MANAGERIAL ECONOMICS
sixth edition

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CHAPTER TWO

OPTIMAL DECISIONS USING
MARGINAL ANALYSIS

OBJECTIVES

1. To introduce the basic economic *model* of the firm (pp. 29-30)
    - The main focus is on determining the firm's profit-maximizing level of output.
    - The main assumption is that there is a single product (or multiple, independent products) with *deterministic* demand and cost.

2. To depict the behavior of price, revenue, cost, and profit as output varies. (pp. 31-36)

3. To explain the notion of marginal profit (including its relationship to calculus) and show that maximum profit occurs at an output such that marginal profit equal zero. (pp. 36-40)

4. To reinterpret the optimality condition in terms of the basic components, marginal revenue and marginal cost. (pp. 40-45)

5. To illustrate the uses of sensitivity analysis (pp. 46-49)

TEACHING SUGGESTIONS

I. **Introduction and Motivation**

A. This is a *"nuts and bolts"* chapter. Because it appears up front in the text, it's important to explain the motivation and assumptions. It is a good idea to remind students of the following points.

1) *The model of the firm is deliberately simplified so that its logic is laid bare.* Many additional complications will be supplied in later chapters. The key simplifications for now are:

- The model is of a generic firm. Although microchips are chosen to make the discussion concrete, there is no description of the kind of market or the nature of competition within it. The description and analysis of different market structures comes in Chapters 7 through 10.

- Profit is the sole goal of the firm; price and output are the sole decision variables.

- The description of demand and cost is as "bare bones" as it gets. The demand curve and cost function are taken as given. (How the firm might estimate these are studied in Chapters 4 through 6.)

B. In general, our policy is to use extended decision examples, *different than the ones in the text,* to illustrate the most important concepts. (Going over the same examples pushes the boredom envelope.) In the present chapter, we make an exception to this rule. It is important to make sure that students with different economic and quantitative backgrounds all get off roughly on the same foot. Reviewing a familiar example (microchips) makes this much easier.

II. **Teaching the "Nuts and Bolts"**

A. **Graphic Overview**. The text presents the revenue, cost, and profit functions in three equivalent forms: in tables, in graphs, and in equations. In our view, the best way to convey the logic of the relationships is via *graphs*. (The student who craves actual numbers can get plenty of them in the text tables.) Here is one strategy for teaching the nuts and bolts:

1. Using the microchip example, depict the demand curve, briefly note its properties and demand equation (in both forms).

2. Next focus on revenue, noting the tradeoff between price and quantity. Present and justify the revenue equation. Graph it and note its properties.

3. Repeat the same process with the cost function (reminding students about fixed versus variable cost). At this point, your blackboard graph should be a copy of Figure 2.8 (p. 44). Steps 1-3 should take no more than 20 minutes.

4. Since the gap between the revenue and cost curves measures profit, one could find the optimal output by carefully measuring the maximum gap (perhaps using calipers). Emphasize that marginal analysis provides a much easier and more insightful approach. Point out the economic meaning of marginal cost and marginal revenue. Note that they are the slopes of the respective curves.

5. Next argue (as on p. 45) that the profit gap increases (with additional output) when MR > MC but narrows when MR < MC. (On the graph, select quantities that are too great or too small to make the point.) Identify Q* where the tangent to the revenue curve is parallel to the slope of the cost function. In short, optimal output occurs where MR = MC.

B. **Other Topics**. The approach in part A provides a simple way of conveying the basic logic of marginal analysis using the components of MR and MC. Once this ground is covered, the instructor should emphasize other basic points:
1. The equivalence between $M\pi = 0$ and MR = MC.
2. Calculus derivations of $M\pi$, MR, and MC.
3. The exact numerical solution for the microchip example.
4. The graphs of MR and MC and an exploration of comparative statics effects (shifts in the curves) and the effects on Q*.

