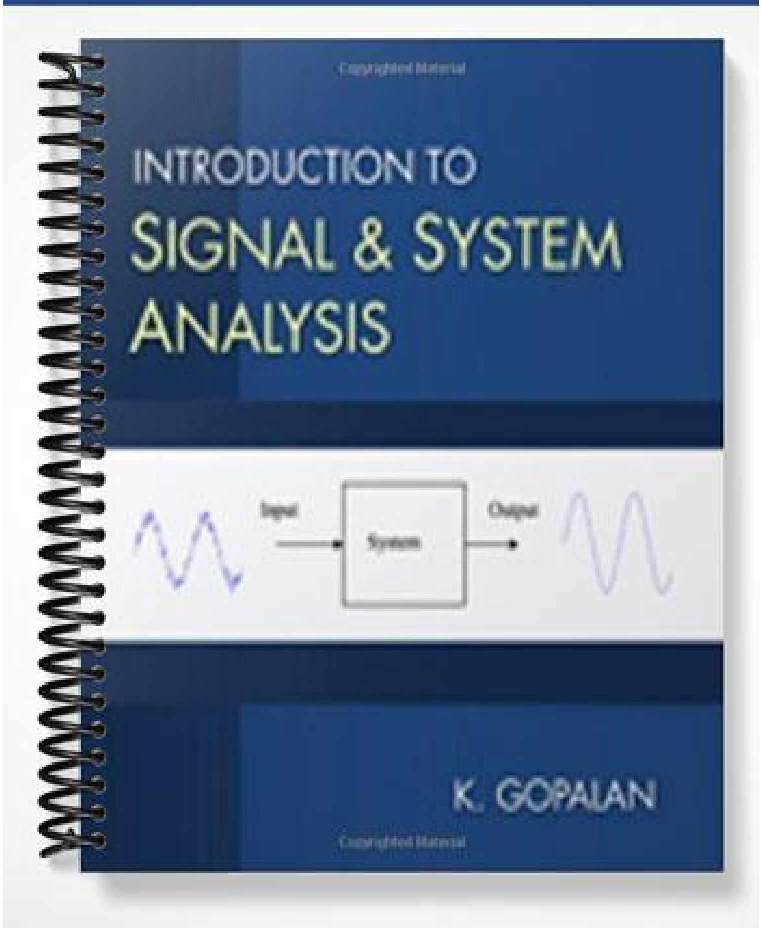
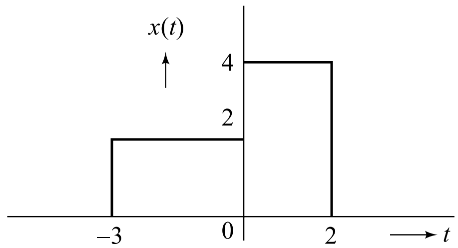
SOLUTIONS MANUAL

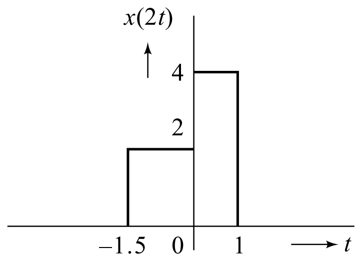


Chapter 2

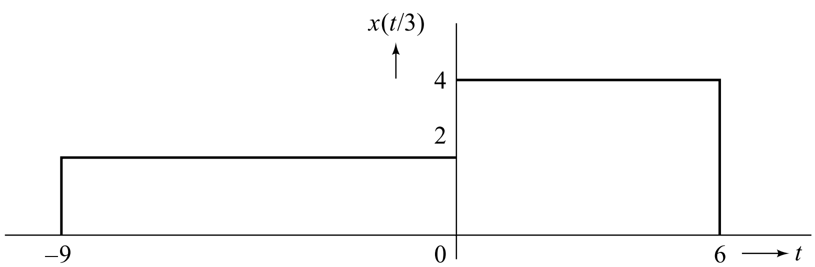
2.1



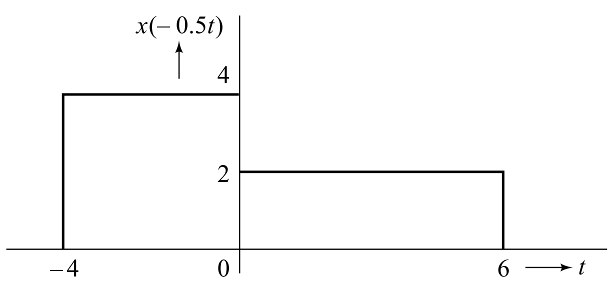
(a)



(b)



(c)

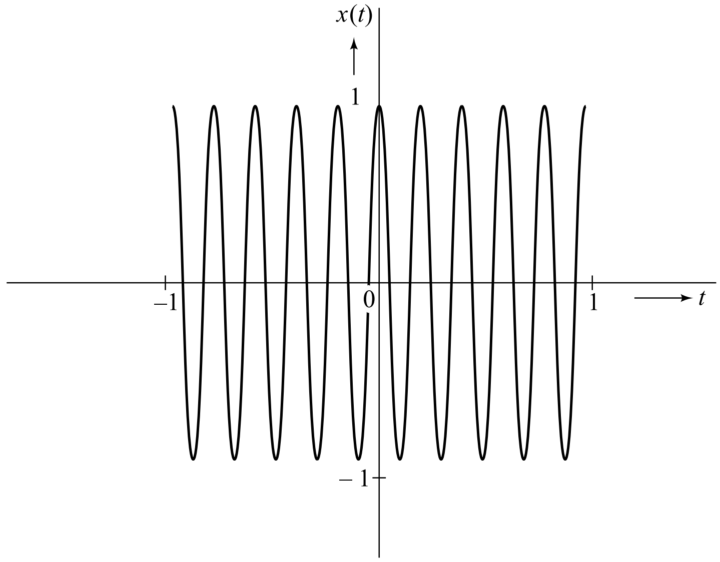


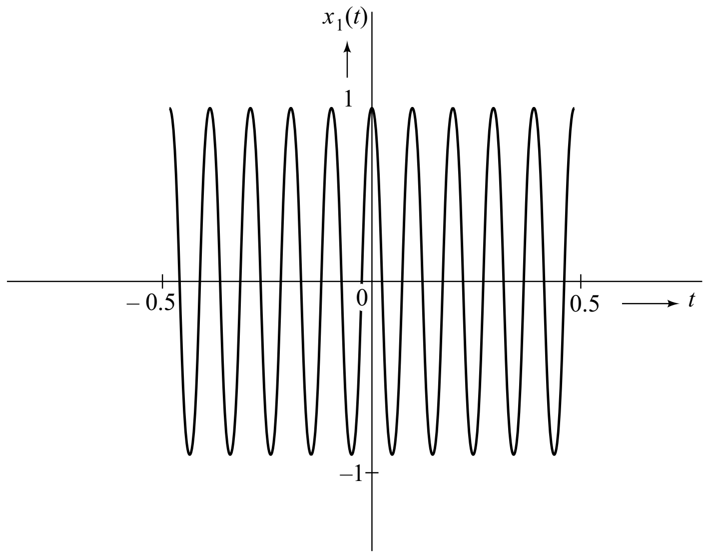
2.2 $x(t) = \cos 10\pi t$, $-1 \le t \le 1$ has 10 cycles of sinusoid.

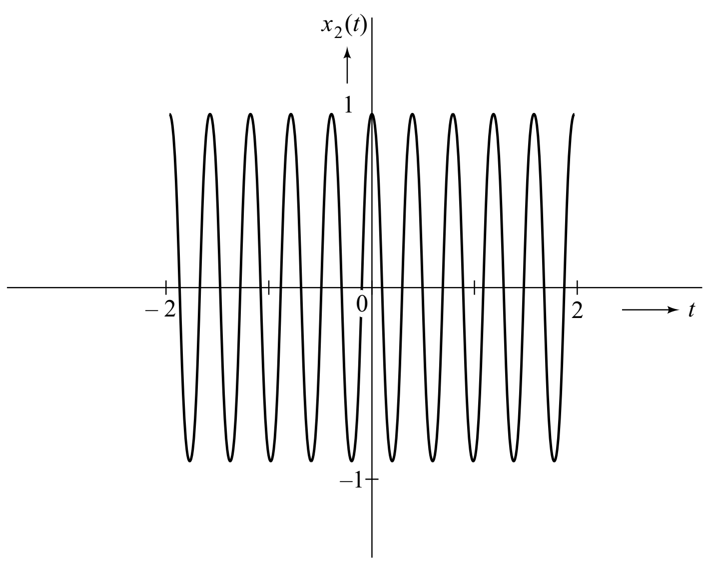
$$x_1(t) = x(2t), \text{ range: } -0.5 \le t \le 0.5$$

