$

b. $(f-g)(0) = f(0) - g(0) = 2(0) - (-0) = 0$

c. $(f \bullet g)(2) = f(2) \bullet g(2) = 2(2) \bullet (-2) = -8$

d. $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{2(1)}{-1} = -2$

2.a. $(f+g)(-1) = f(-1) + g(-1) = \left(1-(-1)^2\right) + \left(-1+1\right) = 0$

b. $(f-g)(0) = f(0) - g(0) = (1-0^2) - (0+1) = 0$

c. $(f \bullet g)(2) = f(2) \bullet g(2) = (1-2^2) \bullet (2+1) = -9$

d. $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{1-1^2}{1+1} = 0$

3.a. $(f+g)(-1) = f(-1) + g(-1) = \frac{1}{\sqrt{-1+2}} + \left(2(-1)+1\right) = 0$

b. $(f-g)(0) = f(0) - g(0) = \frac{1}{\sqrt{0+2}} - \left(2(0)+1\right) = \frac{\sqrt{2}}{2} - 1$

c. $(f \bullet g)(2) = f(2) \bullet g(2) = \frac{1}{\sqrt{2+2}} \bullet \left(2(2)+1\right) = \frac{5}{2}$

d. $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{\frac{1}{\sqrt{1+2}}}{2(1)+1} = \frac{1}{3\sqrt{3}} = \frac{\sqrt{3}}{9}$

4.a. $(f+g)(-1) = f(-1) + g(-1) = \frac{-1}{(-1)^2 - 6(-1) + 8} + \left(3-(-1)\right) = -\frac{1}{15} + 4 = \frac{59}{15}$

b. $(f-g)(0) = f(0) - g(0) = \frac{0}{0^2 - 6(0) + 8} - (3-0) = -3$

c. $(f \bullet g)(2) = f(2) \bullet g(2) = \frac{2}{2^2 - 6(2) + 8} \bullet (3-2) = \frac{2}{0} \bullet 1 \Rightarrow$ the product does not exist.

d. $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{\frac{1}{1^2 - 6(1) + 8}}{3-1} = \frac{\frac{1}{3}}{2} = \frac{1}{6}$

5.a. $(f+g)(-1) = f(-1) + g(-1) = \left(-1^2 + 2(-1)\right) + \left(3-(-1)\right) = -1 + 4 = 3$

b. $(f-g)(0) = f(0) - g(0) = \left(0^2 + 2(0)\right) - \left(3-(0)\right) = -3$

c. $(f \bullet g)(2) = f(2) \bullet g(2) = \left(2^2 + 2(2)\right) \bullet \left(3-(2)\right) = 8$

d. $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{1^2 + 2(1)}{3-1} = \frac{3}{2}$

6.a. $(f+g)(-1) = f(-1) + g(-1) = \left(-1^2 + 3\right) + \left(3(-1)^3 + 24\right) = 4 + 21 = 25$

b. $(f-g)(0) = f(0) - g(0) = \left(0^2 + 3\right) - \left(3(0^3) + 24\right) = -21$

c. $(f \bullet g)(2) = f(2) \bullet g(2) = \left(2^2 + 3\right) \bullet \left(3(2^3) + 24\right) = 336$

d. $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{1^2 + 3}{3(1^3) + 24} = \frac{4}{27}$

7.a. $f + g = x^2 + x - 3$; domain: $(-\infty, \infty)$

b. $f - g = x - 3 - x^2 = -x^2 + x - 3$; domain: $(-\infty, \infty)$

c. $f \bullet g = (x-3)x^2 = x^3 - 3x^2$; domain: $(-\infty, \infty)$

d. $\frac{f}{g} = \frac{x-3}{x^2}$; domain: $(-\infty, 0) \cup (0, \infty)$

e. $\frac{g}{f} = \frac{x^2}{x-3}$; domain: $(-\infty, 3) \cup (3, \infty)$

8.a. $f+g=x^2+2x-1$; domain: $(-\infty,\infty)$

b. $f-g=2x-1-x^2=-x^2+2x-1$; domain: $(-\infty,\infty)$

c. $f\bullet g=(2x-1)x^2=2x^3-x^2$; domain: $(-\infty,\infty)$

d. $\frac{f}{g}=\frac{2x-1}{x^2}$; domain: $(-\infty,0)\cup(0,\infty)$

e. $\frac{g}{f}=\frac{x^2}{2x-1}$; domain: $\left(-\infty,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$

9.a. $f+g=\left(x^3-1\right)+\left(2x^2+5\right)=x^3+2x^2+4$; domain: $(-\infty,\infty)$

b. $f-g=\left(x^3-1\right)-\left(2x^2+5\right)=x^3-2x^2-6$; domain: $(-\infty,\infty)$

c. $f\bullet g=\left(x^3-1\right)\left(2x^2+5\right)$
$=2x^5+5x^3-2x^2-5$; domain: $(-\infty,\infty)$

d. $\frac{f}{g}=\frac{x^3-1}{2x^2+5}$; domain: $(-\infty,\infty)$

e. $\frac{g}{f}=\frac{2x^2+5}{x^3-1}$; domain: $(-\infty,1)\cup(1,\infty)$

10.a. $f+g=\left(x^2-4\right)+\left(x^2-6x+8\right)$
$=2x^2-6x+4$; domain: $(-\infty,\infty)$

b. $f-g=\left(x^2-4\right)-\left(x^2-6x+8\right)=6x-12$; domain: $(-\infty,\infty)$

c. $f\bullet g=\left(x^2-4\right)\left(x^2-6x+8\right)$
$=x^4-6x^3+4x^2+24x-32$; domain: $(-\infty,\infty)$

d. $\frac{f}{g}=\frac{x^2-4}{x^2-6x+8}$; the denominator = 0 if $x=2\cup x=4$, so the domain is $(-\infty,2)\cup(2,4)\cup(4,\infty)$.

e. $\frac{g}{f}=\frac{x^2-6x+8}{x^2-4}$; the denominator = 0 if $x=2\cup x=-2$, so the domain is $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$.

11.a. $f+g=2x-1+\sqrt{x}$; domain: $[0,\infty)$

b. $f-g=2x-1-\sqrt{x}$; domain: $[0,\infty)$

c. $f\bullet g=(2x-1)\sqrt{x}=2x\sqrt{x}-\sqrt{x}$; domain: $[0,\infty)$

d. $\frac{f}{g}=\frac{2x-1}{\sqrt{x}}$; domain: $(0,\infty)$

e. $\frac{g}{f}=\frac{\sqrt{x}}{2x-1}$; the numerator is defined only for $x\ge 0$, while the denominator = 0 when $x=\frac{1}{2}$, so the domain is $\left[0,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$.

12.a. $f+g=\left(1-\frac{1}{x}\right)+\frac{1}{x}=1$. Neither f nor g is defined for $x=0$, so the domain is $(-\infty,0)\cup(0,\infty)$.

b. $f-g=\left(1-\frac{1}{x}\right)-\frac{1}{x}=1-\frac{2}{x}$; domain: $(-\infty,0)\cup(0,\infty)$.

c. $f\bullet g=\left(1-\frac{1}{x}\right)\left(\frac{1}{x}\right)=\frac{1}{x}-\frac{1}{x^2}=\frac{x-1}{x^2}$; domain: $(-\infty,0)\cup(0,\infty)$.

d. $\frac{f}{g}=\frac{1-\frac{1}{x}}{\frac{1}{x}}=\frac{\frac{x-1}{x}}{\frac{1}{x}}=x-1$. Neither f nor g is defined for $x=0$, so the domain is $(-\infty,0)\cup(0,\infty)$.

e. $\frac{g}{f}=\frac{\frac{1}{x}}{1-\frac{1}{x}}=\frac{\frac{1}{x}}{\frac{x-1}{x}}=\frac{1}{x-1}$. Neither f nor g is defined for $x=0$, and $\frac{g}{f}$ is not defined for $x=1$, so the domain is $(-\infty,0)\cup(0,1)\cup(1,\infty)$.

13. $(g\circ f)(x)=2\left(x^2-1\right)+3=2x^2+1$;
$(g\circ f)(2)=2\left(2^2-1\right)+3=9$;
$(g\circ f)(-3)=2\left((-3)^2-1\right)+3=19$

14. $(g\circ f)(x)=3|x+1|^2-1=3(x^2+2x+1)-1$
$=3x^2+6x+2$;
$(g\circ f)(2)=3|2+1|^2-1=26$;
$(g\circ f)(-3)=3|(-3)+1|^2-1=11$

15. $(f \circ g)(2) = 2\left(2(2^2) - 3\right) + 1 = 11$

16. $(g \circ f)(2) = 2\left(2(2) + 1\right)^2 - 3 = 47$

17. $(f \circ g)(-3) = 2\left(2(-3^2) - 3\right) + 1 = 31$

18. $(g \circ f)(-5) = 2\left(2(-5) + 1\right)^2 - 3 = 159$

19. $(f \circ g)(0) = 2\left(2(0^2) - 3\right) + 1 = -5$

20. $(g \circ f)\left(\frac{1}{2}\right) = 2\left(2\left(\frac{1}{2}\right) + 1\right)^2 - 3 = 5$

21. $(f \circ g)(-c) = 2\left(2(-c)^2 - 3\right) + 1 = 4c^2 - 5$

22. $(f \circ g)(c) = 2\left(2c^2 - 3\right) + 1 = 4c^2 - 5$

23. $$\begin{aligned}(g \circ f)(a) &= 2(2a+1)^2 - 3 \\ &= 2\left(4a^2 + 4a + 1\right) - 3 \\ &= 8a^2 + 8a - 1\end{aligned}$$

24. $$\begin{aligned}(g \circ f)(-a) &= 2(2(-a)+1)^2 - 3 \\ &= 2\left(4a^2 - 4a + 1\right) - 3 \\ &= 8a^2 - 8a - 1\end{aligned}$$

25. $(f \circ f)(1) = 2\left(2(1) + 1\right) + 1 = 7$

26. $(g \circ g)(-1) = 2\left(2(-1)^2 - 3\right)^2 - 3 = -1$

27. $(f \circ g)(x) = \dfrac{2}{\frac{1}{x} + 1} = \dfrac{2}{\frac{x+1}{x}} = \dfrac{2x}{x+1}.$

The domain of f is $(-\infty, -1) \cup (-1, \infty)$, while the domain of g is $(-\infty, 0) \cup (0, \infty)$. The domain of $f \circ g$ is $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$.

28. $$(f \circ g)(x) = \dfrac{1}{\frac{2}{x+3} - 1} = \dfrac{1}{\frac{2 - (x+3)}{x+3}} = \dfrac{x+3}{-x-1} = -\dfrac{x+3}{x+1}.$$

The domain of f is $(-\infty, -1) \cup (-1, \infty)$, while the domain of g is $(-\infty, -3) \cup (0, -3)$. The domain of $f \circ g$ is $(-\infty, -3) \cup (-3, -1) \cup (-1, \infty)$.

29. $(f \circ g)(x) = \sqrt{(2 - 3x) - 3} = \sqrt{-1 - 3x}.$

The domain of f is $[3, \infty)$, while the domain of g is $(-\infty, \infty)$. $f \circ g$ is defined for $x \le -\frac{1}{3}$.

So, the domain of $f \circ g$ (the intersection of the three sets listed) is $\left(-\infty, -\frac{1}{3}\right]$.

30. $(f \circ g)(x) = \dfrac{2 + 5x}{(2 + 5x) - 1} = \dfrac{2 + 5x}{1 + 5x}.$

The domain of f is $(-\infty, 1) \cup (1, \infty)$, while the domain of g is $(-\infty, \infty)$. $f \circ g$ is defined for $\left(-\infty, -\frac{1}{5}\right) \cup \left(-\frac{1}{5}, \infty\right)$. The domain of $f \circ g$ (the intersection of the three sets listed) is $\left(-\infty, -\frac{1}{5}\right) \cup \left(-\frac{1}{5}, \infty\right)$.

31. $(f \circ g)(x) = \left|x^2 - 1\right|$; domain: $(-\infty, \infty)$

32. $(f \circ g)(x) = 3|x - 1| - 2$; domain: $(-\infty, \infty)$

33.a. $(f \circ g)(x) = 2(x + 4) - 3 = 2x + 5$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = (2x - 3) + 4 = 2x + 1$; domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 2(2x - 3) - 3 = 4x - 9$; domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = (x + 4) + 4 = x + 8$; domain: $(-\infty, \infty)$

34.a. $(f \circ g)(x) = (3x - 5) - 3 = 3x - 8$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 3(x - 3) - 5 = 3x - 14$; domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = (x - 3) - 3 = x - 6$; domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 3(3x - 5) - 5 = 9x - 20$; domain: $(-\infty, \infty)$

35.a. $(f \circ g)(x) = 1 - 2(1 + x^2) = -2x^2 - 1$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 1 + (1 - 2x)^2 = 4x^2 - 4x + 2$; domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 1 - 2(1 - 2x) = 4x - 1$; domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 1 + (1 + x^2)^2 = x^4 + 2x^2 + 2$; domain: $(-\infty, \infty)$

36.a. $(f \circ g)(x) = 2(2x^2) - 3 = 4x^2 - 3;$
domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 2(2x-3)^2 = 8x^2 - 24x + 18;$
domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 2(2x-3) - 3 = 4x - 9;$
domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 2\left(2x^2\right)^2 = 8x^4;$
domain: $(-\infty, \infty)$

37.a. $(f \circ g)(x) = 2(2x-1)^2 + 3(2x-1)$
$= 2(4x^2 - 4x + 1) + 6x - 3$
$= 8x^2 - 2x - 1;$ domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 2(2x^2 + 3x) - 1 = 4x^2 + 6x - 1;$
domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 2\left(2x^2 + 3x\right)^2 + 3\left(2x^2 + 3x\right)$
$= 2\left(4x^4 + 12x^3 + 9x^2\right) + 6x^2 + 9x$
$= 8x^4 + 24x^3 + 24x^2 + 9x;$
domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 2(2x-1) - 1 = 4x - 3;$
domain: $(-\infty, \infty)$

38.a. $(f \circ g)(x) = (2x)^2 + 3(2x) = 4x^2 + 6x;$
domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 2(x^2 + 3x) = 2x^2 + 6x;$
domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = \left(x^2 + 3x\right)^2 + 3\left(x^2 + 3x\right)$
$= x^4 + 6x^3 + 9x^2 + 3x^2 + 9x$
$= x^4 + 6x^3 + 12x^2 + 9x;$
domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 2(2x) = 4x;$ domain: $(-\infty, \infty)$

39.a. $(f \circ g)(x) = \left(\sqrt{x}\right)^2 = x;$ domain: $[0, \infty)$

b. $(g \circ f)(x) = \sqrt{x^2} = |x|;$ domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = \left(x^2\right)^2 = x^4;$ domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = \sqrt{\sqrt{x}} = \sqrt[4]{x};$ domain: $[0, \infty)$

40.a. $(f \circ g)(x) = \left(\sqrt{x+2}\right)^2 + 2\sqrt{x+2}$
$= x + 2 + \sqrt{x+2};$ domain: $[-2, \infty)$

b. $(g \circ f)(x) = \sqrt{x^2 + 2x + 2};$ domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = (x^2 + 2x)^2 + 2(x^2 + 2x)$
$= x^4 + 4x^3 + 4x^2 + 2x^2 + 4x$
$= x^4 + 4x^3 + 6x^2 + 4x;$
domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = \sqrt{\sqrt{x+2} + 2};$ domain: $[-2, \infty)$

41.a. $(f \circ g)(x) = \dfrac{1}{2\left(\frac{1}{x^2}\right) - 1} = \dfrac{1}{\frac{2 - x^2}{x^2}}$
$= \dfrac{x^2}{2 - x^2} = -\dfrac{x^2}{x^2 - 2}$. The domain of f is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$, while the domain of g is $(-\infty, 0) \cup (0, \infty)$. $-\dfrac{x^2}{x^2 - 2}$ is defined for $\left(-\infty, -\sqrt{2}\right) \cup \left(-\sqrt{2}, \sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$. The domain of $f \circ g$ is the intersection of the sets: $\left(-\infty, -\sqrt{2}\right) \cup \left(-\sqrt{2}, 0\right) \cup \left(0, \sqrt{2}\right)$ $\cup \left(\sqrt{2}, \infty\right)$.

b. $(g \circ f)(x) = \dfrac{1}{\left(\frac{1}{2x-1}\right)^2} = \dfrac{1}{\frac{1}{4x^2 - 4x + 1}}$
$= 4x^2 - 4x + 1$. The domain of g is $(-\infty, 0) \cup (0, \infty)$, while the domain of f is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$. $4x^2 - 4x + 1$ is defined for $(-\infty, \infty)$. The domain of $g \circ f$ is the intersection of the sets:
$\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

c. $(f \circ f)(x) = \dfrac{1}{2\left(\frac{1}{2x-1}\right) - 1} = \dfrac{1}{\frac{2 - 2x + 1}{2x - 1}}$
$= \dfrac{2x-1}{3-2x} = -\dfrac{2x-1}{2x-3}$; The domain of f is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$. $-\dfrac{2x-1}{2x-3}$ is defined

for $\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$, so the domain of $f \circ f$ is the intersection of the sets: $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$.

d. $(g \circ g)(x) = \dfrac{1}{\left(\frac{1}{x^2}\right)} = x^4$. The domain of g is $(-\infty, 0) \cup (0, \infty)$, while x^4 is defined for all real numbers. The domain of $g \circ g$ is the intersection of the sets: $(-\infty, 0) \cup (0, \infty)$.

