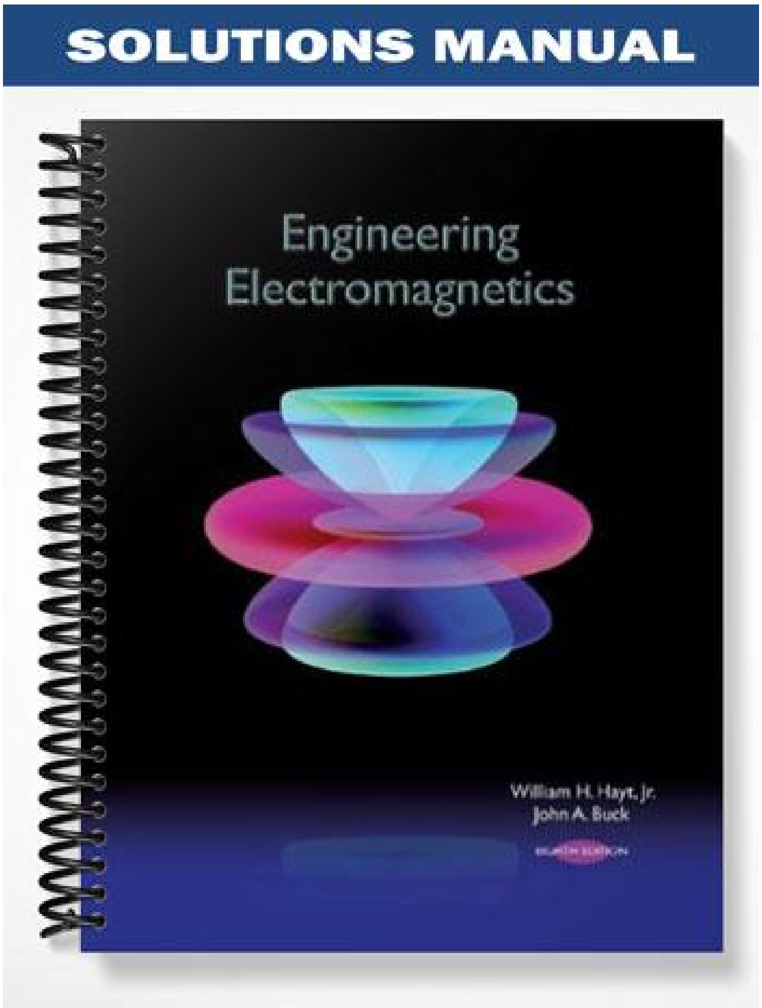
SOLUTIONS MANUAL
Engineering
Electromagnetics
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CHAPTER 2

2.1. Three point charges are positioned in the x-y plane as follows: 5nC at $y = 5$ cm, -10 nC at $y = -5$ cm, 15 nC at $x = -5$ cm. Find the required x-y coordinates of a 20-nC fourth charge that will produce a zero electric field at the origin.

With the charges thus configured, the electric field at the origin will be the superposition of the individual charge fields:

$$\mathbf{E}_0 = \frac{1}{4\pi\epsilon_0}\left[\frac{15}{(5)^2}\,\mathbf{a}_x - \frac{5}{(5)^2}\,\mathbf{a}_y - \frac{10}{(5)^2}\,\mathbf{a}_y\right] = \frac{1}{4\pi\epsilon_0}\left(\frac{3}{5}\right)[\mathbf{a}_x - \mathbf{a}_y] \quad \text{nC/m}$$

The field, $\mathbf{E}_{20}$, associated with the 20-nC charge (evaluated at the origin) must exactly cancel this field, so we write:

$$\mathbf{E}_{20} = \frac{-1}{4\pi\epsilon_0}\left(\frac{3}{5}\right)[\mathbf{a}_x - \mathbf{a}_y] = \frac{-20}{4\pi\epsilon_0\rho^2}\left(\frac{1}{\sqrt{2}}\right)[\mathbf{a}_x - \mathbf{a}_y]$$

From this, we identify the distance from the origin: $\rho = \sqrt{100/(3\sqrt{2})} = 4.85$. The x and y coordinates of the 20-nC charge will both be equal in magnitude to $4.85/\sqrt{2} = 3.43$. The coodinates of the 20-nC charge are then $\underline{(3.43, -3.43)}$.

2.2. Point charges of 1nC and -2nC are located at (0,0,0) and (1,1,1), respectively, in free space. Determine the vector force acting on each charge.

First, the electric field intensity associated with the 1nC charge, evalutated at the -2nC charge location is:

$$\mathbf{E}_{12} = \frac{1}{4\pi\epsilon_0(3)}\left(\frac{1}{\sqrt{3}}\right)(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z) \quad \text{nC/m}$$

in which the distance between charges is $\sqrt{3}$ m. The force on the -2nC charge is then

$$\mathbf{F}_{12} = q_2\mathbf{E}_{12} = \frac{-2}{12\sqrt{3}\,\pi\epsilon_0}(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z) = \underline{\frac{-1}{10.4\,\pi\epsilon_0}(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z) \quad \text{nN}}$$

The force on the 1nC charge at the origin is just the opposite of this result, or

$$\mathbf{F}_{21} = \underline{\frac{+1}{10.4\,\pi\epsilon_0}(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z) \quad \text{nN}}$$

2.3. Point charges of 50nC each are located at $A(1,0,0)$, $B(-1,0,0)$, $C(0,1,0)$, and $D(0,-1,0)$ in free space. Find the total force on the charge at A.

The force will be:

$$\mathbf{F} = \frac{(50 \times 10^{-9})^2}{4\pi\epsilon_0}\left[\frac{\mathbf{R}_{CA}}{|\mathbf{R}_{CA}|^3} + \frac{\mathbf{R}_{DA}}{|\mathbf{R}_{DA}|^3} + \frac{\mathbf{R}_{BA}}{|\mathbf{R}_{BA}|^3}\right]$$

where $\mathbf{R}_{CA} = \mathbf{a}_x - \mathbf{a}_y$, $\mathbf{R}_{DA} = \mathbf{a}_x + \mathbf{a}_y$, and $\mathbf{R}_{BA} = 2\mathbf{a}_x$. The magnitudes are $|\mathbf{R}_{CA}| = |\mathbf{R}_{DA}| = \sqrt{2}$, and $|\mathbf{R}_{BA}| = 2$. Substituting these leads to

$$\mathbf{F} = \frac{(50 \times 10^{-9})^2}{4\pi\epsilon_0}\left[\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} + \frac{2}{8}\right]\mathbf{a}_x = \underline{21.5\mathbf{a}_x \ \mu\text{N}}$$

where distances are in meters.

