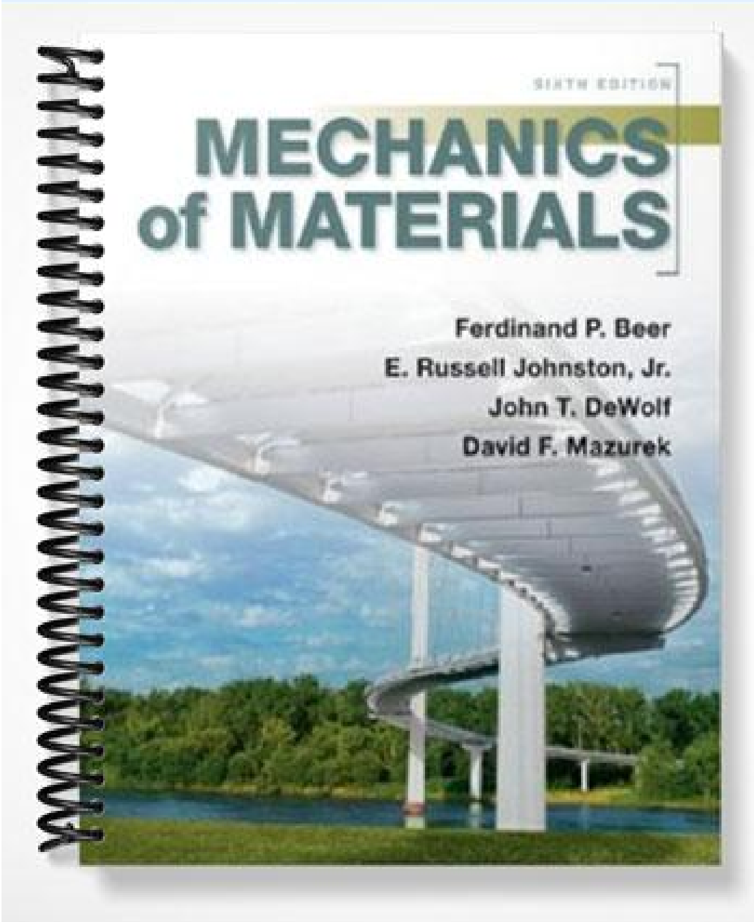
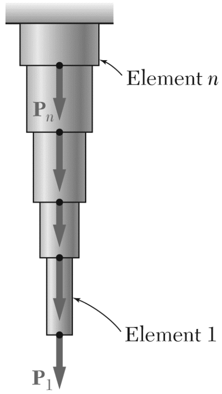
SOLUTIONS MANUAL



COMPUTER PROBLEMS

PROBLEM 1.C1



A solid steel rod consisting of n cylindrical elements welded together is subjected to the loading shown. The diameter of element i is denoted by d_i and the load applied to its lower end by $\mathbf{P}_i$ with the magnitude P_i of this load being assumed positive if $\mathbf{P}_i$ is directed downward as shown and negative otherwise. (*a*) Write a computer program that can be used with either SI or U.S. customary units to determine the average stress in each element of the rod. (*b*) Use this program to solve Problems 1.2 and 1.4.

SOLUTION

<u>Force in element *i*</u>:

It is the sum of the forces applied to that element and all lower ones:

$$F_i = \sum_{k=1}^{i} P_k$$

<u>Average stress in element *i*</u>:

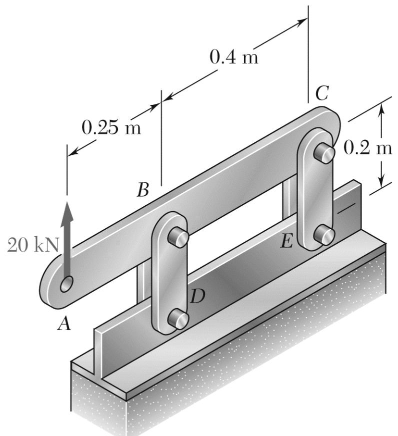
$$\text{Area} = A_i = \frac{1}{4}\pi d_i^2$$

$$\text{Ave stress} = \frac{F_i}{A_i}$$

<u>Program Outputs</u>

Problem 1.2 Element	Stress (ksi)	Problem 1.4 Element	Stress (MPa)
1	42.441	1	12.732
2	38.651	2	– 2.829

PROBLEM 1.C2

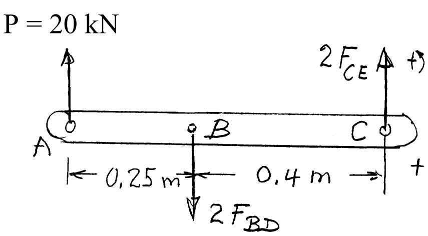


A 20-kN load is applied as shown to the horizontal member *ABC*. Member *ABC* has a 10×50-mm uniform rectangular cross section and is supported by four vertical links, each of 8×36-mm uniform rectangular cross section. Each of the four pins at *A*, *B*, *C*, and *D* has the same diameter *d* and is in double shear. (*a*) Write a computer program to calculate for values of *d* from 10 to 30 mm, using 1-mm increments, (1) the maximum value of the average normal stress in the links connecting Pins *B* and *D*, (2) the average normal stress in the links connecting Pins *C* and *E*, (3) the average shearing stress in Pin *B*, (4) the average shearing stress in Pin *C*, (5) the average bearing stress at *B* in member *ABC*, (6) the average bearing stress at *C* in member *ABC*. (*b*) Check your program by comparing the values obtained for $d = 16$ mm with the answers given for Probs 1.7 and 1.27. (c) Use this program to find the permissible values of the diameter *d* of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 150 MPa, 90 MPa, and 230 MPa. (*d*) Solve Part c, assuming that the thickness of member *ABC* has been reduced from 10 to 8 mm.

