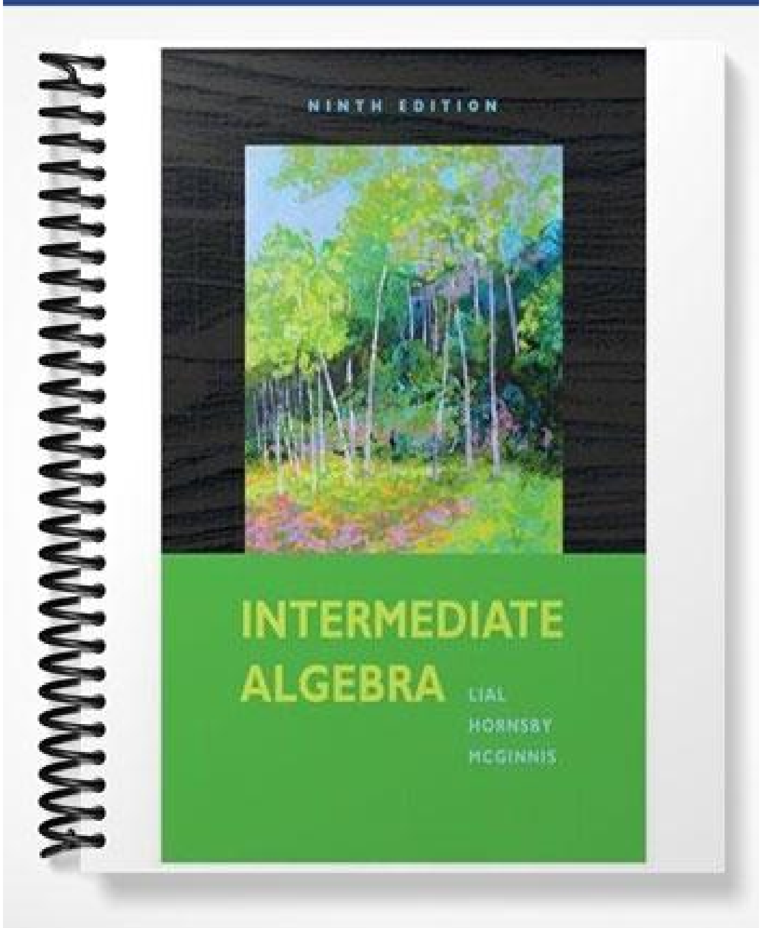
SOLUTIONS MANUAL



CHAPTER 2 LINEAR EQUATIONS AND APPLICATIONS

2.1 Linear Equations in One Variable

2.1 Margin Exercises

1. **(a)** $9x = 10$ is an *equation* because it contains an equals sign.

   (b) $9x + 10$ is an *expression* because it does not contain an equals sign.

   (c) $3 + 5x - 8x + 9$ is an *expression* because it does not contain an equals sign.

   (d) $3 + 5x = -8x + 9$ is an *equation* because it contains an equals sign.

2. To decide if a given number is a solution, substitute that number for the variable in the equation to see if the resulting statement is true or false.

   (a) $3k = 15;\ 5$

   The number 5 is a solution since $3 \cdot 5 = 15$ and $15 = 15$ is true.

   (b) $r + 5 = 4;\ 1$

   The number 1 is not a solution since $1 + 5 = 6$ and $6 = 4$ is false.

   (c) $-8m = 12;\ \frac{3}{2}$

   The number $\frac{3}{2}$ is not a solution since $-8(\frac{3}{2}) = -12$ and $-12 = 12$ is false.

3. **(a)**

$3p + 2p + 1 = -24$	*Original equation*
$5p + 1 = -24$	*Combine terms.*
$5p + 1 - 1 = -24 - 1$	*Subtract 1.*
$5p = -25$	*Combine terms.*
$\frac{5p}{5} = \frac{-25}{5}$	*Divide by 5.*
$p = -5$	*Proposed solution*

   Check by substituting -5 for p in the *original* equation.

$3p + 2p + 1 = -24$	*Original equation*
$3(-5) + 2(-5) + 1 \stackrel{?}{=} -24$	*Let p = −5.*
$-15 - 10 + 1 \stackrel{?}{=} -24$	
$-24 = -24$	*True*

   The true statement indicates that $\{-5\}$ is the solution set.

   (b)

$3p = 2p + 4p + 5$	*Original equation*
$3p = 6p + 5$	*Combine terms.*
$3p - 6p = 6p + 5 - 6p$	*Subtract 6p.*
$-3p = 5$	*Combine terms.*
$\frac{-3p}{-3} = \frac{5}{-3}$	*Divide by −3.*
$p = -\frac{5}{3}$	*Proposed solution*

   Check by substituting $-\frac{5}{3}$ for p in the *original* equation.

$3p = 2p + 4p + 5$	*Original equation*
$3(-\frac{5}{3}) \stackrel{?}{=} 2(-\frac{5}{3}) + 4(-\frac{5}{3}) + 5$	*Let p = $-\frac{5}{3}$.*
$-5 \stackrel{?}{=} -\frac{10}{3} - \frac{20}{3} + 5$	
$-5 \stackrel{?}{=} -\frac{30}{3} + 5$	
$-5 \stackrel{?}{=} -10 + 5$	
$-5 = -5$	*True*

   Solution set: $\{-\frac{5}{3}\}$

   (c)

$4x + 8x = 17x - 9 - 1$	*Original equation*
$12x = 17x - 10$	*Combine terms.*
$12x - 17x = 17x - 10 - 17x$	*Subtract 17x.*
$-5x = -10$	*Combine terms.*
$\frac{-5x}{-5} = \frac{-10}{-5}$	*Divide by −5.*
$x = 2$	*Proposed solution*

   Check by substituting 2 for x in the *original* equation.

$4x + 8x = 17x - 9 - 1$	*Original equation*
$4(2) + 8(2) \stackrel{?}{=} 17(2) - 9 - 1$	*Let x = 2.*
$8 + 16 \stackrel{?}{=} 34 - 9 - 1$	
$24 = 24$	*True*

   Solution set: $\{2\}$

(d) $-7 + 3t - 9t = 12t - 5$

$-7 - 6t = 12t - 5$ *Combine terms.*

$-7 - 6t + 6t + 5 = 12t - 5 + 6t + 5$ *Add 6t; add 5.*

$-2 = 18t$

$\frac{-2}{18} = \frac{18t}{18}$ *Divide by 18.*

$-\frac{1}{9} = t$ *Proposed solution*

We will use the following notation to indicate the value of each side of the original equation after we have substituted the proposed solution and simplified.

Check $t = -\frac{1}{9}$: $-\frac{19}{3} = -\frac{19}{3}$ *True*

Solution set: $\{-\frac{1}{9}\}$

4. **(a)** $5p + 4(3 - 2p) = 2 + p - 10$

$5p + 12 - 8p = 2 + p - 10$ *Distributive property*

$12 - 3p = p - 8$ *Combine terms.*

$12 - 3p + 3p + 8 = p - 8 + 3p + 8$ *Add 3p; add 8.*

$20 = 4p$ *Combine terms.*

$\frac{20}{4} = \frac{4p}{4}$ *Divide by 4.*

$5 = p$ *Proposed solution*

Check $p = 5$: $-3 = -3$ *True*

Solution set: $\{5\}$

(b) $3(z - 2) + 5z = 2$

$3z - 6 + 5z = 2$ *Distributive property*

$8z - 6 = 2$ *Combine terms.*

$8z - 6 + 6 = 2 + 6$ *Add 6.*

$8z = 8$ *Combine terms.*

$\frac{8z}{8} = \frac{8}{8}$ *Divide by 8.*

$z = 1$ *Proposed solution*

Check $z = 1$: $2 = 2$ *True*

Solution set: $\{1\}$

(c) $-2 + 3(x + 4) = 8x$

$-2 + 3x + 12 = 8x$ *Distributive property*

$3x + 10 = 8x$ *Combine terms.*

$3x + 10 - 3x = 8x - 3x$ *Subtract 3x.*

$10 = 5x$ *Combine terms.*

$\frac{10}{5} = \frac{5x}{5}$ *Divide by 5.*

$2 = x$ *Proposed solution*

Check $x = 2$: $16 = 16$ *True*

Solution set: $\{2\}$

(d) $6 - (4 + m) = 8m - 2(3m + 5)$

$6 - 4 - m = 8m - 6m - 10$ *Distributive property*

$2 - m = 2m - 10$ *Combine terms.*

$2 - m + m + 10 = 2m - 10 + m + 10$ *Add m; add 10.*

$12 = 3m$ *Combine terms.*

$\frac{12}{3} = \frac{3m}{3}$ *Divide by 3.*

$4 = m$ *Proposed solution*

Check $m = 4$: $-2 = -2$ *True*

Solution set: $\{4\}$

5. **(a)** $\frac{2p}{7} - \frac{p}{2} = -3$

Multiply each side by the LCD, 14.

$14\left(\frac{2p}{7} - \frac{p}{2}\right) = 14(-3)$

$14\left(\frac{2p}{7}\right) - 14\left(\frac{p}{2}\right) = 14(-3)$ *Distributive property*

$4p - 7p = -42$ *Multiply.*

$-3p = -42$ *Combine terms.*

$\frac{-3p}{-3} = \frac{-42}{-3}$ *Divide by −3.*

$p = 14$ *Proposed solution*

Check $p = 14$: $-3 = -3$ *True*

Solution set: $\{14\}$

(b) $\frac{k+1}{2}+\frac{k+3}{4}=\frac{1}{2}$

Multiply each side by the LCD, 4, and use the distributive property.

$$\begin{aligned} 4\left(\frac{k+1}{2}\right)+4\left(\frac{k+3}{4}\right) &= 4\left(\frac{1}{2}\right) \\ 2(k+1)+1(k+3) &= 2 \\ 2k+2+k+3 &= 2 \\ 3k+5 &= 2 \\ 3k &= -3 && \textit{Subtract 5.} \\ k &= -1 && \textit{Divide by 3.} \end{aligned}$$

Check $k=-1$: $\frac{1}{2}=\frac{1}{2}$ *True*

Solution set: $\{-1\}$

6. $0.04x+0.06(20-x)=0.05(50)$

Multiply each side by 100, and use the distributive property.

$$\begin{aligned} 4x+6(20-x) &= 5(50) \\ 4x+120-6x &= 250 \\ -2x+120 &= 250 \\ -2x &= 130 && \textit{Subtract 120.} \\ x &= -65 && \textit{Divide by } -2. \end{aligned}$$

Check $x=-65$: $2.5=2.5$ *True*

Solution set: $\{-65\}$

7.
$$\begin{aligned} 0.10(x-6)+0.05x &= 0.06(50) \\ 0.10x-0.6+0.05x &= 3 && \textit{Dist. prop.} \\ 0.15x-0.6 &= 3 && \textit{Combine.} \\ 0.15x &= 3.6 && \textit{Add 0.6.} \\ x &= 24 && \textit{Div. by 0.15.} \end{aligned}$$

Check $x=24$: $1.8+1.2=3$ *True*

Solution set: $\{24\}$

8. **(a)**
$$\begin{aligned} 5(x+2)-2(x+1) &= 3x+1 \\ 5x+10-2x-2 &= 3x+1 \\ 3x+8 &= 3x+1 \\ 3x+8-3x &= 3x+1-3x \\ & \quad \textit{Subtract 3x.} \\ 8 &= 1 && \textit{False} \end{aligned}$$

Since the result, $8=1$, is *false*, the equation has no solution and is called a *contradiction.*

Solution set: $\emptyset$

(b) $\frac{x+1}{3}+\frac{2x}{3}=x+\frac{1}{3}$

Multiply each side by the LCD, 3, and use the distributive property.

$$\begin{aligned} 3\left(\frac{x+1}{3}\right)+3\left(\frac{2x}{3}\right) &= 3\left(x+\frac{1}{3}\right) \\ x+1+2x &= 3x+1 \\ 3x+1 &= 3x+1 \end{aligned}$$

This is an *identity.* Any real number will make the equation true.

Solution set: {all real numbers}

(c)
$$\begin{aligned} 5(3x+1) &= x+5 \\ 15x+5 &= x+5 \\ 14x+5 &= 5 && \textit{Subtract x.} \\ 14x &= 0 && \textit{Subtract 5.} \\ x &= 0 && \textit{Divide by 14.} \end{aligned}$$

This is a *conditional equation.*

Check $x=0$: $5=5$ *True*

Solution set: $\{0\}$

2.1 Section Exercises

1. **A.** $3x+x-2=0$ can be written as $4x-2=0$, so it is linear.

C. $9x-4=9$ is in linear form.

2. **B.** $12=x^2$ is not a linear equation because the variable is squared.

D. $\frac{1}{8}x-\frac{1}{x}=0$ is not a linear equation because there is a variable in the denominator of the second term.

3.
$$\begin{aligned} 3(x+4) &= 5x && \textit{Original equation} \\ 3(6+4) &\stackrel{?}{=} 5\cdot 6 && \textit{Let x=6.} \\ 3(10) &\stackrel{?}{=} 30 && \textit{Add.} \\ 30 &= 30 && \textit{True} \end{aligned}$$

Since a true statement is obtained, 6 is a solution.

4.
$$\begin{aligned} 5(x+4)-3(x+6) &= 9(x+1) && \textit{Original equation} \\ 5(-2+4)-3(-2+6) &\stackrel{?}{=} 9(-2+1) && \textit{Let } x=-2. \\ 5(2)-3(4) &\stackrel{?}{=} 9(-1) && \textit{Add.} \\ 10-12 &\stackrel{?}{=} -9 && \textit{Multiply.} \\ -2 &= -9 && \textit{False} \end{aligned}$$

Since a false statement is obtained, -2 is not a solution.

5. Suppose your last name is Lincoln. Then $x=7$ and both sides are evaluated as -48. The equation is an identity, so any number is a solution.

6. The final line of the check, $-11=-11$, does not give the solution, only a confirmation that the solution found, in this case -3, is correct.

7. **(a)** $5x = 10$ is an *equation* because it contains an equals sign.

(b) $5x + 10$ is an *expression* because it does not contain an equals sign.

(c) $5x + 6(x-3) = 12x + 6$ is an *equation* because it contains an equals sign.

(d) $5x + 6(x-3) - (12x+6)$ is an *expression* because it does not contain an equals sign.

8. There is no way to add both 10 and 9 to the same expression $(7x)$ and get equal quantities.

9. The student made a sign error when the distributive property was applied. The left side of the second line should be $8x - 4x + 6$. This gives us $4x + 6 = 3x + 7$ and then $x = 1$. Thus, the correct solution is 1.

10. $-(2m-4) = -1(2m-4)$, so the $-$ sign represents -1.

$$\begin{aligned} &-5m - (2m-4) + 5 \\ &= -5m - 2m + 4 + 5 \\ &= -7m + 9 \end{aligned}$$

In the following exercises, we do not show the checks of the solutions. To be sure that your solutions are correct, check them by substituting into the original equations.

11.
$$\begin{aligned} 9x + 10 &= 1 \\ 9x + 10 - 10 &= 1 - 10 && \textit{Subtract 10.} \\ 9x &= -9 \\ \frac{9x}{9} &= \frac{-9}{9} && \textit{Divide by 9.} \\ x &= -1 \end{aligned}$$

Solution set: $\{-1\}$

12.
$$\begin{aligned} 7x - 4 &= 31 \\ 7x - 4 + 4 &= 31 + 4 && \textit{Add 4.} \\ 7x &= 35 \\ \frac{7x}{7} &= \frac{35}{7} && \textit{Divide by 7.} \\ x &= 5 \end{aligned}$$

Solution set: $\{5\}$

13.
$$\begin{aligned} 5x + 2 &= 3x - 6 \\ 5x + 2 - 3x &= 3x - 6 - 3x && \textit{Subtract 3x.} \\ 2x + 2 &= -6 \\ 2x + 2 - 2 &= -6 - 2 && \textit{Subtract 2.} \\ 2x &= -8 \\ \frac{2x}{2} &= \frac{-8}{2} && \textit{Divide by 2.} \\ x &= -4 \end{aligned}$$

Solution set: $\{-4\}$

14.
$$\begin{aligned} 9p + 1 &= 7p - 9 \\ 9p + 1 - 7p &= 7p - 9 - 7p && \textit{Subtract 7p.} \\ 2p + 1 &= -9 \\ 2p + 1 - 1 &= -9 - 1 && \textit{Subtract 1.} \\ 2p &= -10 \\ \frac{2p}{2} &= \frac{-10}{2} && \textit{Divide by 2.} \\ p &= -5 \end{aligned}$$

Solution set: $\{-5\}$

15.
$$\begin{aligned} 7x - 5x + 15 &= x + 8 \\ 2x + 15 &= x + 8 && \textit{Combine terms.} \\ 2x &= x - 7 && \textit{Subtract 15.} \\ x &= -7 && \textit{Subtract x.} \end{aligned}$$

Solution set: $\{-7\}$

16.
$$\begin{aligned} 2x + 4 - x &= 4x - 5 \\ x + 4 &= 4x - 5 && \textit{Combine terms.} \\ -3x + 4 &= -5 && \textit{Subtract 4x.} \\ -3x &= -9 && \textit{Subtract 4.} \\ x &= 3 && \textit{Divide by –3.} \end{aligned}$$

Solution set: $\{3\}$

17.
$$\begin{aligned} 12w + 15w - 9 + 5 &= -3w + 5 - 9 \\ 27w - 4 &= -3w - 4 && \textit{Combine terms.} \\ 30w - 4 &= -4 && \textit{Add 3w.} \\ 30w &= 0 && \textit{Add 4.} \\ w &= 0 && \textit{Divide by 30.} \end{aligned}$$

Solution set: $\{0\}$

18.
$$\begin{aligned} -4t + 5t - 8 + 4 &= 6t - 4 \\ t - 4 &= 6t - 4 && \textit{Combine terms.} \\ -5t - 4 &= -4 && \textit{Subtract 6t.} \\ -5t &= 0 && \textit{Add 4.} \\ t &= 0 && \textit{Divide by –5.} \end{aligned}$$

Solution set: $\{0\}$

19.
$$\begin{aligned} 3(2t - 4) &= 20 - 2t \\ 6t - 12 &= 20 - 2t && \textit{Distributive property} \\ 8t - 12 &= 20 && \textit{Add 2t.} \\ 8t &= 32 && \textit{Add 12.} \\ t &= 4 && \textit{Divide by 8.} \end{aligned}$$

Solution set: $\{4\}$

20.
$$\begin{aligned} 2(3 - 2x) &= x - 4 \\ 6 - 4x &= x - 4 && \textit{Distributive property} \\ 6 - 5x &= -4 && \textit{Subtract x.} \\ -5x &= -10 && \textit{Subtract 6.} \\ x &= 2 && \textit{Divide by −5.} \end{aligned}$$

