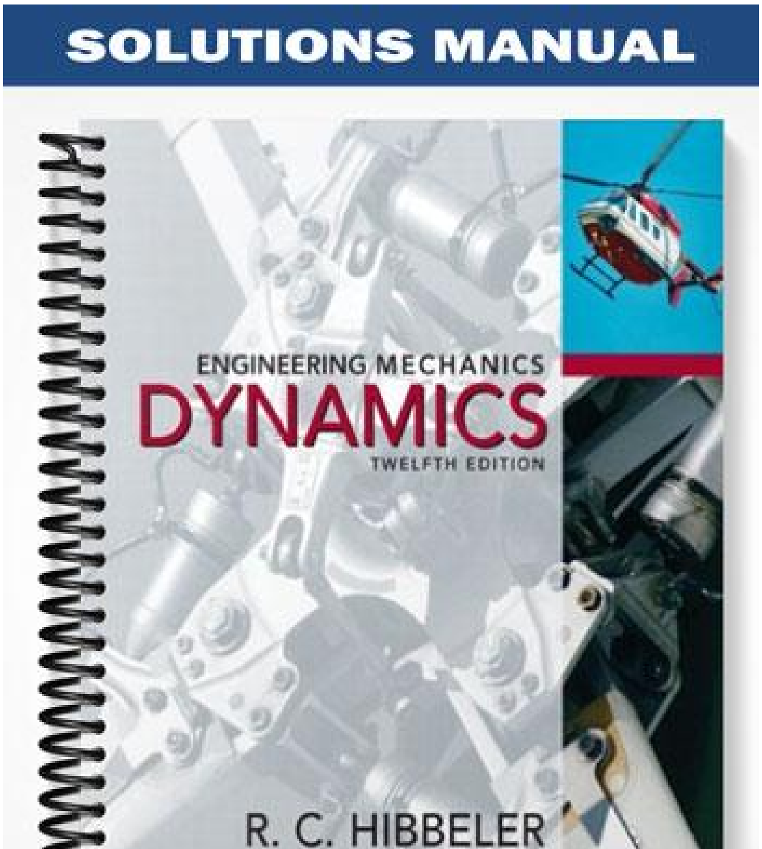
SOLUTIONS MANUAL
ENGINEERING MECHANICS
DYNAMICS
TWELFTH EDITION
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•12–1. A car starts from rest and with constant acceleration achieves a velocity of 15 m/s when it travels a distance of 200 m. Determine the acceleration of the car and the time required.

Kinematics:

$v_0 = 0, v = 15 \text{ m/s}, s_0 = 0, \text{and } s = 200 \text{ m}.$

$(\overset{+}{\rightarrow}) \quad v^2 = v_0{}^2 + 2a_c(s - s_0)$

$15^2 = 0^2 + 2a_c(200 - 0)$

$a_c = 0.5625 \text{ m/s}^2$ **Ans.**

$(\overset{+}{\rightarrow}) \quad v = v_0 + a_c t$

$15 = 0 + 0.5625t$

$t = 26.7 \text{ s}$ **Ans.**

12–2. A train starts from rest at a station and travels with a constant acceleration of 1 m/s^2. Determine the velocity of the train when $t = 30$ s and the distance traveled during this time.

Kinematics:

$a_c = 1 \text{ m/s}^2, v_0 = 0, s_0 = 0, \text{and } t = 30 \text{ s}.$

$(\overset{+}{\rightarrow}) \quad v = v_0 + a_c t$

$= 0 + 1(30) = 30 \text{ m/s}$ **Ans.**

$(\overset{+}{\rightarrow}) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$= 0 + 0 + \frac{1}{2}(1)\left(30^2\right)$

$= 450 \text{ m}$ **Ans.**

12–3. An elevator descends from rest with an acceleration of 5 ft/s^2 until it achieves a velocity of 15 ft/s. Determine the time required and the distance traveled.

Kinematics:

$a_c = 5 \text{ ft/s}^2$, $v_0 = 0$, $v = 15$ ft/s, and $s_0 = 0$.

$(+\downarrow)$ $\quad v = v_0 + a_c t$

$15 = 0 + 5t$

$t = 3\text{s}$ **Ans.**

$(+\downarrow)$ $\quad v^2 = v_0{}^2 + 2a_c(s - s_0)$

$15^2 = 0^2 + 2(5)(s - 0)$

$s = 22.5 \text{ ft}$ **Ans.**

***12–4.** A car is traveling at 15 m/s, when the traffic light 50 m ahead turns yellow. Determine the required constant deceleration of the car and the time needed to stop the car at the light.

Kinematics:

$v_0 = 0$, $s_0 = 0$, $s = 50$ m and $v_0 = 15$ m/s.

$(\overset{+}{\rightarrow})$ $\quad v^2 = v_0{}^2 + 2a_c(s - s_0)$

$0 = 15^2 + 2a_c(50 - 0)$

$a_c = -2.25 \text{ m/s}^2 = 2.25 \text{ m/s}^2 \leftarrow$ **Ans.**

$(\overset{+}{\rightarrow})$ $\quad v = v_0 + a_c t$

$0 = 15 + (-2.25)t$

$t = 6.67 \text{ s}$ **Ans.**

•12–5. A particle is moving along a straight line with the acceleration $a = (12t - 3t^{1/2})\ \text{ft/s}^2$, where t is in seconds. Determine the velocity and the position of the particle as a function of time. When $t = 0$, $v = 0$ and $s = 15\ \text{ft}$.

Velocity:

$(\overset{+}{\rightarrow})$ $$dv = a\,dt$$

$$\int_0^v dv = \int_0^t \left(12t - 3t^{1/2}\right)dt$$

$$v\Big|_0^v = \left(6t^2 - 2t^{3/2}\right)\Big|_0^t$$

$$v = \left(6t^2 - 2t^{3/2}\right)\text{ft/s} \qquad \textbf{Ans.}$$

Position: Using this result and the initial condition $s = 15$ ft at $t = 0$ s,

$(\overset{+}{\rightarrow})$ $$ds = v\,dt$$

$$\int_{15\text{ ft}}^s ds = \int_0^t \left(6t^2 - 2t^{3/2}\right)dt$$

$$s\Big|_{15\text{ ft}}^s = \left(2t^3 - \frac{4}{5}t^{5/2}\right)\Big|_0^t$$

$$s = \left(2t^3 - \frac{4}{5}t^{5/2} + 15\right)\text{ft} \qquad \textbf{Ans.}$$

12–6. A ball is released from the bottom of an elevator which is traveling upward with a velocity of 6 ft/s. If the ball strikes the bottom of the elevator shaft in 3 s, determine the height of the elevator from the bottom of the shaft at the instant the ball is released. Also, find the velocity of the ball when it strikes the bottom of the shaft.

Kinematics: When the ball is released, its velocity will be the same as the elevator at the instant of release. Thus, $v_0 = 6$ ft/s. Also, $t = 3$ s, $s_0 = 0$, $s = -h$, and $a_c = -32.2\ \text{ft/s}^2$.

$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

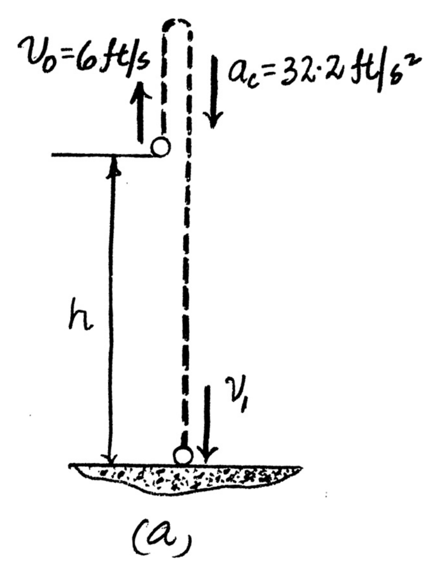
$$-h = 0 + 6(3) + \frac{1}{2}(-32.2)(3^2)$$

$$h = 127\text{ ft} \qquad \textbf{Ans.}$$

$(+\uparrow)$ $$v = v_0 + a_c t$$

$$v = 6 + (-32.2)(3)$$

$$= -90.6\text{ ft/s} = 90.6\text{ ft/s}\ \downarrow \qquad \textbf{Ans.}$$



12–7. A car has an initial speed of 25 m/s and a constant deceleration of 3 m/s^2. Determine the velocity of the car when $t = 4$ s. What is the displacement of the car during the 4-s time interval? How much time is needed to stop the car?

$$v = v_0 + a_c t$$

$$v = 25 + (-3)(4) = 13 \text{ m/s}$$ **Ans.**

$$\Delta s = s - s_0 = v_0 t + \frac{1}{2} a_c t^2$$

$$\Delta s = s - 0 = 25(4) + \frac{1}{2}(-3)(4)^2 = 76 \text{ m}$$ **Ans.**

$$v = v_0 + a_c t$$

$$0 = 25 + (-3)(t)$$

$$t = 8.33 \text{ s}$$ **Ans.**

***12–8.** If a particle has an initial velocity of $v_0 = 12$ ft/s to the right, at $s_0 = 0$, determine its position when $t = 10$ s, if $a = 2$ ft/s^2 to the left.

$(\overset{+}{\rightarrow})$ $$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$= 0 + 12(10) + \frac{1}{2}(-2)(10)^2$$

$$= 20 \text{ ft}$$ **Ans.**

•12–9. The acceleration of a particle traveling along a straight line is $a = k/v$, where k is a constant. If $s = 0$, $v = v_0$ when $t = 0$, determine the velocity of the particle as a function of time t.

Velocity:

$(\overset{+}{\rightarrow})$ $$dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_{v_0}^{v} \frac{dv}{k/v}$$

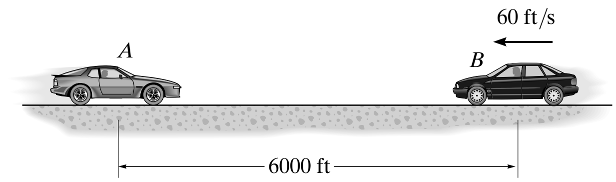
$$\int_0^t dt = \int_{v_0}^{v} \frac{1}{k} v\,dv$$

$$t\Big|_0^t = \frac{1}{2k} v^2 \Big|_{v_0}^{v}$$

$$t = \frac{1}{2k}\left(v^2 - v_0{}^2\right)$$

$$v = \sqrt{2kt + v_0{}^2}$$ **Ans.**

12–10. Car A starts from rest at $t = 0$ and travels along a straight road with a constant acceleration of 6 ft/s^2 until it reaches a speed of 80 ft/s. Afterwards it maintains this speed. Also, when $t = 0$, car B located 6000 ft down the road is traveling towards A at a constant speed of 60 ft/s. Determine the distance traveled by car A when they pass each other.



Distance Traveled: Time for car A to achives $v = 80$ ft/s can be obtained by applying Eq. 12–4.

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$80 = 0 + 6t$$

$$t = 13.33 \text{ s}$$

The distance car A travels for this part of motion can be determined by applying Eq. 12–6.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$80^2 = 0 + 2(6)(s_1 - 0)$$

$$s_1 = 533.33 \text{ ft}$$

For the second part of motion, car A travels with a constant velocity of $v = 80$ ft/s and the distance traveled in $t' = (t_1 - 13.33)$ s (t_1 is the total time) is

$$(\overset{+}{\rightarrow}) \qquad s_2 = vt' = 80(t_1 - 13.33)$$

Car B travels in the opposite direction with a constant velocity of $v = 60$ ft/s and the distance traveled in t_1 is

$$(\overset{+}{\rightarrow}) \qquad s_3 = vt_1 = 60t_1$$

It is required that

$$s_1 + s_2 + s_3 = 6000$$

$$533.33 + 80(t_1 - 13.33) + 60t_1 = 6000$$

$$t_1 = 46.67 \text{ s}$$

The distance traveled by car A is

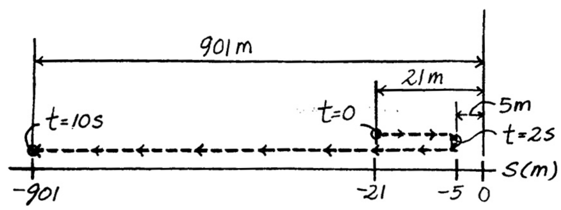
$$s_A = s_1 + s_2 = 533.33 + 80(46.67 - 13.33) = 3200 \text{ ft} \qquad \textbf{Ans.}$$

12–11. A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When $t = 1$ s, the particle is located 10 m to the left of the origin. Determine the acceleration when $t = 4$ s, the displacement from $t = 0$ to $t = 10$ s, and the distance the particle travels during this time period.

$v = 12 - 3t^2$ **(1)**

$a = \frac{dv}{dt} = -6t\Big|_{t=4} = -24\text{ m/s}^2$ **Ans.**

$\int_{-10}^{s} ds = \int_1^t v\,dt = \int_1^t \left(12 - 3t^2\right)dt$



$s + 10 = 12t - t^3 - 11$

$s = 12t - t^3 - 21$

$s|_{t=0} = -21$

$s|_{t=10} = -901$

$\Delta s = -901 - (-21) = -880\text{ m}$ **Ans.**

From Eq. (1):

$v = 0$ when $t = 2s$

$s|_{t=2} = 12(2) - (2)^3 - 21 = -5$

$s_T = (21 - 5) + (901 - 5) = 912\text{ m}$ **Ans.**

***12–12.** A sphere is fired downwards into a medium with an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)\ \text{m/s}^2$, where t is in seconds, determine the distance traveled before it stops.

Velocity: $v_0 = 27$ m/s at $t_0 = 0$ s. Applying Eq. 12–2, we have

$$(+\downarrow) \qquad dv = a\,dt$$

$$\int_{27}^{v} dv = \int_0^t -6t\,dt$$

$$v = \left(27 - 3t^2\right)\ \text{m/s} \qquad \textbf{[1]}$$

At $v = 0$, from Eq.[1]

$$0 = 27 - 3t^2 \qquad t = 3.00\ \text{s}$$

Distance Traveled: $s_0 = 0$ m at $t_0 = 0$ s. Using the result $v = 27 - 3t^2$ and applying Eq. 12–1, we have

$$(+\downarrow) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t \left(27 - 3t^2\right) dt$$

$$s = \left(27t - t^3\right)\ \text{m} \qquad \textbf{[2]}$$

At $t = 3.00$ s, from Eq. [2]

$$s = 27(3.00) - 3.00^3 = 54.0\ \text{m}$$ **Ans.**

•12–13. A particle travels along a straight line such that in 2 s it moves from an initial position $s_A = +0.5$ m to a position $s_B = -1.5$ m. Then in another 4 s it moves from s_B to $s_C = +2.5$ m. Determine the particle's average velocity and average speed during the 6-s time interval.

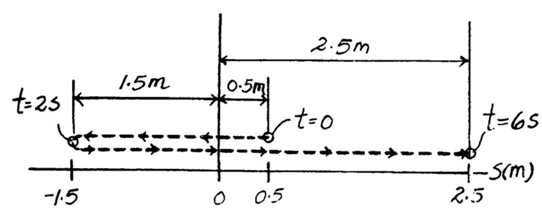
$$\Delta s = (s_C - s_A) = 2\ \text{m}$$

$$s_T = (0.5 + 1.5 + 1.5 + 2.5) = 6\ \text{m}$$

$$t = (2 + 4) = 6\ \text{s}$$

$$v_{avg} = \frac{\Delta s}{t} = \frac{2}{6} = 0.333\ \text{m/s}$$ **Ans.**

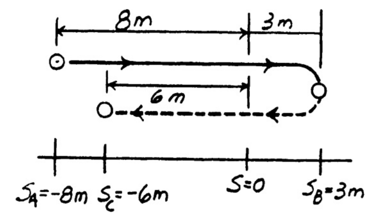
$$(v_{sp})_{avg} = \frac{s_T}{t} = \frac{6}{6} = 1\ \text{m/s}$$ **Ans.**



12–14. A particle travels along a straight-line path such that in 4 s it moves from an initial position $s_A = -8$ m to a position $s_B = +3$ m. Then in another 5 s it moves from s_B to $s_C = -6$ m. Determine the particle's average velocity and average speed during the 9-s time interval.

