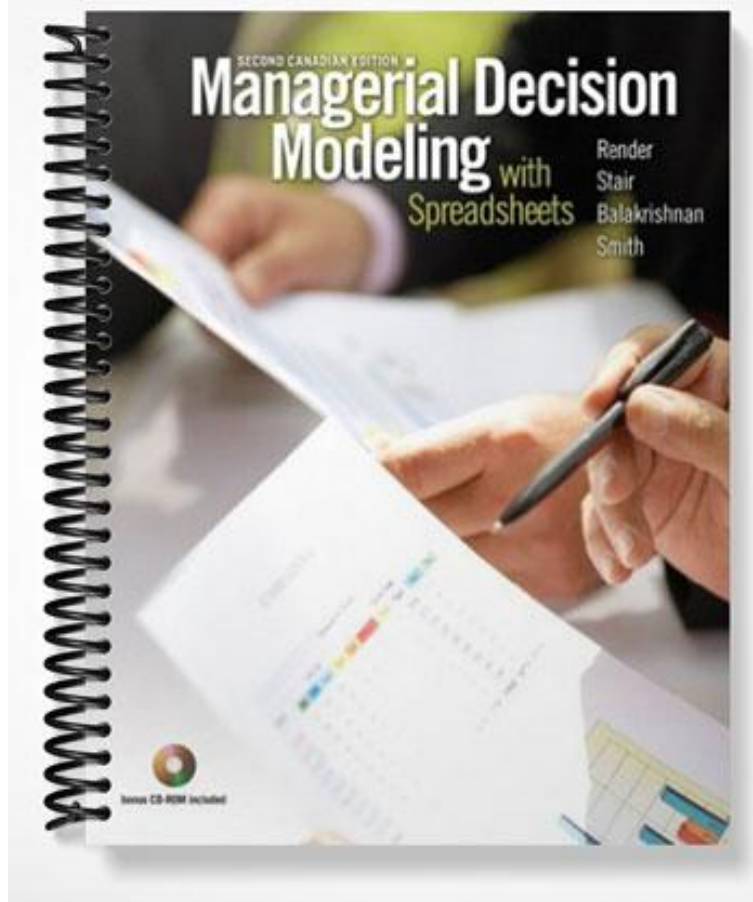


# SOLUTIONS MANUAL



# CHAPTER 2

## SOLUTIONS

### LINEAR PROGRAMMING MODELS: GRAPHICAL AND COMPUTER METHODS

#### SOLUTIONS TO DISCUSSION QUESTIONS AND PROBLEMS

**2-1.** Both minimization and maximization LP problems employ the basic approach of developing a feasible solution region by graphing each of the constraint lines. They can also both be solved by applying the corner point method. The isoprofit line method is used for maximization problems, whereas the isocost line is applied to minimization problems. Conceptually, isoprofit and isocost are the same.

The major differences between minimization and maximization problems deal with the shape of the feasible region and the direction of optimality. In minimization problems, the region must be bounded on the lower left, and the best isocost line is the one closest to the zero origin. The region may be unbounded on the top and right and yet be correctly formulated. A maximization problem must be bounded on the top and to the right. The isoprofit line yielding maximum profit is the one farthest from the zero origin.

**2-2.** The requirements for an LP problem are listed in Section 2.2. It is also assumed that conditions of certainty exist; that is, coefficients in the objective function and constraints are known with certainty and do not change during the period being studied. Another basic assumption that mathematically sophisticated students should be made aware of is *proportionality* in the objective function and constraints. For example, if one product uses 5 hours of a machine resource, then making 10 of that product uses 50 hours of machine time.

LP also assumes *additivity*. This means that the total of all activities equals the sum of each individual activity. For example, if the objective function is to maximize  $P = 6X_1 + 4X_2$ , and if  $X_1 = X_2 = 1$ , the profit contributions of 6 and 4 must add up to produce a sum of 10.

**2-3.** Each LP problem that has been formulated correctly *does* have an infinite number of solutions. Only one of the points in the feasible region usually yields the *optimal* solution, but *all* of the points yield a feasible solution. If we consider the region to be continuous and accept noninteger solutions as valid, there will be an infinite number of feasible combinations of  $X_1$  and  $X_2$ .

**2-4.** If a maximization problem has many constraints, then it can be very time consuming to use the corner point method to solve it. Such an approach would involve using simultaneous equations to solve for each of the feasible region's intersection points. The isoprofit line is much more effective if the problem has numerous constraints.

**2-5.** A problem can have alternative optimal solutions if the isoprofit or isocost line runs parallel to one of the problem's constraint lines (refer to Section 2.7 in the chapter).

**2-6.** This question involves the student using a little originality to develop his or her own LP constraints that fit the three conditions of (1) unboundedness, (2) infeasibility, and (3) redundancy. These conditions are discussed in Section 2.7, but each student's graphical displays should be different.

**2-7.** The manager's statement indeed had merit if the manager understood the deterministic nature of linear programming input data. LP assumes that data pertaining to demand, supply, materials, costs, and resources are known with certainty and are constant during the time period being analyzed. If this production manager operates in a very unstable environment (for example, prices and availability of raw materials change daily, or even hourly), the model's results may be too sensitive and volatile to be trusted. The application of sensitivity analysis might be trusted. The application of sensitivity analysis might be useful to determine whether LP would still be a good approximating tool in decision making.

**2-8.** The objective function is not linear because it contains the product of  $X_1$  and  $X_2$ , making it a second-degree term. The first, second, and fourth constraints are okay as is. The third and fifth constraints are nonlinear because they contain terms to the second degree and one-half degree, respectively.

**2-9.** The computer is valuable in (1) solving LP problems quickly and accurately; (2) solving large problems that might take days or months by hand; (3) performing extensive sensitivity analysis automatically; and (4) allowing a manager to try several ideas, models, or data sets.

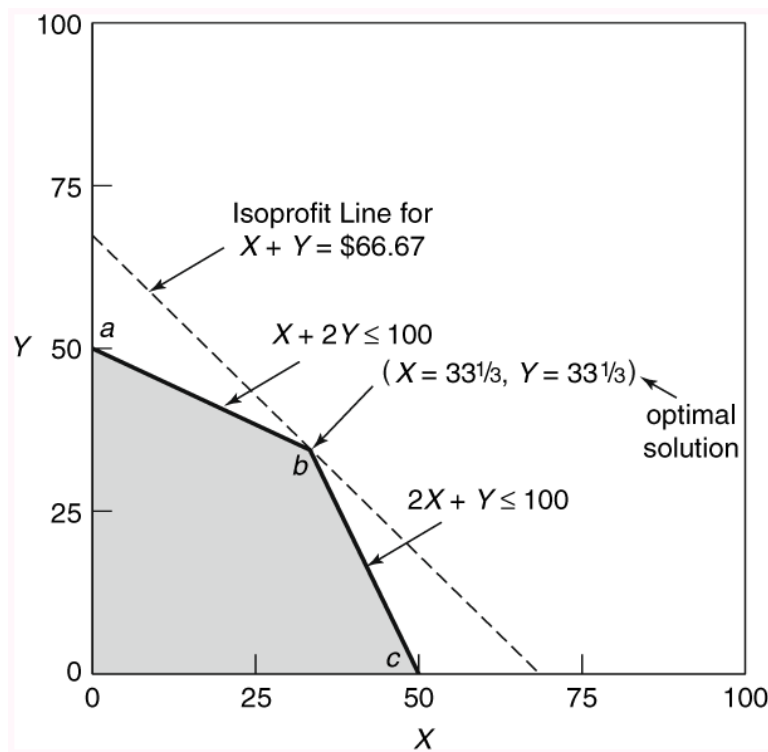
**2-10.** Most managers probably have Excel (or another spreadsheet software) available in their companies, and use it regularly as part of their regular activities. As such, they are likely to be familiar with its usage. In addition, a lot of the data (such as parameter values) required for developing LP models is likely to be available either in some Excel file or in a database file (such as Microsoft Access) from which it is easy to import to Excel. For these reasons, a manager may find the ability to use Excel to set up and solve LP problems very beneficial.

**2-11.** The three components are: target cell (objective function), changing cells (decision variables), and constraints.

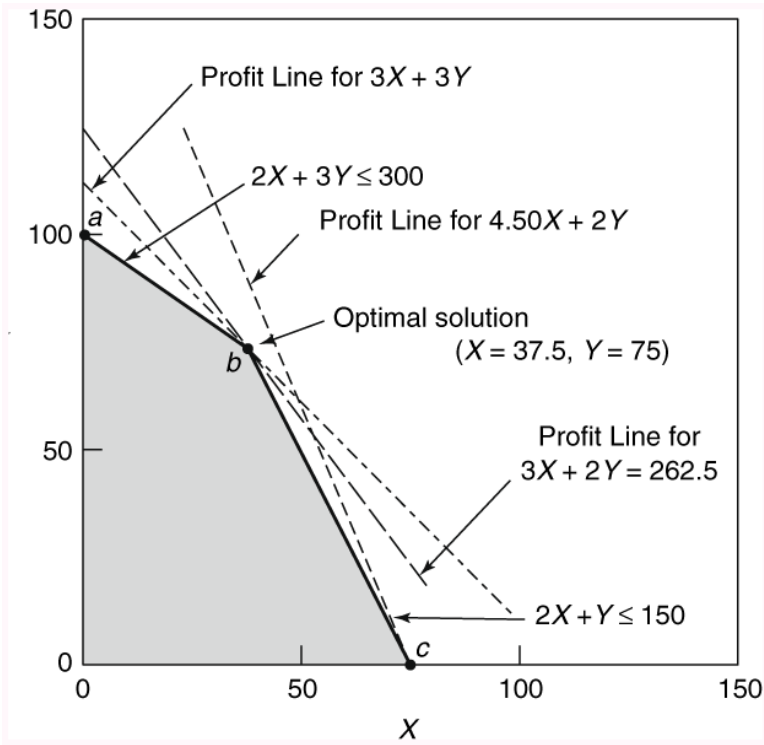
**2-12.** Slack is defined as the RHS minus the LHS value for a  $\leq$  constraint. It may be interpreted as the amount of unused resource described by the constraint. Surplus is defined as the LHS minus the RHS value for a  $\geq$  constraint. It may be interpreted as the amount of over satisfaction of the constraint.

**2-13.** An unbounded solution occurs when the objective of an LP problem can go to infinity (negative infinity for a minimization problem) while satisfying all constraints. Solver indicates an unbounded solution by the message "The Set Cell values do not converge".

