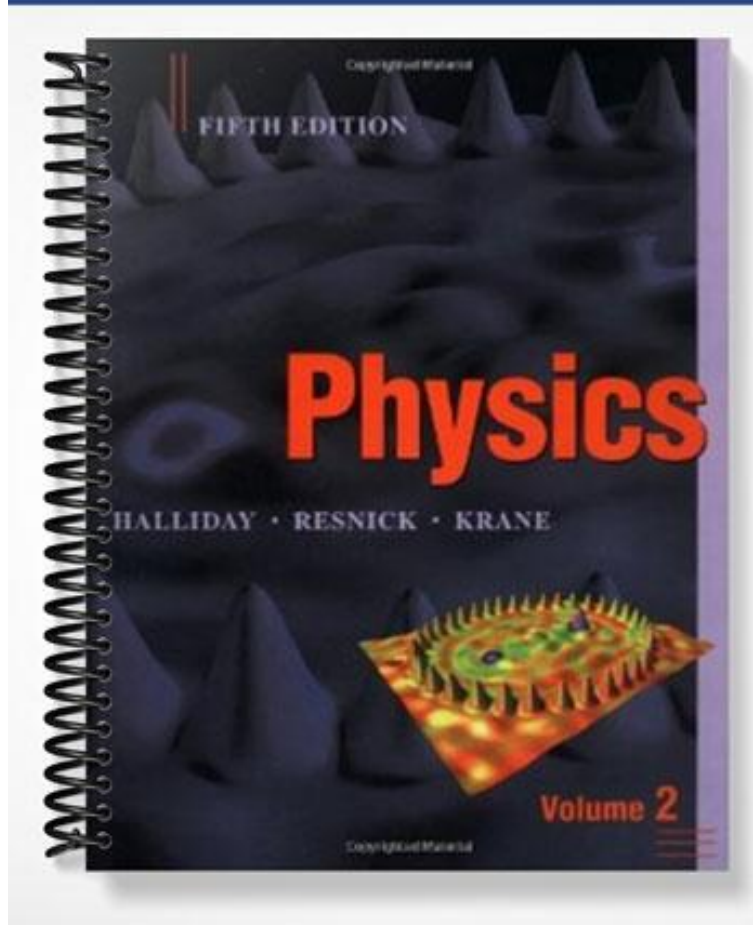


# SOLUTIONS MANUAL



Instructor Solutions Manual  
for  
Physics  
by  
Halliday, Resnick, and Krane

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Volume 2

## A Note To The Instructor...

The solutions here are somewhat brief, as they are designed for the instructor, not for the student. Check with the publishers before electronically posting any part of these solutions; website, ftp, or server access *must* be restricted to your students.

I have been somewhat casual about subscripts whenever it is obvious that a problem is one dimensional, or that the choice of the coordinate system is irrelevant to the *numerical* solution. Although this does not change the validity of the answer, it will sometimes obfuscate the approach if viewed by a novice.

There are some *traditional* formula, such as

$$v_x^2 = v_{0x}^2 + 2a_x x,$$

which are not used in the text. The worked solutions use only material from the text, so there may be times when the solution here seems unnecessarily convoluted and drawn out. Yes, I know an easier approach existed. But if it was not in the text, I did not use it here.

I also tried to avoid reinventing the wheel. There are some exercises and problems in the text which build upon previous exercises and problems. Instead of rederiving expressions, I simply refer you to the previous solution.

I adopt a different approach for rounding of significant figures than previous authors; in particular, I usually round intermediate answers. As such, some of my answers will differ from those in the back of the book.

Exercises and Problems which are enclosed in a box also appear in the Student's Solution Manual with considerably more detail and, when appropriate, include discussion on any physical implications of the answer. These student solutions carefully discuss the steps required for solving problems, point out the relevant equation numbers, or even specify where in the text additional information can be found. When two almost equivalent methods of solution exist, often both are presented. You are encouraged to refer students to the Student's Solution Manual for these exercises and problems. However, the material from the Student's Solution Manual must *not* be copied.

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**E25-1** The charge transferred is

$$Q = (2.5 \times 10^4 \text{ C/s})(20 \times 10^{-6} \text{ s}) = 5.0 \times 10^{-1} \text{ C}.$$

**E25-2** Use Eq. 25-4:

$$r = \sqrt{\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(26.3 \times 10^{-6} \text{ C})(47.1 \times 10^{-6} \text{ C})}{(5.66 \text{ N})}} = 1.40 \text{ m}$$

**E25-3** Use Eq. 25-4:

$$F = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(3.12 \times 10^{-6} \text{ C})(1.48 \times 10^{-6} \text{ C})}{(0.123 \text{ m})^2} = 2.74 \text{ N}.$$

**E25-4** (a) The forces are equal, so  $m_1 a_1 = m_2 a_2$ , or

$$m_2 = (6.31 \times 10^{-7} \text{ kg})(7.22 \text{ m/s}^2) / (9.16 \text{ m/s}^2) = 4.97 \times 10^{-7} \text{ kg}.$$

(b) Use Eq. 25-4:

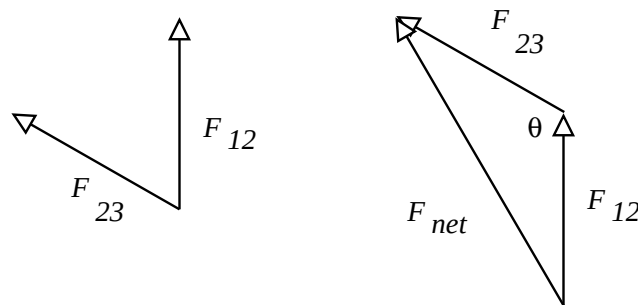
$$q = \sqrt{\frac{(6.31 \times 10^{-7} \text{ kg})(7.22 \text{ m/s}^2)(3.20 \times 10^{-3} \text{ m})^2}{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)}} = 7.20 \times 10^{-11} \text{ C}$$

**E25-5** (a) Use Eq. 25-4,

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} = \frac{1}{4\pi(8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2)} \frac{(21.3 \mu\text{C})(21.3 \mu\text{C})}{(1.52 \text{ m})^2} = 1.77 \text{ N}$$

(b) In part (a) we found  $F_{12}$ ; to solve part (b) we need to first find  $F_{13}$ . Since  $q_3 = q_2$  and  $r_{13} = r_{12}$ , we can immediately conclude that  $F_{13} = F_{12}$ .

We must assess the direction of the force of  $q_3$  on  $q_1$ ; it will be directed along the line which connects the two charges, and will be directed away from  $q_3$ . The diagram below shows the directions.



From this diagram we want to find the magnitude of the *net* force on  $q_1$ . The cosine law is appropriate here:

$$\begin{aligned} F_{\text{net}}^2 &= F_{12}^2 + F_{13}^2 - 2F_{12}F_{13} \cos \theta, \\ &= (1.77 \text{ N})^2 + (1.77 \text{ N})^2 - 2(1.77 \text{ N})(1.77 \text{ N}) \cos(120^\circ), \\ &= 9.40 \text{ N}^2, \\ F_{\text{net}} &= 3.07 \text{ N}. \end{aligned}$$