Solution set: $\{2\}$

21. $-5(x+1)+3x+2=6x+4$

$-5x-5+3x+2=6x+4$ *Distributive property*

$-2x-3=6x+4$ *Combine terms.*

$-3=8x+4$ *Add 2x.*

$-7=8x$ *Subtract 4.*

$-\frac{7}{8}=x$ *Divide by 8.*

Solution set: $\{-\frac{7}{8}\}$

22. $5(x+3)+4x-5=4-2x$

$5x+15+4x-5=4-2x$ *Distributive property*

$9x+10=4-2x$ *Combine terms.*

$11x+10=4$ *Add 2x.*

$11x=-6$ *Subtract 10.*

$x=-\frac{6}{11}$ *Divide by 11.*

Solution set: $\{-\frac{6}{11}\}$

23. $2(x+3)=-4(x+1)$

$2x+6=-4x-4$ *Remove parentheses.*

$6x+6=-4$ *Add 4x.*

$6x=-10$ *Subtract 6.*

$x=\frac{-10}{6}=-\frac{5}{3}$ *Divide by 6.*

Solution set: $\{-\frac{5}{3}\}$

24. $4(t-9)=8(t+3)$

$4t-36=8t+24$ *Remove parentheses.*

$-4t-36=24$ *Subtract 8t.*

$-4t=60$ *Add 36.*

$t=-15$ *Divide by –4.*

Solution set: $\{-15\}$

25. $3(2w+1)-2(w-2)=5$

$6w+3-2w+4=5$ *Remove parentheses.*

$4w+7=5$ *Combine terms.*

$4w=-2$ *Subtract 7.*

$w=\frac{-2}{4}$ *Divide by 4.*

$w=-\frac{1}{2}$

Solution set: $\{-\frac{1}{2}\}$

26. $4(x-2)+2(x+3)=6$

$4x-8+2x+6=6$

$6x-2=6$

$6x=8$

$x=\frac{8}{6}=\frac{4}{3}$

Solution set: $\{\frac{4}{3}\}$

27. $2x+3(x-4)=2(x-3)$

$2x+3x-12=2x-6$

$5x-12=2x-6$

$3x=6$

$x=\frac{6}{3}=2$

Solution set: $\{2\}$

28. $6x-3(5x+2)=4(1-x)$

$6x-15x-6=4-4x$

$-9x-6=4-4x$

$-5x=10$

$x=\frac{10}{-5}=-2$

Solution set: $\{-2\}$

29. $6p-4(3-2p)=5(p-4)-10$

$6p-12+8p=5p-20-10$

$14p-12=5p-30$

$9p=-18$

$p=-2$

Solution set: $\{-2\}$

30. $-2k-3(4-2k)=2(k-3)+2$

$-2k-12+6k=2k-6+2$

$4k-12=2k-4$

$2k=8$

$k=4$

Solution set: $\{4\}$

31. $2[w-(2w+4)+3]=2(w+1)$

$2[w-2w-4+3]=2(w+1)$

$2[-w-1]=2(w+1)$

$-w-1=w+1$ *Divide by 2.*

$-1=2w+1$ *Add w.*

$-2=2w$ *Subtract 1.*

$-1=w$ *Divide by 2.*

Solution set: $\{-1\}$

32. $4[2t-(3-t)+5]=-(2+7t)$

$4[2t-3+t+5]=-(2+7t)$

$4[3t+2]=-(2+7t)$

$12t+8=-2-7t$

$19t+8=-2$ *Add 7t.*

$19t=-10$ *Subtract 8.*

$t=-\frac{10}{19}$ *Divide by 19.*

Solution set: $\{-\frac{10}{19}\}$

33. $-[2z-(5z+2)]=2+(2z+7)$

$-[2z-5z-2]=2+2z+7$

$-[-3z-2]=2+2z+7$

$3z+2=2z+9$

$z=7$

Solution set: $\{7\}$

34.
$$\begin{aligned} -[6x-(4x+8)] &= 9+(6x+3) \\ -[6x-4x-8] &= 9+6x+3 \\ -(2x-8) &= 6x+12 \\ -2x+8 &= 6x+12 \\ -8x &= 4 \\ x &= \tfrac{4}{-8} = -\tfrac{1}{2} \end{aligned}$$

Solution set: $\{-\frac{1}{2}\}$

35.
$$\begin{aligned} -3m+6-5(m-1) &= -5m-(2m-4)+5 \\ -3m+6-5m+5 &= -5m-2m+4+5 \\ -8m+11 &= -7m+9 \\ -m+11 &= 9 \\ -m &= -2 \\ m &= 2 \end{aligned}$$

Solution set: $\{2\}$

36.
$$\begin{aligned} 4(k+2)-8k-5 &= -3k+9-2(k+6) \\ 4k+8-8k-5 &= -3k+9-2k-12 \\ -4k+3 &= -5k-3 \\ k &= -6 \end{aligned}$$

Solution set: $\{-6\}$

37.
$$\begin{aligned} -3(x+2)+4(3x-8) &= 2(4x+7)+2(3x-6) \\ -3x-6+12x-32 &= 8x+14+6x-12 \\ 9x-38 &= 14x+2 \\ -38 &= 5x+2 \\ -40 &= 5x \\ -8 &= x \end{aligned}$$

Solution set: $\{-8\}$

38.
$$\begin{aligned} -7(2x+1)+5(3x+2) &= 6(2x-4)-(12x+3) \\ -14x-7+15x+10 &= 12x-24-12x-3 \\ x+3 &= -27 \\ x &= -30 \end{aligned}$$

Solution set: $\{-30\}$

39. The denominators of the fractions are 4, 3, 6, and 1. The LCD is $(4)(3)(6)(1)=12$, since it is the smallest number into which each denominator can divide without a remainder.

40. Yes, the coefficients will be larger, but you will get the correct solution. As long as you multiply each side of the equation by the *same* nonzero number, the resulting equation is equivalent and the solution does not change.

41. **(a)** We need to make the coefficient of the first term on the left an integer. Since $0.05 = \frac{5}{100}$, we multiply by 10^2 or 100. This will also take care of the second term.

(b) We need to make 0.006, 0.007, and 0.009 integers. These numbers can be written as $\frac{6}{1000}$, $\frac{7}{1000}$, and $\frac{9}{1000}$. Multiplying by 10^3 or 1000 will eliminate the decimal points (the denominators) so that all the coefficients are integers.

42.
$$\begin{aligned} &0.06(10-x)(100) \\ &= 0.06(100)(10-x) \\ &= 6(10-x) \\ &= 60-6x \end{aligned}$$
Choice **B** is correct.

43.
$$\frac{m}{2}+\frac{m}{3} = 10$$

Multiply each side by the LCD, 6.

$$\begin{aligned} 6\left(\frac{m}{2}+\frac{m}{3}\right) &= 6(10) \\ 6\left(\frac{m}{2}\right)+6\left(\frac{m}{3}\right) &= 60 && \textit{Distributive property} \\ 3m+2m &= 60 \\ 5m &= 60 && \textit{Add.} \\ m &= 12 && \textit{Divide by 5.} \end{aligned}$$

Check $m=12$: $6+4=10$ *True*

Solution set: $\{12\}$

44.
$$\frac{x}{5}-\frac{x}{4} = 2$$

Multiply each side by the LCD, 20.

$$\begin{aligned} 20\left(\frac{x}{5}-\frac{x}{4}\right) &= 20(2) \\ 20\left(\frac{x}{5}\right)-20\left(\frac{x}{4}\right) &= 40 && \textit{Distributive property} \\ 4x-5x &= 40 \\ -x &= 40 && \textit{Subtract.} \\ x &= -40 && \textit{Multiply by } -1. \end{aligned}$$

Check $x=-40$: $-8+10=2$ *True*

Solution set: $\{-40\}$

45.
$$\frac{3}{4}x+\frac{5}{2}x = 13$$

Multiply each side by the LCD, 4.

$$\begin{aligned} 4\left(\frac{3}{4}x+\frac{5}{2}x\right) &= 4(13) \\ 4\left(\frac{3}{4}x\right)+4\left(\frac{5}{2}x\right) &= 4(13) && \textit{Distributive property} \\ 3x+10x &= 52 \\ 13x &= 52 && \textit{Combine terms.} \\ x &= 4 && \textit{Divide by 13.} \end{aligned}$$

Check $x=4$: $13=13$ *True*

Solution set: $\{4\}$

46. $$\frac{8}{3}x - \frac{1}{2}x = -13$$

Multiply each side by the LCD, 12.

$$12\left(\frac{8}{3}x - \frac{1}{2}x\right) = 12(-13)$$

$$12\left(\frac{8}{3}x\right) - 12\left(\frac{1}{2}x\right) = 12(-13) \quad \textit{Distributive property}$$

$$32x - 6x = -156$$

$$26x = -156$$

$$x = -6 \quad \textit{Divide by 26.}$$

Check $x = -6$: $-13 = -13$ *True*

Solution set: $\{-6\}$

47. $$\frac{1}{5}x - 2 = \frac{2}{3}x - \frac{2}{5}x$$

Multiply each side by the LCD, 15, and use the distributive property.

$$15\left(\tfrac{1}{5}x\right) - 15(2) = 15\left(\tfrac{2}{3}x\right) - 15\left(\tfrac{2}{5}x\right)$$

$$3x - 30 = 10x - 6x$$

$$3x - 30 = 4x$$

$$-30 = x \quad \textit{Subtract 3x.}$$

Check $x = -30$: $-8 = -8$ *True*

Solution set: $\{-30\}$

48. $$\frac{3}{4}x - \frac{1}{3}x = \frac{5}{6}x - 5$$

Multiply each side by the LCD, 12, and use the distributive property.

$$12\left(\tfrac{3}{4}x\right) - 12\left(\tfrac{1}{3}x\right) = 12\left(\tfrac{5}{6}x\right) - 12(5)$$

$$9x - 4x = 10x - 60$$

$$5x = 10x - 60$$

$$-5x = -60 \quad \textit{Subtract 10x.}$$

$$x = 12 \quad \textit{Divide by } -5.$$

Check $x = 12$: $5 = 5$ *True*

Solution set: $\{12\}$

49. $$\frac{x-8}{5} + \frac{8}{5} = -\frac{x}{3}$$

Multiply each side by the LCD, 15, and use the distributive property.

$$15\left(\frac{x-8}{5}\right) + 15\left(\frac{8}{5}\right) = 15\left(-\frac{x}{3}\right)$$

$$3(x-8) + 3(8) = -5x$$

$$3x - 24 + 24 = -5x$$

$$3x = -5x$$

$$8x = 0 \quad \textit{Add 5x.}$$

$$x = 0 \quad \textit{Divide by 8.}$$

Check $x = 0$: $0 = 0$ *True*

Solution set: $\{0\}$

50. $$\frac{2r-3}{7} + \frac{3}{7} = -\frac{r}{3}$$

Multiply each side by the LCD, 21, and use the distributive property.

$$21\left(\frac{2r-3}{7}\right) + 21\left(\frac{3}{7}\right) = 21\left(-\frac{r}{3}\right)$$

$$3(2r-3) + 3(3) = 7(-r)$$

$$6r - 9 + 9 = -7r$$

$$6r = -7r$$

$$13r = 0 \quad \textit{Add 7r.}$$

$$r = 0 \quad \textit{Divide by 13.}$$

Check $r = 0$: $0 = 0$ *True*

Solution set: $\{0\}$

51. $$\frac{3x-1}{4} + \frac{x+3}{6} = 3$$

Multiply each side by the LCD, 12.

$$12\left(\frac{3x-1}{4} + \frac{x+3}{6}\right) = 12(3)$$

$$3(3x-1) + 2(x+3) = 36$$

$$9x - 3 + 2x + 6 = 36$$

$$11x + 3 = 36$$

$$11x = 33$$

$$x = 3$$

Check $x = 3$: $2 + 1 = 3$ *True*

Solution set: $\{3\}$

52. $$\frac{3x+2}{7} - \frac{x+4}{5} = 2$$

Multiply each side by the LCD, 35.

$$35\left(\frac{3x+2}{7} - \frac{x+4}{5}\right) = 35(2)$$

$$5(3x+2) - 7(x+4) = 70$$

$$15x + 10 - 7x - 28 = 70$$

$$8x - 18 = 70$$

$$8x = 88$$

$$x = 11$$

Check $x = 11$: $5 - 3 = 2$ *True*

Solution set: $\{11\}$

53. $$\frac{4t+1}{3} = \frac{t+5}{6} + \frac{t-3}{6}$$

Multiply each side by the LCD, 6.

$$6\left(\frac{4t+1}{3}\right) = 6\left(\frac{t+5}{6} + \frac{t-3}{6}\right)$$

$$2(4t+1) = (t+5) + (t-3)$$

$$8t + 2 = 2t + 2$$

$$6t = 0$$

$$t = 0$$

Check $t = 0$: $\frac{1}{3} = \frac{5}{6} - \frac{3}{6}$ *True*

Solution set: $\{0\}$

54. $$\frac{2x+5}{5}=\frac{3x+1}{2}+\frac{-x+7}{2}$$

Multiply each side by the LCD, 10.

$$\begin{aligned}10\left(\frac{2x+5}{5}\right)&=10\left(\frac{3x+1}{2}+\frac{-x+7}{2}\right)\\2(2x+5)&=5(3x+1)+5(-x+7)\\4x+10&=15x+5-5x+35\\4x+10&=10x+40\\-6x&=30\\x&=\tfrac{30}{-6}=-5\end{aligned}$$

Check $x=-5$: $-1=-7+6$ *True*

Solution set: $\{-5\}$

55. $0.05x+0.12(x+5000)=940$

Multiply each side by 100.

$$\begin{aligned}5x+12(x+5000)&=100(940)\\5x+12x+60{,}000&=94{,}000\\17x&=34{,}000\\x&=2000\end{aligned}$$

Check $x=2000$: $100+840=940$ *True*

Solution set: $\{2000\}$

56. $$0.09k+0.13(k+300)=61$$

Multiply each side by 100.

$$\begin{aligned}100[0.09k+0.13(k+300)]&=100(61)\\100(0.09k)+100(0.13)(k+300)&=6100\\9k+13(k+300)&=6100\\9k+13k+3900&=6100\\22k&=2200\\k&=\tfrac{2200}{22}=100\end{aligned}$$

Check $k=100$: $9+52=61$ *True*

Solution set: $\{100\}$

57. $0.02(50)+0.08r=0.04(50+r)$

Multiply each side by 100.

$$\begin{aligned}2(50)+8r&=4(50+r)\\100+8r&=200+4r\\4r&=100\\r&=25\end{aligned}$$

Check $r=25$: $1+2=3$ *True*

Solution set: $\{25\}$

58. $$0.20(14{,}000)+0.14t=0.18(14{,}000+t)$$

Multiply each side by 100.

$$\begin{aligned}100[0.20(14{,}000)+0.14t]&=\\&100[0.18(14{,}000+t)]\\20(14{,}000)+14t&=18(14{,}000+t)\\280{,}000+14t&=252{,}000+18t\\28{,}000&=4t\\t&=7000\end{aligned}$$

Check $t=7000$: $2800+980=3780$ *True*

Solution set: $\{7000\}$

59. $0.05x+0.10(200-x)=0.45x$

Multiply each side by 100.

$$\begin{aligned}5x+10(200-x)&=45x\\5x+2000-10x&=45x\\2000-5x&=45x\\2000&=50x\\40&=x\end{aligned}$$

Check $x=40$: $2+16=18$ *True*

Solution set: $\{40\}$

60. $0.08x+0.12(260-x)=0.48x$

Multiply each side by 100.

$$\begin{aligned}8x+12(260-x)&=48x\\8x+3120-12x&=48x\\-4x+3120&=48x\\3120&=52x\\x&=\tfrac{3120}{52}=60\end{aligned}$$

Check $x=60$: $4.8+24=28.8$ *True*

Solution set: $\{60\}$

61. $0.006(x+2)=0.007x+0.009$

Multiply each side by 1000.

$$\begin{aligned}6(x+2)&=7x+9\\6x+12&=7x+9\\3&=x\end{aligned}$$

Check $x=3$: $0.03=0.021+0.009$ *True*

Solution set: $\{3\}$

62. $0.004x+0.006(50-x)=0.004(68)$

Multiply each side by 1000.

$$\begin{aligned}4x+6(50-x)&=4(68)\\4x+300-6x&=272\\-2x+300&=272\\-2x&=-28\\x&=14\end{aligned}$$

Check $x=14$: $0.056+0.216=0.272$ *True*

Solution set: $\{14\}$

63. A conditional equation is true only for certain value(s), an identity has infinitely many solutions, and a contradiction has no solutions.

64. By dividing the equation $8x = 7x$ by x, he possibly divided the equation by 0. He should have subtracted $7x$ from each side of the equation to get $x = 0$. The solution set is $\{0\}$.

65. **(a)** $7 = 7$ is true and the original equation has solution set {all real numbers}, choice **B**.

(b) $x = 0$ indicates the original equation has solution set $\{0\}$, choice **A**.

(c) $7 = 0$ is false and the original equation has solution set $\emptyset$, choice **C**.

66. Each equation in choices **A**, **B**, and **D** is an identity and has {all real numbers} as its solution set. The equation in choice **C**, $4x = 3x$, has $\{0\}$ as its solution set.

67.
$$\begin{aligned} -x + 4x - 9 &= 3(x-4) - 5 \\ 3x - 9 &= 3x - 12 - 5 \\ 3x - 9 &= 3x - 17 \\ -9 &= -17 \quad \textit{False} \end{aligned}$$

The equation is a *contradiction*.

Solution set: $\emptyset$

68.
$$\begin{aligned} -12x + 2x - 11 &= -2(5x-3) + 4 \\ -10x - 11 &= -10x + 6 + 4 \\ -10x - 11 &= -10x + 10 \\ -11 &= 10 \quad \textit{False} \end{aligned}$$

The equation is a *contradiction*.

Solution set: $\emptyset$

69.
$$\begin{aligned} -11x + 4(x-3) + 6x &= 4x - 12 \\ -11x + 4x - 12 + 6x &= 4x - 12 \\ -x - 12 &= 4x - 12 \\ 0 &= 5x \\ 0 &= x \end{aligned}$$

This is a *conditional* equation.

Solution set: $\{0\}$

70.
$$\begin{aligned} 3x - 5(x+4) + 9 &= -11 + 15x \\ 3x - 5x - 20 + 9 &= -11 + 15x \\ -2x - 11 &= -11 + 15x \\ -17x &= 0 \\ x &= 0 \end{aligned}$$

This is a *conditional* equation.

Solution set: $\{0\}$

71.
$$\begin{aligned} -2(t+3) - t - 4 &= -3(t+4) + 2 \\ -2t - 6 - t - 4 &= -3t - 12 + 2 \\ -3t - 10 &= -3t - 10 \end{aligned}$$

The equation is an *identity*.

Solution set: {all real numbers}

72.
$$\begin{aligned} 4(2d+7) &= 2d + 25 + 3(2d+1) \\ 8d + 28 &= 2d + 25 + 6d + 3 \\ 8d + 28 &= 8d + 28 \end{aligned}$$

The equation is an *identity*.

Solution set: {all real numbers}

73.
$$\begin{aligned} 7[2 - (3+4x)] - 2x &= -9 + 2(1 - 15x) \\ 7[2 - 3 - 4x] - 2x &= -9 + 2 - 30x \\ 7[-1 - 4x] - 2x &= -7 - 30x \\ -7 - 28x - 2x &= -7 - 30x \\ -7 - 30x &= -7 - 30x \end{aligned}$$

The equation is an *identity*.

Solution set: {all real numbers}

74.
$$\begin{aligned} 4[6 - (1+2x)] + 10x &= 2(10 - 3x) + 8x \\ 4[6 - 1 - 2x] + 10x &= 20 - 6x + 8x \\ 4(5 - 2x) + 10x &= 20 + 2x \\ 20 - 8x + 10x &= 20 + 2x \\ 20 + 2x &= 20 + 2x \end{aligned}$$

The equation is an *identity*.

Solution set: {all real numbers}

2.2 Formulas and Percent

2.2 Margin Exercises

1. **(a)** To solve $I = prt$ for p, treat p as the only variable.

$$\begin{aligned} I &= prt \\ I &= p(rt) && \textit{Associative property} \\ \frac{I}{rt} &= \frac{p(rt)}{rt} && \textit{Divide by rt.} \\ \frac{I}{rt} &= p, \quad \text{or} \quad p = \frac{I}{rt} \end{aligned}$$

(b) To solve $I = prt$ for r, treat r as the only variable.

$$\begin{aligned} I &= prt \\ \frac{I}{pt} &= \frac{r(pt)}{pt} && \textit{Divide by pt.} \\ \frac{I}{pt} &= r, \quad \text{or} \quad r = \frac{I}{pt} \end{aligned}$$

2. **(a)** Solve $P = a + b + c$ for a.

$$P - (b+c) = a + (b+c) - (b+c) \qquad \textit{Subtract (b + c).}$$
$$P - b - c = a, \quad \text{or} \quad a = P - b - c$$

(b) Solve $V = \frac{1}{3}\pi r^2 h$ for h.

$$3V = \pi r^2 h \qquad \textit{Multiply by 3.}$$
$$\frac{3V}{\pi r^2} = h, \text{ or } h = \frac{3V}{\pi r^2} \qquad \textit{Divide by } \pi r^2.$$

3. Solve $M = \frac{1}{3}(a+b+c)$ for b.

$$3M = a + b + c \qquad \textit{Multiply by 3.}$$
$$3M - a - c = b \qquad \textit{Subtract a \& c.}$$

4. **(a)** Solve each equation for y.

$$2x + 7y = 5$$
$$2x + 7y - 2x = 5 - 2x \qquad \textit{Subtract 2x.}$$
$$7y = 5 - 2x$$
$$\frac{7y}{7} = \frac{5-2x}{7} \qquad \textit{Divide by 7.}$$
$$y = \frac{5-2x}{7}$$

(b) $5x - 6y = 12$

$$-6y = 12 - 5x \qquad \textit{Subtract 5x.}$$
$$y = \frac{12-5x}{-6}, \qquad \textit{Divide by } -6.$$
$$\text{or } y = \frac{5x-12}{6}$$

5. **(a)** Use the formula for the area of a triangle. Solve for h.

$$A = \frac{1}{2}bh$$
$$2A = bh \qquad \textit{Multiply by 2.}$$
$$\frac{2A}{b} = h, \text{ or } h = \frac{2A}{b} \qquad \textit{Divide by b.}$$

Now substitute $A = 36$ and $b = 12$.

$$h = \frac{2(36)}{12} = 6$$

The height is 6 in.

(b) Use $d = rt$. Solve for r.

$$\frac{d}{t} = \frac{rt}{t} \qquad \textit{Divide by t.}$$
$$\frac{d}{t} = r \text{ or } r = \frac{d}{t}$$

Now substitute $d = 500$ and $t = 20$.

$$r = \frac{500}{20} = 25$$

The rate is 25 mph.