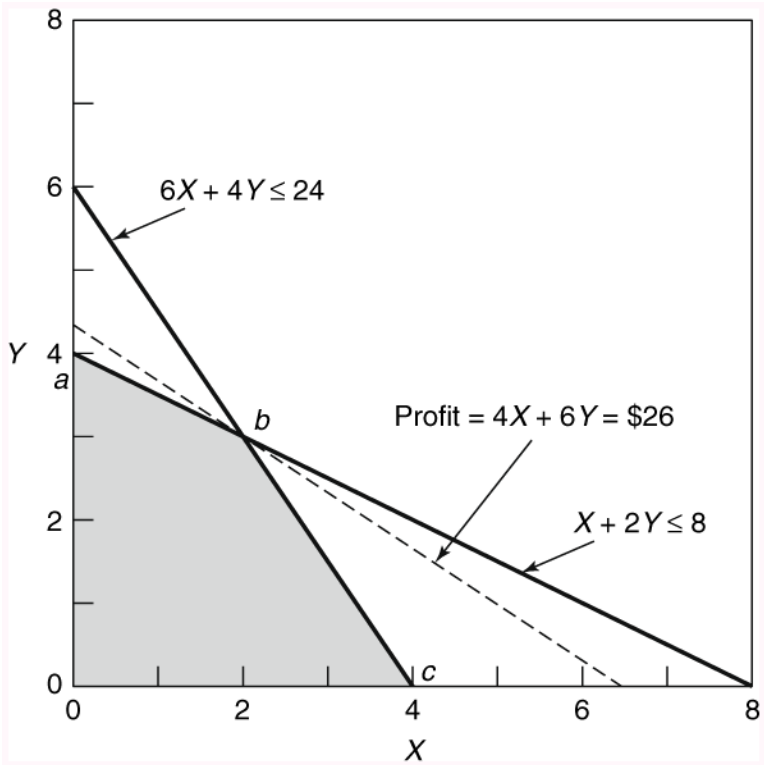
**2-14.**



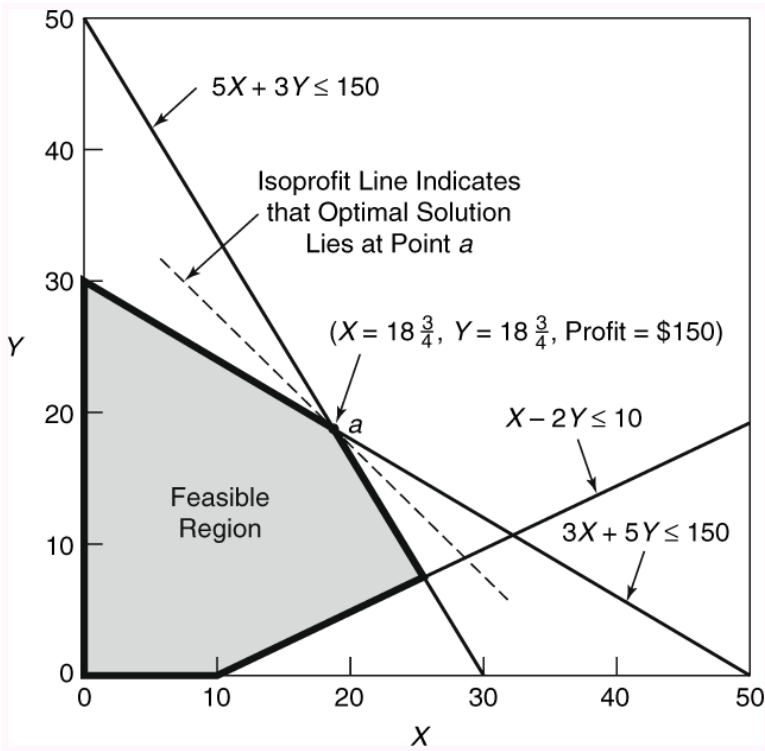
2-15. Using the isoprofit line or corner point method, we see that point  $b$  (where  $X = 37.5$  and  $Y = 75$ ) is optimal.



2-16. The optimal solution of \$26 profit lies at the point  $X = 2, Y = 3$ .

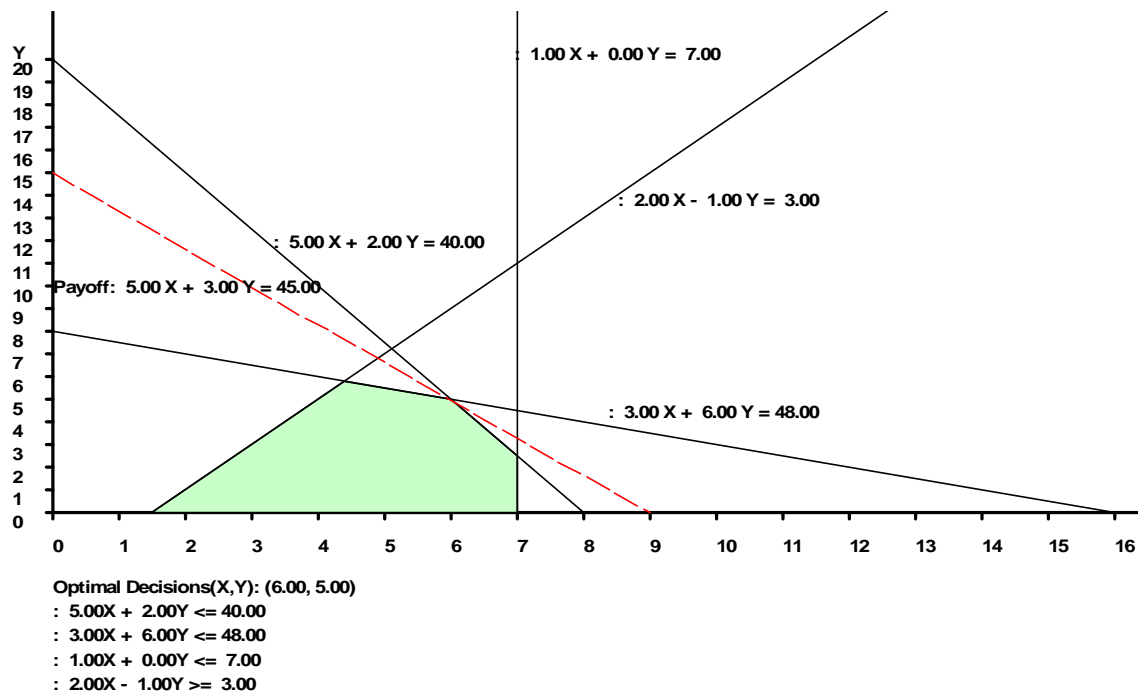


2-17.



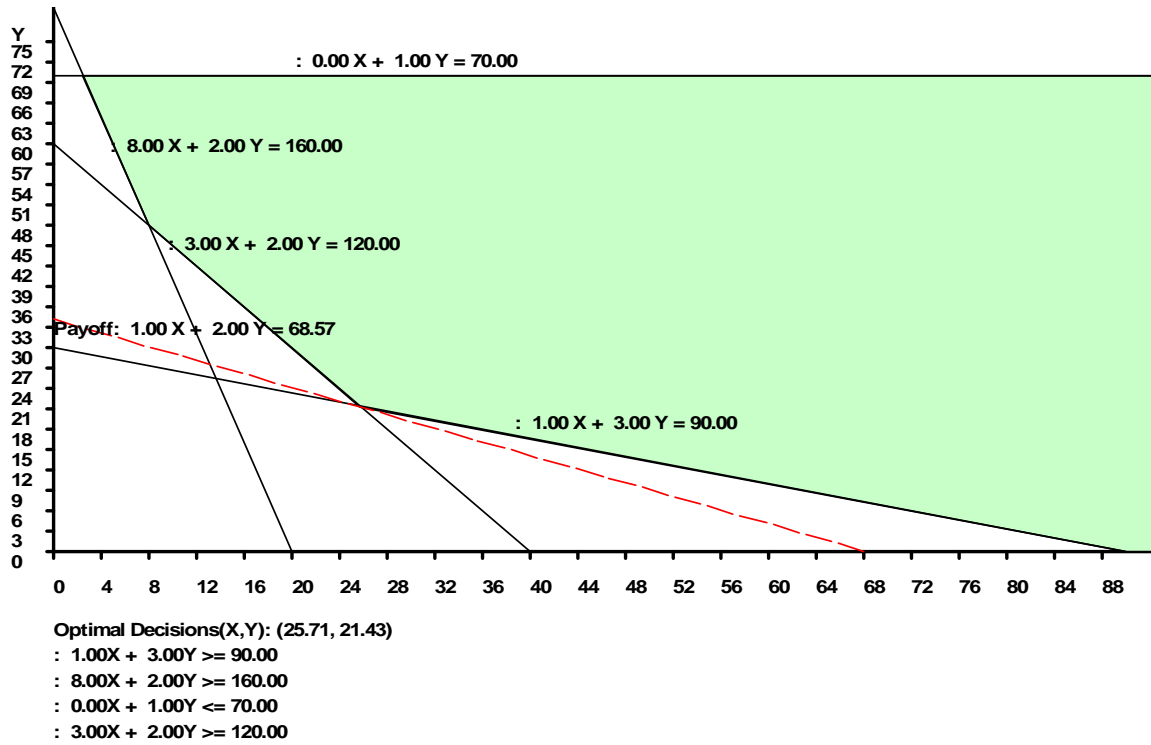
Note that this problem has one constraint with a negative sign. This may cause the beginning student some confusion in plotting the line.

2-18.



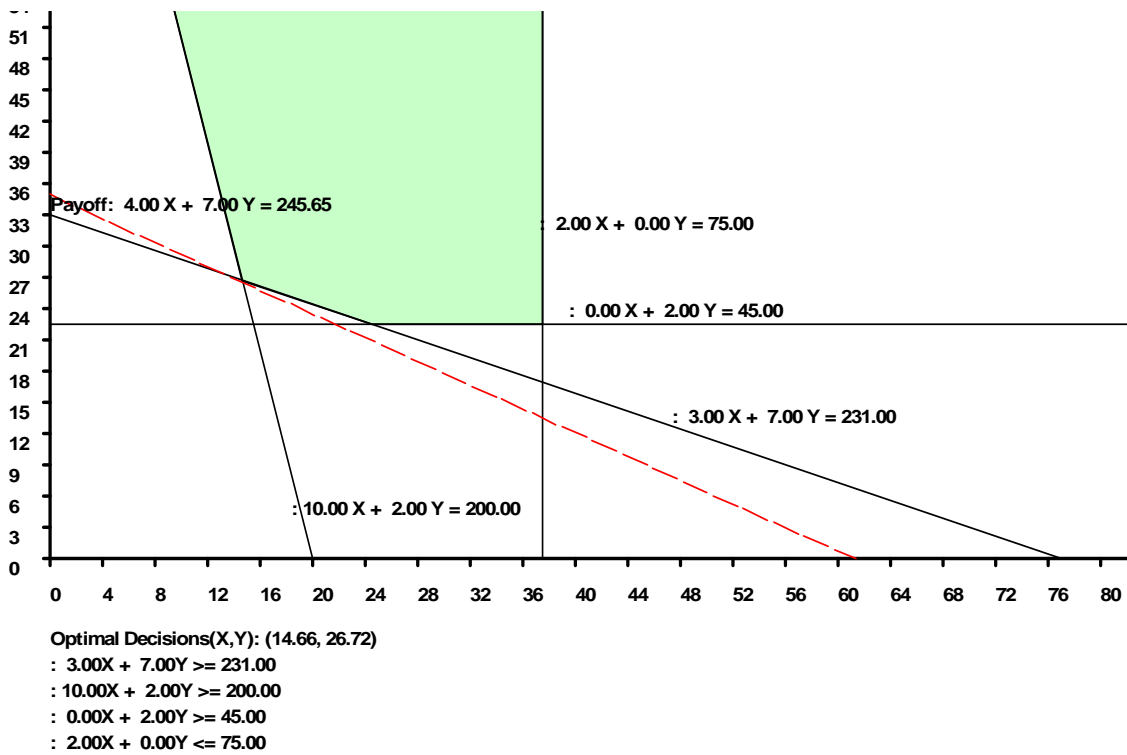
See file P2-18.XLS.

2-19.



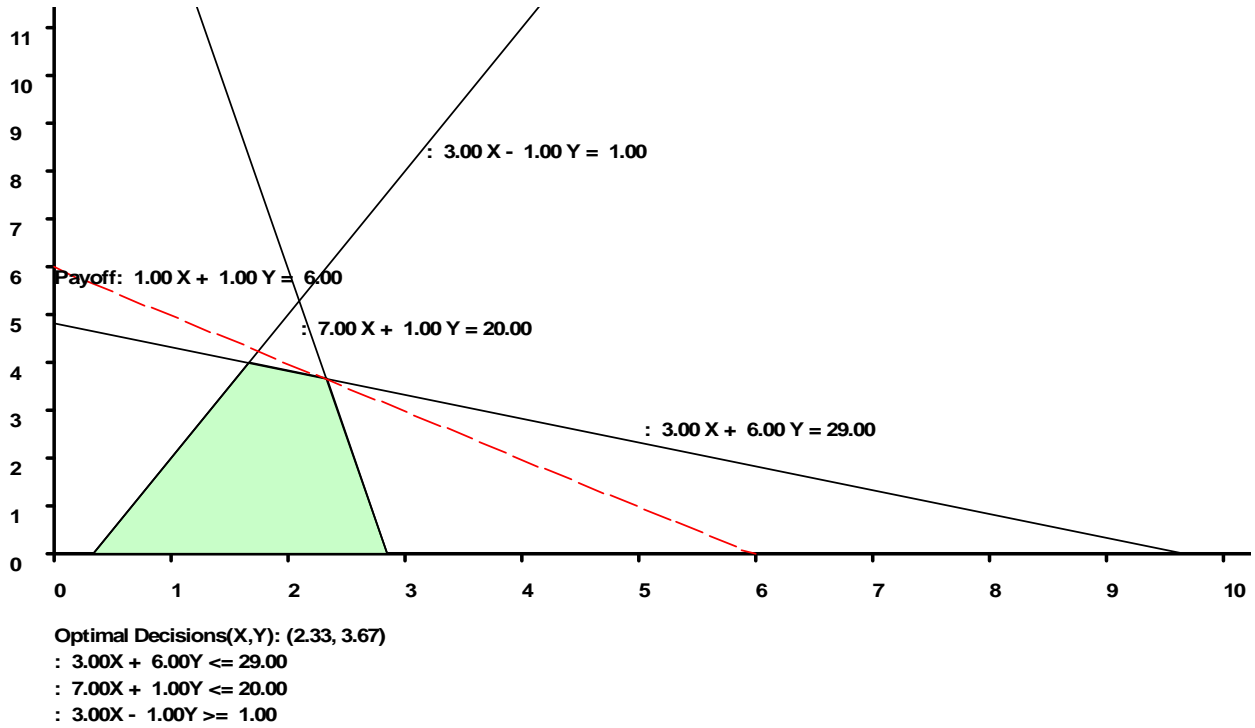
See file P2-19.xls.