$$x_2(t) = x(0.5t), \text{ range: } -2 \le t \le 2$$

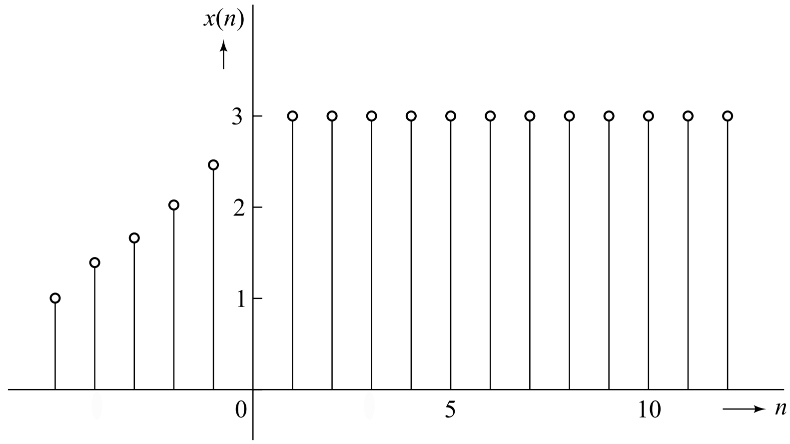
Since the time-compressed $x_1(t) = \cos 20\pi t$, it also has 10 cycles in the compressed time interval $-0.5 \le t \le 0.5$. Similarly, $x_2(t) = x(0.5t) = \cos 5\pi t$, has 10 cycles in the expanded interval $-2 \le t \le 2$. In general, if the time scale (compression/expansion) ratio is an integer, it results in scaled (compressed/expanded) period for each cycle and the same number of cycles appears in the scaled (compressed/expanded) interval.

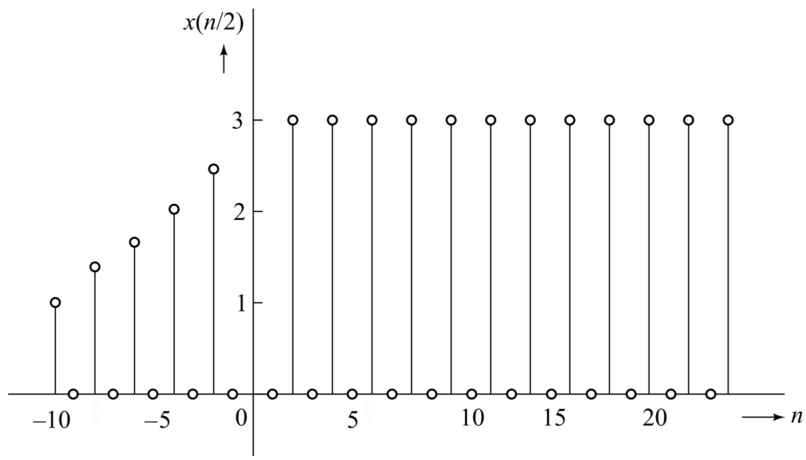

x(t)
1
–1
0
1
t
– 1

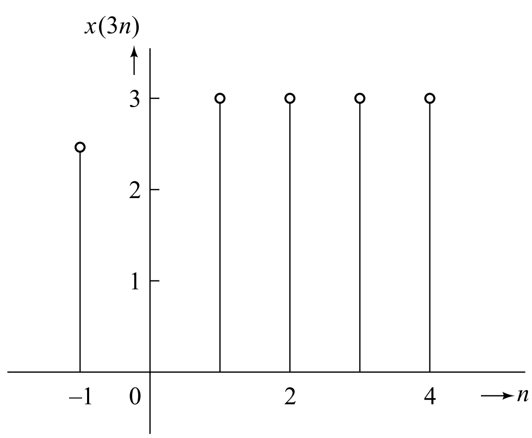

$x_1(t)$
1
– 0.5
0
0.5
t
–1

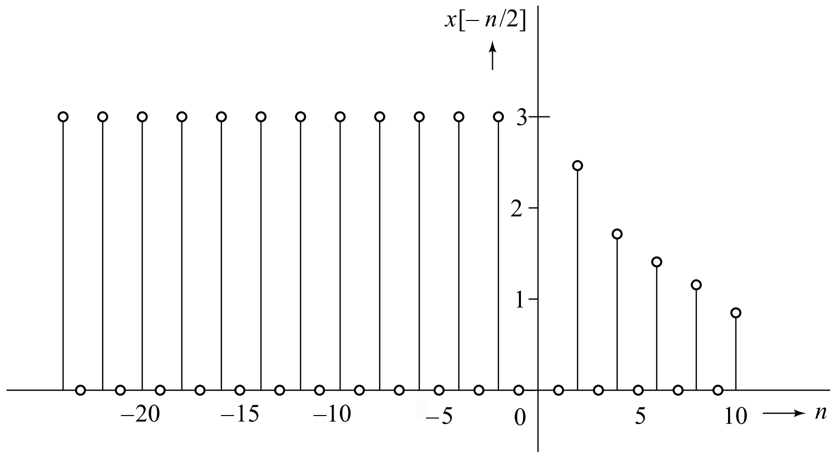

$x_2(t)$
1
– 2
0
2
t
–1

2.3

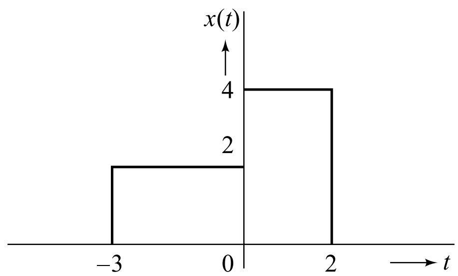


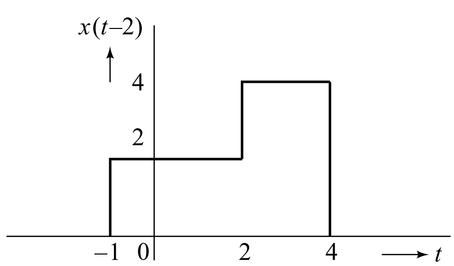


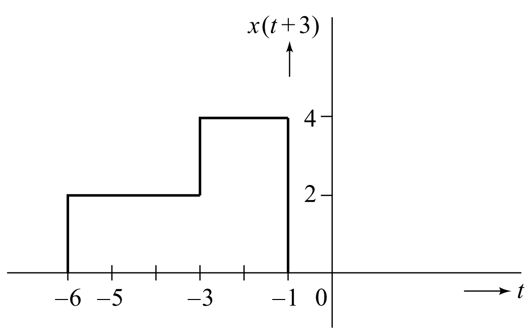




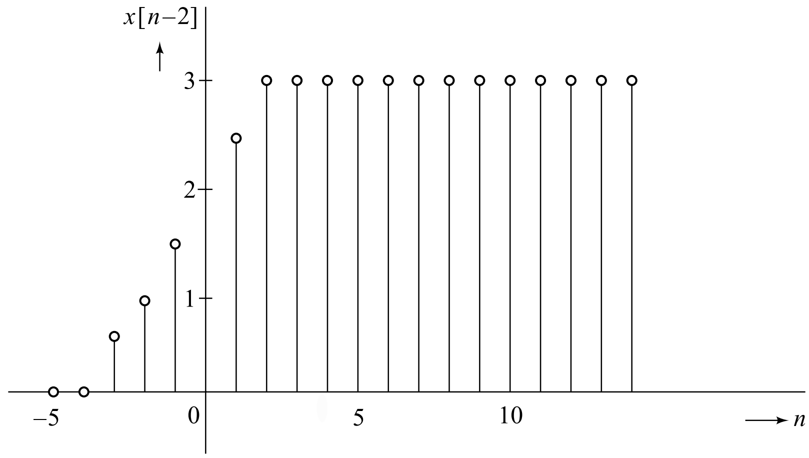
2.4 (a)

