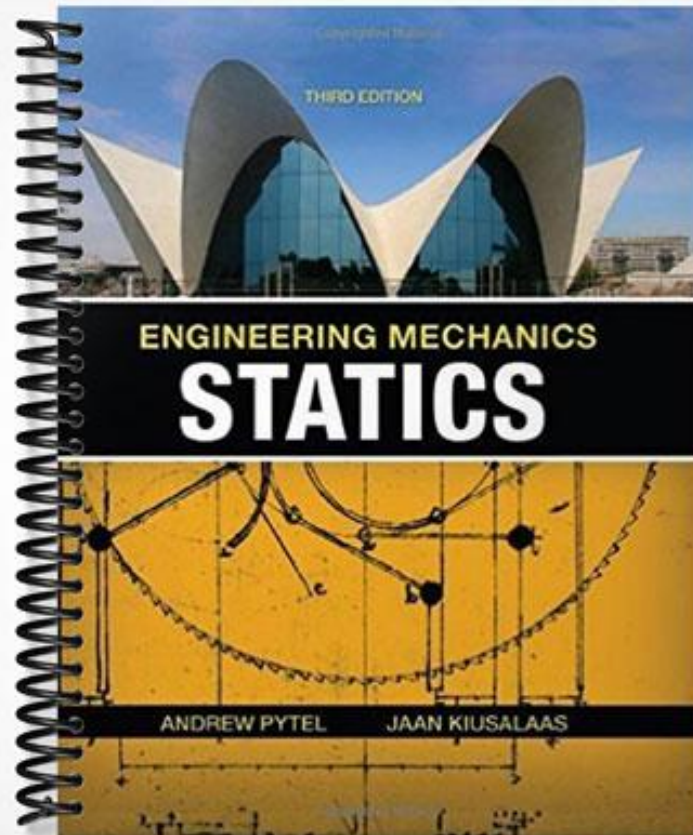


SOLUTIONS MANUAL



Chapter 2

2.1

The resultant of each force system is 500N ↑.

Each resultant force has the same line of action as the the force in (a), except (f) and (h)

Therefore (b), (c), (d), (e) and (g) are equivalent to (a) ◀

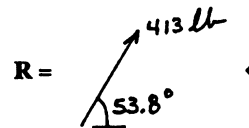
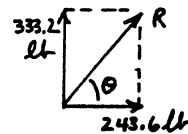
2.2

$$R_x = \Sigma F_x : \rightarrow R_x = 300 \cos 70^\circ + 150 \cos 20^\circ = 243.6 \text{ lb}$$

$$R_y = \Sigma F_y : +\uparrow R_y = 300 \sin 70^\circ + 150 \sin 20^\circ = 333.2 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{243.6^2 + 333.2^2} = 413 \text{ lb}$$

$$\theta = \tan^{-1} \left(\frac{333.2}{243.6} \right) = 53.8^\circ$$



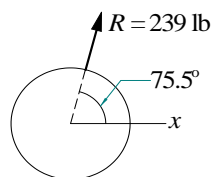
2.3

$$\begin{aligned} R_x &= \Sigma F_x = -T_1 \cos 60^\circ + T_3 \cos 40^\circ \\ &= -110 \cos 60^\circ + 150 \cos 40^\circ = 59.91 \text{ lb} \end{aligned}$$

$$\begin{aligned} R_y &= \Sigma F_y = T_1 \sin 60^\circ + T_2 + T_3 \sin 40^\circ \\ &= 110 \sin 60^\circ + 40 + 150 \sin 40^\circ = 231.7 \text{ lb} \end{aligned}$$

$$R = \sqrt{59.91^2 + 231.7^2} = 239 \text{ lb} \quad \blacktriangleleft$$

$$\theta = \tan^{-1} \frac{231.7}{59.91} = 75.5^\circ \quad \blacktriangleleft$$



2.4

$$\begin{aligned}
 R_x &= \Sigma F_x \quad + \longrightarrow \quad 85 = -30 + P \cos \theta + 40 \cos 60^\circ \\
 \therefore P \cos \theta &= 95.0 \text{ kN} \\
 R_y &= \Sigma F_y \quad + \uparrow \quad 20 = P \sin \theta - 40 \sin 60^\circ \\
 \therefore P \sin \theta &= 54.64 \text{ kN}
 \end{aligned}$$

$$\begin{aligned}
 \therefore P &= \sqrt{95.0^2 + 54.64^2} = 109.6 \text{ kN} \quad \blacktriangleleft \\
 \therefore \theta &= \tan^{-1} \frac{54.64}{95.0} = 29.9^\circ \quad \blacktriangleleft
 \end{aligned}$$

2.5

$$\begin{aligned}
 R_x = \Sigma F_x: \quad + \swarrow \quad R_x &= 40.0 \text{ N} \\
 R_y = \Sigma F_y: \quad \pm \quad R_y &= -60 \frac{120}{\sqrt{100^2 + 120^2}} = -46.1 \text{ N} \\
 R_z = \Sigma F_z: \quad + \uparrow \quad R_z &= 30 + 60 \frac{100}{\sqrt{100^2 + 120^2}} = 68.4 \text{ N} \\
 \therefore \mathbf{R} &= 40.0 \mathbf{i} - 46.1 \mathbf{j} + 68.4 \mathbf{k} \text{ N acting through } (0, 120 \text{ mm}, 0) \quad \blacklozenge
 \end{aligned}$$

2.6

$$\begin{aligned}
 \text{(a) } \mathbf{P}_1 &= 110 \mathbf{j} \text{ lb} \quad \mathbf{P}_2 = -200 \cos 25^\circ \mathbf{i} + 200 \sin 25^\circ \mathbf{j} = -181.26 \mathbf{i} + 84.52 \mathbf{j} \text{ lb} \\
 \mathbf{P}_3 &= -150 \cos 40^\circ \mathbf{i} + 150 \sin 40^\circ \mathbf{k} = -114.91 \mathbf{i} + 96.42 \mathbf{k} \text{ lb} \\
 \mathbf{R} = \Sigma \mathbf{P} &= (-181.26 - 114.91) \mathbf{i} + (110 + 84.52) \mathbf{j} + 96.42 \mathbf{k} \\
 &= -296.17 \mathbf{i} + 194.52 \mathbf{j} + 96.42 \mathbf{k} \text{ lb} \\
 \therefore R &= \sqrt{(-296.17)^2 + 194.52^2 + 96.42^2} = 367.2 \text{ lb} \quad \blacklozenge
 \end{aligned}$$

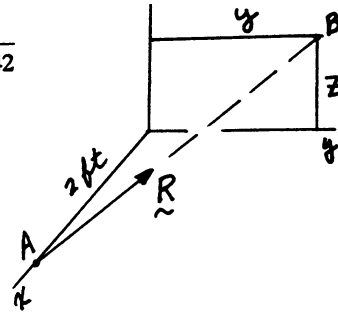
$$\frac{\overline{AB}_x}{|R_x|} = \frac{\overline{AB}_y}{|R_y|} = \frac{\overline{AB}_z}{|R_z|} : \frac{2}{296.17} = \frac{y}{194.52} = \frac{z}{96.42}$$

$$y = \frac{2(194.52)}{296.17} = 1.314 \text{ ft}$$

$$z = \frac{2(96.42)}{296.17} = 0.651 \text{ ft}$$

∴ **R** passes through the point

(b) (0, 1.314 ft, 0.651 ft) ♦



2.7

$$\mathbf{R} = (-P_2 \cos 25^\circ - P_3 \cos 40^\circ)\mathbf{i} + (P_1 + P_2 \sin 25^\circ)\mathbf{j} + P_3 \sin 40^\circ \mathbf{k}$$

$$= -600\mathbf{i} + 500\mathbf{j} + 300\mathbf{k} \text{ lb}$$

Equating like coefficients:

$$0.9063P_2 + 0.7660P_3 = 600 \text{ lb}$$

$$\text{Solving gives: } P_1 = 386.9 \text{ lb } \blacklozenge$$

$$P_1 + 0.4226P_2 = 500 \text{ lb}$$

$$P_2 = 267.6 \text{ lb } \blacklozenge$$

$$0.6428P_3 = 300 \text{ lb}$$

$$P_3 = 466.7 \text{ lb } \blacklozenge$$

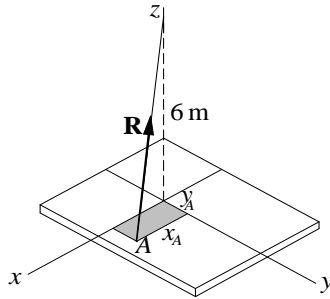
2.8

$$\mathbf{T}_1 = 100 \frac{-\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}}{\sqrt{(-1)^2 + 2^2 + 6^2}} = -15.617\mathbf{i} + 31.23\mathbf{j} + 93.70\mathbf{k} \text{ kN}$$

$$\mathbf{T}_2 = 80 \frac{-2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}}{\sqrt{(-2)^2 + (-3)^2 + 6^2}} = -22.86\mathbf{i} - 34.29\mathbf{j} + 68.57\mathbf{k} \text{ kN}$$

$$\mathbf{T}_3 = 50 \frac{2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}}{\sqrt{2^2 + (-3)^2 + 6^2}} = 14.286\mathbf{i} - 21.43\mathbf{j} + 42.86\mathbf{k} \text{ kN}$$

$$\begin{aligned} \mathbf{R} &= \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3 = (-15.617 - 22.86 + 14.286)\mathbf{i} \\ &\quad + (31.23 - 34.29 - 21.43)\mathbf{j} + (93.70 + 68.57 + 42.86)\mathbf{k} \\ &= -24.19\mathbf{i} - 24.49\mathbf{j} + 205.1\mathbf{k} \text{ kN } \blacktriangleleft \end{aligned}$$



Let \mathbf{R} intersect the plate at point A . Using proportions:

$$\frac{x_A}{-R_x} = \frac{y_A}{-R_y} = \frac{6}{R_z} \quad \frac{x_A}{24.19} = \frac{y_A}{24.49} = \frac{6}{205.1}$$

$$\therefore x_A = 0.708 \text{ m} \quad \blacktriangleleft \quad y_A = 0.716 \text{ m} \quad \blacktriangleleft$$

2.9

$$\mathbf{T}_1 = T_1 \frac{-\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}}{\sqrt{(-1)^2 + 2^2 + 6^2}} = T_1(-0.15617\mathbf{i} + 0.3123\mathbf{j} + 0.9370\mathbf{k})$$

$$\mathbf{T}_2 = T_2 \frac{-2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}}{\sqrt{(-2)^2 + (-3)^2 + 6^2}} = T_2(-0.2857\mathbf{i} - 0.4286\mathbf{j} + 0.8571\mathbf{k})$$

$$\mathbf{T}_3 = T_3 \frac{2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}}{\sqrt{2^2 + (-3)^2 + 6^2}} = T_3(0.2857\mathbf{i} - 0.4286\mathbf{j} + 0.8571\mathbf{k})$$

$$\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3 = \mathbf{R}$$

Equating like components, we get

$$\begin{aligned} -0.15617T_1 - 0.2857T_2 + 0.2857T_3 &= 0 \\ 0.3123T_1 - 0.4286T_2 - 0.4286T_3 &= 0 \\ 0.9370T_1 + 0.8571T_2 + 0.8571T_3 &= 210 \end{aligned}$$

Solution is

$$T_1 = 134.5 \text{ kN} \quad \blacktriangleleft \quad T_2 = 12.24 \text{ kN} \quad \blacktriangleleft \quad T_3 = 85.8 \text{ kN} \quad \blacktriangleleft$$

2.10

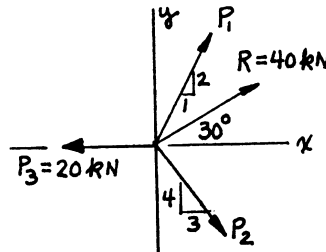
$$R_x = \Sigma F_x: \quad \pm \frac{1}{\sqrt{5}} P_1 + \frac{3}{5} P_2 - 20 = 40 \cos 30^\circ \quad (1)$$

$$R_y = \Sigma F_y: \quad +\uparrow \frac{2}{\sqrt{5}} P_1 - \frac{4}{5} P_2 = 40 \sin 30^\circ \quad (2)$$

Solving (1) and (2) gives:

$$P_1 = 62.3 \text{ kN} \quad \blacklozenge$$

$$P_2 = 44.6 \text{ kN} \quad \blacklozenge$$

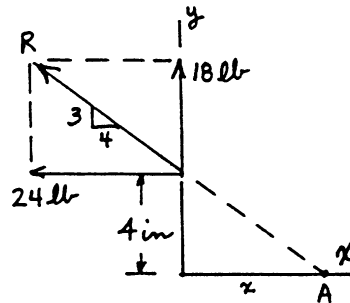


2.11

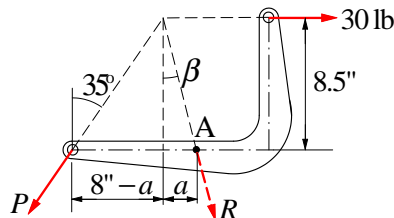
$$R = \sqrt{24^2 + 18^2} = 30 \text{ lb} \quad \blacklozenge$$

Using similar triangles:

$$\frac{3}{4} = \frac{4}{x} \quad \therefore x = \frac{16}{3} = 5.33 \text{ in.} \quad \blacklozenge$$



2.12



First find the direction of \mathbf{R} from geometry (the 3 forces must intersect at a common point).

$$8 - a = 8.5 \tan 35^\circ \quad \therefore a = 2.048 \text{ in.}$$

$$\beta = \tan^{-1} \frac{a}{8.5} = \tan^{-1} \frac{2.048}{8.5} = 13.547^\circ$$

$$R_x = \Sigma F_x \quad + \rightarrow \quad R \sin 13.547^\circ = -P \sin 35^\circ + 30$$

$$R_y = \Sigma F_y \quad + \downarrow \quad R \cos 13.547^\circ = P \cos 35^\circ$$

Solution is

$$P = 38.9 \text{ lb} \quad \blacktriangleleft \quad R = 32.8 \text{ lb} \quad \blacktriangleleft$$

2.13

By inspection:

$$\mathbf{R}_x = \Sigma \mathbf{F}_x = 0 \quad \blacklozenge \quad \mathbf{R}_y = \Sigma \mathbf{F}_y = 0 \quad \blacklozenge$$

$$\mathbf{R}_z = \Sigma \mathbf{F}_z = (20 + 20 + 15 + 15)\sin 75^\circ = 67.6 \text{ lb} \quad \blacklozenge$$