Average Velocity: The displacement from A to C is $\Delta s = s_C - S_A = -6 - (-8) = 2$ m.

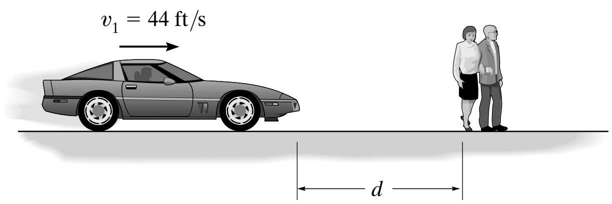
$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{2}{4 + 5} = 0.222 \text{ m/s} \qquad \textbf{Ans.}$$



Average Speed: The distances traveled from A to B and B to C are $s_{A \to B} = 8 + 3 = 11.0$ m and $s_{B \to C} = 3 + 6 = 9.00$ m, respectively. Then, the total distance traveled is $s_{Tot} = s_{A \to B} + s_{B \to C} = 11.0 - 9.00 = 20.0$ m.

$$\left(v_{sp}\right)_{avg} = \frac{s_{Tot}}{\Delta t} = \frac{20.0}{4 + 5} = 2.22 \text{ m/s} \qquad \textbf{Ans.}$$

12–15. Tests reveal that a normal driver takes about 0.75 s before he or she can *react* to a situation to avoid a collision. It takes about 3 s for a driver having 0.1% alcohol in his system to do the same. If such drivers are traveling on a straight road at 30 mph (44 ft/s) and their cars can decelerate at 2 ft/s^2, determine the shortest stopping distance d for each from the moment they see the pedestrians. *Moral*: If you must drink, please don't drive!



Stopping Distance: For normal driver, the car moves a distance of $d' = vt = 44(0.75) = 33.0$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 33.0$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0^2 = 44^2 + 2(-2)(d - 33.0)$$

$$d = 517 \text{ ft} \qquad \textbf{Ans.}$$

For a drunk driver, the car moves a distance of $d' = vt = 44(3) = 132$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 132$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0^2 = 44^2 + 2(-2)(d - 132)$$

$$d = 616 \text{ ft} \qquad \textbf{Ans.}$$

***12–16.** As a train accelerates uniformly it passes successive kilometer marks while traveling at velocities of 2 m/s and then 10 m/s. Determine the train's velocity when it passes the next kilometer mark and the time it takes to travel the 2-km distance.

Kinematics: For the first kilometer of the journey, $v_0 = 2$ m/s, $v = 10$ m/s, $s_0 = 0$, and $s = 1000$ m. Thus,

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$10^2 = 2^2 + 2a_c(1000 - 0)$$

$$a_c = 0.048 \text{ m/s}^2$$

For the second kilometer, $v_0 = 10$ m/s, $s_0 = 1000$ m, $s = 2000$ m, and 0.048 m/s^2. Thus,

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$v^2 = 10^2 + 2(0.048)(2000 - 1000)$$

$$v = 14 \text{ m/s} \qquad \textbf{Ans.}$$

For the whole journey, $v_0 = 2$ m/s, $v = 14$ m/s, and 0.048 m/s^2. Thus,

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$14 = 2 + 0.048t$$

$$t = 250 \text{ s} \qquad \textbf{Ans.}$$

•12–17. A ball is thrown with an upward velocity of 5 m/s from the top of a 10-m high building. One second later another ball is thrown vertically from the ground with a velocity of 10 m/s. Determine the height from the ground where the two balls pass each other.

Kinematics: First, we will consider the motion of ball A with $(v_A)_0 = 5$ m/s, $(s_A)_0 = 0$, $s_A = (h - 10)$ m, $t_A = t'$, and $a_c = -9.81$ m/s^2. Thus,

$$(+\uparrow) \qquad s_A = (s_A)_0 + (v_A)_0 t_A + \frac{1}{2}a_c t_A{}^2$$

$$h - 10 = 0 + 5t' + \frac{1}{2}(-9.81)(t')^2$$

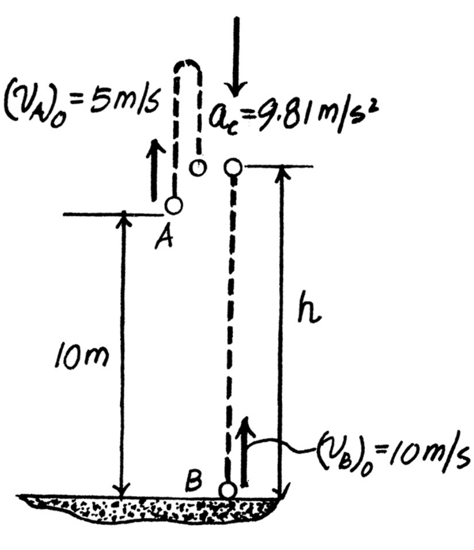
$$h = 5t' - 4.905(t')^2 + 10 \qquad \textbf{(1)}$$

Motion of ball B is with $(v_B)_0 = 10$ m/s, $(s_B)_0 = 0$, $s_B = h$, $t_B = t' - 1$ and $a_c = -9.81$ m/s^2. Thus,

$$(+\uparrow) \qquad s_B = (s_B)_0 + (v_B)_0 t_B + \frac{1}{2}a_c t_B{}^2$$

$$h = 0 + 10(t' - 1) + \frac{1}{2}(-9.81)(t' - 1)^2$$

$$h = 19.81t' - 4.905(t')^2 - 14.905 \qquad \textbf{(2)}$$



(a)

Solving Eqs. (1) and (2) yields

$$h = 4.54 \text{ m} \qquad \textbf{Ans.}$$

$$t' = 1.68 \text{ m}$$

12–18. A car starts from rest and moves with a constant acceleration of 1.5 m/s^2 until it achieves a velocity of 25 m/s. It then travels with constant velocity for 60 seconds. Determine the average speed and the total distance traveled.

Kinematics: For stage (1) of the motion, $v_0 = 0$, $s_0 = 0$, $v = 25$ m/s, and $a_c = 1.5\ \text{m/s}^2$.

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$25 = 0 + 1.5t_1$$

$$t_1 = 16.67 \text{ s}$$

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$25^2 = 0 + 2(1.5)(s_1 - 0)$$

$$s_1 = 208.33 \text{ m}$$

For stage (2) of the motion, $s_0 = 108.22$ ft, $v_0 = 25$ ft/s, $t = 60$ s, and $a_c = 0$. Thus,

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$s = 208.33 + 25(60) + 0$$

$$= 1708.33\text{ft} = 1708 \text{ m} \qquad \textbf{Ans.}$$

The average speed of the car is then

$$v_{avg} = \frac{s}{t_1 + t_2} = \frac{1708.33}{16.67 + 60} = 22.3 \text{ m/s} \qquad \textbf{Ans.}$$

12–19. A car is to be hoisted by elevator to the fourth floor of a parking garage, which is 48 ft above the ground. If the elevator can accelerate at 0.6 ft/s^2, decelerate at 0.3 ft/s^2, and reach a maximum speed of 8 ft/s, determine the shortest time to make the lift, starting from rest and ending at rest.

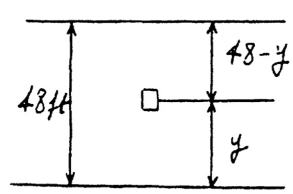
$$+\uparrow \quad v^2 = v_0^2 + 2\,a_c\,(s - s_0)$$

$$v_{max}^2 = 0 + 2(0.6)(y - 0)$$

$$0 = v_{max}^2 + 2(-0.3)(48 - \text{y})$$

$$0 = 1.2\,y - 0.6(48 - y)$$

$$y = 16.0 \text{ ft}, \qquad v_{max} = 4.382 \text{ ft/s} < 8 \text{ ft/s}$$



$$+\uparrow \quad v = v_0 + a_c\,t$$

$$4.382 = 0 + 0.6\,t_1$$

$$t_1 = 7.303 \text{ s}$$

$$0 = 4.382 - 0.3\,t_2$$

$$t_2 = 14.61 \text{ s}$$

$$t = t_1 + t_2 = 21.9 \text{ s} \qquad \textbf{Ans.}$$

***12–20.** A particle is moving along a straight line such that its speed is defined as $v = (-4s^2)$ m/s, where s is in meters. If $s = 2$ m when $t = 0$, determine the velocity and acceleration as functions of time.

$$v = -4s^2$$

$$\frac{ds}{dt} = -4s^2$$

$$\int_2^s s^{-2}\,ds = \int_0^t -4\,dt$$

$$-s^{-1}\big|_2^s = -4t\big|_0^t$$

$$t = \frac{1}{4}\left(s^{-1} - 0.5\right)$$

$$s = \frac{2}{8t + 1}$$

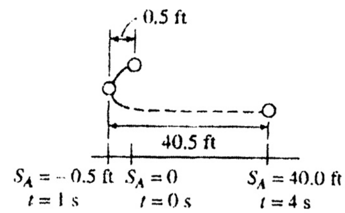
$$v = -4\left(\frac{2}{8t + 1}\right)^2 = \left(-\frac{16}{(8t + 1)^2}\right) \text{ m/s} \qquad \textbf{Ans.}$$

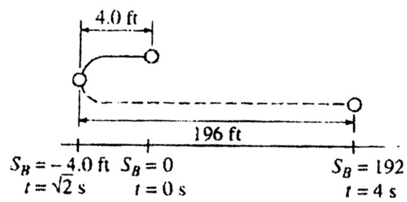
$$a = \frac{dv}{dt} = \frac{16(2)(8t + 1)(8)}{(8t + 1)^4} = \left(\frac{256}{(8t + 1)^3}\right) \text{ m/s}^2 \qquad \textbf{Ans.}$$

•12–21. Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (6t - 3)$ ft/s^2 and $a_B = (12t^2 - 8)$ ft/s^2, where t is in seconds. Determine the distance between them when $t = 4$ s and the total distance each has traveled in $t = 4$ s.

Velocity: The velocity of particles A and B can be determined using Eq. 12-2.

$$dv_A = a_A dt$$
$$\int_0^{v_A} dv_A = \int_0^t (6t - 3)dt$$
$$v_A = 3t^2 - 3t$$
$$dv_B = a_B dt$$
$$\int_0^{v_B} dv_B = \int_0^t (12t^2 - 8)dt$$
$$v_B = 4t^3 - 8t$$





The times when particle A stops are

$3t^2 - 3t = 0 \qquad t = 0$ s and $= 1$ s

The times when particle B stops are

$4t^3 - 8t = 0 \quad t = 0$ s and $t = \sqrt{2}$ s

Position: The position of particles A and B can be determined using Eq. 12-1.

$$ds_A = v_A dt$$
$$\int_0^{s_A} ds_A = \int_0^t (3t^2 - 3t)dt$$
$$s_A = t^3 - \frac{3}{2}t^2$$
$$ds_B = v_B dt$$
$$\int_0^{s_B} ds_B = \int_0^t (4t^3 - 8t)dt$$
$$s_B = t^4 - 4t^2$$

The positions of particle A at $t = 1$ s and 4 s are

$$s_A|_{t=1\,s} = 1^3 - \frac{3}{2}(1^2) = -0.500 \text{ ft}$$
$$s_A|_{t=4\,s} = 4^3 - \frac{3}{2}(4^2) = 40.0 \text{ ft}$$

Particle A has traveled

$$d_A = 2(0.5) + 40.0 = 41.0 \text{ ft}$$ **Ans.**

The positions of particle B at $t = \sqrt{2}$ s and 4 s are

$$s_B|_{t=\sqrt{2}} = (\sqrt{2})^4 - 4(\sqrt{2})^2 = -4 \text{ ft}$$
$$s_B|_{t=4} = (4)^4 - 4(4)^2 = 192 \text{ ft}$$

Particle B has traveled

$$d_B = 2(4) + 192 = 200 \text{ ft}$$ **Ans.**

At $t = 4$ s the distance beween A and B is

$$\Delta s_{AB} = 192 - 40 = 152 \text{ ft}$$ **Ans.**

12–22. A particle moving along a straight line is subjected to a deceleration $a = (-2v^3)\ \text{m/s}^2$, where v is in m/s. If it has a velocity $v = 8\ \text{m/s}$ and a position $s = 10\ \text{m}$ when $t = 0$, determine its velocity and position when $t = 4$ s.

Velocity: The velocity of the particle can be related to its position by applying Eq. 12–3.

$$ds = \frac{vdv}{a}$$

$$\int_{10\text{m}}^{s} ds = \int_{8\text{m/s}}^{v} -\frac{dv}{2v^2}$$

$$s - 10 = \frac{1}{2v} - \frac{1}{16}$$

$$v = \frac{8}{16s - 159} \qquad \textbf{[1]}$$

Position: The position of the particle can be related to the time by applying Eq. 12–1.

$$dt = \frac{ds}{v}$$

$$\int_{0}^{t} dt = \int_{10\text{m}}^{s} \frac{1}{8}(16s - 159)\, ds$$

$$8t = 8s^2 - 159s + 790$$

When $t = 4$ s,

$$8(4) = 8s^2 - 159s + 790$$

$$8s^2 - 159s + 758 = 0$$

Choose the root greater than 10 m $s = 11.94\ \text{m} = 11.9\ \text{m}$ **Ans.**

Substitute $s = 11.94$ m into Eq. [1] yields

$$v = \frac{8}{16(11.94) - 159} = 0.250\ \text{m/s} \qquad \textbf{Ans.}$$

12–23. A particle is moving along a straight line such that its acceleration is defined as $a = (-2v)$ m/s^2, where v is in meters per second. If $v = 20$ m/s when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

$$a = -2v$$

$$\frac{dv}{dt} = -2v$$

$$\int_{20}^{v} \frac{dv}{v} = \int_0^t -2\,dt$$

$$\ln \frac{v}{20} = -2t$$

$$v = (20e^{-2t})\text{m/s} \qquad \textbf{Ans.}$$

$$a = \frac{dv}{dt} = (-40e^{-2t})\text{m/s}^2 \qquad \textbf{Ans.}$$

$$\int_0^s ds = \int_0^t v\,dt = \int_0^t (20e^{-2t})dt$$

$$s = -10e^{-2t}|_0^t = -10(e^{-2t} - 1)$$

$$s = 10(1 - e^{-2t})\text{m} \qquad \textbf{Ans.}$$

***12–24.** A particle starts from rest and travels along a straight line with an acceleration $a = (30 - 0.2v)\ \text{ft/s}^2$, where v is in ft/s. Determine the time when the velocity of the particle is $v = 30$ ft/s.

Velocity:

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{30 - 0.2v}$$

$$t\big|_0^t = -\frac{1}{0.2}\ln(30 - 0.2v)\Big|_0^v$$

$$t = 5\ln\frac{30}{30 - 0.2v}$$

$$t = 5\ln\frac{30}{30 - 0.2(50)} = 1.12\ \text{s} \qquad \textbf{Ans.}$$

•12–25. When a particle is projected vertically upwards with an initial velocity of v_0, it experiences an acceleration $a = -(g + kv^2)$, where g is the acceleration due to gravity, k is a constant and v is the velocity of the particle. Determine the maximum height reached by the particle.

