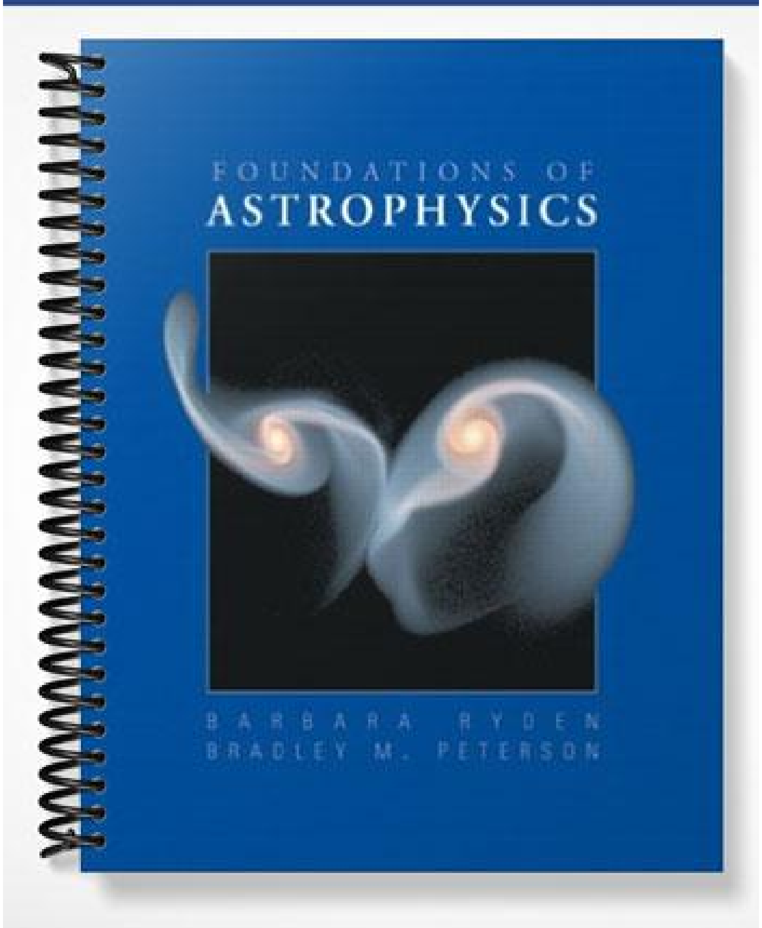
SOLUTIONS MANUAL



INSTRUCTOR SOLUTIONS MANUAL

Foundations of Astrophysics

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Preface

This manual provides complete solutions to the end-of-chapter exercises for *Foundations of Astrophysics* by Barbara Ryden and Bradley M. Peterson, a first course in astrophysics intended primarily for second-year majors in the physical sciences. SI units, augmented when necessary by various units peculiar to astronomy, are used throughout. In the written solutions, units are given whenever they may not be obvious.

Although most of the problems in this book have been heavily field-tested over the years, no doubt some errors, both typographical and conceptual, have eluded our scrutiny. The authors would be pleased to learn of any errors in the textbook or this solutions manual.

This solutions manual is intended to be an evolving document since it is expected to be made available only to instructors via a secure website. It will therefore be updated regularly by the authors, and the revision history will be recorded at the end. Also at the end of this manual will be a list of known errors found in the textbook itself.

We thank Catherine J. Grier for her help in proofreading this manual.

Contents

Chapter 1

Early Astronomy

1.1. The Polynesian inhabitants of the Pacific reportedly held festivals whenever the Sun was at the zenith at local noon. How many times per year was such a festival held? At what time(s) of year was the festival held on Tahiti? At what time(s) of year was it held on Oahu? [Hints: any reputable world atlas will give you the latitude of Tahiti and Oahu. You may also find the information in Figure 1.13 to be useful.]

The latitude of Tahiti = $-17^{\circ}\,37'$. The Sun crosses this declination on approximately 2 February and 1 November.
The latitude of Oahu is $+21^{\circ}\,28'$ and the Sun crosses this declination on approximately May 29 and July 16.
We note in passing that both Tahiti and Oahu extend about $25'$ of latitude in the north–south direction.

