## SOLUTIONS MANUAL



## CHAPTER 2

## <u>Exercises</u>

**E2.1** (a)  $R_2$ ,  $R_3$ , and  $R_4$  are in parallel. Furthermore  $R_1$  is in series with the combination of the other resistors. Thus we have:

$$R_{eq} = R_1 + \frac{1}{1/R_2 + 1/R_3 + 1/R_4} = 3 \Omega$$

(b)  $R_3$  and  $R_4$  are in parallel. Furthermore,  $R_2$  is in series with the combination of  $R_3$ , and  $R_4$ . Finally  $R_1$  is in parallel with the combination of the other resistors. Thus we have:

$$R_{eq} = \frac{1}{1/R_1 + 1/[R_2 + 1/(1/R_3 + 1/R_4)]} = 5 \Omega$$

(c)  $R_1$  and  $R_2$  are in parallel. Furthermore,  $R_3$ , and  $R_4$  are in parallel. Finally, the two parallel combinations are in series.

$$R_{eq} = \frac{1}{1/R_1 + 1/R_2} + \frac{1}{1/R_3 + 1/R_4} = 52.1\,\Omega$$

(d)  $R_1$  and  $R_2$  are in series. Furthermore,  $R_3$  is in parallel with the series combination of  $R_1$  and  $R_2$ .

$$R_{eq} = \frac{1}{1/R_3 + 1/(R_1 + R_2)} = 1.5 \,\mathrm{k}\Omega$$

**E2.2** (a) First we combine  $R_2$ ,  $R_3$ , and  $R_4$  in parallel. Then  $R_1$  is in series with the parallel combination.

$$\mathcal{V}_{s} = \underbrace{\frac{R_{1}}{\lambda_{1}}}_{Q_{0}V} \underbrace{\frac{k_{2}}{\lambda_{1}}}_{R_{2}} \underbrace{\frac{k_{3}}{R_{3}}}_{R_{3}} \underbrace{\frac{k_{4}}{\lambda_{3}}}_{R_{4}} \Rightarrow 20V \underbrace{\frac{R_{1}}{\lambda_{1}}}_{V_{eg}} \underbrace{\frac{R_{eg}}{R_{eg}}}_{P_{eg}} R_{eg}$$

$$\mathcal{R}_{eg} = \frac{1}{1/R_{2} + 1/R_{3} + 1/R_{4}} = 9.231 \Omega \qquad i_{1} = \frac{20V}{R_{1} + R_{eg}} = \frac{20}{10 + 9.231} = 1.04 A$$

$$v_{eg} = R_{eg}i_{1} = 9.600V \qquad i_{2} = v_{eg}/R_{2} = 0.480 A \qquad i_{3} = v_{eg}/R_{3} = 0.320 A$$

$$i_{4} = v_{eg}/R_{4} = 0.240 A$$

(b)  $R_1$  and  $R_2$  are in series. Furthermore,  $R_3$ , and  $R_4$  are in series. Finally, the two series combinations are in parallel.



(c)  $R_3$ , and  $R_4$  are in series. The combination of  $R_3$  and  $R_4$  is in parallel with  $R_2$ . Finally the combination of  $R_2$ ,  $R_3$ , and  $R_4$  is in series with  $R_1$ .



E2.3 (a) 
$$v_1 = v_s \frac{R_1}{R_1 + R_2 + R_3 + R_4} = 10 \text{ V}$$
.  $v_2 = v_s \frac{R_2}{R_1 + R_2 + R_3 + R_4} = 20 \text{ V}$ .  
Similarly, we find  $v_3 = 30 \text{ V}$  and  $v_4 = 60 \text{ V}$ .

(b) First combine  $R_2$  and  $R_3$  in parallel:  $R_{eq} = 1/(1/R_2 + 1/R_3) = 2.917 \Omega$ . Then we have  $v_1 = v_s \frac{R_1}{R_1 + R_{eq} + R_4} = 6.05 \text{ V}$ . Similarly, we find  $v_2 = v_s \frac{R_{eq}}{R_1 + R_{eq} + R_4} = 5.88 \text{ V}$  and  $v_4 = 8.07 \text{ V}$ .

E2.4 (a) First combine 
$$R_1$$
 and  $R_2$  in series:  $R_{eq} = R_1 + R_2 = 30 \Omega$ . Then we have  $i_1 = i_s \frac{R_3}{R_3 + R_{eq}} = \frac{15}{15 + 30} = 1 A$  and  $i_3 = i_s \frac{R_{eq}}{R_3 + R_{eq}} = \frac{30}{15 + 30} = 2 A$ .

(b) The current division principle applies to two resistances in parallel. Therefore, to determine  $i_1$ , first combine  $R_2$  and  $R_3$  in parallel:  $R_{eq} = 1/(1/R_2 + 1/R_3) = 5 \Omega$ . Then we have  $i_1 = i_s \frac{R_{eq}}{R_1 + R_{eq}} = \frac{5}{10 + 5} = 1 A$ . Similarly,  $i_2 = 1 A$  and  $i_3 = 1 A$ .

- **E2.5** Write KVL for the loop consisting of  $v_1$ ,  $v_y$ , and  $v_2$ . The result is  $-v_1 v_y + v_2 = 0$  from which we obtain  $v_y = v_2 v_1$ . Similarly we obtain  $v_z = v_3 v_1$ .
- E2.6 Node 1:  $\frac{v_1 v_3}{R_1} + \frac{v_1 v_2}{R_2} = i_a$  Node 2:  $\frac{v_2 v_1}{R_2} + \frac{v_2}{R_3} + \frac{v_2 v_3}{R_4} = 0$ Node 3:  $\frac{v_3}{R_5} + \frac{v_3 - v_2}{R_4} + \frac{v_3 - v_1}{R_1} + i_b = 0$

$$V_2 = \frac{\begin{vmatrix} -0.2 & 1 \end{vmatrix}}{\begin{vmatrix} 0.7 & -0.2 \\ -0.2 & 0.5 \end{vmatrix}} = \frac{0.7 + 1.2}{0.35 - 0.04} = 6.129 \text{ V}$$

Many other methods exist for solving linear equations.

**E2.8** First write KCL equations at nodes 1 and 2:

Node 1: 
$$\frac{v_1 - 10}{2} + \frac{v_1}{5} + \frac{v_1 - v_2}{10} = 0$$
  
Node 2:  $\frac{v_2 - 10}{10} + \frac{v_2}{5} + \frac{v_2 - v_1}{10} = 0$ 

Then simplify the equations to obtain:  $8v_1 - v_2 = 50$  and  $-v_1 + 4v_2 = 10$ Solving, we find  $v_1 = 6.77$  V and  $v_2 = 4.19$  V.

E2.9 (a) Writing the node equations we obtain:

Node 1: 
$$\frac{v_1 - v_3}{20} + \frac{v_1}{5} + \frac{v_1 - v_2}{10} = 0$$
  
Node 2:  $\frac{v_2 - v_1}{10} + 10 + \frac{v_2 - v_3}{5} = 0$   
Node 3:  $\frac{v_3 - v_1}{20} + \frac{v_3}{10} + \frac{v_3 - v_2}{5} = 0$ 

(b) Simplifying the equations we obtain:

$$\begin{array}{l} 0.35\nu_1-0.10\nu_2-0.05\nu_3=0\\ -0.10\nu_1+0.30\nu_2-0.20\nu_3=-10\\ -0.05\nu_1-0.20\nu_2+0.35\nu_3=0 \end{array}$$

(c) Solving yields  $v_1 = -27.27 \text{ V}$ ,  $v_2 = -72.73 \text{ V}$ , and  $v_3 = -45.45 \text{ V}$ .

(d) Finally, 
$$i_x = (v_1 - v_3)/20 = 0.909 \text{ A}.$$

**E2.10** The equation for the supernode enclosing the 15-V source is:

$$\frac{V_3 - V_2}{R_3} + \frac{V_3 - V_1}{R_1} = \frac{V_1}{R_2} + \frac{V_2}{R_4}$$

This equation can be readily shown to be equivalent to Equation 2.34 in the book. (Keep in mind that  $\nu_3 = -15$  V.)

