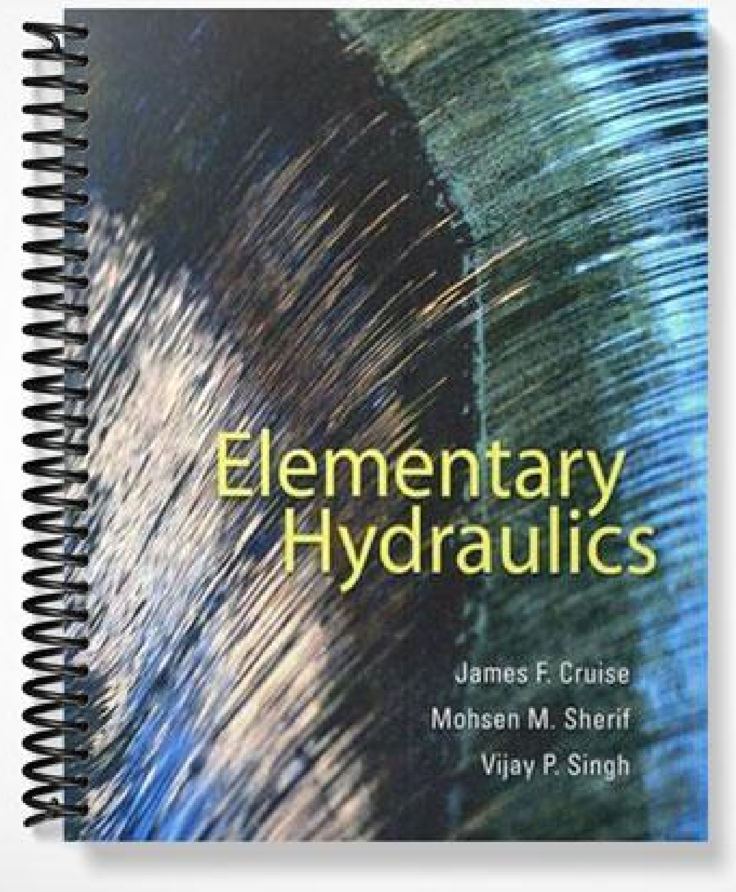
SOLUTIONS MANUAL



CHAPTER 2
FLUID PROPERTIES

2.1. The specific weight of a certain liquid is 90.30 lb/ft^3. Determine its density and specific gravity.

Solution: $\gamma = \rho.g \quad , \quad \rho = \dfrac{\gamma}{g}$

$$\therefore \rho = \frac{90.30}{32.2} = 2.80 \; slug / ft^3$$

$$S.G = \frac{\rho}{\rho_w} = \frac{2.80}{1.95} = 1.44$$

2.2. The density of a jet fuel is 760 kg/m^3. Determine the specific weight and specific gravity of this fuel.

Solution: $\gamma = \rho.g = 760 \times 9.81 = 7455 \; N/m^3 = 7.455 \; kN/m^3$

$$S.G = \frac{\rho}{\rho_w} = \frac{760}{1000} = 0.76$$

2.3. If 1.0 m^3 of oil weighs 890 kg, calculate its specific weight (g), density (r) and specific gravity (S.G.).

Solution:

$$\rho = \frac{m}{V} = \frac{890}{1.0} = 890 \; kg/m^3$$

$$\gamma = \rho g = 890 \times 9.81 = 8730 \; N/m^3$$

$$S.G. = \frac{\gamma_{oil}}{\gamma_{water}} = \frac{8730}{8790} = 0.892$$

2.4. A container has a 6 m^3 volume capacity and weights 1500 N when empty and 56,000 N when filled with a liquid. What is the density of liquid?

Solution: $W = mg = \rho \times V \times g$

$(56000\text{-}1500) = \rho \times 6.0 \times 9.81$

$\rho = 926 \; kg/m^3$

2.5. A cylindrical tank with a diameter of 80 cm is partially filled with SAE 10W oil (ρ = 870 kg/m^3). If the mass of the oil in the tank is 340 kg, determine the height of the oil in the tank.

Solution: $\rho_{oil} = m_{oil}/V_{oil}$

$V_{oil} = m_{oil}/\rho_{oil} = (340 \; kg) / (870/kg/m^3) = 0.39 \; m^3$

Base area of the tank = $\frac{\pi}{4}(0.8)^2 = 0.50265 m^2$

Therefore, height of the oil = 0.39 m^3/0.50265 m^2 = 0.776 m = 77.6 cm

2.6. An overhead tank has length, width and height of 2 m, 2.5 m and 1.5 m, respectively. If the water is filled up to 3-quartes of its height, determine the mass and weight of the water inside the tank.

