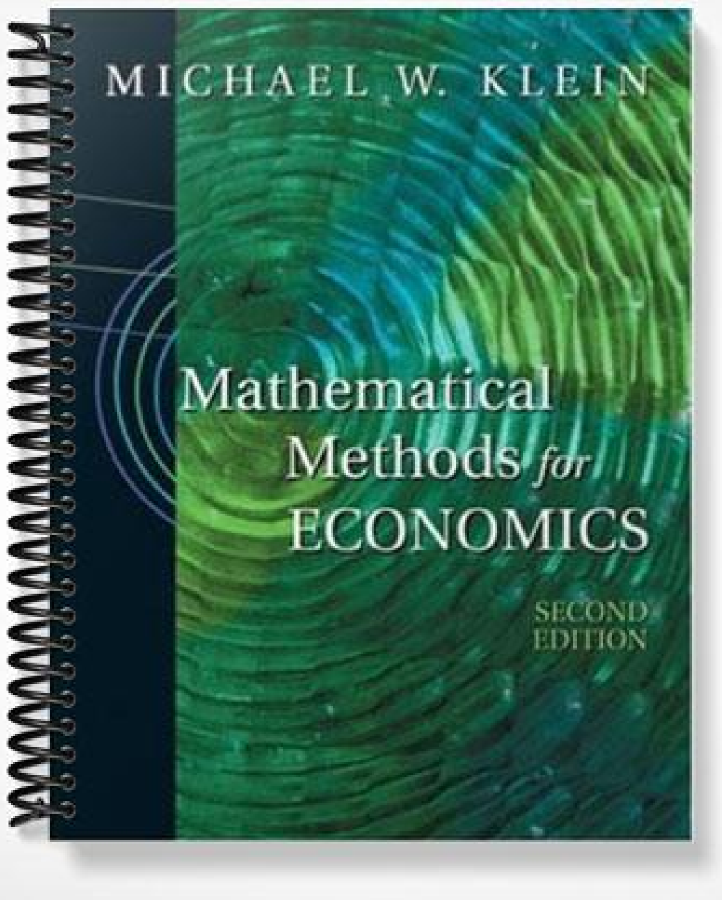
SOLUTIONS MANUAL



Preface

This is a supplement to *Mathematical Methods for Economics,* 2^{nd} edition, by Michael W. Klein.

This manual provides answers to all the end-of-section exercises in the text. Solving the end-of-section exercises is an integral part of the learning process for a course using this textbook. The exercises in *Mathematical Methods for Economics* are designed to develop students' economic and mathematical intuition as well as to hone their skills at practical problem solving. The end-of-section exercises also offer students some additional economic applications. Deirdre M. Savarese developed most of the end-of-section exercises in *Mathematical Methods for Economics*, and her assistance is gratefully acknowledged.

This manual also offers concordances to aid instructors who would like to supplement their microeconomic or macroeconomic courses with *Mathematical Methods for Economics*. The concordances list, for seven leading microeconomic and macroeconomic textbooks, the mathematical tools required in specific chapters of those books and the place where those tools are presented in *Mathematical Methods for Economics*.[1] The concordances also list applications in *Mathematical Methods for Economics* that instructors can use when teaching material from specific chapters in these leading macroeconomic and microeconomic textbooks. A conversion chart is also presented to aid instructors who want to switch from another mathematics for economics textbook to *Mathematical Methods for Economics.* Deirdre M. Savarese put together the concordances and the conversion chart for the 1^{rst} edition of *Mathematical Methods for Economics* and Kieran Brenner updated this material for the 2^{nd} edition.

This manual also offers reproductions of figures from *Mathematical Methods for Economics* and these reproductions can serve as transparency masters. Graphs are important pedagogic tools in a course like this one. Instructors may find transparencies more accurate, easier to use, and less time-consuming to present than figures drawn on a chalkboard in the midst of class.

[1]There are concordances for microeconomic texts by Perloff, Pindyck and Rubinfeld, and Varian, and for macroeconomic texts by Abel and Bernanke, Blanchard, Gordon, and Mankiw.

