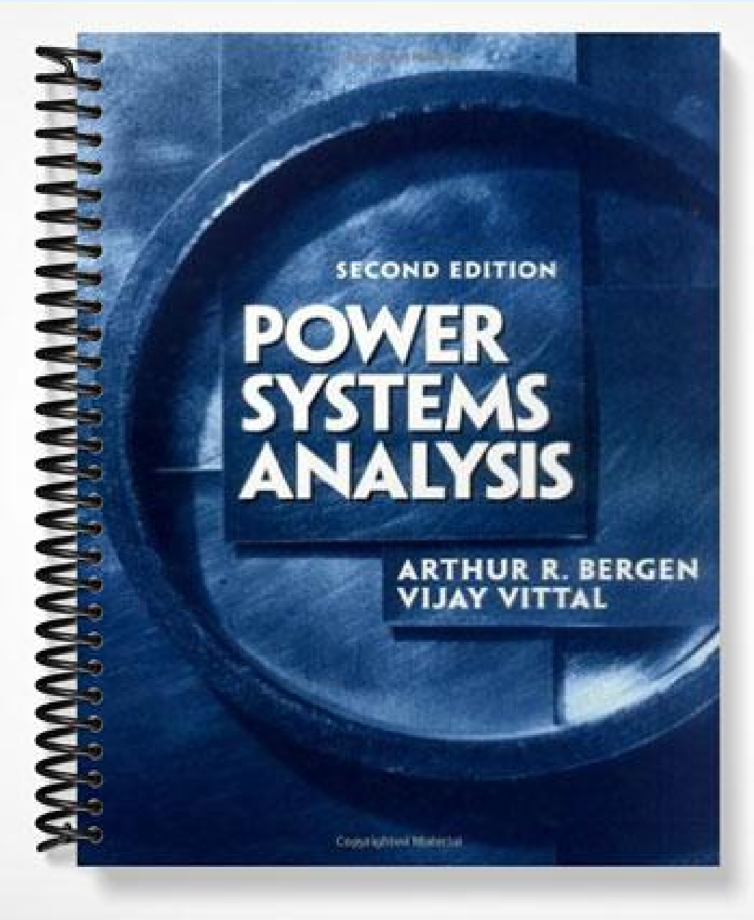
SOLUTIONS MANUAL



SOLUTIONS MANUAL

POWER SYSTEMS ANALYSIS

SECOND EDITION

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VIJAY VITTAL

PART I. SOLUTIONS TO PROBLEM SETS
PART II. DISCUSSION OF SOLUTIONS TO DESIGN EXERCISES

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CONTENTS

PART I. SOLUTIONS TO PROBLEM SETS

2.1 $v = \sqrt{2} \times 120 \cos(\omega t + 30°) \Rightarrow V = 120\angle 30°$ V

$i = \sqrt{2} \times 10 \cos(\omega t - 30°) \Rightarrow I = 10 \angle -30°$ A

(a) $p(t) = |V||I| [\cos\phi + \cos(2\omega t + \angle V + \angle I)]$

$= 600 + 1200 \cos 2\omega t$ W

$S = VI^* = 1200 \angle 60° = P + jQ \Rightarrow P = 600\ W,\ Q = 1039$ VAr

(b) $Z = V/I = 12 \angle 60° = 6 + j10.39 = R + jX \Rightarrow R = 6,\ X = 10.39$

2.2 (a) Using (2.3) we find $p_{max} = 1707 = |V||I|(\cos\phi + 1)$ and $p_{min} = -293 = |V||I|(\cos\phi - 1)$. Then, since $|V| = 100$, we get $|I| = 10$ and $\cos\phi = \pm 45°$. Pick $\phi = 45° \Rightarrow Z = 10\angle 45° = 7.07 + j7.07 = R + jX \Rightarrow R = 7.07,\ X = 7.07$

(b) $S = VI^* = Z|I|^2 = (7.07 + j7.07)\,10^2 \Rightarrow P = 707,\ Q = 707$

(c) For simplicity assume $i(t) = \sqrt{2}\,|I| \cos\omega t$. Then

$p_L(t) = v_L(t)\,i(t) = L\frac{di}{dt}\,i = -2\omega L|I|^2 \cos\omega t \sin\omega t = -\omega L|I|^2 \sin 2\omega t$

$p_{L\,max} = \omega L|I|^2 = 707 = Q$. Thus $p_{L\,max} = Q$. The same!