C. **Applications**. Besides the applications in the text (pp. 45-49), the following problems are recommended: Problem 1 (a quick but important check), Problems 6, 7 and 9 (numerical applications), Problem 13 (the general solution), and Problem 14. (If the class has a good grasp of this last problem, nothing else will seem difficult.) The following question gets students thinking:

1. a. For five years, an oil drilling company has profitably operated in the state of Alaska (the only place it operates). Last year, the state legislature instituted a flat annual tax of $100,000 on any company extracting oil (or natural gas) in Alaska. How would this tax affect the amount of oil the company extracts? Explain.

   b. Suppose instead that the state imposes a well-head tax, let's say a tax of $10.00 on each barrel of oil extracted. Answer the questions of part a.

   c. Finally, suppose that the state levies a proportional income tax (say 10% of net income). Answer the questions of part a. What would be the effect of a progressive tax?

   d. Now suppose that the company has a limited number of drilling rigs extracting oil at Alaskan sites *and* at other sites in the United States. What would be the effect on the company's oil output in Alaska if the state levied a proportional income tax as in part c?

Answer.

a. This tax acts as a fixed cost. As long as it remains profitable to produce in Alaska, the tax has no effect on the firm's optimal output.

b. The well-head tax increases the marginal cost of extraction by $10.00 per barrel. The upward shift in MC means the new intersection of MR and MC occurs at a lower optimal level of output.

c.	The income tax (either proportional or progressive) has no effect on the company's optimal output. For instance, suppose that the company's after-tax income is π = .9(R-C) under a 10% proportional tax. To maximize its after-tax income, the best the company can do is to continue to maximize its before-tax income. Another way of seeing this is to note that the tax causes a 10% downward shift in the firm's MR and MC curves. With the matching shift, the new intersection of MR and MC is at the same optimal quantity as the old intersection.

d.	When the firm operates in multiple states with limited drilling rigs, using a rig in Alaska means less oil is pumped (and lower profit is earned) somewhere else. There is an opportunity cost to Alaskan drilling. Thus, one can argue that before the tax, the company should have allocated rigs so as to equate marginal profits in the different states. With the tax, the marginal profit in Alaska is reduced, prompting the possible switch of rigs from Alaska to other (higher marginal profit) locations.

D. Mini-case: Apple Computer in the Mid-1990s

The mini-case reproduced on the next page provides a hands-on application of profit maximization and marginal analysis.

Answer
	a. Clearly, the period 1994-1995 was marked by a significant adverse shift in demand against Apple due to *major enhancements of competing computers*: lower prices, better interfaces (Windows), sales to order (Dell), and more abundant software.
	b. Setting MR = MC implies 4,500 - .3Q = 1,500, so Q* = 10,000 units and P = $3,000. Given 1994's state of demand, Apple's 1994 production strategy was indeed optimal.
	c. In 1995, demand and MR have declined significantly. Now, setting MR = MC implies 3,900 - .3Q = 1,350, so Q* = 8,500 units and P = $2,625. Apple should cut its price and its planned output.

Apple Computer in the Mid 90s

Between 1991 and 1994, Apple Computer engaged in a holding action in the desktop market dominated by PCs using Intel chips and running Microsoft's operating system.[1] In 1994, Apple's flagship model, the Power Mac, sold roughly 10,000 units per month at an average price of $3,000 per unit. At the time, Apple claimed about a 9% market share of the desktop market (down from greater than 15% in the 1980s).

By the end of 1995, Apple had witnessed a dramatic shift in the competitive environment. In the preceding 18 months, Intel had cut the prices of its top-performing Pentium chip by some 40%. Consequently, Apple's two largest competitors, Compaq and IBM, reduced average PC prices by 15%. Mail-order retailer Dell continued to gain market share via aggressive pricing. At the same time, Microsoft introduced Windows 95, finally offering the PC world the look and feel of the Mac interface. Many software developers began producing applications only for the Windows operating system or delaying development of Macintosh applications until months after Windows versions had been shipped. Overall, fewer users were switching from PCs to Macs.