**E2.11** Write KVL from the reference to node 1 then through the 10 V source to node 2 then back to the reference node:

$$-v_1 + 10 + v_2 = 0$$

Then write KCL equations. First for a supernode enclosing the 10-V source we have:

$$\frac{v_1}{R_1} + \frac{v_1 - v_3}{R_2} + \frac{v_2 - v_3}{R_3} = 1$$

Node 3:

$$\frac{v_3}{R_4} + \frac{v_3 - v_1}{R_2} + \frac{v_3 - v_2}{R_3} = 0$$

Reference node:

$$\frac{v_1}{R_1} + \frac{v_3}{R_4} = 1$$

An independent set consists of the KVL equation and any two of the KCL equations.

E2.12 (a) Select the  
reference node at the  
left-hand end of the  
voltage source as shown  
at right.  
Then write a KCL  
equation at node 1.  

$$\frac{V_1}{R_1} + \frac{V_1 - 10}{R_2} + 1 = 0$$

Substituting values for the resistances and solving, we find  $v_1 = 3.33$  V. Then we have  $i_a = \frac{10 - v_1}{R_2} = 1.333$  A.

(b) Select the reference node and assign node voltages as shown.

Then write KCL equations at nodes 1 and 2.



$$\frac{\frac{v_1 - 25}{R_2} + \frac{v_1}{R_4} + \frac{v_1 - v_2}{R_3} = 0}{\frac{v_2 - 25}{R_1} + \frac{v_2 - v_1}{R_3} + \frac{v_2}{R_5} = 0}$$

Substituting values for the resistances and solving, we find  $v_1 = 13.79$  V and  $v_2 = 18.97$  V. Then we have  $i_b = \frac{v_1 - v_2}{R_a} = -0.259$  A.

E2.13 (a) Select the reference node and node voltage as shown. Then write a KCL equation at node 1, resulting in  $\frac{v_1}{5} + \frac{v_1 - 10}{5} - 2i_x = 0$ 



Then use  $i_x = (10 - v_1)/5$  to substitute and solve. We find  $v_1 = 7.5$  V. Then we have  $i_x = \frac{10 - v_1}{5} = 0.5$  A.

(b) Choose the reference node and node voltages shown:



Then write KCL equations at nodes 1 and 2:

$$\frac{v_1}{5} + \frac{v_1 - 2i_y}{2} + 3 = 0 \qquad \frac{v_2}{5} + \frac{v_2 - 2i_y}{10} = 3$$

Finally use  $i_y = v_2 / 5$  to substitute and solve. This yields  $v_2 = 11.54$  V and  $i_y = 2.31$  A.

- **E2.14** Refer to Figure 2.32b in the book. (a) Two mesh currents flow through  $R_2$ :  $i_1$  flows downward and  $i_4$  flows upward. Thus the current flowing in  $R_2$  referenced upward is  $i_4 i_1$ . (b) Similarly, mesh current  $i_1$  flows to the left through  $R_4$  and mesh current  $i_2$  flows to the right, so the total current referenced to the right is  $i_2 i_1$ . (c) Mesh current  $i_3$  flows downward through  $R_8$  and mesh current  $i_4$  flows upward, so the total current referenced downward is  $i_3 i_4$ . (d) Finally, the total current referenced upward through  $R_8$  is  $i_4 i_3$ .
- E2.15 Refer to Figure 2.32b in the book. Following each mesh current we have

 $\begin{aligned} & \mathcal{R}_{1}i_{1} + \mathcal{R}_{2}(i_{1} - i_{4}) + \mathcal{R}_{4}(i_{1} - i_{2}) - v_{A} = 0\\ & \mathcal{R}_{5}i_{2} + \mathcal{R}_{4}(i_{2} - i_{1}) + \mathcal{R}_{6}(i_{2} - i_{3}) = 0\\ & \mathcal{R}_{7}i_{3} + \mathcal{R}_{6}(i_{3} - i_{2}) + \mathcal{R}_{8}(i_{3} - i_{4}) = 0\\ & \mathcal{R}_{3}i_{4} + \mathcal{R}_{2}(i_{4} - i_{1}) + \mathcal{R}_{8}(i_{4} - i_{3}) = 0 \end{aligned}$ 

**E2.16** We choose the mesh currents as shown:



Then the mesh equations are:

 $5i_1 + 10(i_1 - i_2) = 100$  and  $10(i_2 - i_1) + 7i_2 + 3i_2 = 0$ 

Simplifying and solving these equations, we find that  $i_1 = 10$  A and  $i_2 = 5$  A. The net current flowing downward through the 10- $\Omega$  resistance is  $i_1 - i_2 = 5$  A.

To solve by node voltages, we select the reference node and node voltage shown. (We do not need to assign a node voltage to the connection

between the 7- $\Omega$  resistance and the 3- $\Omega$  resistance because we can treat the series combination as a single 10- $\Omega$  resistance.)



The node equation is  $(v_1 - 10)/5 + v_1/10 + v_1/10 = 0$ . Solving we find that  $v_1 = 50$  V. Thus we again find that the current through the  $10-\Omega$  resistance is  $i = v_1/10 = 5$  A.

Combining resistances in series and parallel, we find that the resistance "seen" by the voltage source is 10  $\Omega$ . Thus the current through the source and 5- $\Omega$  resistance is (100 V)/(10  $\Omega$ ) = 10 A. This current splits equally between the 10- $\Omega$  resistance and the series combination of 7  $\Omega$  and 3  $\Omega$ .

E2.17 First, we assign the mesh currents as shown.



Then we write KVL equations following each mesh current:

$$2(i_1 - i_3) + 5(i_1 - i_2) = 10$$
  

$$5i_2 + 5(i_2 - i_1) + 10(i_2 - i_3) = 0$$
  

$$10i_3 + 10(i_3 - i_2) + 2(i_3 - i_1) = 0$$

Simplifying and solving, we find that  $i_1 = 2.194 \text{ A}$ ,  $i_2 = 0.839 \text{ A}$ , and  $i_3 = 0.581 \text{ A}$ . Thus the current in the 2- $\Omega$  resistance referenced to the right is  $i_1 - i_3 = 2.194 - 0.581 = 1.613 \text{ A}$ .

- **E2.18** Refer to Figure 2.37 in the book. In terms of the mesh currents the current directed to the right in the 5-A current source is  $i_1$ , however by the definition of the current source, the current is 5 A directed to the left. Thus we conclude that  $i_1 = -5 A$ . Then we write a KVL equation following  $i_2$ , which results in  $10(i_2 i_1) + 5i_2 = 100$ .
- E2.19 Refer to Figure 2.38 in the book. First, for the current source, we have

$$i_2 - i_1 = 1$$

Then we write a KVL equation going around the perimeter of the entire circuit:

$$5i_1 + 10i_2 + 20 - 10 = 0$$

Simplifying and solving these equations we obtain  $i_1 = -4/3$  A and  $i_2 = -1/3$  A.

**E2.20** (a) As usual, we select the mesh currents flowing clockwise around the meshes as shown. Then for the current source, we have  $i_2 = -1 A$ . This is because we defined the mesh current  $i_2$  as the current referenced



downward through the current source. However, we know that the current through this source is 1 A flowing upward. Next we write a KVL equation around mesh 1:  $10i_1 - 10 + 5(i_1 - i_2) = 0$ . Solving we find that  $i_1 = 1/3$  A. Referring to Figure 2.29a in the book we see that the value of the current  $i_a$  referenced downward through the 5  $\Omega$  resistance is to be found. In terms of the mesh currents we have  $i_a = i_1 - i_2 = 4/3$  A.



$$-25 + 10(i_1 - i_3) + 10(i_1 - i_2) = 0$$
  

$$10(i_2 - i_1) + 20(i_2 - i_3) + 20i_2 = 0$$
  

$$10(i_3 - i_1) + 5i_3 + 20(i_3 - i_2) = 0$$

Simplifying and solving, we find  $i_1 = 2.3276 \text{ A}$ ,  $i_2 = 0.9483 \text{ A}$ , and  $i_3 = 1.2069 \text{ A}$ . Finally, we have  $i_b = i_2 - i_3 = -0.2586 \text{ A}$ .