42.a. $(f \circ g)(x) = \dfrac{x}{x+1} - 1 = \dfrac{x-(x+1)}{x+1} = -\dfrac{1}{x+1}$. The domain of f is all real numbers, while the domain of g is $(-\infty, -1) \cup (-1, \infty)$. The domain of $f \circ g$ is the intersection of the sets: $(-\infty, -1) \cup (-1, \infty)$.

b. $(g \circ f)(x) = \dfrac{x-1}{(x-1)+1} = \dfrac{x-1}{x}$. The domain of f is all real numbers, while the domain of g is $(-\infty, -1) \cup (-1, \infty)$. $\dfrac{x-1}{x}$ is defined for $(-\infty, 0) \cup (0, \infty)$. The domain of $g \circ f$ is the intersection of the sets: $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$.

c. $(f \circ f)(x) = (x-1) - 1 = x - 2$; domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = \dfrac{\frac{x}{x+1}}{\frac{x}{x+1} + 1} = \dfrac{\frac{x}{x+1}}{\frac{x+x+1}{x+1}} = \dfrac{x}{2x+1}$. The domain of g is $(-\infty, -1) \cup (-1, \infty)$, while $\dfrac{x}{2x+1}$ is defined for $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$. The domain of $g \circ g$ is the intersection of the sets: $(-\infty, -1) \cup \left(-1, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$.

43.a. $(f \circ g)(x) = |-2| = 2$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = -2$; domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = \left|(|x|)\right| = |x|$; domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = -2$; domain: $(-\infty, \infty)$

44.a. $(f \circ g)(x) = 3$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 5$; domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 3$; domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 5$; domain: $(-\infty, \infty)$

45.a. $(f \circ g)(x) = 1 + \dfrac{1}{\frac{1+x}{1-x}} = 1 + \dfrac{1-x}{1+x} = \dfrac{1+x+1-x}{1+x} = \dfrac{2}{1+x}$. The domain of f is $(-\infty, 0) \cup (0, \infty)$, while the domain of g is $(-\infty, 1) \cup (1, \infty)$. $f \circ g$ is defined for $(-\infty, -1) \cup (-1, \infty)$. The domain of $f \circ g$ is the intersection of the sets: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

b. $(g \circ f)(x) = \dfrac{1 + 1 + \frac{1}{x}}{1 - \left(1 + \frac{1}{x}\right)} = \dfrac{\frac{2x+1}{x}}{-\frac{1}{x}} = -2x - 1$. The domain of g is $(-\infty, 1) \cup (1, \infty)$, while the domain of f is $(-\infty, 0) \cup (0, \infty)$. $g \circ f$ is defined for $(-\infty, \infty)$. The domain of $g \circ f$ is the intersection of the sets: $(-\infty, 0) \cup (0, \infty)$.

c. $(f \circ f)(x) = 1 + \dfrac{1}{1 + \frac{1}{x}} = 1 + \dfrac{1}{\frac{x+1}{x}} = 1 + \dfrac{x}{x+1} = \dfrac{2x+1}{x+1}$; The domain of f is $(-\infty, 0) \cup (0, \infty)$. $\dfrac{2x+1}{x+1}$ is defined for $(-\infty, -1) \cup (-1, \infty)$, so the domain of $f \circ f$ is the intersection of the sets: $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$.

d. $(g \circ g)(x) = \dfrac{1 + \frac{1+x}{1-x}}{1 - \frac{1+x}{1-x}} = \dfrac{\frac{2}{1-x}}{\frac{-2x}{1-x}} = -\dfrac{1}{x}$. The domain of g is $(-\infty, 1) \cup (1, \infty)$, while $-\dfrac{1}{x}$ is defined for $(-\infty, 0) \cup (0, \infty)$. The domain of $g \circ g$ is the intersection of the sets: $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$.

46.a. $(f \circ g)(x) = \sqrt[3]{(x^3 + 1) + 1} = \sqrt[3]{x^3 + 2}$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = \left(\sqrt[3]{x+1}\right)^3 + 1 = x + 2;$ domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = \sqrt[3]{\sqrt[3]{x+1} + 1};$ domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = (x^3 + 1)^3 + 1;$ domain: $(-\infty, \infty)$

47. $H(x) = \sqrt{x+2} \Rightarrow f(x) = \sqrt{x}, g(x) = x + 2$

48. $H(x) = |3x+2| \Rightarrow f(x) = |x|, g(x) = 3x + 2$

49. $H(x) = \left(x^2 - 3\right)^{10} \Rightarrow f(x) = x^{10}, g(x) = x^2 - 3$

50. $H(x) = \sqrt{3x^2} + 5 \Rightarrow f(x) = \sqrt{x} + 5, g(x) = 3x^2$

51. $H(x) = \dfrac{1}{3x-5} \Rightarrow f(x) = \dfrac{1}{x}, g(x) = 3x - 5$

52. $H(x) = \dfrac{5}{2x+3} \Rightarrow f(x) = \dfrac{5}{x}, g(x) = 2x + 3$

53. $H(x) = \sqrt[3]{x^2 - 7} \Rightarrow f(x) = \sqrt[3]{x}, g(x) = x^2 - 7$

54. $H(x) = \sqrt[4]{x^2 + x + 1} \Rightarrow f(x) = \sqrt[4]{x},$
$g(x) = x^2 + x + 1$

55. $H(x) = \dfrac{1}{|x^3 - 1|} \Rightarrow f(x) = \dfrac{1}{|x|}, g(x) = x^3 - 1$

56. $H(x) = \sqrt[3]{1 + \sqrt{x}} \Rightarrow f(x) = \sqrt[3]{x}, g(x) = 1 + \sqrt{x}$

2.7 B Exercises: Applying the Concepts

57.a. $f(x)$ is the cost function.

b. $g(x)$ is the revenue function.

c. $h(x)$ is the selling price of x shirts including sales tax.

d. $P(x)$ is the profit function.

58.a.
$$\begin{aligned} C(p) &= C(5000 - 5p) \\ &= 4(5000 - 5p) + 12{,}000 \\ &= 20{,}000 - 20p + 12{,}000 \\ &= 32{,}000 - 20p \end{aligned}$$

b. $R(p) = px = p(5000 - 5p) = 5000p - 5p^2$

c.
$$\begin{aligned} P(p) &= R(p) - C(p) \\ &= 5000p - 5p^2 - (32{,}000 - 20p) \\ &= -5p^2 + 5020p - 32{,}000 \end{aligned}$$

59.a.
$$\begin{aligned} P(x) &= R(x) - C(x) = 25x - (350 + 5x) \\ &= 20x - 350 \end{aligned}$$

b. $P(20) = 20(20) - 350 = 50.$ This represents the profit when 20 radios are sold.

c. $P(x) = 20x - 350; 500 = 20x - 350 \Rightarrow x = 43$

d. $C = 350 + 5x \Rightarrow x = \dfrac{C - 350}{5} = x(C).$

$(R \circ x)(C) = 25\left(\dfrac{C - 350}{5}\right) = 5C - 1750.$

This function represents the revenue in terms of the cost C.

60.a. $g(x) = 0.04x$

b. $h(x)$ is the after tax selling price of merchandise worth x dollars.

c. $f(x) = 0.02x + 3$

d. $T(x)$ represents the total price of merchandise worth x dollars, including the shipping and handling fee.

61.a. $f(x) = 0.7x$

b. $g(x) = x - 5$

c. $(g \circ f)(x) = 0.7x - 5$

d. $(f \circ g)(x) = 0.7(x - 5)$

e.
$$\begin{aligned} (f \circ g) - (g \circ f) &= 0.7(x - 5) - (0.7x - 5) \\ &= 0.7x - 0.35 - 0.7x + 5 \\ &= \$1.50 \end{aligned}$$

62.a. $f(x) = 0.8x$

b. $g(x) = 0.9x$

c. $(g \circ f)(x) = 0.9(0.8x) = 0.72x$

d. $(f \circ g)(x) = 0.8(0.9x) = 0.72x$

e. They are the same.

63.a. $f(x) = 1.1x; g(x) = x + 8$

b. $(f \circ g)(x) = 1.1(x + 8) = 1.1x + 8.8$. This represents a final test score computed by first adding 8 points to the original score and then increasing the total by 10%.

c. $(g \circ f)(x) = 1.1x + 8.$ This represents a final test score computed by first increasing the original score by 10% and then adding 8 points.

d. $(f \circ g)(70) = 1.1(70+8) = 85.8;$
$(g \circ f)(70) = 1.1(70) + 8 = 85.0;$

e. $(f \circ g)(x) \neq (g \circ f)(x)$

f. (i) $(f \circ g)(x) = 1.1x + 8.8 \geq 90 \Rightarrow x \geq 73.82$

(ii) $(g \circ f)(x) = 1.1x + 8 \geq 90 \Rightarrow x \geq 74.55$

64.a. $f(x)$ is a function that models 3% of an amount x.

b. $g(x)$ represents the amount of money that qualifies for a 3% bonus.

c. Her bonus is represented by $(f \circ g)(x)$.

d. $200 + 0.03(17,500 - 8000) = \$485$

e. $521 = 200 + 0.03(x - 8000) \Rightarrow x = \$18,700$

65.a. $f(x) = \pi x^2$

b. $g(x) = \pi(x+30)^2$

c. $g(x) - f(x)$ represents the area between the fountain and the fence.

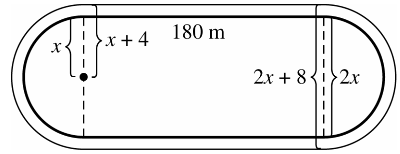
d. The circumference of the fence is $2\pi(x+30)$.
$10.5(2\pi(x+30)) = 4200 \Rightarrow \pi(x+30) = 200 \Rightarrow$
$\pi x + 30\pi = 200 \Rightarrow \pi x = 200 - 30\pi.$
$g(x) - f(x) = \pi(x+30)^2 - \pi x^2$
$= \pi(x^2 + 60x + 900) - \pi x^2$
$= 60\pi x + 900\pi.$ Now substitute $200 - 30\pi$ for πx to compute the estimate:
$1.75[60(200 - 30\pi) + 900\pi]$
$= 1.75(12,000 - 900\pi) \approx \$16,052.$

66.a. $f(x) = 180(2x+8) + \pi(x+4)^2$
$= 1440 + 360x + \pi(x+4)^2$

b. $g(x) = 2x(180) + \pi x^2 = 360x + \pi x^2$

c. $f(x) - g(x)$ represents the area of the track.

d.



(i) First find the radius of the inner track:
$900 = 2\pi x + 360 \Rightarrow \frac{270}{\pi} = x$. Use this value to compute $f(x) - g(x)$:

$$f\left(\frac{270}{\pi}\right) - g\left(\frac{270}{\pi}\right)$$
$$= \left(1440 + 360\left(\frac{270}{\pi}\right) + \pi\left(\frac{270}{\pi} + 4\right)^2\right)$$
$$- \left(360\left(\frac{270}{\pi}\right) + \pi\left(\frac{270}{\pi}\right)^2\right)$$
$$= 1440 + 360\left(\frac{270}{\pi}\right) + \frac{270^2}{\pi} + 2160 + 16\pi$$
$$-360\left(\frac{270}{\pi}\right) - \frac{270^2}{\pi}$$
$$= 3600 + 16\pi \approx 3650.26 \text{ square meters}$$

(ii) The outer perimeter
$$= 360 + 2\pi\left(\frac{270}{\pi} + 4\right) \approx 925.13 \text{ meters}$$

67.a. $(f \circ g)(t) = \pi(2t+1)^2$

b. $A(t) = f(2t+1) = \pi(2t+1)^2$

c. They are the same.

68.a. $(f \circ g)(t) = \frac{4}{3}\pi(2t)^3 = \frac{32}{3}\pi t^3$

b. $V(t) = \frac{4}{3}\pi(2t)^3 = \frac{32}{3}\pi t^3$

c. They are the same.

69.a. $f(x) = 1.7559x$

b. $g(x) = 0.05328x$

c. $(f \circ g)(x)$ first converts pesos to pounds and then pounds to dollars. So this function converts pesos to dollars.

d. $(f \circ g)(1000) = 1.7559(0.05328(1000))$
$= \$93.55$

2.7 C Exercises: Beyond the Basics

70.a. The sum of two even functions is an even function. $f(x)=f(-x)$ and $g(x)=g(-x)\Rightarrow$
$(f+g)(x)=f(x)+g(x)=f(-x)+g(-x)$
$=(f+g)(-x)$.

b. The sum of two odd functions is an odd function.
$f(-x)=-f(x)$ and $g(-x)=-g(x)\Rightarrow$
$(f+g)(-x)=f(-x)+g(-x)=-f(x)-g(x)$
$=-(f+g)(x)$.

c. The sum of an even function and an odd function is neither even nor odd.
$f(x)$ even $\Rightarrow f(x)=f(-x)$ and $g(x)$ odd $\Rightarrow$
$g(-x)=-g(x)\Rightarrow f(-x)+g(-x)=$
$f(x)+(-g(x))$, which is neither even nor odd.

d. The product of two even functions is an even function. $f(x)=f(-x)$ and $g(x)=g(-x)\Rightarrow$
$(f\bullet g)(x)=f(x)\bullet g(x)=f(-x)\bullet(g(-x))$
$=(f\bullet g)(-x)$.

e. The product of two odd functions is an even function.
$f(-x)=-f(x)$ and $g(-x)=-g(x)\Rightarrow$
$(f\bullet g)(-x)=f(-x)\bullet g(-x)=-f(x)\bullet(-g(x))$
$=(f\bullet g)(x)$.

f. The product of an even function and an odd function is an odd function.
$f(x)$ even $\Rightarrow f(x)=f(-x)$ and $g(x)$ odd $\Rightarrow$
$g(-x)=-g(x)\Rightarrow f(-x)\bullet g(-x)=f(x)\bullet(-g(x))$
$=-(f\bullet g)(x)$.

71.a. $f(-x)=-f(x)$ and $g(-x)=-g(x)\Rightarrow$
$(f\circ g)(-x)=f(g(-x))=f(-g(x))=$
$-f(g(x))\Rightarrow(f\circ g)(x)$ is odd.

b. $f(x)=f(-x)$ and $g(x)=g(-x)\Rightarrow$
$(f\circ g)(-x)=f(g(-x))=f(g(x))\Rightarrow$
$(f\circ g)(x)$ is even.

c. $f(x)$ odd $\Rightarrow f(-x)=-f(x)$ and
$g(x)$ even $\Rightarrow g(x)=g(-x)\Rightarrow(f\circ g)(-x)$
$f(g(x))=f(g(-x))\Rightarrow(f\circ g)(x)$ is even.

d. $f(x)$ even $\Rightarrow f(x)=f(-x)$ and $g(x)$ odd $\Rightarrow$
$g(-x)=-g(x)\Rightarrow(f\circ g)(-x)=f(-g(x))$
$=f(g(x))=(f\circ g)(-x)\Rightarrow(f\circ g)(x)$ is even.

72.a. $f(-x)=h(-x)+h(-(-x))=h(-x)+h(x)$
$=f(x)\Rightarrow f(x)$ is an even function.

b. $g(-x)=h(-x)-h(-(-x))=h(-x)-h(x)$
$=-g(x)\Rightarrow g(x)$ is an odd function.

c. $\begin{cases}f(x)=h(x)+h(-x)\\ g(x)=h(x)-h(-x)\Rightarrow\end{cases}$
$f(x)+g(x)=2h(x)\Rightarrow$
$h(x)=\dfrac{f(x)+g(x)}{2}=\dfrac{f(x)}{2}+\dfrac{g(x)}{2}\Rightarrow$
$h(x)$ is the sum of an even function and an odd function.

