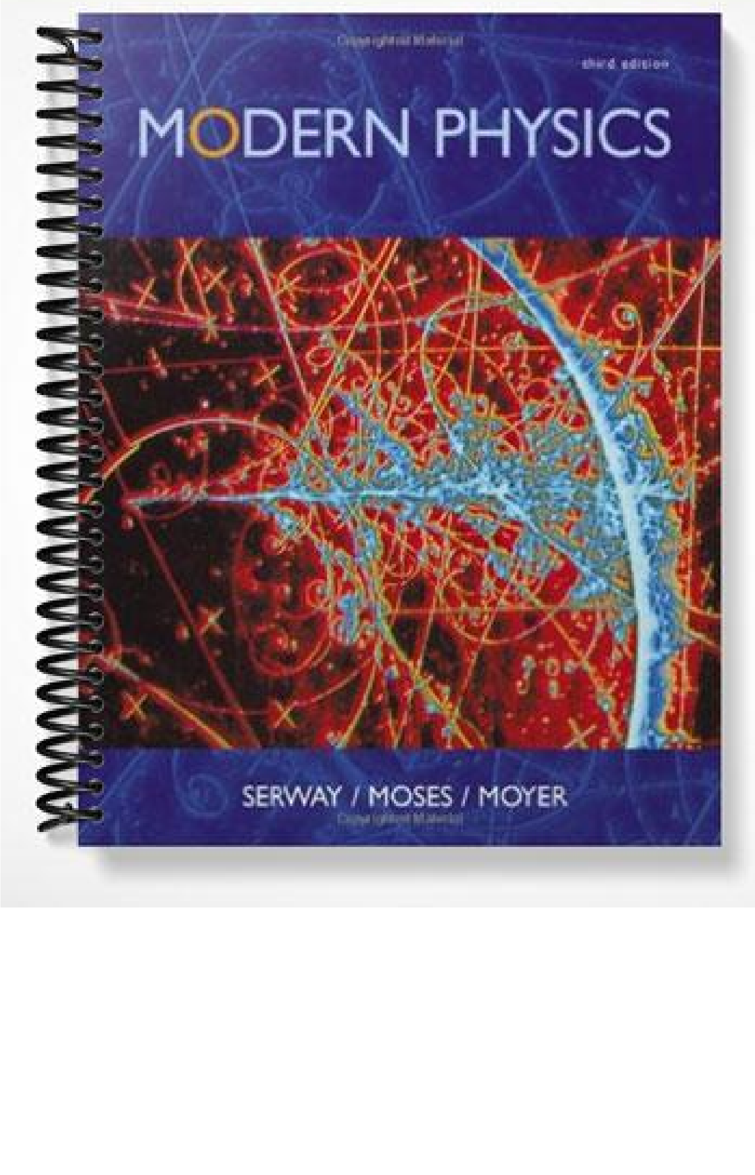
SOLUTIONS MANUAL



2

Relativity II

2-1 $p = \dfrac{mv}{\left[1-\left(v^2/c^2\right)\right]^{1/2}}$

(a) $$p = \frac{\left(1.67\times10^{-27}\ \text{kg}\right)(0.01c)}{\left[1-(0.01c/c)^2\right]^{1/2}} = 5.01\times10^{-21}\ \text{kg}\cdot\text{m/s}$$

(b) $$p = \frac{\left(1.67\times10^{-27}\ \text{kg}\right)(0.5c)}{\left[1-(0.5c/c)^2\right]^{1/2}} = 2.89\times10^{-19}\ \text{kg}\cdot\text{m/s}$$

(c) $$p = \frac{\left(1.67\times10^{-27}\ \text{kg}\right)(0.9c)}{\left[1-(0.9c/c)^2\right]^{1/2}} = 1.03\times10^{-18}\ \text{kg}\cdot\text{m/s}$$

(d) $$\frac{1.00\ \text{MeV}}{c} = \frac{1.602\times10^{-13}\ \text{J}}{2.998\times10^{8}\ \text{m/s}} = 5.34\times10^{-22}\ \text{kg}\cdot\text{m/s} \text{ so for (a)}$$

$$p = \frac{\left(5.01\times10^{-21}\ \text{kg}\cdot\text{m/s}\right)\left(100\ \text{MeV}/c\right)}{5.34\times10^{-22}\ \text{kg}\cdot\text{m/s}} = 9.38\ \text{MeV}/c$$

Similarly, for (b) $p = 540\ \text{MeV}/c$ and for (c) $p = 1\,930\ \text{MeV}/c$.

