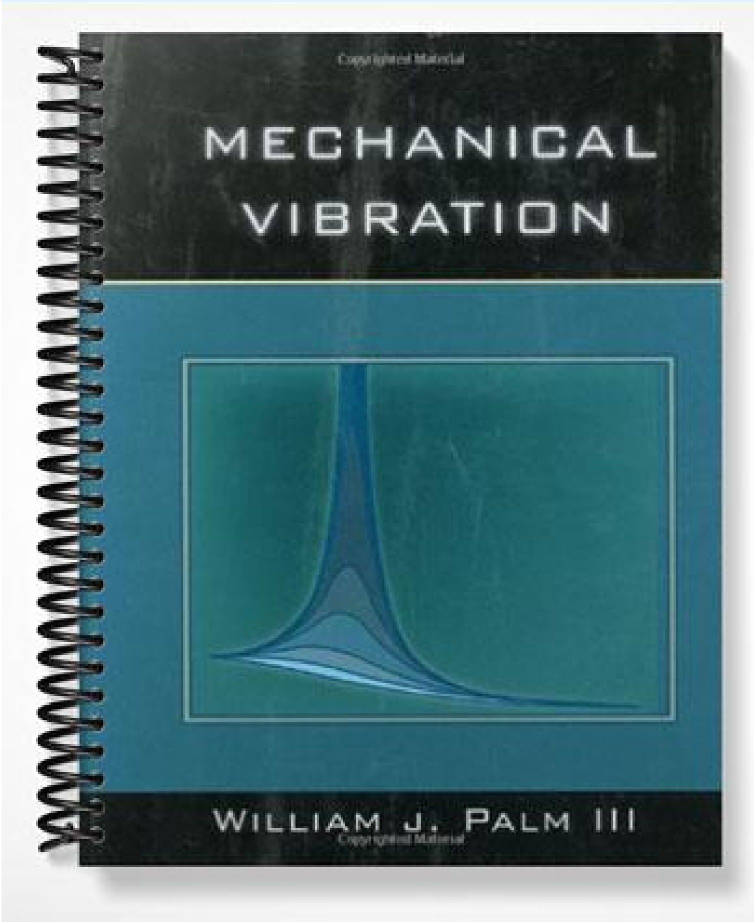
SOLUTIONS MANUAL



Solutions Manual©

to accompany

Mechanical Vibration, First Edition

by

William J. Palm III

University of Rhode Island

Solutions to Problems in Chapter Two

Problem 2.1 The tangential velocity component of the cylinder is $R\dot{\phi}$, and the tangential velocity component of the contact point is $r\dot{\theta}$. If there is no slipping, these two components must be equal. Thus $R\dot{\phi} = r\dot{\theta}$.

Problem 2.2 The tangential velocity component of the cylinder is $R\dot{\phi}$, and the tangential velocity component of the contact point is $r\dot{\theta}$. If there is no slipping, these two components must be equal. Thus $R\dot{\phi} = r\dot{\theta}$.

Problem 2.3 Let θ be the angle of the line BA measured from the vertical. That is, $\theta = 0$ when point A is in contact with the surface. Then $\omega = \dot{\theta}$ and $\alpha = \ddot{\theta}$.

Let s be the linear displacement of point B. Then

$$s = R\theta$$

$$v_B = R\dot{\theta} = R\omega$$

$$a_B = R\ddot{\theta} = R\alpha$$

Establish an xy coordinate system whose origin is located at the point of contact of point A with the surface at time $t = 0$. The coordinate x is positive to the left and y is positive upward. Then the xy coordinates of point A are

$$x = s - R\sin\theta = R(\theta - \sin\theta) \qquad y = R - R\cos\theta$$

$$\dot{x} = R\dot{\theta}(1 - \cos\theta) = v_B(1 - \cos\theta) \qquad \dot{y} = R\dot{\theta}\sin\theta = v_B\sin\theta$$

So the velocity of point A has the components $\dot{x}$ and $\dot{y}$. The magnitude of the velocity is

$$v_A = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{v_B^2(1 - \cos\theta)^2 + v_B^2\sin^2\theta} = v_B\sqrt{2(1 - \cos\theta)}$$