E2.21 (a) KVL mesh 1:  $-10 + 5i_1 + 5(i_1 - i_2) = 0$ For the current source:  $i_2 = -2i_x$ However,  $i_x$  and  $i_1$  are the same current, so we also have  $i_1 = i_x$ . Simplifying and solving, we find  $i_x = i_1 = 0.5$  A.

> (b) First for the current source, we have:  $i_1 = 3 A$ Writing KVL around meshes 2 and 3, we have:

$$2(i_2 - i_1) + 2i_y + 5i_2 = 0$$
  
$$10(i_3 - i_1) + 5i_3 - 2i_y = 0$$



However  $i_3$  and  $i_y$  are the same current:  $i_y = i_3$ . Simplifying and solving, we find that  $i_3 = i_y = 2.31 \text{ A}$ .

**E2.22** Under open-circuit conditions, 5 A circulates clockwise through the current source and the 10- $\Omega$  resistance. The voltage across the 10- $\Omega$  resistance is 50 V. No current flows through the 40- $\Omega$  resistance so the open circuit voltage is  $V_r = 50$  V.

With the output shorted, the 5 A divides between the two resistances in parallel. The short-circuit current is the current through the 40- $\Omega$  resistance, which is  $i_{sc} = 5 \frac{10}{10+40} = 1 A$ . Then the Thévenin resistance is  $R_t = v_{oc} / i_{sc} = 50 \Omega$ .

**E2.23** Choose the reference node at the bottom of the circuit as shown:



Notice that the node voltage is the open-circuit voltage. Then write a KCL equation:

$$\frac{v_{\rm oc} - 20}{5} + \frac{v_{\rm oc}}{20} = 2$$

Solving we find that  $v_{oc}$  = 24 V which agrees with the value found in Example 2.15.

**E2.23** To zero the sources, the voltage sources become short circuits and the current sources become open circuits. The resulting circuits are :







(a) 
$$R_{t} = 10 + \frac{1}{1/5 + 1/20} = 14 \Omega$$
 (b)  $R_{t} = 10 + 20 = 30 \Omega$ 

(c) 
$$R_{t} = \frac{1}{\frac{1}{10} + \frac{1}{6 + \frac{1}{(1/5 + 1/20)}}} = 5 \Omega$$



Then find short-circuit current:



 $I_n = i_{sc} = 10/15 + 1 = 1.67$  A

(b) We cannot find the Thévenin resistance by zeroing the sources because we have a controlled source. Thus we find the open-circuit voltage and the short-circuit current.



$$\frac{\nu_{oc} - 2\nu_{x}}{10} + \frac{\nu_{oc}}{30} = 2 \qquad \nu_{oc} = 3\nu_{x}$$
  
Solving we find  $\nu_{t} = \nu_{oc} = 30$  V.

Now we find the short-circuit current:



$$2v_x + v_x = 0 \implies v_x = 0$$
  
Therefore  $i_{sc} = 2 A$ . Then we have  $R_t = v_{oc} / i_{sc} = 15 \Omega$ .

**E2.26** First we transform the 2-A source and the 5- $\Omega$  resistance into a voltage source and a series resistance:



Then we have  $i_2 = \frac{10+10}{15} = 1.333$  A. From the original circuit, we have  $i_1 = i_2 - 2$ , from which we find  $i_1 = -0.667$  A.

The other approach is to start from the original circuit and transform the  $10-\Omega$  resistance and the 10-V voltage source into a current source and parallel resistance:



Then we combine the resistances in parallel.  $R_{eq} = \frac{1}{1/5 + 1/10} = 3.333 \Omega$ . The current flowing upward through this resistance is 1 A. Thus the voltage across  $R_{eq}$  referenced positive at the bottom is 3.333 V and  $i_1 = -3.333/5 = -0.667 \text{ A}$ . Then from the original circuit we have  $i_2 = 2 + i_1 = 1.333 \text{ A}$ , as before.

- **E2.27** Refer to Figure 2.60b. We have  $i_1 = 15/15 = 1$  A. Refer to Figure 2.60c. Using the current division principle, we have  $i_2 = -2 \times \frac{5}{5+10} = -0.667$  A. (The minus sign is because of the reference direction of  $i_2$ .) Finally, by superposition we have  $i_7 = i_1 + i_2 = 0.333$  A.
- **E2.28** With only the first source active we have:



Then we combine resistances in series and parallel:

$$R_{eq} = 10 + \frac{1}{1/5 + 1/15} = 13.75 \,\Omega$$

Thus,  $i_1 = 20/13.75 = 1.455$  A, and  $v_1 = 3.75i_1 = 5.45$  V.

With only the second source active, we have:



Then we combine resistances in series and parallel:

 $R_{eq2} = 15 + \frac{1}{1/5 + 1/10} = 18.33 \Omega$ Thus,  $i_s = 10/18.33 = 0.546 \text{ A}$ , and  $v_2 = 3.33i_s = 1.818 \text{ V}$ . Then we have  $i_2 = (-v_2)/10 = -0.1818 \text{ A}$ 

Finally we have  $v_{\tau} = v_1 + v_2 = 5.45 + 1.818 = 7.27$  V and  $i_{\tau} = i_1 + i_2 = 1.455 - 0.1818 = 1.27$  A.

**E2.29** First, we replace the controlled source by an independent source denoted as  $I_1$ . Then, we activate one source at a time and solve for  $v_x$ .







Adding the contributions from all three sources, we have  $v_x = -5I_1 - 5 + 25$ Then substituting  $I_1 = 0.6v_x$ , we have  $v_x = -5(0.6v_x) - 5 + 25$ which yields  $v_x = 5$  V. Then, applying Ohm's law and KCL in the original circuit, we readily find that  $i_y = 0.5$  A.

## Problems

**P2.1\*** (a)  $R_{eq} = 20 \Omega$  (b)  $R_{eq} = 23 \Omega$ 

**P2.2** (a)  $R_{eq} = 18 \Omega$  (b)  $R_{eq} = 25 \Omega$ 

**P2.3** (a) 
$$R_{eq} = 25 \Omega$$
 (b)  $R_{eq} = 24 \Omega$ 

**P2.4\*** We have 
$$4 + \frac{1}{1/20 + 1/R_x} = 8$$
 which yields  $R_x = 5 \Omega$ .

**P2.5** We have 
$$\frac{1}{1/120 + 1/R_x} = 48$$
 which yields  $R_x = 80 \Omega$ .

- **P2.6** We have  $R_{eq} = \frac{R(3R)}{R+3R} = \frac{3R}{4}$ . Clearly, for  $R_{eq}$  to be an integer, R must be an integer multiple of 4.
- P2.7  $R_{ab} = 6 \Omega$



- **P2.8** Because the resistances are in parallel, the same voltage v appears across both of them. The current through  $R_1$  is  $i_1 = v/100$ . The current through  $R_2$  is  $i_2 = 2i_1 = 2v/100$ . Finally we have  $R_2 = v/i_2 = v/(2v/100) = 50 \Omega$ .
- **P2.9** Combining the resistances shown in Figure P2.9b, we have

$$\begin{split} \mathcal{R}_{eq} &= 1 + \frac{1}{1 + 1/\mathcal{R}_{eq}} + 1 = 2 + \frac{\mathcal{R}_{eq}}{1 + \mathcal{R}_{eq}} \\ \mathcal{R}_{eq} \left( 1 + \mathcal{R}_{eq} \right) &= 2 \left( 1 + \mathcal{R}_{eq} \right) + \mathcal{R}_{eq} \\ \left( \mathcal{R}_{eq} \right)^2 - 2\mathcal{R}_{eq} - 2 &= 0 \\ \mathcal{R}_{eq} &= 2.732 \ \Omega \\ \left( \mathcal{R}_{eq} \right) &= -0.732 \ \Omega \text{ is another root, but is not physically reasonable.)} \end{split}$$

**P2.10\*** The 12- $\Omega$  and 6- $\Omega$  resistances are in parallel having an equivalent resistance of 4  $\Omega$ . Similarly, the 18- $\Omega$  and 9- $\Omega$  resistances are in parallel

and have an equivalent resistance of 6  $\Omega.\,$  Finally, the two parallel combinations are in series, and we have

$$R_{ab} = 4 + 6 = 10 \ \Omega$$

**P2.11** 
$$R_{eq} = \frac{1}{\frac{1}{1000} + \frac{1}{1000} + \frac{1}{1000} + \dots} = \frac{1}{\frac{n}{1000}} = \frac{1000}{n}$$

P2.12\*



**P2.13** In the lowest power mode, the power is  $P_{lowest} = \frac{120^2}{R_1 + R_2} = 83.33$  W.

For the highest power mode, the two elements should be in parallel with an applied voltage of 240 V. The resulting power is

$$P_{highest} = \frac{240^2}{R_1} + \frac{240^2}{R_2} = 1000 + 500 = 1500$$
 W.