73.a. $h(x)=x^2-2x+3\Rightarrow f(x)=x^2$ (even),
$g(x)=-2x+3$ (odd)

b. $h(x)=[\![x]\!]+x\Rightarrow f(x)=\dfrac{[\![x]\!]+[\![-x]\!]}{2}$ (even),
$g(x)=x+\dfrac{[\![x]\!]-[\![-x]\!]}{2}$ (odd)

2.7 Critical Thinking

74.a. The domain of $f(x)$ is $(-\infty,0)\cup[1,\infty)$.

b. The domain of $g(x)$ is [0, 2].

c. The domain of $f(x)+g(x)$ is [1, 2].

d. The domain of $\dfrac{f(x)}{g(x)}$ is [1, 2).

75.a. The domain of *f* is $(-\infty,0)$. The domain of $f\circ f$ is $\varnothing$ because $f\circ f=\dfrac{1}{\sqrt{-\dfrac{1}{\sqrt{-x}}}}$ and the denominator is the square root of a negative number.

b. The domain of *f* is $(-\infty,1)$. The domain of $f\circ f$ is $(-\infty,0)$ because
$f\circ f=\dfrac{1}{\sqrt{1-\dfrac{1}{\sqrt{1-x}}}}$ and the denominator must be greater than 0. If $x=0$, then the denominator = 0.

Section 2.8 Inverse Functions

2.8 A Exercises: Basic Skills and Concepts

1. Inverse = {(01970, Salem MA), (38736, Doddsville MS), (68102, Omaha NE), (94203, Sacramento, CA), (96772, Naalehu HI). This is a function.

2. Inverse = {(23, Equinox), (26, Malibu), (27, Colorado), (27, Impala), (31, Monte Carlo). This is not a function because there are two different types of Chevrolet corresponding to 27 miles per gallon.

3. Inverse = {(–3, 13), (–2, 8), (–1, –1), (1, –1), (2, –8), (3, –13)}. This is a function.

4. Inverse = {(1, 0), (2, –1), (2, 1), (5, –2), (5, 2), (10, –3), (10, 3)}. This is not a function.

5. one-to-one **6.** not one-to-one

7. not one-to-one **8.** one-to-one

9. not one-to-one **10.** not one-to-one

11. one-to-one **12.** not one-to-one

13. $f(2)=7 \Rightarrow f^{-1}(7)=2$

14. $f^{-1}(4)=-7 \Rightarrow f(-7)=4$

15. $f(-1)=2 \Rightarrow f^{-1}(2)=-1$

16. $f^{-1}(-3)=5 \Rightarrow f(5)=-3$

17. $f(a)=b \Rightarrow f^{-1}(b)=a$

18. $f^{-1}(c)=d \Rightarrow f(d)=c$

19. $\left(f^{-1}\circ f\right)(337)=f^{-1}\left(f(337)\right)=337$

20. $\left(f\circ f^{-1}\right)(25\pi)=f\left(f^{-1}(25\pi)\right)=25\pi$

21. $\left(f\circ f^{-1}\right)(-1580)=f\left(f^{-1}(-1580)\right)=-1580$

22. $\left(f^{-1}\circ f\right)(9728)=f^{-1}\left(f(9728)\right)=9728$

23.a. $f(3)=2(3)-3=3$

b. Using the result from part (a), $f^{-1}(3)=3$

c. $\left(f\circ f^{-1}\right)(19)=f\left(f^{-1}(19)\right)=19$

d. $\left(f\circ f^{-1}\right)(5)=f\left(f^{-1}(5)\right)=5$.

24.a. $f(2)=2^3=8$

b. Using the result from part (a), $f^{-1}(8)=2$.

c. $\left(f\circ f^{-1}\right)(15)=f\left(f^{-1}(15)\right)=15$

d. $\left(f\circ f^{-1}\right)(27)=f\left(f^{-1}(27)\right)=27$

25.a. $f(1)=1^3+1=2$

b. Using the result from part (a), $f^{-1}(2)=1$.

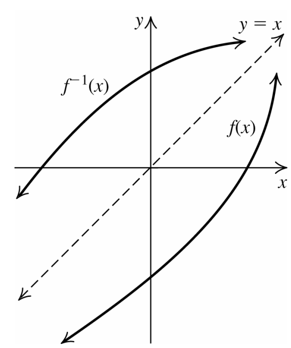
c. $\left(f\circ f^{-1}\right)(269)=f\left(f^{-1}(269)\right)=269$

26.a. $g(1)=\sqrt[3]{2\left(1^3\right)-1}=\sqrt[3]{1}=1$

b. Using the result from part (a), $g^{-1}(1)=1$.

c. $\left(g\circ g^{-1}\right)(135)=g\left(g^{-1}(135)\right)=135$

27.



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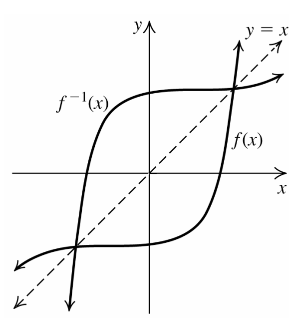

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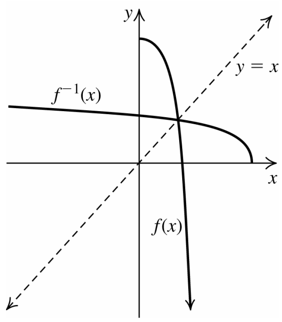

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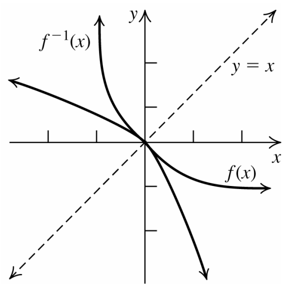

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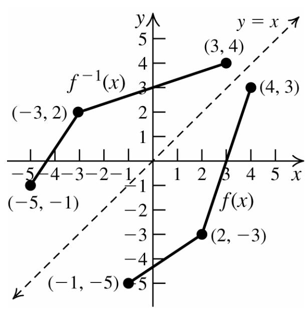

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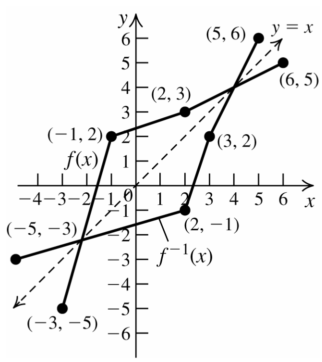


33.

x	3	4	5	6	7
$f^{-1}(x)$	–2	–1	0	1	2

34.

x	–2	0	1	2	3
$f^{-1}(x)$	3	–1	1	–3	5

35. $f\left(g\left(x\right)\right) = 3\left(\frac{x-1}{3}\right) + 1 = x - 1 + 1 = x$

$g\left(f\left(x\right)\right) = \frac{(3x+1)-1}{3} = \frac{3x}{3} = x$

36. $f\left(g\left(x\right)\right) = 2 - 3\left(\frac{2-x}{3}\right) = 2 - 2 + x = x$

$g\left(f\left(x\right)\right) = \frac{2-(2-3x)}{3} = \frac{3x}{3} = x$

37. $f\left(g\left(x\right)\right) = \left(\sqrt[3]{x}\right)^3 = x$

$g\left(f\left(x\right)\right) = \sqrt[3]{x^3} = x$

38. $f\left(g\left(x\right)\right) = g\left(f\left(x\right)\right) = \frac{1}{\frac{1}{x}} = x$

39. $$f\left(g\left(x\right)\right) = \frac{\frac{1+2x}{1-x} - 1}{\frac{1+2x}{1-x} + 2} = \frac{\frac{1+2x-(1-x)}{1-x}}{\frac{1+2x+2(1-x)}{1-x}} = \frac{3x}{3} = x$$

$$g\left(f\left(x\right)\right) = \frac{1 + 2\left(\frac{x-1}{x+2}\right)}{1 - \frac{x-1}{x+2}} = \frac{1 + \frac{2x-2}{x+2}}{1 - \frac{x-1}{x+2}} = \frac{\frac{x+2+2x-2}{x+2}}{\frac{x+2-(x-1)}{x+2}} = \frac{3x}{3} = x$$

40. $$f\left(g\left(x\right)\right) = \frac{3\left(\frac{x+2}{x-3}\right) + 2}{\frac{x+2}{x-3} - 1} = \frac{\frac{3x+6}{x-3} + \frac{2(x-3)}{x-3}}{\frac{x+2}{x-3} - \frac{1(x-3)}{x-3}} = \frac{\frac{3x+6+2x-6}{x-3}}{\frac{x+2-x+3}{x-3}} = \frac{5x}{5} = x$$

$$g\left(f\left(x\right)\right) = \frac{\frac{3x+2}{x-1} + 2}{\frac{3x+2}{x-1} - 3} = \frac{\frac{3x+2}{x-1} + \frac{2(x-1)}{x-1}}{\frac{3x+2}{x-1} - \frac{3(x-1)}{x-1}} = \frac{\frac{3x+2+2x-2}{x-1}}{\frac{3x+2-3x+3}{x-1}} = \frac{5x}{5} = x$$

In exercises 41 and 42, use the fact that the range of f is the same as the domain of f^{-1}.

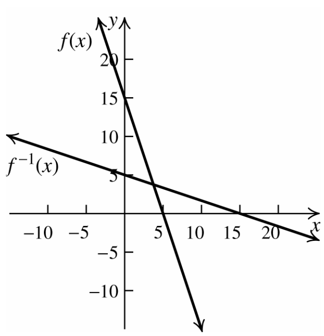
41. Domain: $(-\infty,-2)\cup(-2,\infty)$; range: $(-\infty,1)\cup(1,\infty)$.

42. Domain: $(-\infty,1)\cup(1,\infty)$; range: $(-\infty,3)\cup(3,\infty)$.

43.a. one-to-one

b. $f(x)=y=15-3x$. Interchange the variables and solve for $y: x=15-3y\Rightarrow$
$\dfrac{15-x}{3}=5-\dfrac{1}{3}x=y=f^{-1}(x)$.

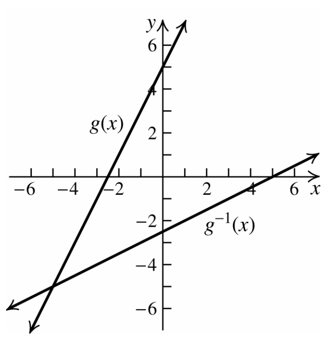
c.



44.a. one-to-one

b. $g(x)=y=2x+5$. Interchange the variables and solve for $y: x=2y+5\Rightarrow$
$y=\dfrac{x-5}{2}=\dfrac{1}{2}x-\dfrac{5}{2}=g^{-1}(x)$.

c.



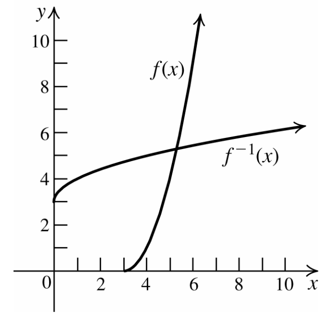
45.a. not one-to-one

46.a. not one-to-one

47.a. one-to-one

b. $f(x)=y=\sqrt{x}+3$ Interchange the variables and solve for $y: x=\sqrt{y}+3\Rightarrow$
$x-3=\sqrt{y}\Rightarrow y=(x-3)^2=f^{-1}(x)$.

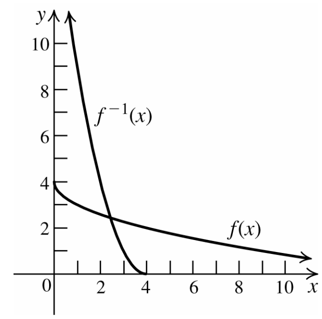
c.



48.a. one-to-one

b. $f(x)=y=4-\sqrt{x}$. Interchange the variables and solve for $y: x=4-\sqrt{y}\Rightarrow$
$-4+x=-\sqrt{y}\Rightarrow y=(x-4)^2=f^{-1}(x)$

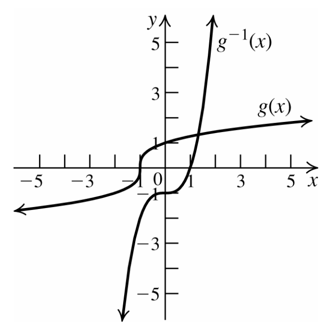
c.



49.a. one-to-one

b. $g(x)=y=\sqrt[3]{x+1}$. Interchange the variables and solve for $y: x=\sqrt[3]{y+1}\Rightarrow$
$x^3=y+1\Rightarrow y=x^3-1=g^{-1}(x)$

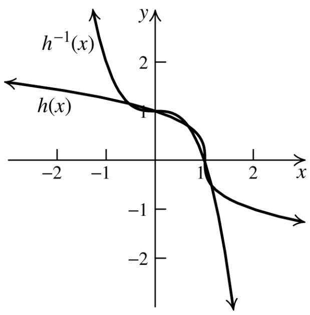
c.



50.a. one-to-one

b. $h(x)=y=\sqrt[3]{1-x}$. Interchange the variables and solve for $y: x=\sqrt[3]{1-y}\Rightarrow$
$x^3=1-y\Rightarrow y=1-x^3=g^{-1}(x)$.

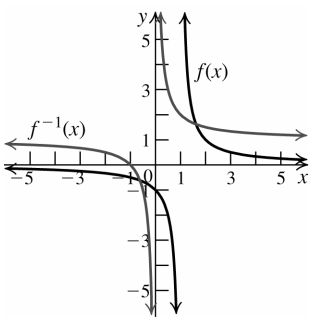
c.



51.a. one-to-one

b. $f(x) = y = \dfrac{1}{x-1}$. Interchange the variables and solve for $y: x = \dfrac{1}{y-1} \Rightarrow x(y-1) = 1 \Rightarrow$ $\dfrac{1}{x} = y - 1 \Rightarrow y = \dfrac{1}{x} + 1 = \dfrac{1+x}{x} = f^{-1}(x).$

c.

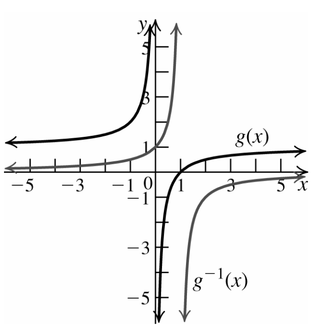


52.a. one-to-one

b. $g(x) = y = 1 - \dfrac{1}{x}$. Interchange the variables and solve for $y: x = 1 - \dfrac{1}{y} \Rightarrow x = \dfrac{y-1}{y} \Rightarrow$

$xy = y - 1 \Rightarrow xy - y = -1 \Rightarrow y(x-1) = -1 \Rightarrow$ $y = -\dfrac{1}{x-1} = \dfrac{1}{1-x} = g^{-1}(x)$

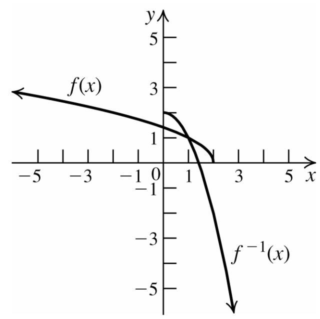
c.



53.a. one-to-one

b. $f(x) = y = \sqrt{2-x}$. Interchange the variables and solve for $y: x = \sqrt{2-y} \Rightarrow x^2 = 2 - y \Rightarrow$ $y = 2 - x^2 = f^{-1}(x).$

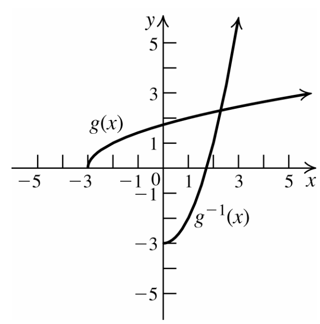
c.



54.a. one-to-one

b. $f(x) = y = \sqrt{3+x}$. Interchange the variables and solve for $y: x = \sqrt{3+y} \Rightarrow x^2 = 3 + y \Rightarrow$ $y = x^2 - 3 = f^{-1}(x).$

c.



