## SOLUTIONS MANUAL



## Chapter 2: Algorithm Discovery and Design

## Solutions to End-of-Chapter Exercises

1. (a) Set the value of area to $1 / 2 b^{*} h$
(b) Set the value of interest to $I^{*} B$

Set the value of FinalBalance to $(1+I) * B$
(c) Set the value of FlyingTime to M/AvgSpeed
2. Algorithm:

Step 1: Get values for $B, I$, and $S$
Step 2: Set the value of FinalBalance to $(1+I / 12)^{12} B$
Step 3: Set the value of Interest to FinalBalance - B
Step 4: Set the value of FinalBalance to FinalBalance - S
Step 5: Print the message "Interest Earned:"
Step 6: Print the value of Interest
Step 7: Print the message "Final Balance"
Step 8: Print the value of FinalBalance
3. Algorithm:

Step 1: Get values for $E 1, E 2, E 3$ and $F$
Step 2: Set the value of Ave to $(E 1+E 2+E 3+2 F) / 5$
Step 3: Print the value of Ave
4. Algorithm:

Step 1: Get values for $P$ and $Q$
Step 2: Set the value of Subtotal to $P^{*} Q$
Step 3: Set the value of TotalCost to (1.06)*Subtotal
Step 4: Print the value of TotalCost
5. (a)

If $y \neq 0$ then Print the value of $(x / y)$

Else
Print the message "Unable to perform the division."
(b)

If $r \geq 1.0$, then
Set the value of Area to $\pi * r^{2}$
Set the value of Circum to $2 * \pi * r$
Else
Set the value of Circum to $2 * \pi * r$
6. Algorithm:

Step 1: Get a value for $B, I$, and $S$
Step 2: Set the value of FinalBalance to $(1+I / 12)^{12} B$
Step 3: Set the value of Interest to FinalBalance - B
Step 4: If $B<1000$ then Set the value of FinalBalance to FinalBalance $-S$
Step 5: Print the message "Interest Earned:"
Step 6: Print the value of Interest
Step 7: Print the message "Final Balance:"
Step 8: Print the value of FinalBalance
7. Algorithm:

Step 1: Set the value of $i$ to 1
Step 2: Set the values of Won, Lost, and Tied all to 0
Step 3: While $i \leq 10$ do
Step 4: $\quad$ Get the value of $C S U_{i}$ and $O P P_{i}$
Step 5: If $C S U_{i}>O P P_{i}$ then
Step 6: $\quad$ Set the value of Won to Won +1

Step 7: Else if $C S U_{i}<O P P_{i}$ then
Step 8: $\quad$ Set the value of Lost to Lost +1
Step 9: Else
Step 10: $\quad$ Set the value of Tied to Tied +1
Step 11: $\quad$ Set the value of $i$ to $i+1$
End of the While loop
Step 12: Print the values of Won, Lost, and Tied
Step 13: If Won $=10$, then
Step 14: Print the message, "Congratulations on your undefeated season."
8. Algorithm:

Step 1: Set the value of $i$ to 1
Step 2: Set the value of Total to 0
Step 3: While $i \leq 14$ do
Step 4: Get the value of $E_{i}$
Step 5: $\quad$ Set the value of Total to Total $+E_{i}$
Step 6: $\quad$ Set the value of $i$ to $i+1$
End of While loop
Step 7: Get the value of $F$
Step 8: Set the value of Total to Total $+2 F$
Step 9: Set the value of Ave to Total / 16
Step 10: Print the value of Ave
9. Algorithm:

Step 1: Set the value of TotalCost to 0
Step 2: Do
Step 3: $\quad$ Get values for $P$ and $Q$