Solution: Height of water = 3/4th of tank height = ¾ × 1.5 = 1.125 m

Volume of water = 1.125 × 2 × 2.5 = 5.625 m^3.

Mass of water = $\rho_{water} \times V_{water}$ = 1000 × 5.625 = 5625 kg

Weight = mg = 5625 x 9.8 = 55125 N = 55.125 kN

2.7. If a similar tank, as in Problem 2.6, is filled to the same level with oil with a mass of 4000 kg, determine the density and specific gravity of the oil.

Solution: $\rho_{oil} = m_{oil}/V$ = 4000/5.625 = 711.1 kg/m^3.

$\gamma = \rho.g$ = 711.1 × 9.81 = 6976 N/m^3.

2.8. If 6.2 m^3 of oil weights 52,980 N, calculate its density and specific gravity.

Solution:

Weight of unit volume = $\rho g = \frac{W}{V} = \frac{52980}{6.2} = 8545.2\ N/m^3$

Density = $\rho = \frac{\rho g}{g} = \frac{8545.2}{9.81} = 871\ kg/m^3$

Specific gravity S.G. = $\frac{\rho_{oil}}{\rho_{water}} = \frac{871}{1000} = 0.871$

2.9. If the density of a certain liquid is 1.82 slug/ft^3, determine its specific weight (γ) in S.I. units and its specific gravity (S.G.).

Solution:

$$\rho_f = \frac{1.82 \times 453.6 \times 32.2}{1000} \times \frac{1}{(30.48/100)^3} = 938.7 kg/m^3$$

Therefore, $\gamma = \rho.g$ = 938.7 × 9.81 = 9209 N/m^3.

S.G. = $\frac{\gamma_{oil}}{\gamma_{water}} = \frac{9209}{9790} = 0.94$

2.10. The specific gravity of mercury at 80^{o}C is 13.4. Determine its density and specific weight at this temperature. Express your answer in both SI and BG units.

Solution: In SI units, $\rho = \rho_w$.S.G = 1000 × 13.4 = 13400 kg/m^3

$\gamma = \rho g$ = 23400 × 9.81 = 131.5 kN/m^3

In BG units, ρ = 13.4 × 1.94 = 26 slug/ft^3

γ = 26 × 32.2 = 837.2 lb/ft^3

2.11. What is the dynamic viscosity of water at 212 ^{o}F? Express your answer in both SI and BG units.

Solution:

$$212\ ^oF = \frac{5}{9}(212 - 32) = 50\ ^oC$$

using Table 2.1, for T = 50 ^{o}C, μ = 5.47 × 10^{-4} N.s/m^2

In BG units,

μ = (5.47 × 10^{-4}) × (2.089 x 10^{-2}) = 1.14 × 10^{-5} lb.s/m^2.

2.12. The kinematic viscosity and specific gravity of liquid are 3.8 × 10^{-4} m^2/s and 0.88, respectively. What is the dynamic viscosity of the liquid in SI?

Solution:

$$v = \frac{\mu}{\rho} \quad , \quad \text{or} \quad \mu = v.\rho$$

$$\rho = \rho_w\ .\text{S.G.} = 0.88 \text{ x } 1000 = 880\ kg/m^3$$

$\therefore$ μ = 880 (kg/m^3) × 3.8 × 10^{-4} (m^2/s) = 0.334 N.s/m^2

2.13. A liquid has a specific weight of 55 lb/ft^3 and a dynamic viscosity of 2.90 lb.s/ft^2. Determine its kinematic viscosity.

Solution:

$$\rho = \frac{\gamma}{g} = \frac{55}{32.2} = 1.71 \text{ slug/ft}^3$$

$$v = \frac{\mu}{\rho} = \frac{2.90}{1.71} = 1.70 \text{ ft}^2\text{/s}$$

2.14. A cylindrical water tank is suspended vertically by its sides. The tank has 2.5 m diameter and is filled with 30 ^{o}C water to 0.75 m in height. Determine the force exerted on the tank bottom.

Solution:

Force on the tank's bottom is equal to the weight of water.

$F = W = mg = \rho \times V \times g = 996 \times \{\pi(2.5)^2/4 \times 0.75\} \times 9.81 = 35{,}972$ N

2.15. The viscosity of a certain fluid is 3.5×10^{-4} poise. Determine its viscosity in both SI and BG units.

Solution: Using the conversion factors,

$\mu = 3.5 \times 10^{-4}$ poise $= (3.5 \times 10^{-4}) \times 10^{-1} = 3.5 \times 10^{-5}$ N.s/m^2

$\mu = (3.5 \times 10^{-5}) \times (2.089 \times 10^{-2}) = 7.31 \times 10^{-7}$ lb.s/ft^2

2.16. The dynamic viscosity (μ) for water at 20 ^oC is 0.01 poise and its specific gravity (S.G.) is 0.98. Find its kinematic viscosity (v).

