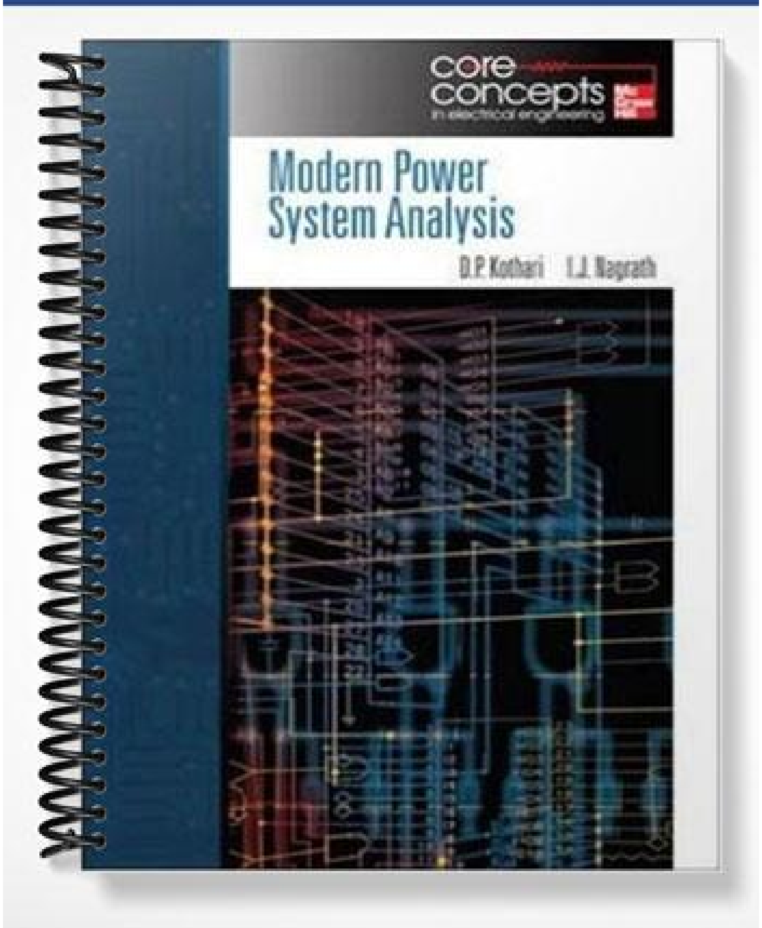
SOLUTIONS MANUAL



SOLUTIONS MANUAL TO ACCOMPANY

MODERN POWER SYSTEM ANALYSIS

3rd Edition

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SOLUTIONS
Chapter 2

2.1

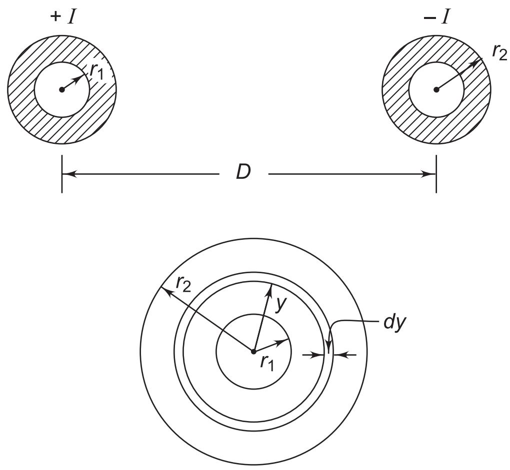

Fig. S-2.1

Assume uniform current density

$$2\pi y\ H_y = I_y$$

$$I_y = \left(\frac{y^2 - r_1^2}{r_1^2 - r_2^2}\right) I$$

$$\therefore \qquad H_y = \left(\frac{y^2 - r_1^2}{r_2^2 - r_1^2}\right) \times \frac{1}{2\pi y}\ I$$

$$d\phi = \mu\ H_y\ d_y$$

$$d\lambda = \left(\frac{y^2 - r_1^2}{r_2^2 - r_1^2}\right) I d\phi$$

$$= \mu \left(\frac{y^2 - r_1^2}{r_2^2 - r_1^2}\right)^2 \frac{I}{2\pi y}\ dy$$

$$= \frac{\mu I}{2\pi} \times \frac{y^3 - 2r_1^2\ y + r_1^4/y}{\left(r_2^2 - r_1^2\right)^2}\ dy$$