Step 4: $\quad$ Set the value of Subtotal to $P^{*} Q$
Step 5: $\quad$ Set the value of TotalCost to TotalCost $+(1.06)^{*}$ Subtotal
While (TotalCost < 1000)
Step 6: Print the value of TotalCost
10. The tricky part is in steps 6 through 9. If you use no more than than 1000 kilowatt hours in the month then you get charged $\$ .06$ for each. If you use more than 1000 , then you get charged $\$ .06$ for the first 1000 hours and $\$ .08$ for each of the remaining hours. There are $M_{i}-1000$ remaining hours, since $M_{i}$ is the number of hours in the $i$ th month. Also, remember that $K W$ Begin ${ }_{i}$ and $K W E n d_{i}$ are meter readings, so we can determine the total kilowatt-hours used for the whole year by subtracting the first meter reading $\left(K W B e g i n_{1}\right)$ from the last $\left(K_{W E n d}^{12}\right)$.

Step 1: Set the value of $i$ to 1
Step 2: Set the value of AnnualCharge to 0
Step 3: While $i \leq 12$ do
Step 4: $\quad$ Get the value of $K W$ Begin $_{i}$ and $K_{W E n d}^{i}$
Step 5: $\quad$ Set the value of $M_{i}$ to $K W E n d_{i}-K W B e g i n ~ i ~$
Step 6: If $M_{i} \leq 1000$ then
Step 7: $\quad$ Set AnnualCharge to AnnualCharge $+\left(.06 M_{i}\right)$
Step 8: Else
Step 9: $\quad$ Set AnnualCharge to AnnualCharge $+(.06) 1000$

$$
+(.08)\left(M_{i}-1000\right)
$$

Step 10: $\quad$ Set the value of $i$ to $i+1$
End of While loop
Step 11: Print the value of AnnualCharge
Step 12: If $\left(\right.$ KWEnd $_{12}-$ KWBegin $\left._{1}\right)<500$, then
Step 13: Print the message "Thank you for conserving electricity."
11. Algorithm:

Step 1: Do

Step 2: $\quad$ Get the values of HoursWorked and PayRate
Step 3: If HoursWorked $>54$ then
Step 4: $\quad D T=$ HoursWorked -54
Step 5: $\quad T H=14$
Step 6: $\quad \operatorname{Reg}=40$
Step 7: Else if HoursWorked $>40$ then
Step 8: $\quad D T=0$
Step 9: $\quad T H=$ HoursWorked -40
Step 10: $\quad \operatorname{Reg}=40$
Step 11: $\quad$ Else (HoursWorked $\leq 40$ )
Step 12: $\quad D T=0$
Step 13: $\quad T H=0$
Step 14: $\quad$ Reg $=$ HoursWorked
Step 15: GrossPay = PayRate * HoursWorked

$$
+1.5 * \text { PayRate } * T H+2 * \text { PayRate } * D T
$$

Step 16: Print the value of GrossPay
Step 17: Print the message "Do you wish to do another computation?"
Step 18: Get the value of Again

> While (Again = yes)
12. Steps $1,2,5,6,7$, and 9 are sequential operations and steps 4 and 8 are conditional operations. After their completion, the algorithm moves on to the step below it, so none of these could cause an infinite loop. Step 3, however, is a while loop, and it could possibly cause an infinite loop. The true/false looping condition is "Found $=$ NO and $i \leq 10000$." If NAME is ever found in the loop then Found gets set to YES and the loop stops, and the algorithm ends after executing steps 8 and 9. If NAME is never found, then 1 is added to $i$ at each iteration of the loop. Since step 2 initializes $i$ to $1, i$ will become 10,001 after the $10,000^{\text {th }}$ iteration of the loop. At this point the loop will halt, steps 8 and 9 will be executed, and the algorithm will end.
13. Algorithm:

Step 1: Get values for $N A M E, N_{1}, \ldots . N_{10000}$, and $T_{1}, \ldots, T_{10000}$
Step 2: Set the value of $i$ to 1 and set the value of NumberFound to 0
Step 3: While ( $i \leq 10000$ ) do steps 4 through 7
Step 4: If NAME equals $N_{i}$ then
Step 5: $\quad$ Print the phone number $T_{i}$
Step 6: $\quad$ Set the value of NumberFound to NumberFound +1
Step 7: $\quad$ Add 1 to the value of $i$
Step 8: Print the message NAME "was found" NumberFound "times"
Step 9: Stop
14. Let's assume that FindLargest is now a primitive to us, and use it to repeatedly remove the largest element from the list until we reach the median.