Solution:

Dynamic viscosity (μ) = 0.01 poise = 0.001 kg/m.s.

$\rho = 0.98 \times 1000 = 980$ kg/m^3

Kinematic viscosity, $v = \dfrac{\mu}{\rho} = \dfrac{0.001}{980} = 1.02 \times 10^{-6}\ \text{m}^2/\text{s}$

2.17. What is kinematic viscosity (v) of a liquid with a viscosity (μ) of 0.006 Pa-s and density of 870 kg/m^3 (Pa is Pascal).

Solution:

$$v = \frac{\mu}{\rho} = \frac{0.006}{870} = 6.897 \times 10^{-3}\ \text{m}^2/\text{sec}$$

2.18. What is the kinematic viscosity of liquid with a viscosity of 0.002 Pa-s and a specific gravity of 0.8?

Solution: S.G. $= \dfrac{\gamma_f}{\gamma_w}$

$\gamma_f = \text{S.G.} \times \gamma_w = 0.8 \times 1000 = 800$ kg/m^3

$$v = \frac{\mu}{\rho} = \frac{\mu g}{\gamma} = \frac{0.002 \times 9.81}{800} = 2.45 \times 10^{-5}\ \text{m}^2/\text{s}$$

2.19. A certain oil flows through a 2.5 in diameter pipe at 60 ^oF with a mean velocity of 6 ft/s. Determine the Reynold's number, R_n, for this flow. The oil has a density of 1.77 slug/ft^3 and a dynamic viscosity of 8×10^{-3} Ib.s/ft^2.

Solution:

$$R_n = \frac{\rho VD}{\mu} = \frac{1.77\ (slug/ft^3) \times 6.0\ (ft/s) \times (2.5/12)\ (ft)}{8 \times 10^{-3}\ Ib.s/ft^2} = 277\ (\text{slug.ft/s}^2)/\text{lb} = 277$$

2.20. A solid cube with a side length of 0.4 ft and a weight of 110 lb slides down a smooth surface that is inclined 45° from the horizontal. Figure P2.20. The block slides on a film of oil with a viscosity of 1.85×10^{-2} Ib.s/ft^2. If the velocity of the block is 1.2 ft/s, determine the thickness of the oil film assuming a linear velocity distribution in the film.

Solution:

Referring to Figure P2.20, P = W sin 45° = 110 × sin 45° = 77.78 lb

$$\tau = \frac{P}{A} = \frac{77.78}{(0.4)^2} = 486\ lb/ft^2$$

$$\tau = \mu \frac{dV}{dy} = \mu . \frac{V}{b}$$

$$476\ (lb/ft^2) = 1.85 \times 10^{-2}\ (lb.s/ft^2) \times \frac{1.2(ft/s)}{b(ft)}$$

The film thickness, b = 4.57×10^{-5} ft.

2.21. Two fixed plates are set a distance of 14 mm apart. A large movable plate with a thickness of 2 mm is located between the two fixed plates a distance of 4 mm from the lower plate and 8 mm from the upper plate as shown in Figure P2.21. The space between the upper fixed plate and the movable plate is filled with a fluid with a dynamic viscosity of 0.03 N.s/m^2. The other space between the lower fixed plate and the movable one is filled with another fluid with a dynamic viscosity of 0.015 N.s/m^2. Determine the magnitude and direction of the shear stress that act on the fixed plates when the speed of the movable plate is 5 m/s. Assume linear velocity distribution in the two oil fields and neglect the thickness of the moving plate.

Solution:

$$\tau = \frac{P}{A} = \mu \frac{dV}{dy} = \mu . \frac{V}{b}$$

For the upper plate, τ_1 = 0.03 × (5/0.008) = 18.75N/m^2, acting to the right.

For the lower plate, τ_2 = 0.015 × (5/0.004) = 18.75N/m^2, acting to the right.

2.22. A plate measuring 20 cm × 20 cm is pulled horizontally through SAE 10 oil at V = 0.15 m/s as shown in Figure P2.22. The oil temperature is 40°C. Find the force F.

Solution:

The dynamic viscosity of SEA 10 oil is
$\mu = 34 \times 10^{-3}$ N.s/m^2

$$\tau = \mu \frac{dV}{dy} = \mu \frac{V}{b}$$

= $34 \times 10^{-3} \times 0.15/0.002 = 2.55$ N/m^2 (Acting on each side of the plate)
Area of plate (one side) = 0.20 × 0.20 = 0.04 m^2

$$\tau = \frac{F}{A} = \frac{F}{2 \text{ x } 0.04} = 2.55 \; N/m^2 \text{ , Get, F = 0.204 N}$$

2.23. The plate in Problem 2.22 is moved with a velocity of 0.15 m/s to the right but the top and bottom plates are moved to the left at 0.10 m/s, as shown in Figure P2.23. Find the force F.