2-2 (a) Scalar equations can be considered in this case because relativistic and classical velocities are in the same direction.

$$p = \gamma\, mv = 1.90mv = \frac{mv}{\left[1-(v/c)^2\right]^{1/2}} \Rightarrow \frac{1}{\left[1-(v/c)^2\right]^{1/2}} = 1.90 \Rightarrow v = \left[1-\left(\frac{1}{1.90}\right)^2\right]^{1/2} c$$
$$= 0.85c$$

(b) No change, because the masses cancel each other.

2-3 As **F** is parallel to **v**, scalar equations are used. Relativistic momentum is given by $p = \gamma\, mv = \dfrac{mv}{\left[1-(v/c)^2\right]^{1/2}}$, and relativistic force is given by

$$F = \frac{dp}{dt} = \frac{d}{dt}\left\{\frac{mv}{\left[1-(v/c)^2\right]^{1/2}}\right\}$$
$$F = \frac{dp}{dt} = \frac{m}{\left[1-\left(v^2/c^2\right)\right]^{3/2}}\left(\frac{dv}{dt}\right)$$

2-4 (a) Using the results of Problem 2-3, $qE = m\left(1-\dfrac{v^2}{c^2}\right)^{-3/2}\left(\dfrac{dv}{dt}\right)$, or $a = \dfrac{dv}{dt} = \left(\dfrac{qE}{m}\right)\left(1-\dfrac{v^2}{c^2}\right)^{3/2}$. Here v is a function of t and q, E, M, and c are parameters.

(b) As expected; as $v \to c$, $a \to 0$ because in general no speed can exceed c, the speed of light.

(c) Separating variables, $\dfrac{dv}{\left(1-v^2/c^2\right)^{3/2}} = \left(\dfrac{qE}{m}\right)dt$, or $\displaystyle\int_0^v \frac{dv}{\left(1-v^2/c^2\right)^{3/2}} = \int_0^t \frac{qE}{m}\,dt$,

$$\left.\frac{v}{\left(1-v^2/c^2\right)^{1/2}}\right|_0^v = \frac{qEt}{m}$$
$$\frac{v^2}{\left(1-v^2/c^2\right)^{1/2}} = \left(\frac{qEt}{m}\right)^2 = \frac{qEt}{m}$$
$$v^2 = \left(\frac{qEt}{m}\right)^2 - \left(\frac{v^2}{c^2}\right)\left(\frac{qEt}{m}\right)^2$$
$$v^2\left[1+\left(\frac{qEt}{m}\right)^2\right] = \left(\frac{qEt}{m}\right)^2$$
$$v^2 = \frac{\left(qEt/mc\right)^2}{1+\left(qEt/mc\right)^2}$$
$$v^2 = \frac{\left(qEct\right)^2}{(mc)^2 + \left(qEt\right)^2}$$

Note that the limiting behavior of v as $t \to 0$ and $t \to \infty$ is reasonable. As $v = \dfrac{dx}{dt} = \dfrac{qEct}{\left[(mc)^2 + \left(qEt\right)^2\right]^{1/2}}$,

$$x = qEc\left[(mc)^2 + (qEt)^2\right]^{1/2}\left[\frac{1}{(qE)^2}\right]\Bigg|_0^t = \frac{c}{qE}\left\{\left[(mc)^2 + (qEt)^2\right]^{1/2} - mc\right\}.$$

As $t \to 0$, $x \to 0$, and $t \to \infty$, $x \to ct$; reasonable results.

2-5 This is the case where we use the relativistic form of Newton's second law, but unlike Problem 2-3 in which **F** is parallel to **v**, here **F** is perpendicular to **v** and $\mathbf{F} = \frac{d\mathbf{p}}{dt}$ so that

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}\left\{\frac{mv}{\sqrt{1-(v/c)^2}}\right\}.$$

Assuming that **B** is perpendicular to the plane of the orbit of q, the force is radially inward, and we find

$$\mathbf{F} = q\mathbf{v}\mathbf{B}\big|_{\text{radial}} = \frac{d}{dt}\left\{\frac{mv}{\sqrt{1-(v/c)^2}}\right\}.$$

As the force is perpendicular to **v**, it does no work on the charge and the magnitude (but not the direction) of **v** remains constant in time. Thus,

$$\frac{d}{dt}\left\{\frac{mv}{\sqrt{1-(v/c)^2}}\right\} = \frac{m}{\sqrt{1-(v/c)^2}}\frac{dv}{dt}.$$