Some other modes and resulting powers are:

 $R_{\rm I}$  operated separately from 240 V yielding 1000 W

- R<sub>2</sub> operated separately from 240 V yielding 500 W
- $R_1$  in series with  $R_2$  operated from 240 V yielding 333.3 W
- $R_1$  operated separately from 120 V yielding 250 W
- P2.14 For operation at the lowest power, we have

$$P = 300 = \frac{120^2}{R_1 + R_2}$$

At the high power setting, we have

$$P = 1200 = \frac{120^2}{R_1} + \frac{120^2}{R_2}$$

Solving these equations we find  $R_1 = R_2 = 24 \Omega$ .

The intermediate power setting is obtained by operating one of the elements from 120 V resulting in a power of 600 W.

**P2.15** By symmetry, we find the currents in the resistors as shown below:



Then, the voltage between terminals a and b is  $v_{ab} = R_{eq} = 1/3 + 1/6 + 1/3 = 5/6$ 

**P2.16\*** The 20- $\Omega$  and 30- $\Omega$  resistances are in parallel and have an equivalent resistance of  $R_{eq1} = 12 \Omega$ . Also the 40- $\Omega$  and 60- $\Omega$  resistances are in parallel with an equivalent resistance of  $R_{eq2} = 24 \Omega$ . Next we see that  $R_{eq1}$  and the 4- $\Omega$  resistor are in series and have an equivalent resistance of  $R_{eq3} = 4 + R_{eq1} = 16 \Omega$ . Finally  $R_{eq3}$  and  $R_{eq2}$  are in parallel and the overall equivalent resistance is

$$R_{ab} = \frac{1}{1/R_{eq1} + 1/R_{eq2}} = 9.6 \,\Omega$$

**P2.17** The 20- $\Omega$  and 30- $\Omega$  resistances are in parallel and have an equivalent resistance of  $R_{eq1} = 12 \Omega$  which in turn is in series with the 8- $\Omega$  resistance resulting in an equivalent resistance of  $R_{eq2} = R_{eq1} + 8 = 20 \Omega$ . Next  $R_{eq2}$  is in parallel with the 60- $\Omega$  resistance resulting in an equivalent resistance  $R_{eq3} = 15 \Omega$  which in turn is in series with the 7- $\Omega$  resistance resulting in an overall equivalent resistance of  $R_{ab} = 22 \Omega$ .

- **P2.18** (a) For a series combination  $G_{eq} = \frac{1}{1/G_1 + 1/G_2} = \frac{G_1G_2}{G_1 + G_2}$ (b) For a parallel combination of conductances  $G_{eq} = G_1 + G_2$
- **P2.19** To supply the loads in such a way that turning one load on or off does not affect the other loads, we must connect the loads in series with a switch in parallel with each load:



To turn a load on, we open the corresponding switch, and to turn a load off, we close the switch.

**P2.20** We have  $R_a + R_b = R_{ab} = 20$ ,  $R_b + R_c = R_{bc} = 18$  and  $R_a + R_c = R_{ca} = 14$ . These equations can be solved to find that  $R_a = 8 \Omega$ ,  $R_b = 12 \Omega$ , and  $R_c = 6 \Omega$ . After shorting terminals *b* and *c*, the equivalent resistance between terminal *a* and the shorted terminals is

$$R_{eq} = R_a + \frac{1}{1/R_b + 1/R_c} = 12 \ \Omega$$

- **P2.21** The steps in solving a circuit by network reduction are:
  - 1. Find a series or parallel combination of resistances.
  - 2. Combine them.
  - 3. Repeat until the network is reduced to a single resistance and a single source (if possible).

The method does not always work because some networks cannot be reduced sufficiently. Then, another method such as node voltages or mesh currents must be used.

P2.22\* We combine resistances in series and parallel until the circuit becomes an equivalent resistance across the voltage source. Then, we solve the simplified circuit and transfer information back along the chain of equivalents until we have found the desired results.



P2.23 Using Ohm's and Kirchhoff's laws, we work from right to left resulting in



- P2.24 The equivalent resistance seen by the current source is  $R_{eq} = 8 + \frac{1}{1/6 + 1/12} + \frac{1}{1/12 + 1/24} = 20 \Omega$ . Then, we have  $\nu = 2R_{eq} = 40 V$ ,  $i_2 = 0.667 A$ , and  $i_1 = 1.333 A$ .
- **P2.25\*** Combining resistors in series and parallel, we find that the equivalent resistance seen by the current source is  $R_{eq} = 17.5 \Omega$ . Thus,  $v = 8 \times 17.5 = 140 \text{ V}$ . Also, i = 1 A.



P2.26\* 
$$i_1 = \frac{10}{R_{eq}} = \frac{10}{10} = 1 \text{ A}$$
  
 $v_x = 4 \text{ V}$   
 $i_2 = \frac{v_x}{8} = 0.5 \text{ A}$ 

**P2.27** The equivalent resistance seen by the voltage source is

$$R_{eq} = \frac{1}{1/28 + 1/(7 + 14)} + 4 = 16 \,\Omega$$

Then, we have

$$i_1 = \frac{16 \text{ V}}{R_{eq}} = 1 \text{ A and } i_2 = i_1 \frac{21}{28 + 21} = 0.4286 \text{ A}$$

**P2.28** The currents through the 3- $\Omega$  resistance and the 4- $\Omega$  resistance are zero because they are series with an open circuit. Similiarly, the 6- $\Omega$  resistance is also in series with the open circuit, and its current is zero. Thus, we can consider the 10- $\Omega$  and the 5- $\Omega$  resistances to be in series. The current circulating clockwise in the left-hand loop is given by  $i_1 = \frac{9}{10+5}$ , and we have  $v_1 = 5i_1 = 3$  V. The current circulating counterclockwise in the right hand loop is 1 A. By Ohm's law, we have  $v_2 = 2$  V. Then, using KVL we have  $v_{ab} = v_1 - v_2 = 1$  V.

P2.29 The equivalent resistance seen by the current source is  

$$R_{eq} = 4 + \frac{1}{1/10 + 1/(5 + 10)} = 10 \Omega$$
  
Then, we have  $v_s = 2R_{eq} = 20 \text{ V}$ ,  $i_2 = 2\frac{15}{10 + 15} = 1.2 \text{ A}$ ,  $i_1 = 2\frac{10}{10 + 15} = 0.8 \text{ A}$   
and  $v_1 = -10i_1 = -8 \text{ V}$ .