Solution: Relative speed of middle plate is, V = 0.10 + 0.15 = 0.25 m/s.

$$\tau_1 = \mu \frac{dV}{dy} = \mu \frac{V}{b} = 34 \times 10^{-3} \times \frac{0.25}{0.002} = 4.25 \; N/m^2$$

$$\tau = \frac{F}{A} = \frac{F}{2 \times 0.04} = 4.25 \; N/m^2, \quad \text{Get F = 0.34 N}$$

2.24. Two parallel plates, one moving at 4 m/s and the other fixed, are separated by a 5 mm thick layer of oil with a specific gravity of 0.8 and a kinematic viscosity of 1.25 × 10^{-4} m^2/s. What is the average shear stress in the oil?

Solution:

$$\tau = \mu \frac{dV}{dy} = \rho v \frac{V}{h} = 0.8\,(998)\,(1.25 \times 10^{-4}) \; \frac{4}{5/1000} = 79.84 \; N/m^2$$

2.25. If the fixed plate in Problem 2.24 is moved with a speed of 2 m/s in the opposite direction of the movement of the second plate, what is the average shear stress in the oil for this case?

Solution:

$$\tau = \mu \frac{dV}{dy} = \rho v \frac{V}{h} = 0.8\,(998)\,(1.25 \times 10^{-4}) \; \frac{4-(-2)}{5/1000} = 119.76 \; N/m^2$$

2.26. Three large plates are separated by thin layers of ethylene glycol and water as shown in Figure P2.26. The top plate moves to right at 1.80 m/s. At what speed and in what direction must the bottom plate be moved to hold the center plate stationary?

Solution:

For water at 20°C, m = 1 × 10^{-3} N.s/m^2
For ethylene glycol at 20°C, m = 1.99 × 10^{-2} N.s/m^2
Shear stress due to motion of upper plate

$$\tau_1 = \mu \frac{dV}{dy} = \mu \frac{V}{b} = 1 \text{ x } 10^{-3} \times \frac{1.80}{0.0015} = 1.20 \; N/m^2$$

To hold the middle plate stationary, the bottom plate must move to left at a speed that produces equal opposite shear,

$$\tau_1 = \tau_2 = 1.20 \; N/m^2$$

$$\tau_1 = \mu \frac{dV}{dy} = \mu \frac{V}{b} = 1.99 \times 10^{-3} \times \frac{V}{0.0025} = 1.20\ N/m^2$$

Get, V = 0.15 m/s

2.27. Oil (μ = 0.0004 Ib.s/ft^2) flows in the boundary layer as shown in Figure P2.27. Calculate the shear stress at (a) the plate surface, (b) 0.01 ft above plate surface.

Solution:

$$\frac{dV}{dy} = (3600 - 9 \times 10^6 y^2)$$

<u>(A): At plate surface</u>: y = 0,

$$\frac{dV}{dy} = 3600$$

$$\tau = \mu \frac{dV}{dy} = 0.0004 \times 3600 = 1.44\ lb/\ ft^2$$

<u>(B): At 0.01 ft above plate surface</u>: y = 0.01 ft,

$$\frac{dV}{dy} = 3600 - 9 \times 106\ (0.01)^2\ = 2700$$

$$\tau = \mu \frac{dV}{dy} = 0.0004 \times 2700 = 1.08\ lb/\ ft^2$$

2.28. A plate is sliding down a plane inclined at an angle of 30° as shown in Figure P2.28. The plate weighs 15 lb, measure 25in × 30in and has a velocity of 0.65 ft/sec. Determine the thickness of the SAE 10 oil between the plate and the plane if the oil temperature is 50°F. Assume linear velocity distribution in the film.

Solution:

For SAE10 oil at 50^{o}F,

μ = 39 × 10^{-6} lb.s/in^2 = 39 × 10^{-6} × (12)^2 = 5.616 × 10^{-3} lb. s/ft^2

F = W.sin30^o = 15 sin30° = 7.5 lb

$$\tau = \frac{F}{A} = \frac{7.5 \times (12)^2}{25 \times 30} = 1.44\ lb/\ ft^2$$

$$\tau = \mu \frac{dV}{dy} = \mu \frac{V}{b}$$

$1.44 = 5616 \times 10^{-3} \times \frac{0.65}{b}$, Get b = 2.54 × 10^{-3} ft.

2.29. A plate, 0.6 mm distance from a fixed plate, move at 0.3 m/s and requires a force per unit area of 4 Pa to maintain this speed. Determine the viscosity of the substance between the two plates.

