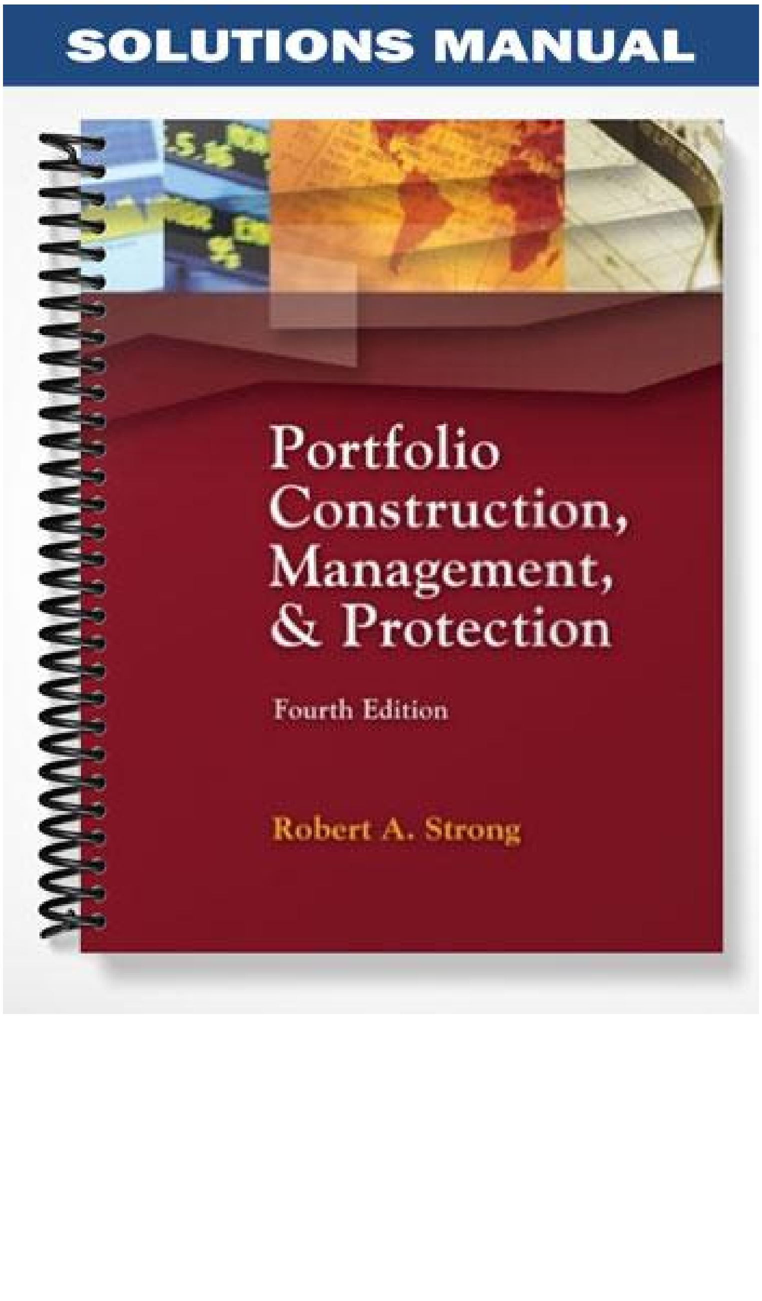
SOLUTIONS MANUAL
Portfolio Construction, Management, & Protection
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Chapter Two

Valuation, Risk, Return, and Uncertainty

KEY POINTS

Most students taking this course will have had a prior course in basic corporate finance. Most also will have had at least one accounting class. Consequently, a good proportion of the material in this chapter should be a review. As the beginning sentence of the chapter states, Chapter 2 functions as a "crash course in the principles of finance and elementary statistics."

Still, almost everyone will learn something from reading this chapter. There is much that instructors inappropriately take for granted. I find this chapter a useful way to resurrect important ideas from previous coursework and get people back into the swing of things before moving on to more difficult material.