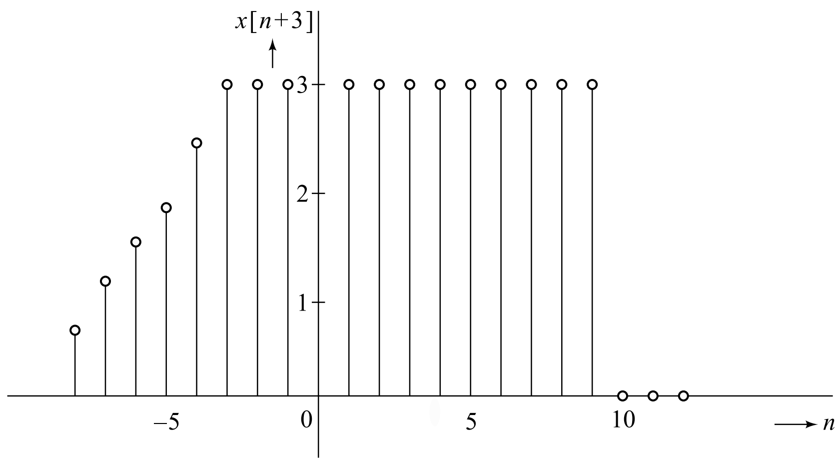




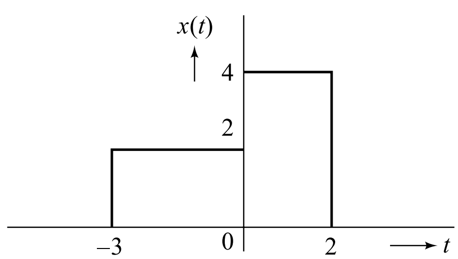


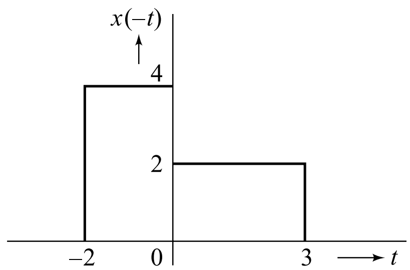
2.4 (b)

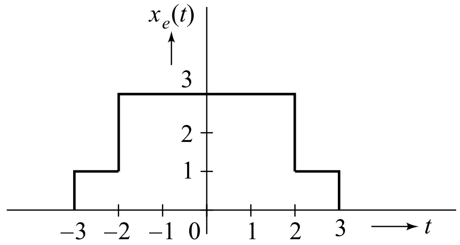


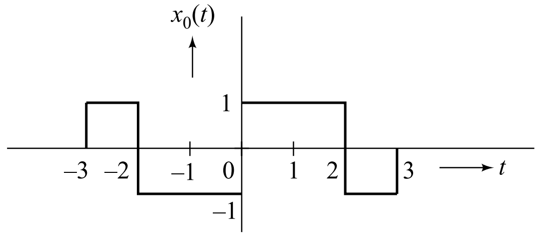


2.5 (a)

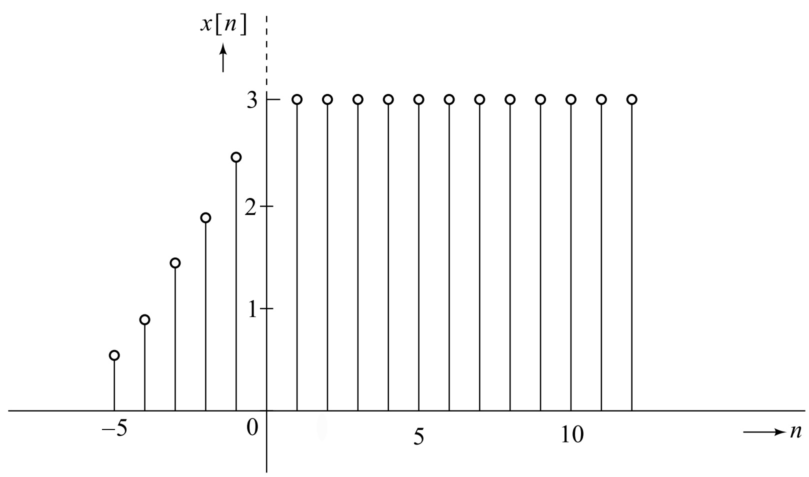


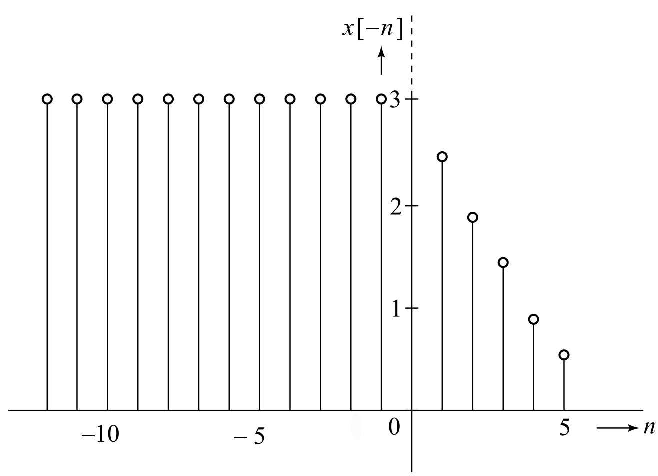


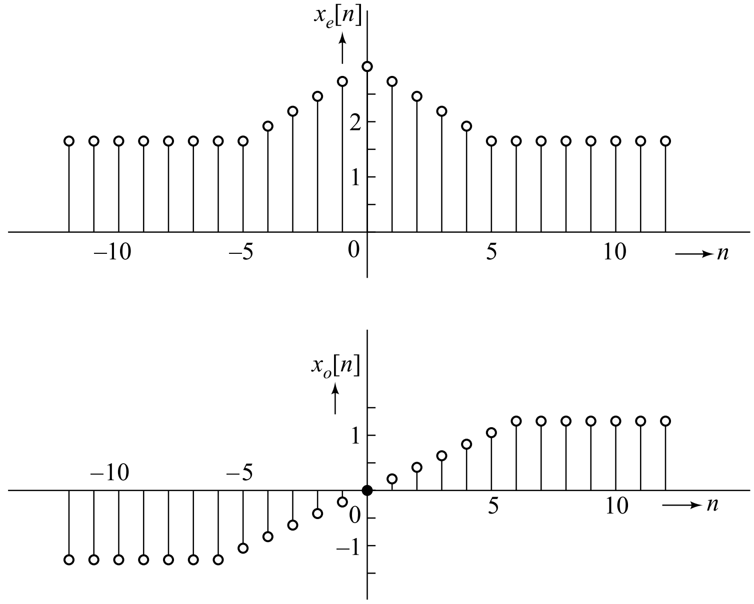




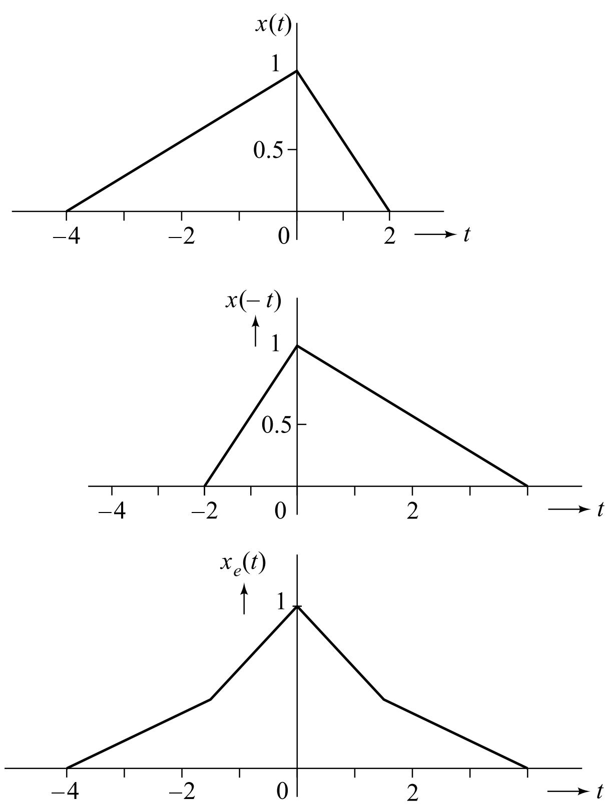
2.5 (b)