Solution:

$$\tau = \mu \frac{V}{y} \text{, or } \mu = \frac{\tau . y}{V} \text{, then}$$

$$\mu = \frac{4 \times (0.6/1000)}{0.3} = 8 \times 10^{-3} \; N.\text{sec}/m^2$$

$$= 8 \times 10^{-3} \times \frac{10^5}{(100)^2} = 0.08 \; dyne.\text{sec}/cm^2 = 0.08 \text{ poise}$$

2.30. If the velocity distribution of a viscous liquid ($\mu = 0.9$ N.s/m^2) over a fixed boundary is given by $v = 0.68\ y - y^2$ in which v is the velocity in m/s at a distance y (m) above the boundary surface. Determine the shear stress at the surface and at y = 0.34 m.

Solution:

$$\tau = \mu \frac{V}{y}$$

$$\frac{dV}{dy} = \frac{d(0.68y - y^2)}{dy} = 0.68 - 2y$$

At the surface, y = 0

$$\tau = 0.9 \times 0.68 = 0.612 \text{ N/m}^2$$

$$\left.\frac{dv}{dy}\right|_{y=0.34} = 0.68 - 2 \times 0.34 = 0.0$$

$$\tau = 0 \text{ at } y = 0.34 \text{ m}$$

2.31. A thin plate is moving vertically between two parallel plates with a constant velocity of 20 m/s. The distance between the boundaries is 0.5 cm and the plate moves at a distance of 0.2 cm from one side of the boundaries. If the area of the plate is one m^2 and $\mu = 0.08$ poise. Find the weight and mass of the plate.

Solution:

$$W = (\tau_1 + \tau_2)A$$

$$W = \left(\mu \frac{V}{y_1} + \mu \frac{V}{y_2}\right) A = \mu A V \left(\frac{1}{y_1} + \frac{1}{y_2}\right)$$

$$W = 0.008 \text{ (kg/m.s)} \times 1 \text{ m}^2 \times 20 \text{ m/s} \times \left(\frac{100}{0.2} + \frac{100}{0.3}\right)\left(\frac{1}{m}\right) = 133.33 \text{ N}$$

$$\text{mass 'm'} = \frac{W}{g} = \frac{133.33}{9.51} = 13.59 \text{ kg}$$

2.32. A piston 11.96 cm diameter and 14 cm long works in a cylinder of 12 cm diameter. If lubricating oil which fills the space between them has a viscosity of 0.65 poise, calculate the speed with which the piston will move through the cylinder when the axial load of 0.86 N is applied.

Solution:

$$F = \tau A$$

$$0.86 = \left(\mu \frac{V}{y}\right) A, \quad = \left(\mu \frac{V}{y}\right)(\pi dL)$$

$$0.86 = \left(0.065 \text{ x } \frac{V}{0.02/100}\right)\left(\pi \times \frac{11.96}{100} \times \frac{14}{100}\right)$$

The speed of the piston, V = 0.05 m/s.

2.33. A 50 mm diameter steel cylinder 600 mm long falls, because of its own gravity force at a uniform rate of 0.2 m/s inside a tube of slightly larger diameter. A castor-oil film of constant thickness fills the space between the cylinder and the tube. Determine the clearance between the tube and the cylinder. The temperature is 38°C. Relative density of steel = 7.85.

Solution:

$$W = \tau . A$$

$$\gamma \frac{\pi D^2}{4} L = \mu \frac{dV}{dy} . \pi DL$$

$$\gamma \frac{D}{4} = \mu \frac{V}{y}$$

at $T = 38\ ^oC$ $\qquad \mu = 0.28 \text{ poise} = 0.028 \text{ N.s/m}^2$

$$7.85 \times 10^3 \times 9.81 \times \frac{50}{1000 \times 4} = 0.028 \times \frac{0.2}{y}$$

$y = 5.81 \times 10^{-6}$ m = 0.0058 mm

2.34. A cylinder of radius 0.2 m and length 1.3 m rotates concentrically inside a fixed cylindrical sleeve of radius 0.21 m, find the viscosity of oil that fills the space between them if a torque of 1.2 N m is required to maintain a rotation of 80 rpm.

Solution:

$$\frac{dV}{dy} = \frac{V}{y}$$

$$T = F R = \tau AR$$

$$= \left(\mu \times \frac{dV}{dy}\right) \times (2\pi Rh) \times R = \mu \times \frac{V}{y} \times 2\pi h \times R^2$$

$$v = \omega R$$

$$T = \mu \times \frac{\omega}{y} 2\pi h \times R^3$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 80}{60} = 8.377 \text{ rad/sec.}$$

$$\therefore 1.2 = \mu \times \frac{8.377}{0.01} \times 2\pi \times 1.3 \times (0.2)^3$$

$$\therefore \mu = 0.0219 \text{ kg/m.s.}$$

2.35. The lower end of a vertical shaft rests on a foot bearing. The shaft is 1.5 m diameter, and is separated from the bearing by an oil film of 0.08 cm thickness and μ = 1.5 poise. What will be the power absorbed by the shaft when it rotates at a speed of 400 rpm.