SOLUTION

<u>Forces in links</u>

F.B. diagram of *ABC*:



$$+\circlearrowleft \Sigma M_C = 0: \quad 2F_{BD}(BC) - P(AC) = 0$$

$$F_{BD} = P(AC)/2(BC) \quad \text{(tension)}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 2F_{CE}(BC) - P(AB) = 0$$

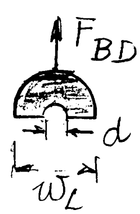
$$F_{CE} = P(AB)/2(BC) \quad \text{(comp.)}$$

(1) <u>Link *BD*</u>

Thickness = t_L

$A_{BD} = t_L(w_L - d)$

$\sigma_{BD} = +F_{BD}/A_{BD}$



(3) <u>Pin *B*</u>

$\tau_B = F_{BD}/(\pi d^2/4)$

(5) <u>Bearing stress at *B*</u>

Thickness of Member $AC = t_{AC}$

Sig Bear $B = F_{BD}/(dt_{AC})$

(6) <u>Bearing stress at *C*</u>

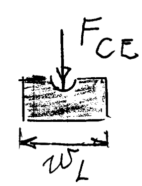
Sig Bear $C = F_{CE}/(dt_{AC})$

(2) <u>Link *CE*</u>

Thickness = t_L

$A_{CE} = t_L w_L$

$\sigma_{CE} = -F_{CE}/A_{CE}$



(4) <u>Pin *C*</u>

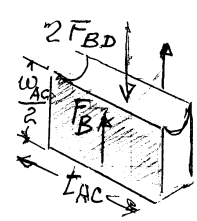
$\tau_C = F_{CE}/(\pi d^2/4)$

<u>Shearing stress in *ABC* under Pin *B*</u>

$F_B = \tau_{AC} t_{AC}(w_{AC}/2)$

$\Sigma F_y = 0: \quad 2F_B = 2F_{BD}$

$$\tau_{AC} = \frac{2F_{BD}}{\tau_{AC} w_{AC}}$$



<u>Program Outputs</u>

Input data for Parts (a), (b), (c):

$$P = 20 \text{ kN}, \quad AB = 0.25 \text{ m}, \quad BC = 0.40 \text{ m}, \quad AC = 0.65 \text{ m},$$
$$TL = 8 \text{ mm}, \quad WL = 36 \text{ mm}, \quad TAC = 10 \text{ mm}, \quad WAC = 50 \text{ mm}$$

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	206.90	79.58	325.00	125.00
11.00	81.25	-21.70	170.99	65.77	295.45	113.64
12.00	84.64	-21.70	143.68	55.26	270.83	104.17
13.00	88.32	-21.70	122.43	47.09	250.00	96.15
14.00	92.33	-21.70	105.56	40.60	232.14	89.29
15.00	96.73	-21.70	91.96	35.37	216.67	83.33
16.00	101.56	-21.70	80.82	31.08	203.12	78.13 ← (b)
17.00	106.91	-21.70	71.59	27.54	191.18	73.53
18.00	112.85	-21.70	63.86	24.56	180.56	69.44
19.00	119.49	-21.70	57.31	22.04	171.05	65.79
20.00	126.95	-21.70	51.73	19.89	162.50	62.50
21.00	135.42	-21.70	46.92	18.04	154.76	59.52
22.00	145.09	-21.70	42.75	16.44	147.73	56.82
23.00	156.25	-21.70	39.11	15.04	141.30	54.35
24.00	169.27	-21.70	35.92	13.82	135.42	52.08
25.00	184.66	-21.70	33.10	12.73	130.00	50.00
26.00	203.13	-21.70	30.61	11.77	125.00	48.08
27.00	225.69	-21.70	28.38	10.92	120.37	46.30
28.00	253.91	-21.70	26.39	10.15	116.07	44.64
29.00	290.18	-21.70	24.60	9.46	112.07	43.10
30.00	338.54	-21.70	22.99	8.84	108.33	41.67

(c) Answer: 16 mm $\leqslant d \leqslant$ 22 mm (c) ◀

Check: For $d = 22$ mm, $\tau_{AC} = 65$ MPa < 90 MPa O.K.

PROBLEM 1.C2 *(Continued)*

Program Outputs ***(Continued)***

Input data for Part (d) $P = 20$ kN,

$AB = 0.25$ m, $BC = 0.40$ m,

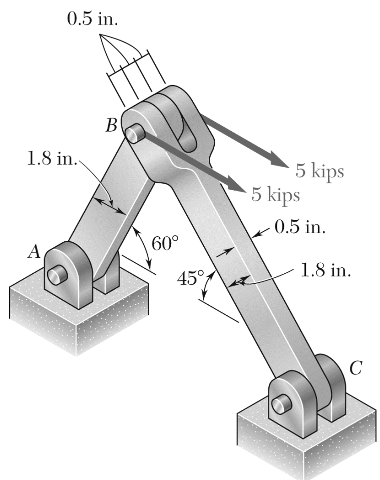
$AC = 0.65$ m, $TL = 8$ mm, $WL = 36$ mm,

