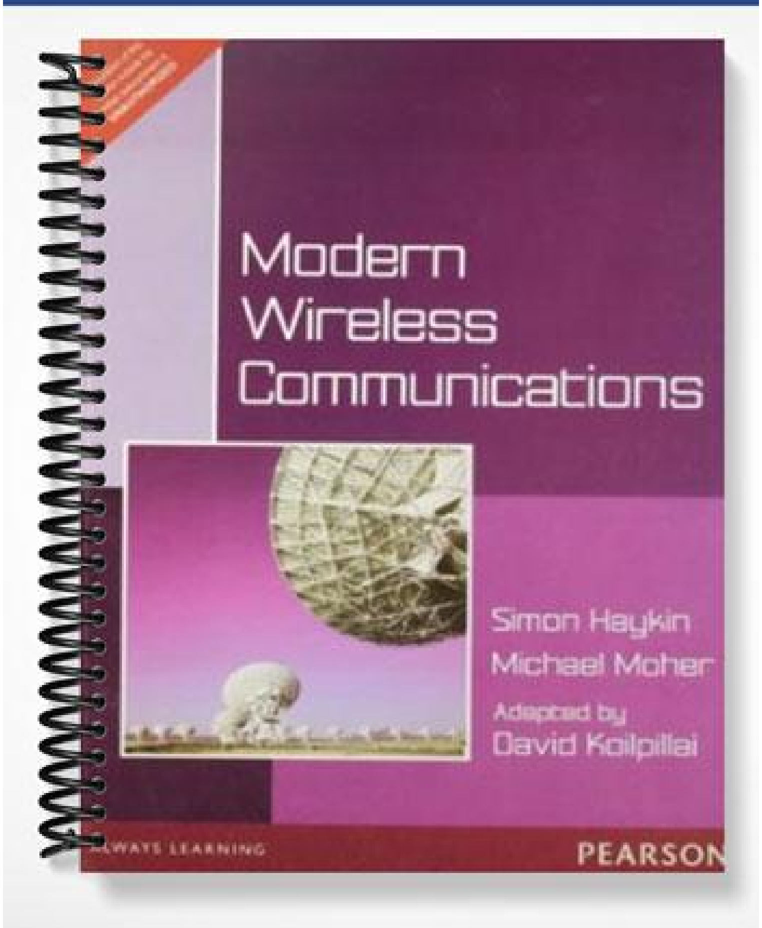
SOLUTIONS MANUAL



CHAPTER 2
Propagation and Noise

Problem 2.1

Early satellite communications systems often used large 20-m diameter parabolic dishes with an efficiency of approximately 60% to receive a signal at 4 GHz. What is the gain of one of these dishes in dB?

Solution

Let D = 20m, η = 60%, and f = 4 GHz. Then from Eq. (2.9)

$$
\begin{aligned}
G &= \eta\left(\frac{\pi D}{\lambda}\right)^2 \\
&= \eta\left(\frac{\pi D f}{c}\right)^2 \\
&= 0.6\left(\frac{\pi \times 20 \times 4 \times 10^9}{3 \times 10^8}\right) \\
&= 4.21 \times 10^5
\end{aligned}
$$

Converting this to decibels

$$
\begin{aligned}
G_{dB} &= 10\log_{10}\left(4.21 \times 10^5\right) \\
&= 56.2 \text{ dB}
\end{aligned}
$$

Problem 2.2

In terrestrial microwave links, line-of-sight transmission limits the separation of transmitters and receivers to about 40 km. If a 100-milliwatt transmitter at 4-GHz is used with transmitting and receiving antennas of 0.5 m^2 effective area, what is the received power level in dBm? If the receiving antenna terminals are matched to a 50 ohm impedance, what voltage would be induced across these terminals by the transmitted signal?

Solution

By the Friis equation Eq.(2.11)

$$
P_R = \frac{G_T G_R P_T}{L_p}
$$

where

$$G_T = G_R = \frac{A_e}{A_{isotropic}}$$

$$= \frac{A_e}{\left(\lambda^2 / 4\pi\right)}$$

$$= 1.12 \times 10^3 \sim 30.5 \text{ dB}$$

$$L_p = \left(\frac{4\pi R}{\lambda}\right)^2 = \left(\frac{4\pi R f}{c}\right)^2$$

$$= 4.50 \times 10^{15} \sim 156.5 \text{ dB}$$

$$P_T = 0.10 \text{ W} \sim 20 \text{ dBm}$$

So using the decibel version of the Friis equation

$$P_R(\text{dBm}) = G_T(\text{dB}) + G_R(\text{dB}) + P_T(\text{dBm}) - L_p(\text{dB})$$

$$= 30.5 + 30.5 + 20 - 156.5$$

$$= -75.5 \text{ dBm}$$

This is equivalent to 28 picowatts. The corresponding rms voltage across a 50-ohm resistor is

$$P_R = \frac{V_{rms}^2}{R} \Rightarrow V_{rms} = \sqrt{P_R R} = 37 \ \mu\text{V}$$

Problem 2.3

Plot and compare the path loss (dB) for the free-space and plane earth models at 800 MHz versus distance on a logarithmic scale for distances from 1 m to 40 km. Assume the antennas are isotropic and have a height of 10 m.

Solution

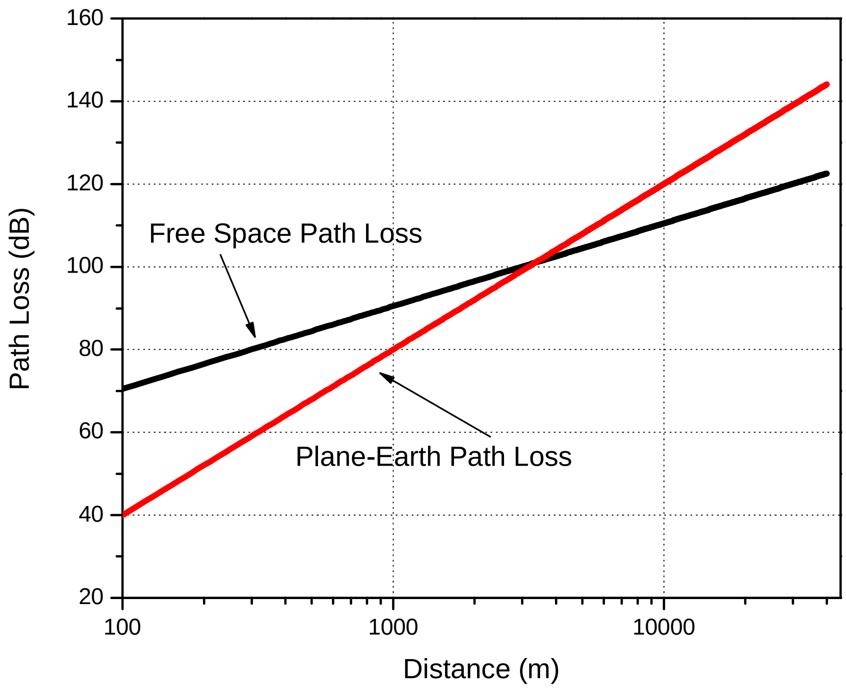
The free-space path loss is

$$L_{space} = \left(\frac{4\pi R}{\lambda}\right)^2$$

and the plane-earth path loss is

$$L_{plane} = \left(\frac{R^2}{h_T h_R}\right)^2$$

These are plotted in the following figure. Note the significantly faster attenuation with the plane-earth model. Note that the plane-earth model applies only for $R >> h_R$, h_T. The plane-earth model shows less loss than free-space at distances less than a kilometer, is this reasonable? How large should R be to apply the plane-Earth model?



Comparison of path losses for Problem 2.3.

Problem 2.4

A company owns two office towers in a city and wants to set up a 4-GHz microwave link between the two towers. The two towers have heights of 100 m and 50 m, respectively, and are separated by 3 km. In the line of sight (LOS) and midway between the two towers is a third tower of height 70 meters. Will line-of-sight transmission be possible between the two towers? Justify your answer. Describe an engineering solution to obtain line-of-sight transmission.

Solution

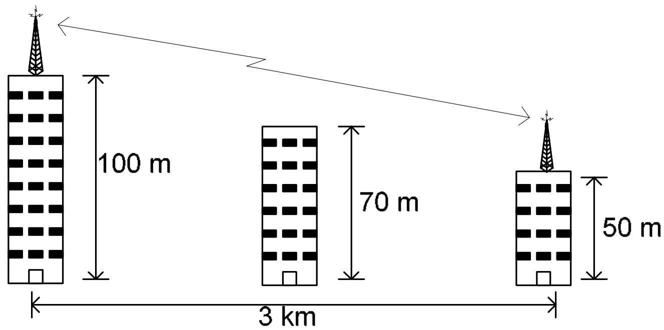
The situation is shown conceptually in the following figure. The radius of the first Fresnel zone is given by Eq. (2.38)

$$\begin{aligned} r_1 &= \sqrt{\frac{\lambda d_1 d_2}{d_1 + d_2}} \\ &= 7.5 \text{ m} \end{aligned}$$

where

$$\lambda = \frac{c}{f} = 0.075 \text{ m}$$

Since the spacing between the centre tower and the line of sight is only 5m (prove using similar triangles), the path does not have a clear first Fresnel zone and some non-line-of-sight effects will be expected. A practical engineering solution would be to raise the height of the antennas on both towers.



Conceptualization of three towers of Problem 2.4.

Problem 2.5

In Problem 2.4, suppose the middle tower was 80 m and the shorter tower was only 30 m. The separation between the two communicating towers is 2 km. What would the increase in path loss be in this case relative to free-space loss? How would the diffraction loss be affected if the transmission frequency is decreased from 4 GHz to 400 MHz?

Solution

Referring to the following figure, the centre office tower now extends 15m above the line of sight. The Fresnel-Kirchhoff diffraction parameter is thus given by

$$\begin{aligned} \nu &= h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}} \\ &= 2.8 \end{aligned}$$

Then from Fig. 2.10 of the text, the corresponding diffraction loss is 22 dB.

If we repeat the calculation at 400 MHz,

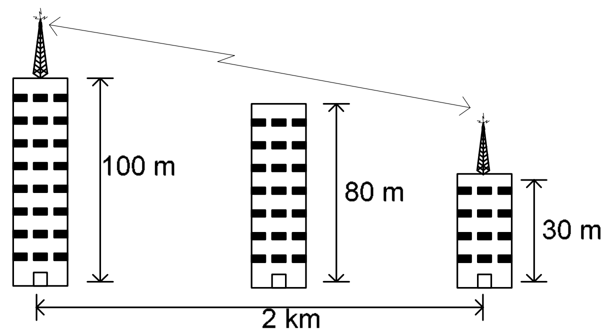
$$\lambda = \frac{3x10^8}{400x10^6} = 0.75 \text{ m}$$

then the Fresnel-Kirchhoff diffraction parameter is

$$\nu = 15\sqrt{\frac{4}{0.75(1500)}} = 0.89\,.$$

and the corresponding diffraction loss is approximately 13 dB.

One must realize that the analysis applies to knife-edge diffraction; in the above example there is liable to be some diffraction of the signal around the sides of the office tower so the above calculations might be treated with some scepticism in practice.



Conceptualization of three towers of Problem 2.5.

Problem 2.6

A brief measurement campaign indicates that the median propagation loss at 420 MHz in a midsize North American city can be modeled with n=2.8 and a fixed loss (β) of 25 dB; that is,

$$L_p = 25\,\text{dB} + 10\log_{10}(r^{2.8})$$

Assuming a cell phone receiver sensitivity of –95 dBm, what transmitter power is required to service a circular area of radius 10 km? Suppose the measurements were optimistic and n = 3.1 is more appropriate, what is the corresponding increase in transmit power that would be required?