**E25-6** Originally  $F_0 = CQ_0^2 = 0.088\text{ N}$ , where  $C$  is a constant. When sphere 3 touches 1 the charge on both becomes  $Q_0/2$ . When sphere 3 touches sphere 2 the charge on each becomes  $(Q_0 + Q_0/2)/2 = 3Q_0/4$ . The force between sphere 1 and 2 is then

$$F = C(Q_0/2)(3Q_0/4) = (3/8)CQ_0^2 = (3/8)F_0 = 0.033\text{ N}.$$

**E25-7** The forces on  $q_3$  are  $\vec{\mathbf{F}}_{31}$  and  $\vec{\mathbf{F}}_{32}$ . These forces are given by the vector form of Coulomb's Law, Eq. 25-5,

$$\begin{aligned}\vec{\mathbf{F}}_{31} &= \frac{1}{4\pi\epsilon_0} \frac{q_3 q_1}{r_{31}^2} \hat{\mathbf{r}}_{31} = \frac{1}{4\pi\epsilon_0} \frac{q_3 q_1}{(2d)^2} \hat{\mathbf{r}}_{31}, \\ \vec{\mathbf{F}}_{32} &= \frac{1}{4\pi\epsilon_0} \frac{q_3 q_2}{r_{32}^2} \hat{\mathbf{r}}_{32} = \frac{1}{4\pi\epsilon_0} \frac{q_3 q_2}{(d)^2} \hat{\mathbf{r}}_{32}.\end{aligned}$$

These two forces are the only forces which act on  $q_3$ , so in order to have  $q_3$  in equilibrium the forces must be equal in magnitude, but opposite in direction. In short,

$$\begin{aligned}\vec{\mathbf{F}}_{31} &= -\vec{\mathbf{F}}_{32}, \\ \frac{1}{4\pi\epsilon_0} \frac{q_3 q_1}{(2d)^2} \hat{\mathbf{r}}_{31} &= -\frac{1}{4\pi\epsilon_0} \frac{q_3 q_2}{(d)^2} \hat{\mathbf{r}}_{32}, \\ \frac{q_1}{4} \hat{\mathbf{r}}_{31} &= -\frac{q_2}{1} \hat{\mathbf{r}}_{32}.\end{aligned}$$

Note that  $\hat{\mathbf{r}}_{31}$  and  $\hat{\mathbf{r}}_{32}$  both point in the same direction and are both of unit length. We then get

$$q_1 = -4q_2.$$

**E25-8** The horizontal and vertical contributions from the upper left charge and lower right charge are straightforward to find. The contributions from the upper left charge require slightly more work. The diagonal distance is  $\sqrt{2}a$ ; the components will be weighted by  $\cos 45^\circ = \sqrt{2}/2$ . The diagonal charge will contribute

$$\begin{aligned}F_x &= \frac{1}{4\pi\epsilon_0} \frac{(q)(2q)}{(\sqrt{2}a)^2} \frac{\sqrt{2}}{2} \hat{\mathbf{i}} = \frac{\sqrt{2}}{8\pi\epsilon_0} \frac{q^2}{a^2} \hat{\mathbf{i}}, \\ F_y &= \frac{1}{4\pi\epsilon_0} \frac{(q)(2q)}{(\sqrt{2}a)^2} \frac{\sqrt{2}}{2} \hat{\mathbf{j}} = \frac{\sqrt{2}}{8\pi\epsilon_0} \frac{q^2}{a^2} \hat{\mathbf{j}}.\end{aligned}$$

(a) The horizontal component of the net force is then

$$\begin{aligned}F_x &= \frac{1}{4\pi\epsilon_0} \frac{(2q)(2q)}{a^2} \hat{\mathbf{i}} + \frac{\sqrt{2}}{8\pi\epsilon_0} \frac{q^2}{a^2} \hat{\mathbf{i}}, \\ &= \frac{4 + \sqrt{2}/2}{4\pi\epsilon_0} \frac{q^2}{a^2} \hat{\mathbf{i}}, \\ &= (4.707)(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.13 \times 10^{-6} \text{ C})^2/(0.152 \text{ m})^2 \hat{\mathbf{i}} = 2.34 \text{ N} \hat{\mathbf{i}}.\end{aligned}$$

(b) The vertical component of the net force is then

$$\begin{aligned}F_y &= -\frac{1}{4\pi\epsilon_0} \frac{(q)(2q)}{a^2} \hat{\mathbf{j}} + \frac{\sqrt{2}}{8\pi\epsilon_0} \frac{q^2}{a^2} \hat{\mathbf{j}}, \\ &= \frac{-2 + \sqrt{2}/2}{8\pi\epsilon_0} \frac{q^2}{a^2} \hat{\mathbf{j}}, \\ &= (-1.293)(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.13 \times 10^{-6} \text{ C})^2/(0.152 \text{ m})^2 \hat{\mathbf{j}} = -0.642 \text{ N} \hat{\mathbf{j}}.\end{aligned}$$

**E25-9** The magnitude of the force on the negative charge from each positive charge is

$$F = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(4.18 \times 10^{-6} \text{ C})(6.36 \times 10^{-6} \text{ C}) / (0.13 \text{ m})^2 = 14.1 \text{ N}.$$

The force from each positive charge is directed along the side of the triangle; but from symmetry only the component along the bisector is of interest. This means that we need to weight the above answer by a factor of  $2 \cos(30^\circ) = 1.73$ . The net force is then 24.5 N.

**E25-10** Let the charge on one sphere be  $q$ , then the charge on the other sphere is  $Q = (52.6 \times 10^{-6} \text{ C}) - q$ . Then

$$\begin{aligned} \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} &= F, \\ (8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) q(52.6 \times 10^{-6} \text{ C} - q) &= (1.19 \text{ N})(1.94 \text{ m})^2. \end{aligned}$$

Solve this quadratic expression for  $q$  and get answers  $q_1 = 4.02 \times 10^{-5} \text{ C}$  and  $q_2 = 1.24 \times 10^{-6} \text{ N}$ .

**E25-11** This problem is similar to Ex. 25-7. There are some additional issues, however. It is easy enough to write expressions for the forces on the third charge

$$\begin{aligned} \vec{\mathbf{F}}_{31} &= \frac{1}{4\pi\epsilon_0} \frac{q_3 q_1}{r_{31}^2} \hat{\mathbf{r}}_{31}, \\ \vec{\mathbf{F}}_{32} &= \frac{1}{4\pi\epsilon_0} \frac{q_3 q_2}{r_{32}^2} \hat{\mathbf{r}}_{32}. \end{aligned}$$

Then

$$\begin{aligned} \vec{\mathbf{F}}_{31} &= -\vec{\mathbf{F}}_{32}, \\ \frac{1}{4\pi\epsilon_0} \frac{q_3 q_1}{r_{31}^2} \hat{\mathbf{r}}_{31} &= -\frac{1}{4\pi\epsilon_0} \frac{q_3 q_2}{r_{32}^2} \hat{\mathbf{r}}_{32}, \\ \frac{q_1}{r_{31}^2} \hat{\mathbf{r}}_{31} &= -\frac{q_2}{r_{32}^2} \hat{\mathbf{r}}_{32}. \end{aligned}$$

The only way to satisfy the *vector* nature of the above expression is to have  $\hat{\mathbf{r}}_{31} = \pm \hat{\mathbf{r}}_{32}$ ; this means that  $q_3$  must be collinear with  $q_1$  and  $q_2$ .  $q_3$  could be between  $q_1$  and  $q_2$ , or it could be on either side. Let's resolve this issue now by putting the values for  $q_1$  and  $q_2$  into the expression:

$$\begin{aligned} \frac{(1.07 \mu\text{C})}{r_{31}^2} \hat{\mathbf{r}}_{31} &= -\frac{(-3.28 \mu\text{C})}{r_{32}^2} \hat{\mathbf{r}}_{32}, \\ r_{32}^2 \hat{\mathbf{r}}_{31} &= (3.07) r_{31}^2 \hat{\mathbf{r}}_{32}. \end{aligned}$$

Since squared quantities are positive, we can only get this to work if  $\hat{\mathbf{r}}_{31} = \hat{\mathbf{r}}_{32}$ , so  $q_3$  is *not* between  $q_1$  and  $q_2$ . We are then left with

$$r_{32}^2 = (3.07) r_{31}^2,$$

so that  $q_3$  is closer to  $q_1$  than it is to  $q_2$ . Then  $r_{32} = r_{31} + r_{12} = r_{31} + 0.618 \text{ m}$ , and if we take the square root of both sides of the above expression,

$$\begin{aligned} r_{31} + (0.618 \text{ m}) &= \sqrt{(3.07)} r_{31}, \\ (0.618 \text{ m}) &= \sqrt{(3.07)} r_{31} - r_{31}, \\ (0.618 \text{ m}) &= 0.752 r_{31}, \\ 0.822 \text{ m} &= r_{31} \end{aligned}$$

**E25-12** The magnitude of the magnetic force between any two charges is  $kq^2/a^2$ , where  $a = 0.153$  m. The force between each charge is directed along the side of the triangle; but from symmetry only the component along the bisector is of interest. This means that we need to weight the above answer by a factor of  $2 \cos(30^\circ) = 1.73$ . The net force on any charge is then  $1.73kq^2/a^2$ .