Point on line of action is the center of the plate \blacklozenge

2.14

$$\mathbf{P}_1 = 100 \frac{3\mathbf{i} + 4\mathbf{k}}{\sqrt{3^2 + 4^2}} = 60\mathbf{i} + 80\mathbf{k} \text{ lb}$$

$$\mathbf{P}_2 = 120 \frac{3\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}}{\sqrt{3^2 + 3^2 + 4^2}} = 61.74\mathbf{i} + 61.74\mathbf{j} + 82.32\mathbf{k} \text{ lb}$$

$$\mathbf{P}_3 = 60\mathbf{j} \text{ lb}$$

$$\mathbf{Q}_1 = Q_1\mathbf{i}$$

$$\mathbf{Q}_2 = Q_2 \frac{-3\mathbf{i} - 3\mathbf{j}}{\sqrt{3^2 + 3^2}} = Q_2 (-0.7071\mathbf{i} - 0.7071\mathbf{j})$$

$$\mathbf{Q}_3 = Q_3 \frac{3\mathbf{j} + 4\mathbf{k}}{\sqrt{3^2 + 4^2}} = Q_3 (0.6\mathbf{j} + 0.8\mathbf{k})$$

Equating similar components of $\Sigma \mathbf{Q} = \Sigma \mathbf{P}$:

$$\begin{aligned} Q_1 - 0.7071Q_2 &= 60 + 61.74 \\ -0.7071Q_2 + 0.6Q_3 &= 61.74 + 60 \\ 0.8Q_3 &= 80 + 82.32 \end{aligned}$$

Solution is

$$Q_1 = 121.7 \text{ lb} \quad \blacktriangleleft \quad Q_2 = 0 \quad Q_3 = 203 \text{ lb} \quad \blacktriangleleft$$

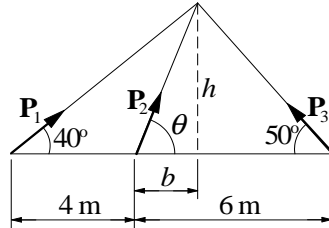
2.15

$$R_x = \Sigma F_x \quad + \longrightarrow \quad 10 = 50 \sin 45^\circ - Q \sin 30^\circ \quad Q = 50.71 \text{ lb}$$

$$R_y = \Sigma F_y \quad + \uparrow \quad 0 = 50 \cos 45^\circ - W + 50.71 \cos 30^\circ$$

$$\therefore W = 79.3 \text{ lb} \quad \blacktriangleleft$$

2.16



The forces must be concurrent. From geometry:

$$\begin{aligned}
 h &= (4 + b) \tan 40^\circ = (6 - b) \tan 50^\circ & \therefore b = 1.8682 \text{ m} \blacktriangleleft \\
 \therefore h &= (4 + 1.8682) \tan 40^\circ = 4.924 \text{ m} \\
 \theta &= \tan^{-1} \frac{h}{b} = \tan^{-1} \frac{4.924}{1.8682} = 69.22^\circ \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{R} = \Sigma \mathbf{F} &= (25 \cos 40^\circ + 60 \cos 69.22^\circ - 80 \cos 50^\circ) \mathbf{i} \\
 &\quad + (25 \sin 40^\circ + 60 \sin 69.22^\circ + 80 \sin 50^\circ) \mathbf{j} \\
 &= -10.99 \mathbf{i} + 133.45 \mathbf{j} \text{ kN} \blacktriangleleft
 \end{aligned}$$

2.17

The three forces intersect at C.

$$h = 1.2 \tan 25^\circ = 0.5596 \text{ m}$$

For the 240-N force :

$$\begin{aligned}
 -240 (\cos 25^\circ \mathbf{i} - \sin 25^\circ \mathbf{k}) &= \\
 -217.5 \mathbf{i} + 101.4 \mathbf{k} \text{ N}
 \end{aligned}$$

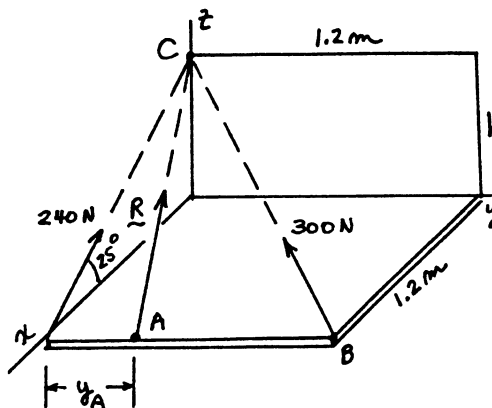
For the 300-N force ($300 \vec{\lambda}_{BC}$):

$$\begin{aligned}
 300 \left(\frac{-1.2 \mathbf{i} - 1.2 \mathbf{j} + 0.5596 \mathbf{k}}{1.787} \right) &= \\
 -201.5 \mathbf{i} - 201.5 \mathbf{j} + 93.95 \mathbf{k} \text{ N}
 \end{aligned}$$

$\mathbf{R} = \Sigma \mathbf{F}$

$$= (-217.5 - 201.5) \mathbf{i} - 201.5 \mathbf{j} + (101.4 + 93.95) \mathbf{k} = -419.0 \mathbf{i} - 201.5 \mathbf{j} + 195.4 \mathbf{k} \text{ N} \blacklozenge$$

Since \mathbf{R} acts along \overline{AC} : $\frac{|R_y|}{y_A} = \frac{|R_x|}{1.2} \quad \therefore y_A = \frac{|R_y|}{|R_x|} (1.2) = \frac{201.5}{419.0} (1.2) = 0.577 \text{ m} \blacklozenge$



2.18

$$\mathbf{T}_1 = 200 \left(\frac{3\mathbf{i} - 2\mathbf{j} - 6\mathbf{k}}{7} \right) = 85.71\mathbf{i} - 57.14\mathbf{j} - 171.43\mathbf{k} \text{ lb}$$

$$\mathbf{T}_2 = 400 \left(\frac{3\mathbf{j} - 6\mathbf{k}}{\sqrt{45}} \right) = 178.89\mathbf{j} - 357.77\mathbf{k} \text{ lb}$$

$$\mathbf{T}_3 = 350 \left(\frac{-4\mathbf{i} - 6\mathbf{k}}{\sqrt{52}} \right) = -194.15\mathbf{i} - 291.22\mathbf{k} \text{ lb}$$

$$\mathbf{R} = \Sigma \mathbf{T} = (85.71 - 194.15)\mathbf{i} + (-57.14 + 178.89)\mathbf{j} + (-171.43 - 357.77 - 291.22)\mathbf{k}$$

$$\therefore \mathbf{R} = -108.4\mathbf{i} + 121.8\mathbf{j} - 820.4\mathbf{k} \text{ lb (acting through point A) } \blacklozenge$$

2.19

(a)

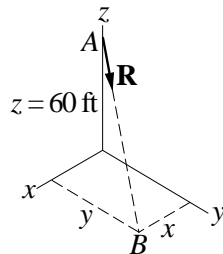
$$R_x = \Sigma F_x = 160 \sin 30^\circ \cos 50^\circ = 51.42 \text{ lb}$$

$$R_y = \Sigma F_y = -80 \sin 30^\circ + 160 \sin 30^\circ \sin 50^\circ = 21.28 \text{ lb}$$

$$R_z = \Sigma F_z = -120 - 80 \cos 30^\circ - 160 \cos 30^\circ = -327.8 \text{ lb}$$

$$R = \sqrt{51.42^2 + 21.28^2 + (-327.8)^2} = 332.5 \text{ lb } \blacktriangleleft$$

(b)

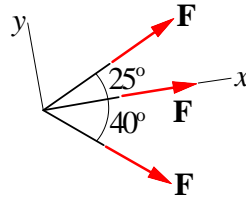


$$\frac{R_x}{x} = \frac{R_y}{y} = \frac{-R_z}{z} \quad \frac{51.42}{x} = \frac{21.28}{y} = \frac{327.8}{60}$$

$$x = \frac{51.42(60)}{327.8} = 9.41 \text{ ft } \blacktriangleleft \quad y = \frac{21.28(60)}{327.8} = 3.90 \text{ ft } \blacktriangleleft$$

2.20

Choose the line of action of the middle force as the x -axis.



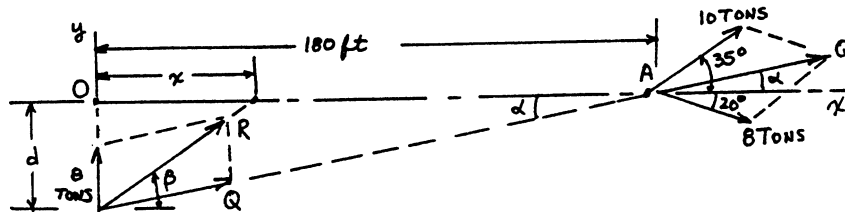
$$R_x = \Sigma F_x = F(\cos 25^\circ + 1 + \cos 40^\circ) = 2.672F$$

$$R_y = \Sigma F_y = F(\sin 25^\circ - \sin 40^\circ) = -0.2202F$$

$$R = F\sqrt{2.672^2 + (-0.2202)^2} = 2.681F$$

$$600 = 2.681F \quad \therefore F = 234 \text{ lb} \quad \blacktriangleleft$$

*2.21



Let Q be the resultant of the two forces at A .

$$\rightarrow Q_x = \Sigma F_x = 10 \cos 35^\circ + 8 \cos 20^\circ = 15.71 \text{ tons}$$

$$+\uparrow Q_y = \Sigma F_y = 10 \sin 35^\circ - 8 \sin 20^\circ = 3.00 \text{ tons}$$

$$\therefore \tan \alpha = Q_y/Q_x = 3.00/15.71 = 0.1910$$

Let R be the resultant of Q and the 8-ton vertical force.

$$\rightarrow R_x = \Sigma F_x = Q_x = 15.71 \text{ tons}$$

$$+\uparrow R_y = \Sigma F_y = 8 + Q_y = 8 + 3 = 11 \text{ tons}$$

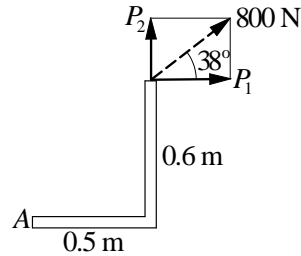
$$\therefore R = 15.71 \mathbf{i} + 11.00 \mathbf{j} \text{ tons} \quad \blacklozenge$$

$$\text{(Note that } \tan \beta = R_y/R_x = 11.00/15.71 = 0.7002)$$

$$\text{To find } x: d = 180 \tan \alpha = 180(0.1910) = 34.38 \text{ ft}$$

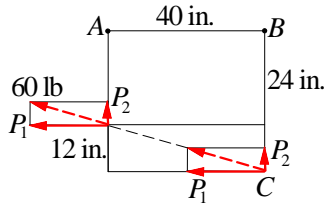
$$x = d/\tan \beta = 34.38/0.7002 = 49.1 \text{ ft} \quad \blacklozenge$$

2.22



$$\begin{aligned}
 + \circlearrowleft M_A &= -0.6P_1 + 0.5P_2 \\
 &= -0.6(800 \cos 38^\circ) + 0.5(800 \sin 38^\circ) = -132.0 \text{ N} \cdot \text{m} \\
 \therefore M_A &= 132.0 \text{ N} \cdot \text{m} \circlearrowright \blacktriangleleft
 \end{aligned}$$

2.23



$$P_1 = 60 \frac{40}{\sqrt{40^2 + 12^2}} = 57.47 \text{ lb}$$

With the force in the original position:

$$M_A = 24P_1 = 24(57.47) = 1379 \text{ lb} \cdot \text{in.} \circlearrowright \blacktriangleleft$$

With the force moved to point C :

$$M_B = 36P_1 = 36(57.47) = 2070 \text{ lb} \cdot \text{in.} \circlearrowright \blacktriangleleft$$

2.24

$$\mathbf{R} = P\mathbf{j} + P \frac{-2.5\mathbf{i} + 3.5\mathbf{j}}{\sqrt{(-2.5)^2 + 3.5^2}} = P(-0.5812\mathbf{i} + 1.8137\mathbf{j})$$

Choosing C as the moment center, the combined moment of the two forces is $M_C = 2.5P$ and the moment of \mathbf{R} is $M_C = R_y b$. Equating the moments, we get

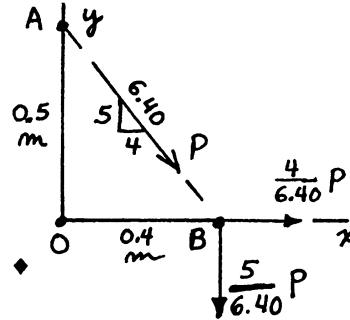
$$2.5P = b(1.8137P) \quad \therefore b = 1.378 \text{ m} \blacktriangleleft$$

2.25

Since M_A and M_B equal zero, the force P passes through A and B, as shown.

$$\begin{aligned} \textcircled{+} M_O &= -\frac{5}{6.40} P(0.4) = -200 \text{ kN}\cdot\text{m} \\ \therefore P &= 640 \text{ N} \end{aligned}$$

$$\mathbf{P} = \frac{4}{6.40}(640)\mathbf{i} - \frac{5}{6.40}(640)\mathbf{j} = 400\mathbf{i} - 500\mathbf{j} \text{ N} \blacklozenge$$



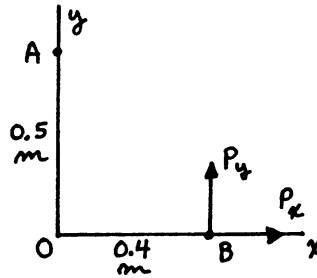
2.26

Since $M_B = 0$, P passes through B.

$$\begin{aligned} \textcircled{+} M_O &= 0.4 P_y = 80 \text{ N}\cdot\text{m} \\ P_y &= 200 \text{ N} \end{aligned}$$

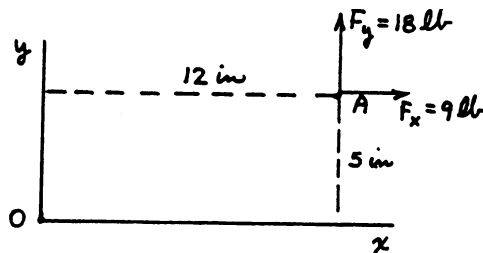
$$\begin{aligned} \textcircled{+} M_A &= 0.4(200) + 0.5 P_x = -200 \text{ N}\cdot\text{m} \\ P_x &= -280/0.5 = -560 \text{ N} \end{aligned}$$

$$\therefore \mathbf{P} = -560\mathbf{i} + 200\mathbf{j} \text{ N} \blacklozenge$$