2.4. Eight identical point charges of Q C each are located at the corners of a cube of side length a, with one charge at the origin, and with the three nearest charges at $(a,0,0)$, $(0,a,0)$, and $(0,0,a)$. Find an expression for the total vector force on the charge at $P(a,a,a)$, assuming free space:

The total electric field at $P(a,a,a)$ that produces a force on the charge there will be the sum of the fields from the other seven charges. This is written below, where the charge locations associated with each term are indicated:

$$\mathbf{E}_{net}(a,a,a) = \frac{q}{4\pi\epsilon_0 a^2}\left[\underbrace{\frac{\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z}{3\sqrt{3}}}_{(0,0,0)} + \underbrace{\frac{\mathbf{a}_y + \mathbf{a}_z}{2\sqrt{2}}}_{(a,0,0)} + \underbrace{\frac{\mathbf{a}_x + \mathbf{a}_z}{2\sqrt{2}}}_{(0,a,0)} + \underbrace{\frac{\mathbf{a}_x + \mathbf{a}_y}{2\sqrt{2}}}_{(0,0,a)} + \underbrace{\mathbf{a}_x}_{(0,a,a)} + \underbrace{\mathbf{a}_y}_{(a,0,a)} + \underbrace{\mathbf{a}_z}_{(a,a,0)}\right]$$

The force is now the product of this field and the charge at (a,a,a). Simplifying, we obtain

$$\mathbf{F}(a,a,a) = q\mathbf{E}_{net}(a,a,a) = \frac{q^2}{4\pi\epsilon_0 a^2}\left[\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{2}} + 1\right](\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z) = \underline{\frac{1.90\,q^2}{4\pi\epsilon_0 a^2}(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)}$$

in which the magnitude is $|\mathbf{F}| = 3.29\,q^2/(4\pi\epsilon_0 a^2)$.

2.5. Let a point charge $Q_1 = 25$ nC be located at $P_1(4,-2,7)$ and a charge $Q_2 = 60$ nC be at $P_2(-3,4,-2)$.

a) If $\epsilon = \epsilon_0$, find $\mathbf{E}$ at $P_3(1,2,3)$: This field will be

$$\mathbf{E} = \frac{10^{-9}}{4\pi\epsilon_0}\left[\frac{25\mathbf{R}_{13}}{|\mathbf{R}_{13}|^3} + \frac{60\mathbf{R}_{23}}{|\mathbf{R}_{23}|^3}\right]$$

where $\mathbf{R}_{13} = -3\mathbf{a}_x + 4\mathbf{a}_y - 4\mathbf{a}_z$ and $\mathbf{R}_{23} = 4\mathbf{a}_x - 2\mathbf{a}_y + 5\mathbf{a}_z$. Also, $|\mathbf{R}_{13}| = \sqrt{41}$ and $|\mathbf{R}_{23}| = \sqrt{45}$. So

$$\mathbf{E} = \frac{10^{-9}}{4\pi\epsilon_0}\left[\frac{25 \times (-3\mathbf{a}_x + 4\mathbf{a}_y - 4\mathbf{a}_z)}{(41)^{1.5}} + \frac{60 \times (4\mathbf{a}_x - 2\mathbf{a}_y + 5\mathbf{a}_z)}{(45)^{1.5}}\right]$$
$$= \underline{4.58\mathbf{a}_x - 0.15\mathbf{a}_y + 5.51\mathbf{a}_z}$$

b) At what point on the y axis is $E_x = 0$? P_3 is now at $(0,y,0)$, so $\mathbf{R}_{13} = -4\mathbf{a}_x + (y+2)\mathbf{a}_y - 7\mathbf{a}_z$ and $\mathbf{R}_{23} = 3\mathbf{a}_x + (y-4)\mathbf{a}_y + 2\mathbf{a}_z$. Also, $|\mathbf{R}_{13}| = \sqrt{65 + (y+2)^2}$ and $|\mathbf{R}_{23}| = \sqrt{13 + (y-4)^2}$. Now the x component of $\mathbf{E}$ at the new P_3 will be:

$$E_x = \frac{10^{-9}}{4\pi\epsilon_0}\left[\frac{25 \times (-4)}{[65 + (y+2)^2]^{1.5}} + \frac{60 \times 3}{[13 + (y-4)^2]^{1.5}}\right]$$

2.5b (continued) To obtain $E_x = 0$, we require the expression in the large brackets to be zero. This expression simplifies to the following quadratic:

$$0.48y^2 + 13.92y + 73.10 = 0$$

which yields the two values: $y = \underline{-6.89, -22.11}$

2.6. Two point charges of equal magnitude q are positioned at $z = \pm d/2$.

a) find the electric field everywhere on the z axis: For a point charge at any location, we have

$$\mathbf{E} = \frac{q(\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^3}$$

In the case of two charges, we would therefore have

$$\mathbf{E}_T = \frac{q_1(\mathbf{r} - \mathbf{r}_1')}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}_1'|^3} + \frac{q_2(\mathbf{r} - \mathbf{r}_2')}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}_2'|^3} \tag{1}$$

In the present case, we assign $q_1 = q_2 = q$, the observation point position vector as $\mathbf{r} = z\mathbf{a}_z$, and the charge position vectors as $\mathbf{r}_1' = (d/2)\mathbf{a}_z$, and $\mathbf{r}_2' = -(d/2)\mathbf{a}_z$ Therefore

$$\mathbf{r} - \mathbf{r}_1' = [z - (d/2)]\mathbf{a}_z, \quad \mathbf{r} - \mathbf{r}_2' = [z + (d/2)]\mathbf{a}_z,$$

then

$$|\mathbf{r} - \mathbf{r}_1|^3 = [z - (d/2)]^3 \quad \text{and} \quad |\mathbf{r} - \mathbf{r}_2|^3 = [z + (d/2)]^3$$

Substitute these results into (1) to obtain:

$$\mathbf{E}_T(z) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{[z - (d/2)]^2} + \frac{1}{[z + (d/2)]^2}\right]\mathbf{a}_z \quad \text{V/m} \tag{2}$$

b) find the electric field everywhere on the x axis: We proceed as in part a, except that now $\mathbf{r} = x\mathbf{a}_x$. Eq. (1) becomes

$$\mathbf{E}_T(x) = \frac{q}{4\pi\epsilon_0}\left[\frac{x\mathbf{a}_x - (d/2)\mathbf{a}_z}{|x\mathbf{a}_x - (d/2)\mathbf{a}_z|^3} + \frac{x\mathbf{a}_x + (d/2)\mathbf{a}_z}{|x\mathbf{a}_x + (d/2)\mathbf{a}_z|^3}\right] \tag{3}$$

where

$$|x\mathbf{a}_x - (d/2)\mathbf{a}_z| = |x\mathbf{a}_x + (d/2)\mathbf{a}_z| = \left[x^2 + (d/2)^2\right]^{1/2}$$