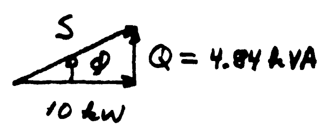
2.3 0.7 PF lagging $\Rightarrow \phi = 45.57°$, $Q = 5.10$ MVAr

0.9 PF lagging $\Rightarrow \phi = 25.84°$, $Q = 2.42$ MVAr. Capacitor must supply $5.10 - 2.42 = 2.68$ MVAr.

2.4 0.707 PF lagging $\Rightarrow S_{3\phi} = 200 + j200$ kVA. Cap supplies 50 kVAr. Resultant $S_{3\phi} = 200 + j150$ kVA $\Rightarrow$ PF = 0.80.

$|S| = \frac{|S_{3\phi}|}{3} = \frac{250 \times 10^3}{3} = |V||I| = \frac{440}{\sqrt{3}}|I| \Rightarrow |I| = 328$ A

2.5 0.9 PF lagging $\Rightarrow \phi = 25.84°$



(a) $S = 10 + j\,4.84$ kVA

(b) $10 \times 10^3 = 416 \times |I| \times 0.9 \Rightarrow |I| = 26.71$ A

(c) Using (2.3), (or first principles) we get

$p(t) = 10 \times 10^3 + 11.11 \times 10^3 \cos(2\omega t + 25.84°)$

Note: the average value of $p(t)$ is 10 kW

2.6 Because system is balanced $V_{ab} = 208\angle 120°$, $V_{bc} = 208\angle 0°$. Using (2.17) or Fig 2.11, $V_{an} = 120\angle 90° \Rightarrow V_{bn} = 120\angle -30°$, $V_{cn} = 120\angle -150°$. Using per phase analysis, $I_a = 12\angle 105° \Rightarrow$ $I_b = 12\angle -15°$, $I_c = 12\angle -135°$.

2.7 $S = VI^* = V(YV)^* = Y^*|V|^2 = Y_C^* + Y_L^* + Y_R^*$

$= -j5 + j10 + 0.1 = 0.1 + j5$

2.8 (a) Using loop or nodal analysis we find, after much work,

$I_a = 0.9123\angle -90.351°$, $I_b = 0.9123\angle -209.65°$, $I_c = 0.9929\angle 30°$.

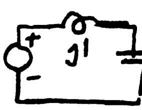
(b) Using per phase analysis $I_a = 1\angle 0° / j1.1 = 0.9091\angle -90°$,

then, $I_b = 0.9091\angle -210°$, $I_c = 0.9091\angle 30°$.

2.9 Proceeding by analogy with 3ϕ, we note

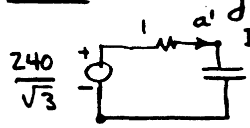
$E_{ab} = E_{an} - E_{bn} = E_{an}(1 - e^{-j\pi/2}) = \sqrt{2}\, E_{an}\, e^{j\pi/4}$.

Thus $E_{an} = \frac{1}{\sqrt{2}} E_{ab}\, e^{-j\pi/4}$, and $E_{an}, E_{bn}, E_{cn}, E_{dn}$ form a pos. seq. set of 4ϕ voltages. Doing per phase (phase a) analysis we have

$\frac{1}{\sqrt{2}}\angle 45°$  $j1$ $-j0.5 \Rightarrow I_a = \dfrac{\frac{1}{\sqrt{2}}\angle -45°}{j0.5} = \sqrt{2}\,\angle -135°$

Then $I_b = \sqrt{2}\angle -225°$, $I_c = \sqrt{2}\angle -315°$, $I_d = \sqrt{2}\angle -405°$