Position:

$$(+\uparrow) \qquad ds = \frac{v\,dv}{a}$$

$$\int_0^s ds = \int_{v_0}^v -\frac{v\,dv}{g + kv^2}$$

$$s\big|_0^s = -\left[\frac{1}{2k}\ln\left(g + kv^2\right)\right]\Big|_{v_0}^v$$

$$s = \frac{1}{2k}\ln\left(\frac{g + kv_0^{\ 2}}{g + kv^2}\right)$$

The particle achieves its maximum height when $v = 0$. Thus,

$$h_{max} = \frac{1}{2k}\ln\left(\frac{g + kv_0^{\ 2}}{g}\right)$$

$$= \frac{1}{2k}\ln\left(1 + \frac{k}{g}v_0^{\ 2}\right) \qquad \textbf{Ans.}$$

12–26. The acceleration of a particle traveling along a straight line is $a = (0.02e^t)\ \text{m/s}^2$, where t is in seconds. If $v = 0$, $s = 0$ when $t = 0$, determine the velocity and acceleration of the particle at $s = 4$ m.

Velocity: $a = 0.02e^{5.329} = 4.13\ \text{m/s}^2$ **Ans.**

$(\overset{+}{\rightarrow})$ $dv = a\,dt$

$$\int_0^v dv = \int_0^t 0.02e^t\,dt$$

$$v\Big|_0^v = 0.02e^t\Big|_0^t$$

$$v = \left[0.02\left(e^t - 1\right)\right]\ \text{m/s} \qquad \textbf{(1)}$$

Position:

$(\overset{+}{\rightarrow})$ $ds = v\,dt$

$$\int_0^s ds = \int_0^t 0.02\left(e^t - 1\right)dt$$

$$s\Big|_0^s = 0.02\left(e^t - t\right)\Big|_0^t$$

$$s = 0.02\left(e^t - t - 1\right)\ \text{m}$$

When $s = 4$ m,

$$4 = 0.02\left(e^t - t - 1\right)$$

$$e^t - t - 201 = 0$$

Solving the above equation by trial and error,

$$t = 5.329\ \text{s}$$

Thus, the velocity and acceleration when $s = 4$ m ($t = 5.329$ s) are

$$v = 0.02\left(e^{5.329} - 1\right) = 4.11\ \text{m/s} \qquad \textbf{Ans.}$$

$$a = 0.02e^{5.329} = 4.13\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–27. A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})\ \text{m/s}^2$, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use Simpson's rule to evaluate the integral.

$$a = \frac{5}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)}$$

$$a\,ds = v\,dv$$

$$\int_1^2 \frac{5\,ds}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)} = \int_0^v v\,dv$$

$$0.8351 = \frac{1}{2}v^2$$

$$v = 1.29\ \text{m/s} \qquad \textbf{Ans.}$$

***12–28.** If the effects of atmospheric resistance are accounted for, a falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]$ m/s^2, where v is in m/s and the positive direction is downward. If the body is released from rest at a very *high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \rightarrow \infty$).

Velocity: The velocity of the particle can be related to the time by applying Eq. 12–2.

$$(+\downarrow) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{9.81[1 - (0.01v)^2]}$$

$$t = \frac{1}{9.81}\left[\int_0^v \frac{dv}{2(1 + 0.01v)} + \int_0^v \frac{dv}{2(1 - 0.01v)}\right]$$

$$9.81t = 50\ln\left(\frac{1 + 0.01v}{1 - 0.01v}\right)$$

$$v = \frac{100(e^{0.1962t} - 1)}{e^{0.1962t} + 1} \qquad \textbf{[1]}$$

a) When $t = 5$ s, then, from Eq. [1]

$$v = \frac{100[e^{0.1962(5)} - 1]}{e^{0.1962(5)} + 1} = 45.5 \text{ m/s} \qquad \textbf{Ans.}$$

b) If $t \rightarrow \infty$, $\frac{e^{0.1962t} - 1}{e^{0.1962t} + 1} \rightarrow 1$. Then, from Eq. [1]

$$v_{max} = 100 \text{ m/s} \qquad \textbf{Ans.}$$

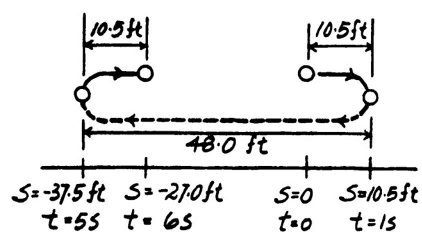
•12–29. The position of a particle along a straight line is given by $s = (1.5t^3 - 13.5t^2 + 22.5t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint:* Plot the path to determine the total distance traveled.

Position: The position of the particle when $t = 6$ s is

$$s|_{t=6s} = 1.5(6^3) - 13.5(6^2) + 22.5(6) = -27.0 \text{ ft} \qquad \textbf{Ans.}$$

Total Distance Traveled: The velocity of the particle can be determined by applying Eq. 12–1.

$$v = \frac{ds}{dt} = 4.50t^2 - 27.0t + 22.5$$



The times when the particle stops are

$$4.50t^2 - 27.0t + 22.5 = 0$$

$$t = 1 \text{ s} \qquad \text{and} \qquad t = 5 \text{ s}$$

The position of the particle at $t = 0$ s, 1 s and 5 s are

$$s\big|_{t=0s} = 1.5(0^3) - 13.5(0^2) + 22.5(0) = 0$$

$$s\big|_{t=1s} = 1.5(1^3) - 13.5(1^2) + 22.5(1) = 10.5 \text{ ft}$$

$$s\big|_{t=5s} = 1.5(5^3) - 13.5(5^2) + 22.5(5) = -37.5 \text{ ft}$$

From the particle's path, the total distance is

$$s_{tot} = 10.5 + 48.0 + 10.5 = 69.0 \text{ ft} \qquad \textbf{Ans.}$$

12–30. The velocity of a particle traveling along a straight line is $v = v_0 - ks$, where k is constant. If $s = 0$ when $t = 0$, determine the position and acceleration of the particle as a function of time.

Position:

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{v_0 - ks}$$

$$t\Big|_0^t = -\frac{1}{k}\ln(v_0 - ks)\Big|_0^s$$

$$t = \frac{1}{k}\ln\left(\frac{v_0}{v_0 - ks}\right)$$

$$e^{kt} = \frac{v_0}{v_0 - ks}$$

$$s = \frac{v_0}{k}\left(1 - e^{-kt}\right)$$ **Ans.**

Velocity:

$$v = \frac{ds}{dt} = \frac{d}{dt}\left[\frac{v_0}{k}\left(1 - e^{-kt}\right)\right]$$

$$v = v_0 e^{-kt}$$

Acceleration:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(v_0 e^{-kt}\right)$$

$$a = -kv_0 e^{-kt}$$ **Ans.**

12–31. The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)\ \text{m/s}^2$, where t is in seconds. If $s = 1$ m and $v = 2$ m/s when $t = 0$, determine the particle's velocity and position when $t = 6$ s. Also, determine the total distance the particle travels during this time period.

$$\int_2^v dv = \int_0^t (2\,t - 1)\,dt$$

$$v = t^2 - t + 2$$

$$\int_1^s ds = \int_0^t (t^2 - t + 2)\,dt$$

$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2\,t + 1$$

When $t = 6$ s,

$$v = 32 \text{ m/s}$$ **Ans.**

$$s = 67 \text{ m}$$ **Ans.**

Since $v \neq 0$ then

$$d = 67 - 1 = 66 \text{ m}$$ **Ans.**

***12–32.** Ball A is thrown vertically upward from the top of a 30-m-high-building with an initial velocity of 5 m/s. At the same instant another ball B is thrown upward from the ground with an initial velocity of 20 m/s. Determine the height from the ground and the time at which they pass.

Origin at roof:

Ball A:

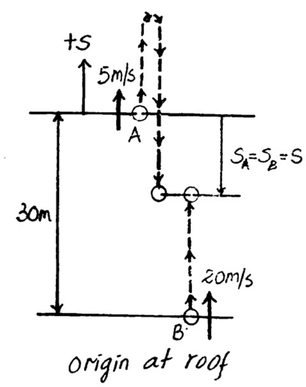
$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-s = 0 + 5t - \frac{1}{2}(9.81)t^2$$

Ball B:

$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-s = -30 + 20t - \frac{1}{2}(9.81)t^2$$



Solving,

$t = 2\text{ s}$ **Ans.**

$s = 9.62\text{ m}$

Distance from ground,

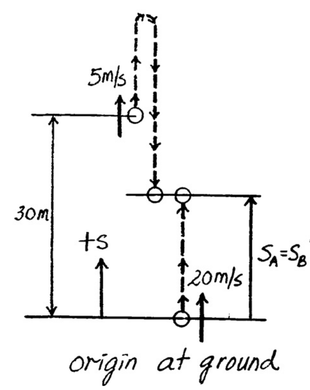
$d = (30 - 9.62) = 20.4\text{ m}$ **Ans.**

Also, origin at ground,

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_A = 30 + 5t + \frac{1}{2}(-9.81)t^2$$

$$s_B = 0 + 20t + \frac{1}{2}(-9.81)t^2$$



Require

$s_A = s_B$

$$30 + 5t + \frac{1}{2}(-9.81)t^2 = 20t + \frac{1}{2}(-9.81)t^2$$

$t = 2\text{ s}$ **Ans.**

$s_B = 20.4\text{ m}$ **Ans.**

•12–33. A motorcycle starts from rest at $t = 0$ and travels along a straight road with a constant acceleration of 6 ft/s^2 until it reaches a speed of 50 ft/s. Afterwards it maintains this speed. Also, when $t = 0$, a car located 6000 ft down the road is traveling toward the motorcycle at a constant speed of 30 ft/s. Determine the time and the distance traveled by the motorcycle when they pass each other.

Motorcycle:

$(\overset{+}{\rightarrow})$ $v = v_0 + a_c t'$

$50 = 0 + 6t'$

$t' = 8.33$ s

$v^2 = v_0^2 + 2a_c(s - s_0)$

$(50)^2 = 0 + 2(6)(s' - 0)$

$s' = 208.33$ ft

In $t' = 8.33$ s car travels

$s'' = v_0 t' = 30(8.33) = 250$ ft

Distance between motorcycle and car:

$6000 - 250 - 208.33 = 5541.67$ ft

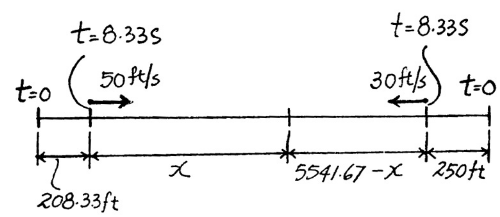
When passing occurs for motorcycle,

$s = v_0 t;$ $x = 50(t'')$

For car:

$s = v_0 t;$ $5541.67 - x = 30(t'')$

Solving,



$x = 3463.54$ ft

$t'' = 69.27$ s

Thus, for the motorcycle,

$t = 69.27 + 8.33 = 77.6$ s **Ans.**

$s_m = 208.33 + 3463.54 = 3.67(10)^3$ ft **Ans.**

12–34. A particle moves along a straight line with a velocity $v = (200s)$ mm/s, where s is in millimeters. Determine the acceleration of the particle at $s = 2000$ mm. How long does the particle take to reach this position if $s = 500$ mm when $t = 0$?

Acceleration:

$$(\overset{+}{\rightarrow}) \qquad \frac{dv}{ds} = 200s$$

Thus, $a = v\dfrac{dv}{ds} = (200s)(200) = 40\left(10^3\right)s \text{ mm/s}^2$

When $s = 2000$ mm,

$$a = 40\left(10^3\right)(2000) = 80\left(10^6\right) \text{ mm/s}^2 = 80 \text{ km/s}^2 \qquad \textbf{Ans.}$$

Position:

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_{500\text{ mm}}^{s} \frac{ds}{200s}$$

$$t\Big|_0^t = \frac{1}{200}\ln s\Big|_{500\text{ mm}}^{s}$$

$$t = \frac{1}{200}\ln\frac{s}{500}$$

At $s = 2000$ mm,

$$t = \frac{1}{200}\ln\frac{2000}{500} = 6.93\left(10^{-3}\right)\text{ s} = 6.93 \text{ ms} \qquad \textbf{Ans.}$$

■**12–35.** A particle has an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)\ \text{m/s}^2$, where t is in seconds, determine its velocity, after it has traveled 10 m. How much time does this take?

Velocity:

$(\overset{+}{\rightarrow})$ $\quad dv = a\,dt$

$$\int_{27}^{t} dv = \int_0^t (-6t)dt$$

$$v\Big|_{27}^{v} = \left(-3t^2\right)\Big|_0^t$$

$$v = (27 - 3t^2)\ \text{m/s}$$

$(\overset{+}{\rightarrow})$ $\quad ds = v\,dt$

$$\int_0^s ds = \int_0^t \left(27 - 3t^2\right)dt$$

$$s\Big|_0^s = \left(27t - t^3\right)\Big|_0^t$$

$$s = (27t - t^3)\ \text{m/s}$$

When $s = 100$ m,

$t = 0.372$ s **Ans.**

$v = 26.6$ m/s **Ans.**

***12–36.** The acceleration of a particle traveling along a straight line is $a = (8 - 2s)\ \text{m/s}^2$, where s is in meters. If $v = 0$ at $s = 0$, determine the velocity of the particle at $s = 2$ m, and the position of the particle when the velocity is maximum.

Velocity:

$(\overset{+}{\rightarrow})$ $\quad v\,dv = a\,ds$

$$\int_0^v v dv = \int_0^s (8 - 2s)\,ds$$

$$\frac{v^2}{2}\Big|_0^v = \left(8s - s^2\right)\Big|_0^s$$

$$v = \sqrt{16s - 2s^2}\ \text{m/s}$$

At $s = 2$ m,

$v\big|_{s=2\text{ m}} = \sqrt{16(2) - 2\left(2^2\right)} = \pm 4.90$ m/s **Ans.**

When the velocity is maximum $\dfrac{dv}{ds} = 0$. Thus,

$$\frac{dv}{ds} = \frac{16 - 4s}{2\sqrt{16s - 2s^2}} = 0$$

$$16 - 4s = 0$$

$s = 4$ m **Ans.**

•12–37. Ball A is thrown vertically upwards with a velocity of v_0. Ball B is thrown upwards from the same point with the same velocity t seconds later. Determine the elapsed time $t < 2v_{0/g}$ from the instant ball A is thrown to when the balls pass each other, and find the velocity of each ball at this instant.