Therefore (3) becomes

$$\mathbf{E}_T(x) = \frac{2qx\,\mathbf{a}_x}{4\pi\epsilon_0\left[x^2 + (d/2)^2\right]^{3/2}}$$

c) repeat parts a and b if the charge at $z = -d/2$ is $-q$ instead of $+q$: The field along the z axis is quickly found by changing the sign of the second term in (2):

$$\mathbf{E}_T(z) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{[z - (d/2)]^2} - \frac{1}{[z + (d/2)]^2}\right]\mathbf{a}_z \quad \text{V/m}$$

In like manner, the field along the x axis is found from (3) by again changing the sign of the second term. The result is

$$\frac{-2qd\,\mathbf{a}_z}{4\pi\epsilon_0\left[x^2 + (d/2)^2\right]^{3/2}}$$

2.7. A 2 μC point charge is located at $A(4,3,5)$ in free space. Find E_ρ, E_ϕ, and E_z at $P(8,12,2)$. Have

$$\mathbf{E}_P = \frac{2\times 10^{-6}}{4\pi\epsilon_0}\frac{\mathbf{R}_{AP}}{|\mathbf{R}_{AP}|^3} = \frac{2\times 10^{-6}}{4\pi\epsilon_0}\left[\frac{4\mathbf{a}_x + 9\mathbf{a}_y - 3\mathbf{a}_z}{(106)^{1.5}}\right] = 65.9\mathbf{a}_x + 148.3\mathbf{a}_y - 49.4\mathbf{a}_z$$

Then, at point P, $\rho = \sqrt{8^2+12^2} = 14.4$, $\phi = \tan^{-1}(12/8) = 56.3^\circ$, and $z = z$. Now,

$$E_\rho = \mathbf{E}_p \cdot \mathbf{a}_\rho = 65.9(\mathbf{a}_x\cdot\mathbf{a}_\rho) + 148.3(\mathbf{a}_y\cdot\mathbf{a}_\rho) = 65.9\cos(56.3^\circ) + 148.3\sin(56.3^\circ) = \underline{159.7}$$

and

$$E_\phi = \mathbf{E}_p \cdot \mathbf{a}_\phi = 65.9(\mathbf{a}_x\cdot\mathbf{a}_\phi) + 148.3(\mathbf{a}_y\cdot\mathbf{a}_\phi) = -65.9\sin(56.3^\circ) + 148.3\cos(56.3^\circ) = \underline{27.4}$$

Finally, $E_z = \underline{-49.4\text{ V/m}}$

2.8. A crude device for measuring charge consists of two small insulating spheres of radius a, one of which is fixed in position. The other is movable along the x axis, and is subject to a restraining force kx, where k is a spring constant. The uncharged spheres are centered at $x = 0$ and $x = d$, the latter fixed. If the spheres are given equal and opposite charges of Q coulombs:

a) Obtain the expression by which Q may be found as a function of x: The spheres will attract, and so the movable sphere at $x = 0$ will move toward the other until the spring and Coulomb forces balance. This will occur at location x for the movable sphere. With equal and opposite forces, we have

$$\frac{Q^2}{4\pi\epsilon_0(d-x)^2} = kx$$

from which $\underline{Q = 2(d-x)\sqrt{\pi\epsilon_0 kx}}$.

b) Determine the maximum charge that can be measured in terms of ϵ_0, k, and d, and state the separation of the spheres then: With increasing charge, the spheres move toward each other until they just touch at $\underline{x_{max} = d - 2a}$. Using the part a result, we find the maximum measurable charge: $Q_{max} = \underline{4a\sqrt{\pi\epsilon_0 k(d-2a)}}$. Presumably some form of stop mechanism is placed at $x = x_{max}^-$ to prevent the spheres from actually touching.

c) What happens if a larger charge is applied? No further motion is possible, so nothing happens.

2.9. A 100 nC point charge is located at $A(-1,1,3)$ in free space.

a) Find the locus of all points $P(x,y,z)$ at which $E_x = 500$ V/m: The total field at P will be:

$$\mathbf{E}_P = \frac{100\times 10^{-9}}{4\pi\epsilon_0}\frac{\mathbf{R}_{AP}}{|\mathbf{R}_{AP}|^3}$$

where $\mathbf{R}_{AP} = (x+1)\mathbf{a}_x + (y-1)\mathbf{a}_y + (z-3)\mathbf{a}_z$, and where $|\mathbf{R}_{AP}| = [(x+1)^2+(y-1)^2+(z-3)^2]^{1/2}$. The x component of the field will be

$$E_x = \frac{100\times 10^{-9}}{4\pi\epsilon_0}\left[\frac{(x+1)}{[(x+1)^2+(y-1)^2+(z-3)^2]^{1.5}}\right] = 500\text{ V/m}$$

And so our condition becomes:

$$\underline{(x+1) = 0.56\,[(x+1)^2+(y-1)^2+(z-3)^2]^{1.5}}$$

2.9b) Find y_1 if $P(-2, y_1, 3)$ lies on that locus: At point P, the condition of part a becomes

$$3.19 = \left[1 + (y_1 - 1)^2\right]^3$$

from which $(y_1 - 1)^2 = 0.47$, or $y_1 = \underline{1.69 \text{ or } 0.31}$

2.10. A charge of -1 nC is located at the origin in free space. What charge must be located at (2,0,0) to cause E_x to be zero at (3,1,1)?

The field from two point charges is given generally by

$$\mathbf{E}_T = \frac{q_1(\mathbf{r} - \mathbf{r}_1')}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}_1'|^3} + \frac{q_2(\mathbf{r} - \mathbf{r}_2')}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}_2'|^3} \tag{1}$$

where we let $q_1 = -1\text{nC}$ and q_2 is to be found. With q_1 at the origin, $\mathbf{r}_1' = 0$. The position vector for q_2 is then $\mathbf{r}_2' = 2\mathbf{a}_x$. The observation point at (3,1,1) gives $\mathbf{r} = 3\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z$. Eq. (1) becomes

$$\frac{1}{4\pi\epsilon_0}\left[\frac{-1(3\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)}{(3^2 + 1 + 1)^{3/2}} + \frac{q_2[(3-2)\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z]}{(1 + 1 + 1)^{3/2}}\right]$$

Requiring the x component to be zero leads to

$$q_2 = \frac{3^{5/2}}{11^{3/2}} = \underline{0.43 \text{ nC}}$$

2.11. A charge Q_0 located at the origin in free space produces a field for which $E_z = 1$ kV/m at point $P(-2, 1, -1)$.

a) Find Q_0: The field at P will be

$$\mathbf{E}_P = \frac{Q_0}{4\pi\epsilon_0}\left[\frac{-2\mathbf{a}_x + \mathbf{a}_y - \mathbf{a}_z}{6^{1.5}}\right]$$