2.27

$$\mathbf{F} = 9\mathbf{i} + 18\mathbf{j} \text{ lb}$$



$$\begin{aligned} \text{(a)} M_O &= \mathbf{r}_{OA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 5 & 0 \\ 9 & 18 & 0 \end{vmatrix} \\ &= \mathbf{k} [18(12) - 5(9)] = 171 \text{ k lb}\cdot\text{in.} \blacklozenge \end{aligned}$$

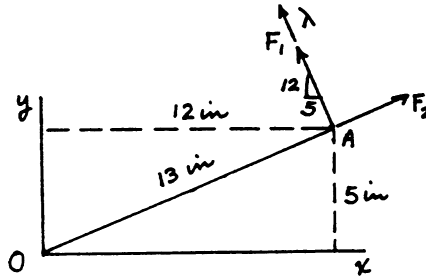
$$\text{(b)} \textcircled{+} M_O = 18(12) - 9(5) = 171 \text{ lb}\cdot\text{in.} \quad \therefore M_O = 171 \text{ lb}\cdot\text{in. CCW} \blacklozenge$$

(c) Unit vector perpendicular to OA is

$$\vec{\lambda} = -\frac{5}{13}\mathbf{i} + \frac{12}{13}\mathbf{j}$$

$$\begin{aligned} F_1 &= \mathbf{F} \cdot \vec{\lambda} \\ &= (9\mathbf{i} + 18\mathbf{j}) \cdot \left(-\frac{5}{13}\mathbf{i} + \frac{12}{13}\mathbf{j}\right) \\ &= \frac{-45 + 216}{13} = 13.15 \text{ lb}\cdot\text{in.} \end{aligned}$$

$$\curvearrowright M_O = 13 F_1 = 13(13.15) = 171 \text{ lb}\cdot\text{in.} \quad \therefore M_O = 171 \text{ lb}\cdot\text{in. CCW} \blacklozenge$$

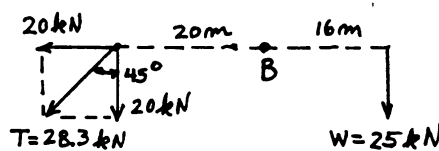


2.28

(a) For T: $\curvearrowright M_B = 20(20) = 400 \text{ kN}\cdot\text{m}$
 $\therefore M_B = 400 \text{ kN}\cdot\text{m CCW} \blacklozenge$

(b) For W: $\curvearrowright M_B = 25(16) = 400 \text{ kN}\cdot\text{m}$
 $\therefore M_B = 400 \text{ kN}\cdot\text{m CW} \blacklozenge$

(c) For T and W: $\curvearrowright \Sigma M_B = +400 - 400 = 0 \blacklozenge$

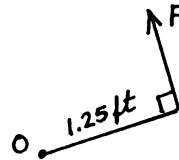


2.29

The moment of \mathbf{F} about O is maximum

when $\theta = 90^\circ \blacklozenge$

$$M_O = F(1.25) = 50 \text{ lb}\cdot\text{ft} \quad \therefore F = \frac{50}{1.25} = 40 \text{ lb} \blacklozenge$$

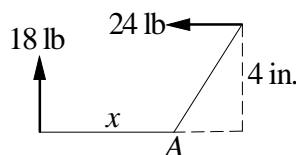


2.30

Note that the horizontal component of \mathbf{P} has no moment about A . Thus the combined moment about A is

$$+\circlearrowleft 30(8.5) - (P \cos 35^\circ)(8) = 0 \quad \therefore P = 38.9 \text{ lb} \blacktriangleleft$$

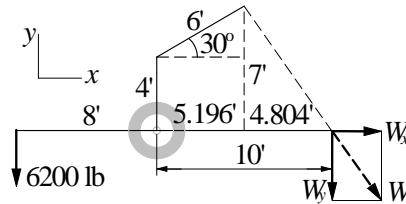
2.31



Because the resultant passes through point A , we have

$$\Sigma M_A = 0 \quad + \circlearrowleft \quad 24(4) - 18x = 0 \quad x = 5.33 \text{ in.} \quad \blacktriangleleft$$

2.32



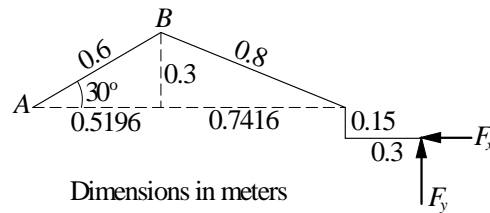
$$W_y = W \frac{7}{\sqrt{7^2 + 4.804^2}} = 0.8245W$$

Largest W occurs when the moment about the rear axle is zero.

$$+ \circlearrowleft \quad M_{\text{axle}} = 6200(8) - (0.8245W)(10) = 0$$

$$\therefore W = 6020 \text{ lb} \quad \blacktriangleleft$$

2.33



$$+ \circlearrowleft \quad M_A = -F_x(0.15) + F_y(0.5196 + 0.7416 + 0.3) \quad (a)$$

$$210 = -0.15F_x + 1.5612F_y$$

$$+ \circlearrowleft \quad M_B = -F_x(0.3 + 0.15) + F_y(0.7416 + 0.3) \quad (b)$$

$$90 = -0.45F_x + 1.0416F_y$$

Solution of Eqs. (a) and (b) is $F_x = 143.2 \text{ N}$ and $F_y = 148.3 \text{ N}$

$$\therefore F = \sqrt{143.2^2 + 148.3^2} = 206 \text{ N} \quad \blacktriangleleft$$

$$\theta = \tan^{-1} \frac{F_x}{F_y} = \tan^{-1} \frac{143.2}{148.3} = 44.0^\circ \quad \blacktriangleleft$$

2.34

$$\mathbf{P} = 200 \frac{-70\mathbf{i} - 100\mathbf{k}}{\sqrt{(-70)^2 + (-100)^2}} = -114.69\mathbf{i} - 163.85\mathbf{k} \text{ N}$$

$$\mathbf{r} = \overrightarrow{AB} = -0.07\mathbf{i} + 0.09\mathbf{j} \text{ m}$$

$$\begin{aligned} \mathbf{M}_A &= \mathbf{r} \times \mathbf{P} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.07 & 0.09 & 0 \\ -114.69 & 0 & -163.85 \end{vmatrix} \\ &= -14.75\mathbf{i} - 11.47\mathbf{j} + 10.32\mathbf{k} \text{ N} \cdot \text{m} \quad \blacktriangleleft \end{aligned}$$

2.35

$$\mathbf{P} = 100 \lambda_{AB} \vec{\lambda}_{AB} = 100 \left(\frac{-0.500\mathbf{i} - 0.600\mathbf{j} + 0.360\mathbf{k}}{0.860} \right) = -58.14\mathbf{i} - 69.77\mathbf{j} + 41.86\mathbf{k} \text{ N}$$

(a) $\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{P}$ $\mathbf{r}_{OB} = 0.360\mathbf{k} \text{ m}$ (\mathbf{r}_{OA} is also convenient)

$$\therefore \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.360 \\ -58.14 & -69.77 & 41.86 \end{vmatrix} = 25.11\mathbf{i} - 20.93\mathbf{j} \text{ N} \cdot \text{m} \quad \blacklozenge$$

(b) $\mathbf{M}_C = \mathbf{r}_{CB} \times \mathbf{P}$ $\mathbf{r}_{CB} = -0.600\mathbf{j} \text{ m}$ (\mathbf{r}_{CA} is also convenient)

$$\therefore \mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.600 & 0 \\ -58.14 & -69.77 & 41.86 \end{vmatrix} = -25.12\mathbf{i} - 34.88\mathbf{k} \text{ N} \cdot \text{m} \quad \blacklozenge$$

2.36

$$\mathbf{Q} = 250 \lambda_{BD} \vec{\lambda}_{BD} = 250 \left(\frac{-0.500\mathbf{i} + 0.360\mathbf{k}}{0.6161} \right) = -202.9\mathbf{i} + 146.1\mathbf{k} \text{ N}$$

(a) $\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{Q}$ $\mathbf{r}_{OB} = 0.360\mathbf{k} \text{ m}$ (\mathbf{r}_{OD} is also convenient)

$$\therefore \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.360 \\ -202.9 & 0 & 146.1 \end{vmatrix} = -73.0\mathbf{j} \text{ N} \cdot \text{m} \quad \blacklozenge$$

(b) $\mathbf{M}_C = \mathbf{r}_{CB} \times \mathbf{Q}$ $\mathbf{r}_{CB} = -0.600\mathbf{j} \text{ m}$ (\mathbf{r}_{CD} is also convenient)

$$\therefore \mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.600 & 0 \\ -202.9 & 0 & 146.1 \end{vmatrix} = -87.7\mathbf{i} - 121.7\mathbf{k} \text{ N} \cdot \text{m} \quad \blacklozenge$$

2.37

$$\mathbf{M}_O = \mathbf{r}_{OC} \times \mathbf{P} \quad \mathbf{r}_{OC} = 2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} \text{ m} \quad \mathbf{P} = P(-\cos 25^\circ \mathbf{i} + \sin 25^\circ \mathbf{k})$$

$$\mathbf{M}_O = P \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & -3 \\ -\cos 25^\circ & 0 & \sin 25^\circ \end{vmatrix} = P(1.6905\mathbf{i} + 1.8737\mathbf{j} + 3.6252\mathbf{k})$$

$$\therefore M_O = P \sqrt{1.6905^2 + 1.8737^2 + 3.6252^2} = 4.417P = 200 \text{ kN}\cdot\text{m}$$

$$\text{which gives } P = 200/4.417 = 45.3 \text{ kN} \quad \blacklozenge$$

2.38

$$\mathbf{P} = 50(-\cos 25^\circ \mathbf{i} + \sin 25^\circ \mathbf{k}) = -45.32\mathbf{i} + 21.13\mathbf{k} \text{ kN}$$

$$\text{(a) } \mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{P} \quad \mathbf{r}_{AC} = 4\mathbf{j} - 3\mathbf{k} \text{ m}$$

$$\therefore \mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -3 \\ -45.32 & 0 & 21.13 \end{vmatrix} = 84.52\mathbf{i} + 135.96\mathbf{j} + 181.28\mathbf{k} \text{ kN}\cdot\text{m}$$

$$\text{(b) } \mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{P} \quad \mathbf{r}_{BC} = 4\mathbf{j} \text{ m}$$

$$\therefore \mathbf{M}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & 0 \\ -45.32 & 0 & 21.13 \end{vmatrix} = 84.52\mathbf{i} + 181.28\mathbf{k} \text{ kN}\cdot\text{m}$$

2.39

$$\mathbf{Q} = 20 \lambda_{AB} \vec{\lambda}_{AB} = 20 \left(\frac{-3\mathbf{i} + 4\mathbf{j} - 8\mathbf{k}}{9.434} \right) = -6.360\mathbf{i} + 8.480\mathbf{j} - 16.96\mathbf{k} \text{ lb}$$

$$\text{(a) } \mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{Q} \quad \mathbf{r}_{OA} = 8\mathbf{k} \text{ ft} \quad (\mathbf{r}_{OB} \text{ is also convenient})$$

$$\therefore \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 8 \\ -6.360 & 8.480 & -16.96 \end{vmatrix} = -67.8\mathbf{i} - 50.9\mathbf{j} \text{ lb}\cdot\text{ft} \quad \blacklozenge$$

(b) $\mathbf{M}_C = \mathbf{r}_{CB} \times \mathbf{Q}$ $\mathbf{r}_{CB} = -3\mathbf{i}$ ft (\mathbf{r}_{CA} is also convenient)

$$\therefore \mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 0 & 0 \\ -6.360 & 8.480 & -16.96 \end{vmatrix} = -50.9\mathbf{j} - 25.4\mathbf{k} \text{ lb}\cdot\text{ft} \quad \blacklozenge$$

2.40

Noting that both \mathbf{P} and \mathbf{Q} pass through A , we have

$$\mathbf{M}_O = \mathbf{r}_{OA}(\mathbf{P} + \mathbf{Q})$$

$$\mathbf{P} = 80 \frac{-4.2\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}}{\sqrt{(-4.2)^2 + (-2)^2 + 2^2}} = -66.36\mathbf{i} - 31.60\mathbf{j} + 31.60\mathbf{k} \text{ lb}$$

$$\mathbf{Q} = 60 \frac{-2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}}{\sqrt{(-2)^2 + (-3)^2 + 2^2}} = -29.10\mathbf{i} - 43.66\mathbf{j} + 29.10\mathbf{k} \text{ lb}$$

$$\mathbf{P} + \mathbf{Q} = -95.46\mathbf{i} - 75.26\mathbf{j} + 60.70\mathbf{k} \text{ lb}$$

$$\therefore \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ -95.46 & -75.26 & 60.70 \end{vmatrix} = 150.5\mathbf{i} - 190.9\mathbf{j} \text{ lb}\cdot\text{ft} \quad \blacktriangleleft$$

2.41

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} \quad \mathbf{r} = -8\mathbf{i} + 12\mathbf{j} \text{ in.} \quad \mathbf{F} = -120\mathbf{k} \text{ lb}$$

$$\therefore \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8 & 12 & 0 \\ 0 & 0 & -120 \end{vmatrix} = -1440\mathbf{i} - 960\mathbf{j} \text{ lb}\cdot\text{in.} = -120\mathbf{i} - 80\mathbf{j} \text{ lb}\cdot\text{ft} \quad \blacklozenge$$

2.42

$$\mathbf{P} = -16 \cos 40^\circ \mathbf{i} + 16 \sin 40^\circ \mathbf{k} = -12.257 \mathbf{i} + 10.285 \mathbf{k} \text{ lb} \quad \mathbf{Q} = -22.00 \mathbf{j} \text{ lb}$$

$$\therefore \mathbf{P} + \mathbf{Q} = -12.257 \mathbf{i} - 22.00 \mathbf{j} + 10.285 \mathbf{k} \text{ lb}$$

$$\mathbf{M}_O = \mathbf{r}_{OA} \times (\mathbf{P} + \mathbf{Q}) \quad \mathbf{r}_{OA} = -(3 + 8 \cos 40^\circ) \mathbf{i} + (8 \sin 40^\circ) \mathbf{k} = -9.128 \mathbf{i} + 5.142 \mathbf{k} \text{ in.}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -9.128 & 0 & 5.142 \\ -12.257 & -22.00 & 10.285 \end{vmatrix} = 113.12 \mathbf{i} + 30.86 \mathbf{j} + 200.82 \mathbf{k} \text{ lb}\cdot\text{in.}$$

$$M_O = \sqrt{113.12^2 + 30.86^2 + 200.82^2} = 232.5 \text{ lb}\cdot\text{in.} \quad \blacklozenge$$

$$\cos \theta_x = \frac{113.12}{232.5} = 0.4865; \quad \cos \theta_y = \frac{30.86}{232.5} = 0.1327; \quad \cos \theta_z = \frac{200.82}{232.5} = 0.8637 \quad \blacklozenge$$