The length of the angle bisector,  $d$ , is given by  $d = a \cos(30^\circ)$ .

The distance from any charge to the center of the equilateral triangle is  $x$ , given by  $x^2 = (a/2)^2 + (d - x)^2$ . Then

$$x = a^2/8d + d/2 = 0.644a.$$

The angle between the strings and the plane of the charges is  $\theta$ , given by

$$\sin \theta = x/(1.17 \text{ m}) = (0.644)(0.153 \text{ m})/(1.17 \text{ m}) = 0.0842,$$

or  $\theta = 4.83^\circ$ .

The force of gravity on each ball is directed vertically and the electric force is directed horizontally. The two must then be related by

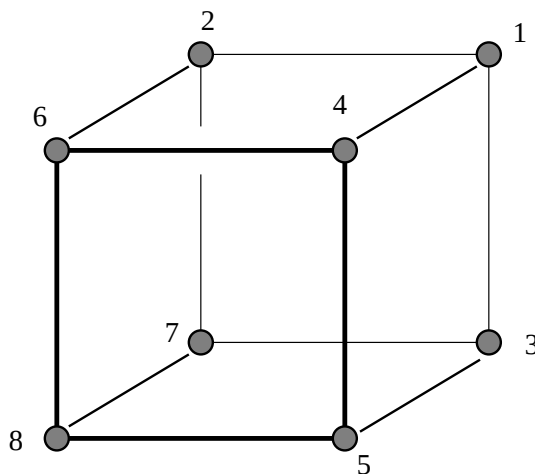
$$\tan \theta = F_E/F_G,$$

so

$$1.73(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)q^2/(0.153 \text{ m})^2 = (0.0133 \text{ kg})(9.81 \text{ m/s}^2) \tan(4.83^\circ),$$

or  $q = 1.29 \times 10^{-7} \text{ C}$ .

**E25-13** On any corner charge there are seven forces; one from each of the other seven charges. The net force will be the sum. Since all eight charges are the same all of the forces will be repulsive. We need to sketch a diagram to show how the charges are labeled.



The magnitude of the force of charge 2 on charge 1 is

$$F_{12} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r_{12}^2},$$

where  $r_{12} = a$ , the length of a side. Since both charges are the same we wrote  $q^2$ . By symmetry we expect that the magnitudes of  $F_{12}$ ,  $F_{13}$ , and  $F_{14}$  will all be the same and they will all be at right angles to each other directed along the edges of the cube. Written in terms of vectors the forces

would be

$$\begin{aligned}\vec{\mathbf{F}}_{12} &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{a^2} \hat{\mathbf{i}}, \\ \vec{\mathbf{F}}_{13} &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{a^2} \hat{\mathbf{j}}, \\ \vec{\mathbf{F}}_{14} &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{a^2} \hat{\mathbf{k}}.\end{aligned}$$

The force from charge 5 is

$$F_{15} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r_{15}^2},$$

and is directed along the side diagonal away from charge 5. The distance  $r_{15}$  is also the side diagonal distance, and can be found from

$$r_{15}^2 = a^2 + a^2 = 2a^2,$$

then

$$F_{15} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{2a^2}.$$

By symmetry we expect that the magnitudes of  $F_{15}$ ,  $F_{16}$ , and  $F_{17}$  will all be the same and they will all be directed along the diagonals of the faces of the cube. In terms of components we would have

$$\begin{aligned}\vec{\mathbf{F}}_{15} &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{2a^2} (\hat{\mathbf{j}}/\sqrt{2} + \hat{\mathbf{k}}/\sqrt{2}), \\ \vec{\mathbf{F}}_{16} &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{2a^2} (\hat{\mathbf{i}}/\sqrt{2} + \hat{\mathbf{k}}/\sqrt{2}), \\ \vec{\mathbf{F}}_{17} &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{2a^2} (\hat{\mathbf{i}}/\sqrt{2} + \hat{\mathbf{j}}/\sqrt{2}).\end{aligned}$$

The last force is the force from charge 8 on charge 1, and is given by

$$F_{18} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r_{18}^2},$$

and is directed along the cube diagonal away from charge 8. The distance  $r_{18}$  is also the cube diagonal distance, and can be found from

$$r_{18}^2 = a^2 + a^2 + a^2 = 3a^2,$$

then in term of components

$$\vec{\mathbf{F}}_{18} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{3a^2} (\hat{\mathbf{i}}/\sqrt{3} + \hat{\mathbf{j}}/\sqrt{3} + \hat{\mathbf{k}}/\sqrt{3}).$$

We can add the components together. By symmetry we expect the same answer for each components, so we'll just do one. How about  $\hat{\mathbf{i}}$ . This component has contributions from charge 2, 6, 7, and 8:

$$\frac{1}{4\pi\epsilon_0} \frac{q^2}{a^2} \left( \frac{1}{1} + \frac{2}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} \right),$$

or

$$\frac{1}{4\pi\epsilon_0} \frac{q^2}{a^2} (1.90)$$

The three components add according to Pythagoras to pick up a final factor of  $\sqrt{3}$ , so

$$F_{\text{net}} = (0.262) \frac{q^2}{\epsilon_0 a^2}.$$



**E25-14** (a) Yes. Changing the sign of  $y$  will change the sign of  $F_y$ ; since this is equivalent to putting the charge  $q_0$  on the “other” side, we would expect the force to also push in the “other” direction.

(b) The equation should look Eq. 25-15, except all  $y$ ’s should be replaced by  $x$ ’s. Then

$$F_x = \frac{1}{4\pi\epsilon_0} \frac{q_0 q}{x\sqrt{x^2 + L^2/4}}.$$

(c) Setting the particle a distance  $d$  away should give a force with the same magnitude as

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_0 q}{d\sqrt{d^2 + L^2/4}}.$$

This force is directed along the  $45^\circ$  line, so  $F_x = F \cos 45^\circ$  and  $F_y = F \sin 45^\circ$ .