Since the z component is of value 1 kV/m, we find $Q_0 = -4\pi\epsilon_0 6^{1.5} \times 10^3 = \underline{-1.63\ \mu\text{C}}$.

b) Find $\mathbf{E}$ at $M(1, 6, 5)$ in cartesian coordinates: This field will be:

$$\mathbf{E}_M = \frac{-1.63 \times 10^{-6}}{4\pi\epsilon_0}\left[\frac{\mathbf{a}_x + 6\mathbf{a}_y + 5\mathbf{a}_z}{[1 + 36 + 25]^{1.5}}\right]$$

or $\mathbf{E}_M = \underline{-30.11\mathbf{a}_x - 180.63\mathbf{a}_y - 150.53\mathbf{a}_z}$.

c) Find $\mathbf{E}$ at $M(1, 6, 5)$ in cylindrical coordinates: At M, $\rho = \sqrt{1 + 36} = 6.08$, $\phi = \tan^{-1}(6/1) = 80.54°$, and $z = 5$. Now

$$E_\rho = \mathbf{E}_M \cdot \mathbf{a}_\rho = -30.11\cos\phi - 180.63\sin\phi = -183.12$$

$$E_\phi = \mathbf{E}_M \cdot \mathbf{a}_\phi = -30.11(-\sin\phi) - 180.63\cos\phi = 0 \quad \text{(as expected)}$$

so that $\mathbf{E}_M = \underline{-183.12\mathbf{a}_\rho - 150.53\mathbf{a}_z}$.

d) Find $\mathbf{E}$ at $M(1, 6, 5)$ in spherical coordinates: At M, $r = \sqrt{1 + 36 + 25} = 7.87$, $\phi = 80.54°$ (as before), and $\theta = \cos^{-1}(5/7.87) = 50.58°$. Now, since the charge is at the origin, we expect to obtain only a radial component of $\mathbf{E}_M$. This will be:

$$E_r = \mathbf{E}_M \cdot \mathbf{a}_r = -30.11\sin\theta\cos\phi - 180.63\sin\theta\sin\phi - 150.53\cos\theta = \underline{-237.1}$$

2.12. Electrons are in random motion in a fixed region in space. During any 1μs interval, the probability of finding an electron in a subregion of volume 10^{-15} m^2 is 0.27. What volume charge density, appropriate for such time durations, should be assigned to that subregion?

The finite probabilty effectively reduces the net charge quantity by the probability fraction. With $e = -1.602 \times 10^{-19}$ C, the density becomes

$$\rho_v = -\frac{0.27 \times 1.602 \times 10^{-19}}{10^{-15}} = \underline{-43.3\ \mu\text{C/m}^3}$$

2.13. A uniform volume charge density of 0.2 μC/m^3 is present throughout the spherical shell extending from $r = 3$ cm to $r = 5$ cm. If $\rho_v = 0$ elsewhere:

a) find the total charge present throughout the shell: This will be

$$Q = \int_0^{2\pi}\int_0^{\pi}\int_{.03}^{.05} 0.2\, r^2 \sin\theta\ dr\, d\theta\, d\phi = \left[4\pi(0.2)\frac{r^3}{3}\right]_{.03}^{.05} = 8.21 \times 10^{-5}\ \mu\text{C} = \underline{82.1\ \text{pC}}$$

b) find r_1 if half the total charge is located in the region $3\,\text{cm} < r < r_1$: If the integral over r in part a is taken to r_1, we would obtain

$$\left[4\pi(0.2)\frac{r^3}{3}\right]_{.03}^{r_1} = 4.105 \times 10^{-5}$$

Thus

$$r_1 = \left[\frac{3 \times 4.105 \times 10^{-5}}{0.2 \times 4\pi} + (.03)^3\right]^{1/3} = \underline{4.24\ \text{cm}}$$

2.14. The electron beam in a certain cathode ray tube possesses cylindrical symmetry, and the charge density is represented by $\rho_v = -0.1/(\rho^2 + 10^{-8})$ pC/m^3 for $0 < \rho < 3 \times 10^{-4}$ m, and $\rho_v = 0$ for $\rho > 3 \times 10^{-4}$ m.

a) Find the total charge per meter along the length of the beam: We integrate the charge density over the cylindrical volume having radius 3×10^{-4} m, and length 1m.

$$q = \int_0^1\int_0^{2\pi}\int_0^{3\times10^{-4}} \frac{-0.1}{(\rho^2 + 10^{-8})}\,\rho\, d\rho\, d\phi\, dz$$

From integral tables, this evaluates as

$$q = -0.2\pi\left(\frac{1}{2}\right)\ln\left(\rho^2 + 10^{-8}\right)\Big|_0^{3\times10^{-4}} = 0.1\pi\ln(10) = \underline{-0.23\pi\ \text{pC/m}}$$

b) if the electron velocity is 5×10^7 m/s, and with one ampere defined as 1C/s, find the beam current:

$$\text{Current} = \text{charge/m} \times v = -0.23\pi\,[\text{pC/m}] \times 5 \times 10^7\,[\text{m/s}] = -11.5\pi \times 10^6\,[\text{pC/s}] = \underline{-11.5\pi\,\mu\text{A}}$$

2.15. A spherical volume having a 2 μm radius contains a uniform volume charge density of 10^5 C/m^3 (not 10^{15} as stated in earlier printings).

a) What total charge is enclosed in the spherical volume?
This will be $Q = (4/3)\pi(2 \times 10^{-6})^3 \times 10^5 = \underline{3.35 \times 10^{-12} \text{ C}}$.

b) Now assume that a large region contains one of these little spheres at every corner of a cubical grid 3mm on a side, and that there is no charge between spheres. What is the average volume charge density throughout this large region? Each cube will contain the equivalent of one little sphere. Neglecting the little sphere volume, the average density becomes

$$\rho_{v,avg} = \frac{3.35 \times 10^{-12}}{(0.003)^3} = \underline{1.24 \times 10^{-4} \text{ C/m}^3}$$

2.16. Within a region of free space, charge density is given as $\rho_v = (\rho_0 r/a)\cos\theta$ C/m^3, where ρ_0 and a are constants. Find the total charge lying within:

a) the sphere, $r \leq a$: This will be

$$Q_a = \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \frac{\rho_0 r}{a} \cos\theta\, r^2 \sin\theta\, dr\, d\theta\, d\phi = 2\pi \int_0^a \frac{\rho_0 r^3}{a}\, dr = \underline{0}$$

b) the cone, $r \leq a$, $0 \leq \theta \leq 0.1\pi$:

$$Q_b = \int_0^{2\pi}\int_0^{0.1\pi}\int_0^{a} \frac{\rho_0 r}{a} \cos\theta\, r^2 \sin\theta\, dr\, d\theta\, d\phi = \pi\frac{\rho_0 a^3}{4}\left[1 - \cos^2(0.1\pi)\right] = \underline{0.024\pi\rho_0 a^3}$$

c) the region, $r \leq a$, $0 \leq \theta \leq 0.1\pi$, $0 \leq \phi \leq 0.2\pi$.