Identifying $\left(\frac{dv}{dt}\right)$ as the centripetal acceleration where the scalar equation $\frac{dv}{dt} = \left(\frac{v^2}{r}\right)_{\text{radial}}$ gives $qvB\big|_{\text{radial}} = \left[\frac{m}{1-v^2/c^2}\right]^{1/2}\left(\frac{v^2}{r}\right)\Bigg|_{\text{radial}}$ or $v = \left(\frac{qBr}{m}\right)\left(1-\frac{v^2}{c^2}\right)^{1/2}$. Finally, the period T is $\frac{2\pi r}{v}$ and $T = \frac{2\pi r}{(qBr/m)\left(1-v^2/c^2\right)^{1/2}} = \frac{2\pi m}{(qB)\left(1-v^2/c^2\right)^{1/2}}$. As $f = \frac{1}{T}$, $f = \left(\frac{qB}{2\pi m}\right)\left(1-\frac{v^2}{c^2}\right)^{1/2}$.

2-6 Using Equation 2.4 $p = e^- BR = \left(1.60\times10^{-19}\ \text{C}\right)BR\ \text{kg}\cdot\text{m/C}\cdot\text{s} = 1.60\times10^{-19}\,BR\ \text{kg}\cdot\text{m/s}$. To convert $\text{kg}\cdot\text{m/s}$ to MeV/c, use

$$1\ \text{MeV}/c = \frac{\left(10^6\right)\left(1.60\times10^{-19}\ \text{C}\right)(1\ \text{J/C})}{3.00\times10^8\ \text{m/s}} = 5.34\times10^{-22}\ \text{kg}\cdot\text{m/s}$$, so that

$$p = \frac{\left(1.60\times10^{-19}\,BR\ \text{kg}\cdot\text{m/s}\right)(1\ \text{MeV}/c)}{5.34\times10^{-22}\ \text{kg}\cdot\text{m/s}} = 300BR\ \text{MeV}/c.$$

2-7 $E = \gamma mc^2$, $p = \gamma mu$; $E^2 = \left(\gamma mc^2\right)^2$; $p^2 = (\gamma mu)^2$;

$$E^2 - p^2c^2 = \left(\gamma mc^2\right)^2 - (\gamma mu)^2 c^2 = \gamma^2 \left\{\left(mc^2\right)^2 - (mc)^2 u^2\right\}$$

$$= \left(mc^2\right)^2 \left(1 - \frac{u^2}{c^2}\right)\left(1 - \frac{u^2}{c^2}\right)^{-1} = \left(mc^2\right)^2 \text{ Q.E.D.}$$

$$E^2 = p^2c^2 + \left(mc^2\right)^2$$

2-8 (a) $E_R = mc^2 = \left(1.67\times10^{-27}\ \text{kg}\right)\left(3\times10^8\ \text{m/s}\right)^2 = 1.503\times10^{-10}\ \text{J} = 939.4\ \text{MeV}$ (Numerical round off gives a slightly larger value for the proton mass)

(b) $E = \gamma mc^2 = \dfrac{1.503\times10^{-10}\ \text{J}}{\left(1-(0.95c/c)^2\right)^{1/2}} = 4.813\times10^{-10}\ \text{J} \approx 3.01\times10^3\ \text{MeV}$

(c) $K = E - mc^2 = 4.813\times10^{-10}\ \text{J} - 1.503\times10^{-10}\ \text{J} = 3.31\times10^{-10}\ \text{J} = 2.07\times10^3\ \text{MeV}$

2-9 (a) When $K = (\gamma - 1)mc^2 = 5mc^2$, $\gamma = 6$ and $E = \gamma mc^2 = 6(0.511\,0\ \text{MeV}) = 3.07\ \text{MeV}$.

(b) $\dfrac{1}{\gamma} = \left[1 - \left(\dfrac{v}{c}\right)^2\right]^{1/2}$ and $v = c\left[1 - \left(\dfrac{1}{\gamma}\right)^2\right]^{1/2} = c\left[1 - \left(\dfrac{1}{6}\right)^2\right]^{1/2} = 0.986c$

2-10 $E = \gamma mc^2$; $1.5mc^2 = \gamma mc^2$; $\gamma = 1.5 \Rightarrow 1.5 = \dfrac{1}{\left[1-\left(v^2/c^2\right)\right]^{1/2}}$; $v = c\left[1 - \left(\dfrac{1}{1.5}\right)^2\right]^{1/2} = 0.75c$