2-20.



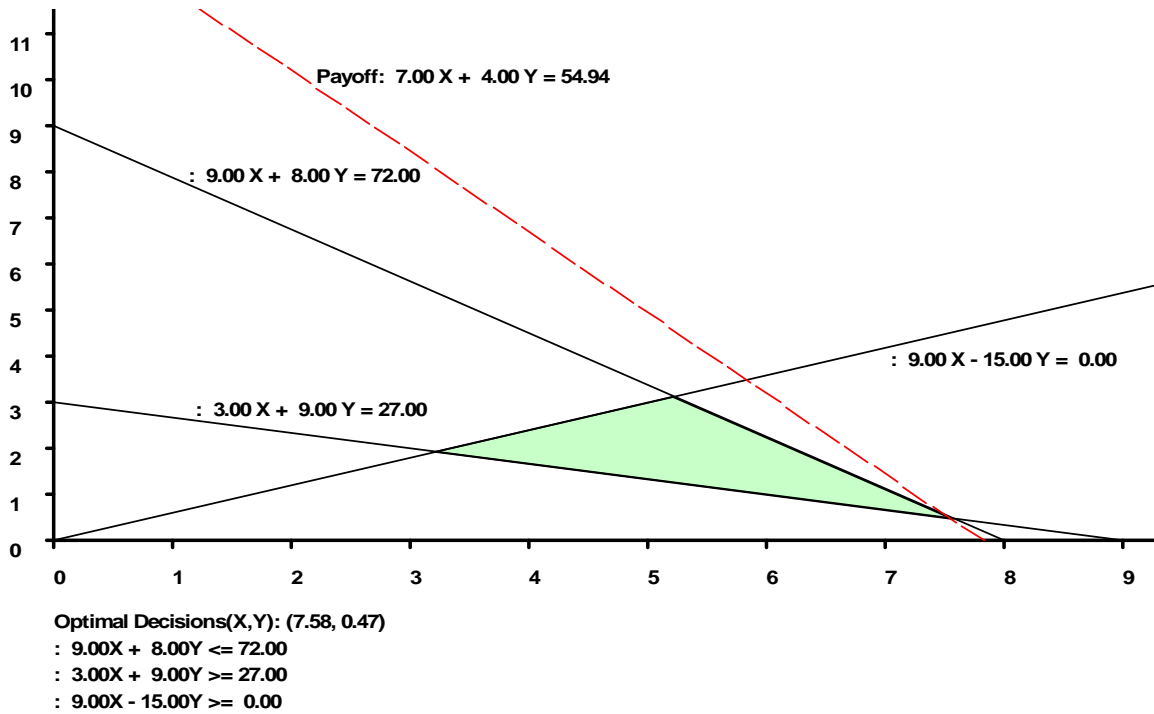
See file P2-20.xls

2-21.



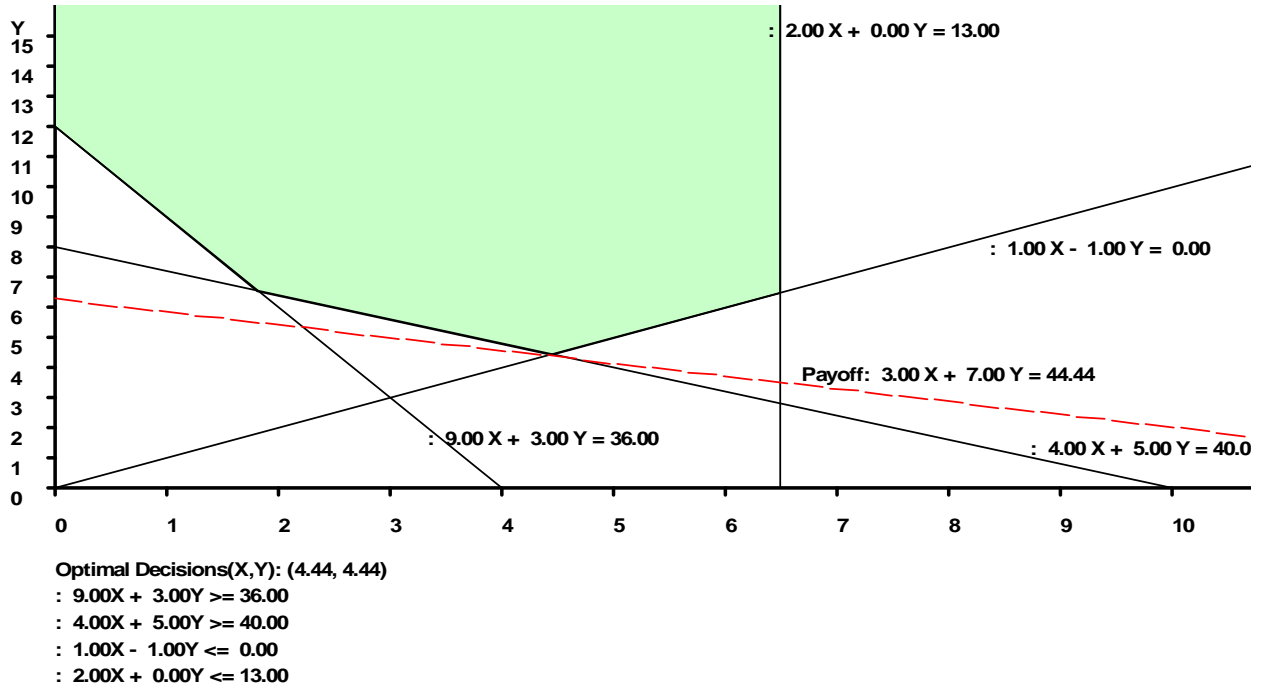
See file P2-21.xls

2-22.



See file P2-22.xls

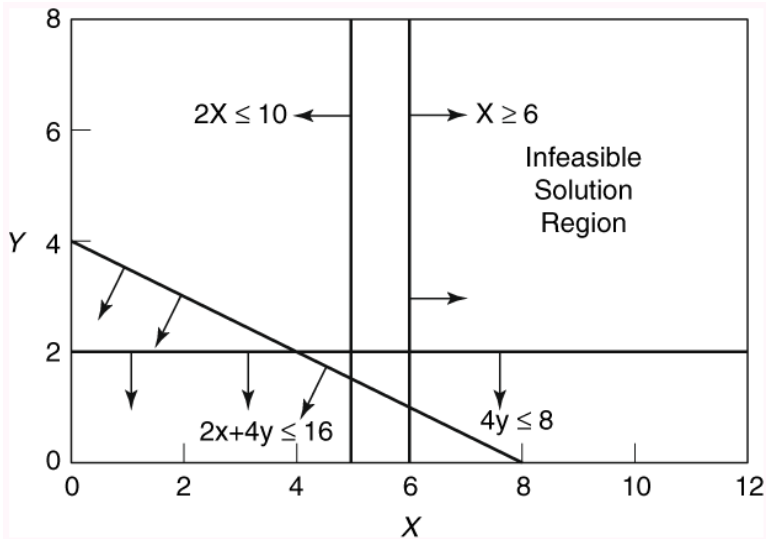
2-23.



See file P2-23.xls

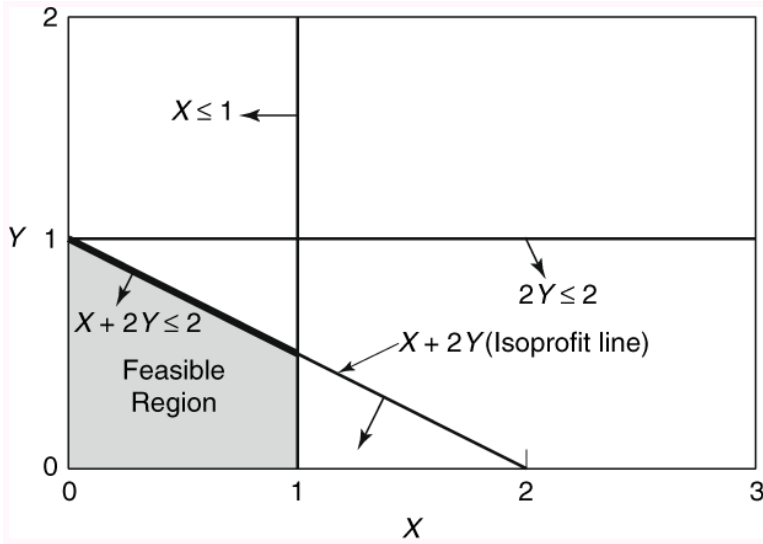
2-24.

Formulation 1:





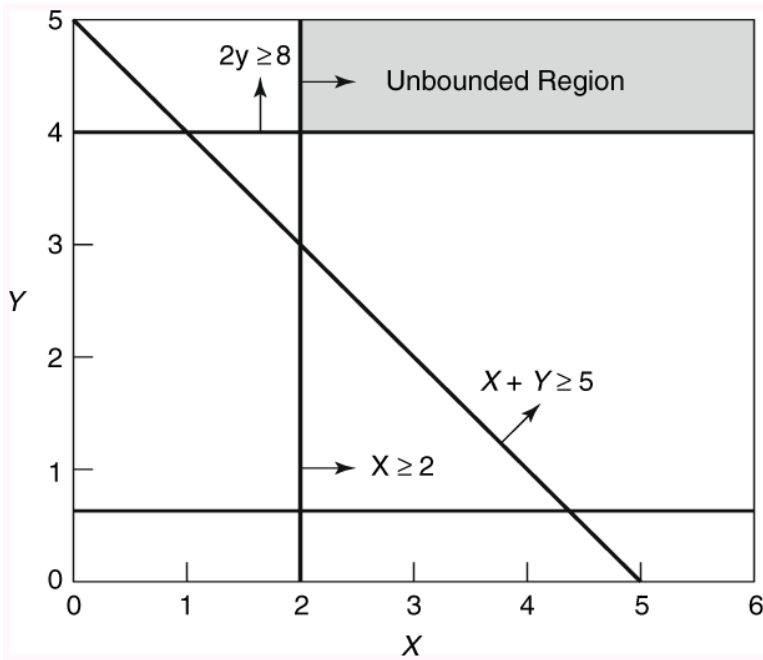
Formulation 2:



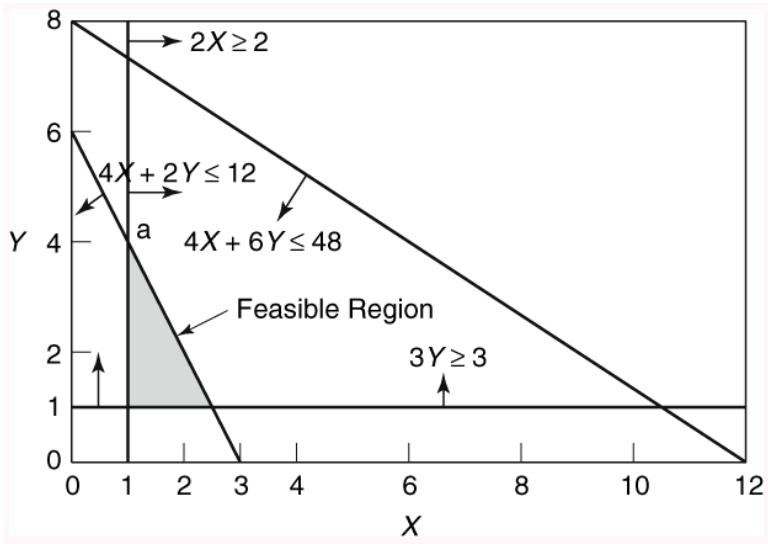
$X + 2Y = 2$  line—this is also on the same slope as the isoprofit line  $X + 2Y$  and hence there will be more than one optional solution.