2.43

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ 50 & -100 & -70 \end{vmatrix} = 100z \mathbf{i} + (70x + 50z) \mathbf{j} - 100x \mathbf{k}$$

Equating the x - and z -components of \mathbf{M}_O to the given values yields

$$\begin{aligned} 100z &= 400 & \therefore z &= 4 \text{ ft} \quad \blacktriangleleft \\ -100x &= -300 & \therefore x &= 3 \text{ ft} \quad \blacktriangleleft \end{aligned}$$

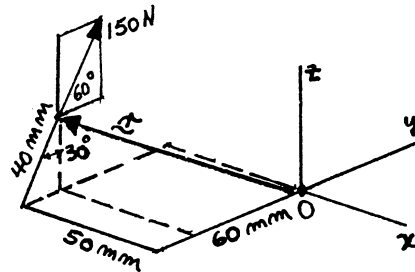
Check y -component:

$$70x + 50z = 70(3) + 50(4) = 410 \text{ lb}\cdot\text{ft} \quad \text{O.K.}$$

2.44

$$\begin{aligned} \mathbf{r} &= -50 \mathbf{i} - (60 - 40 \sin 30^\circ) \mathbf{j} + 40 \cos 30^\circ \mathbf{k} \\ &= -50.00 \mathbf{i} - 40.00 \mathbf{j} + 34.64 \mathbf{k} \text{ mm} \end{aligned}$$

$$\begin{aligned} \mathbf{F} &= -150 \cos 60^\circ \mathbf{j} + 150 \sin 60^\circ \mathbf{k} \\ &= -75.00 \mathbf{j} + 129.90 \mathbf{k} \text{ N} \end{aligned}$$



$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50.00 & -40.00 & 34.64 \\ 0 & -75.00 & 129.90 \end{vmatrix} \left(10^{-3}\right) = -2.598 \mathbf{i} + 6.495 \mathbf{j} - 3.750 \mathbf{k} \text{ N}\cdot\text{m} \quad \blacklozenge$$

$$M_O = \sqrt{(-2.598)^2 + (6.495)^2 + (-3.750)^2} = 7.937 \text{ N}\cdot\text{m} \quad \blacklozenge$$

$$\cos\theta_x = -\frac{2.598}{7.937} = -0.3273; \quad \cos\theta_y = \frac{6.495}{7.937} = 0.8183; \quad \cos\theta_z = -\frac{3.750}{7.937} = -0.4725 \quad \blacklozenge$$

2.45

$$\mathbf{P}_1 = \frac{P}{\sqrt{2}}(\mathbf{j} - \mathbf{k}) \quad \mathbf{r}_1 = -d\mathbf{i} \quad \mathbf{P}_2 = \frac{P}{\sqrt{3}}(\mathbf{i} + \mathbf{j} - \mathbf{k}) \quad \mathbf{r}_2 = (a - d)\mathbf{i}$$

$$\mathbf{M}_A = \mathbf{r}_1 \times \mathbf{P}_1 + \mathbf{r}_2 \times \mathbf{P}_2 = \frac{P}{\sqrt{2}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -d & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix} + \frac{P}{\sqrt{3}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ (a-d) & 0 & 0 \\ 1 & 1 & -1 \end{vmatrix} = \mathbf{0}$$

Canceling P and expanding the determinants gives: $\frac{d}{\sqrt{2}}(-\mathbf{j} - \mathbf{k}) + \frac{a-d}{\sqrt{3}}(\mathbf{j} + \mathbf{k}) = \mathbf{0}$

Equating either the \mathbf{j} -components or the \mathbf{k} -components yields: $\frac{d}{\sqrt{2}} = \frac{a-d}{\sqrt{3}}$

from which we find: $d = \frac{a\sqrt{2}}{\sqrt{2} + \sqrt{3}} = 0.449a \quad \blacklozenge$

2.46

$$\begin{aligned} \mathbf{M}_B &= \mathbf{r}_{BA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 4-y & 2-z \\ -20 & 4 & 6 \end{vmatrix} \\ &= (-6y + 4z + 16)\mathbf{i} - (58 - 20z)\mathbf{j} + (92 - 20y)\mathbf{k} = \mathbf{0} \end{aligned}$$

Setting y - and z -components to zero:

$$58 - 20z = 0 \quad z = 2.9 \text{ ft} \quad \blacktriangleleft$$

$$92 - 20y = 0 \quad y = 4.6 \text{ ft} \quad \blacktriangleleft$$

Check x -component:

$$-6y + 4z + 16 = -6(4.6) + 4(2.9) + 16 = 0 \quad \text{O.K.}$$

2.47

(a)




$$\begin{aligned} M_x &= -75(0.85) = -63.75 \text{ kN} \cdot \text{m} \quad \blacktriangleleft \\ M_y &= 75(0.5) = 37.5 \text{ kN} \cdot \text{m} \quad \blacktriangleleft \\ M_z &= 160(0.5) - 90(0.85) = 3.5 \text{ kN} \cdot \text{m} \quad \blacktriangleleft \end{aligned}$$

(b)

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.5 & 0.85 & 0 \\ 90 & 160 & -75 \end{vmatrix} = -63.75\mathbf{i} + 37.5\mathbf{j} + 3.5\mathbf{k} \text{ kN} \cdot \text{m}$$

The components of \mathbf{M}_O agree with those computed in part (a).

2.48

- (a) moment arm is BF $M_{AB} = 40(0.9) = 36 \text{ kN} \cdot \text{m}$  ♦
- (b) moment arm is DH $M_{CD} = 40(0.9) = 36 \text{ kN} \cdot \text{m}$  ♦
- (c) moment arm is GH $M_{CG} = 40(0.8) = 32 \text{ kN} \cdot \text{m}$  ♦
- (d) moment arm is zero $M_{CH} = 0$ ♦
- (e) force is parallel to EG $M_{EG} = 0$ ♦

2.49

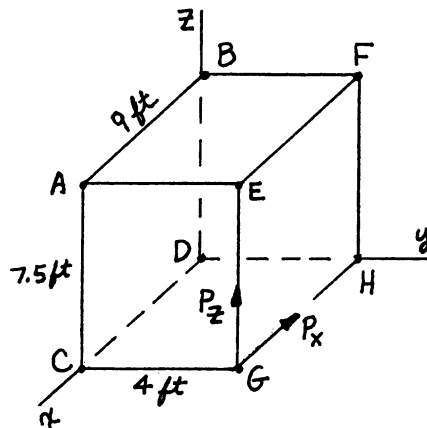
$$\overline{FG} = \sqrt{9^2 + 7.5^2} = 11.715 \text{ ft}$$

$$P_x = 400 \left(\frac{9}{11.715} \right) = 307.3 \text{ lb}$$

$$P_z = 400 \left(\frac{7.5}{11.715} \right) = 256.1 \text{ lb}$$

(a) $M_{AB} = P_z(\overline{AE})\mathbf{i} = 256.1(4)\mathbf{i}$
 $= 1024\mathbf{i} \text{ lb} \cdot \text{ft} \quad \blacklozenge$

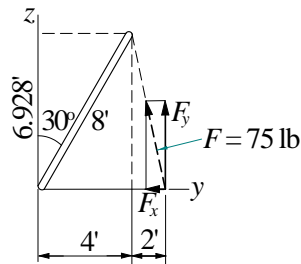
(b) $M_{CD} = P_z(\overline{CG})\mathbf{i} = 256.1(4)\mathbf{i}$
 $= 1024\mathbf{i} \text{ lb} \cdot \text{ft} \quad \blacklozenge$



- (c) $M_{BF} = 0$ (because the force passes through F) ♦
- (d) $M_{DH} = -P_z(\overline{GH})\mathbf{j} = -256.1(9)\mathbf{j} = -2305\mathbf{j} \text{ lb}\cdot\text{ft}$ ♦
- (e) $M_{BD} = P_x(\overline{DH})\mathbf{k} = 307.3(4)\mathbf{k} = 1229\mathbf{k} \text{ lb}\cdot\text{ft}$ ♦

2.50

(a)



Only F_y has a moment about x -axis (since F_x intersects x -axis, it has no moment about that axis).

$$F_y = 75 \frac{6.928}{\sqrt{6.928^2 + 2^2}} = 72.06 \text{ lb}$$

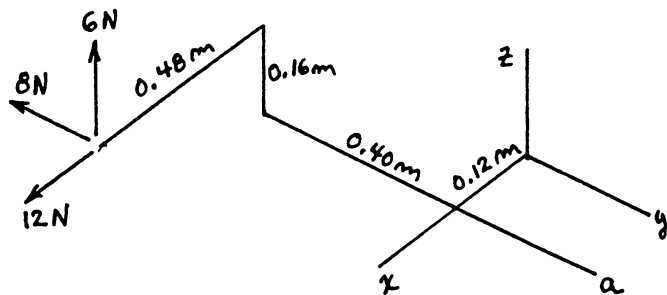
$$+ \circlearrowleft M_x = 6F_y = 6(72.06) = 432 \text{ lb}\cdot\text{ft} \quad \blacktriangleleft$$

(b)

$$\mathbf{F} = 75 \frac{-2\mathbf{i} + 6.928\mathbf{k}}{\sqrt{6.928^2 + 2^2}} = -20.80\mathbf{j} + 72.06\mathbf{k} \text{ lb} \quad \mathbf{r} = 6\mathbf{j} \text{ ft}$$

$$M_x = \mathbf{r} \times \mathbf{F} \cdot \boldsymbol{\lambda} = \begin{vmatrix} 0 & 6 & 0 \\ 0 & -20.80 & 72.06 \\ 1 & 0 & 0 \end{vmatrix} = 432 \text{ lb}\cdot\text{ft} \quad \blacktriangleleft$$

2.51



$$(a) \mathbf{M}_a = [-6(0.480) + 12(0.160)]\mathbf{j} = -0.960\mathbf{j} \text{ N}\cdot\text{m} \blacklozenge$$

$$(b) \mathbf{M}_z = [-8(0.480 + 0.120) + 12(0.4)]\mathbf{k} = \mathbf{0} \blacklozenge$$

2.52

$$M_x = 1080 = F_z(\overline{AC})$$

$$\therefore F_z = 1080/12 = 90 \text{ N}$$

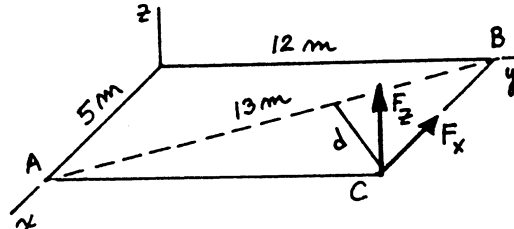
$$M_{AB} = F_z d$$

A convenient way to compute d:

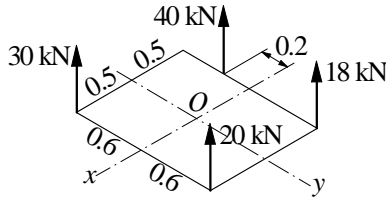
$$\text{area } ABC = \frac{1}{2}(12)(5) = \frac{1}{2}(13)d$$

which gives $d = 4.615 \text{ m}$

$$\therefore M_{AB} = 90(4.615) = 415 \text{ N}\cdot\text{m} \blacklozenge$$



2.53



$$M_x = (20 + 18)(0.6) - 40(0.2) - 30(0.6) = -3.20 \text{ N}\cdot\text{m}$$

$$M_y = (40 + 18)(0.5) - (30 + 20)(0.5) = 4.00 \text{ N}\cdot\text{m}$$

$$\mathbf{M}_O = -3.20\mathbf{i} + 4.00\mathbf{j} \text{ N}\cdot\text{m} \blacktriangleleft$$

2.54

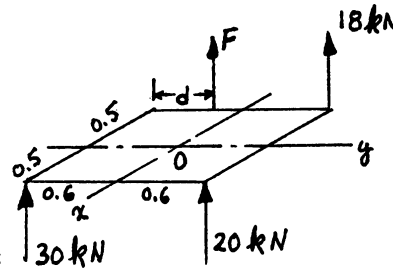
$$M_y = (F + 18)(0.5) - (30 + 20)(0.5) = 0$$

$$\therefore F = \frac{25 - 9}{0.5} = 32.0 \text{ N} \blacklozenge$$

$$M_x = (20 + 18)(0.6) - 30(0.6) - F(0.6 - d) = 0$$

Substituting $F = 32.0 \text{ N}$, and solving for d gives:

$$\therefore d = \frac{-22.8 + 18 + 32.0(0.6)}{32.0} = 0.450 \text{ m} \blacklozenge$$



dimensions in meters

2.55

$$\mathbf{M}_{aa} = 30(4 - y_0) + 20(6 - y_0) - 40y_0 = 0 \quad \text{Solving gives: } y_0 = 2.67 \text{ ft } \blacklozenge$$

$$\mathbf{M}_{bb} = (20 + 40)x_0 - 30(6 - x_0) = 0 \quad \text{Solving gives: } x_0 = 2.00 \text{ ft } \blacklozenge$$

2.56

Only the component T_z has a moment about the y -axis: $M_y = -4T_z$.

$$T_z = T \frac{\overline{AB}_z}{AB} = 40 \frac{3}{\sqrt{4^2 + 4^2 + 3^2}} = 18.741 \text{ lb}$$

$$\therefore M_y = -4(18.741) = -75.0 \text{ lb} \cdot \text{ft} \quad \blacktriangleleft$$

2.57

Only the x -component of each force has a moment about the z -axis.

$$\begin{aligned} \therefore M_z &= (P \cos 30^\circ + Q \cos 25^\circ) 15 \\ &= (32 \cos 30^\circ + 36 \cos 25^\circ) 15 = 905 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft \end{aligned}$$