$TAC = 8$ mm, $WAC = 50$ mm

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	206.90	79.58	406.25	156.25
11.00	81.25	-21.70	170.99	65.77	369.32	142.05
12.00	84.64	-21.70	143.68	55.26	338.54	130.21
13.00	88.32	-21.70	122.43	47.09	312.50	120.19
14.00	92.33	-21.70	105.56	40.60	290.18	111.61
15.00	96.73	-21.70	91.96	35.37	270.83	104.17
16.00	101.56	-21.70	80.82	31.08	253.91	97.66
17.00	106.91	-21.70	71.59	27.54	238.97	91.91
18.00	112.85	-21.70	63.86	24.56	225.69	86.81
19.00	119.49	-21.70	57.31	22.04	213.82	82.24
20.00	126.95	-21.70	51.73	19.89	203.12	78.13
21.00	135.42	-21.70	46.92	18.04	193.45	74.40
22.00	145.09	-21.70	42.75	16.44	184.66	71.02
23.00	156.25	-21.70	39.11	15.04	176.63	67.93
24.00	169.27	-21.70	35.92	13.82	169.27	65.10
25.00	184.66	-21.70	33.10	12.73	162.50	62.50
26.00	203.13	-21.70	30.61	11.77	156.25	60.10
27.00	225.69	-21.70	28.38	10.92	150.46	57.87
28.00	253.91	-21.70	26.39	10.15	145.09	55.80
29.00	290.18	-21.70	24.60	9.46	140.09	53.88
30.00	338.54	-21.70	22.99	8.84	135.42	52.08

(d) Answer: 18 mm $\leqslant d \leqslant$ 22 mm (d) ◄

Check: For $d = 22$ mm, $\tau_{AC} = 81.25$ MPa < 90 MPa O.K.

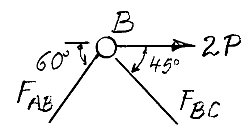


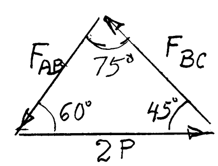
PROBLEM 1.C3

Two horizontal 5-kip forces are applied to Pin B of the assembly shown. Each of the three pins at A, B, and C has the same diameter d and is double shear. (*a*) Write a computer program to calculate for values of d from 0.50 to 1.50 in., using 0.05-in. increments, (1) the maximum value of the average normal stress in member AB, (2) the average normal stress in member BC, (3) the average shearing stress in Pin A, (4) the average shearing stress in Pin C, (5) the average bearing stress at A in member AB, (6) the average bearing stress at C in member BC, (7) the average bearing stress at B in member BC. (*b*) Check your program by comparing the values obtained for $d = 0.8$ in. with the answers given for Problems 1.60 and 1.61. (*c*) Use this program to find the permissible values of the diameter d of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 22 ksi, 13 ksi, and 36 ksi. (*d*) Solve Part *c*, assuming that a new design is being investigated in which the thickness and width of the two members are changed, respectively, from 0.5 to 0.3 in. and from 1.8 to 2.4 in.

SOLUTION

<u>Forces in members AB and BC</u>





Free body: Pin B

From force triangle:

$$\frac{F_{AB}}{\sin 45°} = \frac{F_{BC}}{\sin 60°} = \frac{2P}{\sin 75°}$$

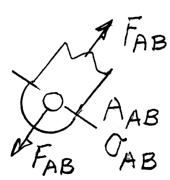
$$F_{AB} = 2P(\sin 45°/\sin 75°)$$

$$F_{BC} = 2P(\sin 60°/\sin 75°)$$

(1) <u>Max. ave. stress in AB</u>

Width $= w$

Thickness $= t$



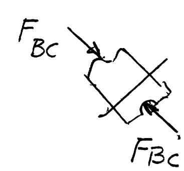
$A_{AB} = (w-d)t$

$\sigma_{AB} = F_{AB}/A_{AB}$

(2) <u>Ave. stress in BC</u>

$A_{BC} = wt$

$\sigma_{BC} = F_{BC}/A_{BC}$



(3) <u>Pin A</u>

$\tau_A = (F_{AB}/2)/(\pi d^2/4)$

(4) <u>Pin C</u>

$\tau_C = (F_{BC}/2)/(\pi d^2/4)$

(5) <u>Bearing stress at A</u>

Sig Bear $A = F_{AB}/dt$

(6) <u>Bearing stress at C</u>

Sig Bear $C = F_{BC}/dt$

(7) <u>Bearing stress at B in member BC</u>

Sig Bear $B = F_{BC}/2dt$

PROBLEM 1.C3 *(Continued)*

Input data for Parts (a), (b), (c):

$P = 5$ kips, $w = 1.8$ in., $t = 0.5$ in.

D in.	SIGAB ksi	SIGBC ksi	TAUA ksi	TAUC ksi	SIGBRGA ksi	SIGBRGC ksi	SIGBRGB ksi	
0.500	11.262	-9.962	18.642	22.831	29.282	35.863	17.932	
0.550	11.713	-9.962	15.406	18.869	26.620	32.603	16.301	
0.600	12.201	-9.962	12.945	15.855	24.402	29.886	14.943	
0.650	12.731	-9.962	11.030	13.510	22.525	27.587	13.793	
0.700	13.310	-9.962	9.511	11.649	20.916	25.616	12.808	
0.750	13.944	-9.962	8.285	10.147	19.521	23.909	11.954	
0.800	14.641	-9.962	7.282	8.918	18.301	22.414	11.207	← (b)
0.850	15.412	-9.962	6.450	7.900	17.225	21.096	10.548	
0.900	16.268	-9.962	5.754	7.047	16.268	19.924	9.962	
0.950	17.225	-9.962	5.164	6.324	15.412	18.875	9.438	
1.000	18.301	-9.962	4.660	5.708	14.641	17.932	8.966	
1.050	19.521	-9.962	4.227	5.177	13.944	17.078	8.539	
1.100	20.916	-9.962	3.852	4.717	13.310	16.301	8.151	
1.150	22.525	-9.962	3.524	4.316	12.731	15.593	7.796	
1.200	24.402	-9.962	3.236	3.964	12.201	14.943	7.471	
1.250	26.620	-9.962	2.983	3.653	11.713	14.345	7.173	
1.300	29.282	-9.962	2.758	3.377	11.262	13.793	6.897	
1.350	32.536	-9.962	2.557	3.132	10.845	13.283	6.641	
1.400	36.603	-9.962	2.378	2.912	10.458	12.808	6.404	
1.450	41.831	-9.962	2.217	2.715	10.097	12.367	6.183	
1.500	48.803	-9.962	2.071	2.537	9.761	11.954	5.977	