**P2.30** In a similar fashion to the solution for Problem P2.9, we can write the following expression for the resistance seen by the 2-V source.

$$R_{eq} = 1 + \frac{1}{1/R_{eq} + 1/2}$$

The solutions to this equation are  $R_{eq} = 2 \Omega$  and  $R_{eq} = -1 \Omega$ . However, we reason that the resistance must be positive and discard the negative

root. Then, we have 
$$i_1 = \frac{2V}{R_{eq}} = 1$$
 A,  $i_2 = i_1 \frac{R_{eq}}{2 + R_{eq}} = \frac{i_1}{2} = 0.5$  A, and  
 $i_3 = \frac{i_1}{2} = 0.5$  A. Similarly,  $i_4 = \frac{i_3}{2} = \frac{i_1}{2^2} = 0.25$  A and  $i_{18} = \frac{i_1}{2^9} = 1.953$  mA

- P2.31  $i_2 = \frac{20 \text{ V}}{5 \Omega} = 4 \text{ A}$   $i_1 = i_2 3 = 1 \text{ A}$   $P_{current-source} = 3 \text{ A} \times 20 \text{ V} = 60 \text{ W}$   $P_{voltage-source} = 20i_1 = 20 \text{ W}$ Power is delivered by both sources.
- **P2.32** With the switch open, the current flowing clockwise in the circuit is given by  $i = \frac{10}{6 + R_2}$ , and we have  $v_2 = R_2 i = \frac{10R_2}{6 + R_2} = 5$ . Solving we find  $R_2 = 6 \Omega$ . With the switch closed,  $R_2$  and  $R_L$  are in parallel with an equivalent resistance given by  $R_{eq} = \frac{1}{1/R_2 + 1/R_L} = \frac{1}{1/6 + 1/R_L}$ . The current through  $R_{eq}$  is given by  $i = \frac{10}{6 + R_{eq}}$  and we have  $v_2 = R_{eq}i = \frac{10R_{eq}}{6 + R_{eq}} = 4$ . Solving, we find  $R_{eq} = 4 \Omega$ . Then, we can write  $R_{eq} = \frac{1}{1/6 + 1/R_L} = 4$ . Solving, we find  $R_L = 12 \Omega$ .
- P2.33\*  $R_{eq} = \frac{1}{1/5 + 1/15} = 3.75 \Omega$   $v_x = 2 A \times R_{eq} = 7.5 V$  $i_1 = v_x/5 = 1.5 A$   $i_2 = v_x/15 = 0.5 A$  $P_{4A} = 4 \times 7.5 = 30 W$  delivering  $P_{2A} = 2 \times 7.5 = 15 W$  absorbing  $P_{5\Omega} = 7.5^2/5 = 11.25 W$  absorbing
  - $P_{15\Omega} = (7.5)^2 / 15 = 3.75 \text{ W}$  absorbing

**P2.34** 
$$i = \frac{P}{v} = \frac{27 \text{ W}}{9 \text{ V}} = 3 \text{ A}$$
  $R_{eq} = R + \frac{1}{1/R + 1/R} = 1.5R$ 

$$i = 3 = \frac{9}{R_{eq}} = \frac{9}{1.5R}$$
  $R = 2 \Omega$ 

P2.35\*



**P2.36\*** 
$$v_1 = \frac{R_1}{R_1 + R_2 + R_3} \times v_s = 5 \text{ V}$$
  $v_2 = \frac{R_2}{R_1 + R_2 + R_3} \times v_s = 7 \text{ V}$   
 $v_3 = \frac{R_3}{R_1 + R_2 + R_3} \times v_s = 13 \text{ V}$ 

**P2.37\*** 
$$i_1 = \frac{R_2}{R_1 + R_2} i_s = 1 A$$
  $i_2 = \frac{R_1}{R_1 + R_2} i_s = 2 A$ 

**P2.38** We have 
$$120 \frac{5}{10+5+R_x} = 20$$
, which yields  $R_x = 15 \Omega$ .

**P2.39** First, we combine the 30  $\Omega$  and 15  $\Omega$  resistances in parallel yielding an equivalent resistance of 10  $\Omega$ , which is in parallel with  $R_x$ . Then, applying the current division principle, we have

$$4\frac{10}{10+R_x}=1$$

which yields  $R_x = 30 \Omega$ .

**P2.40** (a) 
$$R_1 + R_2 = \frac{12 \text{ V}}{0.1 \text{ A}} = 120 \Omega$$
  $\frac{R_2}{R_1 + R_2} \times 12 = 5$   
Solving, we find  $R_2 = 50 \Omega$  and  $R_1 = 70 \Omega$ .

(b)



The equivalent resistance for the parallel combination of  $R_2$  and the load is

$$R_{eq} = \frac{1}{1/50 + 1/200} = 40 \ \Omega$$

Then using the voltage division principle, we have

$$v_o = \frac{R_{eq}}{R_1 + R_{eq}} \times 12 \text{ V} = 4.364 \text{ V}$$

**P2.41\***  $v = 0.1 \, \text{mA} \times R_{w} = 50 \, \text{mV}$ 

$$R_g = \frac{50 \,\mathrm{mV}}{2 \,\mathrm{A} - 0.1 \,\mathrm{mA}} = 25 \,\mathrm{m\Omega}$$

P2.42 The circuit diagram is:



With  $i_{L} = 25 \text{ mA}$  and  $v_{L} = 4.5 \text{ V}$  , we have  $9 - R_{1} (4.5/R_{2} + 25 \text{ mA}) = 4.5$ .

Rearranging, this gives

$$4.5\frac{R_1}{R_2} + R_1 \times 0.025 = 4.5.$$
 (2)

Using Equation (1) to substitute into Equation (2) and solving, we obtain  $R_1 = 36 \Omega$  and  $R_2 = 45 \Omega$ .

Maximum power is dissipated in  $R_1$  for  $i_L = 25 \text{ mA}$ , for which the voltage across  $R_1$  is 4.5 V. Thus,  $P_{\max R_1} = \frac{4.5^2}{36} = 562.5 \text{ mW}$ . Thus,  $R_1$  must be rated for at least 562.5 mW of power dissipation.

Maximum power is dissipated in  $R_2$  for  $i_L = 0$ , in which case the voltage across  $R_2$  is 5 V. Thus,  $P_{\max R^2} = \frac{5^2}{45} = 555.5 \text{ mW}$ . (Standard resistors are available in 1-W ratings and would be suitable for this circuit.)

**P2.43\*** Combining  $R_2$  and  $R_3$ , we have an equivalent resistance

 $R_{eq} = \frac{1}{1/R_2 + 1/R_3} = 10 \ \Omega.$  Then, using the voltage-division principle, we have  $v = \frac{R_{eq}}{R_1 + R_{eq}} \times v_s = \frac{10}{20 + 10} \times 10 = 3.333 \text{ V}.$ 

- **P2.44**  $i_3 = \frac{R_2}{R_2 + R_3} \times i_s = \frac{15}{15 + 5} \times 8 = 6 \text{ A}$
- **P2.45** We need to place a resistor in series with the load and the voltage source as shown



Applying the voltage-division principle, we have  $12\frac{30}{30+R} = 4$ . Solving, we find  $R = 60 \Omega$ .

**P2.46** We have  $P = 32 = I_L^2 R_L = I_L^2 8$ . Solving, we find that the current through the load is  $I_L = 2$  A. Thus, we must place a resistor in parallel with the current source and the load.



To divide the 4 A source current equally between R and  $R_L$ , we need to have  $R = 8 \Omega$ .