55. $f(x) = y = \dfrac{x+1}{x-2}$. Interchange the variables and solve for $y: x = \dfrac{y+1}{y-2} \Rightarrow xy - 2x = y + 1 \Rightarrow$ $xy - y = 2x + 1 \Rightarrow y(x-1) = 2x + 1 \Rightarrow$ $y = \dfrac{2x+1}{x-1} = f^{-1}(x).$ Domain of *f*: $(-\infty, 2) \cup (2, \infty)$; range of *f*: $(-\infty, 1) \cup (1, \infty)$.

56. $g(x) = y = \dfrac{x+2}{x+1}$. Interchange the variables and solve for $y: x = \dfrac{y+2}{y+1} \Rightarrow xy + x = y + 2 \Rightarrow$ $xy - y = -x + 2 \Rightarrow y(x-1) = -x + 2 \Rightarrow$ $y = \dfrac{-x+2}{x-1} = \dfrac{x-2}{1-x} = g^{-1}(x).$ Domain of *f*: $(-\infty, -1) \cup (-1, \infty)$; range of *f*: $(-\infty, 1) \cup (1, \infty)$.

57. $f(x) = y = \frac{1-2x}{1+x}$. Interchange the variables and solve for $y: x = \frac{1-2y}{1+y} \Rightarrow$
$x + xy = 1 - 2y \Rightarrow xy + 2y = 1 - x \Rightarrow$
$y(x+2) = 1 - x \Rightarrow y = \frac{1-x}{x+2} = f^{-1}(x).$
Domain of *f*: $(-\infty, -1) \cup (-1, \infty)$; range of *f*: $(-\infty, -2) \cup (-2, \infty)$.

58. $h(x) = y = \frac{x-1}{x-3}$. Interchange the variables and solve for $y: x = \frac{y-1}{y-3} \Rightarrow xy - 3x = y - 1 \Rightarrow$
$xy - y = 3x - 1 \Rightarrow y(x-1) = 3x - 1 \Rightarrow$
$y = \frac{3x-1}{x-1} = h^{-1}(x).$
Domain of *f*: $(-\infty, 3) \cup (3, \infty)$; range of *f*: $(-\infty, 1) \cup (1, \infty)$.

2.8 B Exercises: Applying the Concepts

59.a. $K(C) = C + 273 \Rightarrow$
$C(K) = K - 273 = K^{-1}(C).$
This represents the Celsius temperature corresponding to a given Kelvin temperature.

b. $C(300) = 300 - 273 = 27°\text{C}$

c. $K(22) = 22 + 273 = 295°\text{K}$

60.a. The two points are (212, 373) and (32, 273). The rate of change is $\frac{373-273}{212-32} = \frac{100}{180} = \frac{5}{9}$.

$273 = \frac{5}{9}(32) + b \Rightarrow b = \frac{2297}{9} \Rightarrow$

$K(F) = \frac{5}{9}F + \frac{2297}{9}.$

b. $K = \frac{5}{9}F + \frac{2297}{9} \Rightarrow K - \frac{2297}{9} = \frac{5}{9}F \Rightarrow$
$9K - 2297 = 5F \Rightarrow \frac{9}{5}K - \frac{2297}{5} = F(K)$
This represents the Fahrenheit temperature corresponding to a given Kelvin temperature.

c. $K(98.6) = \frac{5}{9}(98.6) + \frac{2297}{9} = 310°\text{K}$

61.a.
$$\begin{aligned} F(K(C)) &= \frac{9}{5}(C+273) - \frac{2297}{5} \\ &= \frac{9}{5}C + \frac{9(273)}{5} - \frac{2297}{5} \\ &= \frac{9}{5}C + \frac{160}{5} = \frac{9}{5}C + 32 \end{aligned}$$

b.
$$\begin{aligned} C(K(F)) &= \frac{5}{9}F + \frac{2297}{9} - 273 \\ &= \frac{5}{9}F + \frac{2297 - 2457}{9} \\ &= \frac{5}{9}F - \frac{160}{9} \end{aligned}$$

62.
$$\begin{aligned} F(C(x)) &= \frac{9}{5}\left(\frac{5}{9}x - \frac{160}{9}\right) + 32 \\ &= x - 32 + 32 = x \\ C(F(x)) &= \frac{5}{9}\left(\frac{9}{5}x + 32\right) - \frac{160}{9} \\ &= x + \frac{160}{9} - \frac{160}{9} = x \end{aligned}$$
Therefore, *F* and *C* are inverses of each other.

63.a. $E(x) = 0.75x$ where x represents the number of dollars; $D(x) = 1.25x$ where x represents the number of euros.

b. $E(D(x)) = 0.75(1.25x) = 0.9375x \neq x.$
Therefore, the two functions are not inverses.

c. She loses money either way.

64.a. $w = 4 + 0.05x \Rightarrow w - 4 = 0.05x \Rightarrow$
$x = 20w - 80$. This represents the food sales terms of his hourly wage.

b. $x = 20(12) - 80 = \$160$

65.a. $7 = 4 + 0.05x \Rightarrow x = \$60$. This means that if food sales $\leq \$60$, he will receive the minimum hourly wage. If food sales $> \$60$, his wages will be based on food sales.
$$w = \begin{cases} 4 + 0.05x & \text{if } x > 60 \\ 7 & \text{if } x \leq 60 \end{cases}$$

b. The function does not have an inverse because it is constant on (0, 60), and it is not one-to-one.

c. If the domain is restricted to $[60, \infty)$, the function has an inverse.

66.a. $T = 1.11l \Rightarrow l = \frac{T}{1.11}$. This shows the length as the function of the period.

b. $l = \frac{2}{1.11} \approx 1.8$ ft

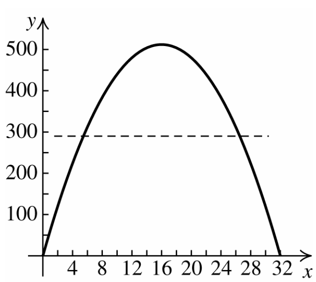
c. $T = 1.11(90) = 99.9$ sec

67.a. $V = 8\sqrt{x} \Rightarrow \frac{V}{8} = \sqrt{x} \Rightarrow \frac{1}{64}V^2 = x = V^{-1}(x)$
This represents the height of the water in terms of the velocity.

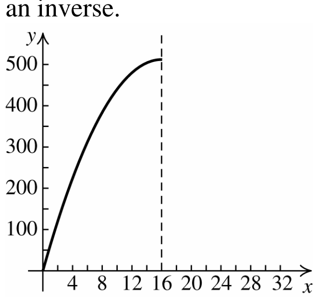
b. (i) $x = \frac{1}{64}\left(30^2\right) = 14.0625$ ft

(ii) $x = \frac{1}{64}\left(20^2\right) = 6.25$ ft

68.a. $y = 64x - 2x^2$ has no inverse because it is not one-to-one across its domain, [0, 32]. (It fails the horizontal line test.)

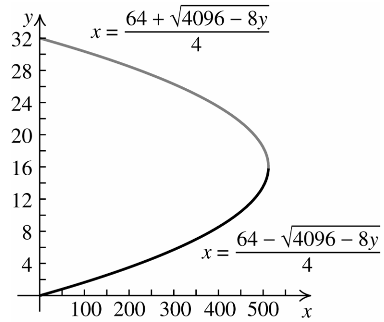


However, if the domain is restricted to [0, 16], the function is one-to-one, and it has an inverse.



$y = 64x - 2x^2 \Rightarrow 2x^2 - 64x + y = 0 \Rightarrow$
$x = \frac{64 \pm \sqrt{64^2 - 8y}}{4} \Rightarrow x = \frac{64 \pm \sqrt{4096 - 8y}}{4}$
$4096 - 8y \ge 0 \Rightarrow 0 \le y \le 512$. (Because y is a number of feet, it cannot be negative.) This is the range of the original function. The domain of the original function is [0, 16], which is the range of the inverse. The range of $x = \frac{64 + \sqrt{4096 - 8y}}{4}$ is [16, 32], so this is not the inverse.

The range of $x = \frac{64 - \sqrt{4096 - 8y}}{4}$ is [0, 16], so this is the inverse.



b. (i) $x = \frac{64 - \sqrt{4096 - 8(32)}}{4} \approx 0.51$ ft

(ii) $x = \frac{64 - \sqrt{4096 - 8(256)}}{4} \approx 4.69$ ft

(iii) $x = \frac{64 - \sqrt{4096 - 8(512)}}{4} \approx 16$ ft

69.a. The function represents the amount she still owes after x months.

b. $y = 36,000 - 600x$. Interchange the variables and solve for y: $x = 36,000 - 600y \Rightarrow$

$600y = 36,000 - x \Rightarrow y = 600 - \frac{x}{600} \Rightarrow$

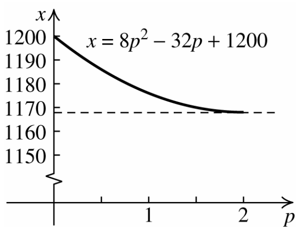
$f^{-1}(x) = 60 - \frac{1}{600}x$. This represents the number of months that have passed from the first payment until the balance due is \$$x$.

c. $y = 60 - \frac{1}{600}(22,000) = 23.33 \approx 24$ months

70.a. To find the inverse, solve $x = 8p^2 - 32p + 1200$ for p:
$8p^2 - 32p + 1200 - x = 0 \Rightarrow$

$$p = \frac{32 \pm \sqrt{(-32)^2 - 4(8)(1200 - x)}}{2(8)} = \frac{32 \pm \sqrt{1024 - 38,400 + 32x}}{16} = \frac{32 \pm \sqrt{32x - 37376}}{16} = \frac{32 \pm 4\sqrt{2x - 2336}}{16} = 2 \pm \frac{1}{4}\sqrt{2x - 2336}$$

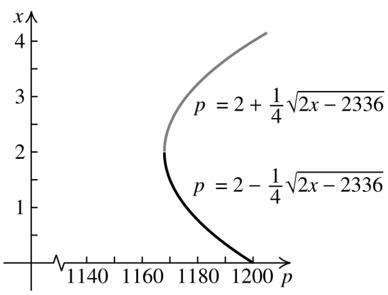
Because the domain of the original function is (0, 2], its range is (1168, 1200].



So the domain of the inverse is (1168, 1200], and its range is (0, 2]. The range of

$p = 2 + \frac{1}{4}\sqrt{2x - 2336}$ is (2, 4], so it is not the inverse. The range of

$p = 2 - \frac{1}{4}\sqrt{2x - 2336}$ is (0, 2], so it is the inverse.



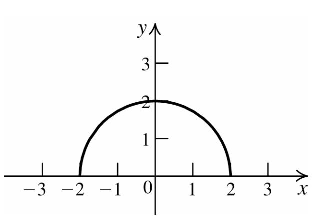
b. $p = 2 - \frac{1}{4}\sqrt{2(1180.5 - 2336} = \$0.75$

2.8 C Exercises: Beyond the Basics

71. $f(g(3)) = f(1) = 3, f(g(5)) = f(3) = 5,$ and $f(g(2)) = f(4) = 2 \Rightarrow f(g(x)) = x$ for each x.
$g(f(1)) = g(3) = 1, g(f(3)) = g(5) = 3,$ and $g(f(4)) = g(2) = 4 \Rightarrow g(f(x)) = x$ for each x.
So, f and g are inverses.

72. $f(g(-2)) = f(1) = -2, f(g(0)) = f(2) = 0,$ $f(g(-3)) = f(3) = -3,$ and $f(g(-2)) = f(1) = -2 \Rightarrow f(g(x)) = x$ for each x.
$g(f(1)) = g(-2) = 1, g(f(2)) = g(0) = 2,$ $g(f(3)) = g(-3) = 3,$ and $g(f(4)) = g(1) = 4$ $\Rightarrow g(f(x)) = x$ for each x.
So f and g are inverses.
So, f and g are inverses.

73.a.



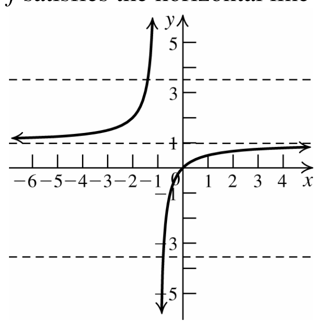
b. f is not one-to-one

c. Domain: [–2, 2]; range: [0, 2]

74.a. Domain: $(-\infty, 2) \cup [3, \infty)$. Note that the domain is not $(-\infty, 2) \cup (2, \infty)$ because $[\![x]\!] = 2$ for $2 \le x \le 3$.

b. The function is not one-to-one. The function is constant on each interval $[n, n + 1)$, n an integer.

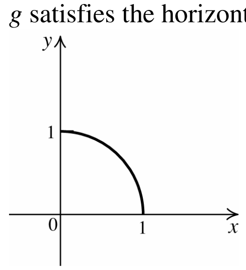
75.a. f satisfies the horizontal line test.



b. $y = 1 - \frac{1}{x+1}$. Interchange the variables and solve for $y: x = 1 - \frac{1}{y+1} \Rightarrow$
$\frac{1}{y+1} = 1 - x \Rightarrow 1 = y + 1 - xy - x \Rightarrow$
$xy - y = -x \Rightarrow y(x-1) = -x \Rightarrow$
$y = f^{-1}(x) = -\frac{x}{x-1} = \frac{x}{1-x}$

c. Domain of f: $(-\infty, -1) \cup (-1, \infty)$; range of f: $(-\infty, 1) \cup (1, \infty)$.

76.a. g satisfies the horizontal line test.



b. $y = \sqrt{1 - x^2}$. Interchange the variables and solve for $y: x = \sqrt{1 - y^2} \Rightarrow$
$x^2 = 1 - y^2 \Rightarrow y^2 = 1 - x^2 \Rightarrow$
$y = g^{-1}(x) = \sqrt{1 - x^2}$

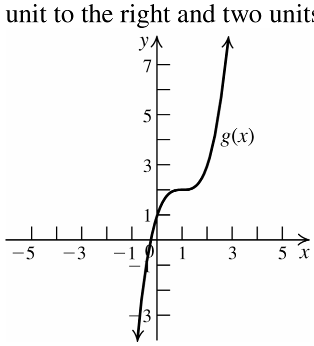
c. Domain of f = range of f: [0, 1]

77.a. $M = \left(\frac{3+7}{2}, \frac{7+3}{2}\right) = (5, 5)$. Since the coordinates of M satisfy the equation $y = x$, it lies on the line.

b. The slope of $y = x$ is 1, while the slope of PQ is $\frac{3-7}{7-3} = -1$. So, $y = x$ is perpendicular to PQ.

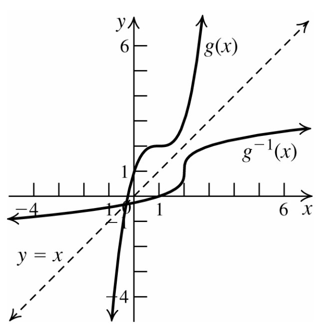
78. $M = \left(\frac{a+b}{2}, \frac{b+a}{2}\right)$. Since the coordinates of M satisfy the equation $y = x$, it lies on the line. The slope of the line segment between the two points is $\frac{b-a}{a-b} = -1$, while the slope of $y = x$ is 1. So the two lines are perpendicular.

79.a. The graph of g is the graph of f shifted one unit to the right and two units up.



b. $g(x) = y = (x-1)^3 + 2$. Interchange the variables and solve for $y: x = (y-1)^3 + 2 \Rightarrow y = g^{-1}(x) = \sqrt[3]{x-2} + 1$.

c.