TEACHING CONSIDERATIONS

The name of the game here is practice with the end of the chapter problems. Students should be encouraged to solve them using the equations presented in the chapter rather than time value of money tables. Students also should be encouraged to develop confidence in the use of a business calculator, such as the Texas Instruments BA II+ that can be acquired for about $35.

Some material here is likely to be new, especially growing annuities, covariance, standard error, R-squared, and the relationship between the arithmetic and geometric mean returns. Table 2-2 is a very handy means of generating class discussion about the nature of risk. Ask for a show of hands regarding students' preference among the four investment alternatives. Be sure to elaborate on the opportunity cost issue associated with picking an investment other choice A.

The notion of fair bets and the diminishing marginal utility of money, and the St. Petersburg paradox also are a good mechanism for prompting student involvement early in the course.

ANSWERS TO QUESTIONS

1. False. Utility measures the combined influences of expected return and risk. A small sum of money to be received for certain has very little utility associated with it, whereas a small investment in a very risky venture, such as a lottery ticket, has considerable utility to some people.

2. The answer depends on the individual, but many people will change their selection if the game can be played repeatedly.

3. The answer depends on the individual. Because you incur the $50 cost despite the choice, it should not necessarily cause a person to change their selection.

4. Yes. Set equations 2-9 and 2-11 equal to each other, cancel out the initial cash flow "C," assume some initial value for "N" or for "g" and solve for the other variable.

5. Mathematically, no, but practically speaking, yes, if the time period is long enough. Depending on the interest rate used, the present value of an annuity approaches some limit as the period increases. If the period is long enough, there is no appreciable difference in the two values.

6. The arithmetic mean will equal the geometric mean only if all the values are identical. Any dispersion will result in the geometric mean being less than the arithmetic mean.

7. No.

8. "Return" is an intuitive idea to most people. It is most commonly associated with annual rates. Clearly 10% per year is different from 10% per week. Combining weekly and annual returns without any adjustment results in meaningless answers.

9. Returns are sometimes multiplied, and if there is an odd number of negative returns, the product is also negative. You cannot take the even root of a negative number, so it may not be possible to calculate the geometric mean unless you eliminate the negative numbers by calculating return relatives first.

10. ROA is net income divided by *total assets*; ROE is net income divided by *equity*. ROE includes the effect of leverage on investment returns.

11. ROA, in general. ROE may be appropriate in situations where shares are bought on margin. The important thing is to ensure that comparisons are valid. Leverage adds to risk, and ideally risk should be held reasonably constant when comparing alternatives.

12. Dispersion on the positive side does not result in investment loss. Investors are not disappointed if their investments show unusually large returns. It is only dispersion on the adverse side that results in a loss of utility.

13. The correlation between a random variable and a constant is mathematically undefined because of division by zero. (See equation 2-10.) Despite this, there are no diversification benefits associated with perfectly correlated investments. They behave as if their correlation coefficient were 1.0.

14. Semi-variance is a concept that has its advocates and its detractors. You never know an outcome until after the outcome has occurred, so the criticism here is a shaky one.

15. Bill only cares which team wins. Joe cares which team wins and whether they beat the spread.

16. Unless the stock is newly issued, the data are sample data from a larger population. If you have the entire history of returns, you could consider them population data.

17. Individual stock returns are usually assumed to be from a univariate distribution.

18. A portfolio of securities generates a return from a multivariate distribution, as the portfolio return depends on a number of subsidiary returns.

19. The geometric mean of logreturns will be less, because logarithms reduce the dispersion.

20. Standard deviations are calculated from the variance, which is calculated from the square of deviations about the mean. Squaring the deviations removes negative signs.