(c) Answer: 0.70 in. $\leqslant d \leqslant$ 1.10 in. (c) ◀

PROBLEM 1.C3 *(Continued)*

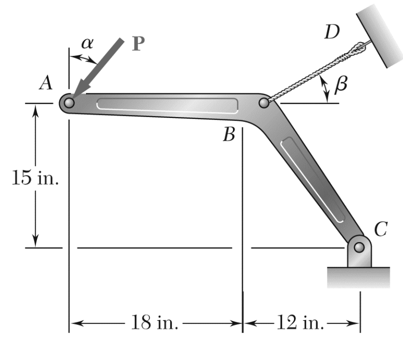
Input data for Part (d),

$$P = 5 \text{ kips}, \quad w = 2.4 \text{ in.}, \quad t = 0.3 \text{ in.}$$

D in.	SIGAB ksi	SIGBC ksi	TAUA ksi	TAUC ksi	SIGBRGA ksi	SIGBRGC ksi	SIGBRGB ksi
0.500	12.843	-12.452	18.642	22.831	48.803	59.772	29.886
0.550	13.190	-12.452	15.406	18.869	44.367	54.338	27.169
0.600	13.556	-12.452	12.945	15.855	40.669	49.810	24.905
0.650	13.944	-12.452	11.030	13.510	37.541	45.978	22.989
0.700	14.354	-12.452	9.511	11.649	34.860	42.694	21.347
0.750	14.789	-12.452	8.285	10.147	32.536	39.848	19.924
0.800	15.251	-12.452	7.282	8.918	30.502	37.357	18.679
0.850	15.743	-12.452	6.450	7.900	28.708	35.160	17.580
0.900	16.268	-12.452	5.754	7.047	27.113	33.206	16.603
0.950	16.829	-12.452	5.164	6.324	25.686	31.459	15.729
1.000	17.430	-12.452	4.660	5.708	24.402	29.886	14.943
1.050	18.075	-12.452	4.227	5.177	23.240	28.463	14.231
1.100	18.771	-12.452	3.852	4.717	22.183	27.169	13.584
1.150	19.521	-12.452	3.524	4.316	21.219	25.988	12.994
1.200	20.335	-12.452	3.236	3.964	20.335	24.905	12.452
1.250	21.219	-12.452	2.983	3.653	19.521	23.909	11.954
1.300	22.183	-12.452	2.758	3.377	18.771	22.989	11.495
1.350	23.240	-12.452	2.557	3.132	18.075	22.138	11.069
1.400	24.402	-12.452	2.378	2.912	17.430	21.347	10.674
1.450	25.686	-12.452	2.217	2.715	16.829	20.611	10.305
1.500	27.113	-12.452	2.071	2.537	16.268	19.924	9.962

(d) Answer: $0.85 \text{ in.} \leqslant d \leqslant 1.25 \text{ in.}$ (d) ◀

PROBLEM 1.C4



A 4-kip force **P** forming an angle α with the vertical is applied as shown to member *ABC*, which is supported by a pin and bracket at *C* and by a cable *BD* forming an angle β with the horizontal. (*a*) Knowing that the ultimate load of the cable is 25 kips, write a computer program to construct a table of the values of the factor of safety of the cable for values of α and β from 0 to 45°, using increments in α and β corresponding to 0.1 increments in tan α and tan β. (*b*) Check that for any given value of α the maximum value of the factor of safety is obtained for $\beta = 38.66°$ and explain why. (*c*) Determine the smallest possible value of the factor of safety for $\beta = 38.66°$, as well as the corresponding value of α, and explain the result obtained.

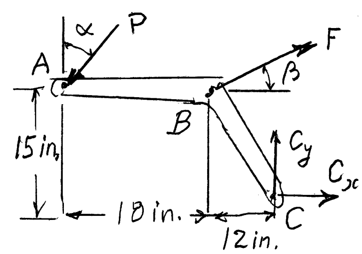
SOLUTION

(a) Draw *F.B.* diagram of *ABC*:

$$+\circlearrowleft \Sigma M_C = 0: \quad (P \sin \alpha)(1.5 \text{ in.}) + (P \cos \alpha)(30 \text{ in.}) - (F \cos \beta)(15 \text{ in.}) - (F \sin \beta)(12 \text{ in.}) = 0$$