(d) Let the distance be  $d = \sqrt{x^2 + y^2}$ , and then use the fact that  $F_x/F = \cos \theta = x/d$ . Then

$$F_x = F \frac{x}{d} = \frac{1}{4\pi\epsilon_0} \frac{x q_0 q}{(x^2 + y^2 + L^2/4)^{3/2}}.$$

and

$$F_y = F \frac{y}{d} = \frac{1}{4\pi\epsilon_0} \frac{y q_0 q}{(x^2 + y^2 + L^2/4)^{3/2}}.$$

**E25-15** (a) The equation *is* valid for both positive and negative  $z$ , so in vector form it would read

$$\vec{\mathbf{F}} = F_z \hat{\mathbf{k}} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q z}{(z^2 + R^2)^{3/2}} \hat{\mathbf{k}}.$$

(b) The equation *is not* valid for both positive and negative  $z$ . Reversing the sign of  $z$  should reverse the sign of  $F_z$ , and one way to fix this is to write  $1 = z/\sqrt{z^2}$ . Then

$$\vec{\mathbf{F}} = F_z \hat{\mathbf{k}} = \frac{1}{4\pi\epsilon_0} \frac{2q_0 q z}{R^2} \left( \frac{1}{\sqrt{z^2}} - \frac{1}{\sqrt{z^2}} \right) \hat{\mathbf{k}}.$$

**E25-16** Divide the rod into small differential lengths  $dr$ , each with charge  $dQ = (Q/L)dr$ . Each differential length contributes a differential force

$$dF = \frac{1}{4\pi\epsilon_0} \frac{q dQ}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2 L} dr.$$

Integrate:

$$\begin{aligned} F &= \int dF = \int_x^{x+L} \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2 L} dr, \\ &= \frac{1}{4\pi\epsilon_0} \frac{qQ}{L} \left( \frac{1}{x} - \frac{1}{x+L} \right) \end{aligned}$$

**E25-17** You must solve Ex. 16 before solving this problem!  $q_0$  refers to the charge that had been called  $q$  in that problem. In either case the distance from  $q_0$  will be the same regardless of the sign of  $q$ ; if  $q = Q$  then  $q$  will be on the right, while if  $q = -Q$  then  $q$  will be on the left.

Setting the forces equal to each other one gets

$$\frac{1}{4\pi\epsilon_0} \frac{qQ}{L} \left( \frac{1}{x} - \frac{1}{x+L} \right) = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2},$$

or

$$r = \sqrt{x(x+L)}.$$

**E25-18** You must solve Ex. 16 and Ex. 17 before solving this problem.

If all charges are positive then moving  $q_0$  off axis will result in a net force away from the axis. That's unstable.

If  $q = -Q$  then both  $q$  and  $Q$  are on the same side of  $q_0$ . Moving  $q_0$  closer to  $q$  will result in the attractive force growing faster than the repulsive force, so  $q_0$  will move away from equilibrium.

**E25-19** We can start with the work that was done for us on Page 577, except since we are concerned with  $\sin \theta = z/r$  we would have

$$dF_x = dF \sin \theta = \frac{1}{4\pi\epsilon_0} \frac{q_0 \lambda dz}{(y^2 + z^2)} \frac{z}{\sqrt{y^2 + z^2}}.$$

We will need to take into consideration that  $\lambda$  changes sign for the two halves of the rod. Then

$$\begin{aligned} F_x &= \frac{q_0 \lambda}{4\pi\epsilon_0} \left( \int_{-L/2}^0 \frac{-z dz}{(y^2 + z^2)^{3/2}} + \int_0^{L/2} \frac{+z dz}{(y^2 + z^2)^{3/2}} \right), \\ &= \frac{q_0 \lambda}{2\pi\epsilon_0} \int_0^{L/2} \frac{z dz}{(y^2 + z^2)^{3/2}}, \\ &= \frac{q_0 \lambda}{2\pi\epsilon_0} \left. \frac{-1}{\sqrt{y^2 + z^2}} \right|_0^{L/2}, \\ &= \frac{q_0 \lambda}{2\pi\epsilon_0} \left( \frac{1}{y} - \frac{1}{\sqrt{y^2 + (L/2)^2}} \right). \end{aligned}$$

**E25-20** Use Eq. 25-15 to find the magnitude of the force from any one rod, but write it as

$$F = \frac{1}{4\pi\epsilon_0} \frac{q Q}{r \sqrt{r^2 + L^2/4}},$$

where  $r^2 = z^2 + L^2/4$ . The component of this along the  $z$  axis is  $F_z = Fz/r$ . Since there are 4 rods, we have

$$F = \frac{1}{\pi\epsilon_0} \frac{q Q z}{r^2 \sqrt{r^2 + L^2/4}}, = \frac{1}{\pi\epsilon_0} \frac{q Q z}{(z^2 + L^2/4) \sqrt{z^2 + L^2/2}},$$

Equating the electric force with the force of gravity and solving for  $Q$ ,

$$Q = \frac{\pi\epsilon_0 m g}{q z} (z^2 + L^2/4) \sqrt{z^2 + L^2/2};$$

putting in the numbers,

$$\frac{\pi(8.85 \times 10^{-12} \text{C}^2/\text{N} \cdot \text{m}^2)(3.46 \times 10^{-7} \text{kg})(9.8 \text{m/s}^2)}{(2.45 \times 10^{-12} \text{C})(0.214 \text{m})} ((0.214 \text{m})^2 + (0.25 \text{m})^2/4) \sqrt{(0.214 \text{m})^2 + (0.25 \text{m})^2/2}$$

so  $Q = 3.07 \times 10^{-6} \text{C}$ .

**E25-21** In each case we conserve charge by making sure that the total number of protons is the same on both sides of the expression. We also need to conserve the number of neutrons.

(a) Hydrogen has one proton, Beryllium has four, so X must have five protons. Then X must be Boron, B.

(b) Carbon has six protons, Hydrogen has one, so X must have seven. Then X is Nitrogen, N.

(c) Nitrogen has seven protons, Hydrogen has one, but Helium has two, so X has  $7 + 1 - 2 = 6$  protons. This means X is Carbon, C.

**E25-22** (a) Use Eq. 25-4:

$$F = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(2)(90)(1.60 \times 10^{-19} \text{ C})^2}{(12 \times 10^{-15} \text{ m})^2} = 290 \text{ N}.$$

(b)  $a = (290 \text{ N}) / (4)(1.66 \times 10^{-27} \text{ kg}) = 4.4 \times 10^{28} \text{ m/s}^2$ .

**E25-23** Use Eq. 25-4:

$$F = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(282 \times 10^{-12} \text{ m})^2} = 2.89 \times 10^{-9} \text{ N}.$$

**E25-24** (a) Use Eq. 25-4:

$$q = \sqrt{\frac{(3.7 \times 10^{-9} \text{ N})(5.0 \times 10^{-10} \text{ m})^2}{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)}} = 3.20 \times 10^{-19} \text{ C}.$$

(b)  $N = (3.20 \times 10^{-19} \text{ C}) / (1.60 \times 10^{-19} \text{ C}) = 2$ .

**E25-25** Use Eq. 25-4,

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} = \frac{(\frac{1}{3} 1.6 \times 10^{-19} \text{ C})(\frac{1}{3} 1.6 \times 10^{-19} \text{ C})}{4\pi(8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2)(2.6 \times 10^{-15} \text{ m})^2} = 3.8 \text{ N}.$$

**E25-26** (a)  $N = (1.15 \times 10^{-7} \text{ C}) / (1.60 \times 10^{-19} \text{ C}) = 7.19 \times 10^{11}$ .

(b) The penny has enough electrons to make a total charge of  $-1.37 \times 10^5 \text{ C}$ . The fraction is then

$$(1.15 \times 10^{-7} \text{ C}) / (1.37 \times 10^5 \text{ C}) = 8.40 \times 10^{-13}.$$

**E25-27** Equate the magnitudes of the forces:

$$\frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} = mg,$$

so

$$r = \sqrt{\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(9.11 \times 10^{-31} \text{ kg})(9.81 \text{ m/s}^2)}} = 5.07 \text{ m}$$

**E25-28**  $Q = (75.0 \text{ kg})(-1.60 \times 10^{-19} \text{ C}) / (9.11 \times 10^{-31} \text{ kg}) = -1.3 \times 10^{13} \text{ C}$ .