2.58

$$\mathbf{P} = 480 \frac{-0.42\mathbf{i} - 0.81\mathbf{j} + 0.54\mathbf{k}}{\sqrt{(-0.42)^2 + (-0.81)^2 + 0.54^2}} = -190.15\mathbf{i} - 366.7\mathbf{j} + 244.5\mathbf{k} \text{ N}$$

$$\mathbf{r}_{CA} = 0.42\mathbf{i} \text{ m} \quad \boldsymbol{\lambda}_{CD} = \frac{0.42\mathbf{i} + 0.54\mathbf{k}}{\sqrt{0.42^2 + 0.54^2}} = 0.6139\mathbf{i} + 0.7894\mathbf{k}$$

$$M_{CD} = \mathbf{r}_{CA} \times \mathbf{P} \cdot \boldsymbol{\lambda}_{CD} = \begin{vmatrix} 0.42 & 0 & 0 \\ -190.15 & -366.7 & 244.5 \\ 0.6139 & 0 & 0.7894 \end{vmatrix} = -121.58 \text{ N} \cdot \text{m}$$

$$\mathbf{M}_{CD} = M_{CD} \boldsymbol{\lambda}_{CD} = -121.58(0.6139\mathbf{i} + 0.7894\mathbf{k}) = -74.6\mathbf{i} - 96.0\mathbf{k} \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

2.59

Let the 20-lb force be \mathbf{Q} :

$$\mathbf{Q} = 20 \vec{\lambda}_{ED} = 20 \left(\frac{-12\mathbf{j} - 4\mathbf{k}}{12.649} \right) = -18.974\mathbf{j} - 6.324\mathbf{k} \text{ lb}$$

$$\mathbf{P} = P \vec{\lambda}_{AF} = P \left(\frac{-4\mathbf{i} + 4\mathbf{k}}{4\sqrt{2}} \right) = P(-0.7071\mathbf{i} + 0.7071\mathbf{k}) \text{ lb}$$

$$\mathbf{M}_{GB} = \mathbf{r}_{BE} \times \mathbf{Q} \cdot \vec{\lambda}_{GB} + \mathbf{r}_{BA} \times \mathbf{P} \cdot \vec{\lambda}_{GB} = 0$$

$$\mathbf{r}_{BE} = 4\mathbf{i} + 4\mathbf{k} \text{ in.} \quad \mathbf{r}_{BA} = 4\mathbf{i} \text{ in.} \quad \vec{\lambda}_{GB} = \frac{12\mathbf{j} - 4\mathbf{k}}{12.649}$$

$$\mathbf{M}_{GB} = \frac{1}{12.649} \begin{vmatrix} 4 & 0 & 4 \\ 0 & -18.974 & -6.324 \\ 0 & 12 & -4 \end{vmatrix} + \frac{P}{12.649} \begin{vmatrix} 4 & 0 & 0 \\ -0.7071 & 0 & 0.7071 \\ 0 & 12 & -4 \end{vmatrix} = 0$$

$$\text{Expanding the determinants gives: } \frac{607.1}{12.649} + \frac{P}{12.649}(-33.94) = 0 \quad \therefore P = 17.89 \text{ lb } \blacklozenge$$

2.60

$$M_{BC} = \mathbf{r}_{BA} \times \mathbf{F} \cdot \lambda_{BC}$$

$$\mathbf{r}_{BA} = 5\mathbf{i} \quad \mathbf{F} = F \frac{-3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}}{\sqrt{(-3)^2 + 3^2 + (-3)^2}} = 0.5774F(-\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$\lambda_{BC} = \frac{4\mathbf{j} - 2\mathbf{k}}{\sqrt{4^2 + (-2)^2}} = 0.8944\mathbf{j} - 0.4472\mathbf{k}$$

$$M_{BC} = \mathbf{r}_{BA} \times \mathbf{F} \cdot \lambda_{BC} = 0.5774F \begin{vmatrix} 5 & 0 & 0 \\ -1 & 1 & -1 \\ 0 & 0.8944 & -0.4472 \end{vmatrix} = 1.2911F$$

$$M_{BC} = 150 \text{ lb} \cdot \text{ft} \quad 1.2911F = 150 \text{ lb} \cdot \text{ft} \quad F = 116.2 \text{ lb } \blacktriangleleft$$

*2.61

Let $\vec{\lambda}$ be a unit vector perpendicular to the plane ABC:

$$\vec{\lambda} = \frac{\vec{AB} \times \vec{AC}}{|\vec{AB} \times \vec{AC}|} \quad \text{where } \vec{AB} = -3\mathbf{j} + 4\mathbf{k} \text{ m and } \vec{AC} = 3\mathbf{i} - 3\mathbf{j} \text{ m}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -3 & 4 \\ 3 & -3 & 0 \end{vmatrix} = 12\mathbf{i} + 12\mathbf{j} + 9\mathbf{k} \text{ m}^2 \quad \therefore \vec{\lambda} = \frac{12\mathbf{i} + 12\mathbf{j} + 9\mathbf{k}}{19.21}$$

$$M_{\lambda} = \mathbf{r}_{OA} \times \mathbf{F} \cdot \vec{\lambda} \quad \mathbf{r}_{OA} = 3\mathbf{j} \text{ m}$$

$$\mathbf{F} = 250 \vec{\lambda}_{AB} = 250 \left(\frac{-3\mathbf{j} + 4\mathbf{k}}{5} \right) = -150\mathbf{j} + 200\mathbf{k} \text{ N}$$

$$\therefore M_{\lambda} = \frac{1}{19.21} \begin{vmatrix} 0 & 3 & 0 \\ 0 & -150 & 200 \\ 12 & 12 & 9 \end{vmatrix} = 374.8 \text{ N}\cdot\text{m}$$

Written in vector form, we have:

$$\mathbf{M}_{\lambda} = M_{\lambda} \vec{\lambda} = 374.8 \left(\frac{12\mathbf{i} + 12\mathbf{j} + 9\mathbf{k}}{19.21} \right) = 234\mathbf{i} + 234\mathbf{j} + 176\mathbf{k} \text{ N}\cdot\text{m} \quad \blacklozenge$$

2.62

$$\mathbf{P} = 240 \vec{\lambda}_{CE} = 240 \left(\frac{-3\mathbf{i} + 2\mathbf{j} - 7\mathbf{k}}{\sqrt{62}} \right) \text{ lb} \quad \vec{\lambda}_{AD} = \frac{-3\mathbf{i} + 6\mathbf{j} + 7\mathbf{k}}{\sqrt{94}}$$

(a) $\mathbf{r} = \mathbf{r}_{AC} = 6\mathbf{j} + 7\mathbf{k} \text{ ft}$

$$M_{AD} = \mathbf{r}_{AC} \times \mathbf{P} \cdot \vec{\lambda}_{AD} = \frac{240}{\sqrt{62}\sqrt{94}} \begin{vmatrix} 0 & 6 & 7 \\ -3 & 2 & -7 \\ -3 & 6 & 7 \end{vmatrix} = \frac{240}{\sqrt{62}\sqrt{94}} (168) = 528 \text{ lb}\cdot\text{ft} \quad \blacklozenge$$

(b) $\mathbf{r} = \mathbf{r}_{DC} = 3\mathbf{i} \text{ ft}$

$$M_{AD} = \mathbf{r}_{DC} \times \mathbf{P} \cdot \vec{\lambda}_{AD} = \frac{240}{\sqrt{62}\sqrt{94}} \begin{vmatrix} 3 & 0 & 0 \\ -3 & 2 & -7 \\ -3 & 6 & 7 \end{vmatrix} = \frac{240}{\sqrt{62}\sqrt{94}} (168) = 528 \text{ lb}\cdot\text{ft} \quad \blacklozenge$$

2.63

(a)

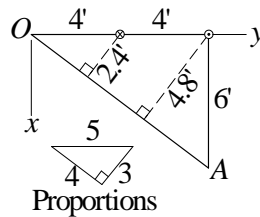
$$\mathbf{P} = -1200\mathbf{k} \text{ lb} \quad \mathbf{Q} = 800\mathbf{k} \text{ lb} \quad \mathbf{r}_{OP} = 4\mathbf{j} \text{ ft} \quad \mathbf{r}_{OQ} = 8\mathbf{j} \text{ ft}$$

$$\lambda_{OA} = \frac{6\mathbf{i} + 8\mathbf{j}}{\sqrt{6^2 + 8^2}} = 0.6\mathbf{i} + 0.8\mathbf{j}$$

$$M_{OA} = \mathbf{r}_{OP} \times \mathbf{P} \cdot \lambda_{OA} + \mathbf{r}_{OQ} \times \mathbf{Q} \cdot \lambda_{OA}$$

$$= \begin{vmatrix} 0 & 4 & 0 \\ 0 & 0 & -1200 \\ 0.6 & 0.8 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 8 & 0 \\ 0 & 0 & 800 \\ 0.6 & 0.8 & 0 \end{vmatrix} = 960 \text{ lb} \cdot \text{ft} \quad \blacktriangleleft$$

(b)



$$M_{OA} = -2.4P + 4.8Q = -2.4(1200) + 4.8(800) = 960 \text{ lb} \cdot \text{ft} \quad \blacktriangleleft$$

2.64

$$\mathbf{M}_{BC} = \mathbf{M}_B \cdot \vec{\lambda}_{BC} = \mathbf{r}_{BD} \times \mathbf{F} \cdot \vec{\lambda}_{BC} = 0 \quad \mathbf{r}_{BD} = -1.6\mathbf{j} - (1.2 - z_D)\mathbf{k} \text{ m}$$

$$\mathbf{F} = F(0.6\mathbf{i} + 0.8\mathbf{j}) \quad \vec{\lambda}_{BC} = \frac{\vec{BC}}{|\vec{BC}|} = \frac{1.2\mathbf{i} - 0.6\mathbf{j} - 1.2\mathbf{k}}{1.8}$$

$$\therefore M_{BC} = \frac{F}{1.8} \begin{vmatrix} 0 & -1.6 & -(1.2 - z_D) \\ 0.6 & 0.8 & 0 \\ 1.2 & -0.6 & -1.2 \end{vmatrix} = 0$$

Expanding the determinant: $1.6(0.6)(-1.2) - (1.2 - z_D)(-0.36 - 0.96) = 0$

which gives: $z_D = 0.327 \text{ m} \quad \blacklozenge$

2.65

$$\vec{\lambda}_{AB} = \frac{-3\mathbf{i} + 4\mathbf{j}}{5} = -0.600\mathbf{i} + 0.800\mathbf{j}$$

For the pulley at A:

$$M_A = M_x = 20(0.5) - 60(0.5) = -20 \text{ kN}\cdot\text{m} \quad \therefore M_A = -20\mathbf{i} \text{ kN}\cdot\text{m}$$

For the pulley at B:

$$M_B = M_y = 40(0.8) - 20(0.8) = 16 \text{ kN}\cdot\text{m} \quad \therefore M_B = 16\mathbf{j} \text{ kN}\cdot\text{m}$$

For both pulleys combined:

$$\begin{aligned} M_{AB} &= (M_A + M_B) \cdot \vec{\lambda}_{AB} = (-20\mathbf{i} + 16\mathbf{j}) \cdot (-0.600\mathbf{i} + 0.800\mathbf{j}) \\ &= 12 + 12.8 = 24.8 \text{ kN}\cdot\text{m} \quad \blacklozenge \end{aligned}$$

2.66

From the figure at the right:

$$x_C = 30 \sin 30^\circ = 15.000 \text{ in.}$$

$$y_C = 30 \cos 30^\circ - 24 = 1.981 \text{ in.}$$

$$x_D = 18 \sin 30^\circ = 9.000 \text{ in.}$$

$$y_D = 24 - 18 \cos 30^\circ = 8.412 \text{ in.}$$

$$(M_B)_x = r_{BC} \times P_C \cdot \mathbf{i} + r_{BD} \times P_D \cdot \mathbf{i}$$

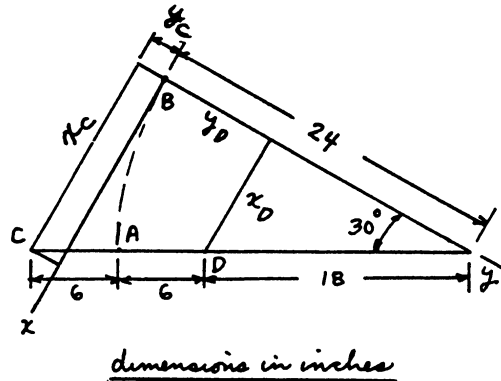
$$P_C = 20 \text{ k lb} \quad P_D = -20 \text{ k lb}$$

$$r_{BC} = x_C \mathbf{i} - y_C \mathbf{j} = 15.000 \mathbf{i} - 1.981 \mathbf{j} \text{ in.}$$

$$r_{BD} = x_D \mathbf{i} + y_D \mathbf{j} = 9.000 \mathbf{i} + 8.412 \mathbf{j} \text{ in.}$$

$$\therefore (M_B)_x = \begin{vmatrix} 15.000 & -1.981 & 0 \\ 0 & 0 & 20 \\ 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 9.000 & 8.412 & 0 \\ 0 & 0 & -20 \\ 1 & 0 & 0 \end{vmatrix} = -39.62 - 168.2 = -208 \text{ lb}\cdot\text{in}$$

Written in vector form: $(M_B)_x = (M_B)_x \mathbf{i} = -208 \mathbf{i} \text{ lb}\cdot\text{in} \quad \blacklozenge$



2.67

(a)

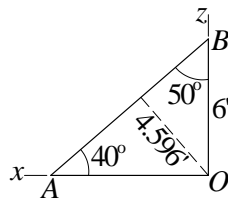
$$\mathbf{F} = 120 \frac{4\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}}{\sqrt{4^2 + 8^2 + 10^2}} = 35.78\mathbf{i} + 71.55\mathbf{j} + 89.44\mathbf{k} \text{ lb}$$

$$\mathbf{r}_{BO} = -6\mathbf{k} \text{ ft} \quad \boldsymbol{\lambda}_{AB} = \frac{(-6 \cot 40^\circ)\mathbf{i} + 6\mathbf{k}}{\sqrt{(-6 \cot 40^\circ)^2 + 6^2}} = -0.7660\mathbf{i} + 0.6428\mathbf{k}$$

$$: 71.5542\mathbf{j} + 89.4427\mathbf{k} + 0.0 + 35.7771\mathbf{i}$$

$$M_{AB} = \mathbf{r}_{BO} \times \mathbf{F} \cdot \boldsymbol{\lambda}_{AB} = \begin{vmatrix} 0 & 0 & -6 \\ 35.78 & 71.55 & 89.44 \\ -0.7660 & 0 & 0.6428 \end{vmatrix} = -329 \text{ lb} \cdot \text{ft} \quad \blacktriangleleft$$

(b)



Note that only $F_y = 71.55$ lb has a moment about AB . From trigonometry, the moment arm is $d = 6 \sin 50^\circ = 4.596$ ft.

$$\therefore M_{AB} = -F_y d = -71.55(4.596) = -329 \text{ lb} \cdot \text{ft} \quad \blacktriangleleft$$

2.68

Assume counterclockwise couples are positive.