2-11 (a) $K = 50\times10^9\ \text{eV}$; $mc^2 = 938.27\ \text{MeV}$;

$$E = K + mc^2 = \left(50\times10^9\ \text{eV}\right) + \left(938.27\times10^6\ \text{eV}\right) = 50\,938.3\ \text{MeV}$$

$$E^2 = p^2c^2 + m^2c^4 \Rightarrow p = \left[\frac{E^2 - m^2c^4}{c^2}\right]^{1/2}$$

$$p = \frac{\left[(50\,938.3\ \text{MeV})^2 - (938.27\ \text{MeV})^2\right]^{1/2}}{c} = 5.09\times10^{10}\ \text{eV}/c$$

$$= \frac{5.09\times10^{10}\ \text{eV}}{3\times10^8\ \text{m/s}}\left(1.6\times10^{-19}\ \text{J/eV}\right) = 2.71\times10^{-17}\ \text{kg}\cdot\text{m/s}$$

(b) $E = \gamma mc^2 = \dfrac{mc^2}{\left[1-(v/c)^2\right]^{1/2}} \Rightarrow v = c\left[1 - \left(\dfrac{mc^2}{E}\right)^2\right]^{1/2}$

$$= \left(3\times10^8\ \text{m/s}\right)\left[1 - \left(\frac{938.27\ \text{MeV}}{50\,938.3\ \text{MeV}}\right)^2\right]^{1/2} = 2.999\,5\times10^8\ \text{m/s}$$

2-12 (a) When $K_e = K_p$, $m_e c^2 (\gamma_e - 1) = m_p c^2 (\gamma_p - 1)$. In this case $m_e c^2 = 0.511\,0$ MeV and $m_p c^2 = 938$ MeV, $\gamma_e = \left[1 - (0.75)^2\right]^{1/2} = 1.511\,9$. Substituting these values into the first equation, we find $\gamma_p = 1 + \frac{m_e c^2 (\gamma - 1)}{m_p c^2} = 1 + \frac{(0.511\,0)(1.511\,9 - 1)}{939} = 1.000\,279$. But $\gamma_p = \frac{1}{\left[1 - (u_p/c)^2\right]^{1/2}}$; therefore $u_p = c\left(1 - \gamma_p^{-2}\right)^{1/2} = 0.023\,6c$.

(b) When $p_e = p_p$, $\gamma_p m_p u_p = \gamma_e m_e u_e$ or $u_p = \left(\frac{\gamma_e}{\gamma_p}\right)\left(\frac{m_e}{m_p}\right) u_e$,

$$u_p = \left(\frac{1.511\,9}{1.000\,279}\right)\left[\frac{0.511\,0/c^2}{939/c^2}\right](0.75c) = 6.17 \times 10^{-4}\, c.$$

2-13 (a)

$$E = 400mc^2 = \gamma\, mc^2$$
$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = 400$$
$$\left(1 - \frac{v^2}{c^2}\right) = \left(\frac{1}{400}\right)^2$$
$$\frac{v}{c} = \left[1 - \frac{1}{400^2}\right]^{1/2}$$
$$v = 0.999\,997c$$

(b) $K = E - mc^2 = (400 - 1)\, mc^2 = 399mc^2 = (399)(938.3 \text{ MeV}) = 3.744 \times 10^5$ MeV

2-14 (a)

$$E = mc^2$$
$$m = \frac{E}{c^2} = \frac{4 \times 10^{26} \text{ J}}{\left(3.0 \times 10^8 \text{ m/s}\right)^2} = 4.4 \times 10^9 \text{ kg}$$

(b) $t = \frac{\left(2.0 \times 10^{30}\right) \text{ kg}}{4.4 \times 10^9 \text{ kg/s}} = 4.5 \times 10^{20} \text{ s} = 1.4 \times 10^{13}$ years

2-15 (a) $K = \gamma\, mc^2 - mc^2 = Vq$ and so, $\gamma^2 = \left(1 + \frac{Vq}{mc^2}\right)^2$ and $\frac{v}{c} = \left\{1 - \left(1 + \frac{Vq}{mc^2}\right)^{-2}\right\}^{1/2}$

$$\frac{v}{c} = \left\{1 - \frac{1}{1 + \left(5.0 \times 10^4 \text{ eV}/0.511 \text{ MeV}\right)^2}\right\}^{1/2} = 0.412\,7$$

or $v = 0.413c$.

(b) $K = \frac{1}{2}mv^2 = Vq$

$$v = \left(\frac{2Vq}{m}\right)^{1/2} = \left\{\frac{2\left(5.0\times10^4 \text{ eV}\right)}{0.511 \text{ MeV}/c^2}\right\}^{1/2} = 0.442c$$

(c) The error in using the classical expression is approximately $\frac{3}{40}\times100\%$ or about 7.5% in speed.