Integrating

$$\lambda_{\text{int}} = \frac{\mu I}{2\pi\,(r_2^2 - r_1^2)^2} \int_{r_1}^{r_2} \left[y^3 - 2r_1^2 y + r_1^4/y\right] dy$$

$$= \frac{\mu I}{2\pi\,(r_2^2 - r_1^2)^2} \left\{ \frac{y^4}{4}\Bigg|_{r_1}^{r_2} - r_1^2 y^2\Big|_{r_1}^{r_2} + r_1^4 \ln y\Big|_{r_1}^{r_2} \right\}$$

$$= \frac{\mu I}{2\pi (r_2^2 - r_1^2)^2} \left\{ \frac{1}{4} (r_2^4 - r_1^4) - r_1^2 (r_2^2 - r_1^2) + r_1^4 \ln \frac{r_2}{r_1} \right\}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m} \qquad \mu_r = 1$$

$$L_{\text{int}} = \frac{\frac{1}{2} \times 10^{-7}}{\left(r_2^2 - r_1^2\right)^2} \left[\left(r_2^4 - r_1^4) - 4r_1^2 \left(r_2^2 - r_1^2\right) + 4r_1^4 \ln \frac{r_2}{r_1} \right) \right]$$

$$L_{\text{ext}} (1) = 2 \times 10^{-7} \ln \frac{D}{r_2} = L_{\text{ext}} (2); \text{ assuming } D >> r_2$$

Line inductance = 2 (L_{int} + L_{ext} (1)) H/m.

2.2.

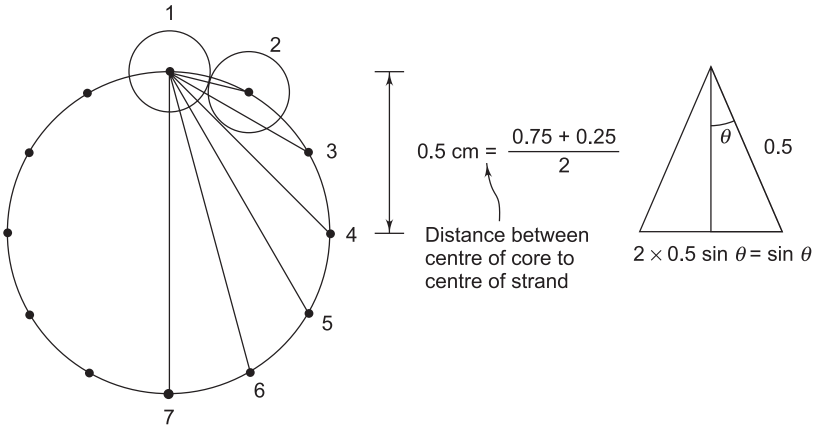

Fig. S-2.2

Diameter of nonconducting core = 1.25 – 2 × (0.25) = 0.75 cm

Note: Core is nonconducting.

D_{12} = sin 15º = 0.259 cm D_{13} = sin 30º = 0.5 cm

D_{14} = sin 45º = 0.707 cm D_{15} = sin 60º = 0.866 cm

D_{16} = sin 75º = 0.965 cm D_{17} = sin 90º = 1.0 cm

$D_{11} = r' = (0.25/2) \times 0.7788 = 0.097$ cm

$$D_s = \{(0.097 \times 1) \times (0.259)^2 \times (0.5)^2 \times (0.707)^2 \times (0.866)^2 \times (0.965)^2\}^{1/12}$$

$= 0.536$ cm

$D_m \approx 1$ m

$$L = 2 \times 0.461 \log \frac{100}{0.536} = 2.094 \text{ mH/km}$$

$$X = 314 \times 2.094 \times 10^{-3} = \mathbf{0.658\ \Omega/km}$$

2.3 $H_y = I/2\pi y$

$$d\phi = \frac{\mu I}{2\pi y}\,dy$$

$$d\lambda = 1 \times d\phi = \frac{\mu I}{2\pi y}\,dy$$

$$\lambda = \frac{\mu}{2\pi} I \int_r^R \frac{dy}{y} = \mu \frac{I}{2\pi} \ln \frac{R}{r}$$

$$L = \frac{\mu}{2\pi} \ln \frac{R}{r} \text{ H/m}$$

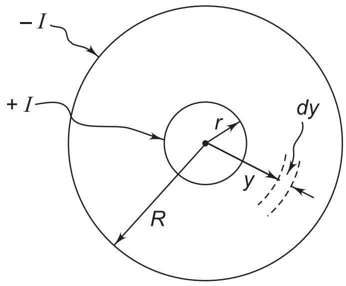

Fig. S-2.3

2.4 Flux linkage of sheath loop due to cable current = $2 \times 2 \times 10^{-7} \times 800 \times \ln \frac{0.5 \times 200}{7.5}$ Wb-T/m