(a) $C = -10(0.6) = -6 \text{ N}\cdot\text{m}$

(f) $C = -5(0.6) - 7.5(0.4) = -6 \text{ N}\cdot\text{m}$

(b) $C = -6 \text{ N}\cdot\text{m}$

(g) $C = -22.5(0.4) + 5(0.6) = -6 \text{ N}\cdot\text{m}$

(c) $C = -15(0.4) = -6 \text{ N}\cdot\text{m}$

(h) $C = -5 + 5(0.3) = -3.5 \text{ N}\cdot\text{m}$

(d) $C = -6 \text{ N}\cdot\text{m}$

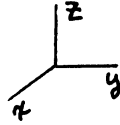
(i) $C = 3 - 4 - 6 + 3 = -4 \text{ N}\cdot\text{m}$

(e) $C = 9 - 3 = 6 \text{ N}\cdot\text{m}$

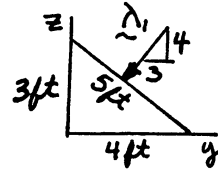
2.69

(a) $\mathbf{C} = -60(5)\mathbf{k} = -300\mathbf{k} \text{ lb}\cdot\text{ft}$

(b) $\mathbf{C} = -75(4)\mathbf{k} = -300\mathbf{k} \text{ lb}\cdot\text{ft}$



(c) $\mathbf{C}_1 = 75(5)\vec{\lambda}_1 = 375\left(-\frac{3}{5}\mathbf{j} - \frac{4}{5}\mathbf{k}\right) = -225\mathbf{j} - 300\mathbf{k} \text{ lb}\cdot\text{ft}$



(d) $\mathbf{C} = 100(3)\mathbf{i} = 300\mathbf{i} \text{ lb}\cdot\text{ft}$

(e) 75-lb forces: $\mathbf{C}_1 = -225\mathbf{j} - 300\mathbf{k} \text{ lb}\cdot\text{ft}$ [as in (c)]

45-lb forces: $\mathbf{C}_2 = 45(5)\mathbf{j} = 225\mathbf{j} \text{ lb}\cdot\text{ft}$

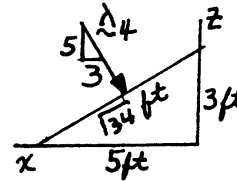
$\mathbf{C}_1 + \mathbf{C}_2 = -300\mathbf{k} \text{ lb}\cdot\text{ft}$

(f) 45-lb forces: $\mathbf{C}_3 = 45(4)\mathbf{i} = 180\mathbf{i} \text{ lb}\cdot\text{ft}$

50-lb forces: $\mathbf{C}_4 = 50(\sqrt{34})\vec{\lambda}_4$

$= 50(\sqrt{34})\left(\frac{-3\mathbf{i} - 5\mathbf{k}}{\sqrt{34}}\right) = -150\mathbf{i} - 250\mathbf{k} \text{ lb}\cdot\text{ft}$

$\mathbf{C}_3 + \mathbf{C}_4 = 30\mathbf{i} - 250\mathbf{k} \text{ lb}\cdot\text{ft}$

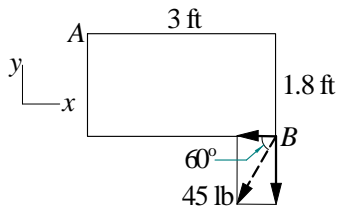


Comparing the above results: (b) and (e) are equivalent to (a). ♦

2.70

$C = (75 \sin 45^\circ)(1.5) - 60 = 19.55 \text{ lb}\cdot\text{ft}$ ◯ ◀

2.71



Moment of a couple is the same about any point. Choosing A as the moment center, we get

+ ◯ $C = M_A = (45 \cos 60^\circ)(1.8) + (45 \sin 60^\circ)(3) = 157.4 \text{ lb}\cdot\text{ft}$

∴ $\mathbf{C} = -157.4\mathbf{k} \text{ lb}\cdot\text{ft}$ ◀

2.72

$$C = -30(5)\mathbf{i} + 60(2)\mathbf{k} = -150\mathbf{i} + 120\mathbf{k} \text{ lb} \cdot \text{ft}$$

$$\therefore C = \sqrt{(-150)^2 + 120^2} = 192.1 \text{ lb} \cdot \text{ft} \quad \blacktriangleleft$$

2.73

$$C = 80 \vec{\lambda}_{DB} = 80 \left(\frac{0.4\mathbf{i} - 0.3\mathbf{j} + 0.4\mathbf{k}}{0.6403} \right) = 49.98\mathbf{i} - 37.48\mathbf{j} + 49.98\mathbf{k} \text{ N} \cdot \text{m}$$

$$\mathbf{P} = -400\mathbf{k} \text{ N} \quad \mathbf{r}_{AD} = -0.4\mathbf{i} \text{ m} \quad \vec{\lambda}_{AB} = \frac{-0.3\mathbf{j} + 0.4\mathbf{k}}{0.5} = -0.6\mathbf{j} + 0.8\mathbf{k}$$

For Couple C:

$$\mathbf{M}_{AB} = C \cdot \vec{\lambda}_{AB} = (49.98\mathbf{i} - 37.48\mathbf{j} + 49.98\mathbf{k}) \cdot (-0.6\mathbf{j} + 0.8\mathbf{k})$$

$$= (-37.48)(-0.6) + (49.98)(0.8) = 62.47 \text{ N} \cdot \text{m}$$

For Force P:

$$\mathbf{M}_{AB} = \mathbf{r}_{AD} \times \mathbf{P} \cdot \vec{\lambda}_{AB} = \begin{vmatrix} -0.4 & 0 & 0 \\ 0 & 0 & -400 \\ 0 & -0.6 & 0.8 \end{vmatrix} = -0.4(-240) = 96.00 \text{ N} \cdot \text{m}$$

$$\therefore \Sigma \mathbf{M}_{AB} = 62.47 + 96.00 = 158.5 \text{ N} \cdot \text{m} \quad \blacklozenge$$

*2.74

$$\mathbf{C}_1 = -200\mathbf{i} \text{ lb} \cdot \text{in.} \quad \mathbf{C}_2 = 140\mathbf{k} \text{ lb} \cdot \text{in.}$$

Identify the three points at the corners of the triangle:

$$A(9 \text{ in.}, 3 \text{ in.}, 6 \text{ in.}); B(3 \text{ in.}, 7 \text{ in.}, 6 \text{ in.}); C(9 \text{ in.}, 7 \text{ in.}, 2 \text{ in.})$$

$\mathbf{C}_3 = 220 \vec{\lambda} \text{ lb} \cdot \text{in.}$ where $\vec{\lambda}$ is the unit vector that is perpendicular to triangle ABC, with its sense consistent with the sense of \mathbf{C}_3 .

$$\vec{\lambda} = \frac{\vec{AC} \times \vec{AB}}{|\vec{AC} \times \vec{AB}|} \quad \text{where } \vec{AC} = 4\mathbf{j} - 4\mathbf{k} \text{ in. and } \vec{AB} = -6\mathbf{i} + 4\mathbf{j} \text{ in.}$$

$$\vec{AC} \times \vec{AB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -4 \\ -6 & 4 & 0 \end{vmatrix} = 16\mathbf{i} + 24\mathbf{j} + 24\mathbf{k} \text{ in.}^2$$

$$\therefore \vec{\lambda} = \frac{16\mathbf{i} + 24\mathbf{j} + 24\mathbf{k}}{37.52} = 0.4264\mathbf{i} + 0.6397\mathbf{j} + 0.6397\mathbf{k}$$

$$\mathbf{C}_3 = 220(0.4264\mathbf{i} + 0.6397\mathbf{j} + 0.6397\mathbf{k}) = 93.81\mathbf{i} + 140.73\mathbf{j} + 140.73\mathbf{k} \text{ lb}\cdot\text{in.}$$

$$\begin{aligned} \therefore \mathbf{C}^R &= \mathbf{C}_1 + \mathbf{C}_2 + \mathbf{C}_3 = -200\mathbf{i} + 140\mathbf{k} + (93.81\mathbf{i} + 140.73\mathbf{j} + 140.73\mathbf{k}) \\ &= -106.2\mathbf{i} + 140.7\mathbf{j} + 280.7\mathbf{k} \text{ lb}\cdot\text{in.} \quad \blacklozenge \end{aligned}$$

2.75

Moment of a couple is the same about any point. Choosing B as the moment center, we have

$$\mathbf{F} = -24\mathbf{i} \text{ kN} \quad \mathbf{r}_{BA} = -1.8\mathbf{j} - 1.2\mathbf{k} \text{ m}$$

$$\mathbf{C} = \mathbf{M}_B = \mathbf{r}_{BA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1.8 & -1.2 \\ -24 & 0 & 0 \end{vmatrix} = 28.8\mathbf{j} - 43.2\mathbf{k} \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

2.76

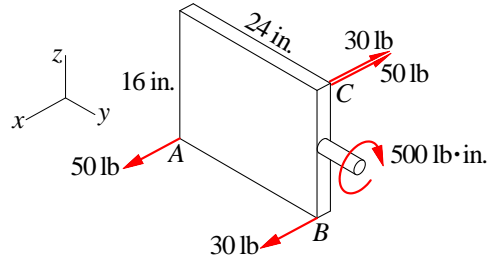
Moment of a couple is the same about any point. Choosing B as the moment center, we have

$$\mathbf{r}_{BA} = 180\mathbf{i} - b\mathbf{j} \text{ mm}$$

$$C_z = (M_B)_z = \mathbf{r}_{BA} \times \mathbf{F} \cdot \mathbf{k} = \begin{vmatrix} 180 & -b & 0 \\ 200 & -110 & -80 \\ 0 & 0 & 1 \end{vmatrix} = 200b - 19\,800 \text{ kN}\cdot\text{mm}$$

$$\therefore 200b - 19\,800 = 0 \quad b = 99.0 \text{ mm} \quad \blacktriangleleft$$

2.77



As seen in the figure, the forces form two couples (30-lb pair and 50-lb pair).

Couple of the 30-lb forces:

$$\mathbf{C}_1 = -30(16)\mathbf{j} = -480\mathbf{j} \text{ lb} \cdot \text{in.}$$

Couple of the 50-lb forces:

$$\mathbf{C}_2 = \mathbf{r}_{AC} \times (-50\mathbf{i}) = (24\mathbf{j} + 16\mathbf{k}) \times (-50\mathbf{i}) = 1200\mathbf{k} - 800\mathbf{j} \text{ lb} \cdot \text{in.}$$

Applied couple:

$$\mathbf{C}_3 = -500\mathbf{j} \text{ lb} \cdot \text{in.}$$

Resultant:

$$\mathbf{C}^R = \Sigma \mathbf{C}_i = -480\mathbf{j} + (1200\mathbf{k} - 800\mathbf{j}) - 500\mathbf{j} = -1780\mathbf{j} + 1200\mathbf{k} \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$

2.78

$$\mathbf{C} = -360 \cos 30^\circ \mathbf{i} - 360 \sin 30^\circ \mathbf{j} = -311.8\mathbf{i} - 180.0\mathbf{j} \text{ lb} \cdot \text{ft}$$

$$\vec{\lambda}_{CD} = -\cos 30^\circ \mathbf{i} - \sin 30^\circ \cos 40^\circ \mathbf{j} + \sin 30^\circ \sin 40^\circ \mathbf{k} = -0.8660\mathbf{i} - 0.3830\mathbf{j} + 0.3214\mathbf{k}$$

$$\therefore M_{CD} = \mathbf{C} \cdot \vec{\lambda}_{CD} = (-311.8)(-0.8660) + (-180.0)(-0.3830) = 339 \text{ lb} \cdot \text{ft} \quad \blacklozenge$$

2.79

$$\vec{\lambda}_{DC} = \sin 30^\circ \sin 40^\circ \mathbf{i} - \sin 30^\circ \cos 40^\circ \mathbf{j} + \cos 30^\circ \mathbf{k} = 0.3214\mathbf{i} - 0.3830\mathbf{j} + 0.8660\mathbf{k}$$

$$\text{(a) } \mathbf{C} = 52 \vec{\lambda}_{DC} = 16.71\mathbf{i} - 19.92\mathbf{j} + 45.03\mathbf{k} \text{ lb} \cdot \text{ft} \quad \blacklozenge$$

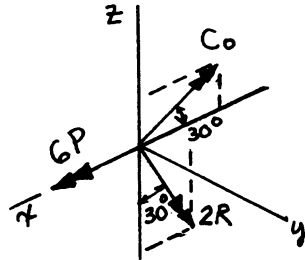
$$\text{(b) } \mathbf{M}_z = \mathbf{C}_z = 45.03\mathbf{k} \text{ lb} \cdot \text{ft} \quad \blacklozenge$$

2.80

$$\mathbf{C}_P = 600(6)\mathbf{i} = 3600\mathbf{i} \text{ lb}\cdot\text{in.}$$

$$\begin{aligned} \mathbf{C}_0 &= -C_0 \cos 30^\circ \mathbf{i} + C_0 \sin 30^\circ \mathbf{k} \\ &= -0.8660 C_0 \mathbf{i} + 0.5000 C_0 \mathbf{k} \text{ lb}\cdot\text{in.} \end{aligned}$$

$$\begin{aligned} \mathbf{C}_R &= -2R \sin 30^\circ \mathbf{i} - 2R \cos 30^\circ \mathbf{k} \\ &= -R \mathbf{i} - 1.7321R \mathbf{k} \text{ lb}\cdot\text{in.} \end{aligned}$$



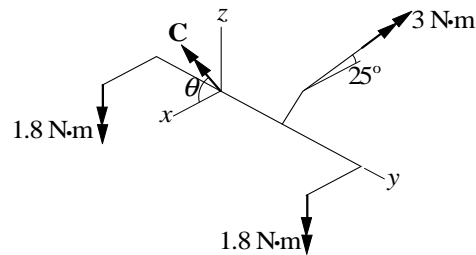
$$\therefore \Sigma \mathbf{C} = 3600\mathbf{i} + (-0.8660 C_0 \mathbf{i} + 0.5000 C_0 \mathbf{k}) + (-R \mathbf{i} - 1.7321R \mathbf{k}) = \mathbf{0}$$

Equating like components: (i) $3600 - 0.8660 C_0 - R = 0$

(k) $0.5000 C_0 - 1.7321R = 0$

Solving gives: $R = 900 \text{ lb}$ ♦ and $C_0 = 3120 \text{ lb}\cdot\text{in.}$ ♦

