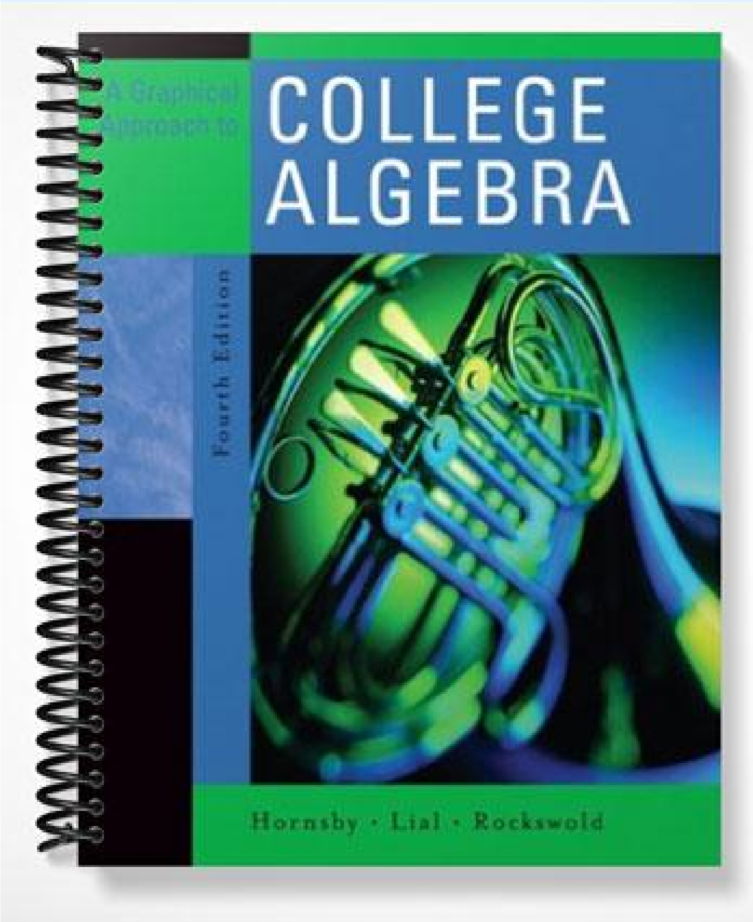
SOLUTIONS MANUAL



Chapter 2: Analysis of Graphs of Functions

2.1: Graphs of Basic Functions and Relations; Symmetry

1. $(-\infty, \infty)$
2. $(-\infty, \infty)$; $[0, \infty)$
3. $(0, 0)$
4. $[0, \infty)$; $[0, \infty)$
5. increases
6. $(-\infty, 0]$; $[0, \infty)$
7. x-axis
8. even
9. odd
10. y-axis; origin
11. The domain can be all real numbers, therefore the function is continuous for the interval: $(-\infty, \infty)$.
12. The domain can be all real numbers, therefore the function is continuous for the interval: $(-\infty, \infty)$.
13. The domain can only be values where $x \geq 0$, therefore the function is continuous for the interval: $[0, \infty)$.
14. The domain can only be values where $x \leq 0$, therefore The function is continuous for the interval: $(-\infty, 0]$.
15. The domain can be all real numbers except -3, therefore the function is continuous for the interval: $(-\infty, -3)$; $(-3, \infty)$.
16. The domain can be all real numbers except 1, therefore the function is continuous for the interval: $(-\infty, 1)$; $(1, \infty)$.
17. (a) The function is increasing for the interval: $[3, \infty)$.
    (b) The function is decreasing for the interval: $(-\infty, 3]$.
    (c) The function is never constant, therefore: none
    (d) The domain can be all real numbers, therefore the interval: $(-\infty, \infty)$.
    (e) The range can only be values where $y \geq 0$, therefore the interval: $[0, \infty)$.
18. (a) The function is increasing for the interval: $[4, \infty)$.
    (b) The function is decreasing for the interval: $(-\infty, -1]$.
    (c) The function is constant for the interval: $[-1, 4]$.
    (d) The domain can be all real numbers, therefore the interval: $(-\infty, \infty)$.
    (e) The range can only be values where $y \geq 3$, therefore the interval: $[3, \infty)$.
19. (a) The function is increasing for the interval: $(-\infty, 1]$.
    (b) The function is decreasing for the interval: $[4, \infty)$.
    (c) The function is constant for the interval: $[1, 4]$.
    (d) The domain can be all real numbers, therefore the interval: $(-\infty, \infty)$.
    (e) The range can only be values where $y \leq 3$, therefore the interval: $(-\infty, 3]$.

20. (a) The function never is increasing, therefore: none

(b) The function is always decreasing, therefore the interval: $(-\infty, \infty)$.

(c) The function is never constant, therefore: none

(d) The domain can be all real numbers, therefore the interval: $(-\infty, \infty)$.

(e) The range can be all real numbers, therefore the interval: $(-\infty, \infty)$.

21. (a) The function never is increasing, therefore: none

(b) The function is decreasing for the intervals: $(-\infty, -2]$; $[3, \infty)$.

(c) The function is constant for the interval: $(-2, 3)$.

(d) The domain can be all real numbers, therefore the interval: $(-\infty, \infty)$.

(e) The range can only be values where $y \le 1.5$ or $y \ge 2$, therefore the interval: $(-\infty, 1.5] \cup [2, \infty)$.

22. (a) The function is increasing for the interval: $(3, \infty)$.

(b) The function is decreasing for the interval: $(-\infty, -3)$.

(c) The function is constant for the interval: $(-3, 3]$.

(d) The domain can be all real numbers except -3, therefore the interval: $(-\infty, -3) \cup (-3, \infty)$.

(e) The range can only be values where $y > 1$, therefore the interval: $(1, \infty)$.

23. Graph $f(x) = x^5$, See Figure 23. As x increases for the interval: $(-\infty, \infty)$, y increases, therefore increasing.

24. Graph $f(x) = -x^3$, See Figure 24. As x increases for the interval: $(-\infty, \infty)$, y decreases, therefore decreasing.

25. Graph $f(x) = x^4$, See Figure 25. As x increases for the interval: $(-\infty, 0]$, y decreases, therefore decreasing.

26. Graph $f(x) = x^4$, See Figure 26. As x increases for the interval: $[0, \infty)$, y increases, therefore increasing.

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

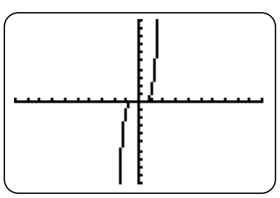


Figure 23

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

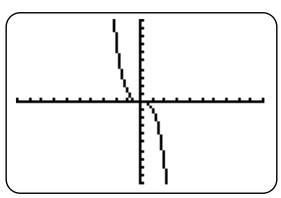


Figure 24

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

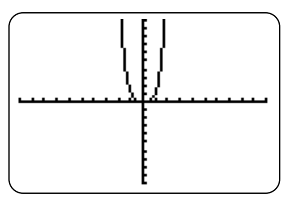


Figure 25

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

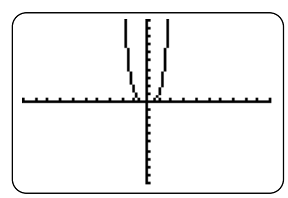


Figure 26

27. Graph $f(x) = -|x|$ See Figure 27. As x increases for the interval: $(-\infty, 0]$, y increases, therefore increasing.

28. Graph $f(x) = -|x|$, See Figure 28. As x increases for the interval: $[0, \infty)$, y decreases, therefore decreasing.

29. Graph $f(x) = -\sqrt[3]{x}$, See Figure 29. As x increases for the interval: $(-\infty, \infty)$, y decreases, therefore decreasing.

30. Graph $f(x) = -\sqrt{x}$, See Figure 30. As x increases for the interval: $[0, \infty)$, y decreases, therefore decreasing.

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

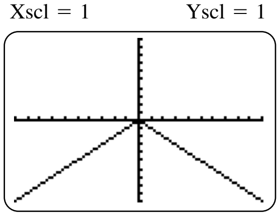


Figure 27

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

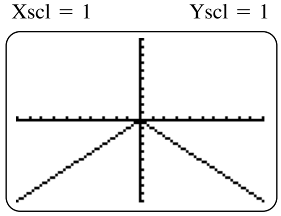


Figure 28

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

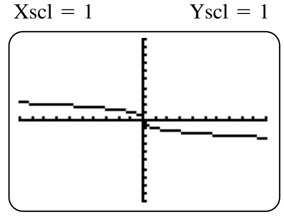


Figure 29

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

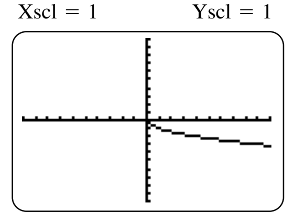


Figure 30

31. Graph $f(x) = 1 - x^3$, See Figure 31. As x increases for the interval: $(-\infty, \infty)$, y decreases, therefore decreasing.

32. Graph $f(x) = x^2 - 2x$, See Figure 32. As x increases for the interval: $[1, \infty)$, y increases, therefore increasing.

33. Graph $f(x) = 2 - x^2$ See Figure 33. As x increases for the interval: $(-\infty, 0]$, y increases, therefore increasing.

34. Graph $f(x) = |x + 1|$, See Figure 34. As x increases for the interval: $(-\infty, -1]$, y decreases, therefore decreasing.

[−10, 10] by [−10, 10]
Xscl = 1 Yscl = 1

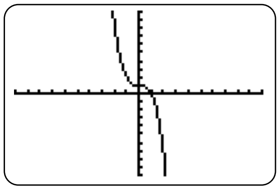


Figure 31

[−10, 10] by [−10, 10]
Xscl = 1 Yscl = 1

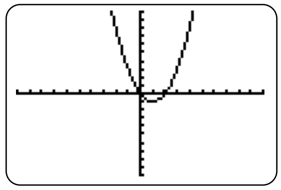


Figure 32

[−10, 10] by [−10, 10]
Xscl = 1 Yscl = 1

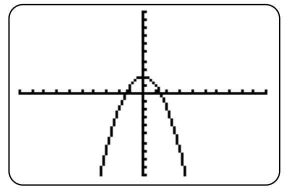


Figure 33

[−10, 10] by [−10, 10]
Xscl = 1 Yscl = 1

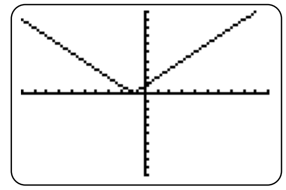


Figure 34

35. (a) No (b) Yes (c) No

36. (a) Yes (b) No (c) No

37. (a) Yes (b) No (c) No

38. (a) No (b) No (c) Yes

39. (a) Yes (b) Yes (c) Yes

40. (a) Yes (b) Yes (c) Yes

41. (a) No (b) No (c) Yes

42. (a) No (b) Yes (c) No

43. If f is an even function then $f(-x) = f(x)$ or opposite domains have the same range. See Figure 43.

44. If g is an odd function then $g(-x) = -g(x)$ or opposite domains have the opposite range. See Figure 44.

x	$f(x)$
–3	21
–2	–12
–1	–25
1	–25
2	–12
3	21

Figure 43

x	$g(x)$
–5	13
–3	1
–2	–5
0	0
2	5
3	–1
5	–13

Figure 44

45. (a) Since $f(-x) = f(x)$, this is an even function and is symmetric with respect to the y-axis. See Figure 45a.

(b) Since $f(-x) = -f(x)$, this is an odd function and is symmetric with respect to the origin. See Figure 45b.

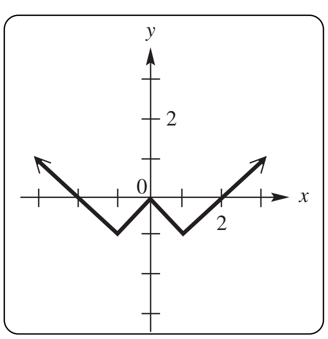


Figure 45a

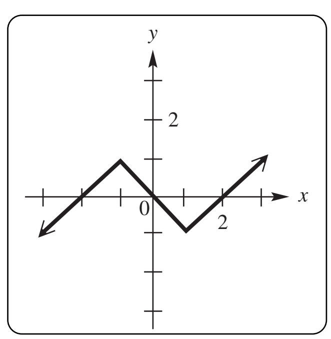


Figure 45b

46. (a) Since this is an odd function the graph is symmetric with respect to the origin. See Figure 46a.

(b) Since this is an even function the graph is symmetric with respect to the y-axis. See Figure 46b.

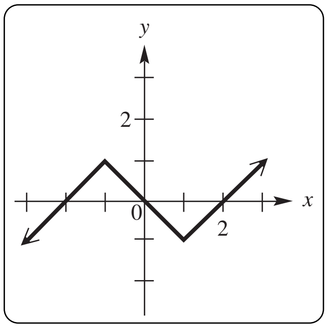


Figure 46a

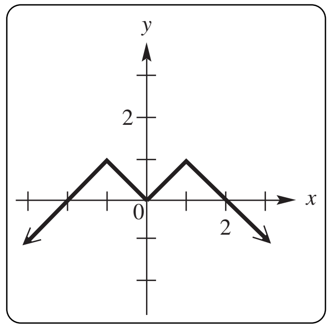


Figure 46b

47. Since $f(-x) = f(x)$, it is even.

48. Since $f(-x) = -f(x)$, it is odd.

49. If $f(x) = x^4 - 7x^2 + 6$, then $f(-x) = (-x)^4 - 7(-x)^2 + 6 \Rightarrow f(-x) = x^4 - 7x^2 + 6$. Since $f(-x) = f(x)$, the function is even.

50. If $f(x) = -2x^6 - 8x^2$, then $f(-x) = -2(-x)^6 - 8(-x)^2 \Rightarrow f(-x) = -2x^6 - 8x^2$. Since $f(-x) = f(x)$, the function is even.

51. If $f(x) = x^6 - 4x^4 + 5$, then $f(-x) = (-x)^6 - 4(-x)^4 + 5 \Rightarrow f(-x) = x^6 - 4x^4 + 5$. Since $f(-x) = f(x)$, the function is even.

52. If $f(x) = 8$, then $f(-x) = 8$. Since $f(-x) = f(x)$, the function is even.

53. If $f(x) = |5x|$, then $f(-x) = |5(-x)| \Rightarrow f(-x) = |5x|$. Since $f(-x) = f(x)$, the function is even.

54. If $f(x) = \sqrt{x^2 + 1}$, then $f(-x) = \sqrt{(-x)^2 + 1} \Rightarrow f(-x) = \sqrt{x^2 + 1}$. Since $f(-x) = f(x)$, the function is even.

55. If $f(x) = 3x^3 - x$, then $f(-x) = 3(-x)^3 - (-x) \Rightarrow f(-x) = -3x^3 + x$ and $-f(x) = -(3x^3 - x) \Rightarrow -f(x) = -3x^3 + x$. Since $f(-x) = -f(x)$, the function is odd.

56. If $f(x) = -x^5 + 2x^3 - 3x$, then $f(-x) = -(-x)^5 + 2(-x)^3 - 3(-x) \Rightarrow f(-x) = x^5 - 2x^3 + 3x$ and $-f(x) = -(-x^5 + 2x^3 - 3x) \Rightarrow -f(x) = x^5 - 2x^3 + 3x$. Since $f(-x) = -f(x)$, the function is odd.

57. If $f(x) = 3x^5 - x^3 + 7x$, then $f(-x) = 3(-x)^5 - (-x)^3 + 7(-x) \Rightarrow f(-x) = -3x^5 + x^3 - 7x$ and $-f(x) = -(3x^5 - x^3 + 7x) \Rightarrow -f(x) = -3x^5 + x^3 - 7x$. Since $f(-x) = -f(x)$, the function is odd.

58. If $f(x) = x^3 - 4x$, then $f(-x) = (-x)^3 - 4(-x) \Rightarrow f(-x) = -x^3 + 4x$ and $-f(x) = -(x^3 - 4x) \Rightarrow -f(x) = -x^3 + 4x$. Since $f(-x) = -f(x)$, the function is odd.

59. If $f(x) = \frac{1}{2x}$, then $f(-x) = \frac{1}{2(-x)} \Rightarrow f(-x) = -\frac{1}{2x}$ and $-f(x) = -\left(\frac{1}{2x}\right) \Rightarrow -f(x) = -\frac{1}{2x}$ Since $f(-x) = -f(x)$, the function is odd.

60. If $f(x) = 4x - \frac{1}{x}$, then $f(-x) = 4(-x) - \frac{1}{(-x)} \Rightarrow f(-x) = -4x + \frac{1}{x}$ and $-f(x) = -\left(4x - \frac{1}{x}\right) \Rightarrow -f(x) = -4x + \frac{1}{x}$. Since $f(-x) = -f(x)$, the function is odd.

61. If $f(x) = -x^3 + 2x$, then $f(-x) = -(-x)^3 + 2(-x) \Rightarrow f(-x) = x^3 - 2x$ and $-f(x) = -(-x^3 + 2x) \Rightarrow -f(x) = x^3 - 2x$. Since $f(-x) = -f(x)$, the function is symmetric with respect to the origin. Graph $f(x) = -x^3 + 2x$. The graph supports symmetry with respect to the origin.

62. If $f(x) = x^5 - 2x^3$, then $f(-x) = (-x)^5 - 2(-x)^3 \Rightarrow f(-x) = -x^5 + 2x^3$ and $-f(x) = -(x^5 - 2x^3) \Rightarrow -f(x) = -x^5 + 2x^3$. Since $f(-x) = -f(x)$, the function is symmetric with respect to the origin. Graph $f(x) = x^5 - 2x^3$. The graph supports symmetry with respect to the origin.

63. If $f(x) = .5x^4 - 2x^2 + 1$, then $f(-x) = .5(-x)^4 - 2(-x)^2 + 1 \Rightarrow f(-x) = .5x^4 - 2x^2 + 1$. Since $f(-x) = f(x)$, the function is symmetric with respect to the y-axis. Graph $f(x) = .5x^4 - 2x^2 + 1$. The graph supports symmetry with respect to the y-axis.

64. If $f(x) = .75x^2 + |x| + 1$, then $f(-x) = .75(-x)^2 + |(-x)| + 1 \Rightarrow f(-x) = .75x^2 + |x| + 1$. Since $f(-x) = f(x)$, the function is symmetric with respect to the y-axis. Graph $f(x) = .75x^4 + |x| + 1$. The graph supports symmetry with respect to the y-axis.

65. If $f(x) = x^3 - x + 3$, then $f(-x) = (-x)^3 - (-x) + 3 \Rightarrow f(-x) = -x^3 + x + 3$ and $-f(x) = -(x^3 - x + 3)$ $\Rightarrow -f(x) = -x^3 + x - 3$. Since $f(x) \neq f(-x) \neq -f(x)$, the function is not symmetric with respect to the y-axis or origin. Graph $f(x) = x^3 - x + 3$. The graph supports no symmetry with respect to the y-axis or origin.

66. If $f(x) = x^4 - 5x + 2$, then $f(-x) = (-x)^4 - 5(-x) + 2 \Rightarrow f(-x) = x^4 + 5x + 2$ and $-f(x) = -(x^4 - 5x + 2)$ $\Rightarrow -f(x) = -x^4 + 5x - 2$. Since $f(x) \neq f(-x) \neq -f(x)$, the function is not symmetric with respect to the y-axis or origin. Graph $f(x) = x^4 - 5x + 2$. The graph supports no symmetry with respect to the y-axis or origin.

67. If $f(x) = x^6 - 4x^3$, then $f(-x) = (-x)^6 - 4(-x)^3 \Rightarrow f(-x) = x^6 + 4x^3$ and $-f(x) = -(x^6 - 4x^3) \Rightarrow$ $-f(x) = -x^6 + 4x^3$ Since $f(x) \neq f(-x) \neq -f(x)$, the function is not symmetric with respect to the y-axis or origin. Graph $f(x) = x^6 - 4x^3$. The graph supports no symmetry with respect to the y-axis or origin.

68. If $f(x) = x^3 - 3x$, then $f(-x) = (-x)^3 - 3(-x) \Rightarrow f(-x) = -x^3 + 3x$ and $-f(x) = -(x^3 - 3x) \Rightarrow -f(x) = -x^3 + 3x$. Since $f(-x) = -f(x)$, the function is symmetric with respect to the origin. Graph $f(x) = x^3 - 3x$. The graph supports symmetry with respect to the origin.

69. If $f(x) = -6$, then $f(-x) = -6$. Since $f(-x) = f(x)$, the function is symmetric with respect to the y-axis. Graph $f(x) = -6$. The graph supports symmetry with respect to the y-axis.

70. If $f(x) = |-x| = |x|$, then $f(-x) = |-(-x)| = |x| \Rightarrow f(-x) = |x|$. Since $f(-x) = f(x)$, the function is symmetric with respect to the y-axis. Graph $f(x) = |-x|$. The graph supports symmetry with respect to the y-axis.

71. If $f(x) = \dfrac{1}{4x^3}$, then $f(-x) = \dfrac{1}{4(-x)^3} \Rightarrow f(-x) = -\dfrac{1}{4x^3}$ and $-f(x) = -\left(\dfrac{1}{4x^3}\right) \Rightarrow -f(x) = -\dfrac{1}{4x^3}$. Since $f(-x) = -f(x)$, the function is symmetric with respect to the origin. Graph $f(x) = \dfrac{1}{4x^3}$. The graph supports symmetry with respect to the origin.

72. If $f(x) = \sqrt{x^2} \Rightarrow f(x) = x$, then $f(-x) = \sqrt{(-x)^2} \Rightarrow f(-x) = \sqrt{x^2} \Rightarrow f(-x) = x$. Since $f(-x) = f(x)$, the function is symmetric with respect to the y-axis. Graph $f(x) = \sqrt{x^2}$. The graph supports symmetry with respect to the y-axis.

73. (a) Functions where $f(-x) = f(x)$ are even, therefore exercises: 63, 64, 69, 70, and 72 are even.

(b) Functions where $f(-x) = -f(x)$ are odd, therefore exercises: 61, 62, 68, and 71 are odd.

(c) Functions where $f(x) \neq f(-x) \neq -f(x)$ are neither odd or even, therefore exercises: 65, 66, and 67 are neither odd or even.

74. Answers may vary. If a function f is even, then $f(x) = f(-x)$ for all x in the domain. Its graph is symmetric with respect to the y-axis. If a function f is odd, then $f(-x) = -f(x)$ for all x in the domain. Its graph is symmetric with respect to the origin.

2.2: Vertical and Horizontal Shifts of Graphs

1. The equation $y = x^2$ shifted 3 units upward is: $y = x^2 + 3$.
2. The equation $y = x^3$ shifted 2 units downward is: $y = x^3 - 2$.
3. The equation $y = \sqrt{x}$ shifted 4 units downward is: $y = \sqrt{x} - 4$.
4. The equation $y = \sqrt[3]{x}$ shifted 6 units upward is: $y = \sqrt[3]{x} + 6$
5. The equation $y = |x|$ shifted 4 units to the right is: $y = |x - 4|$.
6. The equation $y = |x|$ shifted 3 units to the left is: $y = |x + 3|$.
7. The equation $y = x^3$ shifted 7 units to the left is: $y = (x + 7)^3$.
8. The equation $y = \sqrt{x}$ shifted 9 units to the right is: $y = \sqrt{x - 9}$.
9. Shift the graph of f 4 units upward to obtain the graph of g.
10. Shift the graph of f 4 units to the left to obtain the graph of g.
11. The equation $y = x^2 - 3$ is $y = x^2$ shifted 3 units downward, therefore graph B.
12. The equation $y = (x - 3)^2$ is $y = x^2$ shifted 3 units to the right, therefore graph C.
13. The equation $y = (x + 3)^2$ is $y = x^2$ shifted 3 units to the left, therefore graph A.
14. The equation $y = |x| + 4$ is $y = |x|$ shifted 4 units upward, therefore graph A.
15. The equation $y = |x + 4| - 3$ is $y = |x|$ shifted 4 units to the left and 3 units downward, therefore graph B.
16. The equation $y = |x - 4| - 3$ is $y = |x|$ shifted 4 units to the right and 3 units downward, therefore graph C.
17. The equation $y = (x - 3)^3$ is $y = x^3$ shifted 3 units to the right, therefore graph C.
18. The equation $y = (x - 2)^3 - 4$ is $y = x^3$ shifted 2 units to the right and 4 units downward, therefore graph A.
19. The equation $y = (x + 2)^3 - 4$ is $y = x^3$ shifted 2 units to the left and 4 units downward, therefore graph B.
20. If $y = |x - h| + k$ with $h < 0$ and $k < 0$, then the graph of $y = |x|$ is shifted to the left $-h$ units and $-k$ units downward. This would place the vertex or lowest point of the absolute value graph in the third quadrant.
21. For the equation $y = x^2$, the Domain is: $(-\infty, \infty)$ and the Range is: $[0, \infty)$. Shifting this 3 units downward gives us: (a) Domain: $(-\infty, \infty)$ (b) Range: $[-3, \infty)$
22. For the equation $y = x^2$, the Domain is: $(-\infty, \infty)$ and the Range is: $[0, \infty)$. Shifting this 3 units to the right gives us: (a) Domain: $(-\infty, \infty)$ (b) Range: $[0, \infty)$

23. For the equation $y = |x|$, the Domain is: $(-\infty, \infty)$ and the Range is: $[0, \infty)$. Shifting this 4 units to the left and 3 units downward gives us: (a) Domain: $(-\infty, \infty)$ (b) Range: $[-3, \infty)$
24. For the equation $y = |x|$, the Domain is: $(-\infty, \infty)$ and the Range is: $[0, \infty)$. Shifting this 4 units to the right and 3 units downward gives us: (a) Domain: $(-\infty, \infty)$ (b) Range: $[-3, \infty)$
25. For the equation $y = x^3$, the Domain is: $(-\infty, \infty)$ and the Range is: $(-\infty, \infty)$. Shifting this 3 units to the right gives us: (a) Domain: $(-\infty, \infty)$ (b) Range: $(-\infty, \infty)$
26. For the equation $y = x^3$, the Domain is: $(-\infty, \infty)$ and the Range is: $(-\infty, \infty)$. Shifting this 2 units to the right and 4 units downward gives us: (a) Domain: $(-\infty, \infty)$ (b) Range: $(-\infty, \infty)$
27. Using $Y_2 = Y_1 + k$ and $x = 0$, we get $19 = 15 + k \Rightarrow k = 4$.
28. Using $Y_2 = Y_1 + k$ and $x = 0$, we get $-5 = -3 + k \Rightarrow k = -2$.
29. From the graphs $(6, 2)$ is a point on Y_1 and $(6, -1)$ a point on Y_2. Using $Y_2 = Y_1 + k$ and $x = 6$, we get $-1 = 2 + k \Rightarrow k = -3$.
30. From the graphs $(-4, 3)$ is a point on Y_1 and $(-4, 8)$ a point on Y_2. Using $Y_2 = Y_1 + k$ and $x = -4$, we get $8 = 3 + k \Rightarrow k = 5$.
31. The graph of $y = (x - 1)^2$ is the graph of the equation $y = x^2$ shifted 1 unit to the right. See Figure 31.
32. The graph of $y = \sqrt{x + 2}$ is the graph of the equation $y = \sqrt{x}$ shifted 2 units to the left. See Figure 32.
33. The graph of $y = x^3 + 1$ is the graph of the equation $y = x^3$ shifted 1 unit upward. See Figure 33.

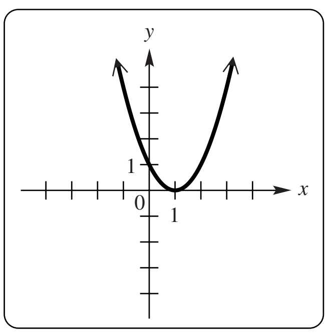


Figure 31

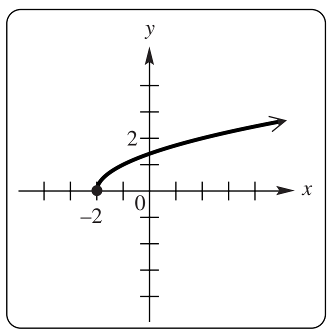


Figure 32

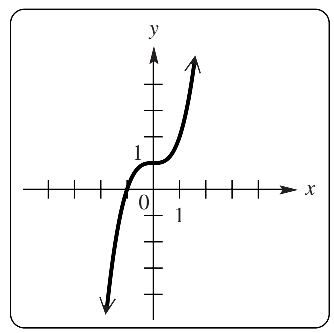


Figure 33

34. The graph of $y = |x + 2|$ is the graph of the equation $y = |x|$ shifted 2 units to the left. See Figure 34.
35. The graph of $y = (x - 1)^3$ is the graph of the equation $y = x^3$ shifted 1 unit to the right. See Figure 35.
36. The graph of $y = |x| - 3$ is the graph of the equation $y = |x|$ shifted 3 units downward. See Figure 36.

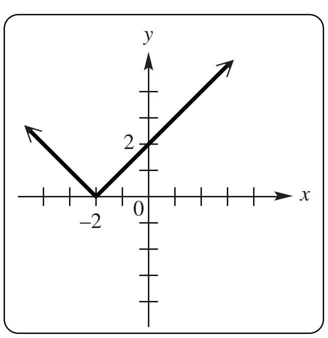


Figure 34

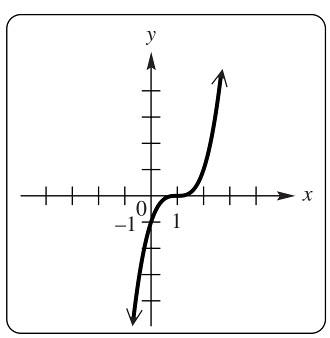


Figure 35

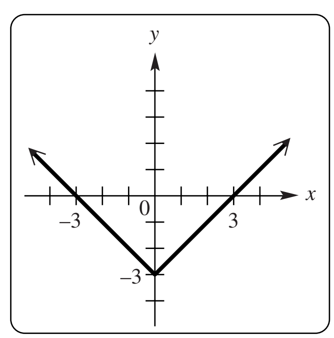


Figure 36

37. The graph of $y = \sqrt{x - 2} - 1$ is the graph of the equation $y = \sqrt{x}$ shifted 2 units to the right and 1 unit downward. See Figure 37.

38. The graph of $y = \sqrt{x + 3} - 4$ is the graph of the equation $y = \sqrt{x}$ shifted 3 units to the left and 4 units downward. See Figure 38.

39. The graph of $y = (x + 2)^2 + 3$ is the graph of the equation $y = x^2$ shifted 2 units to the left and 3 units upward. See Figure 39.

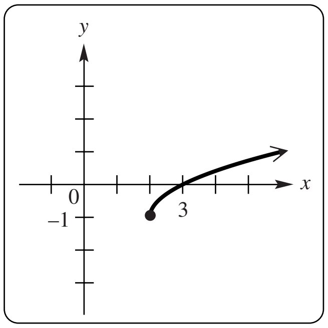


Figure 37

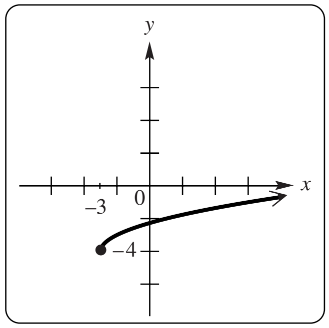


Figure 38

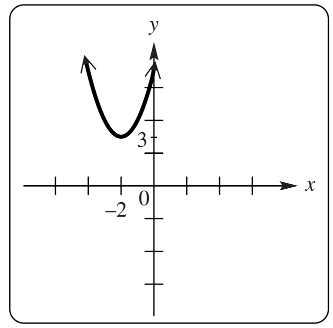


Figure 39

40. The graph of $y = (x - 4)^2 - 4$ is the graph of the equation $y = x^2$ shifted 4 units to the right and 4 units downward. See Figure 40.

41. The graph of $y = |x + 4| - 2$ is the graph of the equation $y = |x|$ shifted 4 units to the left and 2 units downward. See Figure 41.

42. The graph of $y = (x + 3)^3 - 1$ is the graph of the equation $y = x^3$ shifted 3 units to the left and 1 unit downward. See Figure 42.