Voltage induced in sheath = $314 \times 0.32 \ln \frac{100}{7.5}$ V/km

= **260.3 V/km**

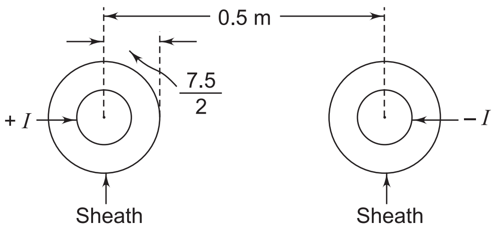

Fig. S-2.4

2.5 $H_P = \frac{I}{2\pi \times 3d} - \frac{I}{2\pi\, d} = \frac{I}{2\pi d}\left(\frac{1}{3} - 1\right) = -\frac{I}{3\pi d}$ AT/m^2

(direction upwards)

2.6

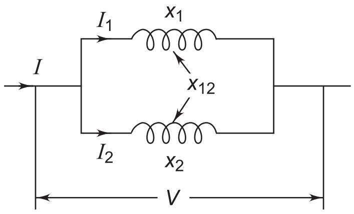

Fig. S-2.6

$$V = j\,X_1\,I_1 + j\,X_{12}\,I_2 = j\,X_2\,I_2 + j\,X_{12}\,I_1$$

$$I = I_1 + I_2\,; \quad I_1 = \frac{V}{j\,(X_1 - X_{12})}; \; I_2 = \frac{V}{j\,(X_2 - X_{12})}$$

$$I = \frac{V}{j}\left[\frac{1}{X_1 - X_{12}} + \frac{1}{X_2 - X_{12}}\right] = \frac{V}{j\,X}$$

$$\therefore \quad X = \frac{(X_1 - X_{12})(X_2 - X_{12})}{X_1 + X_2 - 2X_{12}}$$

2.7

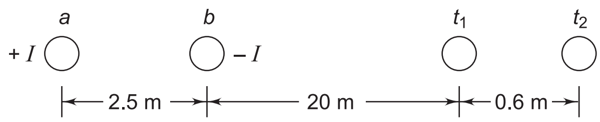

Fig. S-2.7

$$\lambda_{t1} = 2 \times 10^{-7}\left(I \ln\frac{1}{22.5} - I \ln\frac{1}{20}\right)$$

$$= 2 \times 10^{-7} \times 150 \ \ln\frac{20}{22.5}$$

$$= -\,0.353 \times 10^{-5} \text{ Wb-T/m}$$

$$\lambda_{t2} = 2 \times 10^{-7} \times 150 \ \left(\ln\frac{1}{23.1} - \ln\frac{1}{20.6}\right)$$

$$= -\,0.343 \times 10^{-5} \text{ Wb-T/m}$$

$$\lambda_t = \lambda_{t1} - \lambda_{t2} = -\,0.01 \times 10^{-5} \text{ Wb-T/m}$$

Mutual inductance $= (0.01 \times 10^{-5}/150) \times 10^3 \times 10^3$ mH/km

$= 0.00067$ mH/km

Induced voltage in telephone line $= 314 \times 0.01 \times 10^{-5} \times 10^3$

$=$ **0.0314 V/km**

2.8 $I_a = 400 \angle 0°$, $I_b = 400\angle{-120°}$, $I_c = 400\angle 120°$

Using Eq. (2.40)

$$\lambda_t = 2 \times 10^{-7} \times 400 \ \left(\ln\frac{26}{25} + 1\angle{-120°} \times \ln\frac{21}{20} + 1\angle 120° \ln\frac{16}{15}\right) \text{ Wb-T/m}$$

$$= 0.0176 \times 10^{-4} \angle 140° \text{ Wb-T/m}$$

$$\text{Mutual inductance} = \frac{0.0176 \times 10^{-4} \angle 140°}{400} \times 10^6$$

$$= \frac{1.76}{400} \angle 140° \text{ mH/km}$$

$= \mathbf{0.0044\angle 140°}$ **mH/km**

Voltage induced in telephone line $= 314 \times 0.0176 \times 10^{-4} \times 10^3 \angle 140°$

$= \mathbf{0.553\angle 140°}$ **V/km**

2.9 Here $d = 15$ m, $s = 0.5$ m

Using method of GMD

$$D_{ab} = D_{bc} = [d\ (d + s)\ (d - s)d]^{1/4}$$
$$= (15 \times 15.5 \times 14.5 \times 15)^{1/4} = 15 \text{ m}$$
$$D_{ca} = [2d\ (2d + s)\ (2d - s)\ 2d]^{1/4}$$
$$= (30 \times 30.5 \times 29.5 \times 30)^{1/4} = 30 \text{ m}$$
$$D_{eq} = (15 \times 15 \times 30)^{1/3} = 18.89 \text{ m}$$
$$D_s = (r'\ sr'\ s)^{1/4} = (r'\ s)^{1/2}$$
$$= (0.7788 \times 0.015 \times 0.5)^{1/2}$$
$$= 0.0764 \text{ m}$$

Inductive reactance/phase

$$X_L = 314 \times 0.461 \times 10^{-3} \log \frac{18.89}{0.0764}$$
$$= \mathbf{0.346\ \Omega/km}$$