**E25-29** The mass of water is  $(250 \text{ cm}^3)(1.00 \text{ g/cm}^3) = 250 \text{ g}$ . The number of moles of water is  $(250 \text{ g}) / (18.0 \text{ g/mol}) = 13.9 \text{ mol}$ . The number of water molecules is  $(13.9 \text{ mol})(6.02 \times 10^{23} \text{ mol}^{-1}) = 8.37 \times 10^{24}$ . Each molecule has ten protons, so the total positive charge is

$$Q = (8.37 \times 10^{24})(10)(1.60 \times 10^{-19} \text{ C}) = 1.34 \times 10^7 \text{ C}.$$

**E25-30** The total positive charge in 0.250 kg of water is  $1.34 \times 10^7 \text{ C}$ . Mary's imbalance is then

$$q_1 = (52.0)(4)(1.34 \times 10^7 \text{ C})(0.0001) = 2.79 \times 10^5 \text{ C},$$

while John's imbalance is

$$q_2 = (90.7)(4)(1.34 \times 10^7 \text{ C})(0.0001) = 4.86 \times 10^5 \text{ C},$$

The electrostatic force of attraction is then

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \frac{(2.79 \times 10^5)(4.86 \times 10^5)}{(28.0 \text{ m})^2} = 1.6 \times 10^{18} \text{ N}.$$

**E25-31** (a) The gravitational force of attraction between the Moon and the Earth is

$$F_G = \frac{GM_E M_M}{R^2},$$

where  $R$  is the distance between them. If both the Earth and the moon are provided a charge  $q$ , then the electrostatic repulsion would be

$$F_E = \frac{1}{4\pi\epsilon_0} \frac{q^2}{R^2}.$$

Setting these two expression equal to each other,

$$\frac{q^2}{4\pi\epsilon_0} = GM_E M_M,$$

which has solution

$$\begin{aligned} q &= \sqrt{4\pi\epsilon_0 GM_E M_M}, \\ &= \sqrt{4\pi(8.85 \times 10^{-12} \text{C}^2/\text{Nm}^2)(6.67 \times 10^{-11} \text{Nm}^2/\text{kg}^2)(5.98 \times 10^{24} \text{kg})(7.36 \times 10^{22} \text{kg})}, \\ &= 5.71 \times 10^{13} \text{C}. \end{aligned}$$

(b) We need

$$(5.71 \times 10^{13} \text{C}) / (1.60 \times 10^{-19} \text{C}) = 3.57 \times 10^{32}$$

protons on each body. The mass of protons needed is then

$$(3.57 \times 10^{32})(1.67 \times 10^{-27} \text{kg}) = 5.97 \times 10^{65} \text{kg}.$$

Ignoring the mass of the electron (why not?) we can assume that hydrogen is all protons, so we need that much hydrogen.

**P25-1** Assume that the spheres initially have charges  $q_1$  and  $q_2$ . The force of attraction between them is

$$F_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} = -0.108 \text{N},$$

where  $r_{12} = 0.500 \text{m}$ . The net charge is  $q_1 + q_2$ , and after the conducting wire is connected each sphere will get *half* of the total. The spheres will have the same charge, and repel with a force of

$$F_2 = \frac{1}{4\pi\epsilon_0} \frac{\frac{1}{2}(q_1 + q_2)\frac{1}{2}(q_1 + q_2)}{r_{12}^2} = 0.0360 \text{N}.$$

Since we know the separation of the spheres we can find  $q_1 + q_2$  quickly,

$$q_1 + q_2 = 2\sqrt{4\pi\epsilon_0 r_{12}^2 (0.0360 \text{N})} = 2.00 \mu\text{C}$$

We'll put this back into the first expression and solve for  $q_2$ .

$$\begin{aligned} -0.108 \text{N} &= \frac{1}{4\pi\epsilon_0} \frac{(2.00 \mu\text{C} - q_2)q_2}{r_{12}^2}, \\ -3.00 \times 10^{-12} \text{C}^2 &= (2.00 \mu\text{C} - q_2)q_2, \\ 0 &= -q_2^2 + (2.00 \mu\text{C})q_2 + (1.73 \mu\text{C})^2. \end{aligned}$$

The solution is  $q_2 = 3.0 \mu\text{C}$  or  $q_2 = -1.0 \mu\text{C}$ . Then  $q_1 = -1.0 \mu\text{C}$  or  $q_1 = 3.0 \mu\text{C}$ .

**P25-2** The electrostatic force on  $Q$  from each  $q$  has magnitude  $qQ/4\pi\epsilon_0 a^2$ , where  $a$  is the length of the side of the square. The magnitude of the vertical (horizontal) component of the force of  $Q$  on  $Q$  is  $\sqrt{2}Q^2/16\pi\epsilon_0 a^2$ .

(a) In order to have a zero net force on  $Q$  the magnitudes of the two contributions must balance, so

$$\frac{\sqrt{2}Q^2}{16\pi\epsilon_0 a^2} = \frac{qQ}{4\pi\epsilon_0 a^2},$$

or  $q = \sqrt{2}Q/4$ . The charges must actually have opposite charge.

(b) No.

**P25-3** (a) The third charge,  $q_3$ , will be between the first two. The net force on the third charge will be zero if

$$\frac{1}{4\pi\epsilon_0} \frac{q q_3}{r_{31}^2} = \frac{1}{4\pi\epsilon_0} \frac{4q q_3}{r_{32}^2},$$

which will occur if

$$\frac{1}{r_{31}} = \frac{2}{r_{32}}$$

The total distance is  $L$ , so  $r_{31} + r_{32} = L$ , or  $r_{31} = L/3$  and  $r_{32} = 2L/3$ .

Now that we have found the position of the third charge we need to find the magnitude. The second and third charges both exert a force on the first charge; we want this net force on the first charge to be zero, so

$$\frac{1}{4\pi\epsilon_0} \frac{q q_3}{r_{13}^2} = \frac{1}{4\pi\epsilon_0} \frac{q 4q}{r_{12}^2},$$

or

$$\frac{q_3}{(L/3)^2} = \frac{4q}{L^2},$$

which has solution  $q_3 = -4q/9$ . The negative sign is because the force between the first and second charge must be in the opposite direction to the force between the first and third charge.

(b) Consider what happens to the net force on the middle charge if it is displaced a small distance  $z$ . If the charge 3 is moved toward charge 1 then the force of attraction with charge 1 will increase. But moving charge 3 closer to charge 1 means moving charge 3 away from charge 2, so the force of attraction between charge 3 and charge 2 will decrease. So charge 3 experiences more attraction toward the charge that it moves toward, and less attraction to the charge it moves away from. Sounds unstable to me.

**P25-4** (a) The electrostatic force on the charge on the right has magnitude

$$F = \frac{q^2}{4\pi\epsilon_0 x^2},$$

The weight of the ball is  $W = mg$ , and the two forces are related by

$$F/W = \tan \theta \approx \sin \theta = x/2L.$$

Combining,  $2Lq^2 = 4\pi\epsilon_0 mgx^3$ , so

$$x = \left( \frac{q^2 L}{2\pi\epsilon_0} \right)^{1/3}.$$

(b) Rearrange and solve for  $q$ ,

$$q = \sqrt{\frac{2\pi(8.85 \times 10^{-12} \text{C}^2/\text{N} \cdot \text{m}^2)(0.0112 \text{ kg})(9.81 \text{ m/s}^2)(4.70 \times 10^{-2} \text{ m})^3}{(1.22 \text{ m})}} = 2.28 \times 10^{-8} \text{C}.$$

**P25-5** (a) Originally the balls would not repel, so they would move together and touch; after touching the balls would “split” the charge ending up with  $q/2$  each. They would then repel again.