1.2. For what range of latitudes are all the stars of the Big Dipper circumpolar? Use the stars in the following table:

Star	Right Ascension	Declination
Alkaid	13^h48^m	$+49°19'$
Mizar	13^h24^m	$+54°56'$
Alioth	12^h54^m	$+55°58'$
Megrez	12^h15^m	$+57°02'$
Phecda	11^h54^m	$+53°42'$
Merak	11^h02^m	$+56°23'$
Dubhe	11^h04^m	$+61°45'$

For all the stars to be circumpolar, the southernmost star (Alkaid) must be above the horizon at lower transit, as shown in Figure 1.1. Thus the elevation of the North Celestial Pole must be equal to the angle between Alkaid and the

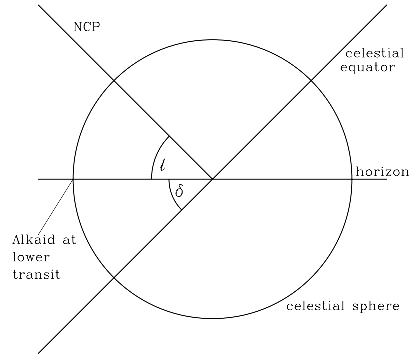

Figure 1.1: Southernmost latitude from which all the stars of the Big Dipper will be circumpolar.

NCP; the elevation is $\ell = 90^o - \delta_{\text{Alkaid}} = 90^o - 49^o 19' = 40^o 41'$. Only for observers at this latitude or higher will all the Big Dipper stars be circumpolar.

What is the southernmost latitude from which all of the stars of the Big Dipper can be seen?

For all the stars to be visible, the northernmost star (Dubhe) must be at the horizon at upper transit, as shown in Figure 1.2. In other words, the NCP is below the horizon by an angle equal to the separation between the NCP and Dubhe, i.e., $90^o - \delta_{\text{Dubhe}} = \ell$. Thus $\ell = \delta_{\text{Dubhe}} - 90^o = 61^o 45' - 90^o = -28^o 15'$. Only observers at or north of latitude $-28^o 15'$ can see all the stars of the Big Dipper.

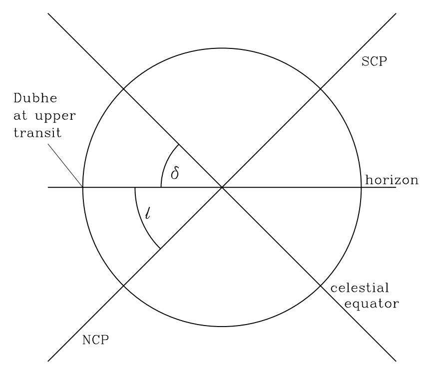

Figure 1.2: Southernmost latitude from which all the stars of the Big Dipper can be seen.

For what range of latitudes are none of the stars of the Big Dipper ever seen above the horizon?

For all of the stars to be below the horizon, the southernmost star must be on the horizon at upper transit, as shown in Figure 1.3. In other words, the NCP must be below the horizon by an angle equal to the distance between the NCP and Alkaid, i.e., $90^\circ - \delta_{\text{Alkaid}} = -\ell$ or $\ell = \delta_{\text{Alkaid}} - 90^\circ = 49^\circ 19' - 90^\circ = -40^\circ 41'$. Observers south of this latitude cannot observe any of the stars of the Big Dipper.

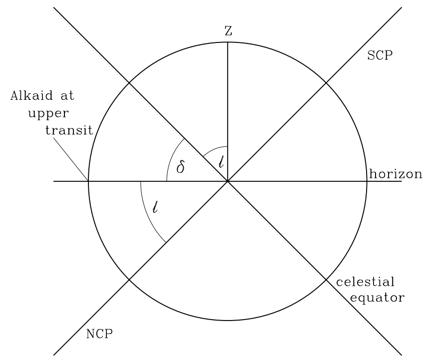

Figure 1.3: Northernmost latitude from which none of the stars of the Big Dipper can be seen.

1.3. Columbus, Ohio, is in the Eastern Time Zone, for which the civil time is equal to the mean solar time along the 75° W meridian of longitude.
(a) Ignoring daylight saving time for the moment, are there any days of the year when civil noon (as shown by a clock) is the same as apparent local noon (as shown by the Sun) in the city of Columbus? If so, what day or days are they?