To find the acceleration, differentiate again to obtain

$$\ddot{x} = \dot{v}_B(1 - \cos\theta) + v_B\dot{\theta}\sin\theta = a_B(1 - \cos\theta) + R\omega^2\sin\theta$$

$$\ddot{y} = \dot{v}_B\sin\theta + v_B\dot{\theta}\cos\theta = a_B\sin\theta + R\omega^2\cos\theta$$

or

$$\ddot{x} = R\alpha(1 - \cos\theta) + R\omega^2\sin\theta$$

$$\ddot{y} = R\alpha\sin\theta + R\omega^2\cos\theta$$

The acceleration magnitude is

$$a_A = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{2R^2\alpha^2 + R^2\omega^4 + 2R^2\alpha\omega^2 - 2R^2\alpha^2\cos\theta}$$

Problem 2.4 Let x be the horizontal displacement of the vehicle measured to the left from directly underneath the pulley. Let y be the height of the block above the ground. Let D be the length of the hypotenuse of the triangle whose sides are x and 3 m. Then, from the Pythagorean theorem,

$$D^2 = x^2 + 3^2 \qquad (1)$$

Differentiate this to obtain

$$D\dot{D} = x\dot{x} \qquad (2)$$

The total cable length is 10 m, so $D + 3 - y = 10$ or

$$D = y + 7 \qquad (3)$$

This gives

$$x^2 = D^2 - 9 = (y + 7)^2 - 9 \qquad (4)$$

and

$$\dot{y} = \dot{D} = \frac{x\dot{x}}{D} = \frac{x\dot{x}}{7 + y} \qquad (5)$$

Substituting the given values of $y = 2$, $\dot{x} = 0.2$ into Equations (4) and (5), we obtain $x = \sqrt{72} = 6\sqrt{2}$ and

$$\dot{y} = \frac{6\sqrt{2}(0.2)}{9} = 0.1885 \ \text{ m/s}$$

This is the velocity of the block after it has been raised 2 m.

To obtain the acceleration, differentiate Equations (2) and (5) to obtain

$$\dot{D}^2 + \dot{D}\ddot{D} = \dot{x}^2 + \dot{x}\ddot{x}$$

$$\ddot{y} = \ddot{D}$$

Solve for $\ddot{D}$:

$$\ddot{D} = \frac{\dot{x}^2 + \dot{x}\ddot{x} - \dot{D}^2}{\dot{D}} = \frac{(0.2)^2 + 0 - (0.1885)^2}{0.1885} = 0.0237 \ \text{ m/s}^2$$

Since $\ddot{y} = \ddot{D}$, the acceleration of the block after it has been raised 2 m is 0.0237 m/s^2.

Problem 2.5 Only one of the pulleys translates, so $v_2 = v_1/2$.

Problem 2.6 Let v_C denote the velocity of the middle pulley. Then $v_C = v_A/2$ and $v_B = v_C/2$. Thus, $v_B = (v_A/2)/2 = v_A/4$.

Problem 2.7

$$M_O = I_O\alpha = I_O\ddot{\theta}$$

$$I_O = I_{RG} + m_R L^2 + m_C L_C^2$$

$$M_O = -m_R g L \sin\theta - m_C g L_C \sin\theta$$

Thus

$$(I_{RG} + m_R L^2 + m_C L_C^2)\ddot{\theta} = -(m_R L + m_C L_C) g \sin\theta$$

If $m_R \approx 0$ and if $I_{RG} \approx 0$, then

$$m_C L_C^2 \ddot{\theta} = -m_C L_C g \sin\theta$$

or

$$L_C \ddot{\theta} = -g \sin\theta$$

Problem 2.8 The tangential velocity component of the cylinder is $R\dot{\phi}$, and the tangential velocity component of the contact point is $r\dot{\theta}$. If there is no slipping, these two components must be equal. Thus $R\dot{\phi} = r\dot{\theta}$ and $R\ddot{\phi} = r\ddot{\theta}$.

$$KE = \frac{1}{2} I_G \dot{\phi}^2 + \frac{1}{2} M \left[(r - R)\dot{\theta} \right]^2$$

$$PE = Mg \left[(r - R) - (r - R) \cos \theta \right] = Mg(r - R)(1 - \cos \theta)$$

Because $KE + PE = \text{constant}$, then $d(KE + PE)/dt = 0$ and

$$I_G \dot{\phi}\ddot{\phi} + M(r - R)^2 \dot{\theta}\ddot{\theta} + Mg(r - R) \sin \theta \, \dot{\theta} = 0$$

But $R\dot{\phi} = r\dot{\theta}$ and $\ddot{\phi} = r\ddot{\theta}/R$. Thus

$$I_G \left(\frac{r}{R} \right)^2 \dot{\theta}\ddot{\theta} + M(r - R)^2 \dot{\theta}\ddot{\theta} + Mg(r - R) \sin \theta \, \dot{\theta} = 0$$

Cancel $\dot{\theta}$ and collect terms to obtain

$$\left[I_G \left(\frac{r}{R} \right)^2 + M(r - R)^2 \right] \ddot{\theta} + Mg(r - R) \sin \theta = 0$$

Problem 2.9 The tangential velocity component of the cylinder is $R\dot{\phi}$, and the tangential velocity component of the contact point is $r\dot{\theta}$. If there is no slipping, these two components must be equal. Thus $R\dot{\phi} = r\dot{\theta}$ and $R\ddot{\phi} = r\ddot{\theta}$.