ANSWERS TO PROBLEMS

1. After the last payment to the custodian, the fund will have a zero balance. This means (PV payments in) – (PV payments out) = 0, or, equivalently,

PV of payments in = PV of payments out

<u>Payments out</u>:

$$PV = \frac{5000}{(1.08)^{26}} + \frac{5000(1.04)}{(1.08)^{27}} + \frac{5000(1.04)^2}{(1.08)^{28}} + \ldots + \frac{5000(1.04)^{14}}{(1.08)^{40}}$$

Multiply both sides of the equation by $(1.08)^{26}$:

$$(1.08)^{26}\text{PV} = 5000 + \frac{5000(1.04)}{(1.08)} + \frac{5000(1.04)^2}{(1.08)^2} + \frac{5000(1.04)^3}{(1.08)^3} \ldots + \frac{5000(1.04)^{14}}{(1.08)^{14}}$$

$$= 5000 + \frac{5000}{1.03846} + \frac{5000}{1.03846^2} + \ldots + \frac{5000}{1.03846^{14}}$$

$$(1.08)^{26} PV = 5000 + 53356.66$$

$$PV = \frac{58356.66}{(1.08)^{26}} = 7889.92$$

<u>Payments in:</u>

Let x = the first payment

$$PV = x + \frac{x(1.04)}{1.08} + \frac{x(1.04)^2}{(1.08)^2} + \ldots + \frac{x(1.04)^{25}}{(1.08)^{25}}$$

$$= x + \frac{x}{1.03846} + \frac{x}{(1.03846)^2} + \ldots + \frac{x}{(1.03846)^{25}}$$

$$PV = x + 15.8795x = 16.8795x$$

Payments out = Payments in

$$16.8795x = 7889.92$$

x **= $467.43**

2. $$PV = \frac{100}{.12 - .05}\left[1 - \left(1.05/1.12\right)^{20}\right] = \mathbf{\$1,035.63}$$

3. $$FV = PV(1+R)^{20} = \$1,035.63(1.12)^{20} = \mathbf{\$9,989.99}$$

4. $C_1 = 200$
PV= 2500
g = 0.03

$$PV = \frac{C_1}{R-g}$$

$$R = \frac{C_1}{PV} + g$$

$$= \frac{200}{2500} + .03 = \mathbf{11.00\%}$$

5. C = 1000 R = .06

$$PV_4 = \frac{C}{R} = \frac{\$1000}{.06} = \$16{,}667.67$$ (This is the value of the perpetuity at time 4.)

$$PV_0 = \frac{\$16{,}667.67}{(1.06)^4} = \mathbf{\$13{,}201.56}$$

6. $$PV = \frac{C_1}{R-g} = \frac{\$1(1.035)}{.14-.035} = \$9.86 \approx \mathbf{9\ 7/8}$$

7. $$PV_9 = \frac{C_{10}}{R-g} = \frac{\$1000}{.07-.04} = \$33{,}333.33$$

$$PV = \frac{PV_9}{(1+R)^9} = \frac{\$33{,}333.33}{(1.07)^9} = \mathbf{\$18{,}131.12}$$

8. This is potentially a complicated problem, depending on how you view it.

Cost = construction cost + maintenance

$$= \$25{,}000 + \frac{\$500}{.12-.05} = \$32{,}142.86$$

500 crypts:

$$\text{Return} = \frac{\text{benefit}}{\text{cost}} = \frac{500X}{32142.86} = .12$$

$$X = \frac{.12(\$32142.86)}{500} = \$7.71$$

To recover costs:

$$\frac{\$32{,}142.86}{500} = \$64.29 \text{ per crypt}$$

To earn a 12% return:

$62.29 + $7.71 = **$72.00** per crypt

9. 264,000 miles = 264000 miles x 5280 ft/mi. x 12 in/ft = 1.672704×10^{10} inches

Let D = number of doublings

$.004 \times 2^D = 1.672704 \times 10^{10}$

$$2^D = \frac{1.672704x10^{10}}{4.00x10^{-3}} = 4.18176x10^{12}$$

$$D = \frac{\ln(4.18176x10^{12})}{\ln(2)} = 41.92 \rightarrow \mathbf{42} \text{ doublings}$$