The longitude of Columbus is $82^\circ 59'$ west. The zone time is set to longitude 75°, so Columbus is behind the zone time by

$$82^\circ 59' - 75^\circ = 7^\circ 59' \left(\frac{12^{\text{h}}}{180^\circ}\right) = 0.53^{\text{h}} \left(\frac{60^{\text{m}}}{1^{\text{h}}}\right) = 31.9^{\text{m}}.$$

Since the amplitude of the Equation of Time is only $\sim 18^{\text{m}}$, the Sun *never* transits the meridian at local noon in Columbus, it always transits $\sim 14^{\text{m}}$ to 50^{m} after noon, zone time.

(b) Daylight savings time advances the clock by one hour from the second Sunday in March to the first Sunday in November ("Spring forward, fall back"). When daylight savings time is in effect, are there

any days of the year when civil noon is the same as apparent local noon in the city of Columbus? If so, what day or days are they?

Since the zone time is advanced an hour, the problem is made worse by daylight savings time. During DST, the Sun crosses the meridian more than an hour after noon, zone time.

1.4. Suppose you've been granted access to a large telescope during the last week in September. One of the two objects you want to observe is in the constellation Virgo; the other is in the constellation Pisces. You only have time to observe one object: which should you choose? Please explain your answer.

The right ascension of Virgo is $\alpha \sim 13^{\rm h}$ and Pisces is at $\sim 0^{\rm h}$. The autumnal equinox is the third week of September: since the vernal equinox is $\alpha = 0^{\rm h}$, the Sun must be at $\alpha \sim 12^{\rm h}$ at the autumnal equinox. Virgo is thus unobservable, only an hour from the Sun. Pisces, however, will be crossing the meridian at midnight.

1.5. In *The Old Man and the Sea*, Hemingway described the old man lying in his boat off the coast of Cuba, looking up at the sky just after sunset: "It was dark now as it becomes dark quickly after the Sun sets in September. He lay against the worn wood of the bow and rested all that he could. The first stars were out. He did not know the name of Rigel but he saw it and knew soon they would all be out and he would have all his distant friends." Explain what is astronomically incorrect about this passage. [Hint: what are the celestial coordinates of the star Rigel?]

The right ascension of Rigel is $\alpha \sim 6^{\rm h}$ and its declination is $\delta \sim -8^{\rm o}$, so it is not circumpolar seen from Cuba. In September, the Sun is at $\alpha \sim 12^{\rm h}$, so at sunset, $\alpha \sim 18^{\rm h}$ is on the meridian. Rigel is thus near the nadir at this time.

1.6. (a) Consider two points on the Earth's surface that are separated by 1 arcsecond as seen from the center of the (assumed to be transparent) Earth. What is the physical distance between the two points?

$$d = \theta R = 1'' \times \left(\frac{\rm rad}{206265''}\right) \times 6378\,{\rm km} \times \left(\frac{10^3\,{\rm m}}{\rm km}\right) \sim 31\,{\rm m}$$

(b) Consider two points on the Earth's equator that are separated by one second of time. What is the physical distance between the two points?

$$\theta = 1\,{\rm sec} \times \left(\frac{1{\rm hr}}{3600\ {\rm sec}}\right) \times \frac{360^{\rm o}}{24^{\rm h}} \times \frac{\pi\,{\rm rad}}{180^{\rm o}} = 7.27 \times 10^{-5}\,{\rm rad}$$

So their physical separation is $d = \theta R = 463.8\,\mathrm{m}$.

1.7. The bright star Mintaka (also known as δ Orionis, the westernmost star of Orion's belt) is extremely close to the celestial equator. Amateur astronomers can determine the field of view of their telescope (that is the angular width of the region that they can see through the telescope) by timing how long it takes Mintaka to drift through the field of view when the telescope is held stationary in hour angle. How long does it take Mintaka to drift through a 1 degree field of view?

The sky appears to rotate westward at the sidereal rate

$$\omega = \frac{360^{\circ}}{24\,\text{sidereal hrs}} = \frac{15^{\circ}}{\text{sidereal hr}}.$$

The time it takes to rotate through an angle θ is

$$t = \frac{\theta}{\omega} = \frac{1^{\circ}}{15^{\circ}\ \text{hr}^{-1}} \times \frac{60^{\text{m}}}{1\ \text{hr}} = 4\,\text{sidereal minutes}$$

In terms of mean solar time,

$$t = 4\,\text{sidereal minutes} \times \frac{23^{\text{h}}56^{\text{m}}\ \text{solar time}}{24^{\text{h}}\ \text{sidereal time}} = 3^{\text{m}}59^{\text{s}}\,\text{solar time}$$

1.8. (a) Imagine that technologically advanced, but highly mischievous, space aliens have reduced the tilt of the Earth's axis from 23°.5 to 0°, while leaving the Earth's orbit unchanged. Sketch the analemma in this case.

