SOLUTIONS MANUAL



CHAPTER 2 KINEMATICS IN ONE DIMENSION

CONCEPTUAL QUESTIONS

- 1. **REASONING AND SOLUTION** The displacement of the honeybee for the trip is *not* the same as the distance traveled by the honeybee. As stated in the question, the distance traveled by the honeybee is 2 km. The displacement for the trip is the shortest distance between the initial and final positions of the honeybee. Since the honeybee returns to the hive, its initial position and final position are the same; therefore, the displacement of the honeybee is zero.
- 2. **REASONING AND SOLUTION** The buses *do not* have equal velocities. Velocity is a vector, with both magnitude and direction. In order for two vectors to be equal, they must have the same magnitude and the same direction. The direction of the velocity of each bus points in the direction of motion of the bus. Thus, the directions of the velocities of the buses are different. Therefore, the velocities are not equal, even though the speeds are the same.
- 3. **REASONING AND SOLUTION** The average speed of a vehicle is defined as the total distance covered by the vehicle divided by the time required for the vehicle to cover the distance. Both distance and time are scalar quantities. Since the average speed is the ratio of two scalar quantities, it is a scalar quantity.
- 4. **SSM REASONING AND SOLUTION** Consider the four traffic lights 1, 2, 3 and 4 shown below. Let the distance between lights 1 and 2 be x_{12} , the distance between lights 2 and 3 be x_{23} , and the distance between lights 3 and 4 be x_{34} .



The lights can be timed so that if a car travels with a constant speed v, red lights can be avoided in the following way. Suppose that at time t = 0 s, light 1 turns green while the rest are red. Light 2 must then turn green in a time t_{12} , where $t_{12} = x_{12}/v$. Light 3 must turn green in a time t_{23} after light 2 turns green, where $t_{23} = x_{23}/v$. Likewise, light 4 must turn

green in a time t_{34} after light 3 turns green, where $t_{34} = x_{34}/v$. Note that the timing of traffic lights is more complicated than indicated here when groups of cars are stopped at light 1. Then the acceleration of the cars, the reaction time of the drivers, and other factors must be considered.

- 5. **REASONING AND SOLUTION** The velocity of the car is a vector quantity with both magnitude and direction. The speed of the car is a scalar quantity and has nothing to do with direction. It is possible for a car to drive around a track at constant speed. As the car drives around the track, however, the car must change direction. Therefore, the direction of the velocity *changes*, and the velocity cannot be constant. The *incorrect* statement is (a).
- 6. **REASONING AND SOLUTION** Answers will vary. One example is a ball that is thrown straight up in the air. When the ball is at its highest point, its velocity is momentarily zero. Since the ball is close to the surface of the earth, its acceleration is nearly constant and is equal to the acceleration due to gravity. Thus, the velocity of the ball is momentarily zero, but since the ball is accelerating, the velocity is changing. An instant later, the velocity of the ball is nonzero as the ball begins to fall. Another example is a swimmer in a race, reversing directions at the end of the pool.
- 7. **REASONING AND SOLUTION** The acceleration of an object is the rate at which its velocity is changing. No information can be gained concerning the acceleration of an object if all that is known is the velocity of the object at a single instant. No conclusion can be reached concerning the accelerations of the two vehicles, so the car does not necessarily have a greater acceleration.
- 8. **REASONING AND SOLUTION** It *is* possible for the instantaneous velocity at any point during a trip to have a negative value, even though the average velocity for the entire trip has a positive value. The average velocity for the trip is the displacement for the trip divided by the elapsed time. It depends only on the initial and final positions, and the time required for the trip. The average velocity contains no information concerning the actual path taken by the object. Let us assume that the object is constrained to move in a straight line with directions designated as positive or negative. The direction of the average velocity is the same as the direction of the displacement, while the direction of the instantaneous velocity is the same as the instantaneous direction of motion. The average velocity will be positive if the displacement vector points in the positive direction. At any point in the trip, the object is moving in the negative direction, its instantaneous velocity is negative. As long as the overall displacement is positive, the average velocity for the trip is positive.
- 9. **REASONING AND SOLUTION** Acceleration is the rate of change of velocity. The average velocity for an object over a time interval Δt is given by $\overline{\mathbf{v}} = \Delta \mathbf{x} / \Delta t$, where $\Delta \mathbf{x}$ is the displacement of the object during the time interval.

A runner runs half the remaining distance to the finish line every ten seconds. In each successive ten-second interval, the distance covered by the runner, and hence the runner's displacement, becomes smaller by a factor of 2. Thus, during each successive ten-second interval, the ratio $\Delta \mathbf{x} / \Delta t$ becomes smaller by a factor of 2. The acceleration, however, is the *change* in velocity per unit time. Assume that the distance to the finish line is 100 m and consider the change in the average velocity between the first and second 10 s intervals; it is $\Delta v_{1,2} = (1/2)(50 \text{ m})/(10 \text{ s}) - (1/2)(100 \text{ m})/(10 \text{ s}) = -2.5 \text{ m/s}$. Now consider the change in velocity between the second and third 10 s average intervals: it is $\Delta v_{2.3} = (1/2)(25 \text{ m})/(10 \text{ s}) - (1/2)(50 \text{ m})/(10 \text{ s}) = -1.25 \text{ m/s}$. Thus, as time passes, the magnitude of the velocity changes by a decreasing amount from one 10 s interval to the next. As a result, the acceleration of the runner does not have a constant magnitude.

- 10. **SSM REASONING AND SOLUTION** An object moving with a constant acceleration will slow down if the acceleration vector points in the opposite direction to the velocity vector; however, if the acceleration remains constant, the object will never come to a permanent halt. As time increases, the magnitude of the velocity will get smaller and smaller. At some time, the velocity will be instantaneously zero. If the acceleration is constant, however, the velocity vector will continue to change at the same rate. An instant after the velocity is zero, the magnitude of the velocity will begin increasing in the same direction as the acceleration. In other words, if the acceleration truly remains constant, the object will slow down, stop *for an instant*, reverse direction and then speed up.
- 11. **REASONING AND SOLUTION** Other than being horizontal, the motion of an experimental vehicle that slows down, comes to a momentary halt, reverses direction and then speeds up with a constant acceleration of 9.80 m/s^2 , is identical to that of a ball that is thrown straight upward near the surface of the earth, comes to a halt, and falls back down. In both cases, the acceleration is constant in magnitude and direction and the objects begin their motion with the acceleration and velocity vectors pointing in opposite directions.
- 12. **REASONING AND SOLUTION** The first ball has an initial speed of zero, since it is dropped from rest. It picks up speed on the way down, striking the ground at a speed v_f . The second ball has a motion that is the reverse of that of the first ball. The second ball starts out with a speed v_f and loses speed on the way up. By symmetry, the second ball will come to a halt at the top of the building. Thus, in approaching the crossing point, the second ball travels faster than the first ball. Correspondingly, the second ball must travel farther on its way to the crossing point than the first ball does. Thus, the crossing point must be located in the upper half of the building.
- 13. **REASONING AND SOLUTION** Two objects are thrown vertically upward, first one, and then, a bit later, the other. The time required for either ball to reach its maximum height can be found from Equation 2.4: $v = v_0 + at$. At the maximum height, v = 0 m/s; solving for t yields $t = -v_0 / a$, where a is the acceleration due to gravity. Clearly, the time required to

reach the maximum height depends on the initial speed with which the object was thrown. Since the second object is launched later, its initial speed must be less than the initial speed of the first object in order that both objects reach their maximum heights at the same instant. The maximum height that each object attains can be found from Equation 2.9: $v^2 = v_0^2 + 2ay$. At the maximum height, v = 0 m/s; solving for y gives $y = -v_0^2 / (2a)$, where a is the acceleration due to gravity. Since the second object has a smaller initial speed v_0 , it will also attain a smaller maximum height. Thus, it is *not* possible for both objects to reach the same maximum height at the same instant.

14. **SSM REASONING AND SOLUTION** The magnitude of the muzzle velocity of the bullet can be found (to a very good approximation) by solving Equation 2.9, $v^2 = v_0^2 + 2ax$, with $v_0 = 0$ m/s; that is

$$v = \sqrt{2ax}$$

where a is the acceleration of the bullet and x is the distance traveled by the bullet before it leaves the barrel of the gun (i.e., the length of the barrel).

Since the muzzle velocity of the rifle with the shorter barrel is greater than the muzzle velocity of the rifle with the longer barrel, the product *ax* must be greater for the bullet in the rifle with the shorter barrel. But *x* is smaller for the rifle with the shorter barrel, thus the *acceleration of the bullet must be larger in the rifle with the shorter barrel*.

CHAPTER 2 *KINEMATICS IN ONE DIMENSION*

PROBLEMS

1. SSM REASONING AND SOLUTION

a. The total distance traveled is found by adding the distances traveled during each segment of the trip.

$$6.9 \text{ km} + 1.8 \text{ km} + 3.7 \text{ km} = |12.4 \text{ km}|$$

b. All three segments of the trip lie along the east-west line. Taking east as the positive direction, the individual displacements can then be added to yield the resultant displacement.

$$6.9 \text{ km} + (-1.8 \text{ km}) + 3.7 \text{ km} = +8.8 \text{ km}$$

The displacement is positive, indicating that it points due east. Therefore,

Displacement of the whale = 8.8 km, due east

2. **REASONING** Since the average speed of the impulse is equal to the distance it travels divided by the elapsed time (see Equation 2.1), the elapsed time is just the distance divided by the average speed.

SOLUTION The time it takes for the impulse to travel from the foot to the brain is

Time =
$$\frac{\text{Distance}}{\text{Average speed}} = \frac{1.8 \text{ m}}{1.1 \times 10^2 \text{ m/s}} = \boxed{1.6 \times 10^{-2} \text{ s}}$$
 (2.1)

3. **SSM** *REASONING AND SOLUTION* The moving plane can be seen through the window of the stationary plane for the time that it takes the moving plane to travel its own length. To find the time, we write Equation 2.1 as

Elapsed time =
$$\frac{\text{Distance}}{\text{Average speed}} = \frac{36 \text{ m}}{45 \text{ m/s}} = \boxed{0.80 \text{ s}}$$

4. **REASONING** The younger (and faster) runner should start the race after the older runner, the delay being the difference between the time required for the older runner to complete the race and that for the younger runner. The time for each runner to complete the race is equal to the distance of the race divided by the average speed of that runner (see Equation 2.1).

SOLUTION The difference in the times for the two runners to complete the race is $t_{50} - t_{18}$, where

 $t_{50} = \frac{\text{Distance}}{(\text{Average Speed})_{50\text{-yr-old}}} \text{ and } t_{18} = \frac{\text{Distance}}{(\text{Average Speed})_{18\text{-yr-old}}}$ (2.1)

The difference in these two times (which is how much later the younger runner should start) is

$$t_{50} - t_{18} = \frac{\text{Distance}}{(\text{Average Speed})_{50\text{-yr-old}}} - \frac{\text{Distance}}{(\text{Average Speed})_{18\text{-yr-old}}}$$
$$= \frac{10.0 \times 10^3 \text{ m}}{4.27 \text{ m/s}} - \frac{10.0 \times 10^3 \text{ m}}{4.39 \text{ m/s}} = \boxed{64 \text{ s}}$$

5. **REASONING** The distance traveled by the Space Shuttle is equal to its speed multiplied by the time. The number of football fields is equal to this distance divided by the length L of one football field.

SOLUTION The number of football fields is

Number =
$$\frac{x}{L} = \frac{vt}{L} = \frac{(7.6 \times 10^3 \text{ m/s})(110 \times 10^{-3} \text{ s})}{91.4 \text{ m}} = 9.1$$

6. **REASONING AND SOLUTION** In 12 minutes the sloth travels a distance of

$$x_{\rm s} = v_{\rm s}t = (0.037 \text{ m/s})(12 \text{ min})\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 27 \text{ m}$$

while the tortoise travels a distance of

$$x_{\rm t} = v_{\rm t} {\rm t} = (0.076 {\rm m/s})(12 {\rm min}) \left(\frac{60 {\rm s}}{1 {\rm min}}\right) = 55 {\rm m}$$

The tortoise goes farther than the sloth by an amount that equals 55 m - 27 m = 28 m

7. **REASONING** In order for the bear to catch the tourist over the distance d, the bear must reach the car at the same time as the tourist. During the time t that it takes for the tourist to reach the car, the bear must travel a total distance of d + 26 m. From Equation 2.1,

$$v_{\text{tourist}} = \frac{d}{t}$$
 (1) and $v_{\text{bear}} = \frac{d+26 \text{ m}}{t}$ (2)

Equations (1) and (2) can be solved simultaneously to find d.