10. GM = $(1 \times 2 \times 3 \times 4 \times 5 \times 6)^{1/6}$ = **2.99**

11. $$mean = \frac{1}{N}\sum_{i=1}^{8} x_i = \frac{1}{8}(.0005 + 0 - .0012 + .0001 - .0010 - .0002 + .0011 + 0)$$

$$= \mathbf{-.0000875}$$

Week	Return	Probability	Return x Probability
1	$(.0005 – \text{mean})^2$	.125	ignore
2	$(0 – \text{mean})^2$	.125	ignore
3	$(-.0012 – \text{mean})^2$	.125	$2.07x10^{-7}$
4	$(.0001 – \text{mean})^2$	.125	ignore
5	$(-.0010 – \text{mean})^2$	.125	$1.48x10^{-7}$
6	$(-.0002 – \text{mean})^2$	.125	$1.03x10^{-8}$
7	$(.0011 – \text{mean})^2$	.125	ignore
8	$(0 – \text{mean})^2$	.125	ignore
			$\mathbf{3.65x10^{-7}}$

12. $E[(\tilde{x}-\bar{x})^2 = E[\tilde{x}^2 - 2\tilde{x}\bar{x} + \bar{x}^2]$ (1)

$= E(\tilde{x}^2) - 2E(\tilde{x}\bar{x}) + E(\bar{x}^2)$ (2)

$E(\tilde{x}) = \bar{x}$ (3)

$E(\tilde{x}\bar{x}) = \bar{x}E(\tilde{x})$ (4)

$E(\bar{x}^2) = \bar{x}^2$ (5)

substitute (4) into (2):

$E(\tilde{x}^2) - 2\bar{x}E(\tilde{x}) + E(\bar{x}^2)$ (6)

substitute (3) and (5) into 6:

$E(\tilde{x}^2) - 2(\bar{x})^2 + \bar{x}^2]$ (7)

$= E(\tilde{x}^2) - \bar{x}^2$ QED (8)

13. $\mathrm{cov}(\tilde{x}, a) = E[(\tilde{x}-\bar{x})(a-\bar{a})]$

Because "a" is a constant, $\bar{a} = a$. Therefore, $(a-\bar{a}) = 0$ and $\mathrm{cov}(\tilde{x}, a) = 0$.

14. $Y = a + b\tilde{x}$

$\sigma^2 = E[(a+b\tilde{x}) - E(a+b\tilde{x})]^2$

$= E[(a+b\tilde{x}) - a - b\bar{x})]^2$

$= E[a + b\tilde{x} - a - b\bar{x}]^2$

$= E[b\tilde{x} - b\bar{x}]^2$

$= E[b(\tilde{x}-\bar{x})]^2$

$= b^2 E[\tilde{x}-\bar{x}]^2$

$= b^2\sigma^2$

15. $\text{cov}(a\tilde{x}, \tilde{y}) = E[(a\tilde{x} - E(a\tilde{x}))(\tilde{y} - E(\tilde{y}))]$

$= E[(a\tilde{x} - aE(\tilde{x}))(\tilde{y} - E(\tilde{y}))]$

$= aE[(\tilde{x} - E(\tilde{x}))(\tilde{y} - E(\tilde{y}))]$

$= aE[(\tilde{x} - \bar{x})(\tilde{y} - \bar{y})]$

$= aCOV(\tilde{x}, \tilde{y})$

16.