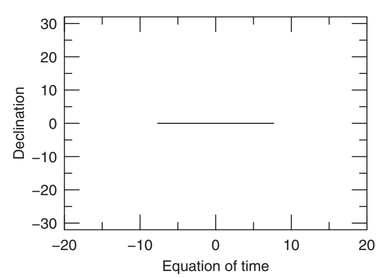

Figure 1.4: The part of the Earth's analemma that is attributable only to the eccentricity of the Earth's orbit. The part due to obliquity has been removed.

(b) Now imagine the aliens have restored the axial tilt to its previous value of 23°.5, but that they have changed the Earth's orbit so that it is a perfect circle, with the Earth's orbital speed being perfectly constant over the course of a year. Sketch the analemma in this case.

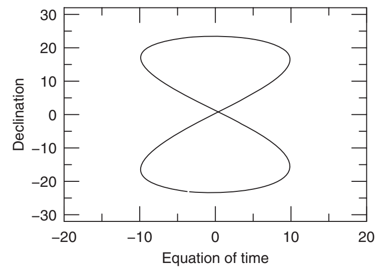

Figure 1.5: The part of the Earth's analemma that is attributable only to the obliquity of the ecliptic. The part due to eccentricity of the Earth's orbit has been removed.

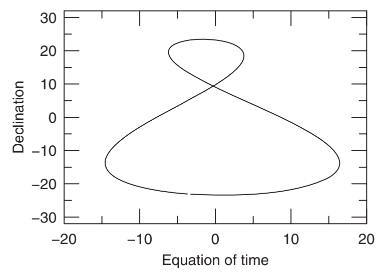

Figure 1.6: The Earth's complete analemma, shown for reference.

(c) The martian analemma is shown in Figure 1.15. What is the tilt of the rotation axis of Mars?

Inspection of the amplitude of the analemma shows that the inclination of Mars must be $\sim 24^o$ relative to its orbital plane.

1.9. How many square degrees are on the complete celestial sphere?

There are 180^o per π radians, so there are 180^2 square degrees in π^2 steradians. Thus, the surface area of the sky in steradians is

$$A = \left(\frac{180^\circ}{\pi \text{ rad}}\right)^2 \times 4\pi \,\text{steradians} = 41,253 \text{ square degrees}$$

Chapter 2

Emergence of Modern Astronomy

2.1. Over the course of the year, which gets more hours of daylight, the Earth's north pole or south pole? [Hint: The Earth is at perihelion in January.]

The Earth is at perihelion in January, so its northern hemisphere winter is shorter, and its southern hemisphere summer is shorter. Consequently, summed over a year, the north pole gets more light.

2.2. On 2003 August 27, Mars was in opposition as seen from the Earth. On 2005 July 14 (687 days later), Mars was in western quadrature as seen from the Earth. What was the distance of Mars from the Sun on these dates, measured in astronomical units (AU)? Is this greater than or less than the semimajor axis length of the Martian orbit? You may assume the Earth's orbit is a perfect circle. [Hint: The sidereal period of Mars is also 687 days.]

The number of orbits Earth makes in 687 days is

$$N_{\mathrm{orbit}} = \frac{687\,\mathrm{days}}{365.24\,\mathrm{days\ per\ orbit}} = 1.881\,\mathrm{orbits}.$$

The angle swept out by the Earth in 0.881 orbits is $\phi = (0.881)(360^{\mathrm{o}}) = 317^{\mathrm{o}}.14$. As per the left diagram in Figure 2.1, θ is the angle between the Earth and Mars as seen from the Sun and is $\theta = 360^{\mathrm{o}} - \phi = 42^{\mathrm{o}}.86$. Simple trigonometry (right diagram in Figure 2.1) gives the distance of Mars from the Sun,

$$c = \frac{a}{\cos\theta} = \frac{1\,\mathrm{AU}}{0.733} = 1.36\,\mathrm{AU},$$

which is less than the length of the semimajor axis of the orbit of Mars. This tells us that the orbit of Mars cannot be circular.