80.a. (i) $f(x) = y = 2x - 1$. Interchange the variables and solve for y: $x = 2y - 1 \Rightarrow y = f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$

(ii) $g(x) = y = 3x + 4$. Interchange the variables and solve for y: $x = 3y + 4 \Rightarrow y = g^{-1}(x) = \frac{1}{3}x - \frac{4}{3}$

(iii) $(f \circ g)(x) = 2(3x+4) - 1 = 6x + 7$

(iv) $(g \circ f)(x) = 3(2x-1) + 4 = 6x + 1$

(v) $(f \circ g)(x) = y = 6x + 7$. Interchange the variables and solve for y: $x = 6y + 7 \Rightarrow (f \circ g)^{-1}(x) = \frac{1}{6}x - \frac{7}{6}$

(vi) $(g \circ f)(x) = y = 6x + 1$. Interchange the variables and solve for y: $x = 6y + 1 \Rightarrow (f \circ g)^{-1}(x) = \frac{1}{6}x - \frac{1}{6}$

(vii) $(f^{-1} \circ g^{-1})(x) = \frac{1}{2}\left(\frac{1}{3}x - \frac{4}{3}\right) + \frac{1}{2}$
$= \frac{1}{6}x - \frac{2}{3} + \frac{1}{2} = \frac{1}{6}x - \frac{1}{6}$

(viii) $(g^{-1} \circ f^{-1})(x) = \frac{1}{3}\left(\frac{1}{2}x + \frac{1}{2}\right) - \frac{4}{3}$
$= \frac{1}{6}x + \frac{1}{6} - \frac{4}{3} = \frac{1}{6}x - \frac{7}{6}$

b. $(f \circ g)^{-1}(x) = \frac{1}{6}x - \frac{7}{6} = (g^{-1} \circ f^{-1})(x)$
$(g \circ f)^{-1}(x) = \frac{1}{6}x - \frac{1}{6} = (f^{-1} \circ g^{-1})(x)$

81.a.(i) $f(x) = y = 2x + 3$. Interchange the variables and solve for y: $x = 2y + 3 \Rightarrow y = f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}$

(ii) $g(x) = y = x^3 - 1$. Interchange the variables and solve for y: $x = y^3 - 1 \Rightarrow y = g^{-1}(x) = \sqrt[3]{x+1}$

(iii) $(f \circ g)(x) = 2(x^3 - 1) + 3 = 2x^3 + 1$

(iv) $(g \circ f)(x) = (2x+3)^3 + 1$
$= 8x^3 + 36x^2 + 54x + 26$

(v) $(f \circ g)(x) = y = 2x^3 + 1$. Interchange the variables and solve for y: $x = 2y^3 + 1 \Rightarrow (f \circ g)^{-1}(x) = \sqrt[3]{\frac{x-1}{2}}$

(vi) $(g \circ f)(x) = y = 8x^3 + 36x^2 + 54x + 26.$
Interchange the variables and solve for y:
$x = 8y^3 + 36y^2 + 54y + 26 \Rightarrow$
$x + 1 = 8y^3 + 36y^2 + 54y + 27 \Rightarrow$
$x + 1 = (2y+3)^3 \Rightarrow \sqrt[3]{x+1} = 2y + 3 \Rightarrow$
$y = (g \circ f)^{-1}(x) = \frac{1}{2}\sqrt[3]{x+1} - \frac{3}{2}$

(vii) $\left(f^{-1} \circ g^{-1}\right)(x) = \frac{1}{2}\left(\sqrt[3]{x+1}\right) - \frac{3}{2}$

(viii) $\left(g^{-1} \circ f^{-1}\right)(x) = \sqrt[3]{\frac{1}{2}x - \frac{3}{2} + 1} = \sqrt[3]{\frac{1}{2}x - \frac{1}{2}}$

b. $(f \circ g)^{-1}(x) = \sqrt[3]{\frac{1}{2}x - \frac{1}{2}} = \left(g^{-1} \circ f^{-1}\right)(x)$

$(g \circ f)^{-1}(x) = \frac{1}{2}\left(\sqrt[3]{x+1}\right) - \frac{3}{2} = \left(f^{-1} \circ g^{-1}\right)(x)$

2.8 Critical Thinking

82. No. For example, $f(x) = x^3 - x$ is odd, but it does not have an inverse, because $f(0) = f(1)$, so it is not one-to-one.

83. Yes. The function $f = \{(0,1)\}$ is even, and it has an inverse: $f^{-1} = \{(1,0)\}$.

84. Yes, because increasing and decreasing functions are one-to-one.

85.a. $R = \{(-1,1), (0,0), (1,1)\}$

b. $R = \{(-1,1), (0,0), (1,2)\}$

Chapter 2: Review Exercises

1. False. The midpoint is $\left(\frac{-3+3}{2}, \frac{1+11}{2}\right) = (0,6).$

2. False. The equation is a circle with center $(-2,-3)$ and radius $\sqrt{5}.$

3. True

4. False. A graph that is symmetric with respect to the origin is the graph of an odd function. A graph that is symmetric with respect to the y-axis is the graph of an even function.

5. False. The slope is 4/3 and the y-intercept is 3.

6. False. The slope of a line that is perpendicular to a line with slope 2 is –1/2.

7. True

8. False. There is no graph because the radius cannot be negative.

9.a. $d(P,Q) = \sqrt{(-1-3)^2 + (3-5)^2} = 2\sqrt{5}$

b. $M = \left(\frac{3+(-1)}{2}, \frac{5+3}{2}\right) = (1,4)$

c. $m = \frac{3-5}{-1-3} = \frac{1}{2}$

10.a. $d(P,Q) = \sqrt{(3-(-3))^2 + (-1-5)^2} = 6\sqrt{2}$

b. $M = \left(\frac{-3+3}{2}, \frac{5+(-1)}{2}\right) = (0,2)$

c. $m = \frac{-1-5}{3-(-3)} = 1$

11.a. $d(P,Q) = \sqrt{(9-4)^2 + (-8-(-3))^2} = 5\sqrt{2}$

b. $M = \left(\frac{4+9}{2}, \frac{-3+(-8)}{2}\right) = \left(\frac{13}{2}, -\frac{11}{2}\right)$

c. $m = \frac{-8-(-3)}{9-4} = -1$

12.a. $d(P,Q) = \sqrt{(-7-2)^2 + (-8-3)^2} = \sqrt{202}$

b. $M = \left(\frac{2+(-7)}{2}, \frac{3+(-8)}{2}\right) = \left(-\frac{5}{2}, -\frac{5}{2}\right)$

c. $m = \frac{-8-3}{-7-2} = \frac{11}{9}$

13.a. $D(P,Q) = \sqrt{(5-2)^2 + (-2-(-7))^2} = \sqrt{34}$

b. $M = \left(\frac{2+5}{2}, \frac{-7+(-2)}{2}\right) = \left(\frac{7}{2}, -\frac{9}{2}\right)$

c. $m = \frac{-2-(-7)}{5-2} = \frac{5}{3}$

14.a. $d(P,Q) = \sqrt{(10-(-5))^2 + (-3-4)^2} = \sqrt{274}$

b. $M=\left(\frac{-5+10}{2},\frac{4+(-3)}{2}\right)=\left(\frac{5}{2},\frac{1}{2}\right)$

c. $m=\frac{-3-4}{10-(-5)}=-\frac{7}{15}$

15. $m_{AB}=\frac{0-5}{3-0}=-\frac{5}{3}; m_{CB}=\frac{0-(-3)}{3-(-2)}=\frac{3}{5}$

$m_{AB}\bullet m_{CB}=-1\Rightarrow AB\perp CB$, so $\triangle ABC$ is a right triangle.

16. $d(A,B)=\sqrt{(4-1)^2+(8-2)^2}=3\sqrt{5}$

$d(C,D)=\sqrt{(10-7)^2+(5-(-1))^2}=3\sqrt{5}$

$d(A,C)=\sqrt{(7-1)^2+(-1-2)^2}=3\sqrt{5}$

$d(B,D)=\sqrt{(10-4)^2+(5-8)^2}=3\sqrt{5}$

The four sides are equal, so the quadrilateral is a rhombus.

17. $A=(-6,3), B=(4,5)$

$d(A,O)=\sqrt{(-6-0)^2+(3-0)^2}=\sqrt{45}$

$d(B,O)=\sqrt{(4-0)^2+(5-0)^2}=\sqrt{41}$

(4, 5) is closer to the origin.

18. $A=(-6,4), B=(5,10), C=(2,3)$

$d(A,C)=\sqrt{(2-(-6))^2+(3-4)^2}=\sqrt{65}$

$d(B,C)=\sqrt{(2-5)^2+(3-10)^2}=\sqrt{58}$

(5, 10) is closer to (2, 3).

19. $A=(-5,3), B=(4,7), C=(x,0)$

$d(A,C)=\sqrt{(x-(-5))^2+(0-3)^2}=\sqrt{(x+5)^2+9}$

$d(B,C)=\sqrt{(x-4)^2+(0-7)^2}=\sqrt{(x-4)^2+49}$

$d(A,C)=d(B,C)\Rightarrow$

$\sqrt{(x+5)^2+9}=\sqrt{(x-4)^2+49}$

$(x+5)^2+9=(x-4)^2+49$

$x^2+10x+34=x^2-8x+65$

$x=\frac{31}{18}\Rightarrow$ The point is $\left(\frac{31}{18},0\right)$.

20. $A=(-3,-2), B(2,-1), C(0,y)$

$d(A,C)=\sqrt{(0-(-3))^2+(y-(-2))^2}=\sqrt{(y+2)^2+9}$

$d(B,C)=\sqrt{(0-(2))^2+(y-(-1))^2}=\sqrt{(y+1)^2+4}$

$d(A,C)=d(B,C)\Rightarrow$

$\sqrt{(y+2)^2+9}=\sqrt{(y+1)^2+4}$

$(y+2)^2+9=(y+1)^2+4$

$y^2+4y+13=y^2+2y+5$

$y=-4\Rightarrow$ The point is $(0,-4)$.

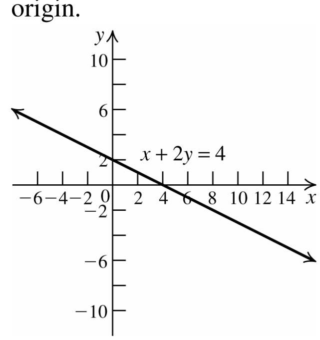
21. Not symmetric with respect to the *x*-axis; symmetric with respect to the *y*-axis; not symmetric with respect to the origin.

22. Not symmetric with respect to the *x*-axis; not symmetric with respect to the *y*-axis; symmetric with respect to the origin.

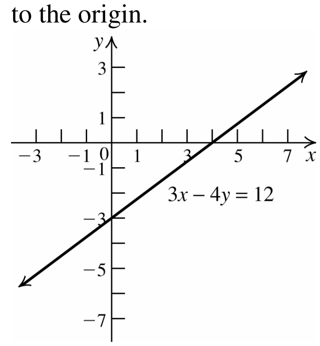
23. Symmetric with respect to the *x*-axis; not symmetric with respect to the *y*-axis; not symmetric with respect to the origin.

24. Symmetric with respect to the *x*-axis; symmetric with respect to the *y*-axis; symmetric with respect to the origin.

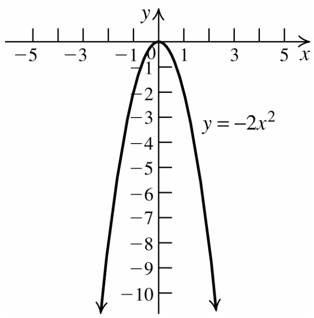
25. *x*-intercept: 4; *y*-intercept: 2; not symmetric with respect to the *x*-axis; not symmetric with respect to the *y*-axis; not symmetric with respect to the origin.



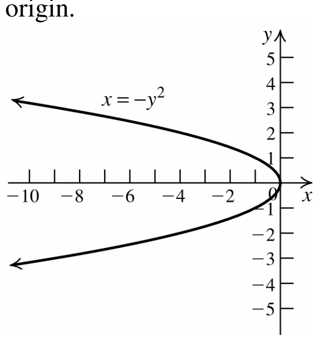
26. *x*-intercept: 4; *y*-intercept: –3; not symmetric with respect to the *x*-axis; not symmetric with respect to the *y*-axis; not symmetric with respect to the origin.



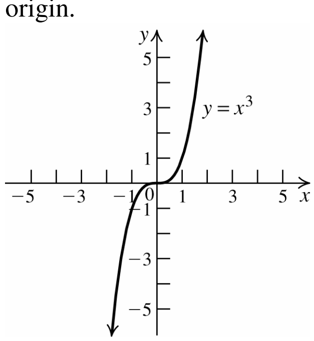
27. *x*-intercept: 0; *y*-intercept: 0; not symmetric with respect to the *x*-axis; symmetric with respect to the *y*-axis; not symmetric with respect to the origin.



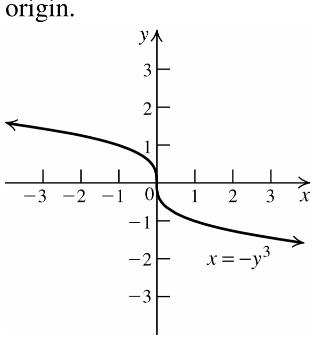
28. x-intercept: 0; y-intercept: 0; symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.



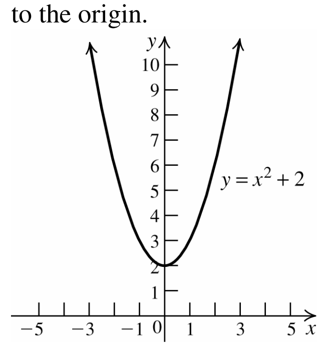
29. x-intercept: 0; y-intercept: 0; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.



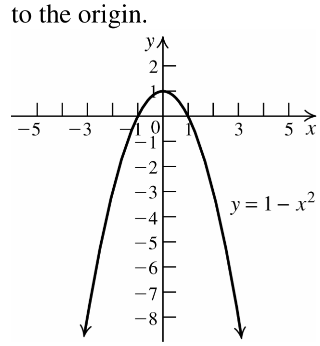
30. x-intercept: 0; y-intercept: 0; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.



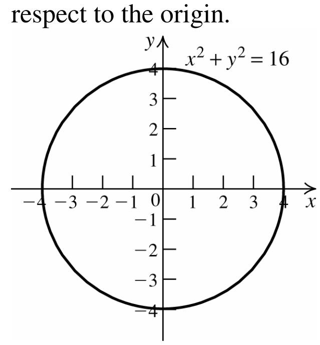
31. No x-intercept; y-intercept: 2; not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.



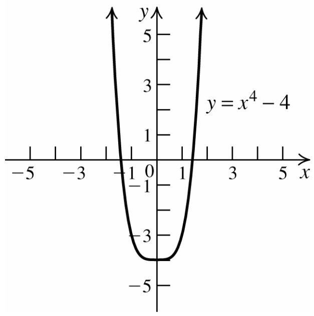
32. x-intercepts: –1, 1; y-intercept: 1; not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.



33. x-intercepts: –4, 4; y-intercepts: –4, 4; symmetric with respect to the x-axis; symmetric with respect to the y-axis; symmetric with respect to the origin.



34. x-intercepts: –2, 2; y-intercept: –4; not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.

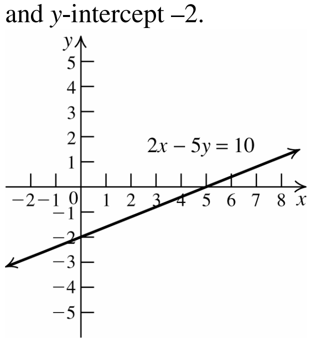


35. $(x-2)^2 + (y+3)^2 = 25$

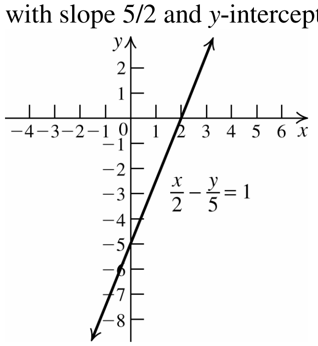
36. The center of the circle is the midpoint of the diameter. $M = \left(\frac{5+(-5)}{2} + \frac{2+4}{2}\right) = (0, 3)$. The length of the radius is the distance from the center to one of the endpoints of the diameter = $\sqrt{(5-0)^2 + (3-2)^2} = \sqrt{26}$. The equation of the circle is $x^2 + (y-3)^2 = 26$.

37. The radius is 2, so the equation of the circle is $(x+2)^2 + (y+5)^2 = 4$.

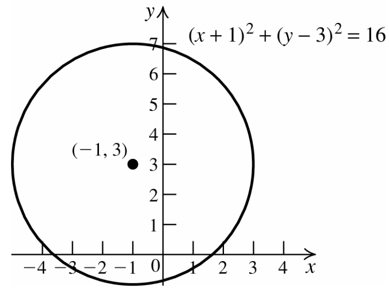
38. $2x - 5y = 10 \Rightarrow \frac{2}{5}x - 2 = y$. Line with slope 2/5 and y-intercept –2.



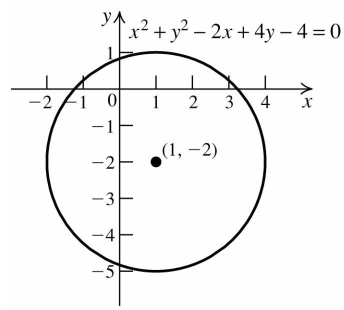
39. $\frac{x}{2} - \frac{y}{5} = 1 \Rightarrow 5x - 2y = 10 \Rightarrow \frac{5}{2}x - 5 = y$. Line with slope 5/2 and y-intercept –5.