Kinematics: First, we will consider the motion of ball A with $(v_A)_0 = v_0$, $(s_A)_0 = 0$, $s_A = h$, $t_A = t'$, and $(a_c)_A = -g$.

$$(+\uparrow) \qquad s_A = (s_A)_0 + (v_A)_0 t_A + \frac{1}{2}(a_c)_A t_A{}^2$$

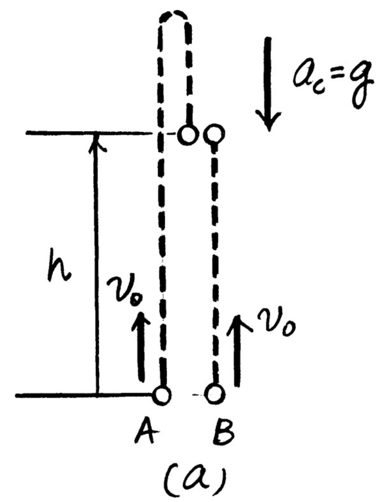
$$h = 0 + v_0 t' + \frac{1}{2}(-g)(t')^2$$

$$h = v_0 t' - \frac{g}{2}t'^2 \qquad \textbf{(1)}$$

$$(+\uparrow) \qquad v_A = (v_A)_0 + (a_c)_A t_A$$

$$v_A = v_0 + (-g)(t')$$

$$v_A = v_0 - gt' \qquad \textbf{(2)}$$



The motion of ball B requires $(v_B)_0 = v_0$, $(s_B)_0 = 0$, $s_B = h$, $t_B = t' - t$, and $(a_c)_B = -g$.

$$(+\uparrow) \qquad s_B = (s_B)_0 + (v_B)_0 t_B + \frac{1}{2}(a_c)_B t_B{}^2$$

$$h = 0 + v_0(t' - t) + \frac{1}{2}(-g)(t' - t)^2$$

$$h = v_0(t' - t) - \frac{g}{2}(t' - t)^2 \qquad \textbf{(3)}$$

$$(+\uparrow) \qquad v_B = (v_B)_0 + (a_c)_B t_B$$

$$v_B = v_0 + (-g)(t' - t)$$

$$v_B = v_0 - g(t' - t) \qquad \textbf{(4)}$$

Solving Eqs. (1) and (3),

$$v_0 t' - \frac{g}{2}t'^2 = v_0(t' - t) - \frac{g}{2}(t' - t)^2$$

$$t' = \frac{2v_0 + gt}{2g} \qquad \textbf{Ans.}$$

Substituting this result into Eqs. (2) and (4),

$$v_A = v_0 - g\left(\frac{2v_0 + gt}{2g}\right)$$

$$= -\frac{1}{2}gt = \frac{1}{2}gt\downarrow \qquad \textbf{Ans.}$$

$$v_B = v_0 - g\left(\frac{2v_0 + gt}{2g} - t\right)$$

$$= \frac{1}{2}gt \uparrow \qquad \textbf{Ans.}$$

12–38. As a body is projected to a high altitude above the earth's *surface*, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R + y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81\ \text{m/s}^2$ and $R = 6356$ km, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \to \infty$.

$$v\,dv = a\,dy$$

$$\int_v^0 v\,dv = -g_0R^2\int_0^\infty \frac{dy}{(R+y)^2}$$

$$\frac{v^2}{2}\bigg|_v^0 = \frac{g_0\,R^2}{R+y}\bigg|_0^\infty$$

$$v = \sqrt{2g_0\,R}$$

$$= \sqrt{2(9.81)(6356)(10)^3}$$

$$= 11167\ \text{m/s} = 11.2\ \text{km/s}$$ **Ans.**

12–39. Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12–38), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12–38.

From Prob. 12–38,

$$(+\uparrow) \qquad a = -g_0\frac{R^2}{(R+y)^2}$$

Since $a\,dy = v\,dv$

then

$$-g_0\,R^2\int_{y_0}^{y}\frac{dy}{(R+y)^2} = \int_0^v v\,dv$$

$$g_0\,R^2\left[\frac{1}{R+y}\right]_{y_0}^{y} = \frac{v^2}{2}$$

$$g_0\,R^2\left[\frac{1}{R+y} - \frac{1}{R+y_0}\right] = \frac{v^2}{2}$$

Thus

$$v = -R\sqrt{\frac{2g_0\,(y_0 - y)}{(R+y)(R+y_0)}}$$

When $y_0 = 500$ km, $\quad y = 0$,

$$v = -6356(10^3)\sqrt{\frac{2(9.81)(500)(10^3)}{6356(6356+500)(10^6)}}$$

$$v = -3016\ \text{m/s} = 3.02\ \text{km/s}\ \downarrow$$ **Ans.**

***12–40.** When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f. If this variation of the acceleration can be expressed as $a = (g/v^2{}_f)(v^2{}_f - v^2)$, determine the time needed for the velocity to become $v = v_f/2$. Initially the particle falls from rest.

$$\frac{dv}{dt} = a = \left(\frac{g}{v_f^2}\right)\left(v_f^2 - v^2\right)$$

$$\int_0^v \frac{dv}{v_f^2 - v^{2'}} = \frac{g}{v_f^2}\int_0^t dt$$

$$\frac{1}{2v_f}\ln\left(\frac{v_f + v}{v_f - v}\right)\Bigg|_0^v = \frac{g}{v_f^2}t$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v}{v_f - v}\right)$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v_{f/2}}{v_f - v_{f/2}}\right)$$

$$t = 0.549\left(\frac{v_f}{g}\right)$$ **Ans.**

•12–41. A particle is moving along a straight line such that its position from a fixed point is $s = (12 - 15t^2 + 5t^3)$ m, where t is in seconds. Determine the total distance traveled by the particle from $t = 1$ s to $t = 3$ s. Also, find the average speed of the particle during this time interval.

Velocity:

$$(\overset{+}{\rightarrow}) \qquad v = \frac{ds}{dt} = \frac{d}{dt}\left(12 - 15t^2 + 5t^3\right)$$

$$v = -30t + 15t^2 \text{ m/s}$$

The velocity of the particle changes direction at the instant when it is momentarily brought to rest. Thus,

$$v = -30t + 15t^2 = 0$$

$$t(-30 + 15t) = 0$$

$$t = 0 \text{ and } 2 \text{ s}$$

Position: The positions of the particle at $t = 0$ s, 1 s, 2 s, and 3 s are

$$s\big|_{t=0\text{ s}} = 12 - 15\left(0^2\right) + 5\left(0^3\right) = 12 \text{ m}$$

$$s\big|_{t=1\text{ s}} = 12 - 15\left(1^2\right) + 5\left(1^3\right) = 2 \text{ m}$$

$$s\big|_{t=2\text{ s}} = 12 - 15\left(2^2\right) + 5\left(2^3\right) = -8 \text{ m}$$

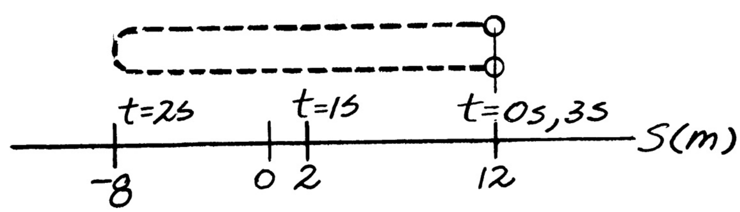
$$s\big|_{t=3\text{ s}} = 12 - 15\left(3^2\right) + 5\left(3^3\right) = 12 \text{ m}$$

Using the above results, the path of the particle is shown in Fig. *a*. From this figure, the distance traveled by the particle during the time interval $t = 1$ s to $t = 3$ s is

$$s_{\text{Tot}} = (2 + 8) + (8 + 12) = 30 \text{ m} \qquad \textbf{Ans.}$$

The average speed of the particle during the same time interval is

$$v_{\text{avg}} = \frac{s_{\text{Tot}}}{\Delta t} = \frac{30}{3 - 1} = 15 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

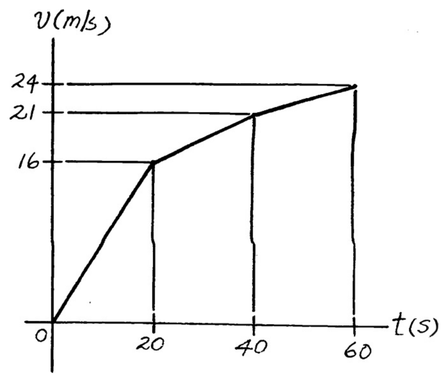
12–42. The speed of a train during the first minute has been recorded as follows:

t (s)	0	20	40	60
v (m/s)	0	16	21	24

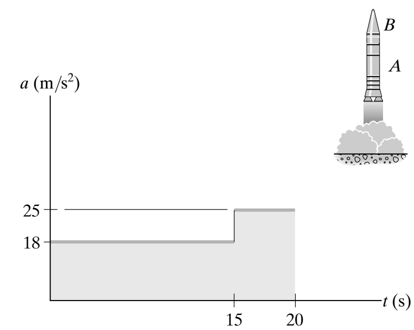
Plot the $v-t$ graph, approximating the curve as straight-line segments between the given points. Determine the total distance traveled.

The total distance traveled is equal to the area under the graph.

$$s_T = \frac{1}{2}(20)(16) + \frac{1}{2}(40 - 20)(16 + 21) + \frac{1}{2}(60 - 40)(21 + 24) = 980 \text{ m}$$ **Ans.**



12–43. A two-stage missile is fired vertically from rest with the acceleration shown. In 15 s the first stage A burns out and the second stage B ignites. Plot the $v-t$ and $s-t$ graphs which describe the two-stage motion of the missile for $0 \leq t \leq 20$ s.



Since $v = \int a\,dt$, the constant lines of the a–t graph become sloping lines for the v–t graph.

The numerical values for each point are calculated from the total area under the a–t graph to the point.

At $t = 15$ s, $\quad v = (18)(15) = 270$ m/s

At $t = 20$ s, $\quad v = 270 + (25)(20 - 15) = 395$ m/s

Since $s = \int v\,dt$, the sloping lines of the v–t graph become parabolic curves for the s–t graph.

The numerical values for each point are calculated from the total area under the v–t graph to the point.

At $t = 15$ s, $\quad s = \frac{1}{2}(15)(270) = 2025$ m

At $t = 20$ s, $\quad s = 2025 + 270(20 - 15) + \frac{1}{2}(395 - 270)(20 - 15) = 3687.5 \text{ m} = 3.69$ km

Also:

$0 \leq t \leq 15$:

$a = 18$

$v = v_0 + a_c t = 0 + 18t$

$s = s_0 + v_0 t + \frac{1}{2} a_c t^2 = 0 + 0 + 9t^2$

At $t = 15$:

$v = 18(15) = 270$

$s = 9(15)^2 = 2025$

$15 \leq t \leq 20$:

$a = 25$

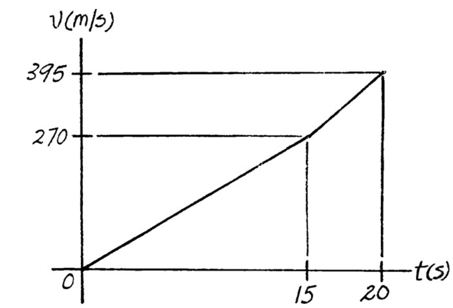
$v = v_0 + a_c t = 270 + 25(t - 15)$

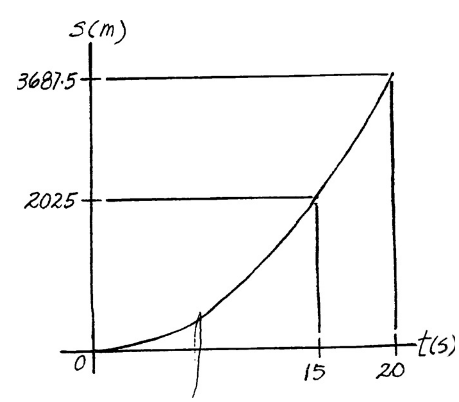
$s = s_0 + v_0 t + \frac{1}{2} a_c t^2 = 2025 + 270(t - 15) + \frac{1}{2}(25)(t - 15)^2$

When $t = 20$:

$v = 395$ m/s

$s = 3687.5 \text{ m} = 3.69$ km



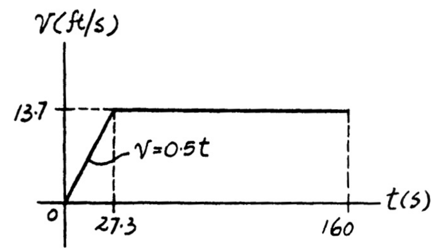


***12–44.** A freight train starts from rest and travels with a constant acceleration of 0.5 ft/s². After a time t' it maintains a constant speed so that when $t = 160$ s it has traveled 2000 ft. Determine the time t' and draw the v–t graph for the motion.

Total Distance Traveled: The distance for part one of the motion can be related to time $t = t'$ by applying Eq. 12–5 with $s_0 = 0$ and $v_0 = 0$.

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(t')^2 = 0.25(t')^2$$



The velocity at time t can be obtained by applying Eq. 12–4 with $v_0 = 0$.

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t = 0 + 0.5t = 0.5t \qquad \textbf{[1]}$$

The time for the second stage of motion is $t_2 = 160 - t'$ and the train is traveling at a constant velocity of $v = 0.5t'$ (Eq. [1]). Thus, the distance for this part of motion is

$$(\overset{+}{\rightarrow}) \qquad s_2 = vt_2 = 0.5t'(160 - t') = 80t' - 0.5(t')^2$$

If the total distance traveled is $s_{\text{Tot}} = 2000$, then

$$s_{\text{Tot}} = s_1 + s_2$$

$$2000 = 0.25(t')^2 + 80t' - 0.5(t')^2$$

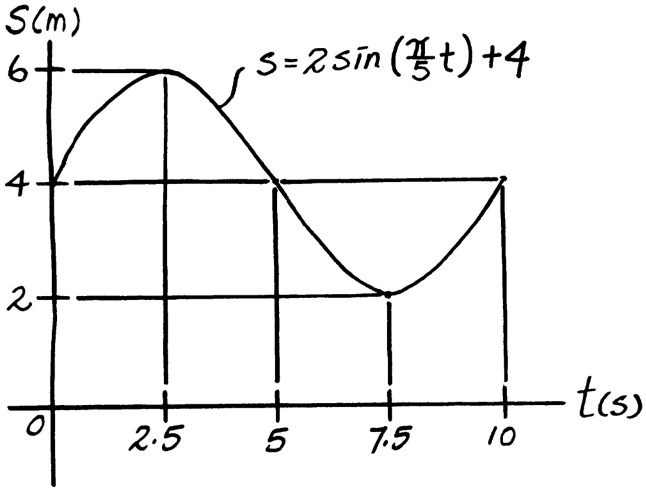
$$0.25(t')^2 - 80t' + 2000 = 0$$

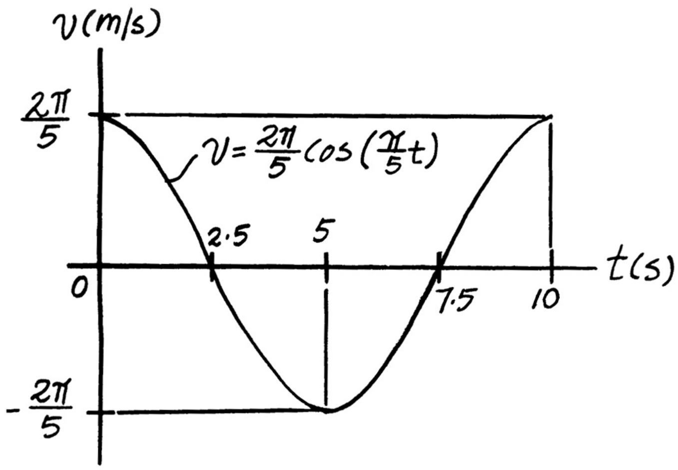
Choose a root that is less than 160 s, then

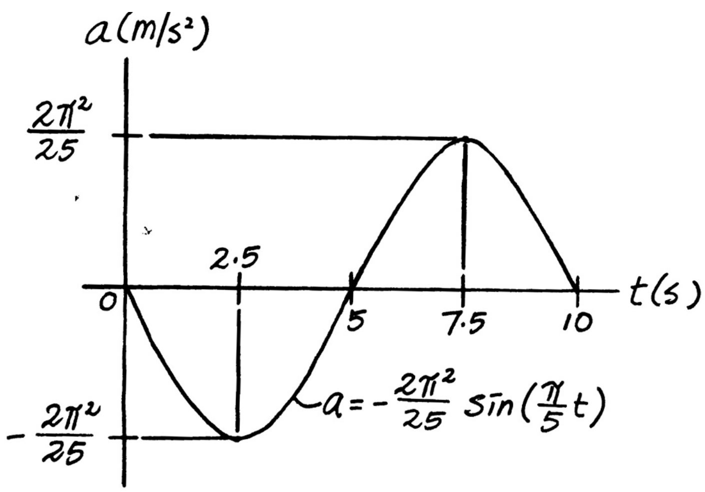
$$t' = 27.34 \text{ s} = 27.3 \text{ s} \qquad \textbf{Ans.}$$

***v* – *t* Graph:** The equation for the velocity is given by Eq. [1]. When $t = t' = 27.34$ s, $v = 0.5(27.34) = 13.7$ ft/s.