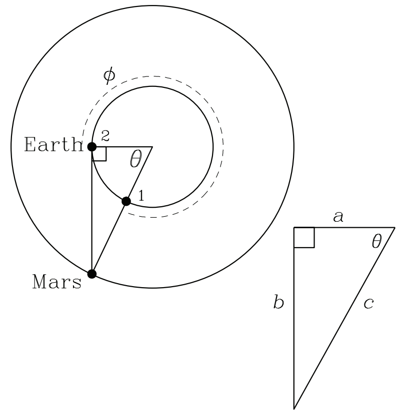

Figure 2.1: When Earth is at point 1 in the left diagram, Mars is at opposition. After one orbit, Mars returns to the same position and Earth is now at point 2, where Mars appears to be at western quadrature; during this time, Earth has swept out an angle $360^o + \phi$, and $\theta = 360^o - \phi$. The triangle from the left diagram is expanded on the right, where a is the Earth–Sun distance, c is the Mars–Sun distance, and b is the Earth–Mars distance when Mars is at western quadrature.

[Aside: In the next Chapter, we introduce the perihelion distance $q = a(1 - e)$. In the case of Mars, the perihelion distance is is $q = (1 - e)a = 1.524\,\mathrm{AU}(1 - 0.093) = 1.382\,\mathrm{AU}$, which is less than the distance of Mars from the Sun on the specified dates. This small error occurs on account of assuming that the Earth's orbit is circular, which it is not.]

2.3. In the 1670s, the astronomer Ole Roemer observed eclipses of the Galilean satellite Io as it plunged through Juptier's shadow once per orbit. He noticed that the time between observed eclipses became shorter as Jupiter came closer to the Earth and longer as Jupiter moved away. Roemer calculated that the eclipses were observed 17 minutes earlier when Jupiter was in opposition than when it was close to conjunction. This was attributed by Roemer to the finite speed of light. From Roemer's data, compute the speed of light, first in $\mathrm{AU\,min^{-1}}$, then in $\mathrm{m\,s^{-1}}$.

The difference in Jupiter's distance from Earth during opposition and conjunction is simply the diameter of the Earth's orbit, $D = 2\,\mathrm{AU}$. The speed of light is thus $c = 2\,\mathrm{AU}/17\,\mathrm{min} = 0.118\,\mathrm{AU/min}$. In SI units, this becomes

$$c = \frac{0.118\,\mathrm{AU}}{\mathrm{min}} \times \frac{1.49 \times 10^{11}\ \mathrm{m\ AU^{-1}}}{60\,\mathrm{s\ min^{-1}}} = 2.92 \times 10^{8}\ \mathrm{m\ s^{-1}}.$$

2.4. In addition to aberration of starlight due to the Earth's orbital motion around the Sun, there should also be diurnal aberration due to the Earth's rotation. Where on the Earth is this effect the largest, and what is its amplitude?

The diurnal effect is largest at the equator where the Earth's rotational speed is greatest,

$$v_{\rm rot} = \frac{2\pi r}{P} = \frac{2\pi \times 6.378 \times 10^6 \,\text{m/sidereal day}}{86,160\,\text{s/sidereal day}} = 465\,\text{m s}^{-1}.$$

The aberration angle will be

$$\theta = \frac{v_{\rm rot}}{c} = \left(\frac{465\text{m s}^{-1}}{3 \times 10^8 \text{m s}^{-1}}\right)\left(\frac{206265''}{\text{rad}}\right) = 0.32\,\text{arcsec}.$$

2.5. A light-year is defined as the distance traveled by light in a vacuum during one tropical year. How many light-years are in a parsec?

$$d = ct = 2.99799 \times 10^8 \text{ m s}^{-1} \times \frac{365.24\,\text{days}}{\text{year}} \times \frac{86,400\,\text{s}}{\text{day}} = 9.461 \times 10^{15}\,\text{m}.$$

Thus,

$$1\,\text{pc} = \frac{3.085678 \times 10^{16}\,\text{m}}{9.461 \times 10^{15}\,\text{m lt-yr}^{-1}} = 3.26\,\text{lt-yr}.$$

2.6. The planets all orbit the Sun in the same sense (counterclockwise as seen from above the Earth's north pole). Imagine a "wrong-way" planet orbiting the Sun in the opposite (clockwise) sense, on an orbit of semimajor axis length $a = 1.3$ AU. What would the sidereal period of this planet be? What would its synodic period be as seen from the Earth? What would its synodic period be as seen from Mars?