(c) Use $d = rt$. Solve for t.

$$\frac{d}{r} = \frac{rt}{r} \qquad \textit{Divide by r.}$$
$$\frac{d}{r} = t \text{ or } t = \frac{d}{r}$$

Now substitute $d = 500$ and $r = 157.085$.

$$t = \frac{500}{157.085} \approx 3.183$$

His time was about 3.183 hr.

6. **(a)** The given amount of mixture is 20 oz. The part that is oil is 1 oz. Thus, the percent of oil is

$$\frac{\text{amount}}{\text{base}} = \frac{1}{20} = 0.05 = 5\%.$$

(b) Let x represent the amount of commission earned.

$$\frac{x}{22{,}000} = 0.06 \qquad \frac{\textit{amount a}}{\textit{base b}} = \textit{percent}$$
$$x = 0.06(22{,}000) \qquad \textit{Multiply by 22,000.}$$
$$x = 1320$$

The salesman earns \$1320.

7. Let x represent the amount spent on pet supplies/medicine.

$$\frac{x}{41.2} = 0.238 \qquad \textit{23.8\% = 0.238}$$
$$x = 0.238(41.2) \qquad \textit{Multiply by 41.2.}$$
$$x = 9.8056$$

Therefore, about \$9.8 billion was spent on pet supplies/medicine.

8. **(a)** Let $x =$ the percent decrease (as a decimal).

$$\text{percent decrease} = \frac{\text{amount of decrease}}{\text{base}}$$
$$x = \frac{80-56}{80}$$
$$x = \frac{24}{80}$$
$$x = 0.3$$

The percent markdown was 30%.

(b) Let $x =$ the percent increase (as a decimal).

$$\text{percent increase} = \frac{\text{amount of increase}}{\text{base}}$$
$$x = \frac{689-650}{650}$$
$$x = \frac{39}{650}$$
$$x = 0.06$$

The percent increase was 6%.

2.2 Section Exercises

1. **(a)**

$$\frac{7x+8}{3} = 12$$
$$3\left(\frac{7x+8}{3}\right) = 3(12)$$
$$7x + 8 = 36$$

(b) $\dfrac{ax+k}{c} = t\,(c \neq 0)$

$c\left(\dfrac{ax+k}{c}\right) = tc$

$ax + k = tc$

2. **(a)** $7x + 8 = 36$

$7x + 8 - 8 = 36 - 8$

(b) $ax + k = tc$

$ax + k - k = tc - k$

3. **(a)** $7x = 28$ **(b)** $ax = tc - k$

4. **(a)** $\dfrac{7x}{7} = \dfrac{28}{7}$

$x = 4$

(b) $\dfrac{ax}{a} = \dfrac{tc-k}{a}$

$x = \dfrac{tc-k}{a}$

5. The restriction $a \neq 0$ must be applied. If $a = 0$, the denominator becomes 0 and division by 0 is undefined.

6. To solve an equation for a particular variable, such as solving the second equation for x, go through the same steps as you would in solving for x in the first equation. Treat all other variables as constants.

7. Solve $A = LW$ for W.

$\dfrac{A}{L} = \dfrac{LW}{L}$ *Divide by L.*

$\dfrac{A}{L} = W$, or $W = \dfrac{A}{L}$

8. Solve $d = rt$ for t.

$\dfrac{d}{r} = \dfrac{rt}{r}$ *Divide by r.*

$\dfrac{d}{r} = t$, or $t = \dfrac{d}{r}$

9. Solve $P = 2L + 2W$ for L.

$P - 2W = 2L$ *Subtract 2W.*

$\dfrac{P-2W}{2} = \dfrac{2L}{2}$ *Divide by 2.*

$\dfrac{P-2W}{2} = L$, or $L = \dfrac{P}{2} - W$

10. Solve $A = bh$ for b.

$\dfrac{A}{h} = \dfrac{bh}{h}$ *Divide by h.*

$\dfrac{A}{h} = b$, or $b = \dfrac{A}{h}$

11. **(a)** Solve for $V = LWH$ for W.

$\dfrac{V}{LH} = \dfrac{LWH}{LH}$

$\dfrac{V}{LH} = W$, or $W = \dfrac{V}{LH}$

(b) Solve for $V = LWH$ for H.

$\dfrac{V}{LW} = \dfrac{LWH}{LW}$

$\dfrac{V}{LW} = H$, or $H = \dfrac{V}{LW}$

12. **(a)** Solve $P = a + b + c$ for b.

$P - (a + c) = a + b + c - (a + c)$

Subtract (a + c).

$P - a - c = b$

(b) Solve $P = a + b + c$ for c.

$P - (a + b) = a + b + c - (a + b)$

Subtract (a + b).

$P - a - b = c$

13. Solve $C = 2\pi r$ for r.

$\dfrac{C}{2\pi} = \dfrac{2\pi r}{2\pi}$ *Divide by 2π.*

$\dfrac{C}{2\pi} = r$

14. Solve $A = \dfrac{1}{2}bh$ for h.

$2A = bh$ *Multiply by 2.*

$\dfrac{2A}{h} = b$, or $b = \dfrac{2A}{h}$ *Divide by h.*

15. **(a)** Solve $A = \dfrac{1}{2}h(b + B)$ for h.

$2A = h(b + B)$ *Multiply by 2.*

$\dfrac{2A}{b+B} = h$ *Divide by $b + B$.*

(b) Solve $A = \dfrac{1}{2}h(b + B)$ for B.

$2A = h(b + B)$ *Multiply by 2.*

$\dfrac{2A}{h} = b + B$ *Divide by h.*

$\dfrac{2A}{h} - b = B$ *Subtract b.*

OR Solve $A = \dfrac{1}{2}h(b + B)$ for B.

$2A = hb + hB$ *Multiply by 2.*

$2A - hb = hB$ *Subtract hb.*

$\dfrac{2A - hb}{h} = B$ *Divide by h.*

16. Solve $V = \pi r^2 h$ for h.

$\dfrac{V}{\pi r^2} = h$, or $h = \dfrac{V}{\pi r^2}$ *Divide by πr^2.*

17. Solve $F = \dfrac{9}{5}C + 32$ for C.

$F - 32 = \frac{9}{5}C$ *Subtract 32.*

$\frac{5}{9}(F - 32) = \frac{5}{9}(\frac{9}{5}C)$ *Multiply by $\frac{5}{9}$.*

$\frac{5}{9}(F - 32) = C$

18. Solve $C = \frac{5}{9}(F - 32)$ for F.

$$\tfrac{9}{5}C = \tfrac{9}{5} \cdot \tfrac{5}{9}(F - 32) \quad \textit{Multiply by } \tfrac{9}{5}.$$
$$\tfrac{9}{5}C = F - 32$$
$$\tfrac{9}{5}C + 32 = F \quad \textit{Add 32.}$$

19.
$$4x + 9y = 11$$
$$4x + 9y - 4x = 11 - 4x \quad \textit{Subtract 4x.}$$
$$9y = 11 - 4x$$
$$\frac{9y}{9} = \frac{11 - 4x}{9} \quad \textit{Divide by 9.}$$
$$y = \frac{11 - 4x}{9}$$

20.
$$-7x + 8y = 11$$
$$8y = 11 + 7x \quad \textit{Add 7x.}$$
$$y = \frac{11 + 7x}{8} \quad \textit{Divide by 8.}$$

21.
$$-3x + 2y = 5$$
$$2y = 5 + 3x \quad \textit{Add 3x.}$$
$$y = \frac{5 + 3x}{2} \quad \textit{Divide by 2.}$$

22.
$$5x - 3y = 12$$
$$-3y = 12 - 5x \quad \textit{Subtract 5x.}$$
$$y = \frac{12 - 5x}{-3}, \quad \textit{Divide by } -3.$$
$$\text{or} \quad y = \frac{5x - 12}{3}$$

23.
$$6x - 5y = 7$$
$$-5y = 7 - 6x \quad \textit{Subtract 6x.}$$
$$y = \frac{7 - 6x}{-5}, \quad \textit{Divide by } -5.$$
$$\text{or} \quad y = \frac{6x - 7}{5}$$

24. Solve $k = dF - DF$ for F.

$$k = F(d - D)$$
Distributive property in reverse
$$\frac{k}{d - D} = F, \quad \text{or} \quad F = \frac{k}{d - D}$$

25. Solve $Mv = mv - Vm$ for m.

$$Mv = m(v - V)$$
Distributive property in reverse
$$\frac{Mv}{v - V} = m, \quad \text{or} \quad m = \frac{Mv}{v - V}$$

26. Solve $A = 2HW + 2LW + 2LH$ for W.

$$A - 2LH = 2HW + 2LW$$
Get the W-terms on one side.
$$A - 2LH = W(2H + 2L)$$
Distributive property in reverse
$$\frac{A - 2LH}{2H + 2L} = W, \quad \text{or} \quad W = \frac{A - 2LH}{2H + 2L}$$

27. Solve $d = rt$ for t.

$$t = \frac{d}{r}$$

To find t, substitute $d = 500$ and $r = 152.672$.

$$t = \frac{500}{152.672} \approx 3.275$$

His time was about 3.275 hours.

28. Solve $d = rt$ for t.

$$t = \frac{d}{r}$$

Replace d by 415 and r by 151.774.

$$t = \frac{415}{151.774} \approx 2.734$$

His time was about 2.734 hours.

29. Use the formula $F = \frac{9}{5}C + 32$.

$$F = \tfrac{9}{5}(45) + 32 \quad \textit{Let } C = 45.$$
$$= 81 + 32$$
$$= 113$$

The corresponding temperature is 113°F.

30. Use the formula $C = \frac{5}{9}(F - 32)$.

$$C = \tfrac{5}{9}(-58 - 32) \quad \textit{Let } F = -58.$$
$$= \tfrac{5}{9}(-90)$$
$$= -50$$

The corresponding temperature is about −50°C.

31. Solve $P = 4s$ for s.

$$s = \frac{P}{4}$$

To find s, substitute 920 for P.

$$s = \frac{920}{4} = 230$$

The length of each side is 230 m.

32. Use $V = \pi r^2 h$.
Replace r by $\frac{35}{2} = 17.5$ and h by 588.

$$\begin{aligned} V &= \pi(17.5^2)(588) \\ &\approx 565{,}722.3 \end{aligned}$$

To the nearest whole number, the volume is $565{,}722$ ft^3.

33. Use the formula $C = 2\pi r$.

$$\begin{aligned} 370\pi &= 2\pi r && \textit{Let } C = 370\pi. \\ \frac{370\pi}{2\pi} &= \frac{2\pi r}{2\pi} && \textit{Divide by } 2\pi. \\ 185 &= r \end{aligned}$$

So the radius of the circle is 185 inches and the diameter is twice that length, that is, 370 inches.

34. $d = 2r = 2(2.5) = 5$
The diameter is 5 inches.

$C = 2\pi r = 2\pi(2.5) = 5\pi$
The circumference is 5π inches.

35. Use $V = LWH$.
Let $V = 187$, $L = 11$, and $W = 8.5$.

$$\begin{aligned} 187 &= 11(8.5)H \\ 187 &= 93.5H \\ 2 &= H && \textit{Divide by 93.5.} \end{aligned}$$

The ream is 2 inches thick.

36. Use $V = LWH$.
Let $V = 238$, $W = 8.5$, and $H = 2$.

$$\begin{aligned} 238 &= L(8.5)(2) \\ 238 &= L(17) \\ 14 &= L && \textit{Divide by 17.} \end{aligned}$$

The length of a legal sheet of paper is 14 inches.

37. The mixture is 36 oz and that part which is alcohol is 9 oz. Thus, the percent of alcohol is

$$\tfrac{9}{36} = \tfrac{1}{4} = \tfrac{25}{100} = 25\%.$$

The percent of water is

$$100\% - 25\% = 75\%.$$

38. Let $x =$ the amount of pure acid in the mixture. Then x can be found by multiplying the total amount of the mixture by the percent of acid given as a decimal (0.35).

$$x = 40(0.35) = 14$$

There are 14 L of pure acid. Since there are 40 L altogether, there are $40 - 14$, or 26 L of pure water in the mixture.

39. Find what percent \$6900 is of \$230,000.

$$\tfrac{6900}{230{,}000} = 0.03 = 3\%$$

The agent received a 3% rate of commission.

40. Solve $I = prt$ for r.

$$\begin{aligned} r &= \frac{I}{pt} \\ r &= \frac{288}{6400(1)} \\ &= 0.045 = 4.5\% \end{aligned}$$

The interest rate on this deposit is 4.5%.

In Exercises 41–44, use the rule of 78:

$$u = f \cdot \frac{k(k+1)}{n(n+1)}$$

41. Substitute 700 for f, 4 for k, and 36 for n.

$$\begin{aligned} u &= 700 \cdot \frac{4(4+1)}{36(36+1)} \\ &= 700 \cdot \frac{4(5)}{36(37)} \approx 10.51 \end{aligned}$$

The unearned interest is \$10.51.

42. Substitute 600 for f, 12 for k, and 36 for n.

$$\begin{aligned} u &= 600 \cdot \frac{12(12+1)}{36(36+1)} \\ &= 600 \cdot \frac{12(13)}{36(37)} \approx 70.27 \end{aligned}$$

The unearned interest is \$70.27.

43. Substitute 380.50 for f, 8 for k, and 24 for n.

$$\begin{aligned} u &= (380.50) \cdot \frac{8(8+1)}{24(24+1)} \\ &= (380.50) \cdot \frac{8(9)}{24(25)} \approx 45.66 \end{aligned}$$

The unearned interest is \$45.66.

44. Substitute 450 for f, 9 for k, and 24 for n.

$$\begin{aligned} u &= 450 \cdot \frac{9(9+1)}{24(24+1)} \\ &= 450 \cdot \frac{9(10)}{24(25)} \approx 67.50 \end{aligned}$$

The unearned interest is \$67.50.

45. **(a)** Detroit:

$$\text{Pct.} = \frac{W}{W+L} = \frac{88}{88+74} = \frac{88}{162} \approx .543$$

(b) Minnesota:

$$\text{Pct.} = \frac{W}{W+L} = \frac{79}{79+83} = \frac{79}{162} \approx .488$$

(c) Chicago:

$$\text{Pct.} = \frac{W}{W+L} = \frac{72}{72+90} = \frac{72}{162} \approx .444$$

(d) Kansas City:

$$\text{Pct.} = \frac{W}{W+L} = \frac{69}{69+93} = \frac{69}{162} \approx .426$$

46. **(a)** Chicago:

$$\text{Pct.} = \frac{W}{W+L} = \frac{85}{85+77} = \frac{85}{162} \approx .525$$

(b) St. Louis:

$$\text{Pct.} = \frac{W}{W+L} = \frac{78}{78+84} = \frac{78}{162} = .481$$

(c) Houston:

$$\text{Pct.} = \frac{W}{W+L} = \frac{73}{73+89} = \frac{73}{162} \approx .451$$

(d) Pittsburgh:

$$\text{Pct.} = \frac{W}{W+L} = \frac{68}{68+94} = \frac{68}{162} \approx .420$$

47. $\frac{57.9 \text{ million}}{111.4 \text{ million}} \approx 0.52$

In 2006, about 52% of the U.S. households that owned at least one TV set owned at least 3 TV sets.

48. $\frac{93.6 \text{ million}}{111.4 \text{ million}} \approx 0.84$

In 2006, about 84% of the U.S. households that owned at least one TV set had a DVD player.

49. $0.34(242{,}070) = 82{,}303.80$

To the nearest dollar, \$82,304 will be spent to provide housing.

50. $0.07(242{,}070) = 16{,}944.90$

To the nearest dollar, \$16,945 will be spent for health care.

51. $\frac{\$41{,}000}{\$242{,}070} \approx 0.1694$

So the food cost is about 17%, which agrees with the percent shown in the graph.

52. $\frac{\$34{,}000}{\$242{,}070} \approx 0.1405$

So the food cost is about 14%, which agrees with the percent shown in the graph.

53. Let $x =$ the percent increase (as a decimal).

$$\begin{aligned} \text{percent increase} &= \frac{\text{amount of increase}}{\text{base}} \\ x &= \frac{11.34 - 10.50}{10.50} \\ x &= \frac{0.84}{10.50} \\ x &= 0.08 \end{aligned}$$

The percent increase was 8%.

54. Let $x =$ the percent decrease (as a decimal).

$$\begin{aligned} \text{percent decrease} &= \frac{\text{amount of decrease}}{\text{base}} \\ x &= \frac{70.00 - 59.50}{70.00} \\ x &= \frac{10.50}{70.00} \\ x &= 0.15 \end{aligned}$$

The percent discount was 15%.

55. Let $x =$ the percent decrease (as a decimal).

$$\begin{aligned} \text{percent decrease} &= \frac{\text{amount of decrease}}{\text{base}} \\ x &= \frac{134{,}953 - 129{,}798}{134{,}953} \\ x &= \frac{5155}{134{,}953} \\ x &= 0.038 \end{aligned}$$

The percent decrease was 3.8%.

56. Let $x =$ the percent increase (as a decimal).

$$\begin{aligned} \text{percent increase} &= \frac{\text{amount of increase}}{\text{base}} \\ x &= \frac{362{,}340 - 320{,}391}{320{,}391} \\ x &= \frac{41{,}949}{320{,}391} \\ x &= 0.131 \end{aligned}$$

The percent increase was 13.1%.

57.

$$\begin{aligned} \text{percent decrease} &= \frac{\text{amount of decrease}}{\text{base}} \\ &= \frac{18.98 - 9.97}{18.98} \\ &= \frac{9.01}{18.98} = 0.475 \end{aligned}$$

The percent discount was 47.5%.

58. $$\begin{aligned}\text{percent decrease} &= \frac{\text{amount of decrease}}{\text{base}}\\ &= \frac{29.99 - 15.99}{29.99}\\ &= \frac{14.00}{29.99} = 0.467\end{aligned}$$

The percent discount was 46.7%.

2.3 Applications of Linear Equations

2.3 Margin Exercises

1. **(a)** "9 added to a number" translates as

$$9 + x, \quad \text{or} \quad x + 9.$$

(b) "The difference between 7 and a number" translates as

$$7 - x.$$

Note: $x - 7$ is the difference between a number and 7.

(c) "Four times a number" translates as

$$4 \cdot x \quad \text{or} \quad 4x.$$

(d) "The quotient of 7 and a nonzero number" translates as

$$\frac{7}{x} \quad (x \neq 0).$$

2. **(a)**

The sum of a number and 6	is	28.
↓	↓	↓
$x + 6$	$=$	28

An equation is $x + 6 = 28$.

(b)

If twice a number	is decreased	by 3,	the result is	17.
↓	↓	↓	↓	↓
$2x$	$-$	3	$=$	17

An equation is $2x - 3 = 17$.

(c)

The product of a number and 7	is	twice the number	plus	12.
↓	↓	↓	↓	↓
$7x$	$=$	$2x$	$+$	12

An equation is $7x = 2x + 12$.

(d)

The quotient of a number and 6,	added to	twice the number,	is	7.
↓	↓	↓	↓	↓
$\frac{x}{6}$	$+$	$2x$	$=$	7

An equation is $\frac{x}{6} + 2x = 7$.

3. **(a)** $5x - 3(x + 2) = 7$ is an *equation* because it has an equals sign.

(b) $5x - 3(x + 2)$ is an *expression* because there is no equals sign.

4. *Step 2*
The length and perimeter are given in terms of the width W. The length L is 5 cm more than the width, so

$$L = W + 5.$$

The perimeter P is 5 times the width, so

$$P = 5W.$$

Step 3
Use the formula for perimeter of a rectangle.

$$\begin{aligned} P &= 2L + 2W \\ 5W &= 2(W + 5) + 2W \quad && \textit{P = 5W; L = W + 5}\end{aligned}$$

Step 4
Solve the equation.

$$\begin{aligned} 5W &= 2W + 10 + 2W \quad && \textit{Distributive property} \\ 5W &= 4W + 10 && \textit{Combine terms.} \\ W &= 10 && \textit{Subtract 4W.}\end{aligned}$$

Step 5
The width is 10 and the length is

$$L = W + 5 = 10 + 5 = 15.$$

The rectangle is 10 cm by 15 cm.

Step 6
15 is 5 more than 10 and $P = 2(10) + 2(15) = 50$ is five times 10, as required.

5. *Step 2*
Let $x =$ the number of RBIs for Rodriguez.
Then $x - 19 =$ the number of RBIs for Holliday.

Step 3
The sum of their RBIs is 293, so an equation is

$$x + (x - 19) = 293.$$

Step 4
Solve the equation.

$$\begin{aligned} 2x - 19 &= 293 \\ 2x &= 312 \quad && \textit{Add 19.} \\ x &= 156 && \textit{Divide by 2.}\end{aligned}$$

Step 5
Rodriguez had 156 RBIs and Holliday had $156 - 19 = 137$ RBIs.