•12–45. If the position of a particle is defined by $s = [2 \sin (\pi/5)t + 4]$ m, where t is in seconds, construct the $s-t$, $v-t$, and $a-t$ graphs for $0 \leq t \leq 10$ s.







12–46. A train starts from station A and for the first kilometer, it travels with a uniform acceleration. Then, for the next two kilometers, it travels with a uniform speed. Finally, the train decelerates uniformly for another kilometer before coming to rest at station B. If the time for the whole journey is six minutes, draw the v–t graph and determine the maximum speed of the train.

For stage (1) motion,

$(\overset{+}{\rightarrow})$ $\quad v_1 = v_0 + (a_c)_1 t$

$$v_{max} = 0 + (a_c)_1 t_1$$

$$v_{max} = (a_c)_1 t_1 \quad \textbf{(1)}$$

$(\overset{+}{\rightarrow})$ $\quad v_1^{\,2} = v_0^{\,2} + 2(a_c)_1(s_1 - s_0)$

$$v_{max}^{\,2} = 0 + 2(a_c)_1(1000 - 0)$$

$$(a_c)_1 = \frac{v_{max}^{\,2}}{2000} \quad \textbf{(2)}$$

Eliminating $(a_c)_1$ from Eqs. (1) and (2), we have

$$t_1 = \frac{2000}{v_{max}} \quad \textbf{(3)}$$

For stage (2) motion, the train travels with the constant velocity of v_{max} for $t = (t_2 - t_1)$. Thus,

$(\overset{+}{\rightarrow})$ $\quad s_2 = s_1 + v_1 t + \frac{1}{2}(a_c)_2 t^2$

$$1000 + 2000 = 1000 + v_{max}(t_2 - t_1) + 0$$

$$t_2 - t_1 = \frac{2000}{v_{max}} \quad \textbf{(4)}$$

For stage (3) motion, the train travels for $t = 360 - t_2$. Thus,

$(\overset{+}{\rightarrow})$ $\quad v_3 = v_2 + (a_c)_3 t$

$$0 = v_{max} - (a_c)_3(360 - t_2)$$

$$v_{max} = (a_c)_3(360 - t_2) \quad \textbf{(5)}$$

$(\overset{+}{\rightarrow})$ $\quad v_3^{\,2} = v_2^{\,2} + 2(a_c)_3(s_3 - s_2)$

$$0 = v_{max}^{\,2} + 2\left[-(a_c)_3\right](4000 - 3000)$$

$$(a_c)_3 = \frac{v_{max}^{\,2}}{2000} \quad \textbf{(6)}$$

Eliminating $(a_c)_3$ from Eqs. (5) and (6) yields

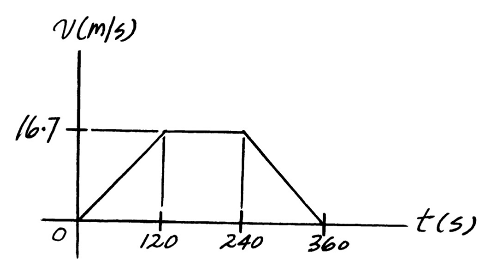
$$360 - t_2 = \frac{2000}{v_{max}} \quad \textbf{(7)}$$

Solving Eqs. (3), (4), and (7), we have

$$t_1 = 120 \text{ s} \qquad t_2 = 240 \text{ s}$$

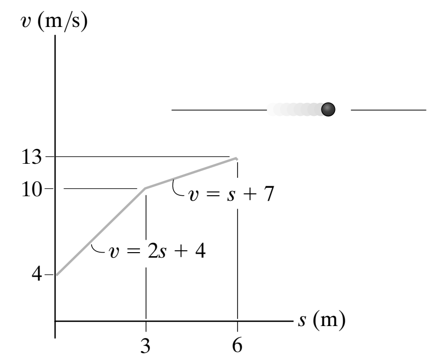
$$v_{max} = 16.7 \text{ m/s} \quad \textbf{Ans.}$$

Based on the above results the $v-t$ graph is shown in Fig. a.



(a)

12–47. The particle travels along a straight line with the velocity described by the graph. Construct the $a-s$ graph.



***a* − *s* Graph:** For $0 \leq s < 3$ m,

$$(\overset{+}{\rightarrow}) \qquad a = v\frac{dv}{ds} = (2s + 4)(2) = (4s + 8)\ \text{m/s}^2$$

At $s = 0$ m and 3 m,

$$a|_{s=0\text{ m}} = 4(0) + 8 = 8\ \text{m/s}^2$$

$$a|_{s=3\text{ m}} = 4(3) + 8 = 20\ \text{m/s}^2$$

For $3\text{m} < s \leq 6$ m,

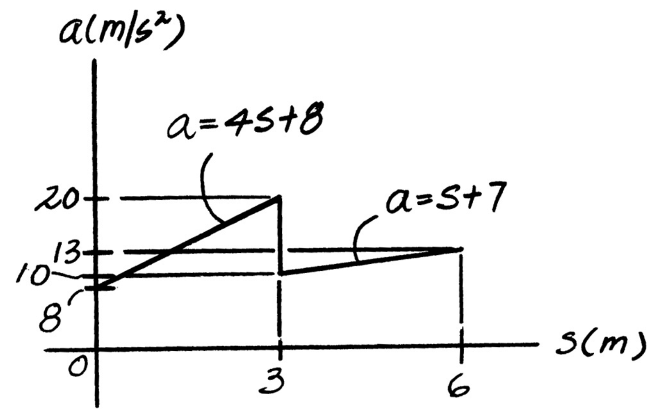
$$(\overset{+}{\rightarrow}) \qquad a = v\frac{dv}{ds} = (s + 7)(1) = (s + 7)\ \text{m/s}^2$$

At $s = 3$ m and 6 m,

$$a|_{s=3\text{ m}} = 3 + 7 = 10\ \text{m/s}^2$$

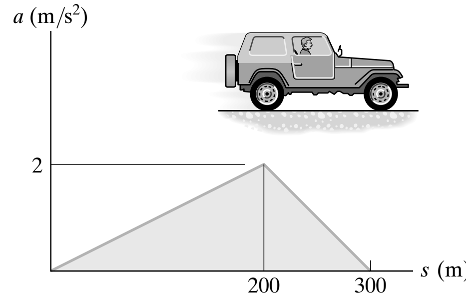
$$a|_{s=6\text{ m}} = 6 + 7 = 13\ \text{m/s}^2$$

The $a-s$ graph is shown in Fig. a.



(a)

***12–48.** The a–s graph for a jeep traveling along a straight road is given for the first 300 m of its motion. Construct the v–s graph. At $s = 0$, $v = 0$.

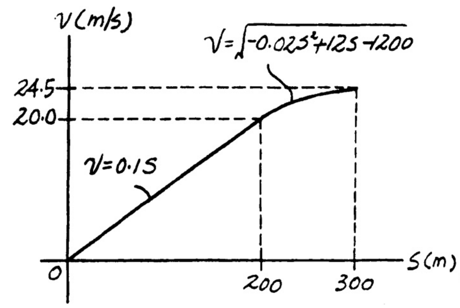


$a-s$ Graph: The function of acceleration a in terms of s for the interval $0 \text{ m} \leq s < 200 \text{ m}$ is

$$\frac{a-0}{s-0} = \frac{2-0}{200-0} \qquad a = (0.01s) \text{ m/s}^2$$

For the interval $200 \text{ m} < s \leq 300 \text{ m}$,

$$\frac{a-2}{s-200} = \frac{0-2}{300-200} \qquad a = (-0.02s + 6) \text{ m/s}^2$$



$v-s$ Graph: The function of velocity v in terms of s can be obtained by applying $vdv = ads$. For the interval $\mathbf{0 \text{ m} \leq s < 200 \text{ m}}$,

$$vdv = ds$$

$$\int_0^v vdv = \int_0^s 0.01sds$$

$$v = (0.1s) \text{ m/s}$$

At $s = 200$ m, $\qquad v = 0.100(200) = 20.0 \text{ m/s}$

For the interval $\mathbf{200 \text{ m} < s \leq 300 \text{ m}}$,

$$vdv = ads$$

$$\int_{20.0\text{m/s}}^v vdv = \int_{200m}^s (-0.02s + 6)ds$$

$$v = \left(\sqrt{-0.02s^2 + 12s - 1200}\right) \text{ m/s}$$

At $s = 300$ m, $\qquad v = \sqrt{-0.02(300^2) + 12(300) - 1200} = 24.5 \text{ m/s}$

•12–49. A particle travels along a curve defined by the equation $s = (t^3 - 3t^2 + 2t)$ m. where t is in seconds. Draw the $s - t$, $v - t$, and $a - t$ graphs for the particle for $0 \le t \le 3$ s.

$$s = t^3 - 3t^2 + 2t$$

$$v = \frac{ds}{dt} = 3t^2 - 6t + 2$$

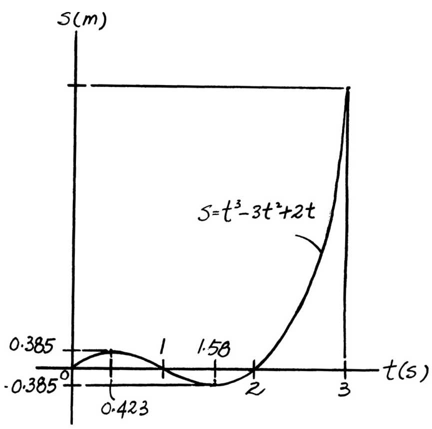
$$a = \frac{dv}{dt} = 6t - 6$$

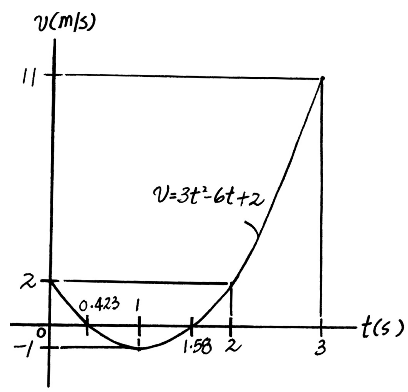
$v = 0$ at $0 = 3t^2 - 6t + 2$

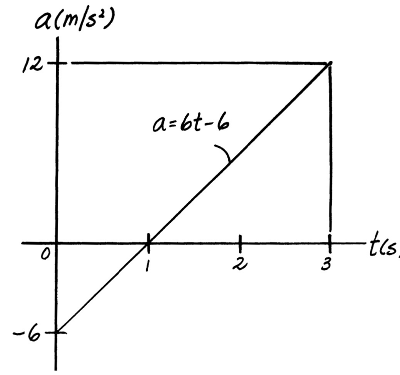
$t = 1.577$ s, and $t = 0.4226$ s,

$s|_{t=1.577} = -0.386$ m

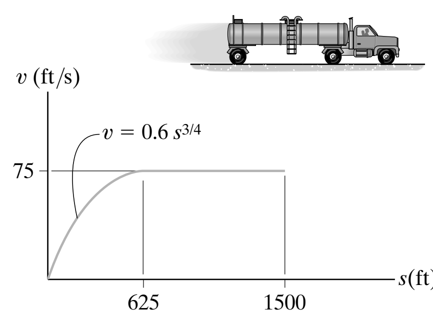
$s|_{t=0.4226} = 0.385$ m







12–50. A truck is traveling along the straight line with a velocity described by the graph. Construct the $a-s$ graph for $0 \le s \le 1500$ ft.



***a* − *s* Graph:** For $0 \le s < 625$ ft,

$$(\overset{+}{\rightarrow}) \qquad a = v\frac{dv}{ds} = \left(0.6s^{3/4}\right)\left[\frac{3}{4}(0.6)s^{-1/4}\right] = \left(0.27s^{1/2}\right)\text{ft}/\text{s}^2$$

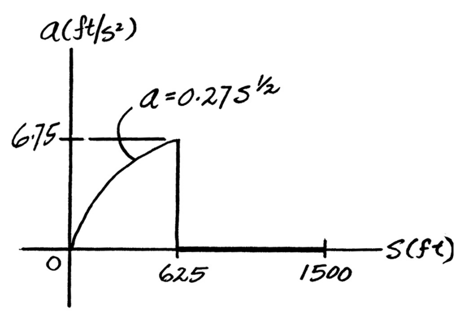
At $s = 625$ft,

$$a|_{s=625\text{ ft}} = 0.27\left(625^{1/2}\right) = 6.75\text{ft/s}^2$$

For $625\text{ ft} < s < 1500\text{ft}$,

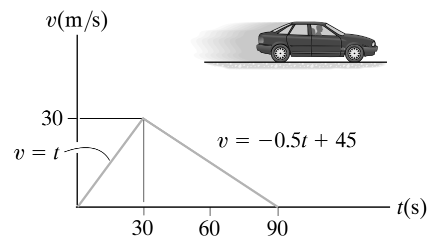
$$(\overset{+}{\rightarrow}) \qquad a = v\frac{dv}{ds} = 75(0) = 0$$

The $a-s$ graph is shown in Fig. a.



(a)

12–51. A car starts from rest and travels along a straight road with a velocity described by the graph. Determine the total distance traveled until the car stops. Construct the s–t and a–t graphs.



$s-t$ Graph: For the time interval $0 \le t < 30$ s, the initial condition is $s = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t t\,dt$$

$$s = \left(\frac{t^2}{2}\right) \text{m}$$

When $t = 30$ s,

$$s = \frac{30^2}{2} = 450 \text{ m}$$

For the time interval $30 \text{ s} < t \le 90$ s, the initial condition is $s = 450$ m when $t = 30$ s.

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

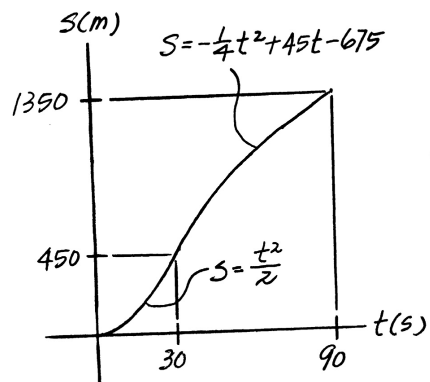
$$\int_{450\text{ m}}^s ds = \int_{30\text{s}}^t (-0.5t + 45)dt$$

$$s = \left(-\frac{1}{4}t^2 + 45t - 675\right) \text{m}$$

When $t = 90$ s,

$$s\big|_{t=90\text{ s}} = -\frac{1}{4}\left(90^2\right) + 45(90) - 675 = 1350 \text{ m} \qquad \textbf{Ans.}$$

The $s-t$ graph shown is in Fig. *a*.