Solution

For isotropic antennas, the relationship between transmitted and receive power is

$$\begin{aligned} P_T(dBm) &= P_R(dBm) + L_p(dB) \\ &= -95 + L_p(dB) \end{aligned}$$

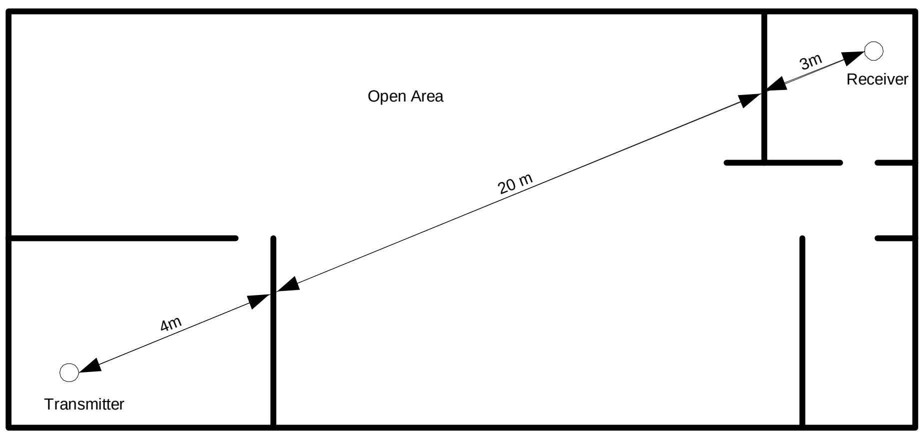
where the second line applies for the above receiver at the edge of coverage (sensitivity threshold). The path loss is

$$\begin{aligned} L_p &= 25 + 28\log_{10} r \\ &= 25 + 28\log_{10}(10^4) \\ &= 137\ \text{dB} \end{aligned}$$

at a distance of 10 km. Consequently, the transmitted power must be 42 dBm or equivalently, 12 dBW. In the second case with n=3.1, the path loss is 149 dB and the transmitted power must by 24 dBW.

Problem 2.7

Using the same model as Example 2.5, predict the path loss for the site geometry shown in the following figure. Assume that the walls cause an attenuation of 5 dB, and floors 10 dB.



Site geometry for Problem 2.7.

Solution

For this scenario we make a link budget as shown below. The total path loss is 87.2 dB and the received power is -67.2 dBm.

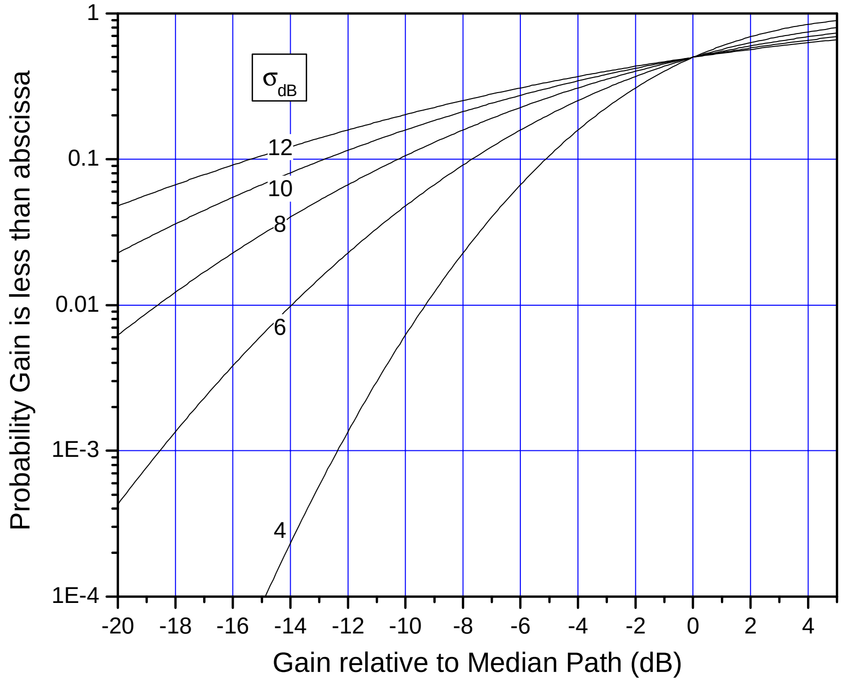
Parameter	Value	Comment
Transmit Power	20 dBm	
Free Space Loss	52.1 dB	$L_p = \left(\frac{4\pi R}{\lambda}\right)^2$
Wall attenuation	5 dB	
Open Area Loss	24.1 dB	$L_p = \left(\frac{24}{4}\right)^{3.1}$
Wall attenuation	5 dB	
Free space loss	1 dB	$L_p = \left(\frac{27}{24}\right)^{2.0}$
Receive Power	-67.2 dBm	

Link budget for Problem 2.7.

An appropriate general question for this type of scenario is when can we assume free space propagation and when should a different model be used? The answer comes from our study of diffraction and Fresnel zones; whenever the first Fresnel zone along the line of sight is unobstructed, it is reasonable to assume free-space propagation.

Problem 2.8
What are the required margins for lognormal and Rayleigh fading in Example 2.6 if the availability requirement is only 90%?

Solution
Due to an unfortunate oversight in the first printing of the text, Fig. 2.11 was wrong. It should have been the following:



Revised Figure 2.11. The lognormal distribution.

With this revised Fig. 2.11 the required margin for log-normal shadowing with $\sigma_{dB} = 6$ is 7.7 dB, and from Fig. 2.14, the required margin for Rayleigh fading is 10 dB.

Problem 2.9 Suppose that the aircraft in Example 2.7 has a satellite receiver operating in the aeronautical mobile-satellite band at 1.5 GHz. What is the Doppler shift observed at this receiver? Assume the geostationary satellite has a 45° elevation with respect to the airport.

Solution
Using Eq. (2.68), the Doppler frequency is

$$\begin{aligned} f_D &= -\frac{f_0}{c} v \cos\alpha \\ &= -\left(\frac{1.5\times10^9}{3\times10^8}\right)\left(\frac{-500}{3.6}\right)\left(\cos 45^\circ\right) \\ &= 491\,\mathrm{Hz} \end{aligned}$$

where v = 500 km/hr.

Problem 2.10

A data signal with a bandwidth of 100 Hz is transmitted at a carrier frequency of 800 MHz. The signal is to be reliably received in vehicles travelling at speeds up to 100 km/hour. What can we say about the minimum bandwidth of the filter at the receiver input?

Solution

The object of this problem is to compute the maximum Doppler shift on the received signal. From Eq. (2.66), this is given by

$$|f_D| = \frac{v}{c} f_0$$

where $f_0 = 800$ MHz. Therefore,

$$\begin{aligned} |f_D| &= \frac{\left(\frac{100}{3.6}\right)}{3\times10^8}\left(8\times10^8\right) \\ &= 74.1\,\text{Hz} \end{aligned}$$

Rounding up, the total bandwidth is the signal bandwidth (100 Hz) plus the maximum Doppler shift (75 Hz). These are the one-sided bandwidths (i.e. the baseband equivalent bandwidth). The RF bandwidth would be twice this.

Problem 2.11

Measurements of a radio channel in the 800 MHz frequency band indicate that the coherence bandwidth is approximately 100 kHz. What is the maximum symbol rate that can be transmitted over this channel that will suffer minimal intersymbol interference?

Solution

From Eq.(2.116), the multipath spread of the channel is approximately

$$\begin{aligned} T_M &\approx \frac{1}{BW_{\text{coh}}} \\ &\approx 10\mu\text{s} \end{aligned}$$

If we assume that spreading of symbol by 10% causes negligible interference into the adjacent symbol, then the maximum symbol period is 100 microseconds. This corresponds to a symbol rate of 10 kHz.

Problem 2.12

Calculate the *rms* delay spread for a HF radio channel for which

$$P(\tau) = .6\delta(\tau) + 0.3\delta(\tau - 0.2) + 0.1\delta(\tau - 0.4)$$

where τ is measured in milliseconds. Assume that signaling with a 5-kHz bandwidth is to use the channel. Will delay spread be a problem, that is, is it likely that some form of compensation (an equalizer) will be necessary?

Solution

From Eq. (2.109) the rms delay spread is given by the square root of μ_2 where

$$\mu_2 = \frac{1}{P_m}\int_0^\infty (\tau - T_D)^2 P_h(\tau)d\tau \tag{1}$$

where the received power is given by Eq. (2.108)

$$\begin{aligned} P_m &= \int_0^\infty P_h(\tau)d\tau \\ &= 0.6 + 0.3 + 0.1 \\ &= 1.0 \end{aligned} \tag{2}$$

and mean delay is given by Eq. (2.107)

$$\begin{aligned} T_D &= \frac{1}{P_m}\int_0^\infty \tau P_h(\tau)d\tau \\ &= 0.6(0) + 0.3(0.2) + 0.1(0.4) \\ &= 0.1\,\text{ms} \end{aligned} \tag{3}$$

Substituting these results in (1), the mean square delay is

$$\begin{aligned} \mu_2 &= (0-0.1)^2 0.6 + (0.2-0.1)^2 0.3 + (0.4-0.1)^2 0.1 \\ &= 0.018\,(\text{ms})^2 \end{aligned} \tag{4}$$

and the corresponding rms delay spread is

$$\begin{aligned} S &= \sqrt{\mu_2} \\ &= 0.134\,\text{ms} \end{aligned} \tag{5}$$

The approximate coherence bandwidth, from Eq. (2.116), is

$$BW_{coh} = \frac{1}{T_m} = \frac{1}{2\sqrt{\mu_2}} = 3.8\,\text{kHz} \tag{6}$$

So some form of equalization will be necessary.

Problem 2.13

Show that the time-varying impulse response of Eq.(2.84) and time-invariant impulse response are related by

$$\tilde{h}_{\text{time-invariant}}(t) = \tilde{h}_{\text{time-varying}}(t,t)$$

Explain, in words, what the preceding equation mean.

Solution

For a time-varying system, the impulse response $h(t,\tau)$ represents the response at time t to an impulse applied at time $t-\tau$. (See Appendix A.2.) The response $h(t,t)$ is therefore the response at time t to an impulse response applied at time zero. Thus, $h(t,t)$ is equivalent to the definition of a *time-invariant* impulse response.

Problem 2.14

What would be the rms voltage observed across a 10-MΩ metallic resistor at room temperature? Suppose the measuring apparatus has a bandwidth of 1 GHz, with an input impedance of 10 MΩ, what voltage would be measured then? Compare your answer with the voltage generated across the antenna terminals by the signal defined in Problem 2.5. Why is the avoidance of large resistors recommended for circuit design? What is the maximum power density (W/Hz) that a thermal resistor therefore delivers to a load?

Solution

Over an infinite bandwidth the rms voltage is given by Eq. (2.117) at 290° K is

$$\begin{aligned}\bar{v}^2 &= \frac{2\pi^2 k^2 T^2}{3h} R \\ &= 1.57\ \text{V}^2 \\ v_{rms} &= 1.3\ \text{volts}\end{aligned}$$

Into a matched load, the noise density due to the resistor is

$$kT = 4\times 10^{-21}\ \text{W/Hz}$$

Over a 1GHz bandwidth, the assicuated power is

$$\begin{aligned}P &= kTB \\ &= 4\times 10^{-12}\ \text{W}\end{aligned}$$

The corresponding rms voltage across a 10 MΩ resistor is

$$\begin{aligned}V &= \sqrt{PR} \\ &= 6\ \text{mV}\end{aligned}$$

In problem 2.5, the voltage across the antenna terminals due to the received signal was 37 μV. Clearly, large resistors have the potential to introduce a lot of noise into the cicuit.