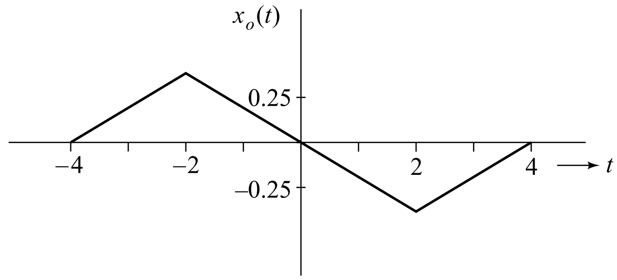





x_e[n]
2
1
−10
−5
0
5
10
n
x_o[n]
1
−10
−5
0
−1
5
10
n

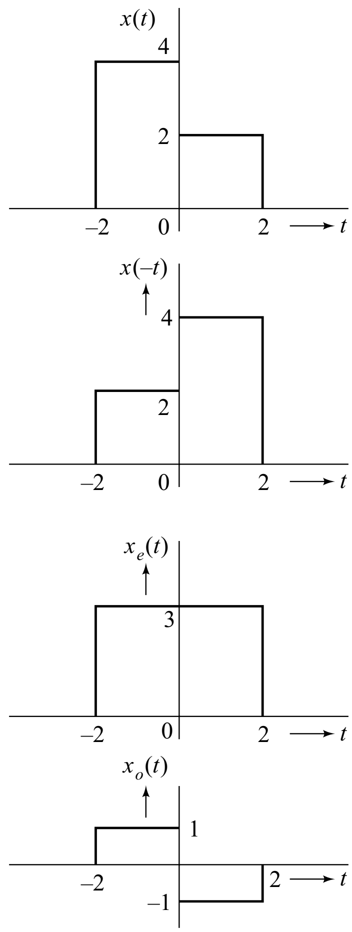
2.6 (a)


x(t)
1
0.5
−4
−2
0
2
t
x(− t)
1
0.5
−4
−2
0
2
t
x_e(t)
1
−4
−2
0
2
t



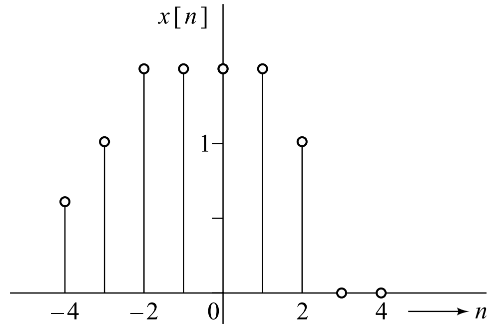
$x_o(t)$
0.25
−4
−2
2
4
t
−0.25

2.6 (b)

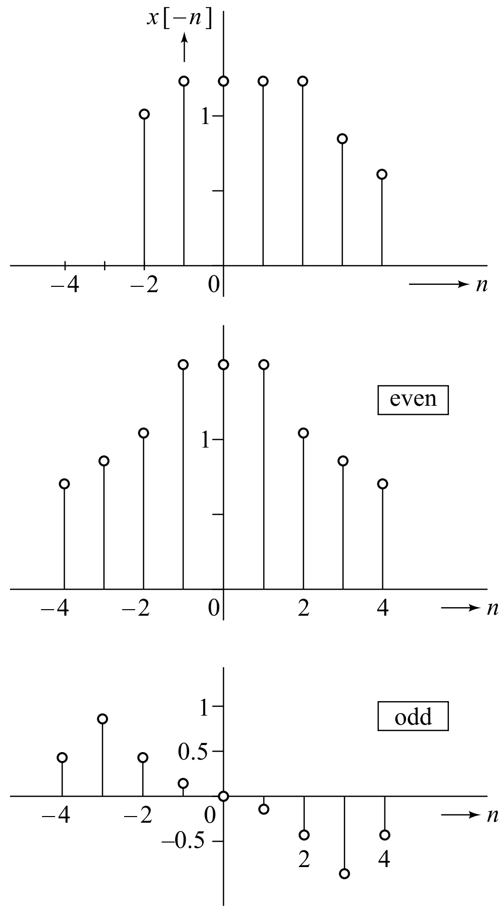


$x(t)$
4
2
−2
0
2
t
$x(-t)$
4
2
−2
0
2
t
$x_e(t)$
3
−2
0
2
t
$x_o(t)$
1
−2
2
t
−1

2.6 (c)

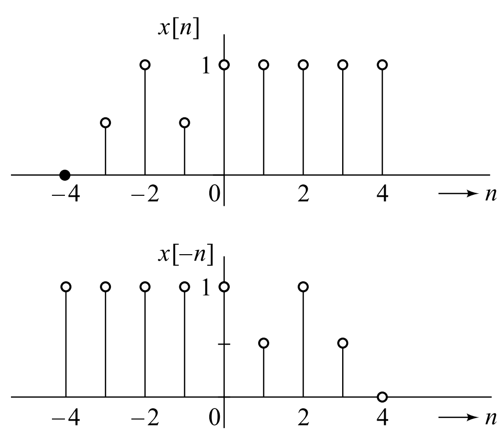


$x[n]$
1
−4
−2
0
2
4
n

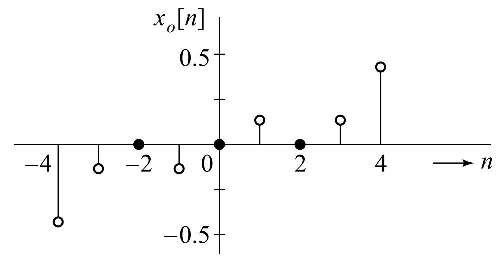


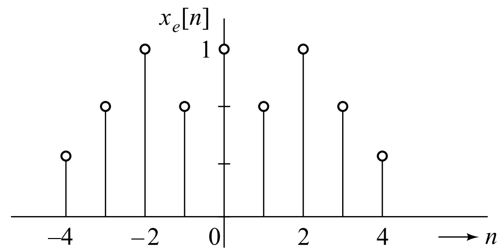
x[−n]
1
−4
−2
0
n
even
1
−4
−2
0
2
4
n
odd
1
0.5
−4
−2
0
−0.5
2
4
n

2.6 (d)



x[n]
1
−4
−2
0
2
4
n
x[−n]
1
−4
−2
0
2
4
n





2.7 $x_3(t) = x_1(t)\, x_2(t)$; $x_1(t) = x_1(-t)$; $x_2(-t) = -x_2(t)$
$x_3(-t) = x_1(-t)\, x_2(-t) = x_1(t)\,(-x_2(t)) = -x_1(t)\, x_2(t)$
$= -x_3(t) \Rightarrow x_3(t)$ is an odd signal

2.8 (a) $x(t) = 2\sin(10\pi t - 30°) \Rightarrow$ periodic

$$10\pi T = 2\pi k \Rightarrow T = \frac{1}{5}\,s$$

(b) $x(t) = 2\cos\sqrt{2}\,\pi t \Rightarrow$ periodic $\Rightarrow T = \sqrt{2}\,s$
(irrational value for T)

(c) periodic $\Rightarrow T = 2\sqrt{\pi}\,s$ (irrational)

(d) $x(t) = \sin^{-1}(10\pi t) \Rightarrow$ aperiodic

(e) $x(t) = 2\cos(t + 60°) = 2\cos((t + T) + 60°) \Rightarrow T = 2\pi\,s$ (irrational)

(f) $x(t) = 2\cos(6\pi t + 60°) + j\sin(6\pi t + 30°) = 2e^{j6\pi t}\, e^{j60°}$

$x(t + T) = 2e^{j6\pi(t+T)}\, e^{j60°} = x(t)$ for $6\pi T = 2\pi k$

or $T = \frac{1}{3}\,s$; periodic

(g) $3e^{j20t}\, e^{j\theta} \Rightarrow T = \frac{\pi}{10}\,s$; (irrational)

(h) $3e^{j(20\pi t + \pi/3)} \Rightarrow$ periodic, $T = \frac{1}{10}\,s$

(i) $3\cos\pi t \Rightarrow T_1 = 2s$; $2\cos t \Rightarrow T_2 = 2\pi s$

$3\cos\pi t + 2\cos t$: No rational value for $\frac{T_1}{T_2} \Rightarrow$ aperiodic

(j) $\cos 100\pi t \Rightarrow T_1 = \frac{1}{50}\,s$; $\sin 200\pi t \Rightarrow T_2 = \frac{1}{100}\,s$

$\frac{T_1}{T_2} = 2 \Rightarrow T = \frac{1}{50}\,s$ for $\cos 100\pi t + \sin 200\pi t$

2.9 (a) $x[n] = 2\cos n\pi = x[n + N] = 2\cos((n + N)\pi)$
periodic, $N = 2$

(b) $3n\cos n\pi \neq 3(n + N)\cos((n + N)\pi)$ for any integer N.
$\therefore$ nonperiodic

(c) $2\cos 10n \neq 2\cos(10(n+N))$ for any integer N.