$$Q_c = \int_0^{0.2\pi}\int_0^{0.1\pi}\int_0^{a} \frac{\rho_0 r}{a} \cos\theta\, r^2 \sin\theta\, dr\, d\theta\, d\phi = 0.024\pi\rho_0 a^3\left(\frac{0.2\pi}{2\pi}\right) = \underline{0.0024\pi\rho_0 a^3}$$

2.17. A uniform line charge of 16 nC/m is located along the line defined by $y = -2$, $z = 5$. If $\epsilon = \epsilon_0$:

a) Find $\mathbf{E}$ at $P(1, 2, 3)$: This will be

$$\mathbf{E}_P = \frac{\rho_l}{2\pi\epsilon_0}\frac{\mathbf{R}_P}{|\mathbf{R}_P|^2}$$

where $\mathbf{R}_P = (1, 2, 3) - (1, -2, 5) = (0, 4, -2)$, and $|\mathbf{R}_P|^2 = 20$. So

$$\mathbf{E}_P = \frac{16 \times 10^{-9}}{2\pi\epsilon_0}\left[\frac{4\mathbf{a}_y - 2\mathbf{a}_z}{20}\right] = \underline{57.5\mathbf{a}_y - 28.8\mathbf{a}_z \text{ V/m}}$$

b) Find $\mathbf{E}$ at that point in the $z = 0$ plane where the direction of $\mathbf{E}$ is given by $(1/3)\mathbf{a}_y - (2/3)\mathbf{a}_z$: With $z = 0$, the general field will be

$$\mathbf{E}_{z=0} = \frac{\rho_l}{2\pi\epsilon_0}\left[\frac{(y+2)\mathbf{a}_y - 5\mathbf{a}_z}{(y+2)^2 + 25}\right]$$

We require $|E_z| = -|2E_y|$, so $2(y+2) = 5$. Thus $y = 1/2$, and the field becomes:

$$\mathbf{E}_{z=0} = \frac{\rho_l}{2\pi\epsilon_0}\left[\frac{2.5\mathbf{a}_y - 5\mathbf{a}_z}{(2.5)^2 + 25}\right] = \underline{23\mathbf{a}_y - 46\mathbf{a}_z}$$

2.18. a) Find **E** in the plane $z = 0$ that is produced by a uniform line charge, ρ_L, extending along the z axis over the range $-L < z < L$ in a cylindrical coordinate system: We find **E** through

$$\mathbf{E} = \int_{-L}^{L} \frac{\rho_L dz(\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}'|^3}$$

where the observation point position vector is $\mathbf{r} = \rho\mathbf{a}_\rho$ (anywhere in the x-y plane), and where the position vector that locates any differential charge element on the z axis is $\mathbf{r}' = z\mathbf{a}_z$. So $\mathbf{r} - \mathbf{r}' = \rho\mathbf{a}_\rho - z\mathbf{a}_z$, and $|\mathbf{r} - \mathbf{r}'| = (\rho^2 + z^2)^{1/2}$. These relations are substituted into the integral to yield:

$$\mathbf{E} = \int_{-L}^{L} \frac{\rho_L dz(\rho\mathbf{a}_\rho - z\mathbf{a}_z)}{4\pi\epsilon_0(\rho^2 + z^2)^{3/2}} = \frac{\rho_L\,\rho\,\mathbf{a}_\rho}{4\pi\epsilon_0}\int_{-L}^{L}\frac{dz}{(\rho^2 + z^2)^{3/2}} = E_\rho\,\mathbf{a}_\rho$$

Note that the second term in the left-hand integral (involving $z\mathbf{a}_z$) has effectively vanished because it produces equal and opposite sign contributions when the integral is taken over symmetric limits (odd parity). Evaluating the integral results in

$$E_\rho = \frac{\rho_L\,\rho}{4\pi\epsilon_0}\,\frac{z}{\rho^2\sqrt{\rho^2 + z^2}}\bigg|_{-L}^{L} = \frac{\rho_L}{2\pi\epsilon_0\rho}\,\frac{L}{\sqrt{\rho^2 + L^2}} = \frac{\rho_L}{2\pi\epsilon_0\rho}\,\frac{1}{\sqrt{1 + (\rho/L)^2}}$$

Note that as $L \to \infty$, the expression reduces to the expected field of the infinite line charge in free space, $\rho_L/(2\pi\epsilon_0\rho)$.

b) if the finite line charge is approximated by an infinite line charge ($L \to \infty$), by what percentage is E_ρ in error if $\rho = 0.5L$? The percent error in this situation will be

$$\%\,\text{error} = \left[1 - \frac{1}{\sqrt{1 + (\rho/L)^2}}\right] \times 100$$

For $\rho = 0.5L$, this becomes $\%\,\text{error} = \underline{10.6\,\%}$

c) repeat b with $\rho = 0.1L$. For this value, obtain $\%\,\text{error} = \underline{0.496\,\%}$.

2.19. A uniform line charge of 2 μC/m is located on the z axis. Find **E** in rectangular coordinates at $P(1, 2, 3)$ if the charge extends from

a) $-\infty < z < \infty$: With the infinite line, we know that the field will have only a radial component in cylindrical coordinates (or x and y components in cartesian). The field from an infinite line on the z axis is generally $\mathbf{E} = [\rho_l/(2\pi\epsilon_0\rho)]\mathbf{a}_\rho$. Therefore, at point P:

$$\mathbf{E}_P = \frac{\rho_l}{2\pi\epsilon_0}\frac{\mathbf{R}_{zP}}{|\mathbf{R}_{zP}|^2} = \frac{(2\times 10^{-6})}{2\pi\epsilon_0}\,\frac{\mathbf{a}_x + 2\mathbf{a}_y}{5} = \underline{7.2\mathbf{a}_x + 14.4\mathbf{a}_y \;\;\text{kV/m}}$$

where $\mathbf{R}_{zP}$ is the vector that extends from the line charge to point P, and is perpendicular to the z axis; i.e., $\mathbf{R}_{zP} = (1, 2, 3) - (0, 0, 3) = (1, 2, 0)$.

b) $-4 \leq z \leq 4$: Here we use the general relation

$$\mathbf{E}_P = \int \frac{\rho_l dz}{4\pi\epsilon_0} \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3}$$

2.19b (continued) where $\mathbf{r} = \mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z$ and $\mathbf{r}' = z\mathbf{a}_z$. So the integral becomes

$$\mathbf{E}_P = \frac{(2 \times 10^{-6})}{4\pi\epsilon_0} \int_{-4}^{4} \frac{\mathbf{a}_x + 2\mathbf{a}_y + (3-z)\mathbf{a}_z}{[5 + (3-z)^2]^{1.5}} \, dz$$

Using integral tables, we obtain:

$$\mathbf{E}_P = 3597 \left[\frac{(\mathbf{a}_x + 2\mathbf{a}_y)(z-3) + 5\mathbf{a}_z}{(z^2 - 6z + 14)} \right]_{-4}^{4} \text{ V/m} = \underline{4.9\mathbf{a}_x + 9.8\mathbf{a}_y + 4.9\mathbf{a}_z \text{ kV/m}}$$

The student is invited to verify that when evaluating the above expression over the limits $-\infty < z < \infty$, the z component vanishes and the x and y components become those found in part a.