Problem 2.15

The noise figure of a cell phone receiver is specified as 16 dB. What is the equivalent noise temperature? Assume that reliable detection of a 30-kHz FM signal by this receiver requires an SNR of 13 dB. What is the receiver sensitivity in dBm?

Solution

The noise temperature is given by Eq. (2.124)

$$\begin{aligned} T_e &= (F-1)T_o \\ &= 11255^{\circ} K \end{aligned}$$

for nominal temperature T_0=290°K. The receiver sensitivity is (see Example 2.14)

$$\begin{aligned} S &= SNR \times N \\ &= SNR \times kT_e B \\ &\approx 13dB + 10\log(kT_e) + 10\log_{10} B \\ &= -130\text{ dBW} \\ &= -100\text{ dBm} \end{aligned}$$

The receiver sensitivity is -100 dBm.

Problem 2.16

Show that the system temperature, in general, is given by Eq.(2.129).

Solution

We may derive Eq. (2.129) from Eq. (2.127) as follows. Let F_1 represents the combined noise figure of the first amplifier and antenna, then Eq. (2.129) is

$$F_{sys} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots$$

We subtract 1 from both sides and multiply by T_0 to obtain

$$(F_{sys} - 1)T_0 = (F_1 - 1)T_0 + \left(\frac{F_2 - 1}{G_1}\right)T_0 + \left(\frac{F_3 - 1}{G_1 G_2}\right)T_0 + \ldots$$

From the relationship of noise temperature and noise figure given by Eq. (2.124) we have the desired result.

$$T_{sys} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \ldots$$

where T_1 represents the combined noise temperature of the first amplifier and antenna.

Problem 2.17

Microwave ovens operate at a natural frequency of the water molecule at approximately 2.45 GHz. This frequency falls in the middle of a band from 2.41 to 2.48 GHz that has been allocated for low-power unlicensed radio use, including wireless local area networks (sometimes referred to as Wi-Fi – see Section 5.16). The oscillators used in some microwave ovens have poor stability and have been observed to vary ±10 MHz around their nominal frequency. Discuss how this would affect the signal design for use of this band.

Solution

Microwave ovens are well shielded such that very little radiation escapes. However, small amounts do escape; not enough to be a biological hazard but enough that it could interfere with radio systems operating on the same frequencies. Consequently, local WiFi systems should avoid operating in the vicinity. Typically these systems have multiple channels and operation in a channel at the same frequency as the interference should be avoided.

Problem 2.18

The minimum tolerable *C*/*I* ratio depends on the modulation and coding strategy and the quality of service required. Suppose a narrowband digital system requires a *C*/*I* of 12 dB. What would be the maximum frequency reuse factor? If the addition of forward error-correction coding would reduce this to 9 dB without increasing the signal bandwidth, what would be the relative improvement in reuse factor. Assume the propagation loss exponent is 2.6.

Solution

From Eq. (2.135), a lower bound on the frequency reuse factor in a cellular FDMA system is

$$N \geq \frac{1}{3}\left[6\left(\frac{C}{I}\right)_{\min}\right]^{2/n}$$

Substituting the given values we obtain

$$\begin{aligned} N &\geq \frac{1}{3}\left[6\times 10^{12/10}\right]^{2/2.6} \\ &= 11.06 \end{aligned}$$

A reuse factor of $N = 12$ is the smallest one that satisfies this.

Similarly for $C/I = 9\,\text{dB}$

$$\begin{aligned} N &\geq \frac{1}{3}\left[6\times 10^{9/10}\right]^{2/2.6} \\ &= 6.5 \end{aligned}$$

A reuse factor of $N = 7$ satisfies this. Thus, forward error correction coding (see Chapter 4) can significantly improve frequency reuse if the coding can be applied without increasing signal bandwidth.

Problem 2.19

A fixed satellite terminal has a 10-m parabolic dish with 60% efficiency and a system noise temperature of 70°K. Find the *G*/*T* ratio of this terminal at 4 GHz. Suppose it was a terrestrial mobile radio using an omni-directional antenna. What would you expect the equivalent noise temperature of the mobile antenna to be?

Solution

For the satellite terminal the antenna gain is

$$G = \eta\left(\frac{\pi D}{\lambda}\right)^2$$
$$= \eta\left(\frac{\pi D}{c/f}\right)^2$$
$$= 0.6\left(\frac{\pi 10}{c/4\times 10^9}\right)^2$$
$$= 1.05\times 10^5$$

The corresponding *G*/*T* is $10\log_{10}(G/T) = 31.8$ dBK^{-1} for a system noise temperature of 70°K. A satellite antenna points skyward ideally and is only marginally influenced by the Earth. The majority of the noise is caused by the receiver. The noise temperature of the sky is typically a few degrees Kelvin, if the antenna is not looking directly at the sun. On the other hand, an omni-directional antenna will "see" the Earth and have a noise temperature that is at least that of the Earth (290°K); this is noise introduced by the environment.

Problem 2.20

For a geostationary satellite at altitude h (36000 km), determine a formula relating the range r from the satellite to an earth station to the satellite elevation ϕ relative to the earth station. (Let R_e =6400 km be the radius of the Earth.)

Solution

Using the cosine law for triangles as applied to the diagram below, we have

$$(h+R_e)^2 = r^2 + R_e^2 - 2rR_e\cos(90° + \phi)$$

Rearranging this as a quadratic equation in r gives

$$r^2 + 2rR_e\sin\phi + \left(R_e^2 - (h+R_e)^2\right) = 0$$

and solving for r, one obtains

$$r = \frac{-2R_e \sin\phi \pm \sqrt{4R_e^2 \sin^2\phi - 4\left(R_e^2 - (h+R_e)^2\right)}}{2}$$
$$= -R_e \sin\phi \pm \sqrt{R_e^2 \sin^2\phi - R_e^2 + (h+R_e)^2}$$
$$= -R_e \sin\phi \pm \sqrt{(h+R_e)^2 - R_e^2 \cos^2\phi}$$

Taking the positive square root gives the range[1]

$$r = -R_e \sin\phi + \sqrt{(h+R_e)^2 - R_e{}^2 \cos^2\phi}$$

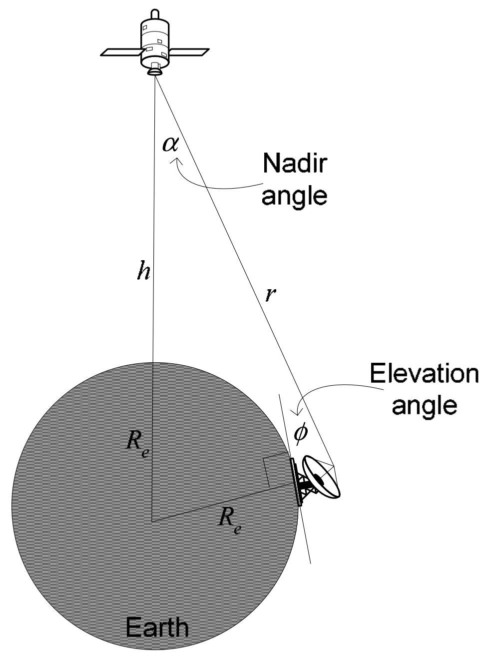


Diagram for Problem 2.20.

Problem 2.21

Numerous other quantities may be included in the satellite link budget. For example if the satellite amplifier (which is typically nonlinear) is shared with a number of carriers, then *intermodulation distortion* will be generated. When there are a large number of equal-power carriers present, intermodulation distortion can be modeled as white noise with spectral density I_o. This will produce a term C/I_o at the satellite that must be combined with the uplink and downlink C/N_o to produce the overall $C/(N_o+I_o)$. Given the uplink and downlink C/N_o of Examples 2.18 and 2.19, what is the overall $C/(N_o+I_o)$ if C/I_o = 50 dB-Hz?

[1] Note that the answer given in the textbook is in error.

Solution

The initial objective is to compute the combination of the thermal noise N_0 and the intermodulation noise I_0, given by $N_0 + I_0$. Since these are not known directly, we compute the scaled equivalent.

$$\frac{N_0 + I_0}{C} = \frac{N_0}{C} + \frac{I_0}{C}$$

Inverting this equation gives

$$\frac{C}{N_0 + I_0} = \left[\left(\frac{C}{N_0} \right)^{-1} + \left(\frac{C}{I_0} \right)^{-1} \right]^{-1}$$

From example 2.19, the overall C/N_o is 47.1 dBHz. Consequently,

$$\begin{aligned} \frac{C}{N_0 + I_0} &= \left[\left(\frac{C}{N_0} \right)^{-1} + \left(\frac{C}{I_0} \right)^{-1} \right]^{-1} \\ &= \left[10^{-4.71} + 10^{-5} \right]^{-1} \\ &= 3.39 \times 10^{4} \end{aligned}$$

where the second line is a conversion from decibels to absolute. Converting the answer to decibels we obtain $C/(N_0+I_0) = 45.3$ dBHz.

<u>Problem 2.22</u>

Repeat the link budget of Table 2.5, analyzing the performance in the city core. Assume that the maximum range within the city core is 2 km, but that the path-loss exponent is 3.5 and the log-normal shadowing deviation is 10 dB. Is this service limited by the receiver sensitivity? What is the expected service availability for the city core?

Solution[2]

In the following link budget, we compute the received signal power based on the propagation losses and the margin required for shadowing. When we compared the received signal power to the signal power required by the modem, we see there is a shortfall of 14.2 dB. This means the system is unable to provide the full 16.5 dB shadowing margin. The shadowing margin is only 2.3 dB. Looking at the revised Fig. 2.11 (see Problem 2.8), this implies that the availability is only 60% in the city core.

The required C/N_0 of the modem and the noise figure of the receiver shown in this table are parameters provided to the propagation analysis.

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[2] There is an error in the answer given in the text.

Parameter	Units		Comments
Base station transmitter			
Transmit Frequency	MHz	705	Mobile public safety band
Tx Power	dBW	15	
Tx Antenna Gain	dBi	2	Uniform radiation in azimuth
Tx EIRP	dBW	17	Maximum EIRP of 30 dBW
Power at 1m (P_0)	dBm	17.6	$P_o = P_T/(4\pi/\lambda)^2$
Losses			
Path loss exponent		3.5	Applicable at edge of coverage
Range (r)	km	2	Range at edge of coverage
Median Path Loss	dB	115	$3.5\text{x}10\log_{10}(r/r_o)$
Log-normal shadowing	dB	10	standard deviation of log-normal shadowing
Shadowing margin	dB	16.5	for 95% availability ($1.65\sigma_{dB}$)
Rx Signal			
Rx Antenna Gain (G_n)	dBi	1.5	Vertically polarized whip antenna
Rx signal strength	dBm	-112.4	$P_R = P_o + G_R - 21\log_{10}(r/r_o)\ - M_{shadow}$
Receiver characteristics			
Required C/N_o	dB-Hz	69.8	from modem characteristics
Boltzmann's constant	dBm-K	-198.6	
Rx Noise Figure	dB	6.0	provided
Rx sensitivity	dBm	-98.2	$S = C/N_o\ + NF +\ kT_o$
Margin	dB	-14.2	$Margin = P_R$ - S

Link budget for Problem 2.22

Problem 2.23

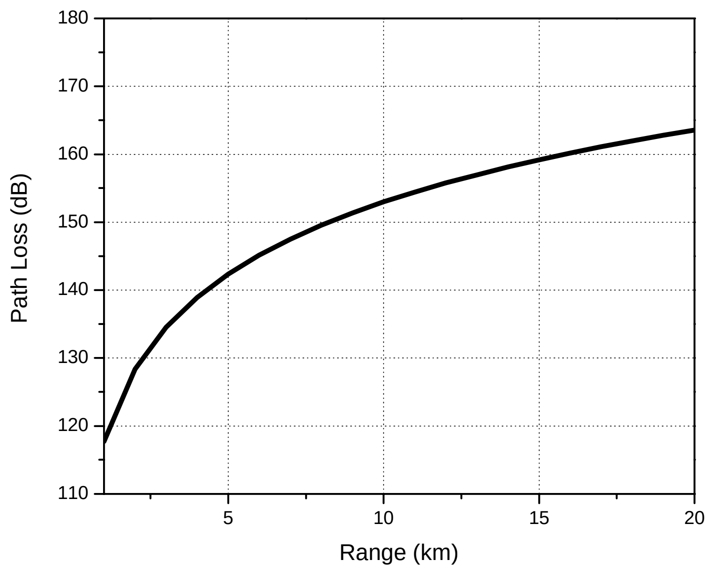
Evaluate the path loss at 900 MHz using the Okumura-Hata model for the suburban environment. Assume that the base and mobile station antenna heights are 30 m and 1 m, respectively.