Apple's top managers grappled with the appropriate pricing response to these competitive events. Driven by the speedy new PowerPC chip, the Power Mac offered capabilities and a user-interface that compared favorably to those of PCs. Analysts expected that Apple could stay competitive by matching its rivals' price cuts. However, John Sculley, Apple's CEO, was adamant about retaining a 50% gross profit margin and maintaining premium prices. He was confident that Apple would remain strong in key market segments – the home PC market, the education market, and desktop publishing.

Questions.

1. What effect (if any) did the events of 1995 have on the demand curve for Power Macs? Should Apple preserve its profit margins or instead cut prices?

2. a) In 1994, the marginal cost of producing the Power Mac was about $1,500 per unit, and a rough estimate of the monthly demand curve was: P = 4,500 - .15Q. At the time, what was Apple's optimal output and pricing policy?

b) By the end of 1995, some analysts estimated that the Power Mac's user value (relative to rival PCs) had fallen by as much as $600 per unit. What does this mean for Apple's new demand curve at end-of-year 1995? How much would sales fall if Apple held to its 1994 price? Assuming a marginal cost reduction to $1,350 per unit, what output and price policy should Apple now adopt?

[1] This account is based on J. Carlton, "Apple's Choice: Preserve Profits or Cut Prices," *The Wall Street Journal*, February 22, 1996, p. B1.

ADDITIONAL MATERIALS

I.   Readings

R. Chittum, "Price Points," *The Wall Street Journal*, October 30, 2006, p. R7. (How providers of consumer services compare extra revenues and extra costs.)

S. Waxman, " 'Crash' Principals Still Await Payments for their Work," *The New York Times*, July 25, 2006, pp. B1, B8.

C. Oggier and E, Fragniere, and J. Stuby, "Nestle Improves its Financial Reporting with Management Science," *Interfaces*, July-August, 2005, pp. 271-280.

J. Thomas, W. Reinartz, and V. Kumar, "Getting the Most out of your Customers," *Harvard Business Review*, July-August, 2004, pp. 117-123.

A. M. Geoffrion and R. Krishnan, "Prospects for Operations Research in the E-Business Era," *Interfaces*, March-April, 2001, pp. 6-36.

R. Fourer and J. P. Goux, "Optimization as an Internet Resource," *Interfaces*, March-April, 2001, pp. 130-150.

D. Ekwurzel and J. McMillan, "Economics Online," *Journal of Economic Literature*, March 2001, pp. 7-10.

"Economics on the Net," *The Economist*, March 13, 1999, p. 7.

W. Biddle, "Skeleton Alleged in the Stealth Bomber's Closet," *Science,* May 12, 1989.

II.   Case

Colgate-Palmolive Co.: The Precision Toothbrush (9-593-064), Harvard Business School, 1993. Teaching Note (5-595-025).   (Explores profit analyses of alternative launch strategies.)

III. Quips and Quotes

Small mistakes are the stepping stones to large failures.

There was an old saying about our small town. Our town's population never changed. Every time a baby was born a man left town. (Does this say something about the balance of marginal changes at an optimum?)

The head of a small commuter plane service reported that as costs rose, the company's breakeven point rose from 6 to 8 to 11 passengers. "I finally figured we were in trouble since our planes only have 9 seats."

Quotes about economics in general:

If you laid all of the economists in the world end to end, they still wouldn't reach a conclusion. (George Bernard Shaw)

An economist is a person who is very good with numbers but who lacks the personality to be an accountant.

The age of chivalry is gone; that of sophisters, economists, and calculators has succeeded. (Edmund Burke)

Please find me a one-armed economist so we will not always hear, "On the other hand . . ." (Herbert Hoover)

2.   The revenue function is $R = 170Q - 20Q^2$. Maximizing revenue means setting marginal revenue equal to zero. Marginal revenue is: $MR = dR/dQ = 170 - 40Q$. Setting $170 - 40Q = 0$ implies $Q = 4.25$ lots. By contrast, profit is maximized by expanding output only to $Q = 3.3$ lots. Although the firm can increase its revenue by expanding output from 3.3 to 4.5 lots, it sacrifices profit by doing so (since the extra revenue gained falls short of the extra cost incurred.)