Step 6
137 is 19 less than 156, and the sum of 137 and 156 is 293.

6. **(a)** Let x be the store's cost, which is increased by 25% of x, or $0.25x$. Then an equation is

$$\begin{aligned} x + 0.25x &= 2375. \\ 1x + 0.25x &= 2375 && \textit{Identity property} \\ 1.25x &= 2375 && \textit{Combine terms} \\ x &= 1900 && \textit{Divide by 1.25.} \end{aligned}$$

The store's cost was \$1900.

(b) Let x be the amount she earned before deductions. Then 10% of x, or $0.10x$, is the amount of her deductions. An equation is

$$\begin{aligned} x - 0.10x &= 162. \\ 1x - 0.10x &= 162 && \textit{Identity property} \\ 0.90x &= 162 && \textit{Combine terms} \\ x &= 180 && \textit{Divide by 0.90.} \end{aligned}$$

She earned \$180 before deductions were made.

7. **(a)** Let $x =$ the amount invested at 5%. Then $72{,}000 - x =$ the amount invested at 3%.

Use $I = prt$ with $t = 1$.
Make a table to organize the information.

Principal	**Rate (as a Decimal)**	**Interest**
x	0.05	$0.05x$
$72{,}000 - x$	0.03	$0.03(72{,}000 - x)$
72,000	← Totals →	3160

The last column gives the equation.

$$\begin{aligned} 0.05x + 0.03(72{,}000 - x) &= 3160 \\ 0.05x + 2160 - 0.03x &= 3160 && \textit{Distributive property} \\ 0.02x + 2160 &= 3160 && \textit{Combine terms.} \\ 0.02x &= 1000 && \textit{Subtract 2160.} \\ x &= 50{,}000 && \textit{Divide by 0.02.} \end{aligned}$$

The woman invested \$50,000 at 5% and $\$72{,}000 - \$50{,}000 = \$22{,}000$ at 3%.

Check 5% of \$50,000 is \$2500 and 3% of \$22,000 is \$660. The sum is \$3160, as required.

(b) Let $x =$ the amount invested at 5%. Then $34{,}000 - x =$ the amount invested at 4%.

Use $I = prt$ with $t = 1$.
Make a table to organize the information.

Principal	**Rate (as a Decimal)**	**Interest**
x	0.05	$0.05x$
$34{,}000 - x$	0.04	$0.04(34{,}000 - x)$
34,000	← Totals →	1545

The last column gives the equation.

$$\begin{aligned} 0.05x + 0.04(34{,}000 - x) &= 1545 \\ 0.05x + 1360 - 0.04x &= 1545 && \textit{Distributive property} \\ 0.01x + 1360 &= 1545 && \textit{Combine terms.} \\ 0.01x &= 185 && \textit{Subtract 1360.} \\ x &= 18{,}500 && \textit{Divide by 0.01.} \end{aligned}$$

The man invested \$18,500 at 5% and $\$34{,}000 - \$18{,}500 = \$15{,}500$ at 4%.

Check 5% of \$18,500 is \$925 and 4% of \$15,500 is \$620. The sum is \$1545, as required.

8. **(a)** Let $x =$ the number of liters of the 10% solution. Then $x + 60 =$ the number of liters of the 15% solution.

Make a table to organize the information.

Number of Liters	**Percent (as a Decimal)**	**Liters of Pure Solution**
x	$10\% = 0.10$	$0.10x$
60	$25\% = 0.25$	$0.25(60)$
$x + 60$	$15\% = 0.15$	$0.15(x + 60)$

The last column gives the equation.

$$\begin{aligned} 0.10x + 0.25(60) &= 0.15(x + 60) \\ 0.10x + 15 &= 0.15x + 9 && \textit{Distributive property} \\ 15 &= 0.05x + 9 && \textit{Subtract 0.10x.} \\ 6 &= 0.05x && \textit{Subtract 9.} \\ 120 &= x && \textit{Divide by 0.05.} \end{aligned}$$

120 L of 10% solution should be used.

Check 10% of 120 L is 12 L and 25% of 60 L is 15 L. The sum is $12 + 15 = 27$ L, which is the same as 15% of 180 L, as required.

(b) Let $x =$ the amount of \$8 per lb candy. Then $x + 100 =$ the amount of \$7 per lb candy.

Make a table to organize the information.

Number of Pounds	**Price per Pound**	**Value**
x	\$8	$8x$
100	\$4	400
$x + 100$	\$7	$7(x + 100)$

The last column gives the equation.

$$\begin{aligned} 8x + 400 &= 7(x + 100) \\ 8x + 400 &= 7x + 700 && \textit{Distributive property} \\ x + 400 &= 700 && \textit{Subtract 7x.} \\ x &= 300 && \textit{Subtract 400.} \end{aligned}$$

300 lb of candy worth \$8 per lb should be used.

Check 300 lb of candy worth \$8 per lb is worth \$2400. 100 lb of candy worth \$4 per lb is worth \$400. The sum is $2400 + 400 = \$2800$, which is the same as 400 lb of candy worth \$7 per lb, as required.

9. **(a)** Let $x =$ the number of liters of pure acid.

Number of Liters	Percent (as a Decimal)	Liters of Pure Acid
x	$100\% = 1$	x
6	$30\% = 0.30$	$0.30(6)$
$x+6$	$50\% = 0.50$	$0.50(x+6)$

The last column gives the equation.

$$\begin{aligned} x + 0.30(6) &= 0.50(x+6) \\ 1x + 1.8 &= 0.5x + 3 \\ 0.5x + 1.8 &= 3 && \textit{Subtract 0.5x.} \\ 0.5x &= 1.2 && \textit{Subtract 1.8.} \\ x &= 2.4 && \textit{Divide by 0.5.} \end{aligned}$$

2.4 L of pure acid are needed.

Check 100% of 2.4 L is 2.4 L and 30% of 6 L is 1.8 L. The sum is $2.4 + 1.8 = 4.2$ L, which is the same as 50% of $2.4 + 6 = 8.4$ L, as required.

(b) Let $x =$ the number of liters of water.

Number of Liters	Percent (as a Decimal)	Liters of Pure Antifreeze
x	$0\% = 0$	0
20	$50\% = 0.50$	$0.50(20)$
$x+20$	$40\% = 0.40$	$0.40(x+20)$

The last column gives the equation.

$$\begin{aligned} 0 + 0.50(20) &= 0.40(x+20) \\ 10 &= 0.4x + 8 \\ 2 &= 0.4x && \textit{Subtract 8.} \\ 5 &= x && \textit{Divide by 0.4.} \end{aligned}$$

5 L of water are needed.

Check 50% of 20 L is 10 L as is 40% of $20 + 5 = 25$ L, as required.

2.3 Section Exercises

1. **(a)** 12 more than a number $\underline{x + 12}$

(b) 12 is more than a number. $\underline{12 > x}$

2. **(a)** 3 less than a number $\underline{x - 3}$

(b) 3 is less than a number. $\underline{3 < x}$

3. **(a)** 4 less than a number $\underline{x - 4}$

(b) 4 is less than a number. $\underline{4 < x}$

4. **(a)** 6 greater than a number $\underline{x + 6}$

(b) 6 is greater than a number. $\underline{6 > x}$

5. 20% can be written as $0.20 = 0.2 = \frac{20}{100} = \frac{2}{10} = \frac{1}{5}$, so "20% of a number" can be written as $0.20x$, $0.2x$, or $\frac{x}{5}$. We see that "20% of a number" cannot be written as $20x$, choice **D**.

6. $24 - x$ is the translation of "x less than 24." The phrase "24 less than a number" translates as $x - 24$.

7. Twice a number, increased by 18 $\underline{2x + 18}$

8. The product of 8 and a number, increased by 14 $\underline{8x + 14}$

9. 15 decreased by four times a number $\underline{15 - 4x}$

10. 12 less than one-third of a number $\underline{\frac{1}{3}x - 12}$

11. The product of 10 and 6 less than a number $\underline{10(x - 6)}$

12. The product of 8 less than a number and 7 more than the number $\underline{(x - 8)(x + 7)}$

13. The quotient of five times a number and 9 $\underline{\frac{5x}{9}}$

14. The quotient of 12 and seven times a nonzero number $\underline{\frac{12}{7x} \quad (x \neq 0)}$

15. The sentence "the sum of a number and 6 is -31" can be translated as

$$\begin{aligned} x + 6 &= -31. \\ x &= -37 && \textit{Subtract 6.} \end{aligned}$$

The number is -37.

16. The sentence "the sum of a number and -4 is 12" can be translated as

$$\begin{aligned} x + (-4) &= 12. \\ x &= 16 && \textit{Add 4.} \end{aligned}$$

The number is 16.

17. The sentence "if the product of a number and -4 is subtracted from the number, the result is 9 more than the number" can be translated as

$$\begin{aligned} x - (-4x) &= x + 9. \\ x + 4x &= x + 9 \\ 4x &= 9 \\ x &= \tfrac{9}{4} \end{aligned}$$

The number is $\frac{9}{4}$.

18. The sentence "if the quotient of a number and 6 is added to twice the number, the result is 8 less than the number" can be translated as

$$2x + \frac{x}{6} = x - 8.$$
$$12x + x = 6x - 48 \quad \textit{Multiply by 6.}$$
$$13x = 6x - 48$$
$$7x = -48$$
$$x = -\tfrac{48}{7}$$

The number is $-\frac{48}{7}$.

19. The sentence "when $\frac{2}{3}$ of a number is subtracted from 12, the result is 10" can be translated as

$$12 - \tfrac{2}{3}x = 10.$$
$$36 - 2x = 30 \quad \textit{Multiply by 3.}$$
$$-2x = -6 \quad \textit{Subtract 36.}$$
$$x = 3 \quad \textit{Divide by } -2.$$

The number is 3.

20. The sentence "when 75% of a number is added to 6, the result is 3 more than the number" can be translated as

$$6 + 0.75x = x + 3.$$
$$600 + 75x = 100x + 300 \quad \textit{Multiply by 100.}$$
$$600 - 25x = 300 \quad \textit{Subtract 100x.}$$
$$-25x = -300 \quad \textit{Subtract 600.}$$
$$x = 12 \quad \textit{Divide by } -25.$$

The number is 12.

21. $5(x+3) - 8(2x-6)$ is an *expression* because there is no equals sign.

22. $-7(y+4) + 13(y-6)$ has no equals sign, so it is an *expression.*

23. $5(x+3) - 8(2x-6) = 12$ has an equals sign, so this represents an *equation.*

24. $-7(y+4) + 13(y-6) = 18$ has an equals sign, so it is an *equation.*

25. $\dfrac{t}{2} - \dfrac{t+5}{6} - 8$ is an *expression* because there is no equals sign.

26. $\dfrac{t}{2} - \dfrac{t+5}{6} = 8$ has an equals sign, so it is an *equation*.

27. *Step 1*
We are asked to find the number of patents each university secured.

Step 2
Let $x =$ the number of patents MIT secured.
Then $x - 38 =$ the number of patents Stanford secured.

Step 3
A total of 230 patents were secured, so

$$x + x - 38 = 230.$$

Step 4

$$2x - 38 = 230$$
$$2x = 268$$
$$x = 134$$

Step 5
MIT secured 134 patents and Stanford secured $134 - 38 =$ 96 patents.

Step 6
The number of Stanford patents was 38 fewer than the number of MIT patents and the total number of patents was $134 +$ 96 $=$ 230.

28. *Step 1*
We are asked to find the number of book buyers at each type of bookstore.

Step 2
Let $x =$ the number of book buyers at large chain bookstores. Then
$x - 70 =$ the number of book buyers at small chain/independent bookstores.

Step 3
A total of 442 book buyers shopped at these two types of stores, so

$$x + (x - 70) = 442.$$

Step 4

$$2x - 70 = 442$$
$$2x = 512$$
$$x = 256$$

Step 5
There were 256 large chain bookstore shoppers and $256 - 70 =$ 186 small chain/independent shoppers.

Step 6
The number of large chain shoppers was 70 more than the number of small chain/independent shoppers, and the total number of these shoppers was $256 +$ 186 $=$ 442.

29. *Step 2*
Let $W =$ the width of the base. Then $2W - 65$ is the length of the base.

Step 3
The perimeter of the base is 860 feet. Using $P = 2L + 2W$ gives us

$$2(2W - 65) + 2W = 860.$$

Step 4

$$\begin{aligned} 4W - 130 + 2W &= 860 \\ 6W - 130 &= 860 \\ 6W &= 990 \\ W &= \tfrac{990}{6} = 165 \end{aligned}$$

Step 5
The width of the base is 165 feet and the length of the base is $2(165) - 65 = 265$ feet.

Step 6
$2L + 2W = 2(265) + 2(165) = 530 + 330 = 860$, which is the perimeter of the base, and the length, 265 ft, is 65 ft less than twice the base, 330 ft.

30. *Step 2*
Let $x =$ the length of one of the sides of equal length.

Step 3
The perimeter of the triangle is 931.5 feet. Using $P = a + b + c$ gives us

$$x + x + 438 = 931.5$$

Step 4

$$\begin{aligned} 2x + 438 &= 931.5 \\ 2x &= 493.5 \quad \textit{Subtract 438.} \\ x &= 246.75 \quad \textit{Divide by 2.} \end{aligned}$$

Step 5
The two walls are each 246.75 feet long.

Step 6
The answer checks since
$246.75 + 246.75 + 438 = 931.75$, which is the correct perimeter.

31. *Step 2*
Let $x =$ the length of the middle side. Then the shortest side is $x - 75$ and the longest side is $x + 375$.

Step 3
The perimeter of the Bermuda Triangle is 3075 miles. Using $P = a + b + c$ gives us

$$x + (x - 75) + (x + 375) = 3075.$$

Step 4

$$\begin{aligned} 3x + 300 &= 3075 \\ 3x &= 2775 \quad \textit{Subtract 300.} \\ x &= 925 \quad \textit{Divide by 3.} \end{aligned}$$

Step 5
The length of the middle side is 925 miles. The length of the shortest side is $x - 75 =$
$925 - 75 = 850$ miles. The length of the longest side is $x + 375 = 925 + 375 = 1300$ miles.

Step 6
$925 + 850 + 1300 = 3075$ miles (the correct perimeter), the shortest side measures 75 miles less than the middle side, and the longest side measures 375 miles more than the middle side, so the answer checks.

32. *Step 2*
Let $L =$ the length of the top floor. Then $\frac{1}{2}L + 20$ is the width of the top floor.

Step 3
The perimeter of the top floor is 520 feet. Using $P = 2L + 2W$ gives us

$$2L + 2(\tfrac{1}{2}L + 20) = 520.$$

Step 4

$$\begin{aligned} 2L + L + 40 &= 520 \\ 3L + 40 &= 520 \\ 3L &= 480 \\ L &= 160 \end{aligned}$$

Step 5
The length of the top floor is 160 feet and the width of the top floor is $\frac{1}{2}(160) + 20 = 100$ feet.

Step 6
$2L + 2W = 2(160) + 2(100) = 320 + 200 = 520$, which is the perimeter of the top floor. Also, the width, 100 ft, is 20 ft more than one-half the length, 80 ft.

33. *Step 2*
Let $x =$ the height of the Eiffel Tower.
Then $x - 804 =$ the height of the Leaning Tower of Pisa.

Step 3
Together these heights are 1164 ft, so

$$x + (x - 804) = 1164.$$

Step 4

$$\begin{aligned} 2x - 804 &= 1164 \\ 2x &= 1968 \\ x &= 984 \end{aligned}$$

Step 5
The height of the Eiffel Tower is 984 feet and the height of the Leaning Tower of Pisa is
$984 - 804 =$ 180 feet.

Step 6
180 feet is 804 feet shorter than 984 feet and the sum of 180 feet and 984 feet is 1164 feet.

34. *Step 2*
Let $x =$ the number of performances of *Cats*.
Then $x - 805 =$ the number of performances of *Les Misérables.*

Step 3
There were 14,165 total performances, so

$$x + (x - 805) = 14{,}165.$$

Step 4
$$\begin{aligned} 2x - 805 &= 14{,}165 \\ 2x &= 14{,}970 \\ x &= 7485 \end{aligned}$$

Step 5
There were 7485 performances of *Cats* and $7485 - 805 = 6680$ performances of *Les Misérables*.

Step 6
The total number of performances is 14,165 and 6680 is 805 fewer than 7485, as required.

35. *Step 2*
Let $x =$ the Yankees' payroll (in millions). Then $x - 70.4 =$ the Tigers' payroll (in millions).

Step 3
The two payrolls totaled \$347.8 million, so

$$x + (x - 70.4) = 347.8$$

Step 4
$$\begin{aligned} 2x - 70.4 &= 347.8 \\ 2x &= 418.2 \\ x &= 209.1 \end{aligned}$$

Step 5
In 2008, the Yankees' payroll was \$209.1 million and the Tigers' payroll was $209.1 - 70.4 =$ \$138.7 million.

Step 6
\$138.7 million is \$70.4 million less than \$209.1 million and the sum of \$138.7 million and \$209.1 million is \$347.8 million.

36. *Step 2*
Let $x =$ the number of hits Williams got. Then $x + 276 =$ the number of hits Hornsby got.

Step 3
Their base hits totaled 5584, so

$$x + (x + 276) = 5584.$$

Step 4
$$\begin{aligned} 2x + 276 &= 5584 \\ 2x &= 5308 \\ x &= 2654 \end{aligned}$$

Step 5
Williams got 2654 base hits, and Hornsby got $2654 + 276 = 2930$ base hits.

Step 6
2930 is 276 more than 2654 and the total is $2654 + 2930 = 5584$.

37. Let $x =$ the 2004 cost. Then

$$\begin{aligned} x + 3.1\%(x) &= 36.78. \\ x + 3.1(0.01)(x) &= 36.78 \\ 1x + 0.031x &= 36.78 \\ 1.031x &= 36.78 \\ x &= \tfrac{36.78}{1.031} \approx 35.67 \end{aligned}$$

The 2004 cost was \$35.67.

38. Let $x =$ the 1987 cost. Then

$$\begin{aligned} x + 37.5\%(x) &= 36.78. \\ x + 37.5(0.01)(x) &= 36.78 \\ 1x + 0.375x &= 36.78 \\ 1.375x &= 36.78 \\ x &= \tfrac{36.78}{1.375} \approx 26.75 \end{aligned}$$

The 1987 cost was \$26.75.

39. Let $x =$ the 2007 population.

The 2007 population was 106.6% of the 2000 population.

$$\begin{aligned} x &= (106.6\%)(237{,}230) \\ &= 1.066(237{,}230) \\ &= 252{,}887.18 \end{aligned}$$

The 2007 population was about 252,887.

40. Let $x =$ the CPI in 2006. Then $0.043x$ represents the 4.3% increase from 2006 to 2007.

The 2007 CPI plus the 4.3% increase equals the 2006 CPI. Thus,

$$\begin{aligned} x + 0.043x &= 210.2. \\ 1.043x &= 210.2 \\ x &= \frac{210.2}{1.043} \\ x &\approx 201.5 \end{aligned}$$

The CPI was approximately 201.5 in 2006.

41. Let $x =$ the amount of the receipts excluding tax. Since the sales tax is 9% of x, the total amount is

$$\begin{aligned} x + 0.09x &= 2725 \\ 1x + 0.09x &= 2725 \\ 1.09x &= 2725 \\ x &= \frac{2725}{1.09} = 2500 \end{aligned}$$

Thus, the tax was $0.09(2500) =$ \$225.

42. Let $x =$ the amount of commission. Since x is 6% of the selling price,

$$x = 0.06(159{,}000) = 9540.$$

So after the agent was paid, he had $159{,}000 - 9540 =$ \$149,460.

43. Let $x=$ the amount invested at 3%. Then $12{,}000-x=$ the amount invested at 4%. Complete the table. Use $I=prt$ with $t=1$.

Principal	Rate (as a Decimal)	Interest
x	0.03	$0.03x$
$12{,}000-x$	0.04	$0.04(12{,}000-x)$
12,000	← Totals →	440

The last column gives the equation.

Interest at 3%	+	interest at 4%	=	total interest.
$0.03x$	+	$0.04(12{,}000-x)$	=	440

$$3x+4(12{,}000-x)=44{,}000 \quad \textit{Multiply by 100.}$$
$$3x+48{,}000-4x=44{,}000$$
$$-x=-4000$$
$$x=4000$$

He should invest \$4000 at 3% and $12{,}000-4000=\$8000$ at 4%.

Check \$4000 @ 3% = \$120 and \$8000 @ 4% = \$320; \$120 + \$320 = \$440.

44. Let $x=$ the amount invested at 2%. Then $60{,}000-x=$ the amount invested at 3%. Complete the table. Use $I=prt$ with $t=1$.