2.20. A line charge of uniform charge density ρ_0 C/m and of length ℓ, is oriented along the z axis at $-\ell/2 < z < \ell/2$.

a) Find the electric field strength, $\mathbf{E}$, in magnitude and direction at any position along the x axis: This follows the method in Problem 2.18. We find $\mathbf{E}$ through

$$\mathbf{E} = \int_{-\ell/2}^{\ell/2} \frac{\rho_0 \, dz(\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^3}$$

where the observation point position vector is $\mathbf{r} = x\mathbf{a}_x$ (anywhere on the x axis), and where the position vector that locates any differential charge element on the z axis is $\mathbf{r}' = z\mathbf{a}_z$. So $\mathbf{r} - \mathbf{r}' = x\mathbf{a}_x - z\mathbf{a}_z$, and $|\mathbf{r} - \mathbf{r}'| = (x^2 + z^2)^{1/2}$. These relations are substituted into the integral to yield:

$$\mathbf{E} = \int_{-\ell/2}^{\ell/2} \frac{\rho_0 dz(x\mathbf{a}_x - z\mathbf{a}_z)}{4\pi\epsilon_0 (x^2 + z^2)^{3/2}} = \frac{\rho_0 \, x \, \mathbf{a}_x}{4\pi\epsilon_0} \int_{-\ell/2}^{\ell/2} \frac{dz}{(x^2 + z^2)^{3/2}} = E_x \, \mathbf{a}_x$$

Note that the second term in the left-hand integral (involving $z\mathbf{a}_z$) has effectively vanished because it produces equal and opposite sign contributions when the integral is taken over symmetric limits (odd parity). Evaluating the integral results in

$$E_x = \frac{\rho_0 \, x}{4\pi\epsilon_0} \frac{z}{x^2\sqrt{x^2 + z^2}} \bigg|_{-\ell/2}^{\ell/2} = \frac{\rho_0}{2\pi\epsilon_0 x} \frac{\ell/2}{\sqrt{x^2 + (\ell/2)^2}} = \frac{\rho_0}{2\pi\epsilon_0 x} \frac{1}{\sqrt{1 + (2x/\ell)^2}}$$

b) with the given line charge in position, find the force acting on an identical line charge that is oriented along the x axis at $\ell/2 < x < 3\ell/2$: The differential force on an element of the x-directed line charge will be $d\mathbf{F} = dq\mathbf{E} = (\rho_0 \, dx)\mathbf{E}$, where $\mathbf{E}$ is the field as determined in part a. The net force is then the integral of the differential force over the length of the horizontal line charge, or

$$\mathbf{F} = \int_{\ell/2}^{3\ell/2} \frac{\rho_0^2}{2\pi\epsilon_0 x} \frac{1}{\sqrt{1 + (2x/\ell)^2}} \, dx \; \mathbf{a}_x$$

This can be re-written and then evaluated using integral tables as

$$\mathbf{F} = \frac{\rho_0^2\,\ell\,\mathbf{a}_x}{4\pi\epsilon_0}\int_{\ell/2}^{3\ell/2}\frac{dx}{x\sqrt{x^2+(\ell/2)^2}} = \frac{-\rho_0^2\,\ell\,\mathbf{a}_x}{4\pi\epsilon_0}\left(\frac{1}{(\ell/2)}\ln\left[\frac{\ell/2+\sqrt{x^2+(\ell/2)^2}}{x}\right]_{\ell/2}^{3\ell/2}\right)$$

$$= \frac{-\rho_0^2\,\mathbf{a}_x}{2\pi\epsilon_0}\ln\left[\frac{(\ell/2)\left(1+\sqrt{10}\right)}{3(\ell/2)\left(1+\sqrt{2}\right)}\right] = \frac{\rho_0^2\,\mathbf{a}_x}{2\pi\epsilon_0}\ln\left[\frac{3(1+\sqrt{2})}{1+\sqrt{10}}\right] = \frac{0.55\rho_0^2}{2\pi\epsilon_0}\,\mathbf{a}_x \text{ N}$$

2.21. Two identical uniform line charges with $\rho_l = 75$ nC/m are located in free space at $x = 0$, $y = \pm 0.4$ m. What force per unit length does each line charge exert on the other? The charges are parallel to the z axis and are separated by 0.8 m. Thus the field from the charge at $y = -0.4$ evaluated at the location of the charge at $y = +0.4$ will be $\mathbf{E} = [\rho_l/(2\pi\epsilon_0(0.8))]\mathbf{a}_y$. The force on a differential length of the line at the positive y location is $d\mathbf{F} = dq\mathbf{E} = \rho_l dz\mathbf{E}$. Thus the force per unit length acting on the line at postive y arising from the charge at negative y is

$$\mathbf{F} = \int_0^1 \frac{\rho_l^2\,dz}{2\pi\epsilon_0(0.8)}\,\mathbf{a}_y = 1.26\times10^{-4}\,\mathbf{a}_y \text{ N/m} = \underline{126\,\mathbf{a}_y\ \mu\text{N/m}}$$

The force on the line at negative y is of course the same, but with $-\mathbf{a}_y$.

2.22. Two identical uniform sheet charges with $\rho_s = 100$ nC/m^2 are located in free space at $z = \pm 2.0$ cm. What force per unit area does each sheet exert on the other?

The field from the top sheet is $\mathbf{E} = -\rho_s/(2\epsilon_0)\,\mathbf{a}_z$ V/m. The differential force produced by this field on the bottom sheet is the charge density on the bottom sheet times the differential area there, multiplied by the electric field from the top sheet: $d\mathbf{F} = \rho_s da\mathbf{E}$. The force per unit area is then just $\mathbf{F} = \rho_s\mathbf{E} = (100\times10^{-9})(-100\times10^{-9})/(2\epsilon_0)\,\mathbf{a}_z = \underline{-5.6\times10^{-4}\,\mathbf{a}_z \text{ N/m}^2}$.