4. a. $\pi = PQ - C$
$$= (120 - .5Q)Q - (420 + 60Q + Q^2)$$
$$= -420 + 60Q - 1.5Q^2.$$
$M\pi = d\pi/dQ = 60 - 3Q = 0$
Solving yields $Q^* = 20$ units.

Revenue $= PQ = (120 - .5Q)Q = 120Q - .5Q^2$; $MR = 120 - Q$
Cost $= -420 + 60Q + Q^2$; therefore, $MC = 60 + 2Q$
Equating marginal revenue and marginal cost yields:
$120 - Q = 60 + 2Q$, or $Q^* = 20$ units.

  b. Here, $R = 120Q$; it follows that $MR = 120$
Equating marginal revenue and marginal cost yields
$120 = 60 + 2Q$ or $Q^* = 30$ units.

6. a. If videos are given away ($P = \$0$), demand is predicted to be: $Q = 1600 - (200)(0) = 1,600$. At this output, firm A's cost is: $1,200 + (2)(1,600) = \$4,400$, and firm B's cost is: $(4)(1,600) = \$6,400$. Firm A is the cheaper option and should be chosen. (In fact, firm A is cheaper as long as $Q > 600$.)

b. To maximize profit, we simply set MR = MC for each supplier and compare the maximum profit attainable from each. We know that MR = 8 - Q/100 and the marginal costs are $MC_A = 2$ and $MC_B = 4$. Thus, for firm A, we find: $8 - Q_A/100 = 2$, and so $Q_A = 600$ and $P_A =$ $5 (from the price equation). For firm B, we find that $Q_B = 400$ and $P_B = \$6$. The station's profit is: 3,000 - [1,200 + (2)(600)] = $600 with firm A. Its profit is 2,400 - 1,600 = $800 with firm B. Thus, an order of 400 videos from firm B (priced at $6 each) is optimal.

8. a. First note that if marginal cost and marginal benefit to consumers both increased by $25, the optimal output would not change since MR(Q*) = MC(Q*) and MR(Q*) + 25 = MC(Q*) + 25 are equivalent. The price would rise by $25 but, since marginal costs rise by $25, the firm's total profits would remain the same. If marginal costs rose by more than $25, profits would fall. Thus the firm should not redesign if the increase in MC is $30.

b. If MC increases by $15 and MR increases by $25, the MR shift is greater than the MC shift and the new intersection of the curves occurs at a higher output. Output and price would both rise. Price, however, would rise by less than $25.

10. The latter view is correct. The additional post-sale revenues increase MR, effectively shifting the MR curve up and to the right. The new intersection of MR and MC occurs at a higher output, which, in turn, implies a cut in price. (Of course, one must discount the additional profit from service and supplies to take into account the time value of money.)

12. a. The MC per passenger is $20. Setting MR = MC, we find 120 - .2Q = 20, so Q = 500 passengers (carried by 5 planes). The fare is $70 and the airline's weekly profit is $35,000 - 10,000 = $25,000.

   b. If it carries the freight, the airline can fly only 4 passenger flights, or 400 passengers. At this lower volume of traffic, it can raise its ticket price to P = $80. Its total revenue is (80)(400) + 4,000 = $36,000. Since this is greater than its previous revenue ($35,000) and its costs are the same, the airline should sign the freight deal.