2.81



The system consists of the four couples shown, where

$$\mathbf{C} = 0.36F(\mathbf{i} \cos \theta + \mathbf{k} \sin \theta) \text{ N}\cdot\text{m}$$

$$\Sigma \mathbf{C} = -2(1.8)\mathbf{k} + 3(-\mathbf{i} \cos 25^\circ + \mathbf{k} \sin 25^\circ) + 0.36F(\mathbf{i} \cos \theta + \mathbf{k} \sin \theta) = \mathbf{0}$$

Equating like components:

$$\begin{aligned} -3 \cos 25^\circ + 0.36F \cos \theta &= 0 \\ -3.6 + 3 \sin 25^\circ + 0.36F \sin \theta &= 0 \end{aligned}$$

$$F \cos \theta = \frac{3 \cos 25^\circ}{0.36} = 7.553$$

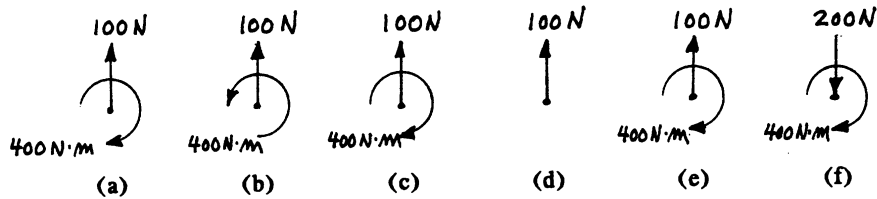
$$F \sin \theta = \frac{3.6 - 3 \sin 25^\circ}{0.36} = 6.478$$

$$\tan \theta = \frac{6.478}{7.553} = 0.8577 \quad \theta = 40.6^\circ \blacktriangleleft$$

$$F = \sqrt{7.553^2 + 6.478^2} = 9.95 \text{ N} \blacktriangleleft$$

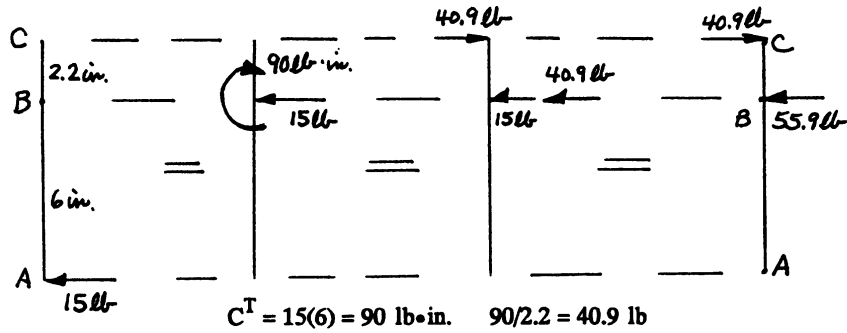
2.82

Represent each of the systems by an equivalent force-couple system with the force acting at the upper left corner of the figure.



By inspection, the systems in (c) and (e) are equivalent to the system in (a). ♦

2.83



Original system

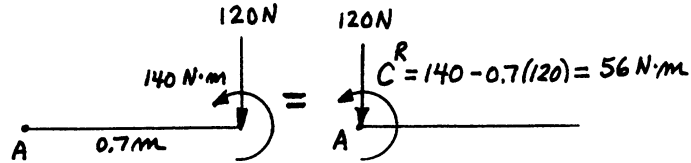
(i) Equivalent system with force at B.

(ii) Equivalent system: one force at B and one force at C.

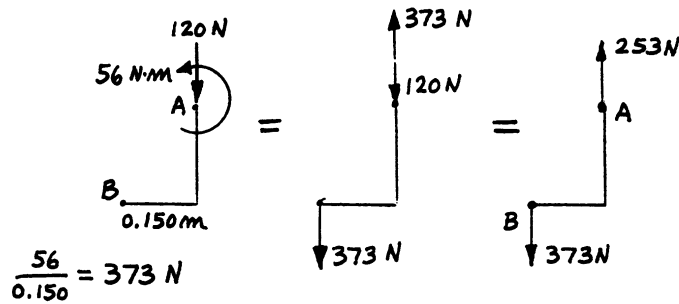
(a) Fig. (i): A 15-lb force acting to the left at B, and a 90 lb-in. clockwise couple. ♦

(b) Fig. (ii): A 55.9-lb force acting to the left at B, and a 40.9-lb force acting to the right at C. ♦

2.84



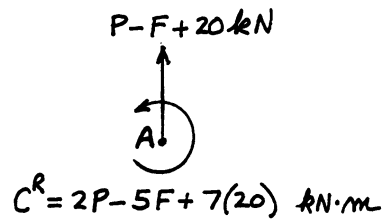
(a) From above figure: a 120-N downward force at A, and a 56 N·m CCW couple. ♦



(b) From above figure: a 253-N upward force at A, and a 373-N downward force at B. ♦

2.85

Moving the three forces to A:



Therefore,

$$P - F + 20 = 50 \text{ N} \quad (1)$$

$$2P - 5F + 140 = 170 \text{ kN}\cdot\text{m} \quad (2)$$

Solving (1) and (2) gives

$$P = 40 \text{ kN and } F = 10 \text{ kN} \quad \blacklozenge$$

2.86

$$\mathbf{R} = -90\mathbf{j} + 50(\mathbf{i} \sin 30^\circ - \mathbf{j} \cos 30^\circ) = 25.0\mathbf{i} - 133.3\mathbf{j} \text{ lb} \quad \blacktriangleleft$$

$$+ \circlearrowleft \quad C^R = 90(9) - 50(12) = 210 \text{ lb}\cdot\text{in.} \quad C^R = 210\mathbf{k} \text{ lb}\cdot\text{in.} \quad \blacktriangleleft$$

2.87

The resultant force R equals V .

$$\therefore V = R = 1200 \text{ lb} \quad \blacktriangleleft$$

$$C^R = \Sigma M_D = 0: \quad 20V - 10H - C = 0$$

$$20(1200) - 10H - 900 \times 12 = 0 \quad H = 1320 \text{ lb} \quad \blacktriangleleft$$

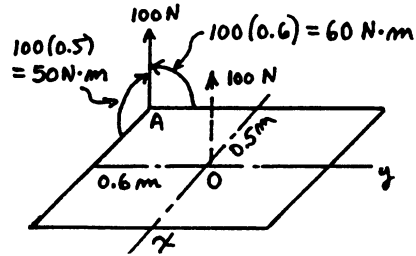
2.88

Transferring force from O to A gives

force = 100 k N ◊

$$(C^T)_x = 60 \text{ N}\cdot\text{m}; (C^T)_y = -50 \text{ N}\cdot\text{m}$$

$$\therefore C^R = 60 \mathbf{i} - 50 \mathbf{j} \text{ N}\cdot\text{m} \quad \blacklozenge$$



2.89

The force acting at O equals \mathbf{F} , and the couple equals the moment about O.

$$\mathbf{F} = 160 \lambda_{AB} \vec{\lambda}_{AB} = 160 \left(\frac{-2.2\mathbf{i} + 2.0\mathbf{j} - 2.0\mathbf{k}}{3.583} \right) = -98.24\mathbf{i} + 89.31\mathbf{j} - 89.31\mathbf{k} \text{ kN} \quad \blacklozenge$$

$$\mathbf{C}^R = \mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 0 \\ -98.24 & 89.31 & -89.31 \end{vmatrix} = -178.6\mathbf{i} + 196.5\mathbf{k} \text{ kN}\cdot\text{m} \quad \blacklozenge$$

2.90

$$40\text{-lb force: } \mathbf{P} = 40 \frac{-3\mathbf{i} - 2\mathbf{k}}{\sqrt{(-3)^2 + (-2)^2}} = -33.28\mathbf{i} - 22.19\mathbf{k} \text{ lb}$$

$$90\text{-lb}\cdot\text{ft couple: } \mathbf{C} = 90 \frac{-3\mathbf{i} - 5\mathbf{j}}{\sqrt{(-3)^2 + (-5)^2}} = -46.30\mathbf{i} - 77.17\mathbf{j} \text{ lb}\cdot\text{ft}$$

$$\mathbf{r}_{OA} = 3\mathbf{i} + 5\mathbf{j} \text{ ft}$$

$$\mathbf{R} = \mathbf{P} = -33.28\mathbf{i} - 22.19\mathbf{k} \text{ lb} \quad \blacktriangleleft$$

$$\begin{aligned} \mathbf{C}^R &= \mathbf{C} + \mathbf{r}_{OA} \times \mathbf{P} = -46.30\mathbf{i} - 77.17\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 5 & 0 \\ -33.28 & 0 & -22.19 \end{vmatrix} \\ &= -157.3\mathbf{i} - 10.6\mathbf{j} + 166.4\mathbf{k} \text{ lb}\cdot\text{ft} \quad \blacktriangleleft \end{aligned}$$

***2.91**

(a)

$$\begin{aligned}\mathbf{R} &= \mathbf{F} = -2800\mathbf{i} + 1600\mathbf{j} + 3000\mathbf{k} \text{ lb} \quad \blacktriangleleft \\ \mathbf{r}_{OA} &= 10\mathbf{i} + 5\mathbf{j} - 4\mathbf{k} \text{ in.} \\ \mathbf{C}^R &= \mathbf{r}_{OA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10 & 5 & -4 \\ -2800 & 1600 & 3000 \end{vmatrix} \\ &= 21\,400\mathbf{i} - 18\,800\mathbf{j} + 30\,000\mathbf{k} \text{ lb} \cdot \text{in.} \quad \blacktriangleleft\end{aligned}$$

(b)

$$\begin{aligned}\text{Normal component of } \mathbf{R} &: P = |R_y| = 1600 \text{ lb} \quad \blacktriangleleft \\ \text{Shear component of } \mathbf{R} &: V = \sqrt{R_x^2 + R_z^2} = \sqrt{(-2800)^2 + 3000^2} = 4100 \text{ lb} \quad \blacktriangleleft\end{aligned}$$

(c)

$$\begin{aligned}\text{Torque: } T &= |C_y^R| = 18\,800 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft \\ \text{Bending moment: } M &= \sqrt{(C_x^R)^2 + (C_z^R)^2} = \sqrt{21\,400^2 + 30\,000^2} \\ &= 36\,900 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft\end{aligned}$$

2.92

$$\begin{aligned}\vec{\lambda}_{DC} &= \sin 30^\circ \sin 40^\circ \mathbf{i} - \sin 30^\circ \cos 40^\circ \mathbf{j} + \cos 30^\circ \mathbf{k} \\ &= 0.3214\mathbf{i} - 0.3830\mathbf{j} + 0.8660\mathbf{k}\end{aligned}$$

The force at O equals the original force:

$$\mathbf{F} = 9.8 \vec{\lambda}_{DC} = 9.8(0.3214\mathbf{i} - 0.3830\mathbf{j} + 0.8660\mathbf{k}) = 3.150\mathbf{i} - 3.753\mathbf{j} + 8.487\mathbf{k} \text{ lb}$$

The given couple is:

$$\mathbf{C} = 52 \vec{\lambda}_{DC} = 52(0.3214\mathbf{i} - 0.3830\mathbf{j} + 0.8660\mathbf{k}) = 16.71\mathbf{i} - 19.92\mathbf{j} + 45.03\mathbf{k} \text{ lb} \cdot \text{ft}$$

Moving the force to O, and letting \mathbf{C}^R be the resultant couple, we have: $\mathbf{C}^R = \mathbf{C} + \mathbf{M}_O$

$$\begin{aligned}\mathbf{M}_O &= \mathbf{r}_{OD} \times \mathbf{F} & \mathbf{r}_{OD} &= -4.2 \sin 40^\circ \mathbf{i} + 4.2 \cos 40^\circ \mathbf{j} + 2.800\mathbf{k} \\ & & &= -2.700\mathbf{i} + 3.217\mathbf{j} + 2.800\mathbf{k} \text{ ft}\end{aligned}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2.700 & 3.217 & 2.800 \\ 3.150 & -3.753 & 8.487 \end{vmatrix} = 37.81\mathbf{i} + 31.73\mathbf{j} \text{ lb}\cdot\text{ft}$$

$$\begin{aligned} \therefore \mathbf{C}^R &= \mathbf{C} + \mathbf{M}_O = (16.71\mathbf{i} - 19.92\mathbf{j} + 45.03\mathbf{k}) + (37.81\mathbf{i} + 31.73\mathbf{j}) \\ &= 54.52\mathbf{i} + 11.81\mathbf{j} + 45.03\mathbf{k} \text{ lb}\cdot\text{ft} \end{aligned}$$

The equivalent force-couple system with the force acting at O is:

$$\text{Force: } 3.150\mathbf{i} - 3.753\mathbf{j} + 8.487\mathbf{k} \text{ lb; Couple: } 54.52\mathbf{i} + 11.81\mathbf{j} + 45.03\mathbf{k} \text{ lb}\cdot\text{ft} \blacklozenge$$

2.93

Original system:

$$\begin{aligned} \mathbf{F} &= 80 \frac{-1.8\mathbf{i} + 0.9\mathbf{k}}{\sqrt{(-1.8)^2 + 0.9^2}} = -71.55\mathbf{i} + 35.78\mathbf{k} \text{ N} \\ \mathbf{C} &= 250 \frac{-1.8\mathbf{i} + 1.3\mathbf{j}}{\sqrt{(-1.8)^2 + 1.3^2}} = -202.7\mathbf{i} + 146.37\mathbf{j} \text{ N}\cdot\text{m} \end{aligned}$$

(a) Since the 80-N force passes through B , it has no moment about B . Thus the equivalent force-couple system at B is

$$\begin{aligned} \mathbf{R} &= \mathbf{F} = -71.55\mathbf{i} + 35.78\mathbf{k} \text{ N} \blacktriangleleft \\ \mathbf{C}^R &= \mathbf{C} = -202.7\mathbf{i} + 146.37\mathbf{j} \text{ N}\cdot\text{m} \blacktriangleleft \end{aligned}$$

(b) The equivalent force-couple system at D is

$$\begin{aligned} \mathbf{R} &= \mathbf{F} = -71.55\mathbf{i} + 35.78\mathbf{k} \text{ N} \blacktriangleleft \\ \mathbf{C}^R &= \mathbf{C} + \mathbf{r}_{DA} \times \mathbf{F} = -202.7\mathbf{i} + 146.37\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.8 & -1.3 & 0 \\ -71.55 & 0 & 35.78 \end{vmatrix} \\ &= -249\mathbf{i} + 82.0\mathbf{j} - 93.0\mathbf{k} \text{ N}\cdot\text{m} \blacktriangleleft \end{aligned}$$