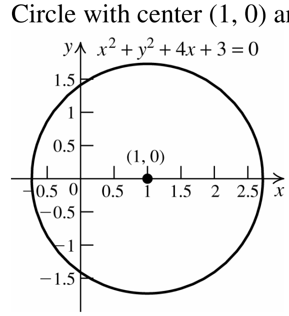
40. Circle with center (–1, 3) and radius 4.



41. $x^2 + y^2 - 2x + 4y - 4 = 0 \Rightarrow$
$x^2 - 2x + 1 + y^2 + 4y + 4 = 4 + 1 + 4 \Rightarrow$
$(x-1)^2 + (y+2)^2 = 9$. Circle with center (1, –2) and radius 3.



42. $3x^2 + 3y^2 - 6x - 6 = 0 \Rightarrow x^2 - 2x + y^2 = 2 \Rightarrow$
$x^2 - 2x + 1 + y^2 = 2 + 1 \Rightarrow (x-1)^2 + y^2 = 3$.
Circle with center (1, 0) and radius $\sqrt{3}$.



43. $y - 2 = -2(x-1) \Rightarrow y = -2x + 4$

44. $m = \frac{5-0}{0-2} = -\frac{5}{2}; y = -\frac{5}{2}x + 5$

45. $m = \frac{7-3}{-1-1} = -2; 3 = -2(1) + b \Rightarrow 5 = b \Rightarrow$
$y = -2x + 5$

46. $y = 2$ **47.** $x = 1$

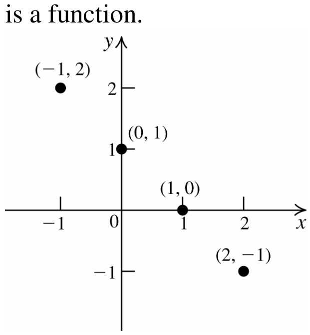
48.a. $y = 3x - 2 \Rightarrow m = 3; y = 3x + 2 \Rightarrow m = 3$. The slopes are equal, so the lines are parallel.

b. $3x - 5y + 7 \Rightarrow m = 3/5$;
$5x - 3y + 2 = 0 \Rightarrow m = 5/3$. The slopes are neither equal nor negative reciprocals, so the lines are neither parallel nor perpendicular.

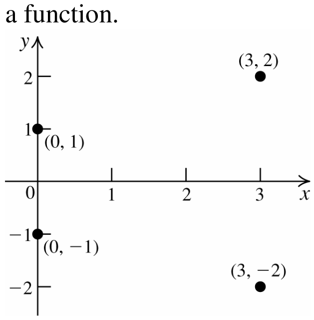
c. $ax + by + c = 0 \Rightarrow m = -a/b$;
$bx - ay + d = 0 \Rightarrow m = b/a$. The slopes are negative reciprocals, so the lines are perpendicular.

d. $y + 2 = \frac{1}{3}(x - 3) \Rightarrow m = \frac{1}{3}$;
$y - 5 = 3(x - 3) \Rightarrow m = 3$. The slopes are neither equal nor negative reciprocals, so the lines are neither parallel nor perpendicular.

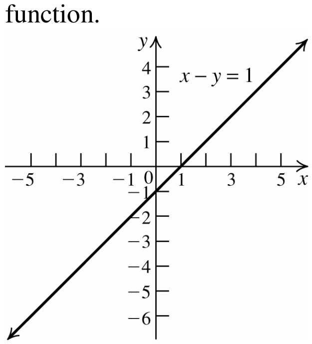
49. Domain: {–1, 0, 1, 2}; range: {–1, 0, 1, 2}. This is a function.



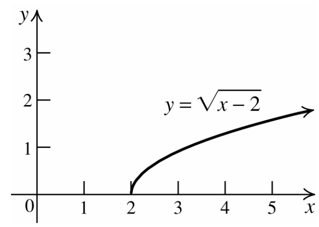
50. Domain: {0,3}; range: {–2, –1, 1, 2}. This is not a function.



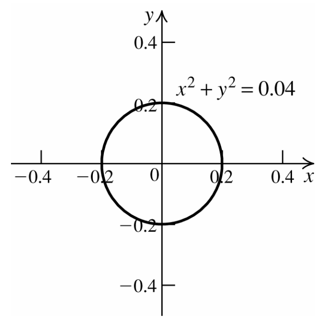
51. Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$. This is a function.



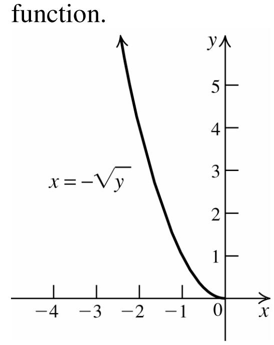
52. Domain: $[2, \infty)$; range: $[0, \infty)$. This is a function.



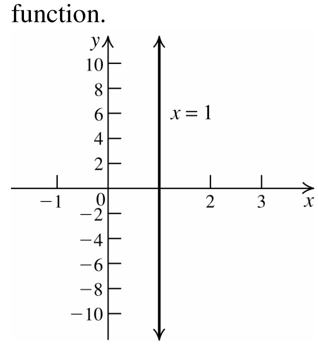
53. Domain: $[-0.2, 0.2]$; range: $[-0.2, 0.2]$. This is not a function.



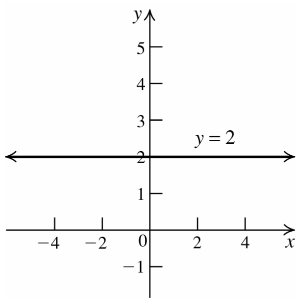
54. Domain: $(-\infty, 0]$; range: $[0, \infty)$. This is a function.



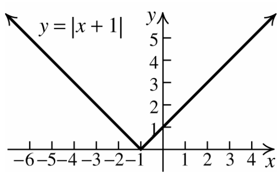
55. Domain: $\{1\}$; range: $(-\infty, \infty)$. This is not a function.



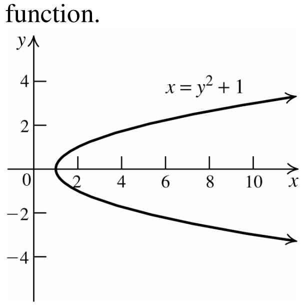
56. Domain: $(-\infty, \infty)$; range: $\{2\}$. This is a function.



57. Domain: $(-\infty, \infty)$; range: $[0, \infty)$. This is a function.



58. Domain: $[1, \infty)$; range: $(-\infty, \infty)$. This is not a function.



59. $f(-2) = 3(-2) + 1 = -5$

60. $g(-2) = (-2)^2 - 2 = 2$

61. $f(x) = 4 \Rightarrow 3x + 1 = 4 \Rightarrow x = 1$

62. $g(x) = 2 \Rightarrow x^2 - 2 = 2 \Rightarrow x = \pm 2$

63.
$$\begin{aligned}(f+g)(1) &= f(1) + g(1) \\ &= \left(3(1)+1\right) + \left(1^2 - 2\right) = 3\end{aligned}$$

64.
$$\begin{aligned}(f-g)(-1) &= f(-1) - g(-1) \\ &= \left(3(-1)+1\right) - \left((-1)^2 - 2\right) = -1\end{aligned}$$

65.
$$\begin{aligned}(f \bullet g)(-2) &= f(-2) \bullet g(-2) \\ &= \left(3(-2)+1\right) \bullet \left((-2)^2 - 2\right) = -10\end{aligned}$$

66.
$$\begin{aligned}(g \bullet f)(0) &= g(0) \bullet f(0) \\ &= \left(0^2 - 2\right) \bullet \left(3(0)+1\right) = -2\end{aligned}$$

67. $(f \circ g)(3) = 3\left(3^2 - 2\right) + 1 = 22$

68. $(g \circ f)(-2) = \left(3(-2)+1\right)^2 - 2 = 23$

69. $(f \circ g)(x) = 3\left(x^2 - 2\right) + 1 = 3x^2 - 5$

70. $(g \circ f)(x) = (3x+1)^2 - 2 = 9x^2 + 6x - 1$

71. $(f \circ f)(x) = 3(3x+1) + 1 = 9x + 4$

72. $(g \circ g)(x) = \left(x^2 - 2\right)^2 - 2 = x^4 - 4x^2 + 2$

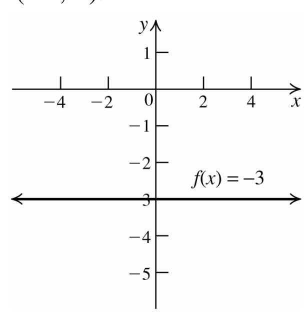
73. $f(a+h) = 3(a+h) + 1 = 3a + 3h + 1$

74. $g(a-h) = (a-h)^2 - 2 = a^2 - 2ah + h^2 - 2$

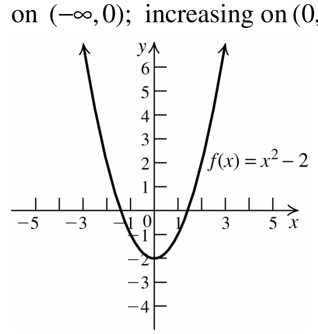
75.
$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{\left(3(x+h)+1\right) - (3x+1)}{h} \\ &= \frac{3x + 3h + 1 - 3x - 1}{h} = \frac{3h}{h} = 3\end{aligned}$$

76.
$$\begin{aligned}\frac{g(x+h) - g(x)}{h} &= \frac{\left((x+h)^2 - 2\right) - (x^2 - 2)}{h} \\ &= \frac{x^2 + 2xh + h^2 - 2 - x^2 + 2}{h} \\ &= \frac{h^2 + 2xh}{h} = h + 2x\end{aligned}$$

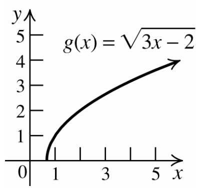
77. Domain: $(-\infty, \infty)$; range: $\{-3\}$. Constant on $(-\infty, \infty)$.



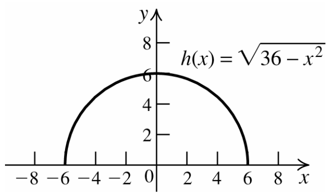
78. Domain: $(-\infty, \infty)$; range: $[-2, \infty)$. Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$.



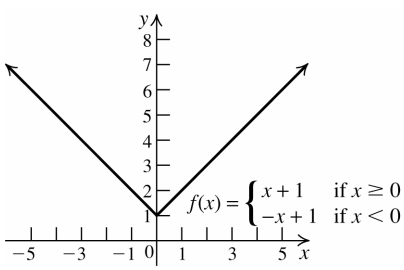
79. Domain: $(-\infty, \infty)$; range: $\{-3\}$. Constant on $(-\infty, \infty)$.



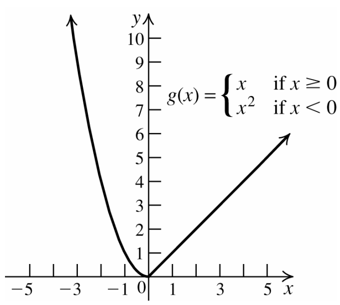
80. Domain: $[-6, 6]$; range: $[0, 6]$. Increasing on $(-6, 0)$; decreasing on $(0, 6)$.



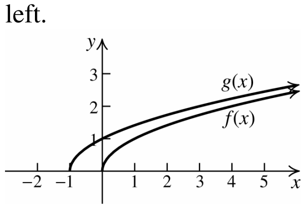
81. Domain: $(-\infty, \infty)$; range: $[1, \infty)$. Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$.



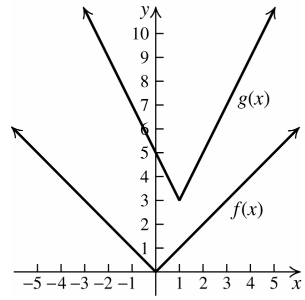
82. Domain: $(-\infty, \infty)$; range: $[0, \infty)$. Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$.



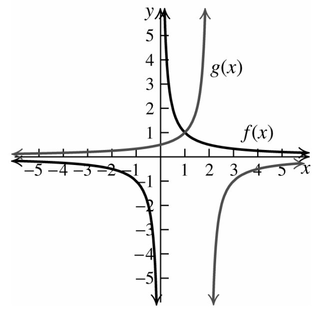
83. The graph of g is the graph of f shifted one unit left.



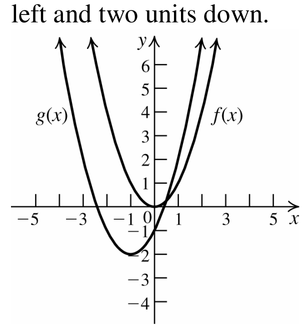
84. The graph of g is the graph of f shifted one unit right, stretched vertically by a factor of 2, and then shifted three units up.



85. The graph of g is the graph of f shifted two units right and then reflected across the x-axis.



86. The graph of g is the graph of f shifted one unit left and two units down.



87. $f(-x) = (-x)^2 + (-x)^4 = x^2 + x^4 = f(x) \Rightarrow$ $f(x)$ is even. Not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.

88. $f(-x) = (-x)^3 + (-x) = -x^3 - x = -f(x) \Rightarrow$ $f(x)$ is odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.

89. $f(-x) = |-x| + 3 = |x| + 3 = f(x) \Rightarrow$ $f(x)$ is even. Not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.

90. $f(-x) = -3x + 5 \neq f(x) \cup f(-x) \Rightarrow f(x)$ is neither even nor odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.

91. $f(-x) = \sqrt{-x} \neq f(x) \cup f(-x) \Rightarrow f(x)$ is neither even nor odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.

92. $f(-x) = -\dfrac{2}{x} = -f(x) \Rightarrow f(x)$ is odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.

93. $f(x) = \sqrt{x^2 - 4} \Rightarrow f(x) = (g \circ h)(x)$ where $g(x) = \sqrt{x}$ and $h(x) = x^2 - 4.$

94. $g(x) = \left(x^2 - x + 2\right)^{50} \Rightarrow g(x) = (f \circ h)(x)$ where $f(x) = x^{50}$ and $h(x) = x^2 - x + 2.$

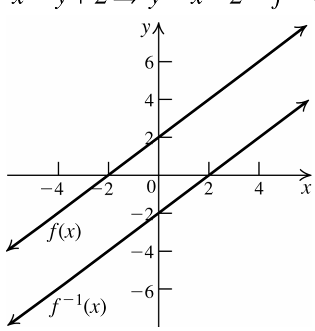
95. $h(x) = \sqrt{\dfrac{x-3}{2x+5}} \Rightarrow h(x) = (f \circ g)(x)$ where $f(x) = \sqrt{x}$ and $g(x) = \dfrac{x-3}{2x+5}.$

96. $H(x) = (2x-1)^3 + 5 \Rightarrow H(x) = (f \circ g)(x)$ where $f(x) = x^3 + 5$ and $g(x) = 2x - 1.$

97. $f(x)$ is one-to-one. $f(x) = y = x + 2.$

Interchange the variables and solve for y:

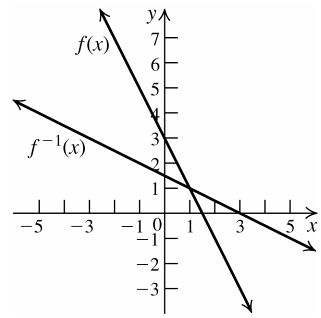
$x = y + 2 \Rightarrow y = x - 2 = f^{-1}(x).$



98. $f(x)$ is one-to-one. $f(x) = y = -2x + 3.$

Interchange the variables and solve for y:

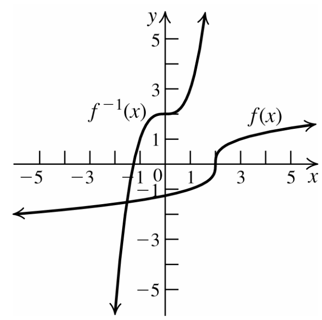
$x = -2y + 3 \Rightarrow y = -\dfrac{1}{2}x + \dfrac{3}{2} = f^{-1}(x).$



99. $f(x)$ is one-to-one. $f(x) = y = \sqrt[3]{x-2}.$

Interchange the variables and solve for y:

$x = \sqrt[3]{y-2} \Rightarrow y = x^3 + 2 = f^{-1}(x).$

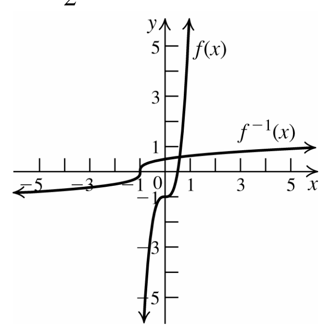


100. $f(x)$ is one-to-one. $f(x) = y = 8x^3 - 1.$

Interchange the variables and solve for y:

$x = 8y^3 - 1 \Rightarrow y = \sqrt[3]{\dfrac{x+1}{8}} \Rightarrow$

$y = \dfrac{1}{2}\sqrt[3]{x+1} = f^{-1}(x).$



101. $f(x) = y = \dfrac{x-1}{x+2}, x \neq 2.$ Interchange the variables and solve for y: $x = \dfrac{y-1}{y+2} \Rightarrow$

$xy + 2x = y - 1 \Rightarrow xy - y = -2x - 1 \Rightarrow$

$y(x-1) = -2x - 1 \Rightarrow y = \dfrac{-2x-1}{x-1} \Rightarrow$

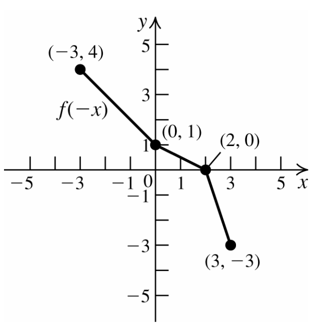
$y = f^{-1}(x) = \dfrac{2x+1}{1-x}.$ Domain of f: $(-\infty, 2) \cup (2, \infty)$; range of f: $(-\infty, 1) \cup (1, \infty).$

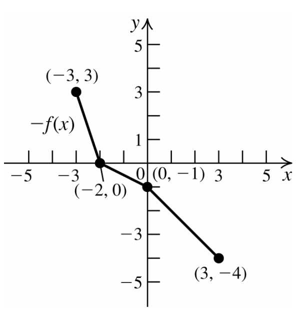
102. $f(x) = y = \dfrac{2x+3}{x-1}, x \neq 1$. Interchange the variables and solve for y: $x = \dfrac{2y+3}{y-1} \Rightarrow$
$xy - x = 2y + 3 \Rightarrow xy - 2y = x + 3 \Rightarrow$
$y(x-2) = x + 3 \Rightarrow y = f^{-1}(x) = \dfrac{x+3}{x-2}$.
Domain of f: $(-\infty, 1) \cup (1, \infty)$;
range of f: $(-\infty, 2) \cup (2, \infty)$.