*14.   The Burger Queen (BQ) facts are P = 3 - Q/800 and MC = .80.
   a. Set MR = 0 to find BQ's revenue-maximizing Q and P. Thus, we have 3 - Q/400 = 0, so Q = 1,200 and P = $1.50. Total revenue is $1,800 and BQ's share is 20% or $360. The franchise's revenue is $1,440, its costs are (.8)(1200) = $960, so its profit is $480.

   b. The franchise maximizes profit by setting MR = MC. Note that the relevant MR is (.8)(3 - Q/400) = 2.4 - Q/500. After setting MR = .8, we find Q = 800. In turn, P = $2.00 and the parties' total profit is (2.00 - .80)(800) = $960, which is considerably larger than $840, the total profit in part a.

   c. Regardless of the exact split, both parties have an interest in maximizing total profit, and this is done by setting (full) MR equal to MC. Thus, we have 3 - Q/400 = .80, so that Q = 880. In turn P = $1.90, and total profit is (1.90 - .80)(880) = $968.

   d. The chief disadvantage of profit sharing is that it is difficult, time-consuming, and expensive for the parent company to *monitor* the reported profits of the numerous franchises. Revenue is relatively easy to check (from the cash register receipts) but costs are another matter. Individual franchises have an incentive to exaggerate the costs they report in order to lower the measured profits from which the parent's split is determined. The difficulty in monitoring cost and profit is the main strike against profit sharing.

Discussion Question

Suppose the firm considers expanding its direct sales force from 20 to, say 23 sales people. Clearly, the firm should be able to estimate the marginal cost of the typical additional sales person (wages plus fringe benefits plus support costs including company vehicle). The additional net profit generated by an additional sales person is a little more difficult to predict. An estimate might be based on the average profitability of its current sales force. A more detailed estimate might judge how many new client contacts a salesperson makes, historically what fraction of these contacts result in new business, what is the average profit of these new accounts, and so on. If the marginal profit of a sales person is estimated to be between $100,000 and $120,000 while the marginal cost is $85,000, then the firm has a clear-cut course of action, namely hire the additional 1, 2, or 3 employees.

ANSWERS TO SPREADSHEET PROBLEMS

(You may download any and all spreadsheets from the John Wiley Samuelson and Marks Website.)

S1. a and b. Setting MR = MC implies: $800 - 4Q = 200 + Q$. Therefore, $Q^* = 120$ and $P^* = 560$.

c. Confirm these values on your spreadsheet by maximizing cell F7 by changing cell B7. Maximum profit in cell F7 is 16,000.

S2. a. Given $\pi = 20[A/(A+8)] - A$, it follows that $M\pi = 20[8/(A+8)^2] - 1$. Setting $M\pi = 0$ implies $(A+8)^2 = 160$, or $A^* = \$4.649$ million.

b. Confirm this value on your spreadsheet by maximizing cell F7 by changing cell C7. Maximum profit in F7 is $2.702 million.

Additional Spreadsheet Problem

The accompanying spreadsheet lists the compass positions of the eight coastal cities of Figure 2.1 and the possible position of the shopping mall. By convention, the city positions are expressed in (x, y) coordinates. All cities are positioned "east-west" along the x-axis. Town A has coordinates (0, 0); Town B is located 3 miles due east, at coordinates (3, 0), and so on. The shopping mall is tentatively located at (5.5, 0), i.e. a mile west of Town C (corresponding to position X in Figure 2.1.)

In the spreadsheet, column M lists the population of shoppers in each of the towns, while column Q computes the distance between the mall and the various towns. For example, cell Q7 contains the distance formula

=((I7-D$7)^2 + (J7-E$7)^2)^.5,

that is, distance is the square root of the sum of squared differences of the x and y coordinates. Finally, cell E11 computes the total travel miles associated with the mall's location.

a. By trial and error, vary the mall's east-west location (cell D7) to minimize total travel miles. Use your spreadsheet's optimizer to confirm the optimal location.

b. More realistically, suppose the towns are scattered north-south as well as east-west. Set the y coordinates of towns B, C, E, F, and H to 11, 7.5, 15, 12, and 8, respectively. Answer again the questions in part a.

c. Column U provides estimates of the fraction of each town's shopping population that will actually frequent the mall. This fraction varies inversely with the square of the distance to the mall: $F = 1 - d^2/500$. Accordingly, the mall attracts 100% of the 10 thousand shoppers from nearby Town C but only 46% of the 15 thousand shoppers from Town H. Cell E14 lists the total number of mall visitors from all towns. (Be sure to check this formula.)