Solution

From Eq. (2.141), the path loss for a suburban environment is given by

$$L_p = A + B\log_{10} r - C \tag{1}$$

Where A, B and C are given by Eq.(2.142). For a medium-sized city the correction factor for mobile antenna height is given by Eq. (2.145). Substituting these in Eq. (1) and plotting the result we obtain the following figure.



Path loss at 900 MHz in a suburban environment for Problem 2.23

Problem 2.24

The 54 Mbps service of IEEE 802.11a uses 64-quadrature-amplitude-modulation (QAM). (This method of modulation is considered in Chapter 3) Suppose the practical E_b/N_o required to achieve a bit error rate of 10^{-6} with 64 QAM is 30 dB. For this data rate, what is the sensitivity of the receiver just discussed?

Solution

Using Eq. (2.149), the sensitivity is given by

$$
\begin{aligned}
S &= \frac{E_s}{N_0} + R_b + N_0 \\
&= 30\ \text{dB} + 10\log\left(54\times 10^6\right) + (-164) \\
&= -56.7\ \text{dBm}
\end{aligned}
$$

With a 3 dB implementation loss, the sensitivity decreases to -53.7 dBm.

Problem 2.25

From the results of Problem 2.24, what is the maximum range expected with the 54-Mbps service?

Solution

At the receiver threshold, with a 200 milliwatt transmitter (23 dBm), the range is determined from Eq. (2.151)

$$
\begin{aligned}
31\log_{10} r &= P_T - P_R - 41 \\
&= 23 - (-53.7) - 41 \\
&= 35.7
\end{aligned}
$$

Solving this for r, one obtains a range of 14 m.

Problem 2.26

Consider a communications link with a geostationary satellite such that the transmitter-receiver separation is 40,000 km. Assume the same transmitter and receiver characteristics as described in Problem 2.2. What is the received power level in dBm? What implications does this power level have on the receiver design? With a land-mobile satellite terminal, the typical antenna gain is 10 dB or less. What does this imply about the data rates that may be supported by such a link?

Solution

The received power is given by Eq. (2.12)

$$P_R(dB) = P_T(dB) + G_R(dB) + G_T(dB) - L_p(dB) \tag{1}$$

with a transmit power of 100 milliwatts (20dBm), a frequency of 4 GHz, and an antenna G_R of 30.5 dB as determined from Problem 2.2. The path loss is

$$\begin{aligned} L_p &= \left(\frac{4\pi r}{\lambda}\right)^2 \\ &= \left(\frac{4\pi 4\times10^6}{c/f}\right)^2 \\ &= 4.5\times10^{19} \ \ (196.5\,\text{dB}) \end{aligned} \tag{2}$$

Combining these observations in Eq.(1), the received power is

$$\begin{aligned} P_R(\text{dBm}) &= 20\,\text{dBm} + 30.5\,\text{dB} + 30.5\,\text{dB} - 196.5\,\text{dB} \\ &= -115.5\,\text{dBm} \end{aligned} \tag{3}$$

For the land-mobile satellite terminal, if the receiver antenna has a G_R of 10 dB then the received power will be

$$\begin{aligned} P_R(\text{dBm}) &= 20\,\text{dBm} + 10\,\text{dB} + 30.5\,\text{dB} - 196.5\,\text{dB} \\ &= -136\,\text{dBm} \end{aligned} \tag{4}$$

Therefore the receiver has to be very sensitive with very little noise. With a low gain antenna, the data rates would also have to be low.

Problem 2.27

Consider a 10-watt transmitter communicating with a mobile receiver having a sensitivity of –100 dBm. Assume that the receiver antenna height is 2 m, and the transmitter and receiver antenna gains are 1 dB. What height of base station antenna would be necessary to provide a service area of radius 10 km? If the receiver is mobile, and the maximum radiated power is restricted by regulation to be 10 watts or less, what realistic options are there for increasing the service area?

Solution

Using the plane-earth model, Eq. (2.30)

$$P_R = P_T G_T G_R \left(\frac{h_T h_R}{R^2} \right)^2$$

and isolating h_T,

$$\begin{aligned} h_T &= \left(\frac{R^2}{h_R} \right) \sqrt{\left(\frac{P_R}{P_T G_T G_R} \right)} \\ &= \left(\frac{10000^2}{2} \right) \sqrt{\left(\frac{1 \times 10^{-13}}{(10)(1.26)(1.26)} \right)} \\ &= 4 \text{ m} \end{aligned}$$

where -100 dBm is equivalent to 10^{-13} watts.

Therefore, the plane earth model indicates the base station antenna would have to be 4 meters in height. Service area may be increased either by improving receiver sensitivity, or boosting the transmitter antenna height, or increasing antenna gain.

Realistically, a 4-meter antenna would be unlikely to provide a line of sight path over a distance of 10 km, thus the plane-earth model would not be applicable. The student is invited to solve this problem using the Okamura-Hata model assuming a transmission frequency of 400 MHz.

Problem 2.28

In Problem 2.26, the satellite-receiver separation was 40,000 km. Assume the altitude of a geostationary satellite was said to be 36,000 km. What is the elevation angle from the receiver to the satellite in Problem 2.26? What was the increased path loss, in decibels, relative to a receiver where the satellite is in the *zenith* position (directly overhead)? If the transmitter-receiver separation in Problem 2.2 had been 20 km, what would the path loss have been? What can be said about comparing the dB path losses in satellite and terrestrial scenarios as a function of absolute distance?

Solution

From Problem 2.20 the range is given by

$$r = -R_e \sin\phi + \sqrt{(R_e + h)^2 - R_e^2 \cos^2\phi}$$

$$(r + R_e \sin\phi)^2 = (R_e + h)^2 - R_e^2 \cos^2\phi$$

$$r^2 + 2rR_e \sin\phi + R_e^2 \sin^2\phi + R_e^2 \cos^2\phi = (R_e + h)^2$$

$$\sin\phi = \frac{h^2 + 2hR_e - r^2}{2rR_e}$$

$$\phi = \sin^{-1}\left(\frac{36000^2 + 2(36000)(6400) - 40000^2}{2(40000)(6400)}\right)$$

$$\phi = 17.8^\circ$$

Therefore the elevation angle is 17.8 degrees.

The increase in path loss relative to the zenith position is

$$\frac{R_{\text{elevation}}^2}{R_{\text{zenith}}^2} = \frac{L_{p\,(\text{elevation})}}{L_{p\,(\text{zenith})}}$$

Expressed in dB, the ratio becomes

$$\begin{aligned}\Delta L_p &= 20\log_{10}\left(\frac{R_{\text{elevation}}}{R_{\text{zenith}}}\right)\\ &= 20\log_{10}\left(\frac{40000}{36000}\right)\\ &= 0.92\ \text{dB}\end{aligned}$$

With the free-space model for propagation, the difference between 40000 km and 20 km is

$$\begin{aligned}&= 20\log_{10}\left(\frac{40000}{20}\right)\\ &= 33\ \text{dB}\end{aligned}$$

The propagation losses are greater in satellite scenarios, but the variation in propagation losses are significantly less.

Problem 2.29

Suppose that, by law, a service operator is not allowed to radiate more than 30 watts of power. From the plane-Earth model, what antenna height is required for a service radius of 1 km? 10 km? Assume the receiver sensitivity is -100 dBm.

Solution

Using the plane earth model for propagation from Eq. (2.30)

$$P_R = P_T G_T G_R \left(\frac{h_T h_R}{R^2} \right)^2 \qquad (1)$$

Assume the transmit and receiver antenna gains are 0 dB, and for a mobile service make the reasonable assumption that the mobile antenna has a height of about 1m.

Solving (1) for h_T

$$h_T = \frac{R^2}{h_R} \sqrt{\frac{P_R}{P_T G_R G_T}}$$

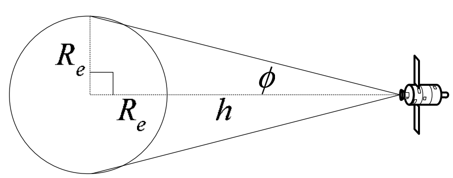
For R = 1 km, h_T = 0.058 m; this is clearly not realistic as this antenna would not have a clear line-of-sight to the receiver. For R = 10km, h_T = 5.8 m; this is a somewhat more realistic antenna height but it is still questionable whether the plane-earth model applies in this situation. The student is invited to solve this problem using the Okamura-Hata model at 800 MHz.

Problem 2.30

A satellite is in a geosynchronous orbit (i.e. an orbit in which the satellite appears to be fixed relative to the Earth). The satellite must be at an altitude of 36,000 km above the equator to achieve this synchrony. Such a satellite may have a global beam that illuminates all of the Earth in view. What is the approximate 3-dB beamwidth of this global beam if the radius of the earth is 6400 km. What is the antenna gain?

Solution

We assume the 3 dB beam width subtends the Earth, as shown in the following figure.



$$\tan\phi = \left(\frac{R_e}{R_e + h} \right)$$

$$\phi = 8.6^\circ$$

Therefore the 3dB beamwidth is 2ϕ, or 17.2 degrees.

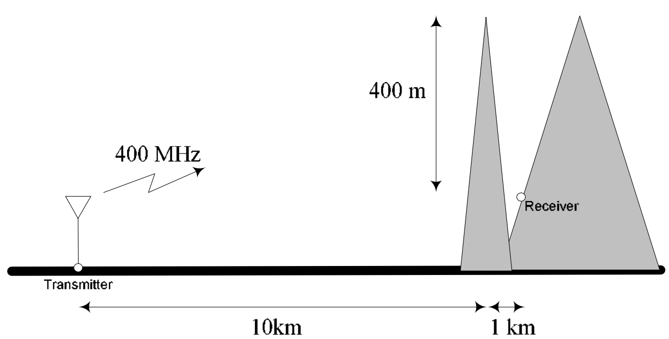
The antenna gain is proportional to the ratio of the power flux density produced by the satellite at the Earth to the power flux density over a sphere at the same distance. Then

$$
\begin{aligned}
G_T &= \frac{\Phi_{Earth}}{\Phi_{sphere}} \\
&\approx \frac{\left(\dfrac{P}{\text{cross - sectional area of Earth}}\right)}{\left(\dfrac{P}{\text{area of sphere}}\right)} \\
&= \frac{\left(\dfrac{P}{\pi R_e{}^2}\right)}{\left(\dfrac{P}{4\pi(R_e+h)^2}\right)} \\
&= 175.6
\end{aligned}
$$

When expressed in decibels, the satellite antenna gain is approximately 23 dB.