Solution:

$$\varpi = \frac{2\pi N}{60} = \frac{2\pi \times 400}{60} = \frac{80}{60}\pi \text{ rad/sec.}$$

Power = Torque × ω

dP = (dT) x ω = (dF r) ω = (τ dA) r ω

$$= \mu \frac{dV}{dy}.(2\pi r dr).r.\omega = \mu \frac{\omega V}{y}.(2\pi r^2 dr), \quad \text{but } V = \omega r$$

$$dP = \mu \frac{\omega^2}{y}.(2\pi r^3 dr)$$

$$\therefore P = 2\pi\mu \frac{\omega^2}{y}.\int r^3 dr = 2\pi\mu \frac{\omega^2}{y}.\frac{r^4}{4}$$

$$\therefore P = \frac{2\pi \times 0.15}{0.08/100}.\left(\frac{80\pi}{6}\right)^2 \times \left(\frac{0.75}{5}\right)^4 = 2554.8W = 3.425\,HP$$

2.36. A piston moves in a cylinder with a clearance of 1 mm around the piston, as shown in Figure 2.36. The oil viscosity is 0.85 kg/m-s. Determine the force required to move the piston with a speed of 6.0 m/s.

Solution:

$$\text{Shear stress } \tau = \mu \frac{dV}{dy} = 0.85 \frac{6}{(10-9.8)/2} \times 100 = 5100\ N/m^2$$

$$F = \tau A = 5100 \times \pi d L = 5100 \times \pi \times \frac{9.8}{100} \times \frac{7.5}{100} = 117.76 \text{ N}$$

2.37. A piston 10 kg in mass is sliding down in a cylinder due to gravity as shown in Figure P2.37. The gap between the piston and cylinder is filled with oil with a viscosity of 0.29 kg/m.s. The piston diameter is 15 cm and the clearance between the piston and the cylinder is 0.4 mm. Determine whether the piston is accelerating or decelerating when it is moving with a speed of a) 5 m/s, and b) 0.5 m/s.

Solution:

a) Shear stress, $\tau = \mu \frac{dV}{dy} = 0.29 \frac{5}{0.4/1000} = 3625\, N/m^2$

Viscous force, F = $\tau . A = 3625 \times \pi \left(\frac{15}{100}\right) \left(\frac{10}{100}\right) = 170.08\, N$

Force balance is given by: ΣF = mg - F = m.a

10 × 9.8 – 170.08 = 10 × a

a = - 7.28 m/s^2

Therefore, the piston is decelerating at the rate of 7.28 m/s^2 when its velocity is 5 m/s.

b) Shear stress $\tau = \mu \frac{dV}{dy} = 0.29 \frac{0.5}{0.4/1000} = 362.5\, N/m^2$

Viscous force F = $\tau . A = 362.5 \times \pi \left(\frac{15}{100}\right)\left(\frac{10}{100}\right) = 17.08\, N$

Force balance is given by ΣF = mg-F = m.a

10 × 9.8 – 17.08 = 10 × a

a = 8.09 m/s^2

Therefore, the piston is accelerating at a rate of 8.09 m/s^2 when its velocity is 0.5 m/s.

2.38. A liquid compressed in a cylinder has a volume of 1.0 lit at pressure of 100 N/m^2 and a volume of 995 cm^3 at a pressure of 200 N/m^2. What is its bulk modulus of elasticity?

Solution:

$$K = - \frac{\text{Change of pressure intensity}}{\text{Volumetric strain}}$$

$$K = - \frac{(P_2 - P_1)}{\frac{(V_2 - V_1)}{V_1}} = - \frac{(200 - 100)}{\frac{(995 - 1000)}{1000}} = 2 \times 10^{-4}\ \text{N/m}^2$$

2.39. A closed steel tank has a volume of 5.0 ft^3. If the bulk modulus of elasticity of water is 300,000 psi, how many pounds of water can the tank hold at 2000 psi?

Solution:

$$K = -\frac{(P_2 - P_1)}{\frac{(V_2 - V_1)}{V_1}}, \text{ then } 300{,}000 = \frac{(2000)}{\frac{(V_2 - 5)}{5}}$$

V_2 = 5.033 ft^3,

Weight of water = $\gamma \overline{V}$ = 62.4 × 5.033 = 314.06 lb

2.40. A liquid compressed in a cylinder has a volume of 995 cm^3 at 2000 KN/m^2 and a volume of 990 cm^3 at 3000 kN/m^2. What is its bulk modulus of elasticity?

