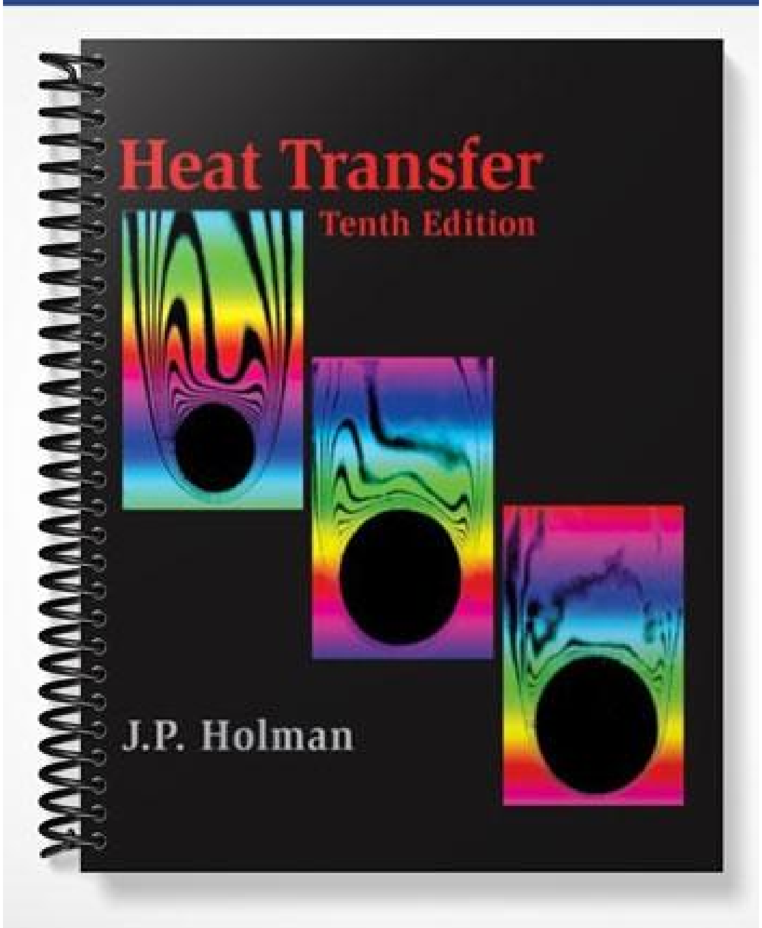
SOLUTIONS MANUAL



Chapter 2

2–1

$$q = \frac{T_i - T_0}{\left.\frac{\Delta x}{kA}\right)_w + \left.\frac{\Delta x}{kA}\right)_{\text{ins}}} \qquad 1830 = \frac{1300 - 30}{\frac{0.02}{1.3} + \frac{\Delta x}{0.35}}$$

$\Delta x = 0.238$ m

2–2

Assume Linear variation: $k = k_0(1 + \beta T)$

$$q = -\frac{k_0 A}{\Delta x}\left[T_3 - T_1 + \frac{\beta}{2}(T_3^2 - T_1^2)\right] = -\frac{k_0 A}{\Delta x/2}\left[T_2 - T_1 + \frac{\beta}{2}(T_2^2 - T_1^2)\right]$$

$T_3 = 95°\text{C},\ T_2 = 62°\text{C},\ T_1 = 35°\text{C},\ \Delta x = 0.025$

$$2\left[62 - 35 + \frac{\beta}{2}(62^2 - 35^2)\right] = \left[95 - 35 + \frac{\beta}{2}(95^2 - 35^2)\right]$$

$\beta = -4.68 \times 10^{-3}$

$$1000 = q = \frac{k_0(0.1)}{0.025}\left[95 - 35 - \frac{4.68 \times 10^{-3}}{2}(95^2 - 35^2)\right]$$

$k_0 = 5.988$

$$k = 5.988(1 - 4.68 \times 10^{-3}\ \text{T})\ \frac{\text{W}}{\text{m} \cdot °\text{C}}$$

2–3

$$\frac{q}{A} = \frac{560}{\frac{0.025}{386} + \frac{3.2\times10^{-3}}{0.16} + \frac{0.05}{0.038}} = 419\ \frac{\text{W}}{\text{m}^2}$$

2–4

$$R = \frac{\Delta x}{kA}$$

$$R_A = \frac{0.025}{(150)(0.1)} = 1.667 \times 10^{-3}$$

$$R_B = \frac{0.075}{(30)(0.05)} = 0.05$$

$$R_C = \frac{0.05}{(50)(0.1)} = 0.01$$

$$R_D = \frac{0.075}{(70)(0.05)} = 0.02143$$

$$R = R_A + R_C + \frac{1}{\frac{1}{R_B} + \frac{1}{R_D}} = 2.667 \times 10^{-2}$$

$$q = \frac{\Delta T}{R} = \frac{370 - 66}{2.667 \times 10^{-2}} = 11{,}400 \text{ W}$$

2–5

$$\frac{44{,}000}{A} = \frac{250 - 35}{\frac{0.05}{386} + \frac{0.025}{0.038}} \qquad A = 134.7 \text{ m}^2$$

2–6

$$\frac{q}{A} = \frac{75}{\frac{0.10}{0.69} + \frac{0.025}{0.05}} = 38.76 \text{ W/m}^2$$

2–7

$$\frac{q}{A} = \frac{\Delta T}{R}$$

$$\frac{300}{A} = \frac{175 - 80}{\frac{0.04}{386} + \frac{0.015}{0.038}}$$

$$A = 1.247 \text{ m}^2$$

2–8

Assume one directional no heat sources

$$q = -kA\frac{dT}{dx} = -k_0 A[1 + \beta T^2]\frac{dT}{dx} = -k_0 A\frac{dT}{dx} - k_0 \beta A T^2 \frac{dT}{dx}$$

Integrating: $q\Delta x = -k_0 A\int_{T_1}^{T_2} dT - Ak_0\beta\int_{T_1}^{T_2} T^2 dT$

$$q = -\frac{k_0 A}{\Delta x}\left[(T_2 - T_1) + \frac{\beta}{3}(T_2^3 - T_1^3)\right]$$

2–9

$$\frac{1}{h_i A_i} = \frac{1}{(1500)\pi(0.03)} = 0.00709 \qquad r_0 = 0.015 + 0.002 = 0.017$$

$$\frac{\ln(r_0/r_i)}{2\pi k} = \frac{\ln(0.017/0.015)}{2\pi(46)} 0.000433$$

$$\frac{1}{h_0 A_0} = \frac{1}{(197)\pi(0.034)} = 0.0475$$

$$\sum R = 0.05502 \ \frac{°\text{C} \cdot \text{m}}{\text{W}}$$

$$\frac{q}{L} = \frac{223 - 57}{0.05502} = 3017 \ \text{W/m}$$

2–10

$$\frac{\partial T}{\partial x} = 300x - 30$$

$$\frac{\partial^2 T}{\partial x^2} = 300 = \frac{1}{\alpha}\frac{\partial T}{\partial \tau} \quad \text{heating up}$$

$$\frac{\partial T}{\partial x} = -30 \quad \text{at} \quad x = 0$$

$$\frac{\partial T}{\partial x} = 60 \quad \text{at} \quad x = 0.3$$

$$\frac{q}{A} = -(0.04)(-30) = +1.2 \ \frac{\text{W}}{\text{m}^2} \ \text{at} \ x = 0$$

$$\frac{q}{A} = -(0.04)(60) = -2.4 \ \frac{\text{W}}{\text{m}^2} \ \text{at} \ x = 0.3$$

2-11

$d = 0.000025$; $k = 16$; $L = 0.8\text{m}$; $h = 500$; $T_\infty = 20°\text{C}$; $T_0 = T_L = 200°\text{C}$

$m = [(500)(4)/(16)(0.000025)]^{1/2} = 2236$

$\theta'_0 = \theta'_L = 200 - 20 = 180$

$q = -2kA(\partial\theta'/\partial x)_0 = -2kA(-m\theta'_0)$

$= (2)(16)\pi(0.000025)^2(2236)(180)/4 = 0.0063 \text{ W}$

2-12

$$R_{Cu} = \frac{0.02}{374} = 5.35 \times 10^{-5}$$
$$R_{As} = \frac{0.003}{0.166} = 0.0181$$
$$R_{Fi} = \frac{0.06}{0.038} = 1.579$$
$$\frac{q}{A} = \frac{500}{\sum R} = 313 \ \frac{\text{W}}{\text{m}^2}$$

2-13

$$R_c = \frac{\frac{6}{12}}{(1.2)(0.5778)} = 0.721$$
$$R_f = \frac{\frac{2}{12}}{(0.038)(0.5778)} = 7.59$$
$$R_g = \frac{\frac{0.375}{12}}{(0.05)(0.5778)} = 1.082$$
$$R_i = \frac{1}{2.0} = 0.5 \qquad R_0 = \frac{1}{7} = 0.143$$
$$\frac{q}{A} = \frac{72 - 20}{\sum R} = 5.18 \ \frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2}$$
$$U = \frac{1}{\sum R} = 0.0996$$

2-14

$$R_{ss} = \frac{\Delta x}{k} = \frac{0.004}{16} = 0.0025$$
$$R_{\text{overall}} = \frac{1}{U} = \frac{1}{120} = 0.00833$$
$$\frac{\Delta T_{ss}}{\Delta T_{\text{overall}}} = \frac{R_{ss}}{R_{\text{overall}}} = \frac{0.0025}{0.00833} = 0.3$$
$$\Delta T_{ss} = (0.3)(60) = 18°\text{C}$$

2-15

Ice at 0°C $\quad \rho = 999.8\ \text{kg/m}^3$

$V = (0.25)(0.4)(1.0) = 0.1\ \text{m}^3$

$m = 100\ \text{kg}$

$q = (100)(330 \times 10^3) = 3.3 \times 10^7\ \text{J}$

$A_i = (2)(0.25)(0.4) + (2)(0.4)(1.0) + (2)(0.25)(1.0) = 1.5\ \text{m}^2$

$A_0 = (2)(0.35)(0.5) + (2)(0.5)(1.1) + (2)(0.35)(1.1) = 2.22\ \text{m}^2$

$A_m = 1.86\ \text{m}^2$

$$R_s = \frac{\Delta x}{kA} = \frac{0.05}{(0.033)(1.86)} = 0.8146$$

$$R_0 = \frac{1}{hA_0} = 0.045$$

$R = 0.8596$

$$\frac{Q}{\Delta T} = \frac{3.3 \times 10^7}{\Delta T} = \frac{25 - 0}{0.8596}$$

$\Delta\tau = 1.135 \times 10^6\ \text{sec}$

$= 315\ \text{hr}$

$= 13\ \text{days}$

2-16

$q\ (\text{no ins.}) = hA(T_w - T_\infty) = (25)(4\pi)(0.5)^2(120 - 15) = 8247\ \text{W}$

$$k_{\text{foam}} = \frac{18\ \text{mW}}{\text{m} \cdot {}^\circ\text{C}}$$

$$q = \frac{4\pi k(T_i - T_0)}{\frac{1}{r_i} - \frac{1}{r_0}} = h4\pi r_0^2(T_0 - T_\infty)$$

$$\frac{(0.018)(120 - 40)}{\frac{1}{0.5} - \frac{1}{r_0}} = (25)r_0^2(40 - 15)$$

$r_0 = 0.5023\ \text{m}$

$\text{thk} = r_0 - r_i = 0.023\ \text{m}$

$q\ (\text{w/ ins}) = (25)(4\pi)(0.5023)^2(40 - 15) = 1982\ \text{W}$

2-17

$$q = \frac{4\pi k(T_i - T_0)}{\frac{1}{r_i} - \frac{1}{r_0}} \qquad k = 204\ \frac{\text{W}}{\text{m} \cdot {}^\circ\text{C}}$$

$$= \frac{(4)\pi(204)(100 - 50)}{\frac{1}{0.02} - \frac{1}{0.04}} = 5127\ \text{W}$$

2-18

$$q = \frac{\Delta T}{\sum R}$$

$$R_{\text{alum}} = \frac{\frac{1}{0.02} - \frac{1}{0.04}}{4\pi(204)} = 9.752 \times 10^{-3}$$

$$R_{\text{ins}} = \frac{\frac{1}{0.04} - \frac{1}{0.05}}{4\pi(0.05)} = 7.958$$

$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(20)(4\pi)(0.05)^2} = 1.592$$

$$q = \frac{100 - 10}{0.00975 + 7.958 + 1.592} = 9.41 \text{ W}$$

2-19

$$d_i = 2.90 \text{ in.} \quad d_0 = 3.50 \text{ in.} \quad k = 43 \ \frac{\text{W}}{\text{m} \cdot {}^\circ\text{C}}$$

$$R_{\text{steel}} = \frac{\ln(3.5/2.9)}{(2\pi)(43)(1)} = 6.96 \times 10^{-4}$$

$$R_{\text{ins}} = \frac{\ln(5.5/3.5)}{(2\pi)(0.06)(1)} = 1.1999$$

$$R_{\text{conv}} = \frac{1}{hA_0} = \frac{1}{(10)\pi(5.5)(0.0254)} = 0.2278$$

$$R_{\text{tot}} = 1.427$$

$$q = \frac{\Delta T}{R} = \frac{250 - 20}{1.427} = 161.1 \ \frac{\text{W}}{\text{m}}$$