2-16 (a) In S the speed of the particle is u and $p = \frac{mu}{\left(1-v^2/c^2\right)^{1/2}}$, $E = \frac{mc^2}{\left(1-u^2/c^2\right)^{1/2}}$, and $\left(E^2 - p^2c^2\right) = m^2c^4$. In S', $u' = \frac{u-v}{1-uv/c^2}$, and

$$p' = \frac{mu'}{\sqrt{1-(u'/c)^2}} = \frac{m\left[(u-v)/\left(1-uv/c^2\right)\right]}{\sqrt{1-\left[(u-v)/\left(1-uv/c^2\right)\right]^2\left(1/c^2\right)}} = \frac{m\left[(u-v)/\left(1-uv/c^2\right)\right]}{\sqrt{1-(u-v)^2/\left(1-uv/c^2\right)^2}}$$

$$E' = \frac{mc^2}{\sqrt{1-(u'/c)^2}} = \frac{mc^2}{\sqrt{1-\left[(u-v)/\left(1-uv/c^2\right)\right]^2\left(1/c^2\right)}} = \frac{mc^2}{\sqrt{1-(u-v)^2/\left(1-uv/c^2\right)^2}}$$

(b) Using these expressions for E' and p', one obtains $\left(E'^2 - p'^2c^2\right) = m^2c^4$, and since $E^2 - p^2c^2 = m^2c^4$, it follows that, $E'^2 - p'^2c^2 = E^2 - p^2c^2$.

2-17 $\Delta m = m_{\text{Ra}} - m_{\text{Rn}} - m_{\text{He}}$ (an atomic unit of mass, the u, is one-twelfth the mass of the ^{12}C atom or $1.660\,54\times10^{-27}$ kg)

$$\Delta m = (226.025\,4 - 22.017\,5 - 4.002\,6) \text{ u} = 0.005\,3 \text{ u}$$
$$E = (\Delta m)(931 \text{ MeV/u}) = (0.005\,3 \text{ u})(931 \text{ MeV/u}) = 4.9 \text{ MeV}$$

2-18 (a) The mass difference of the two nuclei is

$$\Delta m = 54.927\,9 \text{ u} - 54.924\,4 \text{ u} = 0.003\,5 \text{ u}$$
$$\Delta E = (931 \text{ MeV/u})(0.003\,5 \text{ u}) = 3.26 \text{ MeV}.$$

(b) The rest energy for an electron is 0.511 MeV. Therefore,

$$K = 3.26 \text{ MeV} - 0.511 \text{ MeV} = 2.75 \text{ MeV}.$$

2-19 $\Delta m = 6m_p + 6m_n - m_{\text{C}} = [6(1.007\ 276) + 6(1.008\ 665) - 12]\ \text{u} = 0.095\ 646\ \text{u}$,

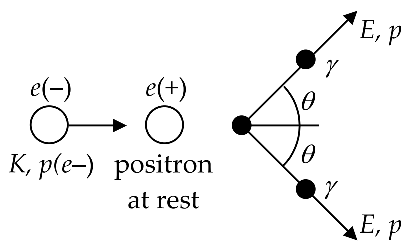
$$\Delta E = (931.49\ \text{MeV/u})(0.095\ 646\ \text{u}) = 89.09\ \text{MeV}.$$

Therefore the energy per nucleon $= \dfrac{89.09\ \text{MeV}}{12} = 7.42\ \text{MeV}$.

2-20 $\Delta m = m\ \ - m_p - m_e = 1.008\ 665\ \text{u} - 1.007\ 276\ \text{u} - 0.000\ 548\ 5\ \text{u} = 8.404 \times 10^{-4}\ \text{u}$

$E = c^2 (8.404 \times 10^{-4}\ \text{u}) = (8.404 \times 10^{-4}\ \text{u})(931.5\ \text{MeV/u}) = 0.783\ \text{MeV}$.