The energy method is easier than the force- moment method here because 1) the motions of the link and cylinder are directly coupled (i.e. if we know the position and velocity of one, we know the position and velocity of the other), 2) the only external force is conservative (gravity), and 3) we need not compute the reaction forces between the link and the cylinder.

$$KE = \frac{1}{2}I_G\dot{\phi}^2 + \frac{1}{2}M\left[(r-R)\dot{\theta}\right]^2 + \frac{1}{2}m\left(\frac{L}{2}\dot{\theta}\right)^2 + \frac{1}{2}\left[I_O + m\left(\frac{L}{2}\right)^2\right]\dot{\theta}^2$$

$$PE = Mg(r-R)\,(1-\cos\,\theta) + mg(r-\frac{L}{2})(1-\cos\,\theta)$$

Because $KE + PE =$ constant, then $d(KE+PE)/dt = 0$, and

$$I_G\dot{\phi}\ddot{\phi} + M(r-R)^2\dot{\theta}\ddot{\theta} + m\frac{L^2}{4}\dot{\theta}\ddot{\theta} + \left[I_O + m\left(\frac{L}{2}\right)^2\right]\dot{\theta}\ddot{\theta} + Mg(r-R)\sin\,\theta\,\dot{\theta} + mg\left(r-\frac{L}{2}\right)\sin\,\theta\,\dot{\theta} = 0$$

But $\dot{\phi} = r\dot{\theta}/R$ and $\ddot{\phi} = r\ddot{\theta}/R$. Substitute these expressions, cancel $\dot{\theta}$, and collect terms to obtain

$$\left[I_G\left(\frac{r}{R}\right)^2 + M(r-R)^2 + m\frac{L^2}{4} + I_O + m\frac{L^2}{4}\right]\ddot{\theta} + \left[Mg(r-R) + mg\left(r-\frac{L}{2}\right)\right]\sin\,\theta = 0$$

Problem 2.10 a) Let point O be the pivot point and G be the center of mass. Let L be the distance from O to G. Treat the pendulum as being composed of three masses:

1) m_1, the rod mass above point O, whose center of mass is 0.03 m above point O;
2) m_2, the rod mass below point O, whose center of mass is 0.045 m below point O, and
3) m_3, the mass of the 4.5 kg block.

Then, summing moments about G gives

$$m_1 g(L + 0.03) - m_2 g(0.045 - L) - m_3 g(0.09 + 0.015 - L) = 0$$

where

$$m_1 = \frac{0.06}{0.15} 1.4 = 0.56 \text{ kg}$$
$$m_2 = \frac{0.09}{0.15} 1.4 = 0.84 \text{ kg}$$
$$m_3 = 4.5 \text{ kg}$$

The factor g cancels out of the equation to give

$$0.56(L + 0.03) - 0.84(0.045 - L) - 4.5(0.105 - L) = 0$$

which gives $L = 0.084$ m.

b) Summing moments about the pivot point O gives

$$I_O \ddot{\theta} = -mgL \sin\theta$$

where m is the total mass. From the parallel-axis theorem, treating the rod as a slender rod, we obtain

$$I_O = \frac{1}{12}(1.4)(0.06 + 0.09)^2 + (1.4)(0.015)^2 + (4.5)(0.09 + 0.015)^2 = 0.0525 \ \text{ kg} \cdot \text{m}^2$$

and $mgL = (1.4 + 4.5)(9.81)(0.084) = 4.862$ N·m. Thus the equation of motion is

$$0.0525\ddot{\theta} = -4.862 \sin\theta$$

or

$$\ddot{\theta} + 92.61 \sin\theta = 0$$

Problem 2.11 See the figure for the coordinate definitions and the definition of the reaction force R. Let P be the point on the axle. Note that $y_P = 0$. The coordinates of the mass center of the rod are $x_G = x_P - (L/2)\sin\theta$ and $y_G = -(L/2)\cos\theta$. Thus

$$\ddot{x}_G = \ddot{x}_P - \frac{L}{2}\ddot{\theta}\cos\theta + \frac{L}{2}\dot{\theta}^2\sin\theta$$

$$\ddot{y}_G = \frac{L}{2}\ddot{\theta}\sin\theta + \frac{L}{2}\dot{\theta}^2\cos\theta$$