$$F = P \frac{15 \sin \alpha + 30 \cos \alpha}{15 \cos \beta + 12 \sin \beta}$$

$$F.S. = F_{ult}/F$$



Output for $P = 4$ kips and $F_{ult} = 20$ kips

VALUES OF FS

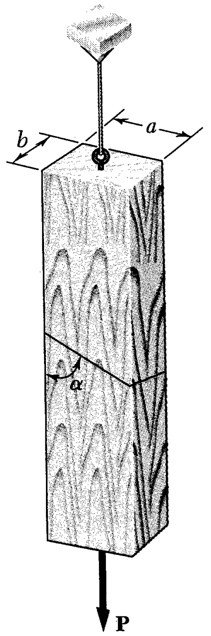
BETA → / ALPHA ↓	0	5.71	11.31	16.70	21.80	26.56	30.96	34.99	38.66	41.99	45.00
0.000	3.125	3.358	3.555	3.712	3.830	3.913	3.966	3.994	4.002	3.995	3.977
5.711	2.991	3.214	3.402	3.552	3.666	3.745	3.796	3.823	3.830	3.824	3.807
11.310	2.897	3.113	3.295	3.441	3.551	3.628	3.677	3.703	3.710	3.704	3.687
16.699	2.837	3.049	3.227	3.370	3.477	3.553	3.600	3.626	3.633	3.627	3.611
21.801	2.805	3.014	3.190	3.331	3.438	3.512	3.560	3.585	3.592	3.586	3.570
26.565	2.795	3.004	3.179	3.320	3.426	3.500	3.547	3.572	3.579	3.573	3.558
30.964	2.803	3.013	3.189	3.330	3.436	3.510	3.558	3.583	3.590	3.584	3.568
34.992	2.826	3.036	3.214	3.356	3.463	3.538	3.586	3.611	3.619	3.612	3.596
38.660	2.859	3.072	3.252	3.395	3.503	3.579	3.628	3.653	3.661	3.655	3.638
41.987	2.899	3.116	3.298	3.444	3.554	3.631	3.680	3.706	3.713	3.707	3.690
45.000	2.946	3.166	3.351	3.499	3.611	3.689	3.739	3.765	3.773	3.767	3.750

↑(b)

(b) When $\beta = 38.66°$; $\tan \beta = 0.8$ and cable *BD* is perpendicular to the lever Arm *BC*.

(c) $F.S. = 3.579$ for $\alpha = 26.6°$; *P* is perpendicular to the lever Arm *AC*.

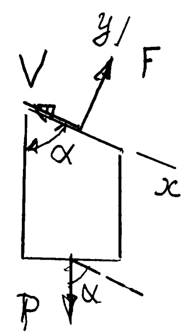
Note: The value $F.S. = 3.579$ is the smallest of the values of *F.S.* corresponding to $\beta = 38.66°$ and the largest of those corresponding to $\alpha = 26.6°$. The point $\alpha = 26.6°$; $\beta = 38.66°$ is a "saddle point", or "minimax" of the function $F.S.(\alpha, \beta)$.



PROBLEM 1.C5

A load **P** is supported as shown by two wooden members of uniform rectangular cross section that are joined by a simple glued scarf splice. (*a*) Denoting by σ_U and τ_U, respectively, the ultimate strength of the joint in tension and in shear, write a computer program which, for given values of *a*, *b*, *P*, σ_U and τ_U, expressed in either SI or U.S. customary units, and for values of α from 5 to 85° at 5° intervals, can be used to calculate (1) the normal stress in the joint, (2) the shearing stress in the joint, (3) the factor of safety relative to failure in tension, (4) the factor of safety relative to failure in shear, (5) the overall factor of safety for the glued joint. (*b*) Apply this program, using the dimensions and loading of the members of Probs 1.29 and 1.31, knowing that $\sigma_U = 1.26$ MP and $\tau_U = 1.50$ MPa for the glue used in Probs 1.29, and that $\sigma_U = 150$ psi and $\tau_U = 214$ psi for the glue used in Probs 1.31. (c) Verify in each of these two cases that the shearing stress is maximum for $a = 45°$.

SOLUTION



<u>(1) and (2)</u> Draw the *F.B.* diagram of lower member:

$$\nwarrow^{+} \Sigma F_x = 0: \quad -V + P\cos\alpha = 0 \qquad V = P\cos\alpha$$

$$^{+}\nearrow \Sigma F_y = 0: \quad F - P\sin\alpha = 0 \qquad F = P\sin\alpha$$

$\text{Area} = ab/\sin\alpha$

Normal stress:

$$\sigma = \frac{F}{\text{Area}} = (P/ab)\sin^2\alpha$$

Shearing stress:

$$\tau = \frac{V}{\text{Area}} = (P/ab)\sin\alpha\cos\alpha$$

(3) *F.S.* for tension (normal stresses)

$$FSN = \sigma_U/\sigma$$

(4) *F.S.* for shear:

$$FSS = \tau_U/\tau$$

(5) Overall *F.S.*:

F.S. = The <u>smaller</u> of *FSN* and *FSS*.

PROBLEM 1.C5 *(Continued)*

Program Outputs

Problem 1.29

$$a = 5 \text{ in.}$$
$$b = 3 \text{ in.}$$
$$P = 1400 \text{ lb}$$
$$\sigma_U = 150 \text{ psi}$$
$$\tau_U = 214 \text{ psi}$$

ALPHA	SIG(psi)	TAU(psi)	FSN	FSS	FS	
5	0.709	8.104	211.574	26.408	26.408	
10	2.814	15.961	53.298	13.408	13.408	
15	6.252	23.333	23.992	9.171	9.171	
20	10.918	29.997	13.739	7.134	7.134	
25	16.670	35.749	8.998	5.986	5.986	
30	23.333	40.415	6.429	5.295	5.295	
35	30.706	43.852	4.885	4.880	4.880	
40	38.563	45.958	3.890	4.656	3.890	
45	46.667	46.667	3.214	4.586	3.214	(c) ◀
50	54.770	45.958	2.739	4.656	2.739	
55	62.628	43.852	2.395	4.880	2.395	
60	70.000	40.415	2.143	5.295	2.143	(b) ◀
65	76.663	35.749	1.957	5.986	1.957	
70	82.415	29.997	1.820	7.134	1.820	
75	87.081	23.333	1.723	9.171	1.723	
80	90.519	15.961	1.657	13.408	1.657	
85	92.624	8.104	1.619	26.408	1.619	

