## SOLUTIONS MANUAL



## Chapter 2 <br> Consumers and Their Preferences

## Lecture Notes

## Experimental Teaser 1. Social Preferences

Consider conducting the divider/receiver experiment. Buy or make "play" ten-dollar bills. Lead the class in discussion about the results. Ask for speculation on what motivated dividers' behaviors. An analysis of the experiment appears in Resolving Teaser 1 at the end of the chapter.

## I. The Consumption Possibility Set

A consumption possibility set is the set of bundles feasible for the agents to consume in a society.

## A. A Primitive State of Nature: People and Goods

Figure 2.1 illustrates the initial primitive state of nature. Remind students that the only institutions are the state, property rights, and economic consulting firms. Apples are measured on the horizontal axis of Figure 2.1 and raspberries are measured on the vertical axis. Even though it is not measured in the figure, leisure time is also an available choice for the two economic agents. Point out bundles $a$ and $b$ and discuss the consumption possibility set.

## B. The Convexity Property of Consumption Possibility Sets

There are three assumptions imposed on the consumption possibility sets of the model's agents:

1. Divisibility assumption states that goods are infinitely divisible
2. Additivity assumption states that it is possible to add consumption bundles
3. Convexity is a property that implies that it is possible to combine two bundles to produce a third by consuming fractions of them

Ask students if they have questions about Example 2.1, Solved Problem 2.1, or Solved Problem 2.2.

## II. Rationality

A complete binary ordering is an assumption on consumer preferences that implies that if any two bundles in the consumption possibility set (hence the term binary) are chosen, say
bundles $a$ and $b$, then the agents will be able to rank them-tell whether they prefer $a$ to $b$ or $b$ to $a$ or whether they consider $a$ to be exactly as good as $b$.

Rationality Assumption 1: A Complete Binary Ordering. For any bundles $a$ and $b$ in a consumption possibility set, $a$ is at least as good as $b, b$ is at least as good as $a$, or $a$ is exactly as good as $b$. When $a$ is exactly as good as $b$, we say that the agent is indifferent between $a$ and $b$.

Counterexample 2.1 is a good example of the limits of models subject to simplifying assumptions. There are real-world comparisons that would be very complex to analyze.

## A. Binary Relationships among Goods

The $R$ relationship is the agents' binary preference relationship over the set of bundles feasible for the agents to consume. R will mean the "at-least-as-good-as relationship." For example, "economics class" R "taking a nap."

Reflexivity is an assumption on consumer preferences that states that any bundle is at least as good as itself. Rationality Assumption 2: Reflexivity. For any bundle $a$, $a$ Ra.

Transitivity is an assumption on consumer preferences, and the property of preference relationships that states that, if agents think that bundle $a$ is at least as good as bundle $b$ and that bundle $b$ is at least as good as bundle $c$, then they also think that bundle $a$ is at least as good as bundle $c$. Rationality Assumption 3: Transitivity. If $a \mathrm{R} b$ and $b \mathrm{R} c$, then $a \mathrm{R} c$.

Select two students to play out Example 2.2. Try it once with transitivity in force and once with intransitivity.

Economics uses many vocabulary terms that are everyday words, but with special meanings. For example, rationality is the assumption that economic agents know what they like and behave accordingly. That is, an agent's preferences exhibit completeness, reflexivity, and transitivity. There is no normative evaluation of each agent's utility.

## III. The Economically Feasible Set

An economically feasible consumption set is the reduced set of consumption bundles economically feasible to consume; that is, each bundle satisfies the budget constraint.

## A. Time Constraints

Figure 2.3 the transformation of an unbounded set to a bounded one. The time constraint line $a b$ represents all feasible consumption bundles when all available time is used. Point $c$ is a bundle that is outside the constraint and cannot be consumed. The shaded triangle $F$ shows the available consumption set.

Ask students for any questions about Solved Problem 2.3.

## B. Income or Budget Constraints

This section is a preview of income or budget constraints and markets. A market is a place where agents can go and exchange one good for another at a fixed price.

Figure 2.4 illustrates an income or budget constraint. Some students may be interested in the algebraic formula for the constraint. Others will prefer the graphical representation. Point out the unattainable point $c$ and attainable point $e$.

What questions do students have about Solved Problem 2.4?

## IV. Rationality and Choice

Given the definition of the economically feasible consumption set and the complete, reflexive, and transitive preference relationship $R$, it can be shown that there exists a set of bundles that are best in the sense that they are at least as good as any other available bundles. In any situation of constrained choice where consumers have to choose a best alternative from a closed and bounded set, agents will have a well-defined best choice or set of choices. The assumptions create the conditions for being able to predict how agents will choose.