Principal	Rate (as a Decimal)	Interest
x	0.02	$0.02x$
$60{,}000-x$	0.03	$0.03(60{,}000-x)$
60,000	← Totals →	1600

The last column gives the equation.

Interest at 2%	+	interest at 3%	=	total interest.
$0.02x$	+	$0.03(60{,}000-x)$	=	1600

$$2x+3(60{,}000-x)=160{,}000 \quad \textit{Multiply by 100.}$$
$$2x+180{,}000-3x=160{,}000$$
$$-x=-20{,}000$$
$$x=20{,}000$$

He invested \$20,000 at 2% and $60{,}000-x=60{,}000-20{,}000=\$40{,}000$ at 3%.

Check \$20,000 @ 2% = \$400 and \$40,000 @ 3% = \$1200; \$400 + \$1200 = \$1600.

45. Let $x=$ the amount invested at 4.5%. Then $2x-1000=$ the amount invested at 3%. Use $I=prt$ with $t=1$. Make a table.

Principal	Rate (as a Decimal)	Interest
x	0.045	$0.045x$
$2x-1000$	0.03	$0.03(2x-1000)$
	Total →	1020

The last column gives the equation.

Interest at 4.5%	+	interest at 3%	=	total interest.
$0.045x$	+	$0.03(2x-1000)$	=	1020

$$45x+30(2x-1000)=1{,}020{,}000 \quad \textit{Multiply by 1000.}$$
$$45x+60x-30{,}000=1{,}020{,}000$$
$$105x=1{,}050{,}000$$
$$x=\frac{1{,}050{,}000}{105}=10{,}000$$

She invested \$10,000 at 4.5% and $2x-1000=2(10{,}000)-1000=\$19{,}000$ at 3%.

Check \$19,000 is \$1000 less than two times \$10,000. \$10,000 @ 4.5% = \$450 and \$19,000 @ 3% = \$570; \$450 + \$570 = \$1020.

46. Let $x=$ the amount invested at 3.5%. Then $3x+5000=$ the amount invested at 4%. Use $I=prt$ with $t=1$. Make a table.

Principal	Rate (as a Decimal)	Interest
x	0.035	$0.035x$
$3x+5000$	0.04	$0.04(3x+5000)$
	Total →	1440

The last column gives the equation.

Interest at 3.5%	+	interest at 4%	=	1440.
$0.035x$	+	$0.04(3x+5000)$	=	1440

$$35x+40(3x+5000)=1{,}440{,}000 \quad \textit{Multiply by 1000.}$$
$$35x+120x+200{,}000=1{,}440{,}000$$
$$155x=1{,}240{,}000$$
$$x=\frac{1{,}240{,}000}{155}=8000$$

He invested \$8000 at 3.5% and $3x+5000=3(8000)+5000=\$29{,}000$ at 4%.

Check \$29,000 is \$5000 more than three times \$8000. \$8000 @ 3.5% = \$280 and \$29,000 @ 4% = \$1160; \$280 + \$1160 = \$1440.

47. Let $x =$ the amount of additional money to be invested at 4%.
Use $I = prt$ with $t = 1$. Make a table.
Use the fact that the total return on the two investments is 6%.

Principal	Rate (as a decimal)	Interest
27,000	0.07	$0.07(27{,}000)$
x	0.04	$0.04x$
$27{,}000 + x$	0.06	$0.06(27{,}000 + x)$

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest at 7\%} & + & \text{interest at 4\%} & = & \text{interest at 6\%}. \\ 0.07(27{,}000) & + & 0.04x & = & 0.06(27{,}000 + x) \end{array}$$

$$\begin{aligned} 7(27{,}000) + 4x &= 6(27{,}000 + x) && \textit{Multiply by 100.} \\ 189{,}000 + 4x &= 162{,}000 + 6x \\ 27{,}000 &= 2x \\ 13{,}500 &= x \end{aligned}$$

They should invest \$13,500 at 4%.

Check \$27,000 @ 7% = \$1890 and \$13,500 @ 4% = \$540;
\$1890 + \$540 = \$2430, which is the same as (\$27,000 + \$13,500) @ 6%.

48. Let $x =$ the amount of additional money to be invested at 5%.
Use $I = prt$ with $t = 1$. Make a table.
Use the fact that the total return on the two investments is 6%.

Principal	Rate (as a decimal)	Interest
17,000	0.065	$0.065(17{,}000)$
x	0.05	$0.05x$
$17{,}000 + x$	0.06	$0.06(17{,}000 + x)$

Write the equation from the last column in the table.

$$\begin{array}{ccccc} \text{Interest at 6.5\%} & + & \text{interest at 5\%} & = & \text{interest at 6\%.} \\ 0.065(17{,}000) & + & 0.05x & = & 0.06(17{,}000 + x) \end{array}$$

$$\begin{aligned} 65(17{,}000) + 50x &= 60(17{,}000 + x) \\ & \qquad \textit{Multiply by 1000.} \\ 1{,}105{,}000 + 50x &= 1{,}020{,}000 + 60x \\ 85{,}000 &= 10x \\ 8500 &= x \end{aligned}$$

She should invest \$8500 at 6%.

Check \$17,000 @ 6.5% = \$1105 and \$8500 @ 5% = \$425;
\$1105 + \$425 = \$1530, which is the same as (\$17,000 + \$8500) @ 6%.

49. Let $x =$ the number of liters of 10% acid solution needed. Make a table.

Liters of Solution	Percent (as a Decimal)	Liters of Pure Acid
10	0.04	$0.04(10) = 0.4$
x	0.10	$0.10x$
$x + 10$	0.06	$0.06(x + 10)$

Write the equation from the last column in the table.

$$\begin{array}{ccccc} \text{Acid in 4\%} & + & \text{acid in 10\%} & = & \text{acid in 6\%.} \\ 0.4 & + & 0.10x & = & 0.06(x + 10) \end{array}$$

$$\begin{aligned} 0.4 + 0.10x &= 0.06x + 0.6 && \textit{Distributive property} \\ 0.04x &= 0.2 && \textit{Subtract 0.06x and 0.4.} \\ x &= 5 && \textit{Divide by 0.04.} \end{aligned}$$

Five liters of the 10% solution are needed.

Check 4% of 10 is 0.4 and 10% of 5 is 0.5; $0.4 + 0.5 = 0.9$, which is the same as 6% of $(10 + 5)$.

50. Let $x =$ the number of liters of 14% alcohol solution needed. Make a chart.

Liters of Solution	Percent (as a Decimal)	Liters of Pure Alcohol
x	0.14	$0.14x$
20	0.50	$0.50(20) = 10$
$x + 20$	0.30	$0.30(x + 20)$

Write the equation from the last column in the table.

$$\begin{array}{ccccc} \text{Alcohol in 14\%} & + & \text{alcohol in 50\%} & = & \text{alcohol in 30\%.} \\ 0.14x & + & 10 & = & 0.30(x + 20) \end{array}$$

$$\begin{aligned} 14x + 1000 &= 30(x + 20) && \textit{Multiply by 100.} \\ 14x + 1000 &= 30x + 600 \\ -16x &= -400 && \textit{Subtract 30x and 1000.} \\ x &= 25 && \textit{Divide by } -16. \end{aligned}$$

25 L of 14% solution must be added.

Check 14% of 25 is 3.5 and 50% of 20 is 10; $3.5 + 10 = 13.5$, which is the same as 30% of $(25 + 20)$.

51. Let $x =$ the number of liters of the 20% alcohol solution. Make a table.

Liters of Solution	Percent (as a Decimal)	Liters of Pure Alcohol
12	0.12	$0.12(12) = 1.44$
x	0.20	$0.20x$
$x + 12$	0.14	$0.14(x + 12)$

Write the equation from the last column in the table.

$$\begin{array}{ccccc} \text{Alcohol} & & \text{alcohol} & & \text{alcohol} \\ \text{in 12\%} & + & \text{in 20\%} & = & \text{in 14\%.} \\ 1.44 & + & 0.20x & = & 0.14(x+12) \end{array}$$

$$\begin{aligned} 144 + 20x &= 14(x+12) && \textit{Multiply by 100.} \\ 144 + 20x &= 14x + 168 && \textit{Distributive property} \\ 6x &= 24 && \textit{Subtract 14x and 144.} \\ x &= 4 && \textit{Divide by 6.} \end{aligned}$$

4L of 20% alcohol solution are needed.

Check 12% of 12 is 1.44 and 20% of 4 is 0.8; $1.44 + 0.8 = 2.24$, which is the same as 14% of $(12 + 4)$.

52. Let $x =$ the number of liters of 10% alcohol solution. Make a chart.

Liters of Solution	Percent (as a Decimal)	Liters of Pure Alcohol
x	0.10	$0.10x$
40	0.50	$0.50(40) = 20$
$x + 40$	0.40	$0.40(x + 40)$

Write the equation from the last column in the table.

$$\begin{array}{ccccc} \text{Alcohol} & & \text{alcohol} & & \text{alcohol} \\ \text{in 10\%} & + & \text{in 50\%} & = & \text{in 40\%.} \\ 0.10x & + & 20 & = & 0.40(x+40) \end{array}$$

$$\begin{aligned} 10x + 2000 &= 40(x+40) && \textit{Multiply by 100.} \\ 10x + 2000 &= 40x + 1600 && \\ -30x &= -400 && \textit{Subtract 40x and 2000.} \\ x &= \tfrac{40}{3} \text{ or } 13\tfrac{1}{3} && \textit{Divide by } -30. \end{aligned}$$

$13\frac{1}{3}$ L of 10% solution should be added.

Check 50% of 40 is 20 and 10% of $\frac{40}{3}$ is $\frac{4}{3}$; $20 + \frac{4}{3} = 21\frac{1}{3}$, which is the same as 40% of $(\frac{40}{3} + 40)$.

53. Let $x =$ the amount of pure dye used (pure dye is 100% dye). Make a table.

Gallons of Solution	Percent (as a Decimal)	Gallons of Pure Dye
x	1	$1x = x$
4	0.25	$0.25(4) = 1$
$x + 4$	0.40	$0.40(x + 4)$

Write the equation from the last column in the table.

$$\begin{aligned} x + 1 &= 0.4(x+4) && \\ x + 1 &= 0.4x + 1.6 && \textit{Distributive property} \\ 0.6x &= 0.6 && \textit{Subtract 0.4x and 1.} \\ x &= 1 && \textit{Divide by 0.6.} \end{aligned}$$

One gallon of pure (100%) dye is needed.

Check 100% of 1 is 1 and 25% of 4 is 1; $1 + 1 = 2$, which is the same as 40% of $(1 + 4)$.

54. Let $x =$ the number of gallons of water. Make a chart.

Gallons of Solution	Percent (as a Decimal)	Gallons of Pure Insecticide
x	0	$0(x) = 0$
6	0.04	$0.04(6) = 0.24$
$x + 6$	0.03	$0.03(x + 6)$

Write the equation from the last column in the table.

$$\begin{array}{ccccc} \text{Insecticide} & & \text{insecticide} & & \text{insecticide} \\ \text{in water} & + & \text{in 4\%} & = & \text{in 3\%.} \\ 0 & + & 0.24 & = & 0.03(x+6) \end{array}$$

$$\begin{aligned} 0 + 24 &= 3(x+6) && \textit{Multiply by 100.} \\ 24 &= 3x + 18 && \textit{Distributive property} \\ 6 &= 3x && \textit{Subtract 18.} \\ 2 &= x && \textit{Divide by 3.} \end{aligned}$$

2 gallons of water should be added.

Check 4% of 6 is 0.24, which is the same as 3% of $(2 + 6)$.

55. Let $x =$ the amount of \$6 per lb nuts. Make a table.

Cost per lb	Pounds of Nuts	Total Cost
\$2	50	$2(50) = 100$
\$6	x	$6x$
\$5	$x + 50$	$5(x + 50)$

The total value of the \$2 per lb nuts and the \$6 per lb nuts must equal the value of the \$5 per lb nuts.

$$\begin{aligned} 100 + 6x &= 5(x+50) \\ 100 + 6x &= 5x + 250 \\ x &= 150 \end{aligned}$$

She should use 150 lb of \$6 nuts.

Check 50 pounds of the \$2 per lb nuts are worth \$100 and 150 pounds of the \$6 per lb nuts are worth \$900; \$100 + \$900 = \$1000, which is the same as $(50 + 150)$ pounds worth \$5 per lb.

56. Let $x =$ the number of ounces of 2¢ per oz tea. Make a table.

Ounces of Tea	Cost per oz	Total Cost
x	2¢ or 0.02	$0.02x$
100	5¢ or 0.05	$0.05(100) = 5$
$x + 100$	3¢ or 0.03	$0.03(x + 100)$

Write the equation from the last column in the table.

$$\begin{array}{ccccc} \text{Cost of} & + & \text{cost of} & = & \text{cost of} \\ 2¢\text{ tea} & & 5¢\text{ tea} & & 3¢\text{ tea.} \\ 0.02x & + & 5 & = & 0.03(x+100) \end{array}$$

$$\begin{aligned} 2x + 500 &= 3(x + 100) && \textit{Multiply by 100.} \\ 2x + 500 &= 3x + 300 && \textit{Distributive property} \\ 200 &= x && \textit{Subtract 2x and 300.} \end{aligned}$$

200 oz of 2¢ per oz tea should be used.

Check 200 oz of 2¢ per oz tea is worth \$4 and 100 oz of 5¢ per oz tea is worth \$5; \$4 + \$5 = \$9, which is the same value as $(200 + 100)$ oz of 3¢ per oz tea.

57. We cannot expect the final mixture to be worth more than each of the ingredients. Answers will vary.

58. Let $x =$ the number of liters of 30% acid solution. Make a chart.

Liters of Solution	Percent (as a Decimal)	Liters of Pure Acid
x	0.30	$0.30x$
15	0.50	$0.50(15) = 7.5$
$x + 15$	0.60	$0.60(x + 15)$

Write the equation from the last column in the table.

$$\begin{array}{ccccc} \text{Acid} & + & \text{acid} & = & \text{acid} \\ \text{in 30\%} & & \text{in 50\%} & & \text{in 60\%.} \\ 0.30x & + & 7.5 & = & 0.60(x+15) \end{array}$$

$$\begin{aligned} 3x + 75 &= 6(x + 15) && \textit{Multiply by 10.} \\ 3x + 75 &= 6x + 90 && \textit{Distributive property} \\ -3x &= 15 && \textit{Subtract 6x and 75.} \\ x &= -5 && \textit{Divide by } -3. \end{aligned}$$

The solution, -5, is impossible since the number of liters of 30% acid solution cannot be negative. Therefore, this problem has no solution.

59. (a) Let $x =$ the amount invested at 5%.
$800 - x =$ the amount invested at 10%.

(b) Let $y =$ the amount of 5% acid used.
$800 - y =$ the amount of 10% acid used.

60. Organize the information in a table.

(a)

Principal	Percent (as a Decimal)	Interest
x	0.05	$0.05x$
$800 - x$	0.10	$0.10(800 - x)$
800	0.0875	$0.0875(800)$

The amount of interest earned at 5% and 10% is found in the last column of the table, $0.05x$ and $0.10(800 - x)$.

(b)

Liters of Solution	Percent (as a Decimal)	Liters of Pure Acid
y	0.05	$0.05y$
$800 - y$	0.10	$0.10(800 - y)$
800	0.0875	$0.0875(800)$

The amount of pure acid in the 5% and 10% mixtures is found in the last column of the table, $0.05y$ and $0.10(800 - y)$.

61. Refer to the tables for Exercise 60. In each case, the last column gives the equation.

(a) $0.05x + 0.10(800 - x) = 0.0875(800)$

(b) $0.05y + 0.10(800 - y) = 0.0875(800)$

62. In both cases, multiply by 10,000 to clear the decimals.

(a)
$$\begin{aligned} 0.05x + 0.10(800 - x) &= 0.0875(800) \\ 500x + 1000(800 - x) &= 875(800) \\ 500x + 800{,}000 - 1000x &= 700{,}000 \\ -500x &= -100{,}000 \\ x &= 200 \end{aligned}$$

Jack invested \$200 at 5% and
$800 - x = 800 - 200 =$ \$600 at 10%.

(b)
$$\begin{aligned} 0.05y + 0.10(800 - y) &= 0.0875(800) \\ 500y + 1000(800 - y) &= 875(800) \\ 500y + 800{,}000 - 1000y &= 700{,}000 \\ -500y &= -100{,}000 \\ y &= 200 \end{aligned}$$

Jill used 200 L of 5% acid solution and
$800 - y = 800 - 200 = 600$ L of 10% acid solution.

63. The processes used to solve Problems A and B were virtually the same. Aside from the variables chosen, the problem information was organized in similar tables and the equations solved were the same. The amounts of money in Problem A correspond to the amounts of solution in Problem B.

2.4 Further Applications of Linear Equations

2.4 Margin Exercises

1. Let $x =$ the number of dimes.
Then $26 - x =$ the number of half-dollars.

	Number of Coins	Denom- ination	Value
Dimes	x	\$0.10	$0.10x$
Halves	$26 - x$	\$0.50	$0.50(26 - x)$
	26	← Totals →	8.60

Multiply the number of coins by the denominations, and add the results to get 8.60.

$$\begin{aligned} 0.10x + 0.50(26 - x) &= 8.60 \\ 1x + 5(26 - x) &= 86 \quad \textit{Multiply by 10.} \\ 1x + 130 - 5x &= 86 \\ -4x &= -44 \\ x &= 11 \end{aligned}$$

The cashier has 11 dimes and $26 - 11 = 15$ half-dollars.

Check The number of coins is $11 + 15 = 26$ and the value of the coins is $\$0.10(11) + \$0.50(15) = \$8.60$, as required.

2. Let $x =$ the amount of time needed for the cars to be 420 mi apart.

Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

	Rate	Time	Distance
Northbound Car	60	x	$60x$
Southbound Car	45	x	$45x$
Total →			420

The total distance traveled is the sum of the distances traveled by each car, since they are traveling in opposite directions. This total is 420 mi.

$$\begin{aligned} 60x + 45x &= 420 \\ 105x &= 420 \\ x &= \tfrac{420}{105} = 4 \end{aligned}$$

The cars will be 420 mi apart in 4 hr.

Check The northbound car travels $60(4) = 240$ miles and the southbound car travels $45(4) = 180$ miles for a total of 420 miles, as required.

3. Let $x =$ the time it takes Clay to catch up to Elayn. Then $x + \frac{1}{2} =$ Elayn's time.

Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

	Rate	Time	Distance
Elayn	3	$x + \frac{1}{2}$	$3(x + \frac{1}{2})$
Clay	5	x	$5x$

The distance traveled by Elayn is equal to the distance traveled by Clay.

$$\begin{aligned} 3(x + \tfrac{1}{2}) &= 5x \\ 3x + \tfrac{3}{2} &= 5x \\ 6x + 3 &= 10x \quad \textit{Multiply by 2.} \\ 3 &= 4x \\ \tfrac{3}{4} &= x \end{aligned}$$

It takes Clay $\frac{3}{4}$ hr or 45 min to catch up to Elayn.

Check Elayn travels $3(\frac{3}{4} + \frac{1}{2}) = \frac{15}{4}$ miles and Clay also travels $5(\frac{3}{4}) = \frac{15}{4}$ miles, as required.

4. Let $x =$ the measure of the second angle.
Then $x + 15 =$ the measure of the first angle, and $2x + 25 =$ the measure of the third angle.

The sum of the three measures must equal 180°.

$$\begin{aligned} x + (x + 15) + (2x + 25) &= 180 \\ 4x + 40 &= 180 \\ 4x &= 140 \\ x &= 35 \end{aligned}$$

The angles measure 35°, $35 + 15 = 50°$, and $2(35) + 25 = 95°$.

Check $35° + 50° + 95° = 180°$, as required.

2.4 Section Exercises

1. The total amount is

$$\begin{aligned} 38(0.05) + 26(0.10) &= 1.90 + 2.60 \\ &= \$4.50. \end{aligned}$$

2. Use $d = rt$, or $t = \dfrac{d}{r}$.
Substitute 7700 for d and 480 for r.

$$t = \frac{7700}{480} \approx 16.04$$

Its travel time is approximately 16 hours.

3. Use $d = rt$, or $r = \dfrac{d}{t}$. Substitute 1320 for d and 24 for t.

$$r = \frac{1320}{24} = 55$$

His rate was 55 mph.

4. Use $P = 4s$ or $s = \dfrac{P}{4}$.
Substitute 40 for P.

$$s = \frac{40}{4} = 10$$

The length of each side of the square is 10 in. This is also the length of each side of the equilateral triangle. To find the perimeter of the equilateral triangle, use $P = 3s$.
Substitute 10 for s.

$$P = 3(10) = 30$$

The perimeter would be 30 inches.