(b) The new equilibrium separation is

$$x' = \left( \frac{(q/2)^2 L}{2\pi\epsilon_0 mg} \right)^{1/3} = \left( \frac{1}{4} \right)^{1/3} x = 2.96 \text{ cm.}$$

**P25-6** Take the time derivative of the expression in Problem 25-4. Then

$$\frac{dx}{dt} = \frac{2}{3} \frac{x}{q} \frac{dq}{dt} = \frac{2}{3} \frac{(4.70 \times 10^{-2} \text{ m})}{(2.28 \times 10^{-8} \text{ C})} (-1.20 \times 10^{-9} \text{ C/s}) = 1.65 \times 10^{-3} \text{ m/s.}$$

**P25-7** The force between the two charges is

$$F = \frac{1}{4\pi\epsilon_0} \frac{(Q-q)q}{r_{12}^2}.$$

We want to maximize this force with respect to variation in  $q$ , this means finding  $dF/dq$  and setting it equal to 0. Then

$$\frac{dF}{dq} = \frac{d}{dq} \left( \frac{1}{4\pi\epsilon_0} \frac{(Q-q)q}{r_{12}^2} \right) = \frac{1}{4\pi\epsilon_0} \frac{Q-2q}{r_{12}^2}.$$

This will vanish if  $Q - 2q = 0$ , or  $q = \frac{1}{2}Q$ .

**P25-8** Displace the charge  $q$  a distance  $y$ . The net restoring force on  $q$  will be approximately

$$F \approx 2 \frac{qQ}{4\pi\epsilon_0} \frac{1}{(d/2)^2} \frac{y}{(d/2)} = \frac{qQ}{4\pi\epsilon_0} \frac{16}{d^3} y.$$

Since  $F/y$  is effectively a force constant, the period of oscillation is

$$T = 2\pi \sqrt{\frac{m}{k}} = \left( \frac{\epsilon_0 m \pi^3 d^3}{qQ} \right)^{1/2}.$$

**P25-9** Displace the charge  $q$  a distance  $x$  toward one of the positive charges  $Q$ . The net restoring force on  $q$  will be

$$\begin{aligned} F &= \frac{qQ}{4\pi\epsilon_0} \left( \frac{1}{(d/2-x)^2} - \frac{1}{(d/2+x)^2} \right), \\ &\approx \frac{qQ}{4\pi\epsilon_0} \frac{32}{d^3} x. \end{aligned}$$

Since  $F/x$  is effectively a force constant, the period of oscillation is

$$T = 2\pi \sqrt{\frac{m}{k}} = \left( \frac{\epsilon_0 m \pi^3 d^3}{2qQ} \right)^{1/2}.$$

**P25-10** (a) Zero, by symmetry.

(b) Removing a positive Cesium ion is equivalent to adding a singly charged negative ion at that same location. The net force is then

$$F = e^2 / 4\pi\epsilon_0 r^2,$$

where  $r$  is the distance between the Chloride ion and the newly placed negative ion, or

$$r = \sqrt{3(0.20 \times 10^{-9} \text{m})^2}$$

The force is then

$$F = \frac{(1.6 \times 10^{-19} \text{C})^2}{4\pi(8.85 \times 10^{-12} \text{C}^2/\text{N} \cdot \text{m}^2)3(0.20 \times 10^{-9} \text{m})^2} = 1.92 \times 10^{-9} \text{N}.$$

**P25-11** We can pretend that this problem is in a single plane containing all three charges. The magnitude of the force on the test charge  $q_0$  from the charge  $q$  on the left is

$$F_1 = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{(a^2 + R^2)}.$$

A force of identical magnitude exists from the charge on the right. we need to add these two forces as vectors. Only the components along  $R$  will survive, and each force will contribute an amount

$$F_1 \sin \theta = F_1 \frac{R}{\sqrt{R^2 + a^2}},$$

so the net force on the test particle will be

$$\frac{2}{4\pi\epsilon_0} \frac{q q_0}{(a^2 + R^2)} \frac{R}{\sqrt{R^2 + a^2}}.$$

We want to find the maximum value as a function of  $R$ . This means take the derivative, and set it equal to zero. The derivative is

$$\frac{2q q_0}{4\pi\epsilon_0} \left( \frac{1}{(a^2 + R^2)^{3/2}} - \frac{3R^2}{(a^2 + R^2)^{5/2}} \right),$$

which will vanish when

$$a^2 + R^2 = 3R^2,$$

a *simple* quadratic equation with solutions  $R = \pm a/\sqrt{2}$ .

**E26-1**  $E = F/q = ma/q$ . Then

$$E = (9.11 \times 10^{-31} \text{ kg})(1.84 \times 10^9 \text{ m/s}^2)/(1.60 \times 10^{-19} \text{ C}) = 1.05 \times 10^{-2} \text{ N/C}.$$

**E26-2** The answers to (a) and (b) are the same!

$$F = Eq = (3.0 \times 10^6 \text{ N/C})(1.60 \times 10^{-19} \text{ C}) = 4.8 \times 10^{-13} \text{ N}.$$

**E26-3**  $F = W$ , or  $Eq = mg$ , so

$$E = \frac{mg}{q} = \frac{(6.64 \times 10^{-27} \text{ kg})(9.81 \text{ m/s}^2)}{2(1.60 \times 10^{-19} \text{ C})} = 2.03 \times 10^{-7} \text{ N/C}.$$

The alpha particle has a positive charge, this means that it will experience an electric force which is in the same direction as the electric field. Since the gravitational force is down, the electric force, and consequently the electric field, must be directed up.

**E26-4** (a)  $E = F/q = (3.0 \times 10^{-6} \text{ N})/(2.0 \times 10^{-9} \text{ C}) = 1.5 \times 10^3 \text{ N/C}$ .

(b)  $F = Eq = (1.5 \times 10^3 \text{ N/C})(1.60 \times 10^{-19} \text{ C}) = 2.4 \times 10^{-16} \text{ N}$ .

(c)  $F = mg = (1.67 \times 10^{-27} \text{ kg})(9.81 \text{ m/s}^2) = 1.6 \times 10^{-26} \text{ N}$ .

(d)  $(2.4 \times 10^{-16} \text{ N})/(1.6 \times 10^{-26} \text{ N}) = 1.5 \times 10^{10}$ .

**E26-5** Rearrange  $E = q/4\pi\epsilon_0 r^2$ ,

$$q = 4\pi(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.750 \text{ m})^2(2.30 \text{ N/C}) = 1.44 \times 10^{-10} \text{ C}.$$

**E26-6**  $p = qd = (1.60 \times 10^{-19} \text{ C})(4.30 \times 10^{-9}) = 6.88 \times 10^{-28} \text{ C} \cdot \text{m}$ .

**E26-7** Use Eq. 26-12 for points along the perpendicular bisector. Then

$$E = \frac{1}{4\pi\epsilon_0} \frac{p}{x^3} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(3.56 \times 10^{-29} \text{ C} \cdot \text{m})}{(25.4 \times 10^{-9} \text{ m})^3} = 1.95 \times 10^4 \text{ N/C}.$$

**E26-8** If the charges on the line  $x = a$  where  $+q$  and  $-q$  instead of  $+2q$  and  $-2q$  then at the center of the square  $E = 0$  by symmetry. This simplifies the problem into finding  $E$  for a charge  $+q$  at  $(a, 0)$  and  $-q$  at  $(a, a)$ . This is a dipole, and the field is given by Eq. 26-11. For this exercise we have  $x = a/2$  and  $d = a$ , so

$$E = \frac{1}{4\pi\epsilon_0} \frac{qa}{[2(a/2)^2]^{3/2}},$$

or, putting in the numbers,  $E = 1.11 \times 10^5 \text{ N/C}$ .

**E26-9** The charges at 1 and 7 are opposite and can be effectively replaced with a single charge of  $-6q$  at 7. The same is true for 2 and 8, 3 and 9, on up to 6 and 12. By symmetry we expect the field to point along a line so that three charges are above and three below. That would mean 9:30.

**E26-10** If both charges are positive then Eq. 26-10 would read  $E = 2E_+ \sin \theta$ , and Eq. 26-11 would look like

$$\begin{aligned} E &= 2 \frac{1}{4\pi\epsilon_0} \frac{q}{x^2 + (d/2)^2} \frac{x}{\sqrt{x^2 + (d/2)^2}}, \\ &\approx 2 \frac{1}{4\pi\epsilon_0} \frac{q}{x^2} \frac{x}{\sqrt{x^2}} \end{aligned}$$

when  $x \gg d$ . This can be simplified to  $E = 2q/4\pi\epsilon_0 x^2$ .