SOLUTION Solving Equation (1) for t and substituting into Equation (2), we find

$$v_{\text{bear}} = \frac{d + 26 \text{ m}}{d / v_{\text{tourist}}} = \frac{(d + 26 \text{ m})v_{\text{tourist}}}{d}$$
$$v_{\text{bear}} = \left(1 + \frac{26 \text{ m}}{d}\right)v_{\text{tourist}}$$

Solving for *d* yields:

$$d = \frac{26 \text{ m}}{\frac{v_{\text{bear}}}{v_{\text{tourist}}} - 1} = \frac{26 \text{ m}}{\frac{6.0 \text{ m/s}}{4.0 \text{ m/s}} - 1} = \boxed{52 \text{ m}}$$

8. **REASONING AND SOLUTION** Let west be the positive direction. The average velocity of the backpacker is

$$v = \frac{x_{\rm W} + x_{\rm e}}{t_{\rm W} + t_{\rm e}}$$
 where $t_{\rm W} = \frac{x_{\rm W}}{v_{\rm W}}$ and $t_{\rm e} = \frac{x_{\rm e}}{v_{\rm e}}$

Combining these equations and solving for x_e (suppressing the units) gives

$$x_{\rm e} = \frac{-(1 - v/v_{\rm w})x_{\rm w}}{(1 - v/v_{\rm e})} = \frac{-[1 - (1.34 \text{ m/s})/(2.68 \text{ m/s})](6.44 \text{ km})}{1 - (1.34 \text{ m/s})/(0.447 \text{ m/s})} = -0.81 \text{ km}$$

The distance traveled is the magnitude of x_{e} , or 0.81 km.

9. SSM REASONING AND SOLUTION

a. The total displacement traveled by the bicyclist for the entire trip is equal to the sum of the displacements traveled during each part of the trip. The displacement traveled during each part of the trip is given by Equation 2.2: $\Delta x = \overline{v} \Delta t$. Therefore,

$$\Delta x_1 = (7.2 \text{ m/s})(22 \text{ min}) \left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 9500 \text{ m}$$
$$\Delta x_2 = (5.1 \text{ m/s})(36 \text{ min}) \left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 11\ 000 \text{ m}$$
$$\Delta x_3 = (13 \text{ m/s})(8.0 \text{ min}) \left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 6200 \text{ m}$$

The total displacement traveled by the bicyclist during the entire trip is then

$$\Delta x = 9500 \text{ m} + 11\ 000 \text{ m} + 6200 \text{ m} = 2.67 \times 10^4 \text{ m}$$

b. The average velocity can be found from Equation 2.2.

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{2.67 \times 10^4 \text{ m}}{(22 \text{ min} + 36 \text{ min} + 8.0 \text{ min})} \left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{6.74 \text{ m/s, due north}}$$

10. **REASONING** The time t_{trip} to make the entire trip is equal to the time t_{cart} that the golfer rides in the golf cart plus the time t_{walk} that she walks; $t_{trip} = t_{cart} + t_{walk}$. Therefore, the time that she walks is

$$t_{\text{walk}} = t_{\text{trip}} - t_{\text{cart}}$$
(1)

The average speed \overline{v}_{trip} for the entire trip is equal to the total distance, $x_{cart} + x_{walk}$, she travels divided by the time to make the entire trip (see Equation 2.1);

$$\overline{v}_{\text{trip}} = \frac{x_{\text{cart}} + x_{\text{walk}}}{t_{\text{trip}}}$$

Solving this equation for t_{trip} and substituting the resulting expression into Equation 1 yields

$$t_{\text{walk}} = \frac{x_{\text{cart}} + x_{\text{walk}}}{\overline{v}_{\text{trip}}} - t_{\text{cart}}$$
(2)

The distance traveled by the cart is $x_{cart} = \overline{v}_{cart} t_{cart}$, and the distance walked by the golfer is $x_{walk} = \overline{v}_{walk} t_{walk}$. Substituting these expressions for x_{cart} and x_{walk} into Equation 2 gives

$$t_{\text{walk}} = \frac{\overline{v}_{\text{cart}} t_{\text{cart}} + \overline{v}_{\text{walk}} t_{\text{walk}}}{\overline{v}_{\text{trip}}} - t_{\text{cart}}$$

The unknown variable t_{walk} appears on both sides of this equation. Algebraically solving for this variable gives

$$t_{\text{walk}} = \frac{\overline{v}_{\text{cart}} t_{\text{cart}} - \overline{v}_{\text{trip}} t_{\text{cart}}}{\overline{v}_{\text{trip}} - \overline{v}_{\text{walk}}}$$

SOLUTION The time that the golfer spends walking is

$$t_{\text{walk}} = \frac{\overline{v}_{\text{cart}} t_{\text{cart}} - \overline{v}_{\text{trip}} t_{\text{cart}}}{\overline{v}_{\text{trip}} - \overline{v}_{\text{walk}}} = \frac{(3.10 \text{ m/s})(28.0 \text{ s}) - (1.80 \text{ m/s})(28.0 \text{ s})}{(1.80 \text{ m/s}) - (1.30 \text{ m/s})} = \boxed{73 \text{ s}}$$

11. **REASONING AND SOLUTION** The upper edge of the wall will disappear after the train has traveled the distance *d* in the figure below.



The distance d is equal to the length of the window plus the base of the 12° right triangle of height 0.90 m.

The base of the triangle is given by

$$b = \frac{0.90 \text{ m}}{\tan 12^\circ} = 4.2 \text{ m}$$
 0.90 m

Thus, d = 2.0 m + 4.2 m = 6.2 m.

The time required for the train to travel 6.2 m is, from the definition of average speed,

$$t = \frac{x}{v} = \frac{6.2 \text{ m}}{3.0 \text{ m/s}} = 2.1 \text{ s}$$

12. **REASONING** We can use the definition of average acceleration $\overline{\mathbf{a}} = (\mathbf{v} - \mathbf{v_0})/(t - t_0)$ (Equation 2.4) to find the sprinter's final velocity \mathbf{v} at the end of the acceleration phase, because her initial velocity ($\mathbf{v_0} = 0$ m/s, since she starts from rest), her average acceleration $\overline{\mathbf{a}}$, and the time interval $t - t_0$ are known.

SOLUTION

a. Since the sprinter has a constant acceleration, it is also equal to her average acceleration, so $\overline{\mathbf{a}} = +2.3 \text{ m/s}^2$ Her velocity at the end of the 1.2-s period is

$$\mathbf{v} = \mathbf{v}_0 + \overline{\mathbf{a}} (t - t_0) = (0 \text{ m/s}) + (+2.3 \text{ m/s}^2)(1.2 \text{ s}) = +2.8 \text{ m/s}$$

b. Since her acceleration is zero during the remainder of the race, her velocity remains constant at $\boxed{+2.8 \text{ m/s}}$.

13. **SSM** *REASONING* Since the velocity and acceleration of the motorcycle point in the same direction, their numerical values will have the same algebraic sign. For convenience, we will choose them to be positive. The velocity, acceleration, and the time are related by Equation 2.4: $v = v_0 + at$.

SOLUTION

a. Solving Equation 2.4 for t we have

$$t = \frac{v - v_0}{a} = \frac{(+31 \text{ m/s}) - (+21 \text{ m/s})}{+2.5 \text{ m/s}^2} = \boxed{4.0 \text{ s}}$$

b. Similarly,

$$t = \frac{v - v_0}{a} = \frac{(+61 \text{ m/s}) - (+51 \text{ m/s})}{+2.5 \text{ m/s}^2} = \boxed{4.0 \text{ s}}$$

14. **REASONING AND SOLUTION** The magnitude of the car's acceleration can be found from Equation 2.4 ($v = v_0 + at$) as

$$a = \frac{v - v_0}{t} = \frac{26.8 \text{ m/s} - 0 \text{ m/s}}{3.275 \text{ s}} = \boxed{8.18 \text{ m/s}^2}$$

15. **SSM** *REASONING AND SOLUTION* The initial velocity of the runner can be found by solving Equation 2.4 ($v = v_0 + at$) for v_0 . Taking west as the positive direction, we have

$$v_0 = v - at = (+4.15 \text{ m/s}) - (+0.640 \text{ m/s}^2)(1.50 \text{ s}) = +3.19 \text{ m/s}$$

Therefore, the initial velocity of the runner is 3.19 m/s, due west .

16. **REASONING AND SOLUTION** The velocity of the automobile for each stage is given by Equation 2.4: $v = v_0 + at$. Therefore,

$$v_1 = v_0 + a_1 t = 0$$
 m/s $+ a_1 t$ and $v_2 = v_1 + a_2 t$

Since the magnitude of the car's velocity at the end of stage 2 is 2.5 times greater than it is at the end of stage 1, $v_2 = 2.5v_1$. Thus, rearranging the result for v_2 , we find

$$a_2 = \frac{v_2 - v_1}{t} = \frac{2.5v_1 - v_1}{t} = \frac{1.5v_1}{t} = \frac{1.5(a_1t)}{t} = 1.5a_1 = 1.5(3.0 \text{ m/s}^2) = \boxed{4.5 \text{ m/s}^2}$$

17. *REASONING* According to Equation 2.4, the average acceleration of the car for the first twelve seconds after the engine cuts out is

$$\overline{a}_1 = \frac{v_{1f} - v_{10}}{\Delta t_1} \tag{1}$$

and the average acceleration of the car during the next six seconds is

$$\overline{a}_{2} = \frac{v_{2f} - v_{20}}{\Delta t_{2}} = \frac{v_{2f} - v_{1f}}{\Delta t_{2}}$$
(2)

The velocity v_{1f} of the car at the end of the initial twelve-second interval can be found by solving Equations (1) and (2) simultaneously.

SOLUTION Dividing Equation (1) by Equation (2), we have

$$\frac{\overline{a}_{1}}{\overline{a}_{2}} = \frac{(v_{1f} - v_{10})/\Delta t_{1}}{(v_{2f} - v_{1f})/\Delta t_{2}} = \frac{(v_{1f} - v_{10})\Delta t_{2}}{(v_{2f} - v_{1f})\Delta t_{1}}$$

Solving for v_{1f} , we obtain

$$v_{1\mathrm{f}} = \frac{\overline{a}_1 \Delta t_1 v_{2\mathrm{f}} + \overline{a}_2 \Delta t_2 v_{10}}{\overline{a}_1 \Delta t_1 + \overline{a}_2 \Delta t_2} = \frac{(\overline{a}_1 / \overline{a}_2) \Delta t_1 v_{2\mathrm{f}} + \Delta t_2 v_{10}}{(\overline{a}_1 / \overline{a}_2) \Delta t_1 + \Delta t_2}$$

$$v_{\rm 1f} = \frac{1.50(12.0 \text{ s})(+28.0 \text{ m/s}) + (6.0 \text{ s})(+36.0 \text{ m/s})}{1.50(12.0 \text{ s}) + 6.0 \text{ s}} = \boxed{+30.0 \text{ m/s}}$$

18. **REASONING AND SOLUTION** During the first phase of the acceleration,

$$a_1 = \frac{v}{t_1}$$

During the second phase of the acceleration,

$$v = (3.4 \text{ m/s}) - (1.1 \text{ m/s}^2)(1.2 \text{ s}) = 2.1 \text{ m/s}$$

Then

$$a_1 = \frac{2.1 \text{ m/s}}{1.5 \text{ s}} = \boxed{1.4 \text{ m/s}^2}$$

19. **REASONING AND SOLUTION** The average acceleration of the basketball player is $\overline{a} = v/t$, so

$$x = \frac{1}{2}\overline{a}t^2 = \frac{1}{2}\left(\frac{6.0 \text{ m/s}}{1.5 \text{ s}}\right)(1.5 \text{ s})^2 = \boxed{4.5 \text{ m}}$$

20. **REASONING** The cart has an initial velocity of $v_0 = +5.0$ m/s, so initially it is moving to the right, which is the positive direction. It eventually reaches a point where the displacement is x = +12.5 m, and it begins to move to the left. This must mean that the cart comes to a momentary halt at this point (final velocity is v = 0 m/s), before beginning to move to the left. In other words, the cart is decelerating, and its acceleration must point opposite to the velocity, or to the left. Thus, the acceleration is negative. Since the initial velocity, the final velocity, and the displacement are known, Equation 2.9 $(v^2 = v_0^2 + 2ax)$ can be used to determine the acceleration.

SOLUTION Solving Equation 2.9 for the acceleration *a* shows that

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(0 \text{ m/s})^2 - (+5.0 \text{ m/s})^2}{2(+12.5 \text{ m})} = \boxed{-1.0 \text{ m/s}^2}$$

21. **SSM** *REASONING* The average acceleration is defined by Equation 2.4 as the change in velocity divided by the elapsed time. We can find the elapsed time from this relation because the acceleration and the change in velocity are given.

SOLUTION

a. The time Δt that it takes for the VW Beetle to change its velocity by an amount $\Delta v = v - v_0$ is (and noting that 0.4470 m/s = 1 mi/h)

$$\Delta t = \frac{v - v_0}{a} = \frac{(60.0 \text{ mi/h}) \left(\frac{0.4470 \text{ m/s}}{1 \text{ mi/h}}\right) - 0 \text{ m/s}}{2.35 \text{ m/s}^2} = \boxed{11.4 \text{ s}}$$

b. From Equation 2.4, the acceleration (in m/s^2) of the dragster is

$$a = \frac{v - v_0}{t - t_0} = \frac{(60.0 \text{ mi/h}) \left(\frac{0.4470 \text{ m/s}}{1 \text{ mi/h}}\right) - 0 \text{ m/s}}{0.600 \text{ s} - 0 \text{ s}} = \boxed{44.7 \text{ m/s}^2}$$

22. REASONING AND SOLUTION

a. From Equation 2.4, the definition of average acceleration, the magnitude of the average acceleration of the skier is

$$\overline{a} = \frac{v - v_0}{t - t_0} = \frac{8.0 \text{ m/s} - 0 \text{ m/s}}{5.0 \text{ s}} = \boxed{1.6 \text{ m/s}^2}$$

b. With *x* representing the displacement traveled along the slope, Equation 2.7 gives:

$$x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(8.0 \text{ m/s} + 0 \text{ m/s})(5.0 \text{ s}) = 2.0 \times 10^1 \text{ m}$$

23. **REASONING** We know the initial and final velocities of the blood, as well as its displacement. Therefore, Equation 2.9 $(v^2 = v_0^2 + 2ax)$ can be used to find the acceleration of the blood. The time it takes for the blood to reach it final velocity can be found by using Equation 2.7 $\left[t = \frac{x}{1+x}\right]$.