PROBLEM 1.C5 *(Continued)*

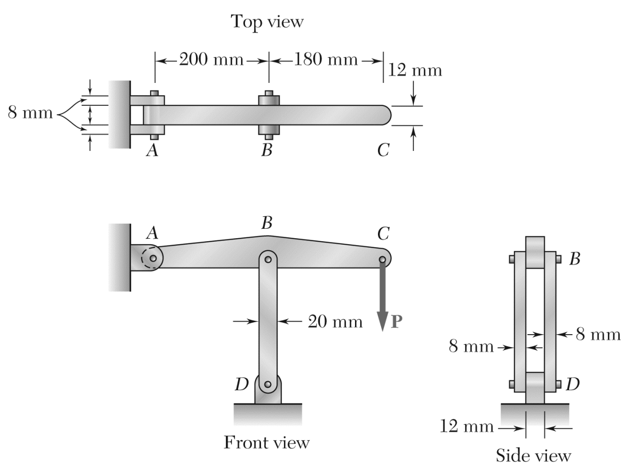
Program Outputs ***(Continued)***

Problem 1.31

$$a = 150 \text{ mm}$$
$$b = 75 \text{ mm}$$
$$P = 11 \text{ kN}$$
$$\sigma_U = 1.26 \text{ MPa}$$
$$\tau_U = 1.50 \text{ MPa}$$

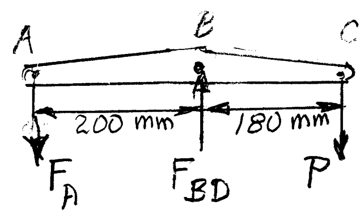
ALPHA	SIG(MPa)	TAU(MPa)	FSN	FSS	FS	
5	0.007	0.085	169.644	17.669	17.669	
10	0.029	0.167	42.736	8.971	8.971	
15	0.065	0.244	19.237	6.136	6.136	
20	0.114	0.314	11.016	4.773	4.773	
25	0.175	0.375	7.215	4.005	4.005	
30	0.244	0.423	5.155	3.543	3.543	
35	0.322	0.459	3.917	3.265	3.265	
40	0.404	0.481	3.119	3.116	3.116	
45	0.489	0.489	2.577	3.068	2.577	(b), (c) ◀
50	0.574	0.481	2.196	3.116	2.196	
55	0.656	0.459	1.920	3.265	1.920	
60	0.733	0.423	1.718	3.543	1.718	
65	0.803	0.375	1.569	4.005	1.569	
70	0.863	0.314	1.459	4.773	1.459	
75	0.912	0.244	1.381	6.136	1.381	
80	0.948	0.167	1.329	8.971	1.329	
85	0.970	0.085	1.298	17.669	1.298	

PROBLEM 1.C6



Member *ABC* is supported by a pin and bracket at *A* and by two links, which are pin-connected to the member at *B* and to a fixed support at *D*. (*a*) Write a computer program to calculate the allowable load P_{all} for any given values of (1) the diameter d_1 of the pin at *A*, (2) the common diameter d_2 of the pins at *B* and *D*, (3) the ultimate normal stress σ_U in each of the two links, (4) the ultimate shearing stress σ_U in each of the three pins, (5) the desired overall factor of safety *F.S.* Your program should also indicate which of the following three stresses is critical: the normal stress in the links, the shearing stress in the pin at *A,* or the shearing stress in the pins at *B* and *D*. (*b* and *c*) Check your program by using the data of Probs 1.55 and 1.56, respectively, and comparing the answers obtained for P_{all} with those given in the text. (*d*) Use your program to determine the allowable load P_{all}, as well as which of the stresses is critical, when $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MP for aluminum links, $\tau_U = 100$ MPa for steel pins, and $F.S. = 3.2$.

SOLUTION



(a) F.B. diagram of *ABC*:

$$\Sigma M_A = 0: \quad P = \frac{200}{380} F_{BD}$$

$$\Sigma M_B = 0: \quad P = \frac{200}{180} F_A$$

(1) For given d_1 of Pin *A*: $F_A = 2(\tau_U/FS)(\pi d_1^2/4)$, $P_1 = \frac{200}{180} F_A$

(2) For given d_2 of Pins *B* and *D*: $F_{BD} = 2(\tau_U/FS)(\pi d_2^2/4)$, $P_2 = \frac{200}{380} F_{BD}$

(3) For ultimate stress in links *BD*: $F_{BD} = 2(\sigma_U/FS)(0.02)(0.008)$, $P_3 = \frac{200}{380} F_{BD}$

(4) For ultimate shearing stress in pins: P_4 is the smaller of P_1 and P_2

(5) For desired overall *F.S.*: P_5 is the smaller of P_3 and P_4

If $P_3 < P_4$, stress is critical in links

If $P_4 < P_3$ and $P_1 < P_2$, stress is critical in Pin *A*

If $P_4 < P_3$ and $P_2 < P_1$, stress is critical in Pins *B* and *D*

PROBLEM 1.C6 *(Continued)*

Program Outputs

(b) Problem 1.53. Data: $d_1 = 8$ mm, $d_2 = 12$ mm, $\sigma_U ,= 250$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.0$

$P_{\text{all}} = 3.72$ kN. Stress in Pin A is critical ◀

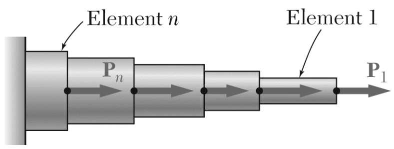
(c) Problem 1.54 Data: $d_1 = 10$ mm, $d_2 = 12$ mm, $\sigma_U = 250$ MPa, $\tau_U = 100$ MPa, $F.S. = 30$

$P_{\text{all}} = 3.97$ kN. Stress in Pins B and D is critical ◀

(d) Data: $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.2$

$P_{\text{all}} = 5.79$ kN. Stress in links is critical ◀

PROBLEM 2.C1



A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to the loading shown. The length of element i is denoted by L_i, its cross-sectional area by A_i, modulus of elasticity by E_i, and the load applied to its right end by P_i, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (*a*) Write a computer program that can be used to determine the average normal stress in each element, the deformation of each element, and the total deformation of the rod. (*b*) Use this program to solve Probs 2.20 and 2.126.