2-21



Conservation of mass-energy requires $K + 2mc^2 = 2E$ where K is the electron's kinetic energy, m is the electron's mass, and E is the gamma ray's energy.

$$E = \frac{K}{2} + mc^2 = (0.500 + 0.511)\ \text{MeV} = 1.011\ \text{MeV}.$$

Conservation of momentum requires that $p_{e^-} = 2p\cos\theta$ where p_{e^-} is the initial momentum of the electron and p is the gamma ray's momentum, $\dfrac{E}{c} = 1.011\ \text{MeV}/c$. Using $E_{e^-}^2 = p_{e^-}^2 c^2 + (mc^2)^2$ where E_{e^-} is the electron's total energy, $E_{e^-} = K + mc^2$, yields

$$p_{e^-} = \frac{1}{c}\sqrt{K^2 + 2Kmc^2} = \frac{\sqrt{(1.00)^2 + 2(1.00)(0.511)}\ \text{MeV}}{c} = 1.422\ \text{MeV}/c.$$

Finally, $\cos\theta = \dfrac{p_{e^-}}{2p} = 0.703$; $\theta = 45.3°$.

2-22 (a) Using the results of Problem 2-6 and substituting numerical values

$$p(\text{in MeV}/c) = 300BR = (300)(2.00\ \text{T})(0.343\ \text{m}) = 206\ \text{MeV}/c.$$

Since the momentum of the K^0 is zero before the decay, conservation of momentum requires the pion momenta to be equal in magnitude and opposite in direction. The pion's speed u may be found by noting that $\frac{p}{E} = \frac{mu/\sqrt{1-u^2/c^2}}{mc^2/\sqrt{1-u^2/c^2}} = \frac{u}{c^2}$ or $\frac{u}{c} = \frac{pc}{E}$ where *p* is the pion momentum and *E* is the pion's total energy. Thus for either pion, $\frac{u}{c} = \frac{pc}{E} = \frac{pc}{\left[p^2c^2 + \left(mc^2\right)^2\right]^{1/2}}$ where *m* is the pion's mass. Finally,

$$\frac{u}{c} = \frac{206 \text{ MeV}}{\sqrt{(206 \text{ MeV})^2 + (104 \text{ MeV})^2}} = 0.827 .$$

(b) Conservation of mass-energy requires that $E_{K^0} = 2E$ where E_{K^0} is the total energy of a pion. As the K^0 pion decays at rest,

$$E_{K^0} = m_{K^0}c^2 = 2\sqrt{p^2c^2 + \left(mc^2\right)^2} = 2\sqrt{(206)^2 + (140)^2} \text{ MeV} = 498 \text{ MeV},$$

or $m_{K^0} = 498 \text{ MeV}/c^2$.

2-23 In this problem, *M* is the mass of the initial particle, m_l is the mass of the lighter fragment, v_l is the speed of the lighter fragment, m_h is the mass of the heavier fragment, and v_h is the speed of the heavier fragment. Conservation of mass-energy leads to

$$Mc^2 = \frac{m_lc^2}{\sqrt{1-v_l^2/c^2}} + \frac{m_hc^2}{\sqrt{1-v_h^2/c^2}}$$

From the conservation of momentum one obtains

$$(m_l)(0.987c)(6.22) = (m_h)(0.868c)(2.01)$$
$$m_l = \frac{(m_h)(0.868c)(2.01)}{(0.987)(6.22)} = 0.284m_h$$

Substituting in this value and numerical quantities in the mass-energy conservation equation, one obtains $3.34 \times 10^{-27} \text{ kg} = 6.22m_l + 2.01m_h$ which in turn gives $3.34 \times 10^{-27} \text{ kg} = (6.22)(0.284)m_l + 2.01m_h$ or $m_h = \frac{3.34 \times 10^{-27} \text{ kg}}{3.78} = 8.84 \times 10^{-28} \text{ kg}$ and $m_l = (0.284)m_h = 2.51 \times 10^{-28} \text{ kg}$.

2-24 The moving observer sees the charge as stationary, so she says it feels no magnetic force.

$$q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = q(\mathbf{E}' + \text{zero}),\ \mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B} .$$

2-25 (a) The x component of the gravitational force between a light particle of mass m and the Sun is given by $F_x = \frac{GM_S m}{r^2}\sin\phi = \frac{GM_S mb}{\left(b^2+y^2\right)^{3/2}}$. The change in momentum in the x direction is given by $\Delta p_x = \int_{-\infty}^{\infty} F_x dt = \int_{-\infty}^{\infty} \frac{GM_S mb}{\left(b^2+y^2\right)^{3/2}} dt$. To convert dt to dy, assume the deflection is very small and that the position of the light particle is given by $y=-ct$ for $x=0$. Thus $dt = -\frac{dy}{c}$ and we get

$$\Delta p_x = -\frac{GM_S mb}{c}\int_{+\infty}^{-\infty}\frac{dy}{\left(b^2+y^2\right)^{3/2}} = \frac{2GM_S mb}{c}\int_0^{+\infty}\frac{dy}{\left(b^2+y^2\right)^{3/2}} = \frac{2GM_S mb}{c}\frac{y}{b^2\left(y^2+b^2\right)^{1/2}}\Bigg|_0^{\infty}$$
$$= \frac{2GM_S mb}{c}\left(\frac{1}{b^2}\right) = \frac{2GM_S m}{cb}$$

From Figure P2.25(b), $\theta \cong \frac{\Delta p_x}{mc}$ so we find $\theta \cong \frac{2GM_S m}{cb(mc)} = \frac{2GM_S}{bc^2}$.