Equation 2.7
$$\left[t = \frac{x}{\frac{1}{2}(v_0 + v)} \right]$$

SOLUTION

a. The acceleration of the blood is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(26 \text{ cm/s})^2 - (0 \text{ cm/s})^2}{2(2.0 \text{ cm})} = \boxed{1.7 \times 10^2 \text{ cm/s}^2}$$

b. The time it takes for the blood, starting from 0 cm/s, to reach a final velocity of +26 cm/s is

$$t = \frac{x}{\frac{1}{2}(v_0 + v)} = \frac{2.0 \text{ cm}}{\frac{1}{2}(0 \text{ cm/s} + 26 \text{ cm/s})} = \boxed{0.15 \text{ s}}$$

24. **REASONING** The cheetah and its prey run the same distance. The prey runs at a constant velocity, so that its distance is the magnitude of its displacement, which is given by Equation 2.2 as the product of velocity and time. The distance for the cheetah can be expressed using Equation 2.8, since the cheetah's initial velocity (zero, since it starts from rest) and the time are given, and we wish to determine the acceleration. The two expressions for the distance can be equated and solved for the acceleration.

SOLUTION We begin by using Equation 2.2 and assuming that the initial position of the prey is $x_0 = 0$ m. The distance run by the prey is

$$\Delta x = x - x_0 = x = v_{\text{Prey}}t$$

The distance run by the cheetah is given by Equation 2.8 as

$$x = v_{0, \text{ Cheetah}}t + \frac{1}{2}a_{\text{Cheetah}}t^2$$

Equating the two expressions for x and using the fact that $v_{0, \text{Cheetab}} = 0 \text{ m/s}$, we find that

$$v_{\text{Prey}}t = \frac{1}{2}a_{\text{Cheetah}}t^2$$

Solving for the acceleration gives

$$a_{\text{Cheetah}} = \frac{2v_{\text{Prey}}}{t} = \frac{2(+9.0 \text{ m/s})}{3.0 \text{ s}} = [+6.0 \text{ m/s}^2]$$

25. SSM WWW REASONING AND SOLUTION

a. The magnitude of the acceleration can be found from Equation 2.4 ($v = v_0 + at$) as

$$a = \frac{v - v_0}{t} = \frac{3.0 \text{ m/s} - 0 \text{ m/s}}{2.0 \text{ s}} = \boxed{1.5 \text{ m/s}^2}$$

b. Similarly the magnitude of the acceleration of the car is

$$a = \frac{v - v_0}{t} = \frac{41.0 \text{ m/s} - 38.0 \text{ m/s}}{2.0 \text{ s}} = \boxed{1.5 \text{ m/s}^2}$$

c. Assuming that the acceleration is constant, the displacement covered by the car can be found from Equation 2.9 ($v^2 = v_0^2 + 2ax$):

$$x = \frac{v^2 - v_0^2}{2a} = \frac{(41.0 \text{ m/s})^2 - (38.0 \text{ m/s})^2}{2(1.5 \text{ m/s}^2)} = 79 \text{ m}$$

Similarly, the displacement traveled by the jogger is

$$x = \frac{v^2 - v_0^2}{2a} = \frac{(3.0 \text{ m/s})^2 - (0 \text{ m/s})^2}{2(1.5 \text{ m/s}^2)} = 3.0 \text{ m}$$

Therefore, the car travels $79 \text{ m} - 3.0 \text{ m} = \boxed{76 \text{ m}}$ further than the jogger.

26. **REASONING** At a constant velocity the time required for Secretariat to run the final mile is given by Equation 2.2 as the displacement (+1609 m) divided by the velocity. The actual time required for Secretariat to run the final mile can be determined from Equation 2.8, since the initial velocity, the acceleration, and the displacement are given. It is the difference between these two results for the time that we seek.

SOLUTION According to Equation 2.2, with the assumption that the initial time is $t_0 = 0$ s, the run time at a constant velocity is

$$\Delta t = t - t_0 = t = \frac{\Delta x}{v} = \frac{+1609 \text{ m}}{+16.58 \text{ m/s}} = 97.04 \text{ s}$$

Solving Equation 2.8 $\left(x = v_0 t + \frac{1}{2}at^2\right)$ for the time shows that

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4(\frac{1}{2}a)(-x)}}{2(\frac{1}{2}a)}$$
$$= \frac{-16.58 \text{ m/s} \pm \sqrt{(+16.58 \text{ m/s})^2 - 4(\frac{1}{2})(+0.0105 \text{ m/s}^2)(-1609 \text{ m})}}{2(\frac{1}{2})(+0.0105 \text{ m/s}^2)} = 94.2 \text{ s}$$

We have ignored the negative root as being unphysical. The acceleration allowed Secretariat to run the last mile in a time that was faster by

97.04 s
$$-$$
 94.2 s $=$ 2.8 s

27. **SSM WWW** *REASONING* Since the belt is moving with constant velocity, the displacement ($x_0 = 0$ m) covered by the belt in a time t_{belt} is giving by Equation 2.2 (with x_0 assumed to be zero) as

$$x = v_{\text{belt}} t_{\text{belt}} \tag{1}$$

Since Clifford moves with constant acceleration, the displacement covered by Clifford in a time t_{Cliff} is, from Equation 2.8,

$$x = v_0 t_{\text{Cliff}} + \frac{1}{2} a t_{\text{Cliff}}^2 = \frac{1}{2} a t_{\text{Cliff}}^2$$
(2)

The speed v_{belt} with which the belt of the ramp is moving can be found by eliminating x between Equations (1) and (2).

SOLUTION Equating the right hand sides of Equations (1) and (2), and noting that $t_{\text{Cliff}} = \frac{1}{4}t_{\text{belt}}$, we have

$$v_{\text{belt}} t_{\text{belt}} = \frac{1}{2} a \left(\frac{1}{4} t_{\text{belt}}\right)^2$$

$$v_{\text{belt}} = \frac{1}{32} a t_{\text{belt}} = \frac{1}{32} (0.37 \text{ m/s}^2)(64 \text{ s}) = 0.74 \text{ m/s}$$

28. **REASONING AND SOLUTION** The car enters the speedway with a speed of

$$v_{01} = a_1 t_1 = (6.0 \text{ m/s}^2)(4.0 \text{ s}) = 24 \text{ m/s}$$

After an additional time, *t*, it will have traveled a distance of

$$x = v_{01}t + a_1t^2/2$$

to overtake the other car. This second car travels the same distance $x = v_{02}t$. Equating and solving for *t* yields

$$t = \frac{2(v_{02} - v_{01})}{a_1} = \frac{2(70.0 \text{ m/s} - 24 \text{ m/s})}{6.0 \text{ m/s}^2} = \boxed{15 \text{ s}}$$

29. **REASONING** The stopping distance is the sum of two parts. First, there is the distance the car travels at 20.0 m/s before the brakes are applied. According to Equation 2.2, this distance is the magnitude of the displacement and is the magnitude of the velocity times the time. Second, there is the distance the car travels while it decelerates as the brakes are

applied. This distance is given by Equation 2.9, since the initial velocity, the acceleration, and the final velocity (0 m/s when the car comes to a stop) are given.

SOLUTION With the assumption that the initial position of the car is $x_0 = 0$ m, Equation 2.2 gives the first contribution to the stopping distance as

$$\Delta x_1 = x_1 = vt_1 = (20.0 \text{ m/s})(0.530 \text{ s})$$

Solving Equation 2.9 $(v^2 = v_0^2 + 2ax)$ for x shows that the second part of the stopping distance is

$$x_2 = \frac{v^2 - v_0^2}{2a} = \frac{(0 \text{ m/s})^2 - (20.0 \text{ m/s})^2}{2(-7.00 \text{ m/s}^2)}$$

Here, the acceleration is assigned a negative value, because we have assumed that the car is traveling in the positive direction, and it is decelerating. Since it is decelerating, its acceleration points opposite to its velocity. The stopping distance, then, is

$$x_{\text{Stopping}} = x_1 + x_2 = (20.0 \text{ m/s})(0.530 \text{ s}) + \frac{(0 \text{ m/s})^2 - (20.0 \text{ m/s})^2}{2(-7.00 \text{ m/s}^2)} = 39.2 \text{ m}$$

30. *REASONING* The drawing shows the two knights, initially separated by the displacement *d*, traveling toward each other. At any moment, Sir George's displacement is x_G and that of Sir Alfred is x_A . When they meet, their displacements are the same, so $x_G = x_A$.



According to Equation 2.8, Sir George's displacement as a function of time is

$$x_{\rm G} = v_{0,\rm G}t + \frac{1}{2}a_{\rm G}t^2 = (0 \text{ m/s})t + \frac{1}{2}a_{\rm G}t^2 = \frac{1}{2}a_{\rm G}t^2$$
(1)

where we have used the fact that Sir George starts from rest $(v_{0,G} = 0 \text{ m/s})$.

Since Sir Alfred starts from rest at x = d at t = 0 s, we can write his displacement as (again, employing Equation 2.8)

$$x_{\rm A} = d + v_{0,\rm A}t + \frac{1}{2}a_{\rm A}t^2 = d + (0 \text{ m/s})t + \frac{1}{2}a_{\rm A}t^2 = d + \frac{1}{2}a_{\rm A}t^2$$
(2)

Solving Equation 1 for $t^2 (t^2 = 2x_G / a_G)$ and substituting this expression into Equation 2 yields

$$x_{\rm A} = d + \frac{1}{2}a_{\rm A} \left(\frac{2x_{\rm G}}{a_{\rm G}}\right) = d + a_{\rm A} \left(\frac{x_{\rm G}}{a_{\rm G}}\right) \tag{3}$$

Noting that $x_A = x_G$ when the two riders collide, we see that Equation 3 becomes

$$x_{\rm G} = d + a_{\rm A} \left(\frac{x_{\rm G}}{a_{\rm G}}\right)$$

Solving this equation for $x_{\rm G}$ gives $x_{\rm G} = \frac{d}{1 - \frac{a_{\rm A}}{a_{\rm F}}}$.

SOLUTION Sir George's acceleration is positive $(a_G = +0.300 \text{ m/s}^2)$ since he starts from rest and moves to the right (the positive direction). Sir Alfred's acceleration is negative $(a_A = -0.200 \text{ m/s}^2)$ since he starts from rest and moves to the left (the negative direction). The displacement of Sir George is, then,

$$x_{\rm G} = \frac{d}{1 - \frac{a_{\rm A}}{a_{\rm G}}} = \frac{88.0 \text{ m}}{1 - \frac{(-0.200 \text{ m/s}^2)}{(+0.300 \text{ m/s}^2)}} = \frac{52.8 \text{ m}}{1 - \frac{(-0.200 \text{ m/s}^2)}{(+0.300 \text{ m/s}^2)}}$$

31. **REASONING** At a constant velocity the time required for the first car to travel to the next exit is given by Equation 2.2 as the magnitude of the displacement $(2.5 \times 10^3 \text{ m})$ divided by the magnitude of the velocity. This is also the travel time for the second car to reach the next exit. The acceleration for the second car can be determined from Equation 2.8, since the initial velocity, the displacement, and the time are known. This equation applies, because the acceleration is constant.

SOLUTION According to Equation 2.2, with the assumption that the initial time is $t_0 = 0$ s, the time for the first car to reach the next exit at a constant velocity is

$$\Delta t = t - t_0 = t = \frac{\Delta x}{v} = \frac{2.5 \times 10^3 \text{ m}}{33 \text{ m/s}} = 76 \text{ s}$$

Remembering that the initial velocity v_0 of the second car is zero, we can solve Equation 2.8 $\left(x = v_0 t + \frac{1}{2}at^2 = \frac{1}{2}at^2\right)$ for the acceleration to show that

$$a = \frac{2x}{t^2} = \frac{2(2.5 \times 10^3 \text{ m})}{(76 \text{ s})^2} = \boxed{0.87 \text{ m/s}^2}$$

Since the second car's speed is increasing, this acceleration must be in the same direction as the velocity.