(a)

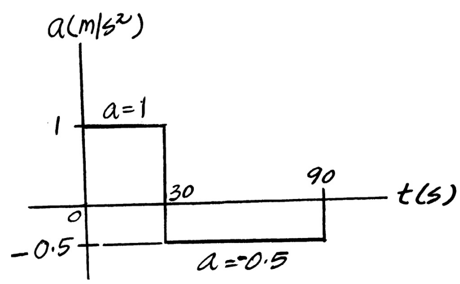
$a-t$ Graph: For the time interval $0 < t < 30$ s,

$$a = \frac{dv}{dt} = \frac{d}{dt}(t) = 1 \text{ m/s}^2$$

For the time interval $30 \text{ s} < t \le 90$ s,

$$a = \frac{dv}{dt} = \frac{d}{dt}(-0.5t + 45) = -0.5 \text{ m/s}^2$$

The $a-t$ graph is shown in Fig. *b*.



(b)

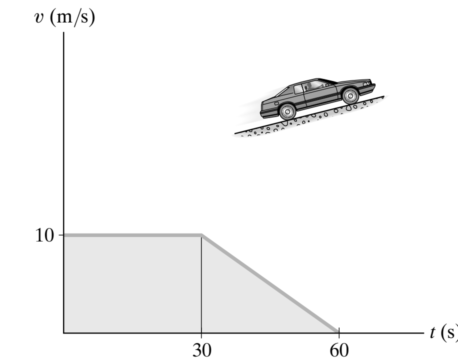
Note: Since the change in position of the car is equal to the area under the $v-t$ graph, the total distance traveled by the car is

$$\Delta s = \int v\,dt$$

$$s\big|_{t=90\text{ s}} - 0 = \frac{1}{2}(90)(30)$$

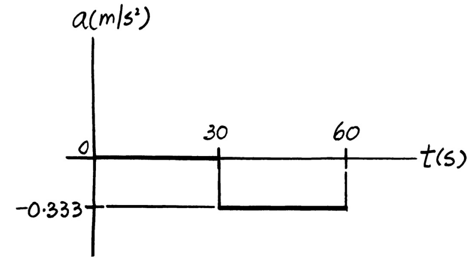
$$s\big|_{t=90\text{ s}} = 1350 \text{ s}$$

***12–52.** A car travels up a hill with the speed shown. Determine the total distance the car travels until it stops ($t = 60$ s). Plot the $a-t$ graph.

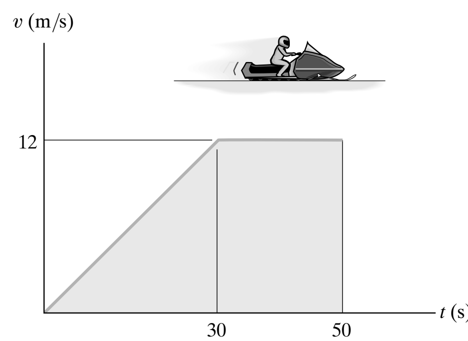


Distance traveled is area under $v-t$ graph.

$$s = (10)(30) + \frac{1}{2}(10)(30) = 450 \text{ m}$$ **Ans.**



•12–53. The snowmobile moves along a straight course according to the v–t graph. Construct the s–t and a–t graphs for the same 50-s time interval. When $t = 0$, $s = 0$.



***s* − *t* Graph:** The position function in terms of time t can be obtained by applying $v = \dfrac{ds}{dt}$. For time interval $0 \text{ s} \le t < 30 \text{ s}$, $v = \dfrac{12}{30}t = \left(\dfrac{2}{5}t\right)$ m/s.

$$ds = vdt$$

$$\int_0^s ds = \int_0^t \frac{2}{5}tdt$$

$$s = \left(\frac{1}{5}t^2\right) \text{m}$$

At $t = 30$ s, $$s = \frac{1}{5}\left(30^2\right) = 180 \text{ m}$$

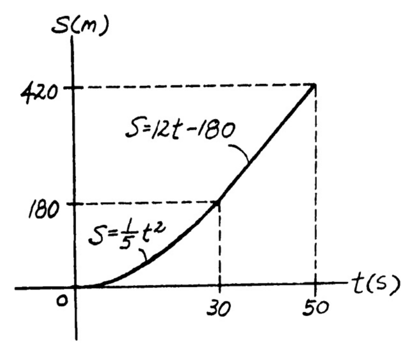
For time interval $\mathbf{30\ s < t \le 50\ s}$,

$$ds = vdt$$

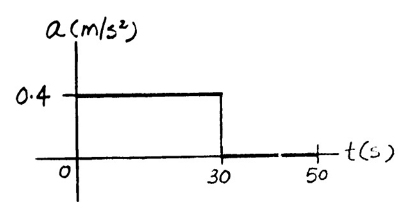
$$\int_{180 \text{ m}}^{s} ds = \int_{30\,s}^{t} 12dt$$

$$s = (12t - 180) \text{ m}$$

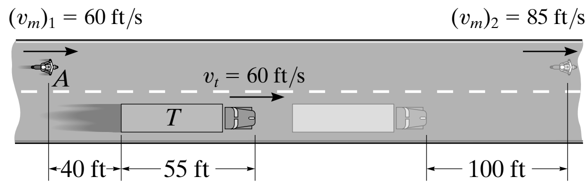
At $t = 50$ s, $$s = 12(50) - 180 = 420 \text{ m}$$



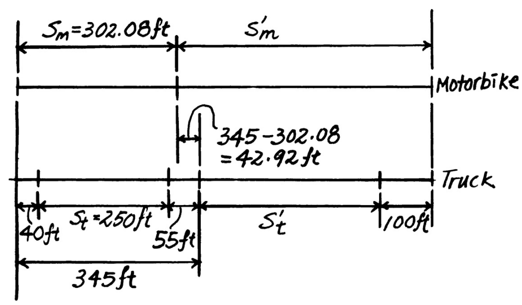
***a* − *t* Graph:** The acceleration function in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$. For time interval $\mathbf{0\ s \le t < 30\ s}$ and $\mathbf{30\ s < t \le 50\ s}$, $a = \dfrac{dv}{dt} = \dfrac{2}{5} = 0.4 \text{ m/s}^2$ and $a = \dfrac{dv}{dt} = 0$, respectively.



12–54. A motorcyclist at A is traveling at 60 ft/s when he wishes to pass the truck T which is traveling at a constant speed of 60 ft/s. To do so the motorcyclist accelerates at 6 ft/s^2 until reaching a maximum speed of 85 ft/s. If he then maintains this speed, determine the time needed for him to reach a point located 100 ft in front of the truck. Draw the $v-t$ and $s-t$ graphs for the motorcycle during this time.



Motorcycle:



Time to reach 85 ft/s,

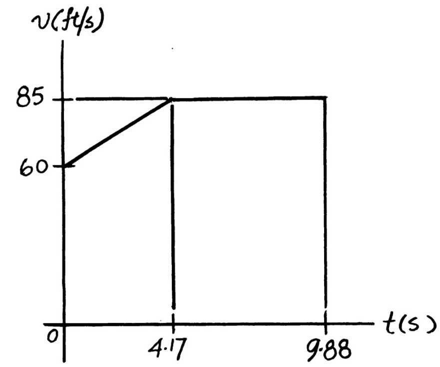
$$v = v_0 + a_c t$$

$$85 = 60 + 6t$$

$$t = 4.167 \text{ s}$$

$$v^2 = v_0^2 + 2a_c\,(s - s_0)$$

Distance traveled,



$$(85)^2 = (60)^2 + 2(6)(s_m - 0)$$

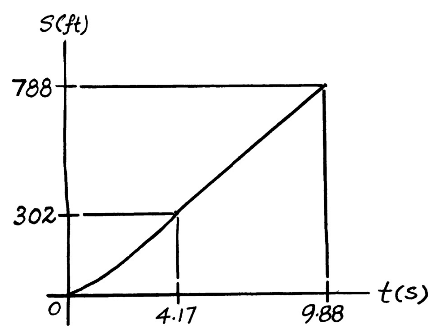
$$s_m = 302.08 \text{ ft}$$

In $t = 4.167$ s, truck travels

$$s_t = 60(4.167) = 250 \text{ ft}$$

Further distance for motorcycle to travel: $40 + 55 + 250 + 100 - 302.08 = 142.92$ ft

Motorcycle:



$$s = s_0 + v_0\,t$$

$$(s + 142.92) = 0 + 85t'$$

Truck:

$$s = 0 + 60t'$$

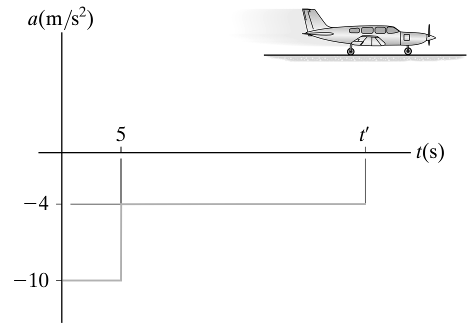
Thus $t' = 5.717$ s

$$t = 4.167 + 5.717 = 9.88 \text{ s}$$ **Ans.**

Total distance motorcycle travels

$$s_T = 302.08 + 85(5.717) = 788 \text{ ft}$$

12–55. An airplane traveling at 70 m/s lands on a straight runway and has a deceleration described by the graph. Determine the time t' and the distance traveled for it to reach a speed of 5 m/s. Construct the v–t and s–t graphs for this time interval, $0 \leq t \leq t'$.



***v* – *t* Graph:** For the time interval $0 \leq t < 5$ s, the initial condition is $v = 70$ m/s when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = adt$$

$$\int_{70\text{ m/s}}^{v} dv = \int_0^t -10dt$$

$$v = (-10t + 70)\text{ m/s}$$

When $t = 5$ s,

$$v|_{t=5\text{ s}} = -10(5) + 70 = 20\text{ m/s}$$

For the time interval $5\text{ s} < t \leq t'$, the initial condition is $v = 20$ m/s when $t = 5$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = adt$$

$$\int_{20\text{ m/s}}^{v} dv = \int_{5\text{ s}}^{t} -4dt$$

$$v = (-4t + 40)\text{ m/s}$$

When $v = 5$ m/s,

$$5 = -4t' + 40 \qquad t' = 8.75\text{ s} \qquad \textbf{Ans.}$$

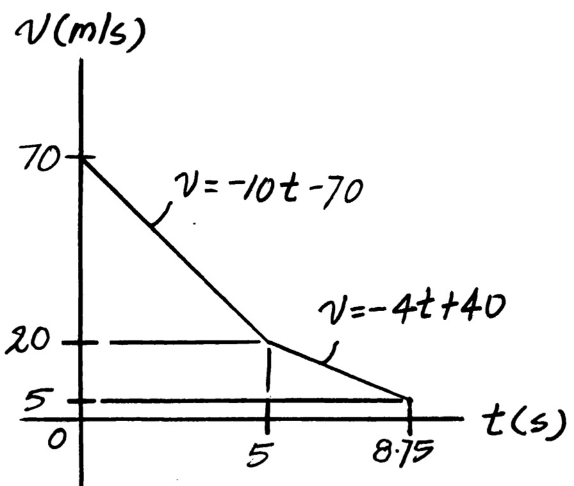
Also, the change in velocity is equal to the area under the a–t graph. Thus,

$$\Delta v = \int adt$$

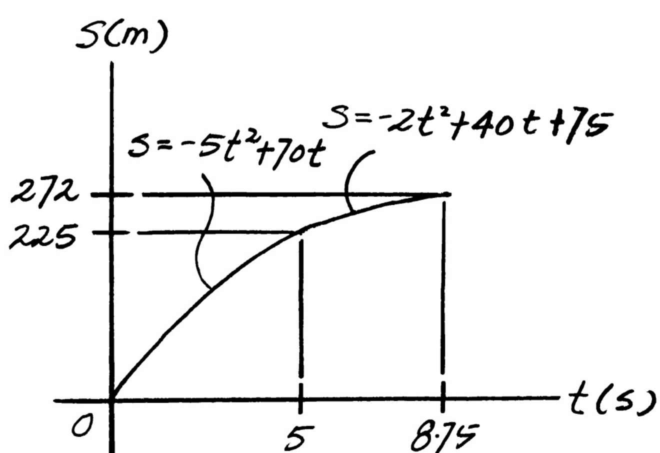
$$5 - 70 = -\left[5(10) + 4(t' - 5)\right]$$

$$t' = 8.75s'$$

The v–t graph is shown in Fig. *a*.



(a)



(b)

12–55. Continued

***s – t* Graph:** For the time interval $0 \leq t < 5$ s, the initial condition is $s = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t (-10t + 70)\,dt$$

$$s = \left(-5t^2 + 70t\right)\text{m}$$

When $t = 5$ s,

$$s\Big|_{t=5\text{ s}} = -5\left(5^2\right) + 70(5) = 225\text{ m}$$

For the time interval $5 < t \leq t' = 8.75$ s the initial condition is $s = 225$ m when $t = 5$ s.

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_{225\text{ m}}^s ds = \int_5^t (-4t + 40)\,dt$$

$$s = \left(-2t^2 + 40t + 75\right)\text{m}$$

When $t = t' = 8.75$ s,

$$s\Big|_{t=8.75\text{ s}} = -2\left(8.75^2\right) + 40(8.75) + 75 = 271.875\text{ m} = 272\text{ m} \qquad \textbf{Ans.}$$

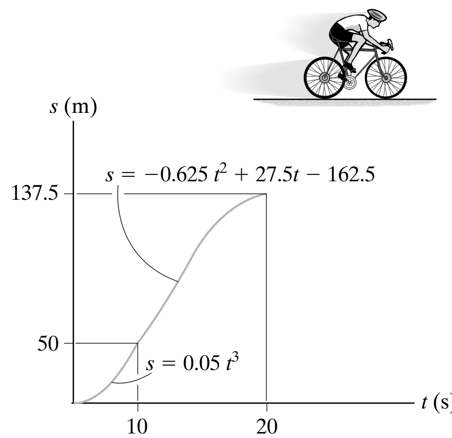
Also, the change in position is equal to the area under the *v–t* graph. Referring to Fig. *a*, we have

$$\Delta s = \int v\,dt$$

$$s\Big|_{t=8.75\text{ s}} - 0 = \frac{1}{2}(70 + 20)(5) + \frac{1}{2}(20 + 5)(3.75) = 271.875\text{ m} = 272\text{ m} \qquad \textbf{Ans.}$$

The *s–t* graph is shown in Fig. *b*.

***12–56.** The position of a cyclist traveling along a straight road is described by the graph. Construct the v–t and a–t graphs.



$v-t$ Graph: For the time interval $0 \leq t < 10$ s,

$$(\xrightarrow{+}) \qquad v = \frac{ds}{dt} = \frac{d}{dt}\left(0.05t^3\right) = \left(0.15t^2\right)\text{ m/s}$$

When $t = 0$ s and 10 s,

$$v\big|_{t=0} = 0.15\left(0^2\right) = 0 \qquad v\big|_{t=10\text{ s}} = 0.15\left(10^2\right) = 15\text{ m/s}$$

For the time interval $10\text{ s} < t \leq 20$ s,

$$(\xrightarrow{+}) \qquad v = \frac{ds}{dt} = \frac{d}{dt}\left(-0.625t^2 + 27.5t - 162.5\right) = (-1.25t + 27.5)\text{ m/s}$$

When $t = 10$ s and 20 s,

$$v\big|_{t=10\text{ s}} = -1.25(10) + 27.5 = 15\text{ m/s}$$

$$v\big|_{t=20\text{ s}} = -1.25(20) + 27.5 = 2.5\text{ m/s}$$

The v–t graph is shown in Fig. *a*.