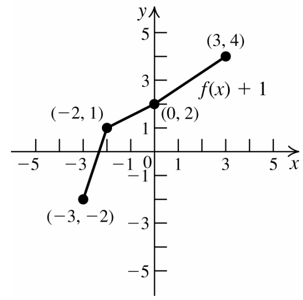
103.a. $A = (-3,-3), B = (-2,0), C = (0,1), D = (3,4)$.
Find the equation of each segment:
$m_{AB} = \dfrac{0-(-3)}{-2-(-3)} = 3. 0 = 3(-2) + b \Rightarrow b = 6$.
The equation of AB is $y = 3x + 6$.
$m_{BC} = \dfrac{1-0}{0-(-2)} = \dfrac{1}{2}; b = 1$. The equation of BC is $y = \dfrac{1}{2}x + 1$.
$m_{CD} = \dfrac{4-1}{3-3} = 1; b = 1$. The equation of CD is $y = x + 1$.
So,
$$f(x) = \begin{cases} 3x+6 & \text{if } -3 \le x \le -2 \\ \frac{1}{2}x + 1 & \text{if } -2 < x < 0 \\ x+1 & \text{if } 0 \le x \le 3 \end{cases}$$

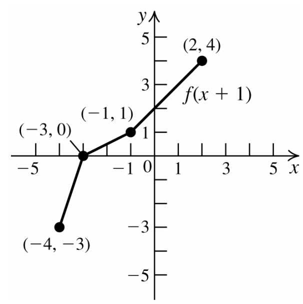
b. Domain: [–3,3]; range: [–3,4]

c. x-intercept: –2; y-intercept: 1

d.


e.


f.


g.


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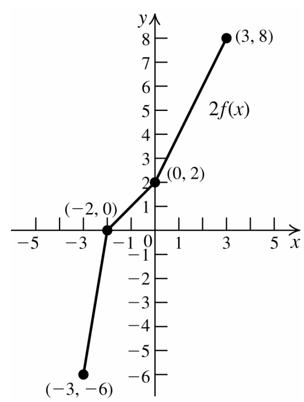
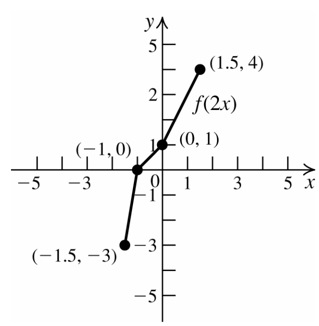
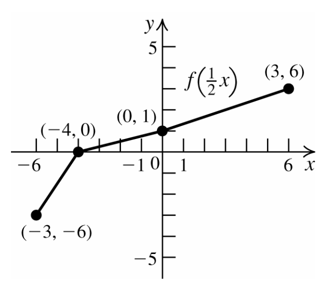
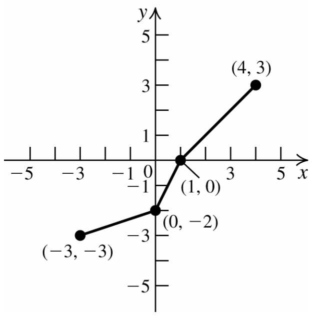


i.


j.


k. f is one-to-one because it satisfies the horizontal line test.

l.



Applying the Concepts

104.a. rate of change (slope) = $\frac{25.95-19.2}{25-10}=0.45.$
$19.2=0.45(10)+b\Rightarrow b=14.7.$ The equation is $P=0.45d+14.7.$

b. The slope represents the amount of increase in pressure (in pounds per square inch) as the diver descends one foot deeper. The *y*-intercept represents the pressure at the surface of the sea.

c. $P=0.45(160)+14.7=86.7\text{ lb/in.}^2$

d. $104.7=0.45d+14.7\Rightarrow 200$ feet

105.a. rate of change (slope) = $\frac{173,000-54,000}{223,000-87,000}$
$=0.875.\ 54,000=0.875(87,000)+b\Rightarrow$
$b=-22,125.$ The equation is
$C=0.875w-22,125.$

b. The slope represents the cost to dispose of one pound of waste. The *x*-intercept represents the amount of waste that can be disposed with no cost. The *y*-intercept represents the fixed cost.

c. $C=0.875(609,000)-22,125=\$510,750$

d. $1,000,000=0.875w-22,125\Rightarrow$
$w=1,168,142.86$ pounds

106.a. At 60 mph = 1 mile per minute, so if the speedometer is correct, the number of minutes elapsed is equal to the number of miles driven.

b. The odometer is based on the speedometer, so if the speedometer is incorrect, so is the odometer.

107.a. $f(2)=100+55(2)-3(2)^2=\$198.$ She started with \$100, so she won \$98.

b. She was winning at a rate of \$49/hour.

c. $0=100+55t-3t^2\Rightarrow(-t+20)(3t+5)\Rightarrow$
$t=20\cup t=-5/3.$ Since *t* represent the amount of time, we reject $t=-5/3.$ Chloe will lose all her money after playing for 20 hours.

d. \$100/20 = \$5/hour.

108. If $100<x\le 500$, then the sales price per case is \$4 – 0.2(4) = \$3.20. The first 100 cases cost \$400.

$$f(x)=\begin{cases}4x & \text{if } 0\le x\le 100\\ 3.2x+80 & \text{if } 100<x\le 500\\ 3x+180 & \text{if } x>500\end{cases}$$

109.a. $(L\circ x)(t)=0.5\sqrt{\left(1+0.002t^2\right)^2+4}$
$=0.5\sqrt{0.000004t^4+0.004t^2+5}$

b. $(L\circ x)(5)=0.5\sqrt{\left(1+0.002(5^2)\right)^2+4}$
$=0.5\sqrt{(1.05)^2+4}=0.5\sqrt{5.1025}$
≈ 1.13

110.a. Revenue = number of units × price per unit:
$x\bullet p=(5000+50t+10t^2)(10+0.5t)$
$=5t^3+125t^2+3000t+50,000$

b. $p=10+0.5t\Rightarrow t=2p-20..$
$x(t)=x(2p-20)$
$=5000+50(2p-20)+10(2p-20)^2$
$=40p^2-700p+8000,$ which is the number of toys made at price *p*. The revenue is $p\left(40p^2-700p+8000\right)=$
$40p^3-700p^2+8000p.$

Chapter 2 Practice Test A

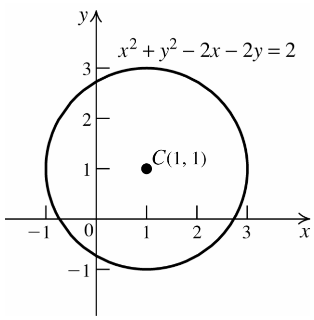
1. To test if the graph is symmetric with respect to the *y*-axis, replace *x* with –*x*:
$3(-x)+2(-x)y^2=1\Rightarrow -3x-2xy^2=1,$ which is not the same as the original equation, so the graph is not symmetric with respect to the *y*-axis. To test if the graph is symmetric with respect to the *x*-axis, replace *y* with –*y*:
$3x+2x(-y)^2=1\Rightarrow 3x+2xy^2=1,$ which is the same as the original equation, so the graph is symmetric with respect to the *x*-axis

.

To test if the graph is symmetric with respect to the origin, replace x with $-x$ and y with $-y$:
$3(-x)+2(-x)(-y)^2=1 \Rightarrow -3x-2xy^2=1,$
which is not the same as the original equation, so the graph is not symmetric with respect to the origin.

2. $0=x^2(x-3)(x+1) \Rightarrow x=0 \cup x=3 \cup x=-1;$
$y=0^2(0-3)(x+1) \Rightarrow y=-3.$ The x-intercepts are 0, 3, and –1; the y-intercept is –3.

3.



4. $7=-1(2)+b \Rightarrow 9=b$. The equation is $y=-x+9.$

5. $8x-2y=7 \Rightarrow y=4x-\frac{7}{2} \Rightarrow$ the slope of the line is 4. $-1=4(2)+b \Rightarrow b=-9.$ So the equation is $y=4x-9.$

6. $(fg)(2)=f(2)\bullet g(2)$
$=(-2(2)+1)(2^2+3(2)+2)$
$=(-3)(12)=-36$

7. $g(f(2))=g(2(2)-3)=g(1)=1-2(1)^2=-1$

8. $(f\circ f)(x)=(x^2-2x)^2-2(x^2-2x)$
$=x^4-4x^3+4x^2-2x^2+4x$
$=x^4-4x^3+2x^2+4x$

9. a. $f(-1)=(-1)^3-2=-3$

b. $f(0)=0^3-2=-2$

c. $f(1)=1-2(1)^2=-1$

10. $1-x>0 \Rightarrow x<1; x$ must also be greater than or equal to 0, so the domain is [0, 1).

11. $x^2+x-6\ge 0 \Rightarrow (x+3)(x-2)\ge 0.$ Test the intervals $(-\infty,-3],[-3,2],$ and $[2,\infty).$

The inequality is true for $(-\infty,-3]$ and $[2,\infty),$ so the domain is $(-\infty,-3]\cup[2,\infty).$

12. $\frac{f(4)-f(1)}{4-1}=\frac{(2(4)+7)-(2(1)+7)}{3}=2$

13. $f(-x)=2(-x)^4-\frac{3}{(-x)^2}=2x^4-\frac{3}{x^2}=f(x)\Rightarrow$
$f(x)$ is even.

14. Increasing on $(-\infty,0)$ and $(2,\infty);$ decreasing on $(0,2).$

15. Shift the graph of f three units to the right.

16. $25=25-(2t-5)^2 \Rightarrow 0=-(2t-5)^2 \Rightarrow$
$0=2t-5 \Rightarrow t=5/2=2.5$ seconds

17. 2

18. $f(x)=y=\frac{2x+1}{x-3}$. Interchange the variables and solve for y: $x=\frac{2y+1}{y-3}\Rightarrow$
$xy-3x=2y+1 \Rightarrow xy-2y=3x+1 \Rightarrow$
$y(x-2)=3x+1 \Rightarrow y=f^{-1}(x)=\frac{3x+1}{x-2}$

19. $A(x)=100x+1000$

20.a. $C(230)=0.25(230)+30=\$87.50$

b. $57.50=0.25m+30 \Rightarrow m=110$ miles

Chapter 2 Practice Test B

1. To test if the graph is symmetric with respect to the y-axis, replace x with $-x$:
$|-x|+2|y|=2 \Rightarrow |x|+2|y|=2,$ which is the same as the original equation, so the graph is symmetric with respect to the y-axis. To test if the graph is symmetric with respect to the x-axis, replace y with $-y$:
$|x|+2|-y|=2 \Rightarrow |x|+2|y|=2,$ which is the same as the original equation, so the graph is symmetric with respect to the x-axis. To test if the graph is symmetric with respect to the origin, replace x with $-x$, and y with $-y$:
$|-x|+2|-y|=2 \Rightarrow |x|+2|y|=2,$ which is the same as the original equation, so the graph is symmetric with respect to the origin. The answer is D.

2. $0 = x^2 - 9 \Rightarrow x = \pm 3;\ y = 0^2 + 9 \Rightarrow y = 9.$ The x-intercepts are ±3; the y-intercept is 9. The answer is B.

3. D **4.** D **5.** C

6. Suppose the coordinates of the second point are (a, b). Then $-\frac{1}{2} = \frac{b-2}{a-3}$. Substitute each of the points given into this equation to see which makes it true. The answer is C.

7. Find the slope of the original line: $6x - 3y = 5 \Rightarrow y = 2x - \frac{5}{3}$. The slope is 2. The equation of the line with slope 2, passing through (–1, 2) is $y - 2 = 2(x+1)$. The answer is D.

8. $(f \circ g)(x) = 3(2 - x^2) - 5 = 1 - 3x^2$. The answer is B.

9. $(f \circ f)(x) = 2(2x^2 - x)^2 - (2x^2 - x)$
$= 8x^4 - 8x^3 + x$. The answer is A.

10. $g(a-1) = \frac{1-(a-1)}{1+(a-1)} = \frac{2-a}{a}$. The answer is C.

11. $1 - x \ge 0 \Rightarrow x \le 1;$ x must also be greater than or equal to 0, so the domain is [0, 1]. The answer is A.

12. $x^2 + 6x - 7 \ge 0 \Rightarrow (x+7)(x-1) \ge 0$. Test the intervals $(-\infty, -7], [-7, 1],$ and $[1, \infty)$. The inequality is true for $(-\infty, -7]$ and $[1, \infty)$, so the answer is B.

13. A **14.** A **15.** B

16. D **17.** C

18. $f(x) = y = \frac{1-3x}{5+2x}$. Interchange the variables and solve for y: $x = \frac{1-3y}{5+2y} \Rightarrow$
$5x + 2xy = 1 - 3y \Rightarrow 2xy + 3y = 1 - 5x$
$y(2x+3) = 1 - 5x \Rightarrow y = f^{-1}(x) = \frac{1-5x}{2x+3}$.
The answer is C.

19. $w = 5x - 190; w = 5(70) - 190 = 160.$ The answer is B.

20. $50 = 0.2m + 25 \Rightarrow m = 125.$ The answer is A.

Cumulative Review Exercises (Chapters P–2)

1.a. $\left(\frac{x^3}{y^2}\right)^2\left(\frac{y^2}{x^3}\right)^3 = \left(\frac{x^6}{y^4}\right)\left(\frac{y^6}{x^9}\right) = \frac{y^2}{x^3}$

b. $\frac{x^{-1}y^{-1}}{x^{-1}+y^{-1}} = \frac{\frac{1}{x}\bullet\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}} = \frac{\frac{1}{xy}}{\frac{y+x}{xy}} = \frac{1}{x+y}$

2.a. $2x^2 + x - 15 = (2x-5)(x+3)$

b. $x^3 - 2x^2 + 4x - 8 = x^2(x-2) + 4(x-2)$
$= (x^2+4)(x-2)$

3.a. $\sqrt{75} + \sqrt{108} - \sqrt{192} = 5\sqrt{3} + 6\sqrt{3} - 8\sqrt{3}$
$= 3\sqrt{3}$

b. $\frac{x-1}{x+1} - \frac{x-2}{x+2} = \frac{(x-1)(x+2)-(x-2)(x+1)}{(x+1)(x+2)}$
$= \frac{(x^2+x-2)-(x^2-x-2)}{(x+1)(x+2)}$
$= \frac{2x}{(x+1)(x+2)}$

4.a. $\frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}}\bullet\frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

b. $\frac{1}{\sqrt{5}-2} = \frac{1}{\sqrt{5}-2}\bullet\frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{5-4} = \sqrt{5}+2$

5.a. $3x - 7 = 5 \Rightarrow 3x = 12 \Rightarrow x = 4$

b. $\frac{1}{x-1} = \frac{3}{x-1} \Rightarrow$ There is no solution.