$\therefore$ nonperiodic

(d) $3\sin 0.2n\pi \Rightarrow$ periodic, $N = 10$

(e) $3\sin 1.2n\pi \Rightarrow$ periodic, $N = \frac{2k}{1.2} \Rightarrow N = 5$ (smallest)

(f) $4e^{jn\pi} = 4e^{j(n+N)\pi} \Rightarrow$ periodic, $N = 2$

(g) $e^{j\frac{n\pi}{2}} = e^{j(n+N)\frac{\pi}{2}} \Rightarrow$ periodic, $N = 4$

(h) $4e^{j(n\pi + 60°)} = 4e^{j((n+N)\pi + 60°)} \Rightarrow$ periodic, $N = 2$

2.10 (a) $2\cos(t + 60°)$: periodic, power signal, $T = 2\pi$

$$P = \frac{1}{2\pi}\int_0^{2\pi} 4\cos^2(t+60°)dt = \frac{1}{\pi}\int_0^{2\pi} [1 + \cos(2(t+60°))]dt$$

$P = 2$

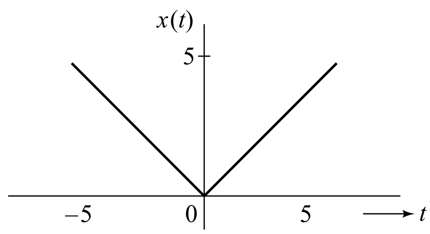
(b) $x(t) = 3e^{j20\pi t} \Rightarrow$ periodic; $T = \frac{1}{10}s$

$$P = \frac{1}{T}\int_0^T x(t)\,x^*(t)dt = 10\int_0^{\frac{1}{10}} 9dt$$

$P = 9$

(c) Energy signal

$$E = 2\int_0^5 t^2\,dt = \frac{250}{3}$$

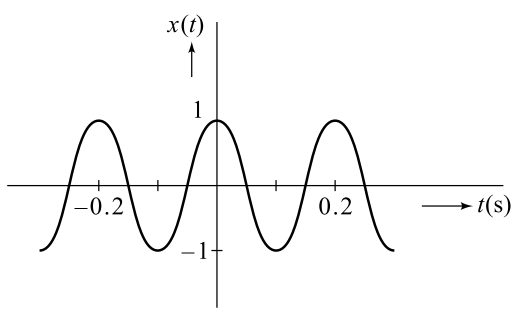


(d) $x(t) = \cos \pi t$, $|t| \leq 0.2$, repeats every $0.4s$

period $= 0.2s \Rightarrow$ power signal

$$P = \frac{1}{0.2}\int_0^{0.2} \cos^2 \pi t\,dt = \frac{1}{0.4}\int_0^{0.2} (1 + \cos 2\pi t)\,dt$$

$P = \frac{1}{2}$



(e) $x[n] = 5$, $n = 0, 2, 5, 7 \Rightarrow$ Energy signal

$E = 4 \times 5^2 = 100$

(f) $x[n] = \cos n\pi \Rightarrow$ periodic; period $N = 2$

$$P = \frac{1}{2}\sum_{n=0}^{1} \cos^2 n\pi = 1$$

2.11 (a) $\int_{-5}^{-2} t\,\delta(t+3)dt = -3$

(b) $\int_{-\infty}^{t} (\alpha+2)\,\delta(\alpha+2)\,u(\alpha)d\alpha = 0;\;\; \alpha = -2 \Rightarrow u(-2) = 0$

(c) $\int_{-\infty}^{t} \delta(t+2)\cos(10t)\,u(t)dt = 0;\;\; t = -2 \Rightarrow u(-2) = 0$

(d) $\int_{0}^{2} \cos 5t\,\delta(t+2)dt = 0$

(e) $\int_{-5}^{5} \cos 5t\,\delta(t+2)dt = \cos(-10)$

(f) $\int_{-\infty}^{\infty} \delta(t-4)\,t\,e^{-at}\,u(t)dt = 4e^{-4a}$

(g) $\int_{-\infty}^{\infty} e^{a\alpha}\,u(-\alpha)\,\delta(a-t_0)d\alpha = \begin{cases} e^{at_0} & , \; t_0 \le 0 \\ 0 & , \; t_0 > 0 \end{cases}$

2.12 $\int_{-\infty}^{\infty} \delta(at-t_0)\,f(t)dt = \int_{-\infty}^{\infty} \delta(\alpha)\,\frac{f}{|a|}(\alpha + t_0/a)\,d\alpha$ using at $- t_0 = \alpha$

$$= \frac{1}{|a|} f\left(\frac{t_0}{a}\right)$$

(a) $\int_{-\infty}^{\infty} e^{at}\,u(t)\,\delta(2t-4)dt = \frac{1}{2}e^{2a}$

(b) $\int_{-\infty}^{\infty} e^{-at}\cos bt\,\delta(2t-5)dt = e^{-\frac{5}{2}a}\cos\frac{5}{2}b$

(c) $\int_{-\infty}^{\infty} r(3t)\,\delta(2t-4)dt = \frac{1r}{2}\left(3\times\frac{4}{2}\right) = \frac{6}{2} = 3$

2.13 (a) $\sum_{n=-\infty}^{\infty} \delta[n+2]e^{-2n} = e^{4}$

(b) $\sum_{n=-\infty}^{\infty} \delta[n-3]\cos\left(\frac{n\pi}{4}\right) = \cos\frac{3\pi}{4}$

(c) $\sum_{n=0}^{\infty} 3(n-2)\,\delta[n-5] = 9$

(d) $\sum_{n=-\infty}^{\infty} \delta[2n-3]\,x[n] = \sum_{l=-\infty}^{\infty} \delta[l]\;x\left[\frac{l+3}{2}\right]$, using $l = 2n-3$

$$= x\left[\frac{3}{2}\right] \Rightarrow \text{undefined}$$

2.14 $y[n] = \frac{1}{3}\,[x[n] + x[n-1] + x[n-2]]$

$y[n-1] = \frac{1}{3}\,[x[n-1] + x[n-2] + x[n-3]]$

$y[n] - y[n-1] = \frac{1}{3}\,[x[n] - x[n-3]]$

2.15 (a) $y(t) = \begin{cases} x(t), & x(t) \ge 0 \\ 0, & x(t) < 0 \end{cases}$

(i) memoryless (ii) noninvertible (iii) nonlinear (iv) time-invariant (v) causal (vi) BIBO-stable.

(iii) $x_1(t) \to y_1(t) = x_1(t)$ for $x_1(t) \ge 0$
$-2x_1(t) \to y_2(t) = 0 \ne -2y_1(t)$ if $x_1(t) \ge 0$

(iv) $x_1(t) \to y_1(t) = x_1(t)$, if $x_1(t) \ge 0$

$x_2(t) = x_1(t-d) \rightarrow y_2(t) = x_2(t) = x_1(t-d) = y_1(t-d)$

(b) $y(t) = \sin(ax(t))$

(i) memoryless (ii) noninvertible (iii) nonlinear (iv) time-invariant (v) causal (vi) stable

(vii) $x_1(t) \rightarrow y_1(t) = \sin(ax_1(t))$

Let $x_2(t) = bx_1(t) \rightarrow y_2(t) = \sin(ax_2(t)) = \sin(abx_1(t))$

$\neq b\sin(ax_1(t))$

(c) $y(t) = x(t)\sin(t+1)$

(i) memoryless $\Rightarrow \sin(t+1)$ calculated

(ii) noninvertible $\Rightarrow x(-1) = \dfrac{y(-1)}{\sin(0)}$, indeterminate

(iii) linear

(iv) time-varying: $x_1(t) \rightarrow y_1(t) = x_1(t)\sin(t+1)$

$x_2(t) = x_1(t-d) \rightarrow y_2(t) = x_2(t)\sin(t+1) = x_1(t-d)\sin(t+1)$

$\neq y_1(t-d) = x_1(t-d)\sin(t+1-d)$

(v) causal

(vi) stable

(d) $y(t) = x(at)$, $a > 0$

(i) memoryless (ii) noninvertible (iii) linear (iv) time-varying:

$x_1(t) \rightarrow y_1(t) = x_1(at)$; $x_2(t) = x_1(t-d) \rightarrow y_2(t) = x_2(at) = x_1(at-d)$

$\neq y_1(t-d) = x_1(a(t-d))$

(v) causal (vi) stable

(e) $y[n] = x[1-n]$

(i) has memory (ii) invertible (iii) linear

(iv) time-varying: $x_1[n] \rightarrow y_1[n] = x_1[1-n]$

$x_2[n] = x_1[n-k] \rightarrow y_2[n] = x_2[1-n] = x_1[1-n-k]$

$\neq y_1[n-k] = x_1[1-n+k]$

(v) causal (vi) stable

(f) $y[n] = x[2n]$

(i) memoryless (ii) noninvertible (iii) linear (iv) time-varying

(v) causal (vi) stable

(g) $y[n] = \sum_{k=-\infty}^{n} x[k]$

(i) has memory (ii) invertible (iii) linear

(iv) $x_1[n] \rightarrow y_1[n] = \sum_{k=-\infty}^{n} x_1[k]$

$x_2[n] = x_1[n-N] \rightarrow y_2[n] = \sum_{k=-\infty}^{n} x_2[k] = \sum_{k=-\infty}^{n} x_1[k-N]$

$y_1[n-N] = \sum_{k=-\infty}^{n-N} x_1[k] \neq y_2[n] \Rightarrow$ time-varying

(v) causal (vi) stable

(h) $y[n] = \sum_{k=n-2}^{n+2} x[k]$

(i) has memory (ii) noninvertible (iii) linear

(iv) time-invariant:

$x_1[n] \rightarrow y_1[n] = \sum_{k=n-2}^{n+2} x_1[k]$

$x_2[n] = x_1[n-N] \rightarrow y_2[n] = \sum_{k=n-2}^{n+2} x_2[k] = \sum_{k=n-2}^{n+2} x_1[k-N]$

$y_1[n-N] = \sum_{k=n-N-2}^{n-N+2} x_1[k] = y_2[n]$ $\therefore$ time invariant

(v) noncausal (vi) stable

2.16 $y_d[n] = Q\{x[n]\}$ is nonlinear

Consider a 4-bit quantizer with the following ranges and their quantized outputs.