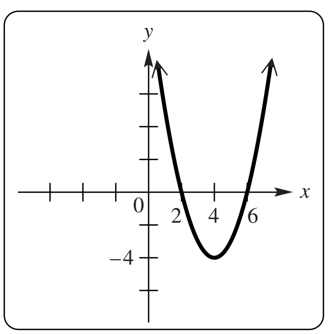


Figure 40

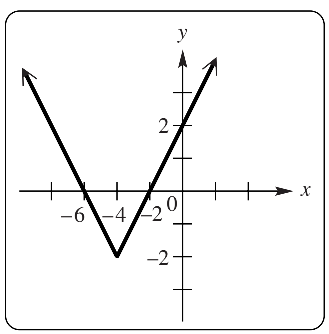


Figure 41

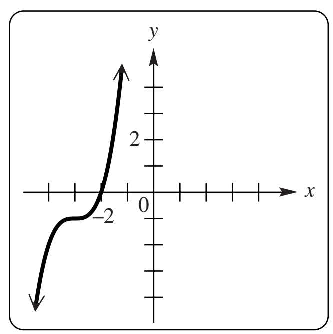


Figure 42

43. Since h and k are positive, the equation $y = (x - h)^2 - k$ is $y = x^2$ shifted to the right and down, therefore: B.

44. Since h and k are positive, the equation $y = (x + h)^2 - k$ is $y = x^2$ shifted to the left and down, therefore: D.

45. Since h and k are positive, the equation $y = (x + h)^2 + k$ is $y = x^2$ shifted to the left and up, therefore: A.

46. Since h and k are positive, the equation $y = (x - h)^2 + k$ is $y = x^2$ shifted to the right and up, therefore: C.

47. The equation $y = f(x) + 2$ is $y = f(x)$ shifted up 2 units or add 2 to the y-coordinate of each point as follows: $(-3, -2) \Rightarrow (-3, 0)$; $(-1, 4) \Rightarrow (-1, 6)$; $(5, 0) \Rightarrow (5, 2)$. See Figure 47.

48. The equation $y = f(x) - 2$ is $y = f(x)$ shifted down 2 units or subtract 2 from the y-coordinate of each point as follows: $(-3, -2) \Rightarrow (-3, -4)$; $(-1, 4) \Rightarrow (-1, 2)$; $(5, 0) \Rightarrow (5, -2)$. See Figure 48.

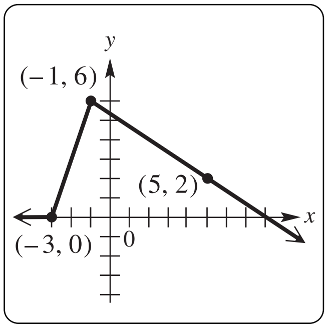


Figure 47

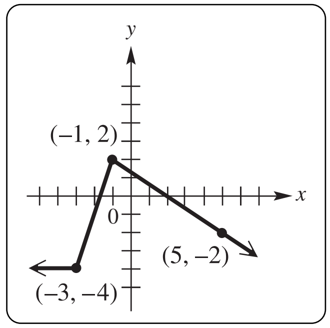


Figure 48

49. The equation $y = f(x + 2)$ is $y = f(x)$ shifted left 2 units or subtract 2 from the x-coordinate of each point as follows: $(-3, -2) \Rightarrow (-5, -2)$; $(-1, 4) \Rightarrow (-3, 4)$; $(5, 0) \Rightarrow (3, 0)$. See Figure 49.

50. The equation $y = f(x - 2)$ is $y = f(x)$ shifted right 2 units or add 2 to the x-coordinate of each point as follows: $(-3, -2) \Rightarrow (-1, -2)$; $(-1, 4) \Rightarrow (1, 4)$; $(5, 0) \Rightarrow (7, 0)$. See Figure 50.

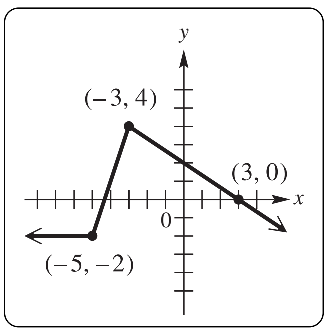


Figure 49

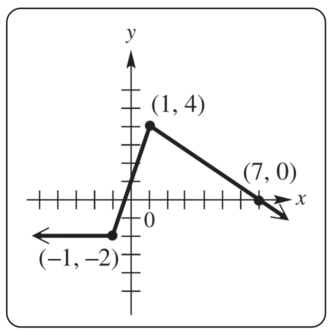


Figure 50

51. The graph is the basic function $y = x^2$ translated 4 units to the left and 3 units up, therefore the new equation is: $y = (x + 4)^2 + 3$. The equation is now increasing for the interval: (a) $[-4, \infty)$ and decreasing for the interval: (b) $(-\infty, -4]$.

52. The graph is the basic function $y = \sqrt{x}$ translated 5 units to the left, therefore the new equation is: $y = \sqrt{x + 5}$. The equation is now increasing for the interval: (a) $[-5, \infty)$ and does not decrease, therefore: (b) none.

53. The graph is the basic function $y = x^3$ translated 5 units down, therefore the new equation is: $y = x^3 - 5$. The equation is now increasing for the interval: (a) $(-\infty, \infty)$ and does not decrease, therefore: (b) none.

54. The graph is the basic function $y = |x|$ translated 10 units to the left, therefore the new equation is: $y = |x + 10|$. The equation is now increasing for the interval: (a) $[-10, \infty)$ and decreasing for the interval: (b) $(-\infty, -10]$.

55. The graph is the basic function $y = \sqrt{x}$ translated 2 units to the right and 1 unit up, therefore the new equation is: $y = \sqrt{x - 2} + 1$. The equation is now increasing for the interval: (a) $[2, \infty)$ and does not decrease, therefore: (b) none.

56. The graph is the basic function $y = x^2$ translated 2 units to the right and 3 units down, therefore the new equation is: $y = (x - 2)^2 - 3$. The equation is now increasing for the interval: (a) $[2, \infty)$ and decreasing for the interval: (b) $(-\infty, 2]$.

57. (a) $f(x) = 0$: $\{3, 4\}$

(b) $f(x) > 0$: for the intervals $(-\infty, 3) \cup (4, \infty)$.

(c) $f(x) < 0$: for the interval $(3, 4)$.

58. (a) $f(x) = 0$: $\{\sqrt{2}\}$

(b) $f(x) > 0$: for the interval $(\sqrt{2}, \infty)$.

(c) $f(x) < 0$: for the interval $(-\infty, \sqrt{2})$.

59. (a) $f(x) = 0$: $\{-4, 5\}$

(b) $f(x) \geq 0$: for the intervals $(-\infty, -4] \cup [5, \infty)$.

(c) $f(x) \leq 0$: for the interval $[-4, 5]$.

60. (a) $f(x) = 0$: never, therefore: $\varnothing$.

(b) $f(x) \geq 0$: for the interval $[1, \infty)$.

(c) $f(x) \leq 0$: never, therefore: $\varnothing$.

61. The translation is 3 units to the left and 1 unit up, therefore the new equation is: $y = |x + 3| + 1$. The form $y = |x - h| + k$ will equal $y = |x + 3| + 1$ when: $h = -3$ and $k = 1$.

62. The equation $y = x^2$ has a Domain: $(-\infty, \infty)$ and a Range: $[0, \infty)$. After the translation the Domain is still: $(-\infty, \infty)$, but now the Range is: $[38, \infty)$, a positive or upward shift of 38 units. Therefore, the horizontal shift can be any number of units, but the vertical shift is up 38. This makes h any real number and $k = 38$.

63. (a) Since 0 corresponds to 1998, our equation using exact years would be: $y = 895.5(x - 1998) + 14{,}709$.

(b) $y = 895.5(2006 - 1998) + 14{,}709 \Rightarrow y = 7164 + 14{,}709 \Rightarrow y = \$21{,}873$

64. (a) Since 0 corresponds to 1998, our equation using exact years would be: $y = 299.8(x - 1998) + 5249.1$.

(b) $y = 299.8(2007 - 1998) + 5249.1 \Rightarrow y = 2698.2 + 5249.1 \Rightarrow y = \$7947.3$ billion.

65. (a) Enter the year in L_1 and enter tuition and fees in L_2. The year 1991 corresponds to $x = 0$ and so on. The regression equation is: $y \approx 190x + 2071.3$.

(b) Since $x = 0$ corresponds to 1991, the equation when the exact year is entered is:

$y \approx 190(x - 1991) + 2071.3$

(c) $y \approx 190(2008 - 1991) + 2071.3 \Rightarrow y \approx 3230 + 2071.3 \Rightarrow y \approx \$5300$

66. (a) Enter the year in L_1 and enter the percent of women in the workforce in L_2. The year 1965 corresponds to $x = 0$ and so on. The regression equation is: $y \approx .6162x + 40.6167$.

(b) Since $x = 0$ corresponds to 1965, the equation when the exact year is entered is:

$y \approx .6162(x - 1965) + 40.6167$

(c) $y \approx .6162(2010 - 1965) + 40.6167 \Rightarrow y \approx 27.729 + 40.6167 \Rightarrow y \approx 68.3\%$

67. See Figure 67.

68. $m = \dfrac{2 - (-2)}{3 - 1} \Rightarrow m = \dfrac{4}{2} = 2$

69. Using slope-intercept form yields: $y_1 - 2 = 2(x - 3) \Rightarrow y_1 - 2 = 2x - 6 \Rightarrow y_1 = 2x - 4$

70. $(1, -2 + 6)$ and $(3, 2 + 6) \Rightarrow (1, 4)$ and $(3, 8)$

71. $m = \dfrac{8 - 4}{3 - 1} \Rightarrow m = \dfrac{4}{2} = 2$

72. Using slope-intercept form yields: $y_2 - 4 = 2(x - 1) \Rightarrow y_2 - 4 = 2x - 2 \Rightarrow y_2 = 2x + 2$

73. Graph $y_1 = 2x - 4$ and $y_2 = 2x + 2$. See Figure 73. The graph y_2 can be obtained by shifting the graph of y_1 upward 6 units. The constant, 6, comes from the 6 we added to each y-value in Exercise 70.

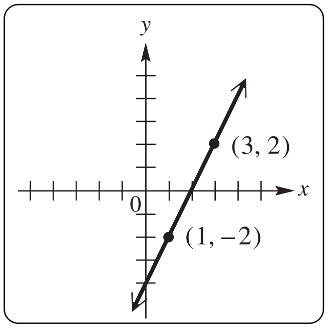


Figure 67

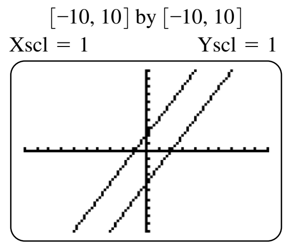


Figure 73

74. c; c; the same as; upward (or positive vertical)

2.3: Stretching, Shrinking, and Reflecting Graphs

1. The function $y = x^2$ vertically stretched by a factor of 2 is: $y = 2x^2$.
2. The function $y = x^3$ vertically shrunk by a factor of $\frac{1}{2}$ is: $y = \frac{1}{2}x^3$.
3. The function $y = \sqrt{x}$ reflected across the y-axis is: $y = \sqrt{-x}$.
4. The function $y = \sqrt[3]{x}$ reflected across the x-axis is: $y = -\sqrt[3]{x}$.
5. The function $y = |x|$ vertically stretched by a factor of 3 and reflected across the x-axis is: $y = -3|x|$.
6. The function $y = |x|$ vertically shrunk by a factor of $\frac{1}{3}$ and reflected across the y-axis is: $y = \frac{1}{3}|-x|$.
7. The function $y = x^3$ vertically shrunk by a factor of .25 and reflected across the y-axis is: $y = .25(-x)^3$ or $y = -.25x^3$.
8. The function $y = \sqrt{x}$ vertically shrunk by a factor of .2 and reflected across the x-axis is: $y = -.2\sqrt{x}$.
9. Graph $y_1 = x$, $y_2 = x + 3$ (y_1 shifted up 3 units), and $y_3 = x - 3$ (y_1 shifted down 3 units). See Figure 9.
10. Graph $y_1 = x^3$, $y_2 = x^3 + 4$ (y_1 shifted up 4 units), and $y_3 = x^3 - 4$ (y_1 shifted down 4 units). See Figure 10.
11. Graph $y_1 = |x|$, $y_2 = |x - 3|$ (y_1 shifted right 3 units), and $y_3 = |x + 3|$ (y_1 shifted left 3 units). See Figure 11.

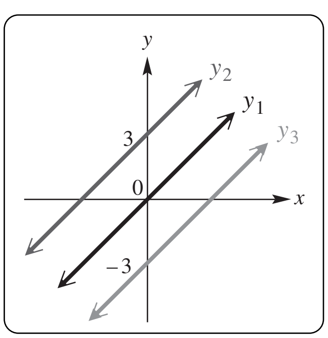


Figure 9

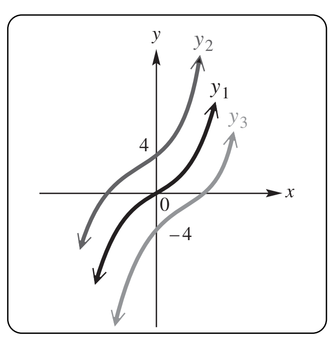


Figure 10

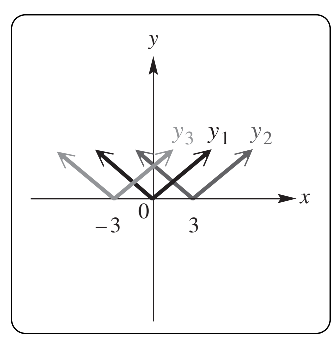


Figure 11

12. Graph $y_1 = |x|$, $y_2 = |x| - 3$ (y_1 shifted down 3 units), and $y_3 = |x| + 3$ (y_1 shifted up 3 units). See Figure 12.

13. Graph $y_1 = \sqrt{x}$, $y_2 = \sqrt{x+6}$ (y_1 shifted left 6 units), and $y_3 = \sqrt{x-6}$ (y_1 shifted right 6 units). See Figure 13.

14. Graph $y_1 = |x|$, $y_2 = 2|x|$ (y_1 stretched vertically by a factor of 2), and $y_3 = 2.5|x|$ (y_1 stretched vertically by a factor of 2.5). See Figure 14.

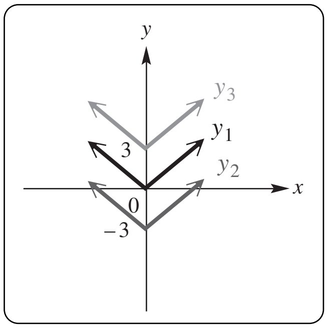


Figure 12

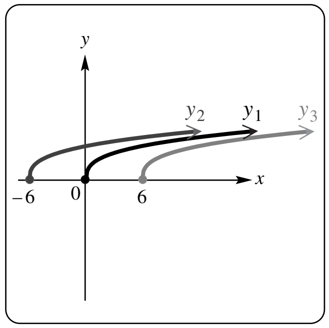


Figure 13

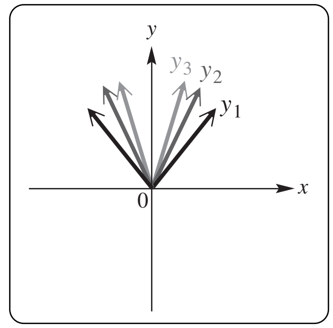


Figure 14

15. Graph $y_1 = \sqrt[3]{x}$, $y_2 = -\sqrt[3]{x}$ (y_1 reflected across the x-axis), and $y_3 = -2\sqrt[3]{x}$ (y_1 reflected across the x-axis and stretched vertically by a factor of 2). See Figure 15.

16. Graph $y_1 = x^2$, $y_2 = (x-2)^2 + 1$ (y_1 shifted right 2 units and up 1 unit), and $y_3 = -(x+2)^2$ (y_1 shifted left 2 units and reflected across the x-axis). See Figure 16.

17. Graph $y_1 = |x|$, $y_2 = -2|x-1| + 1$ (y_1 reflected across the x-axis, stretched vertically by a factor of 2, shifted right 1 unit, and shifted up 1 unit,), and $y_3 = -\frac{1}{2}|x| - 4$ (y_1 reflected across the x-axis, shrunk by factor of $\frac{1}{2}$, and shifted down 4 units). See Figure 17.

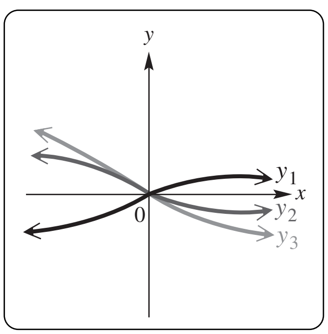


Figure 15

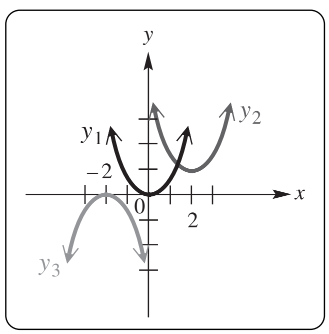


Figure 16

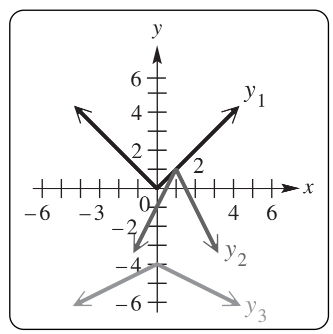


Figure 17

18. Graph $y_1 = \sqrt{x}$, $y_2 = -\sqrt{x}$ (y_1 reflected across the x-axis), and $y_3 = \sqrt{-x}$ (y_1 reflected across the y-axis). See Figure 18.

19. Graph $y_1 = x^2 - 1$ (which is $y = x^2$ shifted down 1 unit), $y_2 = \left(\frac{1}{2}x\right)^2 - 1$ (y_1 shrunk vertically by a factor of $\frac{1}{2}$), and $y_3 = (2x)^2 - 1$ (y_1 stretched vertically by a factor of 2^2 or 4). See Figure 19.

20. Graph $y_1 = 3 - |x|$ (which is $y = |x|$ reflected across the x-axis and shifted up 3 units), $y_2 = 3 - |3x|$ (y_1 stretched vertically by a factor of 3), and $y_3 = 3 - |\frac{1}{3}x|$ (y_1 shrunk vertically by a factor of $\frac{1}{3}$). See Figure 20.

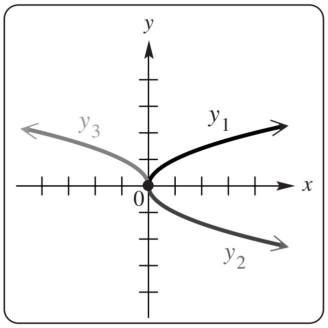


Figure 18

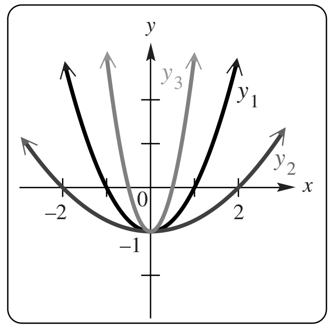


Figure 19

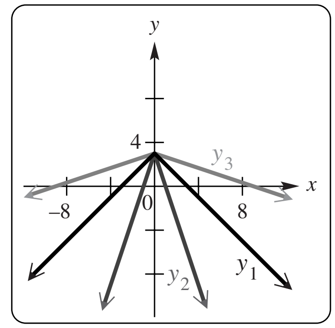


Figure 20

21. Graph $y_1 = \sqrt[3]{x}$, $y_2 = \sqrt[3]{-x}$ (y_1 reflected across the y-axis), and $y_3 = \sqrt[3]{-(x-1)}$ (y_1 reflected across the y-axis and shifted right 1 unit). See Figure 21.

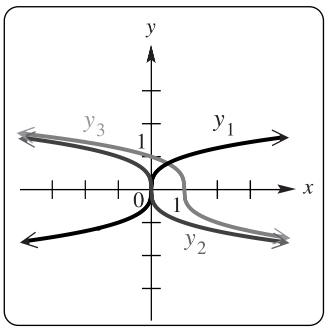


Figure 21

22. Since $y = f(x)$ is symmetric with respect to the y-axis, for every (x, y) on the graph, $(-x, y)$ is also on the graph. Reflection across the y-axis reflect onto itself and will not change the graph. It will be the same.

23. 4; x

24. 6; x

25. 2; left; $\frac{1}{4}$; x; 3; downward (or negative)

26. y; $\frac{2}{5}$; x; 6 ; upward (or positive)

27. 3; right; 6

28. 2; left; .5

29. The function $y = x^2$ is vertically shrunk by a factor of $\frac{1}{2}$ and shifted 7 units down, therefore: $y = \frac{1}{2}x^2 - 7$.

30. The function $y = x^3$ is vertically stretched by a factor of 3, reflected across the x-axis, and shifted 8 units upward, therefore: $y = -3x^3 + 8$.

31. The function $y = \sqrt{x}$ is shifted 3 units right, vertically stretched by a factor of 4.5, and shifted 6 units down, therefore: $y = 4.5\sqrt{x-3} - 6$.

32. The function $y = \sqrt[3]{x}$ is shifted 2 units left, vertically stretched by a factor of 1.5, and shifted 8 units upward, therefore: $y = 1.5\sqrt[3]{x+2} + 8$.

33. The graph $y = f(x) = x^2$ has been reflected across the x-axis, shifted 5 units to the right, and shifted 2 units downward, therefore the equation of $g(x)$ is: $g(x) = -(x - 5)^2 - 2$.

34. The graph $y = f(x) = x^3$ has been shifted 4 units to the right and shifted 3 units upward, therefore the equation of $g(x)$ is: $g(x) = (x - 4)^3 + 3$.

35. The function $f(x) = \sqrt{x - 3} + 2$ is $f(x) = \sqrt{x}$ shifted 3 units right and 2 units upward. See Figure 35.

36. The function $f(x) = |x + 2| - 3$ is $f(x) = |x|$ shifted 2 units left and 3 units downward. See Figure 36.

37. The function $f(x) = \sqrt{2x} = \sqrt{2}\sqrt{x}$ is $f(x) = \sqrt{x}$ stretched vertically by a factor of $\sqrt{2}$. See Figure 37.

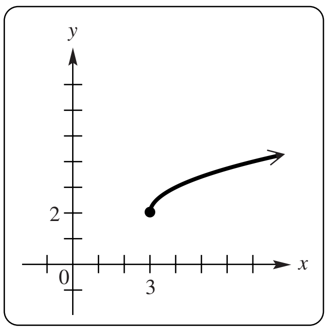


Figure 35

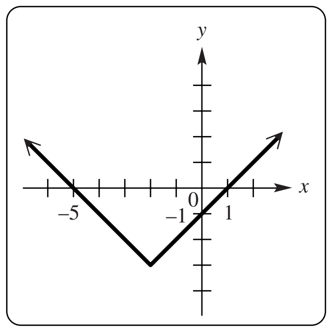


Figure 36

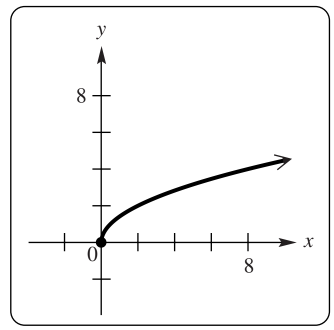


Figure 37

38. The function $f(x) = \frac{1}{2}(x + 2)^2$ is $f(x) = x^2$ shifted 2 units left and shrunk vertically by a factor of $\frac{1}{2}$. See Figure 38.

39. The function $f(x) = |2x| = 2|x|$ is $f(x) = |x|$ stretched vertically by a factor of 2. See Figure 39.

40. The function $f(x) = \frac{1}{2}|x|$ is $f(x) = |x|$ shrunk vertically by a factor of $\frac{1}{2}$. See Figure 40.

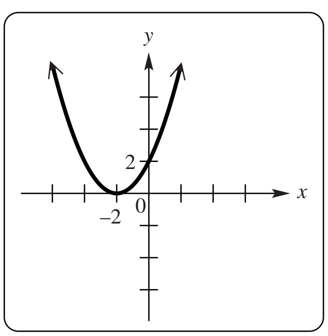


Figure 38

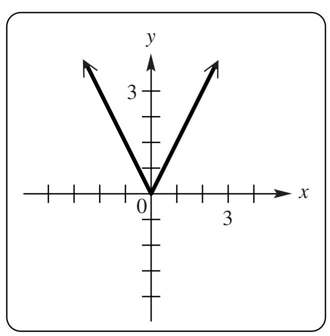


Figure 39

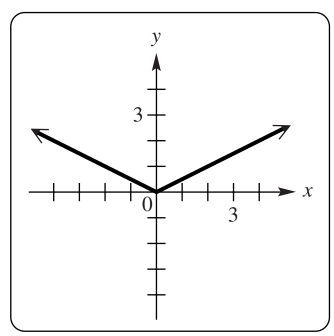


Figure 40

41. The function $f(x) = 1 - \sqrt{x}$ is $f(x) = \sqrt{x}$ reflected across the x-axis and shifted 1 unit upward. See Figure 41.

42. The function $f(x) = 2\sqrt{(x - 2)} - 1$ is $f(x) = \sqrt{x}$ shifted 2 units right, stretched vertically by a factor of 2, and shifted 1 unit downward. See Figure 42.

43. The function $f(x) = -\sqrt{1 - x} = -\sqrt{-(x - 1)}$ is $f(x) = \sqrt{x}$ reflected across both the x-axis and the y-axis and shifted 1 unit right. See Figure 43.

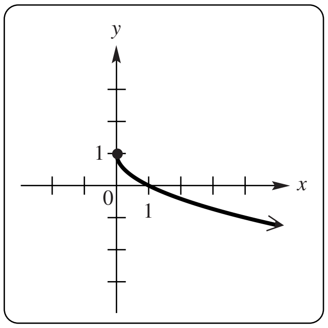


Figure 41

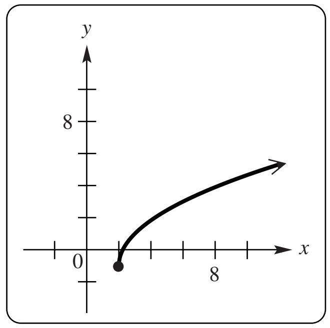


Figure 42

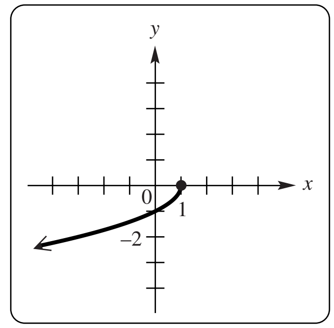


Figure 43

44. The function $f(x) = \sqrt{-x} - 1$ is $f(x) = \sqrt{x}$ reflected across the y-axis and shifted 1 unit downward. See Figure 44.

45. The function $f(x) = \sqrt{-(x + 1)}$ is $f(x) = \sqrt{x}$ reflected across the y-axis and shifted 1 unit left. See Figure 45.

46. The function $f(x) = 2 + \sqrt{-(x - 3)}$ is $f(x) = \sqrt{x}$ reflected across the y-axis, shifted 3 units right, and shifted 2 units upward. See Figure 46.

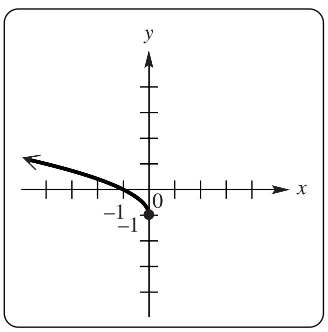


Figure 44

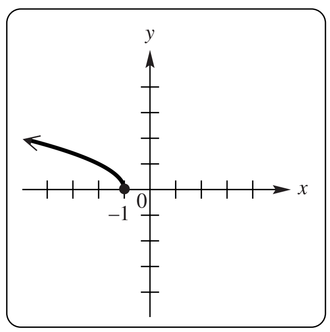


Figure 45

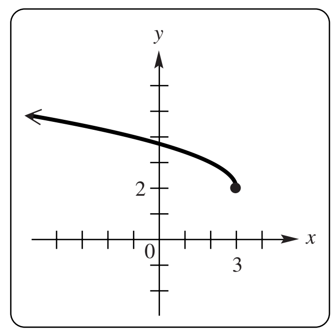


Figure 46

47. The function $f(x) = (x - 1)^3$ is $f(x) = x^3$ shifted 1 unit right. See Figure 47.

48. The function $f(x) = (x + 2)^3$ is $f(x) = x^3$ shifted 2 units left. See Figure 48.

49. The function $f(x) = -x^3$ is $f(x) = x^3$ reflected across the x-axis. See Figure 49.

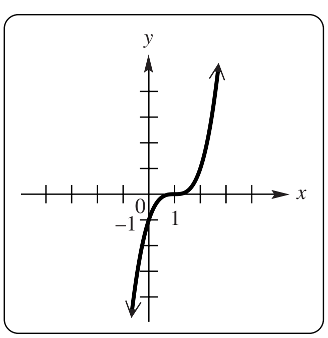


Figure 47

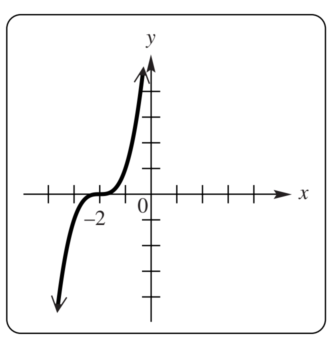


Figure 48

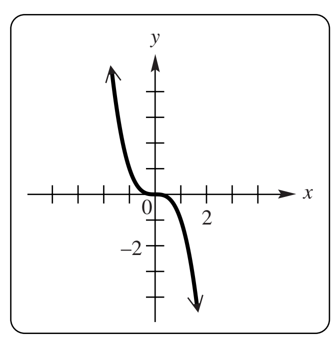


Figure 49

50. The function $f(x) = (-x)^3 + 1$ is $f(x) = x^3$ reflected across the y-axis and shifted 1 unit upward. See Figure 50.

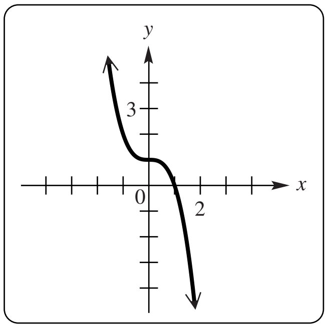


Figure 50

51. (a) The equation $y = -f(x)$ is $y = f(x)$ reflected across the x-axis. See Figure 51a.

(b) The equation $y = f(-x)$ is $y = f(x)$ reflected across the y-axis. See Figure 51b.

(c) The equation $y = 2f(x)$ is $y = f(x)$ stretched vertically by a factor of 2. See Figure 51c.

(d) From the graph $f(0) = 1$.

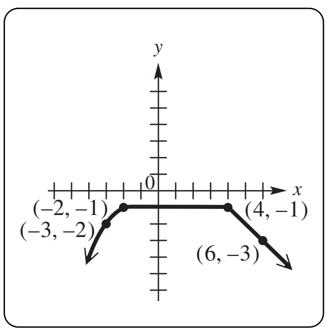


Figure 51a

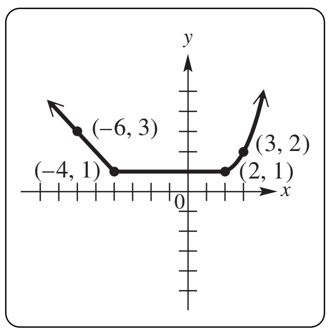


Figure 51b

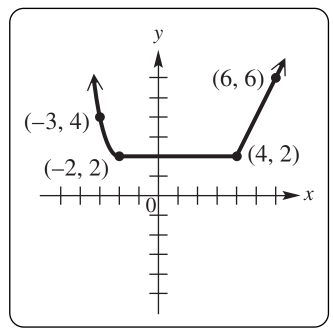


Figure 51c

52. (a) The equation $y = -f(x)$ is $y = f(x)$ reflected across the x-axis. See Figure 52a.

(b) The equation $y = f(-x)$ is $y = f(x)$ reflected across the y-axis. See Figure 52b.

(c) The equation $y = 3f(x)$ is $y = f(x)$ stretched vertically by a factor of 3. See Figure 52c.

(d) From the graph $f(4) = 1$.