2.23. Given the surface charge density, $\rho_s = 2\,\mu$C/m^2, in the region $\rho < 0.2$ m, $z = 0$. Find $\mathbf{E}$ at:

a) $P_A(\rho = 0, z = 0.5)$: First, we recognize from symmetry that only a z component of $\mathbf{E}$ will be present. Considering a general point z on the z axis, we have $\mathbf{r} = z\mathbf{a}_z$. Then, with $\mathbf{r}' = \rho\mathbf{a}_\rho$, we obtain $\mathbf{r} - \mathbf{r}' = z\mathbf{a}_z - \rho\mathbf{a}_\rho$. The superposition integral for the z component of $\mathbf{E}$ will be:

$$E_{z,P_A} = \frac{\rho_s}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{0.2}\frac{z\,\rho\,d\rho\,d\phi}{(\rho^2+z^2)^{1.5}} = -\frac{2\pi\rho_s}{4\pi\epsilon_0}z\left[\frac{1}{\sqrt{z^2+\rho^2}}\right]_0^{0.2}$$

$$= \frac{\rho_s}{2\epsilon_0}z\left[\frac{1}{\sqrt{z^2}} - \frac{1}{\sqrt{z^2+0.04}}\right]$$

With $z = 0.5$ m, the above evaluates as $E_{z,P_A} = \underline{8.1 \text{ kV/m}}$.

b) $P_B(\rho = 0, z = -0.5)$. With z at -0.5 m, we evaluate the expression for E_z to obtain $E_{z,P_B} = \underline{-8.1 \text{ kV/m}}$.

c) Show that the field along the z axis reduces to that of an infinite sheet charge at small values of z: In general, the field can be expressed as

$$E_z = \frac{\rho_s}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+0.04}}\right]$$

At small z, this reduces to $E_z \doteq \rho_s/2\epsilon_0$, which is the infinite sheet charge field.

d) Show that the z axis field reduces to that of a point charge at large values of z: The development is as follows:

$$E_z = \frac{\rho_s}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+0.04}}\right] = \frac{\rho_s}{2\epsilon_0}\left[1 - \frac{z}{z\sqrt{1+0.04/z^2}}\right] \doteq \frac{\rho_s}{2\epsilon_0}\left[1 - \frac{1}{1+(1/2)(0.04)/z^2}\right]$$

where the last approximation is valid if $z >> .04$. Continuing:

$$E_z \doteq \frac{\rho_s}{2\epsilon_0}\left[1 - [1 - (1/2)(0.04)/z^2]\right] = \frac{0.04\rho_s}{4\epsilon_0 z^2} = \frac{\pi(0.2)^2\rho_s}{4\pi\epsilon_0 z^2}$$

This the point charge field, where we identify $q = \pi(0.2)^2\rho_s$ as the total charge on the disk (which now looks like a point).

2.24. a) Find the electric field on the z axis produced by an annular ring of uniform surface charge density ρ_s in free space. The ring occupies the region $z = 0$, $a \le \rho \le b$, $0 \le \phi \le 2\pi$ in cylindrical coordinates: We find the field through

$$\mathbf{E} = \int\!\!\int \frac{\rho_s da(\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}'|^3}$$

where the integral is taken over the surface of the annular ring, and where $\mathbf{r} = z\mathbf{a}_z$ and $\mathbf{r}' = \rho\mathbf{a}_\rho$. The integral then becomes

$$\mathbf{E} = \int_0^{2\pi}\int_a^b \frac{\rho_s\,\rho\,d\rho\,d\phi\,(z\mathbf{a}_z - \rho\mathbf{a}_\rho)}{4\pi\epsilon_0(z^2+\rho^2)^{3/2}}$$

In evaluating this integral, we first note that the term involving $\rho\mathbf{a}_\rho$ integrates to zero over the ϕ integration range of 0 to 2π. This is because we need to introduce the ϕ dependence in $\mathbf{a}_\rho$ by writing it as $\mathbf{a}_\rho = \cos\phi\,\mathbf{a}_x + \sin\phi\,\mathbf{a}_y$, where $\mathbf{a}_x$ and $\mathbf{a}_y$ are invariant in their orientation as ϕ varies. So the integral now simplifies to

$$\mathbf{E} = \frac{2\pi\rho_s\,z\,\mathbf{a}_z}{4\pi\epsilon_0}\int_a^b \frac{\rho\,d\rho}{(z^2+\rho^2)^{3/2}} = \frac{\rho_s\,z\,\mathbf{a}_z}{2\epsilon_0}\left[\frac{-1}{\sqrt{z^2+\rho^2}}\right]_a^b$$

$$= \frac{\rho_s}{2\epsilon_0}\left[\frac{1}{\sqrt{1+(a/z)^2}} - \frac{1}{\sqrt{1+(b/z)^2}}\right]\mathbf{a}_z$$

b) from your part a result, obtain the field of an infinite uniform sheet charge by taking appropriate limits. The infinite sheet is obtained by letting $a \to 0$ and $b \to \infty$, in which case $\mathbf{E} \to \rho_s/(2\epsilon_0)\,\mathbf{a}_z$ as expected.

2.25. Find $\mathbf{E}$ at the origin if the following charge distributions are present in free space: point charge, 12 nC at $P(2,0,6)$; uniform line charge density, 3nC/m at $x = -2$, $y = 3$; uniform surface charge density, 0.2 nC/m^2 at $x = 2$. The sum of the fields at the origin from each charge in order is:

$$\mathbf{E} = \left[\frac{(12\times10^{-9})}{4\pi\epsilon_0}\,\frac{(-2\mathbf{a}_x - 6\mathbf{a}_z)}{(4+36)^{1.5}}\right] + \left[\frac{(3\times10^{-9})}{2\pi\epsilon_0}\,\frac{(2\mathbf{a}_x - 3\mathbf{a}_y)}{(4+9)}\right] - \left[\frac{(0.2\times10^{-9})\mathbf{a}_x}{2\epsilon_0}\right]$$

$$= \underline{-3.9\mathbf{a}_x - 12.4\mathbf{a}_y - 2.5\mathbf{a}_z \text{ V/m}}$$

2.26. Radially-dependent surface charge is distributed on an infinite flat sheet in the xy plane, and is characterized in cylindrical coordinates by surface density $\rho_s = \rho_0/\rho$, where ρ_0 is a constant. Determine the electric field strength, $\mathbf{E}$, everywhere on the z axis.