32. *REASONING AND SOLUTION* The distance covered by the cab driver during the two phases of the trip must satisfy the relation

$$x_1 + x_2 = 2.00 \text{ km} \tag{1}$$

where x_1 and x_2 are the displacements of the acceleration and deceleration phases of the trip, respectively. The quantities x_1 and x_2 can be determined from Equation 2.9 $(v^2 = v_0^2 + 2ax)$:

$$x_1 = \frac{v_1^2 - (0 \text{ m/s})^2}{2a_1} = \frac{v_1^2}{2a_1}$$
 and $x_2 = \frac{(0 \text{ m/s})^2 - v_{02}^2}{2a_2} = -\frac{v_{02}^2}{2a_2}$

with $v_{02} = v_1$ and $a_2 = -3a_1$. Thus,

$$\frac{x_1}{x_2} = \frac{\frac{v_1^2}{2a_1}}{-v_1^2} = 3$$

$$x_1 = 3x_2$$
(2)

so that

Combining (1) and (2), we have,

$$3x_2 + x_2 = 2.00 \text{ km}$$

Therefore, $x_2 = 0.50 \text{ km}$, and from Equation (1), $x_1 = 1.50 \text{ km}$. Thus, the length of the acceleration phase of the trip is $x_1 = \boxed{1.50 \text{ km}}$, while the length of the deceleration phase is $x_2 = \boxed{0.50 \text{ km}}$.

33. REASONING AND SOLUTION

a. The velocity at the end of the first (7.00 s) period is

$$v_1 = v_0 + a_1 = (2.01 \text{ m/s}^2)(7.00 \text{ s})$$

At the end of the second period the velocity is

$$v_2 = v_1 + a_2 t_2 = v_1 + (0.518 \text{ m/s}^2)(6.00 \text{ s})$$

And the velocity at the end of the third (8.00 s) period is

$$v_3 = v_2 + a_3 t_3 = v_2 + (-1.49 \text{ m/s}^2)(8.00 \text{ s}) = +5.26 \text{ m/s}$$

b. The displacement for the first time period is found from

$$x_1 = v_0 t_1 + \frac{1}{2} a_1 t_1^2$$
$$x_1 = (0 \text{ m/s})(7.00 \text{ s}) + \frac{1}{2} (2.01 \text{ m/s}^2)(7.00 \text{ s})^2 = +49.2 \text{ m}$$

Similarly, $x_2 = +93.7$ m and $x_3 = +89.7$ m, so the total displacement of the boat is

$$x = x_1 + x_2 + x_3 = +233 \text{ m}$$

34. **REASONING AND SOLUTION** As the plane decelerates through the intersection, it covers a total distance equal to the length of the plane plus the width of the intersection, so

$$x = 59.7 \text{ m} + 25.0 \text{ m} = 84.7 \text{ m}$$

The speed of the plane as it enters the intersection can be found from Equation 2.9. Solving Equation 2.9 for v_0 gives

$$v_0 = \sqrt{v^2 - 2ax} = \sqrt{(45.0 \text{ m})^2 - 2(-5.70 \text{ m/s}^2)(84.7 \text{ m})} = 54.7 \text{ m/s}$$

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The time required to traverse the intersection can then be found from Equation 2.4. Solving Equation 2.4 for t gives

$$t = \frac{v - v_0}{a} = \frac{45.0 \text{ m/s} - 54.7 \text{ m/s}}{-5.70 \text{ m/s}^2} = \boxed{1.7 \text{ s}}$$

35. **SSM** *REASONING* As the train passes through the crossing, its motion is described by Equations 2.4 ($v = v_0 + at$) and 2.7 $\left[x = \frac{1}{2}(v + v_0)t\right]$, which can be rearranged to give

$$v - v_0 = at$$
 and $v + v_0 = \frac{2x}{t}$

These can be solved simultaneously to obtain the speed v when the train reaches the end of the crossing. Once v is known, Equation 2.4 can be used to find the time required for the train to reach a speed of 32 m/s.

SOLUTION Adding the above equations and solving for v, we obtain

$$v = \frac{1}{2} \left(at + \frac{2x}{t} \right) = \frac{1}{2} \left[(1.6 \text{ m/s}^2)(2.4 \text{ s}) + \frac{2(20.0 \text{ m})}{2.4 \text{ s}} \right] = 1.0 \times 10^1 \text{ m/s}$$

The motion from the end of the crossing until the locomotive reaches a speed of 32 m/s requires a time

$$t = \frac{v - v_0}{a} = \frac{32 \text{ m/s} - 1.0 \times 10^1 \text{ m/s}}{1.6 \text{ m/s}^2} = \boxed{14 \text{ s}}$$

36. *REASONING AND SOLUTION* During the acceleration phase of the motion, the sprinter runs a distance

$$x_1 = \frac{1}{2}at_1^{\ 2} \tag{1}$$

acquiring a final speed

$$v = at_1 \tag{2}$$

During the rest of the race he runs a distance

$$x_2 = vt_2 \tag{3}$$

Adding Equations (1) and (3) gives

$$x = \frac{1}{2}at_1^2 + vt_2$$

where $x = x_1 + x_2$.

or (suppressing the units)

Substituting from Equation (2) and noting that the total time is $t = t_1 + t_2$, we have

$$\frac{1}{2}at_1^2 - att_1 + x = 0$$
$$1.90t_1^2 - 29.9t_1 + 50.0 = 0$$

Solving for t_1 gives 13.9 s and 1.90 s. The first is obviously not a physically realistic solution, since it is larger than the total time for the race so, $t_1 = 1.90$ s. Using this value in Equation (1) gives

$$x_1 = (1/2)(3.80 \text{ m/s}^2)(1.90 \text{ s})^2 = 6.85 \text{ m}$$

37. **SSM** *REASONING AND SOLUTION* When air resistance is neglected, free fall conditions are applicable. The final speed can be found from Equation 2.9;

$$v^2 = v_0^2 + 2ay$$

where v_0 is zero since the stunt man falls from rest. If the origin is chosen at the top of the hotel and the upward direction is positive, then the displacement is y = -99.4 m. Solving for *v*, we have

$$v = -\sqrt{2ay} = -\sqrt{2(-9.80 \text{ m/s}^2)(-99.4 \text{ m})} = -44.1 \text{ m/s}$$

The speed at impact is the magnitude of this result or 44.1 m/s.

38. REASONING AND SOLUTION

a. Once the pebble has left the slingshot, it is subject only to the acceleration due to gravity. Since the downward direction is negative, the acceleration of the pebble is -9.80 m/s^2 . The pebble is not decelerating. Since its velocity and acceleration both point downward, the magnitude of the pebble's velocity is increasing, not decreasing.

b. The displacement y traveled by the pebble as a function of the time t can be found from Equation 2.8. Using Equation 2.8, we have

$$y = v_0 t + \frac{1}{2} a_y t^2 = (-9.0 \text{ m/s})(0.50 \text{ s}) + \frac{1}{2} \left[(-9.80 \text{ m/s}^2)(0.50 \text{ s})^2 \right] = -5.7 \text{ m}$$

Thus, after 0.50 s, the pebble is 5.7 m beneath the cliff-top.

39. **REASONING AND SOLUTION** The figure at the right shows the paths taken by the pellets fired from gun **A** and gun **B**. The two paths differ by the extra distance covered by the pellet from gun **A** as it rises and falls back to the edge of the cliff. When it falls back to the edge of the cliff, the pellet from gun A will have the same speed as the pellet fired from gun B, as Conceptual Example 15 discusses. Therefore, the flight time of pellet **A** will be greater than that of **B** by the amount of time that it takes for pellet **A** to cover the extra distance.



The time required for pellet **A** to return to the cliff edge after being fired can be found from Equation 2.4: $v = v_0 + at$.

If "up" is taken as the positive direction then $v_0 = +30.0$ m/s and v = -30.0 m/s. Solving Equation 2.4 for *t* gives

$$t = \frac{v - v_0}{a} = \frac{(-30.0 \text{ m/s}) - (+30.0 \text{ m/s})}{-9.80 \text{ m/s}^2} = 6.12 \text{ s}$$

Notice that this result is *independent* of the height of the cliff.

40. *REASONING AND SOLUTION* In a time t the card will undergo a vertical displacement *y* given by

$$y = \frac{1}{2}at^2$$

where $a = -9.80 \text{ m/s}^2$. When $t = 60.0 \text{ ms} = 6.0 \times 10^{-2} \text{ s}$, the displacement of the card is 0.018 m, and the distance is the magnitude of this value or $d_1 = 0.018 \text{ m}$.

Similarly, when t = 120 ms, $d_2 = 0.071 \text{ m}$, and when t = 180 ms, $d_3 = 0.16 \text{ m}$.

41. **SSM** *REASONING AND SOLUTION* Equation 2.8 can be used to determine the displacement that the ball covers as it falls halfway to the ground. Since the ball falls from rest, its initial velocity is zero. Taking down to be the negative direction, we have

$$y = v_0 t + \frac{1}{2}at^2 = \frac{1}{2}at^2 = \frac{1}{2}(-9.80 \text{ m/s}^2)(1.2 \text{ s})^2 = -7.1 \text{ m}$$

In falling all the way to the ground, the ball has a displacement of y = -14.2 m. Solving Equation 2.8 with this displacement then yields the time

$$t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-14.2 \text{ m})}{-9.80 \text{ m/s}^2}} = \boxed{1.7 \text{ s}}$$

42. **REASONING** The minimum time that a player must wait before touching the basketball is the time required for the ball to reach its maximum height. The initial and final velocities are known, as well as the acceleration due to gravity, so Equation 2.4 ($v = v_0 + at$) can be used to find the time.

SOLUTION Solving Equation 2.4 for the time yields

$$t = \frac{v - v_0}{a} = \frac{0 \text{ m/s} - 4.6 \text{ m/s}}{-9.8 \text{ m/s}^2} = \boxed{0.47 \text{ s}}$$

43. **REASONING** The initial velocity and the elapsed time are given in the problem. Since the rock returns to the same place from which it was thrown, its displacement is zero (y = 0 m). Using this information, we can employ Equation 2.8 $\left(y = v_0 t + \frac{1}{2}at^2\right)$ to determine the acceleration *a* due to gravity.

SOLUTION Solving Equation 2.8 for the acceleration yields

$$a = \frac{2(y - v_0 t)}{t^2} = \frac{2[0 \text{ m} - (+15 \text{ m/s})(20.0 \text{ s})]}{(20.0 \text{ s})^2} = \boxed{-1.5 \text{ m/s}^2}$$

44. REASONING AND SOLUTION

a.

$$v = \pm \sqrt{(1.8 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-3.0 \text{ m})} = \pm 7.9 \text{ m/s}$$

 $v^2 = v_0^2 + 2ay$

The minus is chosen, since the diver is now moving down. Hence, v = -7.9 m/s.

b. The diver's velocity is zero at his highest point. The position of the diver relative to the board is

$$y = -\frac{v_0^2}{2a} = -\frac{(1.8 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 0.17 \text{ m}$$

The position above the water is 3.0 m + 0.17 m = 3.2 m.

45. **SSM REASONING AND SOLUTION** Since the balloon is released from rest, its initial velocity is zero. The time required to fall through a vertical displacement y can be found from Equation 2.8 $(y = v_0 t + \frac{1}{2}at^2)$ with $v_0 = 0$ m/s. Assuming upward to be the positive direction, we find

$$t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-6.0 \text{ m})}{-9.80 \text{ m/s}^2}} = 1.1 \text{ s}$$

46. **REASONING** Equation 2.9 $(v^2 = v_0^2 + 2ay)$ can be used to find out how far above the cliff's edge the pellet would have gone if the gun had been fired straight upward, provided that we can determine the initial speed imparted to the pellet by the gun. This initial speed can be found by applying Equation 2.9 to the downward motion of the pellet described in the problem statement.

SOLUTION If we assume that upward is the positive direction, the initial speed of the pellet is, from Equation 2.9,

$$v_0 = \sqrt{v^2 - 2ay} = \sqrt{(-27 \text{ m/s})^2 - 2(-9.80 \text{ m/s}^2)(-15 \text{ m})} = 20.9 \text{ m/s}$$

Equation 2.9 can again be used to find the maximum height of the pellet if it were fired straight up. At its maximum height, v = 0 m/s, and Equation 2.9 gives

$$y = \frac{-v_0^2}{2a} = \frac{-(20.9 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = \boxed{22 \text{ m}}$$

47. **REASONING** The initial speed of the ball can be determined from Equation 2.9 $(v^2 = v_0^2 + 2ay)$. Once the initial speed of the ball is known, Equation 2.9 can be used a second time to determine the height above the launch point when the speed of the ball has decreased to one half of its initial value.