5. Let $x =$ the number of pennies. Then x is also the number of dimes, and $44 - 2x$ is the number of quarters.

Number of Coins	Denomination	Value
x	0.01	$0.01x$
x	0.10	$0.10x$
$44 - 2x$	0.25	$0.25(44 - 2x)$
44	← Totals →	4.37

The sum of the values must equal the total value.

$$\begin{aligned} 0.01x + 0.10x + 0.25(44 - 2x) &= 4.37 \\ x + 10x + 25(44 - 2x) &= 437 \\ &\textit{Multiply by 100.} \\ x + 10x + 1100 - 50x &= 437 \\ -39x + 1100 &= 437 \\ -39x &= -663 \\ x &= 17 \end{aligned}$$

There are 17 pennies, 17 dimes, and $44 - 2(17) = 10$ quarters.

Check The number of coins is $17 + 17 + 10 = 44$ and the value of the coins is $\$0.01(17) + \$0.10(17) + \$0.25(10) = \$4.37$, as required.

6. Let $x =$ the number of nickels and the number of quarters. Then $2x$ is the number of half-dollars.

Number of Coins	Denomination	Value
x	0.05	$0.05x$
x	0.25	$0.25x$
$2x$	0.50	$0.50(2x)$
	Total →	2.60

The sum of the values must equal the total value.

$$\begin{aligned} 0.05x + 0.25x + 0.50(2x) &= 2.60 \\ 5x + 25x + 50(2x) &= 260 \\ &\textit{Multiply by 100.} \\ 5x + 25x + 100x &= 260 \\ 130x &= 260 \\ x &= 2 \end{aligned}$$

She found 2 nickels, 2 quarters, and $2(2) = 4$ half-dollars.

Check The number of nickels, 2, is the same as the number of quarters. The number of half-dollars, 4, is twice the number of quarters. The value of the coins is $\$0.05(2) + \$0.25(2) + \$0.50(4) = \$2.60$, as required.

7. Let $x =$ the number of loonies. Then $37 - x$ is the number of toonies.

Number of Coins	Denomination	Value
x	1	$1x$
$37 - x$	2	$2(37 - x)$
37	← Totals →	51

The sum of the values must equal the total value.

$$\begin{aligned} 1x + 2(37 - x) &= 51 \\ x + 74 - 2x &= 51 \\ -x + 74 &= 51 \\ 23 &= x \end{aligned}$$

She has 23 loonies and $37 - 23 = 14$ toonies.

Check The total number of coins is 37 and the value of the coins is $\$1(23) + \$2(14) = \$51$, as required.

8. Let $x =$ the number of \$1 bills. Then $119 - x$ is the number of \$5 bills.

Number of Bills	Denomination	Value
x	1	$1x$
$119 - x$	5	$5(119 - x)$
119	← Totals →	347

The sum of the values must equal the total value.

$$\begin{aligned} 1x + 5(119 - x) &= 347 \\ x + 595 - 5x &= 347 \\ -4x &= -248 \\ x &= 62 \end{aligned}$$

He has 62 \$1 bills and $119 - 62 = 57$ \$5 bills.

Check The value of the bills is $\$1(62) + \$5(57) = \$62 + \$285 = \$347$, as required.

9. Let $x =$ the number of \$10 coins.
Then $53 - x$ is the number of \$20 coins.

Number of Coins	Denomination	Value
x	10	$10x$
$53 - x$	20	$20(53 - x)$
53	← Totals →	780

The sum of the values must equal the total value.

$$\begin{aligned} 10x + 20(53 - x) &= 780 \\ 10x + 1060 - 20x &= 780 \\ -10x &= -280 \\ x &= 28 \end{aligned}$$

He has 28 \$10 coins and $53 - 28 = 25$ \$20 coins.

Check The number of coins is $28 + 25 = 53$ and the value of the coins is
$\$10(28) + \$20(25) = \$780$, as required.

10. Let $x =$ the number of two-cent pieces.
Then $3x$ is the number of three-cent pieces.

Number of Coins	Denomination	Value
x	0.02	$0.02x$
$3x$	0.03	$0.03(3x)$
	Total →	2.42

The sum of the values must equal the total value.

$$\begin{aligned} 0.02x + 0.03(3x) &= 2.42 \\ 2x + 3(3x) &= 242 \quad \textit{Multiply by 100.} \\ 2x + 9x &= 242 \\ 11x &= 242 \\ x &= 22 \end{aligned}$$

She has 22 two-cent pieces and $3(22) = 66$ three-cent pieces.

Check The number of three-cent pieces, 66, is three times the number of two-cent pieces, 22. The value of the coins is
$\$0.02(22) + \$0.03(66) = \$2.42$, as required.

11. Let $x =$ the number of adult tickets sold. Then $2010 - x =$ the number of children and senior tickets sold.

Cost of Ticket	Number Sold	Amount Collected
\$14	x	$14x$
\$11	$2010 - x$	$11(2010 - x)$
Totals	2010	\$24,726

Write the equation from the last column of the table.

$$\begin{aligned} 14x + 11(2010 - x) &= 24{,}726 \\ 14x + 22{,}110 - 11x &= 24{,}726 \\ 3x &= 2616 \\ x &= 872 \end{aligned}$$

There were 872 adult tickets sold and $2010 - 872 = 1138$ children and senior tickets sold.

Check The amount collected was
$\$14(872) + \$11(1138)$
$= \$12{,}208 + \$12{,}518 = \$24{,}726$, as required.

12. Let $x =$ the number of student tickets sold. Then $480 - x =$ the number of nonstudent tickets sold.

Cost of Ticket	Number Sold	Amount Collected
\$5	x	$5x$
\$8	$480 - x$	$8(480 - x)$
Totals	480	\$2895

Write the equation from the last column of the table.

$$\begin{aligned} 5x + 8(480 - x) &= 2895 \\ 5x + 3840 - 8x &= 2895 \\ -3x &= -945 \\ x &= 315 \end{aligned}$$

315 student tickets were sold; $480 - 315 = 165$ nonstudent tickets were sold.

Check The amount collected was
$\$5(315) + \$8(165) = \$1575 + \$1320 = \$2895$, as required.

13. $d = rt$, so

$$r = \frac{d}{t} = \frac{100}{12.37} \approx 8.08$$

Her rate was about 8.08 m/sec.

14. $d = rt$, so

$$r = \frac{d}{t} = \frac{400}{52.82} \approx 7.57$$

Her rate was about 7.57 m/sec.

15. $d = rt$, so

$$r = \frac{d}{t} = \frac{400}{47.63} \approx 8.40$$

His rate was about 8.40 m/sec.

16. $d = rt$, so

$$r = \frac{d}{t} = \frac{400}{44.00} \approx 9.09$$

His rate was about 9.09 m/sec.

17. Let $t =$ the time until they are 110 mi apart.
Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

	Rate	Time	Distance
First Steamer	22	t	$22t$
Second Steamer	22	t	$22t$
Total →			110

The total distance traveled is the sum of the distances traveled by each steamer, since they are traveling in opposite directions. This total is 110 mi.

$$22t + 22t = 110$$
$$44t = 110$$
$$t = \tfrac{110}{44} = \tfrac{5}{2}, \quad \text{or} \quad 2\tfrac{1}{2}$$

It will take them $2\frac{1}{2}$ hr.

Check Each steamer traveled $22(2.5) = 55$ miles for a total of $2(55) = 110$ miles, as required.

18. Let $t =$ the time it takes for the trains to be 315 km apart.
Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

	Rate	Time	Distance
First train	85	t	$85t$
Second train	95	t	$95t$
Total →			315

The total distance traveled is the sum of the distances traveled by each train, since they are traveling in opposite directions. This total is 315 km.

$$85t + 95t = 315$$
$$180t = 315$$
$$t = \tfrac{315}{180} = \tfrac{7}{4}, \quad \text{or} \quad 1\tfrac{3}{4}$$

It will take the trains $1\frac{3}{4}$ hr before they are 315 km apart.

Check The first train traveled $85(1.75) = 148.75$ km and the second train traveled $95(1.75) = 166.25$ km. The sum is 315 km, as required.

19. Let $t =$ Mulder's time.
Then $t - \frac{1}{2} =$ Scully's time.

	Rate	Time	Distance
Mulder	65	t	$65t$
Scully	68	$t - \frac{1}{2}$	$68(t - \frac{1}{2})$

The distances are equal.

$$65t = 68(t - \tfrac{1}{2})$$
$$65t = 68t - 34$$
$$-3t = -34$$
$$t = \tfrac{34}{3}, \quad \text{or} \quad 11\tfrac{1}{3}$$

Mulder's time will be $11\frac{1}{3}$ hr. Since he left at 8:30 A.M., $11\frac{1}{3}$ hr or 11 hr 20 min later is 7:50 P.M.

Check Mulder's distance was $65(\frac{34}{3}) = 736\frac{2}{3}$ miles. Scully's distance was $68(\frac{34}{3} - \frac{1}{2}) = 68(\frac{65}{6}) = 736\frac{2}{3}$, as required.

20. Let $x =$ Lois' travel time.
Since Clark leaves 15 minutes after Lois, and $\frac{15}{60} = \frac{1}{4}$ hr, $x - \frac{1}{4} =$ time for Clark.
Complete the table using the formula $rt = d$.

	Rate	Time	Distance
Lois	35	x	$35x$
Clark	40	$x - \frac{1}{4}$	$40(x - \frac{1}{4})$

Since Lois and Clark are going in opposite directions, we add their distances to get 140 mi.

$$35x + 40(x - \tfrac{1}{4}) = 140$$
$$35x + 40x - 10 = 140$$
$$75x = 150$$
$$x = 2$$

Lois' time will be 2 hours. They will be 140 mi apart at 8 A.M. + 2 hr = 10 A.M.

Check Lois' distance was $35(2) = 70$.
Clark's distance was $40(2 - \frac{1}{4}) = 40(\frac{7}{4}) = 70$.
The sum is 140 miles, as required.

21. Let $x =$ her average speed on Sunday.
Then $x + 5 =$ her average speed on Saturday.

	Rate	Time	Distance
Saturday	$x + 5$	3.6	$3.6(x + 5)$
Sunday	x	4	$4x$

The distances are equal.

$$3.6(x + 5) = 4x$$
$$3.6x + 18 = 4x$$
$$18 = 0.4x \qquad \textit{Subtract 3.6x.}$$
$$x = \tfrac{18}{0.4} = 45$$

Her average speed on Sunday was 45 mph.

Check On Sunday, 4 hours @ 45 mph = 180 miles. On Saturday, 3.6 hours @ 50 mph = 180 miles. The distances are equal.

22. Let $x =$ her biking speed.
Then $x - 7 =$ her walking speed.

	Rate	Time	Distance
Walking	$x - 7$	$\frac{40}{60} = \frac{2}{3}$ hr	$\frac{2}{3}(x - 7)$
Biking	x	$\frac{12}{60} = \frac{1}{5}$ hr	$\frac{1}{5}x$

The distances are equal.

$$\begin{aligned} \tfrac{2}{3}(x-7) &= \tfrac{1}{5}x \\ 10(x-7) &= 3x \quad \textit{Multiply by 15.} \\ 10x - 70 &= 3x \\ 7x &= 70 \\ x &= 10 \end{aligned}$$

The distance from her house to the train station is $\frac{1}{5}x = \frac{1}{5}(10) = 2$ miles.

Check The distance walking is $(3 \text{ mph})(\frac{2}{3} \text{ hr}) = 2$ miles. The distance biking is $(10 \text{ mph})(\frac{1}{5} \text{ hr}) = 2$ miles. The distances are equal.

23. Let $x =$ Anne's time.
Then $x + \frac{1}{2} =$ Johnny's time.

	Rate	Time	Distance
Anne	60	x	$60x$
Johnny	50	$x + \frac{1}{2}$	$50(x + \frac{1}{2})$

The total distance is 80.

$$\begin{aligned} 60x + 50(x + \tfrac{1}{2}) &= 80 \\ 60x + 50x + 25 &= 80 \\ 110x &= 55 \\ x &= \tfrac{55}{110} = \tfrac{1}{2} \end{aligned}$$

They will meet $\frac{1}{2}$ hr after Anne leaves.

Check Anne travels $60(\frac{1}{2}) = 30$ miles. Johnny travels $50(\frac{1}{2} + \frac{1}{2}) = 50$ miles. The sum of the distances is 80 miles, as required.

24. Let $x =$ Heather's rate (speed) during the first part of the trip. Then $x - 25 =$ her rate during rush hour traffic. Make a table using the formula $rt = d$.

	Rate	Time	Distance
First Part	x	2	$2x$
Rush Hour	$x - 25$	$\frac{1}{2}$	$\frac{1}{2}(x - 25)$

The total distance was 125 miles.

$$\begin{aligned} 2x + \tfrac{1}{2}(x - 25) &= 125 \\ 4x + x - 25 &= 250 \quad \textit{Multiply by 2.} \\ 5x &= 275 \\ x &= 55 \end{aligned}$$

The speed during the first part of the trip was 55 mph.

Check The distance traveled during the first part of the trip was $55(2) = 110$ miles. The distance traveled during the second part of the trip was $(55 - 25)(0.5) = 15$ miles. The sum of the distances is 125 miles, as required.

25. The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned} (x - 30) + (2x - 120) + (\tfrac{1}{2}x + 15) &= 180 \\ \tfrac{7}{2}x - 135 &= 180 \\ 7x - 270 &= 360 \\ &\textit{Multiply by 2.} \\ 7x &= 630 \\ x &= 90 \end{aligned}$$

With $x = 90$, the three angle measures become

$$\begin{aligned} (90 - 30)^\circ &= 60^\circ, \\ (2 \cdot 90 - 120)^\circ &= 60^\circ, \\ \text{and} \quad (\tfrac{1}{2} \cdot 90 + 15)^\circ &= 60^\circ. \end{aligned}$$

Check $60^\circ + 60^\circ + 60^\circ = 180^\circ$, as required.

26. The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned} (x + 15) + (10x - 20) + (x + 5) &= 180 \\ 12x &= 180 \\ x &= 15 \end{aligned}$$

With $x = 15$, the three angle measures become

$$\begin{aligned} (15 + 15)^\circ &= 30^\circ, \\ (10 \cdot 15 - 20)^\circ &= 130^\circ, \\ \text{and} \quad (15 + 5)^\circ &= 20^\circ. \end{aligned}$$

Check $30^\circ + 130^\circ + 20^\circ = 180^\circ$, as required.

27. The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned} (3x + 7) + (9x - 4) + (4x + 1) &= 180 \\ 16x + 4 &= 180 \\ 16x &= 176 \\ x &= 11 \end{aligned}$$

With $x = 11$, the three angle measures become

$$\begin{aligned} (3 \cdot 11 + 7)^\circ &= 40^\circ, \\ (9 \cdot 11 - 4)^\circ &= 95^\circ, \\ \text{and} \quad (4 \cdot 11 + 1)^\circ &= 45^\circ. \end{aligned}$$

Check $40^\circ + 95^\circ + 45^\circ = 180^\circ$, as required.

28. The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned}(2x+7)+(x+61)+x&=180\\4x+68&=180\\4x&=112\\x&=28\end{aligned}$$

With $x=28$, the three angle measures become

$$(2\cdot 28+7)^\circ=63^\circ,\quad (28+61)^\circ=89^\circ,\quad\text{and}\quad 28^\circ.$$

Check $63^\circ+89^\circ+28^\circ=180^\circ$, as required.

29. The sum of the measures of the angles of a triangle is 180°.

$$\begin{aligned}x+2x+60&=180\\3x+60&=180\\3x&=120\\x&=40\end{aligned}$$

The measures of the unknown angles are 40° and $2x=80^\circ$.

30. The sum of the measures of the marked angles, $60^\circ+y^\circ$, must equal 180°. Thus, the measure of the unknown angle is 120°.

31. The sum of the measures of the unknown angles in Exercise 29 is $40^\circ+80^\circ=120^\circ$. This is equal to the measure of the angle in Exercise 30.

32. The sum of the measures of angles ① and ② is equal to the measure of angle ③.

33. Vertical angles have equal measure.

$$\begin{aligned}8x+2&=7x+17\\x&=15\end{aligned}$$

$$8\cdot 15+2=122\quad\text{and}\quad 7\cdot 15+17=122.$$

The angles are both 122°.

34. Vertical angles have equal measure.

$$\begin{aligned}9-5x&=25-3x\\9&=25+2x\\-16&=2x\\-8&=x\end{aligned}$$

$$9-5(-8)=49\quad\text{and}\quad 25-3(-8)=49.$$

The angles are both 49°.

35. The sum of the two angles is 90°.

$$\begin{aligned}(5x-1)+2x&=90\\7x-1&=90\\7x&=91\\x&=13\end{aligned}$$

The angles are $(5\cdot 13-1)^\circ=64^\circ$ and $(2\cdot 13)^\circ=26^\circ$.

36. Supplementary angles have an angle measure sum of 180°.

$$\begin{aligned}(3x+5)+(5x+15)&=180\\8x+20&=180\\8x&=160\\x&=20\end{aligned}$$

With $x=20$, the two angle measures become

$$(3\cdot 20+5)^\circ=65^\circ\quad\text{and}\quad(5\cdot 20+15)^\circ=115^\circ.$$

37. Let $x=$ the first consecutive integer. Then $x+1$ will be the second consecutive integer, and $x+2$ will be the third consecutive integer.

The sum of the first and twice the second is 22 more than twice the third.

$$\begin{aligned}x+2(x+1)&=2(x+2)+22\\x+2x+2&=2x+4+22\\3x+2&=2x+26\\x&=24\end{aligned}$$

Since $x=24$, $x+1=25$, and $x+2=26$. The three consecutive integers are 24, 25, and 26.

38. Let $x=$ the first integer. Then $x+1$, $x+2$, and $x+3$ are the next three consecutive integers. The sum of the first three integers is 62 more than the fourth.

$$\begin{aligned}x+(x+1)+(x+2)&=(x+3)+62\\3x+3&=x+65\\2x&=62\\x&=31\end{aligned}$$

The four consecutive integers are 31, 32, 33, and 34.

39. Let $x=$ the current age. Then $x+1$ will be the age next year. The sum of these ages will be 95 years.

$$\begin{aligned}x+(x+1)&=95\\2x+1&=95\\2x&=94\\x&=47\end{aligned}$$

If my current age is 47, in 10 years I will be

$$47+10=57\text{ years old.}$$

40. Let $x =$ the page number on one page. Then $x + 1$ is the page number on the next page. The sum of the page numbers is 365.

$$\begin{aligned} x + (x + 1) &= 365 \\ 2x + 1 &= 365 \\ 2x &= 364 \\ x &= 182 \end{aligned}$$

The page numbers are 182 and 183.

Summary Exercises on Solving Applied Problems

1. Let $x =$ the width of the rectangle. Then $x + 3$ is the length of the rectangle.

If the length were decreased by 2 inches and the width were increased by 1 inch, the perimeter would be 24 inches. Use the formula $P = 2L + 2W$, and substitute 24 for P, $(x + 3) - 2$ or $x + 1$ for L, and $x + 1$ for W.

$$\begin{aligned} P &= 2L + 2W \\ 24 &= 2(x + 1) + 2(x + 1) \\ 24 &= 2x + 2 + 2x + 2 \\ 24 &= 4x + 4 \\ 20 &= 4x \\ 5 &= x \end{aligned}$$

The width of the rectangle is 5 inches, and the length is $5 + 3 = 8$ inches.

2. Let $x =$ the length of the shortest side. Then $2x$ is the length of the middle side and $3x - 2$ is the length of the longest side.

The perimeter is 34 inches. Using $P = a + b + c$ gives us

$$\begin{aligned} x + 2x + (3x - 2) &= 34. \\ 6x - 2 &= 34 \\ 6x &= 36 \\ x &= 6 \end{aligned}$$

The lengths of the three sides are 6 inches, $2(6) = 12$ inches, and $3(6) - 2 = 16$ inches.

Check The sum of the lengths of the three sides is $6 + 12 + 16 = 34$ inches, as required.

3. Let $x =$ the regular price of the item. The sale price after a 37% (or 0.37) discount was \$35.87, so an equation is

$$\begin{aligned} x - 0.37x &= 35.87. \\ 0.63x &= 35.87 \\ x &\approx 56.94 \end{aligned}$$

To the nearest cent, the regular price was \$56.94.

4. Let $x =$ the regular price of the DVD recorder. The sale price after a discount of 40% (or 0.40) was \$255, so an equation is

$$\begin{aligned} x - 0.40x &= 255. \\ 0.60x &= 255 \\ x &= 425 \end{aligned}$$

The regular price of the DVD recorder was \$425.

5. Let $x =$ the amount invested at 4%. Then $2x$ is the amount invested at 5%. Use $I = prt$ with $t = 1$ yr. Make a table.