$x[n] < 0.5 \Rightarrow y_d[n] = 0000$; $0.5 \le x[n] < 1.5 \Rightarrow 0001$;

$1.5 \le x[n] < 2.5$; ... $13.5 \le x[n] < 14.5 \Rightarrow 1110$

$x[n] \ge 14.5 \Rightarrow y_d[n] = 1111$.

For $x_1[n] = 3.4$, $y_{d_1}[n] = 0011$, and for

$x_2[n] = 9.4$, $y_{d_2}[n] = 1001$. But, for

$x_1[n] + x_2[n] = 12.8$, $y_d[n] = 1101 \ne y_{d_1}[n] + y_{d_2}[n]$

The same is true for a 16-bit quantizer.

2.17 Let $x_1(t) \rightarrow y_1(t)$ and $x_2(t) \rightarrow y_2(t)$, each satisfying the model.

For $x_3(t) = a_1x_1(t) + a_2x_2(t) \rightarrow y_3(\text{t})$, assume $y_3(t) = a_1y_1(t) + a_2y_2(t)$. Then, from the model

$$a\frac{dy_3}{dt} + by_3(t) = a\frac{d}{dt}[a_1 y_1(t) + a_2 y_2(t)] + b[a_1 y_1(t) + a_2 y_2(t)]$$
$$= a_1\left[a\frac{dy_1}{dt} + by_1(t)\right] + a_2\left[a\frac{dy_2}{dt} + by_2(t)\right]$$
$$= a_1 x_1(t) + a_2 x_2(t)$$
$$= x_3(t)$$

Hence, a linear system.

2.18 For $x_3[n] = a_1 x_1[n] + a_2 x_2[n] \rightarrow y_3[n] = a_1 y_1[n] + a_2 y_2[n]$

From the model, $y_3[n] + ay_3[n-1] = a_1 y_1[\text{n}] + a_2 y_2[n] + aa_1 y_1[n-1] + aa_2 y_2[n-1]$

$$= a_1[y_1[n] + ay_1[n-1]] + a_2[y_2[n] + ay_2[n-1]]$$
$$= a_1 x_1[n] + a_2 x_2[n]$$

Hence, the system is linear.

2.19 System is nonlinear, e.g., $v_{i_1}(t) = -3 \rightarrow v_{0_1}(t) = -3$; $v_{i_2}(t) = 7 \rightarrow v_{0_2}(t) = 5$; $v_{i_3}(t) = v_{i_1}(t) + v_{i_2}(t) = 4 \rightarrow v_{0_3}(t) = 4 \ne v_{0_1}(t) + v_{0_2}(t)$

Time-invariant: $v_i(t_1 - d) \rightarrow v_0(t_1 - d) = v_0(t_1)|_{t_1 \rightarrow t_1 - d}$

Causal: $v_0(t_1)$ depends only on $v_i(t_1)$, not on $v_i(t_1 + t)$ for any t_1.

2.20 $v_0(t) = -10v_i(t) + 0.05$, $-0.5\text{ V} \le v_i \le 0.5\text{ V}$

Nonlinear due to offset voltage.

e.g., $v_{i_1}(t) = 0.2\text{ V} \rightarrow v_{0_1}(t) = -2 + 0.05 = -1.95\text{ V}$

$v_{i_2}(t) = 0.4\text{ V}$

$= 2v_{i_1}(t) \rightarrow v_{0_2}(t) = -4 + 0.05 = -3.95\text{ V} \ne 2v_{0_1}(t)$

2.21 $y(t) = x(t) \cos w_c t$

linear; causal; BIBO-stable

2.22 $y(t) = x^2(t) \Rightarrow$ nonlinear, causal, BIBO-stable.

2.23 $y(t) = x^*(t)$

Linear: $x_1(t) = a_1(t) + jb_1(t) \rightarrow y_1(t) = a_1(t) - jb_1(t)$

$x_2(t) = a_2(t) + jb_2(t) \rightarrow y_2(t) = a_2(t) - jb_2(t)$

$c_1x_1 + c_2x_2 = c_1a_1(t) + jc_1b_1(t) + c_2a_2(t) + jc_2b_2(t) \rightarrow c_1a_1(t) + c_2a_2(t) - j[c_1b_1(t) + c_2b_2(t)]$

$= c_1y_1(t) + c_2y_2(t)$, for real c_1 and c_2.

Time-invariant: $x_3(t) = x_1(t - d) \rightarrow y_3(t) = y_1(t - d)$

2.24 $x(t) = u(t) \rightarrow y(t) = g(t)$

$f(t) = 3u(t) - 4u(t - 2)$

$\therefore$ Response $= 3g(t) - 4g(t - 2)$

2.25 $a\dfrac{dy}{dt} + by(t) = x(t);\ x(t) = Ae^{st} \rightarrow y(t) = Be^{st}$

Let $x_1(t) = A_1\, e^{s_1t} \rightarrow y_1(t) = B_1\, e^{s_1t}$ so that

$a\dfrac{dy_1}{dt} + by_1(t) = aB_1\, s_1\, e^{s_1t} + bB_1\, e^{s_1t} = A_1\, e^{s_1t}$

and $x_2(t) = A_2\, e^{s_2t} \rightarrow y_2(t) = B_2\, e^{s_2t}$ so that

$a\dfrac{dy_2}{dt} + by_2(t) = aB_2\, s_2\, e^{s_2t} + bB_2\, e^{s_2t} = A_2\, e^{s_2t}$

Then $x_3(t) = c_1\, x_1(t) + c_2\, x_2(t) = c_1 A_1\, e^{s_1t} + c_2 A_2\, e^{s_2t}$ satisfies

$y_3(t) = c_1\, y_1(t) + c_2\, y_2(t)$, for $y_3(t) = c_1\, B_1\, e^{s_1t} + c_2\, B_2\, e^{s_2t}$,

$a\dfrac{dy_3}{dt} + by_3(t) = a\, c_1\, B_1\, s_1\, e^{s_1t} + a\, c_2\, B_2\, s_2\, e^{s_2t} + b\, c_1\, B_1\, e^{s_1t} + bc_2\, B_2\, e^{s_2t}$

$= c_1\, [a\, B_1\, s_1\, e^{s_1t} + b\, B_1\, e^{s_1t}] + c_2\, [a\, B_2\, s_2\, e^{s_1t} + b\, B_2\, e^{s_2t}]$

$= c_1y_1(t) + c_2y_2(t)$

2.26 $y[n] = \text{Real}\,(x[n])$

For $x_1[n] = a_1[n] + jb_1[n] \rightarrow y_1[n] = a_1[n]$

Similarly, for $x_2[n] = a_2[n] + jb_2[n] \rightarrow y_2[n] = a_2[n]$

Then, for $x_3[n] = c_1x_1[n] + c_2x_2[n] = c_1a_1[n] + jc_1b_1[n] + c_2a_2[n] + jc_2b_2[n]$,

$y_3[n] = Re\,(x_3[n]) = c_1a_1[n] + c_2a_2[n]$, for real c_1 and c_2

$= c_1y_1[n] + c_2y_2[n]$

Hence, the system is linear.

For $x_3[n] = x_1[n - d]$, $y_3[n] = Re\,(x_1\,[n - d])$

$= Re\,(a_1\,[n - d] + j\, b_1[n - d])$

$= a_1[n - d]$

$= y_1[n - d]$

Hence, the system is time-invariant.