Problem 2.31

A 100-m base station tower is located on a plateau as depicted in Fig. 1. In one direction there is a valley bounded by two (nonreflective) mountain ranges. If the transmit power is 10 watts, then, assuming ideal diffraction losses, what is the expected received signal strength. Assume free-space loss over the plains and a transmission frequency of 400 MHz.



Site geometry for Problem 2.31.

Solution

We model this problem as a case of knife-edge diffraction. In the absence of the mountain range, the free space loss is

$$L_p = \left(\frac{4\pi R}{\lambda} \right)^2$$

$$L_p = \left(\frac{4\pi(10000)}{\frac{c}{400\times 10^6}} \right)^2$$

$$L_p = 2.8\times 10^{10}$$

Expressed in dB, the free space path loss is 104.5 dB.

The peak that is 400m above the line of sight path implies a Fresnel-Kirchhoff parameter of

$$\begin{aligned} \nu &= h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}} \\ &= (400)\sqrt{\frac{2(10000+1000)}{(10000)(1000)\left(\frac{c}{400\times 10^6}\right)}} \\ &= 21.7 \end{aligned}$$

From Fig. 2.10, we can see that the diffraction parameter is off the scale, and may conclude that the receiver is effectively blocked.

Now consider the case where the first mountain range extends just 50 meters above the receiver. In this case the diffraction parameter becomes $\nu = 2.7$, and the corresponding diffraction loss, (using Fig. 2.10) is approximately 22 dB.

Using the Friis equation Eq. (2.12), the received power is

$$\begin{aligned} P_R(\text{dBm}) &= 40\text{ dBm} + 0\text{ dB} + 0\text{ dB} - 104.5\text{ dB} - 22\text{ dB} \\ &= -86.5\text{ dBm} \end{aligned}$$

Problem 2.32

In Problem 2.6, the estimates were based on median path loss. In the same city, the deviation about the median-path loss was estimated as σ_{dB} = 8 dB. Assuming the log-normal model, how much additional power must be transmitted to cover the same service area with 90% *availability* at the edge of coverage when local shadowing is taken into account?

Solution

From Fig. 2.11 the cumulative log-normal distribution implies just over 10 dB of margin must be included to provide 90% availability. (From the Gaussian distribution with standard deviation σ_{dB}, a 90% margin requires $1.29\sigma_{\text{dB}}$ margin.)

Problem 2.33

In Problem 2.7, we assume that the path-loss exponent depends on distance. For the first 10 m, the path loss exponent is 2, while beyond this it is 3.5. What is the expected path loss in this case?

Solution

Parameter	Value	Comments
Transmit Frequency	2.4 GHz	
Transmit Power	20 dBm	
Free-space loss	52.1 dB	$L_p = \left(\frac{4\pi R}{\lambda}\right)^2$ over 4 m
Wall attenuation	5 dB	
Open area loss 1	8.0 dB	$L_p = (r/r_o)^2$ with r/r_o=10/4
Open area loss 2	13.3 dB	$L_p = (r/r_o)^{3.5}$ with r/r_o=24/10
Wall attenuation	5 dB	
Free space loss	1 dB	$L_p = (27/24)^{3.5}$
Received power	-64.4 dBm	assuming 0dB Rx antenna gain

Link budget for Problem 2.33.

Problem 2.34

Suppose satellites are placed in geosynchronous orbit around the equator at 3° intervals. An Earth station with a 1-m parabolic antenna transmits 100 watts of power at 4 GHz towards the intended satellite. How much power is be radiated toward the adjacent satellites on either side of the intended satellite? What implications does this amount of power have about low-power and high-power signals on adjacent satellites? What does it imply about the diameter of earth station antennas?

Solution

From Fig. 2.4 the gain of the parabolic antenna at 3° off-centre is 10dB below the on-axis gain. Consequently, about 10 watts of power would be radiated toward the adjacent satellite. This implies that a high power signal on one satellite could cause significant interference on an adjacent satellite. This can be remedied if the high power signal uses a larger antenna which focuses the beam more.

Problem 2.35

Prove the Rayleigh density function is given by Eq. (2.55). (*Hint*: Let $Z_i = R\cos\theta$ and $Z_j = R\sin\theta$.)

Solution

The cumulative distribution of the Rayleigh random variable is given by

$$P(r < R) = \iint\limits_{\sqrt{z_i^2+z_r^2}<R} f(z_i) f(z_r) dz_i dz_r$$

where z_i and z_j are Gaussian random variables with common density $f(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z}{2\sigma^2}}$.

Since these random variables are assumed independent, let $z_r = R\cos\theta$ and $z_i = R\sin\theta$, then in polar co-ordinates this becomes

$$P(r < R, \theta < \Theta) = \int_{r<R} \int_0^{\Theta} \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^2 e^{-\frac{r^2}{2\sigma^2}} r dr d\theta$$

with $rdrd\theta$ replacing $dz_i dz_r$. Integrating with respect to θ over (0,2π) gives

$$P(r < R) = \int_{r<R} \frac{1}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} rdr$$
$$= 1 - e^{-\frac{R^2}{2\sigma^2}}$$

This is the cumulative distribution of the Rayleigh random variable. The Rayleigh density function is given by the derivative of this.

$$f_R(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$

Problem 2.36

Suppose the aircraft in Example 2.7 was accelerating at a rate of 0.25 g (i.e., approximately 2.5 *m*/s^2). What would the frequency slew rate be?

Solution

The Doppler frequency is given by

$$f_D = -\frac{v}{c} f_0$$

Hence Doppler slew rate is

$$\frac{df_D}{dt} = -\frac{f_0}{c}\frac{dv}{dt}$$

For the Example 2.7, the transmission frequency is 128 MHz, thus the Doppler slew rate is

$$\frac{df_D}{dt} = -\frac{128\times10^6}{c}(2.5)$$
$$\frac{df_D}{dt} = -1.07 \text{ Hz/s}$$

Problem 2.37

Suppose an aircraft is at an altitude of 4000m in a circular holding pattern above an airport. The radius of the holding pattern is 4 km. If the aircraft speed is 400 km/hr, what is the frequency slew rate, df_r/dt (Hz/s), that an aircraft receiver must be capable of tracking. Assume the transmitter is located at ground level at the centre of the holding pattern. Aircraft safety communications use the VHF frequency band from 118 to 130 MHz.

Solution

In such a situation, the aircraft has no radial velocity with respect to the transmitter so there is no Doppler.

Problem 2.38

A common device used in digital communications over fading channels is an *interleaver*, which is discussed in detail in Chapter 4. It boosts the performance of many forward error-correction codes. An interleaver takes a block of data from the encoder and permutes the order of the bits before transmitting them. At the receiver, the inverse operation, de-interleaving of the bits, is applied before the bits are decoded. The objective of the de-interleaver is to make the fading on adjacent bits (as seen by the decoder) appear uncorrelated. What is the ideal (minimum) spacing of bits in an interleaver to achieve this objective for a terminal moving at 100 km/hr and transmitting at 800 MHz?

Solution

Using the Clarke model for fading, the signal correlation with time is given by Eq. (2.75) and is shown in Fig. 2.19. Looking at Fig. 2.19 we find the signal envelope is first uncorrelated at time offset $\omega_d \tau = 2.2$ or $\tau = \frac{2.2}{2\pi f_D}$. To compute this time, we note that the Doppler frequency under the given conditions is

$$f_D = -\frac{v}{c} f_0$$

therefore the decorrelation time is

$$\tau = -\frac{2.2c}{2\pi f_0 v} = 4.72 \text{ ms}$$

Therefore the interleaver should separate adjacent bits by at least 4.72 ms.

Problem 2.39

From the development of Section 2.6, we could construct a model to simulate a Rayleigh fading process and the effects of fast fading. This model is illustrated below. The outputs of two independent white Gaussian noise generators are added in quadrature and then processed with a filter that approximates the desired fading spectrum. In practice, these are often done digitally. Since the fading process is frequently much slower than desired signalling rates, the spectrum-shaping filter may be very narrowband, requiring many interpolation stages to model accurately in the digital domain. Develop a MATLAB program to generate a Rayleigh fast fading signal.

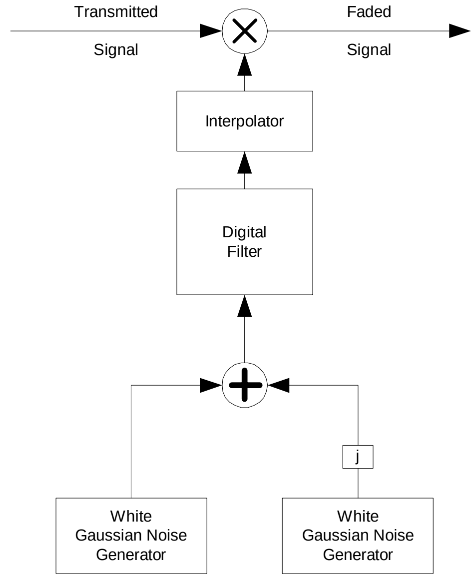


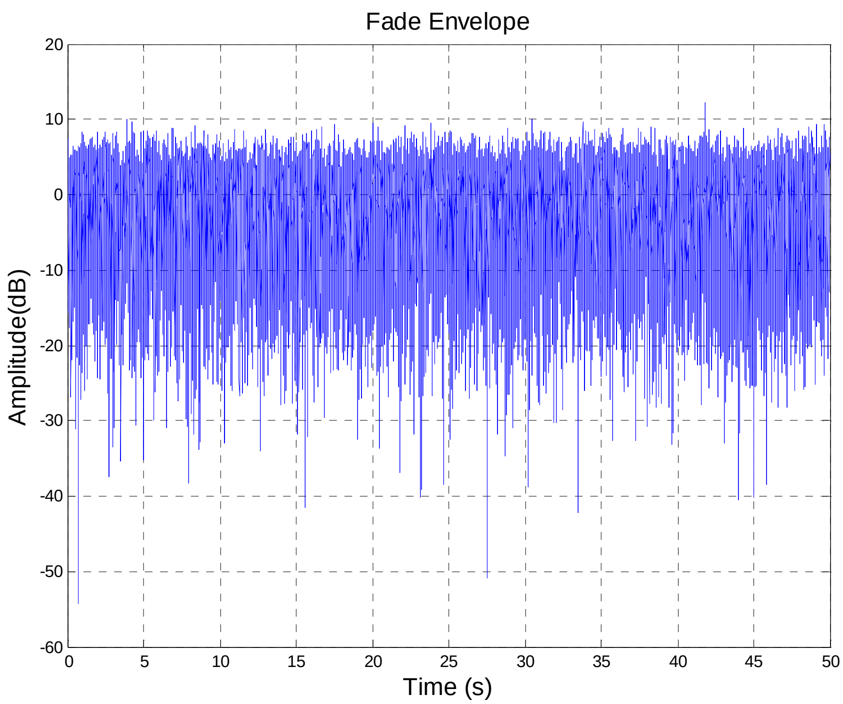
Illustration of digital model for simulating Rayleigh fading for Problem 2.39.