**E26-11** Treat the two charges on the left as one dipole and treat the two charges on the right as a second dipole. Point  $P$  is on the perpendicular bisector of both dipoles, so we can use Eq. 26-12 to find the two fields.

For the dipole on the left  $p = 2aq$  and the electric field due to this dipole at  $P$  has magnitude

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{2aq}{(x+a)^3}$$

and is directed *up*.

For the dipole on the right  $p = 2aq$  and the electric field due to this dipole at  $P$  has magnitude

$$E_r = \frac{1}{4\pi\epsilon_0} \frac{2aq}{(x-a)^3}$$

and is directed *down*.

The net electric field at  $P$  is the sum of these two fields, but since the two component fields point in opposite directions we must actually subtract these values,

$$\begin{aligned} E &= E_r - E_1, \\ &= \frac{2aq}{4\pi\epsilon_0} \left( \frac{1}{(x-a)^3} - \frac{1}{(x+a)^3} \right), \\ &= \frac{aq}{2\pi\epsilon_0} \frac{1}{x^3} \left( \frac{1}{(1-a/x)^3} - \frac{1}{(1+a/x)^3} \right). \end{aligned}$$

We can use the binomial expansion on the terms containing  $1 \pm a/x$ ,

$$\begin{aligned} E &\approx \frac{aq}{2\pi\epsilon_0} \frac{1}{x^3} ((1+3a/x) - (1-3a/x)), \\ &= \frac{aq}{2\pi\epsilon_0} \frac{1}{x^3} (6a/x), \\ &= \frac{3(2qa^2)}{2\pi\epsilon_0 x^4}. \end{aligned}$$

**E26-12** Do a series expansion on the part in the parentheses

$$1 - \frac{1}{\sqrt{1+R^2/z^2}} \approx 1 - \left( 1 - \frac{1}{2} \frac{R^2}{z^2} \right) = \frac{R^2}{2z^2}.$$

Substitute this in,

$$E_z \approx \frac{\sigma}{2\epsilon_0} \frac{R^2}{2z^2} \frac{\pi}{\pi} = \frac{Q}{4\pi\epsilon_0 z^2}.$$

**E26-13** At the surface  $z = 0$  and  $E_z = \sigma/2\epsilon_0$ . Half of this value occurs when  $z$  is given by

$$\frac{1}{2} = 1 - \frac{z}{\sqrt{z^2 + R^2}},$$

which can be written as  $z^2 + R^2 = (2z)^2$ . Solve this, and  $z = R/\sqrt{3}$ .

**E26-14** Look at Eq. 26-18. The electric field will be a maximum when  $z/(z^2 + R^2)^{3/2}$  is a maximum. Take the derivative of this with respect to  $z$ , and get

$$\frac{1}{(z^2 + R^2)^{3/2}} - \frac{3}{2} \frac{2z^2}{(z^2 + R^2)^{5/2}} = \frac{z^2 + R^2 - 3z^2}{(z^2 + R^2)^{5/2}}.$$

This will vanish when the numerator vanishes, or when  $z = R/\sqrt{2}$ .

**E26-15** (a) The electric field strength just above the center surface of a charged disk is given by Eq. 26-19, but with  $z = 0$ ,

$$E = \frac{\sigma}{2\epsilon_0}$$

The surface charge density is  $\sigma = q/A = q/(\pi R^2)$ . Combining,

$$q = 2\epsilon_0 \pi R^2 E = 2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) \pi (2.5 \times 10^{-2} \text{ m})^2 (3 \times 10^6 \text{ N/C}) = 1.04 \times 10^{-7} \text{ C}.$$

Notice we used an electric field strength of  $E = 3 \times 10^6 \text{ N/C}$ , which is the field at air breaks down and sparks happen.

(b) We want to find out how many atoms are on the surface; if  $a$  is the cross sectional area of one atom, and  $N$  the number of atoms, then  $A = Na$  is the surface area of the disk. The number of atoms is

$$N = \frac{A}{a} = \frac{\pi(0.0250 \text{ m})^2}{(0.015 \times 10^{-18} \text{ m}^2)} = 1.31 \times 10^{17}$$

(c) The total charge on the disk is  $1.04 \times 10^{-7} \text{ C}$ , this corresponds to

$$(1.04 \times 10^{-7} \text{ C}) / (1.6 \times 10^{-19} \text{ C}) = 6.5 \times 10^{11}$$

electrons. (We are ignoring the sign of the charge here.) If each surface atom can have at most one excess electron, then the fraction of atoms which are charged is

$$(6.5 \times 10^{11}) / (1.31 \times 10^{17}) = 4.96 \times 10^{-6},$$

which isn't very many.

**E26-16** Imagine switching the positive and negative charges. The electric field would also need to switch directions. By symmetry, then, the electric field can only point vertically down. Keeping only that component,

$$\begin{aligned} E &= 2 \int_0^{\pi/2} \frac{1}{4\pi\epsilon_0} \frac{\lambda d\theta}{r^2} \sin \theta, \\ &= \frac{2}{4\pi\epsilon_0} \frac{\lambda}{r^2}. \end{aligned}$$

But  $\lambda = q/(\pi/2)$ , so  $E = q/\pi^2\epsilon_0 r^2$ .

**E26-17** We want to fit the data to Eq. 26-19,

$$E_z = \frac{\sigma}{2\epsilon_0} \left( 1 - \frac{z}{\sqrt{z^2 + R^2}} \right).$$

There are only two variables,  $R$  and  $q$ , with  $q = \sigma\pi R^2$ .

We can find  $\sigma$  *very* easily if we assume that the measurements have no error because then at the surface (where  $z = 0$ ), the expression for the electric field simplifies to

$$E = \frac{\sigma}{2\epsilon_0}.$$

Then  $\sigma = 2\epsilon_0 E = 2(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(2.043 \times 10^7 \text{ N/C}) = 3.618 \times 10^{-4} \text{ C/m}^2$ .

Finding the radius will take a little more work. We can choose one point, and make that the reference point, and then solve for  $R$ . Starting with

$$E_z = \frac{\sigma}{2\epsilon_0} \left( 1 - \frac{z}{\sqrt{z^2 + R^2}} \right),$$

and then rearranging,

$$\begin{aligned}
\frac{2\epsilon_0 E_z}{\sigma} &= 1 - \frac{z}{\sqrt{z^2 + R^2}}, \\
\frac{2\epsilon_0 E_z}{\sigma} &= 1 - \frac{1}{\sqrt{1 + (R/z)^2}}, \\
\frac{1}{\sqrt{1 + (R/z)^2}} &= 1 - \frac{2\epsilon_0 E_z}{\sigma}, \\
1 + (R/z)^2 &= \frac{1}{(1 - 2\epsilon_0 E_z/\sigma)^2}, \\
\frac{R}{z} &= \sqrt{\frac{1}{(1 - 2\epsilon_0 E_z/\sigma)^2} - 1}.
\end{aligned}$$

Using  $z = 0.03 \text{ m}$  and  $E_z = 1.187 \times 10^7 \text{ N/C}$ , along with our value of  $\sigma = 3.618 \times 10^{-4} \text{ C/m}^2$ , we find

$$\begin{aligned}
\frac{R}{z} &= \sqrt{\frac{1}{(1 - 2(8.854 \times 10^{-12} \text{ C}^2/\text{Nm}^2)(1.187 \times 10^7 \text{ N/C})/(3.618 \times 10^{-4} \text{ C/m}^2))^2} - 1}, \\
R &= 2.167(0.03 \text{ m}) = 0.065 \text{ m}.
\end{aligned}$$