$a-t$ Graph: For the time interval $0 \leq t < 10$ s,

$$(\xrightarrow{+}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}\left(0.15t^2\right) = (0.3t)\text{ m/s}^2$$

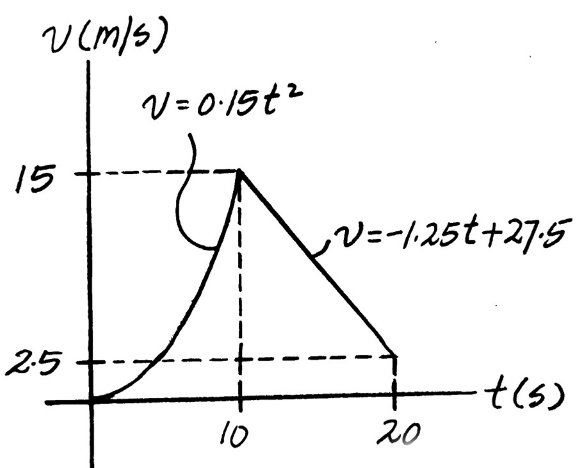
When $t = 0$ s and 10 s,

$$a\big|_{t=0\text{ s}} = 0.3(0) = 0 \qquad a\big|_{t=10\text{ s}} = 0.3(10) = 3\text{ m/s}^2$$

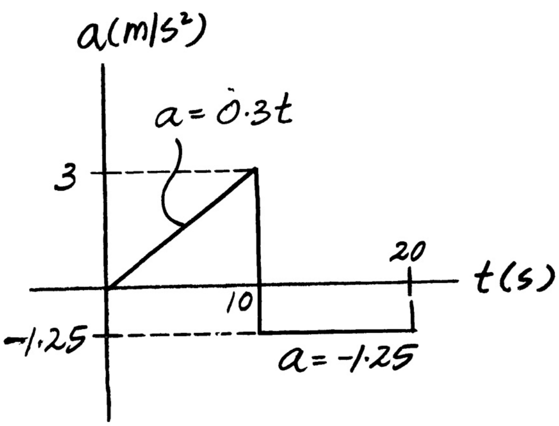
For the time interval $10\text{ s} < t \leq 20$ s,

$$(\xrightarrow{+}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(-1.25t + 27.5) = -1.25\text{ m/s}^2$$

the a–t graph is shown in Fig. *b*.

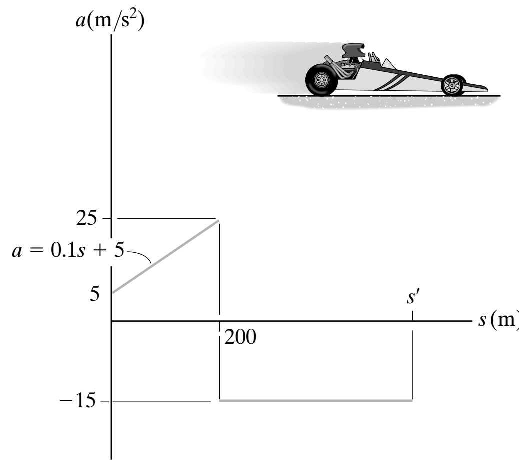


(a)



(b)

•12–57. The dragster starts from rest and travels along a straight track with an acceleration-deceleration described by the graph. Construct the $v-s$ graph for $0 \leq s \leq s'$, and determine the distance s' traveled before the dragster again comes to rest.



$v-s$ Graph: For $0 \leq s < 200$ m, the initial condition is $v = 0$ at $s = 0$.

$$(\overset{+}{\rightarrow}) \qquad v dv = a ds$$

$$\int_0^v v dv = \int_0^s (0.1s + 5) ds$$

$$\left.\frac{v^2}{2}\right|_0^v = \left.\left(0.05s^2 + 5s\right)\right|_0^s$$

$$v = \left(\sqrt{0.1s^2 + 10s}\right) \text{ m/s}$$

At $s = 200$ m,

$$v|_{s=200 \text{ m}} = \sqrt{0.1\left(200^2\right) + 10(200)} = 77.46 \text{ m/s} = 77.5 \text{ m/s}$$

For 200 m $< s \leq s'$, the initial condition is $v = 77.46$ m/s at $s = 200$ m.

$$(\overset{+}{\rightarrow}) \qquad v dv = a ds$$

$$\int_{77.46 \text{ m/s}}^{v} v\, dv = \int_{200 \text{ m}}^{s} -15 ds$$

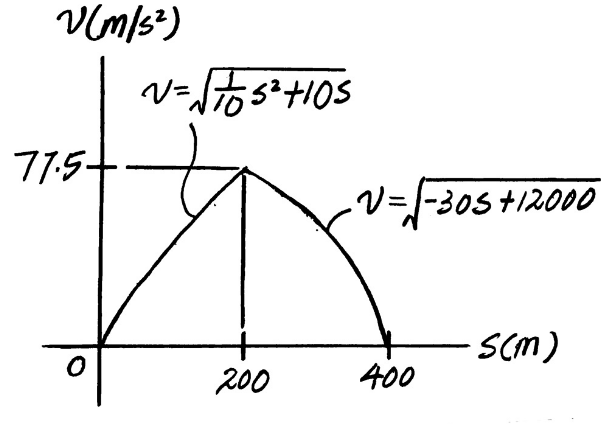
$$\left.\frac{v^2}{2}\right|_{77.46 \text{ m/s}}^{v} = -15s\Big|_{200 \text{ m}}^{s}$$

$$v = \left(\sqrt{-30s + 12000}\right) \text{ m/s}$$

When $v = 0$,

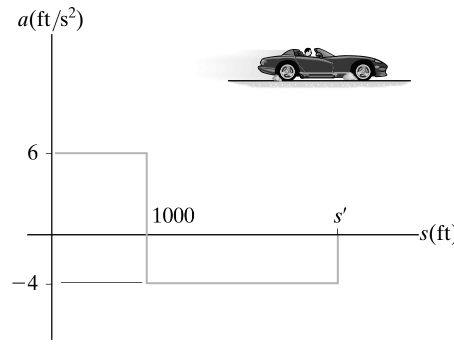
$$0 = \sqrt{-30s' + 12000} \qquad s' = 400 \text{ m} \qquad \textbf{Ans.}$$

The $v–s$ graph is shown in Fig. a.



(a)

12–58. A sports car travels along a straight road with an acceleration-deceleration described by the graph. If the car starts from rest, determine the distance s' the car travels until it stops. Construct the $v-s$ graph for $0 \leq s \leq s'$.



***v* − *s* Graph:** For $0 \leq s < 1000$ ft, the initial condition is $v = 0$ at $s = 0$.

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_0^s 6\,ds$$

$$\frac{v^2}{2} = 6s$$

$$v = \left(\sqrt{12}s^{1/2}\right)\text{ ft/s}$$

When $s = 1000$ ft,

$$v = \sqrt{12}(1000)^{1/2} = 109.54\text{ ft/s} = 110\text{ ft/s}$$

For $1000\text{ ft} < s \leq s'$, the initial condition is $v = 109.54$ ft/s at $s = 1000$ ft.

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_{109.54\text{ ft/s}}^v v\,dv = \int_{1000\text{ ft}}^s -4\,ds$$

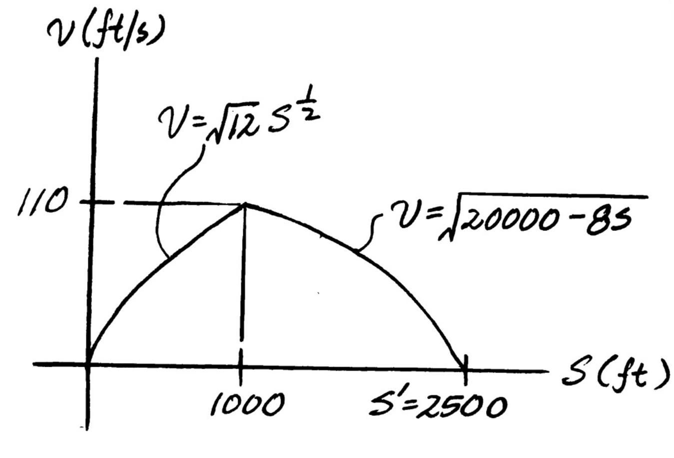
$$\left.\frac{v^2}{2}\right|_{109.54\text{ ft/s}}^v = -4s\Big|_{1000\text{ ft}}^s$$

$$v = \left(\sqrt{20\,000 - 8s}\right)\text{ ft/s}$$

When $v = 0$,

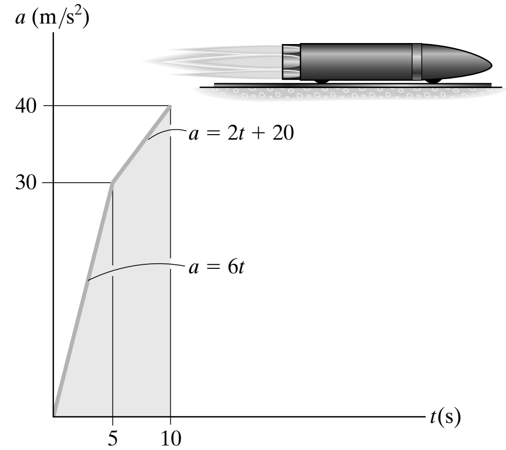
$$0 = \sqrt{20\,000 - 8s'} \qquad\qquad s' = 2500\text{ ft} \qquad\qquad \textbf{Ans.}$$

The v–s graph is shown in Fig. a.



(a)

12–59. A missile starting from rest travels along a straight track and for 10 s has an acceleration as shown. Draw the v–t graph that describes the motion and find the distance traveled in 10 s.



For $t \leq 5$ s,

$$a = 6t$$

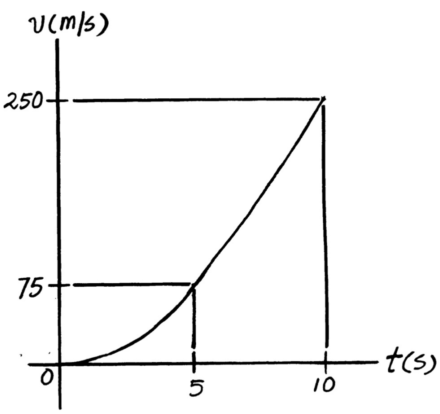
$$dv = a\,dt$$

$$\int_0^v dv = \int_0^t 6t\,dt$$

$$v = 3t^2$$

When $t = 5$ s,

$$v = 75 \text{ m/s}$$



For $5 < t < 10$ s,

$$a = 2t + 20$$

$$dv = a\,dt$$

$$\int_{75}^v dv = \int_5^t (2t + 20)\,dt$$

$$v - 75 = t^2 + 20t - 125$$

$$v = t^2 + 20t - 50$$

When $t = 10$ s,

$$v = 250 \text{ m/s}$$

Distance at $t = 5$ s:

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^5 3t^2\,dt$$

$$s = (5)^3 = 125 \text{ m}$$

Distance at $t = 10$ s:

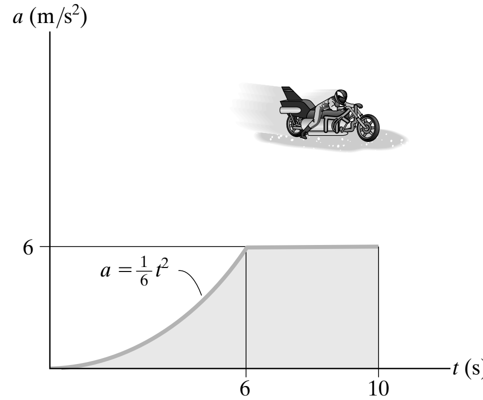
$$ds = v\,dv$$

$$\int_{125}^s ds = \int_5^{10} \left(t^2 + 20t - 50\right)dt$$

$$s - 125 = \frac{1}{3}t^3 + 10t^2 - 50t\Big]_5^{10}$$

$$s = 917 \text{ m}$$ **Ans.**

***12–60.** A motorcyclist starting from rest travels along a straight road and for 10 s has an acceleration as shown. Draw the $v-t$ graph that describes the motion and find the distance traveled in 10 s.



For $0 \leq t < 6$ $\qquad dv = a\,dt$

$$\int_0^v dv = \int_0^t \frac{1}{6}t^2\,dt$$

$$v = \frac{1}{18}t^3$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \frac{1}{18}t^3\,dt$$

$$s = \frac{1}{72}t^4$$

When $t = 6$ s, $\qquad v = 12$ m/s $\qquad s = 18$ m

For $6 < t \leq 10$ $\qquad dv = a\,dt$

$$\int_{12}^v dv = \int_6^t 6\,dt$$

$$v = 6t - 24$$

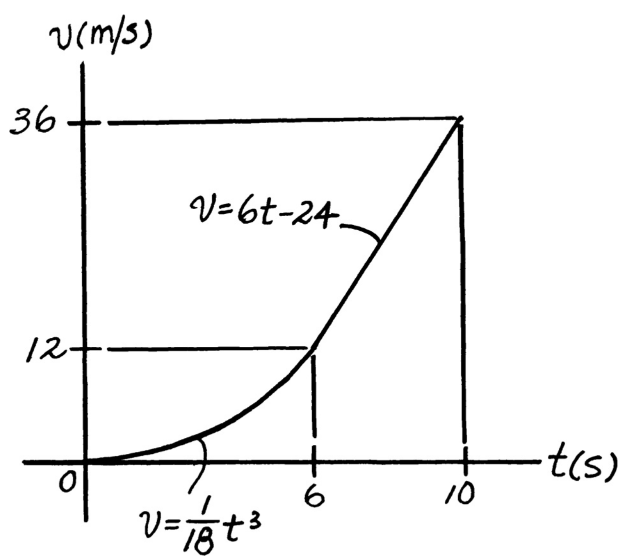
$$ds = v\,dt$$

$$\int_{18}^s ds = \int_6^t (6t - 24)\,dt$$

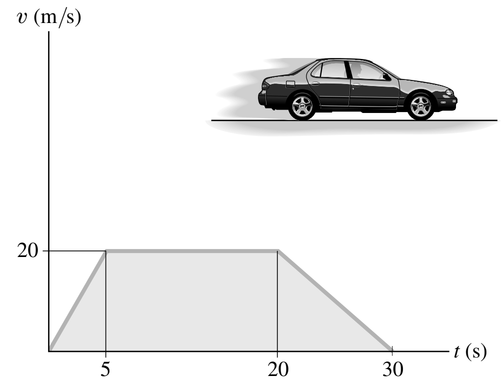
$$s = 3t^2 - 24t + 54$$

When $t = 10$ s, $\qquad v = 36$ m/s

$s = 114$ m **Ans.**



•12–61. The $v-t$ graph of a car while traveling along a road is shown. Draw the $s-t$ and $a-t$ graphs for the motion.