- P2.47\* At node 1 we have:  $\frac{v_1}{20} + \frac{v_1 v_2}{10} = 1$ At node 2 we have:  $\frac{v_2}{5} + \frac{v_2 - v_1}{10} = 2$ In standard form, the equations become  $0.15v_1 - 0.1v_2 = 1$   $-0.1v_1 + 0.3v_2 = 2$ Solving, we find  $v_1 = 14.29$  V and  $v_2 = 11.43$  V. Then we have  $i_1 = \frac{v_1 - v_2}{10} = 0.2857$  A.
- **P2.48\*** Writing a KVL equation, we have  $v_1 v_2 = 10$ . At the reference node, we write a KCL equation:  $\frac{v_1}{5} + \frac{v_2}{10} = 1$ . Solving, we find  $v_1 = 6.667$  and  $v_2 = -3.333$ . Then, writing KCL at node 1, we have  $i_s = \frac{v_2 - v_1}{5} - \frac{v_1}{5} = -3.333$  A.
- P2.49 Writing KCL equations at nodes 1, 2, and 3, we have  $\frac{\frac{v_1}{5} + \frac{v_1 - v_2}{15} + \frac{v_1 - v_3}{15} = 0$   $\frac{\frac{v_2 - v_1}{15} + \frac{v_2 - v_3}{15} = 4$   $\frac{\frac{v_3}{25} + \frac{v_3 - v_2}{15} + \frac{v_3 - v_1}{15} = 0$

In standard form, we have:

$$\begin{array}{c} 0.3333v_1 - 0.06667v_2 - 0.06667v_3 = 0\\ - 0.06667v_1 + 0.1333v_2 - 0.06667v_3 = 4\\ - 0.06667v_1 - 0.06667v_2 + 0.1733v_3 = 0\\ \end{array}$$
  
Solving, we find  
 $v_1 = 15.0$   $v_2 = 50.0$   $v_3 = 25.0$ 

P2.50 Writing KCL equations at nodes 1, 2, and 3, we have  $\frac{\frac{v_1}{10} + \frac{v_1 - v_2}{20} + 2 = 0}{\frac{v_2 - v_1}{20} + \frac{v_2 - v_3}{4} + \frac{v_2}{5}} = 0$   $\frac{\frac{v_3}{5} + \frac{v_3 - v_2}{4}}{2} = 2$ In standard form, we have:  $0.15v_1 - 0.05v_2 = -2$   $-0.05v_1 + 0.5v_2 - 0.25v_3 = 0$   $-0.25v_2 + 0.45v_3 = 2$ Solving, we find  $v_1 = -12.9$  V  $v_2 = 1.29$  V  $v_3 = 5.16$  V

**P2.51** Writing KCL equations at nodes 1 and 2, we have

 $\frac{\frac{\nu_1}{21} + \frac{\nu_1}{28} + \frac{\nu_1 - \nu_2}{9} = 3}{\frac{\nu_2 - \nu_1}{9} + \frac{\nu_2}{6} = -3}$ 

In standard form, we have:

 $0.1944\nu_1 - 0.1111\nu_2 = 3$ 

 $-0.1111\nu_1 + 0.2778\nu_2 = -3$ 

Solving, we find  $v_1 = 12.0$  V and  $v_2 = -6.00$  V.

If the source is reversed, the algebraic signs of the node voltages are reversed.

P2.52 To minimize the number of unknowns, we select the reference node at one end of the voltage source. Then, we define the node voltages and write a KCL equation at each node.



$$\frac{v_1 - 20}{5} + \frac{v_1 - v_2}{2} = 2 \qquad \qquad \frac{v_2 - v_1}{2} + \frac{v_2 - 20}{10} = -3$$

Solving, we find  $v_1 = 18.24$  V and  $v_2 = 13.53$  V. Then, we have  $i_1 = \frac{20 - v_2}{10} = 0.647$  A.

- **P2.53** We must not use all of the nodes (including those that are inside supernodes) in writing KCL equations. Otherwise, dependent equations result.
- P2.54 The circuit with a 1-A source connected is:



$$\frac{\nu_3}{20} + \frac{\nu_3 - \nu_1}{20} + \frac{\nu_3 - \nu_2}{10} = 0$$

Solving, we find  $R_{eq} = v_1 = 13.33$ .

**P2.55\*** First, we can write:  $i_x = \frac{v_1 - v_2}{5}$ . Then, writing KCL equations at nodes 1 and 2, we have:  $\frac{v_1}{10} + i_x = 1$  and  $\frac{v_2}{20} + 0.5i_x - i_x = 0$ Substituting for  $i_x$  and simplifying, we have  $0.3v_1 - 0.2v_2 = 1$   $-0.1v_1 + 0.15v_2 = 0$ Solving, we have  $v_1 = 6$  and  $v_2 = 4$ . Then, we have  $i_x = \frac{v_1 - v_2}{5} = 0.4 \text{ A}$ .

**P2.56** First, we can write  $i_x = \frac{v_1}{10}$ . Then writing KVL, we have  $v_1 - 5i_x - v_2 = 0$ . Writing KCL at the reference node, we have  $i_x + \frac{v_2}{20} = 8$ . Using the first equation to substitute for  $i_x$  and simplifying, we have  $0.5v_1 - v_2 = 0$  $2v_1 + v_2 = 160$ Solving, we find  $v_1 = 64$ ,  $v_2 = 32$ , and  $i_x = \frac{v_1}{10} = 6.4$  A. Finally, the power  $(v_1 - v_1)^2$ 

delivered to the 16- $\Omega$  resistance is  $P = \frac{(v_1 - v_2)^2}{16} = 64$  W.

P2.57\*  $v_x = v_2 - v_1$ Writing KCL at nodes 1 and 2:

$$\frac{v_1}{5} + \frac{v_1 - 2v_x}{15} + \frac{v_1 - v_2}{10} = 1$$
$$\frac{v_2}{5} + \frac{v_2 - 2v_x}{10} + \frac{v_2 - v_1}{10} = 2$$

Substituting and simplifying, we have

 $15v_1 - 7v_2 = 30$  and  $v_1 + 2v_2 = 20$ .

Solving, we find  $v_1 = 5.405$  and  $v_2 = 7.297$ .

**P2.58** The circuit with a 1-A current source connected is



Using the first equation to substitute for  $v_x$  and simplifying, we have  $0.15 v_1 - 0.1 v_2 = 1$ 

 $0.1v_1 + 0.1v_2 = 0$ 

Solving we find  $v_1 = 4$ . However, the equivalent resistance is equal in value to  $v_1$  so we have  $R_{eq} = 4 \Omega$ .

**P2.59** The circuit with a 1-A current source connected is



Using the first equation to substitute for  $i_x$  and simplifying, we have

 $1.2v_1 - v_2 = 1$  $- 3v_1 + 3.5v_2 = 0$ 

Solving we find  $v_1 = 2.917$  V. However, the equivalent resistance is equal in value to  $v_1$  so we have  $R_{eq} = 2.917 \Omega$ .

P2.60 First, we can write:

$$\dot{I}_{x} = \frac{5\dot{I}_{x} - V_{2}}{10}$$

Simplifying, we find  $i_x = -0.2v_2$ .

Then write KCL at nodes 1 and 2:

$$\frac{v_1 - 5i_x}{5} = -3 \qquad \frac{v_2}{20} - i_x = 1$$

Substituting for  $i_x$  and simplifying, we have

 $v_1 - v_2 = -15$  and  $0.25v_2 = 1$ 

which yield  $v_1 = -11 \text{ V}$  and  $v_2 = 4 \text{ V}$ .

**P2.61\*** Writing KVL equations around each mesh, we have  $5i_1 + 15(i_1 - i_2) = 20$  and  $15(i_2 - i_1) + 10i_2 = 10$ Putting the equations into standard from we have  $20i_1 - 15i_2 = 20$  and  $-15i_1 + 25i_2 = 10$ Solving we obtain  $i_1 = 2.364$  A and  $i_2 = 1.818$  A. Then, the power delivered to the 15- $\Omega$  resistor is  $P = (i_1 - i_2)^2 15 = 4.471$ W.

 P2.62
 Writing KVL equations around each mesh, we have

  $5i_1 + 7(i_1 - i_3) + 100 = 0$ 
 $11(i_2 - i_3) + 13i_2 - 100 = 0$ 
 $9i_3 + 11(i_3 - i_2) + 7(i_3 - i_1) = 0$  

 Putting the equations into standard from, we have

  $12i_1 - 7i_3 = -100$ 
 $24i_2 - 11i_3 = 100$ 
 $-7i_1 - 11i_2 + 27i_3 = 0$  

 Solving, we obtain  $i_1 = -8.741$  A,  $i_2 = 3.846$  A, and  $i_3 = -0.6993$ . Then,

the power delivered by the source is  $P = 100(i_1 - i_2) = 1259$  W.

P2.63\*



Writing and simplifying the mesh-current equations, we have:  $28i_1 - 10i_2 = 12$ 

$$-10i_1 + 40i_2 - 30i_3 = 0$$
  
$$-30i_2 + 60i_3 = 0$$

Solving, we obtain

 $i_1 = 0.500$   $i_2 = 0.200$   $i_3 = 0.100$ Thus,  $v_2 = 5i_3 = 0.500$  V and the power delivered by the source is  $P = 12i_1 = 6$  W.