Step 1: Get the values $L_{1}, L_{2}, \ldots, L_{N}$ of the numbers in the list
Step 2: If $N$ is even then

$$
\text { Let } M=N / 2
$$

Else
Let $M=(N+1) / 2$
Step 3: While $(\mathrm{N} \geq \mathrm{M})$ do steps 4 through 9
Step 4: Use FindLargest to find the location of the largest number in the list $L_{1}, L_{2}, \ldots, L_{N}$

Step 5: $\quad$ Exchange $L_{\text {location }}$ and $L_{N}$ as follows
Step 6: $\quad$ Temp $=L_{N}$
Step 7: $\quad L_{N}=L_{\text {location }}$
Step 8: $\quad L_{\text {location }}=$ Temp
Step 9: $\quad$ Set $N$ to $N-1$ and effectively shorten the list

Step 10: Print the message "The median is:"
Step 11: Print the value of $L_{M}$
Step 12: Stop
15. This algorithm will find the first occurrence of the largest element in the collection. This element will become LargestSoFar, and from then on $A_{\mathrm{i}}$ will be tested to see if it is greater than LargestSoFar. Some of the other elements are equal to LargestSoFar but none are greater than it.
16. (a) If $n \leq 2$, then the test would be true, so the loop would be executed. In fact, the test would never become false. Thus the algorithm would either loop forever, or generate an error when referring to an invalid $A_{i}$ value. If $n>2$, then the test would be false the first time through, so the loop would be skipped and $A_{1}$ would be reported as the largest value.
(b) The algorithm would find the largest of the first $n-1$ elements and would not look at the last element, as the loop would exit when $i=n$.
(c) For $n=2$ the loop would execute once, comparing the $A_{1}$ and $A_{2}$ values. Then the loop would quit on the next pass, returning the larger of the first two values. For any other value of $n$, the loop would be skipped, reporting $A_{1}$ as the largest value.
17. (a) The algorithm would still find the largest element in the list, but if the largest were not unique then the algorithm would find the last occurrence of the largest element in the list.
(b) The algorithm would find the smallest element in the list.

The relational operations are very important, and care must be taken to choose the correct one, for changing them can drastically change the output of the algorithm.
18. (a) The algorithm will find the three occurrences of and. First in the word band, second in the word and, and third in the word handle.
(b) We could search for " and ". That is, the word itself surrounded by spaces. Note that the word "and" is special in that it is almost always surrounded by spaces in a sentence. Other words may start or end sentences and be followed by punctuation.
19. It would go into an infinite loop, because $k$ will stay at 1 , and we will never leave the outside while loop. We will keep checking the 1 position over and over again.
20. Step 1: Get the value for $N$

Step 2: Set the value of $i$ to 2
Step 3: Set the value of $R$ to 1 ;
Step 4: While ( $i<N$ and $R \neq 0$ ) do Steps 5-6
Step 5: $\quad$ Set $R$ to the remainder upon computing $N / i$
Step 6: $\quad$ Set the value of $i$ to $i+1$
Step 7: If $R=0$ then
Print the message "not prime"
Else
Print the message "prime"
(This algorithm could be improved upon because it is enough to look for divisors of N less than or equal to $\sqrt{N}$.)
21. Here we assume that we can perform "arithmetic" on characters, so that $\mathrm{m}+3=\mathrm{p}$, for example. Step 4 is the difficult part that must handle the "wraparound" from the end of the alphabet back to the beginning.