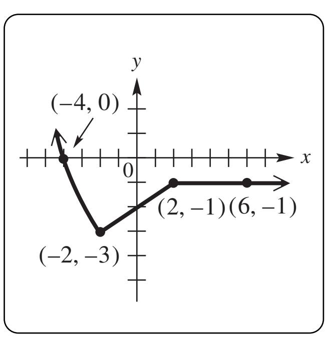


Figure 52a

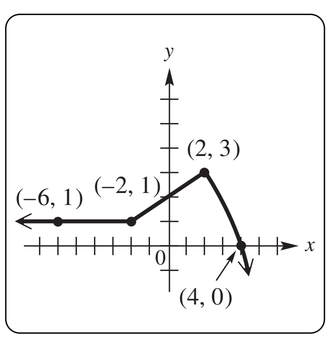


Figure 52b

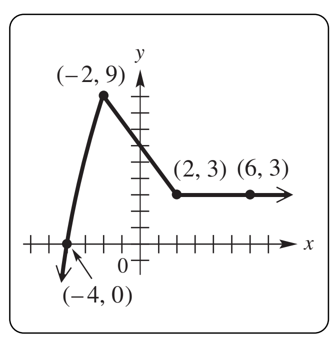


Figure 52c

53. (a) The equation $y = -f(x)$ is $y = f(x)$ reflected across the x-axis. See Figure 53a.
    (b) The equation $y = f(-x)$ is $y = f(x)$ reflected across the y-axis. See Figure 53b.
    (c) The equation $y = f(x + 1)$ is $y = f(x)$ shifted 1 unit to the left. See Figure 53c.
    (d) From the graph, there are two x-intercepts: -1 and 4.

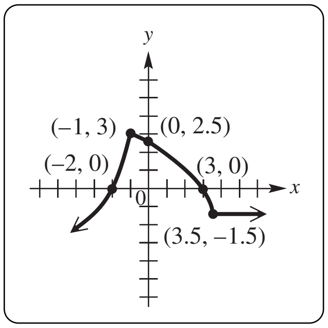


Figure 53a

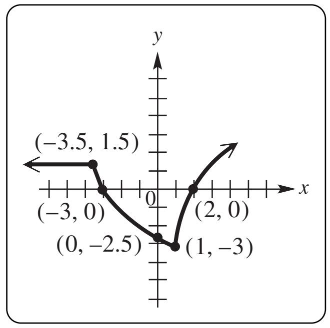


Figure 53b

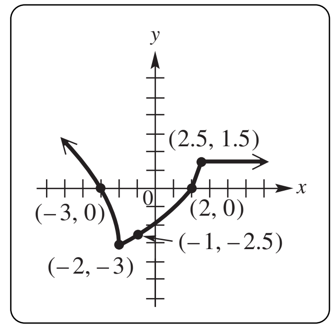


Figure 53c

54. (a) The equation $y = -f(x)$ is $y = f(x)$ reflected across the x-axis. See Figure 54a.
    (b) The equation $y = f(-x)$ is $y = f(x)$ reflected across the y-axis. See Figure 54b.
    (c) The equation $y = \frac{1}{2}f(x)$ is $y = f(x)$ shrunk vertically by a factor of $\frac{1}{2}$. See Figure 54c.
    (d) From the graph $f(x) < 0$ for the interval: $(-\infty, 0)$.

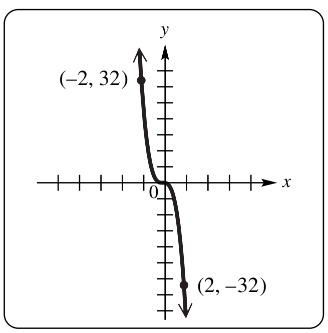


Figure 54a

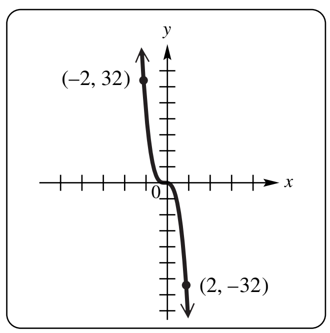


Figure 54b

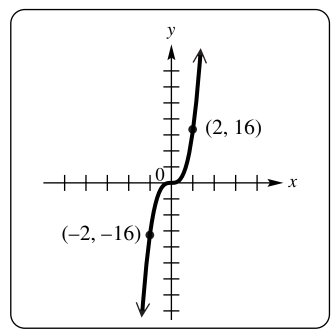


Figure 54c

55. (a) The equation $y = -f(x)$ is $y = f(x)$ reflected across the x-axis. See Figure 55a.
    (b) The equation $y = f\left(\frac{1}{3}x\right)$ is $y = f(x)$ stretched horizontally by a factor of 3. See Figure 55b.
    (c) The equation $y = .5f(x)$ is $y = f(x)$ shrunk vertically by a factor of .5. See Figure 55c.
    (d) From the graph, symmetry with respect to the origin.

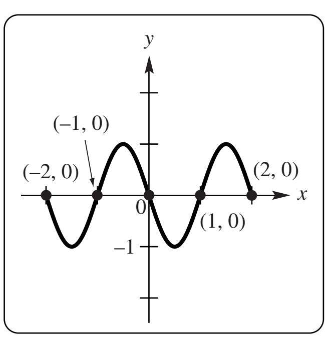


Figure 55a

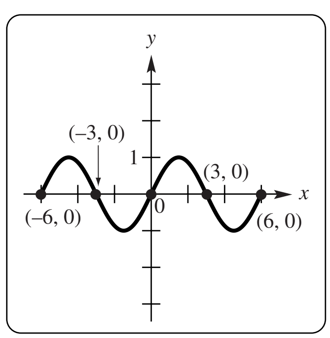


Figure 55b

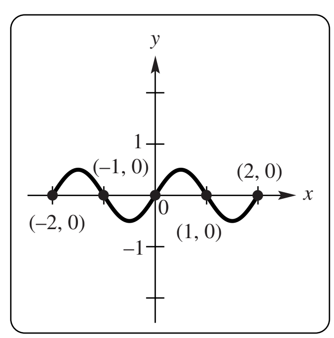


Figure 55c

56. (a) The equation $y = f(2x)$ is $y = f(x)$ stretched horizontally by a factor of $\frac{1}{2}$. See Figure 56a.

(b) The equation $y = f(-x)$ is $y = f(x)$ reflected across the y-axis. See Figure 56b.

(c) The equation $y = 3f(x)$ is $y = f(x)$ stretched vertically by a factor of 3. See Figure 56c.

(d) From the graph, symmetry with respect to the y-axis.

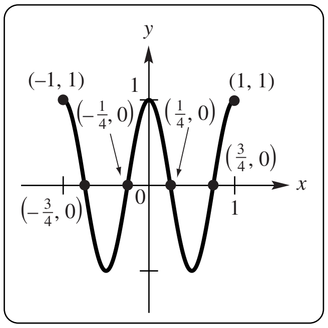


Figure 56a

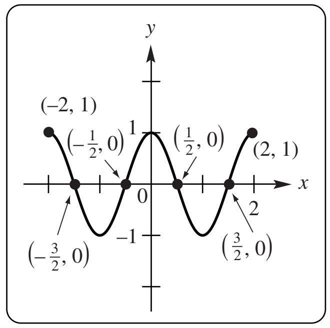


Figure 56b

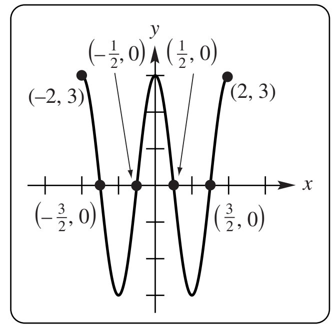


Figure 56c

57. (a) The equation $y = f(x) + 1$ is $y = f(x)$ shifted 1 unit upward. See Figure 57a.

(b) The equation $y = -f(x) - 1$ is $y = f(x)$ reflected across the x-axis and shifted 1 unit down. See Figure 57b.

(c) The equation $y = 2f\left(\frac{1}{2}x\right)$ is $y = f(x)$ stretched vertically by a factor of 2 and horizontally by a factor of 2.

See Figure 57c.

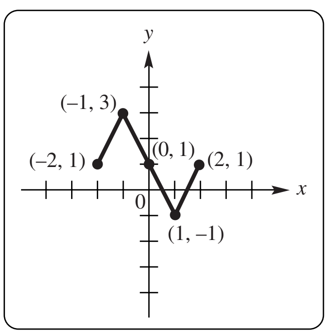


Figure 57a

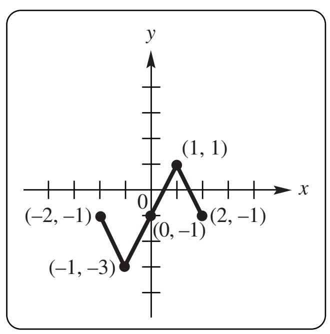


Figure 57b

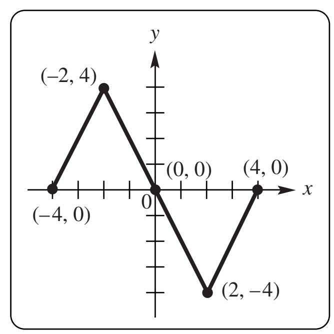


Figure 57c

58. (a) The equation $y = f(x) - 2$ is $y = f(x)$ shifted 2 units downward. See Figure 58a.

(b) The equation $y = f(x - 1) + 2$ is $y = f(x)$ shifted 1 unit right and 2 units upward. See Figure 58b.

(c) The equation $y = 2f(x)$ is $y = f(x)$ stretched vertically by a factor of 2. See Figure 58c.

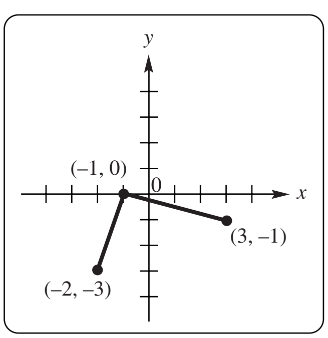


Figure 58a

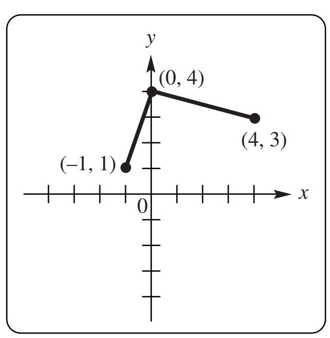


Figure 58b

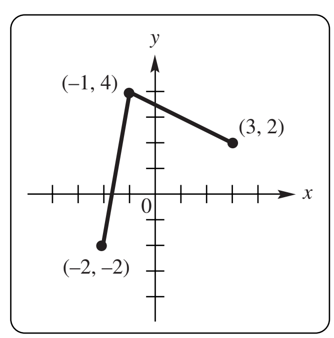


Figure 58c

59. (a) The equation $y = f(2x) + 1$ is $y = f(x)$ shrunk horizontally by a factor of $\frac{1}{2}$ and shifted 1 unit upward. See Figure 59a.

(b) The equation $y = 2f\left(\frac{1}{2}x\right) + 1$ is $y = f(x)$ stretched vertically by a factor of 2, stretched horizontally by a factor of 2, and shifted 1 unit upward. See Figure 59b.

(c) The equation $y = \frac{1}{2}f(x-2)$ is $y = f(x)$ shrunk vertically by a factor of $\frac{1}{2}$ and shifted 2 units to the right. See Figure 59c.

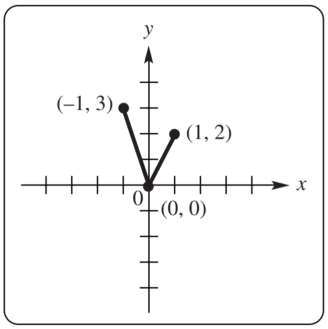


Figure 59a

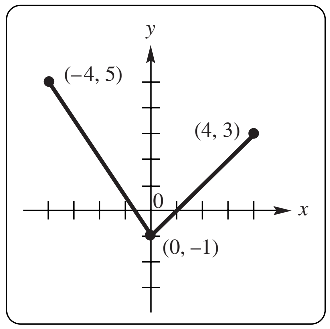


Figure 59b

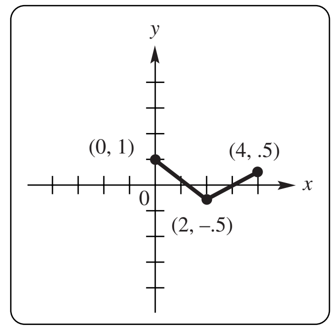


Figure 59c

60. (a) The equation $y = f(2x)$ is $y = f(x)$ shrunk horizontally by a factor of $\frac{1}{2}$. See Figure 60a.

(b) The equation $y = f\left(\frac{1}{2}x\right) - 1$ is $y = f(x)$ stretched horizontally by a factor of 2, and shifted 1 unit downward. See Figure 60b.

(c) The equation $y = 2f(x) - 1$ is $y = f(x)$ stretched vertically by a factor of 2 and shifted 1 unit downward. See Figure 60c.

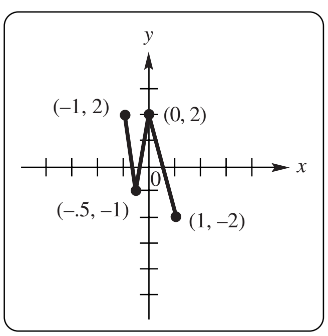


Figure 60a

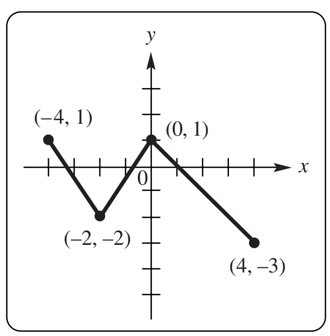


Figure 60b

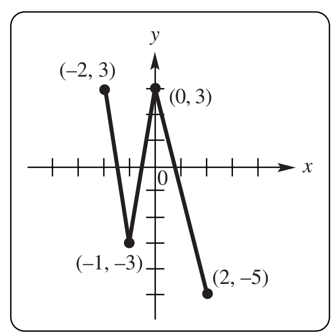


Figure 60c

61. (a) If r is the x-intercept of $y = f(x)$ and $y = -f(x)$ is $y = f(x)$ reflected across the x-axis, then r is also the x-intercept of $y = -f(x)$.

(b) If r is the x-intercept of $y = f(x)$ and $y = f(-x)$ is $y = f(x)$ reflected across the y-axis, then $-r$ is the x-intercept of $y = f(-x)$.

(c) If r is the x-intercept of $y = f(x)$ and $y = -f(-x)$ is $y = f(x)$ reflected across both the x-axis and y-axis, then $-r$ is the x-intercept of $y = -f(-x)$.

62. (a) If b is the y-intercept of $y = f(x)$ and $y = -f(x)$ is $y = f(x)$ reflected across the x-axis, then $-b$ is the y-intercept of $y = -f(x)$.

(b) If b is the y-intercept of $y = f(x)$ and $y = f(-x)$ is $y = f(x)$ reflected across the y-axis, then b is also the y-intercept of $y = f(-x)$.

(c) If b is the y-intercept of $y = f(x)$ and $y = 5f(x)$ is $y = f(x)$ stretched vertically by a factor of 5, then $5b$ is the y-intercept of $y = 5f(x)$.

(d) If b is the y-intercept of $y = f(x)$ and $y = -3f(x)$ is $y = f(x)$ reflected across the x-axis and stretched vertically by a factor of 3, then $-3b$ is the y-intercept of of $y = -3f(x)$.

63. Since $f(x-2)$ is $f(x)$ shifted 2 units to the right, the domain of $f(x-2)$ is: $[-1+2, 2+2]$ or $[1, 4]$; and the range is the same: $[0, 3]$.

64. Since $5f(x+1)$ is $f(x)$ shifted 1 unit to the left, the domain of $5f(x+1)$ is: $[-1-1, 2-1]$ or $[-2, 1]$; and stretched vertically by a factor of 5, the range is: $[5(0), 5(3)]$ or $[0, 15]$.

65. Since $-f(x)$ is $f(x)$ reflected across the x-axis, the domain of $f(x-2)$ is the same: $[-1, 2]$; and the range is: $[-(0), -(3)]$ or $[-3, 0]$.

66. Since $f(x-3)+1$ is $f(x)$ shifted 3 units to the right, the domain of $f(x-3)+1$ is: $[-1+3, 2+3]$ or $[2, 5]$; and shift 1 unit upward, the range is: $[0+1, 3+1]$ or $[1, 4]$.

67. Since $f(2x)$ is $f(x)$ shrunk horizontally by a factor of $\frac{1}{2}$, the domain of $f(2x)$ is: $\left[\frac{1}{2}(-1), \frac{1}{2}(2)\right]$ or $\left[-\frac{1}{2}, 1\right]$; and the range is the same: $[0, 3]$.

68. Since $2f(x-1)$ is $f(x)$ shifted 1 unit to the right, the domain of $2f(x-1)$ is: $[-1+1, 2+1]$ or $[0, 3]$; and stretched vertically by a factor of 2, the range is: $[2(0), 2(3)]$ or $[0, 6]$.

69. Since $3f\left(\frac{1}{4}x\right)$ is $f(x)$ stretched horizontally by a factor of 4, the domain of $3f\left(\frac{1}{4}x\right)$ is: $[4(-1), 4(2)]$ or $[-4, 8]$; and stretched vertically by a factor of 3, the range is: $[3(0), 3(3)]$ or $[0, 9]$.

70. Since $-2f(4x)$ is $f(x)$ shrunk horizontally by a factor of $\frac{1}{4}$, the domain of $-2f(4x)$ is: $\left[\frac{1}{4}(-1), \frac{1}{4}(2)\right]$ or $\left[-\frac{1}{4}, \frac{1}{2}\right]$; and reflected across the x-axis while being stretched vertically by a factor of 2, the range is: $[-2(0), -2(3)] = [0, -6]$ or $[-6, 0]$.

71. Since $f(-x)$ is $f(x)$ reflected across the y-axis, the domain of $f(-x)$ is: $[-(-1), -(2)] = [1, -2]$ or $[-2, 1]$; and the range is the same: $[0, 3]$.

72. Since $-2f(-x)$ is $f(x)$ reflected across the y-axis, the domain of $-2f(-x)$ is: $[-(-1), -(2)] = [1, -2]$ or $[-2, 1]$; and reflected across the x-axis while being stretched vertically by a factor of 2, the range is: $[-2(0), -2(3)] = [0, -6]$ or $[-6, 0]$.

73. Since $f(-3x)$ is $f(x)$ reflected across the y-axis and shrunk horizontally by a factor of $\frac{1}{3}$, the domain of $f(-3x)$ is: $\left[-\frac{1}{3}(-1), -\frac{1}{3}(2)\right] = \left[\frac{1}{3}, -\frac{2}{3}\right]$ or $\left[-\frac{2}{3}, \frac{1}{3}\right]$; and the range is the same: $[0, 3]$.

74. Since $\frac{1}{3}f(x-3)$ is $f(x)$ shifted 3 units to the right, the domain of $\frac{1}{3}f(x-3)$ is: $[-1+3, 2+3]$ or $[2, 5]$; and shrunk vertically by a factor of $\frac{1}{3}$, the range is: $\left[\frac{1}{3}(0), \frac{1}{3}(3)\right]$ or $[0, 1]$.

75. Since $y = \sqrt{x}$ has an endpoint (0, 0), and the graph of $y = 10\sqrt{x-20}+5$ is the graph of $y = \sqrt{x}$ shifted 20 units right, stretched vertically by a factor of 10, and shifted 5 units upward, the endpoint of $y = 10\sqrt{x-20}+5$ is: $(0+20, 10(0)+5)$ or $(20, 5)$. Therefore, the domain is: $[20, \infty)$; and the range is: $[5, \infty)$.

76. Since $y = \sqrt{x}$ has an endpoint (0, 0), and the graph of $y = -2\sqrt{x+15}-18$ is the graph of $y = \sqrt{x}$ shifted 15 units left, reflected across the x-axis, stretched vertically by a factor of 2, and shifted 18 units downward, the endpoint of $y = -2\sqrt{x+15}-18$ is: $(0-15, -2(0)-18)$ or $(-15, -18)$. Therefore, the domain is: $[-15, \infty)$; and the range, because of the reflection across the x-axis, is: $(-\infty, -18]$.

77. Since $y = \sqrt{x}$ has an endpoint (0, 0), and the graph of $y = -.5\sqrt{x+10}+5$ is the graph of $y = \sqrt{x}$ shifted 10 units left, reflected across the x-axis, shrunk vertically by a factor of .5, and shifted 5 units upward, the endpoint of $y = -.5\sqrt{x+10}+5$ is: $(0-10, -.5(0)+5)$ or $(-10, 5)$. Therefore, the domain is: $[-10, \infty)$; and the range, because of the reflection across the x-axis, is: $(-\infty, 5]$.

78. Using ex. 75, the domain is: $[h, \infty)$; and the range is: $[k, \infty)$.

79. The graph of $y = -f(x)$ is $y = f(x)$ reflected across the x-axis, therefore $y = -f(x)$ is decreasing for the interval: $[a, b]$.

80. The graph of $y = f(-x)$ is $y = f(x)$ reflected across the y-axis, therefore $y = f(-x)$ is decreasing for the interval: $[-b, -a]$.

81. The graph of $y = -f(-x)$ is $y = f(x)$ reflected across both the x-axis and y-axis, therefore $y = -f(-x)$ is increasing for the interval: $[-b, -a]$.

82. The graph of $y = -c \cdot f(x)$ is $y = f(x)$ reflected across the x-axis, therefore $y = -c \cdot f(x)$ is decreasing for the interval: $[a, b]$.

83. From the graph,
(a) the function is increasing for the interval: $[-1, 2]$.
(b) the function is decreasing for the interval: $(-\infty, -1]$.
(c) the function is constant for the interval: $[2, \infty)$.

84. From the graph,
(a) the function is increasing for the interval: $(-\infty, -1]$.
(b) the function is decreasing for the interval: $[-1, 2]$.
(c) the function is constant for the interval: $[2, \infty)$.

85. From the graph,
(a) the function is increasing for the interval: $[1, \infty)$.
(b) the function is decreasing for the interval: $[-2, 1]$.
(c) the function is constant for the interval: $(-\infty, -2]$.

86. From the graph,
(a) the function is increasing for the interval: $(-\infty, -3]$.
(b) the function is decreasing for the interval: $[-3, \infty)$.
(c) the function is constant for no interval: none.

87. From the graph, the point on y_2 is approximately: $(8, 10)$.

88. From the graph, the point on y_2 is approximately: $(-27, -15)$.

89. Use two points on the graph to find the slope, two points are: $(-2, -1)$ and $(-1, 1)$, therefore the slope is: $m = \dfrac{1-(-1)}{-1-(-2)} = \dfrac{2}{1} \Rightarrow m = 2$. The stretch factor is 2 and the graph has been shifted 2 units to the left and 1 unit down, therefore the equation is: $y = 2|x + 2| - 1$.

90. Use two points on the graph to find the slope, two points are: $(1, 2)$ and $(5, 0)$, therefore the slope is: $m = \dfrac{0-2}{5-1} = \dfrac{-2}{4} \Rightarrow m = -\dfrac{1}{2}$. The shrinking factor is $\dfrac{1}{2}$, the graph has been reflected across the x-axis, shifted 1 unit to the right, and shifted 2 units upward, therefore the equation is: $y = -\dfrac{1}{2}|x - 1| + 2$.

91. Use two points on the graph to find the slope, two points are: $(0, 2)$ and $(1, -1)$, therefore the slope is: $m = \dfrac{-1-2}{1-0} = \dfrac{-3}{1} \Rightarrow m = -3$. The stretch factor is 3, the graph has been reflected across the x-axis, and shifted 2 units upward, therefore the equation is: $y = -3|x| + 2$.

92. Use two points on the graph to find the slope, two points are: $(-1, -2)$ and $(0, 1)$, therefore the slope is: $m = \dfrac{1-(-2)}{0-(-1)} = \dfrac{3}{1} \Rightarrow m = 3$. The stretch factor is 3 and the graph has been shifted 1 unit to the left and 2 units down, therefore the equation is: $y = 3|x + 1| - 2$.

Reviewing Basic Concepts (Sections 2.1—2.3)

1. (a) If $y = f(x)$ is symmetric with respect to the origin, then another function value is: $f(-3) = -6$.
   (b) If $y = f(x)$ is symmetric with respect to the y-axis, then another function value is: $f(-3) = 6$.
   (c) If $f(-x) = -f(x)$, $y = f(x)$ is symmetric with respect to both the x-axis and y-axis, then another function value is: $f(-3) = -6$.
   (d) If $f(-x) = f(x)$, $y = f(x)$ is symmetric with respect to the y-axis, then another function value is: $f(-3) = 6$.

2. (a) The equation $y = (x - 7)^2$ is $y = x^2$ shifted 7 units to the right: B.
   (b) The equation $y = x^2 - 7$ is $y = x^2$ shifted 7 units downward: D.
   (c) The equation $y = 7x^2$ is $y = x^2$ stretched vertically by a factor of 7: E.
   (d) The equation $y = (x + 7)^2$ is $y = x^2$ shifted 7 units to the left: A.
   (e) The equation $y = \left(\dfrac{1}{3}x\right)^2$ is $y = x^2$ stretched horizontally by a factor of 3: C.

3. (a) The equation $y = x^2 + 2$ is $y = x^2$ shifted 2 units upward: B.
   (b) The equation $y = x^2 - 2$ is $y = x^2$ shifted 2 units downward: A.
   (c) The equation $y = (x + 2)^2$ is $y = x^2$ shifted 2 units to the left: G.
   (d) The equation $y = (x + 2)^2$ is $y = x^2$ shifted 2 units to the right: C.
   (e) The equation $y = 2x^2$ is $y = x^2$ stretched vertically by a factor of 2: F.
   (f) The equation $y = -x^2$ is $y = x^2$ reflected across the x-axis: D.
   (g) The equation $y = (x - 2)^2 + 1$ is $y = x^2$ shifted 2 units to the right and 1 unit upward: H.
   (h) The equation $y = (x + 2)^2 + 1$ is $y = x^2$ shifted 2 units to the left and 1 unit upward: E.

4. (a) The equation $y = |x| + 4$ is $y = |x|$ shifted 4 units upward. See Figure 4a.
   (b) The equation $y = |x + 4|$ is $y = |x|$ shifted 4 units to the left. See Figure 4b.
   (c) The equation $y = |x - 4|$ is $y = |x|$ shifted 4 units to the right. See Figure 4c.
   (d) The equation $y = |x + 2| - 4$ is $y = |x|$ shifted 2 units to the left and 4 units downward. See Figure 4d.
   (e) The equation $y = -|x - 2| + 4$ is $y = |x|$ reflected across the x-axis, shifted 2 units to the right, and 4 units upward. See Figure 4e.

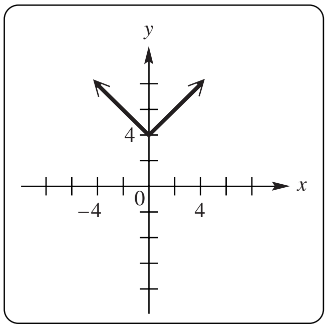


Figure 4a

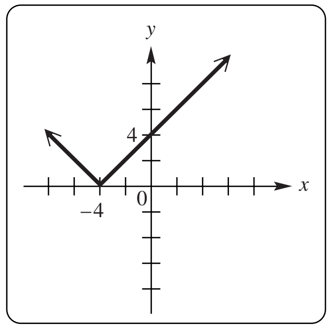


Figure 4b

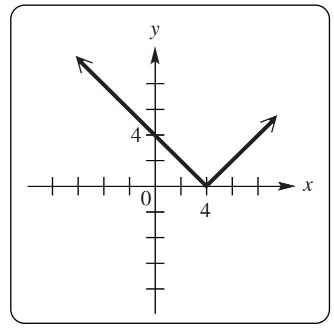


Figure 4c

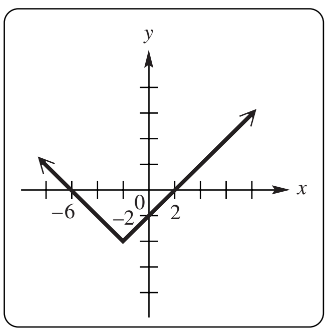


Figure 4d

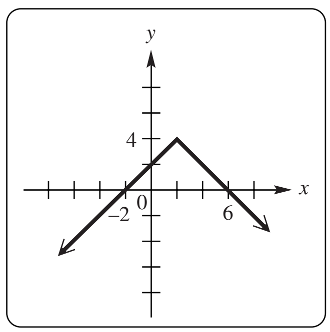


Figure 4e

5. (a) The graph is the function $f(x) = |x|$ reflected across the x-axis, shifted 1 unit left and 3 units upward. Therefore the equation is: $y = -|x + 1| + 3$.
   (b) The graph is the function $g(x) = \sqrt{x}$ reflected across the x-axis, shifted 4 units left and 2 units upward Therefore the equation is: $y = -\sqrt{x + 4} + 2$.
   (c) The graph is the function $g(x) = \sqrt{x}$ stretched vertically by a factor of 2, shifted 4 units left and 4 units downward. Therefore the equation is: $y = 2\sqrt{x + 4} - 4$.
   (d) The graph is the function $f(x) = |x|$ shrunk vertically by a factor of $\frac{1}{2}$, shifted 2 units right and 1 unit downward. Therefore the equation is: $y = \frac{1}{2}|x - 2| - 1$.
6. (a) The graph of $g(x)$ is the graph $f(x)$ shifted 2 units upward. Therefore $c = 2$.
   (b) The graph of $g(x)$ is the graph $f(x)$ shifted 4 units to the left. Therefore $c = 4$.
7. The graph of $y = F(x + h)$ is a horizontal translation of he graph of $y = F(x)$. The graph of $y = F(x) + h$ is not the same as the graph of $y = F(x + h)$ because the graph of $y = F(x) + h$ is a vertical translation of the graph of $y = F(x)$ and $y = F(x + h)$ is a horizontal translation of the graph $y = F(x)$.

8. The effect is either a stretch or a shrink, and perhaps a reflection across the x-axis. If $c > 0$, there is a stretch or shrink by a factor of c. If $c < 0$, there is a stretch or shrink by a factor of $|c|$, and a reflection across the x-axis. If $|c| > 1$, a stretch occurs; when $|c| < 1$, a shrink occurs.
9. (a) If f is even, then $f(x) = f(-x)$. See Figure 9a.
   (b) If f is odd, then $f(-x) = -f(x)$. See Figure 9b.
10. (a) Since $x = 1$ corresponds to 1992, the equation using actual year is: $g(x) = -.279(x - 1992) + 5.532$.
    (b) $g(2006) = -.279(2006 - 1992) + 5.532 \Rightarrow g(2006) = -.279(14) + 5.532 \Rightarrow$
    $g(2006) = 1.626$ ppm.

x	$f(x)$
−3	4
−2	−6
−1	5
1	5
2	−6
3	4

Figure 9a

x	$f(x)$
−3	4
−2	−6
−1	5
1	−5
2	6
3	−4

Figure 9b

2.4: Absolute Value Functions: Graphs, Equations, Inequalities, and Applications

1. If $f(a) = -5$, then $|f(a)| = |-5| = 5$.
2. Since $f(x) = x^2$ is an even function, $f(x) = x^2$ and $f(x) = |x^2|$ are the same graph.
3. If $f(x) = -x^2$, then $y = |f(x)| \Rightarrow y = |-x^2| \Rightarrow y = x^2$. Therefore the range of $y = |f(x)|$ is: $[0, \infty)$.
4. If the range of $y = f(x)$ is $[-2, \infty)$, the range of $y = |f(x)|$ is $[0, \infty)$ since all negative values of y are reflected across the x-axis.
5. If the range of $y = f(x)$ is $(-\infty, -2]$, the range of $y = |f(x)|$ is $[2, \infty)$ since all negative values of y are reflected across the x-axis.
6. $|f(x)|$ is greater than or equal to 0 for any value of x. Since -1 is less than 0, -1 cannot be in the range of f.
7. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 7.
8. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 8.
9. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 9.

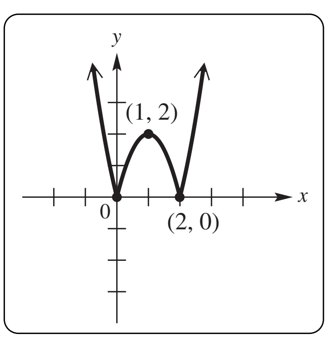


Figure 7

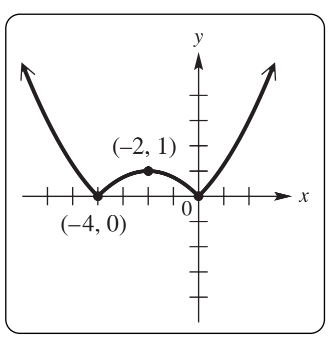


Figure 8

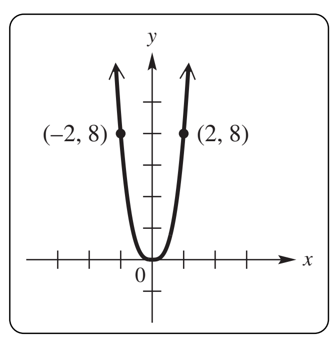


Figure 9

10. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 10.
11. Since for all y, $y \geq 0$, the graph remains unchanged. That is, $y = |f(x)|$ has the same graph as $y = f(x)$.
12. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 12.