As a matter of fact, every point along the heavy line will provide an “alternate optimum.”

Formulation 3:



Formulation 4:



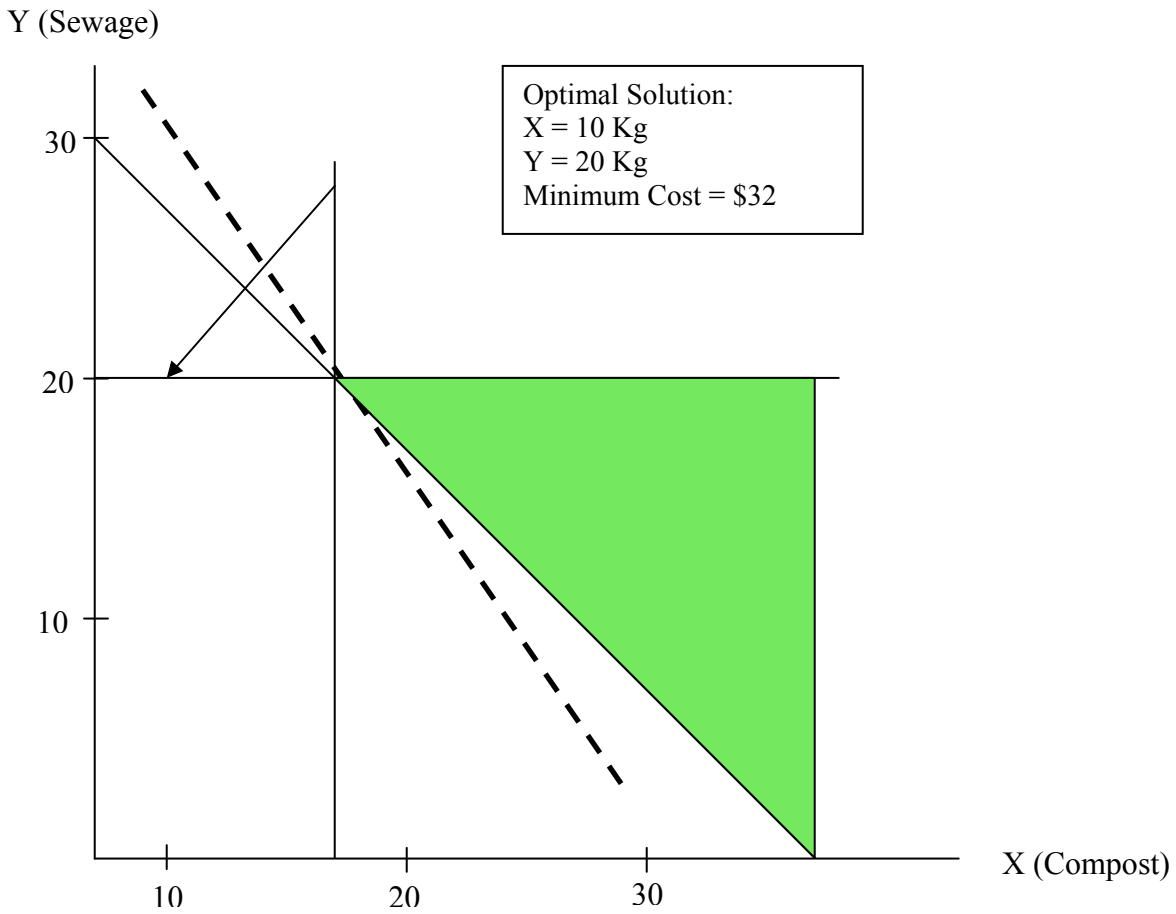
Formulation 4 has a unique optimal solution (point a). Note that the constraint  $4X + 6Y \leq 48$  is redundant.

2-25. Let X = number of kilograms of compost, in each bag, Y = number of kilograms of sewage in each bag.

Objective: Minimize cost =  $\$1.30X + \$0.95Y$

Subject to:

$$\begin{array}{rcll} X + Y & \geq & 30 & \text{Kilograms per bag} \\ X & \geq & 10 & \text{Min compost, kg} \\ Y & \leq & 20 & \text{Max sewage, kg} \\ X, Y & \geq & 0 & \text{Non-negativity} \end{array}$$



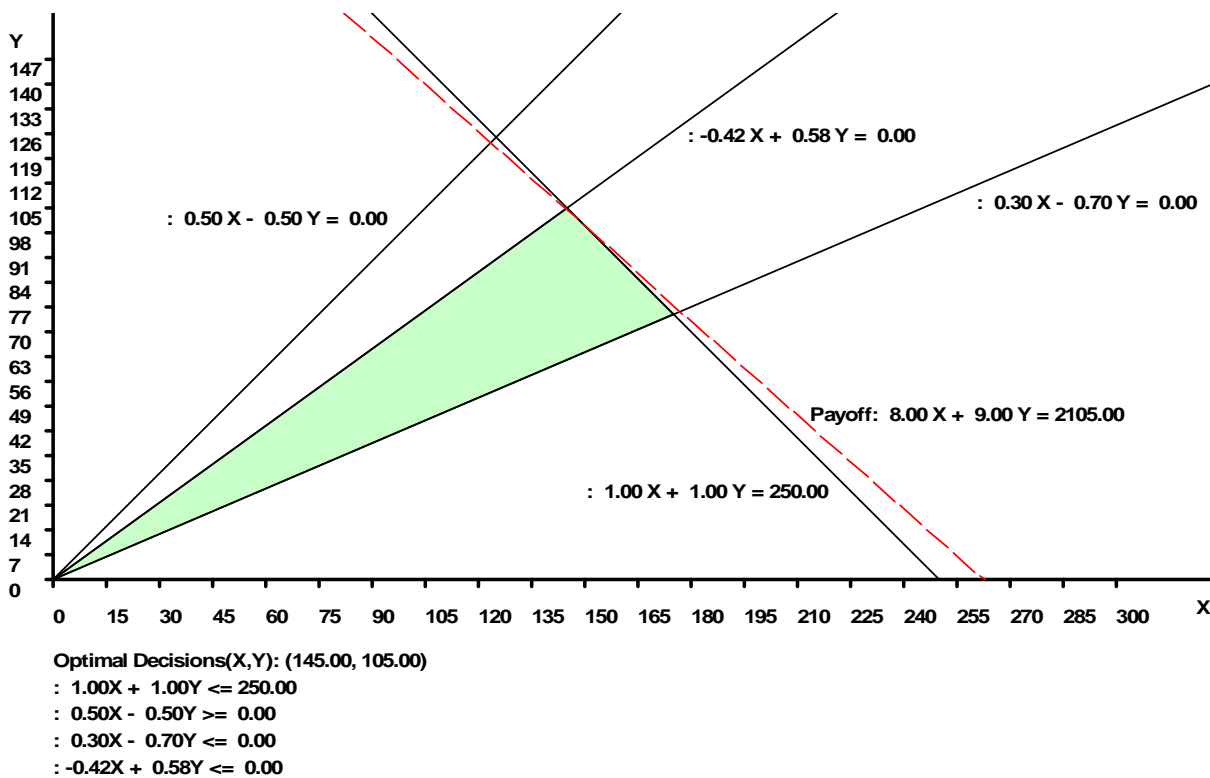
See file P2-25.xls

**2-26.** Let  $X$  = thousands of dollars to invest in provincial bonds,  $Y$  = thousands of dollars to invest in equity account.

Objective: Maximize return =  $8X + 9Y$

Subject to:

$X$	$+ Y$	$\leq$	$\$250\,000$	Amount available
$X$		$\leq$	$0.7(X+Y)$	Max Provincial bonds
$2X$	$+ 3Y$	$\leq$	$2.42(X+Y)$	Max risk score
$X$		$\geq$	$0.5(X+Y)$	Min Provincial bonds
$X,$	$Y$	$\geq$	$0$	Non-negativity



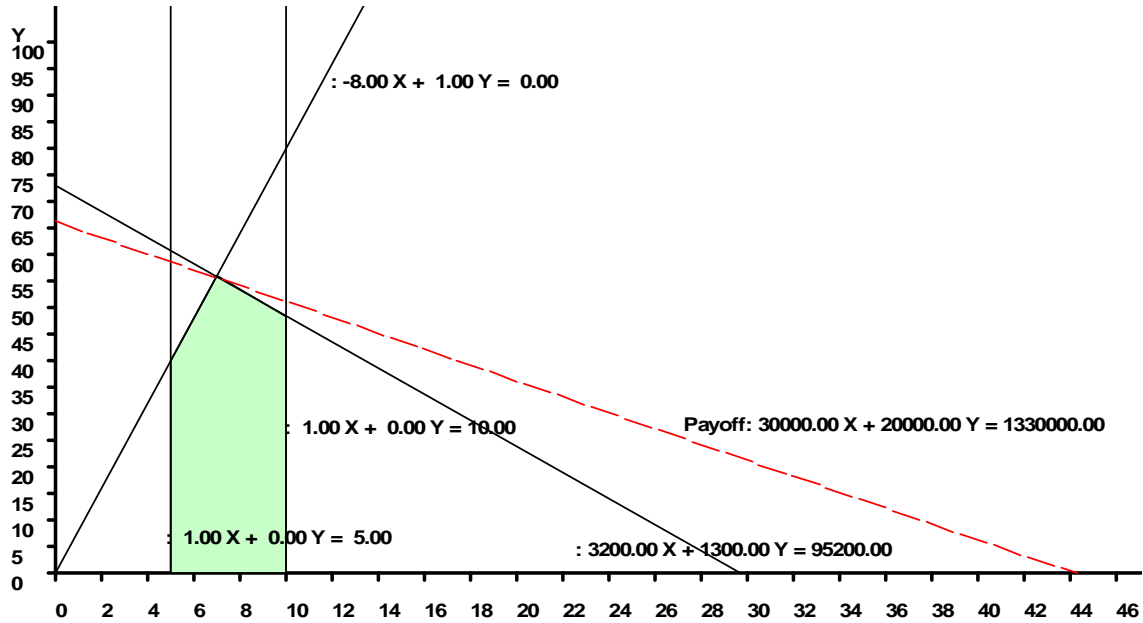
See file P2-26.xls

2-27. Let X = number of TV spots, Y= number of newspaper ads placed.