Solution

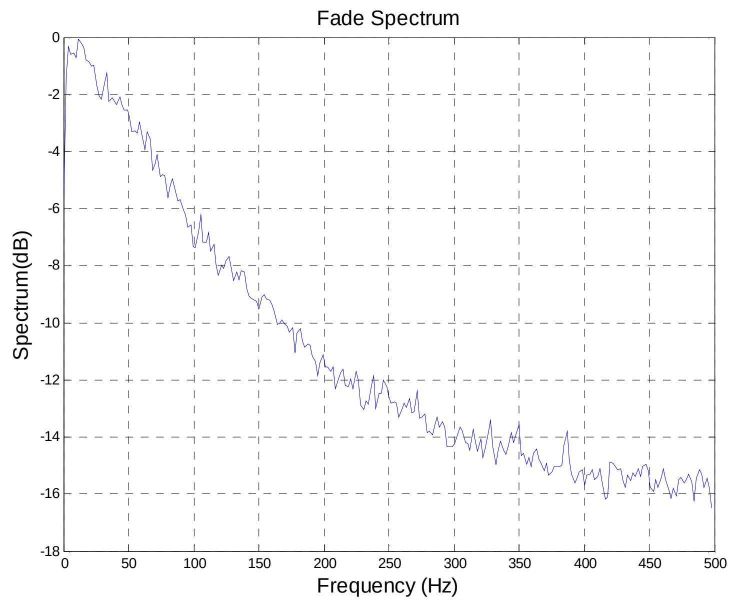
```
%-------------------------------------------------------------
% Problem 2.39  Function to generate Rayleigh fading signal
%-------------------------------------------------------------
function c=Prob2_39;

%set parameters
Fs = 1000;          % sampling rate
nSamples = 50000;    % number of fading samples

Fade = Rayleigh(nSamples);
Fade = Fade/std(Fade);     % normalize rms fade

%--- plot envelope of fading ---------------
figure(1);
sig_envelope = 20*log10(abs(Fade));
plot([1:nSamples]/Fs,sig_envelope);
grid;
title('Fade Envelope');
xlabel('Time (s)')
ylabel('Amplitude(dB)')

%--- plot spectrum of fading ---------------
figure(2);
nFFT   = 512;
[P,F]  = spectrum(Fade,nFFT,nFFT/4,nFFT,Fs);
P(:,1) = P(:,1)/max(P(:,1));  % normalize spectrum
plot(F(1:nFFT/2),10*log10(P(1:nFFT/2,1)))
grid;
title('Fade Spectrum');
xlabel('Frequency (Hz)')
ylabel('Spectrum(dB)')
```

```
%--- Helper functions ------------------
function Fade = Rayleigh(nSamples)
% input parameters
% nSamples - Number of gaussian samples to generate

%--- Define filter
a = [1 -0.7];
b = 0.3;

%generate complex white Gaussian noise samples
GaussianSamples = randn(1,nSamples) + j*randn(1,nSamples);

%filter Gaussian samples
Fade = filter(b,a,GaussianSamples);
```

The fading envelope is show in the figure below.



The spectrum of the fading signal is shown below.



Problem 2.40

Generate a Rayleigh distribution with a 100-Hz fading bandwidth. A suggested approximation is to filter white Gaussian noise samples at 1 kHz with the one-pole filter $y_n = 0.7y_{n-1}+0.3u_n$. For the output samples,

a. plot the distribution of output samples. Does this change with fading rate?
b. plot the spectrum of the output samples. What is the 3-dB fading bandwidth?
c. how much of the time is signal -4dB or lower? How does this time compare with theoretical values?
d. determine the average 3-dB fade duration. That is, each time the signal drops 3 dB below average, how long does it stay 3 dB or more below average?

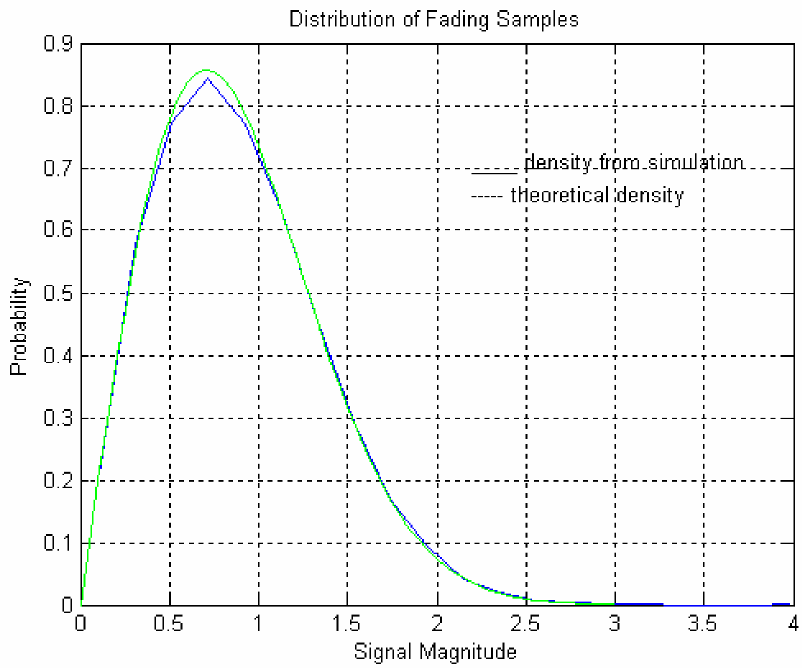
Solution

The MATLAB script is provided below. It uses the Rayleigh function defined in the MATLAB script of Problem 2.39.

```
%-------------------------------------
% Problem 2.40
%-------------------------------------
function c = Prob2_40;
%set parameters
Fs = 1000;          % sample rate
nSamples = 50000;   % number of fading samples

Fade = Rayleigh(nSamples);
Fade = Fade/std(Fade);    % normalize rms fade

%--- Compute distribution of envelope -----------
[N,X] = hist(abs(Fade),20);
N     = N/length(Fade);
DelX  = X(2)-X(1);
figure(1);
plot(X,N/DelX);
grid on;
title('Distribution of Fading Samples');
xlabel('Signal Magnitude');
```

```
ylabel('Probability');
%---- plot theoretical distribution ----
hold on
x   = [0:0.05:3];
pdf = 2*x.*exp(-x.^2);     % normalize Rayleigh pdf with sigma^2 = 0.5
plot(x,pdf,'g-')
hold off

% Determine amount of time signal remains at or below -4dB
%------------------------------------------------------
Sig_envelope = 20*log10(abs(Fade));
time_below   = find(Sig_envelope <= -4);
TimeLess_4dB = (length(time_below)/length(Fade))*100

% Average 3dB fade duration
%------------------------------------------------------
nFade = 0;
kk = 1;
while(kk < length(Sig_envelope)-4)  % leave some margin at end of signal
    if (Sig_envelope(kk)<-3)
       nFade = nFade + 1;
       FadeDur(nFade) = 0;
       while(Sig_envelope(kk) < -3)
          FadeDur(nFade) = FadeDur(nFade)+1;
          kk = kk + 1;
       end
   end
   kk = kk+1;
end

Avg3dBFadeDuration = mean(FadeDur) / Fs  % in seconds
```

The distribution of the output samples is shown below.



The spectrum of the output samples is shown in problem 2.39. From the MATLAB script we see that the Rayleigh signal extends below -4dB approximately 33% of the time, and that the average -3 dB duration is 2.2 ms.

Problem 2.41

Plot the frequency response of the channel $\tilde{h}(t) = \delta(t)-\alpha_2\delta(t-\tau)$ for $\alpha_2 = 0.3j$ and $\tau = 10$ microseconds. From the results obtained, what is the approximate maximum bandwidth over which the channel could be considered frequency-flat?

Solution

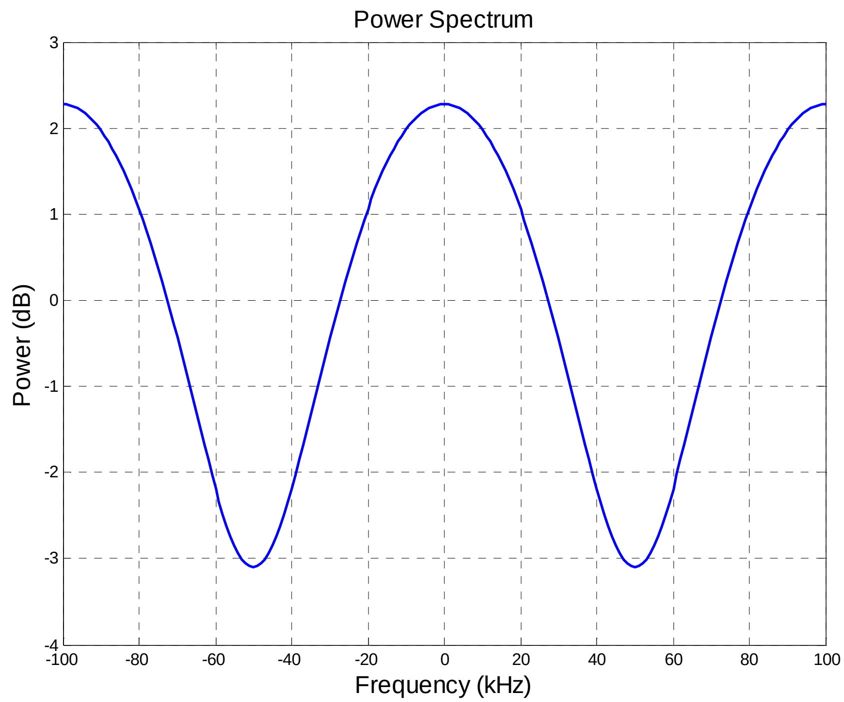
The frequency response for the impulse response is given by its Fourier transform:

$$H(\omega) = \int_{\infty}^{\infty} [\delta(t) - \alpha_2\delta(t-\tau)]e^{j\omega t}\,dt.$$
$$= 1 - \alpha_2 e^{j\omega\tau}$$

Solving for the frequency response, we get

$$\begin{aligned}
|H(2\pi f)|^2 &= H(2\pi f)H^*(2\pi f) \\
&= (1 - 0.3e^{j(\tau(2\pi f)+\pi)})(1 - 0.3e^{-j(\tau(2\pi f)+\pi)}) \\
&= 1.09 - 0.3\left(e^{j(\tau(2\pi f)+\pi)} + e^{-j(\tau(2\pi f)+\pi)}\right) \\
&= 1.09 - 0.6\cos(\tau(2\pi f) + \pi)
\end{aligned}$$

For $\tau = 1\text{x}10^{-5}$, we get the magnitude response shown below. The channel could be considered approximately flat (less than 1dB variation) over 10 kHz.



Magnitude response for Problem 2.41.

Problem 2.42

Assume that a terrestrial radio receiver has a sensitivity of –100 dBm. What other information is necessary to compare this receiver to a satellite receiver that has a *G*/*T* of –22 dB/K.

Solution

The noise figure of the receiver and the antenna gain are required.

Problem 2.43

Suppose the receiver in Problem 2.42 has a noise figure of 6 dB and IF bandwidth of 30 kHz. What is its equivalent *G*/*T* ratio?

Solution

The information is still incomplete, however the system temperature is

$$\begin{aligned} T_e &= (F-1)T_0 \\ &= (10^{0.6}-1)290 \\ &= 865^\circ \text{K} \end{aligned}$$

If one assumes a 0 dB antenna gain, the corresponding *G*/*T* in decibels is

$$10\log_{10}\left(1/865\right) = -29 \text{ dB/K}$$

Problem 2.44

The receiver sensitivity of a commercial GPS receiver is -130 dBm. If the receiver should function within 100 m of a 738-MHz base station transmitting 30 Watts of power, how much attenuation of the second harmonic of the transmitted signal is required. Assume free-space loss between the transmitter and the GPS receiver, and that the GPS receiver requires a carrier-to-interference ratio, *C/I,* of 10 dB.