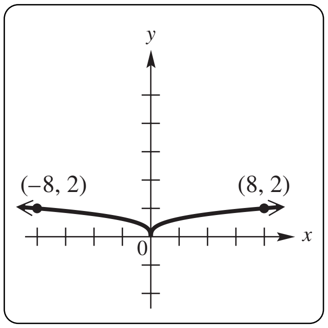


Figure 10

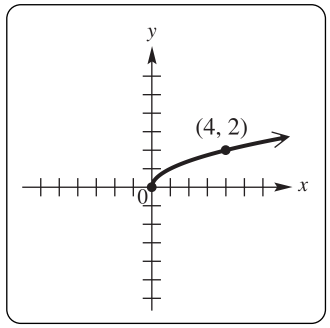


Figure 12

13. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 13.
14. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 14.
15. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 15.

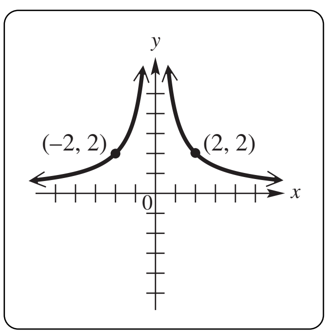


Figure 13

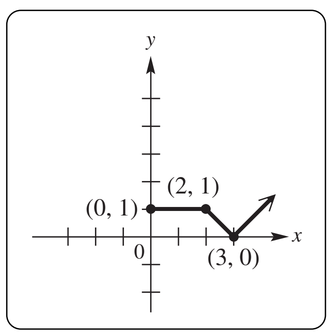


Figure 14

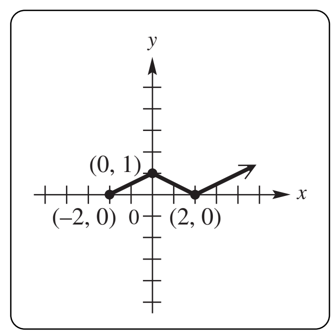


Figure 15

16. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged.
17. From the graph of $y = (x + 1)^2 - 2$ the domain of $f(x)$ is: $(-\infty, \infty)$; and the range is: $[-2, \infty)$.
    From the graph of $y = |(x + 1)^2 - 2|$ the domain of $|f(x)|$ is: $(-\infty, \infty)$; and the range is: $[0, \infty)$.
18. From the graph of $y = 2 - \frac{1}{2}x$ the domain of $f(x)$ is: $(-\infty, \infty)$; and the range is: $[-\infty, \infty)$.
    From the graph of $y = |2 - \frac{1}{2}x|$ the domain of $|f(x)|$ is: $(-\infty, \infty)$; and the range is: $[0, \infty)$.
19. From the graph of $y = -1 - (x - 2)^2$ the domain of $f(x)$ is: $(-\infty, \infty)$; and the range is: $(-\infty, -1]$.
    From the graph of $y = |-1 - (x - 2)^2|$ the domain of $|f(x)|$ is: $(-\infty, \infty)$; and the range is: $[1, \infty)$.

20. From the graph of $y = -|x + 2| - 2$ the domain of $f(x)$ is: $(-\infty, \infty)$; and the range is: $(-\infty, -2]$.
From the graph of $y = |(-|x + 2| - 2)|$ the domain of $|f(x)|$ is: $(-\infty, \infty)$; and the range is: $[2, \infty)$.

21. From the graph, the domain of $f(x)$ is: $[-2, 3]$; and the range is: $[-2, 3]$. For the function $y = |f(x)|$, we reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$ and where $y \geq 0$, the graph remains unchanged. Therefore, the domain of $y = |f(x)|$ is: $[-2, 3]$; and the range is: $[0, 3]$.

22. From the graph, the domain of $f(x)$ is: $[-3, 2]$; and the range is: $[-2, 2]$. For the function $y = |f(x)|$, we reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$ and where $y \geq 0$, the graph remains unchanged. Therefore, the domain of $y = |f(x)|$ is: $[-3, 2]$; and the range is: $[0, 2]$.

23. From the graph, the domain of $f(x)$ is: $[-2, 3]$; and the range is: $[-3, 1]$. For the function $y = |f(x)|$, we reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$ and where $y \geq 0$, the graph remains unchanged. Therefore, the domain of $y = |f(x)|$ is: $[-2, 3]$; and the range is: $[0, 3]$.

24. From the graph, the domain of $f(x)$ is: $[-3, 3]$; and the range is: $[-3, -1]$. For the function $y = |f(x)|$, we reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$ and where $y \geq 0$, the graph remains unchanged. Therefore, the domain of $y = |f(x)|$ is: $[-3, 3]$; and the range is: $[1, 3]$.

25. (a) The function $y = f(-x)$ is the function $y = f(x)$ reflected across the y-axis. See Figure 25a.
(b) The function $y = -f(-x)$ is the function $y = f(x)$ reflected across both the x-axis and y-axis. See Figure 25b.
(c) For the function $y = |-f(-x)|$ we reflect the graph of $y = -f(-x)$ (ex. b) across the x-axis for all points for which $y < 0$ and where $y \geq 0$, the graph remains unchanged. See Figure 25c.

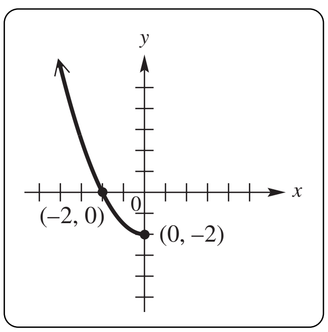


Figure 25a

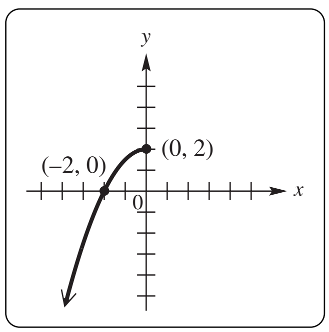


Figure 25b

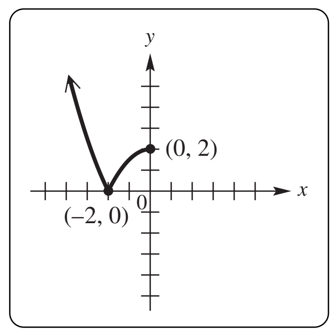


Figure 25c

26. (a) The function $y = f(-x)$ is the function $y = f(x)$ reflected across the y-axis. See Figure 26a.
(b) The function $y = -f(-x)$ is the function $y = f(x)$ reflected across both the x-axis and y-axis. See Figure 26b.
(c) For the function $y = |-f(-x)|$ we reflect the graph of $y = -f(-x)$ (ex. b) across the x-axis for all points for which $y < 0$ and where $y \geq 0$, the graph remains unchanged. See Figure 26c.

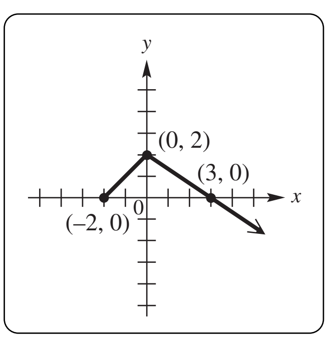


Figure 26a

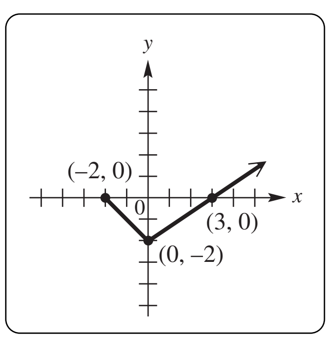


Figure 26b

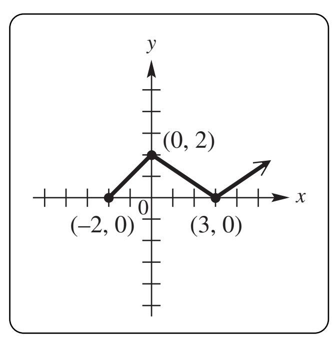


Figure 26c

27. The graph of $y = |f(x)|$ can not be below the x-axis, therefore Figure A shows the graph of $y = f(x)$, while Figure B shows the graph of $y = |f(x)|$.
28. The graph of $y = |f(x)|$ can not be below the x-axis, therefore Figure B shows the graph of $y = f(x)$, while Figure A shows the graph of $y = |f(x)|$.
29. (a) From the graph, $y_1 = y_2$ at the coordinates $(-1, 5)$ and $(6, 5)$, therefore the solution set is: $\{-1, 6\}$.
    (b) From the graph, $y_1 < y_2$ for the interval: $(-1, 6)$.
    (c) From the graph, $y_1 > y_2$ for the intervals: $(-\infty, -1) \cup (6, \infty)$.
30. (a) From the graph, $y_1 = y_2$ at the coordinates $(0, -2)$ and $(8, -2)$, therefore the solution set is: $\{0, 8\}$.
    (b) From the graph, $y_1 < y_2$ for the intervals: $(-\infty, 0) \cup (8, \infty)$.
    (c) From the graph, $y_1 > y_2$ for the intervals: $(0, 8)$.
31. (a) From the graph, $y_1 = y_2$ at the coordinate $(4, 1)$, therefore the solution set is: $\{4\}$.
    (b) From the graph, $y_1 < y_2$ never, therefore the solution set is: $\varnothing$.
    (c) From the graph, $y_1 > y_2$ for all values for x, except 4, therefore for the intervals: $(-\infty, 4) \cup (4, \infty)$.
32. (a) From the graph, $y_1 = y_2$ never, therefore the solution set is: $\varnothing$.
    (b) From the graph, $y_1 < y_2$ for all values for x, therefore for the interval: $(-\infty, \infty)$.
    (c) From the graph, $y_1 > y_2$ never, therefore the solution set is: $\varnothing$.
33. The V-shaped graph is that of $f(x) = |.5x + 6|$, since this is typical of the graphs of absolute value functions of the form $f(x) = |ax + b|$.
34. The straight line graph is that of $g(x) = 3x - 14$, which is a linear function.
35. The graph intersects at $(8, 10)$, so the solution set is: $\{8\}$.
36. From the graph, $f(x) > g(x)$ for the intervals: $(-\infty, 8)$.
37. From the graph, $f(x) < g(x)$ for the intervals: $(8, \infty)$.
38. If $|.5x + 6| - (3x - 14) = 0$ then $|.5x + 6| = 3x - 14$. Therefore the solution is the intersection of the graphs or $\{8\}$.
39. (a) $|x + 4| = 9 \Rightarrow x + 4 = 9$ or $x + 4 = -9 \Rightarrow x = 5$ or $x = -13$. The solution set is: $\{-13, 5\}$; which is supported by the graphs of $y_1 = |x + 4|$ and $y_2 = 9$.
    (b) $|x + 4| > 9 \Rightarrow x + 4 > 9$ or $x + 4 < -9 \Rightarrow x > 5$ or $x < -13$. The solution is: $(-\infty, -13) \cup (5, \infty)$; which is supported by the graphs of $y_1 = |x + 4|$ and $y_2 = 9$.
    (c) $|x + 4| < 9 \Rightarrow -9 < x + 4 < 9 \Rightarrow -13 < x < 5$. The solution is: $(-13, 5)$; which is supported by the graphs of $y_1 = |x + 4|$ and $y_2 = 9$.
40. (a) $|x - 3| = 5 \Rightarrow x - 3 = 5$ or $x - 3 = -5 \Rightarrow x = 8$ or $x = -2$. The solution set is: $\{-2, 8\}$; which is supported by the graphs of $y_1 = |x - 3|$ and $y_2 = 5$.
    (b) $|x - 3| > 5 \Rightarrow x - 3 > 5$ or $x - 3 < -5 \Rightarrow x > 8$ or $x < -2$. The solution is: $(-\infty, -2) \cup (8, \infty)$; which is supported by the graphs of $y_1 = |x - 3|$ and $y_2 = 5$.
    (c) $|x - 3| < 5 \Rightarrow -5 < x - 3 < 5 \Rightarrow -2 < x < 8$. The solution is: $(-2, 8)$; which is supported by the graphs of $y_1 = |x - 3|$ and $y_2 = 5$.

41. (a) $|-2x + 7| = 3 \Rightarrow -2x + 7 = 3$ or $-2x + 7 = -3 \Rightarrow -2x = -4$ or $-2x = -10 \Rightarrow x = 2$ or $x = 5$.
The solution set is: $\{2, 5\}$; which is supported by the graphs of $y_1 = |-2x + 7|$ and $y_2 = 3$.
(b) $|-2x + 7| \geq 3 \Rightarrow -2x + 7 \geq 3$ or $-2x + 7 \leq -3 \Rightarrow -2x \geq -4$ or $-2x \leq -10 \Rightarrow x \leq 2$ or $x \geq 5$.
The solution is: $(-\infty, 2] \cup [5, \infty)$; which is supported by the graphs of $y_1 = |-2x + 7|$ and $y_2 = 3$.
(c) $|-2x + 7| \leq 3 \Rightarrow -3 \leq -2x + 7 \leq 3 \Rightarrow -10 \leq -2x \leq -4 \Rightarrow 5 \geq x \geq 2$ or $2 \leq x \leq 5$.
The solution is: $[2, 5]$; which is supported by the graphs of $y_1 = |-2x + 7|$ and $y_2 = 3$.

42. (a) $|-3x - 9| = 6 \Rightarrow -3x - 9 = 6$ or $-3x - 9 = -6 \Rightarrow -3x = 15$ or $-3x = 3 \Rightarrow x = -5$ or $x = -1$.
The solution set is: $\{-5, -1\}$; which is supported by the graphs of $y_1 = |-3x - 9|$ and $y_2 = 6$.
(b) $|-3x - 9| \geq 6 \Rightarrow -3x - 9 \geq 6$ or $-3x - 9 \leq -6 \Rightarrow -3x \geq 15$ or $-3x \leq 3 \Rightarrow x \leq -5$ or $x \geq -1$.
The solution is: $(-\infty, -5] \cup [-1, \infty)$; which is supported by the graphs of $y_1 = |-3x - 9|$ and $y_2 = 6$.
(c) $|-3x - 9| \leq 6 \Rightarrow -6 \leq -3x - 9 \leq 6 \Rightarrow 3 \leq -3x \leq 15 \Rightarrow -1 \geq x \geq -5$ or $-5 \leq x \leq -1$.
The solution is: $[-5, -1]$; which is supported by the graphs of $y_1 = |-3x - 9|$ and $y_2 = 6$.

43. (a) $|2x + 1| + 3 = 5 \Rightarrow 2x + 1 = 2$ or $2x + 1 = -2 \Rightarrow 2x = 1$ or $2x = -3 \Rightarrow x = \frac{1}{2}$ or $x = -\frac{3}{2}$.
The solution set is: $\left\{-\frac{3}{2}, \frac{1}{2}\right\}$; which is supported by the graphs of $y_1 = |2x + 1| + 3$ and $y_2 = 5$.
(b) $|2x + 1| + 3 \leq 5 \Rightarrow -2 \leq 2x + 1 \leq 2 \Rightarrow -3 \leq 2x \leq 1 \Rightarrow -\frac{3}{2} \leq x \leq \frac{1}{2}$. The solution is:
$\left[-\frac{3}{2}, \frac{1}{2}\right]$; which is supported by the graphs of $y_1 = |2x + 1| + 3$ and $y_2 = 5$.
(c) $|2x + 1| + 3 \geq 5 \Rightarrow 2x + 1 \geq 2$ or $2x + 1 \leq -2 \Rightarrow 2x \geq 1$ or $2x \leq -3 \Rightarrow x \geq \frac{1}{2}$ or $x \leq -\frac{3}{2}$.
The solution is: $\left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$; which is supported by the graphs of $y_1 = |2x + 1| + 3$ and $y_2 = 5$.

44. (a) $|4x + 7| = 0 \Rightarrow 4x + 7 = 0 \Rightarrow 4x = -7 \Rightarrow x = -\frac{7}{4}$. The solution set is: $\left\{-\frac{7}{4}\right\}$; which is supported by the graphs of $y_1 = |4x + 7|$ and $y_2 = 0$.
(b) $|4x + 7| > 0 \Rightarrow 4x + 7 > 0$ or $4x + 7 < 0 \Rightarrow 4x > -7$ or $4x < -7 \Rightarrow x > -\frac{7}{4}$ or $x < -\frac{7}{4}$.
The solution is: $\left(-\infty, -\frac{7}{4}\right) \cup \left(-\frac{7}{4}, \infty\right)$; which is supported by the graphs of $y_1 = |4x + 7|$ and $y_2 = 0$.
(c) Absolute value is always positive, therfore the solution set is: $\varnothing$; which is supported by the graphs of $y_1 = |4x + 7|$ and $y_2 = 0$.

45. (a) $|7x - 5| = 0 \Rightarrow 7x - 5 = 0 \Rightarrow 7x = 5 \Rightarrow x = \frac{5}{7}$. The solution set is: $\left\{\frac{5}{7}\right\}$; which is supported by the graphs of $y_1 = |7x - 5|$ and $y_2 = 0$.
(b) $|7x - 5| \geq 0 \Rightarrow 7x - 5 \geq 0$ or $7x - 5 \leq 0 \Rightarrow 7x \geq 5$ or $7x \leq 5 \Rightarrow x \geq \frac{5}{7}$ or $x \leq \frac{5}{7}$. The solution is: $(-\infty, \infty)$; which is supported by the graphs of $y_1 = |7x - 5|$ and $y_2 = 0$.
(c) $|7x - 5| \leq 0 \Rightarrow 0 \leq 7x - 5 \leq 0 \Rightarrow 5 \leq 7x \leq 5 \Rightarrow \frac{5}{7} \leq x \leq \frac{5}{7}$. The solution set is: $\left\{\frac{5}{7}\right\}$; which is supported by the graphs of $y_1 = |7x - 5|$ and $y_2 = 0$.

46. (a) Absolute value is always positive, therefore the solution set is: $\varnothing$; which is supported by the graphs of $y_1 = |\pi x + 8|$ and $y_2 = -4$.
(b) Absolute value is always positive and cannot be less than -4, therefore the solution set is: $\varnothing$; which is supported by the graphs of $y_1 = |\pi x + 8|$ and $y_2 = -4$.
(c) Absolute value is always positive and is always greater than -4, therefore the solution is: $(-\infty, \infty)$; which is supported by the graphs of $y_1 = |\pi x + 8|$ and $y_2 = -4$.
47. (a) Absolute value is always positive, therefore the solution set is: $\varnothing$; which is supported by the graphs of $y_1 = |\sqrt{2}x - 3.6|$ and $y_2 = -1$.
(b) Absolute value is always positive and cannot be less than or equal to -1, therefore the solution set is: $\varnothing$; which is supported by the graphs of $y_1 = |\sqrt{2}x - 3.6|$ and $y_2 = -1$.
(c) Absolute value is always positive and is always greater than -1, therefore the solution is: $(-\infty, \infty)$; which is supported by the graphs of $y_1 = |\sqrt{2}x - 3.6|$ and $y_2 = -1$.
48. $|2x + 4| + 2 = 10 \Rightarrow |2x + 4| = 8 \Rightarrow 2x + 4 = 8$ or $2x + 4 = -8 \Rightarrow 2x = 4$ or $2x = -12 \Rightarrow x = 2$ or $x = -6$. Therefore, The solution set is: $\{-6, 2\}$.
49. $3|4 - 3x| - 4 = 8 \Rightarrow 3|4 - 3x| = 12 \Rightarrow |4 - 3x| = 4 \Rightarrow 4 - 3x = 4$ or $4 - 3x = -4 \Rightarrow -3x = 0$ or $-3x = -8 \Rightarrow x = 0$ or $x = \frac{8}{3}$. Therefore, the solution set is: $\left\{0, \frac{8}{3}\right\}$.
50. $5|x + 3| - 2 = 18 \Rightarrow 5|x + 3| = 20 \Rightarrow |x + 3| = 4 \Rightarrow x + 3 = 4$ or $x + 3 = -4 \Rightarrow x = 1$ or $x = -7$. Therefore, the solution set is: $\{-7, 1\}$.
51. $\frac{1}{2}|-2x + \frac{1}{2}| = \frac{3}{4} \Rightarrow |-2x + \frac{1}{2}| = \frac{3}{2} \Rightarrow -2x + \frac{1}{2} = \frac{3}{2}$ or $-2x + \frac{1}{2} = -\frac{3}{2} \Rightarrow -2x = 1$ or $-2x = -2 \Rightarrow x = -\frac{1}{2}$ or $x = 1$. Therefore, the solution set is: $\left\{-\frac{1}{2}, 1\right\}$.
52. $|3(x - 5) + 2| + 3 = 9 \Rightarrow |3(x - 5) + 2| = 6 \Rightarrow 3(x - 5) + 2 = 6$ or $3(x - 5) + 2 = -6 \Rightarrow 3(x - 5) = 4$ or $3(x - 5) = -8 \Rightarrow x - 5 = \frac{4}{3}$ or $x - 5 = -\frac{8}{3} \Rightarrow x = \frac{19}{3}$ or $x = \frac{7}{3}$. Therefore, the solution set is: $\left\{\frac{7}{3}, \frac{19}{3}\right\}$.
53. $4.2|.5 - x| + 1 = 3.1 \Rightarrow 4.2|.5 - x| = 2.1 \Rightarrow |.5 - x| = .5 \Rightarrow .5 - x = .5$ or $.5 - x = -.5 \Rightarrow -x = 0$ or $-x = -1 \Rightarrow x = 0$ or $x = 1$. Therefore, the solution set is: $\{0, 1\}$.
54. $|3x - 1| < 8 \Rightarrow -8 < 3x - 1 < 8 \Rightarrow -7 < 3x < 9 \Rightarrow -\frac{7}{3} < x < 3$. Therefore, the solution is: $\left(-\frac{7}{3}, 3\right)$.
55. $|15 - x| < 7 \Rightarrow -7 < 15 - x < 7 \Rightarrow -22 < -x < -8 \Rightarrow 22 > x > 8$ or $8 < x < 22$. Therefore, the solution is: $(8, 22)$.
56. $|7 - 4x| \le 11 \Rightarrow -11 \le 7 - 4x \le 11 \Rightarrow -18 \le -4x \le 4 \Rightarrow \frac{9}{2} \ge x \ge -1$ or $-1 \le x \le \frac{9}{2}$. Therefore, the solution is: $\left[-1, \frac{9}{2}\right]$.
57. $|2x - 3| > 1 \Rightarrow 2x - 3 > 1$ or $2x - 3 < -1 \Rightarrow 2x > 4$ or $2x < 2 \Rightarrow x > 2$ or $x < 1$. Therefore, the solution is: $(-\infty, 1) \cup (2, \infty)$.

58. Absolute value is always positive and is always greater than -2, therefore the solution is: $(-\infty, \infty)$.

59. $|-3x + 8| \geq 3 \Rightarrow -3x + 8 \geq 3$ or $-3x + 8 \leq -3 \Rightarrow -3x \geq -5$ or $-3x \leq -11 \Rightarrow x \leq \frac{5}{3}$ or $x \geq \frac{11}{3}$.

Therefore, the solution is: $\left(-\infty, \frac{5}{3}\right] \cup \left[\frac{11}{3}, \infty\right)$.

60. Absolute value is always positive and is always greater than -1, therefore the solution is: $(-\infty, \infty)$.

61. $|6 - \frac{1}{3}x| > 0 \Rightarrow 6 - \frac{1}{3}x > 0$ or $6 - \frac{1}{3}x < 0 \Rightarrow -\frac{1}{3}x > -6$ or $-\frac{1}{3}x < -6 \Rightarrow x < 18$ or $x > 18$.

Therefore the solution is every real number except 18, the solution is: $(-\infty, 18) \cup (18, \infty)$.

62. Absolute value is always positive and cannot be less than 0, therefore the solution set is: $\emptyset$;

63. Absolute value is always positive and cannot be less than or equal to -6, therefore the solution set is: $\emptyset$;

64. Absolute value is always positive and cannot be less than -4, therefore the solution set is: $\emptyset$;

65. Absolute value is always positive and is always greater than -5, therefore the solution is: $(-\infty, \infty)$.

66. To solve such an equation, we must solve the compound equation $ax + b = cx + d$ or $ax + b = -(cx + d)$.

The solution set consist of the union of the two individual solution sets.

67. (a) $3x + 1 = 2x - 7 \Rightarrow x + 1 = -7 \Rightarrow x = -8$ or $3x + 1 = -(2x - 7) \Rightarrow 3x + 1 = -2x + 7 \Rightarrow$

$5x + 1 = 7 \Rightarrow 5x = 6 \Rightarrow x = \frac{6}{5}$. Therefore the solution set is: $\left\{-8, \frac{6}{5}\right\}$.

(b) Graph $y_1 = |3x + 1|$ and $y_2 = |2x - 7|$. See Figure 67. From the graph, $|f(x)| > |g(x)|$ when

$y_1 > y_2$ which is for the interval: $(-\infty, -8) \cup \left(\frac{6}{5}, \infty\right)$.

(c) Graph $y_1 = |3x + 1|$ and $y_2 = |2x - 7|$. See Figure 67. From the graph, $|f(x)| < |g(x)|$ when

$y_1 < y_2$ which is for the interval: $\left(-8, \frac{6}{5}\right)$.

68. (a) $x - 4 = 7x + 12 \Rightarrow -6x - 4 = 12 \Rightarrow -6x = 16 \Rightarrow x = -\frac{8}{3}$ or $x - 4 = -(7x + 12) \Rightarrow$

$x - 4 = -7x - 12 \Rightarrow 8x - 4 = -12 \Rightarrow 8x = -8 \Rightarrow x = -1$. Therefore, the solution set is: $\left\{-\frac{8}{3}, -1\right\}$.

(b) Graph $y_1 = |x - 4|$ and $y_2 = |7x + 12|$. See Figure 68. From the graph, $|f(x)| > |g(x)|$ when

$y_1 > y_2$ which is for the interval: $\left(-\frac{8}{3}, -1\right)$.

(c) Graph $y_1 = |x - 4|$ and $y_2 = |7x + 12|$. See Figure 68. From the graph, $|f(x)| < |g(x)|$ when

$y_1 < y_2$ which is for the interval: $\left(-\infty, -\frac{8}{3}\right) \cup (-1, \infty)$.

[−20, 20] by [−10, 50]
Xscl = 2 Yscl = 5

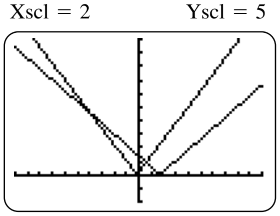


Figure 67

[−10, 10] by [−4, 16]
Xscl = 1 Yscl = 1

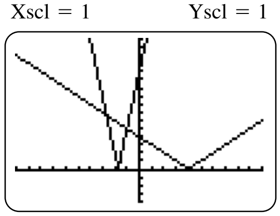


Figure 68

69. (a) $-2x + 5 = x + 3 \Rightarrow -3x = -2 \Rightarrow x = \frac{2}{3}$ or $-2x + 5 = -(x + 3) \Rightarrow -2x + 5 = -x - 3 \Rightarrow$ $-x = -8 \Rightarrow x = 8$. Therefore, the solution set is: $\left\{\frac{2}{3}, 8\right\}$.

(b) Graph $y_1 = |-2x + 5|$ and $y_2 = |x + 3|$. See Figure 69. From the graph, $|f(x)| > |g(x)|$ when $y_1 > y_2$ which is for the interval: $\left(-\infty, \frac{2}{3}\right) \cup (8, \infty)$.

(c) Graph $y_1 = |-2x + 5|$ and $y_2 = |x + 3|$. See Figure 69. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$ which is for the interval: $\left(\frac{2}{3}, 8\right)$.

70. (a) $-5x + 1 = 3x - 4 \Rightarrow -8x = -5 \Rightarrow x = \frac{5}{8}$ or $-5x + 1 = -(3x - 4) \Rightarrow -5x + 1 = -3x + 4 \Rightarrow$ $-2x = 3 \Rightarrow x = -\frac{3}{2}$. Therefore, the solution set is: $\left\{-\frac{3}{2}, \frac{5}{8}\right\}$.

(b) Graph $y_1 = |-5x + 1|$ and $y_2 = |3x - 4|$. See Figure 70. From the graph, $|f(x)| > |g(x)|$ when $y_1 > y_2$ which is for the interval: $\left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{5}{8}, \infty\right)$.

(c) Graph $y_1 = |-5x + 1|$ and $y_2 = |3x - 4|$. See Figure 70. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$ which is for the interval: $\left(-\frac{3}{2}, \frac{5}{8}\right)$.

$[-10, 10]$ by $[-4, 16]$
Xscl = 1 Yscl = 1

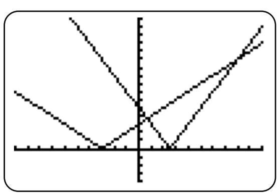


Figure 69

$[-3, 3]$ by $[-4, 16]$
Xscl = 1 Yscl = 2

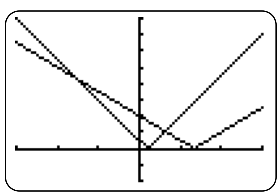


Figure 70

$[-6, 6]$ by $[-2, 10]$
Xscl = 1 Yscl = 1

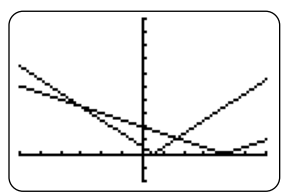


Figure 71

71. (a) $x - \frac{1}{2} = \frac{1}{2}x - 2 \Rightarrow \frac{1}{2}x = -\frac{3}{2} \Rightarrow x = -3$ or $x - \frac{1}{2} = -\left(\frac{1}{2}x - 2\right) \Rightarrow x - \frac{1}{2} = -\frac{1}{2}x + 2 \Rightarrow$ $\frac{3}{2}x = \frac{5}{2} \Rightarrow x = \frac{5}{3}$. Therefore, the solution set is: $\left\{-3, \frac{5}{3}\right\}$.

(b) Graph $y_1 = \left|x - \frac{1}{2}\right|$ and $y_2 = \left|\frac{1}{2}x - 2\right|$. See Figure 71. From the graph, $|f(x)| > |g(x)|$ when $y_1 > y_2$ which is for the interval: $(-\infty, -3) \cup \left(\frac{5}{3}, \infty\right)$.

(c) Graph $y_1 = \left|x - \frac{1}{2}\right|$ and $y_2 = \left|\frac{1}{2}x - 2\right|$. See Figure 71. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$ which is for the interval: $\left(-3, \frac{5}{3}\right)$.

72. (a) $x + 3 = \frac{1}{3}x + 8 \Rightarrow \frac{2}{3}x = 5 \Rightarrow x = \frac{15}{2}$ or $x + 3 = -\left(\frac{1}{3}x + 8\right) \Rightarrow x + 3 = -\frac{1}{3}x - 8 \Rightarrow$ $\frac{4}{3}x = -11 \Rightarrow x = -\frac{33}{4}$. Therefore, the solution set is: $\left\{-\frac{33}{4}, \frac{15}{2}\right\}$.

(b) Graph $y_1 = |x + 3|$ and $y_2 = \left|\frac{1}{3}x + 8\right|$. See Figure 72. From the graph, $|f(x)| > |g(x)|$ when $y_1 > y_2$ which is for the interval: $\left(-\infty, -\frac{33}{4}\right) \cup \left(\frac{15}{2}, \infty\right)$.

(c) Graph $y_1 = |x + 3|$ and $y_2 = \left|\frac{1}{3}x + 8\right|$. See Figure 72. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$ which is for the interval: $\left(-\frac{33}{4}, \frac{15}{2}\right)$.

73. (a) $4x + 1 = 4x + 6 \Rightarrow 1 = 6 \Rightarrow \varnothing$ or $4x + 1 = -(4x + 6) \Rightarrow 4x + 1 = -4x - 6 \Rightarrow$ $8x = -7 \Rightarrow x = -\frac{7}{8}$. Therefore, the solution set is: $\left\{-\frac{7}{8}\right\}$.

(b) Graph $y_1 = |4x + 1|$ and $y_2 = |4x + 6|$. See Figure 73. From the graph, $|f(x)| > |g(x)|$ when $y_1 > y_2$ which is for the interval: $\left(-\infty, -\frac{7}{8}\right)$.

(c) Graph $y_1 = |4x + 1|$ and $y_2 = |4x + 6|$. See Figure 73. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$ which is for the interval: $\left(-\frac{7}{8}, \infty\right)$.