6.a. $x^2 - 3x = 0 \Rightarrow x(x-3) = 0 \Rightarrow x = 0 \cup x = 3$

b. $x^2 + 3x - 10 = 0 \Rightarrow (x+5)(x-2) = 0 \Rightarrow$
$x = -5 \cup x = 2$

7.a. $2x^2 - x + 3 = 0 \Rightarrow x = \frac{1 \pm \sqrt{1-4(2)(3)}}{2(2)} \Rightarrow$
$x = \frac{1 \pm \sqrt{-23}}{4} \Rightarrow x = \frac{1 \pm i\sqrt{23}}{4}$

b. $4x^2 - 12x + 9 = 0 \Rightarrow (2x-3)^2 = 0 \Rightarrow x = \frac{3}{2}$

8.a. $x-6\sqrt{x}+8=0 \Rightarrow \left(\sqrt{x}-4\right)\left(\sqrt{x}-2\right)=0 \Rightarrow$
$\sqrt{x}=4 \Rightarrow x=16 \cup \sqrt{x}=2 \Rightarrow x=4$

b. $\left(x-\frac{1}{x}\right)^2-10\left(x-\frac{1}{x}\right)+21=0.$
Let $u=x-\frac{1}{x}.u^2-10u+21=0 \Rightarrow$
$(u-7)(u-3)=0 \Rightarrow u=7 \cup u=3;$
$x-\frac{1}{x}=7 \Rightarrow x^2-1=7x \Rightarrow$
$x^2-7x-1=0 \Rightarrow x=\frac{7\pm\sqrt{7^2-(4)(-1)}}{2} \Rightarrow$
$x=\frac{7\pm\sqrt{53}}{2};\ x-\frac{1}{x}=3 \Rightarrow x^2-1=3x \Rightarrow$
$x^2-3x-1=0 \Rightarrow x=\frac{3\pm\sqrt{3^2-4(-1)}}{2} \Rightarrow$
$x=\frac{3\pm\sqrt{13}}{2}$. The solution set is
$\left\{\frac{7-\sqrt{53}}{2},\frac{7+\sqrt{53}}{2},\frac{3-\sqrt{13}}{2},\frac{3+\sqrt{13}}{2}\right\}.$

9.a. $\sqrt{3x-1}=2x-1 \Rightarrow 3x-1=(2x-1)^2 \Rightarrow$
$3x-1=4x^2-4x+1 \Rightarrow 4x^2-7x+2=0 \Rightarrow$
$x=\frac{7\pm\sqrt{(-7)^2-4(4)(2)}}{2(4)}=\frac{7\pm\sqrt{17}}{8}$. If
$x=\frac{7-\sqrt{17}}{8},\ \sqrt{3\left(\frac{7-\sqrt{17}}{8}\right)-1}\approx 0.281$ while
$2\left(\frac{7-\sqrt{17}}{8}\right)-1\approx -0.281$, so the solution set
is $\left\{\frac{7+\sqrt{17}}{8}\right\}$.

b.
$$\begin{aligned}
\sqrt{1-x}&=2-\sqrt{2x+1}\\
\left(\sqrt{1-x}\right)^2&=\left(2-\sqrt{2x+1}\right)^2\\
1-x&=4-4\sqrt{2x+1}+2x+1\\
-4-3x&=-4\sqrt{2x+1}\\
(-4-3x)^2&=\left(-4\sqrt{2x+1}\right)^2\\
16+24x+9x^2&=16(2x+1)\\
16+24x+9x^2&=32x+16\\
9x^2-8x=0 &\Rightarrow x(9x-8)=0\\
x=0\cup x&=\frac{8}{9}.
\end{aligned}$$
Check to make sure that neither solution is extraneous. The solution set is $\{0, 8/9\}$.

10.a. $2x-5<11 \Rightarrow x<8 \Rightarrow (-\infty,8)$

b. $-3x+4>-5 \Rightarrow x<3 \Rightarrow (-\infty,3)$

11.a. $x(x-1)>0$. Solve the associated equation to determine the intervals: $x(x-1)=0 \Rightarrow x=0 \cup x=1$. The intervals to be tested are $(-\infty,0), (0,1)$, and $(1,\infty)$.

Interval	Test point	Value of $x(x-1)$	Result
$(-\infty,0)$	–1	2	+
$(0,1)$	1/2	–0.25	–
$(1,\infty)$	2	2	+

The solution set is $(-\infty,0)\cup(1,\infty)$.

b. $(x-2)(x+1)<0$. Solve the associated equation to determine the intervals: $(x-2)(x+1)=0 \Rightarrow x=2 \cup x=-1$. The intervals to be tested are $(-\infty,-1)$, $(-1,2)$, and $(2,\infty)$.

Interval	Test point	Value of $(x-2)(x+1)$	Result
$(-\infty,-1)$	–2	4	+
$(-1,2)$	0	–2	–
$(2,\infty)$	3	4	+

The solution set is $(-1,2)$.

12.a. $\frac{x+1}{x-2}>0$. Solve the associated equations to determine the intervals: $x-2=0 \Rightarrow x=2$, and $x+1=0 \Rightarrow x=-1$. The intervals to be tested are $(-\infty,-1), (-1,2)$, and $(2,\infty)$.

Interval	Test point	Value of $\frac{x+1}{x-2}$	Result
$(-\infty,-1)$	–2	1/4	+
$(-1,2)$	0	–1/2	–
$(2,\infty)$	3	4	+

The solution set is $(-\infty,-1)\cup(2,\infty)$.

b. $\dfrac{x+1}{(x-2)(x-5)} \le 0.$ Solve the associated equations to determine the intervals: $(x-2)(x-5)=0 \Rightarrow x=2 \cup x=5,$ and $x+1=0 \Rightarrow x=-1.$ The intervals to be tested are $(-\infty,-1], [-1,2), (2,5)$ and $(5,\infty)$.

Interval	Test point	Value of $\dfrac{x+1}{(x-2)(x-5)}$	Result
$(-\infty,-1]$	–2	–1/28	–
$[-1,2)$	0	1/10	+
$(2,5)$	3	–2	–
$(5,\infty)$	6	7/4	+

The solution set is $(-\infty,-1] \cup (2,5)$.

13.a. $-3 < 2x-3 < 5 \Rightarrow 0 < 2x < 8 \Rightarrow 0 < x < 4.$ The solution set is (0, 4).

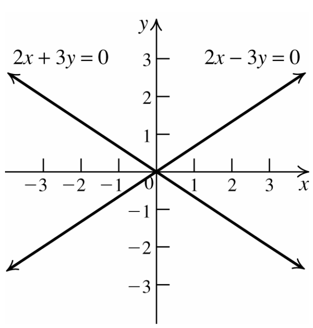
b. $5 \le 1-2x \le 7 \Rightarrow 4 \le -2x \le 6 \Rightarrow -2 \ge x \ge -3.$ The solution set is [–3, –2].

14.a. $|2x-1| \le 7 \Rightarrow 2x-1 \le 7 \Rightarrow x \le 4$ or $2x-1 \ge -7 \Rightarrow x \ge -3.$ The solution set is $[-3,4]$.

b. $|2x-3| \ge 5 \Rightarrow 2x-3 \ge 5 \Rightarrow x \ge 4$ or $2x-3 \le -5 \Rightarrow x \le -1.$ The solution set is $(-\infty,-1] \cup [4,\infty)$.

15. $d(A,C) = \sqrt{(2-5)^2+(2-(-2))^2} = 5;$ $d(B,C) = \sqrt{(2-6)^2+(2-5)^2} = 5.$ Since the lengths of the two sides are equal, the triangle is isosceles.

16.



17. First, find the equation of the circle with center (2, –1) and radius determined by (2, –1) and (–3, –1): $r = \sqrt{2-(-3))^2+(-1-(-1))^2} = 5$. The equation is $(x-2)^2+(y+1)^2=5^2.$ Now check to see if the other three points satisfy the equation: $(2-2)^2+(4+1)^2=5^2 \Rightarrow 5^2=5^2,$ $(5-2)^2+(3+1)^2=5^2 \Rightarrow 3^2+4^2=5^2$ (true because 3, 4, 5 is a Pythagorean triple), and $(6-2)^2+(2+1)^2=5^2 \Rightarrow 4^2+3^2=5^2.$ Since all the points satisfy the equation, they lie on the circle.

18. $x^2+y^2-6x+4y+9=0 \Rightarrow$
$x^2-6x+y^2+4y=-9.$
Now complete both squares:
$x^2-6x+9+y^2+4y+4=-9+9+4 \Rightarrow$
$(x-3)^2+(y+2)^2=4.$ The center is (3, –2) and the radius is 2.

19. $y=-3x+5$

20. The x-intercept is 4, so (4, 0) satisfies the equation. To write the equation in slope-intercept form, find the y-intercept: $0=(4)+b \Rightarrow -8=b.$ The equation is $y=2x-8.$

21. The slope of the perpendicular line is the negative reciprocal of the slope of the original line. The slope of the original line is 2, so the slope of the perpendicular is –1/2. Now find the y-intercept of the perpendicular: $-1=-\frac{1}{2}(2)+b \Rightarrow b=0.$ The equation of the perpendicular is $y=-\frac{1}{2}x.$

22. The slope of the parallel line is the same as the slope of the original line, 2. Now find the y-intercept of the parallel line: $-1=2(2)+b \Rightarrow$ $b=-5.$ The equation of the parallel line is $y=2x-5.$

23. The slope of the perpendicular line is the negative reciprocal of the slope of the original line. The slope of the original line is $\dfrac{7-(-1)}{5-3}=4$, so the slope of the perpendicular is –1/4. The perpendicular bisector passes through the midpoint of the original segment. The midpoint is $\left(\dfrac{3+5}{2}, \dfrac{-1+7}{2}\right)=(4,3).$

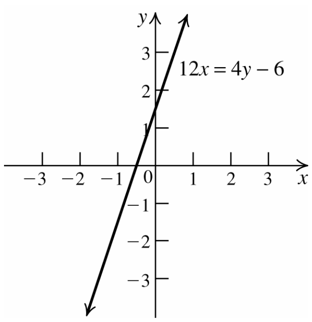
Use this point and the slope to find the *y*-intercept: $3 = -\frac{1}{4}(4) + b \Rightarrow b = 4$. The equation of the perpendicular bisector is $y = -\frac{1}{4}x + 4.$

24. The slope is undefined because the line is vertical. Because it passes through (5, 7), the equation of the line is $x = 5$.

25. Use the slope formula to solve for x:
$$2 = \frac{5-11}{x-5} \Rightarrow 2(x-5) = -6 \Rightarrow 2x - 10 = -6 \Rightarrow x = 2$$

26. The line through (x, 3) and (3, 7) has slope –2 because it is perpendicular to a line with slope 2. Use the slope formula to solve for x:
$$-2 = \frac{3-7}{x-3} \Rightarrow -2(x-3) = -4 \Rightarrow x - 3 = 2 \Rightarrow x = 5$$

27.



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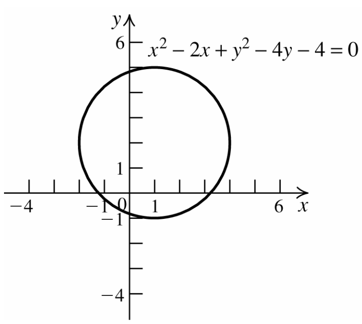

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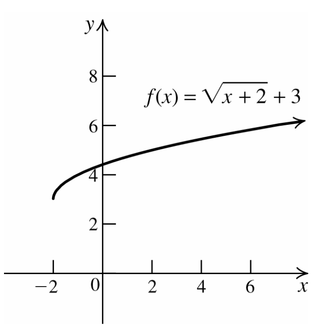
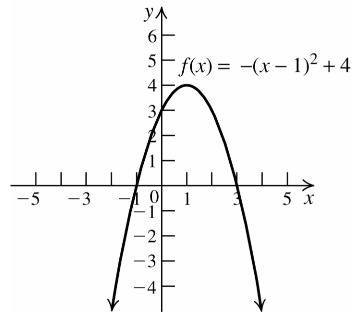


30.



31. Let x = the number of books initially purchased, and $\frac{1650}{x} =$ the cost of each book. Then $x - 16$ = the number of books sold, and $\frac{1650}{x-16} =$ the selling price of each book. The profit = the selling price – the cost, so
$$\frac{1650}{x-16} - \frac{1650}{x} = 10 \Rightarrow$$
$$1650x - 1650(x-16) = 10x(x-16) \Rightarrow$$
$$1650x - 1650x + 26,400 = 10x^2 - 160x \Rightarrow$$
$$10x^2 - 160x - 26,400 = 0 \Rightarrow$$
$$x^2 - 16x - 2640 = 0 \Rightarrow (x-60)(x+44) = 0 \Rightarrow$$
$x = 60 \cup x = -44.$ Reject –44 because there cannot be a negative number of books. So she bought 60 books.

32. Let x = the monthly note on the 1.5 year lease, and $1.5(12)x = 18x$ = the total expense for the 1.5 year lease. Then $x - 250$ = the monthly note on the 2 year lease, and $2(12)(x - 250) = 24x - 6000$ the total expenses for the 2 year lease. Then $18x + 24x - 6000 = 21,000 \Rightarrow 42x = 27,000 \Rightarrow x = 642.86.$ So the monthly note for the 1.5 year lease is \$642.86, and the monthly note for the 2 year lease is \$642.86 – 250 = \$392.86.

33.a. The domain of f is the set of all values of x which make $x + 1 \geq 0$ (because the square root of a negative number is not a real value.) So $x \geq -1$ or $[-1, \infty)$ in interval notation is the domain.

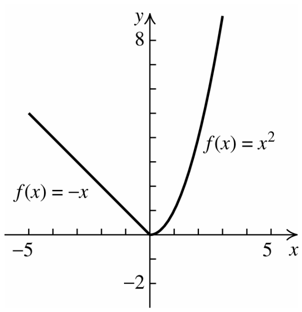
b. $y = \sqrt{0+1} - 3 \Rightarrow y = -2; 0 = \sqrt{x+1} - 3 \Rightarrow 3 = \sqrt{x+1} \Rightarrow 9 = x + 1 \Rightarrow 8 = x.$ The x-intercept is 8, and the y-intercept is –2.

c. $f(-1) = \sqrt{-1+1} - 3 = -3$

d. $f(x) > 0 \Rightarrow \sqrt{x+1} - 3 > 0 \Rightarrow \sqrt{x+1} > 3 \Rightarrow x+1 > 9 \Rightarrow x > 8$. In interval notation, this is $(8, \infty)$.

34.a. $f(-2) = -(-2) = 2;\ f(0) = 0^2 = 0;\ f(2) = 2^2 = 4$

b. f decreases on $(-\infty, 0)$ and increases on $(0, \infty)$.



35.a. $(f \circ g)(x) = \dfrac{1}{\frac{2}{x} - 2} = \dfrac{1}{\frac{2-2x}{x}} = \dfrac{x}{2-2x}$.

Because 0 is not in the domain of g, it must be excluded from the domain of $(f \circ g)$. Because 2 is not in the domain of f, any values of x for which $g(x) = 2$ must also be excluded from the domain of $(f \circ g)$: $\frac{2}{x} = 2 \Rightarrow x = 1$, so 1 is excluded also. The domain of $(f \circ g)$ is $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$.

b. $(g \circ f)(x) = \dfrac{2}{\frac{1}{x-2}} = 2(x-2) = 2x - 4$.

Because 2 is not in the domain of f, it must be excluded from the domain of $(g \circ f)$. Because 0 is not in the domain of g, any values of x for which $f(x) = 0$ must also be excluded from the domain of $(g \circ f)$. However, there is no value for x which makes $f(x) = 0$. So the domain of $(g \circ f)$ is $(-\infty, 2) \cup (2, \infty)$.