2.27 $y[n] = x[n] + by[n - 1];\ y[0] = 0;\ x[n] = K,\ 0 < K < \infty$

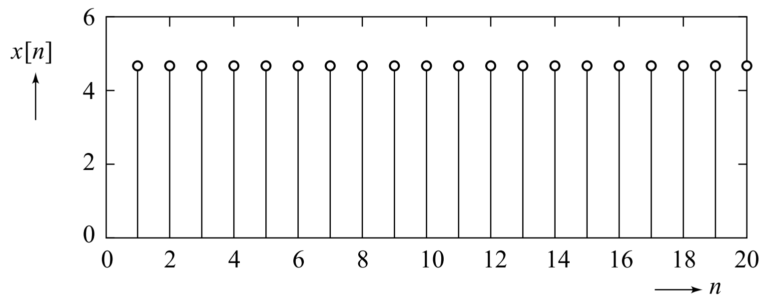
$y[1] = K;\ y[2] = K + bK;\ y[3] = K\,[1 + b + b^2];\ \ldots$

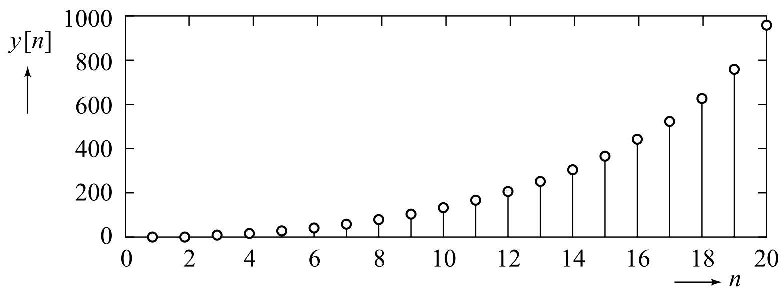
$y[n] = K(1 + b + b^2 + \ldots + b^{n-1})$

For $b > 1$, $y[n] \rightarrow \infty$ as $n \rightarrow \infty$

$\therefore$ BIBO-unstable.

```
K = 5; a = 0.2; N = 20;
b = 1+a;
x = K*ones(N,1);
y0 = 0;
y = zeros(size(x));
y(1) = x(1);
for n = 2:N
  y(n) = x(n) + b*y(n - 1);
end
```





2.28 For inputs $x_1[n] = z_1^n$ and $x_2[n] = z_2^n$, let the outputs be $x_1[n] \rightarrow y_1[n] = Y_1\, z_1^n$ and $x_2[n] \rightarrow y_2[n] = Y_2\, z_2^n$. Then, for $x_3[n] = a_1x_1[n] + a_2x_2[n]$, assume

$y_3[n] = a_1y_1[n] + a_2y_2[n]$.

From the model, the left-hand side gives

$a_1y_1[n] + 1.4a_1y_1[n-1] + 0.48\,a_1y_1[n-2] + a_2y_2[n] + 1.4a_2y_2[n-1] + 0.48a_2y_2[n-2]$

$= a_1x_1[n] + a_2x_2[n] = x_3[n]$

Hence, the model is linear.

2.29 Let the current in the circuit be $i(t)$. Writing the KVL, we have

$$Ri(t) + \frac{1}{C}\int_0^t i\,dt = v_s(t) = Ri(t) + v_0(t)$$

Since $\quad i(t) = \dfrac{v_s(t) - v_0(t)}{R}, \quad$ and $\; v_0(t) = \dfrac{1}{C}\displaystyle\int_0^t i\,dt$

$$\frac{dv_0}{dt} = \frac{1}{C}\frac{[v_s(t) - v_0(t)]}{R}$$

or

$$v_0(t) + RC\,\frac{dv_0}{dt} = v_s(t)$$

Discretization: $v_0[nT] + \dfrac{RC}{T}\,[v_0[nT] - v_0[(n-1)T]] = v_s[nT]$

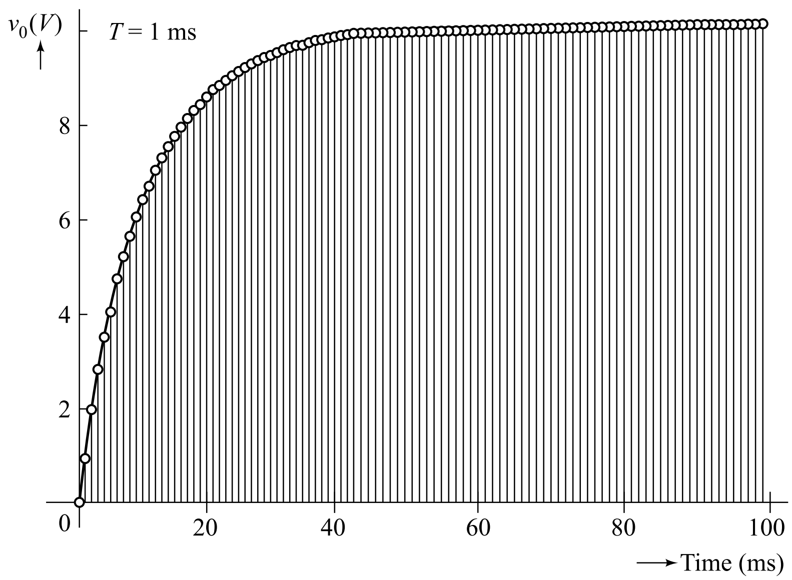
or

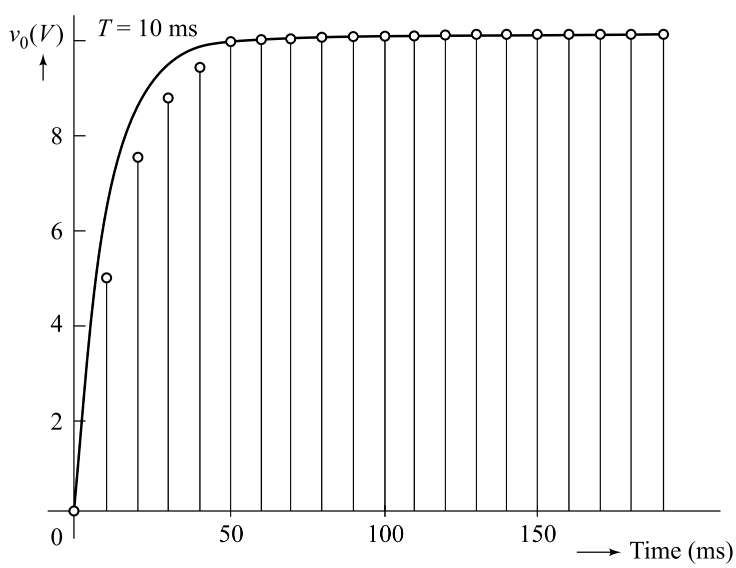
$$v_0[n] = \frac{T}{T + RC}\,v_s[n] + \frac{RC}{T + RC}\,v_0[n-1]$$

```
%Input--10 u(t)
%RCy'(t) + y(t) = x(t)
% y[n] = b1*x[n] + a1*y[n-1], b1 = T/(a+T), a1 = a/(a+T)

%DT-recursive solution
C = 1E-6; R = 1E4; Vi = 10; a = R*C;
T = 1E-3;
Ln = 100;
b1 = T/(a+T); a1 = a/(a + T);
n = 0:Ln-1;
ntime = n/10;
```

```
x = Vi*ones(size(n));
y = zeros(size(x));
y(1) = 0;                              % Initial voltage
for k = 2: Ln
  y(k) = b1*x(k) + a1*y(k-1);      % Response
end
% closed form CT solution
Tc = 1E-4;
Nc = 0:999;
tau = R*C;
yc = Vi*(1-exp(-Nc*Tc/tau));
```

```
Values at t  =     [0.5  1     5     100] ms are
CT:                [0.4877    0.9516     3.9347     9.9995]
DT at T = 1ms:     [------    0.9091     3.7908     9.9993]
DT at T = 10ms:    [------    ------     ------     9.9902]
```





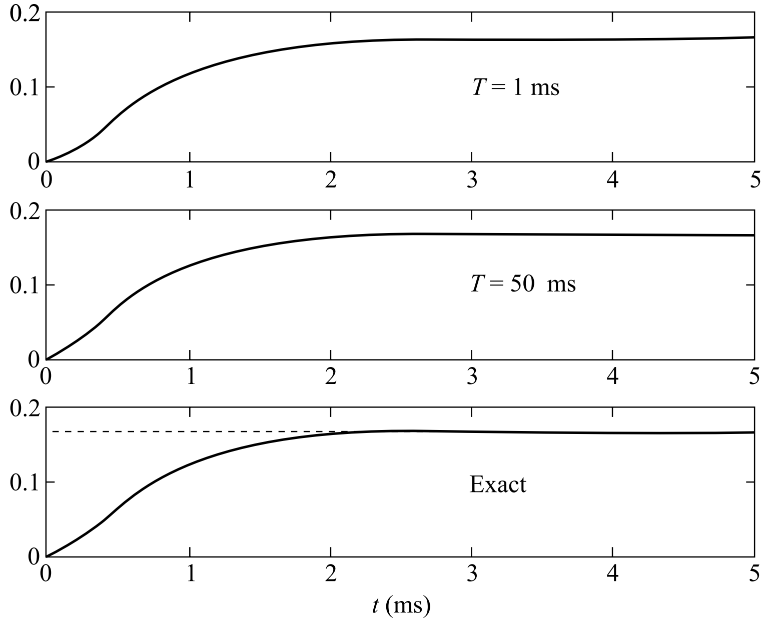
2.30 DT Model: $\dfrac{y[n+2]-2y[n+1]+y[n]}{T^2}+\dfrac{5}{T}\,(y[n+1]-y[n])+6y[n]=x[n]$

or, replacing $n+2$ by n

$$y[n]-\left(\frac{5}{T}-\frac{2}{T^2}\right)y[n-1]+\left(6-\frac{5}{T}+\frac{1}{T^2}\right)y[n-2]=x[n-2]$$