Objective: Maximize exposure = 30 000X + 20 000Y

Subject to:

$\$3200X$	$+$	$\$1300Y$	$\leq$	$\$95\ 200$	Budget available
X			$\leq$	10	Max TV
		Y	$\leq$	8X	Paper vs TV
X			$\geq$	5	Min TV
X,		Y	$\geq$	0	Non-negativity



Optimal Decisions(X,Y): (7.00, 56.00)  
 : 3200.00X + 1300.00Y <= 95200.00  
 : 1.00X + 0.00Y >= 5.00  
 : 1.00X + 0.00Y <= 10.00  
 : -8.00X + 1.00Y <= 0.00

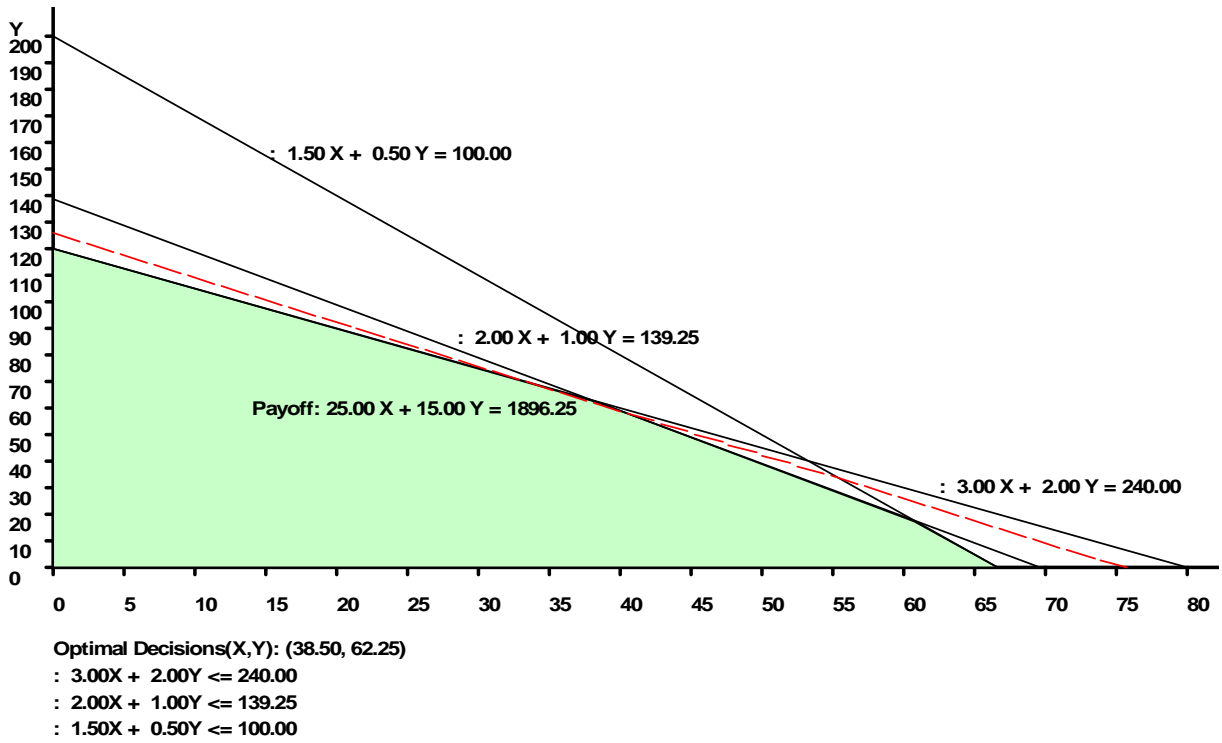
See file P2-27.xls

2-28. Let X = number of air conditioners to produce, Y = number of fans to produce.

Objective: Maximize revenue = \$25X + \$15Y

Subject to:

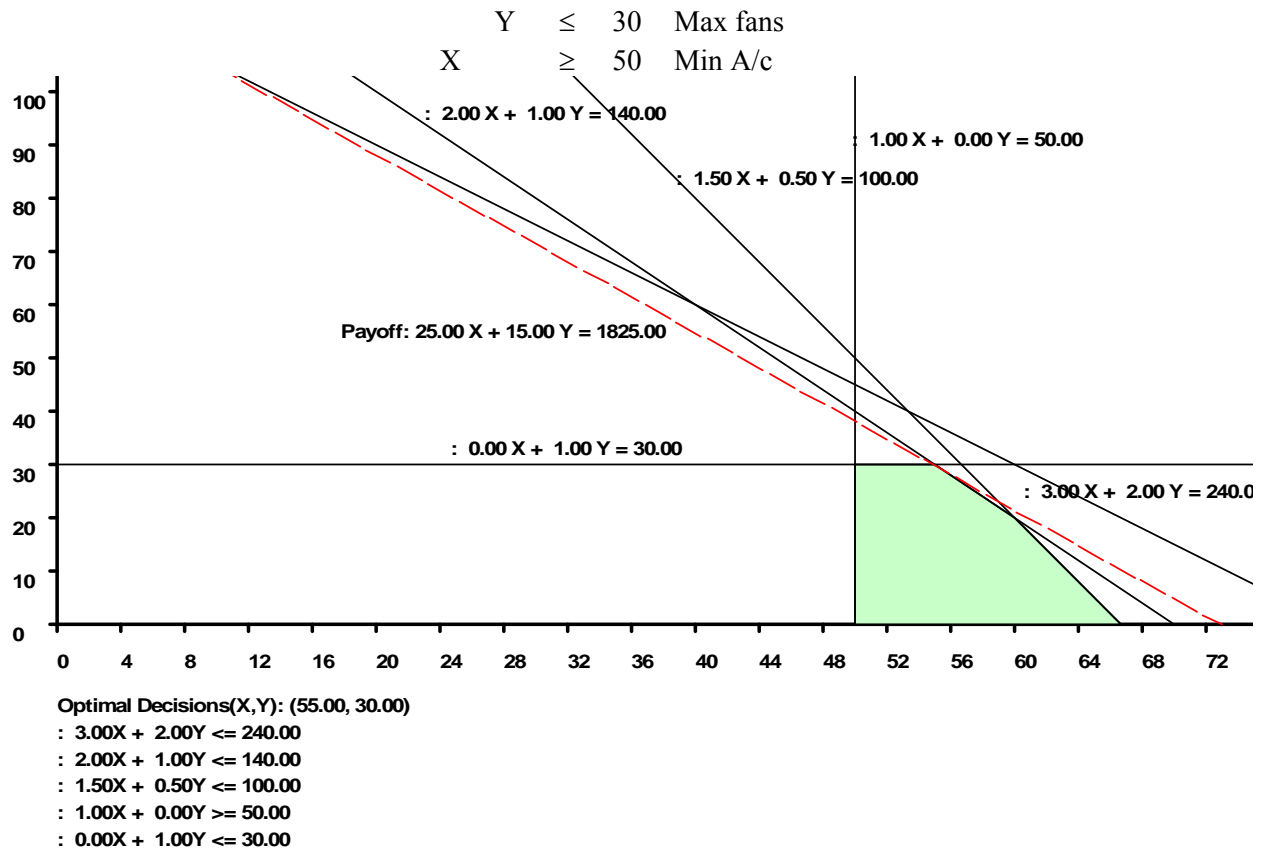
$$\begin{array}{rcll}
 3X & + & 2Y & \leq & 240 & \text{Wiring time} \\
 2X & + & Y & \leq & 140 & \text{Drilling time} \\
 1.5X & + & 0.5Y & \leq & 100 & \text{Assembly time} \\
 X, & Y & & \geq & 0 & \text{Non-negativity}
 \end{array}$$



See file P2-28.xls

2-29. X and Y are defined as in Problem 2-28. Objective remains the same.

Now subject to the following additional constraints:



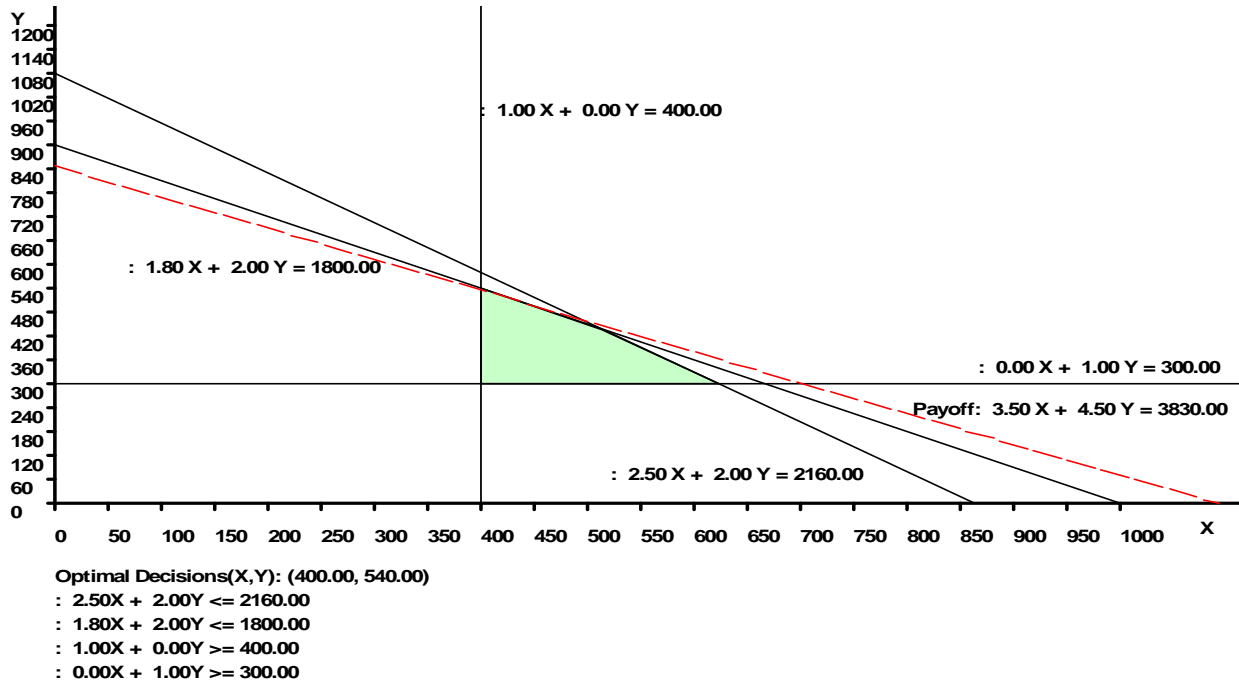
See file P2-29.xls

2-30. Let X = number of copies of Backyard, Y= number of copies of Porch.

Objective: Maximize revenue =  $\$3.50X + \$4.50Y$

Subject to:

$$\begin{array}{rcll} 2.5X + 2Y & \leq & 2160 & \text{Print time, minutes} \\ 1.8X + 2Y & \leq & 1800 & \text{Collate time, minutes} \\ X & \geq & 400 & \text{Non-negativity} \end{array}$$



See file P2-30.xls

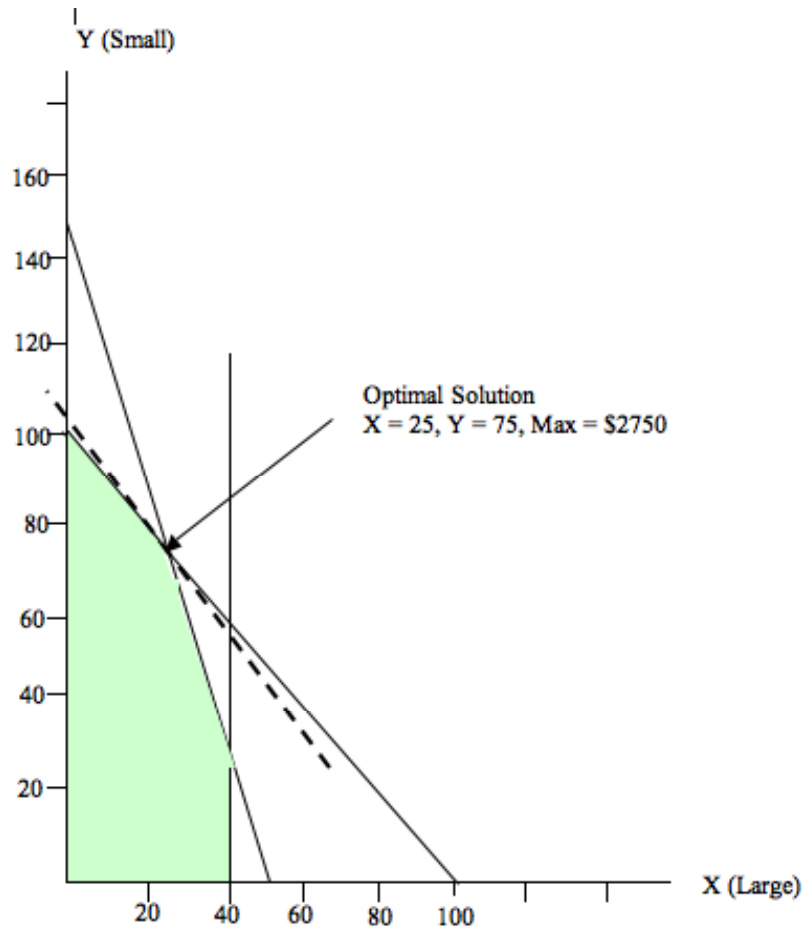


**2-31.** Let  $X$  = number of large sheds to build,  $Y$  = number of small sheds to build.

Objective: Maximize revenue =  $\$50X + \$20Y$

Subject to:

$$\begin{array}{rclcl} X & + & Y & \leq & 100 & \text{Advertising Budget} \\ 15X & + & 5Y & \leq & 750 & \text{Sq metres required} \\ X & & & \leq & 40 & \text{Rental limit} \\ X, & Y & & \geq & 0 & \text{Non-negativity} \end{array}$$