## A. The Need for Utility Functions

Better than binary relationships, an agent's preferences can be represented with a utility function, a representation of an agent's preferences that tells the agent how good a bundle is by assigning it a utility number. The assigned number may be ordinal rather than cardinal. The best bundle to choose from a set of available bundles would then be the one that was assigned the biggest utility number by the agent's utility function or, more precisely, the one that maximized the utility function over the feasible set of consumption bundles.

## B. The Continuity Assumption

Continuity is the assumption on utility functions that states that if two bundles are close to each other in the feasible set, then they will be assigned utility numbers that are close to each other as well. Another way to state this assumption is to say that for every feasible bundle, there exists another feasible bundle that is exactly as good as it is. An agent would be indifferent between the bundles. Solved Problem 2.5 provides a counterexample to continuity.

## C. The Existence of Continuous Utility Functions

It can be proved that if economic agents have preferences that are complete, transitive, reflexive, and continuous, then there exists a continuous utility function that agrees with the underlying preferences of the agent in the following sense: If a consumer prefers bundle $a$ to bundle $b$, then her utility function will assign a utility number to $a$ that is greater than the
utility number assigned to $b$. By using a utility function, all the consumer has to do is to evaluate each bundle separately and tell us how much utility is associated with each bundle. We can then simply select the bundle that yields the greatest utility number.

## D. Additive and Multiplicative Utility Functions

Example 2.3 illustrates an additive utility function. This is a utility function that has the property that the marginal utility of one extra unit of any good consumed is independent of the amount of other goods consumed. The goods enter such utility functions in an additive and separable manner. With these functions, a person need not consume both goods to get positive levels of utility.

Example 2.4 depicts a multiplicative utility function. This is a utility function in which utility is a function of the products of the various units of goods consumed. In such utility functions, the marginal utility of consumption for any good depends on the amount of other goods consumed.

Economics is not definitive about tastes nor which type of utility function is more likely to exist. For a given set of preferences, $R$, there may exist many utility functions that are consistent with it. Still, given any set of bundles, each of these utility functions would yield identical choices. In terms of choice they are equivalent.

## E. Cardinal and Ordinal Utility

Cardinal utility applies if not only the utility numbers assigned to bundles, but also their differences, are meaningful. A CD with a utility number of 30 is three times as preferable as a chocolate bar with a utility number of 10 .

Ordinal utility applies if the utility numbers we assign to objects have no meaning other than to represent the ranking of these goods in terms of a person's preferences. The actual utility numbers assigned are unimportant as long as they preserve the ranking of the objects.

Solved Problem 2.6 illustrates continuous utility functions, both additive and multiplicative. Point out how Commissioners Allen and Baxter had different cardinal results, but identical ordinal results.

## V. Psychological Assumptions

Understanding the psychological make-up of the economic agents will improve comprehension of the choices the agents make and their utility functions.

## A. Psychological Assumption 1: Selfishness

Selfishness is a psychological assumption about agents that states that they are interested only in their own utility or satisfaction and make their choices with just that in mind. The
assumption implies that an agent's utility function does not include the utility of others. See Resolving Teaser 1 at the end of the chapter.

Refer to Example 2.5 and ask students the question at the end: Are the members of the coffee club unselfish or not?

## B. Psychological Assumption 2: Nonsatiation

Nonsatiation is a psychological assumption about consumer preferences that states that more of anything is always better. Visual learners will be able to picture the assumption using Figure 2.6. This assumption refers only to goods and not to economic "bads" that diminish utility.

## C. Psychological Assumption 3: Convexity of Preferences

Convexity of preferences is a psychological assumption about preferences that states that if a consumer is indifferent between a goods bundle $x$ and a goods bundle $y$, then the consumer would prefer (or be indifferent to) a weighted combination of these bundles to either of the original bundles $x$ or $y$. In short, convexity states that mixtures of bundles are at least as good as the indifferent components from which those mixtures were made. Note that the convexity of preferences is not related to the convexity property of the consumption possibility set discussed earlier.

Use Solved Problem 2.7 to test student understanding of the various assumptions.

## VI. Resolving Teaser 1

The divider experiment and the Dictator game provide insight into the selfishness assumption. There are conditions under which selfishness may be mitigated.

## VII. Conclusion

Chapter 2 presents important conditions about preferences in order to lay the groundwork for indifference curves. These curves will then be used to derive demand functions.

## Solutions to Exercises and Problems

1. Which assumption or assumptions about consumer preferences or behavior explain the following phenomena?
a) A person has a wardrobe with shirts of many colors.