SOLUTION

For each element, enter

$$L_i, \quad A_i, \quad E_i$$

Compute deformation

Update axial load $\qquad P = P + P_i$

Compute for each element

$$\sigma_i = P/A_i$$
$$\delta_i = PL_i/A_iE_i$$

Total deformation:

Update through n elements

$$\delta = \delta + \delta_i$$

Program Outputs

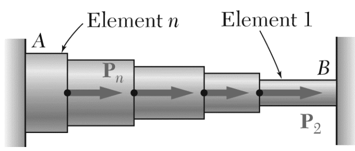
Problem 2.20

Element	Stress (MPa)	Deformation (mm)
1	19.0986	0.1091
2	–12.7324	–0.0909
Total Deformation =		0.0182 mm

Problem 2.126

Element	Stress (ksi)	Deformation (in.)
1	12.7324	0.0176
2	–2.8294	–0.0057
Total Deformation =		0.01190 in.

PROBLEM 2.C2



Rod *AB* is horizontal with both ends fixed; it consists of *n* elements, each of which is homogeneous and of uniform cross section, and is subjected to the loading shown. The length of element *i* is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and the load applied to its right end by $\mathbf{P}_i$, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (Note that $P_1 = 0$.) (*a*) Write a computer program which can be used to determine the reactions at *A* and *B*, the average normal stress in each element, and the deformation of each element. (*b*) Use this program to solve Probs 2.41 and 2.42.

SOLUTION

We Consider the reaction at *B* redundant and release the rod at *B*

Compute δ_B with $R_B = 0$

For each element, enter

$$L_i, \quad A_i, \quad E_i$$

Update axial load

$$P = P + P_i$$

Compute for each element

$$\sigma_i = P/A_i$$
$$\delta_i = PL_i/A_iE_i$$

Update total deformation

$$\delta_B = \delta_B + \delta_i$$

Compute δ_B due to unit load at *B*

$$\text{Unit} \quad \sigma_i = 1/A_i$$
$$\text{Unit} \quad \delta_i = L_i/A_iE_i$$

Update total unit deformation

$$\text{Unit} \quad \delta_B = \text{Unit } \delta_B + \text{Unit } \delta_i$$

Superposition

For total displacement at $B = 0$

$$\delta_B + R_B \quad \text{Unit} \quad \delta_B = 0$$

Solving:

$$R_B = -\delta_B/\text{Unit } \delta_B$$

Then: $$R_A = \Sigma P_i + R_B$$

PROBLEM 2.C2 *(Continued)*

For each element

$$\sigma = \sigma_i + R_B \quad \text{Unit } \sigma_i$$
$$\delta = \delta_i + R_B \quad \text{Unit } \delta_i$$

Program Outputs

Problem 2.41

RA = −62.809 kN

RB = −37.191 kN

Element	Stress (MPa)	Deformation (mm)
1	−52.615	−0.05011
2	3.974	0.00378
3	2.235	0.00134
4	49.982	0.04498

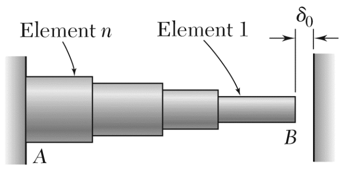
Problem 2.42

RA = −45.479 kN

RB = −54.521 kN

Element	Stress (MPa)	Deformation (mm)
1	−77.131	−0.03857
2	−20.542	−0.01027
3	−11.555	−0.01321
4	36.191	0.06204

PROBLEM 2.C3



Rod *AB* consists of *n* elements, each of which is homogeneous and of uniform cross section. End *A* is fixed, while initially there is a gap δ_0 between end *B* and the fixed vertical surface on the right. The length of element *i* is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and its coefficient of thermal expansion by α_i. After the temperature of the rod has been increased by ΔT, the gap at *B* is closed and the vertical surfaces exert equal and opposite forces on the rod. (*a*) Write a computer program which can be used to determine the magnitude of the reactions at *A* and *B*, the normal stress in each element, and the deformation of each element. (*b*) Use this program to solve Probs 2.51, 2.59, and 2.60.

SOLUTION

We compute the displacements at *B*

Assuming there is no support at *B*:

Enter $L_i,\quad A_i,\quad E_i,\quad \alpha_i$

Enter temperature change *T* compute for each element

$$\delta_i = \alpha_i L_i T$$

Update total deformation

$$\delta_B = \delta_B + \delta_i$$

Compute δ_B due to unit load at *B*

$$\text{Unit}\quad \delta_i = L_i / A_i E_i$$

Update total unit deformation

$$\text{Unit}\quad \delta_B = \text{Unit } \delta_B + \text{Unit } \delta_i$$

Compute Reactions

From superposition

$$R_B = (\delta_B - \delta_0)/\text{Unit } \delta_B$$

Then

$$R_A = -R_B$$

For each element

$$\sigma_i = -R_B / A_i$$
$$\delta_i = \alpha_i L_i T + R_B L_i / A_i E_i$$

PROBLEM 2.C3 *(Continued)*

Program Outputs

Problem 2.51

$R = 125.628$ kN

Element	Stress (MPa)	Deformation (microm)
1	−44.432	0.500
2	−99.972	−0.500

Problem 2.59

$R = 52.279$ kips

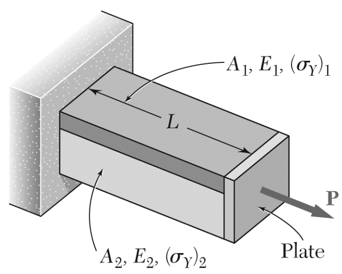
Element	Stress (ksi)	Deformation (10 * −3 in.)
1	−21.783	9.909
2	−18.671	10.091