Solution

If the receiver *C/I* tolerance is 10dB, then the interference must be –140 dBm or less at 100 m. Using the Friis equation and assuming isotropic antennas, then in decibels,

$$P_R = P_T - L_P$$

and

$$\begin{aligned} L_P &= 10\log_{10}\left(\frac{4\pi R}{\lambda}\right)^2 \\ &= 10\log_{10}\left(\frac{4\pi(100)}{\frac{c}{738\times 10^6}}\right)^2 \\ &= 70\text{ dB} \end{aligned}$$

Thus the transmitted power in the second harmonic must be less than

$$\begin{aligned} P_T &\le P_R + L_P \\ &\le -140\text{ dBm} + 70\text{ dB} \\ &= \text{-70 dBm} \end{aligned}$$

Considering that the transmitted power at 738 MHz is equal to 30 Watts or 45 dBm, the relative attenuation of the second harmonic must be $45\text{ dBm} - (-70\text{ dBm}) = 115\text{ dB}$.

Problem 2.45

Suppose a laboratory spectrum analyzer has noise figure of 25 dB. What N_o would you expect to see when measuring a noiseless signal?

Solution

The noise density is

$$\begin{aligned} N_0 &= kT_0F \\ &= -174\text{ dBm/Hz} + 25\text{ dB} \\ &= -149\text{ dBm/Hz} \end{aligned}$$

Problem 2.46

A satellite antenna is installed on the tail of an aircraft and connected to the receiver in an equipment bay located behind the cockpit. The antenna has a noise temperature of 50°K, and the transmission cable connecting the antenna to the low-noise amplifier (LNA) in the equipment bay has a loss of 7 dB. The LNA has a gain of 60 dB and an equivalent noise temperature of 70°K. A secondary gain stage with 40 dB gain and a noise temperature of 1500°K follows. What is the noise temperature of the overall system? Where would be a better location for the LNA? What is the noise temperature of the new system?

Solution

The system noise temperature is given by Eq. (2.129)

$$T_{sys} = T_A + T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \ldots.$$

In this case, the cable adds little additional noise (T_1=0) but since it attenuates the signal, it effectively amplifies the noise of the LNA. That is

$$\begin{aligned} T_{sys} &= 50° + 0 + \frac{70}{0.2} + \frac{1500}{(0.2)(10^6)} + \ldots.. \\ &= 400°K \end{aligned}$$

The LNA would be better positioned next to the antenna. In this case

$$\begin{aligned} T_{sys} &= 50° + 70° + 0 + \frac{1500}{(10^6)(0.2)} \\ &\approx 120\text{ °K} \end{aligned}$$

Problem 2.47

A satellite is in a geosynchronous orbit at an altitude of 36,000 km. The satellite has three spotbeams that illuminate approximately one-third of the visible Earth each. What is the approximate 3-dB beamwidth of the satellite spotbeam if the radius of the Earth is 6400 km. What is the antenna gain? Since the average temperature of the Earth is 290°K, what is the satellite G/T ratio?

Solution

Assuming the beam configuration as shown in above, the approximate diameter of a beam is, R_e, the Earth's radius. Using the approach of Problem 2.30, the half angle subtended by the spotbeam at the satellite is obtained from

$$\tan\phi = \left(\frac{\frac{R_e}{2}}{\frac{R_e}{2} + h} \right)$$

Solving this expression for ϕ gives, $\phi = 4.7^\circ$. Thus, the 3 dB beamwidth of the spotbeam is 9.4 degrees.

As shown in Problem 2.30, the antenna gain is approximately the ratio of the area of a sphere to the area of spotbeam at the same radial distance.

$$G = \frac{4\pi(R_e + h)^2}{\pi\left(\frac{R_e}{2}\right)^2} = 702$$

The satellite G/T ratio is then given by

$$G/T(dB) = 10\log\left(\frac{702}{290}\right) = 3.84 \text{ dB/K}$$

This assumes that the Earth (i.e. antenna noise) is the dominant noise source.

Problem 2.48

A handheld radio has small omni-directional antenna. The first amplifier stage of the radio provides a 20-dB gain and has a noise temperature of 1200°K. The second amplifier stage provides another 20-dB gain and has a noise temperature of 3500°K. What is the combined noise figure of the antenna and first two stages of the radio? If the baseband processing of the radio requires an SNR of 9 dB in 5 kHz, what is the receiver sensitivity?

Solution

$$\begin{aligned} T_{sys} &= T_A + T_1 + \frac{T_2}{G_1} \\ &= 290 + 1200 + \frac{3500}{100} \\ &= 1525\ ^\circ\text{K} \end{aligned}$$

where we assume the antenna temperature is approximately that of the Earth. The corresponding noise figure is

$$\begin{aligned} F &= 1 + \frac{T_{sys}}{T_0} \\ &= 1 + \frac{1525}{290} \\ &= 6.3 \end{aligned}$$

Expressed in decibels, this noise figure is 7.9 dB.

The receiver sensitivity is given by Eq. (2.140) but it is expressed in another form Example 2.14, that is,

$$S(\text{dBm}) = SNR(\text{dB}) + N(\text{dBm})$$

We note that $N = FkTB$ is a measure of the noise in the same bandwidth B. So

$$\begin{aligned} N(\text{dBm}) &= kT(\text{dBm/Hz}) + F(\text{dB}) + B(\text{dBHz}) \\ &= -174 + 7.9 + 10\log_{10}(5\,\text{kHz}) \\ &= -129\,\text{dBm} \end{aligned}$$

and the sensitivity is

$$\begin{aligned} S(\text{dBm}) &= SNR(\text{dB}) + N(\text{dBm}) \\ &= 9 - 129 \\ &= -120\,\text{dBm} \end{aligned}$$

<u>Problem 2.49</u>

When a directional antenna is pointing toward the empty sky, the antenna noise temperature falls to about 3°K at frequencies between 1 GHz and 10 GHz. The 3°K temperature represents the residual background radiation of the universe. Cosmological theory suggests that it is a remnant of the initial Big Bang. If the antenna is connected directly to a low-noise amplifier with a gain of 60 dB and a noise temperature of 100°K, what is system noise temperature? What is the equivalent noise figure for the system?

Solution

The system noise temperature is

$$\begin{aligned} T_{sys} &= T_A + T_1 \\ &= 103° \end{aligned}$$

The equivalent noise figure is

$$F = 1 + \frac{T_{sys}}{T_0}$$
$$= 1 + \frac{103}{290}$$
$$= 1.36$$

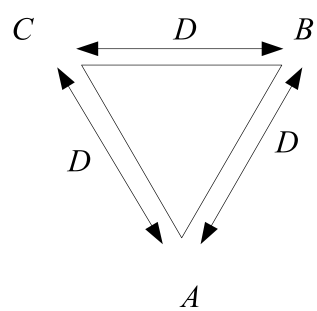
With satellite systems, the noise figure is often close to 1, so these systems are usually specified in terms of their noise temperature rather than noise figure in order to provide better resolution.

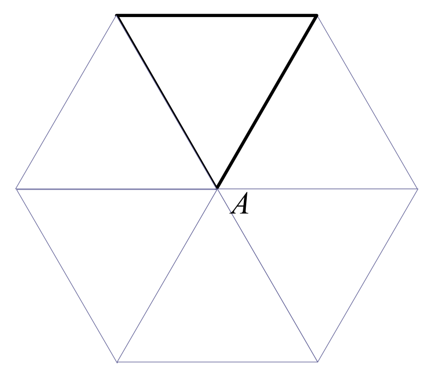
Problem 2.50

Prove that with a 1-in-*N* reuse pattern, the number of cochannel cells at the minimum distance is six.

Solution

Let *D* be the frequency re-use distance and consider three cell centres that are separated by this distance as shown in the left hand figure below. It should be clear that this is the closest spacing of three such cells, and that the three cell centres form an equilateral triangle. Let *A* be the central cell and note that the interior angle of an equilateral triangle is 60°. Then it follows there is a maximum of six non-overlapping triangles with *A* as the vertex as shown in the right hand figure below. Consequently, the number of co-channel cells at the minimum distance is six.





Problem 2.51

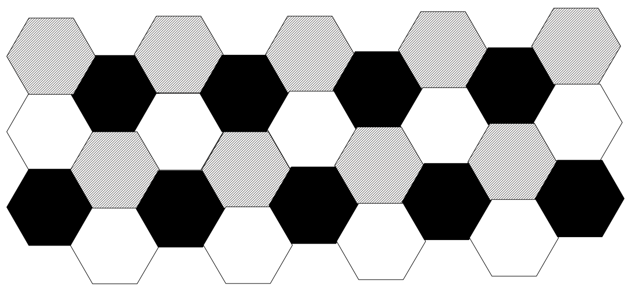
Consider a dense urban environment in which the path-loss exponent is 4.5 and the required *C*/*I* is 13 dB. What reuse factor is permissible? If the cell size is reduced to increase capacity, what other changes would also be required?

Solution

From Eq. (2.135), the frequency re-use factor is

$$
\begin{aligned}
N &\geq \frac{1}{3}\left[6\left(\frac{C}{I}\right)_{\min}\right]^{2/n} \\
&\geq \frac{1}{3}\left[6\left(10^{13/10}\right)\right]^{2/4.5} \\
&\geq 2.8
\end{aligned}
$$

The lowest re-use factor consistent with this is 3, (for i = 1, and j = 1 in $N = i^2 + ij + j^2$). This reuse pattern is illustrated below.



Note how poor propagation characteristics may permit greater frequency reuse (if they can be assumed to be uniform across the service area).

Problem 2.52

Consider the previous problem, 2.51, but in a rolling rural environment with a path-loss exponent of 3. What reuse factor is permissible? What are disadvantages and advantages related to the path-loss exponent?

Solution

In the rural environment the reuse factor is given by

$$
\begin{aligned}
N &\geq \frac{1}{3}\left[6\left(\frac{C}{I}\right)_{\min}\right]^{2/n} \\
&\geq \frac{1}{3}\left[6\left(10^{13/10}\right)\right]^{2/3} \\
&\geq 8.1
\end{aligned}
$$

The lowest reuse factor consistent with this is 9, (for i = 3, and j = 0 in $N = i^2 + ij + j^2$). Better propagation conditions clearly reduce the frequency reuse. Fortunately, the user density tends to be less under these better conditions.

Problem 2.53

Suppose that a particular 100-kbps service requires a bit error rate (BER) of 10^{-5} in AWGN. Could this service be supported by the link budget described in Table 2.3? Justify your answer.

Solution

The C/N_0 produced by Table 2.3 is 52.6 dBHz. The corresponding *SNR* for R = 100 kbps is $52.6 - 10\log_{10} R = 2.6$. This is a very low SNR and unlikely to support a low BER without some very good forward error correction coding.

Problem 2.54

What is the power flux density on the surface of the Earth for Example 2.17, assuming the satellite delivers 50 watts to the antenna terminals. How much power is collected by a 1.8 meter antenna with 50% efficiency? What voltage level would be generated across a 50-ohm resistor by that amount power? What is the equivalent rms thermal noise voltage of this resistor in a 10-kHz bandwidth?