(b) And now find the charge from the charge density and the radius,

$$q = \pi R^2 \sigma = \pi (0.065 \text{ m})^2 (3.618 \times 10^{-4} \text{ C/m}^2) = 4.80 \mu\text{C}.$$

**E26-18** (a)  $\lambda = -q/L$ .

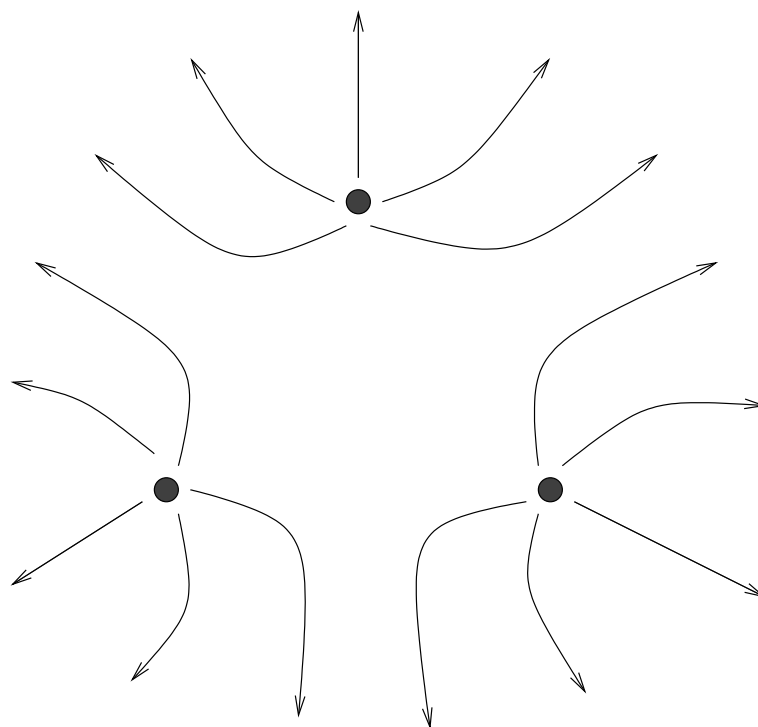
(b) Integrate:

$$\begin{aligned}
E &= \int_a^{L+a} \frac{1}{4\pi\epsilon_0} \lambda dx x^2, \\
&= \frac{\lambda}{4\pi\epsilon_0} \left( \frac{1}{a} - \frac{1}{L+a} \right), \\
&= \frac{q}{4\pi\epsilon_0} \frac{1}{a(L+a)},
\end{aligned}$$

since  $\lambda = q/L$ .

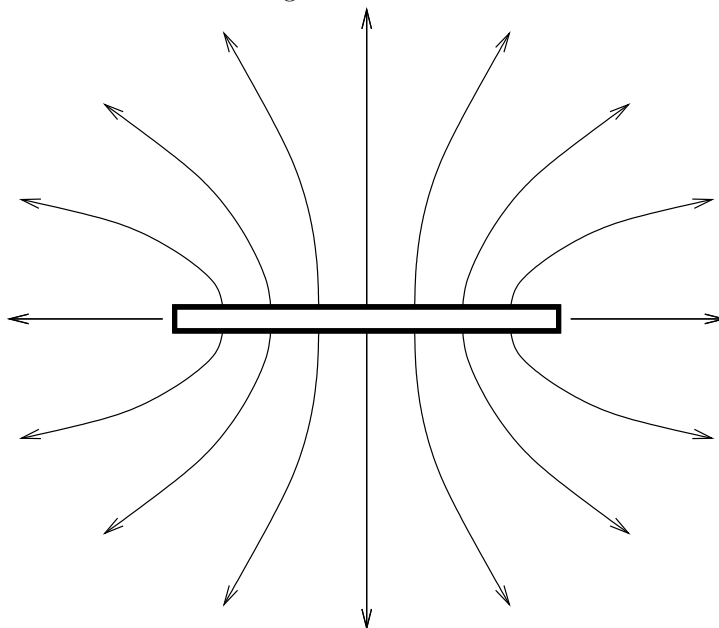
(c) If  $a \gg L$  then  $L$  can be replaced with 0 in the above expression.

**E26-19** A sketch of the field looks like this.

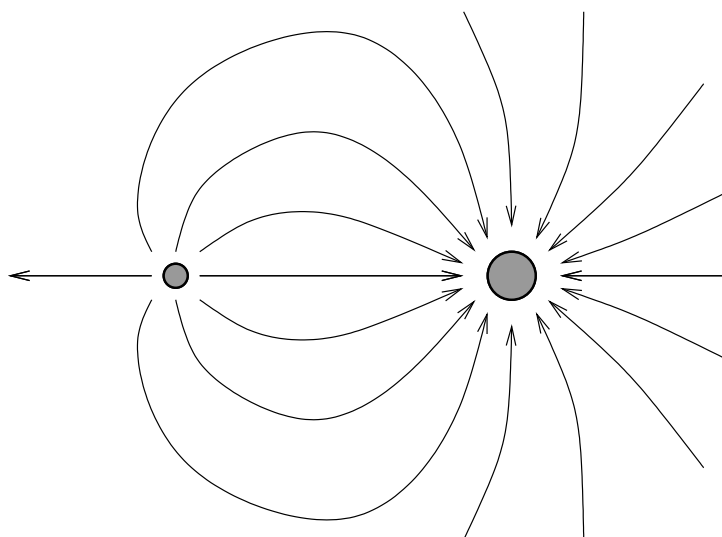


- E26-20** (a)  $F = Eq = (40 \text{ N/C})(1.60 \times 10^{-19} \text{ C}) = 6.4 \times 10^{-18} \text{ N}$   
 (b) Lines are twice as far apart, so the field is half as large, or  $E = 20 \text{ N/C}$ .

**E26-21** Consider a view of the disk on edge.



**E26-22** A sketch of the field looks like this.



**E26-23** To the right.

**E26-24** (a) The electric field is zero nearer to the smaller charge; since the charges have opposite signs it must be to the right of the  $+2q$  charge. Equating the magnitudes of the two fields,

$$\frac{2q}{4\pi\epsilon_0 x^2} = \frac{5q}{4\pi\epsilon_0 (x+a)^2},$$

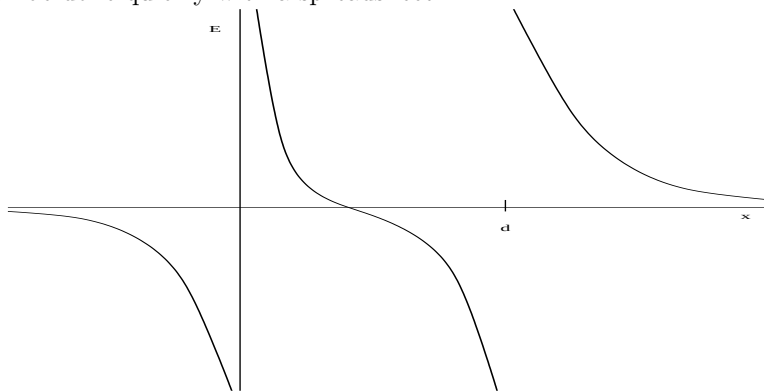
or

$$\sqrt{5}x = \sqrt{2}(x+a),$$

which has solution

$$x = \frac{\sqrt{2}a}{\sqrt{5} - \sqrt{2}} = 2.72a.$$

**E26-25** This can be done quickly with a spreadsheet.



**E26-26** (a) At point A,

$$E = \frac{1}{4\pi\epsilon_0} \left( -\frac{q}{d^2} - \frac{-2q}{(2d)^2} \right) = \frac{1}{4\pi\epsilon_0} \frac{-q}{2d^2},$$