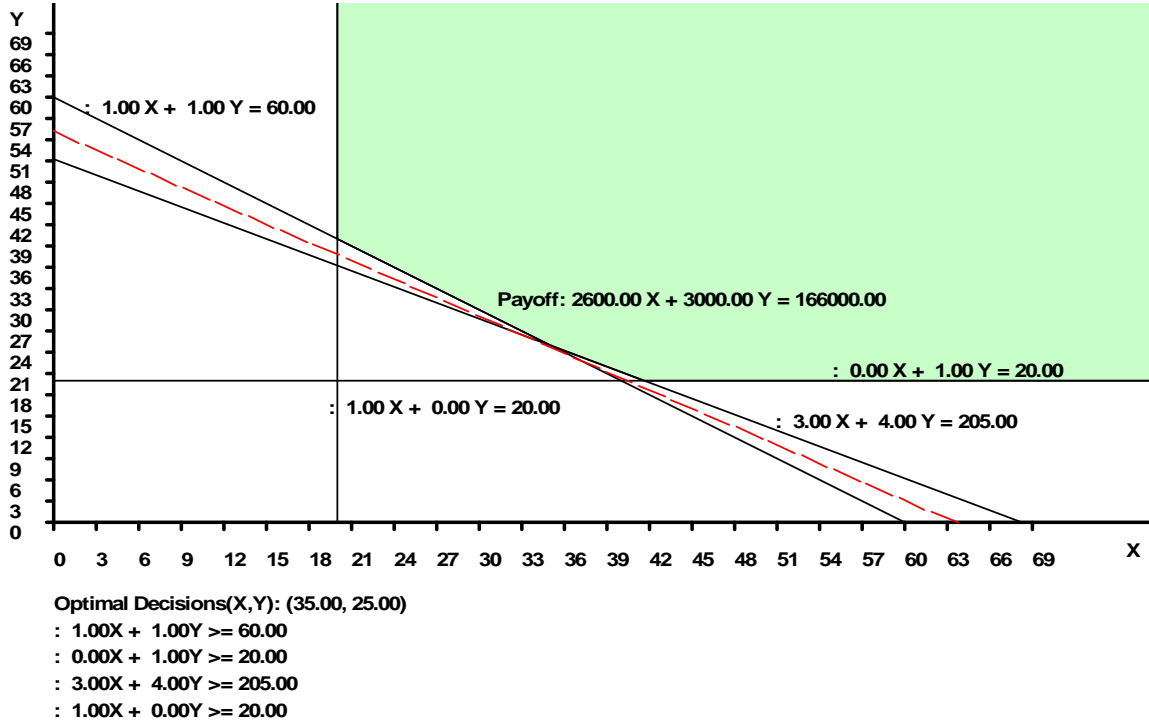
See file P2-31.xls

2-32. Let X = number of core courses, Y = number of elective courses.

Objective: Minimize wages = \$2600X + \$3000Y

Subject to:

X	+	Y	≥	60	Total courses
3X	+	4Y	≥	205	Credit hours
X			≥	20	Min core
		Y	≥	20	Min elective
X, Y			≥	0	Non-negativity



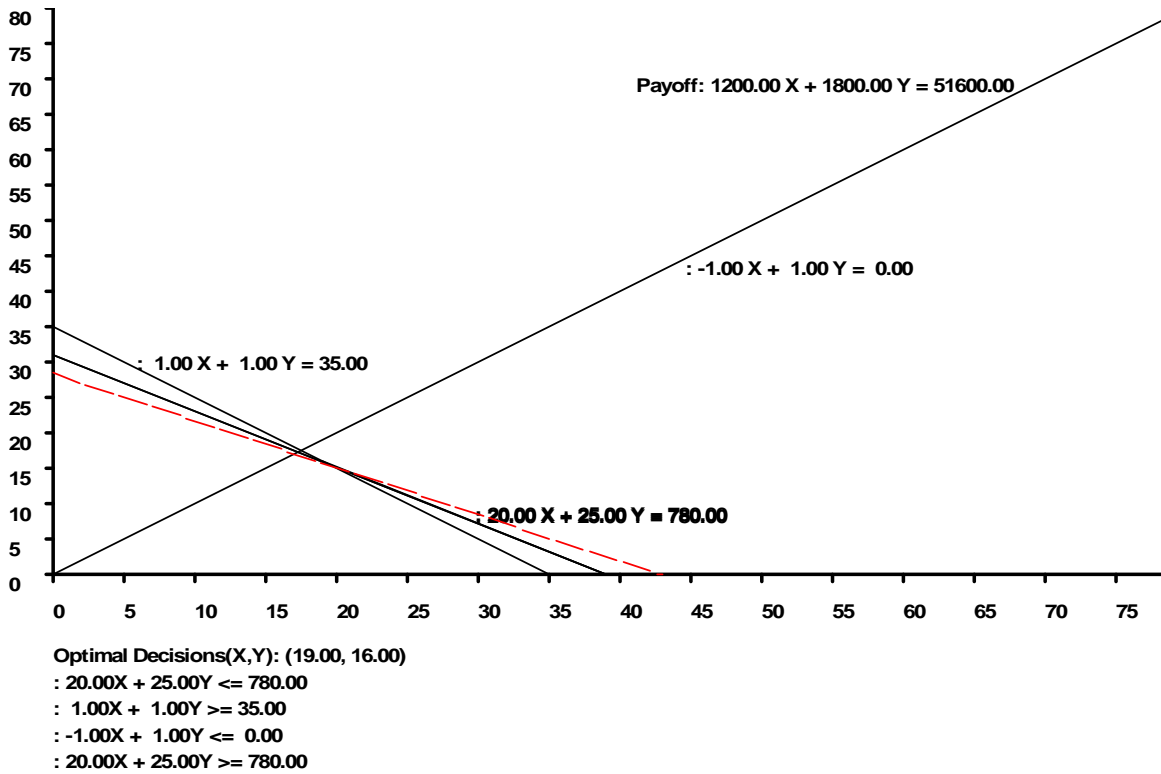
See file P2-32.xls

2-33. Let X = number of Alpha 4 routers to produce, Y = number of Beta 5 routers to produce

Objective: Maximize profit = \$1200X + \$1800Y

Subject to:

$$\begin{array}{rcll}
 20X + 25Y & = & 780 & \text{Labour hours} \\
 X + Y & \geq & 35 & \text{Total routers} \\
 Y & \leq & X & \text{Alpha 4 vs Beta 5} \\
 X, Y & \geq & 0 & \text{Non-negativity}
 \end{array}$$

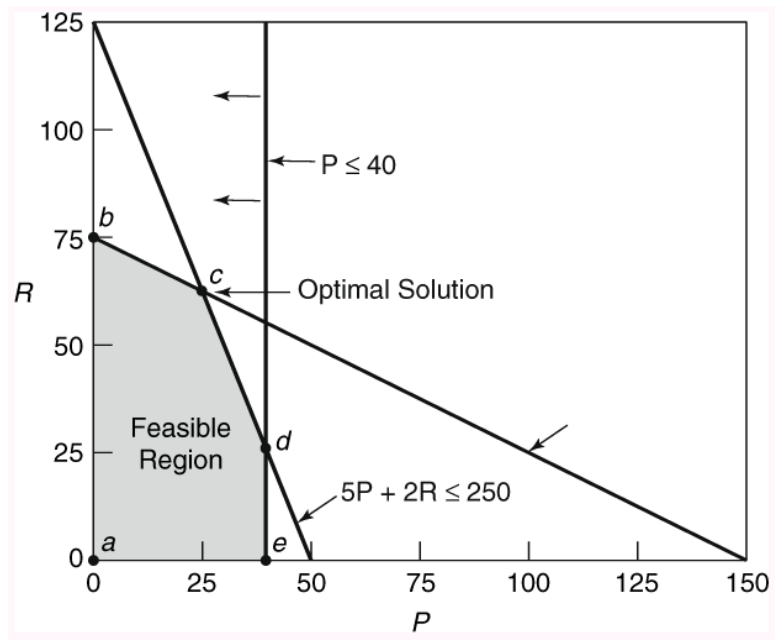


See file P2-33.xls

**2-34.** Let  $P$  = number of barrels of pruned olives  
 $R$  = number of barrels of regular olives

Maximize profit =  $\$20P + \$30R$   
 subject to  $5P + 2R \leq 250$  (labour hours)  
 $\frac{1}{2}P + 1R \leq 75$  (hectares)  
 $P \leq 40$  (barrels)  
 $P, R \geq 0$

- a. Corner point  $a = (P = 0, R = 0)$ , profit = 0  
 Corner point  $b = (P = 0, R = 75)$ , profit =  $\$2,250$   
 Corner point  $c = (P = 25, R = 62.5)$ , profit =  $\$2,375$  ← optimal profit  
 Corner point  $d = (P = 40, R = 25)$ , profit =  $\$1,550$   
 Corner point  $e = (P = 40, R = 0)$ , profit =  $\$800$
- b. Produce 25 barrels of pruned olives and 62 barrels of regular olives.
- c. Devote 12.5 hectares to pruning process and 62.5 hectares to regular process.

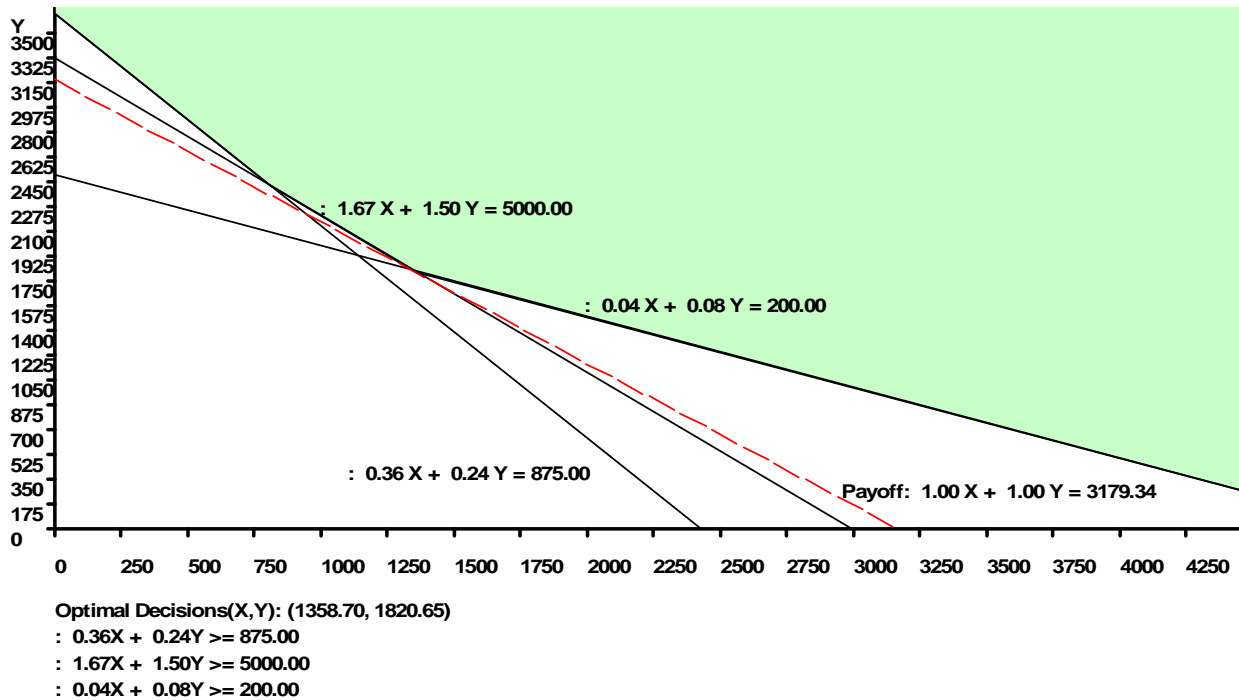


2.35. Let X = dollars to invest in New Brunswick Telecom, Y = dollars to invest in Newfoundland Fishing Co.

Objective: Minimize total investment = X + Y

Subject to:

$$\begin{array}{rcll}
 0.36X + 0.24Y & \geq & 875 & \text{Short term} \\
 1.67X + 1.50Y & \geq & 5000 & \text{Intermediate} \\
 0.04X + 0.08Y & \geq & 200 & \text{Dividend income} \\
 X, Y & \geq & 0 & \text{Non-negativity}
 \end{array}$$