Chapter 2

Section 2.1

1. The intervals are

   (a) $(-5, 0)$
   (b) $[-5, 0)$
   (c) $(-\infty, 100)$
   (d) $(-\infty, 100]$
   (e) $(0, \infty)$
   (f) $(-\infty, \infty)$

2. Do the following represent a function?

   (a) Yes
   (b) No
   (c) Yes
   (d) Yes
   (e) No
   (f) The function y is not defined at $x = 3$.
   (g) No

3. Can the function be defined according to the mapping?

   (a) Yes
   (b) Yes
   (c) No
   (d) No

4. It would not be a function since there would be a mapping between one argument and more than one value.

5. With this cost function

   (a) When $Q = 10, TC = 125$. When $Q = 25, TC = 200$. When there is no production $(Q = 0)\,, TC = 75$.

   (b) Linear graph.

   (c) Domain [0.50] Range $[75, 325]$

6. Four representative ordered pairs are

   (a) $(-2, 140)\,, (-1, 120)\,, (0, 100)\,, (1, 80)$

   (b) $(-1, 2)\,, (0, 0)\,, (1, -2)\,, (2, -10)$

   (c) $(-100, 101)\,, (0, 1)\,, (1, 2)\,, (100, 101)$

7. The limits are

   (a) $\lim\limits_{x \to \infty} = 2$

   (b) $\lim\limits_{x \to 7^+} = \infty$

   (c) $\lim\limits_{x \to 7^+} = 7$

   (d) $\lim\limits_{x \to 1} = 0$

8. Are the functions continuous?

   (a) No

   (b) Yes

   (c) Yes

   (d) Yes

9. The function presented in 8(c)

$$y = -3 + \frac{1}{x + 7}$$

   is not continuous over the domain $(-\infty, 0]$ since the function is undefined at the point $x = -7$.

Section 2.2

1. The respective functions are

   (a) strictly monotonic
   (b) nonmonotonic
   (c) strictly monotonic
   (d) monotonic

2. Are these one-to-one?

   (a) no. An individual may be a citizen of more than one country
   (b) no. While a street address has one zip code, a given zip code applies to more than one street address.
   (c) yes
   (d) no. While one identification number is linked with one course grade, a given course grade may correspond to numerous students' identification numbers.

3. The inverse functions, if they exist, are

   (a) $x = f^{-1}(y) = \frac{y-14}{7}$
   (b) This function does not have an inverse unless we restrict the domain to $y \geq 6$ so that the inverse is $x = f^{-1}(y) = \sqrt{y-6}$
   (c) This function does not have an inverse unless we restrict the domain to $y \geq 0$ so that the inverse is $x = f^{-1}(y) = y^2$
   (d) $x = f^{-1}(y) = y^{\frac{1}{3}}$

4. The inverse is $x = \frac{y}{10} + \frac{1}{2}$. To check this, note that

$$\frac{10x-5}{10} + \frac{1}{2} = x \quad \text{and} \quad 10\left(\frac{y}{10} + \frac{1}{2}\right) - 5 = y.$$

5. For continuous functions with extreme points, the answers are

   (a) No

(b) Yes

(c) Either two minima and one maximum or two maxima and one minimum

6. This is a linear function so the global minimum and maximums are the respective endpoints. The global minimum is $(0, 50)$ and the global maximum is $(100, 100)$

7. The average rates of return are

   (a) Average rate of return $= 12$

   (b) Average rate of return$= 16$

   (c) Average rate of return $= -12$

   (d) Average rate of return $= 0$

8. For a strictly increasing function, $f(x_B) > f(x_A)$ and $x_B > x_A$, so the average rate of change will always be positive since both the numerator and the denominator yield a positive answer. For a strictly decreasing function, the numerator will be negative and the denominator will be positive.

9. The answers are

   (a) $y' = 5$

   (b) The slope of the secant line $\frac{f(x_B)-f(x_A)}{x_B-x_A} = -4$. The value of the slope of the secant line represents the average rate of change of a function over the interval defined by the two endpoints of the secant line.