2.94

$$\mathbf{M}_{AB} = \mathbf{r}_{AO} \times \mathbf{P} \cdot \vec{\lambda}_{AB} = 600 \text{ lb}\cdot\text{ft}$$

$$\mathbf{r}_{AO} = -8\mathbf{j} \text{ ft} \quad \mathbf{P} = P \cos 20^\circ \mathbf{i} + P \sin 20^\circ \mathbf{k} \quad \vec{\lambda}_{AB} = -\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{k}$$

$$\therefore \mathbf{M}_{AB} = \begin{vmatrix} 0 & -8 & 0 \\ P \cos 20^\circ & 0 & P \sin 20^\circ \\ -\cos 30^\circ & 0 & \sin 30^\circ \end{vmatrix} = 8(P \cos 20^\circ \sin 30^\circ + P \sin 20^\circ \cos 30^\circ) = 600 \text{ lb}\cdot\text{ft}$$

$$\text{Solving for } P \text{ gives: } P = 97.9 \text{ lb} \blacklozenge$$

2.95

Given force and couple:

$$\mathbf{F} = 32 \frac{-3\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}}{\sqrt{(-3)^2 + (-4)^2 + 6^2}} = -12.292\mathbf{i} - 16.389\mathbf{j} + 24.58\mathbf{k} \text{ kN}$$

$$\mathbf{C} = 180 \frac{3\mathbf{i} - 4\mathbf{j}}{\sqrt{3^2 + (-4)^2}} = 108.0\mathbf{i} - 144.0\mathbf{j} \text{ kN} \cdot \text{m}$$

Equivalent force-couple system at A:

$$\mathbf{R} = \mathbf{F} = -12.29\mathbf{i} - 16.39\mathbf{j} + 24.6\mathbf{k} \text{ kN} \quad \blacktriangleleft$$

$$\mathbf{C}^R = \mathbf{C} + \mathbf{r}_{AB} \times \mathbf{F} = 108.0\mathbf{i} - 144.0\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 4 & 0 \\ -12.292 & -16.389 & 24.58 \end{vmatrix}$$

$$= 206\mathbf{i} - 70.3\mathbf{j} + 98.3\mathbf{k} \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

2.96

$$\mathbf{T}_1 = T_1 \vec{\lambda}_{AB} = T_1 \left(\frac{-10\mathbf{j} - 12\mathbf{k}}{\sqrt{244}} \right) = T_1 (-0.6402\mathbf{j} - 0.7682\mathbf{k})$$

$$\mathbf{T}_2 = T_2 \vec{\lambda}_{AC} = T_2 \left(\frac{6\mathbf{i} - 12\mathbf{k}}{\sqrt{180}} \right) = T_2 (0.4472\mathbf{i} - 0.8944\mathbf{k})$$

$$\mathbf{T}_3 = T_3 \vec{\lambda}_{AD} = T_3 \left(\frac{-4\mathbf{i} + 3\mathbf{j} - 12\mathbf{k}}{13} \right) = T_3 (-0.3077\mathbf{i} + 0.2308\mathbf{j} - 0.9231\mathbf{k})$$

$\mathbf{R} = \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3$ Equating like components gives:

$$\text{(i-components)} \quad 0.4472T_2 - 0.3077T_3 = 0$$

$$\text{(j-components)} \quad -0.6402T_1 \quad 0.2308T_3 = 0$$

$$\text{(k-components)} \quad -0.7682T_1 - 0.8944T_2 - 0.9231T_3 = -400$$

Solving yields: $T_1 = 79.4 \text{ N}$; $T_2 = 151.6 \text{ N}$; $T_3 = 220.3 \text{ N}$ \blacklozenge

2.97

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b & 0.25 & 0.3 \\ 10 & 20 & -5 \end{vmatrix}$$

$$= -7.25\mathbf{i} + (3 + 5b)\mathbf{j} + (-2.5 + 20b)\mathbf{k} \text{ kN} \cdot \text{m}$$

$$M_y = 3 + 5b = 8 \quad \therefore b = 1.0 \text{ m} \quad \blacktriangleleft$$

$$\mathbf{M}_O = -7.25\mathbf{i} + 8\mathbf{j} + 17.5\mathbf{k} \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

2.98

$$\mathbf{M}_{CD} = \mathbf{r}_{CA} \times \mathbf{P} \cdot \vec{\lambda}_{CD} = 50 \text{ lb}\cdot\text{in.}$$

$$\mathbf{r}_{CA} = 6\mathbf{i} - 2\mathbf{j} \text{ in.} \quad \mathbf{P} = P \vec{\lambda}_{AB} = P \left(\frac{-3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}}{\sqrt{38}} \right) \text{ lb} \quad \vec{\lambda}_{CD} = \frac{-4\mathbf{j} + 5\mathbf{k}}{\sqrt{41}}$$

Using the determinant form of the scalar triple product:

$$\mathbf{M}_{CD} = \frac{P}{\sqrt{38}\sqrt{41}} \begin{vmatrix} 6 & -2 & 0 \\ -3 & -2 & 5 \\ 0 & -4 & 5 \end{vmatrix} = \frac{P}{\sqrt{38}\sqrt{41}} [6(-10 + 20) + 2(-15)] = 50 \text{ lb}\cdot\text{in.}$$

$$\text{Solving for } P \text{ gives: } P = \frac{50\sqrt{38}\sqrt{41}}{30} = 65.8 \text{ lb} \quad \blacklozenge$$

2.99

The resultant couple in Fig. (a) is

$$+ \odot \quad C^R = 400 + 120 - 160(2) = 200 \text{ N}\cdot\text{m}$$

The couple in Fig. (b) must equal C^R :

$$(2 - b)F_y = C^R \quad (2 - b)120 = 200 \quad b = 0.333 \text{ m} \quad \blacktriangleleft$$

2.100

$$\text{(a) } \mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{P} + \mathbf{C}$$

$$\mathbf{r}_{OA} = 4\mathbf{k} \text{ ft} \quad \mathbf{P} = P \vec{\lambda}_{AB} = 500 \left(\frac{3\mathbf{i} - 4\mathbf{k}}{5} \right) = 300\mathbf{i} - 400\mathbf{k} \text{ lb} \quad \mathbf{C} = 1200\mathbf{k} \text{ lb}\cdot\text{ft}$$

$$\therefore \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 300 & 0 & -400 \end{vmatrix} + 1200\mathbf{k} = 1200\mathbf{j} + 1200\mathbf{k} \text{ lb}\cdot\text{ft} \quad \blacklozenge$$

$$\text{The magnitude of the moment about O is: } M_O = \sqrt{1200^2 + 1200^2} = 1697 \text{ lb}\cdot\text{ft}$$

$$\begin{aligned} \text{(b) } M_{OF} &= \mathbf{M}_O \cdot \vec{\lambda}_{OF} = (1200\mathbf{j} + 1200\mathbf{k}) \cdot \left(\frac{3\mathbf{i} + 12\mathbf{j} + 4\mathbf{k}}{13} \right) \\ &= \frac{1200(12)}{13} + \frac{1200(4)}{13} = 1477 \text{ lb}\cdot\text{ft} \quad \blacklozenge \end{aligned}$$

2.101

$$\rightarrow R_x = \Sigma F_x = T_1 \sin 45^\circ - T_3 \sin 30^\circ = 0 \quad (1)$$

$$+\uparrow R_y = \Sigma F_y = T_1 \cos 45^\circ + T_3 \cos 30^\circ + 300 = 500 \text{ lb} \quad (2)$$

Solving (1) and (2) simultaneously gives: $T_1 = 103.5 \text{ lb}$ and $T_3 = 146.4 \text{ lb}$ ♦

2.102

$$(a) \mathbf{F} \cdot \mathbf{C} = 200(-400) + 100(300) + 250(200) = 0$$

Because \mathbf{F} and \mathbf{C} are perpendicular, they can be reduced to a single force. Q.E.D.

(b) Let $A(x, y, 0)$ be the point in the xy plane where the combined moment of \mathbf{F} and \mathbf{C} is zero, and let O be the origin of the coordinate system. Since \mathbf{F} acts at O , we have:

$$\mathbf{M}_A = \mathbf{r}_{AO} \times \mathbf{F} + \mathbf{C} = \mathbf{0} \quad (\text{where } \mathbf{r}_{AO} = -x\mathbf{i} - y\mathbf{j} \text{ in.}).$$

$$\begin{aligned} \therefore \mathbf{M}_A &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -x & -y & 0 \\ 200 & 100 & 250 \end{vmatrix} + (-400\mathbf{i} + 300\mathbf{j} + 200\mathbf{k}) \\ &= (-250y)\mathbf{i} + (250x)\mathbf{j} + (-100x + 200y)\mathbf{k} - 400\mathbf{i} + 300\mathbf{j} + 200\mathbf{k} = \mathbf{0} \end{aligned}$$

Equating the \mathbf{i} - and \mathbf{j} -components to zero gives:

$$\begin{aligned} -250y - 400 &= 0 & 250x + 300 &= 0 \\ y &= -1.60 \text{ in.} & x &= -1.20 \text{ in.} \end{aligned}$$

Check using \mathbf{k} -components:

$$-100(-1.2) + 200(-1.6) + 200 = 0 \quad \text{it checks!}$$

Therefore, the coordinates of point A are $(-1.2 \text{ in.}, -1.6 \text{ in.}, 0)$ ♦

2.103

$$\mathbf{R} = \Sigma \mathbf{F} = 50\mathbf{j} + 30\mathbf{k} \text{ kN} \quad \blacktriangleleft$$

Note that the 30-kN force has a moment only about the x -axis. Also, the 50-kN force has a moment only about the z -axis.

$$\therefore \mathbf{C}^R = 30(4)\mathbf{i} + 50(2)\mathbf{k} = 120\mathbf{i} + 100\mathbf{k} \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

2.104

$$\rightarrow R_x = \Sigma F_x = P - P = 0$$

$$+\uparrow R_y = \Sigma F_y = P$$

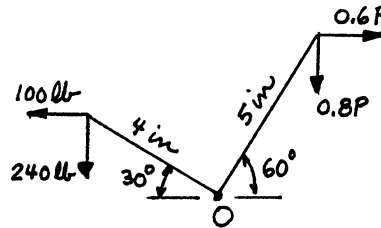
Therefore, the force acting at A is $R = P$ (acting upward) \blacklozenge

Because R passes through point A, the moment of the three forces about A is zero.

$$\curvearrowright \Sigma M_A = P(L - x) - P(L/2) = 0 \quad \text{which gives } x = L/2 \quad \blacklozenge$$

2.105

Because the resultant force passes through O and there is no resultant couple, the combined moment of the two forces about O is zero.



$$\curvearrowright \Sigma M_O = 240(4 \cos 30^\circ) + 100(4 \sin 30^\circ) - 0.8P(5 \cos 60^\circ) - 0.6P(5 \sin 60^\circ) = 0$$

Solving for P gives: $P = 224 \text{ lb}$ \blacklozenge

2.106

$$\vec{BA} = -3\mathbf{i} - 3 \cos 20^\circ \mathbf{j} + (4 - 3 \sin 20^\circ) \mathbf{k} = -3\mathbf{i} - 2.819\mathbf{j} + 2.974\mathbf{k} \text{ lb}$$

$$\vec{CA} = 2\mathbf{i} - 2.819\mathbf{j} + 2.974\mathbf{k} \text{ lb}$$

$$\mathbf{T}_1 = 30 \vec{\lambda}_{BA} = 30 \left(\frac{-3\mathbf{i} - 2.819\mathbf{j} + 2.974\mathbf{k}}{5.0785} \right) = -17.722\mathbf{i} - 16.653\mathbf{j} + 17.568\mathbf{k} \text{ lb}$$

$$\mathbf{T}_2 = 90 \vec{\lambda}_{CA} = 90 \left(\frac{2\mathbf{i} - 2.819\mathbf{j} + 2.974\mathbf{k}}{4.5600} \right) = 39.474\mathbf{i} - 55.638\mathbf{j} + 58.697\mathbf{k} \text{ lb}$$

$$\mathbf{R} = \mathbf{T}_1 + \mathbf{T}_2 = 21.752\mathbf{i} - 72.291\mathbf{j} + 76.265\mathbf{k} \text{ lb}$$

$$\therefore R = \sqrt{21.752^2 + (-72.291)^2 + 76.265^2} = 107.3 \text{ lb} \quad \blacklozenge$$

2.107

$$\mathbf{P} = -300\mathbf{i} + 200\mathbf{j} + 150\mathbf{k} \text{ lb} \quad \mathbf{C} = C \vec{\lambda}_{BE} = C(-0.6\mathbf{j} + 0.8\mathbf{k}) \text{ lb}\cdot\text{ft}$$

$$\mathbf{r}_{DA} = 3\mathbf{j} \text{ ft} \quad \vec{\lambda}_{DE} = -0.6\mathbf{i} + 0.8\mathbf{k}$$

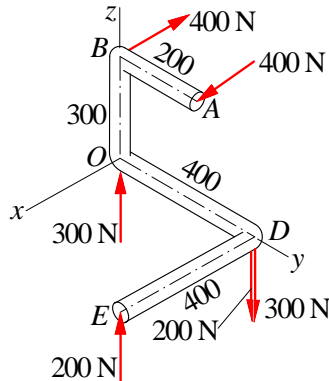
$$\text{For } \mathbf{P}: \mathbf{M}_{DE} = \mathbf{r}_{DA} \times \mathbf{P} \cdot \vec{\lambda}_{DE} = \begin{vmatrix} 0 & 3 & 0 \\ -300 & 200 & 150 \\ -0.6 & 0 & 0.8 \end{vmatrix} = -3(-240 + 90) = 450 \text{ lb}\cdot\text{ft}$$

$$\text{For } \mathbf{C}: \mathbf{M}_{DE} = \mathbf{C} \cdot \vec{\lambda}_{DE} = C(-0.6\mathbf{j} + 0.8\mathbf{k}) \cdot (-0.6\mathbf{i} + 0.8\mathbf{k}) = 0.640C$$

$$\text{Combined moment of } \mathbf{P} \text{ and } \mathbf{C}: \Sigma \mathbf{M}_{DE} = 450 + 0.640C = 800 \text{ lb}\cdot\text{ft}$$

$$\text{which gives: } C = 547 \text{ lb}\cdot\text{ft} \quad \blacklozenge$$

2.108



Split the 500-N force at D into the 200-N and 300-N forces as shown. We now see that the force system consists of three couples.

$$\begin{aligned} \mathbf{C}^R &= \Sigma \mathbf{C} = -300(0.4)\mathbf{i} - 200(0.4)\mathbf{j} - 400(0.2)\mathbf{k} \\ &= -120\mathbf{i} - 80\mathbf{j} - 80\mathbf{k} \text{ N}\cdot\text{m} \quad \blacktriangleleft \end{aligned}$$