SOLUTION When the ball has reached its maximum height, its velocity is zero. If we take upward as the positive direction, we have from Equation 2.9

$$v_0 = \sqrt{v^2 - 2ay} = \sqrt{(0 \text{ m/s})^2 - 2(-9.80 \text{ m/s}^2)(12.0 \text{ m})} = +15.3 \text{ m/s}$$

When the speed of the ball has decreased to one half of its initial value, $v = \frac{1}{2}v_0$, and Equation 2.9 gives

$$y = \frac{v^2 - v_0^2}{2a} = \frac{\left(\frac{1}{2}v_0\right)^2 - v_0^2}{2a} = \frac{v_0^2}{2a} \left(\frac{1}{4} - 1\right) = \frac{(+15.3 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} \left(\frac{1}{4} - 1\right) = \boxed{8.96 \text{ m}}$$

48. **REASONING** The displacement y of the diver is equal to her average velocity \overline{v} multiplied by the time t, or $y = \overline{v}t$. Since the diver has a constant acceleration (the acceleration due to gravity), her average velocity is equal to $\overline{v} = \frac{1}{2}(v_0 + v)$, where v_0 and v are, respectively, the initial and final velocities. Thus, according to Equation 2.7, the displacement of the diver is

$$y = \frac{1}{2} (v_0 + v) t \tag{2.7}$$

The final velocity and the time in this expression are known, but the initial velocity is not. To determine her velocity at the beginning of the 1.20-s period (her initial velocity), we turn to her acceleration. The acceleration is defined by Equation 2.4 as the change in her velocity, $v - v_0$, divided by the elapsed time *t*: $a = (v - v_0)/t$. Solving this equation for the initial velocity v_0 yields

$$v_0 = v - at$$

Substituting this relation for v_0 into Equation 2.7, we obtain

$$y = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(v - at + v)t = vt - \frac{1}{2}at^2$$

SOLUTION The diver's acceleration is that due to gravity, or $a = -9.80 \text{ m/s}^2$. The acceleration is negative because it points downward, and this direction is the negative direction. The displacement of the diver during the last 1.20 s of the dive is

$$y = vt - \frac{1}{2}at^2 = (-10.1 \text{ m/s})(1.20 \text{ s}) - \frac{1}{2}(-9.80 \text{ m/s}^2)(1.20 \text{ s})^2 = -5.06 \text{ m}$$

The displacement of the diver is negative because she is moving downward.

49. **REASONING** To calculate the speed of the raft, it is necessary to determine the distance it travels and the time interval over which the motion occurs. The speed is the distance divided by the time, according to Equation 2.1. The distance is 7.00 m - 4.00 m = 3.00 m. The time is the time it takes for the stone to fall, which can be obtained from Equation 2.8 $\left(y = v_0 t + \frac{1}{2}at^2\right)$, since the displacement y, the initial velocity v_0 , and the acceleration *a* are known.

SOLUTION During the time t that it takes the stone to fall, the raft travels a distance of 7.00 m - 4.00 m = 3.00 m, and according to Equation 2.1, its speed is

speed =
$$\frac{3.00 \text{ m}}{t}$$

The stone falls downward for a distance of 75.0 m, so its displacement is y = -75.0 m, where the downward direction is taken to be the negative direction. Equation 2.8 can be used to find the time of fall. Setting $v_0 = 0$ m/s, and solving Equation 2.8 for the time *t*, we have

$$t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-75.0 \text{ m})}{-9.80 \text{ m/s}^2}} = 3.91 \text{ s}$$

Therefore, the speed of the raft is

speed =
$$\frac{3.00 \text{ m}}{3.91 \text{ s}} = 0.767 \text{ m/s}$$

50. **REASONING AND SOLUTION** The initial speed of either student can be found from Equation (2.9): $v^2 = v_0^2 + 2ay$. Since the speed of either student is zero when the student is at her highest point, v = 0 m/s and

$$v_0 = \sqrt{-2ay}$$

Since Anne bounces twice as high as Joan, $y_A = 2y_J$, and $v_{0A} = \sqrt{2} v_{0J}$.

The time it takes for either student to reach the highest point in her trajectory can be found from Equation 2.4: $v = v_0 + at$. Solving for t with v = 0 m/s gives

$$t = \frac{-v_0}{a}$$

The total time in the air is twice this value. Therefore,

$$\frac{t_{\rm A}}{t_{\rm J}} = \frac{-2v_{\rm 0A}/a}{-2v_{\rm 0J}/a} = \frac{v_{\rm 0A}}{v_{\rm 0J}} = \frac{\sqrt{2}v_{\rm 0J}}{v_{\rm 0J}} = \boxed{\sqrt{2}}$$

51. **SSM** *REASONING AND SOLUTION* The stone will reach the water (and hence the log) after falling for a time *t*, where *t* can be determined from Equation 2.8: $y = v_0 t + \frac{1}{2}at^2$. Since the stone is dropped from rest, $v_0 = 0$ m/s. Assuming that downward is positive and solving for *t*, we have

$$t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(75 \text{ m})}{9.80 \text{ m/s}^2}} = 3.9 \text{ s}$$

During that time, the displacement of the log can be found from Equation 2.8. Since the log moves with constant velocity, $a = 0 \text{ m/s}^2$, and v_0 is equal to the velocity of the log.

$$x = v_0 t = (5.0 \text{ m/s})(3.9 \text{ s}) = 2.0 \times 10^1 \text{ m}$$

Therefore, the horizontal distance between the log and the bridge when the stone is released is 2.0×10^{1} m.

52. REASONING AND SOLUTION

a. We can use Equation 2.9 to obtain the speed acquired as she falls through the distance H. Taking downward as the positive direction, we find

$$v^{2} = v_{0}^{2} + 2ay = (0 \text{ m/s})^{2} + 2aH$$
 or $v = \sqrt{2aH}$

To acquire a speed of twice this value or $2\sqrt{2aH}$, she must fall an additional distance H'. According to Equation 2.9 $(v^2 = v_0^2 + 2ay)$, we have

$$\left(2\sqrt{2aH}\right)^2 = \left(\sqrt{2aH}\right)^2 + 2aH'$$
 or $4\left(2aH\right) = 2aH + 2aH'$

The acceleration due to gravity a can be eliminated algebraically from this result, giving

$$4H = H + H' \qquad \text{or} \qquad H' = 3H$$

b. In the previous calculation the acceleration due to gravity was eliminated algebraically. Thus, a value other than 9.80 m/s² would not have affected the answer to part (a).

SOLUTION When the man first notices the block, it is 14.0 m above the ground and its displacement from the starting point is y = 14.0 m - 53.0 m. Its velocity is given by

^{53.} **SSM** *REASONING* Once the man sees the block, the man must get out of the way in the time it takes for the block to fall through an additional 12.0 m. The velocity of the block at the instant that the man looks up can be determined from Equation 2.9. Once the velocity is known at that instant, Equation 2.8 can be used to find the time required for the block to fall through the additional distance.

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Equation 2.9 $(v^2 = v_0^2 + 2ay)$. Since the block is moving down, its velocity has a negative value,

$$v = -\sqrt{v_0 + 2ay} = -\sqrt{(0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(14.0 \text{ m} - 53.0 \text{ m})} = -27.7 \text{ m/s}$$

The block then falls the additional 12.0 m to the level of the man's head in a time t which satisfies Equation 2.8:

$$y = v_0 t + \frac{1}{2}at^2$$

where y = -12.0 m and $v_0 = -27.7$ m/s. Thus, t is the solution to the quadratic equation

$$4.90t^2 + 27.7t - 12.0 = 0$$

where the units have been suppressed for brevity. From the quadratic formula, we obtain

$$t = \frac{-27.7 \pm \sqrt{(27.7)^2 - 4(4.90)(-12.0)}}{2(4.90)} = 0.40 \text{ s or } -6.1 \text{ s}$$

The negative solution can be rejected as nonphysical, and the time it takes for the block to reach the level of the man is 0.40 s.

54. **REASONING AND SOLUTION** The position of the ball relative to the top of the building at any time *t* is

$$y = v_0 t + \frac{1}{2} a t^2 = v_0 t + \frac{1}{2} (-9.80 \text{ m/s}^2) t^2$$

For the particular time that the ball arrives at the bottom of the building, y = -25.0 m. The equation becomes

$$4.90t^2 - 12.0t - 25.0 = 0$$

The quadratic equation yields solutions t = 3.79 s and -1.34 s. The negative solution is rejected as being non-physical. During time *t* the person has run a distance x = vt so

$$v = \frac{x}{t} = \frac{31.0 \text{ m}}{3.79 \text{ s}} = \boxed{8.18 \text{ m/s}}$$

55. **REASONING AND SOLUTION** The balls pass at a time *t* when both are at a position *y* above the ground. Applying Equation 2.8 to the ball that is dropped from rest, we have

$$y = 24 \text{ m} + v_{01}t + \frac{1}{2}at^2 = 24 \text{ m} + (0 \text{ m/s})t + \frac{1}{2}at^2$$
(1)

Note that we have taken into account the fact that y = 24 m when t = 0 s in Equation (1). For the second ball that is thrown straight upward,

$$y = v_{02}t + \frac{1}{2}at^2$$
(2)

Equating Equations (1) and (2) for y yields

24 m +
$$\frac{1}{2}at^2 = v_{02}t + \frac{1}{2}at^2$$
 or 24 m = $v_{02}t$

Thus, the two balls pass at a time *t*, where

$$t = \frac{24 \text{ m}}{v_{02}}$$

The initial speed v_{02} of the second ball is exactly the same as that with which the first ball hits the ground. To find the speed with which the first ball hits the ground, we take upward as the positive direction and use Equation 2.9 $(v^2 = v_0^2 + 2ay)$. Since the first ball is dropped from rest, we find that

$$v_{02} = v = \sqrt{2ay} = \sqrt{2(-9.80 \text{ m/s}^2)(-24 \text{ m})} = 21.7 \text{ m/s}$$

Thus, the balls pass after a time

$$t = \frac{24 \text{ m}}{21.7 \text{ m/s}} = 1.11 \text{ s}$$

At a time t = 1.11 s, the position of the first ball according to Equation (1) is

$$y = 24 \text{ m} + \frac{1}{2}(-9.80 \text{ m/s}^2)(1.11 \text{ s})^2 = 24 \text{ m} - 6.0 \text{ m}$$

which is 6.0 m below the top of the cliff.

56. **REASONING** To find the initial velocity $v_{0,2}$ of the second stone, we will employ Equation 2.8, $y = v_{0,2}t_2 + \frac{1}{2}at_2^2$. In this expression t_2 is the time that the second stone is in the air, and it is equal to the time t_1 that the first stone is in the air minus the time $t_{3.20}$ it takes for the first stone to fall 3.20 m:

$$t_2 = t_1 - t_{3.20}$$

We can find t_1 and $t_{3,20}$ by applying Equation 2.8 to the first stone.

SOLUTION To find the initial velocity $v_{0,2}$ of the second stone, we employ Equation 2.8, $y = v_{0,2}t_2 + \frac{1}{2}at_2^2$. Solving this equation for $v_{0,2}$ yields

$$v_{0,2} = \frac{y - \frac{1}{2}at_2^2}{t_2}$$

The time t_1 for the first stone to strike the ground can be obtained from Equation 2.8, $y = v_{0,1}t_1 + \frac{1}{2}at_1^2$. Noting that $v_{0,1} = 0$ m/s since the stone is dropped from rest and solving this equation for t_1 , we have

$$t_1 = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-15.0 \text{ m})}{-9.80 \text{ m/s}^2}} = 1.75 \text{ s}$$
 (1)

Note that the stone is falling down, so its displacement is negative (y = -15.0 m). Also, its acceleration *a* is that due to gravity, so a = -9.80 m/s².

The time $t_{3,20}$ for the first stone to fall 3.20 m can also be obtained from Equation 1:

$$t_{3.20} = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-3.20 \text{ m})}{-9.80 \text{ m/s}^2}} = 0.808 \text{ s}$$

The time t_2 that the second stone is in the air is

$$t_2 = t_1 - t_{3.20} = 1.75 \text{ s} - 0.808 \text{ s} = 0.94 \text{ s}$$

The initial velocity of the second stone is

$$v_{0,2} = \frac{y - \frac{1}{2}at_2^2}{t_2} = \frac{(-15.0 \text{ m}) - \frac{1}{2}(-9.80 \text{ m/s}^2)(0.94 \text{ s})^2}{0.94 \text{ s}} = \boxed{-11 \text{ m/s}}$$

57. *REASONING AND SOLUTION* The average acceleration for each segment is the slope of that segment.

$$a_{\rm A} = \frac{40 \text{ m/s} - 0 \text{ m/s}}{21 \text{ s} - 0 \text{ s}} = \boxed{1.9 \text{ m/s}^2}$$
$$a_{\rm B} = \frac{40 \text{ m/s} - 40 \text{ m/s}}{48 \text{ s} - 21 \text{ s}} = \boxed{0 \text{ m/s}^2}$$
$$a_{\rm C} = \frac{80 \text{ m/s} - 40 \text{ m/s}}{60 \text{ s} - 48 \text{ s}} = \boxed{3.3 \text{ m/s}^2}$$

58. **REASONING** The slope of a straight-line segment in a position-versus-time graph is the average velocity. The algebraic sign of the average velocity, therefore, corresponds to the sign of the slope.

SOLUTION

a. The slope, and hence the average velocity, is *positive* for segments A and C, *negative* for segment B, and *zero* for segment D.

b.

$$v_{\rm A} = \frac{1.25 \text{ km} - 0 \text{ km}}{0.20 \text{ h} - 0 \text{ h}} = \boxed{+6.3 \text{ km/h}}$$
$$v_{\rm B} = \frac{0.50 \text{ km} - 1.25 \text{ km}}{0.40 \text{ h} - 0.20 \text{ h}} = \boxed{-3.8 \text{ km/h}}$$
$$v_{\rm C} = \frac{0.75 \text{ km} - 0.50 \text{ km}}{0.80 \text{ h} - 0.40 \text{ h}} = \boxed{+0.63 \text{ km/h}}$$
$$v_{\rm D} = \frac{0.75 \text{ km} - 0.75 \text{ km}}{1.00 \text{ h} - 0.80 \text{ h}} = \boxed{0 \text{ km/h}}$$

59. **SSM** *REASONING* In order to construct the graph, the time for each segment of the trip must be determined.