Solution:

$$K = -\frac{(P_2 - P_1)}{\frac{(V_2 - V_1)}{V_1}} = -\frac{(3-2)\times 10^6}{\frac{(990-995)}{995}} = 199\times 10^6 \text{ N}/m^2$$

2.41. Water has a volume of 1 m^3 at 40 bar and a volume of 0.99 m^3 at 425 bar. Find the bulk modulus of elasticity.

Solution:

$$K = -\frac{(P_2 - P_1)}{\frac{(V_2 - V_1)}{V_1}} = -\frac{(245-40)\times 10^5}{\frac{(0.99-1.0)}{1.0}} = 2.05\times 10^9 \text{ Pa}$$

2.42. Assuming that the bulk modulus of elasticity of water is 1.15 × 10^6 kN/m^2 at standard atmospheric conditions. Determine the increase of pressure necessary to produce 2% reduction in the volume at the same temperature.

Solution:

$$K = -\frac{(P_2 - P_1)}{\frac{(V_2 - V_1)}{V_1}}$$

Reduction of 2% of volume implies that

$$\frac{(\Delta V)}{V_o} = -0.02$$

The increase in pressure is

ΔP = 0.02 K = 0.02 × 1.15 × 10^6 = 23,000 kN/m^2

2.43. Find the change in volume of one cubic meter of water when it is subjected to an increase in pressure of 1840 kN/m^2. Take bulk modulus of elasticity as 2.16×10^6 kN/m^2.

Solution:

$$K = -\frac{(P_2 - P_1)}{\frac{(V_2 - V_1)}{V_1}}, \text{ then } 216 \times 10^6 = \frac{(-1840)}{\frac{(\Delta V)}{V}}$$

$\Delta V = (-8.518 \times 10^{-4}) \times V = -8.5 \times 10^{-4}\ m^3$

2.44. An empty cylinder (filled with air) is sealed at 1 atmospheric pressure during winter when the ambient temperature is –15 ^oC. What will be the pressure inside the cylinder summer when the ambient temperature is 50 ^oC.

Solution:

P_1 = 1 atm = 101.33 kPa, T1 = -15^{o}C = 258.16 K, T_2 = 50 ^{o}C = 323.16 K
Under constant volume of the cylinder, assuming air to be ideal gas:
$P_1/T_1 = P_2/T_2$
Therefore, P_1 = 126.84 kPa = 1.25 atm

2.45. To what height above the reservoir level will water (at 20°C) rise in a glass tube, as shown in Figure P2.45, if the inside diameter of tube is 2 mm.

Solution:

For water at 20^{o}C, σ = 0.073 N/m

By taking the summation of forces in the vertical direction we have, $W-F_o = 0$

$\gamma(h)(\pi d^2/4) - \sigma\pi.d.\cos\theta = 0$, or

$$h = \frac{4\sigma\cos\theta}{\gamma d}, \ (\theta = 0 \text{ for water})$$

$$h = \frac{4 \times 0.073}{9790 \times 2.0 \times 10^{-3}} = 14.9 \times 10^{-3}\, m = 14.9 mm$$

2.46. Estimate the height to which water 26.7^{o}C will rise in a capillary tube of diameter 2.5 mm. Take surface tension (σ) = 0.0718 N/m.

Solution:

Assume angle (θ) = 90^o for a clean tube

$$h = \frac{4\sigma}{\rho g d} = \frac{4 \times 0.0718}{9810 \times 2.5 \times 10^{-3}} == 0.0117\ m = 11.71\ mm$$

2.47. A capillary tube, having an inside diameter of 6 mm is dipped in water at 20°C. Determine the height of water, which will rise in the tube. Take specific weight of water at 20°C = 998 kg/m^3. Take surface tension (σ) = 0.08 gm/cm and angle of contact (α) as 60°.

Solution:

$$h = \frac{4\sigma \sin(\theta)}{\gamma d} = \frac{4 \times 0.08 \times \sin(60)}{0.998 \times 6 \times 10^{-1}} = 4.6\ mm$$

2.48. A U-tube is made up of two capillaries of diameters 1.5 mm and 2.0 mm respectively. The U tube is kept vertically and partially filled with water of surface tension 0.07 N/m and zero contact angle. Calculate the difference in the level caused by the capillary.