2.10 Using per phase circuit we find $I_a = 103.8\angle 41.5°$. Then $|I_b| = 103.8$ A



$Z_\lambda = \frac{1}{3}\frac{1}{j\omega C} = -j0.884$

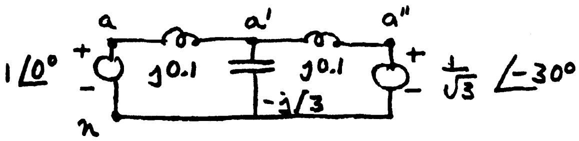
$V_{a'n} = Z_\lambda I_a = 91.76\angle -48.5° \Rightarrow |V_{a'b'}| = \sqrt{3}\cdot 91.76 = 158.9$ V

$S_{load} = V_{a'n} I_a^* = 9524\angle -90°$

$S_{load}^{3\phi} = 3\, S_{load} = 28574\angle -90°$ W

2.11 Assume pos. seq. operation. $V_{a''b''} = V_{a''n} - V_{b''n} = \sqrt{3}\, V_{a''n}\, e^{j\pi/6} \Rightarrow V_{a''n} = \frac{1}{\sqrt{3}} V_{a''b''}\, e^{-j\pi/6} = \frac{1}{\sqrt{3}} \angle{-30°}$

Per Phase Ckt



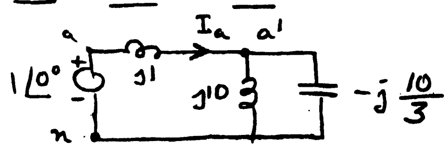
Using superposition & voltage divider law we get

$V_{a'n} = 0.899 \angle{-10.89°} \Rightarrow V_{b'n} = 0.899 \angle{-130.89°}$ and

$V_{c'n} = 0.899 \angle{-250.89°}$. Then $V_{a'b'} = 1.557 \angle{19.11°}$

2.12 Assume pos. seq..

Per Phase Ckt



Combining parallel elements we have $Z_{\parallel} = -j5$. $I_a = 0.25 \angle{90°}$

$V_{a'n} = -j5 \cdot j0.25 = 1.25 \angle{0°}$

$V_{a'b'} = 2.165 \angle{30°} \Rightarrow I_{cap} = .2165 \angle{120°}$

$S_{load}^{3\phi} = 3 V_{a'n} I_a^* = 0.3125 \angle{-90°}$

2.13

(a) $V_{bc} = 208 \angle{-120°}$, $V_{ca} = 208 \angle{120°}$

$V_{an} = \frac{208}{\sqrt{3}} \angle{-30°} \Rightarrow I_a = 1.20 \angle{-90°}$

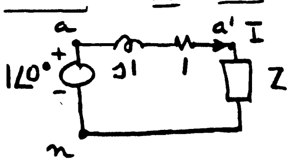
Then $I_b = 1.20 \angle{-210°}$, $I_c = 1.20 \angle{-330°}$

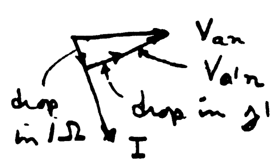
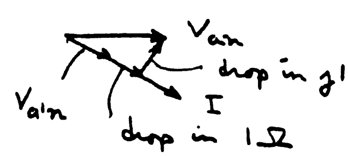
(b) $V_{bc} = 208 \angle{120°}$, $V_{ca} = 208 \angle{-120°}$

$V_{an} = \frac{208}{\sqrt{3}} \angle{30°} \Rightarrow I_a = 1.20 \angle{-30°}$

$I_b = 1.20 \angle{90°}$, $I_c = 1.20 \angle{-150°}$

2.14 Per Phase Ckt. Problem reduces to picking Z so that $|V_{a'n}| > |V_{an}|$. It helps to draw some phasor diagrams.

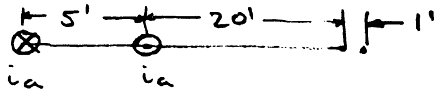


I. $Z = j\omega L$	II $Z = R$	III $Z = -j\frac{1}{\omega C}$
 Clearly $\lvert V_{a'n}\rvert < \lvert V_{an}\rvert$	 Clearly $\lvert V_{a'n}\rvert < \lvert V_{an}\rvert$	For example $Z = -j2$ $I = \frac{1}{\sqrt{2}} \angle{45°}$ $V_{a'n} = \sqrt{2} \angle{-45°}$ $\lvert V_{a'n}\rvert > \lvert V_{an}\rvert$ This is O.K.