$[-40, 20]$ by $[-4, 16]$
Xscl = 4 Yscl = 2

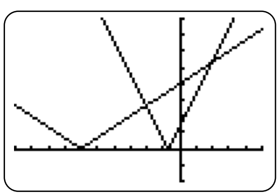


Figure 72

$[-5, 5]$ by $[-4, 16]$
Xscl = 1 Yscl = 2

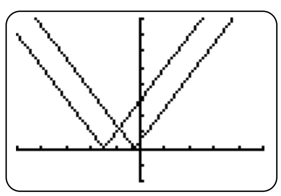


Figure 73

$[-5, 5]$ by $[-4, 16]$
Xscl = 1 Yscl = 2

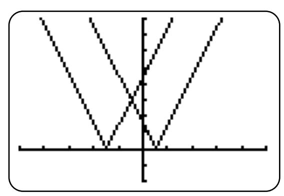


Figure 74

74. (a) $6x + 9 = 6x - 3 \Rightarrow 9 = -3 \Rightarrow \varnothing$ or $6x + 9 = -(6x - 3) \Rightarrow 6x + 9 = -6x + 3 \Rightarrow$ $12x = -6 \Rightarrow x = -\frac{6}{12} = -\frac{1}{2}$. Therefore, the solution set is: $\left\{-\frac{1}{2}\right\}$.

(b) Graph $y_1 = |6x + 9|$ and $y_2 = |6x - 3|$. See Figure 74. From the graph, $|f(x)| > |g(x)|$ when $y_1 > y_2$ which is for the interval: $\left(-\frac{1}{2}, \infty\right)$.

(c) Graph $y_1 = |6x + 9|$ and $y_2 = |6x - 3|$. See Figure 74. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$ which is for the interval: $\left(-\infty, -\frac{1}{2}\right)$.

75. (a) $.25x + 1 = .75x - 3 \Rightarrow -.50x = -4 \Rightarrow x = 8$ or $.25x + 1 = -(.75x - 3) \Rightarrow$ $.25x + 1 = -.75x + 3 \Rightarrow x = 2$. Therefore, the solution set is: $\{2, 8\}$.

(b) Graph $y_1 = |.25x + 1|$ and $y_2 = |.75x - 3|$. See Figure 75. From the graph, $|f(x)| > |g(x)|$ when $y_1 > y_2$ which is for the interval: $(2, 8)$.

(c) Graph $y_1 = |.25x + 1|$ and $y_2 = |.75x - 3|$. See Figure 75. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$ which is for the interval: $(-\infty, 2) \cup (8, \infty)$.

76. (a) $.40x + 2 = .60x - 5 \Rightarrow -.20x = -7 \Rightarrow x = 35$ or $.40x + 2 = -(.60x - 5) \Rightarrow$

$.40x + 2 = -.60x + 5 \Rightarrow x = 3$. Therefore, the solution set is: $\{3, 35\}$.

(b) Graph $y_1 = |.40x + 2|$ and $y_2 = |.60x - 5|$. See Figure 76. From the graph, $|f(x)| > |g(x)|$ when $y_1 > y_2$ which is for the interval: $(3, 35)$.

(c) Graph $y_1 = |.40x + 2|$ and $y_2 = |.60x - 5|$. See Figure 76. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$ which is for the interval: $(-\infty, 3) \cup (35, \infty)$.

$[-20, 20]$ by $[-4, 16]$
Xscl = 2 Yscl = 1

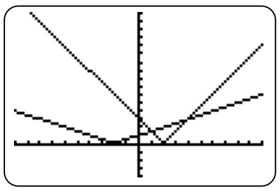


Figure 75

$[-30, 50]$ by $[-5, 30]$
Xscl = 5 Yscl = 5

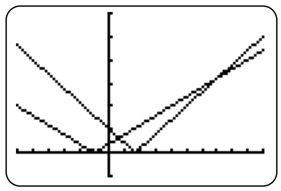


Figure 76

77. Graph $y_1 = |x + 1| + |x - 6|$ and $y_2 = 11$. See Figure 77. From the graph, the lines intersect at: $(-3, 11)$ and $(8, 11)$. Therefore the solution set is: $\{-3, 8\}$.

78. Graph $y_1 = |2x + 2| + |x + 1|$ and $y_2 = 9$. See Figure 78. From the graph, the lines intersect at: $(-4, 9)$ and $(2, 9)$. Therefore the solution set is: $\{-4, 2\}$.

79. Graph $y_1 = |x| + |x - 4|$ and $y_2 = 8$. See Figure 79. From the graph, the lines intersect at: $(-2, 8)$ and $(6, 8)$. Therefore the solution set is: $\{-2, 6\}$.

80. Graph $y_1 = |.5x + 2| + |.25x + 4|$ and $y_2 = 9$. See Figure 80. From the graph, the lines intersect at: $(-20, 9)$ and $(4, 9)$. Therefore the solution set is: $\{-20, 4\}$.

$[-10, 10]$ by $[-4, 16]$
Xscl = 1 Yscl = 1

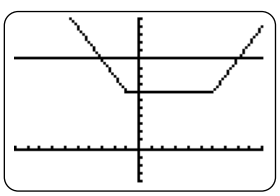


Figure 77

$[-10, 10]$ by $[-4, 16]$
Xscl = 1 Yscl = 1

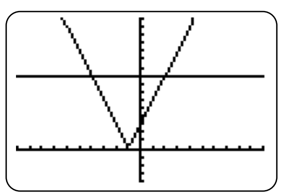


Figure 78

$[-10, 10]$ by $[-4, 16]$
Xscl = 1 Yscl = 1

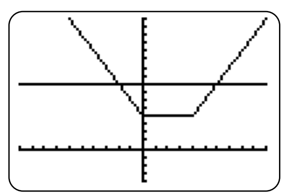


Figure 79

$[-30, 10]$ by $[-4, 16]$
Xscl = 5 Yscl = 1

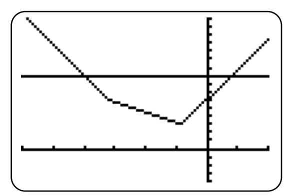


Figure 80

81. (a) $|T - 43| \le 24 \Rightarrow -24 \le T - 43 \le 24 \Rightarrow 19 \le T \le 67$.

(b) The average monthly temperatures in Marquette vary between a low of 19°F and a high of 67°F. The monthly averages are always within 24° of 43°F.

82. (a) $|T - 62| \le 19 \Rightarrow -19 \le T - 62 \le 19 \Rightarrow 43 \le T \le 81$.

(b) The average monthly temperatures in Memphis vary between a low of 43°F and a high of 81°F. The monthly averages are always within 19° of 62°F.

83. (a) $|T - 50| \le 22 \Rightarrow -22 \le T - 50 \le 22 \Rightarrow 28 \le T \le 72$.

(b) The average monthly temperatures in Boston vary between a low of 28°F and a high of 72°F. The monthly averages are always within 22° of 50°F.

84. (a) $|T - 10| \le 36 \Rightarrow -36 \le T - 10 \le 36 \Rightarrow -26 \le T \le 46.$

(b) The average monthly temperatures in Chesterfield vary between a low of −26°F and a high of 46°F. The monthly averages are always within 36° of 10°F.

85. (a) $|T - 61.5| \le 12.5 \Rightarrow -12.5 \le T - 61.5 \le 12.5 \Rightarrow 49 \le T \le 74.$

(b) The average monthly temperatures in Buenos Aires vary between a low of 49°F (possibly in July) and a high of 74°F (possibly in January). The monthly averages are always within 12.5° of 61.5°F.

86. (a) $|T - 43.5| \le 8.5 \Rightarrow -8.5 \le T - 43.5 \le 8.5 \Rightarrow 35 \le T \le 52.$

(b) The average monthly temperatures in Punta Arenas vary between a low of 35°F and a high of 52°F. The monthly averages are always within 8.5° of 43.5°F.

87. $|x - 8.0| \le 1.5 \Rightarrow -1.5 \le x - 8.0 \le 1.5 \Rightarrow 6.5 \le x \le 9.5$, therefore the range is the interval: $[6.5, 9.5]$.

88. If $\dfrac{680 + 780}{2} = 730$ is the midpoint, then $680 \le F \le 780 \Rightarrow 680 - 730 \le F - 730 \le 780 - 730 \Rightarrow$ $-50 \le F - 730 \le 50 \Rightarrow |F - 730| \le 50$ (or $|730 - F| \le 50$).

89. (a) $P_d = |116 - 125| \Rightarrow P_d = |-9| = 9.$

(b) $17 = |P - 130| \Rightarrow P - 130 = 17$ or $P - 130 = -17 \Rightarrow P = 147$ or $P = 113.$

90. If $\dfrac{98 + 148}{2} = 123$ is the midpoint, then $98 \le x \le 148 \Rightarrow 98 - 123 \le x - 123 \le 148 - 123 \Rightarrow$ $-25 \le x - 123 \le 25 \Rightarrow |x - 123| \le 25$ (or $|123 - x| \le 25$); and if $\dfrac{16 + 26}{2} = 21$ is the midpoint, then $16 \le x \le 26 \Rightarrow 16 - 21 \le x - 21 \le 26 - 21 \Rightarrow$ $-5 \le x - 21 \le 5 \Rightarrow |x - 21| \le 5$ (or $|21 - x| \le 5$).

91. If the difference between y and 1 is less than .1, then $|y - 1| < .1 \Rightarrow |2x + 1 - 1| < .1 \Rightarrow |2x| < .1 \Rightarrow$ $-.1 < 2x < .1 \Rightarrow -.05 < x < .05$. The open interval of x is: $(-.05, .05)$.

92. If the difference between y and 2 is less than .01, then $|y - 2| < .1 \Rightarrow |3x - 6 - 2| < .01 \Rightarrow$ $|3x - 8| < .01 \Rightarrow -.01 < 3x - 8 < .01 \Rightarrow 7.99 < 3x < 8.01 \Rightarrow 2.66\overline{3} < x < 2.67.$ The open interval of x is: $(2.66\overline{3}, 2.67)$.

93. If the difference between y and 3 is less than .001, then $|y - 3| < .001 \Rightarrow |4x - 8 - 3| < .001 \Rightarrow$ $|4x - 11| < .001 \Rightarrow -.001 < 4x - 11 < .001 \Rightarrow 10.999 < 4x < 11.001 \Rightarrow 2.74975 < x < 2.75025.$ The open interval of x is: $(2.74975, 2.75025)$.

94. If the difference between y and 4 is less than .0001, then $|y - 4| < .0001 \Rightarrow |5x + 12 - 4| < .0001 \Rightarrow$ $|5x + 8| < .0001 \Rightarrow -.0001 < 5x + 8 < .0001 \Rightarrow -8.0001 < 5x < -7.9999 \Rightarrow$ $-1.60002 < x < -1.59998$. The open interval of x is: $(-1.60002, -1.59998)$.

95. From the chart, 15 mph at 30° is 19°F and 10 mph at −10° is −28°F, Therefore, $|19 - (-28)| = |47| = 47$°F.

96. From the chart, 20 mph at −20° is −48°F and 5 mph at 30° is 25°F, Therefore, $|-48 - 25| = |-73| = 73$°F.

97. From the chart, 30 mph at −30° is −67°F and 15 mph at −20° is −45°F, Therefore, $|-67 - (-45)| = |-22| = 22$°F.

98. From the chart, 40 mph at 40° is 27°F and 25 mph at −30° is −64°F, Therefore, $|27 - (-64)| = |91| = 91$°F.

99. If $|2x + 7| = 6x - 1$ then $|2x + 7| - (6x - 1) = 0$. Graph $y_1 = |2x + 7| - (6x - 1)$,

See Figure 99. The x-intercept is: 2, therefore the solution set is: $\{2\}$.

100. If $-|3x - 12| \geq -x - 1$ then $-|3x - 12| - (-x - 1) \geq 0$. Graph $y_1 = -|3x - 12| - (-x - 1)$,

See Figure 100. The equation is ≥ 0 or the graph intersects or is above the x-axis, for the interval: $[2.75, 6.5]$.

101. If $|x - 4| > .5x - 6$ then $|x - 4| - (.5x - 6) > 0$. Graph $y_1 = |x - 4| - (.5x - 6)$,

See Figure 101. The equation is > 0 or the graph is above the x-axis, for the interval: $(-\infty, \infty)$.

102. If $2x + 8 > -|3x + 4|$ then $2x + 8 - (-|3x + 4|) > 0$. Graph $y_1 = 2x + 8 - (-|3x + 4|)$,

See Figure 102. The equation is > 0 or the graph is above the x-axis, for the interval: $(-\infty, \infty)$.

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

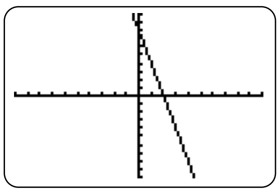


Figure 99

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

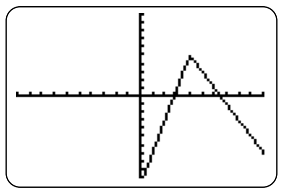


Figure 100

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

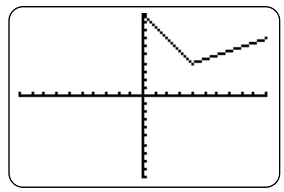


Figure 101

$[-10, 10]$ by $[-4, 16]$
Xscl = 1 Yscl = 1

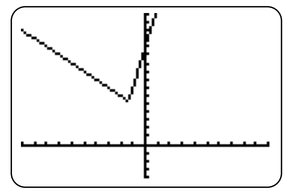


Figure 102

103. If $|3x + 4| < -3x - 14$ then $|3x + 4| - (-3x - 14) < 0$. Graph $y_1 = |3x + 4| - (-3x - 14)$,

See Figure 103. The equation is < 0 or the graph is below the x-axis never, therefore the solution set is: $\varnothing$.

104. If $|x - \sqrt{13}| + \sqrt{6} \leq -x - \sqrt{10}$ then $|x - \sqrt{13}| + \sqrt{6} - (-x - \sqrt{10}) \leq 0$.

Graph $y_1 = |x - \sqrt{13}| + \sqrt{6} - (-x - \sqrt{10})$, See Figure 104. The equation is ≤ 0 or the graph intersects or is below the x-axis never, therefore the solution set is: $\varnothing$.

$[-10, 10]$ by $[-5, 30]$
Xscl = 1 Yscl = 5

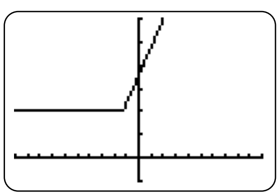


Figure 103

$[-10, 10]$ by $[-5, 30]$
Xscl = 1 Yscl = 5

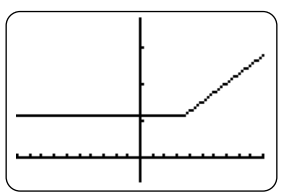


Figure 104

2.5: Piecewise-Defined Functions

1. (a) From the graph, the highest is 55 mph and the lowest is 30 mph.

(b) 3 stretches of 4 miles each or $3 \times 4 = 12$ miles.

(c) From the graph, $f(4) = 40$ mph; $f(12) = 30$ mph; and $f(18) = 55$ mph.

(d) From the graph, the graph is discontinuous at $x = 4, 6, 8, 12$, and 16. The speed limit changes at each discontinuity.

2. (a) From the graph, the initial amount was: \$1000; and the final amount was: \$600.

   (b) From the graph, $f(10) = \$900$; $f(50) = \$600$. The function f is not continuous.

   (c) From the graph, the discontinuity shows 3 drops or withdrawals.

   (d) From the graph, the largest drop or largest withdrawal of \$300 occurred after 15 minute.

   (e) From the graph, the one increase or deposit was \$200.

3. (a) From the graph, the initial amount was: 50,000 gal.; and the final amount was: 30,000 gal.

   (b) From the graph, during the first and fourth days.

   (c) From the graph, $f(2) = 45{,}000$ gal.; $f(4) = 40{,}000$ gal.

   (d) From the graph, between days 1 and 3 the water dropped: $\dfrac{50{,}000 - 40{,}000}{2} = \dfrac{10{,}000}{2} = 5{,}000$ gal./day.

4. (a) From the graph, when $x = 3$ gas was added until 20 gallons was in the tank.

   (b) When the graph is horizontal, the engine is not running; when the graph is decreasing, the engine is burning gasoline; and when the graph is increasing, gasoline is being put into the tank.

   (c) From the graph, the graph deceased the fastest or gasoline was burned the fastest between 1 and 2.9 hours.

5. (a) $f(-5) = 2(-5) = -10$ (b) $f(-1) = 2(-1) = -2$ (c) $f(0) = 0 - 1 = -1$ (d) $f(3) = 3 - 1 = 2$

6. (a) $f(-5) = -5 - 2 = -7$ (b) $f(-1) = -1 - 2 = -3$ (c) $f(0) = 0 - 2 = -2$ (d) $f(3) = 5 - 3 = 2$

7. (a) $f(-5) = 2 + (-5) = -3$ (b) $f(-1) = -(-1) = 1$ (c) $f(0) = -(0) = 0$ (d) $f(3) = 3(3) = 9$

8. (a) $f(-5) = -2(-5) = 10$ (b) $f(-1) = 3(-1) - 1 = -4$ (c) $f(0) = 3(0) - 1 = -1$

   (d) $f(3) = -4(3) = -12$

9. See Figure 9.

10. See Figure 10.

11. See Figure 11.

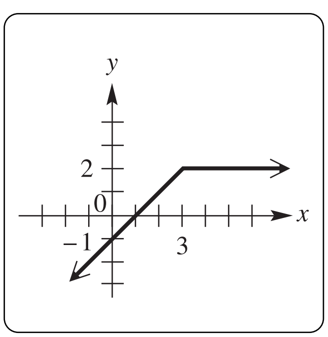


Figure 9

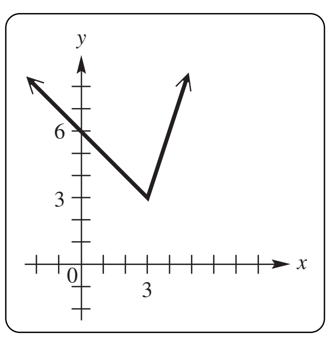


Figure 10

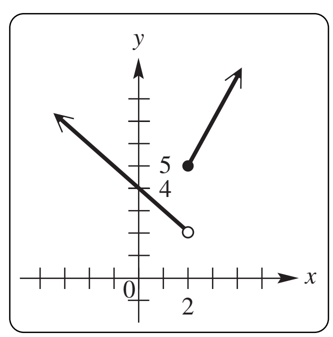


Figure 11

12 See Figure 12.

13. See Figure 13.

14. See Figure 14.

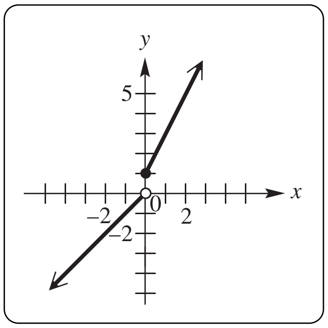


Figure 12

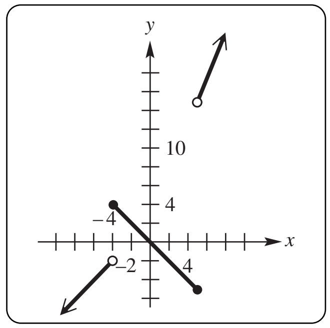


Figure 13

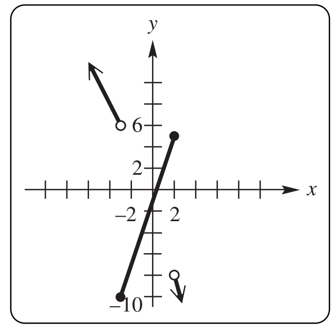


Figure 14

15. See Figure 15.

16. See Figure 16.

17. See Figure 17.

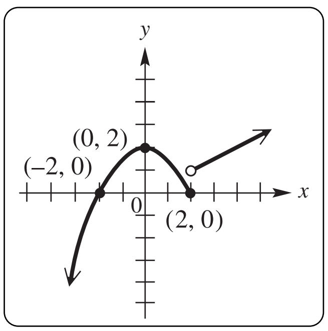


Figure 15

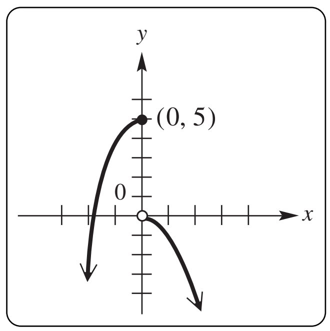


Figure 16

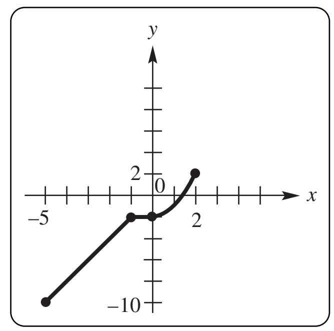


Figure 17

18. See Figure 18.

19. See Figure 19.

20. See Figure 20.

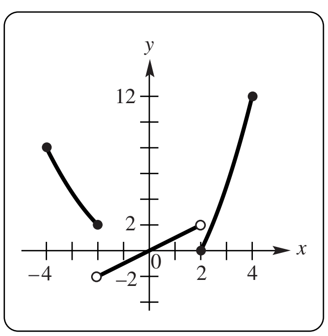


Figure 18

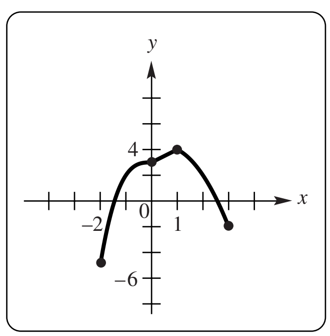


Figure 19

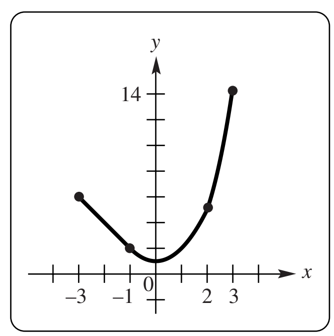


Figure 20

21. Look for a $y = x^2$ graph if $x \geq 0$; and a linear graph if $x < 0$. Therefore: B.

22. Look for a $y = |x|$ graph if $x \geq -1$; and a reflected $y = x^2$ graph if $x < -1$. Therefore: A.

23. Look for a horizontal graph above the x-axis if $x \geq 0$; and a horizontal graph below the x-axis if $x < 0$. Therefore D.

24. Look for a $y = \sqrt{x}$ graph if $x \geq 0$; and a reflected $y = x^2$ graph if $x < 0$. Therefore: C.

25. Graph $y_1 = (x - 1) * (x \leq 3) + (2) * (x > 3)$, See Figure 25.

26. Graph $y_1 = (6 - x) * (x \leq 3) + (3x - 6) * (x > 3)$, See Figure 26.

27. Graph $y_1 = (4 - x) * (x < 2) + (1 + 2x) * (x \geq 2)$, See Figure 27.

28. Graph $y_1 = (2x + 1) * (x \geq 0) + (x) * (x < 0)$, See Figure 28.

$[-4, 6]$ by $[-2, 4]$
Xscl = 1 Yscl = 1

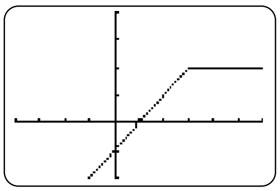


Figure 25

$[-2, 8]$ by $[-2, 10]$
Xscl = 1 Yscl = 1

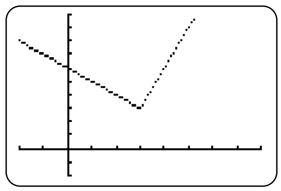


Figure 26

$[-4, 6]$ by $[-2, 8]$
Xscl = 1 Yscl = 1

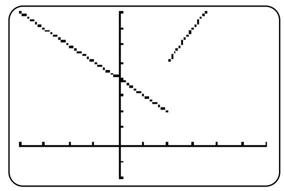


Figure 27

$[-5, 4]$ by $[-3, 8]$
Xscl = 1 Yscl = 1

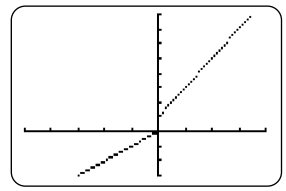


Figure 28

29. Graph $y_1 = (2 + x) * (x < -4) + (-x) * (-4 \leq x \text{ and } x \leq 5) + (3x) * (x > 5)$, See Figure 29.

30. Graph $y_1 = (-2x) * (x < -3) + (3x - 1) * (-3 \leq x \text{ and } x \leq 2) + (-4x) * (x > 2)$, See Figure 30.

31. Graph $y_1 = \left(-\frac{1}{2}x^2 + 2\right) * (x \leq 2) + \left(\frac{1}{2}x\right) * (x > 2)$, See Figure 31.

32. Graph $y_1 = (x^3 + 5) * (x \leq 0) + (-x^2) * (x > 0)$, See Figure 32.

$[-12, 12]$ by $[-6, 20]$
Xscl = 2 Yscl = 2

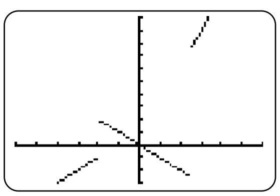


Figure 29

$[-6, 6]$ by $[-10, 8]$
Xscl = 1 Yscl = 1

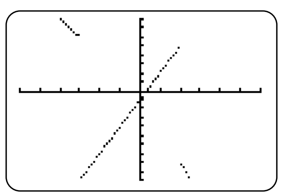


Figure 30

$[-5, 6]$ by $[-2, 4]$
Xscl = 1 Yscl = 1

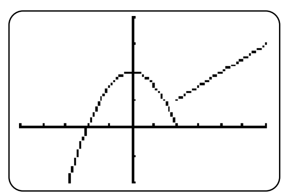


Figure 31

$[-3, 4]$ by $[-3, 6]$
Xscl = 1 Yscl = 1

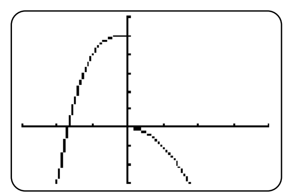


Figure 32

33. From the graph, the function is: $f(x) = \begin{cases} 2 & \text{if } x \leq 0 \\ -1 & \text{if } x > 1 \end{cases}$; domain: $(-\infty, 0] \cup (1, \infty)$; range: $\{-1, 2\}$.

34. From the graph, the function is: $f(x) = \begin{cases} 1 & \text{if } x \leq -1 \\ -1 & \text{if } x > 2 \end{cases}$; domain: $(-\infty, -1] \cup (2, \infty)$; range: $\{-1, 1\}$.

35. From the graph, the function is: $f(x) = \begin{cases} x & \text{if } x \leq 0 \\ 2 & \text{if } x > 0 \end{cases}$; domain: $(-\infty, \infty)$; range: $(-\infty, 0] \cup \{2\}$.

36. From the graph, the function is: $f(x) = \begin{cases} -3 & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \geq 0 \end{cases}$; domain: $(-\infty, \infty)$; range: $\{-3\} \cup [0, \infty)$.

37. From the graph, the function is: $f(x) = \begin{cases} \sqrt[3]{x} & \text{if } x < 1 \\ x + 1 & \text{if } x \geq 1 \end{cases}$; domain: $(-\infty, \infty)$; range: $(-\infty, 1) \cup [2, \infty)$.

38. From the graph, the function is: $f(x) = \begin{cases} 3 & \text{if } x = 2 \\ 2x - 3 & \text{if } x \neq 2 \end{cases}$; domain: $(-\infty, \infty)$; range: $(-\infty, 1) \cup (1, \infty)$.

39. There is an overlap of intervals since the number 4 satisfies both conditions. To be a function, every x-value is used only once.

40. The value $f(4)$ cannot be found since using the first formula, $f(4) = 11$, and using the second formula, $f(4) = 16$. To have two different values violates the definition of function.

41. The graph of $y = [\![x]\!]$ is shifted 1.5 units downward.
42. The graph of $y = [\![x]\!]$ is reflected across the y-axis.
43. The graph of $y = [\![x]\!]$ is reflected across the x-axis.
44. The graph of $y = [\![x]\!]$ is shifted 2 units to the left.
45. Graph $y = [\![x]\!] - 5$, See Figure 45.
46. Graph $y = [\![-x]\!]$, See Figure 46.
47. Graph $y = -[\![x]\!]$, See Figure 47.
48. Graph $y = [\![x + 2]\!]$, See Figure 48.

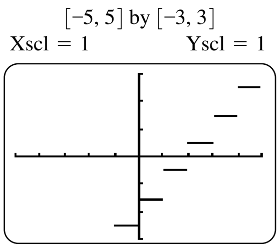


Figure 45

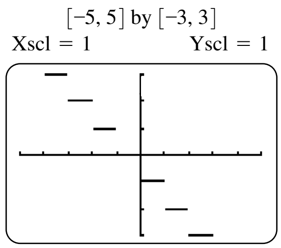


Figure 46

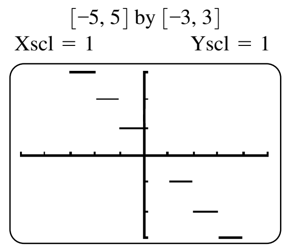


Figure 47

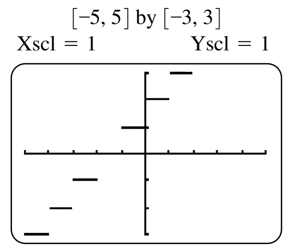


Figure 48

49. When $0 \le x \le 3$, the slope is 5, which means the inlet pipe is open and the outlet pipe is closed; when $3 < x \le 5$, the slope is 2, which means both pipes are open; when $5 < x \le 8$, the slope is 0, which means both pipes are closed; when $8 < x \le 10$, the slope is -3, which means the inlet pipe is closed and the outlet pipe is open.
50. (a) Since for each year given the shoe size is 1 size smaller than his age, the formula is: $y = x - 1$.
    (b) From the table and question, graph the function. See figure 50.
51. (a) If $t = 7 - 6$ then $t = 1$. Then $40(1) + 100 = 140$.
    (b) If $t = 9 - 6$ then $t = 3$. Then $40(3) + 100 = 220$.
    (c) If $t = 10 - 6$ then $t = 4$. Then blood sugar level is: 220.
    (d) If $t = 12 - 6$ then $t = 6$. Then blood sugar level is: 220.
    (e) If $t = (12 - 6) + 2$ then $t = 8$. Then blood sugar level is: 220.
    (f) If $t = (12 - 6) + 5$ then $t = 11$. Then blood sugar level is: 60.
    (g) If $t = (12 - 6) + 12$ then $t = 18$. Then blood sugar level is: 60.
    (h) Graph the function. See Figure 51.
    (i) If f were discontinuous, the insulin level would change instantaneously from one level to a second level.

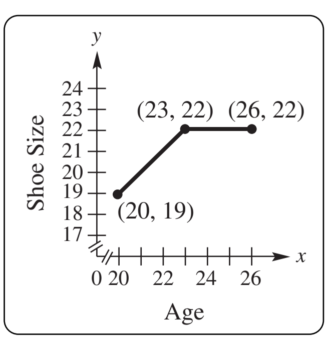


Figure 50

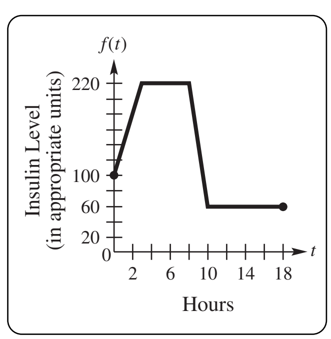


Figure 51

52. (a) Graph the function. See Figure 52.

(b) From the graph, the snow is the deepest in the fourth month: February, when the depth $= 6.5(4) = 26$ in.

(c) From the graph, the snow begins at $x = 0$, the beginning of October; and the snow ends at $x \approx 6.5$, the middle of April.

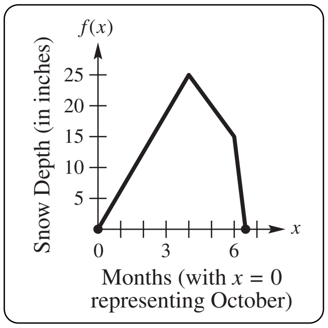


Figure 52

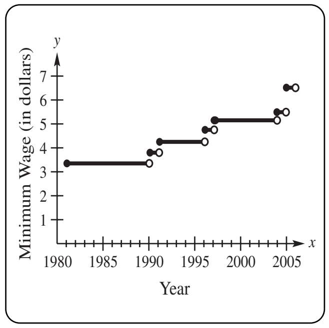


Figure 54

53. (a) From 1997 to 1999, there is a decrease of 700 cases per year. From 1999 to 2001, there is an increase of 200 cases per year.