2-20

$$k_A = 0.166 \qquad k_f = 0.0485 \ \frac{\text{W}}{\text{m} \cdot {}^\circ\text{C}}$$

$$\frac{315 - T_i}{\frac{\ln(31.4/25)}{0.166}} = \frac{T_i - 38}{\frac{\ln(56.4/31.4)}{0.0485}}$$

$$0.7283(315 - T_i) = 0.0828(T_i - 38)$$

$$T_i = 286.7^\circ\text{C}$$

2-21

$$q_r = -k4\pi r^2 \frac{dT}{dr}$$

$$q_r \int_{r_i}^{r_0} \frac{1}{r^2} dr = -k4\pi \int_{T_i}^{T_0} dT$$

$$q_r \left(\frac{1}{r_0} - \frac{1}{r_i} \right) = -4\pi k (T_0 - T_i)$$

$$q = \frac{-4\pi k (T_0 - T_i)}{\left(\frac{1}{r_0} - \frac{1}{r_i} \right)}$$

$$R_{th} = \frac{\left(\frac{1}{r_i} - \frac{1}{r_0} \right)}{4\pi k}$$

2-22

$$r_i = 0.5 \text{ mm} = 5 \times 10^{-4} \text{ m}$$

$$r_0 = \frac{k}{h} = 5 \times 10^{-4} + 2 \times 10^{-4} = 7 \times 10^{-4}$$

$$k = (7 \times 10^{-4})(120) = 0.084 \ \frac{\text{W}}{\text{m} \cdot {}^\circ\text{C}}$$

$$q(\text{bare wire}) = \pi(0.001)(120)(400 - 40) = 135.7 \ \text{W/m}$$

$$q(\text{insulated}) = (135.7)(0.25) = 33.93 \ \text{W/m}$$

$$q = \frac{400 - 40}{\frac{\ln(r_0/5\times10^{-4})}{2\pi(0.084)} + \frac{1}{\pi(2)(120)r_0}} = 33.93$$

By iteration: $r_0 = 135$ mm

thickness $= 134.5$ mm

2-23

$$R_i = \frac{(1)(12)}{(30)\pi(2.067)} = 6.16 \times 10^{-2}$$

$$R_p = \frac{\ln(2.375/2.067)}{2\pi(27)} = 8.188 \times 10^{-4}$$

$$R_i = \frac{\ln(3.375/2.375)}{2\pi(0.023)} = 2.432$$

$$R_0 = \frac{(1)(12)}{2\pi(3.375)} = 5.659 \times 10^{-1}$$

$$\frac{q}{L} = \frac{320 - 70}{\sum R} = 81.7 \ \frac{\text{Btu}}{\text{hr} \cdot \text{ft}}$$

2-24

$$q = -k4\pi r^2 \frac{dT}{dr} = \frac{k4\pi(T_0 - T_i)}{\frac{1}{r_0} - \frac{1}{r_i}} = k4\pi r_0{}^2(T_0 - T_\infty)$$

$$q = \frac{T_i - T_\infty}{\frac{1}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r_0}\right) + \frac{1}{4\pi r_0{}^2 h}}$$

Take $\frac{dq}{dr_0} = 0$

Result is: $r_0 = 2\frac{k}{h}$

2-25

M_w at 90% full $= (0.9)(970)\pi(0.8)^2(2) = 3511$ kg

at 2°C/hr $\quad q = \frac{(3511)(4191)(2)}{3600} = 8174$ W

$A = 2\pi(0.8)^2 + \pi(0.8)(2) = 9.048\ \text{m}^2$

Fiberglass boards with $k = 40$ mW/m · °C

$$\Delta x = \frac{(40\times 10^{-3})(9.048)(80 - 20)}{8174} = 2.66\times 10^{-3}\ \text{m}$$

2-26

For 1 m length

$$R(\text{pipe}) = \frac{\ln(9.1/8)}{2\pi(47)} = 4.363\times 10^{-4}$$

$$R(\text{ins}(1)) = \frac{\ln(27.1/9.1)}{2\pi(0.5)} = 0.3474$$

$$R(\text{ins}(2)) = \frac{\ln(35.1/27.1)}{2\pi(0.25)} = 0.8246$$

$R(\text{tot}) = 1.172$

$$q = \frac{\Delta T}{R} = \frac{250 - 20}{1.172} = 196.2\ \frac{\text{W}}{\text{m}}$$

2-27

Fiberglass	$k = 0.038$	$\Delta x = 1.2$ cm $\times 2$	
Asbestos	$k = 0.154$	$\Delta x = 8.0$ cm	
brick	$k = 0.69$	$\Delta x = 10.0$ cm	$h = 15\ \frac{\text{W}}{\text{m}\cdot°\text{C}}\times 2$

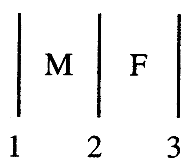
$$U = \frac{1}{\frac{2}{12} + \frac{(2)(0.012)}{0.038} + \frac{0.08}{0.154} + \frac{0.1}{0.69}} = 0.684\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

2-28

$R = \frac{1}{k}$

	k	R
Fiberglass	0.046	21.74
Urethane	0.018	55.6
Mineral Wool	0.091	11.0
Calcium Silicate	0.058	17.2

2-29



$T_1 = 1000°\text{C}$

$T_2 = 400°\text{C}$

$T_3 = 55°\text{C}$

$k_m = 90\ \frac{\text{mW}}{\text{m} \cdot °\text{C}}$ $\qquad h = 15\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$k_F = 42\ \frac{\text{mW}}{\text{m} \cdot °\text{C}}$ $\qquad T_\infty = 40°\text{C}$

$$\frac{q}{A} = h(T_3 - T_\infty) = (15)(55 - 40) = 225\ \frac{\text{W}}{\text{m}^2}$$

$$\frac{q}{A} = k_m \frac{(1000 - 400)}{\Delta x_m} \qquad \Delta x_m = 0.24\ \text{m}$$

$$\frac{q}{A} = k_F \frac{(400 - 55)}{\Delta x_F} \qquad \Delta x_F = 0.0644\ \text{m}$$

2-30

Uniformly distributed heat sources

$$\frac{d^2T}{dx^2}+\frac{\dot{q}}{k}=0 \qquad T=T_1 \quad \text{at} \quad x=-L$$
$$T=T_2 \quad \text{at} \quad x=+L$$

$$T=-\frac{\dot{q}x^2}{2k}+c_1x+c_2$$

$$T_1=-\frac{\dot{q}L^2}{2k}-c_1L+c_2$$

$$T_2=-\frac{\dot{q}L^2}{2k}+c_1L+c_2$$

$$T=\frac{\dot{q}}{2k}(L^2-x^2)+\frac{T_2-T_1}{2L}x+\frac{T_1+T_2}{2}$$

2-31

$$r_1=2.5 \qquad r_2=3.5 \qquad r_3=6.5$$

$$R_1=\frac{\ln(r_2/r_1)}{0.22(2\pi)}=\frac{\ln(3.5/2.5)}{0.22(2\pi)}=0.2433$$

$$R_2=\frac{\ln(r_3/r_2)}{0.06(2\pi)}=1.642$$

$$R_\infty=\frac{1}{hA}=\frac{1}{(60)\pi(2)(0.065)}=0.0408$$

$$\sum R=1.9262\ \frac{^\circ\text{C}\cdot\text{m}}{\text{W}}$$

$$q=\frac{L\Delta T}{R}=\frac{(20)(400-15)}{1.9262}=3997\ \text{W}$$

2-32

$$\frac{d^2T}{dx^2}+\frac{\dot{q}}{k}=0 \qquad \dot{q}=\dot{q}_w[1+\beta(T-T_w)]$$

$T=T_w$ at $x=\pm L$

General solution

$$T-T_w=c_1\left[\cos\left(\sqrt{\frac{\dot{q}_w\beta}{k}}x\right)\right]+c_2\left[\sin\left(\sqrt{\frac{\dot{q}_w\beta}{k}}x\right)\right]-\frac{1}{\beta}$$

From boundary conditions

$$c_2=0 \qquad c_1=\frac{1}{\beta\cos\left(\sqrt{\frac{\dot{q}_w\beta}{k}}L\right)}$$

$$T-T_w=\frac{\cos\sqrt{\frac{\dot{q}_w\beta}{k}}x}{\beta\cos\sqrt{\frac{\dot{q}_w\beta}{k}}L}$$

2-33

$k = 43$; $r_1 = 0.015$; $r_2 = 0.04$; $T_0 = 250°C$; $T_\infty = 35°C$; $h = 43$

$L = 0.025$

$L_c = 0.0255$

$r_{2c} = 0.0405$

$r_{2c}/r_1 = 2.7$

$L_c^{3/2}(h/kA_m)^{1/2} = 0.825$

Fig. 2-12 $\eta_f = 0.59$

$q = (45)(2)\pi(0.0405^2 - 0.015^2)(250 - 35)(0.59) = 5.08$ W

2-34

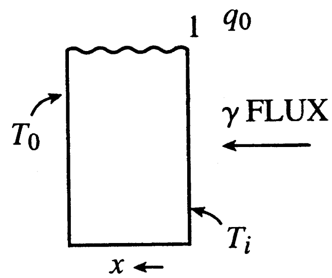
$$\dot{q}=0.30\ \frac{\text{MW}}{\text{m}^3}$$ Same as half of wall 15 cm thick with convection on each side.

$$T_0-T_w=\frac{\dot{q}L^2}{2k}=\frac{(0.30\times10^6)(0.060)^2}{(2)(21)}=25.7°\text{C}$$

$$\dot{q}LA=hA(T_w-T_\infty) \qquad T_w-T_\infty=\frac{(0.30\times10^6)(0.060)}{570}=31.6°\text{C}$$

$$T_0=T_{max}=93+25.7+31.6=150.3°\text{C}$$

2-35



$\dot{q}_x = \dot{q}_0 e^{-ax}$ $\qquad$ $\dfrac{d^2T}{dx^2} = \dfrac{-\dot{q}_0 e^{-ax}}{k}$

$T = c_1 + c_2 x - \dfrac{\dot{q}_0}{a^2 k} e^{-ax}$

Boundary conditions:

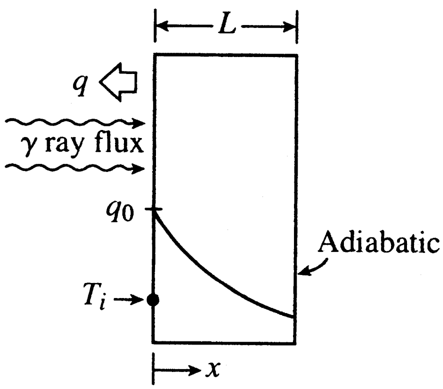
(1) $\quad T = T_i$ at $x = 0$

(2) $\quad T = T_0$ at $x = L$

$c_1 = T_i + \dfrac{\dot{q}_0}{a^2 k}$ $\qquad$ $c_2 = \dfrac{T_0 - T_i - \frac{\dot{q}_0}{a^2 k}(1 - e^{-aL})}{L}$

$$T = T_i + \frac{\dot{q}_0}{a^2 k} + \frac{T_0 - T_i - \frac{\dot{q}_0}{a^2 k}(1 - e^{-aL})x}{L} + \frac{-\dot{q}_0}{a^2 k} e^{-ax}$$

2-36



$$\dot{q} = \dot{q}_0 e^{-ax} \qquad \frac{d^2T}{dx^2} = \frac{-\dot{q}_0}{k} e^{-ax}$$

$$T = c_1 + c_2 x - \frac{\dot{q}_0}{a^2 k} e^{-ax}$$

Boundary conditions:

(1) at $x = L$ $\quad \dfrac{dT}{dx} = 0$ (adiabatic)

(2) at $x = 0$ $\quad T = T_i$

$$c_1 = T_i + \frac{\dot{q}_0}{a^2 k} \qquad c_2 = -\frac{\dot{q}_0}{ak} e^{-aL}$$

$$T = T_i + \frac{\dot{q}_0}{a^2 k} - \frac{q_0 e^{-aL}}{ak} x - \frac{\dot{q}_0}{a^2 k} e^{-ax}$$

2-37

$$T - T_w = c_1 \cos\sqrt{q_w \frac{\beta}{k}}x + c_2 \sin\sqrt{q_w \frac{\beta}{k}}x - \frac{1}{\beta}$$

$T = T_1$ at $x = \pm L$ $\qquad c_2 = 0$

$$-kA\left.\frac{\partial T}{\partial x}\right|_{x=L} = hA(T_1 - T_\infty) \qquad c_1 = \frac{T_1 - T_w + \frac{1}{\beta}}{\cos\sqrt{q_w \frac{\beta}{k}}L}$$

$$T = T_w + \frac{T_1 - T_w + \frac{1}{\beta}}{\cos\sqrt{q_w \frac{\beta}{k}}L}\cos\sqrt{q_w \frac{\beta}{k}}x - \frac{1}{\beta}$$