Principal	Rate (as a Decimal)	Interest
x	0.04	$0.04x$
$2x$	0.05	$0.05(2x) = 0.10x$
	Total →	77

The last column gives the equation.

$$\begin{matrix} \text{Interest at 4\%} & + & \text{interest at 5\%} & = & \text{total interest.} \\ 0.04x & + & 0.10x & = & 77 \end{matrix}$$

$$\begin{aligned} 4x + 10x &= 7700 \quad \textit{Multiply by 100.} \\ 14x &= 7700 \\ x &= 550 \end{aligned}$$

\$550 is invested at 4% and $2(\$550) = \$1100$ is invested at 5%.

Check \$550 @ 4% = \$22 and \$1100 @ 5% = \$55; \$22 + \$55 = \$77

6. Let $x =$ the amount invested at 3%. Then $x + 3000$ is the amount invested at 4%. Use $I = prt$ with $t = 1$ yr. Make a table.

Principal	Rate (as a Decimal)	Interest
x	0.03	$0.03x$
$x + 3000$	0.04	$0.04(x + 3000)$
	Total →	960

The last column gives the equation.

$$\begin{matrix} \text{Interest at 3\%} & + & \text{interest at 4\%} & = & \text{total interest.} \\ 0.03x & + & 0.04(x + 3000) & = & 960 \end{matrix}$$

$$\begin{aligned} 3x + 4(x + 3000) &= 96{,}000 \quad \textit{Multiply by 100.} \\ 3x + 4x + 12{,}000 &= 96{,}000 \\ 7x &= 84{,}000 \\ x &= 12{,}000 \end{aligned}$$

\$12,000 is invested at 3% and \$12,000 + \$3000 = \$15,000 is invested at 4%.

continued

Check \$12,000 @ 3% = \$360 and \$15,000 @ 4% = \$600; \$360 + \$600 = \$960

7. Let $x =$ the number of points he scored in 2005–2006. Then $x - 402 =$ the number of points he scored in 2006–2007. The total number of points he scored was 5262.

$$\begin{aligned} x + (x - 402) &= 5262 \\ 2x - 402 &= 5262 \\ 2x &= 5664 \\ x &= 2832 \end{aligned}$$

He scored in 2832 points in 2005–2006 and $2832 - 402 = 2430$ points 2006–2007.

8. Let $x =$ the amount grossed by *Spider-Man*. Then $x + 29.5 =$ the amount grossed by *Shrek 2* (in millions). Together they grossed \$844.9 million.

$$\begin{aligned} x + (x + 29.5) &= 844.9 \\ 2x + 29.5 &= 844.9 \\ 2x &= 815.4 \\ x &= 407.7 \end{aligned}$$

Spider-Man grossed \$407.7 million and *Shrek 2* grossed $407.7 + 29.5 =$ \$437.2 million.

9. Let $x =$ the side length of the square cut out of each corner. Then the width is $12 - 2x$ and the length is $16 - 2x$. We want the length to be 5 cm less than twice the width.

$$\begin{aligned} \text{length} &= 2(\text{width}) - 5 \\ 16 - 2x &= 2(12 - 2x) - 5 \\ 16 - 2x &= 24 - 4x - 5 \\ 16 - 2x &= 19 - 4x \\ 2x &= 3 \\ x &= \tfrac{3}{2}, \text{ or } 1\tfrac{1}{2} \end{aligned}$$

The square should be $1\frac{1}{2}$ cm on each side.

Check The width is $12 - 2(\frac{3}{2}) = 9$ and the length is $16 - 2(\frac{3}{2}) = 13$. Two times the width is $2(9) = 18$, which is 5 more than the length, 13.

10. Let $t =$ the time it will take until John and Pat meet. Use $d = rt$ and make a table.

	Rate	Time	Distance
John	60	t	$60t$
Pat	28	t	$28t$

The total distance is 440 miles.

$$\begin{aligned} 60t + 28t &= 440 \\ 88t &= 440 \\ t &= 5 \end{aligned}$$

It will take 5 hours for John and Pat to meet.

Check John traveled $60(5) = 300$ miles and Pat traveled $28(5) = 140$ miles; $300 + 140 = 440$, as required.

11. Let $x =$ the number of liters of the 5% drug solution.

Liters of Solution	Percent (as a decimal)	Liters of Pure Drug
20	0.10	$20(0.10) = 2$
x	0.05	$0.05x$
$20 + x$	0.08	$0.08(20 + x)$

$$\begin{array}{ccccc} \text{Drug in 10\%} & + & \text{drug in 5\%} & = & \text{drug in 8\%.} \\ 2 & + & 0.05x & = & 0.08(20 + x) \end{array}$$

$$\begin{aligned} 200 + 5x &= 8(20 + x) && \textit{Multiply by 100.} \\ 200 + 5x &= 160 + 8x \\ 40 &= 3x \\ x &= \tfrac{40}{3}, \text{ or } 13\tfrac{1}{3} \end{aligned}$$

The pharmacist should add $13\frac{1}{3}$ L.

Check 10% of 20 is 2 and 5% of $\frac{40}{3}$ is $\frac{2}{3}$; $2 + \frac{2}{3} = \frac{8}{3}$, which is the same as 8% of $(20 + \frac{40}{3})$.

12. Let $x =$ the number of kilograms of the metal that is 20% tin.

Kilograms of Metal	Percent Tin (as a decimal)	Kilograms of Pure Tin
80	0.70	$80(0.70) = 56$
x	0.20	$0.20x$
$80 + x$	0.50	$0.50(80 + x)$

$$\begin{array}{ccccc} \text{Tin in 70\%} & + & \text{tin in 20\%} & = & \text{tin in 50\%.} \\ 56 & + & 0.20x & = & 0.50(80 + x) \end{array}$$

$$\begin{aligned} 560 + 2x &= 5(80 + x) && \textit{Multiply by 10.} \\ 560 + 2x &= 400 + 5x \\ 160 &= 3x \\ x &= \tfrac{160}{3}, \text{ or } 53\tfrac{1}{3} \end{aligned}$$

$53\frac{1}{3}$ kilograms should be added.

Check 70% of 80 is 56 and 20% of $\frac{160}{3}$ is $\frac{32}{3}$; $56 + \frac{32}{3} = 66\frac{2}{3}$, which is the same as 50% of $(80 + \frac{160}{3})$.

13. Let $x =$ the number of \$5 bills. Then $126 - x$ is the number of \$10 bills.

Number of Bills	Denomination	Value
x	5	$5x$
$126 - x$	10	$10(126 - x)$
126	← Totals →	840

The sum of the values must equal the total value.

$$\begin{aligned} 5x + 10(126 - x) &= 840 \\ 5x + 1260 - 10x &= 840 \\ -5x &= -420 \\ x &= 84 \end{aligned}$$

There are 84 \$5 bills and $126 - 84 = 42$ \$10 bills.

Check The value of the bills is \$5(84) + \$10(42) = \$840, as required.

14. Use the formula for the volume of a box.

$$\begin{aligned} V &= LWH \\ 75 &= 5(1.5)H \\ 75 &= 7.5H \\ 10 &= H \end{aligned}$$

The height is 10 ft.

15. Let $x =$ the least integer. Then $x + 1$ is the middle integer and $x + 2$ is the greatest integer.

"The sum of the least and greatest of three consecutive integers is 45 more than the middle integer" translates to

$$\begin{aligned} x + (x + 2) &= 45 + (x + 1). \\ 2x + 2 &= x + 46 \\ x &= 44 \end{aligned}$$

The three consecutive integers are 44, 45, and 46.

Check The sum of the least and greatest integers is $44 + 46 = 90$, which is the same as 45 more than the middle integer.

16. Let $x =$ the first odd integer. Then $x + 2$ is the next odd integer.

"If the lesser of two consecutive odd integers is doubled, the result is 7 more than the greater of the two integers" translates to

$$\begin{aligned} 2(x) &= 7 + (x + 2). \\ 2x &= x + 9 \\ x &= 9 \end{aligned}$$

The two consecutive odd integers are 9 and 11.

Check Doubling the lesser gives us $2(9) = 18$, which is equal to 7 more than 11.

17. The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned} x + (6x - 50) + (x - 10) &= 180 \\ 8x - 60 &= 180 \\ 8x &= 240 \\ x &= 30 \end{aligned}$$

With $x = 30$, the three angle measures become

$$(6 \cdot 30 - 50)^\circ = 130^\circ,$$
$$(30 - 10)^\circ = 20^\circ, \text{ and } 30^\circ.$$

18. In the figure, the two angles are supplementary, so their sum is 180°.

$$\begin{aligned} (10x + 7) + (7x + 3) &= 180 \\ 17x + 10 &= 180 \\ 17x &= 170 \\ x &= 10 \end{aligned}$$

The two angle measures are $10(10) + 7 = 107°$ and $7(10) + 3 = 73°$.

Chapter 2 Review Exercises

1.
$$\begin{aligned} -(8 + 3x) + 5 &= 2x + 6 \\ -8 - 3x + 5 &= 2x + 6 \\ -3x - 3 &= 2x + 6 \\ -5x &= 9 \\ x &= -\tfrac{9}{5} \end{aligned}$$

Solution set: $\{-\frac{9}{5}\}$

2.
$$\begin{aligned} -(r + 5) - (2 + 7r) + 8r &= 3r - 8 \\ -r - 5 - 2 - 7r + 8r &= 3r - 8 \\ -7 &= 3r - 8 \\ 1 &= 3r \\ \tfrac{1}{3} &= r \end{aligned}$$

Solution set: $\{\frac{1}{3}\}$

3.
$$\frac{m - 2}{4} + \frac{m + 2}{2} = 8$$

Multiply each side by the LCD, 4.

$$\begin{aligned} 4\left(\frac{m - 2}{4} + \frac{m + 2}{2}\right) &= 4(8) \\ (m - 2) + 2(m + 2) &= 32 \\ m - 2 + 2m + 4 &= 32 \\ 3m + 2 &= 32 \\ 3m &= 30 \\ m &= 10 \end{aligned}$$

Solution set: $\{10\}$

4.
$$\begin{aligned} \frac{2q + 1}{3} - \frac{q - 1}{4} &= 0 \\ 4(2q + 1) - 3(q - 1) &= 0 \quad \textit{Multiply by 12.} \\ 8q + 4 - 3q + 3 &= 0 \\ 5q + 7 &= 0 \\ 5q &= -7 \\ q &= -\tfrac{7}{5} \end{aligned}$$

Solution set: $\{-\frac{7}{5}\}$

5.
$$\begin{aligned} 5(2x - 3) &= 6(x - 1) + 4x \\ 10x - 15 &= 6x - 6 + 4x \\ 10x - 15 &= 10x - 6 \\ -15 &= -6 \quad \textit{False} \end{aligned}$$

This is a false statement, so the equation is a *contradiction.*

Solution set: $\emptyset$

6. $-3x + 2(4x + 5) = 10$
$$\begin{aligned} -3x + 8x + 10 &= 10 \\ 5x + 10 &= 10 \\ 5x &= 0 \\ x &= 0 \quad \textit{Divide by 5.} \end{aligned}$$

Solution set: $\{0\}$

7. $\frac{1}{2}x - \frac{3}{8}x = \frac{1}{4}x + 2$
$$\begin{aligned} 4x - 3x &= 2x + 16 \quad \textit{Multiply by 8.} \\ -x &= 16 \qquad \textit{4 − 3 − 2 = −1} \\ x &= -16 \end{aligned}$$

Solution set: $\{-16\}$

8. $0.05x + 0.03(1200 - x) = 42$
Multiply by 100 to clear all decimals.
$$\begin{aligned} 5x + 3(1200 - x) &= 4200 \\ 5x + 3600 - 3x &= 4200 \\ 2x + 3600 &= 4200 \\ 2x &= 600 \\ x &= 300 \end{aligned}$$

Solution set: $\{300\}$

9. Solve each equation.

A. $x - 7 = 7$
$x = 14$ *Add 7.*

Solution set: $\{14\}$

B. $9x = 10x$
$0 = x$ *Subtract 9x.*

Solution set: $\{0\}$

C. $x + 4 = -4$
$x = -8$ *Subtract 4.*

Solution set: $\{-8\}$

D. $8x - 8 = 8$
$8x = 16$ *Add 8.*
$x = 2$ *Divide by 8.*

Solution set: $\{2\}$

Equation **B** has $\{0\}$ as its solution set.

10. Solve $-2x + 5 = 7$.

Begin by subtracting 5 from each side. Then divide each side by -2.

11. $7r - 3(2r - 5) + 5 + 3r = 4r + 20$
$$\begin{aligned} 7r - 6r + 15 + 5 + 3r &= 4r + 20 \\ 4r + 20 &= 4r + 20 \\ 20 &= 20 \quad \textit{True} \end{aligned}$$

This equation is an *identity*.

Solution set: {all real numbers}

12. $8p - 4p - (p - 7) + 9p + 13 = 12p$
$$\begin{aligned} 8p - 4p - p + 7 + 9p + 13 &= 12p \\ 12p + 20 &= 12p \\ 20 &= 0 \quad \textit{False} \end{aligned}$$

This equation is a *contradiction*.

Solution set: $\emptyset$

13. $-2r + 6(r - 1) + 3r - (4 - r) = -(r + 5) - 5$
$$\begin{aligned} -2r + 6r - 6 + 3r - 4 + r &= -r - 5 - 5 \\ 8r - 10 &= -r - 10 \\ 9r &= 0 \\ r &= 0 \end{aligned}$$

This equation is a *conditional* equation.

Solution set: $\{0\}$

14. Solve $V = LWH$ for L.
$$\frac{V}{WH} = \frac{LWH}{WH} \qquad \textit{Divide by WH.}$$
$$\frac{V}{WH} = L, \text{ or } L = \frac{V}{WH}$$

15. Solve $A = \frac{1}{2}h(b + B)$ for b.
$$\begin{aligned} 2A &= h(b + B) \quad \textit{Multiply by 2.} \\ \frac{2A}{h} &= b + B \qquad \textit{Divide by h.} \\ \frac{2A}{h} - B &= b \qquad \textit{Subtract B.} \end{aligned}$$

OR Solve $A = \frac{1}{2}h(b + B)$ for b.
$$\begin{aligned} 2A &= hb + hB \quad \textit{Multiply by 2.} \\ 2A - hB &= hb \qquad \textit{Subtract hB.} \\ \frac{2A - hB}{h} &= b \qquad \textit{Divide by h.} \end{aligned}$$

16. Solve $4x + 7y = 9$ for y.
$$\begin{aligned} 7y &= 9 - 4x \quad \textit{Subtract 4x.} \\ y &= \frac{9 - 4x}{7} \quad \textit{Divide by 7.} \end{aligned}$$

17. Use the formula $V = LWH$ and substitute 180 for V, 9 for L, and 4 for W.
$$\begin{aligned} 180 &= 9(4)H \\ 180 &= 36H \\ 5 &= H \end{aligned}$$

The height is 5 feet.

18.
$$\begin{aligned} \text{percent increase} &= \frac{\text{amount of increase}}{\text{base}} \\ &= \frac{17.5\text{ M} - 15.3\text{ M}}{15.3\text{ M}} \\ &= \frac{2.2\text{ M}}{15.3\text{ M}} \approx 0.144 \end{aligned}$$

The percent increase was 14.4%.

19. Use the formula $I = prt$, and solve for r.

$$\frac{I}{pt} = \frac{prt}{pt}$$
$$\frac{I}{pt} = r$$

Substitute 30,000 for p, 6600 for I, and 4 for t.

$$r = \frac{6600}{30{,}000(4)} = \frac{6600}{120{,}000} = 0.055$$

The rate is 5.5%.

20. Use the formula $C = \frac{5}{9}(F - 32)$ and substitute 77 for F.

$$\begin{aligned} C &= \tfrac{5}{9}(77 - 32) \\ &= \tfrac{5}{9}(45) = 25 \end{aligned}$$

The Celsius temperature is 25°.

21. **(a)** The amount of money spent on Social Security in 2005 was about

$$0.21(\$2500 \text{ billion}) = \$525 \text{ billion}.$$

(b) The amount of money spent on education and social services in 2005 was about

$$0.039(\$2500 \text{ billion}) = \$97.5 \text{ billion}$$

22.

$$\begin{aligned} C &= 2\pi r \\ 200\pi &= 2\pi r && \textit{Substitute } 200\pi \textit{ for C.} \\ \frac{200\pi}{2\pi} &= \frac{2\pi r}{2\pi} && \textit{Divide by } 2\pi. \\ 100 &= r \end{aligned}$$

The radius is 100 mm.

23. "One-fifth of a number, subtracted from 14" is written

$$14 - \tfrac{1}{5}x.$$

24. "The product of 6 and a number, divided by 3 more than the number" is written

$$\frac{6x}{x+3}.$$

25. Let $x =$ the width of the rectangle.
Then $2x - 3 =$ the length of the rectangle.

Use the formula $P = 2L + 2W$ with $P = 42$.

$$\begin{aligned} 42 &= 2(2x - 3) + 2x \\ 42 &= 4x - 6 + 2x \\ 48 &= 6x \\ 8 &= x \end{aligned}$$

The width is 8 meters and the length is $2(8) - 3 = 13$ meters.

26. Let $x =$ the length of each equal side. Then $2x - 15 =$ the length of the third side.

Use the formula $P = a + b + c$ with $P = 53$.

$$\begin{aligned} 53 &= x + x + (2x - 15) \\ 53 &= 4x - 15 \\ 68 &= 4x \\ 17 &= x \end{aligned}$$

The lengths of the three sides are 17 inches, 17 inches, and $2(17) - 15 = 19$ inches.

27. Let $x =$ the number of kilograms of peanut clusters. Then $3x$ is the number of kilograms of chocolate creams. The clerk has a total of 48 kg.

$$\begin{aligned} x + 3x &= 48 \\ 4x &= 48 \\ x &= 12 \end{aligned}$$

The clerk has 12 kilograms of peanut clusters.

28. Let $x =$ the number of liters of the 20% solution. Make a table.

Liters of Solution	Percent (as a decimal)	Liters of Pure Chemical
x	0.20	$0.20x$
15	0.50	$0.50(15) = 7.5$
$x + 15$	0.30	$0.30(x + 15)$

The last column gives the equation.

$$\begin{aligned} 0.20x + 7.5 &= 0.30(x + 15) \\ 0.20x + 7.5 &= 0.30x + 4.5 \\ 3 &= 0.10x \\ 30 &= x \end{aligned}$$

30 L of the 20% solution should be used.

29. Let $x =$ the number of liters of water.

Liters of Solution	Percent (as a decimal)	Liters of Pure Acid
30	0.40	$0.40(30) = 12$
x	0	$0(x) = 0$
$30 + x$	0.30	$0.30(30 + x)$

The last column gives the equation.

$$\begin{aligned} 12 + 0 &= 0.30(30 + x) \\ 12 &= 9 + 0.3x \\ 3 &= 0.3x \\ 10 &= x \end{aligned}$$

10 L of water should be added.

30. Let $x =$ the amount invested at 6%. Then $x - 4000 =$ the amount invested at 4%.

Principal	Rate (as a decimal)	Interest
x	0.06	$0.06x$
$x - 4000$	0.04	$0.04(x - 4000)$
	Total →	\$840

The last column gives the equation.

$$\begin{aligned} 0.06x + 0.04(x - 4000) &= 840 \\ 6x + 4(x - 4000) &= 84{,}000 \quad \textit{Multiply by 100.} \\ 6x + 4x - 16{,}000 &= 84{,}000 \\ 10x &= 100{,}000 \\ x &= 10{,}000 \end{aligned}$$

Anna should invest \$10,000 at 6% and \$10,000 − \$4000 = \$6000 at 4%.

31. Use the formula $d = rt$ or $r = \frac{d}{t}$.
Here, d is about 400 mi and t is about 8 hr.
Since $\frac{400}{8} = 50$, the best estimate is choice **A**.

32. Use the formula $d = rt$.

(a) Here, $r = 53$ mph and $t = 10$ hr.

$$d = 53(10) = 530$$

The distance is 530 miles.

(b) Here, $r = 164$ mph and $t = 2$ hr.

$$d = 164(2) = 328$$

The distance is 328 miles.

33. Let $x =$ the time it takes for the trains to be 297 mi apart.

Make a table. Use the formula $d = rt$.

	Rate	Time	Distance
Passenger Train	60	x	$60x$
Freight Train	75	x	$75x$
Total →			297

The total distance traveled is the sum of the distances traveled by each train.

$$\begin{aligned} 60x + 75x &= 297 \\ 135x &= 297 \\ x &= 2.2 \end{aligned}$$

It will take the trains 2.2 hours before they are 297 miles apart.

Check $2.2(60) + 2.2(75) = 297$

34. Let $x =$ the speed of the faster car and $x - 15 =$ the speed of the slower car.
Make a table. Use the formula $d = rt$.