Answer: The first applicable assumptions are the three psychological assumptions: selfishness, nonsatiation, and convexity of preferences. The person selfishly prefers a variety of colors of shirts, wants more colors rather than less, and is at least as happy
with a wardrobe (bundle) of different colored shirts as a wardrobe of shirts of all one color or the other. Also at work is the convexity property of consumption possibility sets that allows for a bundle consisting of shirts taken from bundles of single-color shirts. Relaxing the assumption that infinite time is available for shopping will impose a time constraint and limit the number of shirts in the wardrobe. Finally, some students may argue that this person illustrates a multiplicative utility function. The marginal utility of a pink shirt depends on the number of other shirts of other colors.
b) In maximizing his utility, given prevailing prices and his income, a consumer exhausts his entire budget.

Answer: Because of nonsatiation, the consumer will want more consumption rather than less. But because of the income or budget constraint, the consumer cannot acquire infinite amounts. Therefore, the consumer will spend all his budget in order to achieve as much consumption as possible.
2. What assumption or assumptions rule out the following phenomenon: Geoffrey has a bundle consisting of 6 apples and 8 raspberries. He states that if he is given 1 more apple, he will ask for 3 more raspberries to keep himself indifferent between his old bundle and the new bundle that he will have after he receives the 1 additional apple.

Answer: If nonsatiation holds, Geoffrey must prefer a bundle of 7 apples and 11 raspberries to a bundle of 6 apples and 8 raspberries because the former bundle has more of each fruit. He cannot be indifferent between the two bundles.
3. Which, if any, of the properties that we discussed in the section titled "The Consumption Possibility Set" are not satisfied by the consumption sets depicted in Figure 2.7? Does the convexity property hold for any of these sets? Explain your answer.

Answer: The left-hand set satisfies the properties of consumption possibility sets. This set is convex, based on the definition given in the text plus the geometric understanding that a set is convex if and only if all the points on a line segment that connects any two points in the set are also contained in the set.

Because of the dip in the upper boundary that appears in the lower left quadrant, the center set does not satisfy the additivity property, nor is it convex.

The right-hand set appears as a spoked wheel with a hollow hub. This set is does not satisfy the additivity property and is not convex.

For further information, see Mathematical Optimization and Economic Theory by Michael D. Intriligator or http://www.colorado.edu/Economics/morey/6808/curve.pdf.
4. We sometimes say that an individual with strong internal conflicts is divided into "several different people." Assume that Geoffrey and Elizabeth fall into such a category.
a) Geoffrey's internal conflicts divide him into three different people-the greedy Geoffrey, the health-conscious Geoffrey, and the diet-conscious Geoffrey. To make up his mind between any pair of goods, Geoffrey takes a majority vote of his three internal selves. Let us say that Geoffrey has three goods to consume-apples, chocolate, and meat-and that his three internal selves have the following binary preferences:

| Greedy Geoffrey: | chocolate $R$ apple $R$ meat |
| :--- | :--- |
| Health-conscious Geoffrey: | apple $R$ meat $R$ chocolate |
| Diet-conscious Geoffrey: | meat $R$ chocolate $R$ apple |

If we consider Geoffrey's preferences after he takes the majority vote of his internal selves, which, if any, of the assumptions that we discussed in the "Rationality" section of this chapter are violated by the ordering that results from the vote?

Answer: There are three possible binary orderings of the three goods: chocolateapple, apple-meat, and chocolate-meat. If students chart each Geoffrey's goods relationships for any one binary ordering, they will find a majority of two Geoffrey's in favor of one of the goods. The same is true for the remaining goods. For the whole, integrated Geoffrey, respectively, the relationships are chocolate $R$ apple, apple $R$ meat, and meat $R$ chocolate. But these three relationships violate the transitivity assumption. See Example 2.2 on p. 27 of the text.
b) Say that Elizabeth is also divided into three internal selves, and her internal selves have the following preferences:

| Greedy Elizabeth: | apple R chocolate R meat |
| :--- | :--- |
| Health-conscious Elizabeth: | apple R meat R chocolate |
| Diet-conscious Elizabeth: | apple $R$ meat $R$ chocolate |

Say that Elizabeth decides between each pair of goods differently than Geoffrey. She uses a unanimity criterion-one good is better than another only if all internal selves believe it to be. Which, if any, of the assumptions that we discussed in the "Rationality" section of this chapter are violated by the ordering that results?

Answer: Clearly, Elizabeth prefers apples to chocolate and apples to meat. But she will not be able to reach a unanimous decision when comparing chocolate and meat. Therefore she will not be able to express a preference for this latter binary ordering. This violates the assumption of a complete binary ordering.