Let m be the mass of the rod. Summing forces in the x direction:

$$m\ddot{x}_G = f \qquad \text{or} \qquad m\left(\ddot{x}_P - \frac{L}{2}\ddot{\theta}\cos\theta + \frac{L}{2}\dot{\theta}^2\sin\theta\right) = f \qquad (1)$$

Summing forces in the y direction:

$$m\ddot{y}_G = R - mg \qquad \text{or} \qquad m\left(\frac{L}{2}\ddot{\theta}\sin\theta + \frac{L}{2}\dot{\theta}^2\cos\theta\right) = R - mg \qquad (2)$$

Summing moments about the mass center: $I_G\ddot{\theta} = (fL/2)\cos\theta - (RL/2)\sin\theta$. Substituting for R from (2) and using the fact that $I_G = mL^2/12$, we obtain

$$\frac{1}{12}mL^2\ddot{\theta} = \frac{fL}{2}\cos\theta - \frac{mgL}{2}\sin\theta - \frac{mL^2}{4}\sin^2\theta\,\ddot{\theta} - \frac{mL^2}{4}\dot{\theta}^2\sin\theta\cos\theta \qquad (3)$$

The model consists of (1) and (3) with $m = 20$ kg and $L = 1.4$ m.

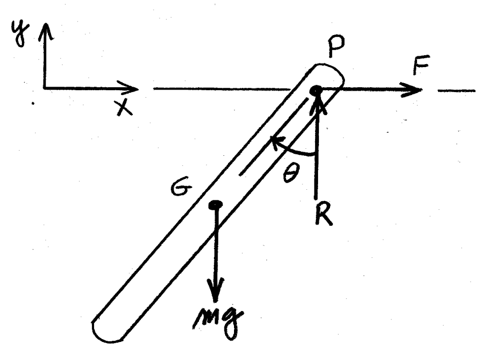


Figure : for Problem 2.11

Problem 2.12 a) $2kx = mg$, so $x = mg/2k$.

b)

$$T_0 = 0 \qquad V_0 = \frac{1}{2}k(0.06)^2$$

$$T_1 = \frac{1}{2}mv^2 \qquad V_1 = \frac{1}{2}k[0.06 + 2(0.09)]^2 - mg(0.09)$$

From conservation of energy,

$$T_0 + V_0 = T_1 + V_1$$

Thus

$$\frac{1}{2}mv^2 = T_0 + V_0 - V_1 = \frac{1}{2}k(0.06)^2 - \frac{1}{2}k[0.06 + 2(0.09)]^2 + mg(0.09)$$

Solve for v using $m = 30$ and $k = 600$ to obtain $v = 0.828$ m/s.

Problem 2.13 a) From conservation of energy,

$$T_0 + V_0 - W_{01} = T_1 + V_1$$

Thus

$$0 + mgd \sin\theta - \mu mgd \cos\theta = \frac{1}{2}mv^2 + 0$$

Thus

$$v = \sqrt{2gd(\sin\theta - \mu\cos\theta)} = 2.04 \ \ \mathrm{m/s}$$

b) From conservation of energy,

$$T_1 + V_1 - W_{12} = T_2 + V_2$$

Thus

$$\frac{1}{2}mv_1^2 + mgx \sin\theta - \mu mgx \cos\theta = 0 + \frac{1}{2}kx^2$$

This gives

$$6.2421 + 20.807x - 8.3228x = 500x^2$$

which has the roots $x = 0.1249$ and $x = -0.0999$. Choosing the positive root, we see that the spring is compressed by 0.1249 m.

Problem 2.14

$$W = fx - \frac{1}{2}k\left[(x+x_0)^2 - x_0^2\right] - mgx\,\sin\,\theta$$

or

$$W = 400(2) - \frac{1}{2}44\left[(2.5)^2 - (0.5)^2\right] - 11(9.81)2(0.5) = 560.09 \quad \mathrm{N}\cdot\mathrm{m}$$

Problem 2.15 Let x_1 be the initial stretch in the spring from its free length. Then

$$(L + x_1)^2 = (D_1 - r)^2 + D_2^2$$

where r is the radius of the cylinder. This gives $x_1 = 1.828$ m.