Week	**Return**	**Return Relative**	**Log of Return Relative**
1	0.0005	1.0005	5.00×10^{-4}
2	0	1.0000	0
3	-0.0012	0.9988	-1.20×10^{-3}
4	0.0001	1.0001	1.00×10^{-4}
5	-0.0010	0.9990	-1.00×10^{-3}
6	-0.0002	0.9998	-2.00×10^{-4}
7	0.0011	1.0011	1.10×10^{-3}
8	0	1.0000	0

17. a.

$$GM = \left[\prod_{i=1}^{n}(1+R_i)\right]^{\frac{1}{n}} - 1 =$$

$$[(1.0005)(1.0000)(.9988)(1.0001)(.9990)(.9998)(1.0011)(1.000)]^{\frac{1}{8}} - 1 = .999298^{.125} - 1 = -.0000877$$

b.

$$\mathbf{GM} = \mathbf{e}^{\frac{1}{n}\sum \mathbf{LN}(1+\mathbf{R_i})} - \mathbf{1} = \mathbf{e}^{\frac{1}{8}(.0005+0-.0012+.0001-.001-.0002+.0011+0)} - \mathbf{1} = \mathbf{e}^{.125(-.0007)} - \mathbf{1} = \mathbf{e}^{-.00008775} - \mathbf{1} = -\mathbf{.0000877}$$

18.

Observation	$\tilde{\mathbf{x}}$	$\mathbf{2\tilde{x}}$	$\mathbf{(2\tilde{x}-\bar{x})^2}$
1	2	4	1.96
2	-1	-2	21.16
3	4	8	29.16
4	1	2	0.36
5	2	4	1.96
6	-2	-4	43.56
7	5	10	54.76
8	-1	-2	21.16
9	0	0	6.76
10	3	6	11.56
Average		2.6	19.24

$$\sigma^2(\mathbf{a\tilde{x}}) = \mathbf{19.24}$$
$$\mathbf{a}^2\sigma_{\mathbf{x}}^2 = \mathbf{2^2\ 4.81} = \mathbf{19.24}$$

19. Standard error = $\dfrac{\sigma}{\sqrt{n}} = \dfrac{6.97x10^{-4}}{\sqrt{8}} = 2.46\text{x}10^{-4}$

20. 1994 – 1995 return: $\dfrac{24-20.5+.23}{20.5} = .1820$

1995 – 1996 return: $\dfrac{36.25-24+.25}{24} = .5208$

1996 – 1997 return: $\dfrac{43-36.25+.27}{36.25} = .1937$

1997 – 1998 return: $\dfrac{56.5-43+.31}{43} = .3212$

$[(.1820)(.5208)(.1937)(.3212)]^{1/4} = .2311$ **23.11%**

21. 1994 – 1995 change: $\frac{.23-.23}{.23}=0$

1995 – 1996 change: $\frac{.25-.23}{.23}=.0870$

1996 – 1997 change: $\frac{.27-.25}{.25}=.0800$

1997 – 1998 change: $\frac{.31-.27}{.27}=.1481$

a. Arithmetic mean: (0 + .0870 + .0800 + .1481)/4 = .0788 = **7.88%**

b. Geometric mean: $[(1.000)(1.0870)(1.0800)(1.1481)]^{1/4}$ = 1.0775 → **7.75%**

22. $HPR=\frac{P_2-P_1+D}{P_1}=\frac{56.5-20.5+(.23+.25+.27+.31)}{20.5}=1.8078$ → **180.78%**

23. $$P_0=\frac{D_1}{R-g}=\frac{D_0(1+g)}{R-g}$$

$$R=\frac{D_0(1+g)}{P_0}+g$$

$$R=\frac{\$0.31(1+.0775)}{\$56.5}+.0775=\ \mathbf{8.34\%}$$

24. The 95% confidence interval is about two standard deviations either side of the mean. The standard deviation of this distribution is the square root of 2.56, or 1.60. The 95% confidence interval is then 23.2 +/- 2(1.60) = 20.00 to 26.4. **Technique B** lies outside this range, so it is unlikely to have happened by chance.

25. a. This is true. The order of their raises does not matter: by laws of algebra, abc = cab.

b. This is true. He earns more money sooner, and dollars today are worth more than dollars tomorrow.