(b) For $b = R_S = 6.96\times10^8$ m and $M_S = 1.99\times10^{30}$ kg

$$\theta = \frac{2\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(1.99\times10^{30}\ \mathrm{kg}\right)}{\left(6.96\times10^8\ \mathrm{m}\right)\left(3.00\times10^8\ \mathrm{m/s}\right)^2} = 4.24\times10^{-6}\ \mathrm{rad} = 2.43\times10^{-4}\ \mathrm{deg}$$

2-26 Energy conservation: $\frac{1}{\sqrt{1-0^2}}1\,400\ \mathrm{kg}c^2 + \frac{900\ \mathrm{kg}c^2}{\sqrt{1-0.85^2}} = \frac{Mc^2}{\sqrt{1-v^2/c^2}}$; $3\,108\ \mathrm{kg}\sqrt{1-\frac{v^2}{c^2}} = M$.

Momentum conservation: $0 + \frac{900\ \mathrm{kg}\,(0.85c)}{\sqrt{1-0.85^2}} = \frac{Mv}{\sqrt{1-v^2/c^2}}$; $1\,452\ \mathrm{kg}\sqrt{1-\frac{v^2}{c^2}} = \frac{Mv}{c}$.

(a) Dividing gives $\frac{v}{c} = \frac{1\,452}{3\,108} = 0.467$ $\qquad v = 0.467c$.

(b) Now by substitution $3\,108\ \mathrm{kg}\sqrt{1-0.467^2} = M = 2.75\times10^3\ \mathrm{kg}$.

2-27 If the energy required to remove a mass m from the surface is equal to its rest energy mc^2, then $\frac{GM_S m}{R_g} = mc^2$ and $R_g = \frac{GM_S}{c^2} = \frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(1.99\times10^{30}\ \mathrm{kg}\right)}{\left(3.00\times10^8\ \mathrm{m/s}\right)^2}$,

$R_g = 1.47\times10^3\ \mathrm{m} = 1.47\ \mathrm{km}$.

2-28 (a) The charged battery stores energy

$$E = \mathscr{P}t = (1.20\ \mathrm{J/s})(50\ \mathrm{min})(60\ \mathrm{s/min}) = 3\,600\ \mathrm{J}$$

so its mass excess is $\Delta m = \frac{E}{c^2} = \frac{3\,600 \text{ J}}{\left(3\times 10^8 \text{ m/s}\right)^2} = 4.00\times 10^{-14} \text{ kg}$.

(b) $\frac{\Delta m}{m} = \frac{4.00\times 10^{-14} \text{ kg}}{25\times 10^{-3} \text{ kg}} = 1.60\times 10^{-12}$ too small to measure.

2-29 The energy of the first fragment is given by $E_1^2 = p_1^2c^2 + \left(m_1c^2\right)^2 = (1.75 \text{ MeV})^2 + (1.00 \text{ MeV})^2$; $E_1 = 2.02 \text{ MeV}$. For the second, $E_2^2 = (2.00 \text{ MeV})^2 + (1.50 \text{ MeV})^2$; $E_2 = 2.50 \text{ MeV}$.

(a) Energy is conserved, so the unstable object had $E = 4.52 \text{ MeV}$. Each component of momentum is conserved, so the original object moved with

$$p^2 = p_x^2 + p_y^2 = \left(\frac{1.75 \text{ MeV}}{c}\right)^2 + \left(\frac{2.00 \text{ MeV}}{c}\right)^2.$$

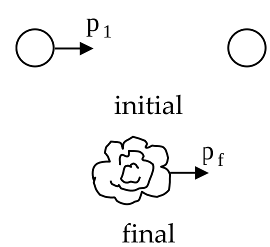
Then for $(4.52 \text{ MeV})^2 = (1.75 \text{ MeV})^2 + (2.00 \text{ MeV})^2 + \left(mc^2\right)^2$; $m = 3.65 \text{ MeV}/c^2$.