   (c) The function is strictly convex over the given interval since the secant line lies wholly above the function.

10. The answers are

   (a) $x' = 2.8$; $f(x') = 28.16$

   (b) $y' = 26$

   (c) The function is strictly concave since $f(x') > y'; 28.16 > 26$

11. The answers are

    (a) $A \Leftarrow B$
    (b) $A \Leftarrow B$
    (c) $A \Rightarrow B$
    (d) $A \Leftrightarrow B$ This set satisfies the necessary and sufficient condition

Section 2.3

1. These expressions can be written as

    (a) x^{-1}
    (b) $(xy)^4$
    (c) x^{20}
    (d) x^3y^2 (no further simplification possible)
    (e) $\left(\frac{1}{xy}\right)^6$

2. These expressions can be condensed as

    (a) $x^{(a+b+c-d)}$
    (b) $x^{\frac{5}{3}}$
    (c) $x^{\frac{31}{12}}$
    (d) x^2y^2

3. Simplifying the expressions, we get

    (a) 8
    (b) $2^{12} = 4096$
    (c) $\frac{1}{2}$
    (d) $\frac{x+3}{x+2}$
    (e) $(x+1)^{25}$

4. The quadrants through in which the graphs of the functions appear are
   (a) Quadrants I and II
   (b) Quadrants I and III
   (c) Quadrants I and II
   (d) Quadrants I and III

5. The roots are
   (a) $x = \frac{6}{5}$
   (b) $x = 1, -6$
   (c) $x = -3$ (two equal roots)
   (d) $f(x) = (x+1)(x^2 - 3x + 2) = (x+1)(x-1)(x-2) = 0$ so $x = (-1, 1, 2)$

6. The root is $x = \frac{-q \pm \sqrt{q^2 - pr}}{p}$

7. Some points of these functions are

x	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$2x^2$	0	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{9}{8}$	2
2^x	1	1.19	1.41	1.68	2

   The two functions do not share a y-intercept but do have a common value when $x = 1$.

8. The average rate of change for the function $y = 2x^2$ is 10. The average rate of change for the function $y = 2^x$ is $\frac{14}{3}$. Both functions are strictly convex such that $f(x') < y'$.

9. Some points of these functions are

x	0	1	2	3	4
$\left(\frac{1}{2}\right) 2^x$	$\frac{1}{2}$	1	2	4	8
$\left(\frac{1}{2}\right) 4^x$	$\frac{1}{2}$	2	8	32	128

   The two curves share a y-intercept since in both cases when $x = 0$, $y = \frac{1}{2}$. When $A = \frac{1}{4}$, the curves no longer share a common y value since the graph of the first function shifts down.

10. When the domain of x is restricted, some points of these functions are

x	-2	-1	0
$\left(\frac{1}{2}\right)2^x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$
$\left(\frac{1}{2}\right)4^x$	$\frac{1}{32}$	$\frac{1}{8}$	$\frac{1}{2}$

The functions are graphed in Quadrant II. The y-intercept is still at $\frac{1}{2}$ but the curve slopes down and to the left, asymptotically approaching the x-axis.

11. Matching the functions to the economic relations, we have

    (a) $a = iii$

    (b) $b = v$

    (c) $c = i$

    (d) $d = ii$

    (e) $e = iv$

12. a. There are two tax rates consistent with raising \$60 billion, $t = 0.3$, $t = 0.4$. These are found by using the quadratic formula.

$$t_1, t_2 = \frac{-350 \pm \sqrt{(350)^2 - 4 \cdot (-60) \cdot (-500)}}{2 \cdot (-500)}$$

b. There is one tax rate consistent with raising \$61.25 billion, $t = 0.35$, since, using quadratic formula.

$$t = \frac{-350 \pm \sqrt{(350)^2 - 4 \cdot (-61.25) \cdot (-500)}}{2 \cdot (-500)} = \frac{-350}{2 \cdot (-500)} = 0.35$$

This is a case of a single root to a quadratic equation.