SOLUTION From the definition of average velocity

$$\Delta t = \frac{\Delta x}{\overline{v}}$$

Therefore,

$$\Delta t_1 = \left(\frac{10.0 \text{ km}}{15.0 \text{ km/h}}\right) \left(\frac{60 \text{ min}}{1.0 \text{ h}}\right) = 40 \text{ min}$$

$$\Delta t_2 = \left(\frac{15.0 \text{ km}}{10.0 \text{ km/h}}\right) \left(\frac{60 \text{ min}}{1.0 \text{ h}}\right) = 90 \text{ min}$$

Thus, the second time interval ends $40 \min + 90 \min = 130 \min$ after the trip begins.

$$\Delta t_3 = \left(\frac{15.0 \text{ km}}{5.0 \text{ km/h}}\right) \left(\frac{60 \text{ min}}{1.0 \text{ h}}\right) = 180 \text{ min}$$

and the third time interval ends $130 \min + 180 \min = 310 \min$ after the trip begins.



Note that the slope of each segment of the graph gives the average velocity during that interval.

60. *REASONING* The average velocity for each segment is the slope of the line for that segment.

SOLUTION Taking the direction of motion as positive, we have from the graph for segments A, B, and C,

$$v_{\rm A} = \frac{10.0 \text{ km} - 40.0 \text{ km}}{1.5 \text{ h} - 0.0 \text{ h}} = \boxed{-2.0 \times 10^1 \text{ km/h}}$$
$$v_{\rm B} = \frac{20.0 \text{ km} - 10.0 \text{ km}}{2.5 \text{ h} - 1.5 \text{ h}} = \boxed{1.0 \times 10^1 \text{ km/h}}$$
$$v_{\rm C} = \frac{40.0 \text{ km} - 20.0 \text{ km}}{3.0 \text{ h} - 2.5 \text{ h}} = \boxed{40 \text{ km/h}}$$

61. **SSM REASONING** The average acceleration is given by Equation 2.4 as $\overline{a} = (v_{\rm C} - v_{\rm A})/\Delta t$. The velocities $v_{\rm A}$ and $v_{\rm C}$ can be found from the slopes of the position-time graph for segments A and C.

SOLUTION The average velocities in the segments A and C are

$$v_{\rm A} = \frac{24 \text{ km} - 0 \text{ km}}{1.0 \text{ h} - 0 \text{ h}} = 24 \text{ km/h}$$

 $v_{\rm C} = \frac{27 \text{ km} - 33 \text{ km}}{3.5 \text{ h} - 2.2 \text{ h}} = -5 \text{ km/h}$

From the definition of average acceleration,

 $\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_{\rm C} - v_{\rm A}}{\Delta t} = \frac{(-5 \text{ km/h}) - (24 \text{ km/h})}{3.5 \text{ h} - 0 \text{ h}} = \frac{-8.3 \text{ km/h}^2}{-8.3 \text{ km/h}^2}$

62. **REASONING** The graph that accompanies the problem shows that the motion consists of three segments: $0 \rightarrow 2.0$ s, $2.0 \text{ s} \rightarrow 6.0$ s, and $6.0 \text{ s} \rightarrow 12.0$ s. The displacements of the helicopter for these segments are, respectively, y_1 , y_2 , and y_3 . The height y of the helicopter after 12.0 s is the magnitude of the total displacement of the three segments, or $y = y_1 + y_2 + y_3$.

From the graph we see that the initial and final velocities of each segment, as well as the elapsed time, are known. Thus, we may employ the relation $y = \frac{1}{2}(v_0 + v)t$, Equation 2.7, to find the displacement of each segment.

SOLUTION The displacements of the three segments of the motion are:

| 1st segment: | $y = \frac{1}{2} (v_0 + v) t = \frac{1}{2} (0 \text{ m/s} + 6.0 \text{ m/s}) (2.0 \text{ s}) = 6.0 \text{ m}$ | |
|--|---|--|
| 2nd segment: | $y = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(6.0 \text{ m/s} + 6.0 \text{ m/s})(6.0 \text{ s} - 2.0 \text{ s}) = 24 \text{ m}$ | |
| 3rd segment: | $y = \frac{1}{2} (v_0 + v) t = \frac{1}{2} (6.0 \text{ m/s} + 0 \text{ m/s}) (12.0 \text{ s} - 6.0 \text{ s}) = 18 \text{ m}$ | |
| The height of the helicenter after 12.0 g is | | |

The height of the helicopter after 12.0 s is

$$y = y_1 + y_2 + y_3 = 6.0 \text{ m} + 24 \text{ m} + 18 \text{ m} = 48 \text{ m}$$

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63. **REASONING** The two runners start one hundred meters apart and run toward each other. As stated in the problem, each runs ten meters during the first second and, during each second thereafter, each runner runs ninety percent of the distance he ran in the previous second. While the velocity of each runner changes from second to second, it remains constant during any one second. The following table shows the distance covered during each second for one of the runners, and the position at the end of each second (assuming that he begins at the origin) for the first eight seconds.

| Time t (s) | Distance covered (m) | Position x (m) |
|------------|----------------------|----------------|
| | | |
| 0.00 | | 0.00 |
| 1.00 | 10.00 | 10.00 |
| 2.00 | 9.00 | 19.00 |
| 3.00 | 8.10 | 27.10 |
| 4.00 | 7.29 | 34.39 |
| 5.00 | 6.56 | 40.95 |
| 6.00 | 5.90 | 46.85 |
| 7.00 | 5.31 | 52.16 |
| 8.00 | 4.78 | 56.94 |
| | | |

The following graph is the position-time graph constructed from the data in the table above.



a. Since the two runners are running toward each other in exactly the same way, they will meet halfway between their respective starting points. That is, they will meet at x = 50.0 m. According to the graph, therefore, this position corresponds to a time of 6.6 s.

b. Since the runners collide during the seventh second, the speed at the instant of collision can be found by taking the slope of the position-time graph for the seventh second. The speed of either runner in the interval from t = 6.00 s to t = 7.00 s is

$$v = \frac{\Delta x}{\Delta t} = \frac{52.16 \text{ m} - 46.85 \text{ m}}{7.00 \text{ s} - 6.00 \text{ s}} = 5.3 \text{ m/s}$$

Therefore, at the moment of collision, the speed of either runner is 5.3 m/s.

64. **REASONING AND SOLUTION** Since $v = v_0 + at$, the acceleration is given by $a = (v - v_0)/t$. Since the direction of travel is in the negative direction throughout the problem, all velocities will be negative.

a.
$$a = \frac{(-29.0 \text{ m/s}) - (-27.0 \text{ m/s})}{5.0 \text{ s}} = \boxed{-0.40 \text{ m/s}^2}$$

Since the acceleration is negative, it is in the same direction as the velocity and the car is speeding up.

b.
$$a = \frac{(-23.0 \text{ m/s}) - (-27.0 \text{ m/s})}{5.0 \text{ s}} = \boxed{+0.80 \text{ m/s}^2}$$

Since the acceleration is positive, it is in the opposite direction to the velocity and the car is slowing down or decelerating.

65. **SSM REASONING AND SOLUTION** The speed of the penny as it hits the ground can be determined from Equation 2.9: $v^2 = v_0^2 + 2ay$. Since the penny is dropped from rest, $v_0 = 0$ m/s. Solving for *v*, with downward taken as the positive direction, we have

$$v = \sqrt{2(9.80 \text{ m/s}^2)(427 \text{ m})} = 91.5 \text{ m/s}$$

66. **REASONING** In a race against el-Guerrouj, Bannister would run a distance given by his average speed times the time duration of the race (see Equation 2.1). The time duration of the race would be el-Guerrouj's winning time of 3:43.13 (223.13 s). The difference between Bannister's distance and the length of the race is el-Guerrouj's winning margin.

SOLUTION From the table of conversion factors on the page facing the front cover, we find that one mile corresponds to 1609 m. According to Equation 2.1, Bannister's average speed is

Average speed =
$$\frac{\text{Distance}}{\text{Elapsed time}} = \frac{1609 \text{ m}}{239.4 \text{ s}}$$

Had he run against el-Guerrouj at this average speed for the 223.13-s duration of the race, he would have traveled a distance of

Distance = Average speed × Time =
$$\left(\frac{1609 \text{ m}}{239.4 \text{ s}}\right)$$
 (223.13 s)

while el-Guerrouj traveled 1609 m. Thus, el-Guerrouj would have won by a distance of

$$1609 \text{ m} - \left(\frac{1609 \text{ m}}{239.4 \text{ s}}\right) (223.13 \text{ s}) = \boxed{109 \text{ m}}$$

67. **SSM REASONING AND SOLUTION** The average acceleration of the plane can be found by solving Equation 2.9 $(v^2 = v_0^2 + 2ax)$ for *a*. Taking the direction of motion as positive, we have

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(+6.1 \text{ m/s})^2 - (+69 \text{ m/s})^2}{2(+750 \text{ m})} = \boxed{-3.1 \text{ m/s}^2}$$

The minus sign indicates that the direction of the acceleration is opposite to the direction of motion, and the plane is slowing down.

68. **REASONING** The initial velocity of the compass is +2.50 m/s. The initial position of the compass is 3.00 m and its final position is 0 m when it strikes the ground. The displacement of the compass is the final position minus the initial position, or y = -3.00 m. As the compass falls to the ground, its acceleration is the acceleration due to gravity, $a = -9.80 \text{ m/s}^2$. Equation 2.8 $\left(y = v_0 t + \frac{1}{2} a t^2\right)$ can be used to find how much time elapses before the compass hits the ground.

SOLUTION Starting with Equation 2.8, we use the quadratic equation to find the elapsed time.

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{1}{2}a\right)(-y)}}{2\left(\frac{1}{2}a\right)} = \frac{-(2.50 \text{ m/s}) \pm \sqrt{(2.50 \text{ m/s})^2 - 4\left(-4.90 \text{ m/s}^2\right)\left[-(-3.00 \text{ m})\right]}}{2\left(-4.90 \text{ m/s}^2\right)}$$

There are two solutions to this quadratic equation, $t_1 = 1.08 \text{ s}$ and $t_2 = -0.568 \text{ s}$. The second solution, being a negative time, is discarded.

69. **REASONING** The average speed is the distance traveled divided by the elapsed time (Equation 2.1). Since the average speed and distance are known, we can use this relation to find the time.

SOLUTION The time it takes for the continents to drift apart by 1500 m is

Elapsed time =
$$\frac{\text{Distance}}{\text{Average speed}} = \frac{1500 \text{ m}}{\left(3\frac{\text{cm}}{\text{y}}\right)\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)} = 5 \times 10^4 \text{ y}$$

70. **REASONING** The average acceleration is defined by Equation 2.4 as the change in velocity divided by the elapsed time. We can find the elapsed time from this relation because the acceleration and the change in velocity are given. Since the acceleration of the spacecraft is constant, it is equal to the average acceleration.

SOLUTION

a. The time Δt that it takes for the spacecraft to change its velocity by an amount $\Delta v = +2700$ m/s is

$$\Delta t = \frac{\Delta v}{a} = \frac{+2700 \text{ m/s}}{+9.0 \frac{\text{m/s}}{\text{day}}} = \boxed{3.0 \times 10^2 \text{ days}}$$

b. Since 24 hr = 1 day and 3600 s = 1 hr, the acceleration of the spacecraft (in m/s^2) is

71. **REASONING** At time *t* both rockets return to their starting points and have a displacement of zero. This occurs, because each rocket is decelerating during the first half of its journey. However, rocket A has a smaller initial velocity than rocket B. Therefore, in order for rocket B to decelerate and return to its point of origin in the same time as rocket A, rocket B must have a deceleration with a greater magnitude than that for rocket A. Since we know that the displacement of each rocket is zero at time *t*, since both initial velocities are given, and since we seek information about the acceleration, we begin our solution with Equation 2.8, for it contains just these variables.

SOLUTION Applying Equation 2.8 to each rocket gives

$$x_{A} = v_{0A}t + \frac{1}{2}a_{A}t^{2} \qquad x_{B} = v_{0B}t + \frac{1}{2}a_{B}t^{2}$$

$$0 = v_{0A}t + \frac{1}{2}a_{A}t^{2} \qquad 0 = v_{0B}t + \frac{1}{2}a_{B}t^{2}$$

$$0 = v_{0A} + \frac{1}{2}a_{A}t \qquad 0 = v_{0B} + \frac{1}{2}a_{B}t$$

$$t = \frac{-2v_{0A}}{a_{A}} \qquad t = \frac{-2v_{0B}}{a_{B}}$$

The time for each rocket is the same, so that we can equate the two expressions for t, with the result that

$$\frac{-2v_{0A}}{a_{A}} = \frac{-2v_{0B}}{a_{B}} \quad \text{or} \quad \frac{v_{0A}}{a_{A}} = \frac{v_{0B}}{a_{B}}$$

Solving for $a_{\rm B}$ gives

$$a_{\rm B} = \frac{a_{\rm A}}{v_{0\rm A}} v_{0\rm B} = \frac{-15 \text{ m/s}^2}{5800 \text{ m/s}} (8600 \text{ m/s}) = \boxed{-22 \text{ m/s}^2}$$

As expected, the magnitude of the acceleration for rocket B is greater than that for rocket A.