Solution:

Surface tension in the small tube is equal to $\pi\sigma d_1$ upward

Surface tension in the other tube is equal to $\pi\sigma d_2$ upward

Difference in the two surface tension forces = weight of the column of water in the small tube

$$\pi\sigma d_1 - \pi\sigma d_2 = \frac{\pi d_1^2}{4} h\rho g$$

$$h = \frac{4(d_2 - d_1)\sigma}{d_2^2 \rho g}$$

$$= \frac{4(0.002 - 0.0015) \times 0.07}{(0.0015)^2 \times 9810} = 6.343 \times 10^{-3}\ m\ = 6.343\ mm$$

2.49. Calculate the capillary effect in mm in a glass tube of 6-mm diameter when immersed in (i) water, and (ii) mercury, both liquids being at 20°C. Assume σ to be 73 x 10^{-3} N/m for water and 0.5 N/m for mercury. The contact angles for water and mercury are zero and 130°, respectively.

Solution:

$$h = \frac{4\sigma \cos(\theta)}{\gamma d}$$

$$h_{water} = \frac{4 \times 73 \times 10^{-3} \cos(180 - 130)}{9806 \times 6 \times 10^{-3}} = 4.96\ \text{mm}$$

$$h_{hg} = \frac{4 \times 0.5 \times \cos(0)}{13.6 \times 9806 \times 10^{-3}} = 2.5\ \text{mm}$$

2.50. Show that the gauge pressure within a liquid droplet varies inversely with diameter of the droplet.

Solution:

$$\pi d\sigma = p \times \frac{\rho d^2}{4}$$

$$p = \frac{4\sigma}{d}$$

2.51. A small drop of water is in contact with air and has diameter of 0.06 mm. If the pressure within the droplet is 466 Pa greater than the atmosphere, what is the value of surface tension?

Solution:

$$\sigma = \frac{pD}{4} = \frac{1}{4} \times (466) \times \frac{0.06}{1000} = 0.007 \text{ N/m}$$

2.52. Calculate the internal pressure of 25-mm diameter soap bubble if the surface tension in the soap film is 0.5 N/m.

Solution:

$$\Delta p \times A = \sigma \times (2\pi r \cos\theta)$$

$$\Delta p \times \frac{\pi D^2}{4} = \sigma \times \pi D \cos\theta$$

$$\Delta p = \frac{4\sigma}{D} = \frac{4 \times 0.5}{25 \times 10^{-3}} = 80 \text{ N/m}^2$$

2.53. Calculate the gauge pressure and the absolute pressure within: a) a droplet of water 0.3 cm in diameter, and b) a jet of water 0.3 cm in diameter. Assume the surface tension of water 0.064 N/m.

Solution:

a) For the droplet (assume it to be spherical)

$$p_{gauge} = \frac{4\sigma}{d} = \frac{4 \times 0.064}{0.003} = 85.33 \text{ N/m}^2$$

$$p_{abs} = 10.33 \times 9810 + 85.33 = 101{,}422.6 \text{ N/m}^2 = 101.4 \text{ kN/m}^2$$

b) For the jet (assume it to be cylindrical)

$$p_{gauge} = \frac{2\sigma}{d} = \frac{2 \times 0.064}{0.003} = 42.67 \text{ N/m}^2$$

$$p_{abs} = 10.33 \times 9810 + 42.67 = 101{,}380 \text{ N/m}^2 = 101.38 \text{ kN/m}^2$$

2.54. What is the pressure intensity within as free jet of water 0.02-inch diameter, if the surface tension of water is 0.005 lb/ft?

Solution:

$$\Delta p \times A = \sigma \times (2L) + W$$

where, W is weight of water. Consider unit length.

$$\Delta p \times (D \times 1.0) = \sigma \times (2 \text{ x } 1.0) + \frac{\gamma \pi D^2}{8}$$

$$\Delta p = \frac{2\sigma}{D} + \frac{\gamma \times \pi \times D}{8}$$

$$\Delta p = \frac{2 \times 0.005}{(0.02/12)} + \frac{62.4 \times \pi \times 0.02}{12 \times 8} = 6.0408 \text{ lb/ft}^2$$

2.55. Calculate the maximum capillary rise in 1 mm diameter glass tube for: (a) Water at 20°C, and (b) Kerosene at 20°C (0 = 5°).

Solution:

(a) For water at 20°C, $\sigma = 0.073$ N/m, $\theta = 0$, $\gamma = 9790$ N/m^3

$$h = \frac{4\sigma}{\gamma d} = \frac{4 \times 0.073}{9790 \times 0.001} = 0.0298 \text{ m} = 29.8 \text{ mm}$$

(b) For kerosene at 20°C, $\sigma = 0.029$ N/m, $\theta = 5$, $\gamma = 7985$ N/m^3

$$h = \frac{4\sigma \cos\theta}{\gamma d} = \frac{4 \times 0.029 \times \cos 5^\circ}{7985 \times 0.001} = 0.0145 \text{ m} = 14.5 \text{ mm}$$