Solution

From Example 2.17, the 50 watts of power would be spread over an approximate area of πR_e^2, so the flux density is

$$\begin{aligned}\Phi &= \frac{50}{\pi R_e^2} \\ &= 0.4\,\mathrm{pW/m^2}\end{aligned}$$

The power collected by a 1.8 meter antenna is, from Eq. (2.2)

$$\begin{aligned}P &= \eta A \Phi \\ &= \eta\left(\frac{\pi D^2}{4}\right)\Phi \\ &= 0.5\,\mathrm{pW}\end{aligned}$$

From the power relation $P = \frac{V_{rms}^2}{R}$, the rms voltage this received power would create across 50 Ω is

$$\begin{aligned}V_{rms} &= \sqrt{PR} \\ &= 5\,\mu\mathrm{V}\end{aligned}$$

To compute the noise voltage, recall that $kT_0 = -174\,\mathrm{dBm/Hz}$, and the equivalent thermal noise in 10 kHz is $N = kT_0 B$ or in decibels

$$\begin{aligned}N &= kT_0 + 10\log_{10} B \\ &= -134\,\mathrm{dBm} \\ &= \text{-}164\,\mathrm{dBW} \\ &\approx 39.8\times 10^{\text{-}18}\ \mathrm{W}\end{aligned}$$

The corresponding noise voltage of the 50 Ω resistor is

$$
\begin{aligned}
V_{noise} &= \sqrt{NR} \\
&= 44 \text{ nV}
\end{aligned}
$$

Problem 2.55[3]

Construct a link budget for the return link from mobile to base of a commercial cell phone that transmits at most 600 milliwatts. Find the range of the service for the propagation models shown in Table 2.6, assuming that a service availability of 95% is required. Assume that the base station receiver noise figure is 3 dB and the required SNR is 8 dB for a 9.6-kbps service.

n	***β*(dB)**	***σ*_{dB}**
2	40	8
3	20	8
3.5	10	10
4	6	12

Propagation models for Problem 2.55.

How do these results change if the required availability is 90%? What additional things could be done at the base station to help close the return link at greater distances?

Solution

The first step is to determine the receiver sensitivity. From Example 2.14, we know that sensitivity can be expressed as

$$S(\text{dBm}) = SNR(\text{dB}) + N(\text{dBm}) \tag{1}$$

where

$$
\begin{aligned}
N(\text{dBm}) &= kT(\text{dBm/Hz}) + F(\text{dB}) + B(\text{dBHz}) \\
&= -174 + 3 + 10\log_{10}(9600) \\
&= -131 \text{ dBm}
\end{aligned} \tag{2}
$$

Consequently, substituting this result in Eq.(1) gives a sensitivity of -123 dBm.

The second step is to use the link budget equation, Eq.(2.12), and apply it in the case where the received power is equal to the sensitivity plus the shadowing margin

$$P_R(dBm) = P_T(dBm) + G_T(dB) + G_L(dB) - L_p(dB) \tag{3}$$

Isolating the path loss in this equation we obtain

[3] Note there was a typo in Table 2.6 in the first printing of the text.

$$
\begin{aligned}
L_p &= P_T - P_R + G_T + G_R \\
&= 28 - (-123 + M_{shadow}) + 0 + 0 \\
&= 154 \text{ dB} - M_{shadow}
\end{aligned} \tag{4}
$$

where we have assumed a transmit and receive antenna gain of 0 dB, and used the fact that the transmit power is 600 milliwatts (28 dBm). M_{shadow} is the shadowing margin.

In the third step, we use the propagation result, Eq.(2.44), for the median path loss

$$
L_p(dB) = \beta_0(dB) + 10n\log_{10}(r/r_0) \tag{5}
$$

and isolate the range, r,

$$
\log_{10}(r/r_0) = \frac{L_p - \beta_0}{10n} \tag{6}
$$

The ranges are computed in the following table assuming r_0 is 1 meter, and using the shadowing losses computed according: 95% availability implies $1.65\sigma_{\text{dB}}$ margin and 90% availability implies $1.29\sigma_{\text{dB}}$ margin. Clearly, the first case is very similar to free-space propagation.

n	***β(dB)***	***σ_{dB}***	***M_{shadow}(dB) (95%)***	***Range (r) (km)***	***M_{shadow}(dB) (90%)***	***Range(r) (km)***
2	40	8	13.2	109.6	10.3	153.1
3	20	8	13.2	10.6	10.3	13.3
3.5	10	10	16.5	4.4	12.9	5.6
4	6	12	19.8	1.6	15.5	2.1

Propagation results for Problem 2.55.

Problem 2.56

Consider the design of a radio-controlled model airplane with a maximum range of 300 m. The receiver requires a C/N_o ratio of 47 dB-Hz. Due to poor isolation from the aircraft engine, the receiver has a noise figure of 22 dB. What EIRP would have to be transmitted to achieve the maximum range? Assume line-of-sight transmission at 45 MHz, and assume that transmit and receive antennas have gains of –3 dB relative to an isotropic antenna.

Solution

The first step is to compute the receiver sensitivity. From Eq.(2.149) noting that $C=E_bR_b$, we have

$$
\begin{aligned}
S(\text{dBm}) &= C/N_0(\text{dBHz}) + N_0(\text{dBm/Hz}) \\
&= 47 + 10\log 10(kT_0F) \\
&= -105 \text{ dBm}
\end{aligned}
$$

With free-space propagation, the path loss over 300 meters is determined from Eq.(2.6)

$$
\begin{aligned}
L_p &= \left(\frac{4\pi R}{\lambda}\right)^2 \\
&= \left(\frac{4\pi Rf}{c}\right)^2 \\
&= 3.2x10^5 \quad (\sim 55\,\mathrm{dB})
\end{aligned}
$$

with f = 45 MHz. The maximum transmit power then determined by setting the received power to the threshold in the link budget equation, Eq.(2.12)

$$
\begin{aligned}
&P_R(dBm) = P_T(dBm) + G_T(dB) + G_L(dB) - L_p(dB) \\
&-105 = P_T - 3 - 3 - 55
\end{aligned}
$$

Isolating P_T, we obtain

$$P_T = -44 \text{ dBm}$$

The transmit EIRP is P_T+G_T = -47 dBm, a very small amount of power.

Problem 2.57

A typical 1.5 Volt "AA" cell has 1.5 ampere-hours (Ah) of energy. Assuming that a 3-V transmitter in the model airplane of the Problem 2.56 has a 50% efficiency in converting input energy into EIRP, how long would a pair of batteries last? How long would they last if the transmitter had a 10% duty cycle?

Solution

Assuming continuous transmission, then transmitter consumes -44 dBm assuming a 50% efficiency of converting input power to EIRP. This corresponds to 0.04 microwatts. The pair of batteries provides 3x1.5 = 4.5 watt-hours of energy. Consequently, the lifetime of the pair of batteries is determined by

$$4.5(\text{watt} - \text{hours}) = 4\times10^{-8}(\text{watts})\times T(\text{hours})$$

Solving for T gives a lifetime of 112 million hours. In practice, such a system would likely be designed with tens of dBs of margin, and transmit powers on the order of milliwatts are more likely. However, this still implies that batteries would degrade faster due to aging than due to actual usage.

Problem 2.58

You are a delegate to an ITU-R meeting representing your country. Your country's objective is have the frequency band from 1910 to 1930 MHz designated worldwide for personal communication services (PCS). You have prepared submissions detailing the suitability of this band for this application, bearing in mind propagation characteristics, technology, and the need for the designation of this band, based on commercial projections of growth in the PCS market. The proposal has support from a number of participants at the meeting, but a number of countries that have existing fixed microwave services operating in the band from 1910 to 1920 MHz object to the proposal. Suggest a compromise proposal that may achieve your long-term objectives.

Solution

The PCS network could be designated as a secondary user of the 1910-1920 MHz band while the existing microwave network could be classified as the primary user. With this arrangement, the PCS network might use very small cells so that the interference levels to the primary user were kept low. In the long term, it may be possible to shift the operating band of the current microwave network to an unused location.

Problem 2.59

When G. Marconi made the first radio transmission in 1899 across the Atlantic Ocean, he used all of the spectrum available worldwide to transmit a few bits per second. It has been suggested that, in the period since then, spectrum usage (bits/s/Hz worldwide) has increased by a factor of a million. List the factors that have resulted in this substantial increase. Which factor will likely result in the largest increase in the future?

Solution

The following factors are mainly responsible for the increase in spectrum usage.

Factor	**Comment**
Antenna gain	More directionality means more frequency reuse.
Modulation	Increased spectral efficiency leads to a more compact spectrum and less interference with adjacent channel users. Constant envelope schemes are immune to amplifier nonlinearities, therefore less spectral regrowth.
Filters	Match filtering improvements: allow for steeper band rolloffs and a more compact spectral shape
Oscillators	Crystal oscillators allow operation in the high GHz range. Stability and accuracy of oscillators also help improve compactness of spectrum.
Semiconductor technology	Improved switching speeds and good immunity to RFI and EMI. Circuit design methods have improved, thus reducing susceptibility to interference.
Digital processing	Advanced algorithms have been developed to track phase and gain with great efficiency resulting in low implementation losses. Also, advanced algorithms allow operation in environments with high levels of interference; thereby increasing the amount of frequency reuse.
Coding	Error correcting codes such as Viterbi and Turbo Codes allow large improvements in spectral efficiency. Receivers that use error coding can operate at lower power and in the presence of more interference than those receivers without error correcting codes.

Many of the above factors have permitted spectral reuse at different frequencies (frequency reuse) and by sharing spectrum in time (temporal reuse) and by reusing spectrum in different areas (spatial reuse). These are the subthemes of this book and it is likely the spatial reuse of spectrum that will see the greatest growth in the near future.

Problem 2.60

In Example 2.23, to what would the transmit power need to be reduced in order to produce an interference level equivalent to the noise floor of the 800 MHz receiver? If the transmit power remains at one milliwatt, what is the minimum separation of the impulse radio from the 800 MHz radio to produce equivalent interference and noise floor densities? Assume free-space propagation conditions.

Solution

Using Eq. (2.159), the noise floor of the 800 MHz radio, with a 10 dB noise figure is

$$\begin{aligned} N_0 &= kTF \\ &\approx -174\ \text{dBm/Hz} + 10\ \text{dB} \\ &= \text{-}164\ \text{dBm/Hz} \end{aligned}$$

Let the interference density at the narrowband receiver input, J_0, equal the 800 MHz receiver noise floor, N_0, and solve for I_0.

$$\begin{aligned} J_0 &= N_0 \\ &= -164\ \text{dBm/Hz} \\ &= I_0 - L_P \end{aligned}$$

where I_0 is the narrowband interference density of the impulse radio, and L_P is the free-space path loss at 10 meters (L_P = 50.5 dB).

Solving for I_0,

$$\begin{aligned} I_0 &= J_0 + L_P \\ &= -164\ \text{dBm/Hz} + 50.5\ \text{dB} \\ &= -113.5\ \text{dBm/Hz} \end{aligned}$$

Since the average power of the impulse radio is spread over a bandwidth of 1 GHz, we may calculate the peak power of the impulse radio by solving

$$\begin{aligned} P_I &= I_0(\text{dB}) + B(\text{dB}) + 10\log_{10}(10^9) \\ &= -23.5\ \text{dBm} \\ &\sim 4.5\ \mu\text{W} \end{aligned}$$

Thus the maximum power that may be transmitted from the impulse radio, if the interference level is to equal the narrowband receiver's noise floor, is approximately 4.5 microwatts.

If the power level of the impulse radio remains at 1 milliwatt, then the minimum separation between the impulse radio and narrowband receiver may be determined by calculating the minimum required path loss.

$$
\begin{aligned}
J_0 &= N_0 \\
&= -164\ \text{dBm/Hz} \\
&= I_0 - L_P \\
&= -90\ \text{dBm/Hz} - L_P
\end{aligned}
$$

Solving for L_P, we get $L_P = 74\text{dB}$.

Then, rearranging Eq. (2.157), and solving for R, we get

$$
\begin{aligned}
R &= \frac{c\sqrt{L_P}}{4\pi f} \\
&= \frac{c\sqrt{10^{7.4}}}{4\pi\left(800\times 10^{6}\right)} \\
&= 149.5\ \text{m}
\end{aligned}
$$

So the impulse radio would have to be located at least 150 meters from the narrowband receiver if the impulse radio is to transmit at 1 milliwatt, and cause interference at a level similar to the receiver's noise floor.