By trial and error and using your spreadsheet's optimizer, find the mall location that maximizes total visitors (and thus revenue) for the town locations of part a and part b.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1																						
2		Locating a Shopping Mall																				
3																						
4				Location					Location			Population			Distance				Fraction who			
5				(x , y)			Towns		(x , y)			of Shoppers			from Mall				Visit Mall			
6																						
7		Mall		5.5	0		A		0.0	0		15			5.50				0.94			
8																						
9							B		3.0	0		10			2.50				0.99			
10																						
11		Total Travel Miles:			742.5		C		6.5	0		10			1.00				1.00			
12																						
13							D		8.5	0		10			3.00				0.98			
14		Total Visitors:			78.4																	
15							E		11.0	0		5			5.50				0.94			
16																						
17							F		15.5	0		20			10.0				0.80			
18																						
19							G		17.5	0		10			12.0				0.71			
20																						
21							H		22.0	0		15			16.5				0.46			
22																						

Answer

a. Using trial and error or the spreadsheet optimizer, we confirm that the optimal location for the mall is Town E, at coordinates (11, 0).

b. Using the spreadsheet optimizer, we now find the optimal mall location to be (11.14, 7.3).

c. Now the number of visitors varies with the square of the distance to the mall. Using the spreadsheet optimizer, our goal is to find the location that maximizes cell E14. For coastal cities, the optimal location is (11.05, 0). This is the mean of the one-dimensional distribution. For cities scattered north-west and east-south the optimal location is (11.05, 6.5).

ANSWERS TO APPENDIX PROBLEMS

1. When tax rates become very high, individuals will make great efforts to shield their income from taxes. Furthermore, higher taxes will discourage the taxed activities altogether. (In the extreme case of a 100% tax, there is no point in undertaking income-generating activities.) Thus, a higher tax rate mean a smaller tax base. Increasing the tax rate from zero, the revenue curve first increases, eventually peaks, and then falls to zero (at a 100% tax). Thus, the curve is shaped like an upside-down U.

2. a. $B(t) = 80 - 100t$. Therefore, $R = 80t - 100t^2$. Setting $MR = dR/dt = 0$, we find: $80 - 200t = 0$, or $t = .4$.

   b. $B(t) = 80 - 240t^2$. Therefore, $R = 80t - 240t^3$. Setting $MR = dR/dt = 0$, we find: $80 - 720t^2 = 0$. Therefore, $t^2 = 1/9$, or $t = 1/3$.

   c. $B(t) = 80 - 80t^{.5}$. Therefore, $R = 80t - 80t^{1.5}$. Setting $MR = dR/dt = 0$, we find: $80 - 120t^{.5} = 0$. Thus, $t^{.5} = 2/3$, or $t = 4/9$.

3. a. $\pi = 20x - x^2 + 16y - 2y^2$. Setting $d\pi/dx = 0$ and $d\pi/dy = 0$ implies $x = 10$ and $y = 4$.

   b. The Lagrangian is $L = \pi + z(8 - x - y)$. Therefore, the optimality conditions are: $20 - 2x - z = 0$, $16 - 4y - z = 0$, and $x + y = 8$. The solution is $x = 6$, $y = 2$, and $z = 8$.

   c. The Lagrangian is $L = \pi + z(7.5 - x - .5y)$. Therefore, the optimality conditions are: $20 - 2x - z = 0$, $16 - 4y - .5z = 0$, and $x + .5y = 7.5$. The solution is $x = 6$, $y = 3$, and $z = 8$.