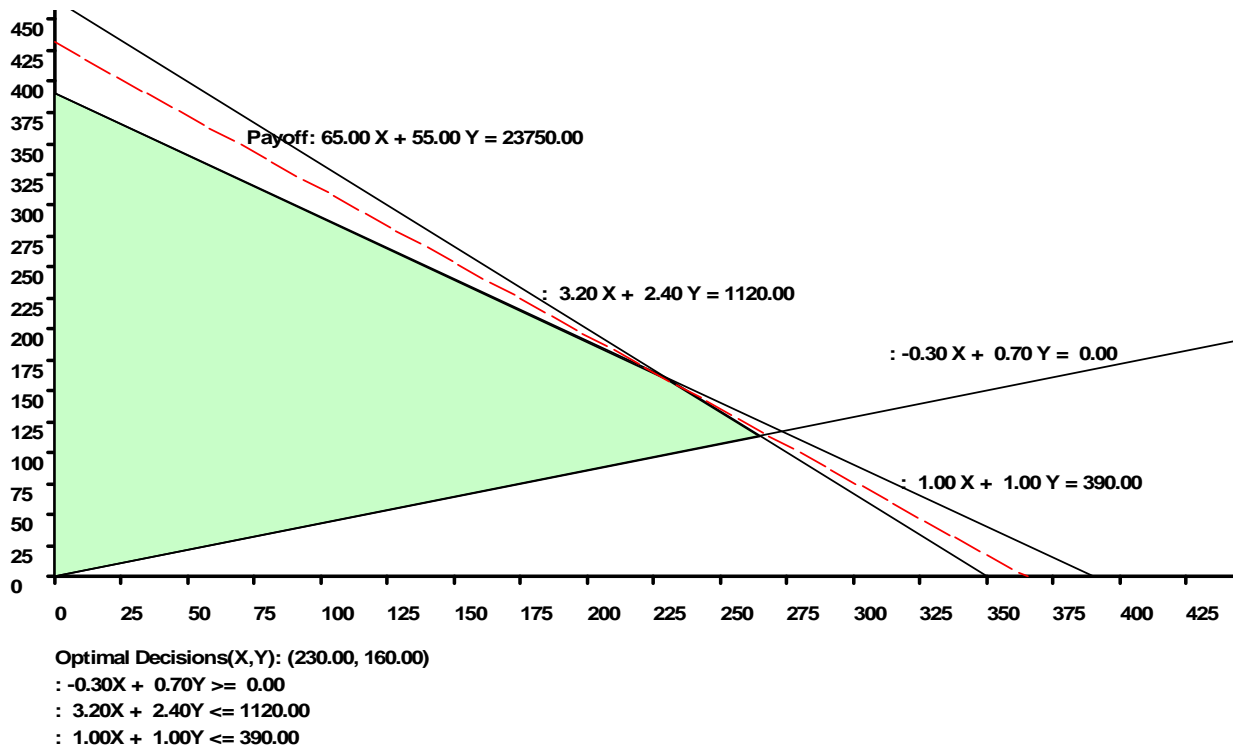
See file P2-35.xls

2.36. Let X = number of boys' bikes to produce, Y = number of girls' bikes to produce.

Objective: Maximize profit =  $(225 - 101.25 - 38.75 - 20)X + (175 - 70 - 30 - 20)Y = \$65X + \$55Y$

Subject to:

$$\begin{array}{rcll} X & + & Y & \leq & 390 & \text{Production limit} \\ 3.2X & + & 2.4Y & \leq & 1120 & \text{Labour hours} \\ & & Y & \geq & 0.3(X+Y) & \text{Min Girls' bikes} \\ X, & Y & & \geq & 0 & \text{Non-negativity} \end{array}$$



See file P2-36.xls

### 2-37

a. Let: R = number of CMC regular modems made and sold in November  
I = number of CMC intelligent modems made and sold in November

Data needed for variable costs and contribution margin are shown in the following table:

Table for Problem 2-35(a)

	CMC REGULAR MODEM		CMC INTELLIGENT MODEM	
	Total	Per Unit	Total	Per Unit
Net sales	\$424,000	\$47.11	\$613,000	\$58.94
Variable costs <sup>a</sup>				
Direct labour	60,000	6.67	76,800	7.38
Indirect labour	9,000	1.00	11,520	1.11
Materials	90,000	10.00	128,000	12.31
General expenses	30,000	3.33	35,000	3.37
Sales commissions	31,000	3.44	60,000	5.76
Total variable costs	\$220,000	\$24.44	\$311,320	\$29.93
Contribution margin	\$204,000	\$22.67	\$301,680	\$29.01

<sup>a</sup>Depreciation, fixed general expense, and advertising are excluded from the calculations.

Hours needed to produce each modem:

$$\text{CMC regular} = \frac{5,000 \text{ hours}}{9,000 \text{ modems}} = 0.555 \text{ hour / modem}$$

$$\text{CMC intelligent} = \frac{10,400 \text{ hours}}{10,400 \text{ modems}} = 1.0 \text{ hour / modem}$$

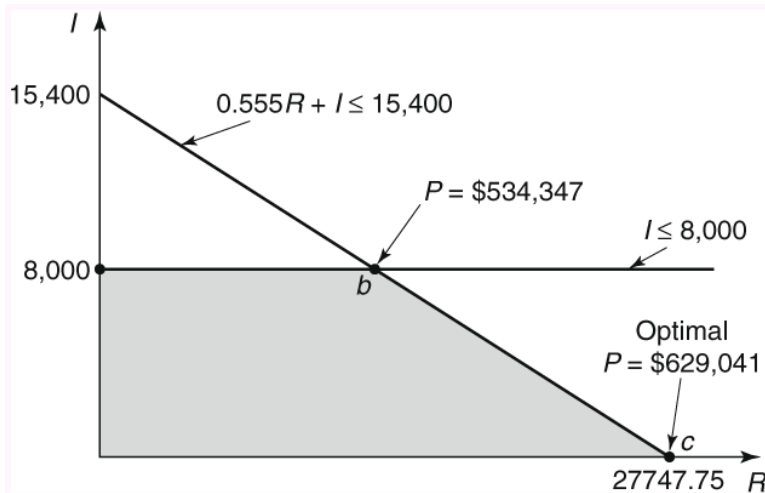
Maximize profit =  $\$22.67R + \$29.01I$

subject to  $0.555R + 1.0I \leq 15,400$  (direct labour hours)

$I \leq 8,000$  (intelligent modems)

$R, I \geq 0$

b.



c. The optimal solution suggests making all CMC regular modems. Students should discuss the implications of shipping no CMC intelligent modems.

d. See file P2-37 for the Excel solution.

**2-38.** Let  $X$  = number of columns of large mailboxes,  $Y$  = number of columns of small mailboxes.

Each column of large mailboxes contains 12 boxes, while each column of small mailboxes contains 18 boxes.

Objective: Maximize total number of mailboxes =  $12X + 18Y$

Subject to:

$$\begin{array}{rcll} 25X + 12Y & \leq & 1200 & \text{Total width} \\ 4500X & \geq & 108\,000 & \text{Large area} \\ X, Y & \geq & 0 & \text{Non-negativity} \end{array}$$

Explanation of constraints:

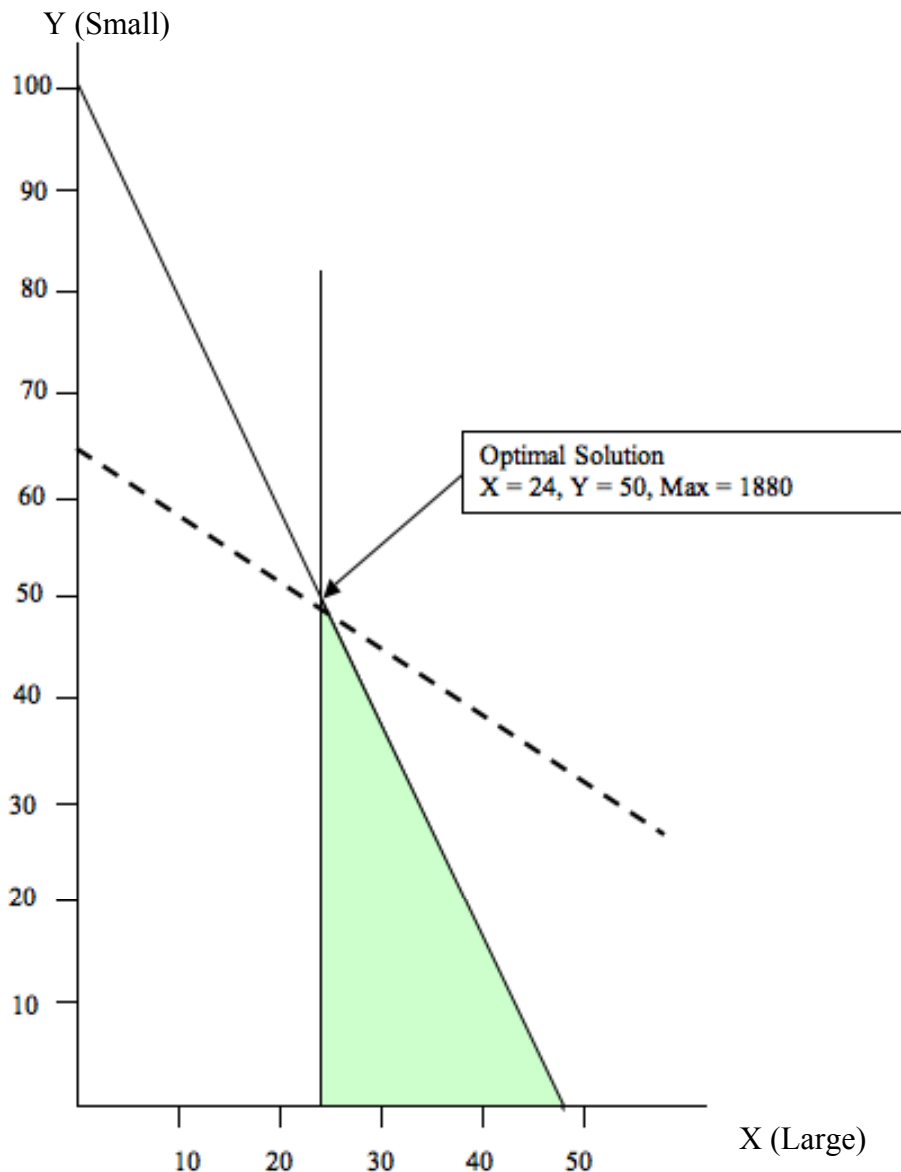
### Total width

The mailroom is 12 m wide, i.e., 1200 cm.

Each column of large mailboxes takes up 25 cm and each column of small mailboxes takes up 12 cm. Therefore the total width used by X large columns and Y small columns is  $25X + 12Y$ . It follows that  $25X + 12Y \leq 1200$ .