From conservation of energy,

$$T_1 + V_1 = T_2 + V_2$$

or

$$0 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}kx_2^2$$

where $v = R\omega$ and $x_2 = D_1 - r - L = 1$ m. This gives

$$0 + 41.77 = \frac{1}{2}10(0.25\omega)^2 + \frac{1}{2}0.4\omega^2 + 12.5$$

or $\omega = 7.56$ rad/s.

Problem 2.16 The velocity of the mass is zero initially and also when the maximum compression is attained. Therefore $\Delta T = 0$ and we have $\Delta T + \Delta V = \Delta T + \Delta V_s + \Delta V_g = 0$ or $\Delta V_s + \Delta V_g = 0$. That is, if the mass is dropped from a height h above the middle spring and if we choose the gravitational potential energy to be zero at that height, then the maximum spring compression x can be found by adding the change in the gravitational potential energy $0 - W(h+x) = -W(h+x)$ to the change in potential energy stored in the springs. Thus, letting $W = mg$,

$$\frac{1}{2}k_1(x^2 - 0) + [0 - W(h+x)] \qquad \text{if } x < d$$

where d is the difference in the spring lengths ($d = 0.1$ m). This gives the following quadratic equation to solve for x:

$$\frac{1}{2}k_1 x^2 - Wx - Wh = 0 \qquad \text{if } x < d \qquad (1)$$

If $x \geq d$, $\Delta V_s + \Delta V_g = 0$ gives

$$\frac{1}{2}k_1(x^2 - 0) + \frac{1}{2}(2k_2)\left[(x-d)^2 - 0\right] + [0 - W(h+x)] = 0 \qquad \text{if } x \geq d$$

which gives the following quadratic equation to solve for x.

$$(k_1 + 2k_2)x^2 - (2W + 4k_2 d)x + 2k_2 d^2 - 2Wh = 0 \qquad \text{if } x \geq d \qquad (2)$$

For the given values, equation (1) becomes

$$10^4 x^2 - 200(9.81)x - 200(9.81)(0.75) = 0 \qquad \text{if } x < 0.1$$

which has the roots $x = 0.494$, which is greater than 0.1, and $x = -0.2978$, which is not feasible. Thus, since there is no solution for which $x < 0.1$, the side springs will also be compressed. From equation (2)

$$2.6 \times 10^4 x^2 - (1962 + 3200)x + 160 - 1471.5 = 0$$

which has the solutions: $x = 0.344$ and $x = -0.146$. We discard the second solution because it is negative. So the outer springs will be compressed by $0.344 - 0.1 = 0.244$ m and the middle spring will be compressed 0.344 m.

Problem 2.17 From conservation of angular momentum,

$$H_{O1} = H_{O2}$$

or

$$mv_1(0.5) = \left[I_G + m_2(1)^2 + m_1(0.5)^2\right]\omega$$

or

$$0.8(10)v_1(0.5) = \left[\frac{1}{12}(4)(1)^2 + 4 + 0.8(0.5)^2\right]\omega$$

Solve for ω:

$$\omega = 0.882 \quad \text{rad/s}$$

Problem 2.18 From the figure,

$$v_{B1} = 10 \cos 30^\circ$$

$$e = 1 = \frac{v_{A2} - v_{B2}}{v_{B1} - v_{A1}} = \frac{0.6\omega - v_{B2}}{v_{B1} - 0}$$

Thus

$$v_{B1} = 10 \cos 30^0 = 0.6\omega - v_{B2}$$

and

$$v_{B2} = 0.6\omega - 8.66$$

From conservation of momentum,

$$H_{O1} = H_{O2}$$

or

$$m_1 v_{B1}(0.6) = I_O\omega + m_1 v_{B2}(0.6)$$

where

$$I_O = I_G + m_2(0.6)^2 = \frac{1}{12}m_2(0.8)^2 + 4(0.36) = 1.653$$

Thus

$$0.8(8.66)(0.6) = 1.653\omega + 0.8(0.6)(0.6\omega - 8.66)$$

This gives

$$\omega = 4.28 \ \ \text{rad/s}$$

Problem 2.19 From conservation of angular momentum,

$$H_{O1} = H_{O2}$$

or

$$mv_1(2L) = \left[I_G + m_2L^2 + m_1(2L)^2\right]\omega$$

where

$$I_G = \frac{1}{12}m_2(2L)^2 = \frac{1}{12}4.5(2.4)^2 = 2.16$$

Thus

$$0.005(365)(2.4) = \left[2.16 + 4.5(1.2)^2 + 0.005(2.4)^2\right]\omega$$

Solve for ω:

$$\omega = 0.505 \quad \text{rad/s}$$

Problem 2.20 Let f_n be the reaction force on the pendulum from the pivot in the normal direction. When $\theta = \pi/2$, f_n is vertical and is positive upward. Let f_t be the reaction force on the pendulum from the pivot in the tangential direction. When $\theta = \pi/2$, f_t is horizontal and is positive to the right.