Solving for $\left.\frac{\partial T}{\partial x}\right|_{x=L}$ and substituting in above equation:

$$T_1 = \frac{1}{1-h}\left[T_w - \frac{1}{\beta} - \frac{hT_\infty}{k\sqrt{q_w \frac{\beta}{k}}\tan\sqrt{q_w \frac{\beta}{k}}L}\right]$$

2-38

$$\dot{q}AL = hPL(T_w - T_\infty)$$

$$(35.3 \times 10^6)(0.025)^2 = (4000)(4)(0.025)(T_w - 20)$$

$$T_w = 75.16°\text{C}$$

2-39

$$\frac{d^2T}{dx^2} + \frac{\dot{q}_0 \cos(ax)}{k} = 0 \qquad \frac{dT}{dx} = \frac{-\dot{q}_0}{ak}\sin(ax) + c_1$$

$$T = T_w \text{ at } x = \pm L \qquad \therefore c_1 = 0 \qquad T = \frac{\dot{q}_0}{a^2k}\cos(ax) + c_1x + c_2$$

$$T_w = \frac{\dot{q}_0}{a^2k}\cos(aL) + c_2 \qquad T - T_w = \frac{\dot{q}_0}{a^2k}[\cos(ax) - \cos(aL)]$$

$$\frac{q}{A} = -2k\left.\frac{dT}{dx}\right|_{x=L} = -2k\frac{\dot{q}_0}{ak}[-\sin(aL)] = \frac{2k\dot{q}_0}{ak}\sin(aL)$$

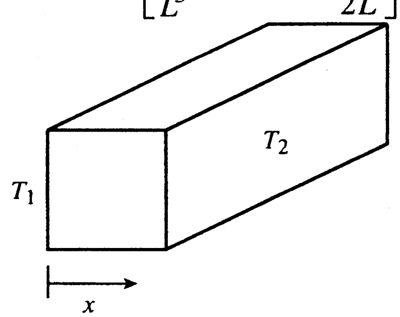
2-40

$$k = 0.0124\ \frac{\text{W}}{\text{cm}\cdot{}^\circ\text{C}} = 1.24\ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$$
$$\rho = 1.5\times10^{-3}\ \Omega\cdot\text{cm}$$
$$R = (1.5\times10^{-3})\left(\frac{3}{1}\right) = 4.5\times10^{-3}$$
$$q = I^2R = (50)^2(4.5\times10^{-3}) = 11.25\text{ W}$$
$$q = \frac{q}{V} = \frac{11.25}{3\times10^{-6}} = 3.75\ \frac{\text{MW}}{\text{m}^3} \qquad T = -\frac{\dot{q}}{2k}x^2 + c_1x + c_2$$
$L = 1.5\text{ cm} = 0.015\text{ m}$ $\qquad T = 300$ at $x = -0.015$
$T = 100$ at $x = +0.015$
$300 - 100 = c_1(-0.015 - 0.015)$ $\qquad c_1 = -6667$
$$300 = \frac{(-3.75\times10^6)(0.015)^2}{(2)(1.24)} - (6667)(-0.015) + c_2 \qquad c_2 = 540.2$$
at $x = 0$ $\qquad T = c_2 = 540.2°\text{C}$

2-41

$k =$ constant $\dot{q} = \dot{q}_0$ at $x = 0$ Assume one directional with no heat storage.

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \qquad T - T_1 = (T_2 - T_1)(c_1 + c_2x^2 + c_3x^3)$$
$$T = T_1 + (T_2 - T_1)(c_1 + c_2x^2 + c_3x^3)$$
$$\frac{d^2T}{dx^2} = (T_2 - T_1)(2c_2 + 3c_3x)$$
$T = T_1$ at $x = 0$ $\qquad T = T_2$ at $x = L$ $\qquad \dot{q} = \dot{q}_0$ at $x = 0$
$$q = -k(T_2 - T_1)(2c_2 + 3c_3x)$$
so: $c_1 = 0 \qquad c_2 = -\dfrac{\dot{q}_0}{2k(T_2 - T_1)} \qquad c_3 = \dfrac{1}{L^3} + \dfrac{\dot{q}_0}{2kL(T_2 - T_1)}$
$$\dot{q}_x = \dot{q}_0 - \left[\frac{3k}{L^3}(T_2 - T_1) + \frac{3\dot{q}_0}{2L}\right]x$$



2-42

$k = 2.5$ $\quad h_1 = 75$ (left) $\quad h_2 = 50$ (right)

$T_{1\infty} = 50°C \quad T_{2\infty} = 30°C$

$$T = -\frac{\dot{q}x^2}{2k} + c_1 x + c_2$$

$T = T_1$ at $x = -0.04$; $T = T_2$ at $x = +0.04$

$$\frac{\partial T}{\partial x} = -\frac{\dot{q}x}{k} + c_1$$

$$T = T_{max} = 300 \text{ at } x = c_1 \frac{k}{\dot{q}} \quad (1)$$

$$h_1(T_{1\infty} - T_1) = -k\left.\frac{\partial T}{\partial x}\right|_{x=-0.04} \quad (2)$$

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=+0.04} = h_2(T_2 - T_{2\infty}) \quad (3)$$

$$300 = -\frac{\dot{q}}{2k}\left[c_1 \frac{k}{\dot{q}}\right]^2 + c_1\left[c_1 \frac{k}{\dot{q}}\right] + c_2 \quad (1)$$

$$75\left[50 + \frac{\dot{q}}{2k}(0.04)^2 + c_1(0.04) - c_2\right] = -k\left(\frac{+\dot{q}(0.04)}{2k}\right) \quad (2)$$

$$-k\left[\frac{-\dot{q}(0.04)}{2k}\right] = 50\left[\frac{-\dot{q}(0.04)^2}{2k} + c_1(0.04) + c_2 - 30\right] \quad (3)$$

3 Equations, 3 Unknowns, c_1, c_2, $\dot{q}$

Solve for $\dot{q} = 2.46 \times 10^5 \ \text{W/m}^3$

2-43

d = 0.05; L = 0.02; ΔT = 200°C; k = 204

shim 0.025 mm = 0.001 inch

Table 2-3 ; $1/h_c = 3.52 \times 10^{-4}$

Bars: $\Delta x/kA = (4)(0.02)/\pi(0.05)^2 = 10.19$

Joint: $1/h_c A = 0.179$

$\Sigma R = (2)(10.19) + 0.179 = 20.559$

$\Delta T_{joint} = (200)(0.179/20.559) = 1.74°C$

2-44

Use solution from Prob. 2–28

$T = T_0$ at $x = 0$

$$T_0 = \frac{\dot{q}}{2k}L^2 + \frac{T_1 + T_2}{2} = \frac{(5\times 10^5)(0.015)^2}{(2)(16)} + \frac{220+45}{2} = 136°\text{C}$$

2-45

Use solution from Prob. 2–28

$$T_0 = \frac{\dot{q}}{2k}L^2 + \frac{T_1 + T_2}{2} = \frac{(500\times 10^6)(0.005)^2}{(2)(20)} + \frac{100+200}{2} = 462.5°\text{C}$$

2-46

Behaves like half a plate having a thickness of 8 mm. Max Temp is at $x=0$

$$T_0 = \frac{\dot{q}L^2}{2k} + T_w$$

$L = 0.004$ m $\quad T_w = 100°\text{C}$

$$T_0 = \frac{(200\times10^6)(0.004)}{(2)(25)} + 100 = 164°\text{C}$$

2-47

$$\dot{q}\pi r^2 L = \frac{E^2}{R} = \frac{(10)^2\pi(0.16)^2}{(7\times10^{-6})(30)}$$

$$\dot{q} = \frac{(10)^2\pi(0.16)^2}{(7\times10^{-6})(30)(0.3)\pi(1.6\times10^{-3})^2} = 1587\ \frac{\text{MW}}{\text{m}^3}$$

$$T_0 = \frac{\dot{q}r^2}{4k} + T_w = \frac{(1.587\times10^9)(1.6\times10^{-3})^2}{(4)(22.5)} + 93 = 138.1°\text{C}$$

2-48

$$(200)^2(0.099) = (5700)\pi(3\times10^{-3})(1)(T_w - 93)$$

$T_w = 166.7°\text{C} \qquad T_0 = 16.6 + 166.7 = 183.3°\text{C}$

2-49

$$q = EI = \dot{q}\pi(r_0^2 - r_i^2)L = h2\pi rL(T_i - T_f)$$

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{\dot{q}}{k} = 0 \quad T = \frac{-\dot{q}r}{4k} + c_1\ln r + c_2$$

$$T = T_0 \text{ at } r = r_0 \qquad \frac{dT}{dr} = 0 \text{ at } r = r_0 \qquad T_0 = \frac{-\dot{q}r}{4k} + c_1\ln r_0 + c_2$$

$$\frac{dT}{dr} = 0 = \frac{-\dot{q}r}{2k} + \frac{c_1}{r_0} \qquad c_1 = \frac{\dot{q}r_0^2}{2k} \qquad T_i = \frac{-\dot{q}r}{4k} + \frac{\dot{q}r_0^2}{2k}\ln r_1 + c_2$$

$$T_0 = \frac{-\dot{q}r_0^2}{4k} + \frac{\dot{q}r_0^2}{2k}\ln r_0 + c_2 \quad T_i = T_0 - \frac{\dot{q}}{4k}(r_1^2 - r_0^2) + \frac{\dot{q}r_0^2}{2k}\ln\left(\frac{r_1}{r_0}\right) \quad \text{(a)}$$

$$\dot{q} = \frac{EI}{\pi(r_0^2 - r_i^2)L} \quad \text{(b)}$$

Insert (a) and (b) in $EI = 2\pi rLh(T_i - T_f)$ and solve for h.

2-50

$\dot{q}$ uniform $T = T_w$ at $r = R$ steady state, T varies only with r.

$$\frac{1}{r}\frac{\partial^2(rT)}{\partial r^2} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial\theta^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

this reduces to:

$$\frac{1}{r}\frac{\partial^2(rT)}{\partial r^2} + \frac{\dot{q}}{k} = 0 \qquad \frac{\partial^2(rT)}{\partial r^2} = \frac{-\dot{q}r}{k} \qquad \text{Integrating } T = \frac{-\dot{q}r^2}{6k} + c_1 + \frac{c_2}{r}$$

Boundary conditions: $\dot{q}\frac{4}{3}\pi R^3 = -k4\pi R^2 \left.\frac{dT}{dr}\right|_{r=R}$

(1) $\frac{dT}{dr} = -\frac{\dot{q}r}{3k}$ (2) $T = T_w$ at $r = R$

(3) $\left.\frac{dT}{dr}\right|_{r=R} = 0$ then $c_1 = T_w + \frac{\dot{q}R^2}{6k}$ $\qquad c_2 = 0$

$$T - T_w = \frac{\dot{q}}{6k}(R^2 - r^2)$$

2-51

From Prob. 2-~~47~~ 50

$$T - T_w = \frac{\dot{q}}{6k}(R^2 - r^2)$$

$$T_0 - T_w = \frac{(1\times 10^6)(0.02)^2}{(6)(16)} = 4.17°\text{C}$$

$$q = \dot{q}V = \dot{q}\frac{4}{3}\pi R^3 = h4\pi r^2(T_w - T_\infty)$$

$$T_w - T_\infty = \frac{(1\times 10^6)(0.02)}{(3)(15)} = 444.4°\text{C}$$

$$T_0 - T_\infty = 444.4 + 4.17 = 448.6°\text{C}$$

$$T_0 = 448.6 + 20 = 468.6°\text{C}$$

2-52

$$\frac{R}{L} = \frac{\rho}{\pi r_0^2} = \frac{2.9\times 10^{-6}}{\pi(1.5)^2} = 4.1\times 10^{-7}\ \frac{\Omega}{\text{cm}} = 4.1\times 10^{-5}\ \frac{\Omega}{\text{m}}$$

$$\dot{q} = \frac{I^2 R}{V} = \frac{(230)^2(4.1\times 10^{-5})}{\pi(0.015)^2} = 3.07\times 10^3\ \frac{\text{W}}{\text{m}^3}$$

$$T_0 = \frac{(3.07\times 10^3)(0.015)^2}{(4)(190)} + 180 = 180.0009°\text{C}$$

2-53

$$\dot{q} = a + br \qquad \frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{q}r}{k} = -\frac{ar + br^2}{k}$$

$$r\frac{dT}{dr} = -\frac{1}{k}\left(\frac{ar^2}{2} + \frac{br^3}{3}\right) + c_1 \qquad T = -\frac{1}{k}\left(\frac{ar^2}{4} + \frac{br^3}{9}\right) + c_1 \ln r + c_2$$

$$\frac{dT}{dr} = -\frac{1}{k}\left(\frac{ar}{2} + \frac{br^2}{3}\right) + \frac{c_1}{r}$$