2.10 $$X_L = 314 \times 0.921 \times 10^{-3} \log \frac{D}{0.01} = 31.4/50$$

$\therefore$ $$D = \mathbf{1.48} \text{ m (maximum permissible)}$$

2.11

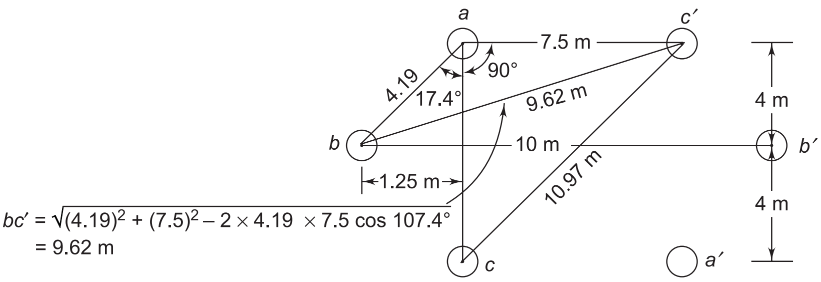

Fig. S-2.11

In section 1 of transposition cycle

$$D_{ab} = \sqrt{1.19 \times 9.62} = 6.35;\ D_{bc} = \sqrt{4.19 \times 9.62} = 6.35$$
$$D_{ca} = \sqrt{7.5 \times 8} = 7.746$$
$$D_{eq} = \sqrt[3]{6.35 \times 6.35 \times 7.746} = 6.78$$
$$D_{sa} = \sqrt{0.01 \times 10.97} = 0.3312 = D_{sc}$$
$$D_{sb} = \sqrt{0.01 \times 10} = 0.3162$$
$$D_s = \sqrt[3]{0.3312 \times 0.3312 \times 0.3162} = 0.326 \text{ m}$$
$$X = 0.314 \times 0.461 \log \frac{6.78}{0.326} = \mathbf{0.191}\ \Omega\text{/km/phase}$$

2.12 $$r' = 0.7788 \times 1.5 \times 10^{-2} = 0.0117 \text{ m}$$

$$D_{ab} = \sqrt[4]{1 \times 4 \times 1 \times 2}\ ;\ D_{bc} = \sqrt[4]{1 \times 4 \times 1 \times 2}\ ;\ D_{ca} = \sqrt[4]{2 \times 1 \times 2 \times 5}$$

$$D_m = \sqrt[3]{D_{ab}D_{bc}D_{ca}} = \sqrt[12]{1280} = 1.815 \text{ m}$$

$$D_{sa} = D_{sb} = D_{sc} = \sqrt{0.0117 \times 3} = 0.187$$

$$\therefore \quad D_s = 0.187 \text{ m}$$

$$L = 0.461 \log \frac{1.815}{0.187} = \mathbf{0.455} \text{ mH/km/phase}$$

2.13

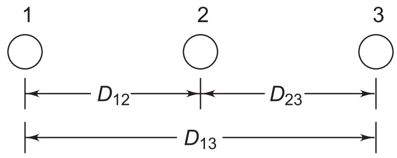

Fig. S-2.13

$$D_{13} = 2D_{12} = 2D_{23} = 2d$$

$$\sqrt[3]{2d \times d \times d} = 3$$

$$\sqrt[3]{2}\ d = 3 \qquad \therefore \quad d = \mathbf{2.38\ m}$$

2.14 Refer to Fig. 2.16 of the text book.

Case (i) $\quad 2\pi r^2 = A$

$$r = (A/2\pi)^{1/2} \quad \therefore \quad r' = 0.7788\ (A/2\pi)^{1/2}$$

$$\text{Self G.M.D} = \sqrt{r'd} = \sqrt{(0.7788)\, d\, (A/2\pi)^{1/2}}$$

$$= \mathbf{0.557\ d^{1/2}\ A^{1/4}}$$

Case (ii) $\quad 3\pi r^2 = A \quad \therefore \quad r = \sqrt{A/3\pi}$

$$\text{Self GMD} = (r'dd)^{1/3} = \sqrt{(0.7788)^{1/3}\ (A/3\pi)^{1/6}\ d^{2/3}}$$

$$= \mathbf{0.633\ d^{2/3}\ A^{1/6}}$$

Case (iii) $\quad 4\pi r^2 = A \quad \therefore \quad r = \sqrt{A/4\pi}$

$$\text{Self GMD} = \sqrt[4]{r'\, dd\, 2^{1/2} d}$$

$$= 1.09\ \sqrt[4]{r'\, d^3}$$

$$= 1.09\ (0.7788)^{1/4} \left(\frac{A}{4\pi}\right)^{1/8} d^{3/4}$$

$$= \mathbf{0.746\ d^{3/4}\ A^{1/8}}$$