Summing moments about the pivot at point O gives the equation of motion of the pendulum.

$$I_O\ddot{\theta} = mg\frac{L}{2}\cos\theta$$

Summing moments about the mass center gives

$$I_G\ddot{\theta} = f_t\frac{L}{2}$$

Comparing these two expressions, we find that

$$f_t = \frac{mgI_G}{I_O}\cos\theta$$

Thus the tangential reaction force is $f_t = 0$ when $\theta = \pi/2$.

To compute the normal reaction force, sum forces in the vertical direction to obtain

$$ma_n = f_n - mg$$

where the normal acceleration is given by the following expression for circular motion.

$$a_n = \frac{L}{2}\omega^2$$

Thus

$$f_n = mg + ma_n = mg + m\frac{L}{2}\omega^2$$

We can compute ω either from energy conservation or by integrating the equation of motion. With the energy method we use the fact that the kinetic energy at $\theta = 90°$ equals the original potential energy at $\theta = 0$. Thus

$$\frac{1}{2}I_O\omega^2 = mg\frac{L}{2}$$

Since $I_O = mL^2/3$ for a slender rod, this gives $\omega^2 = 3g/L$.

(Continued on the next page)

Problem 2.20 (continued)

To integrate the equation of motion, note that

$$\ddot{\theta} = \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \omega\frac{d\omega}{d\theta}$$

Thus

$$\omega\frac{d\omega}{d\theta} = \frac{mgL}{2I_O}\cos\theta$$

or

$$\omega\, d\omega = \frac{mgL}{2I_O}\cos\theta\, d\theta$$

Integrating gives

$$\int_0^\omega \omega d\omega = \frac{mgL}{2I_O}\int_0^{\pi/2}\cos\theta\, d\theta = \frac{mgL}{2I_O}\sin\frac{\pi}{2} = \frac{mgL}{2I_O}$$

This gives

$$\frac{\omega^2}{2} = \frac{mgL}{2I_O}$$

Thus the normal reaction force is

$$f_n = mg + m\frac{L}{2}\omega^2 = m\left(g + \frac{mgL^2}{I_O}\right)$$

For a slender rod,

$$I_O = \frac{1}{3}mL^2$$

Thus

$$f_n = m\left(g + \frac{3g}{2}\right) = 2.5mg$$

which is 2.5 times the weight of the rod.

Problem 2.21 The equivalent mass is $m_e = m_w + I_w/R^2 = 80 + 3/(0.3)^2 = 113.33$ kg. The equation of motion is $m_e\dot{v} = 400 - m_w g \sin 25° = 68.33$ or $\dot{v} = 0.603$. Thus $v(t) = 0.603t$ and $v(60) = 36.18$ m/s. In addition, $\omega(60) = v(60)/R = 36.18/0.3 = 120.6$ rad/s.

Problem 2.22 The equivalent mass is $m_e = 30 + 0.23(15) = 33.45$ kg. From statics, $k(0.003) = 30(9.81)$, or $k = 9.81 \times 10^4$ N/m.

Problem 2.23 The equivalent mass is $m_e = 10 + 0.38(2) = 10.76$ kg. From statics, $k(0.02) = 10(9.81)$ or $k = 4905$ N/m.

Problem 2.24 Equate the kinetic energies of m and the fictitious inertia I, and note that $\dot{x} = R\dot{\theta}$.

$$\text{kinetic energy} = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}I\dot{\theta}^2$$

Thus

$$I = m\left(\frac{\dot{x}}{\dot{\theta}}\right)^2 = mR^2$$

Problem 2.25 Let r_2 be the radius of pulley 2. The equivalent inertia felt on shaft 1 is

$$I_e = I_1 + \frac{1}{N^2} I_2 + \frac{1}{N^2} m_2 r_2^2 + \frac{1}{N^2} m_3 r_2^2$$

With $N = 2$,

$$I_e = I_1 + \frac{1}{4}\left(I_2 + m_2 r_2^2 + m_3 r_2^2\right)$$

The equation of motion is

$$I_e \dot{\omega}_1 = T_1 - \frac{m_2 g r_2}{N} + \frac{m_3 g r_2}{N} = T_1 - \frac{g r_2}{2}\left(m_3 - m_2\right)$$

Problem 2.26 With $I_1 = I_2 = I_3 = 0$, the total kinetic energy is

$$KE = \frac{1}{2} I_4 \omega_1^2 + \frac{1}{2} I_5 \omega_3^2$$

Substituting $\omega_2 = 1.4\omega_3$ and $\omega_1 = 1.4\omega_2 = (1.4)^2\omega_3 = 1.96\omega_3$, and $I_4 = 0.02$, $I_5 = 0.1$, we obtain

$$KE = \frac{1}{2}(0.177)\omega_3^2$$

and the equivalent inertia is $I_e = 0.177$ kg·m^2.