$T = T_i$ at $r = r_1$ $\qquad$ $T = T_0$ at $r = r_0$

Solving for constants gives:

$$c_1 = \frac{T_i - T_0 - \frac{1}{k}\left[\frac{a(r_0{}^2 - r_i{}^2)}{4} + \frac{b(r_0{}^3 - r_i{}^3)}{9}\right]}{\ln\left(\frac{r_i}{r_0}\right)}$$

$$c_2 = T_0 + \frac{1}{k}\left(\frac{ar_0{}^2}{4} + \frac{br_0{}^3}{9}\right) + \frac{T_i - T_0 - \frac{1}{k}\left[\frac{a(r_0{}^2 - r_i{}^2)}{4} + \frac{b(r_0{}^3 - r_i{}^3)}{9}\right]}{\ln r_0}$$

Also $\dot{q} = \dot{q}_i$ at $r = r_i = a + br_i$

2-54

$d = 2$ mm $\qquad T_\infty = 100°\text{C} \qquad h = 5000 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$T_0 = 150°\text{C} \qquad \rho_e = 1.67\ \mu\Omega \cdot \text{cm} \qquad k = 386 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}}$

$$T_0 - T_w = \frac{\dot{q}R^2}{4k}$$

$$q = \dot{q}(\pi R^2)L = h(2\pi RL)(T_w - T_\infty)$$

$$T_w - T_\infty = \frac{\dot{q}R^2}{2h}$$

$$T_0 - T_\infty = \frac{\dot{q}R^2}{4k} + \frac{\dot{q}R}{2h}$$

$$150 - 100 = \dot{q}\left[\frac{0.001^2}{(4)(386)} + \frac{0.001}{(2)(5000)}\right] \qquad \dot{q} = 4.97 \times 10^8 \ \frac{\text{W}}{\text{m}^3}$$

$$q = I^2 \rho_e \frac{L}{A} = \dot{q}AL$$

$$I^2 = \frac{\dot{q}A^2}{\rho_e} = \frac{(4.97 \times 10^8)(\pi)^2(0.001)^4}{1.67 \times 10^{-8}}$$

$I = 542$ amp

2-55

$r_i = 0.0125$ m $\qquad r_0 = 0.0129$ m

Assume inner surface is insulated

$$\frac{dT}{dr} = -\frac{\dot{q}r}{2k} + \frac{c_1}{r} = 0 \text{ at } r = r_i$$

$$c_1 = \frac{\dot{q}r_i^2}{2k} \qquad \text{(a)}$$

$$T = \frac{-\dot{q}r^2}{4k} + c_1 \ln r + c_2 \qquad T_i = \frac{-\dot{q}r_i^2}{4k} c_1 \ln r_i + c_2$$

$$T_0 = \frac{-\dot{q}r_0^2}{4k} c_1 \ln r_0 + c_2$$

$$T_i - T_0 = \frac{-\dot{q}}{4k}(r_0^2 - r_i^2) + c_1 \ln\left(\frac{r_i}{r_0}\right) \qquad \text{(b)}$$

Heat Transfer is:

$$q = \dot{q}V = \dot{q}\pi(r_0^2 - r_i^2) = h\pi(2r_0)(T_0 - T_\infty) \qquad \text{(c)}$$

Inserting (a) in (b) gives

$$T_i - T_0 = \frac{q}{4k}(r_0^2 - r_i^2) + \frac{\dot{q}r_i^2}{2k} \ln\left(\frac{r_i}{r_0}\right) \qquad \text{(d)}$$

We take: $T_i = 250°\text{C}$ $\quad h = 100 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$ $\qquad T_\infty = 40°\text{C}$ $\qquad k = 24 \ \frac{\text{W}}{\text{m} \cdot °\text{C}}$

Inserting the numerical values in Equations (c) and (d) and solving gives:

$$\dot{q} = 53.26 \ \frac{\text{MW}}{\text{m}^3}$$

$$T_0 = 249.76°\text{C}$$

2-56

$$k = 43 \ \frac{\text{W}}{\text{m} \cdot °\text{C}}$$

$$U_i = \frac{1}{\frac{1}{500} + \frac{\ln(1.45/1.25)\pi(0.025)}{(2\pi)(43)} + \frac{0.025}{0.029}\left(\frac{1}{12}\right)} = \frac{1}{2 \times 10^{-3} + 4.31 \times 10^{-5} + 71.84 \times 10^{-3}}$$

$$= 13.54 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

2-57

$$r_0 = \frac{k}{h} = \frac{0.18}{12} = 0.015 \text{ m} = 1.5 \text{ cm}$$

(a) $\quad r_0 = 1.25 + 0.05 = 1.3$ cm $\qquad$ Increased

(b) $\quad r_0 = 1.25 + 1.0 = 2.25$ cm $\qquad$ Decreased

2-58

$$U = \frac{1}{R} = \frac{1}{3.114 \times 10^{-2}} = 32.11 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

2-59

$$A = 130.4 \text{ m}^2$$
$$U = \frac{q}{A\Delta T} = \frac{44{,}000}{(130.4)(260-38)} = 1.52 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

2-60

For $L = 1$ m $\quad \dfrac{1}{h_i A_i} = \dfrac{1}{(65)\pi(0.025)} = 0.1959$

$$\frac{\ln(r_0/r_i)}{2\pi k} = \frac{\ln(2.58/2.5)}{2\pi(18)} = 2.79 \times 10^{-4}$$
$$\frac{1}{h_0 A_0} = \frac{1}{(6.5)\pi(0.0258)} = 1.898$$
$$UA = \frac{1}{\sum R} = 2.094$$
$$\frac{q}{L} = (2.094)(120-15) = 219.9 \ \frac{\text{W}}{\text{m}}$$

2-61

$A = 1 \text{ m}^2$

$$R_{\text{glass}} = \frac{\Delta x}{k} = \frac{0.005}{0.78} = 6.41 \times 10^{-3}$$
$$R_{\text{air}} = \frac{\Delta x}{k} = \frac{0.004}{0.026} = 0.1538$$
$$R_{\text{conv}_1} = \frac{1}{h} = \frac{1}{12} = 0.0833$$
$$R_{\text{conv}_2} = \frac{1}{50} = 0.02$$

$$U = \frac{1}{(2)(6.41 \times 10^{-3}) + 0.1538 + 0.0833 + 0.02} = \frac{1}{0.2699} = 3.705 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

$R = 0.2699$

single glass plate: $R = 6.41 \times 10^{-3} + 0.0833 + 0.02 = 0.1097$

$$U = \frac{1}{R} = 9.11 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

2-62

$$R_{Cu} = \frac{0.001}{386} = 2.59\times10^{-6} \qquad \Delta T_{Cu} = (52)(2.59\times10^{-6}) = 1.35\times10^{-4}\,°\mathrm{C}$$

$$R_{St} = \frac{0.004}{43} = 9.3\times10^{-5} \qquad \Delta T_{St} = (52)(9.3\times10^{-5}) = 4.84\times10^{-3}\,°\mathrm{C}$$

$$R_{As} = \frac{0.01}{0.166} = 0.0602 \qquad \Delta T_{As} = (52)(0.0602) = 3.13°\mathrm{C}$$

$$R_F = \frac{0.1}{0.038} = 2.632 \qquad \Delta T_F = (52)(2.632) = 136.9°\mathrm{C}$$

$$\sum R = 2.692 \qquad U = \frac{1}{R} = 0.371\ \frac{\mathrm{W}}{\mathrm{m}^2\cdot°\mathrm{C}}$$

$$q = U\Delta T = (0.371)(150-10) = 52\ \frac{\mathrm{W}}{\mathrm{m}^2}$$

Inside of copper = 150°C

2-63

$$r_1 = 0.015 \qquad L_c = 0.02 + 0.00035 = 0.02035$$

$$r_{2c} = 0.03535 \qquad r_{2c}/r_1 = 2.357$$

$$L_c^{3/2}\left[\frac{h}{kA_m}\right]^{1/2} = (0.02035)^{3/2}\left[\frac{524}{(386)(0.0007)(0.02035)}\right]^{1/2} = 0.896$$

$$\eta_f = 0.55$$

$$q = \eta_f hA\theta_0 = (0.55)\pi(0.03535^2 - 0.015^2)(2)(524)(200-100) = 186\ \mathrm{W}$$

2-64

The general solution of eq. 2–19 (b) is:

$\theta = T - T_\infty = c_1 e^{-mx} + c_2 e^{mx}$ (1)

boundary conditions:

(1) at $x = 0$ $\theta = \theta_1 = T_1 - T_\infty$

(2) at $x = L$ $\theta = \theta_2 = T_2 - T_\infty$

from:

(1) $\theta_1 = c_1 + c_2 \quad c_1 = \theta_1 - c_2$

(2) $\theta_2 = c_1 e^{-mL} + c_2 e^{mL}$

$$\theta_2 = (\theta_1 - c_2)e^{-mL} + c_2 e^{mL} = \theta_1 e^{-mL} - c_2(e^{-mL} - e^{mL})$$

$$c_2 = \frac{\theta_2 - \theta_1 e^{-mL}}{e^{mL} - e^{-mL}} \quad (2)$$

$$c_1 = \theta_1 - c_2 = \theta_1 - \frac{\theta_2 - \theta_1 e^{-mL}}{e^{mL} - e^{-mL}}$$

$$c_1 = \frac{\theta_2 - \theta_1 e^{mL}}{e^{-mL} - e^{mL}} \quad (3)$$

Then eq. (1) becomes

$$\theta = \frac{\theta_2 - \theta_1 e^{mL}}{e^{-mL} - e^{mL}} e^{-mx} + \frac{\theta_2 - \theta_1 e^{-mL}}{e^{mL} - e^{-mL}} e^{mx}$$

$$\theta = \frac{e^{-mx}(\theta_2 - \theta_1 e^{mL}) + e^{mx}(\theta_1 e^{-mL} - \theta_2)}{e^{-mL} - e^{mL}}$$

Part heat lost by rod:

$$q = -kA\left.\frac{d\theta}{dx}\right|_{x=0} + kA\left.\frac{d\theta}{dx}\right|_{x=L}$$

$$\frac{d\theta}{dx} = m\left[\frac{-e^{-mx}(\theta_2 - \theta_1 e^{mL}) + e^{mx}(\theta_1 e^{-mL} - \theta_2)}{e^{-mL} - e^{mL}}\right]$$

$$q = \frac{kAm[-e^{-mL}(\theta_2 - \theta_1 e^{mL}) + e^{mL}(\theta_1 e^{-mL} - \theta_2)]}{e^{-mL} - e^{mL}}$$

$$+ \frac{kAm[-(\theta_2 - \theta_1 e^{mL}) + (\theta_1 e^{-mL} - \theta_2)]}{e^{-mL} - e^{mL}}$$

$$q = \frac{kAm[(\theta_2 - \theta_1 e^{mL})(1 - e^{-mL}) + (\theta_2 - \theta_1 e^{-mL})(1 - e^{mL})]}{e^{-mL} - e^{mL}}$$

2-64

Part A:

$$\dot{q}A = -kA\frac{d^2T}{dx^2} + hP(T - T_\infty)$$

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) + \frac{\dot{q}}{k} = 0$$

let $\theta = T - T_\infty$

$$\left(D^2 - \frac{hP}{kA}\right)\theta = \frac{-\dot{q}}{k}$$

$$\theta = c_1 e^{\sqrt{hP/kA}x} + c_2 e^{-\sqrt{hP/kA}x} + \frac{\dot{q}A}{hP}$$

let $\sqrt{\dfrac{hP}{kA}} = m$

$\theta = \theta_0$ at $x = 0$ $\qquad \therefore\ c_1 = \theta_0 - \dfrac{\dot{q}A}{hP} - c_2$

$$-kA\left.\frac{dT}{dx}\right|_{x=L} = hA(T - T_\infty)\Big|_{x=L} = hA\theta_L$$

$$\theta = \frac{\left[e^{-mL}\left(\theta_0 - \frac{\dot{q}A}{hP}\right) - \frac{h\theta_L}{km}\right]e^{mx}}{e^{mL} + e^{-mL}} + \frac{\left[e^{mL}\left(\theta_0 - \frac{\dot{q}A}{hP}\right) + \frac{h\theta_L}{km}\right]e^{-mx}}{e^{mL} + e^{-mL}} + \frac{\dot{q}A}{hP}$$

Part B:

$$q = \int_0^L [hP(T - T_\infty)]dx + hA\theta_L$$

$$q = \frac{hP}{m}\left\{-\frac{h\theta_L}{kL} + \frac{\left(\theta_0 - \frac{\dot{q}A}{hP}\right)(e^{mL} - e^{-mL}) + \dfrac{2h\theta_L}{km}}{e^{mL} + e^{-mL}}\right\} + \dot{q}AL + hA\theta_L$$