2.15 Since $E_a + E_b + E_c = 0$, neutrals are at same potential. $Z = E_a / I_a = \sqrt{2}\ \angle 55°$. For each Z, $S = VI^* = |V|^2/Z^*$. thus

$$S^{3\phi} = \frac{(\sqrt{2})^2 + 1^2 + 1^2}{\sqrt{2}\ \angle -55°} = 2\sqrt{2}\ \angle 55°.$$

3.1



$\frac{\mu_0}{2\pi} = 2\times 10^{-7}$, $|I_a| = 100$

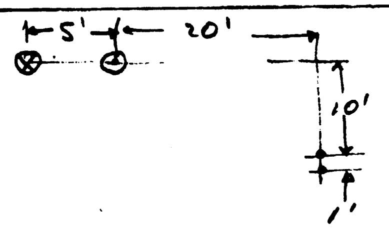
(Upward) flux linkages of telephone wires

$$\lambda = 2\times10^{-7}\left[i_a \ln\frac{26}{25} - i_a \ln\frac{21}{20}\right] = -.00957\times2\times10^{-7}\, i_a$$

$$v_{tel} = \frac{d\lambda}{dt} \Rightarrow V_{tel} = j\omega \times (-0.00957\times2\times10^{-7})\times 100 \quad V/\text{meter}$$

Since 1 mile = 1.609 km, $|V_{tel}| = 0.116$ V/mile

3.2

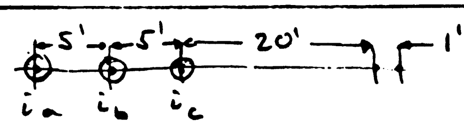


$$\lambda = 2\times10^{-7}\, i_a\left[\ln\frac{\sqrt{25^2+11^2}}{\sqrt{25^2+10^2}} - \ln\frac{\sqrt{20^2+11^2}}{\sqrt{20^2+10^2}}\right]$$

$$= -0.066294\times2\times10^{-7}\, i_a$$

$$\Rightarrow |V_{tel}| = 0.076 \text{ V/mile}$$

3.3



$$\lambda = 2\times10^{-7}\left[i_a \ln\frac{31}{30} + i_b \ln\frac{26}{25} + i_c \ln\frac{21}{20}\right]$$

$$= 2\times10^{-7}\left[0.0328\, i_a + 0.0392\, i_b + 0.0488\, i_c\right]$$

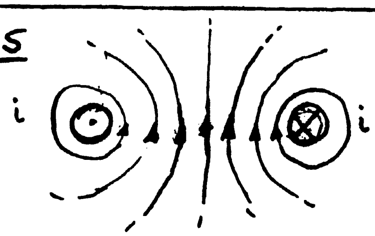
$$V_{tel} = j\omega\times2\times10^{-7}\left[0.0328\, I_a + 0.0392\, I_b + 0.0488\, I_c\right]$$

$$|V_{tel}| = 377\times2\times10^{-7}\times100\times\left|0.0328 + 0.0392\ \angle -120° + 0.0488\ \angle 120°\right|$$

$$= 1048\times10^{-7}\ V/\text{meter} = 0.1692\ V/\text{mile}$$

3.4 The telephone wires are transposed every 1000 ft. Cancellation occurs in all but 720' of line $\Rightarrow$ 0.0158 V.