The equation of motion is $I_e \dot{\omega}_3 = (1.4)^2 T$, or $0.177\dot{\omega}_3 = 1.96T$

Problem 2.27 Note that $y = x/2$. Assuming that x and y are measured from the equilibrium positions, the static spring forces will cancel the gravity force mg. Let T be the dynamic tension in the cable. Then

$$m\ddot{x} = -T$$

and $2T = ky = kx/2$. Thus

$$m\ddot{x} = -\frac{k}{4}x$$

Problem 2.28 Assume that x and y are measured from the equilibrium positions. Note that the simultaneous translation and rotation of the pulley reduces the stretch in spring k_2 by $2y$; that is, by the pulley translation distance y and the cable motion y due to the rotation. Thus the dynamic stretch in spring k_2 is $x - 2y$.

Let T be the tension force in the cable, which must equal the force in spring k_2. Thus

$$T = k_2(x - 2y)$$

The dynamic force balance on the massless pulley gives

$$k_1 y = k_2(x - 2y) + T = 2k_2(x - 2y)$$

Thus

$$y = \frac{2k_2}{k_1 + 4k_2} x$$

Also

$$m\ddot{x} = -T = -k_2(x - 2y) = -\frac{k_1 k_2}{k_1 + 4k_2} x$$

The equation of motion is

$$m\ddot{x} + \frac{k_1 k_2}{k_1 + 4k_2} x = 0$$

Problem 2.29 Assuming that x is measured from the equilibrium position, the static spring force will cancel the gravity force $mg\ \sin\theta$. Let T_1 be the dynamic tension force in the lower cable that is attached to the cart, and let T_2 be the dynamic spring force.

From the massless-pulley relation, $T_1 = 2T_2$. From Newton's law applied to the cart,

$$m\ddot{x} = -T_1 - T_2 = -3T_2$$

Because only one of the pulleys translates, the dynamic spring stretch is $x - x/2 = x/2$ and the dynamic spring force is $T_2 = kx/2$. Thus

$$m\ddot{x} = -\frac{3}{2}kx$$

Problem 2.30 Assuming that x is measured from the equilibrium position, the moment about the pivot due to the static spring force will cancel the gravity moment mgD_1. Summing the dynamic moments about the pivot point, we obtain

$$mD_1^2\ddot{\theta} = -k(D_2\theta)D_2$$

where $\theta = x/D_1$. Thus

$$mD_1^2\ddot{x} = -kD_2^2 x$$

Problem 2.31 Assuming that x is measured from the equilibrium position, the moment about the pulley center due to the static spring force will cancel the gravity moment mgR_1. Treating the entire system as a single mass and summing the dynamic moments about the pulley center, we obtain

$$mR_1^2\ddot{\theta} = -k(R_2\theta)R_2 = -kR_2^2\theta$$

where $\theta = x/R_1$. Thus

$$mR_1^2\ddot{x} = -kR_2^2x$$

Problem 2.32 a) Newton's law for rotation applied to a fictitious inertia I located at the pivot gives

$$I\ddot{\theta} = fL_1 - k_1(L_2\theta)L_2 - k(L_3\theta)L_3$$

Set $I = 0$ to obtain the model.

$$fL_1 - k_1L_2^2\theta - kL_3^2\theta = 0$$

Note that the L_2^2 and L_3^3 terms are due to the fact that the translational displacement of the springs are $L_2\theta$ and $L_3\theta$. Thus the spring forces are $kL_2\theta$ and $kL_3\theta$. The moments due to these forces are obtained by multiplying by the moment arms L_2 and L_3.

b) From Newton's law for rotation:

$$I\ddot{\theta} = \sum \text{moments about the pivot} = fL_1 - k(L_2\theta)L_2 - c(L_2\dot{\theta})L_2$$

or

$$I\ddot{\theta} + cL_2^2\dot{\theta} + kL_2^2\theta = fL_1$$

where $\omega = \dot{\theta}$. Note that the L_2^2 terms are due to the fact that the translational displacement of the spring is $L_2\theta$ and the translational velocity of the damper is $L_2\dot{\theta}$. Thus the spring force is $k(L_2\theta)$ and the damper force is $c(L_2\dot{\theta})$. The moments due to these forces are obtained by multiplying by the moment arm L_2.