Step 1: Get the values for nextChar and $k$
Step 2: While (nextChar $\neq \$$ ) do steps 3 through 5
Step 3: $\quad$ Set the value of outChar to nextChar $+k$
Step 4: If outChar $>\mathrm{z}$ then
Set the value of outChar to (outChar-26)
Step 5: $\quad$ Print outChar
22. Step 1: Get the values for $k$ and $N_{1}, N_{2}, \ldots, N_{\mathrm{k}}$

Step 2: Set the value of front to 1
Step 3: Set the value of back to $k$
Step 4: While (front $\leq$ back) do steps 5 through 9
Step 5: $\quad$ Set the value of $\operatorname{Temp}$ to $N_{\text {back }}$

Step 6: $\quad$ Set the value of $N_{b a c k}$ to $N_{\text {front }}$
Step 7: $\quad$ Set the value of $N_{\text {front }}$ to Temp
Step 8: $\quad$ Set front $=$ front +1
Step $9 \quad$ Set back $=$ back -1
23. Step 1: Get the values for $N_{1}, N_{2}, \ldots, N_{\mathrm{k}}$, and $S U M$

Step 2: Set the value of $i$ to 1
Step 3: Set the value of $j$ to 2
Step 4: While $(i<k)$ do steps 5 through 11
Step 5: $\quad$ While $(j \leq k)$ do steps 6 through 9
Step 6: $\quad$ If $N_{\mathrm{i}}+N_{\mathrm{j}}=S U M$ then
Step 7
Step 8:
$\operatorname{Print}\left(N_{\mathrm{i}}, N_{\mathrm{k}}\right)$
Stop
Else
Step 9: $\quad$ Set the value of $j$ to $j+1$
Step 10: $\quad$ Set the value of $i$ to $i+1$
Step 11: $\quad$ Set the value of $j$ to $i+1$
Step 12: Print the message "Sorry, there is no such pair of values."

## Discussion of Challenge Work

1. The general algorithm is fairly clear, in English, in the text.

Step 1: Read values for start, step, and accuracy
Step 2: While $\mid$ step $\mid>$ accuracy do steps 3 through 9
Step 3: $\quad$ If $f($ start $)>0$ then set FirstSign to +
Step 4: $\quad$ Else set FirstSign to -

Step 5: Do steps 6 through 8
Step 6: $\quad$ Set the value of start to start + step
Step 7: $\quad$ If $f($ start $)>0$ then set the value of Sign to +
Step 8: $\quad$ Else set the value of Sign to -
while $($ Sign $=$ FirstSign $)$
Step 9: $\quad$ If $\mid$ step $\mid \geq$ accuracy then set the value of step to (-0.1)step
Step 10: Set the value of root to start - step/2
Step 11: Print the value of root.
2. Many excellent simulations of sorting algorithms are available on the Web, suggest students examine them if they have questions about this algorithm.

The Find Largest algorithm given in the book always searches the whole list. First, we should create a variation that takes, in addition to the list of values, two indices which bound the range of the list that should be searched. Also, it is easiest to return the location of the largest value, for use in the sort algorithm. Below is a sketch of how it should change:

FindLargest(A, start, end)
Step 1: Set the value of loc to start
Step 2: Set the value of $i$ to start +1
Step 3: While ( $i \leq e n d$ ) do
Step 4: If $A_{i}>A_{l o c}$ then
Step 5: $\quad$ Set loc to $i$
Step 6: $\quad$ Add 1 to the value of $i$
Step 7: End of the loop
Step 8: Return the value loc
The Selection Sort algorithm is quite simple, once we have a suitable form for the Find Largest portion of it.

Step 1: $\operatorname{SelectionSort}(A, n)$
Step 2: Set lastpos to $n$

Step 3: While (lastpos $\geq 1$ ) do
Step 4: $\quad$ Set biggestpos to FindLargest(A, 1, lastpos)
Step 5: $\quad$ Swap $A_{\text {lastpos }}$ and $A_{\text {biggestpos }}$
Step 6: $\quad$ Subtract 1 from lastpos
Step 7: End of loop
3. Students should be provided with concrete leads to reference materials about nonEuropean mathematicians, including references to online resources.