P2.64 First, we select the mesh currents and then write three equations.



Mesh 1:  $12i_1 + 24(i_1 - i_3) = 0$ Mesh 2:  $12i_2 + 6(i_2 - i_3) = 0$ However by inspection, we have  $i_3 = 2$ . Solving, we obtain  $i_1 = 1.333$  A and  $i_2 = 0.6667$  A.

P2.65 Writing and simplifying the mesh equations yields:  $14i_1 - 8i_2 = 10$  $-8i_1 + 16i_2 = 0$ Solving, we find  $i_1 = 1.000$  and  $i_2 = 0.500$ .



Finally, the power delivered by the source is  $P = 10i_1 = 10$  W.



 $4i_{A} + 28(i_{A} - i_{B}) = 16$  $28(i_{B} - i_{A}) + 7i_{B} + 14i_{B} = 0$ 

Solving we find  $i_A = 1 A$  and  $i_B = 0.5714 A$ . Then we have  $i_1 = i_A = 1 A$  and  $i_2 = i_A - i_B = 0.4286 A$ .

**P2.67** Mesh A:  $10i_{A} + 5i_{A} + 10(i_{A} - i_{B}) = 0$ By inspection:  $i_{B} = 2$ 



Solving, we find  $i_A = 0.8 \text{ A}$ . Then we have  $i_1 = i_A = 0.8 \text{ A}$  and  $i_2 = i_B - i_A = 1.2 \text{ A}$ .

P2.68 First we select mesh-current variables as shown.



Then, we can write

$$(R_{w} + R_{n} + R_{1})i_{1} - R_{n}i_{2} - R_{1}i_{3} = 120$$
  
- R\_{n}i\_{1} + (R\_{w} + R\_{n} + R\_{2})i\_{2} - R\_{2}i\_{3} = 120  
- R\_{1}i\_{1} - R\_{2}i\_{2} + (R\_{1} + R\_{2} + R\_{3})i\_{3} = 0

Substituting values for the resistances and solving we find  $i_1 = i_2 = 40.58$  A and  $i_3 = 28.99$  A. Then, the voltages across  $R_1$  and  $R_2$  are both  $10(i_1 - i_3) = 115.9$  V and the voltage across  $R_3$  is  $8i_3 = 231.9$  V. The current through the neutral wire is  $i_1 - i_2 = 0$ .

P2.69

Writing and simplifying the mesh equations, we obtain:  $40i_1 - 20i_2 = 10$   $-20i_1 + 40i_2 = 0$ Solving, we find  $i_1 = 0.3333$  and  $i_2 = 0.1667$ . Thus,  $v = 20(i_1 - i_2) = 3.333$  V.

**P2.70** The mesh currents and corresponding equations are:



$$i_1 = 8 A$$
  $15(i_2 - i_1) + 5i_2 = 0$ 

Solving, we find  $i_2 = 6 A$ .

However,  $i_3$  shown in Figure P2.44 is the same as  $i_2$ , so the answer is  $i_3 = 6$ A.

P2.71\* Because of the current sources, two of the mesh currents are known.



Writing a KVL equation around the middle loop we have  $20(i_1 - 1) + 10i_1 + 5(i_1 + 2) = 0$ 

Solving, we find  $i_1 = 0.2857$  A.

P2.72



Current source in terms of mesh currents:  $-i_1 + i_2 = 4$ KVL for mesh 3:  $-15i_1 - 15i_2 + 45i_3 = 0$ KVL around outside of network:  $5i_1 + 25i_2 + 15i_3 = 0$ Solving, we find  $i_1 = -3.000$ ,  $i_2 = 1.000$  and  $i_3 = -0.6667$ Then, we have  $v_3 = 25i_2 = 25V$ 

P2.73





P2.74



Thus,  $R_{f} = \frac{1}{1/10 + 1/5} = 3.333 \,\Omega$ . The Thévenin and Norton equivalents are:



**P2.76** First, we solve the network with a short circuit:



Zeroing the source, we have:



Combining resistances in series and parallel we find  $R_t = 6.563 \Omega$ . Then the Thévenin voltage is  $v_t = i_{sc}R_t = 9.00 V$ .







P2.78 With open-circuit conditions:



Notice the source polarity relative to terminals a and b.

**P2.79** The  $10-\Omega$  resistor has no effect on the equivalent circuits because the voltage across the 12-V source is independent of the resistor value.



**P2.80** The Thévenin voltage is equal to the open-circuit voltage which is 12.6 V. The equivalent circuit with the  $0.1-\Omega$  load connected is:



We have  $12.6/(R_t + 0.1) = 100$  from which we find  $R_t = 0.026 \Omega$ . The Thévenin and Norton equivalent circuits are:



Because no energy is converted from chemical form to heat in a battery under open-circuit conditions, the Thévenin equivalent seems more realistic from an energy conversion standpoint.

**P2.81** The Thévenin voltage is equal to the open-circuit voltage which is 20 V. The circuit with the load attached is:



We have  $i_{L} = \frac{5}{1000} = 5 \text{ mA}$  and  $v_{x} = V_{t} - 5 = 15 \text{ V}$ . Thus, the Thévenin resistance is  $R_{t} = \frac{15 \text{ V}}{5 \text{ mA}} = 3 \text{ k}\Omega$ .

**P2.82** The equivalent circuit with a load attached is:



For a load of 7  $\Omega$ , we have  $i_L = 7/7 = 1 A$ , and we can write  $v_L = V_t - R_t i_L$ . Substituting values this becomes  $7 = V_t - R_t$  (1) Similarly, for the 10- $\Omega$  load we obtain  $8 = V_t - 0.8R_t$  (2) Solving Equations (1) and (2), we find  $V_t = 12 V$  and  $R_t = 5 \Omega$ . P2.83 Open-circuit conditions:

$$i_x = \frac{20 - v_{oc}}{5}$$

$$\frac{v_{oc}}{10} - i_x + 0.5i_x = 0 \quad \text{Solving,}$$
we find  $v_{oc} = 10 \text{ V}$ .



5r

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Short-circuit conditions:

$$i_x = 20/5 = 4 A$$
  
 $i_{sc} = i_x - 0.5 i_x = 2 A$ 

Then, we have  $R_t = v_{oc}/i_{sc} = 5 \Omega$ . Thus



20 V





Then maximum power is obtained for a load resistance equal to the Thévenin resistance.

$$P_{\rm max} = \frac{(v_{\rm f}/2)^2}{R_{\rm f}} = 3.333 \, {\rm W}$$

P2.85 As in Problem P2.78, we find the Thévenin equivalent:



Then, maximum power is obtained for a load resistance equal to the Thévenin resistance.

$$P_{\max} = \frac{(v_{t}/2)^{2}}{R_{t}} = 1.667 \text{ W}$$

**P2.86\*** To maximize the power to  $R_{L}$ , we must maximize the voltage across it. Thus we need to have  $R_{t} = 0$ . The maximum power is

$$P_{\max}=\frac{20^2}{5}=80 \text{ W}$$

P2.87 The circuit is



By the current division principle:

$$\dot{I}_{L} = I_{n} \frac{R_{t}}{R_{L} + R_{t}}$$

The power delivered to the load is

$$P_{L} = (i_{L})^{2} R_{L} = (I_{n})^{2} \frac{(R_{t})^{2} R_{L}}{(R_{L} + R_{t})^{2}}$$

Taking the derivative and setting it equal to zero, we have

$$\frac{dP_{L}}{dR_{L}} = 0 = (I_{n})^{2} \frac{(R_{t})^{2} (R_{t} + R_{L})^{2} - 2(R_{t})^{2} R_{L} (R_{t} + R_{L})}{(R_{t} + R_{L})^{4}}$$

which yields  $R_L = R_t$ .

The maximum power is  $P_{L_{\text{max}}} = (I_n)^2 R_r / 4$ .