### Area constraint

Each column of large mailboxes contains 12 boxes. Each large mailbox uses an area of  $25 \times 15 = 375 \text{ cm}^2$ . Therefore the total area used by a column of large mailboxes is  $12 \times 375 = 4500 \text{ cm}^2$ . Since large mailboxes must account for at least half of the total area of the mailroom wall ( $1/2 \times 1200 \times 180 = 108\,000 \text{ cm}^2$ ), it follows that  $4500X \geq 108\,000$ .



See file P2-38.xls.



**2-39.**

- a. Let:  $X$  = number of kg of stock  $X$  purchased per cow each month  
 $Y$  = number of kg of stock  $Y$  purchased per cow each month  
 $Z$  = number of kg of stock  $Z$  purchased per cow each month

Four kilograms of ingredient  $A$  per cow can be transformed to:

$$4 \text{ kg} \times (1000 \text{ g/kg}) = 4000 \text{ g/cow}$$

$$5 \text{ kg} = 5000 \text{ g of Ingredient B}$$

$$1 \text{ kg} = 1000 \text{ g of Ingredient C}$$

$$8 \text{ kg} = 8000 \text{ g of Ingredient D}$$

$$300X + 200Y + 400Z \geq 4000 \text{ (ingredient A requirement)}$$

$$200X + 300Y + 100Z \geq 5000 \text{ (ingredient B requirement)}$$

$$100X + 0Y + 200Z \geq 1000 \text{ (ingredient C requirement)}$$

$$600X + 800Y + 400Z \geq 8000 \text{ (ingredient D requirement)}$$

$$Z \leq 5 \text{ (stock Z limitation)}$$

$$X, Y, Z \geq 0$$

Minimize cost =  $\$4X + \$8Y + \$5Z$

- b. See File P2-39.XLS for the Excel Solution.

$$\text{Cost} = \$100$$

$$X = 25 \text{ Kg of } X$$

$$Y = 0 \text{ Kg of } Y$$

$$Z = 0 \text{ Kg of } Z$$

**2.40.** Let  $R$  = number of Rocket printers to produce,  $O$ ,  $A$  defined similarly.

Objective: Maximize profit =  $\$60R + \$90O + \$73A$

Subject to:

$$2.9R + 3.7O + 3.0A \leq 4,000 \quad \text{Assembly time}$$

$$1.4R + 2.1O + 1.7A \leq 2,000 \quad \text{Testing time}$$

$$O \geq 0.15(R + O + A) \quad \text{Min Omega}$$

$$R + O \geq 0.40(R + O + A) \quad \text{Min Rocket \& Omega}$$

$$R, O, A \geq 0 \quad \text{Non-negativity}$$

See file P2-40.xls

**2.41.** Let  $J$  = number of units of XJ201 to produce,  $M$ ,  $T$ ,  $B$  defined similarly.

Objective: Maximize profit =  $\$9J + \$12M + \$15T + \$11B$

Subject to:

$$0.5J + 1.5M + 1.5T + 1.0B \leq 15\,000 \quad \text{Wiring time}$$

$$0.3J + 1.0M + 2.0T + 3.0B \leq 17\,000 \quad \text{Drilling time}$$

$$0.2J + 4.0M + 1.0T + 2.0B \leq 10\,000 \quad \text{Assembly time}$$

$$0.5J + 1.0M + 0.5T + 0.5B \leq 12\,000 \quad \text{Inspection time}$$

$$J \geq 150 \quad \text{Minimum XJ201}$$

$$M \geq 100 \quad \text{Minimum XM897}$$

$$T \geq 300 \quad \text{Minimum TR29}$$

$$B \geq 400 \quad \text{Minimum BR788}$$

$$J, M, T, B \geq 0 \quad \text{Non-negativity}$$

See file P2-41.xls

**2-42.** Let  $M_1$  = number of X4509 valves to produce,  $M_2, M_3, M_4$  defined similarly.

Objective: Maximize profit =  $\$16M_1 + \$12M_2 + \$13M_3 + \$8M_4$

Subject to:

$$\begin{array}{rccccrcr}
 0.40M_1 & + & 0.30M_2 & + & 0.45M_3 & + & 0.35M_4 & \leq & 700 & \text{Drilling time} \\
 0.60M_1 & + & 0.65M_2 & + & 0.52M_3 & + & 0.48M_4 & \leq & 890 & \text{Milling time} \\
 1.20M_1 & + & 0.60M_2 & + & 0.50M_3 & + & 0.70M_4 & \leq & 1,200 & \text{Lathe time} \\
 0.25M_1 & + & 0.25M_2 & + & 0.25M_3 & + & 0.25M_4 & \leq & 525 & \text{Inspection time} \\
 M_1 & & & & & & & \geq & 200 & \text{Minimum X4509} \\
 & & M_2 & & & & & \geq & 250 & \text{Minimum X3125} \\
 & & & & M_3 & & & \geq & 600 & \text{Minimum X4950} \\
 & & & & & & M_4 & \geq & 450 & \text{Minimum X2173} \\
 M_1, & M_2, & M_3, & M_4 & & & & \geq & 0 & \text{Non-negativity}
 \end{array}$$

See file P2-42.xls.

**2.43.** Let  $P$  = cans of Plain nuts to produce.  $M, PR$  defined similarly

Objective: Maximize revenue =  $\$2.25P + \$3.37M + \$6.49R$

Subject to:

$$\begin{array}{rccccrcr}
 360P & + & 225M & & & & & \leq & 225\ 000 & \text{Peanuts} \\
 90P & + & 135M & + & 135PR & & & \leq & 125\ 000 & \text{Cashews} \\
 & & + & 45M & + & 135PR & & \leq & 45\ 000 & \text{Almonds} \\
 & & + & 45M & + & 180PR & & \leq & 36\ 000 & \text{Walnuts} \\
 & & & & PR & & & \geq & 100 & \text{Min Premium} \\
 P & & & & -2PR & & & \geq & 0 & \text{Plain vs Premium} \\
 P, & M, & PR & & & & & \geq & 0 & \text{Non-negativity}
 \end{array}$$

See file P2-43.xls

**2.44** Let  $P$  = cans of Plain nuts to produce.  $M, PR$  defined similarly

Objective: Maximize revenue =  $\$2.25P + \$3.37M + \$6.49R$

Subject to:

$$\begin{array}{rccccrcr}
 360P & + & 225M & & & & & \leq & 225\ 000 & \text{Peanuts} \\
 90P & + & 135M & + & 135PR & & & \leq & 125\ 000 & \text{Cashews} \\
 & & + & 45M & + & 135PR & & \leq & 450\ 00 & \text{Almonds} \\
 & & + & 45M & + & 180PR & & \leq & 36\ 000 & \text{Walnuts} \\
 P & + & M & + & PR & & & = & 525 & \text{Cans available} \\
 P & & & & -2PR & & & = & 0 & \text{Plain} = 2 \text{ Premium} \\
 P & & -0.5M & & & & & = & 0 & \text{Plain} = \frac{1}{2} \text{ Mixed} \\
 P, & M, & PR & & & & & \geq & 0 & \text{Non-negativity}
 \end{array}$$

See file P2-44.xls.

**2.45.** Let B = dollars invested in B&O. S, R defined similarly.

Objective: Minimize investment = B + S + R

Subject to:

0.39B	+ 0.26S	+ 0.42R	≥	\$1000	Short term growth
1.59B	+ 1.70S	+ 1.55R	≥	\$6000	Intermediate growth
0.08B	+ 0.04S	+ 0.06R	≥	\$250	Dividend income
B,	S,	R	≥	0	Non-negativity

See file P2-45.xls

## SOLUTIONS TO INTERACTIONS 98 CASE

See PDF file: 8a99E18\_Interactions 98.pdf.

## SOLUTIONS TO GOLDING LANDSCAPING CASE

Minimize cost  $24X_1 + 18X_2 + 22X_3 + 8X_4$

$$\begin{aligned} \text{subject to} \quad & X_1 + X_2 + X_3 + X_4 = 50 \\ & X_4 \geq 7.5 \\ & X_1 + X_2 \geq 22.5 \\ & X_2 + X_3 \leq 15.0 \\ & X_1, X_2, X_3, X_4 \geq 0 \end{aligned}$$

*Solution:* (See file P2-Golding.XLS)

$$X_1 = 7.5 \text{ kilos of C-30}$$

$$X_2 = 15 \text{ kilos of C-92}$$

$$X_3 = 0 \text{ kilos of D-21}$$

$$X_4 = 27.5 \text{ kilos of E-11}$$

Cost = \$6.70.

## SOLUTIONS TO INTERNATIONAL PHARMA INC. CASE

1) The optimal number of subjects, per region, is shown in the following table:

	North Africa & Middle East	Central & Southern Africa	Asia Pacific	South America	Western Europe	Central & Eastern Europe	North America
# of Subjects	500	500	500	3,220	2,500	9,280	3,500

Costs are \$487,737,400 for this least cost solution.

2) While the results here are integer, and thus acceptable, this is actually an integer programming application. The number of subjects must be an integer value, so the LP requirement of divisibility is violated. However, the standard approach to solving IP problems is to see if the LP relaxation yields integer results. Since this occurs here, the solution is acceptable and optimal.

3) Depending upon whether or not the class has covered sensitivity, two approaches can be taken here. Looking at the Sensitivity Report and shadow prices, one sees that CRAs have the greatest impact on cost. All the shadow prices are valid over a substantial range

This could also be determined by repeatedly resolving the LP and adjusting individual RHS values by one unit.

## Appendix 1 Problem Formulation

Let  $x(i)$ ,  $i = 1-7$  be the number of subjects in each of the seven geographical regions

### Objective Function:

Minimize  $z = 12570x_1 + 9764x_2 + 12680x_3 + 11590x_4 + 42980x_5 + 24145x_6 + 28970x_7$

### Subject to:

#### Resources

- 1) # CRA  $0.06x_1 + 0.078x_2 + 0.053x_3 + 0.043x_4 + 0.010x_5 + 0.018x_6 + 0.0140x_7 \leq 475$
- 2) # medical kits  $4x_1 + 9x_2 + 4x_3 + 6x_4 + 1x_5 + 3x_6 + 10x_7 \leq 65000$
- 3) # of subjects  $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \geq 20000$

#### Area Minimums

- 4) NA&ME  $x_1 \geq 500$
- 5) C&SA  $x_2 \geq 500$
- 6) AP  $x_3 \geq 500$
- 7) SA  $x_4 \geq 500$
- 8) WE  $x_5 \geq 2500$
- 9) C&EE  $x_6 \geq 500$
- 10) NA  $x_7 \geq 3500$

Non-negativity:  $x(i) \geq 0$

## Appendix 1 Solver Solution

### Problem Data

Country	North Africa and Middle East	Central and Southern Africa	Asia Pacific	South America	Western Europe	Central and Eastern Europe	North America		RHS
Cost/Subject	12570	9764	12680	11590	42980	24145	28970		
Staff (CRA)	0.06	0.078	0.053	0.043	0.01	0.018	0.014	<=	475
Medical Supply Kits	4	9	4	6	1	3	1	<=	65000
Subjects	1	1	1	1	1	1	1	>=	20000
# from North Africa and Middle East	1	0	0	0	0	0	0	>=	500
# from Central and Southern Africa	0	1	0	0	0	0	0	>=	500
# from Asia Pacific	0	0	1	0	0	0	0	>=	500
# from South America	0	0	0	1	0	0	0	>=	500
# from Western Europe	0	0	0	0	1	0	0	>=	2500
# from Central and Eastern Europe	0	0	0	0	0	1	0	>=	500
# from North America	0	0	0	0	0	0	1	>=	3500

Decision Variables	North Africa and Middle East	Central and Southern Africa	Asia Pacific	South America	Western Europe	Central and Eastern Europe	North America
Number of Subjects	500	500	500	3220	2500	9280	3500

**Cost** 487737400

Constraints	Amount	Slack/Surplus
Staff	475	0
Medical Supply Kits	61660	-3340
Total Subjects	20000	0
# from North Africa and Middle	500	0

East		
# from Central and Southern Africa	500	0
# from Asia Pacific	500	0
# from South America	3220	2720
# from Western Europe	2500	0
# from Central and Eastern Europe	9280	8780
# from North America	3500	0