Problem 2.60

$R = 232.390$ kN

Element	Stress (MPa)	Deformation (microm)
1	−116.195	363.220
2	−290.487	136.780

PROBLEM 2.C4



Bar AB has a length L and is made of two different materials of given cross-sectional area, modulus of elasticity, and yield strength. The bar is subjected as shown to a load **P** which is gradually increased from zero until the deformation of the bar has reached a maximum value δ_m and then decreased back to zero. (*a*) Write a computer program which, for each of 25 values of δ_m equally spaced over a range extending from 0 to a value equal to 120% of the deformation causing both materials to yield, can be used to determine the maximum value P_m of the load, the maximum normal stress in each material, the permanent deformation δ_P of the bar, and the residual stress in each material. (*b*) Use this program to solve Probs 2.111 and 2.112.

SOLUTION

Note: The following assumes $(\sigma_Y)_1 < (\sigma_Y)_2$

Displacement increment

$$\delta_m = 0.05(\sigma_Y)_2 L/E_2$$

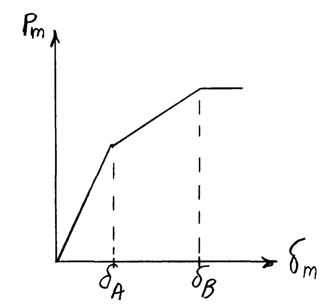
Displacements at yielding

$$\delta_A = (\sigma_Y)_1 L/E_1 \quad \delta_B = (\sigma_Y)_2 L/E_2$$

For each displacement

If $\delta_m < \delta_A$:

$$\sigma_1 = \delta_m E_1/L$$
$$\sigma_2 = \delta_m E_2/L$$
$$P_m = (\delta_m/L)(A_1E_1 + A_2E_2)$$



If $\delta_A < \delta_m < \delta_B$:

$$\sigma_1 = (\sigma_Y)_1$$
$$\sigma_2 = \delta_m E_2/L$$
$$P_m = A_1\sigma_1 + (\delta_m/L)A_2E_2$$

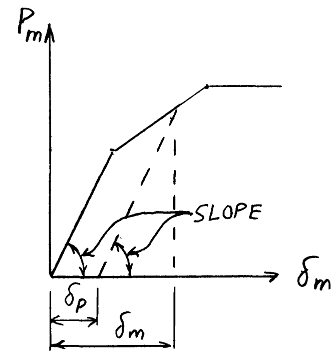
If $\delta_m > \delta_B$:

$$\sigma_1 = (\sigma_Y)_1$$
$$\sigma_2 = (\sigma_Y)_2$$
$$P_m = A_1\sigma_1 + A_2\sigma_2$$

PROBLEM 2.C4 *(Continued)*

Permanent deformations, residual stresses

Slope of first (elastic) segment



$$\text{Slope} = (A_1E_1 + A_2E_2)/L$$

$$\delta_P = \delta_m - (P_m/\text{Slope})$$

$$(\sigma_1)_{\text{res}} = \sigma_1 - (E_1P_m/(L\ \text{Slope}))$$

$$(\sigma_2)_{\text{res}} = \sigma_2 - (E_2P_m/(L\ \text{Slope}))$$

Program Outputs

Problems 2.111 and 2.112

DM 10** – 3 in.	PM kips	SIGM (1) ksi	SIGM (2) ksi	DP 10** – 3 in.	SIGR (1) ksi	SIG (2) ksi	
0.000	0.000	0.000	0.000	0.000	0.000	0.000	
2.414	8.750	5.000	5.000	0.000	0.000	0.000	
4.828	17.500	10.000	10.000	0.000	0.000	0.000	
7.241	26.250	15.000	15.000	0.000	0.000	0.000	
9.655	35.000	20.000	20.000	0.000	0.000	0.000	
12.069	43.750	25.000	25.000	0.000	0.000	0.000	
14.483	52.500	30.000	30.000	0.000	0.000	0.000	
16.897	61.250	35.000	35.000	0.000	0.000	0.000	
19.310	70.000	40.000	40.000	0.000	0.000	0.000	
21.724	78.750	45.000	45.000	0.000	0.000	0.000	
24.138	87.500	50.000	50.000	0.000	0.000	0.000	
26.552	91.250	50.000	55.000	1.379	–2.143	2.857	
28.966	95.000	50.000	60.000	2.759	–4.286	5.714	
31.379	98.750	50.000	65.000	4.138	–6.429	8.571	2.112 ◀
33.793	102.500	50.000	70.000	5.517	–8.571	11.429	
36.207	106.250	50.000	75.000	6.897	–10.714	14.286	
38.621	110.000	50.000	80.000	8.276	–12.857	17.143	
41.034	113.750	50.000	85.000	9.655	–15.000	20.000	2.111 ◀
43.448	117.500	50.000	90.000	11.034	–17.143	22.857	
45.862	121.250	50.000	95.000	12.414	–19.286	25.714	
48.276	125.000	50.000	100.000	13.793	–21.429	28.571	
50.690	125.000	50.000	100.000	16.207	–21.429	28.571	
53.103	125.000	50.000	100.000	18.621	–21.429	28.571	
55.517	125.000	50.000	100.000	21.034	–21.429	28.571	
57.931	125.000	50.000	100.000	23.448	–21.429	28.571	