72. *REASONING AND SOLUTION* The speed of the car at the end of the first (402 m) phase can be obtained as follows:

$$v_1^2 = v_0^2 + 2a_1x_1$$

 $v_1 = \sqrt{2(17.0 \text{ m/s}^2)(402 \text{ m})}$

The speed after the second phase $(3.50 \times 10^2 \text{ m})$ can be obtained in a similar fashion.

$$v_2^2 = v_{02}^2 + 2a_2x_2$$

 $v_2 = \sqrt{v_1^2 + 2(-6.10 \text{ m/s}^2)(3.50 \times 10^2 \text{ m})}$
 $v_2 = \boxed{96.9 \text{ m/s}}$

73. **SSM WWW** *REASONING* Since the woman runs for a known distance at a known constant speed, we can find the time it takes for her to reach the water from Equation 2.1.

We can then use Equation 2.1 to determine the total distance traveled by the dog in this time.

SOLUTION The time required for the woman to reach the water is

Elapsed time =
$$\frac{d_{\text{woman}}}{v_{\text{woman}}} = \left(\frac{4.0 \text{ km}}{2.5 \text{ m/s}}\right) \left(\frac{1000 \text{ m}}{1.0 \text{ km}}\right) = 1600 \text{ s}$$

In 1600 s, the dog travels a total distance of

$$d_{\text{dog}} = v_{\text{dog}}t = (4.5 \text{ m/s})(1600 \text{ s}) = 2.2 \times 10^3 \text{ m}$$

74. **REASONING** The definition of average velocity is given by Equation 2.2 as Average velocity = Displacement/(Elapsed time). The displacement in this expression is the total displacement, which is the sum of the displacements for each part of the trip. Displacement is a vector quantity, and we must be careful to account for the fact that the displacement in the first part of the trip is north, while the displacement in the second part is south.

SOLUTION According to Equation 2.2, the displacement for each part of the trip is the average velocity for that part times the corresponding elapsed time. Designating north as the positive direction, we find for the total displacement that

Displacement =
$$\underbrace{(27 \text{ m/s})t_{\text{North}}}_{\text{Northward}}$$
 + $\underbrace{(-17 \text{ m/s})t_{\text{South}}}_{\text{Southward}}$

where t_{North} and t_{South} denote, respectively, the times for each part of the trip. Note that the minus sign indicates a direction due south. Noting that the total elapsed time is $t_{\text{North}} + t_{\text{South}}$, we can use Equation 2.2 to find the average velocity for the entire trip as follows:

Average velocity =
$$\frac{\text{Displacement}}{\text{Elapsed time}} = \frac{(27 \text{ m/s})t_{\text{North}} + (-17 \text{ m/s})t_{\text{South}}}{t_{\text{North}} + t_{\text{South}}}$$

= $(27 \text{ m/s})\left(\frac{t_{\text{North}}}{t_{\text{North}} + t_{\text{South}}}\right) + (-17 \text{ m/s})\left(\frac{t_{\text{South}}}{t_{\text{North}} + t_{\text{South}}}\right)$
But $\left(\frac{t_{\text{North}}}{t_{\text{North}} + t_{\text{South}}}\right) = \frac{3}{4}$ and $\left(\frac{t_{\text{South}}}{t_{\text{North}} + t_{\text{South}}}\right) = \frac{1}{4}$. Therefore, we have that

Average velocity =
$$(27 \text{ m/s})\left(\frac{3}{4}\right) + (-17 \text{ m/s})\left(\frac{1}{4}\right) = \boxed{+16 \text{ m/s}}$$

The plus sign indicates that the average velocity for the entire trip points north.

75. **SSM WWW REASONING AND SOLUTION** The stone requires a time, t_1 , to reach the bottom of the hole, a distance y below the ground. Assuming downward to be the positive direction, the variables are related by Equation 2.8 with $v_0 = 0$ m/s:

$$y = \frac{1}{2}at_1^2 \tag{1}$$

The sound travels the distance y from the bottom to the top of the hole in a time t_2 . Since the sound does not experience any acceleration, the variables y and t_2 are related by Equation 2.8 with a = 0 m/s² and v_{sound} denoting the speed of sound:

$$y = v_{\text{sound}} t_2 \tag{2}$$

Equating the right hand sides of Equations (1) and (2) and using the fact that the total elapsed time is $t = t_1 + t_2$, we have

 $\frac{1}{2}at_1^2 = v_{\text{sound}}t_2$ or $\frac{1}{2}at_1^2 = v_{\text{sound}}(t - t_1)$

$$\frac{1}{2}at_1^2 + v_{\text{sound}}t_1 - v_{\text{sound}}t = 0$$

Substituting values and suppressing units for brevity, we obtain the following quadratic equation for t_1 :

$$4.90t_1^2 + 343t_1 - 514 = 0$$

From the quadratic formula, we obtain

Rearranging gives

$$t_1 = \frac{-343 \pm \sqrt{(343)^2 - 4(4.90)(-514)}}{2(4.90)} = 1.47 \text{ s or } -71.5 \text{ s}$$

The negative time corresponds to a nonphysical result and is rejected. The depth of the hole is then found using Equation 2.8 with the value of t_1 obtained above:

$$y = v_0 t_1 + \frac{1}{2} a t_1^2 = (0 \text{ m/s})(1.47 \text{ s}) + \frac{1}{2}(9.80 \text{ m/s}^2)(1.47 \text{ s})^2 = 10.6 \text{ m}$$

76. **REASONING** The displacement x of your car is equal to its average velocity \overline{v} multiplied by the time t, or $x = \overline{v}t$ (See Equation 2.2). Since the car has a constant acceleration, its average velocity is equal to $\overline{v} = \frac{1}{2}(v_0 + v)$, where v_0 and v are, respectively, the initial and final velocities. Thus, the displacement of the car can be written as (see Equation 2.7)

$$x = \frac{1}{2} (v_0 + v) t \tag{2.7}$$

In this expression the final velocity v and the time t are known, but the initial velocity is not. To determine the velocity at the beginning of the 3.00-s period, we note that the acceleration is defined by Equation 2.4 as the change in the car's velocity, $v - v_0$, divided by the elapsed time t: $a = (v - v_0)/t$. Solving this equation for the initial velocity v_0 yields

$$v_0 = v - at$$

Substituting this expression for v_0 into Equation 1 gives

$$x = \frac{1}{2} (v_0 + v)t = \frac{1}{2} (v - at + v)t = vt - \frac{1}{2}at^2$$

SOLUTION We assume that the car is moving in the +x direction. The car's acceleration is $a = -2.70 \text{ m/s}^2$, negative because the car is slowing down so the acceleration must point in a direction opposite to that of the velocity. The displacement of the car during the 3.00-s time interval is

$$x = vt - \frac{1}{2}at^2 = (+4.50 \text{ m/s})(3.00 \text{ s}) - \frac{1}{2}(-2.70 \text{ m/s}^2)(3.00 \text{ s})^2 = +25.7 \text{ m}$$

77. **REASONING** We choose due north as the positive direction. Our solution is based on the fact that when the police car catches up, both cars will have the same displacement, relative to the point where the speeder passed the police car. The displacement of the speeder can be obtained from the definition of average velocity given in Equation 2.2, since the speeder is moving at a constant velocity. During the 0.800-s reaction time of the policeman, the police car is also moving at a constant velocity. Once the police car begins to accelerate, its displacement can be expressed as in Equation 2.8 $(x = v_0 t + \frac{1}{2}at^2)$, because the initial velocity v_0 and the acceleration *a* are known and it is the time *t* that we seek. We will set the displacements of the speeder and the police car equal and solve the resulting equation for the time *t*.

SOLUTION Let *t* equal the time during the accelerated motion of the police car. Relative to the point where he passed the police car, the speeder then travels a time of t + 0.800 s

before the police car catches up. During this time, according to the definition of average velocity given in Equation 2.2, his displacement is

$$x_{\text{Speeder}} = v_{\text{Speeder}} (t + 0.800 \text{ s}) = (42.0 \text{ m/s})(t + 0.800 \text{ s})$$

The displacement of the police car consists of two contributions, the part due to the constant-velocity motion during the reaction time and the part due to the accelerated motion. Using Equation 2.2 for the contribution from the constant-velocity motion and Equation 2.9 for the contribution from the accelerated motion, we obtain

$$x_{\text{Police car}} = \underbrace{v_{0, \text{ Police car}}(0.800 \text{ s})}_{\text{Constant velocity motion, Equation 2.2}} + \underbrace{v_{0, \text{ Police car}}t + \frac{1}{2}at^{2}}_{\text{Accelerated motion, Equation 2.8}}$$
$$= (18.0 \text{ m/s})(0.800 \text{ s}) + (18.0 \text{ m/s})t + \frac{1}{2}(5.00 \text{ m/s}^{2})t^{2}$$

Setting the two displacements equal we obtain

$$\underbrace{(42.0 \text{ m/s})(t+0.800 \text{ s})}_{\text{Displacement of speeder}} = \underbrace{(18.0 \text{ m/s})(0.800 \text{ s}) + (18.0 \text{ m/s})t + \frac{1}{2}(5.00 \text{ m/s}^2)t^2}_{\text{Displacement of police car}}$$

Rearranging and combining terms gives this result in the standard form of a quadratic equation:

$$(2.50 \text{ m/s}^2)t^2 - (24.0 \text{ m/s})t - 19.2 \text{ m} = 0$$

Solving for *t* shows that

$$t = \frac{-(-24.0 \text{ m/s}) \pm \sqrt{(-24.0 \text{ m/s})^2 - 4(2.50 \text{ m/s}^2)(-19.2 \text{ m})}}{2(2.50 \text{ m/s}^2)} = 10.3 \text{ s}$$

We have ignored the negative root, because it leads to a negative value for the time, which is unphysical. The total time for the police car to catch up, including the reaction time, is

$$0.800 \text{ s} + 10.3 \text{ s} = 11.1 \text{ s}$$

78. **REASONING AND SOLUTION** We measure the positions of the balloon and the pellet relative to the ground and assume up to be positive. The balloon has no acceleration, since it travels at a constant velocity $v_{\rm B}$, so its displacement in time *t* is $v_{\rm B}t$. Its position above the ground, therefore, is

$$y_{\rm B} = H_0 + v_{\rm B}t$$

where $H_0 = 12$ m. The pellet moves under the influence of gravity (a = -9.80 m/s²), so its position above the ground is given by Equation 2.8 as

$$y_{\rm P} = v_0 t + \frac{1}{2}at^2$$

But $y_{\rm P} = y_{\rm B}$ at time *t*, so that

$$v_0 t + \frac{1}{2}at^2 = H_0 + v_{\rm B}t$$

Rearranging this result and suppressing the units gives

$$\frac{1}{2}at^{2} + (v_{0} - v_{B})t - H_{0} = \frac{1}{2}(-9.80)t^{2} + (30.0 - 7.0)t - 12.0 = 0$$
$$4.90t^{2} - 23.0t + 12.0 = 0$$
$$t = \frac{23.0 \pm \sqrt{23.0^{2} - 4(4.90)(12.0)}}{2(4.90)} = 4.09 \text{ s or } 0.602 \text{ s}$$

Substituting each of these values in the expression for $y_{\rm B}$ gives

$$y_{\rm B} = 12.0 \text{ m} + (7.0 \text{ m/s})(4.09 \text{ s}) = 41 \text{ m}$$

 $y_{\rm B} = 12.0 \text{ m} + (7.0 \text{ m/s})(0.602 \text{ s}) = 16 \text{ m}$

79. **SSM** *REASONING* Since the car is moving with a constant velocity, the displacement of the car in a time t can be found from Equation 2.8 with $a = 0 \text{ m/s}^2$ and v_0 equal to the velocity of the car: $x_{car} = v_{car}t$. Since the train starts from rest with a constant acceleration, the displacement of the train in a time t is given by Equation 2.8 with $v_0 = 0$ m/s:

$$x_{\text{train}} = \frac{1}{2}a_{\text{train}}t^2$$

At a time t_1 , when the car just reaches the front of the train, $x_{car} = L_{train} + x_{train}$, where L_{train} is the length of the train. Thus, at time t_1 ,

$$v_{\rm car}t_1 = L_{\rm train} + \frac{1}{2}a_{\rm train}t_1^2 \tag{1}$$

At a time t_2 , when the car is again at the rear of the train, $x_{car} = x_{train}$. Thus, at time t_2

$$v_{\rm car}t_2 = \frac{1}{2}a_{\rm train}t_2^2\tag{2}$$

Equations (1) and (2) can be solved simultaneously for the speed of the car v_{car} and the acceleration of the train a_{train} .

SOLUTION

a. Solving Equation (2) for a_{train} we have

$$a_{\text{train}} = \frac{2v_{\text{car}}}{t_2} \tag{3}$$

Substituting this expression for a_{train} into Equation (1) and solving for v_{car} , we have

$$v_{\text{car}} = \frac{L_{\text{train}}}{t_1 \left(1 - \frac{t_1}{t_2}\right)} = \frac{92 \text{ m}}{(14 \text{ s}) \left(1 - \frac{14 \text{ s}}{28 \text{ s}}\right)} = 13 \text{ m/s}$$

b. Direct substitution into Equation (3) gives the acceleration of the train:

$$a_{\text{train}} = \frac{2v_{\text{car}}}{t_2} = \frac{2(13 \text{ m/s})}{28 \text{ s}} = \boxed{0.93 \text{ m/s}^2}$$

80. *REASONING AND SOLUTION* Both motorcycles have the same velocity *v* at the end of the four second interval. Now

$$v = v_{0A} + a_A t$$
$$v = v_{0B} + a_B t$$

for motorcycle A and

$$v_{0A} - v_{0B} = (4.0 \text{ m/s}^2 - 2.0 \text{ m/s}^2)(4 \text{ s}) = |+8.0 \text{ m/s}|$$

The positive result indicates that motorcycle A was initially traveling faster.