3.5



The flux linkages due to the current in the left conductor is ≈

$$\lambda_1 = \frac{\mu_0}{2\pi} i \left[\frac{\mu_r}{4} + \ln\frac{D}{r}\right] = i\,\frac{\mu_0}{2\pi}\ln\frac{D}{r'}$$

Here we are neglecting partial flux linkages of the far (right) conductor. The current in the right conductor contributes an equal number of flux linkages. Thus (at least approximately),

$$l = 2\lambda_1/i = \frac{\mu_0}{\pi}\ln\frac{D}{r'} = 4\times10^{-7}\ln\frac{D}{r'} \quad \text{H/meter}$$

3.6 If each conductor is hollow then there are no partial flux linkages and in (3.13) the term involving μ_r is absent. Then $r' = r$ and

$$l = \frac{\mu_0}{2\pi}\ln\frac{D}{r} \qquad \text{H/m}$$

3.7 The hint is misleading. A better hint for combining the 7 (unequal) inductances $l_1, l_2, \cdots, l_7$ would be to use the average inductance i.e. let

$$l_a = \frac{l_{av}}{7} = \frac{l_1 + l_2 + \cdots + l_7}{7^2}$$

Returning to the problem, paralleling the steps which lead to (3.27) we have

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[\frac{i_a}{7}\left(\ln\frac{1}{d_{11}} + \ln\frac{1}{d_{12}} + \cdots + \ln\frac{1}{d_{17}}\right)\right.$$

$$+ \frac{i_b}{7}\left(\ln\frac{1}{d_{18}} + \ln\frac{1}{d_{19}} + \cdots + \ln\frac{1}{d_{1,21}}\right)$$

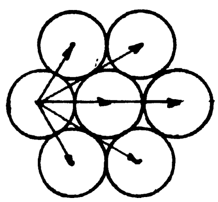
$$\left. + \frac{i_c}{7}\left(\ln\frac{1}{d_{1,15}} + \ln\frac{1}{d_{1,16}} + \cdots + \ln\frac{1}{d_{1,21}}\right)\right]$$

$$\approx \frac{\mu_0}{2\pi} i_a \ln\frac{D}{(d_{11}\cdots d_{17})^{1/7}} \Rightarrow l_1 = 7\times\frac{\mu_0}{2\pi}\ln\frac{D}{(d_{11}\cdots d_{17})^{1/7}}$$

$$l_a = \frac{l_{av}}{7} = \frac{l_1 + \cdots + l_7}{7^2} = \frac{\mu_0}{2\pi} \ln \frac{D}{R_s}$$

where $R_s \triangleq \left[(d_{11} \cdots d_{17})^{1/7} \cdots (d_{71} \cdots d_{77})^{1/7}\right]^{1/7}$

3.8

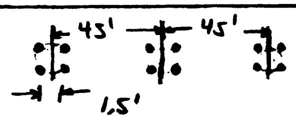


Six of the product terms are the same; the relevant distances are shown on the left. The different geometry is for the center wire to the outside wires. Let $d = 0.0876$ in. be the wire diameter. Noting that $d_{ii} = 0.7788 \cdot \frac{d}{2}$ we have

$$R_s = \left[\left(\left(0.7788\,\frac{d}{2}\right) \cdot d^3 \cdot 2d \cdot (\sqrt{3}d)^2\right)^6 \left(0.7788\,\frac{d}{2} \cdot d^6\right)\right]^{1/7^2}$$

$= 1.088\,d = 0.0943$ in. $= 0.00786$ ft.. very close!

3.9



$D_m = (45' \times 45' \times 90')^{1/3} = 56.70$ ft

$R_b = (0.0479 \times 1.5^2 \times \sqrt{2} \times 1.5)^{1/4} = .6915'$

$$l = 2 \times 10^{-7} \ln \frac{D_m}{R_b} = 8.81 \times 10^{-7} \text{ H/m}$$

3.10 Using $r' = 0.7788 \times \frac{1.424}{2} \times \frac{1}{12} = 0.0462'$ (instead of GMR $= 0.0479$) we get $R_b = 0.6853'$.

$$l = 2 \times 10^{-7} \ln \frac{D_m}{R_b} = 8.83 \text{ H/m}.$$ $\approx 0.23\%$ error.