Part C:

$$-kA\left.\frac{dT}{dx}\right|_{x=0} = 0 = q_0$$

$$0 = [c_1 m e^{mx} - c_2 m e^{-mx}]_{x=0}$$

$$\therefore\ c_1 = c_2$$

$$2c_1 = \theta_0 - \frac{\dot{q}A}{hP}$$

$$\dot{q} = \frac{hP}{A}\left[\theta_0 + \frac{2h\theta_L}{km(e^{mL} - e^{-mL})}\right]$$

2-66

$$\frac{d^2\theta}{dx^2} - \frac{hP}{kA}\theta = 0 \text{ let } m = \sqrt{\frac{hP}{kA}}$$

$$T_\infty = 38 \qquad d = 12.5 \text{ mm} \quad L = 30 \text{ cm} \qquad h = 17$$

$$\theta = c_1 e^{mx} + c_2 e^{-mx} \text{ at } x = 0 \quad \theta = 200 - 38 = 162 \qquad k = 386$$

$$P = \pi d \qquad A = \frac{\pi d^2}{4} \qquad x = 0.3 \qquad \theta = 93 - 38 = 55$$

$$m = \left[\frac{(17)\pi(0.0125)(4)}{(386)\pi(0.0125)^2}\right]^{1/2} = 3.754 \qquad 162 = c_1 + c_2$$

$$55 = 3.084c_1 + 0.324c_2 \qquad c_1 = 0.91 \qquad c_2 = 161.09$$

$$\theta = 0.91e^{mx} + 161.09e^{-mx}$$

$$q\int_0^L hP\theta\, dx = hP\frac{1}{m}[0.91e^{mx} - 161.09e^{-mx}]_0^L = \sqrt{hPkA}[0.91e^{mx} - 161.09e^{-mx}]_0^{0.3}$$

$$= [(17)\pi(0.0125)(386)\pi(0.0125)^2]^{1/2} \times [0.91e^{mx} - 161.09e^{-mx}]_0^{0.3}$$

$$= 122.7 \text{ W}$$

2-68

$$k = 204 \ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}} \qquad L_c = L + \frac{d}{4} = 12 + \frac{2}{4} = 12.5 \quad T_0 = 250^\circ\text{C} \quad T_\infty = 15^\circ\text{C}$$

$$h = 12 \ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}} \qquad A = \frac{\pi d^2}{4} \qquad P = \pi d$$

$$m = \sqrt{\frac{hP}{kA}} = \left[\frac{(12)\pi(0.02)(4)}{(204)\pi(0.02)^2}\right]^{1/2} = 3.43$$

$$mL_c = (3.43)(0.125) = 0.429 \qquad q = \sqrt{hPkA}\theta_0 \tanh(mL_c)$$

$$q = \left[(12)\pi(0.02)(204)\pi\frac{(0.02)^2}{4}\right]^{1/2} (250 - 15)\tanh(0.429) = 20.89 \text{ W}$$

2-69

$$q = \int_0^\infty hP(T - T_\infty)dx = \int_0^\infty hP\theta\, dx = hP\int_0^\infty e^{-mx}dx = \frac{hP\theta_0}{-m}[e^{-mx}]_0^\infty = \sqrt{hPkA\theta_0}$$

$$= \frac{hP\theta_0}{-\sqrt{\frac{hP}{kA}}}\left[\underset{0}{\cancel{e^{-\sqrt{\frac{hP}{kA}}L}}} - 1\right]$$

$$q = \frac{hP\theta_0}{\sqrt{\frac{hP}{kA}}} = \sqrt{hPkA\theta_0}$$

2-70

$\theta = c_1 e^{-mx} + c_2 e^{mx}$ In case II the end of the fin is insulated.

$\left.\dfrac{dT}{dx}\right|_{x=L} = 0$ the boundary conditions are $\theta = \theta_0$ at $x = 0$

$\dfrac{d\theta}{dx} = 0$ at $x = L$ then $\theta = \theta_0 \dfrac{\cosh[m(L-x)]}{\cosh(mL)}$

$dq_{\text{conv}} = hP\,dx(T - T_\infty)$ or $hPdx\theta$

$$q_{\text{conv}} = \int_0^L hP\,dx = \int_0^L \frac{hP\theta \cosh[m(L-x)]dx}{\cosh(mL)}$$

$$q_{\text{conv}} = \frac{hP\theta_0}{\cosh(mL)}\frac{1}{m}[\sinh(m(L-x))]_0^L = \sqrt{hPkA}\theta_0 \tanh(mL)$$

2-71

$$q = \sqrt{hPkA}\theta_0 = \left[\frac{(20)\pi(0.0005)(372)\pi(0.0005)^2}{4}\right]^{1/2}(120-20) = 0.152 \text{ W}$$

2-72

$$q = \sqrt{hPkA}\theta_0 = \left[\frac{(3.5)\pi(0.025)(372)\pi(0.025)^2}{4}\right]^{1/2}(90-40) = 11.2 \text{ W}$$

2-73

$T_0 = 150°\text{C}$ $T_\infty = 15°\text{C}$ $r_1 = 1.35$ cm $L = 6.0$ mm $t = 1.5$ mm

$h = 20 \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$ $k = 210 \dfrac{\text{W}}{\text{m} \cdot °\text{C}}$

$L_c = L + \dfrac{t}{2} = 6.0 + 0.75 = 6.75$ mm

$r_{2e} = r_1 + L_c = 1.35 + 0.675 = 2.025$ cm $\dfrac{r_{2c}}{r_1} = 1.50$

$A_m = t(r_{2c} - r_1) = (0.0015)(0.00675) = 1.012 \times 10^{-5}\ \text{m}^2$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.00675)^{3/2}\left[\frac{20}{(210)(1.012\times10^{-5})}\right]^{1/2} = 0.0538$$

From Fig. 2–11 $\eta_f = 97\%$

$q_{\max} = 2h\pi(r_{2c}^2 - r_1^2)(T_0 - T_\infty) = 3.86$ W

$q = (0.97)(3.86) = 3.75$ W

2-74

$L_c = 23 + 1 = 24$ mm

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.024)^{3/2}\left[\frac{25}{(14)(0.002)(0.024)}\right]^{1/2} = 0.717$$

$\eta_f = 0.77$

$q = \eta_f hA\theta_0 = (0.77)(25)(0.024)(2)(220 - 23) = 182\ \text{W/m}$

2-75

$L_c = 3 + 0.1 = 3.1$ cm $\qquad r_{2c} = 1.5 + 3.1 = 4.6$ cm

$r_1 = 1.5 \qquad \dfrac{r_{2c}}{r_1} = 3.067$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.031)^{3/2}\left[\frac{68}{(55)(0.002)(0.031)}\right]^{1/2} = 0.77$$

$\eta_f = 0.58$

$q = \eta_f hA\theta_0 = (0.58)(68)(2\pi)[0.046^2 - 0.015^2](100 - 20) = 37.49$ W

2-76

η = total efficiency $\quad A_f$ = surface area of all fins $\quad \eta_f$ = fin efficiency
A = total heat transfer area including fins and exposed tube or other surface.
T_0 = base temp $\qquad T_\infty$ = environment temp

$q_{\text{act}} = h(A - A_f)(T_0 - T_\infty) + \eta_f A_f h(T_0 - T_\infty)$

$q_{\text{ideal}} = hA(T_0 - T_\infty)$

$$\eta_t = \frac{q_{\text{act}}}{q_{\text{ideal}}} = \frac{A - A_c + A_f\eta_f}{A} = 1 - \frac{A_f}{A}(1 - \eta_f)$$

2-77

$T_0 = 460 \qquad t = 6.4$ mm $\qquad L = 2.5$ cm $\qquad T_\infty = 93°\text{C} \qquad h = 28$

$k = 16.3 \qquad A_m = L\left(\dfrac{t}{2}\right) = 8 \times 10^{-5}\ \text{m}^2 \qquad L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2} = 0.579$

$\eta_f = 0.85$

$q = (0.85)(28)(1)(2)(0.025)(460 - 93) = 437$ W

2-78

$T_0 = 200°C \quad T_\infty = 93°C \quad L = 12.5 \text{ mm} \quad t = 0.8 \text{ mm} \quad r_1 = 1.25 \text{ cm}$

$k = 204 \quad L_c = 12.9 \text{ mm} \quad h = 110 \quad r_{2c} = 2.54 \quad \dfrac{r_{2c}}{r_1} = 2.03$

$$A_m = 1.03 \times 10^{-5} \text{ m}^2 \qquad L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.335 \qquad \eta_f = 0.87$$

$$\text{No. of Fins} = \frac{1.0}{0.0095} = 105.3$$

Tube surface area $= (105.3)\pi(0.025)(9.5 - 0.8)(10^{-3}) = 0.0719 \text{ m}^2$

Tube heat transfer $= (110)(0.0719)(200 - 93) = 846.6 \text{ W}$

$$\frac{q}{\text{fin}} = (0.87)(2)\pi(110)(0.0254^2 - 0.0125^2)(200 - 93) = 31.46 \text{ W}$$

Total fin heat transfer $= (31.46)(105.3) = 3312 \text{ W}$

Total heat transfer $= 846.6 + 3312 = 4159 \text{ W}$

2-79

$r_1 = 1.0 \text{ cm} \quad L = 5 \text{ mm} \quad t = 2.5 \text{ mm} \quad h = 25 \quad T_0 = 260°C$

$T_\infty = 93°C \quad k = 43 \quad L_c = 5 + 1.25 = 6.25 \text{ mm} \quad r_{2c} = 1.625 \text{ cm}$

$\dfrac{r_{2c}}{r_1} = 1.625 \quad A_m = (0.0025)(0.00625) = 1.56 \times 10^{-5} \text{ m}^2$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.00625^{3/2}\left[\frac{25}{(43)(1.56 \times 10^{-5})}\right]^{1/2} = 0.095 \qquad \eta_f = 97\%$$

$q = (0.97)(25)(2)\pi(0.01625^2 - 0.01^2)(260 - 93) = 4.17 \text{ W}$

2-80

$k = 43 \quad t = 2 \text{ cm} \quad h = 20 \quad L_c = 15 \text{ cm}$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.723 \qquad \eta_f = 0.75$$

$q = (0.75)(20)(2)(0.15)(200 - 15) = 833 \text{ W/m depth}$

2-81

$t = 1.6$ mm $\quad r_1 = 1.25$ cm $\quad L = 12.5$ mm $\quad T_0 = 200°C$

$T_\infty = 20°C \quad h = 60\ \dfrac{W}{m^2 \cdot °C} \quad k = 204\ \dfrac{W}{m \cdot °C} \quad L_c = 13.3$ mm

$r_{2c} = 2.58$ cm $\quad \dfrac{r_{2c}}{r_1} = 2.064 \quad A_m = (0.0016)(0.0133) = 2.128 \times 10^{-5}\ m^2$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.18 \qquad \eta_f = 95\%$$

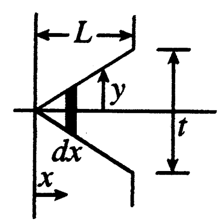
$$q = (0.95)(60)(2)\pi(0.0258^2 - 0.0125^2)(200 - 20) = 32.84\ W$$

2-83

$$A = 2y = \frac{tx}{L} \qquad y = \frac{\frac{t}{2}}{L}x = \frac{tx}{2L}$$

$$-kA\frac{dT}{dx} = hPdx(T - T_\infty) - \left[kA\frac{dT}{dx} + \frac{d}{dx}\left(kA\frac{dT}{dx}\right)dx\right]$$

$$\frac{d}{dx}\left(kA\frac{dT}{dx}\right) - hP(T - T_\infty) = 0 \qquad \theta = T - T_\infty$$



$$\frac{ktx}{L}\frac{d^2\theta}{dx^2} + \frac{kt}{L}\frac{d\theta}{dx} - hP\theta = 0$$

$$x\frac{d^2\theta}{dx^2} + \frac{d\theta}{dx} - \frac{hPL}{kt}\theta = 0$$

2-84

$r_1 = 0.05 \quad r_2 = 0.2 \quad L = 0.15$

$L_c = 0.1 + 0.001 = 0.101 \quad r_{2c} = 0.201$

$$\frac{r_{2c}}{r_1} = 4$$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.101)^{3/2}\left[\frac{60}{(120)(0.101)(0.002)}\right]^{1/2} = 2.388$$

$\eta_f = 0.16$

$$q = \eta_f hA\theta_0 = (0.16)(60)\pi(0.201^2 - 0.05^2)(2)(120 - 23) = 222\ W$$

2-85

$k = 16 \quad h = 40 \quad T_0 = 250°C \quad T_\infty = 90°C$

$P = (4)(0.0125) = 0.05$ m $\quad A = (0.0125)^2 = 1.565 \times 10^{-4}\ m^2$

$$q = \sqrt{hPkA}\,\theta_0 = [(40)(0.05)(16)(1.565 \times 10^{-4})]^{1/2}(250 - 90) = 11.31\ W$$

2-86

$t = 2.1$ mm $\quad L = 17$ mm $\quad h = 75 \quad k = 164 \quad T_0 = 100°C$

$T_\infty = 30°C \quad L_c = 17 + 1.05 = 18.05$ mm

$A_m = (0.0021)(0.01805) = 3.79 \times 10^{-5}\ m^2$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.01805)^{3/2}\left[\frac{75}{(164)(3.79 \times 10^{-5})}\right] = 0.266 \qquad \eta_f = 94\%$$

$q = (0.94)(75)(2)(0.01805)(100 - 30) = 178.2$ W

2-87

$L_c = 0.0574$ ft $\qquad r_{2c} = 2.688$ in.