(b) For the first line of the piecewise function, $m = -700$ (a decrease of 700 cases per year) and a starting coordinate of (0, 8100), gives us the equation: $y = -700x + 8100$ for the first 2 years or $0 \le x \le 2$. The second line of the function has $m = 200$ (increase of 200 cases per year) and a starting coordinate of (2, 6700), for an equation: $y - 6700 = 200(x - 2) \Rightarrow y - 6700 = 200x - 400 \Rightarrow y = 200x + 6300$ for years 2 through 4 or $2 < x \le 4$. Therefore the function is: $f(x) = \begin{cases} -700x + 8100 & \text{if } 0 \le x \le 2 \\ 200x + 6300 & \text{if } 2 < x \le 4 \end{cases}$.

54. From the table, graph the piecewise function. See Figure 54.

55. (a) From the table, graph the piecewise function. See Figure 55.

(b) The likelihood of being a victim peaks from age 16 up to age 20, then decreases.

56. (a) From the table, graph the piecewise function. See Figure 56.

(b) Housing starts increased and then decreased, with the maximum occurring during the 1960's.

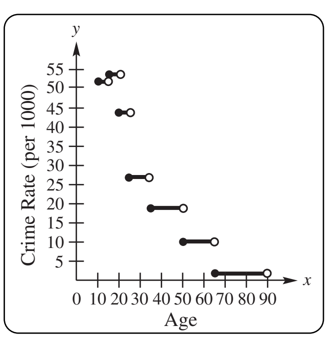


Figure 55

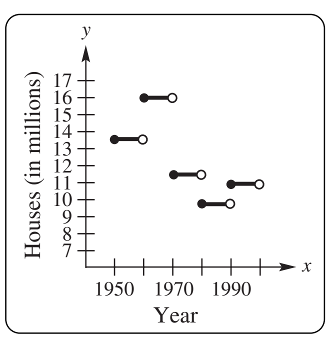


Figure 56

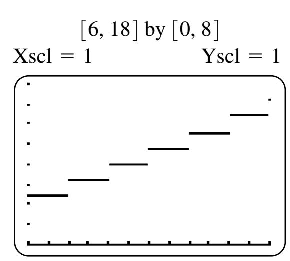


Figure 58

57. (a) A 3.5 minute call would round up to 4 minutes. A 4 minute call would cost: $.50 + 3(.25) = \$1.25$.

(b) We use a piecewise defined function where the cost increases after each whole number as follows:

$$f(x) = \begin{cases} .50 & \text{if } 0 < x \le 1 \\ .75 & \text{if } 1 < x \le 2 \\ 1.00 & \text{if } 2 < x \le 3 \\ 1.25 & \text{if } 3 < x \le 4 \\ 1.50 & \text{if } 4 < x \le 5 \end{cases}. \quad \text{Another possibility is } f(x) = \begin{cases} .50 & \text{if } 0 < x \le 1 \\ .50 - .25[\![1 - x]\!] & \text{if } 1 < x \le 5 \end{cases}.$$

58. (a) Since cost is rounded down to the nearest 2 foot interval, we can use the greatest integer function.

The function is: $f(x) = .80\left[\!\!\left[\frac{x}{2}\right]\!\!\right]$ from $6 \le x \le 18$.

(b) Graph $f(x) = .80\left[\!\!\left[\frac{x}{2}\right]\!\!\right]$ from $6 \le x \le 18$. See Figure 58.

(c) $f(8.5) = .80\left[\!\!\left[\frac{8.5}{2}\right]\!\!\right] = .80[\![4.25]\!] = .80(4) \Rightarrow f(8.5) = \$3.20$.

$f(15.2) = .80\left[\!\!\left[\frac{15.2}{2}\right]\!\!\right] = .80[\![7.6]\!] = .80(7) \Rightarrow f(8.5) = \$5.60$.

59. Sketch a piecewise function that fills a tank at a rate of 5 gallons a minute for the first 20 minutes (the time it takes to fill the 100 gallon tank) and then drains the tank at a rate of 2 gallons per minute for 50 minutes (the time it takes to drain the 100 gallon tank). See Figure 59.

60. Sketch a piecewise function that measures miles over minutes. The first piece increases at a rate of 40 mph or $\frac{40}{60} = \frac{2}{3}$ miles per minute, for 30 minutes (the time it takes to travel 20 miles at that rate). The second piece stays at a constant distance of 20 miles for the 2 hour period at the park. The third piece decreases (returns home) at a rate of 20 mph or $\frac{20}{60} = \frac{1}{3}$ miles per minute, for 60 minutes (the time it takes to travel 20 miles at that rate). See Figure 60.

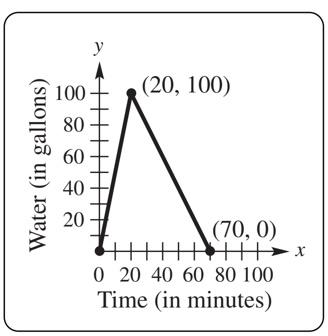


Figure 59

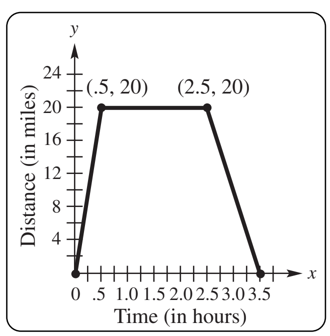


Figure 60

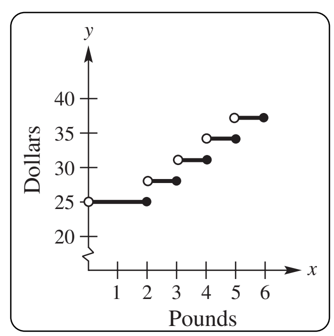


Figure 61

61. For x in the interval $(0, 2]$, $y = 25$. For x in the interval $(2, 3]$, $y = 25 + 3 = 28$. For x in the interval $(3, 4]$, $y = 28 + 3 = 31$ and so on. The graph is a step function. In this case, the first step has a different width. See Figure 61.

62. (a) For the first line of the piecewise function, $m = \frac{72 - 73}{6 - 3} = \frac{-1}{3} \Rightarrow m = -\frac{1}{3}$ (a decrease of $\frac{1}{3}$% per year) and a starting coordinate of (3, 73), gives us the equation: $y - 73 = -\frac{1}{3}(x - 3) \Rightarrow y - 73 = -\frac{1}{3}x + 1 \Rightarrow$ $y = -\frac{1}{3}x + 74$ for the 3 years 1993 to 1996 or $3 \leq x \leq 6$. The second line of the function has $m = \frac{1999 - 1996}{69 - 72} = \frac{3}{-3} \Rightarrow m = -1$ (decrease of 1% per year) and a starting coordinate of (6, 72), for an equation: $y - 72 = -(x - 6) \Rightarrow y - 72 = -x + 6 \Rightarrow y = -x + 78$ for years 1996 through 1999 or $6 < x \leq 9$.

(b) Therefore the piecewise-defined function is: $f(x) = \begin{cases} -\frac{1}{3}x + 74 & \text{if } 3 \leq x \leq 6 \\ -x + 78 & \text{if } 6 < x \leq 9 \end{cases}$.

2.6: Operations and Composition

1. $x^2 + (2x - 5) = x^2 + 2x - 5 \Rightarrow$ E.
2. $x^2 - (2x - 5) = x^2 - 2x + 5 \Rightarrow$ B.
3. $x^2(2x - 5) = 2x^3 - 5x^2 \Rightarrow$ F.
4. $\frac{x^2}{2x - 5} \Rightarrow$ D.
5. $(2x - 5)^2 = 4x^2 - 20x + 25 \Rightarrow$ A.
6. $(2(x^2) - 5) = 2x^2 - 5 \Rightarrow$ C.
7. $4(8x + 1)^2 - 2(8x + 1) = 4(64x^2 + 16x + 1) - 16x - 2 = 256x^2 + 64x + 4 - 16x - 2 =$ $256x^2 + 48x + 2 \Rightarrow 256(3)^2 + 48(3) + 2 = 256(9) + 144 + 2 = 2304 + 146 = 2450$
8. $8(4x^2 - 2x) + 1 = 32x^2 - 16x + 1 \Rightarrow 32(-2)^2 - 16(-2) + 1 = 32(4) + 32 + 1 = 128 + 33 = 161$
9. $4(8x + 1)^2 - 2(8x + 1) = 4(64x^2 + 16x + 1) - 16x - 2 = 256x^2 + 64x + 4 - 16x - 2 =$ $256x^2 + 48x + 2$
10. $8(4x^2 - 2x) + 1 = 32x^2 - 16x + 1$
11. $(4x^2 - 2x) + (8x + 1) = 4x^2 + 6x + 1 = 4(3)^2 + 6(3) + 1 = 4(9) + 18 + 1 = 36 + 19 = 55$
12. $(4x^2 - 2x) + (8x + 1) = 4x^2 + 6x + 1 = 4(-5)^2 + 6(-5) + 1 = 4(25) - 30 + 1 = 100 - 30 + 1 = 71$
13. $(4x^2 - 2x)(8x + 1) = 32x^3 + 4x^2 - 16x^2 - 2x = 32x^3 - 12x^2 - 2x = 32(4)^3 - 12(4)^2 - 2(4) =$ $32(64) - 12(16) - 8 = 2048 - 192 - 8 = 1848$
14. $(4x^2 - 2x)(8x + 1) = 32x^3 + 4x^2 - 16x^2 - 2x = 32x^3 - 12x^2 - 2x = 32(-3)^3 - 12(-3)^2 - 2(-3) =$ $32(-27) - 12(9) + 6 = -864 - 108 + 6 = -966$
15. $\frac{4x^2 - 2x}{8x + 1} = \frac{4(-1)^2 - 2(-1)}{8(-1) + 1} = \frac{4(1) + 2}{-8 + 1} = \frac{6}{-7} = -\frac{6}{7}$
16. $\frac{4x^2 - 2x}{8x + 1} = \frac{4(4)^2 - 2(4)}{8(4) + 1} = \frac{4(16) - 8}{32 + 1} = \frac{64 - 8}{33} = \frac{56}{33}$

17. $4(8x + 1)^2 - 2(8x + 1) = 4(64x^2 + 16x + 1) - 16x - 2 = 256x^2 + 64x + 4 - 16x - 2 =$
$256x^2 + 48x + 2 \Rightarrow 256(2)^2 + 48(2) + 2 = 256(4) + 96 + 2 = 1024 + 98 = 1122$

18. $4(8x + 1)^2 - 2(8x + 1) = 4(64x^2 + 16x + 1) - 16x - 2 = 256x^2 + 64x + 4 - 16x - 2 =$
$256x^2 + 48x + 2 \Rightarrow 256(-5)^2 + 48(-5) + 2 = 256(25) - 240 + 2 = 6400 - 238 = 6162$

19. $8(4x^2 - 2x) + 1 = 32x^2 - 16x + 1 \Rightarrow 32(2)^2 - 16(2) + 1 = 32(4) - 32 + 1 = 128 - 31 = 97$

20. $8(4x^2 - 2x) + 1 = 32x^2 - 16x + 1 \Rightarrow 32(-5)^2 - 16(-5) + 1 = 32(25) + 80 + 1 = 800 + 81 = 881$

21. (a) $(f + g)(x) = (4x - 1) + (6x + 3) = 10x + 2$
$(f - g)(x) = (4x - 1) - (6x + 3) = -2x - 4$
$(fg)(x) = (4x - 1)(6x + 3) = 24x^2 + 12x - 6x - 3 = 24x^2 + 6x - 3$
(b) All values can replace x in all three equations, therefore: Domain is $(-\infty, \infty)$ in all cases.
(c) $\left(\frac{f}{g}\right)(x) = \frac{4x - 1}{6x + 3}$; all values can replace x, except $-\frac{1}{2}$, therefore the domain is: $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$.
(d) $(f \circ g)(x) = f[g(x)] = 4(6x + 3) - 1 = 24x + 12 - 1 = 24x + 11$; all values can be input for x, therefore the domain is: $(-\infty, \infty)$.
(e) $(g \circ f)(x) = g[f(x)] = 6(4x - 1) + 3 = 24x - 6 + 3 = 24x - 3$; all values can replace x, therefore the domain is: $(-\infty, \infty)$.

22. (a) $(f + g)(x) = (9 - 2x) + (-5x + 2) = -7x + 11$
$(f - g)(x) = (9 - 2x) - (-5x + 2) = 3x + 7$
$(fg)(x) = (9 - 2x)(-5x + 2) = -45x + 18 + 10x^2 - 4x = 10x^2 - 49x + 18$
(b) All values can replace x in all three equations, therefore: Domain is $(-\infty, \infty)$ in all cases.
(c) $\left(\frac{f}{g}\right)(x) = \frac{9 - 2x}{-5x + 2}$; all values can replace x, except $\frac{2}{5}$, therefore the domain is: $\left(-\infty, \frac{2}{5}\right) \cup \left(\frac{2}{5}, \infty\right)$.
(d) $(f \circ g)(x) = f[g(x)] = 9 - 2(-5x + 2) = 9 + 10x - 4 = 10x + 5$; all values can replace x, therefore the domain is: $(-\infty, \infty)$.
(e) $(g \circ f)(x) = g[f(x)] = -5(9 - 2x) + 2 = -45 + 10x + 2 = 10x - 43$; all values can replace x, therefore the domain is: $(-\infty, \infty)$.

23. (a) $(f + g)(x) = |x + 3| + 2x$
$(f - g)(x) = |x + 3| - 2x$
$(fg)(x) = |x + 3|(2x)$
(b) All values can replace x in all three equations, therefore: Domain is $(-\infty, \infty)$ in all cases.
(c) $\left(\frac{f}{g}\right)(x) = \frac{|x + 3|}{2x}$; all values can replace x, except 0, therefore the domain is: $(-\infty, 0) \cup (0, \infty)$.
(d) $(f \circ g)(x) = f[g(x)] = |(2x) + 3| = |2x + 3|$ all values can replace x, therefore the domain is: $(-\infty, \infty)$.
(e) $(g \circ f)(x) = g[f(x)] = 2(|x + 3|) = 2|x + 3|$; all values can replace x, therefore the domain is: $(-\infty, \infty)$.

24. (a) $(f+g)(x) = |2x-4| + (x+1) = |2x-4| + x + 1$

$(f-g)(x) = |2x-4| - (x+1) = |2x-4| - x - 1$

$(fg)(x) = |2x-4|(x+1) = (x+1)|2x-4|$

(b) All values can replace x in all three equations, therefore: The domain is $(-\infty, \infty)$ in all cases.

(c) $\left(\frac{f}{g}\right)(x) = \frac{|2x-4|}{x+1}$; all values replace x, except -1, therefore the domain is: $(-\infty, -1) \cup (-1, \infty)$.

(d) $(f \circ g)(x) = f[g(x)] = |2(x+1) - 4| = |2x-2|$; all values can replace x, so the domain is: $(-\infty, \infty)$.

(e) $(g \circ f)(x) = g[f(x)] = |2x-4| + 1$; all values can replace x, so the domain is: $(-\infty, \infty)$.

25. (a) $(f+g)(x) = \sqrt[3]{x+4} + (x^3+5) = \sqrt[3]{x+4} + x^3 + 5$

$(f-g)(x) = \sqrt[3]{x+4} - (x^3+5) = \sqrt[3]{x+4} - x^3 - 5$

$(fg)(x) = (\sqrt[3]{x+4})(x^3+5)$

(b) All values can replace x in all three equations, therefore: The domain is $(-\infty, \infty)$ in all cases.

(c) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt[3]{x+4}}{x^3+5}$; all values can replace x, except $\sqrt[3]{-5}$, so the domain is: $(-\infty, \sqrt[3]{-5}) \cup (\sqrt[3]{-5}, \infty)$.

(d) $(f \circ g)(x) = f[g(x)] = \sqrt[3]{(x^3+5)+4} = \sqrt[3]{x^3+9}$; all values can replace x, so the domain is: $(-\infty, \infty)$.

(e) $(g \circ f)(x) = g[f(x)] = (\sqrt[3]{x+4})^3 + 5 = x + 4 + 5 = x + 9$; all values can replace x, so the domain is: $(-\infty, \infty)$.

26. (a) $(f+g)(x) = \sqrt[3]{6-3x} + (2x^3+1) = \sqrt[3]{6-3x} + 2x^3 + 1$

$(f-g)(x) = \sqrt[3]{6-3x} - (2x^3+1) = \sqrt[3]{6-3x} - 2x^3 - 1$

$(fg)(x) = (\sqrt[3]{6-3x})(2x^3+1)$

(b) All values can replace x in all three equations, therefore: Domain is $(-\infty, \infty)$ in all cases.

(c) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt[3]{6-3x}}{2x^3+1}$; all values can replace x, except $\sqrt[3]{-\frac{1}{2}}$, so the domain is: $\left(-\infty, \sqrt[3]{-\frac{1}{2}}\right) \cup \left(\sqrt[3]{-\frac{1}{2}}, \infty\right)$.

(d) $(f \circ g)(x) = f[g(x)] = \sqrt[3]{6 - 3(2x^3+1)} = \sqrt[3]{-6x^3+3}$; all values can replace x, so the domain is: $(-\infty, \infty)$.

(e) $(g \circ f)(x) = g[f(x)] = 2(\sqrt[3]{6-3x})^3 + 1 = 2(6-3x) + 1 = -6x + 13$; all values can replace x, therefore the domain is: $(-\infty, \infty)$.

27. (a) $(f+g)(x) = \sqrt{x^2+3} + (x+1) = \sqrt{x^2+3} + x + 1$

$(f-g)(x) = \sqrt{x^2+3} - (x+1) = \sqrt{x^2+3} - x - 1$

$(fg)(x) = (\sqrt{x^2+3})(x+1)$

(b) All values can replace x in all three equations, therefore: Domain is $(-\infty, \infty)$ in all cases.

(c) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x^2+3}}{x+1}$; all values can replace x, except -1, therefore the domain is: $(-\infty, -1) \cup (-1, \infty)$.

(d) $(f \circ g)(x) = f[g(x)] = \sqrt{(x+1)^2+3} = \sqrt{x^2+2x+1+3} = \sqrt{x^2+2x+4}$; all values can replace x, therefore the domain is: $(-\infty, \infty)$.

(e) $(g \circ f)(x) = g[f(x)] = (\sqrt{x^2+3}) + 1 = \sqrt{x^2+3} + 1$; all values can replace x, therefore the domain is: $(-\infty, \infty)$.

28. (a) $(f + g)(x) = \sqrt{2 + 4x^2} + (x) = \sqrt{2 + 4x^2} + x.$
$(f - g)(x) = \sqrt{2 + 4x^2} - (x) = \sqrt{2 + 4x^2} - x.$
$(fg)(x) = (\sqrt{2 + 4x^2})(x) = x\sqrt{2 + 4x^2}.$
(b) All values can replace x in all three equations, therefore: Domain is $(-\infty, \infty)$ in all cases.
(c) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{2 + 4x^2}}{x}$; all values can replace x, except 0, therefore the domain is: $(-\infty, 0) \cup (0, \infty)$.
(d) $(f \circ g)(x) = f[g(x)] = \sqrt{2 + 4(x)^2} = \sqrt{2 + 4x^2}$; all values can replace x, therefore the domain is: $(-\infty, \infty)$.
(e) $(g \circ f)(x) = g[f(x)] = (\sqrt{2 + 4x^2}) = \sqrt{2 + 4x^2}$; all values can replace x, therefore the domain is: $(-\infty, \infty)$.

29. (a) From the graph, $4 + (-2) = 2$.
(b) From the graph, $1 - (-3) = 4$.
(c) From the graph, $(0)(-4) = 0$.
(d) From the graph, $\frac{1}{-3} = -\frac{1}{3}$.

30. (a) From the graph, $0 + 2 = 2$.
(b) From the graph, $-2 - 1 = -3$.
(c) From the graph, $(2)(1) = 2$.
(d) From the graph, $\frac{4}{-2} = -2$.

31. (a) From the graph, $0 + 3 = 3$.
(b) From the graph, $-1 - 4 = -5$.
(c) From the graph, $(1)(2) = 2$.
(d) From the graph, $\frac{3}{0} =$ undefined.

32. (a) From the graph, $-3 + 1 = -2$.
(b) From the graph, $-2 - 0 = -2$.
(c) From the graph, $(-3)(-1) = 3$.
(d) From the graph, $\frac{-3}{1} = -3$.

33. (a) From the table, $7 + (-2) = 5$.
(b) From the table, $10 - 5 = 5$.
(c) From the table, $(0)(6) = 0$.
(d) From the table, $\frac{5}{0} =$ undefined.

34. (a) From the table, $5 + 4 = 9$.
(b) From the table, $0 - 0 = 0$.
(c) From the table, $(-4)(2) = -8$.
(d) From the table, $\frac{8}{-1} = -8$.

35. See Figure 35.

36. See Figure 36.

x	$(f+g)(x)$	$(f-g)(x)$	$(fg)(x)$	$\left(\frac{f}{g}\right)(x)$
-2	6	-6	0	0
0	5	5	0	undefined
2	5	9	-14	-3.5
4	15	5	50	2

Figure 35

x	$(f+g)(x)$	$(f-g)(x)$	$(fg)(x)$	$\left(\frac{f}{g}\right)(x)$
-2	-2	-6	-8	-2
0	7	9	-8	-8
2	9	1	20	1.25
4	0	0	0	undefined

Figure 36

37. From the graph, $G(1996) \approx 7.7$; $B(1996) \approx 11.8$; and $T(1996) \approx 19.5$.

38. From the graph, $G(1991) \approx 6.3$; $B(1991) \approx 8.2$; and $T(1991) \approx 14.5$.

39. From the graph, The slope from 1978-1991 is: $m = \dfrac{14.5 - 8}{1991 - 1978} = \dfrac{6.5}{13} \Rightarrow m = \dfrac{1}{2}$. The slope from 1991-1996 is: $m = \dfrac{19.5 - 14.5}{1996 - 1991} = \dfrac{5}{5} \Rightarrow m = 1$. Because it's slope is higher, the period from 1991-1996 increased more rapidly.

40. For a given x-coordinate, add the y-coordinates on the graph of $B(x)$ and $G(x)$ to obtain the corresponding y-coordinate on the graph of $T(x)$.

41. $(T - S)(2000) \approx 19 - 13 \approx 6$; It represents the dollars in billions spent for general science in 2000.

42. $(T - G)(2005) \approx 23 - 8 \approx 15$; It represents the dollars in billions spent for space and other technologies in 2005.

43. From the graph, the only level part of the graph is: space and other technologies from 1995-2000.

44. From the graph, space and other technologies from 2000-2005, increases the most.

45. (a) $(f \circ g)(4) = f[g(4)]$, so from the graph find $g(4) = 0$. Now find $f(0) = -4$, therefore $(f \circ g)(4) = -4$.

(b) $(g \circ f)(3) = g[f(3)]$, so from the graph find $f(3) = 2$. Now find $g(2) = 2$, therefore $(g \circ f)(3) = 2$.

(c) $(f \circ f)(2) = f[f(2)]$, so from the graph find $f(2) = 0$. Now find $f(0) = -4$, therefore $(f \circ f)(2) = -4$.

46. (a) $(f \circ g)(2) = f[g(2)]$, so from the graph find $g(2) = -2$. Now find $f(-2) = -4$, therefore $(f \circ g)(2) = -4$.

(b) $(g \circ g)(0) = g[g(0)]$, so from the graph find $g(0) = 2$. Now find $g(2) = -2$, therefore $(g \circ g)(0) = -2$.

(c) $(g \circ f)(4) = g[f(4)]$, so from the graph find $f(4) = 2$. Now find $g(2) = -2$, therefore $(g \circ f)(4) = -2$.

47. (a) $(f \circ g)(1) = f[g(1)]$, so from the graph find $g(1) = 2$. Now find $f(2) = -3$, therefore $(f \circ g)(1) = -3$.

(b) $(g \circ f)(-2) = g[f(-2)]$, so from the graph find $f(-2) = -3$. Now find $g(-3) = -2$, therefore $(g \circ f)(-2) = -2$.

(c) $(g \circ g)(-2) = g[g(-2)]$, so from the graph find $g(-2) = -1$. Now find $g(-1) = 0$, therefore $(g \circ g)(-1) = 0$.

48. (a) $(f \circ g)(-2) = f[g(-2)]$, so from the graph find $g(-2) = 4$. Now find $f(4) = 2$, therefore $(f \circ g)(-2) = 2$.

(b) $(g \circ f)(1) = g[f(1)]$, so from the graph find $f(1) = 1$. Now find $g(1) = 1$, therefore $(g \circ f)(1) = 1$.

(c) $(f \circ f)(0) = f[f(0)]$, so from the graph find $f(0) = 0$. Now find $f(0) = 0$, therefore $(f \circ f)(0) = 0$.

49. (a) $(g \circ f)(1) = g[f(1)]$, so from the table find $f(1) = 4$. Now find $g(4) = 5$, therefore $(g \circ f)(1) = 5$.

(b) $(f \circ g)(4) = f[g(4)]$, so from the table find $g(4) = 5$. Now we find $f(5)$ is undefined, therefore $(f \circ g)(4)$ is undefined.

(c) $(f \circ f)(3) = f[f(3)]$, so from the table find $f(3) = 1$. Now find $f(1) = 4$, therefore $(f \circ f)(3) = 4$.

50. (a) $(g \circ f)(1) = g[f(1)]$, so from the table find $f(1) = 2$. Now find $g(2) = 4$, therefore $(g \circ f)(1) = 4$.

(b) $(f \circ g)(4) = f[g(4)]$, so from the table we find $g(4)$ is undefined, therefore $(f \circ g)(4)$ is undefined.

(c) $(f \circ f)(3) = f[f(3)]$, so from the table find $f(3) = 6$. Now find $f(6) = 7$, therefore $(f \circ f)(3) = 7$.

51. From the table, $g(3) = 4$ and $f(4) = 2$.

52. From the table, $f(6) = 7$ and $g(7) = 0$.

53. Since $Y_3 = Y_1 \circ Y_2$ and $X = -1$, we solve $Y_1[Y_2(-1)]$. First solve $Y_2 = (-1)^2 = 1$, now solve $Y_1 = 2(1) - 5 = -3$, therefore $Y_3 = -3$.

54. Since $Y_3 = Y_1 \circ Y_2$ and $X = -2$, we solve $Y_1[Y_2(-2)]$. First solve $Y_2 = (-2)^2 = 4$, now solve $Y_1 = 2(4) - 5 = 3$, therefore $Y_3 = 3$.

55. Since $Y_3 = Y_1 \circ Y_2$ and $X = 7$, we solve $Y_1[Y_2(7)]$. First solve $Y_2 = (7)^2 = 49$, now solve $Y_1 = 2(49) - 5 = 93$, therefore $Y_3 = 93$.

56. Since $Y_3 = Y_1 \circ Y_2$ and $X = 8$, we solve $Y_1[Y_2(8)]$. First solve $Y_2 = (8)^2 = 64$, now solve $Y_1 = 2(64) - 5 = 123$, therefore $Y_3 = 123$.

57. (a) $(f \circ g)(x) = f[g(x)] = (x^2 + 3x - 1)^3$; all values can be input for x, therefore the domain is: $(-\infty, \infty)$.

(b) $(g \circ f)(x) = g[f(x)] = (x^3)^2 + 3(x^3) - 1 = x^6 + 3x^3 - 1$; all values can be input for x, therefore the domain is: $(-\infty, \infty)$.

(c) $(f \circ f)(x) = f[f(x)] = (x^3)^3 = x^9$; all values can be input for x, therefore the domain is: $(-\infty, \infty)$.

58. (a) $(f \circ g)(x) = f[g(x)] = 2 - \left(\frac{1}{x^2}\right) = 2 - \frac{1}{x^2}$; all values can be input for x, except 0, therefore the domain is: $(-\infty, 0) \cup (0, \infty)$.

(b) $(g \circ f)(x) = g[f(x)] = \frac{1}{(2 - x)^2}$; all values can be input for x, except 2, therefore the domain is: $(-\infty, 2) \cup (2, \infty)$.

(c) $(f \circ f)(x) = f[f(x)] = 2 - (2 - x) = x$; all values can be input for x, therefore the domain is: $(-\infty, \infty)$.

59. (a) $(f \circ g)(x) = f[g(x)] = (\sqrt{1 - x})^2 = 1 - x$; all values less than 1 can be input for x, therefore the domain is: $(-\infty, 1]$.

(b) $(g \circ f)(x) = g[f(x)] = \sqrt{1 - (x^2)} = \sqrt{1 - x^2}$; only values where $x^2 \le 1$ can be input for x, therefore the domain is: $[-1, 1]$.

(c) $(f \circ f)(x) = f[f(x)] = (x^2)^2 = x^4$; all values can be input for x, therefore the domain is: $(-\infty, \infty)$.

60. (a) $(f \circ g)(x) = f[g(x)] = (x^4 + x^2 - 3x - 4) + 2 = x^4 + x^2 - 3x - 2$; all values can be input for x, therefore the domain is: $(-\infty, \infty)$.

(b) $(g \circ f)(x) = g[f(x)] = (x + 2)^4 + (x + 2)^2 - 3(x + 2) - 4$; all values can be input for x, therefore the domain is: $(-\infty, \infty)$.

(c) $(f \circ f)(x) = f[f(x)] = (x + 2) + 2 = x + 4$; all values can be input for x, therefore the domain is: $(-\infty, \infty)$.

61. (a) $(f \circ g)(x) = f[g(x)] = \dfrac{1}{(5x) + 1} = \dfrac{1}{5x + 1}$; all values can be input for x, except $-\dfrac{1}{5}$, therefore the domain is: $\left(-\infty, -\dfrac{1}{5}\right) \cup \left(-\dfrac{1}{5}, \infty\right)$.

(b) $(g \circ f)(x) = g[f(x)] = 5\left(\dfrac{1}{x + 1}\right) = \dfrac{5}{x + 1}$; all values can be input for x, except -1, therefore the domain is: $(-\infty, -1) \cup (-1, \infty)$.

(c) $(f \circ f)(x) = f[f(x)] = \dfrac{1}{\left(\frac{1}{x+1}\right) + 1} = \dfrac{1}{\frac{1}{x+1} + \frac{x+1}{x+1}} = \dfrac{1}{\frac{x+2}{x+1}} = \dfrac{x + 1}{x + 2}$; all values can be input for x, except those that make $\dfrac{x + 2}{x + 1} = 0$ or undefined. That would be -1 and -2, therefore the domain is: $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$.

62. (a) $(f \circ g)(x) = f[g(x)] = (\sqrt{4 - x^2}) + 4 = \sqrt{4 - x^2} + 4$; only values where $x^2 \le 4$ can be input for x, therefore the domain is: $[-2, 2]$.

(b) $(g \circ f)(x) = g[f(x)] = \sqrt{4 - (x + 4)^2}$; only values where $(x + 4)^2 \le 4$ can be input for x, therefore the domain is: $[-6, -2]$

(c) $(f \circ f)(x) = f[f(x)] = (x + 4) + 4 = x + 8$; all values can be input for x, therefore the domain is: $(-\infty, \infty)$.

63. (a) $(f \circ g)(x) = f[g(x)] = 2(4x^3 - 5x^2) + 1 = 8x^3 - 10x^2 + 1$; all values can be input for x, therefore the domain is: $(-\infty, \infty)$.

(b) $(g \circ f)(x) = g[f(x)] = 4(2x + 1)^3 - 5(2x + 1)^2 = 4(8x^3 + 12x^2 + 6x + 1) - 5(4x^2 + 4x + 1) =$ $32x^3 + 48x^2 + 24x + 4 - (20x^2 + 20x + 5) = 32x^3 + 28x^2 + 4x - 1$; all values can be input for x, therefore the domain is: $(-\infty, \infty)$.

(c) $(f \circ f)(x) = f[f(x)] = 2(2x + 1) + 1 = 4x + 3$; all values can be input for x, therefore the domain is: $(-\infty, \infty)$.

64. (a) $(f \circ g)(x) = f[g(x)] = \dfrac{(2x + 3) - 3}{2} = \dfrac{2x}{2} = x$; all values can be input for x, therefore the domain is: $(-\infty, \infty)$.

(b) $(g \circ f)(x) = g[f(x)] = 2\left(\dfrac{x - 3}{2}\right) + 3 = (x - 3) + 3 = x$; all values can be input for x, therefore the domain is: $(-\infty, \infty)$.