Problem 2.33 a) Let f_c be the force in the cable. Assume that the cable does not slip on the pulley. Note that for the massless pulley 2, $kx_2 = 2f_c$ and $x_1 = 2x_2$. From Newton's law,

$$m_1\ddot{x}_1 = f - f_c$$

Substituting for f_c and x_2, we obtain

$$m_1\ddot{x}_1 = f - \frac{k}{4}x_1$$

b) Let f_{c1} be the force in the cable on the m_1 side of pulley 1, and f_{c2} be the force in the cable on the pulley 2 side of pulley 1. Assume that the cable does not slip on the pulley. Newton's law applied to m_1 gives

$$m_1\ddot{x}_1 = f - f_{c1} \qquad (1)$$

Summing moments about the axis of rotation of pulley 1, we obtain

$$I_p\dot{\omega} = (f_{c1} - f_{c2})R \qquad (2)$$

Note that for the massless pulley 2, $kx_2 = 2f_{c2}$ and $x_1 = 2x_2$. Solve (2) for f_{c1}:

$$f_{c1} = f_{c2} + \frac{I_p}{R}\dot{\omega} = \frac{k}{2}x_2 + \frac{I_p}{R}\dot{\omega} = \frac{k}{4}x_1 + \frac{I_p}{R}\dot{\omega}$$

Substitute the above expression for f_{c1} into (1) to obtain

$$m_1\ddot{x}_1 = f - \frac{k}{4}x_1 - \frac{I_p}{R}\dot{\omega} \qquad (3)$$

Note that $R\omega = \dot{\ }_1x$ Thus $\dot{\omega} = \ddot{x}_1/R$. Substitute this into (3) to obtain

$$\left(m_1 + \frac{I_p}{R^2}\right)\ddot{x}_1 + \frac{k}{4}x_1 = f$$

Problem 2.34 Assume that x is measured from the equilibrium position. Note that the total dynamic stretch in the spring is $3x$. To see this, note that if the wheel moves down a distance x, the spring stretches by $2x$, But if the wheel rotates without slipping, the additional stretch caused by the rotation is $R\theta = x$. Thus the total dynamic stretch in the spring is $2x + x = 3x$.

Using the energy approach, we have

$$KE + PE = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}k(3x)^2$$

Since $\omega = \dot{x}/R$, this becomes

$$KE + PE = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\frac{\dot{x}^2}{R^2} + \frac{1}{2}9kx^2$$

Note that gravitational potential energy is not used because x is measured from equilibrium. Differentiating the previous equation and canceling $\dot{x}$ gives

$$\left(m + \frac{I}{R^2}\right)\ddot{x} + 9kx = 0$$

Problem 2.35 Let T be the tension in the cable acting on mass m_1 (positive to the right). Then, assuming that x_1 and x_2 are measured from the equilibrium positions, we have

$$m_1\ddot{x}_1 = -T - kx_1$$

and

$$m_2\ddot{x}_2 = -2T$$

Eliminate T from these equations and use the fact that $x_1 = -2x_2$ to obtain

$$(4m_1 + m_2)\ddot{x}_2 + 4kx_2 = 0 \qquad (1)$$

Using the energy approach instead, we have (since $x_1 = -2x_2$ and $\dot{x}_1 = -2\dot{x}_2$)

$$KE + PE = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}\left(4m_1 + m_2\right)\dot{x}_2^2 + \frac{1}{2}4kx_2^2$$

Note that gravitational potential energy is not used because x_1 and x_2 are measured from equilibrium. Differentiating the previous equation and canceling $\dot{x}_2$ gives Equation (1).

Problem 2.36 Assume that x and y are measured from the equilibrium positions. Note that $\dot{x} = -3\dot{y}$, we have

$$KE + PE = \frac{1}{2}m_A\dot{x}^2 + \frac{1}{2}m_B\dot{y}^2 + \frac{1}{2}kx^2 = \frac{1}{2}\left(m_A + \frac{m_B}{9}\right)\dot{x}_2^2 + \frac{1}{2}kx^2$$

Note that gravitational potential energy is not used because x and y are measured from equilibrium. Differentiating the previous equation and canceling $\dot{x}$ gives

$$\left(m_A + \frac{m_B}{9}\right)\ddot{x} + kx = 0$$