From Kepler's Third Law, the sidereal period of planet is $P_p = (1.3)^{3/2} = 1.48$ years. From Figure 2.2, we see that

$$\vec{\omega_p} = \vec{\omega}_E + \vec{\omega}_s,$$

or $-\omega_s = -\omega_p - \omega_E$, which leads to

$$\frac{1}{S} = \frac{1}{P_p} + \frac{1}{P_E} = \frac{1}{1.48} + \frac{1}{1},$$

or $S = 0.597\,\text{yr}$.

As seen from Mars (see Figure 2.3), $-\omega_s = -\omega_p - \omega_{\rm Mars}$. Using the sidereal period of Mars $P_{\rm Mars} = (1.54)^{3/2} = 1.91$ years, we solve for the synodic period of the planet using

$$\frac{1}{S} = \frac{1}{P_p} + \frac{1}{P_{\rm Mars}} = \frac{1}{1.48} + \frac{1}{1.91}$$

and find that $S = 0.833\,\text{yr} = 305\,\text{days}$.

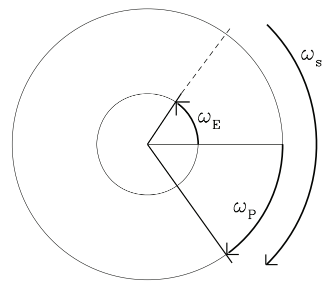

Figure 2.2: Angular speeds of Earth (ω_E) and the "wrong-way planet" (ω_p) in the sidereal reference frame, and the angular speed of the planet in a reference frame that co-rotates with the Earth–Sun line (ω_s).

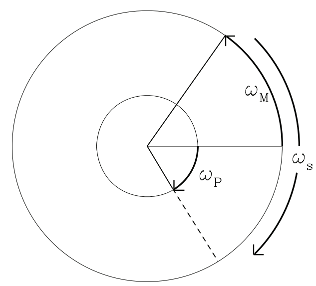

Figure 2.3: Angular speeds of Mars (ω_M) and the "wrong-way planet" (ω_p) in the sidereal reference frame, and the angular speed of the planet in a reference frame that co-rotates with the Mars–Sun line (ω_s).

2.7. Consider a football thrown directly northward at a latitude 40° N. The distance of the quarterback from the receiver is 20 yards (18.5 m), and the speed of the thrown ball is 25 m s^{-1}. Does the Coriolis force deflect the ball to the right or to the left? By what amount (in meters) is the ball deflected? Does the receiver need to worry about correcting for the deflection, or should he be more worried about being nailed by the free safety? [Hint: Remember that the angular velocity $\vec{\omega}$ of the Earth's rotation is parallel to the rotation axis.]

The Coriolis acceleration is given by equation (2.23),

$$\vec{a} = 2(\vec{v} \times \vec{\omega}).$$

The velocity of the football is: $|\vec{v}| = 25\,\text{m s}^{-1}$ and ω is the angular rotation speed of the Earth,

$$\omega \approx \frac{2\pi\,\text{rad}}{\text{day}} \times \frac{1\,\text{day}}{86,400\,\text{s}} \approx 7.27 \times 10^{-5}\,\text{rad s}^{-1}.$$

The ball will be deflected to the right in the northern hemisphere, by an amount

$$\Delta d \approx \frac{1}{2} a (\Delta t)^2,$$

where Δt is the time of flight, given by $\Delta t = D/v$. Thus,

$$d = \left(\frac{1}{2}\right)(2v\omega \sin \ell)\left(\frac{D^2}{v^2}\right) = \frac{D^2 \omega \sin \ell}{v}.$$

For $\ell = 40^{\rm o}$, $\sin \ell = 0.64$, this becomes

$$d = \frac{(18.5)^2 \times 7.27 \times 10^{-5} \times 0.64}{25} = 6.4 \times 10^{-4}\,\mathrm{m} = 0.64\,\mathrm{mm}.$$

Look out for the free safety!