(c) $(f \circ f)(x) = f[f(x)] = \dfrac{\left(\frac{x-3}{2}\right) - 3}{2} = \dfrac{\left(\frac{x-3}{2} - \frac{6}{2}\right)}{2} = \dfrac{\left(\frac{x-9}{2}\right)}{2} = \dfrac{x - 9}{4}$; all values can be input for x, therefore the domain is: $(-\infty, \infty)$.

65. $(f \circ g)(x) = f[g(x)] = 4\left(\dfrac{1}{4}(x - 2)\right) + 2 = x - 2 + 2 = x$

$(g \circ f)(x) = g[f(x)] = \dfrac{1}{4}((4x + 2) - 2) = \dfrac{1}{4}(4x) = x$

66. $(f \circ g)(x) = f[g(x)] = -3\left(-\frac{1}{3}x\right) = x$

$(g \circ f)(x) = g[f(x)] = -\frac{1}{3}(-3x) = x$

67. $(f \circ g)(x) = f[g(x)] = \sqrt[3]{5\left(\frac{1}{5}x^3 - \frac{4}{5}\right) + 4} = \sqrt[3]{(x^3 - 4) + 4} = \sqrt[3]{x^3} = x$

$(g \circ f)(x) = g[f(x)] = \frac{1}{5}(\sqrt[3]{5x + 4})^3 - \frac{4}{5} = \frac{1}{5}(5x + 4) - \frac{4}{5} = x + \frac{4}{5} - \frac{4}{5} = x$

68. $(f \circ g)(x) = f[g(x)] = \sqrt[3]{(x^3 - 1) + 1} = \sqrt[3]{x^3} = x$

$(g \circ f)(x) = g[f(x)] = (\sqrt[3]{x + 1})^3 - 1 = x + 1 - 1 = x$

69. Graph $y_1 = \sqrt[3]{x - 6}$, $y_2 = x^3 + 6$, and $y_3 = x$ in the same viewing window. See Figures 69. The graph of y_2 can be obtained by *reflecting* the graph of y_1 across the line $y_3 = x$.

70. Graph $y_1 = 5x - 3$, $y_2 = \frac{1}{5}(x + 3)$, and $y_3 = x$ in the same viewing window. See Figures 70. The graph of y_2 can be obtained by *reflecting* the graph of y_1 across the line $y_3 = x$.

[−10, 10] by [−10, 10]
Xscl = 1 Yscl = 1

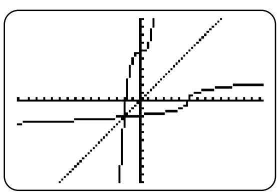


Figure 69

[−10, 10] by [−10, 10]
Xscl = 1 Yscl = 1

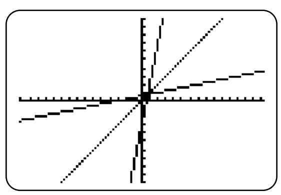


Figure 70

71. Using $\frac{f(x + h) - f(x)}{h}$ gives: $\frac{4(x + h) + 3 - (4x + 3)}{h} = \frac{4x + 4h + 3 - 4x - 3}{h} = \frac{4h}{h} = 4.$

72. Using $\frac{f(x + h) - f(x)}{h}$ gives: $\frac{5(x + h) - 6 - (5x - 6)}{h} = \frac{5x + 5h - 6 - 5x + 6}{h} = \frac{5h}{h} = 5.$

73. Using $\frac{f(x + h) - f(x)}{h}$ gives: $\frac{-6(x + h)^2 - (x + h) + 4 - (-6x^2 - x + 4)}{h} =$

$\frac{-6(x^2 + 2xh + h^2) - x - h + 4 + 6x^2 + x - 4}{h} = \frac{-6x^2 - 12xh - 6h^2 - x - h + 4 + 6x^2 + x - 4}{h} =$

$\frac{-12xh - 6h^2 - h}{h} = -12x - 6h - 1.$

74. Using $\frac{f(x + h) - f(x)}{h}$ gives: $\frac{\frac{1}{2}(x + h)^2 + 4(x + h) - (\frac{1}{2}x^2 + 4x)}{h} =$

$\frac{\frac{1}{2}(x^2 + 2xh + h^2) + 4x + 4h - \frac{1}{2}x^2 - 4x}{h} = \frac{\frac{1}{2}x^2 + xh + \frac{1}{2}h^2 + 4x + 4h - \frac{1}{2}x^2 - 4x}{h} =$

$\frac{xh + \frac{1}{2}h^2 + 4h}{h} = x + \frac{1}{2}h + 4.$

75. Using $\frac{f(x+h)-f(x)}{h}$ gives: $\frac{(x+h)^3-x^3}{h}=\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}=\frac{3x^2h+3xh^2+h^3}{h}=$ $3x^2+3xh+h^2$.

76. Using $\frac{f(x+h)-f(x)}{h}$ gives: $\frac{-2(x+h)^3-(-2x^3)}{h}=\frac{-2(x^3+3x^2h+3xh^2+h^3)+2x^3}{h}=$ $\frac{-2x^3-6x^2h-6xh^2-2h^3+2x^3}{h}=\frac{-6x^2h-6xh^2-2h^3}{h}=-6x^2-6xh-2h^2$.

77. One possible solution is: $f(x)=x^2$ and $g(x)=6x-2$. Then $(f\circ g)(x)=f[g(x)]=(6x-2)^2$.

78. One possible solution is: $f(x)=x^2$ and $g(x)=11x^2+12x$. Then $(f\circ g)(x)=f[g(x)]=(11x^2+12x)^2$.

79. One possible solution is: $f(x)=\sqrt{x}$ and $g(x)=x^2-1$. Then $(f\circ g)(x)=f[g(x)]=\sqrt{x^2-1}$.

80. One possible solution is: $f(x)=x^3$ and $g(x)=2x-3$. Then $(f\circ g)(x)=f[g(x)]=(2x-3)^3$.

81. One possible solution is: $f(x)=\sqrt{x}+12$ and $g(x)=6x$. Then $(f\circ g)(x)=f[g(x)]=\sqrt{6x}+12$.

82. One possible solution is: $f(x)=\sqrt[3]{x}-4$ and $g(x)=2x+3$. Then $(f\circ g)(x)=f[g(x)]=\sqrt[3]{2x+3}-4$.

83. (a) With a cost of \$10 to produce each item and a fixed cost of \$500, the cost function is: $C(x)=10x+500$.

(b) With a selling price of \$35 for each item, the revenue function is: $R(x)=35x$.

(c) The profit function is $P(x)=R(x)-C(x)\Rightarrow P(x)=35x-(10x+500)\Rightarrow P(x)=25x-500$.

(d) A profit is shown when $P(x)>0\Rightarrow 25x-500>0\Rightarrow 25x>500\Rightarrow x>20$. Therefore, 21 items must be produced and sold to realize a profit.

(e) Graph $y_1=25x-500$, the smallest whole number for which $P(x)>0$ is 21. Use a window of $[0, 30]$ by $[-1000, 500]$, for example.

84. (a) With a cost of \$11 to produce each item and a fixed cost of \$180, the cost function is: $C(x)=11x+180$.

(b) With a selling price of \$20 for each item, the revenue function is: $R(x)=20x$.

(c) The profit function is $P(x)=R(x)-C(x)\Rightarrow P(x)=20x-(11x+180)\Rightarrow P(x)=9x-180$.

(d) A profit is shown when $P(x)>0\Rightarrow 9x-180>0\Rightarrow 9x>180\Rightarrow x>20$. Therefore, 21 items must be produced and sold to realize a profit.

(e) Graph $y_1=9x-180$, the smallest whole number for which $P(x)>0$ is 21. Use a window of $[-5, 30]$ by $[-200, 200]$, for example.

85. (a) With a cost of \$100 to produce each item and a fixed cost of \$2700, the cost function is: $C(x)=100x+2700$.

(b) With a selling price of \$280 for each item, the revenue function is: $R(x)=280x$.

(c) The profit function is $P(x)=R(x)-C(x)\Rightarrow P(x)=280x-(100x+2700)\Rightarrow P(x)=180x-2700$.

(d) A profit is shown when $P(x)>0\Rightarrow 180x-2700>0\Rightarrow 180x>2700\Rightarrow x>15$. Therefore, 16 items must be produced and sold to realize a profit.

(e) Graph $y_1=180x-2700$, the smallest whole number for which $P(x)>0$ is 16. Use a window of $[0, 30]$ by $[-3000, 500]$, for example.

86. (a) With a cost of \$200 to produce each item and a fixed cost of \$1000, the cost function is:

$C(x) = 200x + 1000$.

(b) With a selling price of \$240 for each item, the revenue function is: $R(x) = 240x$.

(c) The profit function is $P(x) = R(x) - C(x) \Rightarrow P(x) = 240x - (200x + 1000) \Rightarrow P(x) = 40x - 1000$.

(d) A profit is shown when $P(x) > 0 \Rightarrow 40x - 1000 > 0 \Rightarrow 40x > 1000 \Rightarrow x > 25$. Therefore, 26 items must be produced and sold to realize a profit.

(e) Graph $y_1 = 40x - 1000$, the smallest whole number for which $P(x) > 0$ is 26. Use a window of $[-5, 40]$ by $[-1200, 600]$, for example.

87. (a) If $V(r) = \frac{4}{3}\pi r^3$, then a 3 inch increase would be: $V(r) = \frac{4}{3}\pi(r + 3)^3$, and the volume gained would be:

$V(r) = \frac{4}{3}\pi(r + 3)^3 - \frac{4}{3}\pi r^3$.

(b) Graph $y_1 = \frac{4}{3}\pi(x + 3)^3 - \frac{4}{3}\pi x^3$ in the window $[0, 10]$ by $[0, 1500]$. See Figure 87. Although this appears to be a portion of a parabola, it is actually a cubic function.

(c) From the graph in exercise 87b, an input value of $x = 4$ results in a gain of: $y \approx 1168.67$.

(d) $V(4) = \frac{4}{3}\pi(4 + 3)^3 - \frac{4}{3}\pi(4)^3 = \frac{4}{3}\pi(343) - \frac{4}{3}\pi(64) = \frac{1372}{3}\pi - \frac{256}{3}\pi = \frac{1116}{3}\pi = 372\pi \approx 1168.67$.

88. If $S(r) = 4\pi r^2$, then doubling the radius would give us a surface area gained function of:

$S(r) = 4\pi(2r)^2 - 4\pi r^2 = 16\pi r^2 - 4\pi r^2 = 12\pi r^2$.

89. (a) If x = width, then $2x$ = length. Since the perimeter formula is: $P = 2W + 2L$ our perimeter function is:

$P(x) = 2(x) + 2(2x) = 2x + 4x \Rightarrow P(x) = 6x$. This is a linear function.

(b) Graph $P(x) = 6x$ in the window $[0, 10]$ by $[1, 100]$. See Figure 89b. From the graph when $x = 4$, $y = 24$. The 4 represent the width of a rectangle and 24 represents the perimeter.

(c) If $x = 4$ is the width of a rectangle then $2x = 8$ is the length. See Figure 89c. Using the standard perimeter formula yields: $P = 2(4) + 2(8) = 24$. This compares favorably with the graph result in part b.

(d) (Answers may vary.) If the perimeter y of a rectangle satisfying the given conditions is 36, then the width x is 6. See Figure 89d.

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

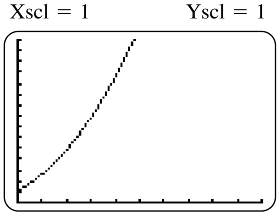


Figure 87

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

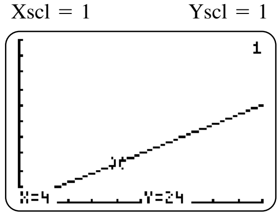


Figure 89b

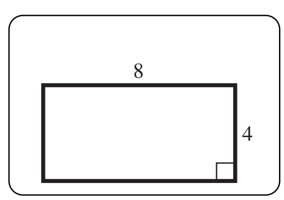


Figure 89c

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

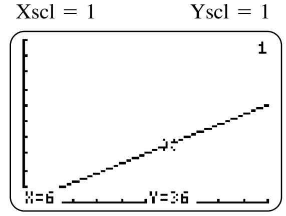


Figure 89d

90. (a) If $x = 4s$ then $s = \frac{x}{4}$.

(b) If $y = s^2$ and $s = \frac{x}{4}$, then $y(x) = \left(\frac{x}{4}\right)^2 \Rightarrow y(x) = \frac{x^2}{16}$.

(c) Sine x is perimeter and $x = 6$, $y(6) = \frac{(6)^2}{16} = \frac{36}{16} = \frac{9}{4} = 2.25$.

(d) Show that the point $(6, 2.25)$ is on the graph $y = \frac{x^2}{16}$. A square with perimeter 6 will have area 2.25 square units.

91. (a) $A(2x) = \frac{\sqrt{3}}{4}(2x)^2 = \frac{\sqrt{3}}{4}(4x^2) \Rightarrow A(2x) = \sqrt{3}x^2$

(b) $A(x) = \frac{\sqrt{3}}{4}(16)^2 = \frac{\sqrt{3}}{4}(256) \Rightarrow A(x) = 64\sqrt{3}$ square units.

(c) On the graph of $y = \frac{\sqrt{3}}{4}x^2$, locate the point where $x = 16$ to find $y \approx 110.85$, an approximation for $64\sqrt{3}$.

92. (a) If $A(r) = \pi r^2$ and $r(t) = 2t$ then $(A \circ r)(t) = A[r(t)] = A[2t] = \pi(2t)^2 = 4\pi t^2$.

(b) $(A \circ r)(t)$ is a composite function that expresses the area of the circular region covered by the pollutants as a function of time t (in hours).

(c) Since $t = 0$ is 8 A.M., noon would be $t = 4$. $(A \circ r)(4) = 4\pi(4)^2 = 64\pi$ mi^2.

(d) Graph $y_1 = 4\pi x^2$ and show that for $x = 4$, $y \approx 201$ (an approximation for 64π).

93. (a) The function h is the addition of functions f and g.

x	1999	2000	2001	2002	2003
$h(x)$	76	82	79	89	103

(b) The function h is the addition of functions f and g. Therefore. $h(x) = f(x) + g(x)$.

94. (a) The domain is the years. See Table. Therefore: $D = \{1998, 1999, 2000, 2001, 2002\}$.

x	1998	1999	2000	2001	2002
$h(x)$	94.2	95.3	99.3	106.4	93.2

(b) The function h computes the value of animals produced in the U.S. and sold in billions of dollars.

95. (a) $(f + g)(1970) = 32.4 + 17.6 = 50.0$

(b) The function $(f + g)(x)$ computes the total SO_2 emissions from burning coal and oil during year x.

(c) Add functions f and g.

x	1860	1900	1940	1970	2000
$(f + g)(x)$	2.4	12.8	26.5	50.0	78.0

96. (a) The function h is the addition of functions f and g.

x	1990	2000	2010	2020	2030
$h(x)$	32	35.5	39	42.5	46

(b) The function h is the addition of functions f and g. Therefore. $h(x) = f(x) + g(x)$.

97. (a) The function h is the subtraction of function f from g. Therefore. $h(x) = g(x) - f(x)$.

(b) $h(1996) = g(1996) - f(1996) = 841 - 694 = 147$

$h(2006) = g(2006) - f(2006) = 1165 - 1012 = 153$

(c) Using the points $(1996, 147)$ and $(2006, 153)$ from part b, the slope is: $m = \frac{153 - 147}{2006 - 1996} = \frac{6}{10} = .6$.

Now using point slope form: $y - 147 = .6(x - 1996) \Rightarrow y = .6(x - 1996) + 147$.

98. (a) Graph $h(x) = \dfrac{1900(x - 1982)^2 + 619}{3200(x - 1982)^2 + 1586}$, in the window $[1982, 1994]$ by $[0, 1]$. See Figure 98a.

Approximately 59% of the people who contracted AIDS during this time period died.

(b) Divide number of deaths by number of cases for each year. The results compare favorably with the graph.

See Figure 98b.

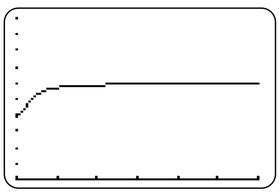


Figure 98a

Year	1982	1984	1986	1988	1990	1992	1994
Ratio	.39	.51	.59	.58	.61	.60	.61

Figure 98b

Reviewing Basic Concepts (Sections 2.4—2.6)

1. (a) $\left|\frac{1}{2}x + 2\right| = 4 \Rightarrow \frac{1}{2}x + 2 = 4 \Rightarrow \frac{1}{2}x = 2 \Rightarrow x = 4$ or $\frac{1}{2}x + 2 = -4 \Rightarrow \frac{1}{2}x = -6 \Rightarrow x = -12.$

Therefore, the solution set is: $\{-12, 4\}$.

(b) $\left|\frac{1}{2}x + 2\right| > 4 \Rightarrow \frac{1}{2}x + 2 > 4 \Rightarrow \frac{1}{2}x > 2 \Rightarrow x > 4$ or $\frac{1}{2}x + 2 < -4 \Rightarrow \frac{1}{2}x < -6 \Rightarrow x < -12.$

Therefore, the solution interval is: $(-\infty, -12) \cup (4, \infty)$.

(c) $\left|\frac{1}{2}x + 2\right| \le 4 \Rightarrow -4 \le \frac{1}{2}x + 2 \le 4 \Rightarrow -6 \le \frac{1}{2}x \le 2 \Rightarrow -12 \le x \le 4.$

Therefore, the solution interval is: $[-12, 4]$.

2. For the graph of $y = |f(x)|$, we reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \ge 0$, the graph remains unchanged. See Figure 2.

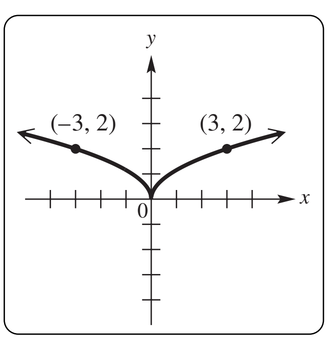


Figure 2

3. (a) The range of values for $|R_L - 26.75| \le 1.42$ is: $-1.42 \le R_L - 26.75 \le 1.42 \Rightarrow 25.33 \le R_L \le 28.17$.

The range of values for $|R_E - 38.75| \le 2.17$ is: $-2.17 \le R_E - 38.75 \le 2.17 \Rightarrow 36.58 \le R_E \le 40.92$.

(b) If $T_L = 225(R_L)$ then the range for T_L is: $225(25.33 \le T_L \le 28.17) = 5699.25 \le T_L \le 6338.25$.

If $T_E = 225(R_E)$ then the range for T_L is: $225(36.58 \le T_E \le 40.92) = 8230.5 \le T_E \le 9207$.

4. (a) $f(-3) = 2(-3) + 3 = -3$ (b) $f(0) = (0)^2 + 4 = 4$ (c) $f(2) = (2)^2 + 4 = 8$

5. (a) See Figure 5a.

(b) Graph $y_1 = (-x^2) * (x \le 0) + (x - 4) * (x > 0)$ in the window $[-10, 10]$ by $[-10, 10]$. See Figure 5b.

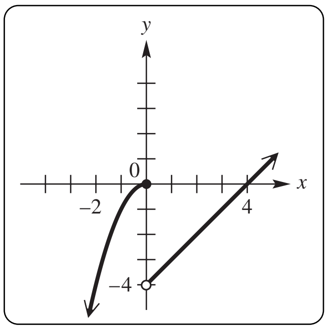


Figure 5a

$[-10, 10]$ by $[-10, 10]$

Xscl = 1 Yscl = 1

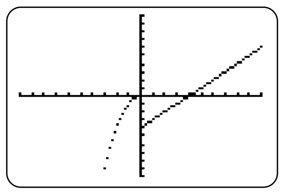


Figure 5b

$[0, 1000]$ by $[-20, 100]$

Xscl = 100 Yscl = 10

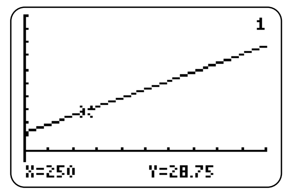


Figure 9

6. (a) $(f + g)(x) = (-3x - 4) + (x^2) = x^2 - 3x - 4$. Therefore, $(f + g)(1) = (1)^2 - 3(1) - 4 = -6$.

(b) $(f - g)(x) = (-3x - 4) - (x^2) = -x^2 - 3x - 4$. Therefore, $(f - g)(3) = -(3)^2 - 3(3) - 4 = -22$.

(c) $(fg)(x) = (-3x - 4)(x^2) = -3x^3 - 4x^2$. Therefore, $(fg)(-2) = -3(-2)^3 - 4(-2)^2 = 24 - 16 = 8$.

(d) $\left(\frac{f}{g}\right)(x) = \frac{-3x - 4}{x^2}$. Therefore, $\left(\frac{f}{g}\right)(-3) = \frac{-3(-3) - 4}{(-3)^2} = \frac{5}{9}$.

(e) $(f \circ g)(x) = f[g(x)] = -3(x^2) - 4 \Rightarrow (f \circ g)(x) = -3x^2 - 4$

(f) $(g \circ f)(x) = g[f(x)] = (-3x - 4)^2 \Rightarrow (g \circ f)(x) = 9x^2 + 24x + 16$

7. One of many possible solutions for $(f \circ g)(x) = h(x)$ is: $f(x) = x^4$ and $g(x) = x + 2$. Then $(f \circ g)(x) = f[g(x)] = (x + 2)^4$.

8. $$\frac{-2(x + h)^2 + 3(x + h) - 5 - (-2x^2 + 3x - 5)}{h} = \frac{-2(x^2 + 2xh + h^2) + 3x + 3h - 5 + 2x^2 - 3x + 5}{h} =$$
$$\frac{-2x^2 - 4xh - 2h^2 + 3x + 3h - 5 + 2x^2 - 3x + 5}{h} = \frac{-4xh - 2h^2 + 3h}{h} = -4x - 2h + 3.$$

9. (a) At 4% simple interest the equation for interest earned is: $y_1 = .04x$.

(b) If he invested x dollars in the first account, then he invested $x + 500$ in the second account. The equation for the amount of interest earned on this account is: $y_2 = .025(x + 500) \Rightarrow y_2 = .025x + 12.5$.

(c) It represents the total interest earned in both accounts for 1 year.

(d) Graph $y_1 + y_2 = .04x + (.025x + 12.5) \Rightarrow y_1 + y_2 = .04x + .025x + 12.5$ in the window $[0, 1000]$ by $[0, 100]$. See Figure 9. An input value of $x = 250$, results in \$28.75 earned interest.

(e) At $x = 250$, $y_1 + y_2 = .04(250) + .025(250) + 12.5 = 10 + 6.25 + 12.5 = \$28.75$.

10. If the radius is r, then the height is $2r$ and the equation is

$S = \pi r\sqrt{r^2 + (2r)^2} = \pi r\sqrt{r^2 + 4r^2} = \pi r\sqrt{5r^2} \Rightarrow S = \pi r^2\sqrt{5}$.

Chapter 2 Review Exercises

The graphs for exercises 1–10 can be found in the "Function Capsule" boxes located in section 2.1 in the text.

1. True. Both $f(x) = x^2$ and $f(x) = |x|$ have the interval: $[0, \infty)$ as the range.
2. True. Both $f(x) = x^2$ and $f(x) = |x|$ increase on the interval: $[0, \infty)$.
3. False. The function $f(x) = \sqrt{x}$ has the domain: $[0, \infty)$ and $f(x) = \sqrt[3]{x}$ the domain: $(-\infty, \infty)$.
4. False. The function $f(x) = \sqrt[3]{x}$ increases on its entire domain.
5. True. The function $f(x) = x$ has a domain and range of: $(-\infty, \infty)$.
6. False. The function $f(x) = \sqrt{x}$ is not defined on $(-\infty, 0)$, so certainly cannot be continuous.
7. True. All of the functions show increases on the interval: $[0, \infty)$.
8. True. Both $f(x) = x$ and $f(x) = x^3$ have graphs that are symmetric with respect to the origin.
9. True. Both $f(x) = x^2$ and $f(x) = |x|$ have graphs that are symmetric with respect to the y-axis.
10. True. No graphs are symmetric with respect to the x-axis.
11. Only values where $x \geq 0$ can be input for x, therefore the domain of $f(x) = \sqrt{x}$ is: $[0, \infty)$.
12. Only positive solutions are possible in absolute value functions, therefore the range of $f(x) = \sqrt{x}$ is: $[0, \infty)$.
13. All solutions are possible in cube root functions, therefore the range of $f(x) = \sqrt[3]{x}$ is: $(-\infty, \infty)$.
14. All values can be input for x, therefore the domain of $f(x) = x^2$ is: $(-\infty, \infty)$.
15. The function $f(x) = \sqrt[3]{x}$ increases for all inputs for x, therefore the interval is: $(-\infty, \infty)$.
16. The function $f(x) = |x|$ increases for all inputs where $x \geq 0$, therefore the interval is: $[0, \infty)$.
17. The equation $y^2 = x$ is the equation $y = \sqrt{x}$. Only values where $x \geq 0$ can be input for x, therefore the domain of $y = \sqrt{x}$ is: $[0, \infty)$.
18. The equation $y^2 = x$ is the equation $y = \sqrt{x}$. Square root functions have both positive and negative solutions and all solution are possible, therefore the range of $y = \sqrt{x}$ is: $(-\infty, \infty)$.
19. The graph of $f(x) = (x + 3) - 1$ is the graph $y = x$ shifted 3 units to the left and 1 unit downward. See Figure 19.
20. The graph of $f(x) = -\frac{1}{2}x + 1$ is the graph $y = x$ reflected across the x-axis, vertically shrunk by a factor of $\frac{1}{2}$, and shifted 1 unit upward. See Figure 20.
21. The graph of $f(x) = (x + 1)^2 - 2$ is the graph $y = x^2$ shifted 1 unit to the left and 2 units downward. See Figure 21.

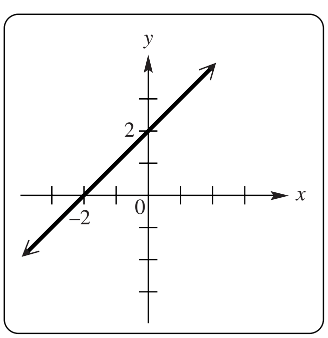


Figure 19

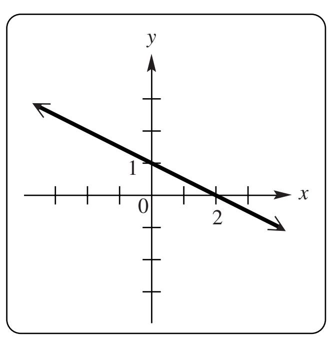


Figure 20

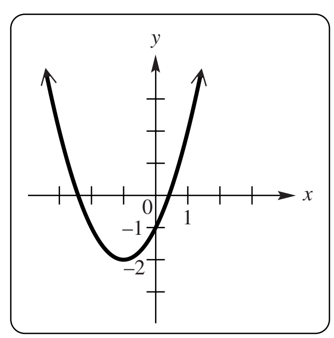


Figure 21

22. The graph of $f(x) = -2x^2 + 3$ is the graph $y = x^2$ reflected across the x-axis, vertically stretched by a factor of 2, and shifted 3 units upward. See Figure 22.

23. The graph of $f(x) = -x^3 + 2$ is the graph $y = x^3$ reflected across the x-axis and shifted 2 units upward. See Figure 23.

24. The graph of $f(x) = (x - 3)^3$ is the graph $y = x^3$ shifted 3 units to the right. See Figure 24.

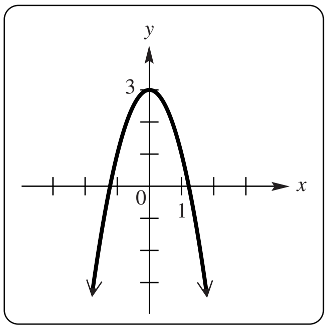


Figure 22

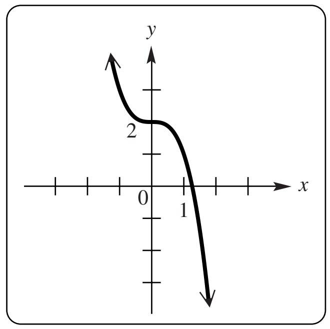


Figure 23

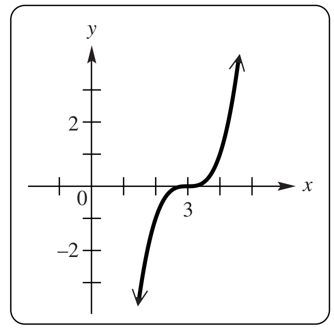


Figure 24

25. The graph of $f(x) = \sqrt{\frac{1}{2}x}$ is the graph $y = \sqrt{x}$ horizontally stretched by a factor of 2. See Figure 25.

26. The graph of $f(x) = \sqrt{x - 2} + 1$ is the graph $y = \sqrt{x}$ shifted 2 units to the right and 1 unit upward. See Figure 26.

27. The graph of $f(x) = 2\sqrt[3]{x}$ is the graph $y = \sqrt[3]{x}$ vertically stretched by a factor of 2. See Figure 27.

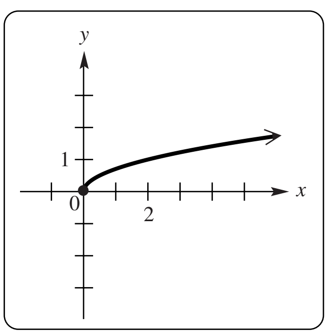


Figure 25

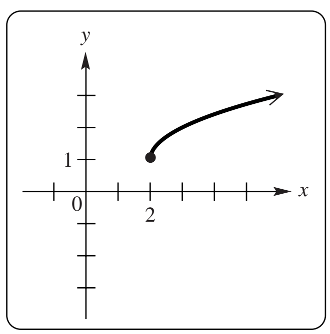


Figure 26

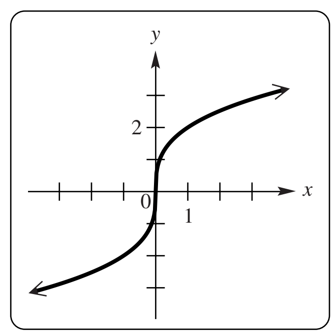


Figure 27

28. The graph of $f(x) = \sqrt[3]{x} - 2$ is the graph $y = \sqrt[3]{x}$ shifted 2 units downward. See Figure 28.

29. The graph of $f(x) = |x - 2| + 1$ is the graph $y = |x|$ shifted 2 units right and 1 unit upward. See Figure 29.

30. The graph of $f(x) = |-2x + 3|$ is the graph $y = |x|$ horizontally shrunk by a factor of $\frac{1}{2}$, shifted $\left(\frac{1}{2}\right)(3)$ or $\frac{3}{2}$ units to the left, and reflected across the y-axis. See Figure 30.