$A_m = (0.125)(0.688) = 0.081\ in^2 = 5.97 \times 10^{-4}\ ft^2$

$$\frac{r_{2c}}{r_1} = 1.34 \qquad L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.329 \qquad \eta_f = 87\%$$

$$q_{max} = 2h\pi(r_{2c}^2 - r_1^2)(450 - 100) = 591\ \frac{\text{Btu}}{\text{hr}}$$

$$q = (0.87)(591) = 514\ \frac{\text{Btu}}{\text{hr}}$$

2-88

Calculate heat lost (not temp. at tip)

$d = 1.5$ mm $\quad k = 19 \quad L = 12$ mm $\quad T_0 = 45°C \quad T_\infty = 20°C$

$h = 500$

Use insulated tip solution

$$L_c = L + \frac{d}{4} = 12 + 0.375 = 12.375\ \text{mm}$$

$$m = \left(\frac{hP}{kA}\right)^{1/2} = \left[\frac{(500)\pi(0.0015)}{(19)\pi(0.0015)^2(4)}\right]^{1/2} = 264.9$$

$mL_c = (0.012375)(264.9) = 3.278$

$q = \sqrt{hPkA}\theta_0 \tanh(mL)$

$$= \left[(500)\pi(0.0015)(19)\pi(0.0015)\left(\frac{2}{4}\right)\right]^{1/2}(45 - 20)\tanh(3.278) = 0.177\ \text{W}$$

For $h = 200 \qquad mL_c = 2.073 \qquad \tanh(mL_c) = 0.969$

$$q = \left(\frac{200}{500}\right)^{1/2}(0.177)\left(\frac{0.969}{0.997}\right) = 0.109\ \text{W}$$

For $h = 1500 \qquad mL_c = 5.677 \qquad \tanh(mL_c) = 1.0$

$$q = \left(\frac{1500}{500}\right)^{1/2}(0.177)\left(\frac{1.0}{0.997}\right) = 0.307\ \text{W}$$

2-89

$k = 204$; $T_\infty = 20°C$; $T_0 = 70°C$

$L = 25$ mm

$h = 13.2$; $d = 2$ mm; $N = 225$ pins

$m = [(13.2)(4)/(204)(0.002)]^{1/2} = 11.38$

$L_c = 0.025 + 0.002/4 = 0.0255$

$q/\text{pin} = (hPkA)^{1/2}\, \theta_0 \tanh(mL_c)$

$= [(13.2)\pi(0.002)(204)\pi(0.001)^2]^{1/2}\, (70 - 20)\tanh[(11.38)(0.0255)]$

$= 0.1029$ W/ pin fin

total = (225)(0.1029) = 23.1`5 W

2-90

$k = 204$; $N = 8$; $T_0 = 100°C$; $T_\infty = 30°C$; $h = 15$; $L = 0.02$; $t = 0.002$

$P = (2)(0.15 + 0.002) = 0.304$

$A = (0.002)(0.15) = 0.0003$

$m = [(15)(0.304)/(294)(0.0003)]^{1/2} = 8.632$

$L_c = 0.02 + 0.001 = 0.021$

$q/\text{fin} = (hPkA)^{1/2}\, \theta_0 \tanh(mL_c)$

$= [(15)(0.304)(204)(0.0003)]^{1/2}(100 - 30) \tanh[(8.632)(0.021)]$

$= 6.62$ W/fin

Total = (8)(6.62) = 53 W

2-91

Surface area from Prob. 2-90 = (8)(0.304)(0.02 = 0.04864

Area per circular fin = $(2)\pi(0.0325^2 - 0.0125^2) = 0.00565$

Number of circular fins = 0.04865/0.0565 = 8.6 Round off to 9 fins

$r_1 = 0.0125$; $r_2 = 0.0325$; $L_c = 0.02 + 0.001 = 0.021$

$r_{2c} = 0.0335$; $r_{2c}/r_1 = 2.68$

$L_c^{3/2}(h/kA_m)^{1/2} = 0.1273$

$\eta_f = 0.98$

For 9 fins;

$q = (9)(0.98)((15)\pi(0.0335^2 - 0.0125^2)(100 - 30)(2)$

$= 56.2$ W

2-92

$\theta = c_1 e^{-mx} + c_2 e^{+mx}$

$\theta = 100 - 20 = 80$ at $x = 0$

$\theta = 35 - 20 = 15$ at $x = 0.06$

$$m = \sqrt{\frac{hP}{kA}}$$

$$80 = c_1 + c_2 \qquad (1)$$

$$15 = c_1 e^{-m(0.06)} + c_2 e^{+m(0.06)} \qquad (2)$$

$$-k[c_1 e^{-m(0.06)}(-m) + c_2 e^{+m(0.06)}(+m)] = h(15) \qquad (3)$$

$$m = \left[\frac{h\pi(0.02)(4)}{k(0.02)^2}\right]^{1/2} \qquad (4)$$

4 Equations, 4 unknowns, c_1, c_2, m, h.
Solve, and then evaluate q from Eq. (2–37) or (2–36) using L_c

2-93

$L = 2.5$ cm $\qquad t = 1.5$ mm $\qquad k = 50 \ \frac{\text{W}}{\text{m} \cdot {}^\circ\text{C}}$ $\qquad T_\infty = 20^\circ\text{C}$ $\qquad h = 500$

$T_0 = 200^\circ\text{C}$

$L_c = 0.025 + 0.00075 = 0.02575$

$q_{\max} = (2)(500)(0.02595)(200 - 20) = 4635$ W/m

$A_m = (0.0015)(0.02575) = 3.863 \times 10^{-5}$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right) = (0.02578)^{3/2}\left[\frac{500}{(50)(3.863 \times 10^{-5})}\right]^{1/2} = 2.1$$

$\eta_f = 0.36$

$q = (0.36)(4635) = 1669$ W

2-94

$L = 3.5$ cm $\qquad t = 1.4$ mm $\qquad L_c = 3.57$ cm $\qquad k = 55$

$q_{\max} = hA\theta_0 = (500)(2)(0.0357)(150 - 20) = 4641$ W/m

$$mL_c = \left(\frac{2h}{kA_m}\right)^{1/2} L_c^{3/2} = 4.068$$

$$\eta_f = \frac{\tanh(mL_c)}{mL_c} = 0.246 \qquad q_{act} = (0.246)(4641) = 1140 \text{ W/m}$$

2-95

$k = 43$ $h = 100$ $r_1 = 2.5$ cm $r_2 = 7.5$ cm $L = 5$ cm
$T_\infty = 20°\text{C}$ $t = 2$ mm $r_{2c} = 7.51$ cm $L_c = 5.1$ cm

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 1.74 \qquad \frac{r_{2c}}{r_1} \cong 3 \qquad \eta_f = 0.27$$

$$q = \eta_f 2h\pi(r_{2c}^2 - r_1^2)\theta_0 = (0.27)(100)(2)\pi(0.0751^2 - 0.025^2)(150-20)$$
$$= 110.6 \text{ W}$$

2-96

$r_1 = 1.5$ cm $L = 2$ cm $r_2 = 3.5$ cm $t = 1$ mm $h = 80$
$k = 200$ $L_c = 2.05$ cm $r_{2c} = 3.55$ cm

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.41 \qquad \frac{r_{2c}}{r_1} = 2.37 \qquad \eta_f = 0.81$$

$$q = (80)\pi(0.0355^2 - 0.015^2)(2)(200-20)(0.81) = 75.9 \text{ W}$$

2-97

$h = 50$ $k = 20$

$$m = \left(\frac{hP}{kA}\right)^{1/2} = \left[\frac{(50)\pi(0.01)(4)}{(20)\pi(0.01)^2}\right]^{1/2} = 31.62$$

$mL = (0.2)(31.62) = 6.324$

$e^{mL} = 557.8$ $e^{-mL} = 0.00179$

$e^{mx} = 2362$ $e^{-mx} = 0.0423$

$\theta_1 = 50 - 20 = 30$ $\theta_2 = 100 - 20 = 80$

Using solution from Prob 2–61

$$\theta_{(x=10\text{ cm})} = \frac{(0.0423)[80-(30)(557.8)] + (23.62)[(30)(0.00179) - 80]}{0.00179 - 557.8}$$
$$= \frac{-704.46 - 1888.33}{-557.8}$$
$$= 4.65°\text{C}$$

$T = 20 + 4.65 = 24.65°\text{C}$

2-98

$k = 386$ $\quad r_1 = 0.625\text{ cm}$ $\quad L = 0.6\text{ cm}$ $\quad h = 55$

$t = 0.3\text{ mm}$ $\quad L_c = 0.6 + 0.015 = 0.615\text{ cm}$

$r_{2c} = 0.625 + 0.615 = 1.24$

$$\frac{r_{2c}}{r_1} = \frac{1.24}{0.625} \cong 20$$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.00615)^{3/2}\left[\frac{55}{(386)(0.00615)(0.0003)}\right]^{1/2} = 0.134$$

$\eta_f = 0.95$

$$q = \eta_f hA\theta_0 = (0.95)(55)\pi(2)(0.0124^2 - 0.00625^2)(100 - 20) = 3.012\text{ W}$$

2-99

$t = 2\text{ cm}$ $\quad L = 17\text{ cm}$ $\quad k = 43$ $\quad h = 23$ $\quad L_c = 18\text{ cm}$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.93 \qquad \eta_f = 0.64$$

$$q = (0.64)(2)(23)(0.18)(230 - 25) = 1086\text{ W/m}$$

2-100

$L = 5\text{ cm}$ $\quad L_c = 5\text{ cm}$ $\quad t = 4\text{ mm}$ $\quad k = 23$ $\quad h = 20$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 1.042 \qquad \eta_f = 0.68 \qquad q = \eta_f hA\theta_0$$

$$A = (2)(0.002^2 + 0.05^2)^{1/2} = 0.10008\ \frac{\text{m}^2}{\text{m depth}}$$

$$q = (0.68)(20)(0.10008)(200 - 40) = 217.8\text{ W/m}$$

2-101

$t = 1.0\text{ mm}$ $\quad r_1 = 1.27\text{ cm}$ $\quad L = 1.27\text{ cm}$ $\quad L_c = 1.32\text{ cm}$ $\quad r_2 = 2.54\text{ cm}$

$r_{2c} = 2.59\text{ cm}$ $\quad h = 56$ $\quad k = 204$ $\quad L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.219$

$$\frac{r_{2c}}{r_1} = 2.04 \qquad \eta_f = 0.93$$

$$q = (0.93)(2)\pi(0.0259^2 - 0.0127^2)(56)(125 - 30) = 15.84\text{ W}$$

2-102

$t = 2$ mm $\quad r_1 = 2.0$ cm $\quad r_2 = 10.0$ cm $\quad L = 8$ cm $\quad L_c = 8.1$ cm

$r_{2c} = 10.2$ cm $\quad h = 20 \quad k = 17 \quad L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2} = 1.96$

$\dfrac{r_{2c}}{r_1} = 5.1 \quad \eta_f = 0.19$

$q = (0.19)(20)\pi(0.102^2 - 0.02^2)(2)(135 - 15) = 28.7$ W

2-103

$L = 2.5$ cm $\quad t = 1.1$ mm $\quad k = 55 \quad h = 500 \quad L_c = 2.555$ cm

$L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2} = 2.32 \quad \eta_f = 0.33$

$q = (0.33)(2)(0.02555)(500)(125 - 20) = 885$ W/m

2-104

$t = 1.0$ mm $\quad r_1 = 1.25$ cm $\quad r_2 = 2.5$ cm $\quad r_{2c} = 2.55$ cm

$h = 25 \quad k = 204 \quad L = 1.25$ cm $\quad L_c = 1.3$ cm

$L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2} = 0.249 \quad \eta_f = 0.91$

$q = (0.91)(2)(75)\pi(0.0255^2 - 0.0125^2)(100 - 30) = 4.94$ W

2-105

$d = 1$ cm $\quad L = 5$ cm $\quad h = 20 \quad k = 0.78 \quad T_0 = 180$

$T_\infty = 20$

$L_c = L + \dfrac{d}{4} = 5 + 0.25 = 5.25$ cm

$$\left(\frac{hP}{kA}\right)^{1/2} = m = \left[\frac{(20)\pi(0.01)(4)}{(0.78)\pi(0.01)^2}\right]^{1/2} = 101.3$$

$mL_c = (101.3)(0.0525) = 5.317$

$\tanh(5.317) = 1.0$

$$q = (hPkA)^{1/2}\theta_0 \tanh(mL_c) = \left[\frac{(20)\pi(0.01)(0.78)\pi(0.01)^2}{4}\right]^{1/2}(180 - 20)(1.0)$$

$$= 0.993 \text{ W}$$

2-106

$A = 1 \times 1 \text{ cm}^2$ $\quad L = 8 \text{ cm}$ $\quad L_c = 8.5 \text{ cm}$ $\quad m = \left[\frac{hP}{kA}\right]^{1/2} = 31.62$