Exact solution: $y(t)=\dfrac{1}{6}-\dfrac{1}{2}e^{-2t}+\dfrac{1}{3}e^{-3t}$, $t\geq 0$

```
% Second order diff. eq.--discretized to DT model
% Input--u(t)
% y''(t) + 5y'(t) + 6y(t) = x(t) = u(t)
% y[n] = T^2*u[n-2] - (5*T-2)*y[n-1] - (6*T^2+1-5*T)*y[n-2]
% a = 5*T-2; b = 6*T^2+1-5*T

% DT-recursive solution
T1 = 1E-3;
a1 = 5*T1-2; b1 = 6*T1^2+1-5*T1;
Ln1 = 5000;

n1 = 0:Ln1-1;
x1 = ones(size(n1));
y1 = zeros(size(x1));
% initial voltage
y10 = 0;
y1(1) = 0;
y1(2) = T1^2;
for k = 3: Ln1
  y1(k) = T1^2*x1(k)-a1*y1(k-1)-b1*y1(k-2); % Response
end

T2 = 50E-3;
a2 = 5*T2-2; b2 = 6*T2^2+1-5*T2;
Ln2 = 5000;

n2 = 0:Ln2-1;
x2 = ones(size(n2));
y2 = zeros(size(x2));
% initial voltage
y20 = 0;
y2(1) = 0;
y2(2) = T2^2;
for k = 3: Ln2
  y2(k) = T2^2*x2(k)-a2*y2(k-1)-b2*y2(k-2); % Response
end

% closed form CT solution
Tc = 5E-4;
Nc = 0:9999;
LNc = length(Nc);
mode1 = 0.5*exp(-2*Tc*Nc);
mode2 = (1/3)*exp(-3*Tc*Nc);
yc = (1/6)*ones(size(mode1))-mode1 + mode2;
```



2.31 DT Model: $(L/T)\,(y[n] - y[n-1]) + (T/2C)\sum_{k=0}^{n-1}(y[k] + y[k+1)] + Ry[n] = x[n]$

For n–1

$$(L/T)\,(y[n-1] - y[n-2]) + (T/2C)\sum_{k=0}^{n-2}(y[k] + y[k+1]) + Ry[n-1] = x[n-1]$$

Using the above

$$(T/2C)\sum_{k=0}^{n-2}(y[k] + y[k+1]) = x[n-1] - (L/T)\,(y[n-1] - y[n-2]) - Ry[n-1]$$

Hence, the model equation becomes

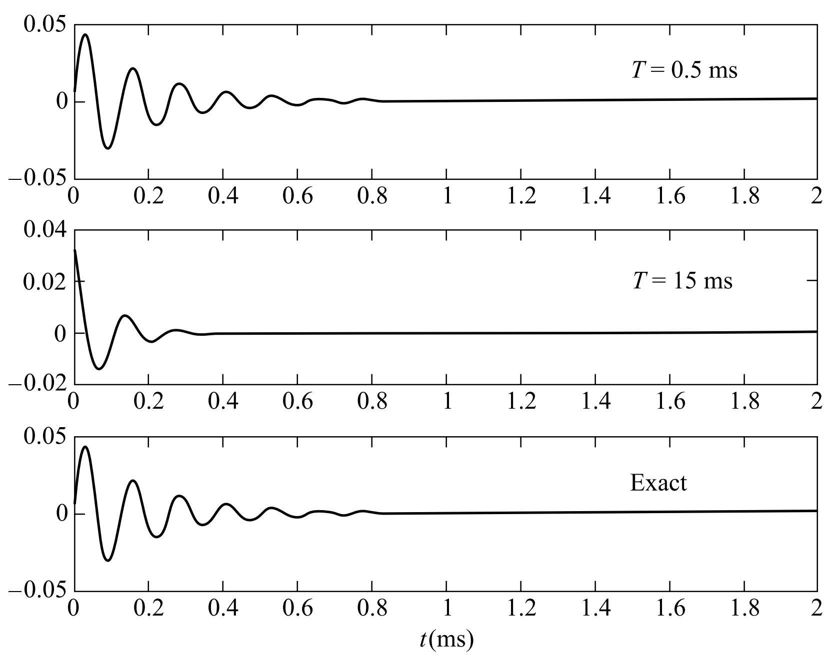
$(L/T)\,(y[n] - y[n-1]) + (T/2C)\,(y[n-1] + y[n]) + Ry[n] + x[n-1] - (L/T\,)\,(y[n-1]$
$- y[n-2]) - Ry[n-1]\;x[n]$

Or

$$((L/T) + T/2C + R)\,y[n] = x[n] - x[n-1] + (R + 2L/T - T/2C)\,y[n-1] - (L/T)\,y[n-2]$$

```
% Integro-diff. eq. discretized to DT model
% Input--u(t)
% Ly′(t) + Ry(t) + (1/C)integ(y(t)) = x(t) = u(t)
% y[n] = (1/a)*[x[n]-x[n-1]+b*y[n-1]-c*y[n-2]]
% a = L/T + (T/(2*C) + R;
% b = R – T/(2*C) + 2*L/T;
% c = L/T
R = 4; L = 0.4; C = 0.001;
% DT-recursive solution
T1 = 0.5E-3;
a1 = (T1/(2*C) + L/T1 + R);
b1 = R – T1/(2*C) + 2*L/T1;
c1 = L/T1;
Ln1 = 5000;
n1 = 1:Ln1-1;
x1 = ones(size(n1));
y1 = zeros(size(x1));
% initial conditions—unused
y1m1 = 0; y1m2 = 0;
y10 = 1/a1;
```

```
y1(1) = y10*b1/a1;
y1(2) = y1(1)*b1/a1-y10*c1/a1;
for k = 3: Ln1-1
  y1(k) = y1(k-1)*b1/a1 - y1(k-2)*c1/a1;
end
% Response
resp1 = [y10y1]; time 1 = [0 n1]*T1;
% Change time sample
T2 = 15E-3;
a2 = (T2/(2*C) + L/T2 + R);
b2 = R-T2/(2*C) + 2*L/T2;
c2 = L/T2;
Ln2 = 500;
n2 = 1:Ln2-1;
x2 = ones(size(n2));
y2 = zeros(size(x2));
%initial conditions--unused
y2m1 = 0; y2m2 = 0;
y20 = 1/a2;
y2(1) = y20*b2/a2;
y2(2) = y2(1)*b2/a2 - y20*c2/a2;
for m = 3:Ln2-1
  y2(m) = y2(m-1)*b2/a2 - y2(m-2)*c2/a2;
end
% Response
resp2 = [y20 y2]; time2 = [0 n2]*T2;

% closed form CT solution
Tc = 1E-4;
Nc = 0:99999;
LNc = length(Nc);
yc = 0.0502*exp(-5*Tc*Nc).*sin(49.7494*Tc*Nc);
```



2.32 $i_c = I_S e^{\frac{v_{BE}}{V_T}}$, $I_S = 1E - 15$ A, and $V_T = 25$ mV

(a) At $V_{BE} = 0.73$ V, $I_C = 4.8017$ mA. Hence, the Q-point is (0.73 V, 4.8017 mA)

(b) For v_{BE} varying about the bias point by a small amount

$$i_c = I_c + i_c = I_S e^{\frac{v_{BE}}{V_T}} = I_S e^{\frac{v_{BE} + v_{be}}{V_T}} = I_S\ e^{\frac{V_{BE}}{V_T}}\ e^{\frac{v_{be}}{V_T}} = I_c\ e^{\frac{v_{be}}{V_T}}$$

Expanding $i_c = I_c + i_c = I_c e^{\frac{v_{BE}}{V_T}} \approx I_c\left(1 + \frac{v_{be}}{V_T}\right)$, for $| v_{be} | << V_T$.

Hence, the incremental current is related to the incremental voltage by

$i_c \approx \frac{I_c}{V_T} v_{be}$, for $| v_{be} | << V_T$.

(c) $v_c(t) = 15 - i_c R_c \rightarrow V_c + v_c = 15 - (I_c + i_c)R_c$

Hence, $v_c = i_c R_c \approx \frac{I_c}{V_T} v_{be} R_c$, and the incremental gain is given by

$$\frac{\partial v_c}{\partial v_{be}} = -\frac{I_c}{V_T} R_c = -\ 960.35$$