81. **CONCEPT QUESTION** The displacement is a vector that points from an object's initial position to its final position. If the final position is greater than the initial position, the

displacement is positive. On the other hand, if the final position is less than the initial position, the displacement is negative

- (a) The final position is greater than the initial position, so the displacement is positive.
- (b) The final position is less than the initial position, so the displacement is negative.
- (c) The final position is greater than the initial position, so the displacement is positive.

SOLUTION The displacement is defined as Displacement = $x - x_0$, where x is the final position and x_0 is the initial position. The displacements for the three cases are:

- (a) Displacement = 6.0 m 2.0 m = |+4.0 m|
- (b) Displacement = 2.0 m 6.0 m = -4.0 m
- (c) Displacement = 7.0 m (-3.0 m) = |+10.0 m|

82. CONCEPT QUESTION

a. According to Equation 2.2, the direction of the car's average velocity is the same as its displacement, which is equal to the difference between the final and initial positions. Therefore, the answers to this Concept Question are the same as those to the Concept Question in problem 81.

b. The average velocities are: (a) positive, (b) negative, (c) positive.

SOLUTION The average velocity is equal to the displacement divided by the elapsed time (Equation 2.2), where the displacement is equal to the final position minus the initial position;

$$\overline{v} = \frac{x - x_0}{t - t_0}$$

The average velocities for the three cases are:

- (a) Average velocity = (6.0 m 2.0 m)/(0.50 s) = |+8.0 m/s|
- (b) Average velocity = (2.0 m 6.0 m)/(0.50 s) = |-8.0 m/s|
- (c) Average velocity = [7.0 m (-3.0 m)]/(0.50 s) = +20.0 m/s

83. CONCEPT QUESTIONS

a. The average acceleration is defined by Equation 2.4 as the change in velocity divided by the elapsed time. The change in velocity is equal to the final velocity minus the initial velocity. Therefore, the change in velocity, and hence the acceleration, is positive if the final velocity is greater than the initial velocity. The acceleration is negative if the final velocity is less than the initial velocity.

b. The directions of the average accelerations are as follows:

(a) The final velocity is greater than the initial velocity, so the acceleration is positive.

(b) The final velocity is less than the initial velocity, so the acceleration is negative.

(c) The final velocity is greater than the initial velocity (-3.0 m/s is greater than -6.0 m/s), so the acceleration is positive.

(d) The final velocity is less than the initial velocity, so the acceleration is negative.

SOLUTION The average acceleration is equal to the change in velocity divided by the elapsed time (Equation 2.4), where the change in velocity is equal to the final velocity minus the initial velocity;

$$\overline{a} = \frac{v - v_0}{t - t_0}$$

The average accelerations are:

(a)
$$\overline{a} = (5.0 \text{ m/s} - 2.0 \text{ m/s})/(2.0 \text{ s}) = +1.5 \text{ m/s}^2$$

(b)
$$\overline{a} = (2.0 \text{ m/s} - 5.0 \text{ m/s})/(2.0 \text{ s}) = -1.5 \text{ m/s}^2$$

(c)
$$\overline{a} = [-3.0 \text{ m/s} - (-6.0 \text{ m/s})]/(2.0 \text{ s}) = +1.5 \text{ m/s}^2$$

(d)
$$\overline{a} = (-4.0 \text{ m/s} - 4.0 \text{ m/s})/(2.0 \text{ s}) = \left| -4.0 \text{ m/s}^2 \right|$$

84. **CONCEPT QUESTIONS** When the velocity and acceleration vectors are in the same direction, the speed of the object increases in time. When the velocity and acceleration vectors are in opposite directions, the speed of the object decreases in time.

(a) The initial velocity and acceleration are in the same direction, so the speed is increasing.

(b) The initial velocity and acceleration are in opposite directions, so the speed is decreasing.

(c) The initial velocity and acceleration are in opposite directions, so the speed is decreasing.

(d) The initial velocity and acceleration are in the same direction, so the speed is increasing.

SOLUTION The final velocity v is related to the initial velocity v_0 , the acceleration a, and the elapsed time t through Equation 2.4, $v = v_0 + a t$. The final velocities and speeds for the four moving objects are:

a.
$$v = 12 \text{ m/s} + (3.0 \text{ m/s}^2)(2.0 \text{ s}) = 18 \text{ m/s}$$
. The final speed is 18 m/s .
b. $v = 12 \text{ m/s} + (-3.0 \text{ m/s}^2)(2.0 \text{ s}) = 6.0 \text{ m/s}$. The final speed is 6.0 m/s .
c. $v = -12 \text{ m/s} + (3.0 \text{ m/s}^2)(2.0 \text{ s}) = -6.0 \text{ m/s}$. The final speed is 6.0 m/s .
d. $v = -12 \text{ m/s} + (-3.0 \text{ m/s}^2)(2.0 \text{ s}) = -18 \text{ m/s}$. The final speed is 18 m/s .

85. CONCEPT QUESTIONS

a. No. A zero acceleration means that the velocity is constant, but not necessarily zero.

b. No, because the direction of the car changes and so the final velocity is not equal to the initial velocity. The change in the velocity is not zero, so the average acceleration is not zero.

SOLUTION

The average acceleration is equal to the change in velocity divided by the elapsed time,

$$\overline{a} = \frac{v - v_0}{t - t_0} \tag{2.4}$$

a. The initial and final velocities are +82 m/s and +82 m/s. The average acceleration is

 $\overline{a} = (82 \text{ m/s} - 82 \text{ m/s})/(t - t_0) = 0 \text{ m/s}^2$

b. The initial velocity is + 82 m/s and the final velocity is -82 m/s. The average acceleration is

 $\overline{a} = (-82 \text{ m/s} - 82 \text{ m/s})/(12 \text{ s}) = -14 \text{ m/s}^2$

86. CONCEPT QUESTIONS

a. The acceleration of the ball does not reverse direction on the downward part of the trip. The acceleration is the same for both the upward and downward parts, namely -9.80 m/s^2 .

b. The displacement is y = 0 m, since the final and initial positions of the ball are the same.

SOLUTION The displacement of the ball, the acceleration due to gravity, and the elapsed time are known. We may use Equation 2.8, $y = v_0 t + \frac{1}{2}at^2$, to find the initial velocity of the ball. Solving this equation for the initial velocity gives

$$v_0 = \frac{y - \frac{1}{2}at^2}{t} = \frac{0 \text{ m} - \frac{1}{2} \left(-9.80 \text{ m/s}^2\right) \left(8.0 \text{ s}\right)^2}{8.0 \text{ s}} = \boxed{+39 \text{ m/s}}$$

87. CONCEPT QUESTIONS

a. Car B's final velocity is greater than car A's constant velocity. This must be so, because B starts from rest and moves more slowly than A at the beginning. The only way in which B can cover the same distance in the same time as A and move more slowly at the beginning is to move more quickly at the end.

b. Car B's average velocity is the same as car A's constant velocity. This follows from the definition of average velocity given in Equation 2.2 as the displacement divided by the elapsed time. When the velocity is constant, as it is for car A, the average velocity is the same as the constant velocity. Since both displacement and time are the same for each car, this equation gives the same value for car B's average velocity and car A's constant velocity.

c. Car B's constant acceleration can be calculated from Equation 2.4 ($v_{\rm B} = v_{\rm B0} + a_{\rm B}t$), which is one of the equations of kinematics and gives the acceleration as $[a_{\rm B} = (v_{\rm B} - v_{\rm B0})/t]$. Since car B starts from rest, we know that $v_{\rm B0} = 0$ m/s. Furthermore, *t* is given. Therefore, calculation of the acceleration $a_{\rm B}$ requires that we first determine the final velocity $v_{\rm B}$.

SOLUTION

a. According to Equation 2.2, the velocity of car A is the displacement L divided by the time t. Thus, we obtain

$$v_{\rm A} = \frac{L}{t} = \frac{460 \text{ m}}{210 \text{ s}} = \boxed{2.2 \text{ m/s}}$$

b. Since the acceleration of car B is constant, we know that its average velocity is given by Equation 2.6 as $\overline{v}_{\rm B} = \frac{1}{2} (v_{\rm B} + v_{\rm B0})$, where $v_{\rm B}$ is the final velocity and $v_{\rm B0}$ is the initial velocity. Solving for the final velocity and using the fact that car B starts from rest $(v_{\rm B0} = 0 \text{ m/s})$ gives

$$v_{\rm B} = 2\overline{v}_{\rm B} - v_{\rm B0} = 2\overline{v}_{\rm B} \tag{1}$$

As discussed in the answer to Concept Question (b), the average velocity of car B is equal to the constant velocity of car A. Substituting this result into Equation (1), we find that

$$v_{\rm B} = 2\overline{v}_{\rm B} = 2v_{\rm A} = 2(2.2 \text{ m/s}) = 4.4 \text{ m/s}$$

As expected, car B's final velocity is greater than car A's constant velocity.

c. Solving Equation 2.4 ($v_{\rm B} = v_{\rm B0} + a_{\rm B}t$) for the acceleration shows that

$$a_{\rm B} = \frac{v_{\rm B} - v_{\rm B0}}{t} = \frac{4.4 \text{ m/s} - 0 \text{ m/s}}{210 \text{ s}} = \boxed{0.021 \text{ m/s}^2}$$

88. CONCEPT QUESTIONS

a. The stone that is thrown upward loses speed on the way up. The initial velocity points upward, while the acceleration due to gravity points downward. Under these circumstances, the stone decelerates on the way up. In other words, it loses speed.

b. The stone that is thrown downward gains speed on the way down. The initial velocity points downward, in the same direction as the acceleration due to gravity. Under these circumstances, the stone accelerates on the way down and gains speed.

c. The stones cross paths below the point that corresponds to half the height of the cliff. To see why, consider where they would cross paths if they each maintained their initial speed as the moved. Then, they would cross paths exactly at the halfway point. However, the stone traveling upward begins immediately to lose speed, while the stone traveling downward immediately gains speed. Thus, the upward moving stone travels more slowly than the downward moving stone. Consequently, the stone thrown downward has traveled farther when it reaches the crossing point than the stone thrown upward. The crossing point, then, is below the halfway point.

SOLUTION The initial velocity v_0 is known for both stones, as is the acceleration *a* due to gravity. In addition, we know that at the crossing point the stones are at the same place at the same time *t*. Furthermore, the position of each stone is specified by its displacement *y* from its starting point. The equation of kinematics that relates the variables v_0 , *a*, *t* and *y* is Equation 2.8 ($y = v_0 t + \frac{1}{2}at^2$), and we will use it in our solution. In using this equation, we will assume upward to be the positive direction. Applying Equation 2.8 to each stone, we have

$$\underbrace{y_{\text{up}} = v_0^{\text{up}} t + \frac{1}{2}at^2}_{\text{Upward moving stone}} \quad \text{and} \quad \underbrace{y_{\text{down}} = v_0^{\text{down}} t + \frac{1}{2}at^2}_{\text{Downward moving stone}}$$

In these expressions t is the time it takes for either stone to reach the crossing point, and a is the acceleration due to gravity. Note that y_{up} is the displacement of the upward moving stone above the base of the cliff, y_{down} is the displacement of the downward moving stone below the top of the cliff, and H is the displacement of the clifftop above the base of the cliff, as the drawing shows. The



distances above and below the crossing point must add to equal the height of the cliff, so we have

$$y_{\rm up} - y_{\rm down} = H$$

where the minus sign appears because the displacement y_{down} points in the negative direction. Substituting the two expressions for y_{up} and y_{down} into this equation gives

$$v_0^{\text{up}} t + \frac{1}{2}at^2 - \left(v_0^{\text{down}} t + \frac{1}{2}at^2\right) = H$$

This equation can be solved for t to show that the travel time to the crossing point is

$$t = \frac{H}{v_0^{\rm up} - v_0^{\rm down}}$$

Substituting this result into the expression from Equation 2.8 for y_{up} gives

$$y_{up} = v_0^{up} t + \frac{1}{2} a t^2 = v_0^{up} \left(\frac{H}{v_0^{up} - v_0^{down}} \right) + \frac{1}{2} a \left(\frac{H}{v_0^{up} - v_0^{down}} \right)^2$$
$$= (9.00 \text{ m/s}) \left[\frac{6.00 \text{ m}}{9.00 \text{ m/s} - (-9.00 \text{ m/s})} \right] + \frac{1}{2} (-9.80 \text{ m/s}^2) \left[\frac{6.00 \text{ m}}{9.00 \text{ m/s} - (-9.00 \text{ m/s})} \right]^2$$
$$= 2.46 \text{ m}$$

Thus, the crossing is located a distance of 2.46 m above the base of the cliff, which is below the halfway point of 3.00 m, as expected.