$0 \le t \le 5 \qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{20}{5} = 4\ \text{m/s}^2$

$5 \le t \le 20 \qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{20 - 20}{20 - 5} = 0\ \text{m/s}^2$

$20 \le t \le 30 \qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{0 - 20}{30 - 20} = -2\ \text{m/s}^2$

From the v–t graph at $t_1 = 5\ s$, $t_2 = 20\ s$, and $t_3 = 30$ s,

$s_1 = A_1 = \dfrac{1}{2}(5)(20) = 50\ \text{m}$

$s_2 = A_1 + A_2 = 50 + 20(20 - 5) = 350\ \text{m}$

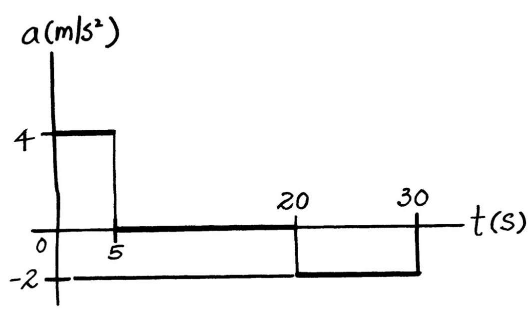
$s_3 = A_1 + A_2 + A_3 = 350 + \dfrac{1}{2}(30 - 20)(20) = 450\ \text{m}$

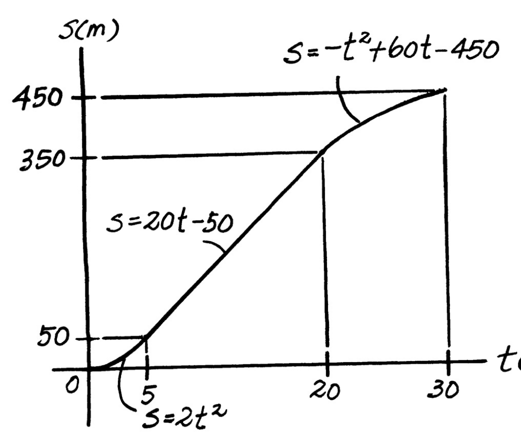
The equations defining the portions of the s–t graph are

$0 \le t \le 5\ \text{s} \qquad v = 4t; \qquad ds = v\,dt; \qquad \int_0^s ds = \int_0^t 4t\,dt; \qquad s = 2t^2$

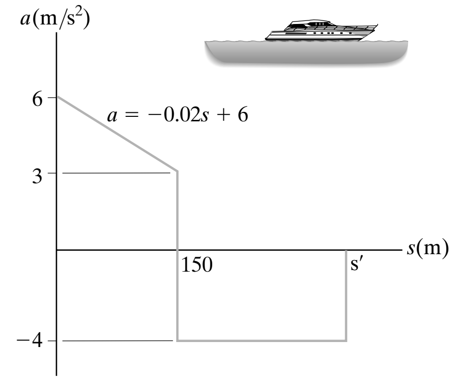
$5 \le t \le 20\ \text{s} \qquad v = 20; \qquad ds = v\,dt; \qquad \int_{50}^s ds = \int_5^t 20\,dt; \qquad s = 20t - 50$

$20 \le t \le 30\ \text{s} \qquad v = 2(30 - t); \qquad ds = v\,dt; \qquad \int_{350}^s ds = \int_{20}^t 2(30 - t)\,dt; \qquad s = -t^2 + 60t - 450$





12–62. The boat travels in a straight line with the acceleration described by the $a-s$ graph. If it starts from rest, construct the $v-s$ graph and determine the boat's maximum speed. What distance s' does it travel before it stops?



***v − s* Graph:** For $0 \leq s < 150$ m, the initial condition is $v = 0$ at $s = 0$.

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_0^s (-0.02s + 6)\,ds$$

$$\left.\frac{v^2}{2}\right|_0^v = \left(-0.01s^2 + 6s\right)\Big|_0^s$$

$$v = \left(\sqrt{-0.02s^2 + 12s}\right) \text{ m/s}$$

The maximum velocity of the boat occurs at $s = 150$ m, where its acceleration changes sign. Thus,

$$v_{max} = v\big|_{s=150\text{ m}} = \sqrt{-0.02\left(150^2\right) + 12(150)} = 36.74 \text{ m/s} = 36.7 \text{ m/s} \qquad \textbf{Ans.}$$

For 150 m $< s < s'$, the initial condition is $v = 36.74$ m/s at $s = 150$ m.

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_{36.74\text{ m/s}}^v v\,dv = \int_{150\text{ m}}^s -4\,ds$$

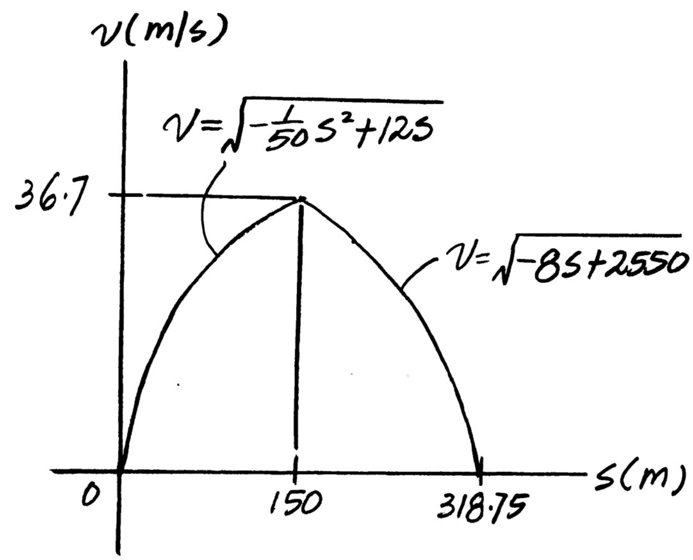
$$\left.\frac{v^2}{2}\right|_{36.74\text{ m/s}}^v = -4s\Big|_{150\text{ m}}^s$$

$$v = \sqrt{-8s + 2550} \text{ m/s}$$

Thus, when $v = 0$,

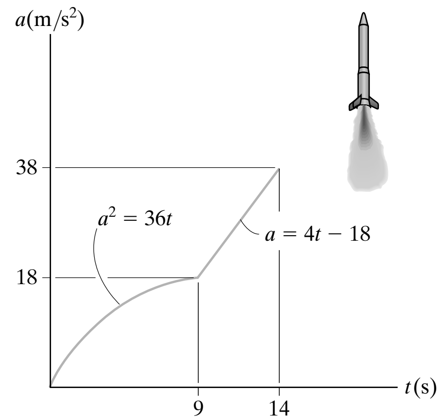
$$0 = \sqrt{-8s' + 2550} \qquad s' = 318.7 \text{ m} = 319 \text{ m} \qquad \textbf{Ans.}$$

The v–s graph is shown in Fig. a.



(a)

12–63. The rocket has an acceleration described by the graph. If it starts from rest, construct the $v-t$ and $s-t$ graphs for the motion for the time interval $0 \le t \le 14\,\text{s}$.



$v-t$ Graph: For the time interval $0 \le t < 9$ s, the initial condition is $v = 0$ at $s = 0$.

$$(+\uparrow) \qquad dv = a\,dt$$

$$\int_0^v dv = \int_0^t 6t^{1/2}dt$$

$$v = \left(4t^{3/2}\right)\text{m/s}$$

When $t = 9$ s,

$$v\big|_{t=9\,\text{s}} = 4\left(9^{3/2}\right) = 108\text{ m/s}$$

The initial condition is $v = 108$ m/s at $t = 9$ s.

$$(+\uparrow) \qquad dv = a\,dt$$

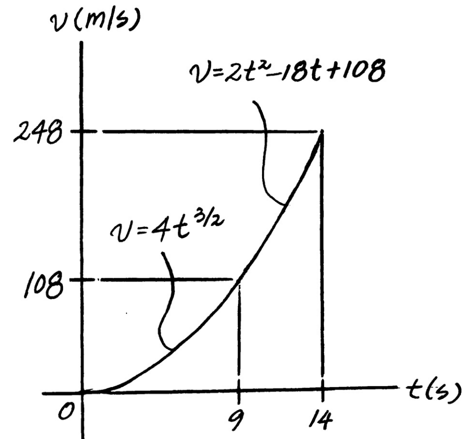
$$\int_{108\text{ m/s}}^v dv = \int_{9\text{ s}}^t (4t - 18)dt$$

$$v = \left(2t^2 - 18t + 108\right)\text{ m/s}$$

When $t = 14$ s,

$$v\big|_{t=14\,\text{s}} = 2\left(14^2\right) - 18(14) + 108 = 248\text{ m/s}$$

The v–t graph is shown in Fig. *a*.



(a)

$s-t$ Graph: For the time interval $0 \le t < 9$ s, the initial condition is $s = 0$ when $t = 0$.

$$(+\uparrow) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t 4t^{3/2}\,dt$$

$$s = \frac{8}{5}t^{5/2}$$

When $t = 9$ s,

$$s\big|_{t=9\,\text{s}} = \frac{8}{5}\left(9^{5/2}\right) = 388.8\text{ m}$$

For the time interval $9\text{ s} < t \le 14$ s, the initial condition is $s = 388.8$ m when $t = 9$ s.

$$(+\uparrow) \qquad ds = v\,dt$$

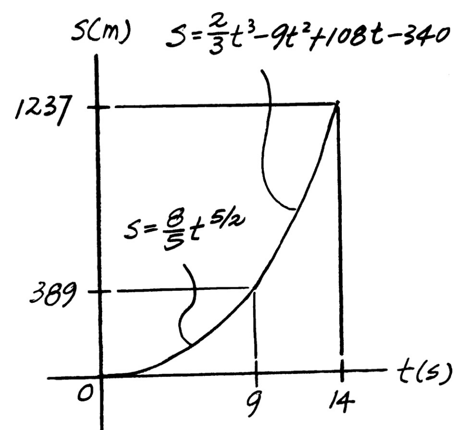
$$\int_{388.8\text{ m}}^s ds = \int_{9\text{ s}}^t \left(2t^2 - 18t + 108\right)dt$$

$$s = \left(\frac{2}{3}t^3 - 9t^2 + 108t - 340.2\right)\text{ m}$$

When $t = 14$ s,

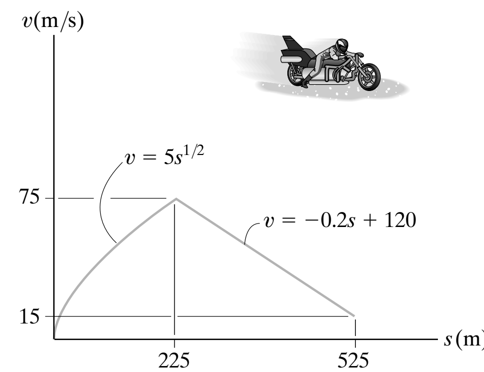
$$s\big|_{t=14\,\text{s}} = \frac{2}{3}\left(14^3\right) - 9\left(14^2\right) + 108(14) - 340.2 = 1237\text{ m}$$

The s–t graph is shown in Fig. *b*.



(b)

***12–64.** The jet bike is moving along a straight road with the speed described by the $v-s$ graph. Construct the $a-s$ graph.



$a - s$ Graph: For $0 \leq s < 225$ m,

$$(\xrightarrow{+}) \qquad a = v\frac{dv}{ds} = \left(5s^{1/2}\right)\left(\frac{5}{2}s^{-1/2}\right) = 12.5 \text{ m/s}^2$$

For $225 \text{ m} < s \leq 525$ m,

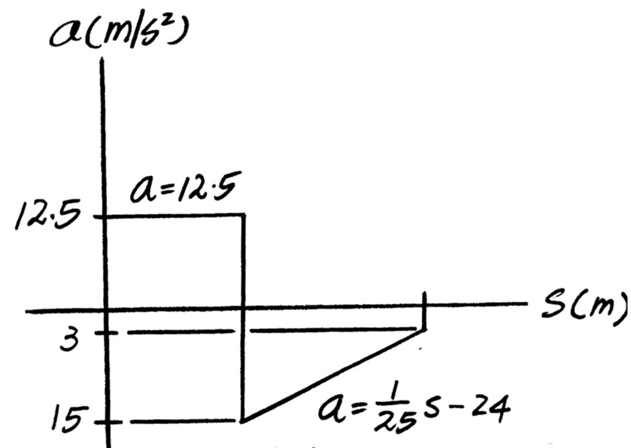
$$(\xrightarrow{+}) \qquad a = v\frac{dv}{ds} = (-0.2s + 120)(-0.2) = (0.04s - 24) \text{ m/s}^2$$

At $s = 225$ m and 525 m,

$$a\big|_{s=225 \text{ m}} = 0.04(225) - 24 = -15 \text{ m/s}^2$$

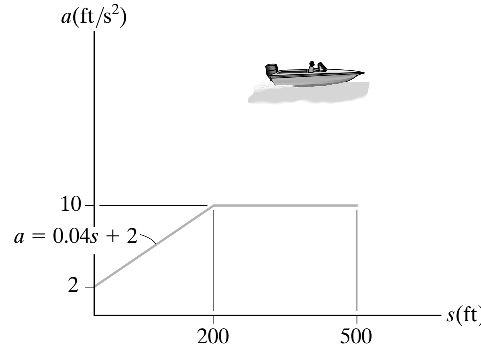
$$a\big|_{s=525 \text{ m}} = 0.04(525) - 24 = -3 \text{ m/s}^2$$

The a–s graph is shown in Fig. a.



(a)

•12–65. The acceleration of the speed boat starting from rest is described by the graph. Construct the $v-s$ graph.



$v-s$ Graph: For $0 \leq s < 200$ ft, the initial condition is $v = 0$ at $s = 0$.

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_0^s (0.04s + 2)\,ds$$

$$\left.\frac{v^2}{2}\right|_0^v = 0.02s^2 + 2s\Big|_0^s$$

$$v = \sqrt{0.04s^2 + 4s}\ \text{ft/s}$$

At $s = 200$ ft,

$$v\big|_{s=200\text{ ft}} = \sqrt{0.04\left(200^2\right) + 4(200)} = 48.99\text{ ft/s} = 49.0\text{ ft/s}$$

For 200 ft $< s \leq$ 500 ft, the initial condition is $v = 48.99$ ft/s at $s = 200$ ft.

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_{48.99\text{ ft/s}}^v v\,dv = \int_{200\text{ ft}}^s 10\,ds$$

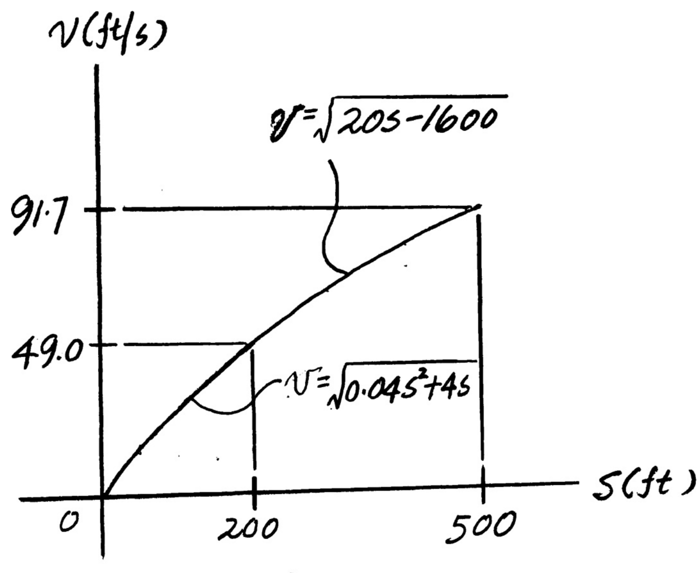
$$\left.\frac{v^2}{2}\right|_{48.99\text{ ft/s}}^v = 10s\Big|_{200\text{ ft}}^s$$

$$v = \sqrt{20s - 1600}\ \text{ft/s}$$

At $s = 500$ ft,

$$v\big|_{s=500\text{ ft}} = \sqrt{20(500) - 1600} = 91.65\text{ ft/s} = 91.7\text{ ft/s}$$

The v–s graph is shown in Fig. *a*.



(a)