3.11 $l = 9.47 \times 10^{-7}$ H/m

$X_L = 1609 \times 377 \times 9.47 \times 10^{-7} = 0.574\ \Omega\text{s/mile}$

3.12 $D_m = (26' \times 26' \times 52')^{1/3}$, $R_b = (0.0386' \times 1.5)^{1/2} = 0.2406'$

$$l = 2 \times 10^{-7} \ln \frac{D_m}{R_b} = 9.83 \times 10^{-7} \text{ H/m}$$

3.13 Using $r' = 0.7788 \times \frac{1.165}{2} \times \frac{1}{12} = 0.0378'$ we get $R_b = 0.2381'$

$l = 2 \times 10^{-7} \ln \frac{D_m}{R_b} = 9.85$ H/m. $\approx$ 0.20% error.

3.14 $X_L = 1609 \times 377 \times 9.83 \times 10^{-7} = 0.596\,\Omega$ / mile

3.15 $D_m = (45' \times 45' \times 90')^{1/3} = 56.70'$

$r = \frac{1.424''}{2} \times \frac{1}{12} = 0.0593'$, $R_b^c = (0.0593 \times 1.5^2 \times \sqrt{2}\, 1.5)^{1/4} = .07295'$

$$c = \frac{2\pi \cdot 8.854 \cdot 10^{-12}}{\ln \frac{D_m}{R_b^c}} = 12.78 \times 10^{-12} \text{ F/m (to neutral)}$$

3.16 $B_c = 1609 \times 377 \times 12.78 \times 10^{-12} = 7.75 \times 10^{-6}\ \mho$/mile

$|X_c| = 1/B_c = 0.129$ MΩ/mile

3.17 $D_m = (26' \times 26' \times 52')^{1/3} = 32.76'$

$r = \frac{1.165''}{2} \times \frac{1}{12} = 0.0485'$

$R_b^c = (0.0485' \times 1.5')^{1/2} = 0.2698'$

$c = 11.59 \times 10^{-12}$ F/m (to neutral)

3.18 $B_c = 1609 \times 377 \times 11.59 \times 10^{-12} = 7.03 \times 10^{6}\ \mho$/mi.

$|X_c| = 1/B_c = 0.142$ MΩ /mile

3.19 $l\,c = 8.81 \times 10^{-7} \times 12.78 \times 10^{-12} = 11.259 \times 10^{-18}$

$\mu_0 \epsilon_0 = 4\pi \times 10^{-7} \times 8.854 \times 10^{-12} = 11.126 \times 10^{-18}$

3.20 $\ell c = 9.83 \times 10^{-7} \times 11.59 \times 10^{-12} = 11.393 \times 10^{-18}$

$\mu_o \epsilon_o = 111.126 \times 10^{-18}$

Note: $\mu_o \epsilon_o$ is a universal constant.

Velocity of light in vacuum "c" $= \frac{1}{\sqrt{\mu_o \epsilon_o}} \approx 2.998 \times 10^8$ m/sec.

3.21

$r_a = r_b = r_c = 0.145$ Ω/mi, from Table A8.1, for Grosbeak, at 25°C.

$GMR_a = GMR_b = GMR_c = 0.0355$ ft. (From Table A8.1).

Assume $\rho = 100$ Ω-m (as in Example 3.6), $f = 60$ Hz, then

$$D_e = 2160\sqrt{\rho/f} = 2160\sqrt{100/60} = 2790 \text{ ft.}$$

At 60 Hz, $r_d = 9.869 \times 10^{-7} \times f \quad \Omega/\text{m}$

Using the conversion factor: 1 mile = 1.609 km we get

$r_d = 0.09528 \ \Omega/\text{mi}$

Then $z_{aa} = z_{bb} = z_{cc} = r_a + r_d + j\omega \times 2 \times 10^{-7} \ell n \dfrac{D_e}{GMR_i} \quad \Omega/\text{m}$

$$= (0.145 + 0.09528) + j(377 \times 2 \times 10^{-7})\ell n \frac{2790}{0.0355} \times 1.609 \times 10^3 = 0.2403 + j1.3675 \ \Omega/\text{mi}$$