$mL_c = 2.688$ $\quad \eta_f = \frac{\tanh(mL_c)}{mL_c} = 0.369$

$q = (0.369)(45)(0.085)(4)(0.01)(300 - 50) = 14.11 \text{ W}$

2-107

$t = 1.0 \text{ mm}$ $\quad r_1 = 1.25 \text{ cm}$ $\quad L = 12 \text{ mm}$ $\quad T_0 = 275°\text{C}$

$T_\infty = 25°\text{C}$ $\quad h = 120$ $\quad k = 386$

$L_c = 0.012 + 0.0005 = 0.0125$ $\quad r_{2c} = 0.025$ $\quad \frac{r_{2c}}{r_1} = 2.0$

$A_m = (0.0125)(0.001) = 1.25 \times 10^{-5}$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.0125)^{3/2}\left[\frac{120}{(386)(1.25 \times 10^{-5})}\right]^{1/2} = 0.22$$

$\eta_f = 0.93$

$q = (0.93)(120)\pi(0.025^2 - 0.0125^2)(2)(275 - 25) = 82.12 \text{ W}$

2-108

$k = 17$ $\quad h = 47$ $\quad L = 5 \text{ cm}$ $\quad t = 2.5 \text{ cm}$ $\quad L_c = 6.25 \text{ cm}$

$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.657$ $\quad \eta_f = 0.8$

$q = (0.8)(47)(2)(0.0625)(100 - 20) = 376 \text{ W/m}$

2-109

$r_1 = 1.5 \text{ cm}$ $\quad L = 2 \text{ cm}$ $\quad r_2 = 3.5 \text{ cm}$ $\quad t = 1 \text{ mm}$ $\quad r_{2c} = 3.55 \text{ cm}$

$h = 80$ $\quad k = 204$ $\quad L_c = 2.05 \text{ cm}$ $\quad L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.453$

$\frac{r_{2c}}{r_1} = 2.37$ $\quad \eta_f = 0.79$

$q = (0.79)(80)(2)\pi(0.0355^2 - 0.015^2)(200 - 20) = 74 \text{ W}$

2-110

$r_1 = 1.5\text{ cm} \qquad r_2 = 4.5\text{ cm} \qquad t = 1.0\text{ mm} \qquad h = 50 \qquad r_{2c} = 4.55\text{ cm}$

$L_c = 3.05\text{ cm} \qquad k = 204 \qquad \eta_f = 0.6 \qquad \dfrac{r_{2c}}{r_1} = 3$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.78$$

$$k = \frac{1}{(0.001)(0.0305)}(0.0305)^3\frac{50}{(0.78)^2} = 76.5\ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$$

2-111

$t = 1.0\text{ mm} \qquad L = 2.0\text{ cm} \qquad r_1 = 1.0\text{ cm} \qquad h = 150 \qquad k = 204$

$L_c = 2.05\text{ cm} \qquad r_{2c} = 3.05\text{ cm} \qquad \dfrac{r_{2c}}{r_1} = 3.05 \qquad T_0 = 150$

$T_\infty = 20$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.0205)^{3/2}\left[\frac{150}{(204)(0.001)(0.0205)}\right]^{1/2} = 0.556$$

$\eta_f = 0.75$

$$q = (0.75)(150)(2)\pi(0.0305^2 - 0.01^2)(150 - 20) = 76.3\text{ W}$$

2-112

$k_A = k_B = 17 \quad A_c = 0.001A \quad \Delta T = 300°\text{C} \quad A = \dfrac{\pi d^2}{4} = 5.067\times10^{-4}\text{ m}^2$

$\dfrac{L_g}{2} = 1.3\times10^{-6}\text{ m} \qquad k_f = 0.035 \qquad L_A = L_B = 7.5\text{ cm}$

$$h_c = \frac{1}{L_g}\left[\frac{A_c}{A}\left(\frac{2k_Ak_B}{k_A + k_B}\right) + \frac{A_v}{A}k_f\right] = 19{,}986\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$\frac{1}{h_cA} = \frac{1}{(19{,}986)(5.067\times10^{-4})} = 0.0987\ {}^\circ\text{C/W}$$

$$q = \frac{300}{\frac{(0.075)(2)}{(17)(5.067\times10^{-4})} + 0.0987} = 17.31\text{ W} \quad \text{w/no contact resistance}$$

$$q = kA\frac{\Delta T}{\Delta x} = \frac{(17)(5.067\times10^{-4})(300)}{0.15} = 17.228\text{ W}$$

2-114

$$R_{th} = \frac{\Delta x}{kA} = \frac{5\times 10^{-3}}{204} = 2.45\times 10^{-5} \qquad R_c = \frac{1}{h_c A} = 0.88\times 10^{-4}$$

$$\sum R_{th} = (2)(2.45\times 10^{-5}) + 0.88 + 10^{-4} = 1.37\times 10^{-4}$$

$$\Delta T_c = \frac{(80)(0.88\times 10^{-4})}{1.37\times 10^{-4}} = 51.4°\text{C}$$

2-115

$t = 1\times 10^{-3}$ $\qquad r_i = 0.0125$ $\qquad L = 0.0125$ $\qquad L_c = 0.0130$

$r_{2c} = 0.0255$ $\qquad \frac{r_0}{r_i} = 2.04$ $\qquad A_m = (0.001)(0.0130) = 1.3\times 10^{-5}\ \text{m}^2$

$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.322$ $\qquad \eta_f = 0.86$ $\qquad \frac{1}{h_c} = 0.88\times 10^{-4}$ $\qquad T_0 = 200°\text{C}$

$T_\infty = 20°\text{C}$

$$R_{\text{fin}} = \frac{1}{2\eta_f \pi (r_{2c}^2 - r_1^2)h} = 2.997\ °\text{C/W}$$

$$R_c = \frac{1}{h_c A} = 1.1205\ °\text{C/W}$$

$$q = \frac{\theta_0}{R_{\text{fin}} + R_c} = 43.72\ \text{W} \qquad \text{w/o contact resistance}$$

$$q = \frac{\theta_0}{R_{\text{fin}}} = 60.06\ \text{W} \qquad \%\ \text{Reduction} = \frac{60.06 - 43.72}{60.06}\times 100\% = 27.2\%$$

2-116

$\frac{1}{h_c} = 0.9\times 10^{-4}$ $\qquad A_c = 0.5\ \text{cm}^2$ $\qquad q = 300\ \text{mW}$

Assume fin at 27°C

$$q = (300\times 10^{-3}) = \frac{1}{0.9\times 10^{-4}}(0.5)(10^{-4})(T_t - 27)$$

$$T_t = 27.54°\text{C}$$

2-117

$2L = 20\text{ cm} = 0.2\text{ m} \qquad T_\infty = 50°\text{C}$

$$\frac{q}{A} = \dot{q}(2L) = (2\times10^5)(-0.2) = 40{,}000\ \text{W/m}^2 = 2h(T_w - T_\infty) = (400)(T_w - 50)(2)$$

$T_w = 100°\text{C}$

$$T_0 - T_w = \frac{\dot{q}L^2}{2k} = \frac{(2\times10^5)(0.1)^2}{(2)(20)} = 50°\text{C}$$

$T_0 = 150°\text{C}$

2-118 2

$\frac{1}{2}$ heat generated, max temperature at insulated surface which is the same as in Problem 2–~~111~~ 117.

2-119

$L_c = 0.03 + 0.001 = 0.031 \qquad k = 204 \qquad h = 220$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.031)^{3/2}\left[\frac{220}{(204)(0.031)(0.002)}\right]^{1/2} = 0.715$$

$\eta_f = 0.69$

$$q = \eta_f hA\theta_0 = (0.69)\pi(0.061^2 - 0.03^2)(2)(220)(120-20) = 269\ \text{W}$$

2-120

$L_c = 0.008 \qquad k = 204\ \dfrac{\text{W}}{\text{m}\cdot°\text{C}} \qquad h = 45\ \dfrac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.008)^{3/2}\left[\frac{(45)(2)}{(204)(0.008)(0.002)}\right]^{1/2} = 0.119$$

$\eta_f = 0.97$

$$q = (2)(0.97)(0.008^2 + 0.001^2)^{1/2}(45)(200-25) = 123.2\ \text{W/m}$$

2-121

$r_1 = 1.25$ cm $\quad r_2 = 2.25$ cm $\quad t = 2.0$ mm $\quad r_{2c} = 2.35$ cm

$\frac{r_{2c}}{r_1} = 1.88 \quad L_c = 1.1$ cm $\quad T_0 = 180°C \quad T_\infty = 20°C$

$h = 50 \ \frac{W}{m^2 \cdot °C} \quad k = 204 \ \frac{W}{m \cdot °C}$

L of tube $= 1.0$ m

bare length $= 1.0 - (100)(0.002) = 0.8$ m

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.011)^{3/2}\left[\frac{50}{(204)(0.002)(0.011)}\right]^{1/2} = 0.122$$

$\eta_f = 0.95$

$q(100 \text{ fins}) = (100)(2)(0.95)\pi(0.0235^2 - 0.0125^2)(50)(180 - 20) = 1869$ W

$q(\text{bare tube}) = (50)\pi(0.025)(0.8)(180 - 20) = 503$ W

$q(\text{total}) = 1869 + 503 = 2372$ W

2-122

$k = 100 \ \frac{W}{m \cdot °C} \quad q = kA\frac{\Delta T}{\Delta x}$

$A = (1.7 - 1.5) \text{ cm} = \frac{0.2}{100} = 0.002 \ m^2/m$

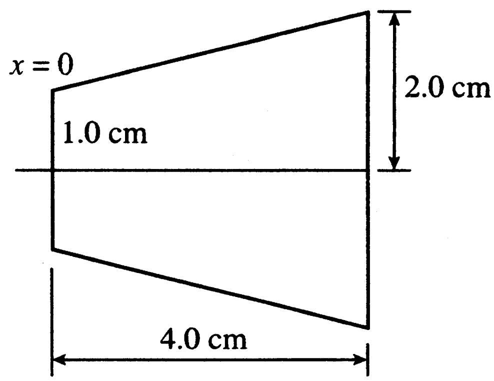
$\Delta x =$ circumference at $r = 1.6$ cm for $\frac{\pi}{4}$

$\Delta x = (2)(1.6)\left(\frac{\pi}{4}\right) = 2.513 \text{ cm} = 0.02513$ m

$$R = \frac{\Delta x}{kA} = \frac{0.02513}{(100)(0.002)} = 0.1256°C^{-1}$$

$$\frac{q}{L} = \frac{50}{0.1256} = 398 \ W/m$$

2-123



$k = 386 \dfrac{\text{W}}{\text{m} \cdot {}^\circ\text{C}}$ $\qquad r = ax + b \qquad$ $r = 0.01$ at $x = 0$

$r = 0.02$ at $x = 0.04$

$0.01 = b \qquad a = 0.25$

$0.02 = (0.04)a + 0.01$

$r = 0.25x + 0.01 \qquad A = 2\pi r(0.0005)$

$$q = -k2\pi(0.25x + 0.01)(0.0005)\frac{dT}{dx}$$

$$\int_0^{0.04} \frac{dx}{0.25x + 0.01} = \int -2\pi k(0.0005)\frac{dT}{q}$$

$$\frac{1}{0.25}\ln\left[\frac{(0.25)(0.04) + 0.01}{0.01}\right] = 2.773 = 2\pi(386)(0.0005)\frac{\Delta T}{q} = 1.213\frac{\Delta T}{q}$$

$q = \dfrac{\Delta T}{2.287} \qquad R = 2.287$

For $\Delta T = 300°\text{C} \qquad q = \dfrac{300}{2.287} = 131.2\ \text{W}$

2-124

Tube $d_i = 1.25 \text{ cm} = 12.5 \text{ mm}$ $\quad T_{\text{inside}} = 100°\text{C}$ $\quad \text{thk} = 0.8 \text{ mm}$

$d_0 = 14.1 \text{ mm}$ $\quad T_\infty = 20°\text{C}$

Fins: $T = 0.3 \text{ mm}$ $\quad L = 3 \text{ mm}$ $\quad L_c = 3 + 0.15 = 3.15 \text{ mm}$

$r_1 = 6.25 + 0.8 = 7.05 \text{ mm}$ $\quad r_{2c} = 7.05 + 3.15 = 10.20 \text{ mm}$

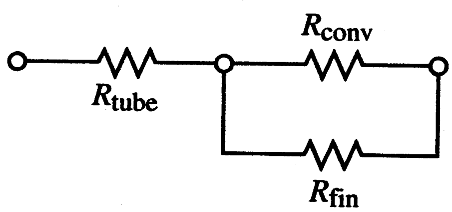
$h = 50 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$ $\quad k = 386 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}}$