	Rate	Time	Distance
Faster Car	x	2	$2x$
Slower Car	$x - 15$	2	$2(x - 15)$
Total →			230

The total distance traveled is the sum of the distances traveled by each car.

$$\begin{aligned} 2x + 2(x - 15) &= 230 \\ 2x + 2x - 30 &= 230 \\ 4x &= 260 \\ x &= 65 \end{aligned}$$

The faster car travels at 65 km per hr, while the slower car travels at $65 - 15 = 50$ km per hr.

Check $2(65) + 2(50) = 230$

35. Let $x =$ amount of time spent averaging 45 miles per hour. Then $4 - x =$ amount of time at 50 mph.

	Rate	Time	Distance
First Part	45	x	$45x$
Second Part	50	$4 - x$	$50(4 - x)$
Total →			195

From the last column:

$$\begin{aligned} 45x + 50(4 - x) &= 195 \\ 45x + 200 - 50x &= 195 \\ -5x &= -5 \\ x &= 1 \end{aligned}$$

The automobile averaged 45 mph for 1 hour.

Check 45 mph for 1 hour = 45 miles and 50 mph for 3 hours = 150 miles; $45 + 150 = 195$.

36. Let $x =$ the average speed for the first hour. Then $x - 7 =$ the average speed for the second hour. Using $d = rt$, the distance traveled for the first hour is $x(1)$ miles, for the second hour is $(x - 7)(1)$ miles, and for the whole trip, 85 miles.

$$\begin{aligned} x + (x - 7) &= 85 \\ 2x - 7 &= 85 \\ 2x &= 92 \\ x &= 46 \end{aligned}$$

The average speed for the first hour was 46 mph.

Check 46 mph for 1 hour = 46 miles and $46 - 7 = 39$ mph for 1 hour = 39 miles; $46 + 39 = 85$.

37. **[2.1]**

$$\begin{aligned} (7 - 2k) + 3(5 - 3k) &= k + 8 \\ 7 - 2k + 15 - 9k &= k + 8 \\ -11k + 22 &= k + 8 \\ -12k + 22 &= 8 \\ -12k &= -14 \\ k &= \tfrac{-14}{-12} = \tfrac{7}{6} \end{aligned}$$

Solution set: $\{\frac{7}{6}\}$

38. **[2.1]**
$$\frac{4x+2}{4}+\frac{3x-1}{8}=\frac{x+6}{16}$$
Clear fractions by multiplying by the LCD, 16.
$$4(4x+2)+2(3x-1)=x+6$$
$$16x+8+6x-2=x+6$$
$$22x+6=x+6$$
$$21x=0$$
$$x=0$$
Solution set: $\{0\}$

39. **[2.1]**
$$-5(6p+4)-2p=-32p+14$$
$$-30p-20-2p=-32p+14$$
$$-32p-20=-32p+14$$
$$-20=14 \quad \textit{False}$$
The equation is a *contradiction.*

Solution set: $\emptyset$

40. **[2.1]**
$$0.08x+0.04(x+200)=188$$
$$8x+4(x+200)=18{,}800 \quad \textit{Multiply by 100.}$$
$$8x+4x+800=18{,}800$$
$$12x+800=18{,}800$$
$$12x=18{,}000$$
$$x=1500$$
Solution set: $\{1500\}$

41. **[2.1]**
$$5(2r-3)+7(2-r)=3(r+2)-7$$
$$10r-15+14-7r=3r+6-7$$
$$3r-1=3r-1$$
$$3r=3r$$
$$0=0 \quad \textit{True}$$
Solution set: {all real numbers}

42. **[2.2]** $Ax+By=C$ for x
$$Ax=C-By \quad \textit{Subtract By.}$$
$$x=\frac{C-By}{A} \quad \textit{Divide by A.}$$

43. **[2.3]** Let $x=$ the length of each side of the original square;
$x+4=$ the length of each side of the enlarged square.

The original perimeter is $4x$. The perimeter of the enlarged square is $4(x+4)$. The perimeter of the enlarged square is 8 in. less than twice the perimeter of the original square.
$$4(x+4)=2(4x)-8$$
$$4x+16=8x-8$$
$$16=4x-8$$
$$24=4x$$
$$6=x$$
The length of a side of the original square is 6 in.

44. **[2.4]** Let $x=$ the time traveled by eastbound car.
Then $x-1=$ the time traveled by westbound car.

	Rate	Time	Distance
Eastbound Car	40	x	$40x$
Westbound Car	60	$x-1$	$60(x-1)$

Their total distance is 240 mi.
$$40x+60(x-1)=240$$
$$40x+60x-60=240$$
$$100x-60=240$$
$$100x=300$$
$$x=3$$
The eastbound car traveled for 3 hr and the westbound car traveled for $3-1=$ 2 hr.

45. **[2.3]** *Step 2*
Let $x=$ the number of visits to the Golden Gate National Recreation Area (in millions).
Then $x+5.46=$ the number of visits to the Blue Ridge Parkway (in millions).

Step 3
The total number of visits was 32.44 million, so
$$x+(x+5.46)=32.44$$
Step 4
$$2x+5.46=32.44$$
$$2x=26.98$$
$$x=13.49$$
Step 5
In 2006, there were 13.49 million visits to the Golden Gate National Recreation Area and $13.49+5.46=18.95$ million visits to the Blue Ridge Parkway.

Step 6
18.95 million is 5.46 million more than 13.49 million and the sum of 13.49 million and 18.95 million is 32.44 million.

46. **[2.3]** Let $x=$ the amount invested at 3%.
Then $x+600=$ the amount invested at 5%.

Principal	Rate (as a Decimal)	Interest
x	0.03	$0.03x$
$x+600$	0.05	$0.05(x+600)$

The total interest is $126.
$$0.03x+0.05(x+600)=126$$
$$0.03x+0.05x+30=126$$
$$0.08x+30=126$$
$$0.08x=96$$
$$x=1200$$
$1200 was invested at 3% and
1200 + 600 = $1800 was invested at 5%.

Check 5% of $1800 is $90 and 3% of $1200 is $36. The sum is $126, as required.

Chapter 2 Test

1. $3(2x-2)-4(x+6)=4x+8$
$6x-6-4x-24=4x+8$
$2x-30=4x+8$
$-2x-30=8$
$-2x=38$
$x=-19$

Check $x=-19$: $-120+52=-68$ *True*

Solution set: $\{-19\}$

2. $0.08x+0.06(x+9)=1.24$
$8x+6(x+9)=124$
Multiply each side by 100 to eliminate the decimals.
$8x+6x+54=124$
$14x+54=124$
$14x=70$
$x=5$

Check $x=5$: $0.40+0.84=1.24$ *True*

Solution set: $\{5\}$

3. $\dfrac{x+6}{10}+\dfrac{x-4}{15}=1$
Multiply each side by the LCD, 30.
$3(x+6)+2(x-4)=30$
$3x+18+2x-8=30$
$5x+10=30$
$5x=20$
$x=4$

Check $x=4$: $1+0=1$ *True*

Solution set: $\{4\}$

4. **(a)** $3x-(2-x)+4x+2=8x+3$
$3x-2+x+4x+2=8x+3$
$8x=8x+3$
$0=3$ *False*

The false statement indicates that the equation is a *contradiction*.

Solution set: $\emptyset$

(b) $\dfrac{x}{3}+7=\dfrac{5x}{6}-2-\dfrac{x}{2}+9$
Multiply each side by the LCD, 6.
$2x+42=5x-12-3x+54$
$2x+42=2x+42$
$0=0$ *True*

The true statement indicates that the equation is an *identity*.

Solution set: $\{\text{all real numbers}\}$

(c) $-4(2x-6)=5x+24-7x$
$-8x+24=-2x+24$
$24=6x+24$
$0=6x$
$0=x$

This is a *conditional equation.*

Check $x=0$: $24=0+24-0$ *True*

Solution set: $\{0\}$

5. Solve $S=-16t^2+vt$ for v.
$S+16t^2=vt$ *Add 16t^2.*
$\dfrac{S+16t^2}{t}=v,$ *Divide by t.*
or $v=\dfrac{S+16t^2}{t}$

6. Solve $-3x+2y=6$ for y.
$2y=6+3x$ *Add 3x.*
$y=\dfrac{6+3x}{2}$ *Divide by 2.*

7. Solve $d=rt$ for t and substitute 500 for d and 149.335 for r.

$$t=\frac{d}{r}=\frac{500}{149.335}\approx 3.348$$

Harvik's time was about 3.348 hr.

8. Use $I=Prt$ and substitute \$1733.75 for I, \$36,500 for P, and 1 for t.

$$1733.75=36{,}500r(1)$$
$$r=\frac{1733.75}{36{,}500}=0.0475$$

The rate of interest is 4.75%.

9. $\dfrac{27{,}318}{36{,}826}\approx 0.742$

About 74.2% were classified as post offices.

10. Let $x=$ the amount invested at 3%.
Then $32{,}000-x=$ the amount invested at 5%.

Principal	Rate (as a Decimal)	Interest
x	0.03	$0.03x$
$32{,}000-x$	0.05	$0.05(32{,}000-x)$
\$32,000	← Totals →	\$1320

We can write an equation from the last column.

$$\begin{aligned} 0.03x + 0.05(32{,}000 - x) &= 1320 \\ 3x + 5(32{,}000 - x) &= 132{,}000 \quad \textit{Multiply each side by 100.} \\ 3x + 160{,}000 - 5x &= 132{,}000 \\ -2x &= -28{,}000 \\ x &= 14{,}000 \end{aligned}$$

He invested \$14,000 at 3% and \$32,000 − \$14,000 = \$18,000 at 5%.

11. Let $x =$ the speed of the faster car.
Then $x - 15 =$ the speed of the slower car.

Make a table. Use the formula $d = rt$.

	Rate	Time	Distance
Slower Car	$x - 15$	6	$6(x - 15)$
Faster Car	x	6	$6x$
Total →			630

The total distance traveled is the sum of the distances traveled by each car.

$$\begin{aligned} 6(x - 15) + 6x &= 630 \\ 6x - 90 + 6x &= 630 \\ 12x &= 720 \\ x &= 60 \end{aligned}$$

The faster car traveled at 60 mph, while the slower car traveled at $60 - 15 = 45$ mph.

12. The sum of the three angle measures is 180°.

$$\begin{aligned} (2x + 20) + x + x &= 180 \\ 4x + 20 &= 180 \\ 4x &= 160 \\ x &= 40 \end{aligned}$$

The three angle measures are 40°, 40°, and $(2 \cdot 40 + 20)° = 100°$.

13. $$\begin{aligned} A &= \frac{24f}{b(p+1)} \\ A &= \frac{24(200)}{1920(24+1)} \quad \textit{Let } f = 200,\ b = 1920, \textit{ and } p = 24. \\ &= \frac{4800}{48{,}000} \\ &= 0.1 \end{aligned}$$

The approximate annual interest rate is 10%.

14. $$\begin{aligned} A &= \frac{24f}{b(p+1)} \\ A &= \frac{24(740)}{3600(36+1)} \quad \textit{Let } f = 740,\ b = 3600, \textit{ and } p = 36. \\ &= \frac{17{,}760}{133{,}200} \\ &\approx 0.1333 \end{aligned}$$

The approximate annual interest rate is 13.33%.

15. 21% of $5000 = 0.21(5000) = 1050$

We would expect 1050 white-collar workers in a group of 5000 stockholders.

Cumulative Review Exercises (Chapters 1–2)

Exercises 1–6 refer to set A.
Let $A = \{-8, -\frac{2}{3}, -\sqrt{6}, 0, \frac{4}{5}, 9, \sqrt{36}\}$.

Note that $\sqrt{36} = 6$.

1. The elements 9 and 6 are natural numbers.

2. The elements 0, 9, and 6 are whole numbers.

3. The elements -8, 0, 9, and 6 are integers.

4. The elements $-8, -\frac{2}{3}, 0, \frac{4}{5}, 9,$ and 6 are rational numbers.

5. The element $-\sqrt{6}$ is an irrational number.

6. All the elements in set A are real numbers.

7. $$\begin{aligned} -\frac{4}{3} - \left(-\frac{2}{7}\right) &= -\frac{4}{3} + \frac{2}{7} \\ &= -\frac{28}{21} + \frac{6}{21} \\ &= -\frac{22}{21} \end{aligned}$$

8. $$\begin{aligned} |-4.2| + |5.6| - |-1.9| &= 4.2 + 5.6 - 1.9 \\ &= 9.8 - 1.9 \\ &= 7.9 \end{aligned}$$

9. $(-2)^4 + (-2)^3 = 16 + (-8) = 8$

10. $$\begin{aligned} \sqrt{25} - \frac{\sqrt{100}}{2} &= 5 - \frac{10}{2} \\ &= 5 - 5 \\ &= 0 \end{aligned}$$

11. $(-3)^5 = (-3)(-3)(-3)(-3)(-3) = -243$

12. $\left(\frac{6}{7}\right)^3 = \frac{6}{7} \cdot \frac{6}{7} \cdot \frac{6}{7} = \frac{216}{343}$

13. $4^6 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4096$

14. $-4^6 = -(4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4) = -4096$

15. $-\sqrt{49} = -(6) = -6$, which *is* a real number.

$\sqrt{-49}$ *is not* a real number.

16. $\frac{4-4}{4+4} = \frac{0}{8} = 0$

$\frac{4+4}{4-4} = \frac{8}{0}$, which is *undefined*.

For Exercises 17–20, let $a = 2$, $b = -3$, and $c = 4$.

17. $$\begin{aligned} -3a + 2b - c &= -3(2) + 2(-3) - 4 \\ &= -6 - 6 - 4 \\ &= -16 \end{aligned}$$

18. $$\begin{aligned} -2b^2 - c^2 &= -2(-3)^2 - 4^2 \\ &= -2(9) - 16 \\ &= -18 - 16 \\ &= -34 \end{aligned}$$

19. $$\begin{aligned} -8(a^2 + b^3) &= -8[2^2 + (-3)^3] \\ &= -8[4 + (-27)] \\ &= -8(-23) \\ &= 184 \end{aligned}$$

20. $$\begin{aligned} \frac{3a^3 - b}{4 + 3c} &= \frac{3(2)^3 - (-3)}{4 + 3(4)} \\ &= \frac{3(8) - (-3)}{4 + 3(4)} \\ &= \frac{24 + 3}{4 + 12} \\ &= \frac{27}{16} \end{aligned}$$

21. $$\begin{aligned} &-7r + 5 - 13r + 12 \\ &= -7r - 13r + 5 + 12 \\ &= (-7 - 13)r + (5 + 12) \\ &= -20r + 17 \end{aligned}$$

22. $$\begin{aligned} &-(3k + 8) - 2(4k - 7) + 3(8k + 12) \\ &= -3k - 8 - 8k + 14 + 24k + 36 \\ &= -3k - 8k + 24k - 8 + 14 + 36 \\ &= 13k + 42 \end{aligned}$$

23. $(a + b) + 8 = 8 + (a + b)$

The order of the terms $(a + b)$ and 8 have been reversed. This is an illustration of the *commutative property*.

24. $5x + 13x = (5 + 13)x$

The common variable, x, has been removed from each term. This is an illustration of the *distributive property*.

25. $-13 + 13 = 0$

The sum of a number and its opposite is equal to 0. This is an illustration of the *inverse property*.

26. $$\begin{aligned} -4x + 7(2x + 3) &= 7x + 36 \\ -4x + 14x + 21 &= 7x + 36 \\ 10x + 21 &= 7x + 36 \\ 3x &= 15 \\ x &= 5 \end{aligned}$$

Solution set: $\{5\}$

27. $$\begin{aligned} -\frac{3}{5}x + \frac{2}{3}x &= 2 \\ 3(-3x) + 5(2x) &= 15(2) \quad \textit{Multiply by 15.} \\ -9x + 10x &= 30 \\ x &= 30 \end{aligned}$$

Solution set: $\{30\}$

28. $$\begin{aligned} 0.06x + 0.03(100 + x) &= 4.35 \\ 6x + 3(100 + x) &= 435 \quad \textit{Multiply by 100.} \\ 6x + 300 + 3x &= 435 \\ 9x &= 135 \\ x &= 15 \end{aligned}$$

Solution set: $\{15\}$

29. Solve $P = a + b + c$ for c.

$$P - (a + b) = a + b + c - (a + b)$$

Subtract (a + b).

$$P - a - b = c$$

30. $$\begin{aligned} 4(2x - 6) + 3(x - 2) &= 11x + 1 \\ 8x - 24 + 3x - 6 &= 11x + 1 \\ 11x - 30 &= 11x + 1 \\ -30 &= 1 \quad \textit{False} \end{aligned}$$

Solution set: $\emptyset$

31. $$\begin{aligned} \frac{2}{3}x + \frac{5}{8}x &= \frac{31}{24}x \\ 8(2x) + 3(5x) &= 31x \quad \textit{Multiply by the LCD, 24.} \\ 16x + 15x &= 31x \\ 31x &= 31x \quad \textit{True} \end{aligned}$$

Solution set: $\{$all real numbers$\}$

32. Let $x =$ the amount of pure alcohol that should be added.

Liters of Solution	Percent (as a Decimal)	Liters of Pure Alcohol
x	1.00	$1.00x$
7	0.10	$0.10(7)$
$x + 7$	0.30	$0.30(x + 7)$

The last column gives the equation.

$$\begin{aligned} 1.00x + 0.10(7) &= 0.30(x + 7) \\ 10x + 1(7) &= 3(x + 7) \quad \textit{Multiply by 10.} \\ 10x + 7 &= 3x + 21 \\ 7x &= 14 \\ x &= 2 \end{aligned}$$

2 L of pure alcohol should be added to the solution.

33. Let $x =$ the number of nickels. Then $x - 4 =$ the number of quarters. The collection contains 29 coins, so the number of pennies is

$$29 - x - (x - 4) = 33 - 2x.$$

	Number of Coins	Denomination	Value
Pennies	$33 - 2x$	0.01	$0.01(33 - 2x)$
Nickels	x	0.05	$0.05x$
Quarters	$x - 4$	0.25	$0.25(x - 4)$
	29		\$2.69

From the last column:

$$\begin{aligned} 0.01(33 - 2x) + 0.05x + 0.25(x - 4) &= 2.69 \\ 1(33 - 2x) + 5x + 25(x - 4) &= 269 \\ &\textit{Multiply by 100.} \\ 33 - 2x + 5x + 25x - 100 &= 269 \\ 28x - 67 &= 269 \\ 28x &= 336 \\ x &= 12 \end{aligned}$$

There are $33 - 2(12) = 9$ pennies, 12 nickels, and $12 - 4 = 8$ quarters.

34. Let $x =$ the amount invested at 5%. Then $x + 2000 =$ the amount invested at 6%.

Principal	Rate (as a Decimal)	Interest
x	0.05	$0.05x$
$x + 2000$	0.06	$0.06(x + 2000)$

The total interest is \$670.

$$\begin{aligned} 0.05x + 0.06(x + 2000) &= 670 \\ 0.05x + 0.06x + 120 &= 670 \\ 0.11x + 120 &= 670 \\ 0.11x &= 550 \\ x &= \frac{550}{0.11} = 5000 \end{aligned}$$

\$5000 was invested at 5% and
$5000 + 2000 =$ \$7000 was invested at 6%.

35. Let $x =$ the time for Jack to be $\frac{1}{4}$ mile ahead of Jill.

	Rate	Time	Distance
Jack	7	x	$7x$
Jill	5	x	$5x$

Jack's distance is $\frac{1}{4}$ mile more than Jill's distance.

$$\begin{aligned} 7x &= 5x + \tfrac{1}{4} \\ 2x &= \tfrac{1}{4} \\ x &= \tfrac{1}{8} \end{aligned}$$

Jack will be $\frac{1}{4}$ mile ahead of Jill in $\frac{1}{8}$ hr.

36. Clark's rule:

$$\frac{\text{Weight of child in pounds}}{150} \times \begin{matrix}\text{adult}\\ \text{dose}\end{matrix} = \begin{matrix}\text{child's}\\ \text{dose}\end{matrix}$$

If the child weighs 55 lb and the adult dosage is 120 mg, then

$$\frac{55}{150} \times 120 = 44.$$

The child's dosage is 44 mg.

37. 5 feet, 8 inches $= 5(12) + 8 = 68$ inches

$$\begin{aligned} \text{BMI} &= \frac{704 \times (\text{weight in pounds})}{(\text{height in inches})^2} \\ &= \frac{704 \times 160}{68^2} = \frac{112{,}640}{4624} \approx 24.4 \end{aligned}$$

His BMI is about 24.4.

38. **(a)** 1975: 1756
2005: 1452

$1756 - 1452 = 304$

The number decreased by 304 newspapers.

(b) $\frac{304}{1756} \approx 0.173$ or 17.3%.

The number decreased by approximately 17.3%.