(b) Now $E = \gamma mc^2$ gives $4.52 \text{ MeV} = \frac{1}{\sqrt{1 - v^2/c^2}} 3.65 \text{ MeV}$; $1 - \frac{v^2}{c^2} = 0.654$ and $v = 0.589c$.

2-30 Take the two colliding protons as the system

$E_1 = K + mc^2$
$E_2 = mc^2$

$E_1^2 = p_1^2c^2 + m^2c^4$
$p_2 = 0.$

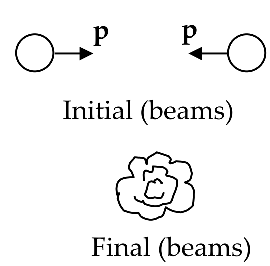


In the final state, $E_f = K_f + Mc^2$: $E_f^2 = p_f^2c^2 + M^2c^4$.

By energy conservation, $E_1 + E_2 = E_f$, so

$$E_1^2 + 2E_1E_2 + E_2^2 = E_f^2$$

$$p_1^2c^2 + m^2c^4 + 2\left(K + mc^2\right)mc^2 + m^2c^4 = p_f^2c^2 + M^2c^4$$



By conservation of momentum, $p_1 = p_f$.

Then $M^2c^4 = 2Kmc^2 + 4m^2c^4 = \frac{4Km^2c^4}{2mc^2} + 4m^2c^4$

$$Mc^2 = 2mc^2\sqrt{1+\frac{K}{2mc^2}}\ .$$

By contrast, for colliding beams in the original state, we have $E_1 = K + mc^2$ and $E_2 = K + mc^2$. In the final state, $E_f = Mc^2$

$$E_1 + E_2 = E_f:\ K + mc^2 + K + mc^2 = Mc^2$$

$$Mc^2 + 2mc^2\left(1+\frac{K}{2mc^2}\right).$$

2-31 Conservation of momentum γmu:

$$\frac{mu}{\sqrt{1-u^2/c^2}} + \frac{m(-u)}{3\sqrt{1-u^2/c^2}} = \frac{Mv_f}{\sqrt{1-v_f^2/c^2}} = \frac{2mu}{3\sqrt{1-u^2/c^2}}\ .$$

Conservation of energy γmc^2:

$$\frac{mc^2}{\sqrt{1-u^2/c^2}} + \frac{mc^2}{3\sqrt{1-u^2/c^2}} = \frac{Mc^2}{\sqrt{1-v_f^2/c^2}} = \frac{4mc^2}{3\sqrt{1-u^2/c^2}}\ .$$

To start solving we can divide: $v_f = \frac{2u}{4} = \frac{u}{2}$. Then

$$\frac{M}{\sqrt{1-u^2/4c^2}} = \frac{4m}{3\sqrt{1-u^2/c^2}} = \frac{M}{(1/2)\sqrt{4-u^2/c^2}}$$

$$M = \frac{2m\sqrt{4-u^2/c^2}}{3\sqrt{1-u^2/c^2}}$$

Note that when $v \ll c$, this reduces to $M = \frac{4m}{3}$, in agreement with the classical result.

2-32 (a) $\rho = \frac{\text{energy}}{\Delta t} = \frac{2\ \text{J}}{100\times10^{-15}\ \text{s}} = 2.00\times10^{13}\ \text{W}$

(b) The kinetic energy of one electron with $v = 0.999\,9c$ is

$$(\gamma-1)\,mc^2 = \left(\frac{1}{\sqrt{1-0.999\,9^2}}\right)\left(9.11\times10^{-31}\ \text{kg}\right)\left(3\times10^8\ \text{m/s}\right)^2 = 69.7\left(8.20\times10^{-14}\ \text{J}\right)$$

$$= 5.72\times10^{-12}\ \text{J}$$

Then we require $\frac{0.01}{100}(2\text{ J}) = N\left(5.72\times10^{-12}\text{ J}\right)$

$$N = \frac{2\times10^{-4}\text{ J}}{5.72\times10^{-12}\text{ J}} = 3.50\times10^{7}.$$

2-33 The energy that arrives in one year is

$$E = \mathscr{P}\,\Delta t = \left(1.79\times10^{17}\text{ J/s}\right)\left(3.16\times10^{7}\text{ s}\right) = 5.66\times10^{24}\text{ J}.$$

Thus, $m = \frac{E}{c^2} = \frac{5.66\times10^{24}\text{ J}}{\left(3.00\times10^{8}\text{ m/s}\right)^2} = 6.28\times10^{7}\text{ kg}$.