**P2.88** For maximum power conditions, we have  $R_{L} = R_{T}$ . The power taken from the voltage source is

$$P_{s} = \frac{\left(V_{t}\right)^{2}}{R_{t} + R_{L}} = \frac{\left(V_{t}\right)^{2}}{2R_{t}}$$

Then half of  $V_{\tau}$  appears across the load and the power delivered to the load is

$$P_L = \frac{(0.5V_t)^2}{R_t}$$

Thus the percentage of the power taken from the source that is delivered to the load is

$$\eta = \frac{P_L}{P_s} \times 100\% = 50\%$$

On the other hand, for  $R_L = 9R_f$ , we have

$$P_{s} = \frac{(V_{t})^{2}}{R_{t} + R_{L}} = \frac{(V_{t})^{2}}{10R_{t}}$$
$$P_{L} = \frac{(0.9V_{t})^{2}}{9R_{t}}$$
$$\eta = \frac{P_{L}}{P_{s}} \times 100\% = 90\%$$

Thus, design for maximum power transfer is relatively inefficient. Thus, systems in which power efficiency is important are almost never designed for maximum power transfer.

**P2.89\*** First, we zero the current source and find the current due to the voltage source.



Then, we zero the voltage source and use the current-division principle to find the current due to the current source.



$$i_c = 3\frac{10}{5+10} = 2 A$$

Finally, the total current is the sum of the contributions from each source.

$$i = i_v + i_c = 4 A$$

P2.90 Zero the 2 A source and use the current-division principle:



Then zero the 1 A source and use the current-division principle:



Finally,

$$\dot{i_1} = \dot{i_{1,1A}} + \dot{i_{1,2A}} = 0.2857 \, \text{A}$$

**P2.91** The circuits with only one source active at a time are:



Finally, we add the components to find the current with both sources active.

$$i_1 = i_{1,4A} + i_{1,2A} = 1.5 \text{ A}$$

P2.92\* The circuits with only one source active at a time are:



Then the total current due to both sources is  $i_s = i_{s,v} + i_{s,c} = -3.333 \text{ A}$ .

**P2.93** The circuit, assuming that  $v_2 = 1$  V is:



$$i_2 = (v_2/5) = 0.2 \text{ A}$$
  
 $v_1 = 30i_2 = 6 \text{ V}$   
 $i_{10} = v_1/10 = 0.6 \text{ A}$   
 $i_{30} = v_1/30 = 0.2 \text{ A}$   
 $i_s = i_2 + i_{10} + i_{30} = 1 \text{ A}$   
 $v_c = 12i_c + v_1 + 6i_c = 24 \text{ V}$ 

We have established that for  $\nu_s=24~\rm V$  , we have  $\nu_2=1~\rm V$  . Thus, for  $\nu_s=12~\rm V$  , we have:

$$v_2 = 1 \times \frac{12}{24} = 0.5 \text{ V}$$

**P2.94** We start by assuming i = 1 A and work back through the circuit to determine the value of  $v_s$ . This results in:



However, the circuit actually has  $v_s = 70$  V, so the actual value of *i* is  $\frac{70}{140} \times (1 \text{ A}) = 0.5 \text{ A}.$ 

**P2.95** We start by assuming  $i_2 = 1$  A and work back through the circuit to determine the value of  $v_s$ . The results are shown on the circuit diagram.



However, the circuit actually has  $v_s = 10$  V, so the actual value of  $i_2$  is

$$\frac{10}{20}$$
 × (1 A) = 0.5 A.

**P2.96** (a) With only the 2-A source activated, we have  $i_2 = 2$  and  $v_2 = 2(i_2)^3 = 16$  V

(b) With only the 1-A source activated, we have  $i_1 = 1$  A and  $v_1 = 2(i_1)^3 = 2$  V

(c) With both sources activated, we have

$$i = 3$$
 A and  $v = 2(i)^3 = 54$  V

Superposition does not apply because device A has a nonlinear relationship between v and i.

**P2.97** 1. Replace the controlled source with an independent source of unknown value.

2. Use superposition to determine the total response for the controlling current or voltage in terms of the unknown source value and other circuit parameters.

3. Substitute the expression for the value of contolled source for the unknown dependent source and solve for the controlling current or voltage. Then, compute the value for unknown independent source.

4. Use superposition to solve for any other currents or voltages of interest.

**P2.98\*** Replacing the dependent current source with an independent current source and turning on one source at a time, we have



 $i_{xa} = 1 \frac{10}{10 + 25} = 0.2857$   $v_{1a} = 25i_{xa} = 7.143$ 



Adding the results from the two analyses, we have  $i_x = i_{xa} + i_{xb} = 0.2857 + 0.5714I$ Now, we substitute  $I = 0.5i_x$  which yields  $i_x = 0.2857 + 0.5714(0.5i_x)$ Solving, we find  $i_x = 0.4000$  A. Then, we have  $I = 0.5i_x = 0.2$  and  $v_1 = v_{1a} + v_{1b} = 7.143 - 5.714I = 6.000$  V.

**P2.99** Following the same procedure used in P2.98, we eventually have  $i_x = 1.333 + 0.6667I$ 

Then, substituting  $I = 0.5i_x$  and solving, we find  $i_x = 2$  A and I = 1 A. Also, we have

$$v_{ab} = 20 \frac{10}{10+5} - I \frac{1}{1/5+1/10} = 10 V$$

P2.100 First, we replace the controlled source with an independent voltage source of unknown value V. Then, we use superposition to find expressions for the controlling current:

$$v_x = 4\frac{6}{6+12} + V\frac{4}{18} = 1.333 + 0.2222V$$

Then, we substitute  $V = 2v_x$  and solve, resulting in  $v_x = 2.400$  V and V = 4.800 V. Finally, we use superpositon to solve for the current of interest:

$$i_1 = \frac{12}{18} - \frac{V}{18} = 0.4$$
 A

**P2.101** From Equation 2.82, we have

(a) 
$$R_x = \frac{R_2}{R_1}R_3 = \frac{1\,\mathrm{k}\Omega}{10\,\mathrm{k}\Omega} \times 3419 = 341.9\,\Omega$$

(b) 
$$R_{x} = \frac{R_2}{R_1} R_3 = \frac{100 \text{ k}\Omega}{10 \text{ k}\Omega} \times 3419 = 34.19 \text{ k}\Omega$$

P2.102\* (a) Rearranging Equation 2.82, we have

$$R_3 = \frac{R_1}{R_2} R_x = \frac{10^4}{10^4} \times 5932 = 5932 \Omega$$

(b) The circuit is:



The Thévenin resistance is

$$R_{r} = \frac{1}{1/R_{3} + 1/R_{1}} + \frac{1}{1/R_{2} + 1/R_{x}} = 7447 \ \Omega$$

The Thévenin voltage is

$$\boldsymbol{v}_{t} = \boldsymbol{v}_{s} \frac{\boldsymbol{R}_{3}}{\boldsymbol{R}_{1} + \boldsymbol{R}_{3}} - \boldsymbol{v}_{s} \frac{\boldsymbol{R}_{x}}{\boldsymbol{R}_{x} + \boldsymbol{R}_{2}}$$

 $= 0.3939 \, \text{mV}$ 

Thus, the equivalent circuit is:

Thus, the detector must be sensitive to very small currents if the bridge is to be accurately balanced.

**P2.103** If  $R_1$  and  $R_3$  are too small, large currents are drawn from the source. If the source were a battery, it would need to be replaced frequently. Large power dissipation could occur, leading to heating of the components and inaccuracy due to changes in resistance values with temperature.

If  $R_1$  and  $R_3$  are too large, we would have very small detector current when the bridge is not balanced, and it would be difficult to balance the bridge accurately.

**P2.104** With the source replaced by a short circuit and the detector removed, the Wheatstone bridge circuit becomes



The Thévenin resistance seen looking back into the detector terminals is  $R_t = \frac{1}{1/R_3 + 1/R_1} + \frac{1}{1/R_2 + 1/R_x}$ 

The Thévenin voltage is zero when the bridge is balanced.