We find the field through

$$\mathbf{E} = \int\int \frac{\rho_s da(\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^3}$$

where the integral is taken over the surface of the annular ring, and where $\mathbf{r} = z\mathbf{a}_z$ and $\mathbf{r}' = \rho\mathbf{a}_\rho$. The integral then becomes

$$\mathbf{E} = \int_0^{2\pi}\int_0^{\infty} \frac{(\rho_0/\rho)\,\rho\,d\rho\,d\phi\,(z\mathbf{a}_z - \rho\mathbf{a}_\rho)}{4\pi\epsilon_0(z^2+\rho^2)^{3/2}}$$

In evaluating this integral, we first note that the term involving $\rho\mathbf{a}_\rho$ integrates to zero over the ϕ integration range of 0 to 2π. This is because we need to introduce the ϕ dependence in $\mathbf{a}_\rho$ by writing it as $\mathbf{a}_\rho = \cos\phi\,\mathbf{a}_x + \sin\phi\,\mathbf{a}_y$, where $\mathbf{a}_x$ and $\mathbf{a}_y$ are invariant in their orientation as ϕ varies. So the integral now simplifies to

$$\mathbf{E} = \frac{2\pi\rho_s\,z\,\mathbf{a}_z}{4\pi\epsilon_0}\int_0^{\infty}\frac{d\rho}{(z^2+\rho^2)^{3/2}} = \frac{\rho_s\,z\,\mathbf{a}_z}{2\epsilon_0}\left[\frac{\rho}{z^2\sqrt{z^2+\rho^2}}\right]_{\rho=0}^{\infty} = \frac{\rho_s}{2\epsilon_0 z}\mathbf{a}_z$$

2.27. Given the electric field $\mathbf{E} = (4x - 2y)\mathbf{a}_x - (2x + 4y)\mathbf{a}_y$, find:

a) the equation of the streamline that passes through the point $P(2, 3, -4)$: We write

$$\frac{dy}{dx} = \frac{E_y}{E_x} = \frac{-(2x+4y)}{(4x-2y)}$$

Thus

$$2(x\,dy + y\,dx) = y\,dy - x\,dx$$

or

$$2\,d(xy) = \frac{1}{2}\,d(y^2) - \frac{1}{2}\,d(x^2)$$

So

$$C_1 + 2xy = \frac{1}{2}y^2 - \frac{1}{2}x^2$$

or

$$y^2 - x^2 = 4xy + C_2$$

Evaluating at $P(2, 3, -4)$, obtain:

$$9 - 4 = 24 + C_2, \text{ or } C_2 = -19$$

Finally, at P, the requested equation is

$$\underline{y^2 - x^2 = 4xy - 19}$$

b) a unit vector specifying the direction of $\mathbf{E}$ at $Q(3, -2, 5)$: Have $\mathbf{E}_Q = [4(3) + 2(2)]\mathbf{a}_x - [2(3) - 4(2)]\mathbf{a}_y = 16\mathbf{a}_x + 2\mathbf{a}_y$. Then $|\mathbf{E}| = \sqrt{16^2+4} = 16.12$ So

$$\mathbf{a}_Q = \frac{16\mathbf{a}_x + 2\mathbf{a}_y}{16.12} = \underline{0.99\mathbf{a}_x + 0.12\mathbf{a}_y}$$

2.28 An electric dipole (discussed in detail in Sec. 4.7) consists of two point charges of equal and opposite magnitude $\pm Q$ spaced by distance d. With the charges along the z axis at positions $z = \pm d/2$ (with the positive charge at the positive z location), the electric field in spherical coordinates is given by $\mathbf{E}(r, \theta) = \left[Qd/(4\pi\epsilon_0 r^3)\right][2\cos\theta\mathbf{a}_r + \sin\theta\mathbf{a}_\theta]$, where $r >> d$. Using rectangular coordinates, determine expressions for the vector force on a point charge of magnitude q:

a) at $(0, 0, z)$: Here, $\theta = 0$, $\mathbf{a}_r = \mathbf{a}_z$, and $r = z$. Therefore

$$\mathbf{F}(0, 0, z) = \frac{qQd\,\mathbf{a}_z}{4\pi\epsilon_0 z^3}\ \text{N}$$

b) at $(0, y, 0)$: Here, $\theta = 90°$, $\mathbf{a}_\theta = -\mathbf{a}_z$, and $r = y$. The force is

$$\mathbf{F}(0, y, 0) = \frac{-qQd\,\mathbf{a}_z}{4\pi\epsilon_0 y^3}\ \text{N}$$

2.29. If $\mathbf{E} = 20e^{-5y}(\cos 5x\mathbf{a}_x - \sin 5x\mathbf{a}_y)$, find:

a) $|\mathbf{E}|$ at $P(\pi/6, 0.1, 2)$: Substituting this point, we obtain $\mathbf{E}_P = -10.6\mathbf{a}_x - 6.1\mathbf{a}_y$, and so $|\mathbf{E}_P| = \underline{12.2}$.

b) a unit vector in the direction of $\mathbf{E}_P$: The unit vector associated with $\mathbf{E}$ is $(\cos 5x\mathbf{a}_x - \sin 5x\mathbf{a}_y)$, which evaluated at P becomes $\mathbf{a}_E = \underline{-0.87\mathbf{a}_x - 0.50\mathbf{a}_y}$.

c) the equation of the direction line passing through P: Use

$$\frac{dy}{dx} = \frac{-\sin 5x}{\cos 5x} = -\tan 5x \quad \Rightarrow \quad dy = -\tan 5x\,dx$$

Thus $y = \frac{1}{5}\ln\cos 5x + C$. Evaluating at P, we find $C = 0.13$, and so

$$\underline{y = \frac{1}{5}\ln\cos 5x + 0.13}$$

2.30. For fields that do not vary with z in cylindrical coordinates, the equations of the streamlines are obtained by solving the differential equation $E_\rho/E_\phi = d\rho(\rho d\phi)$. Find the equation of the line passing through the point $(2, 30°, 0)$ for the field $\mathbf{E} = \rho\cos 2\phi\,\mathbf{a}_\rho - \rho\sin 2\phi\,\mathbf{a}_\phi$:

$$\frac{E_\rho}{E_\phi} = \frac{d\rho}{\rho d\phi} = \frac{-\rho\cos 2\phi}{\rho\sin 2\phi} = -\cot 2\phi \;\Rightarrow\; \frac{d\rho}{\rho} = -\cot 2\phi\,d\phi$$

Integrate to obtain

$$2\ln\rho = \ln\sin 2\phi + \ln C = \ln\left[\frac{C}{\sin 2\phi}\right] \;\Rightarrow\; \rho^2 = \frac{C}{\sin 2\phi}$$

At the given point, we have $4 = C/\sin(60°) \;\Rightarrow\; C = 4\sin 60° = 2\sqrt{3}$. Finally, the equation for the streamline is $\underline{\rho^2 = 2\sqrt{3}/\sin 2\phi}$.