$d_{ab} = \sqrt{4^2 + 5.5^2} = 6.807$ ft, $d_{ca} = 4$ ft, $d_{bc} = 5.5$ ft.

$$z_{ab} = r_d + j\omega \times 2 \times 10^{-7} \ell n \frac{D_e}{d_{ab}} \quad \Omega/\text{m}$$

$$= 0.09528 + j(377 \times 2 \times 10^{-7})\ell n \frac{2790}{6.8007} \times 1.609 \times 10^3 = 0.09528 + j0.7299 \ \Omega/\text{mi}$$

Similarly using $d_{bc} = 5.5$ ft. and $d_{ca} = 4$ ft., we get

$z_{bc} = 0.09528 + j0.7557$ Ω/mi, $z_{ca} = 0.09528 + j0.7943$ Ω/mi

For 30 miles of line we multiply the above values by 30 to write, in matrix notation

$$Z_{abc} = \begin{bmatrix} (7.209 + j41.025) & (2.858 + j21.8970) & (2.858 + j23.8290) \\ (2.858 + j21.8970) & (7.209 + j41.025) & (2.858 + j22.6710) \\ (2.858 + j23.8290) & (2.858 + j22.6710) & (7.209 + j41.025) \end{bmatrix} \Omega$$

3.22

$r_a = r_b = r_c = 0.306$ Ω/mi (From Table A8.1, for Ostrich, at 25°).

$GMR_a = GMR_b = GMR_c = 0.0230$ ft. (From Table A8.1).

Assume $\rho = 100$ Ω-m as in Example 3.6, $f = 60$ Hz, then

$$D_e = 2160 \sqrt{\rho/f} \text{ ft} = 2160 \sqrt{100/60} = 2790 \text{ ft.}$$

At 60 Hz, $r_d = 9.869 \times 10^{-7} \times f \quad \Omega / \text{m}$

Using the conversion factor: 1 mile = 1.609 km we get

$r_d = 0.09528 \ \Omega / \text{mi}$

Then $z_{aa} = z_{bb} = z_{cc} = r_a + r_d + j\omega \times 2 \times 10^{-7} \ell n \dfrac{D_e}{GMR_i} \ \Omega / \text{m}$

$$= (0.306 + 0.09528) + j\left(377 \times 2 \times 10^{-7}\right) \ell n \frac{2790}{0.0230} \times 1.609 \times 10^3 \ \Omega/\text{mi}$$

$$d_{ab} = d_{bc} = \sqrt{4^2 + 1.5^2} = 4.2720 \text{ ft}, \ d_{ca} = 8.0 \text{ ft}.$$

$$z_{ab} = r_d + j\omega \times 2 \times 10^{-7} \ell n \frac{D_e}{d_{ab}} \ \Omega/\text{m}$$

$$= 0.09528 + j\left(377 \times 2 \times 10^{-7}\right) \ell n \frac{2790}{4.2720} \times 1.609 \times 10^3 = 0.09528 + j0.7864 \ \Omega/\text{mi}$$

$z_{bc} = z_{ab} = 0.09528 + j0.7864 \ \Omega/\text{mi}.$

Similarly using d_{ca} = 8.0 ft. we get

$z_{ca} = 0.09528 + j0.7102 \ \Omega/\text{mi}.$

For 40 miles of line we multiply the above values by 40 to write, in matrix notation

$$Z_{abc} = \begin{bmatrix} (16.0512 + j56.804) & (3.8112 + j31.456) & (3.8112 + j28.408) \\ (3.8112 + j31.456) & (16.0512 + j56.804) & (3.8112 + j31.456) \\ (3.8112 + j28.408) & (3.8112 + j31.456) & (16.0512 + j56.804) \end{bmatrix} \Omega$$