No. of fins $= \dfrac{300}{6} = 50$ for 30 cm length

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.00315)^{3/2}\left[\frac{50}{(386)(0.00315)(0.0003)}\right]^{1/2} = 0.0654$$

$$\frac{r_{2c}}{r_1} = \frac{10.2}{7.05} = 1.447 \qquad \eta_f = 0.98$$

$$\frac{R_{\text{tube}}}{L} = \frac{\ln(14.1/12.5)}{2\pi(386)} = 4.97 \times 10^{-5} \qquad \frac{R_{\text{conv}}}{L} = \frac{1}{(50)\pi(0.0141)} = 0.4515$$



For 6 mm length

$L_{\text{tube}} = 6 - 0.3 = 5.7 \text{ mm}$

$$R_{\text{tube}} = \frac{4.97 \times 10^{-5}}{0.0057} = 8.72 \times 10^{-3} \qquad R_{\text{conv}} = \frac{0.4515}{0.0057} = 79.2$$

$$q_{\text{fin}} = \frac{\Delta T}{R_{\text{fin}}} = (0.98)(50)(2\pi)(0.0102^2 - 0.00705^2)\Delta T = 0.01672\Delta T$$

$R_{\text{fin}} = 59.78$

$$R_{\text{overall}} = 8.72 \times 10^{-3} + \frac{1}{\frac{1}{79.2} + \frac{1}{59.78}} = 34.07$$

$$q = \frac{100 - 20}{34.07} = 2.348 \ \text{W/6 mm length}$$

For 30 cm length

$$q = (2.348)\left(\frac{300}{6}\right) = 117.4 \text{ W} \qquad R = \frac{100 - 20}{117.4} = 0.6814$$

For 1 m length

$$q = (117.4)\left(\frac{1}{0.3}\right) = 392 \text{ W}$$

2-125

For 6 mm length total surface area

$$= A_{\text{tube}} + A_{\text{fin}} = \pi(0.0141)(0.0057) + (2)\pi(0.0102^2 - 0.00705^2)$$
$$= 5.939 \times 10^{-4}\ \text{m}^2$$
$$\frac{q}{A} = \frac{2.348}{5.939 \times 10^{-4}} = 3953\ \text{W/m}^2$$
$$= \frac{\Delta T}{R\text{-value}}$$
$$R\text{-value} = \frac{100-20}{3953} = 0.0202$$

2-126

$$k = 204\ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}} \qquad L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.0654)\left(\frac{386}{204}\right)^{1/2} = 0.09$$
$$\eta_f = 0.96$$

For 6 mm length

$$R_{\text{tube}} = 8.72 \times 10^{-3}$$
$$R_{\text{conv}} = 79.2$$
$$R_{\text{fin}} = (59.78)\left(\frac{0.98}{0.96}\right) = 61.03$$
$$R_{\text{overall}} = 8.72 \times 10^{-3} + \frac{1}{\frac{1}{79.2} + \frac{1}{61.03}} = 34.47$$
$$q = \frac{100-20}{34.47} = 2.32\ \text{W/ 6 mm length}$$

For 30 cm length

$$q = (2.32)\left(\frac{300}{6}\right) = 116\ \text{W}$$
$$R = \frac{100-20}{116} = 0.6896$$

For 1 m length $q = (116)\left(\frac{1}{0.3}\right) = 387\ \text{W}$

2-127

$$\frac{q}{A} = \frac{2.32}{5.939 \times 10^{-4}} = 3906\ \text{W/m}^2 = \frac{\Delta T}{R\text{-value}}$$
$$R\text{-value} = \frac{110-20}{3906} = 0.0205$$

2-128

$h = 100 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$ $\quad d = 2 \text{ mm} \quad T_\infty = 20°\text{C} \quad T_0 = 100°\text{C} \quad L = 10 \text{ cm}$

$k = 16 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}}$ $\quad \theta = c_1 e^{-mx} + c_2 e^{mx}$

$$m = \left(\frac{hP}{kA}\right)^{1/2} = \left[\frac{(100)\pi(0.002)}{(16)\pi(0.001)^2}\right]^{1/2} = 111.8$$

$\theta = 100 - 20 = 80$ at $x = 0$ and $x = 0.1$

$80 = c_1 + c_2$

$80 = 1.395 \times 10^{-5} c_1 + 71{,}682 c_2$

$c_1 = 79.999 \quad c_2 = 1.117 \times 10^{-3}$

$q = \sqrt{hPkA}[-c_1 e^{-mx} + c_2 e^{mx}]$

$\sqrt{hPkA} = [(100)\pi(0.002)(16)\pi(0.001)^2]^{1/2} = 5.62 \times 10^{-3}$

$q_0 = -(5.62 \times 10^{-3})(-79.999 + 1.12 \times 10^{-3}) = 0.45 \text{ W} = -q_L$

$q_{\text{total}} = (2)(0.45) = 0.9 \text{ W}$

2-129

$T_\infty = 20°\text{C} \quad h = 100 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \quad k = 16 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}} \quad d = 2 \text{ mm}$

$T_0 = 100°\text{C}$

$\theta = c_1 e^{-mx} + c_2 e^{mx}$

For very long rod $c_2 = 0$

Acts like <u>two</u> long rods 100°C on each end

$q = 2\sqrt{hPkA}\theta_0 = (2)[(100)\pi(0.002)(16)\pi(0.001)^2]^{1/2}(100 - 20) = 0.9 \text{ W}$

2-130

$A = (0.017 - 0.015)(1) = 0.002 \ \text{m}^2/\text{m}$ depth

$k = 100 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}} \quad h = 75 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \quad T_\infty = 30°\text{C} \quad T_0 = 100°\text{C}$

$T_L = 50°\text{C} \quad L = \dfrac{2\pi}{4}(0.016) = 0.02513 \text{ m}$

$\theta = T - T_\infty = c_1 e^{-mx} + c_2 e^{mx}$

$$m = \left(\frac{hP}{kA}\right)^{1/2} = \left(\frac{2h}{kt}\right)^{1/2} = \left[\frac{(2)(75)}{(100)(0.002)}\right]^{1/2} = 27.39$$

$mL = 0.6882$

$100 - 30 = c_1 + c_2$

$50 - 30 = c_1 e^{-0.6882} + c_2 e^{0.6882}$

$c_1 = 80.2 \qquad c_2 = -10.2$

$q_x = -kAm(-c_1e^{-mx} + c_2e^{mx})$

$q_{x=0} = -(100)(0.002)(27.39)(-80.2 - 10.2) = 495 \text{ W}$

$q_{x=L} = -(100)(0.002)(27.39)[-(80.2)(0.5024) - (10.2)(1.99)] = 332 \text{ W}$

$q_{\text{net lost}} = 495 - 332 = 163 \text{ W/m depth}$

2-131

$t = 3.0 \text{ mm} \qquad k = 204 \dfrac{\text{W}}{\text{m}\cdot°\text{C}} \qquad h = 50 \dfrac{\text{W}}{\text{m}^2\cdot°\text{C}} \qquad \rho = 2707 \text{ kg/m}^3$

Take $L = 2$ cm

Rect. Fin $\qquad L_c = 2 + 0.15 = 2.15$ cm

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.0215)^{3/2}\left[\frac{50}{(204)(0.003)(0.0215)}\right]^{1/2} = 0.1943$$

$\eta_f = 0.96$

$A_{\text{surf}} = (2)L_c = 0.043$

For same weight A_m = same

$$t(0.0215) = \frac{t}{2}L(\text{triang})$$

$$L(\text{triang}) = 0.043$$

$$(\text{triang})L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.5496$$

$\eta_f = 0.85$

$q \sim A_{\text{surf}}\eta_f$

$A_{\text{surf}}(\text{triang}) = 2(4.3^2 + 0.15^2)^{1/2} = 8.6 = 0.086 \text{ m}^2$

$A_{\text{surf}}\eta(\text{triangle}) = (0.086)(0.85) = 0.0731$

$A_{\text{surf}}\eta(\text{rect}) = (0.043)(0.96) = 0.0413$

Triangle fin produces more heat transfer for given weight.

2-132

$r_1 = 1.0 \qquad r_2 = 2.0 \text{ cm} \qquad h = 160 \dfrac{\text{W}}{\text{m}^2\cdot°\text{C}} \qquad k = 204 \dfrac{\text{W}}{\text{m}\cdot°\text{C}}$

1.0 mm Fin

$L_c = 1.05 \text{ cm}$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.0105)^{3/2}\left[\frac{160}{(204)(0.001)(0.0105)}\right]^{1/2} = 0.294$$

$\eta_f = 0.88$

$q = (6)(160)\pi(0.0205^2 - 0.01^2)(2)(\Delta T)(0.88) = 1.7\Delta T$

2.0 mm Fin

$L_c = 1.1 \text{ cm} \qquad L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2} = 0.218 \qquad \eta_f = 0.92$

$q = (3)(160)\pi(0.021^2 - 0.01^2)(2)\Delta T(0.92) = 0.95\Delta T$

3.0 mm Fin

$L_c = 1.15 \text{ cm} \qquad L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2} = 0.186 \qquad \eta_f = 0.95$

$q = (2)(160)\pi(0.0215^2 - 0.01^2)(2)\Delta T(0.95) = 0.69\Delta T$

Conclusion: Several thin fins are better than a few thick fins. More heat transfer for the same weight of fins.

2-133

$L = 5 \text{ cm} \qquad d = 2,\ 5,\ 10 \text{ mm} \qquad T_\infty = 20°\text{C} \qquad h = 40 \dfrac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$k = 204 \dfrac{\text{W}}{\text{m}\cdot°\text{C}} \qquad T_0 = 200°\text{C} \qquad L_c = L + \dfrac{d}{4}$

2 mm pin

$L_c = 5 + 0.05 = 5.05 \text{ cm} = 0.0505 \text{ m}$

$$m = \left(\frac{hP}{kA}\right)^{1/2} = \left(\frac{h\pi d}{k\pi \frac{d^2}{4}}\right)^{1/2} = \left(\frac{4h}{kd}\right)^{1/2} = \left[\frac{(4)(40)}{(204)(0.002)}\right]^{1/2} = 19.8$$

$mL_c = (19.8)(0.0505) = 1.0$

$$\eta = \frac{\tanh(mL_c)}{mL_c} = 0.762$$

$q = (0.762)(40)\pi(0.002)(0.0505)(200 - 20) = 1.74 \text{ W}$

5 mm pin

$L_c = 5 + 0.125 \text{ cm} = 0.05125 \text{ m} \qquad m = 12.52 \qquad mL_c = 0.6419$

$$\eta = \frac{\tanh(mL_c)}{mL_c} = 0.882$$

$q = (0.882)(40)\pi(0.005)(0.05125)(200 - 20) = 5.11 \text{ W}$

<u>10 mm pin</u>

$L_c = 5 + 0.25 \text{ cm} = 0.0525 \text{ m} \qquad m = 8.856 \qquad mL_c = 0.46495$

$$\eta = \frac{\tanh(mL_c)}{mL_c} = 0.934$$

$q = (0.934)(40)\pi(0.01)(0.0525)(200 - 20) = 11.09 \text{ W}$

d (mm)	q (W)	$\frac{q}{d \text{ (mm)}}$	$\frac{q}{d^2}$ (per weight)
2	1.74	0.87	0.435
5	5.11	1.022	0.2044
10	11.09	1.109	0.1109

<u>Conclusion:</u> Smaller pins produce more heat transfer per unit weight.

2-134

See <u>conclusion</u> at end of Problem 2–1~~27~~33.

2-137

Once an insulation material is selected for this problem a commercial vendor must be consulted to determine the cost, as no cost figures are given in the problem statement. Cost figures will also have to be determined for the material with reflective coating. When the reflective material is installed it may have an emissivity of 0.1, but after a period of time the surface may oxidize or become coated with foreign matter such that its emissivity will increase. In that case the economic benefit of the coated material will be reduced.

2-138

For this problem, the net heat generated in the tube will be equal to the heat which will be delivered to the fluid by convection. The temperature gradient at the other surface of the tube will be zero. The problem does not state whether the fluid is on the outside or inside of the tube so both cases must be examined. In either case the maximum tube temperature will occur at the insulated surface. To effect the design one might first assume a surface temperature for the tube surface in contact with the fluid. This will then determine the surface area. Suitable combinations of tube length and diameter may then be examined to equal the total surface area. The heat generation equation for a hollow cylinder may then be solved for the other tube surface temperature if a tube wall thickness is assumed (i.e., establishing the other diameter). The resultant value of temperature must be reasonable, i.e., low enough. Obviously, there are many combinations which will be satisfactory.

2-145

$d^2T/dx^2 - (\varepsilon\sigma P/kA)(T^4 - T_s^4) = 0$

Boundary conditions:

Base: $T = 0$ at $x = 0$

Tip: $-kA(dT/dx)_{x=L} = \sigma\varepsilon A(T^4_{x=L} - T_s^4)$

2-146

Insulated tip: $dT/dx = 0$ at $x = L$

Very long fin: $T \rightarrow T_s$ as $x \rightarrow \infty$

2-147

Set $T_s = 0$ in differential equations