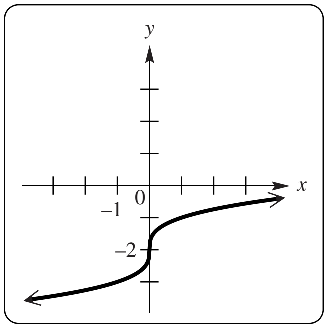


Figure 28

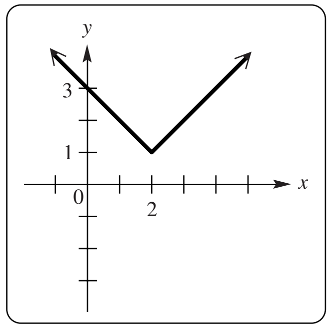


Figure 29

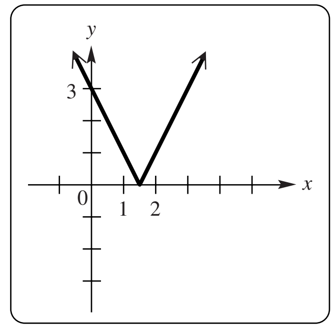


Figure 30

31. (a) From the graph, the function is continuous for the intervals: $(-\infty, -2)$, $[-2, 1]$, and $(1, \infty)$.
    (b) From the graph, the function is increasing for the interval: $[-2, 1]$.
    (c) From the graph, the function is decreasing for the interval: $(-\infty, -2)$.
    (d) From the graph, the function is constant for the interval: $(1, \infty)$.
    (e) From the graph, all values can be input for x, therefore the domain is: $(-\infty, \infty)$.
    (f) From the graph, the possible values of y or the range is: $\{-2\} \cup [-1, 1] \cup (2, \infty)$.
32. $x = y^2 - 4 \Rightarrow y^2 = x + 4 \Rightarrow y = \sqrt{x + 4}$ and $y = -\sqrt{x + 4}$
33. From the graph, the relation is symmetric with respect to the x-axis, y-axis, and origin. The relation is not a function since some inputs x have two outputs y.
34. If $F(x) = x^3 - 6$, then $F(-x) = (-x)^3 - 6 \Rightarrow F(-x) = -x^3 - 6$ and $-F(x) = -(x^3 - 6) \Rightarrow -F(x) = -x^3 + 6$. Since $F(x) \neq F(-x) \neq -F(x)$, the function has no symmetry and is neither an even nor an odd function.
35. If $f(x) = |x| + 4$, then $f(-x) = |(-x)| + 4 \Rightarrow f(-x) = |x| + 4$ and $-f(x) = -|x| - 4$. Since $f(-x) = f(x)$, the function is symmetric with respect to the y-axis and is an even function.
36. If $f(x) = \sqrt{x - 5}$, then $f(-x) = \sqrt{(-x) - 5}$ and $-f(x) = -\sqrt{x - 5}$. Since $f(x) \neq f(-x) \neq -f(x)$, the function has no symmetry and is neither an even nor an odd function.
37. If $y^2 = x - 5$ then $y = \pm\sqrt{x - 5}$. Since $f(x) = -\sqrt{x - 5}$ is the reflection of $f(x) = \sqrt{x - 5}$ across the x-axis, the relation has symmetry with respect to the x-axis. Also, one x input can produce two y outputs. The relation is not a function.
38. If $f(x) = 3x^4 + 2x^2 + 1$, then $f(-x) = 3(-x)^4 + 2(-x)^2 + 1 \Rightarrow f(-x) = 3x^4 + 2x^2 + 1$ and $-f(x) = -3x^4 - 2x^2 - 1$. Since $f(-x) = f(x)$, the function is symmetric with respect to the y-axis and is an even function.
39. True, a graph that is symmetrical with respect to the x-axis means that for every (x, y) there is also $(x, -y)$, which is not a function.
40. True, since an even function and one that is symmetric with respect to the y-axis both contain the points (x, y) and $(-x, y)$.
41. True, since an odd function and one that is symmetric with respect to the origin both contain the points (x, y) and $(-x, -y)$.

42. False, for an even function, if (a, b) is on the graph, then $(-a, b)$ is on the graph and not $(a, -b)$.
For example, $f(x) = x^2$ is even, and $(2, 4)$ is on the graph, but $(2, -4)$ is not.

43. False, for an odd function, if (a, b) is on the graph, then $(-a, -b)$ is on the graph and not $(-a, b)$.
For example, $f(x) = x^3$ is odd, and $(2, 8)$ is on the graph, but $(-2, 8)$ is not.

44. True, if $(x, 0)$ is on the graph of $f(x) = 0$, then $(-x, 0)$ is on the graph.

45. The graph of $y = -3(x + 4)^2 - 8$ is the graph of $y = x^2$ shifted 4 units to the left, vertically stretched by a factor of 3, reflected across the x-axis, and shifted 8 units downward.

46. The equation $y = \sqrt{x}$ reflected across the y-axis is: $y = \sqrt{-x}$, then reflected across the x-axis is: $y = -\sqrt{-x}$, now vertically shrunk by a factor of $\frac{2}{3}$ is: $y = -\frac{2}{3}\sqrt{-x}$, and finally shifted 4 units upward is: $y = -\frac{2}{3}\sqrt{-x} + 4$.

47. Shift the function f upward 3 units. See Figure 47.

48. Shift the function f to the right 2 units. See Figure 48.

49. Shift the function f to the left 3 units and downward 2 units. See Figure 49.

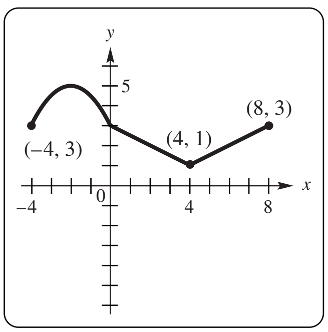


Figure 47

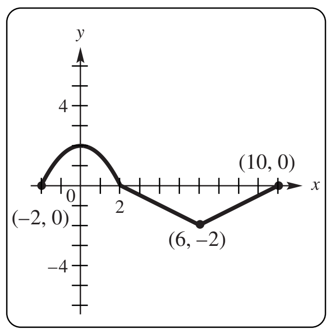


Figure 48

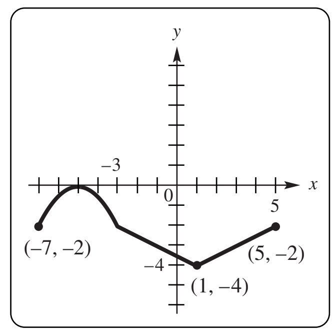


Figure 49

50. For values where $f(x) > 0$ the graph remains the same. For values where $f(x) < 0$ reflect the graph across the x-axis. See Figure 50.

51. Horizontally shrink the function f by a factor of $\frac{1}{4}$. See Figure 51.

52. Horizontally stretch the function f by a factor of 2. See Figure 52.

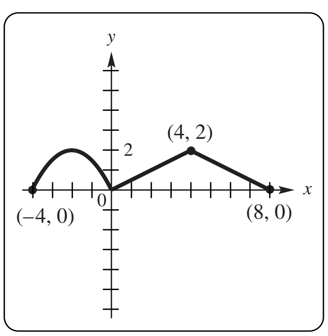


Figure 50

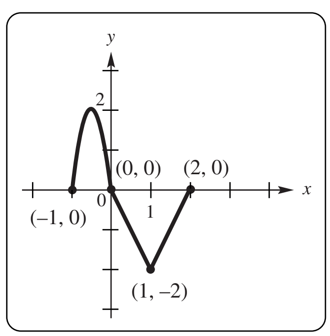


Figure 51

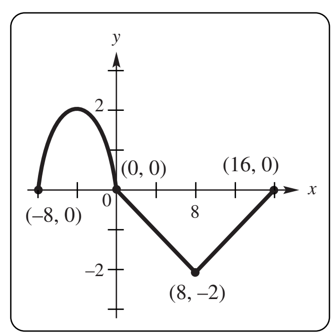


Figure 52

53. The function is shifted upward 4 units, therefore the domain remains the same: $[-3, 4]$ and the range is increased by 4 and is: $[2, 9]$.

54. The function is shifted left 10 units, therefore the domain is decreased by 10 and is: $[-13, -6]$; and the function is stretched vertically by a factor of 5, therefore the range is multiplied by 5 and is: $[-10, 25]$.

55. The function is horizontally shrunk by a factor of $\frac{1}{2}$, therefore the domain is divided by 2 and is: $\left[-\frac{3}{2}, 2\right]$; and the function is reflected across the x-axis, therefore the range is opposite of the original and is: $[-5, 2]$.

56. The function is shifted right 1 unit, therefore the domain is increased by 1 and is: $[-2, 5]$; and the function is also shifted upward 3 units, therefore the range is increased by 3 and is: $[1, 8]$.

57. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 57.

58. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 58.

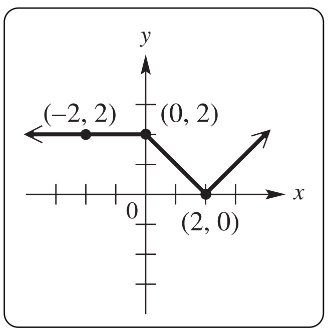


Figure 57

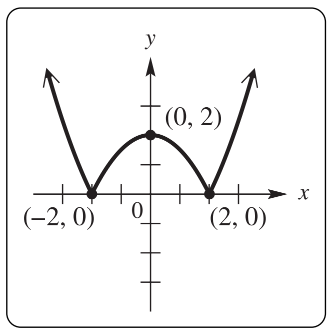


Figure 58

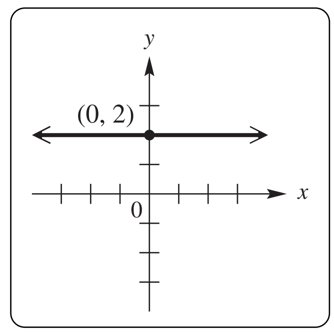


Figure 60

59. Since the range is $\{2\}$, $y \geq 0$, so the graph remains unchanged.

60. Since the range is $\{-2\}$, $y < 0$, so we reflect the graph across the x-axis. See Figure 60.

61. $|4x + 3| = 12 \Rightarrow 4x + 3 = 12 \Rightarrow 4x = 9 \Rightarrow x = \frac{9}{4}$ or $4x + 3 = -12 \Rightarrow 4x = -15 \Rightarrow x = -\frac{15}{4}$, therefore the solution set is: $\left\{-\frac{15}{4}, \frac{9}{4}\right\}$.

62. $|-2x - 6| + 4 = 1 \Rightarrow |-2x - 6| = -3$. Since an absolute value equation can not have a solution less than zero, the solution set is: $\varnothing$.

63. $|5x + 3| = |x + 11| \Rightarrow 5x + 3 = x + 11 \Rightarrow 4x = 8 \Rightarrow x = 2$ or $5x + 3 = -(x + 11) \Rightarrow 6x = -14 \Rightarrow x = -\frac{14}{6} = -\frac{7}{3}$, therefore the solution set is: $\left\{-\frac{7}{3}, 2\right\}$.

64. $|2x + 5| = 7 \Rightarrow 2x + 5 = 7 \Rightarrow 2x = 2 \Rightarrow x = 1$ or $2x + 5 = -7 \Rightarrow 2x = -12 \Rightarrow x = -6$, therefore the solution set is: $\{-6, 1\}$.

65. $|2x + 5| \leq 7 \Rightarrow -7 \leq 2x + 5 \leq 7 \Rightarrow -12 \leq 2x \leq 2 \Rightarrow -6 \leq x \leq 1$, therefore the interval is: $[-6, 1]$.

66. $|2x + 5| \geq 7 \Rightarrow 2x + 5 \geq 7 \Rightarrow 2x \geq 2 \Rightarrow x \geq 1$ or $2x + 5 \leq -7 \Rightarrow 2x \leq -12 \Rightarrow x \leq -6$, therefore the solution is the interval: $(-\infty, -6] \cup [1, \infty)$.

67. $|5x - 12| > 0 \Rightarrow 5x - 12 > 0 \Rightarrow 5x > 12 \Rightarrow x > \frac{12}{5}$ or $5x - 12 < 0 \Rightarrow 5x = 12 \Rightarrow x < \frac{12}{5}$, therefore the solution is the interval: $\left(-\infty, \frac{12}{5}\right) \cup \left(\frac{12}{5}, \infty\right)$ or $\left\{x \mid x \neq \frac{12}{5}\right\}$.

68. Since an absolute value equation can not have a solution less than zero, the solution set is: $\varnothing$.

69. $2|3x - 1| + 1 = 21 \Rightarrow 2|3x - 1| = 20 \Rightarrow |3x - 1| = 10 \Rightarrow 3x - 1 = 10 \Rightarrow 3x = 11 \Rightarrow x = \frac{11}{3}$ or $3x - 1 = -10 \Rightarrow 3x = -9 \Rightarrow x = -3$, therefore the solution set is: $\left\{-3, \frac{11}{3}\right\}$.

70. $|2x + 1| = |-3x + 1| \Rightarrow 2x + 1 = -3x + 1 \Rightarrow 5x = 0 \Rightarrow x = 0$ or $2x + 1 = -(-3x + 1) \Rightarrow -x = -2 \Rightarrow x = 2$, therefore the solution set is: $\{0, 2\}$.

71. The x-coordinates of the points of intersection of the graphs are -6 and 1. Thus, $\{-6, 1\}$ is the solution set of $y_1 = y_2$. The graph of y_1 lies on or below the graph of y_2 between -6 and 1, so the solution set of $y_1 \leq y_2$ is $[-6, 1]$. The graph of y_1 lies above the graph of y_2 everywhere else, so the solution set of $y_1 \geq y_2$ is $(-\infty, -6] \cup [1, \infty)$.

72. Graph $y_1 = |x + 1| + |x - 3|$ and $y_2 = 8$. See Figure 72. The intersections are $x = -3$ and $x = 5$, therefore the solution set is: $\{-3, 5\}$.

Check: $|(-3) + 1| + |(-3) - 3| = 8 \Rightarrow |-2| + |-6| = 8 \Rightarrow 2 + 6 = 8 \Rightarrow 8 = 8$ and
$|(5) + 1| + |(5) - 3| = 8 \Rightarrow |6| + |2| = 8 \Rightarrow 6 + 2 = 8 \Rightarrow 8 = 8$

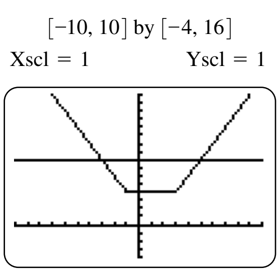


Figure 72

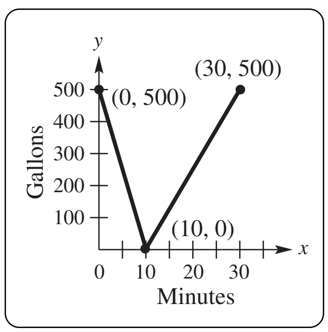


Figure 74

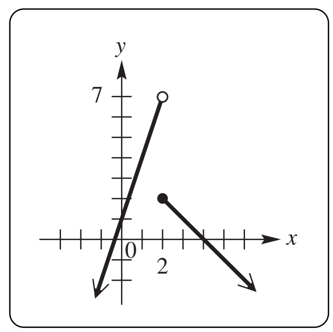


Figure 75

73. Initially, the car is at home. After traveling 30 mph for 1 hr, the car is 30 mi away from home. During the second hour the car travels 20 mph until it is 50 mi away. During the third hour the car travels toward home at 30 mph until it is 20 mi away. During the fourth hour the car travels away from home at 40 mph until it is 60 mi away from home. During the last hour, the car travels 60 mi at 60 mph until it arrives home.

74. See Figure 74.

75. See Figure 75.

76. See Figure 76.

77. Graph $y_1 = (3x + 1) * (x < 2) + (-x + 4) * (x \geq 2)$ in the window $[-10, 10]$ by $[-10, 10]$. See Figure 77.

78. See Figure 78.

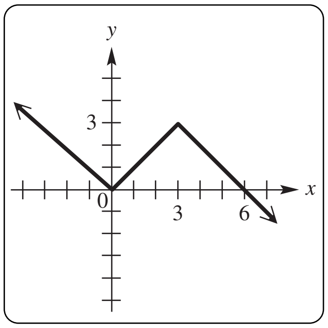


Figure 76

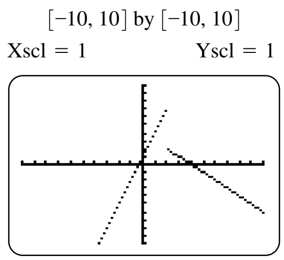


Figure 77

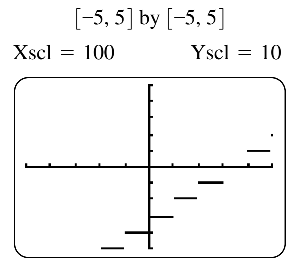


Figure 78

79. From the graphs $(f + g)(1) = 2 + 3 = 5$

80. From the graphs $(f - g)(0) = 1 - 4 = -3$

81. From the graphs $(fg)(-1) = (0)(3) = 0$

82. From the graphs $\left(\frac{f}{g}\right)(2) = \frac{3}{2}$

83. From the graphs $(f \circ g)(2) = f[g(2)] = f(2) = 3$

84. From the graphs $(g \circ f)(2) = g[f(2)] = g(3) = 2$

85. From the graphs $(g \circ f)(-4) = g[f(-4)] = g(2) = 2$

86. From the graphs $(f \circ g)(-2) = f[g(-2)] = f(2) = 3$

87. From the table $(f + g)(1) = 7 + 1 = 8$

88. From the table $(f - g)(3) = 9 - 9 = 0$

89. From the table $(fg)(-1) = (3)(-2) = -6$

90. From the table $\left(\frac{f}{g}\right)(0) = \frac{5}{0}$, which is undefined.

91. From the tables $(g \circ f)(-2) = g[f(-2)] = g(1) = 2$

92. From the graphs $(f \circ g)(3) = f[g(3)] = f(-2) = 1$

93. $\frac{2(x + h) + 9 - (2x + 9)}{h} = \frac{2x + 2h + 9 - 2x - 9}{h} = \frac{2h}{h} = 2$

94. $\frac{(x + h)^2 - 5(x + h) + 3 - (x^2 - 5x + 3)}{h} = \frac{x^2 + 2xh + h^2 - 5x - 5h + 3 - x^2 + 5x - 3}{h} =$

$\frac{2xh + h^2 - 5h}{h} = 2x + h - 5$

95. One of many possible solutions for $(f \circ g)(x) = h(x)$ is: $f(x) = x^2$ and $g(x) = x^3 - 3x$. Then $(f \circ g)(x) = f[g(x)] = (x^3 - 3x)^2$.

96. One of many possible solutions for $(f \circ g)(x) = h(x)$ is: $f(x) = \frac{1}{x}$ and $g(x) = x - 5$. Then

$(f \circ g)(x) = f[g(x)] = \frac{1}{x - 5}$.

97. If $V(r) = \frac{4}{3}\pi r^3$, then a 4 inch increase would be: $V(r) = \frac{4}{3}\pi(r + 4)^3$, and the volume gained would be:

$V(r) = \frac{4}{3}\pi(r + 4)^3 - \frac{4}{3}\pi r^3$.

98. (a) Since $h = d$, $r = \frac{d}{2}$, and the formula for the volume of a can is: $V = \pi r^2 h$, the function is:

$$V(d) = \pi\left(\frac{d}{2}\right)^2 d \Rightarrow V(d) = \frac{\pi d^3}{4}.$$

(b) Since $h = d$, $r = \frac{d}{2}$, $c = 2\pi r$, and the formula for the surface area of a can is: $A = 2\pi rh + 2\pi r^2$, the function is: $S(d) = 2\pi\left(\frac{d}{2}\right)d + 2\pi\left(\frac{d}{2}\right)^2 \Rightarrow S(d) = \pi d^2 + \frac{\pi d^2}{2} \Rightarrow S(d) = \frac{3\pi d^2}{2}$.

99. The function for changing yards to inches is: $f(x) = 36x$ and the function for changing miles to yards is: $g(x) = 1760x$. The composition of this which would change miles into inches is: $f[g(x)] = 36[1760(x)] \Rightarrow (f \circ g)(x) = 63{,}360x$.

100. If $x =$ width, then length $= 2x$. A formula for Perimeter can now be written as: $P = x + 2x + x + 2x$ and the function is: $P(x) = 6x$. This is a linear function.

Chapter 2 Test

1. (a) D, only values where $x \geq 0$ can be input into a square root function.

(b) D, only values where $y \geq 0$ can be the range of a square root function.

(c) C, all values can be input for x in a squaring function.

(d) B, only values where $y \geq 3$ can be the range of $f(x) = x^2 + 3$.

(e) C, all values can be input for x in a cube root function.

(f) C, all values can be the range of a cube root function.

(g) C, all values can be input for x in an absolute value function.

(h) D, only values where $y \geq 0$ can be the range to an absolute value function.

(i) D, if $x = y^2$ then $y = \sqrt{x}$ and only values where $x \geq 0$ can be input into a square root function.

(j) C, all values can be the range in this function.

2. (a) This is $f(x)$ shifted 2 units upward. See Figure 2a.

(b) This is $f(x)$ shifted 2 units to the left. See Figure 2b.

(c) This is $f(x)$ reflected across the x-axis. See Figure 2c.

(d) This is $f(x)$ reflected across the y-axis. See Figure 2d.

(e) This is $f(x)$ vertically stretched by a factor of 2. See Figure 2e.

(f) We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 2f.

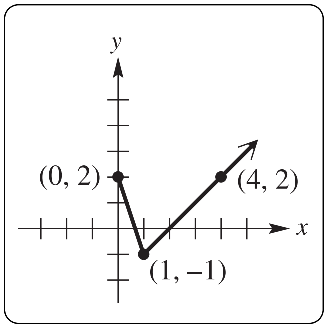


Figure 2a

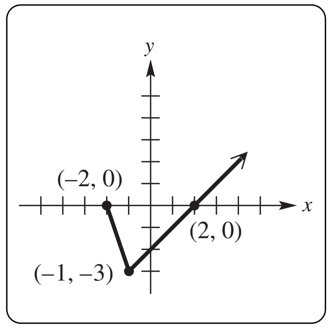


Figure 2b

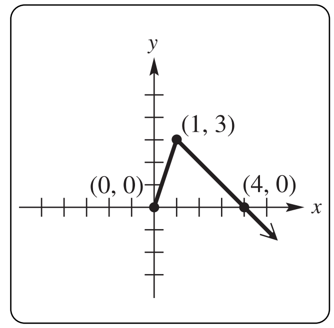


Figure 2c

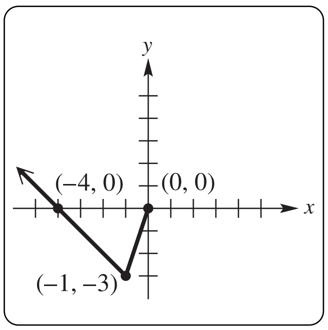


Figure 2d

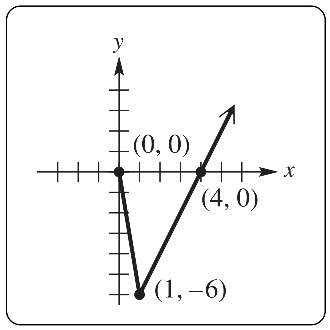


Figure 2e

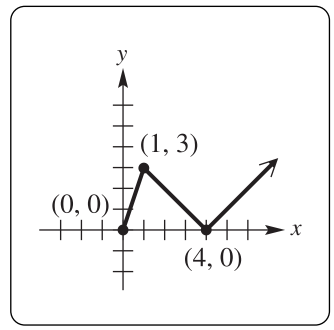


Figure 2f

3. (a) Since $y = f(2x)$ is $y = f(x)$ horizontally shrunk by a factor of $\frac{1}{2}$, the point $(-2, 4)$ on $y = f(x)$ becomes the point $(-1, 4)$ on the graph of $y = f(2x)$.

(b) Since $y = f\left(\frac{1}{2}x\right)$ is $y = f(x)$ horizontally stretched by a factor of 2, the point $(-2, 4)$ on $y = f(x)$ becomes the point $(-4, 4)$ on the graph of $y = f\left(\frac{1}{2}x\right)$.

4. (a) The graph of $f(x) = -(x - 2)^2 + 4$ is the basic graph $f(x) = x^2$ reflected across the x-axis, shifted 2 units to the right, and shifted 4 units upward. See Figure 4a.

(b) The graph of $f(x) = -2\sqrt{-x}$ is the basic graph $f(x) = \sqrt{x}$ reflected across the y-axis and vertically stretched by a factor of 2. See Figure 4b.

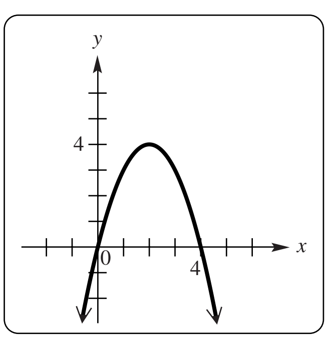


Figure 4a

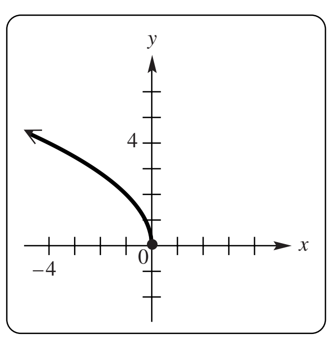


Figure 4b

5. (a) If the graph is symmetric with respect to the y-axis, then $(x, y) \Rightarrow (-x, y)$, therefore $(3, 6) \Rightarrow (-3, 6)$.

(b) If the graph is symmetric with respect to the x-axis, then $(x, y) \Rightarrow (-x, -y)$, therefore $(3, 6) \Rightarrow (-3, -6)$.

(c) See Figure 5. We give an actual screen here. The drawing should resemble it.

$[-4, 4]$ by $[0, 8]$
Xscl = 1 Yscl = 1

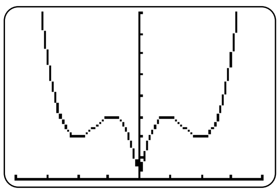


Figure 5

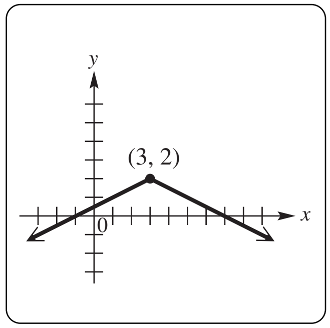


Figure 6

6. (a) Shift the graph of $y = \sqrt[3]{x}$ to the left 2 units, vertically stretch by a factor of 4, and shift 5 units downward.

(b) Graph $y = |x|$ reflected across the x-axis, vertically shrunk by a factor of $\frac{1}{2}$, shifted 3 units to the right, and shifted up 2 units. See Figure 6. From the graph the domain is: $(-\infty, \infty)$; and the range is: $(-\infty, 2]$.

7. (a) From the graph, the function is increasing for the interval: $(-\infty, -3)$.

(b) From the graph, the function is decreasing for the interval: $(4, \infty)$.

(c) From the graph, the function is constant for the interval: $[-3, 4]$.

(d) From the graph, the function is continuous for the intervals: $(-\infty, -3), [-3, 4], (4, \infty)$.

(e) From the graph, the domain is: $(-\infty, \infty)$.

(f) From the graph, the range is: $(-\infty, 2)$.

8. (a) $|4x + 8| = 4 \Rightarrow 4x + 8 = 4 \Rightarrow 4x = -4 \Rightarrow x = -1$ or $4x + 8 = -4 \Rightarrow 4x = -12 \Rightarrow x = -3$, therefore the solution set is: $\{-3, -1\}$. From the graph, the x-coordinates of the points of intersection of the graphs of Y_1 and Y_2 are -3 and -1. See Figure 8.

(b) $|4x + 8| < 4 \Rightarrow -4 < 4x + 8 < 4 \Rightarrow -12 < 4x < -4 \Rightarrow -3 < x < -1$, therefore the solution is: $(-3, -1)$. From the graph, See Figure 8, the graphs of Y_1 lies below the graph of Y_2 for x-values between -3 and -1.

(c) $|4x + 8| > 4 \Rightarrow 4x + 8 > 4 \Rightarrow 4x > -4 \Rightarrow x > -1$ or $4x + 8 < -4 \Rightarrow 4x < -12 \Rightarrow x < -3$, therefore the solution is: $(-\infty, -3) \cup (-1, \infty)$. From the graph, See Figure 8, the graph of Y_1 lies above the graph of Y_2 for x-values less than -3 or for x-values greater than -1.

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

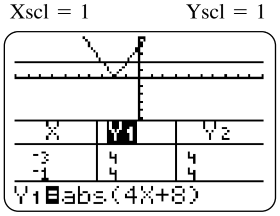


Figure 8

9. (a) $(f - g)(x) = 2x^2 - 3x + 2 - (-2x + 1) \Rightarrow (f - g)(x) = 2x^2 - x + 1$

(b) $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x^2 - 3x + 2}{-2x + 1}$

(c) The domain can be all values for x, except any that make $g(x) = 0$. Therefore $-2x + 1 \neq 0 \Rightarrow -2x \neq -1$ $\Rightarrow x \neq \dfrac{1}{2}$ or the interval: $\left(-\infty. \dfrac{1}{2}\right) \cup \left(\dfrac{1}{2}, \infty\right)$.

(d) $(f \circ g)(x) = f[g(x)] = 2(-2x + 1)^2 - 3(-2x + 1) + 2 = 2(4x^2 - 4x + 1) + 6x - 3 + 2 =$ $8x^2 - 8x + 2 + 6x - 3 + 2 = 8x^2 - 2x + 1$

(e) $\dfrac{2(x + h)^2 - 3(x + h) + 2 - (2x^2 - 3x + 2)}{h} = \dfrac{2(x^2 + 2xh + h^2) - 3x - 3h + 2 - 2x^2 + 3x - 2}{h} =$ $\dfrac{2x^2 + 4xh + 2h^2 - 3x - 3h + 2 - 2x^2 + 3x - 2}{h} = \dfrac{4xh + 2h^2 - 3h}{h} = 4x + 2h - 3$

10. (a) See Figure 10a.

(b) Graph $y_1 = (-x^2 + 3) * (x \leq 1) + (\sqrt[3]{x} + 2) * (x > 1)$ in the window $[-4.7, 4.7]$ by $[-5.1, 5.1]$. See Figure 10b.

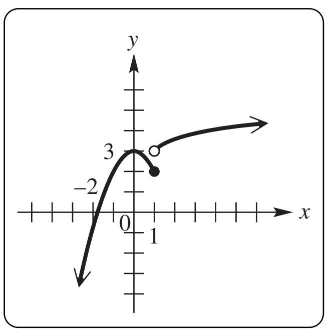


Figure 10a

[−4.7, 4.7] by [−5.1, 5.1]
Xscl = 1 Yscl = 1

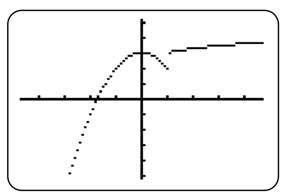


Figure 10b

[0, 10] by [0, 6]
Xscl = 1 Yscl = 1

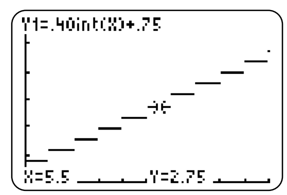


Figure 11

11. (a) See Figure 11.

(b) Set $x = 5.5$, then \$2.75 is the cost of a 5.5-minute call. See the display at the bottom of the screen.

12. (a) With an initial set-up cost of \$3300 and a production cost of \$4.50 the function is: $C(x) = 3300 + 4.50x$

(b) With a selling price of \$10.50 the revenue function is: $R(x) = 10.50x$

(c) $P(x) = R(x) - C(x) \Rightarrow P(x) = 10.50x - (3300 + 4.50x) \Rightarrow P(x) = 6x - 3300$

(d) To make a profit $P(x) > 0$, therefore $6x - 3300 > 0 \Rightarrow 6x > 3300 \Rightarrow x > 550$.

Tyler needs to sell 551before he earns a profit.

(e) Graph $y_1 = 6x - 3300$, See Figure 12. The first integer x-value for which $P(x) > 0$ is 551.

[0, 1000] by [−4000, 4000]
Xscl = 50 Yscl = 500

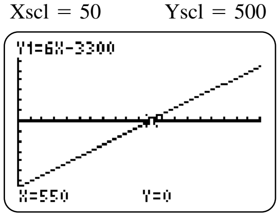


Figure 12

Chapter 2 Project

1. Since the front is moving at 40 mph for 4 hr the front moved 160 miles with each unit representing 100 miles. This is a shift of 1.6 units south or downward. The function $f(x) = \frac{1}{20}x^2$ shifted 1.6 units downward is: $y = \frac{1}{20}x^2 - 1.6$.

2. (a) Because the front has moved 250 south and 210 miles east, graph the shifted equation: $y = \frac{1}{20}(x - 2.1)^2 - 2.5$. Plot the point $(5.5, -.8)$ for Columbus, Ohio. Here we see that the front has reached the city. See Figure 2a.

   (b) Because the front has moved 250 south and 210 miles east, graph the shifted equation: $y = \frac{1}{20}(x - 2.1)^2 - 2.5$. Plot the point $(1.9, -4.3)$ for Memphis, Tennessee. Here we see that the front has not reached the city. See Figure 2b.

   (c) Because the front has moved 250 south and 210 miles east, graph the shifted equation: $y = \frac{1}{20}(x - 2.1)^2 - 2.5$. Plot the point $(4.2, -2.3)$ for Louisville, Kentucky. Although the graph is difficult to read, repeated zooming will show that the front has not reached the city. See Figure 2c.

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

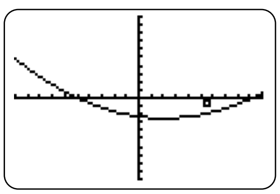


Figure 2a

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

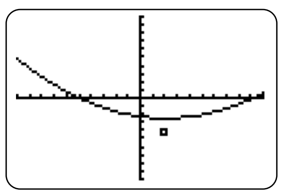


Figure 2b

$[-10, 10]$ by $[-10, 10]$
Xscl = 1 Yscl = 1

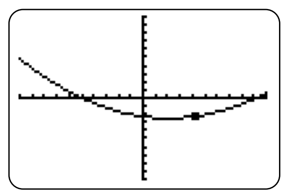


Figure 2c